

Mathematica 11.3 Integration Test Results

on the problems in "4 Trig functions\4.5 Secant"

Test results for the 299 problems in "4.5.0 (a sec)^m (b trg)^n.m"

- **Problem 1: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[a + b x] dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b}$$

Result (type 3, 68 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b}$$

- **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[a + b x]^3 dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{2 b} + \frac{\text{Sec}[a + b x] \text{Tan}[a + b x]}{2 b}$$

Result (type 3, 69 leaves):

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right] + \text{Sec}[a + b x] \text{Tan}[a + b x]}{2 b}$$

- **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int (\text{Sec}[x]^2)^{3/2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$\frac{1}{2} \text{ArcSinh}[\text{Tan}[x]] + \frac{1}{2} \sqrt{\text{Sec}[x]^2} \text{Tan}[x]$$

Result (type 3, 52 leaves) :

$$\frac{1}{2} \cos[x] \sqrt{\sec[x]^2} \left(-\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \sec[x] \tan[x] \right)$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[x]^2} dx$$

Optimal (type 3, 3 leaves, 2 steps) :

$$\text{ArcSinh}[\tan[x]]$$

Result (type 3, 44 leaves) :

$$\cos[x] \left(-\log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) \sqrt{\sec[x]^2}$$

■ **Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[c+dx]} \sqrt{b \sec[c+dx]} dx$$

Optimal (type 3, 33 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[\sin[c+dx]] \sqrt{b \sec[c+dx]}}{d \sqrt{\sec[c+dx]}}$$

Result (type 3, 75 leaves) :

$$\frac{\left(-\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sqrt{b \sec[c+dx]}}{d \sqrt{\sec[c+dx]}}$$

■ **Problem 146: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \sec[c+dx])^{3/2}}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 34 leaves, 2 steps) :

$$\frac{b \text{ArcTanh}[\sin[c+dx]] \sqrt{b \sec[c+dx]}}{d \sqrt{\sec[c+dx]}}$$

Result (type 3, 75 leaves) :

$$\frac{\left(-\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) (b \sec[c+dx])^{3/2}}{d \sec[c+dx]^{3/2}}$$

■ **Problem 156: Result more than twice size of optimal antiderivative.**

$$\int \frac{(b \operatorname{Sec}[c + d x])^{5/2}}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps) :

$$\frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{b \operatorname{Sec}[c + d x]}}{d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{\left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) (b \operatorname{Sec}[c + d x])^{5/2}}{d \operatorname{Sec}[c + d x]^{5/2}}$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2}}{\sqrt{b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 33 leaves, 2 steps) :

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{\operatorname{Sec}[c + d x]}}{d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{\left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) \sqrt{\operatorname{Sec}[c + d x]}}{d \sqrt{b \operatorname{Sec}[c + d x]}}$$

■ **Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2}}{(b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps) :

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]] \sqrt{\operatorname{Sec}[c + d x]}}{b d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 3, 75 leaves) :

$$\frac{\left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) \operatorname{Sec}[c + d x]^{3/2}}{d (b \operatorname{Sec}[c + d x])^{3/2}}$$

■ **Problem 178: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{7/2}}{(b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]] \sqrt{\text{Sec}[c + d x]}}{b^2 d \sqrt{b \text{Sec}[c + d x]}}$$

Result (type 3, 78 leaves):

$$\frac{(-\text{Log}[\text{Cos}[\frac{1}{2}(c + d x)]] - \text{Sin}[\frac{1}{2}(c + d x)]) + \text{Log}[\text{Cos}[\frac{1}{2}(c + d x)] + \text{Sin}[\frac{1}{2}(c + d x)]]}{b^2 d \sqrt{b \text{Sec}[c + d x]}} \sqrt{\text{Sec}[c + d x]}$$

■ **Problem 230: Result unnecessarily involves higher level functions.**

$$\int (d \text{Csc}[a + b x])^{9/2} \sqrt{c \text{Sec}[a + b x]} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$-\frac{4 c d^3 (d \text{Csc}[a + b x])^{3/2}}{7 b \sqrt{c \text{Sec}[a + b x]}} - \frac{2 c d (d \text{Csc}[a + b x])^{7/2}}{7 b \sqrt{c \text{Sec}[a + b x]}} + \frac{4 d^4 \sqrt{d \text{Csc}[a + b x]} \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \text{Sec}[a + b x]} \sqrt{\text{Sin}[2 a + 2 b x]}}{7 b}$$

Result (type 5, 122 leaves):

$$\frac{1}{7 b (-2 + \text{Csc}[a + b x]^2)} 2 d^4 \text{Cos}[2(a + b x)] \text{Cot}[a + b x] \sqrt{d \text{Csc}[a + b x]} \sqrt{c \text{Sec}[a + b x]}$$

$$\left((-2 + \text{Cos}[2(a + b x)]) \text{Csc}[a + b x]^4 - 2 (-\text{Cot}[a + b x]^2)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \text{Csc}[a + b x]^2\right] \text{Sec}[a + b x]^2 \right)$$

■ **Problem 232: Result unnecessarily involves higher level functions.**

$$\int (d \text{Csc}[a + b x])^{5/2} \sqrt{c \text{Sec}[a + b x]} dx$$

Optimal (type 4, 93 leaves, 4 steps):

$$-\frac{2 c d (d \text{Csc}[a + b x])^{3/2}}{3 b \sqrt{c \text{Sec}[a + b x]}} + \frac{2 d^2 \sqrt{d \text{Csc}[a + b x]} \text{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \text{Sec}[a + b x]} \sqrt{\text{Sin}[2 a + 2 b x]}}{3 b}$$

Result (type 5, 109 leaves):

$$-\frac{1}{3 b (-2 + \text{Csc}[a + b x]^2)} d (\text{Cos}[a + b x] + \text{Cos}[3(a + b x)]) (d \text{Csc}[a + b x])^{3/2}$$

$$\left(\text{Cot}[a + b x]^2 + (-\text{Cot}[a + b x]^2)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \text{Csc}[a + b x]^2\right] \right) \text{Sec}[a + b x]^2 \sqrt{c \text{Sec}[a + b x]}$$

■ **Problem 234: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{b}$$

Result (type 5, 68 leaves):

$$\frac{1}{b} \left(-\operatorname{Cot}[a + b x]^2\right)^{7/4} \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \sqrt{c \operatorname{Sec}[a + b x]} \operatorname{Tan}[a + b x]^3$$

■ **Problem 235: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \operatorname{Sec}[a + b x]}}{\sqrt{d \operatorname{Csc}[a + b x]}} dx$$

Optimal (type 3, 270 leaves, 12 steps):

$$\frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{c \operatorname{Sec}[a + b x]}}{\sqrt{2} b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} + \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{c \operatorname{Sec}[a + b x]}}{\sqrt{2} b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} +$$

$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{c \operatorname{Sec}[a + b x]}}{2 \sqrt{2} b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{c \operatorname{Sec}[a + b x]}}{2 \sqrt{2} b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}}$$

Result (type 5, 69 leaves):

$$\frac{\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a + b x]^2\right] \sqrt{c \operatorname{Sec}[a + b x]} \operatorname{Sin}[2(a + b x)]}{3 b \left(\operatorname{Cos}[a + b x]^2\right)^{1/4} \sqrt{d \operatorname{Csc}[a + b x]}}$$

■ **Problem 236: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \operatorname{Sec}[a + b x]}}{(d \operatorname{Csc}[a + b x])^{3/2}} dx$$

Optimal (type 4, 93 leaves, 4 steps):

$$-\frac{c}{b d \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}} + \frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{2 b d^2}$$

Result (type 5, 80 leaves):

$$-\frac{1}{2 b c d \sqrt{d \operatorname{Csc}[a + b x]}} \left(1 + \operatorname{Cos}[2(a + b x)] + \left(-\operatorname{Cot}[a + b x]^2\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right]\right) (c \operatorname{Sec}[a + b x])^{3/2}$$

■ **Problem 237: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{c \operatorname{Sec}[a + b x]}}{(d \operatorname{Csc}[a + b x])^{5/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\begin{aligned} & - \frac{c}{2 b d (d \operatorname{Csc}[a + b x])^{3/2} \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{3 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{c \operatorname{Sec}[a + b x]}}{4 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} + \frac{3 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{c \operatorname{Sec}[a + b x]}}{4 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} + \\ & \frac{3 \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{c \operatorname{Sec}[a + b x]}}{8 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} - \frac{3 \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{c \operatorname{Sec}[a + b x]}}{8 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}} \end{aligned}$$

Result (type 5, 87 leaves):

$$\frac{\left(\left(\operatorname{Cos}[a + b x]^2\right)^{1/4} - \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a + b x]^2\right]\right) \sqrt{c \operatorname{Sec}[a + b x]} \operatorname{Sin}[2(a + b x)]}{4 b d^2 \left(\operatorname{Cos}[a + b x]^2\right)^{1/4} \sqrt{d \operatorname{Csc}[a + b x]}}$$

■ **Problem 239: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Csc}[a + b x])^{7/2} (c \operatorname{Sec}[a + b x])^{3/2} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\begin{aligned} & \frac{24 c d^5 \sqrt{c \operatorname{Sec}[a + b x]}}{5 b (d \operatorname{Csc}[a + b x])^{3/2}} - \frac{12 c d^3 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}}{5 b} - \\ & \frac{2 c d (d \operatorname{Csc}[a + b x])^{5/2} \sqrt{c \operatorname{Sec}[a + b x]}}{5 b} - \frac{24 c^2 d^4 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{5 b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}} \end{aligned}$$

Result (type 5, 87 leaves):

$$\begin{aligned} & \frac{1}{5 b} 2 c d (d \operatorname{Csc}[a + b x])^{5/2} \sqrt{c \operatorname{Sec}[a + b x]} \\ & \left(2 - 3 \operatorname{Cos}[2(a + b x)] - 6 \left(-\operatorname{Cot}[a + b x]^2\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \operatorname{Sin}[a + b x]^2\right) \end{aligned}$$

■ **Problem 241: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{3/2} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$\frac{4 c d^3 \sqrt{c \operatorname{Sec}[a + b x]}}{b (d \operatorname{Csc}[a + b x])^{3/2}} - \frac{2 c d \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}}{b} - \frac{4 c^2 d^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 66 leaves) :

$$-\frac{1}{b} 2 c d \sqrt{d \operatorname{Csc}[a+b x]} \left(-1 + (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a+b x]}$$

■ **Problem 243: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \operatorname{Sec}[a+b x])^{3/2}}{\sqrt{d \operatorname{Csc}[a+b x]}} dx$$

Optimal (type 4, 89 leaves, 4 steps) :

$$\frac{2 c d \sqrt{c \operatorname{Sec}[a+b x]}}{b (d \operatorname{Csc}[a+b x])^{3/2}} - \frac{2 c^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 70 leaves) :

$$-\frac{\operatorname{Cot}[a+b x] \left(-2 + (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) (c \operatorname{Sec}[a+b x])^{3/2}}{b \sqrt{d \operatorname{Csc}[a+b x]}}$$

■ **Problem 244: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \operatorname{Sec}[a+b x])^{3/2}}{(d \operatorname{Csc}[a+b x])^{3/2}} dx$$

Optimal (type 3, 327 leaves, 13 steps) :

$$\frac{2 c \sqrt{c \operatorname{Sec}[a+b x]}}{b d \sqrt{d \operatorname{Csc}[a+b x]}} + \frac{c^2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{\sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{c^2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{\sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a+b x]}}$$

Result (type 5, 86 leaves) :

$$\frac{2 c \left((\operatorname{Cos}[a+b x]^2)^{3/4} - \operatorname{Cos}[a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a+b x]}}{b d (\operatorname{Cos}[a+b x]^2)^{3/4} \sqrt{d \operatorname{Csc}[a+b x]}}$$

■ **Problem 245: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \operatorname{Sec}[a+b x])^{3/2}}{(d \operatorname{Csc}[a+b x])^{5/2}} dx$$

Optimal (type 4, 94 leaves, 4 steps) :

$$\frac{2 c \sqrt{c \operatorname{Sec}[a+b x]}}{b d (d \operatorname{Csc}[a+b x])^{3/2}} - \frac{3 c^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 79 leaves):

$$\frac{1}{2 b d^3} c \sqrt{d \operatorname{Csc}[a+b x]} \left(5 + \operatorname{Cos}[2(a+b x)] - 3 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a+b x]}$$

■ **Problem 246: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Csc}[a+b x])^{9/2} (c \operatorname{Sec}[a+b x])^{5/2} dx$$

Optimal (type 4, 166 leaves, 6 steps):

$$\frac{40 c d^5 (c \operatorname{Sec}[a+b x])^{3/2}}{21 b \sqrt{d \operatorname{Csc}[a+b x]}} - \frac{20 c d^3 (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}}{21 b} + \frac{2 c d (d \operatorname{Csc}[a+b x])^{7/2} (c \operatorname{Sec}[a+b x])^{3/2}}{7 b} + \frac{40 c^2 d^4 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{21 b}$$

Result (type 5, 92 leaves):

$$-\frac{1}{21 b \sqrt{d \operatorname{Csc}[a+b x]}} + \frac{2 c d^5 \left(-7 + \operatorname{Cot}[a+b x]^2 (13 + 3 \operatorname{Csc}[a+b x]^2) + 20 (-\operatorname{Cot}[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) (c \operatorname{Sec}[a+b x])^{3/2}}{21 b}$$

■ **Problem 248: Result unnecessarily involves higher level functions.**

$$\int (d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{5/2} dx$$

Optimal (type 4, 131 leaves, 5 steps):

$$\frac{4 c d^3 (c \operatorname{Sec}[a+b x])^{3/2}}{3 b \sqrt{d \operatorname{Csc}[a+b x]}} - \frac{2 c d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}}{3 b} + \frac{4 c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{3 b}$$

Result (type 5, 87 leaves):

$$-\frac{1}{3 b \sqrt{c \operatorname{Sec}[a+b x]}} 2 c^3 d (d \operatorname{Csc}[a+b x])^{3/2} \left(-1 + \operatorname{Cot}[a+b x]^2 + 2 (-\operatorname{Cot}[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \operatorname{Tan}[a+b x]^2$$

■ **Problem 250: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{5/2} dx$$

Optimal (type 4, 93 leaves, 4 steps) :

$$\frac{2 c d (c \operatorname{Sec}[a+b x])^{3/2}}{3 b \sqrt{d \operatorname{Csc}[a+b x]}} + \frac{2 c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{3 b}$$

Result (type 5, 68 leaves) :

$$\frac{2 c d \left(-1 + \left(-\operatorname{Cot}[a+b x]^2\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]\right) (c \operatorname{Sec}[a+b x])^{3/2}}{3 b \sqrt{d \operatorname{Csc}[a+b x]}}$$

■ **Problem 252: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \operatorname{Sec}[a+b x])^{5/2}}{(d \operatorname{Csc}[a+b x])^{3/2}} dx$$

Optimal (type 4, 98 leaves, 4 steps) :

$$\frac{2 c (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d \sqrt{d \operatorname{Csc}[a+b x]}} - \frac{c^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}{3 b d^2}$$

Result (type 5, 70 leaves) :

$$\frac{c \left(2 + \left(-\operatorname{Cot}[a+b x]^2\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]\right) (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d \sqrt{d \operatorname{Csc}[a+b x]}}$$

■ **Problem 253: Result unnecessarily involves higher level functions.**

$$\int \frac{(c \operatorname{Sec}[a+b x])^{5/2}}{(d \operatorname{Csc}[a+b x])^{5/2}} dx$$

Optimal (type 3, 329 leaves, 13 steps) :

$$\frac{2 c (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d (d \operatorname{Csc}[a+b x])^{3/2}} + \frac{c^2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} - \frac{c^2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} - \frac{c^2 \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \frac{c^2 \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}$$

Result (type 5, 88 leaves) :

$$\frac{2 c \left(\left(\operatorname{Cos}[a+b x]^2\right)^{1/4} - \operatorname{Cos}[a+b x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a+b x]^2\right]\right) (c \operatorname{Sec}[a+b x])^{3/2}}{3 b d \left(\operatorname{Cos}[a+b x]^2\right)^{1/4} (d \operatorname{Csc}[a+b x])^{3/2}}$$

■ **Problem 255: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a + b x])^{7/2}}{\sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 4, 128 leaves, 5 steps):

$$-\frac{4 c d^3 \sqrt{d \operatorname{Csc}[a + b x]}}{5 b (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{2 c d (d \operatorname{Csc}[a + b x])^{5/2}}{5 b (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{4 d^4 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{5 b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 79 leaves):

$$-\frac{1}{5 b c} 2 d^3 \sqrt{d \operatorname{Csc}[a + b x]} \left(\operatorname{Cot}[a + b x]^2 + (-\operatorname{Cot}[a + b x])^2 \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \sqrt{c \operatorname{Sec}[a + b x]}$$

■ **Problem 257: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a + b x])^{3/2}}{\sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 4, 89 leaves, 4 steps):

$$-\frac{2 c d \sqrt{d \operatorname{Csc}[a + b x]}}{b (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{2 d^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 65 leaves):

$$-\frac{d (-\operatorname{Cot}[a + b x])^2)^{1/4} \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \sqrt{c \operatorname{Sec}[a + b x]}}{b c}$$

■ **Problem 258: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{d \operatorname{Csc}[a + b x]}}{\sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 3, 270 leaves, 12 steps):

$$-\frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}}{\sqrt{2} b \sqrt{c \operatorname{Sec}[a + b x]}} + \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]} \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{2 \sqrt{2} b \sqrt{c \operatorname{Sec}[a + b x]}} + \frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{2 \sqrt{2} b \sqrt{c \operatorname{Sec}[a + b x]}}$$

Result (type 5, 66 leaves):

$$\frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sin[a + b x]^2\right] \sin[2(a + b x)]}{b (\cos[a + b x]^2)^{3/4} \sqrt{c \operatorname{Sec}[a + b x]}}$$

- **Problem 259: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$\frac{\operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\sin[2a + 2bx]}}$$

Result (type 5, 81 leaves):

$$-\frac{1}{2 b c d} \sqrt{d \operatorname{Csc}[a + b x]} \left(1 + \cos[2(a + b x)] - (-\cot[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right]\right) \sqrt{c \operatorname{Sec}[a + b x]}$$

- **Problem 260: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \operatorname{Csc}[a + b x])^{3/2} \sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\begin{aligned} & -\frac{c}{2 b d \sqrt{d \operatorname{Csc}[a + b x]} (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\tan[a + b x]}}{4 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a + b x]}} + \\ & \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\tan[a + b x]}}{4 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[a + b x]} + \tan[a + b x]\right] \sqrt{\tan[a + b x]}}{8 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a + b x]}} + \\ & \frac{\sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[a + b x]} + \tan[a + b x]\right] \sqrt{\tan[a + b x]}}{8 \sqrt{2} b d^2 \sqrt{c \operatorname{Sec}[a + b x]}} \end{aligned}$$

Result (type 5, 82 leaves):

$$-\frac{\cot[a + b x] \left((\cos[a + b x]^2)^{3/4} - \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sin[a + b x]^2\right] \right)}{2 b (\cos[a + b x]^2)^{3/4} (d \operatorname{Csc}[a + b x])^{3/2} \sqrt{c \operatorname{Sec}[a + b x]}}$$

- **Problem 261: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \operatorname{Csc}[a + b x])^{5/2} \sqrt{c \operatorname{Sec}[a + b x]}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$-\frac{c}{3 b d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} + \frac{\operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{2 b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\sin[2 a + 2 b x]}}$$

Result (type 5, 99 leaves):

$$\left(2 (-4 + \cos[2(a+b x)]) \cot[a+b x] + 3 (-\cot[a+b x]^2)^{1/4} \operatorname{Csc}[a+b x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \operatorname{Sec}[a+b x] \right) / \left(12 b d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \right)$$

■ **Problem 263: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a+b x])^{9/2}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\frac{2 d^3 (d \operatorname{Csc}[a+b x])^{3/2}}{21 b c \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{2 d (d \operatorname{Csc}[a+b x])^{7/2}}{7 b c \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{2 d^4 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\sin[2 a + 2 b x]}}{21 b c^2}$$

Result (type 5, 119 leaves):

$$-\left(d^3 \cos[2(a+b x)] (d \operatorname{Csc}[a+b x])^{3/2} \left((5 + \cos[2(a+b x)]) \operatorname{Csc}[a+b x]^4 - 2 (-\cot[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \operatorname{Sec}[a+b x]^2 \right) \right) / \left(21 b c (-2 + \operatorname{Csc}[a+b x]^2) \sqrt{c \operatorname{Sec}[a+b x]} \right)$$

■ **Problem 265: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a+b x])^{5/2}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 98 leaves, 4 steps):

$$\frac{2 d (d \operatorname{Csc}[a+b x])^{3/2}}{3 b c \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{d^2 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a - \frac{\pi}{4} + b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\sin[2 a + 2 b x]}}{3 b c^2}$$

Result (type 5, 105 leaves):

$$-\left(d \cos[2(a+b x)] (d \operatorname{Csc}[a+b x])^{3/2} \left(2 \cot[a+b x]^2 - (-\cot[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \operatorname{Sec}[a+b x]^3 \right) \right) / \left(3 b (-2 + \operatorname{Csc}[a+b x]^2) (c \operatorname{Sec}[a+b x])^{3/2} \right)$$

■ **Problem 266: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a+b x])^{3/2}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 3, 327 leaves, 13 steps):

$$\frac{-\frac{2 d \sqrt{d \operatorname{Csc}[a+b x]}}{b c \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{d^2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} - \frac{d^2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{\sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}}{\frac{d^2 \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \frac{d^2 \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{2 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}}$$

Result (type 5, 95 leaves):

$$-\left(2(d \operatorname{Csc}[a+b x])^{3/2} \left(3(\operatorname{Cos}[a+b x]^2)^{1/4} \operatorname{Csc}[a+b x]^2 + \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sin}[a+b x]^2\right]\right) \operatorname{Sin}[a+b x]^3\right) / \left(3 b c (\operatorname{Cos}[a+b x]^2)^{1/4} \sqrt{c \operatorname{Sec}[a+b x]}\right)$$

■ **Problem 267: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{d \operatorname{Csc}[a+b x]}}{(c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{d}{b c \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{\sqrt{d \operatorname{Csc}[a+b x]} \operatorname{EllipticF}\left[a-\frac{\pi}{4}+b x, 2\right] \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a+2 b x]}}{2 b c^2}$$

Result (type 5, 84 leaves):

$$\frac{d(1+\operatorname{Cos}[2(a+b x)] - (-\operatorname{Cot}[a+b x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right]) \operatorname{Sec}[a+b x]^3}{2 b \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}}$$

■ **Problem 268: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\frac{d}{2 b c (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{c \operatorname{Sec}[a+b x]}}{4 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} + \frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{8 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}} - \frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}+\operatorname{Tan}[a+b x]\right] \sqrt{c \operatorname{Sec}[a+b x]}}{8 \sqrt{2} b c^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}$$

Result (type 5, 80 leaves):

$$\frac{d \left(3 \left(\cos [a + b x]^2 \right)^{1/4} + \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sin [a + b x]^2 \right] \right)}{6 b c \left(\cos [a + b x]^2 \right)^{1/4} \left(d \csc [a + b x] \right)^{3/2} \sqrt{c \sec [a + b x]}}$$

■ **Problem 269: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \csc [a + b x])^{3/2} (c \sec [a + b x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$-\frac{c}{3 b d \sqrt{d \csc [a + b x]} (c \sec [a + b x])^{5/2}} + \frac{1}{6 b c d \sqrt{d \csc [a + b x]} \sqrt{c \sec [a + b x]}} + \frac{\sqrt{d \csc [a + b x]} \text{EllipticF} \left[a - \frac{\pi}{4} + b x, 2 \right] \sqrt{c \sec [a + b x]} \sqrt{\sin [2 a + 2 b x]}}{12 b c^2 d^2}$$

Result (type 5, 89 leaves):

$$\frac{-2 \cos [2 (a + b x)] + \frac{\csc [a + b x]^2 \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc [a + b x]^2 \right]}{(-\cot [a + b x]^2)^{1/4}}}{12 b c d \sqrt{d \csc [a + b x]} \sqrt{c \sec [a + b x]}}$$

■ **Problem 270: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \csc [a + b x])^{5/2} (c \sec [a + b x])^{3/2}} dx$$

Optimal (type 3, 371 leaves, 14 steps):

$$-\frac{c}{4 b d (d \csc [a + b x])^{3/2} (c \sec [a + b x])^{5/2}} + \frac{3}{16 b c d (d \csc [a + b x])^{3/2} \sqrt{c \sec [a + b x]}} - \frac{3 \text{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [a + b x]} \right] \sqrt{c \sec [a + b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{d \csc [a + b x]} \sqrt{\tan [a + b x]}} + \frac{3 \text{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [a + b x]} \right] \sqrt{c \sec [a + b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{d \csc [a + b x]} \sqrt{\tan [a + b x]}} + \frac{3 \text{Log} \left[1 - \sqrt{2} \sqrt{\tan [a + b x]} + \tan [a + b x] \right] \sqrt{c \sec [a + b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{d \csc [a + b x]} \sqrt{\tan [a + b x]}} - \frac{3 \text{Log} \left[1 + \sqrt{2} \sqrt{\tan [a + b x]} + \tan [a + b x] \right] \sqrt{c \sec [a + b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{d \csc [a + b x]} \sqrt{\tan [a + b x]}}$$

Result (type 5, 93 leaves):

$$\frac{\left(\cos [a + b x]^2 \right)^{1/4} (1 - 2 \cos [2 (a + b x)]) + \text{Hypergeometric2F1} \left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sin [a + b x]^2 \right]}{16 b c d \left(\cos [a + b x]^2 \right)^{1/4} \left(d \csc [a + b x] \right)^{3/2} \sqrt{c \sec [a + b x]}}$$

■ **Problem 272: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \csc [a + b x])^{7/2}}{(c \sec [a + b x])^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps) :

$$\frac{6 d^3 \sqrt{d \operatorname{Csc}[a + b x]}}{5 b c (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{2 d (d \operatorname{Csc}[a + b x])^{5/2}}{5 b c (c \operatorname{Sec}[a + b x])^{3/2}} + \frac{6 d^4 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{5 b c^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 82 leaves) :

$$-\frac{1}{5 b c^3} d^3 \sqrt{d \operatorname{Csc}[a + b x]} \left(2 \operatorname{Cot}[a + b x]^2 - 3 (-\operatorname{Cot}[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a + b x]}$$

■ **Problem 273: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a + b x])^{5/2}}{(c \operatorname{Sec}[a + b x])^{5/2}} dx$$

Optimal (type 3, 329 leaves, 13 steps) :

$$\begin{aligned} & -\frac{2 d (d \operatorname{Csc}[a + b x])^{3/2}}{3 b c (c \operatorname{Sec}[a + b x])^{3/2}} + \frac{d^2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}}{\sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} - \\ & \frac{d^2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{\operatorname{Tan}[a + b x]}}{\sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} + \frac{d^2 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{2 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} - \\ & \frac{d^2 \sqrt{d \operatorname{Csc}[a + b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{2 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} \end{aligned}$$

Result (type 5, 88 leaves) :

$$-\frac{2 d (d \operatorname{Csc}[a + b x])^{3/2} \left((\operatorname{Cos}[a + b x]^2)^{3/4} + 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a + b x]^2\right] \operatorname{Sin}[a + b x]^2 \right)}{3 b c (\operatorname{Cos}[a + b x]^2)^{3/4} (c \operatorname{Sec}[a + b x])^{3/2}}$$

■ **Problem 274: Result unnecessarily involves higher level functions.**

$$\int \frac{(d \operatorname{Csc}[a + b x])^{3/2}}{(c \operatorname{Sec}[a + b x])^{5/2}} dx$$

Optimal (type 4, 94 leaves, 4 steps) :

$$-\frac{2 d \sqrt{d \operatorname{Csc}[a + b x]}}{b c (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{3 d^2 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{b c^2 \sqrt{d \operatorname{Csc}[a + b x]} \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 79 leaves) :

$$\frac{1}{2 b c^3} d \sqrt{d \operatorname{Csc}[a + b x]} \left(1 + \operatorname{Cos}[2 (a + b x)] - 3 (-\operatorname{Cot}[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a + b x]}$$

■ **Problem 275: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{d} \operatorname{Csc}[a + b x]}{(c \operatorname{Sec}[a + b x])^{5/2}} dx$$

Optimal (type 3, 322 leaves, 13 steps):

$$\frac{d}{2 b c \sqrt{d} \operatorname{Csc}[a + b x] (c \operatorname{Sec}[a + b x])^{3/2}} - \frac{3 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d} \operatorname{Csc}[a + b x] \sqrt{\operatorname{Tan}[a + b x]}}{4 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} +$$

$$\frac{3 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]}\right] \sqrt{d} \operatorname{Csc}[a + b x] \sqrt{\operatorname{Tan}[a + b x]}}{4 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} - \frac{3 \sqrt{d} \operatorname{Csc}[a + b x] \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{8 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}} +$$

$$\frac{3 \sqrt{d} \operatorname{Csc}[a + b x] \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a + b x]} + \operatorname{Tan}[a + b x]\right] \sqrt{\operatorname{Tan}[a + b x]}}{8 \sqrt{2} b c^2 \sqrt{c \operatorname{Sec}[a + b x]}}$$

Result (type 5, 87 leaves):

$$\frac{\sqrt{d} \operatorname{Csc}[a + b x] \left((\operatorname{Cos}[a + b x]^2)^{3/4} + 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a + b x]^2\right] \right) \operatorname{Sin}[2(a + b x)]}{4 b c^2 (\operatorname{Cos}[a + b x]^2)^{3/4} \sqrt{c \operatorname{Sec}[a + b x]}}$$

■ **Problem 276: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{d} \operatorname{Csc}[a + b x] (c \operatorname{Sec}[a + b x])^{5/2}} dx$$

Optimal (type 4, 95 leaves, 4 steps):

$$\frac{d}{3 b c (d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{3/2}} + \frac{\operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{2 b c^2 \sqrt{d} \operatorname{Csc}[a + b x] \sqrt{c \operatorname{Sec}[a + b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}$$

Result (type 5, 91 leaves):

$$-\frac{1}{24 b c^3 d}$$

$$\sqrt{d} \operatorname{Csc}[a + b x] \left(5 + 6 \operatorname{Cos}[2(a + b x)] + \operatorname{Cos}[4(a + b x)] - 6 (-\operatorname{Cot}[a + b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a + b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a + b x]}$$

■ **Problem 277: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \operatorname{Csc}[a + b x])^{3/2} (c \operatorname{Sec}[a + b x])^{5/2}} dx$$

Optimal (type 3, 371 leaves, 14 steps):

$$\begin{aligned}
& - \frac{c}{4 b d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{7/2}} + \frac{1}{16 b c d \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} - \\
& \frac{3 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \frac{3 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}} - \\
& \frac{3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
& \frac{3 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{64 \sqrt{2} b c^2 d^2 \sqrt{c \operatorname{Sec}[a+b x]}}
\end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{\operatorname{Cot}[a+b x] \left((\operatorname{Cos}[a+b x]^2)^{3/4} (1 + 2 \operatorname{Cos}[2(a+b x)]) - 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \right)}{16 b c^2 (\operatorname{Cos}[a+b x]^2)^{3/4} (d \operatorname{Csc}[a+b x])^{3/2} \sqrt{c \operatorname{Sec}[a+b x]}}$$

■ **Problem 278: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$\begin{aligned}
& - \frac{c}{5 b d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{7/2}} + \\
& \frac{1}{10 b c d (d \operatorname{Csc}[a+b x])^{3/2} (c \operatorname{Sec}[a+b x])^{3/2}} + \frac{3 \operatorname{EllipticE}\left[a - \frac{\pi}{4} + b x, 2\right]}{20 b c^2 d^2 \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{c \operatorname{Sec}[a+b x]} \sqrt{\operatorname{Sin}[2 a + 2 b x]}}
\end{aligned}$$

Result (type 5, 91 leaves):

$$\frac{1}{160 b c^3 d^3} \sqrt{d \operatorname{Csc}[a+b x]} \left(-12 - 13 \operatorname{Cos}[2(a+b x)] + \operatorname{Cos}[6(a+b x)] + 12 (-\operatorname{Cot}[a+b x]^2)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \operatorname{Csc}[a+b x]^2\right] \right) \sqrt{c \operatorname{Sec}[a+b x]}$$

■ **Problem 279: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d \operatorname{Csc}[a+b x])^{7/2} (c \operatorname{Sec}[a+b x])^{5/2}} dx$$

Optimal (type 3, 406 leaves, 15 steps):

$$\begin{aligned}
& - \frac{c}{6 b d (d \operatorname{Csc}[a+b x])^{5/2} (c \operatorname{Sec}[a+b x])^{7/2}} - \frac{5 c}{48 b d^3 \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{7/2}} + \\
& \frac{5}{192 b c d^3 \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2}} - \frac{5 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
& \frac{5 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]}\right] \sqrt{d \operatorname{Csc}[a+b x]} \sqrt{\operatorname{Tan}[a+b x]}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}} - \frac{5 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{256 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}} + \\
& \frac{5 \sqrt{d \operatorname{Csc}[a+b x]} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[a+b x]} + \operatorname{Tan}[a+b x]\right] \sqrt{\operatorname{Tan}[a+b x]}}{256 \sqrt{2} b c^2 d^4 \sqrt{c \operatorname{Sec}[a+b x]}}
\end{aligned}$$

Result (type 5, 106 leaves):

$$\begin{aligned}
& \left(-2 (\operatorname{Cos}[a+b x]^2)^{3/4} (9 + 10 \operatorname{Cos}[2(a+b x)] - 4 \operatorname{Cos}[4(a+b x)]) + 30 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sin}[a+b x]^2\right] \right) / \\
& \left(384 b c d^3 (\operatorname{Cos}[a+b x]^2)^{3/4} \sqrt{d \operatorname{Csc}[a+b x]} (c \operatorname{Sec}[a+b x])^{3/2} \right)
\end{aligned}$$

■ **Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+f x]^n \operatorname{Sec}[e+f x]^m dx$$

Optimal (type 5, 81 leaves, 2 steps):

$$\frac{(\operatorname{Cos}[e+f x]^2)^{\frac{1+m}{2}} \operatorname{Csc}[e+f x]^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+f x]^2\right] \operatorname{Sec}[e+f x]^{1+m}}{f(1-n)}$$

Result (type 6, 2840 leaves):

$$\begin{aligned}
& - \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Csc}[e+f x]^{-1+2n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^m \operatorname{Sec}[e+f x]^m \right. \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^m \right) / \left(f(-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. m \operatorname{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) \right) \\
& \left(\left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Cos}[e+f x] \operatorname{Csc}[e+f x]^n \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^m \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \Big/ \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) - \\
& \left(m (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Csc}[e+fx]^{-1+n} \right. \\
& \quad \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \\
& \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right. \\
& \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) - \\
& \left((-3+n) \operatorname{Csc}[e+fx]^{-1+n} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} \right. \right. \\
& \quad \left. \left. m \left(\frac{1}{2}-\frac{n}{2} \right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \\
& \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right. \\
& \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) + \\
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Csc}[e+fx]^{-1+n} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \left(-2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}} (1-m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \\
& \quad \left. \left. \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - 2 \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \\
& \left((-1+m+n) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (2-m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \quad \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \\
& \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (1-m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, \right. \right. \\
& \quad \left. \left. 2-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} (1+m) \left(\frac{3}{2}-\frac{n}{2}\right) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. m \text{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(m (-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Csc}[e+fx]^{-1+n} \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \\
& \quad \left. \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^{-1+m} \left(-\text{Cos}\left[\frac{1}{2}(e+fx)\right] \text{Sec}[e+fx] \text{Sin}\left[\frac{1}{2}(e+fx)\right] + \text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \text{Tan}[e+fx] \right) \right) \Big/ \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right.
\end{aligned}$$

$$2 \left((-1+m+n) \operatorname{AppellF1} \left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \right. \\ \left. \operatorname{AppellF1} \left[\frac{3-n}{2}, 1+m, 1-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right)$$

■ **Problem 281: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^n (a \operatorname{Sec}[e+fx])^m dx$$

Optimal (type 5, 86 leaves, 2 steps):

$$\frac{(\operatorname{Cos}[e+fx]^2)^{\frac{1+m}{2}} \operatorname{Csc}[e+fx]^{-1+n} \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2 \right] (a \operatorname{Sec}[e+fx])^{1+m}}{a f (1-n)}$$

Result (type 6, 2842 leaves):

$$- \left(\left((-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Csc}[e+fx]^{-1+2n} \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m (a \operatorname{Sec}[e+fx])^m \right. \right. \\ \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^m \right) / \left(f (-1+n) \left((-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\ \left. \left. 2 \left((-1+m+n) \operatorname{AppellF1} \left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\ \left. \left. m \operatorname{AppellF1} \left[\frac{3-n}{2}, 1+m, 1-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \\ \left(\left((-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Cos}[e+fx] \operatorname{Csc}[e+fx]^n \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \right. \right. \\ \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^m \right) / \left((-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\ \left. \left. 2 \left((-1+m+n) \operatorname{AppellF1} \left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\ \left. \left. m \operatorname{AppellF1} \left[\frac{3-n}{2}, 1+m, 1-m-n, \frac{5-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) - \right. \\ \left. \left(m (-3+n) \operatorname{AppellF1} \left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Csc}[e+fx]^{-1+n} \right. \right. \\ \left. \left. \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^m \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \right)$$

$$\begin{aligned}
& \left((-1+m+n) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (2-m-n) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}} (1-m-n) \left(\frac{3}{2}-\frac{n}{2} \right) \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 1+m, \right. \right. \\
& \quad \left. \left. 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} (1+m) \left(\frac{3}{2}-\frac{n}{2} \right) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) / \\
& \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. m \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) - \right. \\
& \left(m (-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Csc} [e+fx]^{-1+n} \left(\text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^m \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \right)^{-1+m} \left(-\cos \left[\frac{1}{2} (e+fx) \right] \text{Sec} [e+fx] \sin \left[\frac{1}{2} (e+fx) \right] + \cos \left[\frac{1}{2} (e+fx) \right]^2 \text{Sec} [e+fx] \tan [e+fx] \right) \right) \right) / \\
& \left((-1+n) \left((-3+n) \text{AppellF1} \left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left((-1+m+n) \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + m \right. \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) /
\end{aligned}$$

■ **Problem 282: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \text{Csc} [e+fx])^n \text{Sec} [e+fx]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{b \left(\cos[e + f x]^2 \right)^{\frac{1+m}{2}} (b \csc[e + f x])^{-1+n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e + f x]^2\right] \sec[e + f x]^{1+m}}{f(1-n)}$$

Result (type 6, 2848 leaves):

$$\begin{aligned} & - \left(\left((-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\ & \quad \left. \left. \csc[e+fx]^{-1+n} (b \csc[e+fx])^n \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \sec[e+fx]^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \right) \right) / \\ & \left(f(-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ & \quad \left. \left. \left. m \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \\ & \left(\left((-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \csc[e+fx]^n \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \right. \\ & \quad \left. \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \right) \right) / \left((-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ & \quad \left. \left. \left. m \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\ & \left(m(-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \csc[e+fx]^{-1+n} \right. \\ & \quad \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \tan\left[\frac{1}{2}(e+fx)\right] \right) / \\ & \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1}{2} - \frac{n}{2}, m, 1-m-n, \frac{3}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, m, 2-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right. \right. \right. \\ & \quad \left. \left. \left. \text{AppellF1}\left[\frac{3}{2} - \frac{n}{2}, 1+m, 1-m-n, \frac{5}{2} - \frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) - \\ & \left((-3+n) \csc[e+fx]^{-1+n} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^m \left(-\frac{1}{\frac{3}{2} - \frac{n}{2}} (1-m-n) \left(\frac{1}{2} - \frac{n}{2} \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{5-n}{2}, 2+m, 1-m-n, \frac{7-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right]\right)\right)\right)\right)\right) / \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. m \text{AppellF1}\left[\frac{3-n}{2}, 1+m, 1-m-n, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \right. \\
& \left(m (-3+n) \text{AppellF1}\left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Csc}[e+fx]^{-1+n} \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^{-1+m} \left(-\cos\left[\frac{1}{2}(e+fx)\right] \text{Sec}[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \tan[e+fx] \right) \right) \right) / \\
& \left((-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1}\left[\frac{3-n}{2}, 1+m, 1-m-n, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \right) /
\end{aligned}$$

■ **Problem 283: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \text{Csc}[e+fx])^n (a \text{Sec}[e+fx])^m dx$$

Optimal (type 5, 89 leaves, 2 steps):

$$\frac{1}{af(1-n)} b (\cos[e+fx]^2)^{\frac{1+m}{2}} (b \text{Csc}[e+fx])^{-1+n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e+fx]^2\right] (a \text{Sec}[e+fx])^{1+m}$$

Result (type 6, 2850 leaves):

$$\begin{aligned}
& - \left(\left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right. \\
& \quad \left. \text{Csc}[e+fx]^{-1+n} (b \text{Csc}[e+fx])^n \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^m (a \text{Sec}[e+fx])^m \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^m \right) \right) / \\
& \left(f (-1+n) \left((-3+n) \text{AppellF1}\left[\frac{1-n}{2}, m, 1-m-n, \frac{3-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 2 \left((-1+m+n) \text{AppellF1}\left[\frac{3-n}{2}, m, 2-m-n, \frac{5-n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m \left(-2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]^2 + m \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. (-3+n) \left(-\frac{1}{\frac{3}{2}-\frac{n}{2}}(1-m-n) \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}-\frac{n}{2}} m \left(\frac{1}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) - 2 \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \left(-1+m+n\right) \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}}(2-m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, m, 3-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}} m \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) + m \left(-\frac{1}{\frac{5}{2}-\frac{n}{2}}(1-m-n) \left(\frac{3}{2}-\frac{n}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 1+m, \right. \right. \right. \\
& \quad \left. \left. 2-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}-\frac{n}{2}}(1+m) \left(\frac{3}{2}-\frac{n}{2}\right) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}-\frac{n}{2}, 2+m, 1-m-n, \frac{7}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right)\right) \right) \Big/ \\
& \quad \left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]^2 + \right. \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) - \\
& \quad \left(m (-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Csc}[e+fx]^{-1+n} \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^m \right)
\end{aligned}$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+m} \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) /$$

$$\left((-1+n) \left((-3+n) \operatorname{AppellF1}\left[\frac{1}{2}-\frac{n}{2}, m, 1-m-n, \frac{3}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right.$$

$$2 \left((-1+m+n) \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, m, 2-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + m \right.$$

$$\left. \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-\frac{n}{2}, 1+m, 1-m-n, \frac{5}{2}-\frac{n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \right)$$

■ **Problem 284: Result more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Csc}[e+fx])^n \operatorname{Sec}[e+fx]^5 dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{(b \operatorname{Csc}[e+fx])^{5+n} \operatorname{Hypergeometric2F1}\left[3, \frac{5+n}{2}, \frac{7+n}{2}, \operatorname{Csc}[e+fx]^2\right]}{b^5 f (5+n)}$$

Result (type 5, 139 leaves):

$$-\frac{1}{f(-3+n)(-1+n)} b (b \operatorname{Csc}[e+fx])^{-1+n} (\operatorname{Sec}[e+fx]^2)^{\frac{1-n}{2}} \left((-3+n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{3-n}{2}, -\tan[e+fx]^2\right] + \right.$$

$$\left. (-1+n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{3-n}{2}, \frac{5-n}{2}, -\tan[e+fx]^2\right] \tan[e+fx]^2 \right)$$

■ **Problem 290: Result more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Csc}[e+fx])^n \operatorname{Sec}[e+fx]^6 dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{b \sqrt{\cos[e+fx]^2} (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin[e+fx]^2\right] \operatorname{Sec}[e+fx]}{f(1-n)}$$

Result (type 5, 192 leaves):

$$-\frac{1}{f(-5+n)(-3+n)(-1+n)} \\ (b \operatorname{Csc}[e+fx])^n (\operatorname{Sec}[e+fx]^2)^{-n/2} \operatorname{Tan}[e+fx] \left((15-8n+n^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right] + \right. \\ \left. (-1+n) \operatorname{Tan}[e+fx]^2 \left(2(-5+n) \operatorname{Hypergeometric2F1}\left[\frac{3}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{5}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right] + \right. \right. \\ \left. \left. (-3+n) \operatorname{Hypergeometric2F1}\left[\frac{5}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{7}{2}-\frac{n}{2}, -\operatorname{Tan}[e+fx]^2\right] \operatorname{Tan}[e+fx]^2 \right) \right)$$

■ **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+fx]^2 (b \operatorname{Csc}[e+fx])^n dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{b \operatorname{Cos}[e+fx] (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2\right]}{f(1-n) \sqrt{\operatorname{Cos}[e+fx]^2}}$$

Result (type 5, 165 leaves):

$$-\frac{1}{f(-1+n)} 2 (b \operatorname{Csc}[e+fx])^n \\ \left(\operatorname{Hypergeometric2F1}\left[1-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 4 \operatorname{Hypergeometric2F1}\left[2-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ \left. 4 \operatorname{Hypergeometric2F1}\left[3-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]$$

■ **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+fx]^4 (b \operatorname{Csc}[e+fx])^n dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{b \operatorname{Cos}[e+fx] (b \operatorname{Csc}[e+fx])^{-1+n} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Sin}[e+fx]^2\right]}{f(1-n) \sqrt{\operatorname{Cos}[e+fx]^2}}$$

Result (type 5, 246 leaves):

$$\begin{aligned}
& - \frac{1}{f(-1+n)} 2 (b \operatorname{Csc}[e+fx])^n \\
& \left(\operatorname{Hypergeometric2F1}\left[1-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 8 \operatorname{Hypergeometric2F1}\left[2-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& \quad \left. 3 \operatorname{Hypergeometric2F1}\left[3-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 4 \operatorname{Hypergeometric2F1}\left[4-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& \quad \left. 2 \operatorname{Hypergeometric2F1}\left[5-n, \frac{1}{2}-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]
\end{aligned}$$

Test results for the 46 problems in "4.5.10 (c+d x)^m (a+b sec)^n.m"

- Problem 6: Result more than twice size of optimal antiderivative.

$$\int (c+dx)^3 (a+a \operatorname{Sec}[e+fx])^2 dx$$

Optimal (type 4, 371 leaves, 17 steps):

$$\begin{aligned}
& - \frac{i a^2 (c+dx)^3}{f} + \frac{a^2 (c+dx)^4}{4d} - \frac{4 i a^2 (c+dx)^3 \operatorname{ArcTan}\left[e^{i(e+fx)}\right]}{f} + \frac{3 a^2 d (c+dx)^2 \operatorname{Log}\left[1+e^{2i(e+fx)}\right]}{f^2} + \\
& \frac{6 i a^2 d (c+dx)^2 \operatorname{PolyLog}\left[2, -i e^{i(e+fx)}\right]}{f^2} - \frac{6 i a^2 d (c+dx)^2 \operatorname{PolyLog}\left[2, i e^{i(e+fx)}\right]}{f^2} - \frac{3 i a^2 d^2 (c+dx) \operatorname{PolyLog}\left[2, -e^{2i(e+fx)}\right]}{f^3} - \\
& \frac{12 a^2 d^2 (c+dx) \operatorname{PolyLog}\left[3, -i e^{i(e+fx)}\right]}{f^3} + \frac{12 a^2 d^2 (c+dx) \operatorname{PolyLog}\left[3, i e^{i(e+fx)}\right]}{f^3} + \frac{3 a^2 d^3 \operatorname{PolyLog}\left[3, -e^{2i(e+fx)}\right]}{2 f^4} - \\
& \frac{12 i a^2 d^3 \operatorname{PolyLog}\left[4, -i e^{i(e+fx)}\right]}{f^4} + \frac{12 i a^2 d^3 \operatorname{PolyLog}\left[4, i e^{i(e+fx)}\right]}{f^4} + \frac{a^2 (c+dx)^3 \operatorname{Tan}[e+fx]}{f}
\end{aligned}$$

Result (type 4, 1027 leaves):

$$\begin{aligned}
& \frac{1}{16} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 + \\
& \left(\operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 \left(c^3 \operatorname{Sin}\left[\frac{f x}{2}\right] + 3 c^2 d x \operatorname{Sin}\left[\frac{f x}{2}\right] + 3 c d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + d^3 x^3 \operatorname{Sin}\left[\frac{f x}{2}\right] \right) \right) / \\
& \left(4 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right] \right) \right) + \\
& \left(\operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 \left(c^3 \operatorname{Sin}\left[\frac{f x}{2}\right] + 3 c^2 d x \operatorname{Sin}\left[\frac{f x}{2}\right] + 3 c d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + d^3 x^3 \operatorname{Sin}\left[\frac{f x}{2}\right] \right) \right) / \\
& \left(4 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right] \right) \right) - \\
& \frac{1}{8 f^4} i \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 \left(6 c^2 d f^3 x + 6 c d^2 f^3 x^2 + 2 d^3 f^3 x^3 + 8 c^3 f^3 \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + \right. \\
& 24 c^2 d f^3 x \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + 24 c d^2 f^3 x^2 \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + \\
& 8 d^3 f^3 x^3 \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + 6 i c^2 d f^2 \operatorname{Log}[1 + \operatorname{Cos}[2(e + f x)] + i \operatorname{Sin}[2(e + f x)]] + \\
& 12 i c d^2 f^2 x \operatorname{Log}[1 + \operatorname{Cos}[2(e + f x)] + i \operatorname{Sin}[2(e + f x)]] + 6 i d^3 f^2 x^2 \operatorname{Log}[1 + \operatorname{Cos}[2(e + f x)] + i \operatorname{Sin}[2(e + f x)]] + \\
& 12 d f^2 (c + d x)^2 \operatorname{PolyLog}[2, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] - 12 d f^2 (c + d x)^2 \operatorname{PolyLog}[2, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] + \\
& 6 c d^2 f \operatorname{PolyLog}[2, -\operatorname{Cos}[2(e + f x)] - i \operatorname{Sin}[2(e + f x)]] + 6 d^3 f x \operatorname{PolyLog}[2, -\operatorname{Cos}[2(e + f x)] - i \operatorname{Sin}[2(e + f x)]] + \\
& 24 i c d^2 f \operatorname{PolyLog}[3, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] + 24 i d^3 f x \operatorname{PolyLog}[3, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] - \\
& 24 i c d^2 f \operatorname{PolyLog}[3, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] - 24 i d^3 f x \operatorname{PolyLog}[3, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] + \\
& 3 i d^3 \operatorname{PolyLog}[3, -\operatorname{Cos}[2(e + f x)] - i \operatorname{Sin}[2(e + f x)]] - 24 d^3 \operatorname{PolyLog}[4, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] + \\
& \left. 24 d^3 \operatorname{PolyLog}[4, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] + 6 i c^2 d f^3 x \operatorname{Tan}[e] + 6 i c d^2 f^3 x^2 \operatorname{Tan}[e] + 2 i d^3 f^3 x^3 \operatorname{Tan}[e] \right)
\end{aligned}$$

■ **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 (a + a \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 4, 262 leaves, 14 steps):

$$\begin{aligned}
& -\frac{i a^2 (c + d x)^2}{f} + \frac{a^2 (c + d x)^3}{3 d} - \frac{4 i a^2 (c + d x)^2 \operatorname{ArcTan}\left[e^i (e + f x)\right]}{f} + \frac{2 a^2 d (c + d x) \operatorname{Log}\left[1 + e^{2 i (e + f x)}\right]}{f^2} + \\
& \frac{4 i a^2 d (c + d x) \operatorname{PolyLog}\left[2, -i e^i (e + f x)\right]}{f^2} - \frac{4 i a^2 d (c + d x) \operatorname{PolyLog}\left[2, i e^i (e + f x)\right]}{f^2} - \frac{i a^2 d^2 \operatorname{PolyLog}\left[2, -e^{2 i (e + f x)}\right]}{f^3} - \\
& \frac{4 a^2 d^2 \operatorname{PolyLog}\left[3, -i e^i (e + f x)\right]}{f^3} + \frac{4 a^2 d^2 \operatorname{PolyLog}\left[3, i e^i (e + f x)\right]}{f^3} + \frac{a^2 (c + d x)^2 \operatorname{Tan}[e + f x]}{f}
\end{aligned}$$

Result (type 4, 685 leaves):

$$\frac{1}{12} x (3 c^2 + 3 c d x + d^2 x^2) \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 +$$

$$\frac{\operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 (c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right])}{4 f (\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]) (\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right])} +$$

$$\frac{\operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \operatorname{Sec}[e + f x])^2 (c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right])}{4 f (\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]) (\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right])} - \frac{1}{4 f^3} i \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4$$

$$(a + a \operatorname{Sec}[e + f x])^2 (2 c d f^2 x + d^2 f^2 x^2 + 4 c^2 f^2 \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + 8 c d f^2 x \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] +$$

$$4 d^2 f^2 x^2 \operatorname{ArcTan}[\operatorname{Cos}[e + f x] + i \operatorname{Sin}[e + f x]] + 2 i c d f \operatorname{Log}[1 + \operatorname{Cos}[2 (e + f x)] + i \operatorname{Sin}[2 (e + f x)]] +$$

$$2 i d^2 f x \operatorname{Log}[1 + \operatorname{Cos}[2 (e + f x)] + i \operatorname{Sin}[2 (e + f x)]] + 4 d f (c + d x) \operatorname{PolyLog}[2, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] -$$

$$4 d f (c + d x) \operatorname{PolyLog}[2, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] + d^2 \operatorname{PolyLog}[2, -\operatorname{Cos}[2 (e + f x)] - i \operatorname{Sin}[2 (e + f x)]] +$$

$$4 i d^2 \operatorname{PolyLog}[3, i \operatorname{Cos}[e + f x] - \operatorname{Sin}[e + f x]] - 4 i d^2 \operatorname{PolyLog}[3, -i \operatorname{Cos}[e + f x] + \operatorname{Sin}[e + f x]] + 2 i c d f^2 x \operatorname{Tan}[e] + i d^2 f^2 x^2 \operatorname{Tan}[e])$$

■ **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) (a + a \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$\frac{a^2 (c + d x)^2}{2 d} - \frac{4 i a^2 (c + d x) \operatorname{ArcTan}\left[e^{i (e + f x)}\right]}{f} + \frac{a^2 d \operatorname{Log}[\operatorname{Cos}[e + f x]]}{f^2} +$$

$$\frac{2 i a^2 d \operatorname{PolyLog}[2, -i e^{i (e + f x)}]}{f^2} - \frac{2 i a^2 d \operatorname{PolyLog}[2, i e^{i (e + f x)}]}{f^2} + \frac{a^2 (c + d x) \operatorname{Tan}[e + f x]}{f}$$

Result (type 4, 728 leaves):

$$\begin{aligned}
& \frac{x \cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 (2 c f \cos[e] + d f x \cos[e] + 2 d \sin[e])}{8 f (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right])} + \frac{1}{4 f^2 (\cos[e]^2 + \sin[e]^2)} \\
& d \cos[e + f x]^2 \sec[e] \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 (\cos[e] \log[\cos[e] \cos[f x] - \sin[e] \sin[f x]] + f x \sin[e]) + \\
& \frac{i c \operatorname{ArcTan}\left[\frac{-i \sin[e] - i \cos[e] \tan\left[\frac{f x}{2}\right]}{\sqrt{\cos[e]^2 + \sin[e]^2}}\right] \cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2}{f \sqrt{\cos[e]^2 + \sin[e]^2}} + \frac{1}{2 f^2} d \cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 \\
& \left(-1 / \left(\sqrt{1 + \cot[e]^2} \right) \operatorname{Csc}[e] \left((f x - \operatorname{ArcTan}[\cot[e]]) \left(\log\left[1 - e^{i (f x - \operatorname{ArcTan}[\cot[e]])}\right] - \log\left[1 + e^{i (f x - \operatorname{ArcTan}[\cot[e]])}\right] \right) + \right. \right. \\
& \left. \left. i \left(\operatorname{PolyLog}\left[2, -e^{i (f x - \operatorname{ArcTan}[\cot[e]])}\right] - \operatorname{PolyLog}\left[2, e^{i (f x - \operatorname{ArcTan}[\cot[e]])}\right] \right) \right) + \frac{2 \operatorname{ArcTan}[\cot[e]] \operatorname{ArcTanh}\left[\frac{\sin[e] + \cos[e] \tan\left[\frac{f x}{2}\right]}{\sqrt{\cos[e]^2 + \sin[e]^2}}\right]}{\sqrt{\cos[e]^2 + \sin[e]^2}} \right) + \\
& \frac{\cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 (c \sin\left[\frac{f x}{2}\right] + d x \sin\left[\frac{f x}{2}\right])}{4 f (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right])} + \\
& \frac{\cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 (c \sin\left[\frac{f x}{2}\right] + d x \sin\left[\frac{f x}{2}\right])}{4 f (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right])} - \\
& \frac{d x \cos[e + f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e + f x])^2 \tan[e]}{4 f}
\end{aligned}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^2}{a + a \sec[e + f x]} dx$$

Optimal (type 4, 119 leaves, 8 steps):

$$\frac{i (c + d x)^2}{a f} + \frac{(c + d x)^3}{3 a d} - \frac{4 d (c + d x) \log\left[1 + e^{i (e + f x)}\right]}{a f^2} + \frac{4 i d^2 \operatorname{PolyLog}\left[2, -e^{i (e + f x)}\right]}{a f^3} - \frac{(c + d x)^2 \tan\left[\frac{e}{2} + \frac{f x}{2}\right]}{a f}$$

Result (type 4, 528 leaves):

$$\frac{2 x \left(3 c^2 + 3 c d x + d^2 x^2\right) \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Sec}[e + f x]}{3 \left(a + a \operatorname{Sec}[e + f x]\right)} -$$

$$\frac{8 c d \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x] \left(\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \operatorname{Sin}\left[\frac{e}{2}\right]\right)}{f^2 \left(a + a \operatorname{Sec}[e + f x]\right) \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)} -$$

$$\left(8 d^2 \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]} f^2 x^2 - 1 / \left(\sqrt{1 + \operatorname{Cot}\left[\frac{e}{2}\right]^2}\right) \operatorname{Cot}\left[\frac{e}{2}\right]\right.\right.$$

$$\left.\left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]\right) - \pi \operatorname{Log}\left[1 + e^{-i f x}\right] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]\right) \operatorname{Log}\left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right)}\right]\right] + \pi \operatorname{Log}\left[\operatorname{Cos}\left[\frac{f x}{2}\right]\right] -\right.$$

$$\left.\left.2 \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{e}{2}\right]\right)}\right]\right]\right) \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x] \Bigg/$$

$$\left(f^3 \left(a + a \operatorname{Sec}[e + f x]\right) \sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2\right)} - \frac{2 \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x] \left(c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right]\right)}{f \left(a + a \operatorname{Sec}[e + f x]\right)}\right)$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 4, 288 leaves, 19 steps):

$$\frac{5 i (c + d x)^3}{3 a^2 f} + \frac{(c + d x)^4}{4 a^2 d} - \frac{10 d (c + d x)^2 \operatorname{Log}\left[1 + e^{i (e + f x)}\right]}{a^2 f^2} + \frac{4 d^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]\right]}{a^2 f^4} +$$

$$\frac{20 i d^2 (c + d x) \operatorname{PolyLog}\left[2, -e^{i (e + f x)}\right]}{a^2 f^3} - \frac{20 d^3 \operatorname{PolyLog}\left[3, -e^{i (e + f x)}\right]}{a^2 f^4} - \frac{d (c + d x)^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{2 a^2 f^2} +$$

$$\frac{2 d^2 (c + d x) \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{a^2 f^3} - \frac{5 (c + d x)^3 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f} + \frac{(c + d x)^3 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{6 a^2 f}$$

Result (type 4, 1455 leaves):

$$\begin{aligned}
& \frac{1}{3 f^4 (a + a \operatorname{Sec}[e + f x])^2} 20 d^3 e^{-\frac{ie}{2}} \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \\
& \left(i f^2 x^2 (e^{ie} f x + 3 i (1 + e^{ie}) \operatorname{Log}[1 + e^{i(e+fx)}]) + 6 i (1 + e^{ie}) f x \operatorname{PolyLog}[2, -e^{i(e+fx)}] - 6 (1 + e^{ie}) \operatorname{PolyLog}[3, -e^{i(e+fx)}]) \operatorname{Sec}\left[\frac{e}{2}\right] \right. \\
& \left. \operatorname{Sec}[e + f x]^2 + \left(16 d^3 \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \operatorname{Sin}\left[\frac{e}{2}\right] \right) \right) \right) / \\
& \left(f^4 (a + a \operatorname{Sec}[e + f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2 \right) \right) - \\
& \left(40 c^2 d \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \operatorname{Sin}\left[\frac{e}{2}\right] \right) \right) \right) / \\
& \left(f^2 (a + a \operatorname{Sec}[e + f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2 \right) \right) - \left(80 c d^2 \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]]} f^2 x^2 - 1 / \left(\sqrt{1 + \operatorname{Cot}\left[\frac{e}{2}\right]^2} \right) \operatorname{Cot}\left[\frac{e}{2}\right] \right. \right. \\
& \left. \left. \left(\frac{1}{2} i f x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right) - \pi \operatorname{Log}[1 + e^{-i f x}] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right) \operatorname{Log}\left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right)} \right] + \pi \operatorname{Log}\left[\operatorname{Cos}\left[\frac{f x}{2}\right]\right] - \right. \right. \\
& \left. \left. 2 \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right)} \right] \right) \right) \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \right) / \\
& \left(f^3 (a + a \operatorname{Sec}[e + f x])^2 \sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2 \right)} \right) + \frac{1}{24 f^3 (a + a \operatorname{Sec}[e + f x])^2} \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \\
& \left(-24 c^2 d f \operatorname{Cos}\left[\frac{f x}{2}\right] - 48 c d^2 f x \operatorname{Cos}\left[\frac{f x}{2}\right] + 36 c^3 f^3 x \operatorname{Cos}\left[\frac{f x}{2}\right] - 24 d^3 f x^2 \operatorname{Cos}\left[\frac{f x}{2}\right] + 54 c^2 d f^3 x^2 \operatorname{Cos}\left[\frac{f x}{2}\right] + 36 c d^2 f^3 x^3 \operatorname{Cos}\left[\frac{f x}{2}\right] + \right. \\
& 9 d^3 f^3 x^4 \operatorname{Cos}\left[\frac{f x}{2}\right] - 24 c^2 d f \operatorname{Cos}\left[e + \frac{f x}{2}\right] - 48 c d^2 f x \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 36 c^3 f^3 x \operatorname{Cos}\left[e + \frac{f x}{2}\right] - 24 d^3 f x^2 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + \\
& 54 c^2 d f^3 x^2 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 36 c d^2 f^3 x^3 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 9 d^3 f^3 x^4 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 12 c^3 f^3 x \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + 18 c^2 d f^3 x^2 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + \\
& 12 c d^2 f^3 x^3 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + 3 d^3 f^3 x^4 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + 12 c^3 f^3 x \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + 18 c^2 d f^3 x^2 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + \\
& 12 c d^2 f^3 x^3 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + 3 d^3 f^3 x^4 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + 96 c d^2 \operatorname{Sin}\left[\frac{f x}{2}\right] - 72 c^3 f^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 96 d^3 x \operatorname{Sin}\left[\frac{f x}{2}\right] - \\
& 216 c^2 d f^2 x \operatorname{Sin}\left[\frac{f x}{2}\right] - 216 c d^2 f^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right] - 72 d^3 f^2 x^3 \operatorname{Sin}\left[\frac{f x}{2}\right] - 48 c d^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 48 c^3 f^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] - \\
& 48 d^3 x \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 144 c^2 d f^2 x \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 144 c d^2 f^2 x^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 48 d^3 f^2 x^3 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 48 c d^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - \\
& \left. 40 c^3 f^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 48 d^3 x \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 120 c^2 d f^2 x \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 120 c d^2 f^2 x^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 40 d^3 f^2 x^3 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] \right)
\end{aligned}$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^2}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 4, 229 leaves, 17 steps):

$$\frac{5 i (c + d x)^2}{3 a^2 f} + \frac{(c + d x)^3}{3 a^2 d} - \frac{20 d (c + d x) \operatorname{Log}[1 + e^{i(e + f x)}]}{3 a^2 f^2} + \frac{20 i d^2 \operatorname{PolyLog}[2, -e^{i(e + f x)}]}{3 a^2 f^3} -$$

$$\frac{d (c + d x) \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{3 a^2 f^2} + \frac{2 d^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f^3} - \frac{5 (c + d x)^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f} + \frac{(c + d x)^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]}{6 a^2 f}$$

Result (type 4, 925 leaves):

$$- \left(80 c d \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \operatorname{Sin}\left[\frac{e}{2}\right] \right) \right) /$$

$$\left(3 f^2 (a + a \operatorname{Sec}[e + f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2 \right) \right) - \left(80 d^2 \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]]} f^2 x^2 - 1 / \left(\sqrt{1 + \operatorname{Cot}\left[\frac{e}{2}\right]^2} \right) \operatorname{Cot}\left[\frac{e}{2}\right] \right. \right.$$

$$\left. \left. \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right) - \pi \operatorname{Log}[1 + e^{-i f x}] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right) \operatorname{Log}\left[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right)} \right] + \pi \operatorname{Log}\left[\operatorname{Cos}\left[\frac{f x}{2}\right]\right] - \right. \right.$$

$$\left. \left. 2 \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]]\right]\right] + i \operatorname{PolyLog}\left[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\operatorname{Cot}[\frac{e}{2}]] \right)} \right] \right) \right) \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2 \right) /$$

$$\left(3 f^3 (a + a \operatorname{Sec}[e + f x])^2 \sqrt{\operatorname{Csc}\left[\frac{e}{2}\right]^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]^2 + \operatorname{Sin}\left[\frac{e}{2}\right]^2 \right)} \right) + \frac{1}{6 f^3 (a + a \operatorname{Sec}[e + f x])^2}$$

$$\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2$$

$$\left(-4 c d f \operatorname{Cos}\left[\frac{f x}{2}\right] - 4 d^2 f x \operatorname{Cos}\left[\frac{f x}{2}\right] + 9 c^2 f^3 x \operatorname{Cos}\left[\frac{f x}{2}\right] + 9 c d f^3 x^2 \operatorname{Cos}\left[\frac{f x}{2}\right] + 3 d^2 f^3 x^3 \operatorname{Cos}\left[\frac{f x}{2}\right] - 4 c d f \operatorname{Cos}\left[e + \frac{f x}{2}\right] - \right.$$

$$4 d^2 f x \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 9 c^2 f^3 x \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 9 c d f^3 x^2 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 3 d^2 f^3 x^3 \operatorname{Cos}\left[e + \frac{f x}{2}\right] + 3 c^2 f^3 x \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] +$$

$$3 c d f^3 x^2 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + d^2 f^3 x^3 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] + 3 c^2 f^3 x \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + 3 c d f^3 x^2 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] +$$

$$d^2 f^3 x^3 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] + 8 d^2 \operatorname{Sin}\left[\frac{f x}{2}\right] - 18 c^2 f^2 \operatorname{Sin}\left[\frac{f x}{2}\right] - 36 c d f^2 x \operatorname{Sin}\left[\frac{f x}{2}\right] - 18 d^2 f^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right] -$$

$$4 d^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 12 c^2 f^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 24 c d f^2 x \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 12 d^2 f^2 x^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] +$$

$$4 d^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 10 c^2 f^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 20 c d f^2 x \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 10 d^2 f^2 x^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] \left. \right)$$

■ **Problem 24: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 (a + b \operatorname{Sec}[e + f x]) dx$$

Optimal (type 4, 227 leaves, 11 steps):

$$\frac{a (c + d x)^4}{4 d} - \frac{2 i b (c + d x)^3 \operatorname{ArcTan}\left[e^{i (e + f x)}\right]}{f} + \frac{3 i b d (c + d x)^2 \operatorname{PolyLog}\left[2, -i e^{i (e + f x)}\right]}{f^2} - \frac{3 i b d (c + d x)^2 \operatorname{PolyLog}\left[2, i e^{i (e + f x)}\right]}{f^2} - \frac{6 b d^2 (c + d x) \operatorname{PolyLog}\left[3, -i e^{i (e + f x)}\right]}{f^3} + \frac{6 b d^2 (c + d x) \operatorname{PolyLog}\left[3, i e^{i (e + f x)}\right]}{f^3} - \frac{6 i b d^3 \operatorname{PolyLog}\left[4, -i e^{i (e + f x)}\right]}{f^4} + \frac{6 i b d^3 \operatorname{PolyLog}\left[4, i e^{i (e + f x)}\right]}{f^4}$$

Result (type 4, 474 leaves):

$$\frac{1}{4 f^4} \left(4 a c^3 f^4 x + 6 a c^2 d f^4 x^2 + 4 a c d^2 f^4 x^3 + a d^3 f^4 x^4 - 8 i b c^3 f^3 \operatorname{ArcTan}\left[e^{i (e + f x)}\right] + 12 b c^2 d f^3 x \operatorname{Log}\left[1 - i e^{i (e + f x)}\right] + 12 b c d^2 f^3 x^2 \operatorname{Log}\left[1 - i e^{i (e + f x)}\right] + 4 b d^3 f^3 x^3 \operatorname{Log}\left[1 - i e^{i (e + f x)}\right] - 12 b c^2 d f^3 x \operatorname{Log}\left[1 + i e^{i (e + f x)}\right] - 12 b c d^2 f^3 x^2 \operatorname{Log}\left[1 + i e^{i (e + f x)}\right] - 4 b d^3 f^3 x^3 \operatorname{Log}\left[1 + i e^{i (e + f x)}\right] + 12 i b d f^2 (c + d x)^2 \operatorname{PolyLog}\left[2, -i e^{i (e + f x)}\right] - 12 i b d f^2 (c + d x)^2 \operatorname{PolyLog}\left[2, i e^{i (e + f x)}\right] - 24 b c d^2 f \operatorname{PolyLog}\left[3, -i e^{i (e + f x)}\right] - 24 b d^3 f x \operatorname{PolyLog}\left[3, -i e^{i (e + f x)}\right] + 24 b c d^2 f \operatorname{PolyLog}\left[3, i e^{i (e + f x)}\right] + 24 b d^3 f x \operatorname{PolyLog}\left[3, i e^{i (e + f x)}\right] - 24 i b d^3 \operatorname{PolyLog}\left[4, -i e^{i (e + f x)}\right] + 24 i b d^3 \operatorname{PolyLog}\left[4, i e^{i (e + f x)}\right] \right)$$

■ **Problem 26: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) (a + b \operatorname{Sec}[e + f x]) dx$$

Optimal (type 4, 93 leaves, 7 steps):

$$\frac{a (c + d x)^2}{2 d} - \frac{2 i b (c + d x) \operatorname{ArcTan}\left[e^{i (e + f x)}\right]}{f} + \frac{i b d \operatorname{PolyLog}\left[2, -i e^{i (e + f x)}\right]}{f^2} - \frac{i b d \operatorname{PolyLog}\left[2, i e^{i (e + f x)}\right]}{f^2}$$

Result (type 4, 236 leaves):

$$a c x + \frac{1}{2} a d x^2 - \frac{b c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right]}{f} + \frac{b c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right]}{f} + \frac{1}{f^2} b d \left(\left(-e + \frac{\pi}{2} - f x\right) \left(\operatorname{Log}\left[1 - e^{i \left(-e + \frac{\pi}{2} - f x\right)}\right] - \operatorname{Log}\left[1 + e^{i \left(-e + \frac{\pi}{2} - f x\right)}\right]\right) - \left(-e + \frac{\pi}{2}\right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x\right)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i \left(-e + \frac{\pi}{2} - f x\right)}\right] - \operatorname{PolyLog}\left[2, e^{i \left(-e + \frac{\pi}{2} - f x\right)}\right]\right) \right)$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 (a + b \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 4, 364 leaves, 17 steps):

$$\begin{aligned}
& - \frac{i b^2 (c+dx)^3}{f} + \frac{a^2 (c+dx)^4}{4d} - \frac{4 i a b (c+dx)^3 \text{ArcTan}[e^{i(e+fx)}]}{f} + \frac{3 b^2 d (c+dx)^2 \text{Log}[1+e^{2i(e+fx)}]}{f^2} + \\
& \frac{6 i a b d (c+dx)^2 \text{PolyLog}[2, -i e^{i(e+fx)}]}{f^2} - \frac{6 i a b d (c+dx)^2 \text{PolyLog}[2, i e^{i(e+fx)}]}{f^2} - \frac{3 i b^2 d^2 (c+dx) \text{PolyLog}[2, -e^{2i(e+fx)}]}{f^3} \\
& \frac{12 a b d^2 (c+dx) \text{PolyLog}[3, -i e^{i(e+fx)}]}{f^3} + \frac{12 a b d^2 (c+dx) \text{PolyLog}[3, i e^{i(e+fx)}]}{f^3} + \frac{3 b^2 d^3 \text{PolyLog}[3, -e^{2i(e+fx)}]}{2 f^4} - \\
& \frac{12 i a b d^3 \text{PolyLog}[4, -i e^{i(e+fx)}]}{f^4} + \frac{12 i a b d^3 \text{PolyLog}[4, i e^{i(e+fx)}]}{f^4} + \frac{b^2 (c+dx)^3 \text{Tan}[e+fx]}{f}
\end{aligned}$$

Result (type 4, 1700 leaves):

$$\begin{aligned}
& \frac{a^2 x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) \text{Cos}[e+fx]^2 (a+b \text{Sec}[e+fx])^2}{4 (b+a \text{Cos}[e+fx])^2} + \\
& \frac{1}{2 (1+e^{2ie}) f^4 (b+a \text{Cos}[e+fx])^2} b \text{Cos}[e+fx]^2 (-12 i b c^2 d e^{2ie} f^3 x - 12 i b c d^2 e^{2ie} f^3 x^2 - 4 i b d^3 e^{2ie} f^3 x^3 - 8 i a c^3 f^3 \text{ArcTan}[e^{i(e+fx)}] - \\
& 8 i a c^3 e^{2ie} f^3 \text{ArcTan}[e^{i(e+fx)}] + 12 a c^2 d f^3 x \text{Log}[1-i e^{i(e+fx)}] + 12 a c^2 d e^{2ie} f^3 x \text{Log}[1-i e^{i(e+fx)}] + 12 a c d^2 f^3 x^2 \text{Log}[1-i e^{i(e+fx)}] + \\
& 12 a c d^2 e^{2ie} f^3 x^2 \text{Log}[1-i e^{i(e+fx)}] + 4 a d^3 f^3 x^3 \text{Log}[1-i e^{i(e+fx)}] + 4 a d^3 e^{2ie} f^3 x^3 \text{Log}[1-i e^{i(e+fx)}] - 12 a c^2 d f^3 x \text{Log}[1+i e^{i(e+fx)}] - \\
& 12 a c^2 d e^{2ie} f^3 x \text{Log}[1+i e^{i(e+fx)}] - 12 a c d^2 f^3 x^2 \text{Log}[1+i e^{i(e+fx)}] - 12 a c d^2 e^{2ie} f^3 x^2 \text{Log}[1+i e^{i(e+fx)}] - \\
& 4 a d^3 f^3 x^3 \text{Log}[1+i e^{i(e+fx)}] - 4 a d^3 e^{2ie} f^3 x^3 \text{Log}[1+i e^{i(e+fx)}] + 6 b c^2 d f^2 \text{Log}[1+e^{2i(e+fx)}] + 6 b c^2 d e^{2ie} f^2 \text{Log}[1+e^{2i(e+fx)}] + \\
& 12 b c d^2 f^2 x \text{Log}[1+e^{2i(e+fx)}] + 12 b c d^2 e^{2ie} f^2 x \text{Log}[1+e^{2i(e+fx)}] + 6 b d^3 f^2 x^2 \text{Log}[1+e^{2i(e+fx)}] + 6 b d^3 e^{2ie} f^2 x^2 \text{Log}[1+e^{2i(e+fx)}] + \\
& 12 i a d (1+e^{2ie}) f^2 (c+dx)^2 \text{PolyLog}[2, -i e^{i(e+fx)}] - 12 i a d (1+e^{2ie}) f^2 (c+dx)^2 \text{PolyLog}[2, i e^{i(e+fx)}] - \\
& 6 i b c d^2 f \text{PolyLog}[2, -e^{2i(e+fx)}] - 6 i b c d^2 e^{2ie} f \text{PolyLog}[2, -e^{2i(e+fx)}] - 6 i b d^3 f x \text{PolyLog}[2, -e^{2i(e+fx)}] - \\
& 6 i b d^3 e^{2ie} f x \text{PolyLog}[2, -e^{2i(e+fx)}] - 24 a c d^2 f \text{PolyLog}[3, -i e^{i(e+fx)}] - 24 a c d^2 e^{2ie} f \text{PolyLog}[3, -i e^{i(e+fx)}] - \\
& 24 a d^3 f x \text{PolyLog}[3, -i e^{i(e+fx)}] - 24 a d^3 e^{2ie} f x \text{PolyLog}[3, -i e^{i(e+fx)}] + 24 a c d^2 f \text{PolyLog}[3, i e^{i(e+fx)}] + \\
& 24 a c d^2 e^{2ie} f \text{PolyLog}[3, i e^{i(e+fx)}] + 24 a d^3 f x \text{PolyLog}[3, i e^{i(e+fx)}] + 24 a d^3 e^{2ie} f x \text{PolyLog}[3, i e^{i(e+fx)}] + \\
& 3 b d^3 \text{PolyLog}[3, -e^{2i(e+fx)}] + 3 b d^3 e^{2ie} \text{PolyLog}[3, -e^{2i(e+fx)}] - 24 i a d^3 \text{PolyLog}[4, -i e^{i(e+fx)}] - \\
& 24 i a d^3 e^{2ie} \text{PolyLog}[4, -i e^{i(e+fx)}] + 24 i a d^3 \text{PolyLog}[4, i e^{i(e+fx)}] + 24 i a d^3 e^{2ie} \text{PolyLog}[4, i e^{i(e+fx)}]) (a+b \text{Sec}[e+fx])^2 + \\
& \left(\text{Cos}[e+fx]^2 (a+b \text{Sec}[e+fx])^2 \left(b^2 c^3 \text{Sin}\left[\frac{fx}{2}\right] + 3 b^2 c^2 d x \text{Sin}\left[\frac{fx}{2}\right] + 3 b^2 c d^2 x^2 \text{Sin}\left[\frac{fx}{2}\right] + b^2 d^3 x^3 \text{Sin}\left[\frac{fx}{2}\right] \right) \right) / \\
& \left(f (b+a \text{Cos}[e+fx])^2 \left(\text{Cos}\left[\frac{e}{2}\right] - \text{Sin}\left[\frac{e}{2}\right] \right) \left(\text{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right] \right) \right) + \\
& \left(\text{Cos}[e+fx]^2 (a+b \text{Sec}[e+fx])^2 \left(b^2 c^3 \text{Sin}\left[\frac{fx}{2}\right] + 3 b^2 c^2 d x \text{Sin}\left[\frac{fx}{2}\right] + 3 b^2 c d^2 x^2 \text{Sin}\left[\frac{fx}{2}\right] + b^2 d^3 x^3 \text{Sin}\left[\frac{fx}{2}\right] \right) \right) / \\
& \left(f (b+a \text{Cos}[e+fx])^2 \left(\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right] \right) \left(\text{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^2 (a+b \text{Sec}[e+fx])^2 dx$$

Optimal (type 4, 257 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{i b^2 (c+d x)^2}{f} + \frac{a^2 (c+d x)^3}{3 d} - \frac{4 i a b (c+d x)^2 \operatorname{ArcTan}\left[e^{i(e+f x)}\right]}{f} + \frac{2 b^2 d (c+d x) \operatorname{Log}\left[1+e^{2 i(e+f x)}\right]}{f^2} + \\
 & \frac{4 i a b d (c+d x) \operatorname{PolyLog}\left[2,-i e^{i(e+f x)}\right]}{f^2} - \frac{4 i a b d (c+d x) \operatorname{PolyLog}\left[2,i e^{i(e+f x)}\right]}{f^2} - \frac{i b^2 d^2 \operatorname{PolyLog}\left[2,-e^{2 i(e+f x)}\right]}{f^3} - \\
 & \frac{4 a b d^2 \operatorname{PolyLog}\left[3,-i e^{i(e+f x)}\right]}{f^3} + \frac{4 a b d^2 \operatorname{PolyLog}\left[3,i e^{i(e+f x)}\right]}{f^3} + \frac{b^2 (c+d x)^2 \operatorname{Tan}[e+f x]}{f}
 \end{aligned}$$

Result (type 4, 703 leaves):

$$\begin{aligned}
 & \frac{a^2 x \left(3 c^2 + 3 c d x + d^2 x^2\right) \operatorname{Cos}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^2}{3 (b+a \operatorname{Cos}[e+f x])^2} + \\
 & \frac{\operatorname{Cos}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^2 \left(b^2 c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 b^2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + b^2 d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right]\right)}{f (b+a \operatorname{Cos}[e+f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} + \\
 & \frac{\operatorname{Cos}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^2 \left(b^2 c^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 2 b^2 c d x \operatorname{Sin}\left[\frac{f x}{2}\right] + b^2 d^2 x^2 \operatorname{Sin}\left[\frac{f x}{2}\right]\right)}{f (b+a \operatorname{Cos}[e+f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} + \\
 & \frac{1}{f^3 (b+a \operatorname{Cos}[e+f x])^2} b \operatorname{Cos}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^2 \\
 & \left(-2 i b c d f^2 x - i b d^2 f^2 x^2 - 4 i a c^2 f^2 \operatorname{ArcTan}[\operatorname{Cos}[e+f x] + i \operatorname{Sin}[e+f x]] - 8 i a c d f^2 x \operatorname{ArcTan}[\operatorname{Cos}[e+f x] + i \operatorname{Sin}[e+f x]] - \right. \\
 & \left. 4 i a d^2 f^2 x^2 \operatorname{ArcTan}[\operatorname{Cos}[e+f x] + i \operatorname{Sin}[e+f x]] + 2 b c d f \operatorname{Log}[1+\operatorname{Cos}[2(e+f x)] + i \operatorname{Sin}[2(e+f x)]] + \right. \\
 & \left. 2 b d^2 f x \operatorname{Log}[1+\operatorname{Cos}[2(e+f x)] + i \operatorname{Sin}[2(e+f x)]] - 4 i a d f (c+d x) \operatorname{PolyLog}[2, i \operatorname{Cos}[e+f x] - \operatorname{Sin}[e+f x]] + \right. \\
 & \left. 4 i a d f (c+d x) \operatorname{PolyLog}[2, -i \operatorname{Cos}[e+f x] + \operatorname{Sin}[e+f x]] - i b d^2 \operatorname{PolyLog}[2, -\operatorname{Cos}[2(e+f x)] - i \operatorname{Sin}[2(e+f x)]] + \right. \\
 & \left. 4 a d^2 \operatorname{PolyLog}[3, i \operatorname{Cos}[e+f x] - \operatorname{Sin}[e+f x]] - 4 a d^2 \operatorname{PolyLog}[3, -i \operatorname{Cos}[e+f x] + \operatorname{Sin}[e+f x]] + 2 b c d f^2 x \operatorname{Tan}[e] + b d^2 f^2 x^2 \operatorname{Tan}[e]\right)
 \end{aligned}$$

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int (c+d x) (a+b \operatorname{Sec}[e+f x])^2 dx$$

Optimal (type 4, 131 leaves, 9 steps):

$$\begin{aligned}
 & \frac{a^2 (c+d x)^2}{2 d} - \frac{4 i a b (c+d x) \operatorname{ArcTan}\left[e^{i(e+f x)}\right]}{f} + \frac{b^2 d \operatorname{Log}[\operatorname{Cos}[e+f x]]}{f^2} + \\
 & \frac{2 i a b d \operatorname{PolyLog}\left[2,-i e^{i(e+f x)}\right]}{f^2} - \frac{2 i a b d \operatorname{PolyLog}\left[2,i e^{i(e+f x)}\right]}{f^2} + \frac{b^2 (c+d x) \operatorname{Tan}[e+f x]}{f}
 \end{aligned}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& \frac{1}{2 f^2} \left(-a^2 (e + f x) (-2 c f + d (e - f x)) + \right. \\
& 2 b \left(-2 a d (e + f x) \left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) + a (d e - c f) \left(e + f x + \operatorname{Log}\left[-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - \right. \\
& \quad \left. \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) - a (d e - c f) \left(e + f x - \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] + \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) + \\
& b d \left(-\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]\right]^2 + \operatorname{Log}\left[-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] + \operatorname{Log}\left[4 \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] \right) - \\
& 2 i a d \left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] - \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] - \right. \\
& \quad \left. \operatorname{Log}\left[\frac{1}{2} \left((1 + i) - (1 - i) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] + \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) + \\
& \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] - \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] - \\
& \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] + \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right)\right] \left. \right) + \\
& \frac{2 b^2 f (c + d x) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]}{\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]} + \frac{2 b^2 f (c + d x) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]}{\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]} \left. \right)
\end{aligned}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{a + b \operatorname{Sec}[e + f x]} dx$$

Optimal (type 4, 526 leaves, 14 steps):

$$\begin{aligned}
& \frac{(c + d x)^4}{4 a d} + \frac{i b (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f} - \frac{i b (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f} + \\
& \frac{3 b d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^2} - \frac{3 b d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^2} + \frac{6 i b d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^3} - \\
& \frac{6 i b d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^3} - \frac{6 b d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^4} + \frac{6 b d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a \sqrt{-a^2 + b^2} f^4}
\end{aligned}$$

Result (type 4, 1356 leaves):

$$\begin{aligned}
& \frac{x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) (b + a \operatorname{Cos}[e + f x]) \operatorname{Sec}[e + f x]}{4 a (a + b \operatorname{Sec}[e + f x])} + \\
& \frac{1}{a \sqrt{a^2 - b^2} \sqrt{(-a^2 + b^2) e^{2 i e}} f^4 (a + b \operatorname{Sec}[e + f x])} b (b + a \operatorname{Cos}[e + f x]) \left(2 i c^3 \sqrt{(-a^2 + b^2) e^{2 i e}} f^3 \operatorname{ArcTan}\left[\frac{b + a e^{i (e + f x)}}{\sqrt{a^2 - b^2}}\right] + \right. \\
& 3 i \sqrt{a^2 - b^2} c^2 d e^{i e} f^3 x \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + 3 i \sqrt{a^2 - b^2} c d^2 e^{i e} f^3 x^2 \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& i \sqrt{a^2 - b^2} d^3 e^{i e} f^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - 3 i \sqrt{a^2 - b^2} c^2 d e^{i e} f^3 x \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& 3 i \sqrt{a^2 - b^2} c d^2 e^{i e} f^3 x^2 \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - i \sqrt{a^2 - b^2} d^3 e^{i e} f^3 x^3 \operatorname{Log}\left[1 + \frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& 3 \sqrt{a^2 - b^2} d e^{i e} f^2 (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - 3 \sqrt{a^2 - b^2} d e^{i e} f^2 (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& 6 i \sqrt{a^2 - b^2} c d^2 e^{i e} f \operatorname{PolyLog}\left[3, -\frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + 6 i \sqrt{a^2 - b^2} d^3 e^{i e} f x \operatorname{PolyLog}\left[3, -\frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& 6 i \sqrt{a^2 - b^2} c d^2 e^{i e} f \operatorname{PolyLog}\left[3, -\frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - 6 i \sqrt{a^2 - b^2} d^3 e^{i e} f x \operatorname{PolyLog}\left[3, -\frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& \left. 6 \sqrt{a^2 - b^2} d^3 e^{i e} \operatorname{PolyLog}\left[4, -\frac{a e^{i (2 e + f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + 6 \sqrt{a^2 - b^2} d^3 e^{i e} \operatorname{PolyLog}\left[4, -\frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] \right) \operatorname{Sec}[e + f x]
\end{aligned}$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^3}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 4, 1523 leaves, 36 steps):

$$\begin{aligned}
& - \frac{i b^2 (c + d x)^3}{a^2 (a^2 - b^2) f} + \frac{(c + d x)^4}{4 a^2 d} + \frac{3 b^2 d (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^2} + \frac{3 b^2 d (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^2} - \\
& \frac{i b^3 (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f} + \frac{2 i b (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f} + \frac{i b^3 (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f} - \\
& \frac{2 i b (c + d x)^3 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f} - \frac{6 i b^2 d^2 (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^3} - \frac{6 i b^2 d^2 (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^3} - \\
& \frac{3 b^3 d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^2} + \frac{6 b d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^2} + \frac{3 b^3 d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^2} - \\
& \frac{6 b d (c + d x)^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^2} + \frac{6 b^2 d^3 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^4} + \frac{6 b^2 d^3 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) f^4} - \\
& \frac{6 i b^3 d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^3} + \frac{12 i b d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^3} + \frac{6 i b^3 d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^3} - \\
& \frac{12 i b d^2 (c + d x) \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^3} + \frac{6 b^3 d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^4} - \frac{12 b d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^4} - \\
& \frac{6 b^3 d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^4} + \frac{12 b d^3 \operatorname{PolyLog}\left[4, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^4} + \frac{b^2 (c + d x)^3 \operatorname{Sin}[e + f x]}{a (a^2 - b^2) f (b + a \operatorname{Cos}[e + f x])}
\end{aligned}$$

Result (type 4, 9003 leaves): Display of huge result suppressed!

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^2}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 4, 1117 leaves, 30 steps):

$$\begin{aligned}
& - \frac{i b^2 (c + d x)^2}{a^2 (a^2 - b^2) f} + \frac{(c + d x)^3}{3 a^2 d} + \frac{2 b^2 d (c + d x) \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b - i \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) f^2} + \frac{2 b^2 d (c + d x) \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b + i \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) f^2} - \frac{i b^3 (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f} + \\
& \frac{2 i b (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f} + \frac{i b^3 (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f} - \frac{2 i b (c + d x)^2 \operatorname{Log}\left[1 + \frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f} - \frac{2 i b^2 d^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b - i \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) f^3} \\
& \frac{2 i b^2 d^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b + i \sqrt{a^2 - b^2}}\right]}{a^2 (a^2 - b^2) f^3} - \frac{2 b^3 d (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^2} + \frac{4 b d (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^2} + \\
& \frac{2 b^3 d (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^2} - \frac{4 b d (c + d x) \operatorname{PolyLog}\left[2, -\frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^2} - \frac{2 i b^3 d^2 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^3} + \\
& \frac{4 i b d^2 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b - \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^3} + \frac{2 i b^3 d^2 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} f^3} - \frac{4 i b d^2 \operatorname{PolyLog}\left[3, -\frac{a e^{i(e+f x)}}{b + \sqrt{-a^2 + b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} f^3} + \frac{b^2 (c + d x)^2 \operatorname{Sin}[e + f x]}{a (a^2 - b^2) f (b + a \operatorname{Cos}[e + f x])}
\end{aligned}$$

Result (type 4, 5576 leaves):

$$\begin{aligned}
& \frac{x (3 c^2 + 3 c d x + d^2 x^2) (b + a \operatorname{Cos}[e + f x])^2 \operatorname{Sec}[e + f x]^2}{3 a^2 (a + b \operatorname{Sec}[e + f x])^2} - \frac{1}{(a^2 - b^2)^{3/2} f^2 (a + b \operatorname{Sec}[e + f x])^2} \\
& 4 b c d (b + a \operatorname{Cos}[e + f x])^2 \left(2 (e + f x) \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] - 2 \left(e + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \left(\operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cos}[e + f x]}}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cos}[e + f x]}}\right] - \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b - i \sqrt{a^2 - b^2}) (a + b - \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right])}{a (a + b + \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right])}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] - \text{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] \right) \\
& \text{Sec}[e+fx]^2 + \frac{1}{a^2(a^2-b^2)^{3/2} f^2 (a+b \text{Sec}[e+fx])^2} - 2b^3 c d (b+a \text{Cos}[e+fx])^2 \\
& \left(2(e+fx) \text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - 2\left(e + \text{ArcCos}\left[-\frac{b}{a}\right]\right) \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) + \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] - 2i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2}i(e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Cos}[e+fx]}}\right] + \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Cos}[e+fx]}}\right] - \\
& \left(\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b-i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& \left(-\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] - \text{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \tan\left[\frac{1}{2}(e+fx)\right]\right)}\right] \right) \\
& \text{Sec}[e+fx]^2 - \left(2bd^2 e^{ie} (b+a \text{Cos}[e+fx])^2 \right) \left(-2fx \text{PolyLog}\left[2, -\frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2)} e^{2ie}}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& i \left(f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2) e^{2ie}}} \right] - f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] + 2 i f x \operatorname{PolyLog} \left[2, -\frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] + \right. \\
& \quad \left. 2 \operatorname{PolyLog} \left[3, -\frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2) e^{2ie}}} \right] - 2 \operatorname{PolyLog} \left[3, -\frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] \right) \operatorname{Sec}[e+fx]^2 \Big/ \\
& \left((a^2-b^2) \sqrt{(-a^2+b^2) e^{2ie}} f^3 (a+b \operatorname{Sec}[e+fx])^2 \right) + \left(b^3 d^2 e^{ie} (b+a \operatorname{Cos}[e+fx])^2 \left(-2 f x \operatorname{PolyLog} \left[2, -\frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2) e^{2ie}}} \right] - \right. \right. \\
& \quad \left. \left. i \left(f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2) e^{2ie}}} \right] - f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] + 2 i f x \operatorname{PolyLog} \left[2, -\frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] + \right. \right. \\
& \quad \left. \left. 2 \operatorname{PolyLog} \left[3, -\frac{a e^{i(2e+fx)}}{b e^{ie} - \sqrt{(-a^2+b^2) e^{2ie}}} \right] - 2 \operatorname{PolyLog} \left[3, -\frac{a e^{i(2e+fx)}}{b e^{ie} + \sqrt{(-a^2+b^2) e^{2ie}}} \right] \right) \operatorname{Sec}[e+fx]^2 \Big/ \right. \\
& \quad \left. (a^2 (a^2-b^2) \sqrt{(-a^2+b^2) e^{2ie}} f^3 (a+b \operatorname{Sec}[e+fx])^2 \right) - \frac{4 i b c^2 \operatorname{ArcTan} \left[\frac{-i a \operatorname{Sin}[e] - i (-b+a \operatorname{Cos}[e]) \operatorname{Tan} \left[\frac{fx}{2} \right]}{\sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} \right] (b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e+fx]^2}{(a^2-b^2) f (a+b \operatorname{Sec}[e+fx])^2 \sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} + \\
& \quad \frac{2 i b^3 c^2 \operatorname{ArcTan} \left[\frac{-i a \operatorname{Sin}[e] - i (-b+a \operatorname{Cos}[e]) \operatorname{Tan} \left[\frac{fx}{2} \right]}{\sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} \right] (b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e+fx]^2}{a^2 (a^2-b^2) f (a+b \operatorname{Sec}[e+fx])^2 \sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} + \\
& \quad \left(2 b^2 c d (b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e] \operatorname{Sec}[e+fx]^2 \right. \\
& \quad \left. \left(a \operatorname{Cos}[e] \operatorname{Log}[b+a \operatorname{Cos}[e] \operatorname{Cos}[fx] - a \operatorname{Sin}[e] \operatorname{Sin}[fx]] + a f x \operatorname{Sin}[e] - \frac{2 i a b \operatorname{ArcTan} \left[\frac{-i a \operatorname{Sin}[e] - i (-b+a \operatorname{Cos}[e]) \operatorname{Tan} \left[\frac{fx}{2} \right]}{\sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} \right] \operatorname{Sin}[e]}{\sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}} \right) \right) \Big/ \\
& \quad (a (a^2-b^2) f^2 (a+b \operatorname{Sec}[e+fx])^2 (a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2)) - \frac{1}{a (a^2-b^2) f (a+b \operatorname{Sec}[e+fx])^2} \\
& \quad 2 b^2 d^2 (b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e] \operatorname{Sec}[e+fx]^2
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{x^2 \sin[e]}{2a} - \frac{1}{af} x \left(\cos[e] \log[b + a \cos[e + fx]] + fx \sin[e] + \frac{b \operatorname{ArcTan} \left[\frac{(i \cos[e] + \sin[e]) (a \sin[e] + (-b + a \cos[e]) \tan[\frac{fx}{2}])}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}} \right]}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}} (2 \sin[e]^2 + i \sin[2e]) \right) \right) + \\
& \frac{1}{af} \left(\frac{(e + fx) \cos[e] \log[b + a \cos[e + fx]]}{f} + \frac{1}{f} \right. \\
& a \cos[e] \left(- \frac{(e + fx) \log[b + a \cos[e + fx]]}{a} + 1/a \left(\frac{1}{2} i (e + fx)^2 - 4 i \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[\frac{(-a + b) \tan[\frac{1}{2}(e + fx)]}{\sqrt{-a^2 + b^2}} \right] - \right. \\
& \left. \left(e + fx + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \right) \log \left[1 + \frac{(b - \sqrt{-a^2 + b^2}) e^{i(e+fx)}}{a} \right] - \left(e + fx - 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \right) \log \left[1 + \frac{(b + \sqrt{-a^2 + b^2}) e^{i(e+fx)}}{a} \right] + \right. \\
& \left. \left. (e + fx) \log[b + a \cos[e + fx]] + i \left(\operatorname{PolyLog} \left[2, - \frac{(b - \sqrt{-a^2 + b^2}) e^{i(e+fx)}}{a} \right] + \operatorname{PolyLog} \left[2, - \frac{(b + \sqrt{-a^2 + b^2}) e^{i(e+fx)}}{a} \right] \right) \right) \right) \right) + \\
& \frac{b x \operatorname{ArcTan} \left[\frac{(i \cos[e] + \sin[e]) (a \sin[e] + (-b + a \cos[e]) \tan[\frac{fx}{2}])}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}} \right] (2 \sin[e]^2 + i \sin[2e])}{\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2}} - \frac{1}{2 \sqrt{a^2 - b^2} f (\cos[e] - i \sin[e])^2} \\
& b \left(2 (e + fx) \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2}(e + fx)]}{\sqrt{a^2 - b^2}} \right] - 2 \left(e + \operatorname{ArcCos} \left[-\frac{b}{a} \right] \right) \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2}(e + fx)]}{\sqrt{a^2 - b^2}} \right] + \right. \\
& \left. \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] - 2 i \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2}(e + fx)]}{\sqrt{a^2 - b^2}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2}(e + fx)]}{\sqrt{a^2 - b^2}} \right] \right) \log \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + fx]}} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cos}[e+fx]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(a-b-i\sqrt{a^2-b^2}\right) \left(1+i \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(-ia+ib+\sqrt{a^2-b^2}\right) \left(i+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{\left(b-i\sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] - \operatorname{PolyLog}\left[2, \right. \\
& \left. \frac{\left(b+i\sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] \right) \left(i \operatorname{Cos}[e] + \operatorname{Sin}[e] \right) \left(2 \operatorname{Sin}[e]^2 + i \operatorname{Sin}[2e] \right) \Bigg) + \\
& \left((b+a \operatorname{Cos}[e+fx]) \operatorname{Sec}[e+fx]^2 \left(b^3 c^2 \operatorname{Sin}[e] + 2b^3 c dx \operatorname{Sin}[e] + b^3 d^2 x^2 \operatorname{Sin}[e] - a b^2 c^2 \operatorname{Sin}[fx] - 2a b^2 c dx \operatorname{Sin}[fx] - a b^2 d^2 x^2 \operatorname{Sin}[fx] \right) \right) / \\
& \left(a^2 (-a+b) (a+b) f (a+b \operatorname{Sec}[e+fx])^2 \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right] \right) \right) + \\
& \frac{1}{a^2 (a^2-b^2)^{3/2} f^3 (a+b \operatorname{Sec}[e+fx])^2} 2b^3 d^2 (b+a \operatorname{Cos}[e+fx])^2 \\
& \left(2(e+fx) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - 2 \left(e + \operatorname{ArcCos}\left[-\frac{b}{a}\right] \right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2}i(e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cos}[e+fx]}}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(e+fx)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cos}[e+fx]}}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b-i\sqrt{a^2-b^2}) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2}) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2}) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] - \operatorname{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2}) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right)}\right] \right) \\
& \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e] + \frac{4i b^3 c d \operatorname{ArcTan}\left[\frac{-i a \operatorname{Sin}[e] - i(-b+a \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{fx}{2}\right]}{\sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}}\right] (b+a \operatorname{Cos}[e+fx])^2 \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e]}{a^2 (a^2-b^2) f^2 (a+b \operatorname{Sec}[e+fx])^2 \sqrt{-b^2+a^2 \operatorname{Cos}[e]^2+a^2 \operatorname{Sin}[e]^2}}
\end{aligned}$$

Test results for the 83 problems in "4.5.11 (e x)^m (a+b sec(c+d x^n))^p.m"

- Problem 5: Result more than twice size of optimal antiderivative.

$$\int x (a+b \operatorname{Sec}[c+d x^2]) dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{a x^2}{2} + \frac{b \operatorname{ArcTanh}[\operatorname{Sin}[c+d x^2]]}{2 d}$$

Result (type 3, 91 leaves):

$$\frac{a x^2}{2} - \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x^2}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d} + \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x^2}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d}$$

- Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^3 (a+b \operatorname{Sec}[c+d x^2])^2 dx$$

Optimal (type 4, 133 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 i a b x^2 \operatorname{ArcTan}\left[e^{i(c+dx^2)}\right]}{d} + \frac{b^2 \operatorname{Log}\left[\operatorname{Cos}\left[c+dx^2\right]\right]}{2d^2} + \frac{i a b \operatorname{PolyLog}\left[2, -i e^{i(c+dx^2)}\right]}{d^2} - \frac{i a b \operatorname{PolyLog}\left[2, i e^{i(c+dx^2)}\right]}{d^2} + \frac{b^2 x^2 \operatorname{Tan}\left[c+dx^2\right]}{2d}$$

Result (type 4, 677 leaves):

$$\begin{aligned} & \frac{x^2 \operatorname{Cos}\left[c+dx^2\right]^2 (a+b \operatorname{Sec}\left[c+dx^2\right])^2 (a^2 dx^2 \operatorname{Cos}[c] + 2b^2 \operatorname{Sin}[c])}{4d (b+a \operatorname{Cos}\left[c+dx^2\right])^2 (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right])} + \\ & \left(b^2 \operatorname{Cos}\left[c+dx^2\right]^2 \operatorname{Sec}[c] (a+b \operatorname{Sec}\left[c+dx^2\right])^2 (\operatorname{Cos}[c] \operatorname{Log}\left[\operatorname{Cos}[c] \operatorname{Cos}\left[dx^2\right] - \operatorname{Sin}[c] \operatorname{Sin}\left[dx^2\right]\right] + dx^2 \operatorname{Sin}[c]) \right) / \\ & \left(2d^2 (b+a \operatorname{Cos}\left[c+dx^2\right])^2 (\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2) \right) + \frac{1}{d^2 (b+a \operatorname{Cos}\left[c+dx^2\right])^2} a b \operatorname{Cos}\left[c+dx^2\right]^2 (a+b \operatorname{Sec}\left[c+dx^2\right])^2 \\ & \left(-1 / \left(\sqrt{1 + \operatorname{Cot}[c]^2} \right) \operatorname{Csc}[c] \left((dx^2 - \operatorname{ArcTan}[\operatorname{Cot}[c]]) (\operatorname{Log}\left[1 - e^{i(dx^2 - \operatorname{ArcTan}[\operatorname{Cot}[c])}\right]) - \operatorname{Log}\left[1 + e^{i(dx^2 - \operatorname{ArcTan}[\operatorname{Cot}[c])}\right]\right) + \right. \\ & \left. i (\operatorname{PolyLog}\left[2, -e^{i(dx^2 - \operatorname{ArcTan}[\operatorname{Cot}[c])}\right]) - \operatorname{PolyLog}\left[2, e^{i(dx^2 - \operatorname{ArcTan}[\operatorname{Cot}[c])}\right]) \right) + \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[c] + \operatorname{Cos}[c] \operatorname{Tan}\left[\frac{dx^2}{2}\right]}{\sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}\right]}{\sqrt{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}} \right) + \\ & \frac{b^2 x^2 \operatorname{Cos}\left[c+dx^2\right]^2 (a+b \operatorname{Sec}\left[c+dx^2\right])^2 \operatorname{Sin}\left[\frac{dx^2}{2}\right]}{2d (b+a \operatorname{Cos}\left[c+dx^2\right])^2 (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx^2}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx^2}{2}\right]\right)} + \\ & \frac{b^2 x^2 \operatorname{Cos}\left[c+dx^2\right]^2 (a+b \operatorname{Sec}\left[c+dx^2\right])^2 \operatorname{Sin}\left[\frac{dx^2}{2}\right]}{2d (b+a \operatorname{Cos}\left[c+dx^2\right])^2 (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx^2}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx^2}{2}\right]\right)} - \\ & \frac{b^2 x^2 \operatorname{Cos}\left[c+dx^2\right]^2 (a+b \operatorname{Sec}\left[c+dx^2\right])^2 \operatorname{Tan}[c]}{2d (b+a \operatorname{Cos}\left[c+dx^2\right])^2} \end{aligned}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int x (a+b \operatorname{Sec}\left[c+dx^2\right])^2 dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\frac{a^2 x^2}{2} + \frac{a b \operatorname{ArcTanh}\left[\operatorname{Sin}\left[c+dx^2\right]\right]}{d} + \frac{b^2 \operatorname{Tan}\left[c+dx^2\right]}{2d}$$

Result (type 3, 92 leaves) :

$$\frac{1}{2d} \left(a \left(a c + a d x^2 - 2 b \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x^2) \right] - \sin \left[\frac{1}{2} (c + d x^2) \right] \right] + 2 b \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x^2) \right] + \sin \left[\frac{1}{2} (c + d x^2) \right] \right] \right) + b^2 \operatorname{Tan} [c + d x^2] \right)$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3}{a + b \operatorname{Sec} [c + d x^2]} dx$$

Optimal (type 4, 261 leaves, 11 steps) :

$$\frac{x^4}{4a} + \frac{i b x^2 \operatorname{Log} \left[1 + \frac{a e^{i(c+dx^2)}}{b - \sqrt{-a^2+b^2}} \right]}{2 a \sqrt{-a^2+b^2} d} - \frac{i b x^2 \operatorname{Log} \left[1 + \frac{a e^{i(c+dx^2)}}{b + \sqrt{-a^2+b^2}} \right]}{2 a \sqrt{-a^2+b^2} d} + \frac{b \operatorname{PolyLog} \left[2, -\frac{a e^{i(c+dx^2)}}{b - \sqrt{-a^2+b^2}} \right]}{2 a \sqrt{-a^2+b^2} d^2} - \frac{b \operatorname{PolyLog} \left[2, -\frac{a e^{i(c+dx^2)}}{b + \sqrt{-a^2+b^2}} \right]}{2 a \sqrt{-a^2+b^2} d^2}$$

Result (type 4, 845 leaves) :

$$\begin{aligned}
& \frac{1}{4 a (a + b \operatorname{Sec}[c + d x^2])} (b + a \operatorname{Cos}[c + d x^2]) \\
& \left(x^4 - \frac{1}{\sqrt{a^2 - b^2} d^2} 2 b \left(2 (c + d x^2) \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] - 2 \left(c + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + \right. \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (c + d x^2)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cos}[c + d x^2]}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a + b) \operatorname{Cot}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (c + d x^2)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \operatorname{Cos}[c + d x^2]}}\right] - \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) (a - b - i \sqrt{a^2 - b^2}) (1 + i \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}\right] - \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right]}{\sqrt{a^2 - b^2}}\right] \right) \operatorname{Log}\left[\frac{(a + b) (-i a + i b + \sqrt{a^2 - b^2}) (i + \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}\right] + \right. \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(b - i \sqrt{a^2 - b^2}) (a + b - \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}\right] - \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(b + i \sqrt{a^2 - b^2}) (a + b - \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}{a (a + b + \sqrt{a^2 - b^2} \operatorname{Tan}\left[\frac{1}{2} (c + d x^2)\right])}\right] \right) \right) \operatorname{Sec}[c + d x^2]
\end{aligned}$$

■ **Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{Sec}[c + d \sqrt{x}]}{\sqrt{x}} dx$$

Optimal (type 3, 26 leaves, 4 steps) :

$$2 a \sqrt{x} + \frac{2 b \operatorname{ArcTanh}\left[\operatorname{Sin}[c + d \sqrt{x}]\right]}{d}$$

Result (type 3, 84 leaves) :

$$\frac{1}{d} 2 \left(a (c + d \sqrt{x}) - b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d \sqrt{x}) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d \sqrt{x}) \right] \right] + b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d \sqrt{x}) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d \sqrt{x}) \right] \right] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d \sqrt{x}])^2}{\sqrt{x}} dx$$

Optimal (type 3, 47 leaves, 5 steps) :

$$2 a^2 \sqrt{x} + \frac{4 a b \operatorname{ArcTanh}[\operatorname{Sin}[c + d \sqrt{x}]]}{d} + \frac{2 b^2 \operatorname{Tan}[c + d \sqrt{x}]}{d}$$

Result (type 3, 102 leaves) :

$$\frac{1}{d} 2 \left(a \left(a c + a d \sqrt{x} - 2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d \sqrt{x}) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d \sqrt{x}) \right] \right] + 2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d \sqrt{x}) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d \sqrt{x}) \right] \right] \right) + b^2 \operatorname{Tan}[c + d \sqrt{x}] \right)$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int (e x)^{-1+n} (a + b \operatorname{Sec}[c + d x^n]) dx$$

Optimal (type 3, 44 leaves, 5 steps) :

$$\frac{a (e x)^n}{e n} + \frac{b x^{-n} (e x)^n \operatorname{ArcTanh}[\operatorname{Sin}[c + d x^n]]}{d e n}$$

Result (type 3, 89 leaves) :

$$\frac{1}{d e n} x^{-n} (e x)^n \left(a (c + d x^n) - b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x^n) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x^n) \right] \right] + b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x^n) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x^n) \right] \right] \right)$$

■ **Problem 74: Unable to integrate problem.**

$$\int (e x)^{-1+3 n} (a + b \operatorname{Sec}[c + d x^n]) dx$$

Optimal (type 4, 235 leaves, 11 steps) :

$$\frac{a (e x)^{3 n}}{3 e n} - \frac{2 i b x^{-n} (e x)^{3 n} \operatorname{ArcTan}\left[e^{i(c+d x^n)}\right]}{d e n} + \frac{2 i b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -i e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 i b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, i e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -i e^{i(c+d x^n)}\right]}{d^3 e n} + \frac{2 b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, i e^{i(c+d x^n)}\right]}{d^3 e n}$$

Result (type 8, 24 leaves) :

$$\int (e x)^{-1+3 n} (a + b \operatorname{Sec}[c + d x^n]) dx$$

■ **Problem 76: Result more than twice size of optimal antiderivative.**

$$\int (e x)^{-1+2 n} (a+b \operatorname{Sec}[c+d x^n])^2 dx$$

Optimal (type 4, 221 leaves, 11 steps):

$$\frac{a^2 (e x)^{2 n}}{2 e n} - \frac{4 i a b x^{-n} (e x)^{2 n} \operatorname{ArcTan}\left[e^{i(c+d x^n)}\right]}{d e n} + \frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[\operatorname{Cos}[c+d x^n]]}{d^2 e n} +$$

$$\frac{2 i a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -i e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{2 i a b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, i e^{i(c+d x^n)}\right]}{d^2 e n} + \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Tan}[c+d x^n]}{d e n}$$

Result (type 4, 769 leaves):

$$\frac{x^{1-n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 (a+b \operatorname{Sec}[c+d x^n])^2 \left(a^2 d x^n \operatorname{Cos}[c]+2 b^2 \operatorname{Sin}[c]\right)}{2 d n (b+a \operatorname{Cos}[c+d x^n])^2 \left(\operatorname{Cos}\left[\frac{c}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}\right]\right)} +$$

$$\left(b^2 x^{1-2 n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 \operatorname{Sec}[c] (a+b \operatorname{Sec}[c+d x^n])^2 \left(\operatorname{Cos}[c] \operatorname{Log}[\operatorname{Cos}[c] \operatorname{Cos}[d x^n]-\operatorname{Sin}[c] \operatorname{Sin}[d x^n]]+d x^n \operatorname{Sin}[c]]\right) / \right.$$

$$\left.\left(d^2 n (b+a \operatorname{Cos}[c+d x^n])^2 \left(\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2\right)\right) + \frac{1}{d^2 n (b+a \operatorname{Cos}[c+d x^n])^2} 2 a b x^{1-2 n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 (a+b \operatorname{Sec}[c+d x^n])^2 \right.$$

$$\left. \left(-1 / \left(\sqrt{1+\operatorname{Cot}[c]^2}\right) \operatorname{Csc}[c] \left(\left(d x^n-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right) \left(\operatorname{Log}\left[1-e^{i\left(d x^n-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]-\operatorname{Log}\left[1+e^{i\left(d x^n-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]\right) + \right.$$

$$\left. i \left(\operatorname{PolyLog}\left[2,-e^{i\left(d x^n-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]-\operatorname{PolyLog}\left[2,e^{i\left(d x^n-\operatorname{ArcTan}[\operatorname{Cot}[c]]\right)}\right]\right) + \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[c]] \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[c]+\operatorname{Cos}[c] \operatorname{Tan}\left[\frac{d x^n}{2}\right]}{\sqrt{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}\right]}{\sqrt{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}\right) +$$

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 (a+b \operatorname{Sec}[c+d x^n])^2 \operatorname{Sin}\left[\frac{d x^n}{2}\right]}{d n (b+a \operatorname{Cos}[c+d x^n])^2 \left(\operatorname{Cos}\left[\frac{c}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}+\frac{d x^n}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}+\frac{d x^n}{2}\right]\right)} +$$

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 (a+b \operatorname{Sec}[c+d x^n])^2 \operatorname{Sin}\left[\frac{d x^n}{2}\right]}{d n (b+a \operatorname{Cos}[c+d x^n])^2 \left(\operatorname{Cos}\left[\frac{c}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}+\frac{d x^n}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}+\frac{d x^n}{2}\right]\right)} -$$

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Cos}[c+d x^n]^2 (a+b \operatorname{Sec}[c+d x^n])^2 \operatorname{Tan}[c]}{d n (b+a \operatorname{Cos}[c+d x^n])^2}$$

■ **Problem 77: Unable to integrate problem.**

$$\int (e x)^{-1+3 n} (a+b \operatorname{Sec}[c+d x^n])^2 dx$$

Optimal (type 4, 390 leaves, 16 steps):

$$\frac{a^2 (e x)^{3 n}}{3 e n} - \frac{i b^2 x^{-n} (e x)^{3 n}}{d e n} - \frac{4 i a b x^{-n} (e x)^{3 n} \operatorname{ArcTan}\left[e^{i(c+d x^n)}\right]}{d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1+e^{2 i(c+d x^n)}\right]}{d^2 e n} +$$

$$\frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,-i e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,i e^{i(c+d x^n)}\right]}{d^2 e n} - \frac{i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,-e^{2 i(c+d x^n)}\right]}{d^3 e n} -$$

$$\frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,-i e^{i(c+d x^n)}\right]}{d^3 e n} + \frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,i e^{i(c+d x^n)}\right]}{d^3 e n} + \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Tan}[c+d x^n]}{d e n}$$

Result (type 8, 26 leaves):

$$\int (e x)^{-1+3 n} (a+b \operatorname{Sec}[c+d x^n])^2 dx$$

■ **Problem 79: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^{-1+2 n}}{a+b \operatorname{Sec}[c+d x^n]} dx$$

Optimal (type 4, 328 leaves, 12 steps):

$$\frac{(e x)^{2 n}}{2 a e n} + \frac{i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1+\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} - \frac{i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1+\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} +$$

$$\frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,-\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} - \frac{b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2,-\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n}$$

Result (type 4, 861 leaves):

$$\begin{aligned}
& \frac{1}{2 a e^n (a+b \operatorname{Sec}[c+d x^n])} (e x)^{2 n} (b+a \operatorname{Cos}[c+d x^n]) \\
& \left(1 - \frac{1}{\sqrt{a^2-b^2} d^2} 2 b x^{-2 n} \left(2 (c+d x^n) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] - 2 \left(c + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i (c+d x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cos}[c+d x^n]}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i (c+d x^n)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Cos}[c+d x^n]}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(a-b-i \sqrt{a^2-b^2}\right) \left(1+i \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[\frac{(a+b) \left(-i a+i b+\sqrt{a^2-b^2}\right) \left(i+\operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{\left(b-i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}\right] - \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{\left(b+i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]\right)}\right] \right) \right) \right) \operatorname{Sec}[c+d x^n]
\end{aligned}$$

■ **Problem 80: Unable to integrate problem.**

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Sec}[c+d x^n]} dx$$

Optimal (type 4, 485 leaves, 14 steps):

$$\frac{(e x)^{3 n}}{3 a e n} + \frac{i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} - \frac{i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d e n} + \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} - \frac{2 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^2 e n} + \frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n} - \frac{2 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a \sqrt{-a^2+b^2} d^3 e n}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{a+b \operatorname{Sec}[c+d x^n]} dx$$

■ **Problem 82: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e x)^{-1+2 n}}{(a+b \operatorname{Sec}[c+d x^n])^2} dx$$

Optimal (type 4, 757 leaves, 23 steps):

$$\frac{(e x)^{2 n}}{2 a^2 e n} - \frac{i b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} + \frac{2 i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} + \frac{i b^3 x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d e n} - \frac{2 i b x^{-n} (e x)^{2 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d e n} + \frac{b^2 x^{-2 n} (e x)^{2 n} \operatorname{Log}[b+a \operatorname{Cos}[c+d x^n]]}{a^2 (a^2-b^2) d^2 e n} - \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} + \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} + \frac{b^3 x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} d^2 e n} - \frac{2 b x^{-2 n} (e x)^{2 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} d^2 e n} + \frac{b^2 x^{-n} (e x)^{2 n} \operatorname{Sin}[c+d x^n]}{a (a^2-b^2) d e n (b+a \operatorname{Cos}[c+d x^n])}$$

Result (type 4, 2450 leaves):

$$-\frac{1}{(a^2-b^2)^{3/2} d^2 n (a+b \operatorname{Sec}[c+d x^n])^2} 2 b x^{1-2 n} (e x)^{-1+2 n} (b+a \operatorname{Cos}[c+d x^n])^2$$

$$\left(2 (c+d x^n) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] - 2 \left(c + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x^n)\right]}{\sqrt{a^2-b^2}}\right] \right) +$$

$$\begin{aligned}
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2}i(c+dx^n)}}{\sqrt{2}\sqrt{a}\sqrt{b+a}\operatorname{Cos}[c+dx^n]}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(c+dx^n)}}{\sqrt{2}\sqrt{a}\sqrt{b+a}\operatorname{Cos}[c+dx^n]}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}{a(a+b+\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}{a(a+b+\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}\right] + \\
& i \left(\operatorname{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}{a(a+b+\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}\right] - \operatorname{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}{a(a+b+\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}\right] \right) \Bigg) \\
& \operatorname{Sec}[c+dx^n]^2 + \frac{1}{a^2(a^2-b^2)^{3/2}d^2n(a+b\operatorname{Sec}[c+dx^n])^2} b^3 x^{1-2n} (ex)^{-1+2n} (b+a\operatorname{Cos}[c+dx^n])^2
\end{aligned}$$

$$\begin{aligned}
& \left(2(c+dx^n)\operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] - 2\left(c+\operatorname{ArcCos}\left[-\frac{b}{a}\right]\right)\operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) + \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2}i(c+dx^n)}}{\sqrt{2}\sqrt{a}\sqrt{b+a}\operatorname{Cos}[c+dx^n]}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \left(\operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2}i(c+dx^n)}}{\sqrt{2}\sqrt{a}\sqrt{b+a}\operatorname{Cos}[c+dx^n]}\right] - \\
& \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2i \operatorname{ArcTanh}\left[\frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \operatorname{Log}\left[1 - \frac{(b-i\sqrt{a^2-b^2})(a+b-\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}{a(a+b+\sqrt{a^2-b^2}\operatorname{Tan}\left[\frac{1}{2}(c+dx^n)\right])}\right] +
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}\left[-\frac{b}{a}\right] + 2i \text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]}{\sqrt{a^2-b^2}}\right] \right) \text{Log}\left[1 - \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}\right] + \\
& i \left(\text{PolyLog}\left[2, \frac{(b-i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}\right] - \text{PolyLog}\left[2, \frac{(b+i\sqrt{a^2-b^2})\left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}(c+dx^n)\right]\right)}\right] \right) \\
& \text{Sec}[c+dx^n]^2 + \frac{x^{1-n} (ex)^{-1+2n} (b+a \text{Cos}[c+dx^n])^2 \text{Sec}[c+dx^n]^2 (a^2 dx^n \text{Cos}[c] - b^2 dx^n \text{Cos}[c] + 2b^2 \text{Sin}[c])}{2a^2(a-b)(a+b)dn(a+b \text{Sec}[c+dx^n])^2 (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right])} + \\
& \left(b^2 x^{1-2n} (ex)^{-1+2n} (b+a \text{Cos}[c+dx^n])^2 \text{Sec}[c] \text{Sec}[c+dx^n]^2 \right. \\
& \left. \left(a \text{Cos}[c] \text{Log}[b+a \text{Cos}[c] \text{Cos}[dx^n] - a \text{Sin}[c] \text{Sin}[dx^n]] + a dx^n \text{Sin}[c] - \frac{2iab \text{ArcTan}\left[\frac{-ia \text{Sin}[c] - i(-b+a \text{Cos}[c]) \text{Tan}\left[\frac{dx^n}{2}\right]}{\sqrt{-b^2+a^2 \text{Cos}[c]^2+a^2 \text{Sin}[c]^2}}\right]}{\sqrt{-b^2+a^2 \text{Cos}[c]^2+a^2 \text{Sin}[c]^2}} \text{Sin}[c] \right) \right) / \\
& \frac{(a(a^2-b^2) d^2 n (a+b \text{Sec}[c+dx^n])^2 (a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2)) + b^2 x^{1-n} (ex)^{-1+2n} (b+a \text{Cos}[c+dx^n]) \text{Sec}[c+dx^n]^2 (b \text{Sin}[c] - a \text{Sin}[dx^n])}{a^2(-a+b)(a+b)dn(a+b \text{Sec}[c+dx^n])^2 (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right])} + \\
& \frac{b^2 x^{1-n} (ex)^{-1+2n} (b+a \text{Cos}[c+dx^n])^2 \text{Sec}[c+dx^n]^2 \text{Tan}[c]}{a^2(-a^2+b^2)dn(a+b \text{Sec}[c+dx^n])^2} - \\
& \frac{2iab^3 x^{1-2n} (ex)^{-1+2n} \text{ArcTan}\left[\frac{b+a \text{Cos}[c+dx^n]+ia \text{Sin}[c+dx^n]}{\sqrt{a^2-b^2}}\right] (b+a \text{Cos}[c+dx^n])^2 \text{Sec}[c+dx^n]^2 \text{Tan}[c]}{a^2(a^2-b^2)^{3/2} d^2 n (a+b \text{Sec}[c+dx^n])^2}
\end{aligned}$$

■ **Problem 83: Unable to integrate problem.**

$$\int \frac{(ex)^{-1+3n}}{(a+b \text{Sec}[c+dx^n])^2} dx$$

Optimal (type 4, 1384 leaves, 32 steps):

$$\begin{aligned}
& \frac{(e x)^{3 n}}{3 a^2 e n} - \frac{i b^2 x^{-n} (e x)^{3 n}}{a^2 (a^2 - b^2) d e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} + \frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} - \\
& \frac{i b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} + \frac{2 i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} + \frac{i b^3 x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} - \\
& \frac{2 i b x^{-n} (e x)^{3 n} \operatorname{Log}\left[1 + \frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} - \frac{2 i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \frac{2 i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \\
& \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} + \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} + \frac{2 b^3 x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} - \\
& \frac{4 b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} - \frac{2 i b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} + \frac{4 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} + \\
& \frac{2 i b^3 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} - \frac{4 i b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3, -\frac{a e^{i(c+d x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} + \frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Sin}[c + d x^n]}{a (a^2 - b^2) d e n (b + a \operatorname{Cos}[c + d x^n])}
\end{aligned}$$

Result (type 8, 26 leaves):

$$\int \frac{(e x)^{-1+3 n}}{(a + b \operatorname{Sec}[c + d x^n])^2} dx$$

Test results for the 879 problems in "4.5.1.2 (d sec)^n (a+b sec)^m.m"

- Problem 1: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[c + d x]^4 (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{3 a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a \operatorname{Tan}[c + d x]}{d} + \frac{3 a \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{a \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$\begin{aligned}
& - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
& \frac{a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\
& \frac{3 a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 a \operatorname{Tan}[c+d x]}{3 d} + \frac{a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}
\end{aligned}$$

■ **Problem 2: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^3 (a+a \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} + \frac{a \operatorname{Tan}[c+d x]}{d} + \frac{a \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d} + \frac{a \operatorname{Tan}[c+d x]^3}{3 d}$$

Result (type 3, 163 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 a \operatorname{Tan}[c+d x]}{3 d} + \frac{a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}
\end{aligned}$$

■ **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^2 (a+a \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} + \frac{a \operatorname{Tan}[c+d x]}{d} + \frac{a \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}$$

Result (type 3, 138 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{a \operatorname{Tan}[c+d x]}{d}
\end{aligned}$$

■ **Problem 4: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x] (a+a \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{a \operatorname{Tan}[c + d x]}{d}$$

Result (type 3, 81 leaves) :

$$-\frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a \operatorname{Tan}[c + d x]}{d}$$

■ **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 16 leaves, 2 steps) :

$$a x + \frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d}$$

Result (type 3, 73 leaves) :

$$a x - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d}$$

■ **Problem 10: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^4 (a + a \operatorname{Sec}[c + d x])^2 dx$$

Optimal (type 3, 122 leaves, 7 steps) :

$$\frac{3 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{4 d} + \frac{9 a^2 \operatorname{Tan}[c + d x]}{5 d} + \frac{3 a^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{4 d} + \frac{a^2 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{2 d} + \frac{a^2 \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d} + \frac{3 a^2 \operatorname{Tan}[c + d x]^3}{5 d}$$

Result (type 3, 487 leaves) :

$$\begin{aligned}
& - \frac{1}{640 d} a^2 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \\
& \left(75 \operatorname{Cos}[2 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 75 \operatorname{Cos}[4 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\
& 15 \operatorname{Cos}[4 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 15 \operatorname{Cos}[6 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \\
& 150 \operatorname{Cos}[d x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 150 \operatorname{Cos}[2 c + d x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\
& 75 \operatorname{Cos}[2 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 75 \operatorname{Cos}[4 c + 3 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \\
& 15 \operatorname{Cos}[4 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 15 \operatorname{Cos}[6 c + 5 d x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 400 \operatorname{Sin}[d x] + \\
& \left. 80 \operatorname{Sin}[2 c + d x] - 140 \operatorname{Sin}[c + 2 d x] - 140 \operatorname{Sin}[3 c + 2 d x] - 240 \operatorname{Sin}[2 c + 3 d x] - 30 \operatorname{Sin}[3 c + 4 d x] - 30 \operatorname{Sin}[5 c + 4 d x] - 48 \operatorname{Sin}[4 c + 5 d x] \right)
\end{aligned}$$

■ **Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^3 (a + a \operatorname{Sec}[c + d x])^2 dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{7 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{2 a^2 \operatorname{Tan}[c + d x]}{d} + \frac{7 a^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a^2 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{2 a^2 \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 877 leaves):

$$\begin{aligned}
& - \frac{7 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{32 d} + \\
& \frac{7 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{32 d} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{64 d\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^4} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{12 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(29 \cos \left[\frac{c}{2}\right]-13 \sin \left[\frac{c}{2}\right]\right)}{192 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{3 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} - \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{64 d\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^4} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{12 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-29 \cos \left[\frac{c}{2}\right]-13 \sin \left[\frac{c}{2}\right]\right)}{192 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{3 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^2 (a+a \operatorname{Sec}[c+d x])^2 dx$$

Optimal (type 3, 74 leaves, 6 steps):

$$\frac{a^2 \operatorname{ArcTanh}\left[\sin [c+d x]\right]}{d} + \frac{5 a^2 \tan [c+d x]}{3 d} + \frac{a^2 \operatorname{Sec}[c+d x] \tan [c+d x]}{d} + \frac{a^2 \operatorname{Sec}[c+d x]^2 \tan [c+d x]}{3 d}$$

Result (type 3, 318 leaves):

$$\begin{aligned}
& - \frac{1}{24 d} \\
& a^2 \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \left(3 \cos [2 c+3 d x] \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] + 3 \cos [4 c+3 d x] \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& 9 \cos [d x] \left(\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& 9 \cos [2 c+d x] \left(\operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] \right) - \\
& 3 \cos [2 c+3 d x] \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] - 3 \cos [4 c+3 d x] \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] - \\
& \left. 24 \sin [d x] + 6 \sin [2 c+d x] - 6 \sin [c+2 d x] - 6 \sin [3 c+2 d x] - 10 \sin [2 c+3 d x] \right)
\end{aligned}$$

■ **Problem 13: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + a \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$\frac{3 a^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} + \frac{2 a^2 \text{Tan}[c + d x]}{d} + \frac{a^2 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 d}$$

Result (type 3, 219 leaves):

$$\begin{aligned} & \frac{1}{16 d} a^2 (1 + \text{Cos}[c + d x])^2 \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \left(-6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ & \left. 6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \frac{1}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{1}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \right. \\ & \left. (8 \text{Sin}[d x]) / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) \right) \end{aligned}$$

■ **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$a^2 x + \frac{2 a^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{a^2 \text{Tan}[c + d x]}{d}$$

Result (type 3, 171 leaves):

$$\begin{aligned} & \frac{1}{4 d} a^2 (1 + \text{Cos}[c + d x])^2 \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \left(d x - 2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ & \left. \text{Sin}[d x] / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) \right) \end{aligned}$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x] (a + a \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$2 a^2 x + \frac{a^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{a^2 \text{Sin}[c + d x]}{d}$$

Result (type 3, 106 leaves):

$$2 a^2 x - \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a^2 \cos[dx] \sin[c]}{d} + \frac{a^2 \cos[c] \sin[dx]}{d}$$

■ **Problem 20: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^3 (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 114 leaves, 11 steps):

$$\frac{13 a^3 \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{4 a^3 \tan[c + dx]}{d} + \frac{13 a^3 \sec[c + dx] \tan[c + dx]}{8 d} + \frac{3 a^3 \sec[c + dx]^3 \tan[c + dx]}{4 d} + \frac{5 a^3 \tan[c + dx]^3}{3 d} + \frac{a^3 \tan[c + dx]^5}{5 d}$$

Result (type 3, 487 leaves):

$$-\frac{1}{3840 d} a^3 \sec[c] \sec[c + dx]^5 \left(975 \cos[2c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 975 \cos[4c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 195 \cos[4c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 195 \cos[6c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 1950 \cos[dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + 1950 \cos[2c + dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - 975 \cos[2c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 975 \cos[4c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 195 \cos[4c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 195 \cos[6c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 4640 \sin[dx] + 1440 \sin[2c + dx] - 1500 \sin[c + 2dx] - 1500 \sin[3c + 2dx] - 3040 \sin[2c + 3dx] - 390 \sin[3c + 4dx] - 390 \sin[5c + 4dx] - 608 \sin[4c + 5dx] \right)$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^2 (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 93 leaves, 11 steps):

$$\frac{15 a^3 \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{4 a^3 \tan[c + dx]}{d} + \frac{15 a^3 \sec[c + dx] \tan[c + dx]}{8 d} + \frac{a^3 \sec[c + dx]^3 \tan[c + dx]}{4 d} + \frac{a^3 \tan[c + dx]^3}{d}$$

Result (type 3, 877 leaves):

$$\begin{aligned}
& - \frac{15 \operatorname{Cos}[c + d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3}{64 d} + \\
& \frac{15 \operatorname{Cos}[c + d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3}{64 d} + \\
& \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3}{128 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^4} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{16 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} + \\
& \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \left(19 \operatorname{Cos}\left[\frac{c}{2}\right] - 11 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{128 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2} + \frac{3 \operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{8 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} - \\
& \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3}{128 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^4} + \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{16 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} + \\
& \frac{\operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \left(-19 \operatorname{Cos}\left[\frac{c}{2}\right] - 11 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{128 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2} + \frac{3 \operatorname{Cos}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{8 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^3 dx$$

Optimal (type 3, 72 leaves, 9 steps):

$$\frac{5 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{4 a^3 \operatorname{Tan}[c + d x]}{d} + \frac{3 a^3 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d} + \frac{a^3 \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 733 leaves):

Result (type 3, 211 leaves) :

$$\frac{1}{8} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left(3x - \frac{3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{\cos[dx] \sin[c]}{d} + \frac{\cos[c] \sin[dx]}{d} + \frac{\sin\left[\frac{dx}{2}\right]}{d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} + \frac{\sin\left[\frac{dx}{2}\right]}{d (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} \right)$$

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^2 (a + a \sec[c + dx])^4 dx$$

Optimal (type 3, 111 leaves, 13 steps) :

$$\frac{7 a^4 \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{8 a^4 \tan[c + dx]}{d} + \frac{7 a^4 \sec[c + dx] \tan[c + dx]}{2d} + \frac{a^4 \sec[c + dx]^3 \tan[c + dx]}{d} + \frac{8 a^4 \tan[c + dx]^3}{3d} + \frac{a^4 \tan[c + dx]^5}{5d}$$

Result (type 3, 498 leaves) :

$$-\frac{1}{960d} a^4 \sec[c] \sec[c + dx]^5$$

$$\left(525 \cos[2c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 525 \cos[4c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 105 \cos[4c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 105 \cos[6c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 1050 \cos[dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + 1050 \cos[2c + dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - 525 \cos[2c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 525 \cos[4c + 3dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 105 \cos[4c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 105 \cos[6c + 5dx] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 2360 \sin[dx] + 960 \sin[2c + dx] - 660 \sin[c + 2dx] - 660 \sin[3c + 2dx] - 1600 \sin[2c + 3dx] + 60 \sin[4c + 3dx] - 210 \sin[3c + 4dx] - 210 \sin[5c + 4dx] - 332 \sin[4c + 5dx] \right)$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + a \sec[c + dx])^4 dx$$

Optimal (type 3, 96 leaves, 12 steps) :

$$\frac{35 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{8 a^4 \operatorname{Tan}[c + d x]}{d} + \frac{27 a^4 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a^4 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{4 a^4 \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 877 leaves) :

$$\begin{aligned} & - \frac{35 \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4}{128 d} + \\ & \frac{35 \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4}{128 d} + \\ & \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4}{256 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^4} + \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{d x}{2}\right]}{24 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} + \\ & \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \left(97 \operatorname{Cos}\left[\frac{c}{2}\right] - 65 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{768 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2} + \frac{5 \operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{d x}{2}\right]}{12 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} - \\ & \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4}{256 d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^4} + \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{d x}{2}\right]}{24 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} + \\ & \frac{\operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \left(-97 \operatorname{Cos}\left[\frac{c}{2}\right] - 65 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{768 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2} + \frac{5 \operatorname{Cos}[c + d x]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}\left[\frac{d x}{2}\right]}{12 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} \end{aligned}$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^4 dx$$

Optimal (type 3, 91 leaves, 6 steps) :

$$a^4 x + \frac{6 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^4 \operatorname{Tan}[c + d x]}{d} + \frac{(a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{4 (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 773 leaves) :

$$\frac{1}{16} x \cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 - \frac{3 \cos [c+d x]^4 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4}{8 d} +$$

$$\frac{3 \cos [c+d x]^4 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4}{8 d} +$$

$$\frac{\cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \sin \left[\frac{d x}{2} \right]}{96 d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^3} + \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \left(13 \cos \left[\frac{c}{2} \right] - 11 \sin \left[\frac{c}{2} \right] \right)}{192 d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^2} +$$

$$\frac{5 \cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \sin \left[\frac{d x}{2} \right]}{12 d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)} + \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \sin \left[\frac{d x}{2} \right]}{96 d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^3} +$$

$$\frac{\cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \left(-13 \cos \left[\frac{c}{2} \right] - 11 \sin \left[\frac{c}{2} \right] \right)}{192 d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)^2} + \frac{5 \cos [c+d x]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^4 \sin \left[\frac{d x}{2} \right]}{12 d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right)}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x] (a+a \sec [c+d x])^4 dx$$

Optimal (type 3, 73 leaves, 8 steps):

$$4 a^4 x + \frac{13 a^4 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{a^4 \sin [c+d x]}{d} + \frac{4 a^4 \tan [c+d x]}{d} + \frac{a^4 \sec [c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 272 leaves):

$$\frac{1}{64} a^4 (1+\cos [c+d x])^4 \sec \left[\frac{1}{2} (c+d x) \right]^8$$

$$\left(16 x - \frac{26 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right]}{d} + \frac{26 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right]}{d} + \frac{4 \cos [d x] \sin [c]}{d} + \right.$$

$$\frac{4 \cos [c] \sin [d x]}{d} + \frac{1}{d \left(\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right)^2} + \frac{16 \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right)}$$

$$\left. \frac{1}{d \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right)^2} + \frac{16 \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right)} \right)$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+a \sec [c+d x])^4 dx$$

Optimal (type 3, 73 leaves, 8 steps):

$$\frac{13 a^4 x}{2} + \frac{4 a^4 \operatorname{ArcTanh}[\sin[c + d x]]}{d} + \frac{4 a^4 \sin[c + d x]}{d} + \frac{a^4 \cos[c + d x] \sin[c + d x]}{2 d} + \frac{a^4 \tan[c + d x]}{d}$$

Result (type 3, 241 leaves):

$$\frac{1}{64} a^4 (1 + \cos[c + d x])^4 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^8$$

$$\left(26 x - \frac{16 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{16 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{16 \cos[d x] \sin[c]}{d} + \right.$$

$$\left. \frac{\cos[2 d x] \sin[2 c]}{d} + \frac{16 \cos[c] \sin[d x]}{d} + \frac{\cos[2 c] \sin[2 d x]}{d} + \frac{4 \sin\left[\frac{d x}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} + \right.$$

$$\left. \frac{4 \sin\left[\frac{d x}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)} \right)$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^5}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$-\frac{3 \operatorname{ArcTanh}[\sin[c + d x]]}{2 a d} + \frac{4 \tan[c + d x]}{a d} - \frac{3 \operatorname{Sec}[c + d x] \tan[c + d x]}{2 a d} - \frac{\operatorname{Sec}[c + d x]^3 \tan[c + d x]}{d (a + a \operatorname{Sec}[c + d x])} + \frac{4 \tan[c + d x]^3}{3 a d}$$

Result (type 3, 760 leaves):

$$\begin{aligned}
& \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c+dx]}{d(a+a\sec[c+dx])} - \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c+dx]}{d(a+a\sec[c+dx])} + \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c+dx] \sin\left[\frac{dx}{2}\right]}{d(a+a\sec[c+dx])} + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \sin\left[\frac{dx}{2}\right]}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \left(-\cos\left[\frac{c}{2}\right] + 2\sin\left[\frac{c}{2}\right]\right)}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{10 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \sin\left[\frac{dx}{2}\right]}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \sin\left[\frac{dx}{2}\right]}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \left(\cos\left[\frac{c}{2}\right] + 2\sin\left[\frac{c}{2}\right]\right)}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \frac{10 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[c+dx] \sin\left[\frac{dx}{2}\right]}{3d(a+a\sec[c+dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^4}{a+a\sec[c+dx]} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{3 \operatorname{ArcTanh}[\sin[c+dx]]}{2ad} - \frac{2 \tan[c+dx]}{ad} + \frac{3 \sec[c+dx] \tan[c+dx]}{2ad} - \frac{\sec[c+dx]^2 \tan[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 3, 250 leaves):

$$\begin{aligned}
& \frac{1}{2ad(1+\sec[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx] \\
& \left(-4 \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \cos\left[\frac{1}{2}(c+dx)\right] \left(-6 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 6 \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\
& \left. \frac{1}{\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{1}{\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \right. \\
& \left. (4 \sin[dx]) / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a d} + \frac{\text{Tan}[c + d x]}{a d} + \frac{\text{Tan}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 3, 194 leaves):

$$\frac{1}{a d (1 + \text{Sec}[c + d x])} 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Sec}[c + d x] \\ \left(\text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + \text{Cos}\left[\frac{1}{2} (c + d x)\right] \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \right. \\ \left. \text{Sin}[d x] / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) \right)$$

■ **Problem 45: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^2}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a d} - \frac{\text{Tan}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 3, 109 leaves):

$$-\frac{1}{a d (1 + \text{Sec}[c + d x])} 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Sec}[c + d x] \\ \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] \right)$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c + d x]}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$-\frac{x}{a} + \frac{2 \text{Sin}[c + d x]}{a d} - \frac{\text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 3, 89 leaves):

$$\frac{1}{4 a d} \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2} (c + d x)\right] \left(-2 d x \text{Cos}\left[\frac{d x}{2}\right] - 2 d x \text{Cos}\left[c + \frac{d x}{2}\right] + 5 \text{Sin}\left[\frac{d x}{2}\right] + \text{Sin}\left[c + \frac{d x}{2}\right] + \text{Sin}\left[c + \frac{3 d x}{2}\right] + \text{Sin}\left[2 c + \frac{3 d x}{2}\right] \right)$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^5}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 123 leaves, 7 steps):

$$\frac{7 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 a^2 d} - \frac{16 \text{Tan}[c + d x]}{3 a^2 d} + \frac{7 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 a^2 d} - \frac{8 \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} - \frac{\text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 300 leaves):

$$\begin{aligned} & \frac{1}{3 a^2 d (1 + \text{Sec}[c + d x])^2} \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Sec}[c + d x]^2 \left(-2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] - 40 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + \right. \\ & \left. 3 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \left(-14 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 14 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + \\ & \frac{1}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} - \frac{1}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} - (8 \text{Sin}[d x]) / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right] \right) \right. \\ & \left. \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) \right) - 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Tan}\left[\frac{c}{2}\right] \end{aligned}$$

■ **Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^4}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{2 \text{ArcTanh}[\text{Sin}[c + d x]]}{a^2 d} + \frac{4 \text{Tan}[c + d x]}{3 a^2 d} + \frac{2 \text{Tan}[c + d x]}{a^2 d (1 + \text{Sec}[c + d x])} - \frac{\text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 247 leaves):

$$\begin{aligned} & \frac{1}{3 a^2 d (1 + \text{Sec}[c + d x])^2} 2 \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Sec}[c + d x]^2 \left(\text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 14 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + \right. \\ & \left. 6 \text{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \left(2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \text{Sin}[d x] / \left(\left(\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right] \right) \right. \right. \right. \\ & \left. \left. \left(\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) \right) \right) + \text{Cos}\left[\frac{1}{2} (c + d x)\right] \text{Tan}\left[\frac{c}{2}\right] \end{aligned}$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 66 leaves, 4 steps) :

$$\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a^2 d} - \frac{5 \text{Tan}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} + \frac{\text{Tan}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 160 leaves) :

$$-\frac{1}{3 a^2 d (1 + \text{Sec}[c + d x])^2} + 2 \text{Cos}\left[\frac{1}{2}(c + d x)\right] \text{Sec}[c + d x]^2 \left(6 \text{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) + \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + 8 \text{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] + \text{Cos}\left[\frac{1}{2}(c + d x)\right] \text{Tan}\left[\frac{c}{2}\right]\right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c + d x]}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 72 leaves, 5 steps) :

$$-\frac{2 x}{a^2} + \frac{10 \text{Sin}[c + d x]}{3 a^2 d} - \frac{2 \text{Sin}[c + d x]}{a^2 d (1 + \text{Sec}[c + d x])} - \frac{\text{Sin}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 151 leaves) :

$$\frac{1}{48 a^2 d} \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \left(-36 d x \text{Cos}\left[\frac{d x}{2}\right] - 36 d x \text{Cos}\left[c + \frac{d x}{2}\right] - 12 d x \text{Cos}\left[c + \frac{3 d x}{2}\right] - 12 d x \text{Cos}\left[2 c + \frac{3 d x}{2}\right] + 66 \text{Sin}\left[\frac{d x}{2}\right] - 30 \text{Sin}\left[c + \frac{d x}{2}\right] + 41 \text{Sin}\left[c + \frac{3 d x}{2}\right] + 9 \text{Sin}\left[2 c + \frac{3 d x}{2}\right] + 3 \text{Sin}\left[2 c + \frac{5 d x}{2}\right] + 3 \text{Sin}\left[3 c + \frac{5 d x}{2}\right]\right)$$

■ **Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^6}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 3, 162 leaves, 8 steps) :

$$\frac{13 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 a^3 d} - \frac{152 \text{Tan}[c + d x]}{15 a^3 d} + \frac{13 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 a^3 d} - \frac{\text{Sec}[c + d x]^4 \text{Tan}[c + d x]}{5 d (a + a \text{Sec}[c + d x])^3} - \frac{11 \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{15 a d (a + a \text{Sec}[c + d x])^2} - \frac{76 \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{15 d (a^3 + a^3 \text{Sec}[c + d x])}$$

Result (type 3, 351 leaves) :

$$\frac{1}{480 a^3 d (1 + \operatorname{Sec}[c + dx])^3} \cos\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx]^3 \left(24960 \cos\left[\frac{1}{2}(c + dx)\right]^5 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \left(-1235 \sin\left[\frac{dx}{2}\right] + 3805 \sin\left[\frac{3dx}{2}\right] - 4329 \sin\left[c - \frac{dx}{2}\right] + 1989 \sin\left[c + \frac{dx}{2}\right] - 3575 \sin\left[2c + \frac{dx}{2}\right] - 475 \sin\left[c + \frac{3dx}{2}\right] + 2005 \sin\left[2c + \frac{3dx}{2}\right] - 2275 \sin\left[3c + \frac{3dx}{2}\right] + 2673 \sin\left[c + \frac{5dx}{2}\right] + 105 \sin\left[2c + \frac{5dx}{2}\right] + 1593 \sin\left[3c + \frac{5dx}{2}\right] - 975 \sin\left[4c + \frac{5dx}{2}\right] + 1325 \sin\left[2c + \frac{7dx}{2}\right] + 255 \sin\left[3c + \frac{7dx}{2}\right] + 875 \sin\left[4c + \frac{7dx}{2}\right] - 195 \sin\left[5c + \frac{7dx}{2}\right] + 304 \sin\left[3c + \frac{9dx}{2}\right] + 90 \sin\left[4c + \frac{9dx}{2}\right] + 214 \sin\left[5c + \frac{9dx}{2}\right] \right) \right)$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^5}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 3, 128 leaves, 7 steps) :

$$-\frac{3 \operatorname{ArcTanh}[\sin[c + dx]]}{a^3 d} + \frac{9 \tan[c + dx]}{5 a^3 d} - \frac{\operatorname{Sec}[c + dx]^3 \tan[c + dx]}{5 d (a + a \operatorname{Sec}[c + dx])^3} - \frac{3 \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{5 a d (a + a \operatorname{Sec}[c + dx])^2} + \frac{3 \tan[c + dx]}{d (a^3 + a^3 \operatorname{Sec}[c + dx])}$$

Result (type 3, 294 leaves) :

$$\frac{1}{5 a^3 d (1 + \operatorname{Sec}[c + dx])^3} 2 \cos\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx]^3 \left(\operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 8 \cos\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 76 \cos\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 20 \cos\left[\frac{1}{2}(c + dx)\right]^5 \left(3 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - 3 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \sin[dx] / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) + \cos\left[\frac{1}{2}(c + dx)\right] \tan\left[\frac{c}{2}\right] + 8 \cos\left[\frac{1}{2}(c + dx)\right]^3 \tan\left[\frac{c}{2}\right] \right)$$

■ **Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^7}{(a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 3, 193 leaves, 9 steps) :

$$\frac{21 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^4 d} - \frac{576 \operatorname{Tan}[c + d x]}{35 a^4 d} + \frac{21 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a^4 d} - \frac{43 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{35 a^4 d (1 + \operatorname{Sec}[c + d x])^2} - \frac{288 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{35 a^4 d (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Sec}[c + d x]^5 \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^4} - \frac{2 \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 a d (a + a \operatorname{Sec}[c + d x])^3}$$

Result (type 3, 403 leaves):

$$-\frac{1}{2240 a^4 d (1 + \operatorname{Sec}[c + d x])^4} \cos\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^4 \left(376320 \cos\left[\frac{1}{2}(c + d x)\right]^7 \left(\log\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]\right) + \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \left(-24402 \sin\left[\frac{d x}{2}\right] + 55556 \sin\left[\frac{3 d x}{2}\right] - 61054 \sin\left[c - \frac{d x}{2}\right] + 33614 \sin\left[c + \frac{d x}{2}\right] - 51842 \sin\left[2c + \frac{d x}{2}\right] - 12460 \sin\left[c + \frac{3 d x}{2}\right] + 33716 \sin\left[2c + \frac{3 d x}{2}\right] - 34300 \sin\left[3c + \frac{3 d x}{2}\right] + 39788 \sin\left[c + \frac{5 d x}{2}\right] - 2940 \sin\left[2c + \frac{5 d x}{2}\right] + 26068 \sin\left[3c + \frac{5 d x}{2}\right] - 16660 \sin\left[4c + \frac{5 d x}{2}\right] + 21351 \sin\left[2c + \frac{7 d x}{2}\right] + 1295 \sin\left[3c + \frac{7 d x}{2}\right] + 14911 \sin\left[4c + \frac{7 d x}{2}\right] - 5145 \sin\left[5c + \frac{7 d x}{2}\right] + 7329 \sin\left[3c + \frac{9 d x}{2}\right] + 1225 \sin\left[4c + \frac{9 d x}{2}\right] + 5369 \sin\left[5c + \frac{9 d x}{2}\right] - 735 \sin\left[6c + \frac{9 d x}{2}\right] + 1152 \sin\left[4c + \frac{11 d x}{2}\right] + 280 \sin\left[5c + \frac{11 d x}{2}\right] + 872 \sin\left[6c + \frac{11 d x}{2}\right]\right)$$

■ **Problem 71: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^6}{(a + a \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 159 leaves, 8 steps):

$$-\frac{4 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^4 d} + \frac{244 \operatorname{Tan}[c + d x]}{105 a^4 d} - \frac{88 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{105 a^4 d (1 + \operatorname{Sec}[c + d x])^2} + \frac{4 \operatorname{Tan}[c + d x]}{a^4 d (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^4} - \frac{12 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{35 a d (a + a \operatorname{Sec}[c + d x])^3}$$

Result (type 3, 349 leaves):

$$\frac{1}{1680 a^4 d (1 + \operatorname{Sec}[c + d x])^4} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^4 \left(107520 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^7 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \left(-10780 \operatorname{Sin}\left[\frac{d x}{2}\right] + 18788 \operatorname{Sin}\left[\frac{3 d x}{2}\right] - 20524 \operatorname{Sin}\left[c - \frac{d x}{2}\right] + 14644 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 16660 \operatorname{Sin}\left[2c + \frac{d x}{2}\right] - 4690 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 14378 \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] - 9100 \operatorname{Sin}\left[3c + \frac{3 d x}{2}\right] + 11668 \operatorname{Sin}\left[c + \frac{5 d x}{2}\right] - 630 \operatorname{Sin}\left[2c + \frac{5 d x}{2}\right] + 9358 \operatorname{Sin}\left[3c + \frac{5 d x}{2}\right] - 2940 \operatorname{Sin}\left[4c + \frac{5 d x}{2}\right] + 4228 \operatorname{Sin}\left[2c + \frac{7 d x}{2}\right] + 315 \operatorname{Sin}\left[3c + \frac{7 d x}{2}\right] + 3493 \operatorname{Sin}\left[4c + \frac{7 d x}{2}\right] - 420 \operatorname{Sin}\left[5c + \frac{7 d x}{2}\right] + 664 \operatorname{Sin}\left[3c + \frac{9 d x}{2}\right] + 105 \operatorname{Sin}\left[4c + \frac{9 d x}{2}\right] + 559 \operatorname{Sin}\left[5c + \frac{9 d x}{2}\right] \right) \right)$$

■ **Problem 77: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 111 leaves, 5 steps):

$$\frac{x}{a^4} - \frac{11 \operatorname{Tan}[c + d x]}{21 a^4 d (1 + \operatorname{Sec}[c + d x])^2} - \frac{32 \operatorname{Tan}[c + d x]}{21 a^4 d (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^4} - \frac{2 \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^3}$$

Result (type 3, 224 leaves):

$$\frac{1}{2688 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^7 \left(735 d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 735 d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 441 d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + 441 d x \operatorname{Cos}\left[2c + \frac{3 d x}{2}\right] + 147 d x \operatorname{Cos}\left[2c + \frac{5 d x}{2}\right] + 147 d x \operatorname{Cos}\left[3c + \frac{5 d x}{2}\right] + 21 d x \operatorname{Cos}\left[3c + \frac{7 d x}{2}\right] + 21 d x \operatorname{Cos}\left[4c + \frac{7 d x}{2}\right] - 1988 \operatorname{Sin}\left[\frac{d x}{2}\right] + 1652 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 1428 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 756 \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] - 560 \operatorname{Sin}\left[2c + \frac{5 d x}{2}\right] + 168 \operatorname{Sin}\left[3c + \frac{5 d x}{2}\right] - 104 \operatorname{Sin}\left[3c + \frac{7 d x}{2}\right] \right)$$

■ **Problem 78: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]}{(a + a \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$-\frac{4x}{a^4} + \frac{664 \operatorname{Sin}[c + d x]}{105 a^4 d} - \frac{88 \operatorname{Sin}[c + d x]}{105 a^4 d (1 + \operatorname{Sec}[c + d x])^2} - \frac{4 \operatorname{Sin}[c + d x]}{a^4 d (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Sin}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^4} - \frac{12 \operatorname{Sin}[c + d x]}{35 a d (a + a \operatorname{Sec}[c + d x])^3}$$

Result (type 3, 263 leaves):

$$\begin{aligned}
& - \frac{1}{26\,880\,a^4\,d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7 \left(29\,400\,dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 29\,400\,dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 17\,640\,dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + \right. \\
& \quad 17\,640\,dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 5\,880\,dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + 5\,880\,dx \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + 840\,dx \operatorname{Cos}\left[3c + \frac{7dx}{2}\right] + 840\,dx \operatorname{Cos}\left[4c + \frac{7dx}{2}\right] - \\
& \quad 60\,830 \operatorname{Sin}\left[\frac{dx}{2}\right] + 46\,130 \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 46\,116 \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 18\,060 \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 19\,292 \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + \\
& \quad \left. 2100 \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 3\,791 \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 735 \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] - 105 \operatorname{Sin}\left[4c + \frac{9dx}{2}\right] - 105 \operatorname{Sin}\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^7}{(a+a\operatorname{Sec}[c+dx])^5} dx$$

Optimal (type 3, 200 leaves, 9 steps):

$$\begin{aligned}
& - \frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{a^5 d} + \frac{181 \operatorname{Tan}[c+dx]}{63 a^5 d} - \frac{\operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx]}{9 d (a+a\operatorname{Sec}[c+dx])^5} - \\
& \frac{5 \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx]}{21 a d (a+a\operatorname{Sec}[c+dx])^4} - \frac{29 \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{63 a^2 d (a+a\operatorname{Sec}[c+dx])^3} - \frac{67 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{63 a^3 d (a+a\operatorname{Sec}[c+dx])^2} + \frac{5 \operatorname{Tan}[c+dx]}{d (a^5 + a^5 \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 3, 401 leaves):

$$\begin{aligned}
& \frac{1}{2016 a^5 d (1 + \operatorname{Sec}[c+dx])^5} \\
& \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx]^5 \left(322\,560 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^9 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\
& \quad \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \left(-33\,978 \operatorname{Sin}\left[\frac{dx}{2}\right] + 52\,002 \operatorname{Sin}\left[\frac{3dx}{2}\right] - 56\,952 \operatorname{Sin}\left[c - \frac{dx}{2}\right] + 43\,722 \operatorname{Sin}\left[c + \frac{dx}{2}\right] - \right. \\
& \quad 47\,208 \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 18\,144 \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 41\,796 \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 28\,350 \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] + 34\,578 \operatorname{Sin}\left[c + \frac{5dx}{2}\right] - \\
& \quad 5691 \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 28\,719 \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 11\,550 \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] + 15\,517 \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] - \\
& \quad 504 \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] + 13\,186 \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] - 2835 \operatorname{Sin}\left[5c + \frac{7dx}{2}\right] + 4149 \operatorname{Sin}\left[3c + \frac{9dx}{2}\right] + 252 \operatorname{Sin}\left[4c + \frac{9dx}{2}\right] + \\
& \quad \left. \left. 3582 \operatorname{Sin}\left[5c + \frac{9dx}{2}\right] - 315 \operatorname{Sin}\left[6c + \frac{9dx}{2}\right] + 496 \operatorname{Sin}\left[4c + \frac{11dx}{2}\right] + 63 \operatorname{Sin}\left[5c + \frac{11dx}{2}\right] + 433 \operatorname{Sin}\left[6c + \frac{11dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]}{(a+a\operatorname{Sec}[c+dx])^5} dx$$

Optimal (type 3, 159 leaves, 8 steps) :

$$-\frac{5x}{a^5} + \frac{496 \operatorname{Sin}[c+dx]}{63 a^5 d} - \frac{\operatorname{Sin}[c+dx]}{9 d (a+a \operatorname{Sec}[c+dx])^5} - \frac{5 \operatorname{Sin}[c+dx]}{21 a d (a+a \operatorname{Sec}[c+dx])^4} - \frac{29 \operatorname{Sin}[c+dx]}{63 a^2 d (a+a \operatorname{Sec}[c+dx])^3} - \frac{67 \operatorname{Sin}[c+dx]}{63 a^3 d (a+a \operatorname{Sec}[c+dx])^2} - \frac{5 \operatorname{Sin}[c+dx]}{d (a^5+a^5 \operatorname{Sec}[c+dx])}$$

Result (type 3, 319 leaves) :

$$-\frac{1}{64 512 a^5 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^9$$

$$\left(79 380 dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 79 380 dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 52 920 dx \operatorname{Cos}\left[c + \frac{3 dx}{2}\right] + 52 920 dx \operatorname{Cos}\left[2c + \frac{3 dx}{2}\right] + 22 680 dx \operatorname{Cos}\left[2c + \frac{5 dx}{2}\right] + 22 680 dx \operatorname{Cos}\left[3c + \frac{5 dx}{2}\right] + 5670 dx \operatorname{Cos}\left[3c + \frac{7 dx}{2}\right] + 5670 dx \operatorname{Cos}\left[4c + \frac{7 dx}{2}\right] + 630 dx \operatorname{Cos}\left[4c + \frac{9 dx}{2}\right] + 630 dx \operatorname{Cos}\left[5c + \frac{9 dx}{2}\right] - 175 014 \operatorname{Sin}\left[\frac{dx}{2}\right] + 143 010 \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 138 726 \operatorname{Sin}\left[c + \frac{3 dx}{2}\right] + 73 290 \operatorname{Sin}\left[2c + \frac{3 dx}{2}\right] - 70 389 \operatorname{Sin}\left[2c + \frac{5 dx}{2}\right] + 20 475 \operatorname{Sin}\left[3c + \frac{5 dx}{2}\right] - 21 141 \operatorname{Sin}\left[3c + \frac{7 dx}{2}\right] + 1575 \operatorname{Sin}\left[4c + \frac{7 dx}{2}\right] - 3091 \operatorname{Sin}\left[4c + \frac{9 dx}{2}\right] - 567 \operatorname{Sin}\left[5c + \frac{9 dx}{2}\right] - 63 \operatorname{Sin}\left[5c + \frac{11 dx}{2}\right] - 63 \operatorname{Sin}\left[6c + \frac{11 dx}{2}\right] \right)$$

- **Problem 94: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 37 leaves, 2 steps) :

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d}$$

Result (type 4, 331 leaves) :

$$\begin{aligned}
& -\frac{1}{d} 8 (-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec\left[\frac{1}{2}(c + dx)\right] \sec[c + dx] \sqrt{a(1 + \sec[c + dx])} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}
\end{aligned}$$

- **Problem 95: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] \sqrt{a + a \sec[c + dx]} dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$\frac{\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{d} + \frac{a \sin[c + dx]}{d \sqrt{a + a \sec[c + dx]}}$$

Result (type 4, 389 leaves):

$$\frac{\sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \left(-\frac{1}{2}\sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}\sin\left[\frac{3}{2}(c+dx)\right]\right)}{d} -$$

$$\frac{1}{d} 4(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2 \sec\left[\frac{1}{2}(c+dx)\right] \sec[c+dx] \sqrt{a(1+\sec[c+dx])} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 \sqrt{a+a\sec[c+dx]} dx$$

Optimal (type 3, 102 leaves, 4 steps):

$$\frac{3\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{4d} + \frac{3a\sin[c+dx]}{4d\sqrt{a+a\sec[c+dx]}} + \frac{a\cos[c+dx]\sin[c+dx]}{2d\sqrt{a+a\sec[c+dx]}}$$

Result (type 4, 401 leaves):

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{8} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]\right)}{d} +$$

$$\frac{1}{d} 3 \left(2 + \frac{3}{\sqrt{2}}\right) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2} + (10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1+\sqrt{2} - (-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 97: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{5\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{5a \operatorname{Sin}[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{5a \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 417 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{11}{48} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{24} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]\right) +$$

$$\frac{1}{4d} 5(4+3\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

- **Problem 98: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 \sqrt{a+a\operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{35\sqrt{a}\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{64d} + \frac{35a\operatorname{Sin}[c+dx]}{64d\sqrt{a+a\operatorname{Sec}[c+dx]}} +$$

$$\frac{35a\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{96d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{7a\operatorname{Cos}[c+dx]^2\operatorname{Sin}[c+dx]}{24d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{a\operatorname{Cos}[c+dx]^3\operatorname{Sin}[c+dx]}{4d\sqrt{a+a\operatorname{Sec}[c+dx]}}$$

Result (type 4, 431 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(-\frac{41}{384} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{11}{48} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{15}{128} \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{48} \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + \frac{1}{64} \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]\right) + \\
& \frac{1}{(-64+48\sqrt{2})d} 35 \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

■ **Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+dx])^{3/2} dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2 a^2 \operatorname{Tan}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 374 leaves):

$$\begin{aligned}
& -\frac{1}{d} 4 (-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \text{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \text{Sec}[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \text{Tan}\left[\frac{1}{4}(c + dx)\right]^2} + } \\
& \frac{\cos[c + dx] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \text{Sec}[c + dx]))^{3/2} \text{Tan}\left[\frac{1}{2}(c + dx)\right]}{d}
\end{aligned}$$

- **Problem 104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + a \text{Sec}[c + dx])^{3/2} dx$$

Optimal (type 3, 65 leaves, 5 steps):

$$\frac{3 a^{3/2} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{d} + \frac{a^2 \text{Sin}[c + dx]}{d \sqrt{a + a \text{Sec}[c + dx]}}$$

Result (type 4, 393 leaves):

$$\frac{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(-\frac{1}{4}\sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4}\sin\left[\frac{3}{2}(c + dx)\right]\right)}{d}$$

$$\frac{1}{d} 6(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}$$

- **Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^{3/2} dx$$

Optimal (type 3, 106 leaves, 5 steps):

$$\frac{7 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{4 d} + \frac{7 a^2 \sin[c + dx]}{4 d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{a^2 \cos[c + dx] \sin[c + dx]}{2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 4, 407 leaves):

$$\frac{1}{d} \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \left(-\frac{5}{16} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{3}{8} \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{16} \sin\left[\frac{5}{2}(c + dx)\right]\right) +$$

$$\frac{1}{4d} 7(4 + 3\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^{3/2} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\frac{11 a^{3/2} \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{8 d} + \frac{11 a^2 \sin[c + dx]}{8 d \sqrt{a + a \sec[c + dx]}} + \frac{11 a^2 \cos[c + dx] \sin[c + dx]}{12 d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 \cos[c + dx]^2 \sin[c + dx]}{3 d \sqrt{a + a \sec[c + dx]}}$$

Result (type 4, 421 leaves):

$$\frac{1}{d} \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \left(-\frac{17}{96} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{7}{24} \sin\left[\frac{3}{2}(c + dx)\right] + \frac{3}{32} \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{48} \sin\left[\frac{7}{2}(c + dx)\right]\right) +$$

$$\frac{1}{8d} 11(4 + 3\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 111: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c + dx])^{5/2} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\frac{2 a^{5/2} \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{d} + \frac{14 a^3 \tan[c + dx]}{3 d \sqrt{a + a \sec[c + dx]}} + \frac{2 a^2 \sqrt{a + a \sec[c + dx]} \tan[c + dx]}{3 d}$$

Result (type 4, 407 leaves):

$$\frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \left(\frac{4}{3} \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{6} \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right]\right) -$$

$$\frac{1}{d} 2(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \cos[c+dx]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 112: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] (a+a \sec[c+dx])^{5/2} dx$$

Optimal (type 3, 94 leaves, 4 steps):

$$\frac{5 a^{5/2} \text{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{d} - \frac{a^3 \sin[c+dx]}{d \sqrt{a+a \sec[c+dx]}} + \frac{2 a^2 \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 4, 399 leaves):

$$\frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5\left(a\left(1+\operatorname{Sec}[c+d x]\right)\right)^{5 / 2}\left(\frac{3}{8} \sin \left[\frac{1}{2}(c+d x)\right]+\frac{1}{8} \sin \left[\frac{3}{2}(c+d x)\right]\right)}{d} +$$

$$\frac{1}{d} 5\left(2+\frac{3}{\sqrt{2}}\right) \cos \left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}}\left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5\left(a\left(1+\operatorname{Sec}[c+d x]\right)\right)^{5 / 2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}$$

■ **Problem 113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2\left(a+a \operatorname{Sec}[c+d x]\right)^{5 / 2} d x$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{19 a^{5 / 2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{4 d} + \frac{9 a^3 \sin [c+d x]}{4 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 \cos [c+d x] \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{2 d}$$

Result (type 4, 415 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(-\frac{9}{32} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{5}{16} \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{32} \sin\left[\frac{5}{2}(c + dx)\right]\right) +$$

$$\frac{1}{8d} 19(4 + 3\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \cos[c + dx]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 114: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^{5/2} dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{25 a^{5/2} \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{8 d} + \frac{25 a^3 \sin[c + dx]}{8 d \sqrt{a + a \sec[c + dx]}} + \frac{13 a^3 \cos[c + dx] \sin[c + dx]}{12 d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 \cos[c + dx]^2 \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{3 d}$$

Result (type 4, 429 leaves):

$$\frac{1}{d} \cos[c+dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}$$

$$\left(-\frac{47}{192} \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} \sin\left[\frac{3}{2}(c+dx)\right] + \frac{5}{64} \sin\left[\frac{5}{2}(c+dx)\right] + \frac{1}{96} \sin\left[\frac{7}{2}(c+dx)\right]\right) +$$

$$\frac{1}{8d} 25 \left(2 + \frac{3}{\sqrt{2}}\right) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2} + (10-7\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \cos[c+dx]$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \sqrt{3-2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 115: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+a \operatorname{Sec}[c+dx])^{5/2} dx$$

Optimal (type 3, 182 leaves, 6 steps):

$$\frac{163 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64 d} + \frac{163 a^3 \sin[c+dx]}{64 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{163 a^3 \cos[c+dx] \sin[c+dx]}{96 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{17 a^3 \cos[c+dx]^2 \sin[c+dx]}{24 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 \cos[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} \sin[c+dx]}{4 d}$$

Result (type 4, 443 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(-\frac{265 \sin\left[\frac{1}{2}(c + dx)\right]}{1536} + \frac{55}{192} \sin\left[\frac{3}{2}(c + dx)\right] + \frac{47}{512} \sin\left[\frac{5}{2}(c + dx)\right] + \frac{5}{192} \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{256} \sin\left[\frac{9}{2}(c + dx)\right] \right) + \frac{1}{64d} 163 \left(2 + \frac{3}{\sqrt{2}}\right) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \cos[c + dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a - a \sec[c + dx]} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a - a \sec[c + dx]}}\right]}{d}$$

Result (type 3, 214 leaves):

$$-\left(\cos[c + dx] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \left(\operatorname{ArcTanh}\left[\left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right]\right) \sqrt{(1 + e^{2ix}) \cos[c] + i(-1 + e^{2ix}) \sin[c]}\right] + \operatorname{Log}\left[2 \left(e^{ix} \cos\left[\frac{c}{2}\right] + i e^{ix} \sin\left[\frac{c}{2}\right] + \sqrt{(1 + e^{2ix}) \cos[c] + i(-1 + e^{2ix}) \sin[c]}\right)\right]\right) \sqrt{a - a \sec[c + dx]} \left(\cos\left[\frac{dx}{2}\right] + i \sin\left[\frac{dx}{2}\right]\right) \right) / \left(d \sqrt{(1 + e^{2ix}) \cos[c] + i(-1 + e^{2ix}) \sin[c]}\right)$$

- **Problem 118: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] \sqrt{a - a \sec[c + dx]} dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a \operatorname{Sin}[c+dx]}{d \sqrt{a-a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 348 leaves):

$$\frac{1}{2 d \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}} \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]$$

$$\sqrt{a-a \operatorname{Sec}[c+dx]} \left(\operatorname{Cos}\left[\frac{dx}{2}\right] \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right]+i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right] + \right.$$

$$\left. \operatorname{ArcTanh}\left[\left(\operatorname{Cos}\left[\frac{c}{2}\right]+i \operatorname{Sin}\left[\frac{c}{2}\right]\right) \sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right] \left(\operatorname{Cos}\left[\frac{dx}{2}\right]+i \operatorname{Sin}\left[\frac{dx}{2}\right]\right) + \right.$$

$$\left. i \operatorname{Log}\left[2\left(e^{i d x} \operatorname{Cos}\left[\frac{c}{2}\right]+i e^{i d x} \operatorname{Sin}\left[\frac{c}{2}\right]+\sqrt{(1+e^{2 i d x}) \operatorname{Cos}[c]+i(-1+e^{2 i d x}) \operatorname{Sin}[c]}\right)\right] \operatorname{Sin}\left[\frac{dx}{2}\right] - \right.$$

$$\left. 2 \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{\operatorname{Cos}[c+dx] (\operatorname{Cos}[dx]+i \operatorname{Sin}[dx])}\right)$$

■ **Problem 141: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 113 leaves):

$$-\frac{i \sqrt{2} (-1+e^{i(c+dx)}) \left(\operatorname{Log}[1-e^{i(c+dx)}] - \operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2} \sqrt{1+e^{2i(c+dx)}}] \right)}{d \sqrt{1+e^{2i(c+dx)}} \sqrt{a-a \operatorname{Sec}[c+dx]}}$$

■ **Problem 142: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a-a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 157 leaves):

$$- \left(i \left(-1 + e^{i(c+dx)} \right) \left(-i dx + \text{ArcSinh} \left[e^{i(c+dx)} \right] + \sqrt{2} \text{Log} \left[1 - e^{i(c+dx)} \right] + \text{Log} \left[1 + \sqrt{1 + e^{2i(c+dx)}} \right] - \sqrt{2} \text{Log} \left[1 + e^{i(c+dx)} + \sqrt{2} \sqrt{1 + e^{2i(c+dx)}} \right] \right) \right) / \left(d \sqrt{1 + e^{2i(c+dx)}} \sqrt{a - a \text{Sec}[c + dx]} \right)$$

■ **Problem 143: Result unnecessarily involves higher level functions.**

$$\int \text{Sec}[c + dx]^3 (a + a \text{Sec}[c + dx])^{2/3} dx$$

Optimal (type 4, 383 leaves, 7 steps):

$$\begin{aligned} & - \frac{9 (a + a \text{Sec}[c + dx])^{2/3} \text{Tan}[c + dx]}{40 d} + \frac{57 (a + a \text{Sec}[c + dx])^{2/3} \text{Tan}[c + dx]}{80 d (1 + \text{Sec}[c + dx])} + \\ & \frac{3 (a + a \text{Sec}[c + dx])^{5/3} \text{Tan}[c + dx]}{8 a d} - \left(19 \times 3^{3/4} \text{EllipticF} \left[\text{ArcCos} \left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) \\ & \left((a + a \text{Sec}[c + dx])^{2/3} (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \text{Sec}[c + dx])^{1/3} + (1 + \text{Sec}[c + dx])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2}} \text{Tan}[c + dx] \right) / \\ & \left(80 \times 2^{1/3} d (1 - \text{Sec}[c + dx]) (1 + \text{Sec}[c + dx]) \sqrt{-\frac{(1 + \text{Sec}[c + dx])^{1/3} (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 105 leaves):

$$\begin{aligned} & \frac{1}{80 d} (a (1 + \text{Sec}[c + dx]))^{2/3} \left(57 \text{Tan} \left[\frac{1}{2} (c + dx) \right] + \right. \\ & \left. 19 \times 2^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \left(\frac{1}{1 + \text{Sec}[c + dx]} \right)^{2/3} \text{Tan} \left[\frac{1}{2} (c + dx) \right] + 6 (2 + 5 \text{Sec}[c + dx]) \text{Tan}[c + dx] \right) \end{aligned}$$

■ **Problem 144: Result unnecessarily involves higher level functions.**

$$\int \text{Sec}[c + dx]^2 (a + a \text{Sec}[c + dx])^{2/3} dx$$

Optimal (type 4, 353 leaves, 6 steps):

$$\frac{3 (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d} + \frac{3 (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d (1 + \operatorname{Sec}[c + d x])} -$$

$$\left(3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) /$$

$$\left(5 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 97 leaves):

$$\frac{1}{5 d} (a (1 + \operatorname{Sec}[c + d x]))^{2/3}$$

$$\left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\cos[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 \left(\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Tan}[c + d x]\right) \right)$$

■ **Problem 145: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^{2/3} dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\frac{3 (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{2 d (1 + \operatorname{Sec}[c + d x])} -$$

$$\left(3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \right.$$

$$\left. \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) /$$

$$\left(2 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 75 leaves) :

$$\frac{1}{2d} (a (1 + \text{Sec}[c + dx]))^{2/3} \left(3 + 2^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \left(\frac{1}{1 + \text{Sec}[c + dx]} \right)^{2/3} \right) \text{Tan} \left[\frac{1}{2} (c + dx) \right]$$

■ **Problem 146: Result more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sec}[c + dx])^{2/3} dx$$

Optimal (type 6, 77 leaves, 3 steps) :

$$\frac{3\sqrt{2} \text{AppellF1} \left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, \frac{1}{2} (1 + \text{Sec}[c + dx]), 1 + \text{Sec}[c + dx] \right] (a + a \text{Sec}[c + dx])^{2/3} \text{Tan}[c + dx]}{7d\sqrt{1 - \text{Sec}[c + dx]}}$$

Result (type 6, 1575 leaves) :

$$\begin{aligned} & \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] (1 + \text{Sec}[c + dx])^{2/3} (a (1 + \text{Sec}[c + dx]))^{2/3} \text{Sin}[c + dx] \right) / \\ & \left(d \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + \right. \\ & 2 \left(-3 \text{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \\ & \left. \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right) \left(\left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \text{Cos}[c + dx] (1 + \text{Sec}[c + dx])^{2/3} \right) / \right. \\ & \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + 2 \left(-3 \text{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \\ & \left. \left. 2 \text{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right) + \\ & \left(9 (1 + \text{Sec}[c + dx])^{2/3} \text{Sin}[c + dx] \left(-\frac{1}{3} \text{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[\frac{1}{2} (c + dx) \right] + \right. \right. \\ & \left. \left. \frac{2}{9} \text{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \text{Tan} \left[\frac{1}{2} (c + dx) \right] \right) \right) / \\ & \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + 2 \left(-3 \text{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \\ & \left. \left. 2 \text{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right) - \\ & \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] (1 + \text{Sec}[c + dx])^{2/3} \text{Sin}[c + dx] \right. \\ & \left. \left(2 \left(-3 \text{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \Bigg) + \\
& \left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\
& \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) \\
& \left(-\cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \tan[c+dx] \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 148: Result unnecessarily involves higher level functions.**

$$\int \sec[c+dx]^3 (a+a \sec[c+dx])^{5/3} dx$$

Optimal (type 4, 413 leaves, 8 steps):

$$\begin{aligned}
& \frac{147 a (a + a \sec[c + dx])^{2/3} \tan[c + dx]}{440 d} + \frac{1029 a (a + a \sec[c + dx])^{2/3} \tan[c + dx]}{880 d (1 + \sec[c + dx])} - \\
& \frac{9 (a + a \sec[c + dx])^{5/3} \tan[c + dx]}{88 d} + \frac{3 (a + a \sec[c + dx])^{8/3} \tan[c + dx]}{11 a d} - \\
& \left(343 \times 3^{3/4} a \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \sec[c + dx])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \sec[c + dx])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \sec[c + dx])^{2/3} (2^{1/3} - (1 + \sec[c + dx])^{1/3}) \right. \\
& \left. \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \sec[c + dx])^{1/3} + (1 + \sec[c + dx])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \sec[c + dx])^{1/3})^2}} \tan[c + dx] \right) / \\
& \left(880 \times 2^{1/3} d (1 - \sec[c + dx]) (1 + \sec[c + dx]) \sqrt{-\frac{(1 + \sec[c + dx])^{1/3} (2^{1/3} - (1 + \sec[c + dx])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \sec[c + dx])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 116 leaves):

$$\frac{1}{880 d} a (a (1 + \operatorname{Sec}[c + d x]))^{2/3} \left(1029 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 343 \times 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 (74 + 65 \operatorname{Sec}[c + d x] + 40 \operatorname{Sec}[c + d x]^2) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 149: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^{5/3} dx$$

Optimal (type 4, 383 leaves, 7 steps):

$$\frac{3 a (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{8 d} + \frac{21 a (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{16 d (1 + \operatorname{Sec}[c + d x])} + \frac{3 (a + a \operatorname{Sec}[c + d x])^{5/3} \operatorname{Tan}[c + d x]}{8 d} - \left(7 \times 3^{3/4} a \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right) \left((a + a \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \left(16 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 104 leaves):

$$\frac{1}{16 d} a (a (1 + \operatorname{Sec}[c + d x]))^{2/3} \left(21 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 7 \times 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 (2 + \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 150: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^{5/3} dx$$

Optimal (type 4, 356 leaves, 6 steps):

$$\frac{3 a (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d} + \frac{21 a (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{10 d (1 + \operatorname{Sec}[c + d x])} -$$

$$\left(7 \times 3^{3/4} a \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right/$$

$$\left(10 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 96 leaves):

$$\frac{1}{10 d} a (a (1 + \operatorname{Sec}[c + d x]))^{2/3}$$

$$\left(21 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 7 \times 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 \operatorname{Tan}[c + d x] \right)$$

■ **Problem 151: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{5/3} dx$$

Optimal (type 6, 86 leaves, 3 steps):

$$\frac{1}{13 d \sqrt{1 - \operatorname{Sec}[c + d x]}} 3 \sqrt{2} a \operatorname{AppellF1}\left[\frac{13}{6}, \frac{1}{2}, 1, \frac{19}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]$$

Result (type 6, 3988 leaves):

$$\frac{3 ((1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x])^{2/3} (a (1 + \operatorname{Sec}[c + d x]))^{5/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{2 d (1 + \operatorname{Sec}[c + d x])^{5/3}} +$$

$$\left(5 \operatorname{Cos}[c + d x] \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x] \right)^{5/3} (a (1 + \operatorname{Sec}[c + d x]))^{5/3} \right.$$

$$\left. \left(\frac{3}{4} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 (1 + \operatorname{Sec}[c + d x])^{2/3} + \frac{1}{2} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 (1 + \operatorname{Sec}[c + d x])^{2/3} \right) \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right/ \right.$$

$$\left. \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \right. \right.$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int \cos[c + dx] (a + a \sec[c + dx])^{5/3} dx$$

Optimal (type 6, 86 leaves, 3 steps):

$$-\frac{1}{13 d \sqrt{1 - \sec[c + dx]}} \\ 3 \sqrt{2} a \operatorname{AppellF1}\left[\frac{13}{6}, \frac{1}{2}, 2, \frac{19}{6}, \frac{1}{2} (1 + \sec[c + dx]), 1 + \sec[c + dx]\right] (1 + \sec[c + dx]) (a + a \sec[c + dx])^{2/3} \tan[c + dx]$$

Result (type 6, 4011 leaves):

$$\frac{((1 + \cos[c + dx]) \sec[c + dx])^{2/3} (a (1 + \sec[c + dx]))^{5/3} (\sin[c + dx] - \tan[\frac{1}{2} (c + dx)])}{d (1 + \sec[c + dx])^{5/3}} - \\ \left(2^{2/3} \cos[c + dx] \left(\cos\left[\frac{1}{2} (c + dx)\right]^2 \sec[c + dx]\right)^{5/3} (a (1 + \sec[c + dx]))^{5/3} \right. \\ \left.\left(\frac{2}{3} \sec\left[\frac{1}{2} (c + dx)\right]^2 (1 + \sec[c + dx])^{2/3} + \frac{5}{6} \cos[c + dx] \sec\left[\frac{1}{2} (c + dx)\right]^2 (1 + \sec[c + dx])^{2/3}\right) \right. \\ \left.\tan\left[\frac{1}{2} (c + dx)\right] \left(-\left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right]\right) / \right. \right. \\ \left.\left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, \right. \right. \right. \right. \\ \left.\left.\left.-\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right]\right) \tan\left[\frac{1}{2} (c + dx)\right]^2\right) \right. \\ \left.\left(5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] \tan\left[\frac{1}{2} (c + dx)\right]^2\right) / \right. \\ \left.\left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + 2 \left(-3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, \right. \right. \right. \right. \\ \left.\left.\left.-\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right]\right) \tan\left[\frac{1}{2} (c + dx)\right]^2\right) \right) \right) / \\ \left(3 d (1 + \sec[c + dx])^{5/3} \left(-\frac{1}{3 \times 2^{1/3}} \cos[c + dx] \sec\left[\frac{1}{2} (c + dx)\right]^2 \left(\cos\left[\frac{1}{2} (c + dx)\right]^2 \sec[c + dx]\right)^{5/3} \right. \right. \\ \left.\left(-\left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right]\right) / \right. \right. \\ \left.\left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, \right. \right. \right. \right. \\ \left.\left.\left.-\tan\left[\frac{1}{2} (c + dx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2} (c + dx)\right]^2, -\tan\left[\frac{1}{2} (c + dx)\right]^2\right]\right) \tan\left[\frac{1}{2} (c + dx)\right]^2\right) \right. \\ \left.\left. - \tan\left[\frac{1}{2} (c + dx)\right]^2\right) \right) \right)$$

$$\begin{aligned}
& 2 \left(-\frac{5}{7} \operatorname{AppellF1} \left[\frac{7}{2}, \frac{5}{3}, 2, \frac{9}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{25}{21} \operatorname{AppellF1} \left[\frac{7}{2}, \frac{8}{3}, 1, \frac{9}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \Bigg) \Bigg) \Bigg) / \\
& \left(15 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2 - \frac{5}{9} 2^{2/3} \\
& \operatorname{Cos}[c+dx] \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \right)^{2/3} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(- \left(9 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \Bigg) \Bigg) / \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \frac{2}{9} \left(-3 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) + \\
& \left(5 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) \Bigg) / \\
& \left(15 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) \Bigg) \\
& \left(-\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sec}[c+dx] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 153: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c+dx]^4}{(a+a \operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\frac{99 \operatorname{Tan}[c + d x]}{80 d (a + a \operatorname{Sec}[c + d x])^{1/3}} + \frac{3 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{8 d (a + a \operatorname{Sec}[c + d x])^{1/3}} - \frac{3 (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{40 a d} +$$

$$\left(37 \times 3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right/$$

$$\left(80 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 108 leaves):

$$\frac{1}{80 a d} (a (1 + \operatorname{Sec}[c + d x]))^{2/3} \left(129 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - \right.$$

$$\left. 37 \times 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 (-6 + 5 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 154: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 4, 336 leaves, 6 steps):

$$-\frac{9 \operatorname{Tan}[c + d x]}{10 d (a + a \operatorname{Sec}[c + d x])^{1/3}} + \frac{3 (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 a d} -$$

$$\left(7 \times 3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right/$$

$$\left(10 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 98 leaves) :

$$\frac{1}{10 a d} (a (1 + \operatorname{Sec}[c + d x]))^{2/3} \left(-9 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 7 \times 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 6 \operatorname{Tan}[c + d x] \right)$$

■ **Problem 155: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 4, 306 leaves, 5 steps) :

$$\frac{3 \operatorname{Tan}[c + d x]}{2 d (a + a \operatorname{Sec}[c + d x])^{1/3}} + \left(3^{3/4} \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\ \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\ \left(2 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 78 leaves) :

$$-\frac{1}{2 a d} (a (1 + \operatorname{Sec}[c + d x]))^{2/3} \left(-3 + 2^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{2/3} \right) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]$$

■ **Problem 156: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 4, 276 leaves, 4 steps) :

$$\begin{aligned}
& - \left(3^{3/4} \text{EllipticF} \left[\text{ArcCos} \left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \text{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + d x])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right. \\
& \quad \left. (2^{1/3} - (1 + \text{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \text{Sec}[c + d x])^{1/3} + (1 + \text{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + d x])^{1/3})^2}} \text{Tan}[c + d x] \right) / \\
& \quad \left(2^{1/3} d (1 - \text{Sec}[c + d x]) (a + a \text{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \text{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \text{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{1}{a d} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \left(\text{Cos}[c + d x] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right)^{2/3} (a (1 + \text{Sec}[c + d x]))^{2/3} \text{Tan} \left[\frac{1}{2} (c + d x) \right]$$

■ **Problem 157: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \text{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 6, 75 leaves, 3 steps):

$$\frac{3 \sqrt{2} \text{AppellF1} \left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2} (1 + \text{Sec}[c + d x]), 1 + \text{Sec}[c + d x] \right] \text{Tan}[c + d x]}{d \sqrt{1 - \text{Sec}[c + d x]} (a + a \text{Sec}[c + d x])^{1/3}}$$

Result (type 6, 733 leaves):

$$\begin{aligned}
& \left(45 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Cos}[c+dx]^2 (1+\operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 (a(1+\operatorname{Sec}[c+dx]))^{2/3} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \quad \left(9 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(3 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Big/ \\
& \left(a d \left(135 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right]^2 (1+2\operatorname{Cos}[c+dx]+3\operatorname{Cos}[2(c+dx)]) + \right. \right. \\
& \quad \frac{3}{2} \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \\
& \quad \left(15 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] (8+15\operatorname{Cos}[c+dx]+4\operatorname{Cos}[2(c+dx)]-3\operatorname{Cos}[3(c+dx)]) + \right. \\
& \quad \left. 5 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] (8+15\operatorname{Cos}[c+dx]+4\operatorname{Cos}[2(c+dx)]-3\operatorname{Cos}[3(c+dx)]) - 96 \right. \\
& \quad \left(9 \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{3}, 3, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 3 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Cos}[c+dx] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + \\
& \quad \left. 40 \left(3 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right)^2 \right. \\
& \quad \left. \left. \operatorname{Cos}[c+dx] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^4 \right) \right) \Big)
\end{aligned}$$

■ **Problem 158: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]}{(a+a\operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 6, 75 leaves, 3 steps):

$$\frac{3\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{6}, \frac{1}{2}, 2, \frac{7}{6}, \frac{1}{2} (1+\operatorname{Sec}[c+dx]), 1+\operatorname{Sec}[c+dx] \right] \operatorname{Tan}[c+dx]}{d\sqrt{1-\operatorname{Sec}[c+dx]} (a+a\operatorname{Sec}[c+dx])^{1/3}}$$

Result (type 6, 1986 leaves):

$$\frac{((1+\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx])^{2/3} (1+\operatorname{Sec}[c+dx])^{1/3} (\operatorname{Sin}[c+dx] - \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right])}{d(a(1+\operatorname{Sec}[c+dx]))^{1/3}} +$$

$$\begin{aligned}
& \frac{2}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \Bigg) + \\
& 6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-3 \left(-\frac{10}{7} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{2}{3}, 3, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \right. \\
& \quad \left. \frac{10}{21} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{3}, 2, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) + \\
& 2 \left(-\frac{5}{7} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{5}{3}, 2, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{25}{21} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{8}{3}, 1, \frac{9}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(45 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 + \\
& \left(20 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \operatorname{Tan}[c+dx] \right) \Bigg) / \\
& \left(3 (1 + \operatorname{Sec}[c+dx])^{1/3} \left(45 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad 6 \left(-3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{2}{3}, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{3}, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 159: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c+dx]^4}{(a+a \operatorname{Sec}[c+dx])^{5/3}} dx$$

Optimal (type 4, 766 leaves, 9 steps):

$$\begin{aligned}
& - \frac{33 \operatorname{Tan}[c + d x]}{28 d (a + a \operatorname{Sec}[c + d x])^{5/3}} + \frac{3 \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{4 d (a + a \operatorname{Sec}[c + d x])^{5/3}} + \\
& \frac{135 \operatorname{Tan}[c + d x]}{14 a d (a + a \operatorname{Sec}[c + d x])^{2/3}} + \frac{375 (1 + \sqrt{3}) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{28 a^2 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})} - \\
& \left(375 \times 3^{1/4} \operatorname{EllipticE} \left[\operatorname{ArcCos} \left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) / \\
& \left(14 \times 2^{2/3} a^2 d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) - \\
& \left(125 \times 3^{3/4} (1 - \sqrt{3}) \operatorname{EllipticF} \left[\operatorname{ArcCos} \left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) / \\
& \left(28 \times 2^{2/3} a^2 d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 5, 137 leaves):

$$\begin{aligned}
& \left(\frac{3}{2} \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \operatorname{Sec}[c + d x]^2 \left(3 \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] - 12 \operatorname{Sin} \left[\frac{3}{2} (c + d x) \right] - 23 \operatorname{Sin} \left[\frac{5}{2} (c + d x) \right] \right) + \right. \\
& \left. 125 \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \left(\operatorname{Cos}[c + d x] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right)^{1/3} \operatorname{Tan}[c + d x] \right) / (28 a d (a (1 + \operatorname{Sec}[c + d x]))^{2/3})
\end{aligned}$$

■ **Problem 160: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + a \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 4, 731 leaves, 8 steps) :

$$\frac{3 \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^{5/3}} - \frac{36 \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^{2/3}} - \frac{57 (1 + \sqrt{3}) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{7 a^2 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})} +$$

$$\left(57 \times 2^{1/3} 3^{1/4} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(7 a^2 d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) +$$

$$\left(19 \times 3^{3/4} (1 - \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(7 \times 2^{2/3} a^2 d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 5, 135 leaves) :

$$\left((1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \right.$$

$$\left. \left(3 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \left(3 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 4 \operatorname{Sin}\left[\frac{3}{2} (c + d x)\right] \right) - 38 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right. \right.$$

$$\left. \left. \left(\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right) \right) / (14 a d (a (1 + \operatorname{Sec}[c + d x]))^{2/3})$$

■ **Problem 161: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 4, 731 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^{5/3}} + \frac{15 \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^{2/3}} + \frac{15 (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})} - \\ & \left(15 \times 2^{1/3} 3^{1/4} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (1 + \operatorname{Sec}[c + d x])^{1/3} \right. \\ & \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x]} \right) / \\ & \left(7 a d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) - \\ & \left(5 \times 3^{3/4} (1 - \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (1 + \operatorname{Sec}[c + d x])^{1/3} \right. \\ & \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x]} \right) / \\ & \left(7 \times 2^{2/3} a d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 86 leaves):

$$\frac{-3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 5 \times 2^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{1/3} \operatorname{Tan}[c + d x]}{7 a d (a (1 + \operatorname{Sec}[c + d x]))^{2/3}}$$

■ **Problem 162: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + a \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 4, 744 leaves, 8 steps):

$$\begin{aligned} & \frac{6 \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^{2/3}} + \frac{3 \operatorname{Tan}[c + d x]}{7 a d (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3}} + \frac{6 (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{7 a d (a + a \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})} - \\ & \left(6 \times 2^{1/3} 3^{1/4} \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (1 + \operatorname{Sec}[c + d x])^{1/3} \right. \\ & \quad \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x]} \right) / \\ & \left(7 a d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) - \\ & \left(2^{1/3} 3^{3/4} (1 - \sqrt{3}) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (1 + \operatorname{Sec}[c + d x])^{1/3} \right. \\ & \quad \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x]} \right) / \\ & \left(7 a d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 111 leaves):

$$\left(\left(3 \operatorname{Cos}[c + d x] + 4 \times 2^{1/3} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \left(\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) \right. \\ \left. \operatorname{Sec}[c + d x] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right) / (7 a d (a (1 + \operatorname{Sec}[c + d x]))^{2/3})$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 6, 90 leaves, 3 steps):

$$\frac{3\sqrt{2} \operatorname{AppellF1}\left[-\frac{7}{6}, \frac{1}{2}, 1, -\frac{1}{6}, \frac{1}{2}(1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{7 a d \sqrt{1 - \operatorname{Sec}[c + d x]} (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{2/3}}$$

Result (type 6, 4638 leaves):

$$\frac{1}{d (a (1 + \operatorname{Sec}[c + d x]))^{5/3}} \left((1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \right)^{1/3} (1 + \operatorname{Sec}[c + d x])^{5/3} \left(\frac{27}{7} \operatorname{Sin}[c + d x] - \frac{30}{7} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + \frac{3}{14} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right) + \left(2^{1/3} \operatorname{Cos}[c + d x] \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}[c + d x] \right)^{1/3} (1 + \operatorname{Sec}[c + d x])^{5/3} \left(\frac{16}{7} (1 + \operatorname{Sec}[c + d x])^{1/3} - \frac{27}{7} \operatorname{Cos}[c + d x] (1 + \operatorname{Sec}[c + d x])^{1/3} \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \left(-27 - \left(5 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \right) \right) / \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) - \left(45 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) / \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + 2 \left(3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) \right) / \left(7 d (a (1 + \operatorname{Sec}[c + d x]))^{5/3} \left(\frac{1}{7 \times 2^{2/3}} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}[c + d x] \right)^{1/3} \right) \right) \left(-27 - \left(5 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \right) / \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) \right) - \left(45 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) / \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + 2 \left(3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) \right) \right) /$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{4}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right] \right) \right) \right) \right) / \left(\left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right. \\
& \left. \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) + \\
& \left(45 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(2 \left(3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \quad \left. \left. 15 \left(-\frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \frac{1}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) + \\
& \quad 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{5}{7} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}, 2, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad \left. \frac{20}{21} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{7}{3}, 1, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. 3 \left(-\frac{10}{7} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{3}, 3, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{5}{21} \operatorname{AppellF1}\left[\frac{7}{2}, \frac{4}{3}, 2, \frac{9}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right) \right) \right) / \\
& \left(\left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \left(-15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 \left(3 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \operatorname{AppellF1}\left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right) + \frac{1}{21 \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \right)^{2/3}} 2^{1/3} \cos[c+dx] \\
& \tan\left[\frac{1}{2}(c+dx)\right] \left(-27 - \left(5 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right) / \left(\left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right. \\
& \quad \left(\operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{2}{9} \left(-3 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 \left(3 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - \operatorname{AppellF1} \left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \Bigg) - \frac{1}{63 \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec} [c + d x] \right)^{2/3}} \\
& 5 \times 2^{1/3} \operatorname{Cos} [c + d x] \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \left(-33 - \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) / \right. \\
& \quad \left(\left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \left(\operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + \frac{2}{9} \right. \right. \right. \\
& \quad \left. \left. \left(-3 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) - \\
& \quad \left(55 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) / \left(\left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right)^2 \\
& \quad \left(-15 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + 2 \left(3 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{3}, 2, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - \operatorname{AppellF1} \left[\frac{5}{2}, \frac{4}{3}, 1, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Bigg) \\
& \quad \left(-\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] \operatorname{Sec} [c + d x] \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x] \right) \Bigg)
\end{aligned}$$

■ **Problem 172: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec} [c + d x]^{5/2} (a + a \operatorname{Sec} [c + d x])^2 dx$$

Optimal (type 4, 187 leaves, 9 steps):

$$\begin{aligned}
& -\frac{12 a^2 \sqrt{\operatorname{Cos} [c + d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{5 d} + \frac{8 a^2 \sqrt{\operatorname{Cos} [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{7 d} + \\
& \frac{12 a^2 \sqrt{\operatorname{Sec} [c + d x]} \operatorname{Sin} [c + d x]}{5 d} + \frac{8 a^2 \operatorname{Sec} [c + d x]^{3/2} \operatorname{Sin} [c + d x]}{7 d} + \frac{4 a^2 \operatorname{Sec} [c + d x]^{5/2} \operatorname{Sin} [c + d x]}{5 d} + \frac{2 a^2 \operatorname{Sec} [c + d x]^{7/2} \operatorname{Sin} [c + d x]}{7 d}
\end{aligned}$$

Result (type 5, 287 leaves):

$$\frac{1}{70 d} a^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 (1+\operatorname{Sec}[c+d x])^2 \left(-1 / (-1+e^{2 i c}) 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \operatorname{Cos}[c+d x]^2 \left(21(1+e^{2 i(c+d x)})+21(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \right. \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]+10 e^{i(c+d x)}(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) + \right. \\ \left. \frac{42 \operatorname{Cos}[d x] \operatorname{Csc}[c]+(15+14 \operatorname{Cos}[c+d x]+10 \operatorname{Cos}[2(c+d x)]) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{\operatorname{Sec}[c+d x]^{3 / 2}} \right)$$

■ **Problem 173: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec}[c+d x]^{3 / 2}(a+a \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 161 leaves, 8 steps):

$$-\frac{16 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{5 d} + \frac{4 a^2 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d} + \\ \frac{16 a^2 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{5 d} + \frac{4 a^2 \operatorname{Sec}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 a^2 \operatorname{Sec}[c+d x]^{5 / 2} \operatorname{Sin}[c+d x]}{5 d}$$

Result (type 5, 269 leaves):

$$\frac{1}{30 d} a^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 (1+\operatorname{Sec}[c+d x])^2 \left(-1 / (-1+e^{2 i c}) 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \operatorname{Cos}[c+d x]^2 \left(12(1+e^{2 i(c+d x)})+12(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \right. \right. \\ \left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]+5 e^{i(c+d x)}(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) + \frac{24 \operatorname{Cos}[d x] \operatorname{Csc}[c]+(10+3 \operatorname{Sec}[c+d x]) \operatorname{Tan}[c+d x]}{\operatorname{Sec}[c+d x]^{3 / 2}} \right)$$

■ **Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+d x]}(a+a \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 131 leaves, 7 steps):

$$-\frac{4 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{8 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{4 a^2 \sqrt{\sec [c+d x]} \sin [c+d x]}{d} + \frac{2 a^2 \sec [c+d x]^{3/2} \sin [c+d x]}{3 d}$$

Result (type 5, 264 leaves):

$$\frac{1}{3} a^2 \sec \left[\frac{1}{2}(c+d x) \right]^4 (1 + \sec [c+d x])^2 \left(-\frac{1}{d(-1+e^{2ic})} i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c+d x]^2 \left(3(1+e^{2i(c+dx)}) + 3(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) + \frac{6 \cos [d x] \operatorname{Csc}[c] + \tan [c+d x]}{2 d \sec [c+d x]^{3/2}} \right)$$

■ **Problem 176: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c+d x])^2}{\sec [c+d x]^{3/2}} dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$\frac{4 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{8 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a^2 \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{3 d \sqrt{\sec [c+d x]}} a^2 \left(\cos \left[\frac{c}{2} \right] - i \sin \left[\frac{c}{2} \right] \right) \left(-i \cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(12 - \frac{24 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} + 8 \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec [c+d x] + 2 i \sin [c+d x] \right)$$

■ **Problem 177: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c+d x])^2}{\sec [c+d x]^{5/2}} dx$$

Optimal (type 4, 135 leaves, 7 steps):

$$\frac{16 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{4 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a^2 \sin [c+d x]}{5 d \sec [c+d x]^{3 / 2}} + \frac{4 a^2 \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 136 leaves):

$$\frac{1}{30 d \sqrt{\sec [c+d x]}} a^2 \left(-96 i + \frac{192 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} - \right.$$

$$\left. 40 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x] + 40 \sin [c+d x] + 6 \sin [2(c+d x)] \right)$$

■ **Problem 178: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^2}{\sec [c+d x]^{7 / 2}} dx$$

Optimal (type 4, 161 leaves, 8 steps):

$$\frac{12 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{8 a^2 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{7 d} + \frac{2 a^2 \sin [c+d x]}{7 d \sec [c+d x]^{5 / 2}} + \frac{4 a^2 \sin [c+d x]}{5 d \sec [c+d x]^{3 / 2}} + \frac{8 a^2 \sin [c+d x]}{7 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 149 leaves):

$$\frac{1}{140 d \sqrt{\sec [c+d x]}} a^2 \left(\frac{672 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1+e^{2 i(c+d x)}}} + 2 \left(-168 i - 80 i \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \sec [c+d x] + \right.$$

$$\left. 85 \sin [c+d x] + 28 \sin [2(c+d x)] + 5 \sin [3(c+d x)] \right) \right)$$

■ **Problem 179: Result unnecessarily involves higher level functions.**

$$\int \sec [c+d x]^{3 / 2} (a+a \sec [c+d x])^3 dx$$

Optimal (type 4, 187 leaves, 16 steps):

$$\begin{aligned}
& - \frac{28 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} + \frac{52 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} + \\
& \frac{28 a^3 \sqrt{\sec[c+dx]} \sin[c+dx]}{5d} + \frac{52 a^3 \sec[c+dx]^{3/2} \sin[c+dx]}{21d} + \frac{6 a^3 \sec[c+dx]^{5/2} \sin[c+dx]}{5d} + \frac{2 a^3 \sec[c+dx]^{7/2} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 5, 287 leaves):

$$\begin{aligned}
& \frac{1}{420d} a^3 \sec\left[\frac{1}{2}(c+dx)\right]^6 (1 + \sec[c+dx])^3 \\
& \left(-1 / (-1 + e^{2ic}) 2i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos[c+dx]^3 \left(147 (1 + e^{2i(c+dx)}) + 147 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \right) \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 65 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) + \\
& \quad \left. \frac{294 \cos[dx] \operatorname{Csc}[c] + (80 + 63 \cos[c+dx] + 65 \cos[2(c+dx)]) \sec[c+dx]^2 \tan[c+dx]}{\sec[c+dx]^{5/2}} \right)
\end{aligned}$$

■ **Problem 180: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^3 dx$$

Optimal (type 4, 157 leaves, 14 steps):

$$\begin{aligned}
& - \frac{36 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} + \frac{4 a^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{d} + \\
& \frac{36 a^3 \sqrt{\sec[c+dx]} \sin[c+dx]}{5d} + \frac{2 a^3 \sec[c+dx]^{3/2} \sin[c+dx]}{d} + \frac{2 a^3 \sec[c+dx]^{5/2} \sin[c+dx]}{5d}
\end{aligned}$$

Result (type 5, 267 leaves):

$$\begin{aligned}
& \frac{1}{20d} a^3 \sec\left[\frac{1}{2}(c+dx)\right]^6 (1 + \sec[c+dx])^3 \left(-1 / (-1 + e^{2ic}) 2i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \right. \\
& \quad \left. \cos[c+dx]^3 \left(9 (1 + e^{2i(c+dx)}) + 9 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \right) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \\
& \quad \left. 5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) + \frac{18 \cos[dx] \operatorname{Csc}[c] + (5 + \sec[c+dx]) \tan[c+dx]}{\sec[c+dx]^{5/2}}
\end{aligned}$$

■ **Problem 181: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 131 leaves, 12 steps):

$$\begin{aligned} & -\frac{4 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \\ & \frac{20 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{6 a^3 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d} + \frac{2 a^3 \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 d} \end{aligned}$$

Result (type 5, 187 leaves):

$$\begin{aligned} & \frac{1}{3 d} a^3 e^{-2 i(c+d x)} \operatorname{Sec}[c + d x]^{3/2} \\ & \left(-6 - 6 \operatorname{Cos}[2(c + d x)] + 6 e^{-2 i(c+d x)} (1 + e^{2 i(c+d x)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 20 \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Cos}[c + d x] \right. \\ & \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] + 2 i \operatorname{Sin}[c + d x] + 9 i \operatorname{Sin}[2(c + d x)] \right) (-i \operatorname{Cos}[2(c + d x)] + \operatorname{Sin}[2(c + d x)]) \end{aligned}$$

■ **Problem 182: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 131 leaves, 12 steps):

$$\begin{aligned} & \frac{4 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \\ & \frac{20 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a^3 \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a^3 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d} \end{aligned}$$

Result (type 5, 169 leaves):

$$\begin{aligned} & \frac{1}{3 d \sqrt{\operatorname{Sec}[c + d x]}} a^3 \left(\operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\frac{24 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]}{\sqrt{1 + e^{2 i(c+d x)}}} + \right. \\ & \left. 2 \left(-6 i - 10 i \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \operatorname{Sec}[c + d x] + \operatorname{Sin}[c + d x] + 3 \operatorname{Tan}[c + d x] \right) \right) \end{aligned}$$

■ **Problem 183: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 131 leaves, 12 steps):

$$\frac{36 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \frac{4 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{2 a^3 \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 a^3 \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 171 leaves):

$$\frac{1}{10 d \sqrt{\operatorname{Sec}[c + d x]}} a^3 \left(\operatorname{Cos}\left[\frac{c}{2}\right] - i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + i \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\frac{144 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right]}{\sqrt{1 + e^{2 i (c + d x)}}} + 2 \left(-36 i - 20 i \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)}\right] \operatorname{Sec}[c + d x] + 10 \operatorname{Sin}[c + d x] + \operatorname{Sin}[2 (c + d x)] \right) \right)$$

■ **Problem 184: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 161 leaves, 14 steps):

$$\frac{28 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \frac{52 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{2 a^3 \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{6 a^3 \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{52 a^3 \operatorname{Sin}[c + d x]}{21 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 146 leaves):

$$\frac{1}{420 d \sqrt{\operatorname{Sec}[c + d x]}} a^3 \left(-2352 i + \frac{4704 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right]}{\sqrt{1 + e^{2 i (c + d x)}}} - 1040 i \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)}\right] \operatorname{Sec}[c + d x] + 1070 \operatorname{Sin}[c + d x] + 252 \operatorname{Sin}[2 (c + d x)] + 30 \operatorname{Sin}[3 (c + d x)] \right)$$

■ **Problem 185: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 187 leaves, 16 steps):

$$\frac{68 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} + \frac{44 a^3 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} +$$

$$\frac{2 a^3 \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{6 a^3 \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{68 a^3 \operatorname{Sin}[c + d x]}{45 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{44 a^3 \operatorname{Sin}[c + d x]}{21 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{2520 d \sqrt{\operatorname{Sec}[c + d x]}} a^3$$

$$\left(-11424 i + \frac{22848 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]}{\sqrt{1 + e^{2 i (c+d x)}}} - 5280 i \sqrt{1 + e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c+d x)}\right] \operatorname{Sec}[c + d x] + \right.$$

$$\left. 5820 \operatorname{Sin}[c + d x] + 2044 \operatorname{Sin}[2(c + d x)] + 540 \operatorname{Sin}[3(c + d x)] + 70 \operatorname{Sin}[4(c + d x)] \right)$$

■ **Problem 186: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^4 dx$$

Optimal (type 4, 213 leaves, 21 steps):

$$-\frac{152 a^4 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} +$$

$$\frac{32 a^4 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{7 d} + \frac{152 a^4 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \frac{32 a^4 \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{7 d} +$$

$$\frac{122 a^4 \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{45 d} + \frac{8 a^4 \operatorname{Sec}[c + d x]^{7/2} \operatorname{Sin}[c + d x]}{7 d} + \frac{2 a^4 \operatorname{Sec}[c + d x]^{9/2} \operatorname{Sin}[c + d x]}{9 d}$$

Result (type 5, 396 leaves):

$$\begin{aligned}
& - \frac{1}{105 \sqrt{2} d (-1 + e^{2 i c})} \\
& i e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2 i(c+dx)}}} \operatorname{Cos}[c + dx]^4 \left(133 (1 + e^{2 i(c+dx)}) + 133 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] + \right. \\
& \left. 60 e^{i(c+dx)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 + \frac{1}{\operatorname{Sec}[c + dx]^{7/2}} \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 \left(\frac{19 \operatorname{Cos}[dx] \operatorname{Csc}[c]}{30 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^4 \operatorname{Sin}[dx]}{72 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 (7 \operatorname{Sin}[c] + 36 \operatorname{Sin}[dx])}{504 d} + \right. \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (180 \operatorname{Sin}[c] + 427 \operatorname{Sin}[dx])}{2520 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx] (427 \operatorname{Sin}[c] + 720 \operatorname{Sin}[dx])}{2520 d} + \frac{2 \operatorname{Tan}[c]}{7 d} \right)
\end{aligned}$$

■ **Problem 187: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^4 dx$$

Optimal (type 4, 187 leaves, 18 steps):

$$\begin{aligned}
& - \frac{64 a^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{5 d} + \frac{136 a^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{21 d} + \\
& \frac{64 a^4 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{5 d} + \frac{94 a^4 \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{21 d} + \frac{8 a^4 \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{5 d} + \frac{2 a^4 \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{7 d}
\end{aligned}$$

Result (type 5, 279 leaves):

$$\begin{aligned}
& \frac{1}{840 d} a^4 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^8 (1 + \operatorname{Sec}[c + dx])^4 \\
& \left(-1 / (-1 + e^{2 i c}) 4 i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2 i(c+dx)}}} \operatorname{Cos}[c + dx]^4 \left(168 (1 + e^{2 i(c+dx)}) + 168 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \right. \right. \\
& \left. \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] + 85 e^{i(c+dx)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+dx)}\right] \right) + \right. \\
& \left. \frac{672 \operatorname{Cos}[dx] \operatorname{Csc}[c] + (235 + 84 \operatorname{Sec}[c + dx] + 15 \operatorname{Sec}[c + dx]^2) \operatorname{Tan}[c + dx]}{\operatorname{Sec}[c + dx]^{7/2}} \right)
\end{aligned}$$

■ **Problem 188: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^4}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 4, 161 leaves, 16 steps):

$$\frac{56 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{32 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{66 a^4 \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{8 a^4 \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 d} + \frac{2 a^4 \sec [c+d x]^{5 / 2} \sin [c+d x]}{5 d}$$

Result (type 5, 286 leaves):

$$\frac{1}{240 d} a^4 \sec \left[\frac{1}{2}(c+d x) \right]^8 (1 + \sec [c+d x])^4 \left(-1 / (-1 + e^{2 i c}) 8 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \cos [c+d x]^4 \left(21 (1 + e^{2 i(c+d x)}) + 21 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \right) \right. \\ \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 20 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) + \frac{-3(-61 + 5 \cos [2 c]) \cos [d x] \csc [c] + 30 \cos [c] \sin [d x] + 2(20 + 3 \sec [c+d x]) \tan [c+d x]}{\sec [c+d x]^{7 / 2}} \left. \right)$$

■ **Problem 190: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c+d x])^4}{\sec [c+d x]^{5 / 2}} dx$$

Optimal (type 4, 159 leaves, 15 steps):

$$\frac{56 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{32 a^4 \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} + \frac{2 a^4 \sin [c+d x]}{5 d \sec [c+d x]^{3 / 2}} + \frac{8 a^4 \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}} + \frac{2 a^4 \sqrt{\sec [c+d x]} \sin [c+d x]}{d}$$

Result (type 5, 184 leaves):

$$\frac{1}{30 d \sqrt{\sec[c+dx]}} a^4 \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \left(-336 i + \frac{672 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 320 i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec[c+dx] + 80 \sin[c+dx] + 3 \sec[c+dx] \sin[3(c+dx)] + 63 \tan[c+dx] \right)$$

■ **Problem 191: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx])^4}{\sec[c+dx]^{7/2}} dx$$

Optimal (type 4, 161 leaves, 16 steps):

$$\frac{64 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5 d} + \frac{136 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21 d} + \frac{2 a^4 \sin[c+dx]}{7 d \sec[c+dx]^{5/2}} + \frac{8 a^4 \sin[c+dx]}{5 d \sec[c+dx]^{3/2}} + \frac{94 a^4 \sin[c+dx]}{21 d \sqrt{\sec[c+dx]}}$$

Result (type 5, 180 leaves):

$$\frac{1}{420 d \sqrt{\sec[c+dx]}} a^4 \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \left(-5376 i + \frac{10752 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 2720 i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec[c+dx] + 1910 \sin[c+dx] + 336 \sin[2(c+dx)] + 30 \sin[3(c+dx)] \right)$$

■ **Problem 192: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx])^4}{\sec[c+dx]^{9/2}} dx$$

Optimal (type 4, 187 leaves, 18 steps):

$$\frac{152 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15 d} + \frac{32 a^4 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{7 d} + \frac{2 a^4 \sin[c+dx]}{9 d \sec[c+dx]^{7/2}} + \frac{8 a^4 \sin[c+dx]}{7 d \sec[c+dx]^{5/2}} + \frac{122 a^4 \sin[c+dx]}{45 d \sec[c+dx]^{3/2}} + \frac{32 a^4 \sin[c+dx]}{7 d \sqrt{\sec[c+dx]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{2520 d \sqrt{\text{Sec}[c + d x]}}$$

$$a^4 \left(-25536 i + \frac{51072 i \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1+e^{2i(c+dx)}}} - 11520 i \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right.$$

$$\left. \text{Sec}[c + d x] + 12240 \text{Sin}[c + d x] + 3556 \text{Sin}[2(c + d x)] + 720 \text{Sin}[3(c + d x)] + 70 \text{Sin}[4(c + d x)] \right)$$

■ **Problem 193: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \text{Sec}[c + d x])^4}{\text{Sec}[c + d x]^{11/2}} dx$$

Optimal (type 4, 213 leaves, 21 steps):

$$\frac{128 a^4 \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]} + 904 a^4 \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{15 d} + \frac{231 d}{11 d \text{Sec}[c + d x]^{9/2}} + \frac{8 a^4 \text{Sin}[c + d x]}{9 d \text{Sec}[c + d x]^{7/2}} + \frac{150 a^4 \text{Sin}[c + d x]}{77 d \text{Sec}[c + d x]^{5/2}} + \frac{128 a^4 \text{Sin}[c + d x]}{45 d \text{Sec}[c + d x]^{3/2}} + \frac{904 a^4 \text{Sin}[c + d x]}{231 d \sqrt{\text{Sec}[c + d x]}}$$

Result (type 5, 306 leaves):

$$-\frac{1}{1774080 d \text{Sec}[c + d x]^{7/2}}$$

$$i a^4 e^{-6i(c+dx)} \left(-315 - 3080 e^{i(c+dx)} - 14760 e^{2i(c+dx)} - 48664 e^{3i(c+dx)} - 137055 e^{4i(c+dx)} + 427504 e^{5i(c+dx)} + 518672 e^{7i(c+dx)} + \right.$$

$$137055 e^{8i(c+dx)} + 48664 e^{9i(c+dx)} + 14760 e^{10i(c+dx)} + 3080 e^{11i(c+dx)} + 315 e^{12i(c+dx)} -$$

$$946176 e^{5i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] +$$

$$\left. 433920 e^{6i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \text{Sec}\left[\frac{1}{2}(c + d x)\right]^8 (1 + \text{Sec}[c + d x])^4$$

■ **Problem 194: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Sec}[c + d x]^{7/2}}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 164 leaves, 8 steps):

$$\frac{3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{5 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3ad} -$$

$$\frac{3 \sqrt{\sec[c+dx]} \sin[c+dx]}{ad} + \frac{5 \sec[c+dx]^{3/2} \sin[c+dx]}{3ad} - \frac{\sec[c+dx]^{5/2} \sin[c+dx]}{d(a+a \sec[c+dx])}$$

Result (type 5, 291 leaves):

$$\frac{1}{3ad(1+\sec[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]$$

$$\left(\frac{1}{-1+e^{2ic}} 2i\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left(9(1+e^{2i(c+dx)}) + 9(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right.$$

$$\left. \left. 5e^{i(c+dx)}(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) - \right.$$

$$\left. \left. \sqrt{\sec[c+dx]} \left(18 \cos[dx] \operatorname{Csc}[c] + \sec[c+dx] \left(-5 \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{3}{2}(c+dx)\right] + \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right)$$

■ **Problem 195: Result unnecessarily involves higher level functions.**

$$\int \frac{\sec[c+dx]^{5/2}}{a+a \sec[c+dx]} dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{3 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} -$$

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{3 \sqrt{\sec[c+dx]} \sin[c+dx]}{ad} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{d(a+a \sec[c+dx])}$$

Result (type 5, 262 leaves):

$$\frac{1}{a(1+\sec[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \left(-1/(d(-1+e^{2ic})) 2i\sqrt{2} e^{-i(c+dx)} \right.$$

$$\left. \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left(3(1+e^{2i(c+dx)}) + 3(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right.$$

$$\left. \left. e^{i(c+dx)}(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) + \frac{\sqrt{\sec[c+dx]} (6 \cos[dx] \operatorname{Csc}[c] - 2 \tan\left[\frac{1}{2}(c+dx)\right])}{d} \right)$$

■ **Problem 196: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Sec}[c + d x]^{3/2}}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 110 leaves, 6 steps):

$$\frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} + \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} - \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 5, 201 leaves):

$$-\left(2 i e^{-i(c+dx)} \text{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \left(1 + e^{2i(c+dx)} - (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + e^{i(c+dx)} (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \text{Sec}[c + d x]^{3/2} \right) / (a d (1 + e^{i(c+dx)}) (1 + \text{Sec}[c + d x]))$$

■ **Problem 197: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\text{Sec}[c + d x]}}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 110 leaves, 6 steps):

$$-\frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} + \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} + \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 5, 202 leaves):

$$-\left(2 i e^{-i(c+dx)} \text{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \left(-1 - e^{2i(c+dx)} + (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + e^{i(c+dx)} (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \text{Sec}[c + d x]^{3/2} \right) / (a d (1 + e^{i(c+dx)}) (1 + \text{Sec}[c + d x]))$$

■ **Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\text{Sec}[c + d x]} (a + a \text{Sec}[c + d x])} dx$$

Optimal (type 4, 112 leaves, 6 steps):

$$\frac{3 \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} - \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} - \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 5, 317 leaves):

$$\frac{1}{a(1 + \operatorname{Sec}[c + dx])}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \left(\frac{1}{d(-1 + e^{2ic})} 2i\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left(3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) - \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right) + e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) -$$

$$1 / (2d) \left(\operatorname{Cos}\left[\frac{1}{2}(c - dx)\right] + 2 \operatorname{Cos}\left[\frac{1}{2}(3c + dx)\right] + 2 \operatorname{Cos}\left[\frac{1}{2}(c + 3dx)\right] + \operatorname{Cos}\left[\frac{1}{2}(5c + 3dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{\operatorname{Sec}[c + dx]} \right) \operatorname{Sec}[c + dx]$$

■ **Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 140 leaves, 7 steps):

$$-\frac{3\sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{ad} + \frac{5\sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3ad} + \frac{5 \operatorname{Sin}[c + dx]}{3ad \sqrt{\operatorname{Sec}[c + dx]}} - \frac{\operatorname{Sin}[c + dx]}{d \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])}$$

Result (type 5, 380 leaves):

$$-\left(2i\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(9(1 + e^{2i(c+dx)}) + 9(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 5e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + dx] \right) / (3d(-1 + e^{2ic})(a + a \operatorname{Sec}[c + dx])) +$$

$$\frac{1}{a + a \operatorname{Sec}[c + dx]} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx]^{3/2} \left(\frac{(2 + \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{2 \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{3d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} - \frac{4 \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{2 \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{3d} - \frac{2 \operatorname{Tan}\left[\frac{c}{2}\right]}{d} \right)$$

- **Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\text{Sec}[c + d x]^{5/2} (a + a \text{Sec}[c + d x])} dx$$

Optimal (type 4, 168 leaves, 8 steps):

$$\frac{21 \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]} - 5 \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{5 a d} + \frac{7 \text{Sin}[c + d x]}{5 a d \text{Sec}[c + d x]^{3/2}} - \frac{5 \text{Sin}[c + d x]}{3 a d \sqrt{\text{Sec}[c + d x]}} - \frac{\text{Sin}[c + d x]}{d \text{Sec}[c + d x]^{3/2} (a + a \text{Sec}[c + d x])}$$

Result (type 5, 347 leaves):

$$\frac{1}{60 a d (1 + \text{Sec}[c + d x])} \text{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \text{Sec}[c + d x] \left(\frac{1}{-1 + e^{2 i c}} 8 i \sqrt{2} e^{-i(c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \left(63 (1 + e^{2 i(c + d x)}) + 63 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 25 e^{i(c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c + d x)}\right] \right) - \sqrt{\text{Sec}[c + d x]} \left(18 (17 + 11 \text{Cos}[2 c]) \text{Cos}[d x] \text{Csc}[c] + 4 \left(10 \text{Cos}[2 d x] \text{Sin}[2 c] - 3 \text{Cos}[3 d x] \text{Sin}[3 c] - 30 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c + d x)\right] \text{Sin}\left[\frac{d x}{2}\right] - 99 \text{Cos}[c] \text{Sin}[d x] + 10 \text{Cos}[2 c] \text{Sin}[2 d x] - 3 \text{Cos}[3 c] \text{Sin}[3 d x] - 30 \text{Tan}\left[\frac{c}{2}\right] \right) \right) \right)$$

- **Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{9/2}}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 4, 202 leaves, 9 steps):

$$\frac{7 \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]} - 10 \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a^2 d} + \frac{7 \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{a^2 d} + \frac{10 \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 a^2 d} - \frac{7 \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} - \frac{\text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 5, 451 leaves):

$$\frac{1}{d (a + a \operatorname{Sec}[c + d x])^2} 7 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right]$$

$$\left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 +$$

$$\frac{20 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c]}{3 d (a + a \operatorname{Sec}[c + dx])^2} + \frac{1}{(a + a \operatorname{Sec}[c + dx])^2}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c + dx]^{5/2} \left(-\frac{14 \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] (-3 \operatorname{Sin}\left[\frac{c}{2}\right] + 5 \operatorname{Sin}\left[\frac{3c}{2}\right])}{3 d}\right) +$$

$$\left(\frac{32 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{3 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d}\right)$$

■ **Problem 202: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + dx]^{7/2}}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 176 leaves, 8 steps):

$$-\frac{4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a^2 d} - \frac{5 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3 a^2 d} +$$

$$\frac{4 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{a^2 d} - \frac{5 \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{3 a^2 d (1 + \operatorname{Sec}[c + dx])} - \frac{\operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{3 d (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 5, 259 leaves):

$$-\frac{1}{6 a^2 d (1 + \operatorname{Sec}[c + dx])^2}$$

$$e^{-i(2c+dx)} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx]^{5/2} \left(12 i e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] +\right.$$

$$40 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right) -$$

$$\left. i (29 + 50 \operatorname{Cos}[c + dx] + 17 \operatorname{Cos}[2(c + dx)] - 12 i \operatorname{Sin}[c + dx] - 7 i \operatorname{Sin}[2(c + dx)])\right) \left(\operatorname{Cos}\left[\frac{1}{2}(3c + dx)\right] + i \operatorname{Sin}\left[\frac{1}{2}(3c + dx)\right]\right)$$

■ **Problem 203: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2}}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{a^2 d (1 + \sec[c+dx])} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{3 d (a + a \sec[c+dx])^2}$$

Result (type 5, 249 leaves):

$$\frac{1}{6 a^2 d (1 + \sec[c+dx])^2} e^{-i(2c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]^{5/2} \left(3 i e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 16 \cos\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) - i (5 + 14 \cos[c+dx] + 5 \cos[2(c+dx)] - i \sin[2(c+dx)]) \right) \left(\cos\left[\frac{1}{2}(3c+dx)\right] + i \sin\left[\frac{1}{2}(3c+dx)\right] \right)$$

■ **Problem 205: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\sec[c+dx]}}{(a + a \sec[c+dx])^2} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$-\frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} + \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{a^2 d (1 + \sec[c+dx])} - \frac{\sec[c+dx]^{3/2} \sin[c+dx]}{3 d (a + a \sec[c+dx])^2}$$

Result (type 5, 247 leaves):

$$\frac{1}{6 a^2 d (1 + \sec[c+dx])^2} e^{-i(2c+dx)} \cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]^{5/2} \left(16 \cos\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right] + i \sin\left[\frac{1}{2}(c+dx)\right] \right) - i \left(-7 - 10 \cos[c+dx] - 7 \cos[2(c+dx)] + 3 e^{-2i(c+dx)} (1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - i \sin[2(c+dx)] \right) \right) \left(\cos\left[\frac{1}{2}(3c+dx)\right] + i \sin\left[\frac{1}{2}(3c+dx)\right] \right)$$

- **Problem 206: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sec[c+dx]} (a+a\sec[c+dx])^2} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\frac{4 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} - \frac{5 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} - \frac{5 \sqrt{\sec[c+dx]} \sin[c+dx]}{3 a^2 d (1+\sec[c+dx])} - \frac{\sqrt{\sec[c+dx]} \sin[c+dx]}{3 d (a+a\sec[c+dx])^2}$$

Result (type 5, 430 leaves):

$$\frac{1}{d (a+a\sec[c+dx])^2} 4 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \\ \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \sec\left[\frac{c}{2}\right] \sec[c+dx]^2 - \\ \frac{10 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{5/2} \sin[c]}{3 d (a+a\sec[c+dx])^2} + \frac{1}{(a+a\sec[c+dx])^2} \\ \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^{5/2} \left(-\frac{2(3+\cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{d} + \frac{28 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{3 d} - \right. \\ \left. \frac{2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{8 \cos[c] \sin[dx]}{d} + \frac{28 \tan\left[\frac{c}{2}\right]}{3 d} - \frac{2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3 d} \right)$$

- **Problem 207: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sec[c+dx]^{3/2} (a+a\sec[c+dx])^2} dx$$

Optimal (type 4, 178 leaves, 8 steps):

$$-\frac{7 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \frac{10 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 a^2 d} + \\ \frac{10 \sin[c+dx]}{3 a^2 d \sqrt{\sec[c+dx]}} - \frac{7 \sin[c+dx]}{3 a^2 d \sqrt{\sec[c+dx]} (1+\sec[c+dx])} - \frac{\sin[c+dx]}{3 d \sqrt{\sec[c+dx]} (a+a\sec[c+dx])^2}$$

Result (type 5, 278 leaves):

$$\frac{1}{3 a^2 (1 + \operatorname{Sec}[c + d x])^2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^4 \left(\frac{40 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \right. \\ \left. 1 / (d (1 + e^{i(c+d x)})^3) 2 i e^{-2 i(c+d x)} \left(1 + 33 e^{i(c+d x)} + 73 e^{2 i(c+d x)} + 87 e^{3 i(c+d x)} + 81 e^{4 i(c+d x)} + 53 e^{5 i(c+d x)} + 9 e^{6 i(c+d x)} - e^{7 i(c+d x)} - \right. \right. \\ \left. \left. 42 e^{i(c+d x)} (1 + e^{i(c+d x)})^3 \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right]\right) \sqrt{\operatorname{Sec}[c + d x]} \right) \operatorname{Sec}[c + d x]^2$$

■ **Problem 208: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 200 leaves, 9 steps):

$$\frac{56 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - 5 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 a^2 d} + \\ \frac{56 \operatorname{Sin}[c + d x]}{15 a^2 d \operatorname{Sec}[c + d x]^{3/2}} - \frac{5 \operatorname{Sin}[c + d x]}{a^2 d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{3 \operatorname{Sin}[c + d x]}{a^2 d \operatorname{Sec}[c + d x]^{3/2} (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Sin}[c + d x]}{3 d \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 5, 500 leaves):

$$\frac{1}{5 d (a + a \operatorname{Sec}[c + d x])^2} 56 \sqrt{2} e^{-i(2c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \\ \left(1 + e^{2 i(c+d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 - \\ \frac{10 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c]}{d (a + a \operatorname{Sec}[c + d x])^2} + \\ \frac{1}{(a + a \operatorname{Sec}[c + d x])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}[c + d x]^{5/2} \\ \left(- \frac{(151 + 73 \operatorname{Cos}[2 c]) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{10 d} - \frac{8 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{3 d} + \frac{2 \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{5 d} + \frac{52 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right]}{3 d} - \right. \\ \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \operatorname{Sin}\left[\frac{d x}{2}\right]}{3 d} + \frac{146 \operatorname{Cos}[c] \operatorname{Sin}[d x]}{5 d} - \frac{8 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{3 d} + \frac{2 \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{5 d} + \frac{52 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right)$$

- **Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{11/2}}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 247 leaves, 10 steps):

$$\frac{119 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{10 a^3 d} +$$

$$\frac{11 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{2 a^3 d} - \frac{119 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{10 a^3 d} +$$

$$\frac{11 \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{2 a^3 d} - \frac{\operatorname{Sec}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Sec}[c + dx])^3} - \frac{2 \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{3 a d (a + a \operatorname{Sec}[c + dx])^2} - \frac{119 \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{30 d (a^3 + a^3 \operatorname{Sec}[c + dx])}$$

Result (type 5, 516 leaves):

$$\frac{1}{5 d (a + a \operatorname{Sec}[c + dx])^3} 119 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right]$$

$$\left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 +$$

$$\frac{22 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c]}{d (a + a \operatorname{Sec}[c + dx])^3} +$$

$$\frac{1}{(a + a \operatorname{Sec}[c + dx])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c + dx]^{7/2} \left(-\frac{238 \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} +$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] (-25 \operatorname{Sin}\left[\frac{c}{2}\right] + 33 \operatorname{Sin}\left[\frac{3c}{2}\right])}{3 d} + \frac{116 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{52 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{15 d} +$$

$$\frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \frac{16 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{3 d} + \frac{52 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}\right)$$

- **Problem 210: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + dx]^{9/2}}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$\frac{49 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{10 a^3 d} - \frac{13 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{6 a^3 d} +$$

$$\frac{49 \sqrt{\sec[c+dx]} \sin[c+dx]}{10 a^3 d} - \frac{\sec[c+dx]^{7/2} \sin[c+dx]}{5 d (a + a \sec[c+dx])^3} - \frac{8 \sec[c+dx]^{5/2} \sin[c+dx]}{15 a d (a + a \sec[c+dx])^2} - \frac{13 \sec[c+dx]^{3/2} \sin[c+dx]}{6 d (a^3 + a^3 \sec[c+dx])}$$

Result (type 5, 371 leaves):

$$\frac{1}{15 a^3 d (1 + \sec[c+dx])^3} 2 \cos\left[\frac{1}{2}(c+dx)\right]^6$$

$$\left(-\frac{1}{-1 + e^{2ic}} 2i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left(147 (1 + e^{2i(c+dx)}) + 147 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right.$$

$$\left. 65 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) +$$

$$\frac{1}{32} \left(1284 \cos\left[\frac{1}{2}(c-dx)\right] + 921 \cos\left[\frac{1}{2}(3c+dx)\right] + 1243 \cos\left[\frac{1}{2}(c+3dx)\right] + 374 \cos\left[\frac{1}{2}(5c+3dx)\right] + 670 \cos\left[\frac{1}{2}(3c+5dx)\right] + \right.$$

$$\left. 65 \cos\left[\frac{1}{2}(7c+5dx)\right] + 147 \cos\left[\frac{1}{2}(5c+7dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} \operatorname{Sec}[c+dx]^3$$

■ **Problem 211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{7/2}}{(a + a \sec[c+dx])^3} dx$$

Optimal (type 4, 195 leaves, 8 steps):

$$\frac{9 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{10 a^3 d} + \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{2 a^3 d} -$$

$$\frac{\sec[c+dx]^{5/2} \sin[c+dx]}{5 d (a + a \sec[c+dx])^3} - \frac{2 \sec[c+dx]^{3/2} \sin[c+dx]}{5 a d (a + a \sec[c+dx])^2} - \frac{9 \sqrt{\sec[c+dx]} \sin[c+dx]}{10 d (a^3 + a^3 \sec[c+dx])}$$

Result (type 5, 474 leaves):

$$\frac{1}{5 d (a + a \operatorname{Sec}[c + d x])^3} 9 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right]$$

$$\left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 +$$

$$\frac{2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c]}{d (a + a \operatorname{Sec}[c + dx])^3} + \frac{1}{(a + a \operatorname{Sec}[c + dx])^3}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c + dx]^{7/2} \left(-\frac{18 \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} +$$

$$\frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \frac{4 \operatorname{Tan}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}\right)$$

■ **Problem 212: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2}}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 195 leaves, 8 steps):

$$\frac{\sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{10 a^3 d} + \frac{\sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{6 a^3 d} -$$

$$\frac{\operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Sec}[c + dx])^3} - \frac{4 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Sec}[c + dx])^2} + \frac{\sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{6 d (a^3 + a^3 \operatorname{Sec}[c + dx])}$$

Result (type 5, 371 leaves):

$$\frac{1}{15 a^3 d (1 + \operatorname{Sec}[c + dx])^3} 2 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left(\frac{1}{-1 + e^{2ic}} 2 i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \left(3 (1 + e^{2i(c+dx)}) + 3 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] -$$

$$5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) -$$

$$\frac{1}{32} \left(36 \operatorname{Cos}\left[\frac{1}{2}(c - dx)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(3c + dx)\right] + 7 \operatorname{Cos}\left[\frac{1}{2}(c + 3dx)\right] + 26 \operatorname{Cos}\left[\frac{1}{2}(5c + 3dx)\right] + 10 \operatorname{Cos}\left[\frac{1}{2}(3c + 5dx)\right] +$$

$$5 \operatorname{Cos}\left[\frac{1}{2}(7c + 5dx)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(5c + 7dx)\right]\right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sec}[c + dx]^3$$

■ **Problem 213: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{Sec}[c + d x]^{3/2}}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 4, 195 leaves, 8 steps):

$$\begin{aligned} & - \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{10 a^3 d} + \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{6 a^3 d} + \\ & \frac{\text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{5 d (a + a \text{Sec}[c + d x])^3} - \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{15 a d (a + a \text{Sec}[c + d x])^2} + \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{6 d (a^3 + a^3 \text{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 371 leaves):

$$\begin{aligned} & \frac{1}{15 a^3 d (1 + \text{Sec}[c + d x])^3} 2 \text{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \\ & \left(- \frac{1}{-1 + e^{2 i c}} 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left(3 (1 + e^{2 i(c+d x)}) + 3 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) + \right. \\ & \quad \left. 5 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) + \\ & \frac{1}{32} \left(36 \text{Cos}\left[\frac{1}{2}(c - d x)\right] + 9 \text{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 17 \text{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 16 \text{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 20 \text{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] - \right. \\ & \quad \left. 5 \text{Cos}\left[\frac{1}{2}(7 c + 5 d x)\right] + 3 \text{Cos}\left[\frac{1}{2}(5 c + 7 d x)\right] \right) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \sqrt{\text{Sec}[c + d x]} \Bigg) \text{Sec}[c + d x]^3 \end{aligned}$$

■ **Problem 214: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\text{Sec}[c + d x]}}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 4, 195 leaves, 8 steps):

$$\begin{aligned} & - \frac{9 \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{10 a^3 d} + \frac{\sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{2 a^3 d} - \\ & \frac{\text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{5 d (a + a \text{Sec}[c + d x])^3} + \frac{2 \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{5 a d (a + a \text{Sec}[c + d x])^2} + \frac{\sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{2 d (a^3 + a^3 \text{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 474 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \operatorname{Sec}[c + d x])^3} 9 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 + \\
& \frac{2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c]}{d (a + a \operatorname{Sec}[c + dx])^3} + \frac{1}{(a + a \operatorname{Sec}[c + dx])^3} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c + dx]^{7/2} \left(\frac{18 \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} - \frac{12 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} + \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \frac{12 \operatorname{Tan}\left[\frac{c}{2}\right]}{d} + \frac{16 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

■ **Problem 215: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 195 leaves, 8 steps):

$$\begin{aligned}
& \frac{49 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{10 a^3 d} - \frac{13 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{6 a^3 d} - \\
& \frac{\sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Sec}[c + dx])^3} - \frac{8 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Sec}[c + dx])^2} - \frac{13 \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{6 d (a^3 + a^3 \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 386 leaves):

$$\frac{1}{15 a^3 d (1 + \operatorname{Sec}[c + d x])^3} 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6$$

$$\left(\frac{1}{-1 + e^{2 i c}} 2 i \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1 + e^{2 i(c+d x)}}} \left(147 (1 + e^{2 i(c+d x)}) + 147 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + \right. \right.$$

$$\left. 65 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) -$$

$$\frac{1}{32} \left(1134 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 1071 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 923 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 694 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 470 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] + \right.$$

$$\left. 265 \operatorname{Cos}\left[\frac{1}{2}(7 c + 5 d x)\right] + 117 \operatorname{Cos}\left[\frac{1}{2}(5 c + 7 d x)\right] + 30 \operatorname{Cos}\left[\frac{1}{2}(9 c + 7 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sec}[c + d x]^3$$

- **Problem 216: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$-\frac{119 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} + \frac{11 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{2 a^3 d} + \frac{11 \operatorname{Sin}[c + d x]}{2 a^3 d \sqrt{\operatorname{Sec}[c + d x]}} -$$

$$\frac{\operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3} - \frac{2 \operatorname{Sin}[c + d x]}{3 a d \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2} - \frac{119 \operatorname{Sin}[c + d x]}{30 d \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 529 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (a + a \operatorname{Sec}[c + d x])^3} 119 \sqrt{2} e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 + \\
& \frac{22 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c]}{d (a + a \operatorname{Sec}[c + dx])^3} + \frac{1}{(a + a \operatorname{Sec}[c + dx])^3} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c + dx]^{7/2} \left(\frac{2(89 + 30 \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 d} + \frac{8 \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{3 d} - \right. \\
& \frac{172 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d} + \frac{88 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \\
& \left. \frac{48 \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{8 \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{3 d} - \frac{172 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{88 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right)
\end{aligned}$$

■ **Problem 217: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 247 leaves, 10 steps):

$$\begin{aligned}
& \frac{231 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{10 a^3 d} - \\
& \frac{21 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{2 a^3 d} + \frac{77 \operatorname{Sin}[c + dx]}{10 a^3 d \operatorname{Sec}[c + dx]^{3/2}} - \frac{21 \operatorname{Sin}[c + dx]}{2 a^3 d \sqrt{\operatorname{Sec}[c + dx]}} - \\
& \frac{\operatorname{Sin}[c + dx]}{5 d \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} - \frac{4 \operatorname{Sin}[c + dx]}{5 a d \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^2} - \frac{63 \operatorname{Sin}[c + dx]}{10 d \operatorname{Sec}[c + dx]^{3/2} (a^3 + a^3 \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 332 leaves):

$$\begin{aligned}
& - \frac{1}{5 a^3 d (1 + e^{i(c+dx)})^5 (1 + \operatorname{Sec}[c + dx])^3} \\
& 2 i e^{-3 i(c+dx)} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^6 \left(-1 + 5 e^{i(c+dx)} + 369 e^{2 i(c+dx)} + 1505 e^{3 i(c+dx)} + 2900 e^{4 i(c+dx)} + 3590 e^{5 i(c+dx)} + 3340 e^{6 i(c+dx)} + 2182 e^{7 i(c+dx)} + \right. \\
& 805 e^{8 i(c+dx)} + 93 e^{9 i(c+dx)} - 5 e^{10 i(c+dx)} + e^{11 i(c+dx)} - 210 i e^{3 i(c+dx)} (1 + e^{i(c+dx)})^5 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] - \\
& \left. 462 e^{2 i(c+dx)} (1 + e^{i(c+dx)})^5 \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] \right) \operatorname{Sec}[c + dx]^{7/2}
\end{aligned}$$

- **Problem 218: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{5/2} \sqrt{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$\frac{3 \sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4 d} + \frac{3 a \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{4 d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{a \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 434 leaves):

$$\begin{aligned}
& \frac{1}{32 \sqrt{2} d \sqrt{\operatorname{Sec}[c + dx]}} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a (1 + \operatorname{Sec}[c + dx])} \\
& \left(-12 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}\right] - 12 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}\right] \right) + \\
& \operatorname{Sec}[c + dx]^2 \left(6 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 3 \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& 3 \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \\
& \left. \operatorname{Cos}[2(c + dx)] \left(6 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 3 \left(\operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \operatorname{Log}\left[\right. \right. \right. \\
& \left. \left. \left. 2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) + 4 \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 12 \sqrt{2} \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right] \right)
\end{aligned}$$

- **Problem 219: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$\frac{\sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 306 leaves):

$$\frac{1}{4 \sqrt{2} d \sqrt{\operatorname{Sec}[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \\ \left(-2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] - 2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \right) + \\ 2 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\ \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 4 \sqrt{2} \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \Bigg)$$

- **Problem 220: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d}$$

Result (type 3, 283 leaves):

$$\frac{1}{2 \sqrt{2} d \sqrt{\operatorname{Sec}[c+dx]}} \left(-2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] - 2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \right) + \\ 2 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \\ \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \Bigg) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

- **Problem 225: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^{3/2} dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{11 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8 d} + \frac{11 a^2 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{8 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{11 a^2 \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{12 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{3 d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 456 leaves):

$$\frac{1}{384 \sqrt{2} d \sqrt{\operatorname{Sec}[c+dx]}} a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

$$\left(-264 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] - 264 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] + \right.$$

$$\operatorname{Sec}[c+dx]^3 \left(99 \operatorname{Cos}[c+dx] \left(2 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$\left. \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$33 \operatorname{Cos}[3(c+dx)] \left(2 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$\left. \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 4 \sqrt{2} \left(54 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 11 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 33 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) \right)$$

- **Problem 226: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} dx$$

Optimal (type 3, 120 leaves, 5 steps):

$$\frac{7 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4 d} + \frac{7 a^2 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{4 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{2 d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 432 leaves):

$$\begin{aligned}
& - \frac{1}{32 \sqrt{2} d \sqrt{\sec[c+dx]}} a \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \\
& \left(28 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + 28 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + \right. \\
& \sec[c+dx]^2 \left(7 \cos[2(c+dx)] \left(-2 \log\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \\
& \left. \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + 12 \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right] + 7 \left(-2 \log\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log\left[\right. \right. \\
& \left. \left. 2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] - 4 \sqrt{2} \sin\left[\frac{3}{2}(c+dx)\right] \right) \left. \right) \left. \right)
\end{aligned}$$

- **Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^{3/2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{3 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{d} + \frac{a^2 \sec[c+dx]^{3/2} \sin[c+dx]}{d \sqrt{a+a \sec[c+dx]}}$$

Result (type 3, 307 leaves):

$$\begin{aligned}
& \frac{1}{4 \sqrt{2} d \sqrt{\sec[c+dx]}} a \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \\
& \left(-6 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - 6 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + \right. \\
& 6 \log\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] - 3 \log\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] - \\
& \left. 3 \log\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4 \sqrt{2} \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

- **Problem 228: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c+dx])^{3/2}}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} + \frac{2 a^2 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 301 leaves):

$$\frac{1}{2 \sqrt{2} d \sqrt{\operatorname{Sec}[c+d x]}} a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1+\operatorname{Sec}[c+d x])} \\ \left(-2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] - 2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] \right) + \\ 2 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \\ \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 4 \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] \right)$$

- **Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{5/2} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{163 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{64 d} + \frac{163 a^3 \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{64 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \\ \frac{163 a^3 \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{96 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{17 a^3 \operatorname{Sec}[c+d x]^{7/2} \operatorname{Sin}[c+d x]}{24 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 \operatorname{Sec}[c+d x]^{7/2} \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}$$

Result (type 3, 1098 leaves):

$$\frac{163 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]-\sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right]+\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]}\right]}{512\sqrt{2}d\operatorname{Sec}[c+dx]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} -$$

$$\frac{163 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right]+\sin\left[\frac{1}{4}(c+dx)\right]-\sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right]+\sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]}\right]}{512\sqrt{2}d\operatorname{Sec}[c+dx]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} +$$

$$\frac{163 i \operatorname{Log}\left[\sqrt{2}+2\sin\left[\frac{1}{2}(c+dx)\right]\right]}{256\left((-1+i)+\sqrt{2}\right)\left((1+i)+\sqrt{2}\right)} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} -$$

$$\frac{163 \operatorname{Log}\left[2-\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right]-\sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right]}{1024\sqrt{2}d\operatorname{Sec}[c+dx]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} -$$

$$\frac{163 \operatorname{Log}\left[2+\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right]-\sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right]}{1024\sqrt{2}d\operatorname{Sec}[c+dx]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} +$$

$$\frac{23 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}}{384d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{163 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}}{512d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

$$\frac{23 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}}{384d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{163 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}}{512d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{64d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{43 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{256d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{64d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{43 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{256d\operatorname{Sec}[c+dx]^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

■ **Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+a\operatorname{Sec}[c+dx])^{5/2} dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$\frac{25 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8 d} + \frac{25 a^3 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{8 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{13 a^3 \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{12 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 \operatorname{Sec}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d}$$

Result (type 3, 458 leaves):

$$\frac{1}{384 \sqrt{2} d \sqrt{\operatorname{Sec}[c+dx]}} a^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

$$\left(-600 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] - 600 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] + \right.$$

$$\operatorname{Sec}[c+dx]^3 \left(225 \operatorname{Cos}[c+dx] \left(2 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$\left. \left. \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right.$$

$$75 \operatorname{Cos}[3(c+dx)] \left(2 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$\left. \left. \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 4 \sqrt{2} \left(114 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 7 \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 75 \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) \right)$$

■ **Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{19 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4 d} + \frac{9 a^3 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{4 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2 d}$$

Result (type 3, 435 leaves):

$$\begin{aligned}
& - \frac{1}{32 \sqrt{2} d \sqrt{\sec[c+dx]}} a^2 \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \\
& \left(76 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + 76 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \right) + \\
& \sec[c+dx]^2 \left(-38 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] + 19 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 19 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \left. \left. \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 19 \cos[2(c+dx)] \left(-2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \\
& \left. \left. \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + 28 \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right] - 44 \sqrt{2} \sin\left[\frac{3}{2}(c+dx)\right] \right) \Bigg)
\end{aligned}$$

- **Problem 236: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c+dx])^{5/2}}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 3, 112 leaves, 4 steps):

$$\frac{5 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{d} + \frac{a^3 \sqrt{\sec[c+dx]} \sin[c+dx]}{d \sqrt{a+a \sec[c+dx]}} + \frac{a^2 \sqrt{\sec[c+dx]} \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 3, 309 leaves):

$$\begin{aligned}
& \frac{1}{4 \sqrt{2} d \sqrt{\sec[c+dx]}} a^2 \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \\
& \left(-10 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - 10 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \right) + \\
& 10 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] - 5 \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] - \\
& 5 \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4 \sqrt{2} \sec[c+dx] \sin\left[\frac{3}{2}(c+dx)\right] \Bigg)
\end{aligned}$$

- **Problem 237: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c+dx])^{5/2}}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 3, 118 leaves, 4 steps):

$$\frac{2 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} + \frac{14 a^3 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{2 a^2 \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 3, 320 leaves):

$$\frac{1}{6 \sqrt{2} d \sqrt{\operatorname{Sec}[c+d x]}} a^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1+\operatorname{Sec}[c+d x])} \\ \left(-6 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] - 6 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] \right) + \\ 6 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 3 \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \\ 3 \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 30 \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 2 \sqrt{2} \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] \right)$$

- **Problem 243: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[e+f x]} \sqrt{a+a \operatorname{Sec}[e+f x]} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+a \operatorname{Sec}[e+f x]}}\right]}{f}$$

Result (type 3, 283 leaves):

$$\frac{1}{2 \sqrt{2} f \sqrt{\operatorname{Sec}[e+f x]}} \left(-2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(e+f x)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(e+f x)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e+f x)\right]}\right] - 2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(e+f x)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(e+f x)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e+f x)\right]}\right] \right) + \\ 2 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] - \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] - \\ \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{a(1+\operatorname{Sec}[e+f x])}$$

- **Problem 244: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{-\operatorname{Sec}[e+f x]} \sqrt{a-a \operatorname{Sec}[e+f x]} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2\sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a-a \operatorname{Sec}[e+fx]}}\right]}{f}$$

Result (type 3, 299 leaves):

$$\frac{1}{2\sqrt{2} f \sqrt{-\operatorname{Sec}[e+fx]}}$$

$$\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \left(-2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(e+fx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(e+fx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]}\right] + 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(e+fx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(e+fx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]}\right] - \right.$$

$$4 \operatorname{ArcTanh}\left[\sqrt{2} \cos\left[\frac{1}{4}(2e+fx)\right] \operatorname{Sec}\left[\frac{fx}{4}\right] + \operatorname{Tan}\left[\frac{fx}{4}\right]\right] + \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(e+fx)\right]\right] -$$

$$\left. \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(e+fx)\right]\right] \right) \sqrt{a - a \operatorname{Sec}[e+fx]}$$

■ **Problem 245: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2}}{\sqrt{a+a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 128 leaves, 6 steps):

$$-\frac{\operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d} + \frac{\operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 362 leaves):

$$\frac{1}{2\sqrt{2} d \sqrt{a(1+\operatorname{Sec}[c+dx])}} \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\operatorname{Sec}[c+dx]}$$

$$\left(2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - \right.$$

$$4\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] + 4\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] - 2 \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[\right.$$

$$\left. 2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4\sqrt{2} \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \left. \right)$$

- **Problem 246: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2}}{\sqrt{a + a \text{Sec}[c + d x]}} dx$$

Optimal (type 3, 95 leaves, 5 steps) :

$$\frac{2 \text{ArcSinh}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \text{Sec}[c+dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 394 leaves) :

$$\frac{1}{((-1+i)+\sqrt{2}) d \sqrt{a} (1+\text{Sec}[c+dx])} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \cos\left[\frac{1}{2}(c+dx)\right] \left(2 \left((-1-i) + \sqrt{2} \right) \text{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) + \right. \\ \left. 2 \left((-1-i) + \sqrt{2} \right) \text{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] - \right. \\ \left. i \left((-4 + (2+2i)\sqrt{2}) \text{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] + (4 - (2+2i)\sqrt{2}) \text{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \right) - \right. \\ \left. \left((-1-i) + \sqrt{2} \right) \left(2 \text{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] - \text{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] - \right. \right. \\ \left. \left. \text{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \right) \right) \sqrt{\text{Sec}[c+dx]}$$

- **Problem 251: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{7/2}}{(a + a \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 174 leaves, 7 steps) :

$$-\frac{3 \text{ArcSinh}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{a^{3/2} d} + \frac{9 \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \text{Sec}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\text{Sec}[c+dx]^{5/2} \text{Sin}[c+dx]}{2 d (a + a \text{Sec}[c+dx])^{3/2}} + \frac{3 \text{Sec}[c+dx]^{3/2} \text{Sin}[c+dx]}{2 a d \sqrt{a + a \text{Sec}[c+dx]}}$$

Result (type 3, 457 leaves) :

$$\frac{1}{2 d (a (1 + \operatorname{Sec}[c + d x]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Sec}[c + d x]^{3/2}$$

$$\left(6 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] + 6 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] - \right.$$

$$18 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] + 18 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] -$$

$$6 \sqrt{2} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 3 \sqrt{2} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] +$$

$$3 \sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right)^2} -$$

$$\left. \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right)^2} + \frac{4}{\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]} - \frac{4}{\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}\right)$$

- **Problem 252: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2}}{(a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 134 leaves, 6 steps):

$$\frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{a^{3/2} d} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{2 d (a + a \operatorname{Sec}[c + d x])^{3/2}}$$

Result (type 3, 437 leaves):

$$\frac{1}{2 d (a (1 + \operatorname{Sec}[c + d x]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Sec}[c + d x]^{3/2}$$

$$\left(\frac{(4 + 4 i) \left((-1 - i) + \sqrt{2} \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right]}{(-1 + i) + \sqrt{2}} - 4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] \right) +$$

$$10 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] - 10 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] + 4 \sqrt{2} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] -$$

$$2 \sqrt{2} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \frac{(2 - 2 i) \left((-1 - i) + \sqrt{2} \right) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right]}{(-1 + i) + \sqrt{2}} -$$

$$\left. \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right)^2} + \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right)^2} \right)$$

- **Problem 258: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{9/2}}{(a + a \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 3, 214 leaves, 8 steps):

$$-\frac{5 \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{a^{5/2} d} + \frac{115 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{\operatorname{Sec}[c + d x]^{7/2} \operatorname{Sin}[c + d x]}{4 d (a + a \operatorname{Sec}[c + d x])^{5/2}} - \frac{15 \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{16 a d (a + a \operatorname{Sec}[c + d x])^{3/2}} + \frac{35 \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{16 a^2 d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 1042 leaves):

$$\begin{aligned}
& \frac{10 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]-\sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right]+\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]}\right]}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2} + \\
& \frac{10 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right]+\sin\left[\frac{1}{4}(c+dx)\right]-\sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right]+\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]}\right]}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2} - \\
& \frac{115 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{4 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{115 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]+\sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{4 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} - \\
& \frac{40 i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\sqrt{2}+2 \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{\left((-1+i)+\sqrt{2}\right)\left((1+i)+\sqrt{2}\right) d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{5 \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{5 \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]\right)^4} + \frac{19 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]-\sin\left[\frac{1}{4}(c+dx)\right]\right)^2} - \\
& \frac{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]+\sin\left[\frac{1}{4}(c+dx)\right]\right)^4} - \frac{19 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]+\sin\left[\frac{1}{4}(c+dx)\right]\right)^2} + \\
& \frac{4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2}\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)}
\end{aligned}$$

■ **Problem 259: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{7/2}}{(a+a \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} - \frac{43 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{\operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{4 d(a+a \operatorname{Sec}[c+dx])^{5/2}} - \frac{11 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{16 a d(a+a \operatorname{Sec}[c+dx])^{3/2}}$$

Result (type 3, 925 leaves) :

$$\begin{aligned}
 & \frac{4 i \sqrt{2} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]-\sqrt{2} \sin\left[\frac{1}{4}(c+d x)\right]}{-\cos\left[\frac{1}{4}(c+d x)\right]+\sqrt{2} \cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]}\right] \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2}}{d(a(1+\operatorname{Sec}[c+d x]))^{5/2}} + \\
 & \left((4+4 i)\left((-1-i)+\sqrt{2}\right) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right]+\sin\left[\frac{1}{4}(c+d x)\right]-\sqrt{2} \sin\left[\frac{1}{4}(c+d x)\right]}{\cos\left[\frac{1}{4}(c+d x)\right]+\sqrt{2} \cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]}\right] \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2} \right) / \\
 & \left(\left((-1+i)+\sqrt{2} \right) d(a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) + \frac{43 \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^{5/2}}{4 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}} - \\
 & \frac{43 \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+d x)\right]+\sin\left[\frac{1}{4}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^{5/2}}{4 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}} + \\
 & \frac{4 \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\sqrt{2}+2 \sin\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^{5/2}}{d(a(1+\operatorname{Sec}[c+d x]))^{5/2}} - \\
 & \frac{2 \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[2-\sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \sin\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^{5/2}}{d(a(1+\operatorname{Sec}[c+d x]))^{5/2}} + \\
 & \left((2-2 i)\left((-1-i)+\sqrt{2}\right) \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[2+\sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \sin\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^{5/2} \right) / \\
 & \left(\left((-1+i)+\sqrt{2} \right) d(a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) - \frac{\cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}\left(\cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]\right)^4} - \\
 & \frac{11 \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}\left(\cos\left[\frac{1}{4}(c+d x)\right]-\sin\left[\frac{1}{4}(c+d x)\right]\right)^2} + \frac{\cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}\left(\cos\left[\frac{1}{4}(c+d x)\right]+\sin\left[\frac{1}{4}(c+d x)\right]\right)^4} + \\
 & \frac{11 \cos\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x]^{5/2}}{8 d(a(1+\operatorname{Sec}[c+d x]))^{5/2}\left(\cos\left[\frac{1}{4}(c+d x)\right]+\sin\left[\frac{1}{4}(c+d x)\right]\right)^2}
 \end{aligned}$$

■ **Problem 265: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{7/2}}{\sqrt{1+\operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 126 leaves, 7 steps) :

$$-\frac{\sqrt{2} \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[c+d x]}{1+\operatorname{Sec}[c+d x]}\right]}{d} + \frac{7 \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[c+d x]}{\sqrt{1+\operatorname{Sec}[c+d x]}}\right]}{4 d} - \frac{\operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{4 d \sqrt{1+\operatorname{Sec}[c+d x]}} + \frac{\operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{2 d \sqrt{1+\operatorname{Sec}[c+d x]}}$$

Result (type 3, 542 leaves) :

$$\frac{1}{32 d} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\left(-28 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-(-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right]-28 i \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\left(1+\sqrt{2}\right) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{\left(-1+\sqrt{2}\right) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right]\right)+$$

$$\operatorname{Sec}[c+d x]^2\left(16 \sqrt{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right]-16 \sqrt{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right]\right)+$$

$$14 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-7 \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]+\operatorname{Cos}[2(c+d x)]$$

$$\left(16 \sqrt{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right]-16 \sqrt{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right]+14 \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-$$

$$7 \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-7 \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right)-$$

$$7 \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]+20 \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]-4 \sqrt{2} \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right]\right)$$

■ **Problem 266: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2}}{\sqrt{1+\operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 85 leaves, 6 steps) :

$$\frac{\sqrt{2} \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[c+d x]}{1+\operatorname{Sec}[c+d x]}\right]}{d}-\frac{\operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[c+d x]}{\sqrt{1+\operatorname{Sec}[c+d x]}}\right]}{d}+\frac{\operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{d \sqrt{1+\operatorname{Sec}[c+d x]}}$$

Result (type 3, 349 leaves) :

$$\frac{1}{4d} \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec\left[\frac{1}{2}(c+dx)\right]^2} \left(2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - 4\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] + 4\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] - 2 \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] + 4\sqrt{2} \operatorname{Sec}[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 267: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2}}{\sqrt{1+\sec[c+dx]}} dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$-\frac{\sqrt{2} \operatorname{ArcSinh}\left[\frac{\tan[c+dx]}{1+\sec[c+dx]}\right]}{d} + \frac{2 \operatorname{ArcSinh}\left[\frac{\tan[c+dx]}{\sqrt{1+\sec[c+dx]}}\right]}{d}$$

Result (type 3, 384 leaves):

$$\frac{1}{((-1+i)+\sqrt{2})d} \left(\frac{1}{2} + \frac{i}{2}\right) \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1+\cos[c+dx]}} \left(2 \left((-1-i) + \sqrt{2} \right) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] + 2 \left((-1-i) + \sqrt{2} \right) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2})\sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2})\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - i \left((-4 + (2+2i)\sqrt{2}) \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] + (4 - (2+2i)\sqrt{2}) \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] - \left((-1-i) + \sqrt{2} \right) \left(2 \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \right)$$

■ **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]}}{\sqrt{1+\sec[c+dx]}} dx$$

Optimal (type 3, 27 leaves, 2 steps) :

$$\frac{\sqrt{2} \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[c+dx]}{1+\operatorname{Sec}[c+dx]}\right]}{d}$$

Result (type 3, 78 leaves) :

$$\frac{1}{d} 2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1+\operatorname{Cos}[c+dx]}} \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] \right)$$

■ **Problem 272: Result unnecessarily involves higher level functions.**

$$\int (e \operatorname{Sec}[c+dx])^{4/3} \sqrt{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 325 leaves, 4 steps) :

$$\frac{6 a e (e \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{5 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\left(4 \times 3^{3/4} \sqrt{2+\sqrt{3}} a^2 e \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3}) e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3}}\right], -7-4\sqrt{3}\right] (e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3}) \right.$$

$$\left. \frac{\sqrt{e^{2/3} + e^{1/3} (e \operatorname{Sec}[c+dx])^{1/3} + (e \operatorname{Sec}[c+dx])^{2/3}} \operatorname{Tan}[c+dx]}{\left((1+\sqrt{3}) e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3}\right)^2} \right) /$$

$$\left(5 d (a - a \operatorname{Sec}[c+dx]) \sqrt{a+a \operatorname{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \operatorname{Sec}[c+dx])^{1/3}\right)^2}} \right)$$

Result (type 5, 160 leaves) :

$$-\frac{1}{10 d (1+e^{i(c+dx)})} 3 i e \left(-4 + 4 e^{i(c+dx)} + 4 (1+e^{2i(c+dx)})^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{5}{6}, \frac{7}{6}, -e^{2i(c+dx)}\right] + \right. \\ \left. e^{i(c+dx)} (1+e^{2i(c+dx)})^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{5}{6}, \frac{5}{3}, -e^{2i(c+dx)}\right] \right) (e \operatorname{Sec}[c+dx])^{1/3} \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

■ **Problem 273: Result unnecessarily involves higher level functions.**

$$\int (e \operatorname{Sec}[c+dx])^{1/3} \sqrt{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 280 leaves, 3 steps) :

$$\left(2 \times 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}{(1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}\right], -7 - 4 \sqrt{3}\right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c + d x])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c + d x])^{1/3} + (e \text{Sec}[c + d x])^{2/3}}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}\right)^2}} \text{Tan}[c + d x] \right) / \\ \left(d (a - a \text{Sec}[c + d x]) \sqrt{a + a \text{Sec}[c + d x]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c + d x])^{1/3})}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}\right)^2}} \right)$$

Result (type 5, 153 leaves):

$$-\frac{1}{2 \times 2^{2/3} d (1 + e^{i(c+dx)})} 3 i \left(\frac{e^{e^{i(c+dx)}}}{1 + e^{2i(c+dx)}} \right)^{1/3} (1 + e^{2i(c+dx)})^{5/6} \\ \left(4 \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{5}{6}, \frac{7}{6}, -e^{2i(c+dx)}\right] + e^{i(c+dx)} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{5}{6}, \frac{5}{3}, -e^{2i(c+dx)}\right] \right) \sqrt{a(1 + \text{Sec}[c + d x])}$$

■ **Problem 274: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \text{Sec}[c + d x]} dx}{(e \text{Sec}[c + d x])^{2/3}}$$

Optimal (type 4, 326 leaves, 4 steps):

$$\frac{3 a \text{Tan}[c + d x]}{2 d (e \text{Sec}[c + d x])^{2/3} \sqrt{a + a \text{Sec}[c + d x]}} + \\ \left(3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 - \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}{(1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}\right], -7 - 4 \sqrt{3}\right] (e^{1/3} - (e \text{Sec}[c + d x])^{1/3}) \right. \\ \left. \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c + d x])^{1/3} + (e \text{Sec}[c + d x])^{2/3}}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}\right)^2}} \text{Tan}[c + d x] \right) / \\ \left(2 d e (a - a \text{Sec}[c + d x]) \sqrt{a + a \text{Sec}[c + d x]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c + d x])^{1/3})}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}\right)^2}} \right)$$

Result (type 5, 231 leaves):

$$\frac{1}{16 \times 2^{2/3} d e} 3 e^{-2 i (c+d x)} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{1/3} \left(4 \left(-1 + e^{i (c+d x)} - e^{2 i (c+d x)} + e^{3 i (c+d x)} \right) + 4 e^{i (c+d x)} \left(1 + e^{2 i (c+d x)} \right)^{5/6} \text{Hypergeometric2F1} \left[\frac{1}{6}, \frac{5}{6}, \frac{7}{6}, -e^{2 i (c+d x)} \right] + e^{2 i (c+d x)} \left(1 + e^{2 i (c+d x)} \right)^{5/6} \text{Hypergeometric2F1} \left[\frac{2}{3}, \frac{5}{6}, \frac{5}{3}, -e^{2 i (c+d x)} \right] \right) \sqrt{a (1 + \text{Sec}[c + d x])} \left(-i + \text{Tan} \left[\frac{1}{2} (c + d x) \right] \right)$$

■ **Problem 275: Result unnecessarily involves higher level functions.**

$$\int (e \text{Sec}[c + d x])^{8/3} \sqrt{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 716 leaves, 7 steps):

$$\frac{60 a e^2 (e \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x]}{91 d \sqrt{a + a \text{Sec}[c + d x]}} + \frac{6 a e (e \text{Sec}[c + d x])^{5/3} \text{Tan}[c + d x]}{13 d \sqrt{a + a \text{Sec}[c + d x]}} - \frac{240 a e^3 \text{Tan}[c + d x]}{91 d \sqrt{a + a \text{Sec}[c + d x]} \left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3} \right)} + \left(120 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^2 e^{7/3} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}{(1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c + d x])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c + d x])^{1/3} + (e \text{Sec}[c + d x])^{2/3}}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3} \right)^2}} \text{Tan}[c + d x]} \right) / \\ \left(91 d (a - a \text{Sec}[c + d x]) \sqrt{a + a \text{Sec}[c + d x]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c + d x])^{1/3})}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3} \right)^2}} \right) - \\ \left(80 \sqrt{2} 3^{3/4} a^2 e^{7/3} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}}{(1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c + d x])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c + d x])^{1/3} + (e \text{Sec}[c + d x])^{2/3}}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3} \right)^2}} \text{Tan}[c + d x]} \right) / \\ \left(91 d (a - a \text{Sec}[c + d x]) \sqrt{a + a \text{Sec}[c + d x]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c + d x])^{1/3})}{\left((1 + \sqrt{3}) e^{1/3} - (e \text{Sec}[c + d x])^{1/3} \right)^2}} \right)$$

Result (type 5, 221 leaves):

$$\frac{1}{91 d} 3^i e^{-\frac{5}{2} i (c+dx)} \left(10 - 5 e^{i(c+dx)} + 22 e^{2i(c+dx)} - 22 e^{3i(c+dx)} + 5 e^{4i(c+dx)} - 10 e^{5i(c+dx)} - 10 (1 + e^{2i(c+dx)})^{13/6} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, -e^{2i(c+dx)}\right] + 5 e^{i(c+dx)} (1 + e^{2i(c+dx)})^{13/6} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{4}{3}, -e^{2i(c+dx)}\right] \right) \text{Sec}\left[\frac{1}{2}(c+dx)\right] (e \text{Sec}[c+dx])^{5/3} \sqrt{a(1+\text{Sec}[c+dx])}$$

■ **Problem 276: Result unnecessarily involves higher level functions.**

$$\int (e \text{Sec}[c+dx])^{5/3} \sqrt{a+a \text{Sec}[c+dx]} dx$$

Optimal (type 4, 673 leaves, 6 steps):

$$\frac{6 a e (e \text{Sec}[c+dx])^{2/3} \text{Tan}[c+dx]}{7 d \sqrt{a+a \text{Sec}[c+dx]}} - \frac{24 a e^2 \text{Tan}[c+dx]}{7 d \sqrt{a+a \text{Sec}[c+dx]} \left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)} + \left(12 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^2 e^{4/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}\right], -7-4\sqrt{3}\right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ \left(7 d (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right) - \\ \left(8 \sqrt{2} 3^{3/4} a^2 e^{4/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}\right], -7-4\sqrt{3}\right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ \left(7 d (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right)$$

Result (type 5, 192 leaves):

$$\frac{1}{7d} 3 i e^{-\frac{3}{2} i (c+dx)} \left(2 - e^{i(c+dx)} + e^{2i(c+dx)} - 2 e^{3i(c+dx)} - 2 (1 + e^{2i(c+dx)})^{7/6} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, -e^{2i(c+dx)}\right] + e^{i(c+dx)} (1 + e^{2i(c+dx)})^{7/6} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{4}{3}, -e^{2i(c+dx)}\right] \right) \text{Sec}\left[\frac{1}{2}(c+dx)\right] (e \text{Sec}[c+dx])^{2/3} \sqrt{a(1 + \text{Sec}[c+dx])}$$

■ **Problem 277: Result unnecessarily involves higher level functions.**

$$\int (e \text{Sec}[c+dx])^{2/3} \sqrt{a+a \text{Sec}[c+dx]} dx$$

Optimal (type 4, 624 leaves, 5 steps):

$$\begin{aligned} & -\frac{6 a e \text{Tan}[c+dx]}{d \sqrt{a+a \text{Sec}[c+dx]} \left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)} + \\ & \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^2 e^{1/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}\right], -7-4\sqrt{3}\right] \right. \\ & \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ & \left(d (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right) - \\ & \left(2 \sqrt{2} 3^{3/4} a^2 e^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}\right], -7-4\sqrt{3}\right] \right. \\ & \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ & \left(d (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 158 leaves):

$$\left(3 i e \left(2 - 2 e^{i(c+dx)} - 2 \left(1 + e^{2i(c+dx)} \right)^{1/6} \text{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, -e^{2i(c+dx)} \right] + \right. \right. \\ \left. \left. e^{i(c+dx)} \left(1 + e^{2i(c+dx)} \right)^{1/6} \text{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) \sqrt{a(1+\text{Sec}[c+dx])} \right) / \left(d \left(1 + e^{i(c+dx)} \right) \left(e \text{Sec}[c+dx] \right)^{1/3} \right)$$

■ **Problem 278: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a \text{Sec}[c+dx]}}{(e \text{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 4, 662 leaves, 6 steps):

$$\frac{3 a \text{Tan}[c+dx]}{d (e \text{Sec}[c+dx])^{1/3} \sqrt{a+a \text{Sec}[c+dx]}} + \frac{3 a \text{Tan}[c+dx]}{d \sqrt{a+a \text{Sec}[c+dx]} \left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)} - \\ \left(3 \times 3^{1/4} \sqrt{2-\sqrt{3}} a^2 \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}} \right], -7-4\sqrt{3} \right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ \left(2 d e^{2/3} (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right) + \\ \left(\sqrt{2} 3^{3/4} a^2 \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1-\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}}{(1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3}} \right], -7-4\sqrt{3} \right] \right. \\ \left. (e^{1/3} - (e \text{Sec}[c+dx])^{1/3}) \sqrt{\frac{e^{2/3} + e^{1/3} (e \text{Sec}[c+dx])^{1/3} + (e \text{Sec}[c+dx])^{2/3}}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \text{Tan}[c+dx]} \right) / \\ \left(d e^{2/3} (a - a \text{Sec}[c+dx]) \sqrt{a+a \text{Sec}[c+dx]} \sqrt{\frac{e^{1/3} (e^{1/3} - (e \text{Sec}[c+dx])^{1/3})}{\left((1+\sqrt{3}) e^{1/3} - (e \text{Sec}[c+dx])^{1/3} \right)^2}} \right)$$

Result (type 5, 153 leaves):

$$- \left(3 i \left(1 + e^{2 i (c+d x)} \right)^{1/6} \left(-2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, -e^{2 i (c+d x)} \right] + e^{i (c+d x)} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{4}{3}, -e^{2 i (c+d x)} \right] \right) \right. \\ \left. \sqrt{a \left(1 + \operatorname{Sec}[c+d x] \right)} \right) / \left(2 \times 2^{1/3} d \left(1 + e^{i (c+d x)} \right) \left(\frac{e^{e^{i (c+d x)}}}{1 + e^{2 i (c+d x)}} \right)^{1/3} \right)$$

■ **Problem 279: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c+d x]}}{\left(e \operatorname{Sec}[c+d x] \right)^{4/3}} dx$$

Optimal (type 4, 715 leaves, 7 steps):

$$\frac{3 a \operatorname{Tan}[c+d x]}{4 d \left(e \operatorname{Sec}[c+d x] \right)^{4/3} \sqrt{a + a \operatorname{Sec}[c+d x]}} + \frac{15 a \operatorname{Tan}[c+d x]}{8 d e \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \sqrt{a + a \operatorname{Sec}[c+d x]}} + \frac{15 a \operatorname{Tan}[c+d x]}{8 d e \sqrt{a + a \operatorname{Sec}[c+d x]} \left(\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)} - \left(15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} a^2 \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\left(1 - \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3}}{\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right. \\ \left. \left(e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right) \sqrt{\frac{e^{2/3} + e^{1/3} \left(e \operatorname{Sec}[c+d x] \right)^{1/3} + \left(e \operatorname{Sec}[c+d x] \right)^{2/3}}{\left(\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)^2}} \operatorname{Tan}[c+d x]} \right) / \left(16 d e^{5/3} \left(a - a \operatorname{Sec}[c+d x] \right) \sqrt{a + a \operatorname{Sec}[c+d x]} \sqrt{\frac{e^{1/3} \left(e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)}{\left(\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)^2}} \right) + \left(5 \times 3^{3/4} a^2 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(1 - \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3}}{\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3}} \right], -7 - 4 \sqrt{3} \right] \right. \\ \left. \left(e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right) \sqrt{\frac{e^{2/3} + e^{1/3} \left(e \operatorname{Sec}[c+d x] \right)^{1/3} + \left(e \operatorname{Sec}[c+d x] \right)^{2/3}}{\left(\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)^2}} \operatorname{Tan}[c+d x]} \right) / \left(4 \sqrt{2} d e^{5/3} \left(a - a \operatorname{Sec}[c+d x] \right) \sqrt{a + a \operatorname{Sec}[c+d x]} \sqrt{\frac{e^{1/3} \left(e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)}{\left(\left(1 + \sqrt{3} \right) e^{1/3} - \left(e \operatorname{Sec}[c+d x] \right)^{1/3} \right)^2}} \right)$$

Result (type 5, 248 leaves) :

$$\left(3 e^{-2i(c+dx)} \left(2 \left(-1 + e^{i(c+dx)} - e^{2i(c+dx)} + e^{3i(c+dx)} \right) - 10 e^{i(c+dx)} \left(1 + e^{2i(c+dx)} \right)^{1/6} \text{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{6}, \frac{5}{6}, -e^{2i(c+dx)} \right] + \right. \right. \\ \left. \left. 5 e^{2i(c+dx)} \left(1 + e^{2i(c+dx)} \right)^{1/6} \text{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) \sqrt{a(1+\text{Sec}[c+dx])} \left(-i + \text{Tan} \left[\frac{1}{2}(c+dx) \right] \right) \right) / \\ (16 \times 2^{5/6} d \left(e^{-i(c+dx)} \left(1 + e^{2i(c+dx)} \right) \right)^{1/6} \text{Cos}[c+dx]^{5/6} \left(e \text{Sec}[c+dx] \right)^{4/3}$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \text{Sec}[c+dx])^{2/3}}{\sqrt{a+a \text{Sec}[c+dx]}} dx$$

Optimal (type 6, 78 leaves, 4 steps) :

$$\frac{3 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \text{Sec}[c+dx], -\text{Sec}[c+dx] \right] (e \text{Sec}[c+dx])^{2/3} \text{Tan}[c+dx]}{2 d \sqrt{1-\text{Sec}[c+dx]} \sqrt{a+a \text{Sec}[c+dx]}}$$

Result (type 6, 1975 leaves) :

$$\left(9 \times 2^{5/6} \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] (e \text{Sec}[c+dx])^{2/3} \left(1 + \text{Sec}[c+dx] \right)^{1/6} \text{Tan} \left[\frac{1}{2}(c+dx) \right] \right) / \\ \left(d \left(\text{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right)^{1/3} \sqrt{a(1+\text{Sec}[c+dx])} \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \right. \right. \\ \left. \left(-2 \text{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, \frac{4}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \text{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, \frac{1}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right) \right. \\ \left. \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right) \left(\left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \left(\text{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right)^{2/3} \left(1 + \text{Sec}[c+dx] \right)^{1/6} \right) \right) / \\ \left(2^{1/6} \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \left(-2 \text{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, \frac{4}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \text{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, \frac{1}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right) \right) - \\ \left(3 \times 2^{5/6} \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \left(1 + \text{Sec}[c+dx] \right)^{1/6} \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right) / \\ \left(\left(\text{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right)^{1/3} \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \left(-2 \text{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, \frac{4}{3}, \frac{5}{2}, \right. \right. \right. \right. \\ \left. \left. \left. \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] + \text{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, \frac{1}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right) \right) \right) + \\ \left(9 \times 2^{5/6} \left(1 + \text{Sec}[c+dx] \right)^{1/6} \text{Tan} \left[\frac{1}{2}(c+dx) \right] \left(-\frac{1}{9} \text{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, \frac{4}{3}, \frac{5}{2}, \text{Tan} \left[\frac{1}{2}(c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \text{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right. \right.$$

$$\int \frac{(e \operatorname{Sec}[c + d x])^{1/3}}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 6, 76 leaves, 4 steps):

$$\frac{3 \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \operatorname{Sec}[c + d x], -\operatorname{Sec}[c + d x]\right] (e \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{d \sqrt{1 - \operatorname{Sec}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 6, 749 leaves):

$$\begin{aligned} & \left(720 e \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \right. \\ & \quad \left. (1 + \operatorname{Cos}[c + d x])^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - \right. \right. \\ & \quad \left. \left. \left(4 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, \frac{5}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \right. \right. \\ & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) / \left(d (e \operatorname{Sec}[c + d x])^{2/3} \sqrt{a (1 + \operatorname{Sec}[c + d x])} \right) \\ & \quad \left(4320 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 (-1 + 4 \operatorname{Cos}[c + d x]) + \right. \\ & \quad \left. 160 \left(4 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, \frac{5}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right)^2 \right. \\ & \quad \left. \operatorname{Cos}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^4 + 12 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^2 \right. \\ & \quad \left. \left(20 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{6}, \frac{5}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] (7 + 14 \operatorname{Cos}[c + d x] + 5 \operatorname{Cos}[2(c + d x)] - 2 \operatorname{Cos}[3(c + d x)]) + \right. \right. \\ & \quad \left. \left. 5 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] (7 + 14 \operatorname{Cos}[c + d x] + 5 \operatorname{Cos}[2(c + d x)] - 2 \operatorname{Cos}[3(c + d x)]) - \right. \right. \\ & \quad \left. \left. 24 \left(40 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{6}, \frac{8}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + 8 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{6}, \frac{5}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] - 5 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{6}, \frac{2}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) \operatorname{Cos}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) \right) \end{aligned}$$

■ **Problem 282: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \operatorname{Sec}[c + d x])^{1/3} \sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 6, 76 leaves, 4 steps):

$$\frac{3 \operatorname{AppellF1}\left[-\frac{1}{3}, \frac{1}{2}, 1, \frac{2}{3}, \operatorname{Sec}[c+dx], -\operatorname{Sec}[c+dx]\right] \operatorname{Tan}[c+dx]}{d \sqrt{1-\operatorname{Sec}[c+dx]} (e \operatorname{Sec}[c+dx])^{1/3} \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 6, 5151 leaves): Display of huge result suppressed!

■ **Problem 283: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \operatorname{Sec}[c+dx])^{2/3} \sqrt{a+a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 6, 78 leaves, 4 steps):

$$\frac{3 \operatorname{AppellF1}\left[-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, \operatorname{Sec}[c+dx], -\operatorname{Sec}[c+dx]\right] \operatorname{Tan}[c+dx]}{2 d \sqrt{1-\operatorname{Sec}[c+dx]} (e \operatorname{Sec}[c+dx])^{2/3} \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 6, 2819 leaves):

$$\begin{aligned} & \frac{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx]^2 \left(-\frac{3}{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{3}{2} \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{d (e \operatorname{Sec}[c+dx])^{2/3} \sqrt{a(1+\operatorname{Sec}[c+dx])}} + \\ & \left(\left(-\frac{3}{4 \operatorname{Sec}[c+dx]^{1/6}} + \frac{1}{4} \operatorname{Sec}[c+dx]^{5/6} \right) \operatorname{Sec}[c+dx]^{7/6} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \right)^{5/6} \right. \\ & \quad \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-\operatorname{Cos}[c+dx]^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\ & \quad \left(10 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\ & \quad \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(-4 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{6}, \frac{5}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 5 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{6}, \frac{2}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) / \\ & \left(d \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{2/3} (e \operatorname{Sec}[c+dx])^{2/3} \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(\frac{1}{2} \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{1/3} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \right)^{5/6} \right. \right. \right. \\ & \quad \left(-\operatorname{Cos}[c+dx]^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\ & \quad \left(10 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \\ & \quad \left(15 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{6}, \frac{2}{3}, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(-4 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{6}, \frac{5}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 5 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{6}, \frac{2}{3}, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \left. \right) - \end{aligned}$$

$$\frac{2^{5/6} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}, \frac{3}{2}, 1 - \text{Sec}[c + d x], \frac{1}{2} (1 - \text{Sec}[c + d x])\right] (a + a \text{Sec}[c + d x])^{1/3} \text{Tan}[c + d x]}{d (1 + \text{Sec}[c + d x])^{5/6}}$$

Result (type 6, 1982 leaves):

$$\frac{3 \text{Sec}[c + d x]^{1/3} ((1 + \text{Cos}[c + d x]) \text{Sec}[c + d x])^{1/3} (a (1 + \text{Sec}[c + d x]))^{1/3} \text{Sin}[c + d x]}{2 d (1 + \text{Sec}[c + d x])^{1/3}} +$$

$$\left(3 (a (1 + \text{Sec}[c + d x]))^{1/3} \left(-\frac{(1 + \text{Sec}[c + d x])^{1/3}}{\text{Sec}[c + d x]^{2/3}} + \frac{1}{2} \text{Sec}[c + d x]^{1/3} (1 + \text{Sec}[c + d x])^{1/3} \right) \right.$$

$$\text{Tan}\left[\frac{1}{2} (c + d x)\right] \left(-1 + \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) \right) /$$

$$\left(9 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] - 2 \left(2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, \frac{5}{3}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right.$$

$$\left. \left. -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \text{HypergeometricPFQ}\left[\left\{\frac{2}{3}, \frac{3}{4}\right\}, \left\{\frac{7}{4}\right\}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4\right] \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) /$$

$$\left(2^{2/3} d \left(\text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)^{2/3} \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}[c + d x] \right)^{1/3} (1 + \text{Sec}[c + d x])^{1/3} \right.$$

$$\left. \left(3 \left(\text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)^{1/3} \left(-1 + \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) \right) / \left(9 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \right. \right.$$

$$\left. \left. \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] - 2 \left(2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, \frac{5}{3}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \right. \right.$$

$$\left. \left. \text{HypergeometricPFQ}\left[\left\{\frac{2}{3}, \frac{3}{4}\right\}, \left\{\frac{7}{4}\right\}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4\right] \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) / \left(2 \times 2^{2/3} \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}[c + d x] \right)^{1/3} \right) -$$

$$\left(2^{1/3} \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \left(-1 + \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) \right) /$$

$$\left(9 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] - 2 \left(2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, \frac{5}{3}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right.$$

$$\left. \left. -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \text{HypergeometricPFQ}\left[\left\{\frac{2}{3}, \frac{3}{4}\right\}, \left\{\frac{7}{4}\right\}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^4\right] \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) /$$

$$\left(\left(\text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)^{2/3} \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}[c + d x] \right)^{1/3} \right) + \frac{1}{2^{2/3} \left(\text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)^{2/3} \left(\text{Cos}\left[\frac{1}{2} (c + d x)\right]^2 \text{Sec}[c + d x] \right)^{1/3}}$$

$$3 \text{Tan}\left[\frac{1}{2} (c + d x)\right] \left(3 \left(-\frac{2}{9} \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, \frac{5}{3}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \text{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right. \right.$$

$$\left(-\cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \tan[c+dx] \right) / \left(2^{2/3} \left(\sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{2/3} \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \right)^{4/3} \right)$$

- **Problem 285: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^{4/3} (a + a \sec[c+dx])^{2/3} dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{2 \times 2^{1/6} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \sec[c+dx], \frac{1}{2}(1 - \sec[c+dx])\right] (a + a \sec[c+dx])^{2/3} \tan[c+dx]}{d (1 + \sec[c+dx])^{7/6}}$$

Result (type 6, 3726 leaves):

$$\frac{\sec[c+dx]^{1/3} ((1 + \cos[c+dx]) \sec[c+dx])^{2/3} (a (1 + \sec[c+dx]))^{2/3} \tan\left[\frac{1}{2}(c+dx)\right]}{d (1 + \sec[c+dx])^{2/3}} +$$

$$\left(15 \times 2^{2/3} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right] (a (1 + \sec[c+dx]))^{2/3} \right.$$

$$\left. \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] - 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, \frac{4}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \right. \right.$$

$$\left. \left. \tan\left[\frac{1}{4}(c+dx)\right]^2 + 4 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, \frac{1}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \tan\left[\frac{1}{4}(c+dx)\right]^2 \right) \tan\left[\frac{1}{2}(c+dx)\right] \right) /$$

$$\left(d \sec[c+dx]^{2/3} \left(9 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] - \right. \right.$$

$$\left. 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, \frac{4}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] - 2 \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, \frac{1}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \right) \right.$$

$$\left. \tan\left[\frac{1}{4}(c+dx)\right]^2 \right) \left(405 \times 2^{2/3} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right]^2 \cos\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$90 \times 2^{2/3} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, \frac{4}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \right.$$

$$\left. \sec\left[\frac{1}{4}(c+dx)\right]^2 \sin\left[\frac{1}{2}(c+dx)\right] \tan\left[\frac{1}{4}(c+dx)\right] - 180 \times 2^{2/3} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, \frac{1}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \sec\left[\frac{1}{4}(c+dx)\right]^2 \sin\left[\frac{1}{2}(c+dx)\right] \tan\left[\frac{1}{4}(c+dx)\right] - \right.$$

$$90 \times 2^{2/3} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \operatorname{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, \frac{4}{3}, \frac{5}{2}, \tan\left[\frac{1}{4}(c+dx)\right]^2, -\tan\left[\frac{1}{4}(c+dx)\right]^2\right] \right)$$

$$\begin{aligned}
& \left. \frac{\left(\frac{1}{3} - \frac{i}{3}\right) \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, \frac{4}{3}, \frac{1-i}{1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} \right/ \\
& \left(\left(\frac{-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \left(\frac{i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \right) + \left(\text{AppellF1}\left[-\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{1-i}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \right. \\
& \left. \left(-\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right])}{2 \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2 \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} \right) \right) \Big/ \\
& \left(3 \left(\frac{-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{4/3} \left(\frac{i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \right) + \left(\text{AppellF1}\left[-\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{1-i}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \right. \\
& \left. \left(-\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (i + \text{Tan}\left[\frac{1}{2}(c+dx)\right])}{2 \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2 \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)} \right) \right) \Big/ \\
& \left(3 \left(\frac{-i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \left(\frac{i + \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)^{4/3} \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 287: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + dx])^{4/3}}{\text{Sec}[c + dx]^{1/3}} dx$$

Optimal (type 6, 80 leaves, 3 steps):

$$\frac{1}{d (1 + \text{Sec}[c + dx])^{5/6}} 2 \times 2^{5/6} a \text{AppellF1}\left[\frac{1}{2}, \frac{4}{3}, -\frac{5}{6}, \frac{3}{2}, 1 - \text{Sec}[c + dx], \frac{1}{2} (1 - \text{Sec}[c + dx])\right] (a + a \text{Sec}[c + dx])^{1/3} \text{Tan}[c + dx]$$

Result (type 6, 2325 leaves):

$$- \left(\left(3 (a (1 + \text{Sec}[c + dx]))^{4/3} \left(\frac{(1 + \text{Sec}[c + dx])^{1/3}}{\text{Sec}[c + dx]^{1/3}} + \text{Sec}[c + dx]^{2/3} (1 + \text{Sec}[c + dx])^{1/3} \right) \right) \right)$$

$$\begin{aligned}
& \left(-8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{\operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3} \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3}} \right. \\
& \operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \\
& \left. \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \right) / \left(4 \times 2^{2/3} d \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{1/3} (1+\operatorname{Sec}[c+dx])^{4/3}\right) \\
& \left(\left(\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(-8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{\operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3} \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3}} \right. \\
& \operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{1-i}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \\
& \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \right) / \left(4 \times 2^{2/3} \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{1/3} - \frac{1}{4 \times 2^{2/3} \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{1/3}} 3 \left(-4 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-\left(\frac{4}{3} - \frac{4i}{3}\right) \operatorname{AppellF1}\left[-\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right) \right) / \right. \\
& \left. \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 - \left(\left(\frac{4}{3} + \frac{4i}{3}\right) \operatorname{AppellF1}\left[-\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) / \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \right) / \left(\left(\frac{-i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3} \left(\frac{i+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{2/3}\right) + \\
& \left. \left(\operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1+i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right) / \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \right) - \text{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{1-i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) - \\
& \left(2 \text{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1+i}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2(-1 + \tan\left[\frac{1}{2}(c+dx)\right])} - \right. \right. \\
& \left. \left. \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2(-i + \tan\left[\frac{1}{2}(c+dx)\right])}{2(-1 + \tan\left[\frac{1}{2}(c+dx)\right])^2} \right) \right) / \left(3 \left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{5/3} \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \right) - \\
& \left(2 \text{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, -\frac{1+i}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]}, -\frac{1-i}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left(\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2(-1 + \tan\left[\frac{1}{2}(c+dx)\right])} - \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2(i + \tan\left[\frac{1}{2}(c+dx)\right])}{2(-1 + \tan\left[\frac{1}{2}(c+dx)\right])^2} \right) \right) / \\
& \left(3 \left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3} \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{-1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{5/3} \right) - \left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \\
& \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(\frac{\left(\frac{4}{3} + \frac{4i}{3}\right) \text{AppellF1}\left[-\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1-i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} + \right. \\
& \left. \frac{\left(\frac{4}{3} - \frac{4i}{3}\right) \text{AppellF1}\left[-\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}, \frac{1-i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} \right) - \frac{1}{3 \left(\frac{-i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3}} \\
& \text{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{1-i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1 + \tan\left[\frac{1}{2}(c+dx)\right]}\right] \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{1/3} \\
& \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \left(-\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2(-i + \tan\left[\frac{1}{2}(c+dx)\right])}{2\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)} \right) - \frac{1}{3 \left(\frac{i + \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]} \right)^{2/3}}
\end{aligned}$$

$$\text{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{1-i}{1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}, \frac{1+i}{1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right] \left(\frac{-i+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]}\right)^{1/3} \\ \left(\left(1+\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(-\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (i+\text{Tan}\left[\frac{1}{2}(c+dx)\right])}{2(1+\text{Tan}\left[\frac{1}{2}(c+dx)\right])^2} + \frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2(1+\text{Tan}\left[\frac{1}{2}(c+dx)\right])}\right)\right)^{1/3}$$

■ **Problem 288: Unable to integrate problem.**

$$\int \text{Sec}[e+fx]^n (a+a \text{Sec}[e+fx])^4 dx$$

Optimal (type 5, 304 leaves, 8 steps):

$$\frac{a^4 (30+21n+4n^2) \text{Sec}[e+fx]^{1+n} \text{Sin}[e+fx]}{f(1+n)(2+n)(3+n)} + \\ \frac{\text{Sec}[e+fx]^{1+n} (a^2+a^2 \text{Sec}[e+fx])^2 \text{Sin}[e+fx]}{f(3+n)} + \frac{2(4+n) \text{Sec}[e+fx]^{1+n} (a^4+a^4 \text{Sec}[e+fx]) \text{Sin}[e+fx]}{f(2+n)(3+n)} - \\ \frac{a^4 (3+24n+8n^2) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \text{Cos}[e+fx]^2\right] \text{Sec}[e+fx]^{-1+n} \text{Sin}[e+fx]}{f(1-n)(1+n)(3+n) \sqrt{\text{Sin}[e+fx]^2}} + \\ \frac{4a^4 (3+2n) \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \text{Cos}[e+fx]^2\right] \text{Sec}[e+fx]^n \text{Sin}[e+fx]}{fn(2+n) \sqrt{\text{Sin}[e+fx]^2}}$$

Result (type 8, 23 leaves):

$$\int \text{Sec}[e+fx]^n (a+a \text{Sec}[e+fx])^4 dx$$

■ **Problem 289: Unable to integrate problem.**

$$\int \text{Sec}[e+fx]^n (a+a \text{Sec}[e+fx])^3 dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$\frac{a^3 (5 + 2n) \operatorname{Sec}[e + f x]^{1+n} \operatorname{Sin}[e + f x]}{f (1+n) (2+n)} + \frac{\operatorname{Sec}[e + f x]^{1+n} (a^3 + a^3 \operatorname{Sec}[e + f x]) \operatorname{Sin}[e + f x]}{f (2+n)} -$$

$$\frac{a^3 (1 + 4n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^{-1+n} \operatorname{Sin}[e + f x]}{f (1-n^2) \sqrt{\operatorname{Sin}[e + f x]^2}} +$$

$$\frac{a^3 (7 + 4n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^n \operatorname{Sin}[e + f x]}{f n (2+n) \sqrt{\operatorname{Sin}[e + f x]^2}}$$

Result (type 8, 23 leaves):

$$\int \operatorname{Sec}[e + f x]^n (a + a \operatorname{Sec}[e + f x])^3 dx$$

■ **Problem 290: Unable to integrate problem.**

$$\int \operatorname{Sec}[e + f x]^n (a + a \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 5, 172 leaves, 6 steps):

$$\frac{a^2 \operatorname{Sec}[e + f x]^{1+n} \operatorname{Sin}[e + f x]}{f (1+n)} - \frac{a^2 (1 + 2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^{-1+n} \operatorname{Sin}[e + f x]}{f (1-n^2) \sqrt{\operatorname{Sin}[e + f x]^2}} +$$

$$\frac{2 a^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^n \operatorname{Sin}[e + f x]}{f n \sqrt{\operatorname{Sin}[e + f x]^2}}$$

Result (type 8, 23 leaves):

$$\int \operatorname{Sec}[e + f x]^n (a + a \operatorname{Sec}[e + f x])^2 dx$$

■ **Problem 291: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^n (a + a \operatorname{Sec}[e + f x]) dx$$

Optimal (type 5, 132 leaves, 5 steps):

$$\frac{a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^{-1+n} \operatorname{Sin}[e + f x]}{f (1-n) \sqrt{\operatorname{Sin}[e + f x]^2}} +$$

$$\frac{a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \operatorname{Cos}[e + f x]^2\right] \operatorname{Sec}[e + f x]^n \operatorname{Sin}[e + f x]}{f n \sqrt{\operatorname{Sin}[e + f x]^2}}$$

Result (type 6, 3990 leaves):

$$\frac{2}{3} \left(n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (1+n) \operatorname{AppellF1} \left[\frac{3}{2}, 2+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{2}{3} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \left(n \left(-\frac{3}{5} (1-n) \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{3}{5} (1+n) \operatorname{AppellF1} \left[\frac{5}{2}, 2+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + (1+n) \left(\frac{3}{5} n \operatorname{AppellF1} \left[\frac{5}{2}, 2+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{3}{5} (2+n) \operatorname{AppellF1} \left[\frac{5}{2}, 3+n, -n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \left(\operatorname{AppellF1} \left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \frac{2}{3} \left(n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (1+n) \operatorname{AppellF1} \left[\frac{3}{2}, 2+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right)$$

■ **Problem 292: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+fx]^n}{a+a \operatorname{Sec}[e+fx]} dx$$

Optimal (type 5, 174 leaves, 6 steps):

$$\frac{\operatorname{Sec}[e+fx]^n \operatorname{Sin}[e+fx]}{f(a+a \operatorname{Sec}[e+fx])} + \frac{(1-n) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \operatorname{Cos}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^{-2+n} \operatorname{Sin}[e+fx]}{a f (2-n) \sqrt{\operatorname{Sin}[e+fx]^2}} - \frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^{-1+n} \operatorname{Sin}[e+fx]}{a f \sqrt{\operatorname{Sin}[e+fx]^2}}$$

Result (type 8, 23 leaves):

$$\int \frac{\operatorname{Sec}[e+fx]^n}{a+a \operatorname{Sec}[e+fx]} dx$$

■ **Problem 293: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+fx]^n}{(a+a \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 5, 217 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2(2-n) \operatorname{Sec}[e+fx]^{1+n} \operatorname{Sin}[e+fx]}{3a^2 f (1+\operatorname{Sec}[e+fx])} - \frac{\operatorname{Sec}[e+fx]^{1+n} \operatorname{Sin}[e+fx]}{3f (a+a \operatorname{Sec}[e+fx])^2} - \\
& \frac{(3-2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[e+fx]^2\right] \operatorname{Sec}[e+fx]^{-1+n} \operatorname{Sin}[e+fx]}{3a^2 f \sqrt{\operatorname{Sin}[e+fx]^2}} + \\
& \frac{2(2-n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \operatorname{Cos}[e+fx]^2\right] \operatorname{Sec}[e+fx]^n \operatorname{Sin}[e+fx]}{3a^2 f \sqrt{\operatorname{Sin}[e+fx]^2}}
\end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{\operatorname{Sec}[e+fx]^n}{(a+a \operatorname{Sec}[e+fx])^2} dx$$

■ **Problem 294: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^n (1+\operatorname{Sec}[e+fx])^{5/2} dx$$

Optimal (type 5, 162 leaves, 4 steps):

$$\begin{aligned}
& \frac{2(7+4n) \operatorname{Sec}[e+fx]^{1+n} \operatorname{Sin}[e+fx]}{f(1+2n)(3+2n)\sqrt{1+\operatorname{Sec}[e+fx]}} + \frac{2 \operatorname{Sec}[e+fx]^{1+n} \sqrt{1+\operatorname{Sec}[e+fx]} \operatorname{Sin}[e+fx]}{f(3+2n)} + \\
& \frac{2(3+24n+16n^2) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1-\operatorname{Sec}[e+fx]\right] \operatorname{Tan}[e+fx]}{f(1+2n)(3+2n)\sqrt{1+\operatorname{Sec}[e+fx]}}
\end{aligned}$$

Result (type 5, 433 leaves):

$$\begin{aligned}
& - \frac{1}{f \operatorname{Sec}[e+fx]^{5/2}} i 2^{-\frac{5}{2}+n} e^{-i\left(\frac{1}{2}+n\right)(e+fx)} \left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}} \right)^{\frac{1}{2}+n} (1+e^{2i(e+fx)})^{\frac{1}{2}+n} \\
& \left(\frac{10 e^{i(2+n)(e+fx)} \operatorname{Hypergeometric2F1}\left[1+\frac{n}{2}, \frac{5}{2}+n, 2+\frac{n}{2}, -e^{2i(e+fx)}\right]}{2+n} + \frac{5 e^{i(4+n)(e+fx)} \operatorname{Hypergeometric2F1}\left[2+\frac{n}{2}, \frac{5}{2}+n, 3+\frac{n}{2}, -e^{2i(e+fx)}\right]}{4+n} + \right. \\
& \frac{e^{i n(e+fx)} \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{5}{2}+n, 1+\frac{n}{2}, -e^{2i(e+fx)}\right]}{n} + \frac{5 e^{i(1+n)(e+fx)} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{5}{2}+n, \frac{3+n}{2}, -e^{2i(e+fx)}\right]}{1+n} + \\
& \frac{10 e^{i(3+n)(e+fx)} \operatorname{Hypergeometric2F1}\left[\frac{5}{2}+n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2i(e+fx)}\right]}{3+n} + \\
& \left. \frac{e^{i(5+n)(e+fx)} \operatorname{Hypergeometric2F1}\left[\frac{5}{2}+n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2i(e+fx)}\right]}{5+n} \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^5 (1+\operatorname{Sec}[e+fx])^{5/2}
\end{aligned}$$

- **Problem 295: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^n (1 + \text{Sec}[e + f x])^{3/2} dx$$

Optimal (type 5, 98 leaves, 4 steps):

$$\frac{2 \text{Sec}[e + f x]^{1+n} \text{Sin}[e + f x]}{f (1 + 2n) \sqrt{1 + \text{Sec}[e + f x]}} + \frac{2 (1 + 4n) \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \text{Sec}[e + f x]\right] \text{Tan}[e + f x]}{f (1 + 2n) \sqrt{1 + \text{Sec}[e + f x]}}$$

Result (type 5, 6367 leaves): Display of huge result suppressed!

- **Problem 296: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^n \sqrt{1 + \text{Sec}[e + f x]} dx$$

Optimal (type 5, 45 leaves, 2 steps):

$$\frac{2 \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \text{Sec}[e + f x]\right] \text{Tan}[e + f x]}{f \sqrt{1 + \text{Sec}[e + f x]}}$$

Result (type 5, 211 leaves):

$$-\frac{1}{f n (1+n)} i 2^{-\frac{1}{2}+n} e^{\frac{1}{2} i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)^{-\frac{1}{2}+n} \left(e^{-i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)\right)^{\frac{1}{2}-n} \text{Cos}[e + f x]^{\frac{1}{2}+n}$$

$$\left((1+n) \text{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2} + n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f x)} n \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right)$$

$$\text{Sec}\left[\frac{1}{2} (e + f x)\right] \text{Sec}[e + f x]^n \sqrt{1 + \text{Sec}[e + f x]}$$

- **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]^n}{\sqrt{1 + \text{Sec}[e + f x]}} dx$$

Optimal (type 6, 59 leaves, 3 steps):

$$\frac{\text{AppellF1}\left[\frac{1}{2}, 1 - n, 1, \frac{3}{2}, 1 - \text{Sec}[e + f x], \frac{1}{2} (1 - \text{Sec}[e + f x])\right] \text{Tan}[e + f x]}{f \sqrt{1 + \text{Sec}[e + f x]}}$$

Result (type 6, 2938 leaves):

$$\left(3 \sqrt{2} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \right.$$

$$\left. \left(\text{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \right)^n \text{Sec}[e + f x]^{-\frac{1}{2} + \frac{1}{2} (-1+2n)} \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \text{Sec}[e + f x] \right)^n \text{Tan}\left[\frac{1}{2} (e + f x)\right] \right) /$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(3\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\
& \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^n \sqrt{1+\sec[e+fx]} \tan\left[\frac{1}{2}(e+fx)\right] \right) \\
& \left(\left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\right) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. 3\left(-\frac{1}{3}(1-n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3}\left(-\frac{1}{2}+n\right) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\
& \quad \left. \left. \left(2(-1+n) \left(-\frac{3}{5}(2-n) \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}+n, 3-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \frac{3}{5}\left(-\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+n, 2-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \\
& \quad \left. \left. (-1+2n) \left(-\frac{3}{5}(1-n) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+n, 2-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \frac{3}{5}\left(\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+n, 1-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \right) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 + \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^n \tan\left[\frac{1}{2}(e+fx)\right] \tan[e+fx] \right) / \\
& \left(\sqrt{2} \sqrt{1+\sec[e+fx]} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) +
\end{aligned}$$

$$\left(3 \sqrt{2} {}_n\text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sqrt{1+\sec[e+fx]} \tan\left[\frac{1}{2}(e+fx)\right] \right. \\ \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) / \\ \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ \left. \left(2(-1+n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-1+2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^n}{(1+\sec[e+fx])^{3/2}} dx$$

Optimal (type 6, 62 leaves, 3 steps):

$$\frac{\text{AppellF1}\left[\frac{1}{2}, 1-n, 2, \frac{3}{2}, 1-\sec[e+fx], \frac{1}{2}(1-\sec[e+fx])\right] \tan[e+fx]}{2f\sqrt{1+\sec[e+fx]}}$$

Result (type 6, 2990 leaves):

$$\left(6 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ \left. \sec[e+fx]^{\frac{1}{2}+\frac{1}{2}(-3+2n)} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{\frac{3}{2}+n} \tan\left[\frac{1}{2}(e+fx)\right] \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right) / \\ \left(f (1+\sec[e+fx])^{3/2} \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left(2(-1+n) \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-3+2n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \\ \left(\left(12 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \right. \right)$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \left(2 (-1 + n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2} + n, 2 - n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + (-3 + 2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 1 - n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 + \\
& \left(6 \left(\frac{3}{2} + n \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Cos} [e + f x] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \right)^{\frac{1}{2} + n} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 \right. \\
& \quad \left. \left(-\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] \operatorname{Sec} [e + f x] \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \operatorname{Tan} [e + f x] \right) \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left(2 (-1 + n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2} + n, 2 - n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-3 + 2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 1 - n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

- **Problem 299: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\operatorname{Sec} [e + f x])^n (1 + \operatorname{Sec} [e + f x])^{3/2} dx$$

Optimal (type 5, 117 leaves, 4 steps):

$$\frac{2 (-\operatorname{Sec} [e + f x])^n \operatorname{Tan} [e + f x]}{f (1 + 2n) \sqrt{1 + \operatorname{Sec} [e + f x]}} - \frac{(1 + 4n) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, n, 1 + n, \operatorname{Sec} [e + f x] \right] (-\operatorname{Sec} [e + f x])^n \operatorname{Tan} [e + f x]}{f n (1 + 2n) \sqrt{1 - \operatorname{Sec} [e + f x]} \sqrt{1 + \operatorname{Sec} [e + f x]}}$$

Result (type 5, 6381 leaves): Display of huge result suppressed!

- **Problem 300: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\operatorname{Sec} [e + f x])^n \sqrt{1 + \operatorname{Sec} [e + f x]} dx$$

Optimal (type 5, 64 leaves, 2 steps):

$$-\frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, n, 1 + n, \operatorname{Sec} [e + f x] \right] (-\operatorname{Sec} [e + f x])^n \operatorname{Tan} [e + f x]}{f n \sqrt{1 - \operatorname{Sec} [e + f x]} \sqrt{1 + \operatorname{Sec} [e + f x]}}$$

Result (type 5, 213 leaves):

$$\begin{aligned}
& - \frac{1}{f n (1+n)} i 2^{-\frac{1}{2}+n} e^{\frac{1}{2} i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)^{-\frac{1}{2}+n} \left(e^{-i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)\right)^{\frac{1}{2}-n} \operatorname{Cos}[e+f x]^{\frac{1}{2}+n} \\
& \left((1+n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] (-\operatorname{Sec}[e+f x])^n \sqrt{1+\operatorname{Sec}[e+f x]}
\end{aligned}$$

■ **Problem 301: Result more than twice size of optimal antiderivative.**

$$\int \frac{(-\operatorname{Sec}[e+f x])^n}{\sqrt{1+\operatorname{Sec}[e+f x]}} dx$$

Optimal (type 6, 73 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 1, 1+n, \operatorname{Sec}[e+f x], -\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f n \sqrt{1-\operatorname{Sec}[e+f x]} \sqrt{1+\operatorname{Sec}[e+f x]}}$$

Result (type 6, 2951 leaves):

$$\begin{aligned}
& \left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \\
& \left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n (-\operatorname{Sec}[e+f x])^n \operatorname{Sec}[e+f x]^{-\frac{1}{2}-n+\frac{1}{2}(-1+2n)} \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right) / \\
& \left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Cos}[e+f x] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^{1+n} \right. \right. \\
& \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \sqrt{1+\operatorname{Sec}[e+f x]}\right) / \left(\sqrt{2} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \right. \\
& \left. \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \right. \\
& \left. \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)\right) - \\
& \left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \right. \\
& \left. \sqrt{1+\operatorname{Sec}[e+f x]} \operatorname{Sin}[e+f x] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{5} \left(-\frac{1}{2} + n \right) \text{AppellF1} \left[\frac{5}{2}, \frac{1}{2} + n, 2 - n, \frac{7}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \text{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + \\
& (-1 + 2n) \left(-\frac{3}{5} (1 - n) \text{AppellF1} \left[\frac{5}{2}, \frac{1}{2} + n, 2 - n, \frac{7}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \text{Tan} \left[\frac{1}{2} (e + f x) \right] + \right. \\
& \left. \frac{3}{5} \left(\frac{1}{2} + n \right) \text{AppellF1} \left[\frac{5}{2}, \frac{3}{2} + n, 1 - n, \frac{7}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \text{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \Bigg) \Bigg) / \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 + \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \left. \left(\text{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \right)^n \text{Tan} \left[\frac{1}{2} (e + f x) \right] \text{Tan}[e + f x] \right) \Bigg) / \\
& \left(\sqrt{2} \sqrt{1 + \text{Sec}[e + f x]} \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
& \left. \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
& \left. \left. (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) + \\
& \left(3 \sqrt{2} n \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \text{Cos}[e + f x] \left(\text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \left(\text{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \right)^{-1+n} \sqrt{1 + \text{Sec}[e + f x]} \text{Tan} \left[\frac{1}{2} (e + f x) \right] \right. \\
& \left. \left. \left(-\text{Cos} \left[\frac{1}{2} (e + f x) \right] \text{Sec}[e + f x] \text{Sin} \left[\frac{1}{2} (e + f x) \right] + \text{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \text{Tan}[e + f x] \right) \right) \right) \Bigg) / \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \left. \left. (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 302: Result more than twice size of optimal antiderivative.**

$$\int \frac{(-\text{Sec}[e + f x])^n}{(1 + \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 6, 73 leaves, 2 steps):

$$\frac{\text{AppellF1}\left[n, \frac{1}{2}, 2, 1+n, \text{Sec}[e+fx], -\text{Sec}[e+fx]\right] (-\text{Sec}[e+fx])^n \text{Tan}[e+fx]}{f n \sqrt{1-\text{Sec}[e+fx]} \sqrt{1+\text{Sec}[e+fx]}}$$

Result (type 6, 3003 leaves):

$$\begin{aligned} & \left(6 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ & \quad \left. (-\text{Sec}[e+fx])^n \text{Sec}[e+fx]^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)} \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx]\right)^{\frac{3}{2}+n} \text{Tan}\left[\frac{1}{2}(e+fx)\right] \left(-1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2\right) / \\ & \left(f (1+\text{Sec}[e+fx])^{3/2} \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left(2(-1+n) \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-3+2n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\ & \left(\left(12 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Cos}[e+fx] \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \right. \\ & \quad \left. \left. \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx]\right)^{\frac{3}{2}+n} \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) \right) / \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(2(-1+n) \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (-3+2n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Cos}[e+fx] \left(\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \\ & \quad \left. \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx]\right)^{\frac{3}{2}+n} \left(-1+\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\ & \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(2(-1+n) \text{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (-3+2n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \end{aligned}$$

$$\begin{aligned}
& \left(\left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (-3+2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \right. \\
& \quad 3 \left(-\frac{1}{3}(1-n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \frac{1}{3} \left(-\frac{3}{2}+n \right) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) + \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \\
& \quad \left(2(-1+n) \left(-\frac{3}{5}(2-n) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{3}{2}+n, 3-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} \left(-\frac{3}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) \right) + \\
& \quad \left. (-3+2n) \left(-\frac{3}{5}(1-n) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} \left(-\frac{1}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) \right) \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + (-3+2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right)^2 + \\
& \left(6 \left(\frac{3}{2}+n \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Cos}[e+fx] \left(\operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \right)^n \right. \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^{\frac{1}{2}+n} \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \left(-1 + \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right)^2 \right. \\
& \quad \left. \left. \left(-\operatorname{Cos} \left[\frac{1}{2}(e+fx) \right] \operatorname{Sec}[e+fx] \operatorname{Sin} \left[\frac{1}{2}(e+fx) \right] + \operatorname{Cos} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \\
& \quad \left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-3+2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right) \Bigg)
\end{aligned}$$

■ **Problem 303: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\operatorname{d} \operatorname{Sec}[e + f x])^n (1 + \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 5, 117 leaves, 4 steps):

$$\frac{2 (\operatorname{d} \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{1 + \operatorname{Sec}[e + f x]}} - \frac{(1 + 4 n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, 1 + n, \operatorname{Sec}[e + f x]\right] (\operatorname{d} \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f n (1 + 2 n) \sqrt{1 - \operatorname{Sec}[e + f x]} \sqrt{1 + \operatorname{Sec}[e + f x]}}$$

Result (type 5, 6381 leaves): Display of huge result suppressed!

■ **Problem 304: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\operatorname{d} \operatorname{Sec}[e + f x])^n \sqrt{1 + \operatorname{Sec}[e + f x]} dx$$

Optimal (type 5, 64 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, 1 + n, \operatorname{Sec}[e + f x]\right] (\operatorname{d} \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f n \sqrt{1 - \operatorname{Sec}[e + f x]} \sqrt{1 + \operatorname{Sec}[e + f x]}}$$

Result (type 5, 213 leaves):

$$-\frac{1}{f n (1 + n)} i 2^{-\frac{1}{2} + n} e^{\frac{1}{2} i (e + f x)} (1 + e^{2 i (e + f x)})^{-\frac{1}{2} + n} (e^{-i (e + f x)} (1 + e^{2 i (e + f x)}))^{\frac{1}{2} - n} \operatorname{Cos}[e + f x]^{\frac{1}{2} + n} \\ \left((1 + n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2} + n, \frac{2 + n}{2}, -e^{2 i (e + f x)}\right] + e^{i (e + f x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1 + n}{2}, \frac{3 + n}{2}, -e^{2 i (e + f x)}\right] \right) \\ \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] (\operatorname{d} \operatorname{Sec}[e + f x])^n \sqrt{1 + \operatorname{Sec}[e + f x]}$$

■ **Problem 305: Result more than twice size of optimal antiderivative.**

$$\int \frac{(\operatorname{d} \operatorname{Sec}[e + f x])^n}{\sqrt{1 + \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 6, 73 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 1, 1 + n, \operatorname{Sec}[e + f x], -\operatorname{Sec}[e + f x]\right] (\operatorname{d} \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f n \sqrt{1 - \operatorname{Sec}[e + f x]} \sqrt{1 + \operatorname{Sec}[e + f x]}}$$

Result (type 6, 2951 leaves):

$$\left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \right. \\ \left. \left(\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \right)^n \operatorname{Sec}[e + f x]^{-\frac{1}{2} - n + \frac{1}{2} (-1 + 2n)} (\operatorname{d} \operatorname{Sec}[e + f x])^n \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x] \right)^n \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] \right) / \\ \left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] + \left(2 (-1 + n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \right. \right. \right. \right.$$

$$\left(\left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sqrt{1+\sec[e+fx]} \tan\left[\frac{1}{2}(e+fx)\right] \right. \\ \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) / \\ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)$$

■ **Problem 306: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \sec[e+fx])^n}{(1+\sec[e+fx])^{3/2}} dx$$

Optimal (type 6, 73 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 2, 1+n, \sec[e+fx], -\sec[e+fx]\right] (d \sec[e+fx])^n \tan[e+fx]}{f n \sqrt{1-\sec[e+fx]} \sqrt{1+\sec[e+fx]}}$$

Result (type 6, 3003 leaves):

$$\left(6 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ \left. \sec[e+fx]^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)} (d \sec[e+fx])^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^{\frac{3}{2}+n} \tan\left[\frac{1}{2}(e+fx)\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\ \left(f (1+\sec[e+fx])^{3/2} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\ \left. \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \right. \right. \\ \left. \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^{\frac{3}{2}+n} \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \right) \right) /$$

$$\begin{aligned}
& \left(6 \left(\frac{3}{2} + n \right) \text{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \cos [e + f x] \left(\sec \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \quad \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x] \right)^{\frac{1}{2} + n} \tan \left[\frac{1}{2} (e + f x) \right] \left(-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 \\
& \quad \left. \left(-\cos \left[\frac{1}{2} (e + f x) \right] \sec [e + f x] \sin \left[\frac{1}{2} (e + f x) \right] + \cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x] \tan [e + f x] \right) \right) / \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{3}{2} + n, 2 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-3 + 2 n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right)
\end{aligned}$$

- **Problem 307: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [e + f x]^n (a + a \sec [e + f x])^{5/2} dx$$

Optimal (type 5, 177 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 a^3 (7 + 4 n) \sec [e + f x]^{1+n} \sin [e + f x]}{f (1 + 2 n) (3 + 2 n) \sqrt{a + a \sec [e + f x]}} + \frac{2 a^2 \sec [e + f x]^{1+n} \sqrt{a + a \sec [e + f x]} \sin [e + f x]}{f (3 + 2 n)} + \\
& \frac{2 a^3 (3 + 24 n + 16 n^2) \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \sec [e + f x] \right] \tan [e + f x]}{f (1 + 2 n) (3 + 2 n) \sqrt{a + a \sec [e + f x]}}
\end{aligned}$$

Result (type 5, 435 leaves):

$$\begin{aligned}
& - \frac{1}{f \operatorname{Sec}[e + f x]^{5/2}} i 2^{-\frac{5}{2}+n} e^{-i \left(\frac{1}{2}+n\right) (e+f x)} \left(\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}} \right)^{\frac{1}{2}+n} \left(1 + e^{2 i (e+f x)} \right)^{\frac{1}{2}+n} \\
& \left(\frac{10 e^{i (2+n) (e+f x)} \operatorname{Hypergeometric2F1}\left[1 + \frac{n}{2}, \frac{5}{2} + n, 2 + \frac{n}{2}, -e^{2 i (e+f x)}\right]}{2 + n} + \frac{5 e^{i (4+n) (e+f x)} \operatorname{Hypergeometric2F1}\left[2 + \frac{n}{2}, \frac{5}{2} + n, 3 + \frac{n}{2}, -e^{2 i (e+f x)}\right]}{4 + n} \right. \\
& \frac{e^{i n (e+f x)} \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{5}{2} + n, 1 + \frac{n}{2}, -e^{2 i (e+f x)}\right]}{n} + \frac{5 e^{i (1+n) (e+f x)} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{5}{2} + n, \frac{3+n}{2}, -e^{2 i (e+f x)}\right]}{1 + n} + \\
& \frac{10 e^{i (3+n) (e+f x)} \operatorname{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2 i (e+f x)}\right]}{3 + n} + \\
& \left. \frac{e^{i (5+n) (e+f x)} \operatorname{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2 i (e+f x)}\right]}{5 + n} \right) \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^5 (a (1 + \operatorname{Sec}[e + f x]))^{5/2}
\end{aligned}$$

- **Problem 308: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^n (a + a \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 5, 108 leaves, 4 steps):

$$\frac{2 a^2 \operatorname{Sec}[e + f x]^{1+n} \operatorname{Sin}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 (1 + 4 n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \operatorname{Sec}[e + f x]\right] \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 5, 6369 leaves): Display of huge result suppressed!

- **Problem 309: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^n \sqrt{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{2 a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \operatorname{Sec}[e + f x]\right] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 5, 213 leaves):

$$\begin{aligned}
& - \frac{1}{f n (1 + n)} i 2^{-\frac{1}{2}+n} e^{\frac{1}{2} i (e+f x)} \left(1 + e^{2 i (e+f x)} \right)^{-\frac{1}{2}+n} \left(e^{-i (e+f x)} \left(1 + e^{2 i (e+f x)} \right) \right)^{\frac{1}{2}-n} \operatorname{Cos}[e + f x]^{\frac{1}{2}+n} \\
& \left((1 + n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2} + n, \frac{2 + n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1 + n}{2}, \frac{3 + n}{2}, -e^{2 i (e+f x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}[e + f x]^n \sqrt{a (1 + \operatorname{Sec}[e + f x])}
\end{aligned}$$

■ **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]^n}{\sqrt{a + a \text{Sec}[e + f x]}} dx$$

Optimal (type 6, 61 leaves, 4 steps) :

$$\frac{\text{AppellF1}\left[\frac{1}{2}, 1 - n, 1, \frac{3}{2}, 1 - \text{Sec}[e + f x], \frac{1}{2}(1 - \text{Sec}[e + f x])\right] \text{Tan}[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]}}$$

Result (type 6, 2964 leaves) :

$$\begin{aligned} & \left(3 \sqrt{2} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right. \\ & \quad \left. \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^n \text{Sec}[e + f x]^{-\frac{1}{2} + \frac{1}{2}(-1 + 2n)} \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^n \sqrt{1 + \text{Sec}[e + f x]} \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \\ & \left(f \sqrt{a(1 + \text{Sec}[e + f x])} \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \right. \\ & \quad \left. \left(2(-1 + n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-1 + 2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \\ & \left(\left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \text{Cos}[e + f x] \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{1+n} \right. \right. \\ & \quad \left. \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^n \sqrt{1 + \text{Sec}[e + f x]} \right) / \left(\sqrt{2} \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \right. \\ & \quad \left. \left(2(-1 + n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (-1 + 2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) - \\ & \left(3 \sqrt{2} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^n \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^n \right. \\ & \quad \left. \sqrt{1 + \text{Sec}[e + f x]} \text{Sin}[e + f x] \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \left(3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\ & \quad \left(2(-1 + n) \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\ & \quad \left. \left. (-1 + 2n) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{3}{5} \left(\frac{1}{2} + n \right) \text{AppellF1} \left[\frac{5}{2}, \frac{3}{2} + n, 1 - n, \frac{7}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \tan \left[\frac{1}{2} (e + f x) \right] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 + \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \right)^n \tan \left[\frac{1}{2} (e + f x) \right] \tan[e + f x] \right) \Bigg) \Bigg) / \\
& \left(\sqrt{2} \sqrt{1 + \text{Sec}[e + f x]} \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
& \quad \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) + \\
& \left(3 \sqrt{2} n \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \cos[e + f x] \left(\text{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \quad \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \right)^{-1+n} \sqrt{1 + \text{Sec}[e + f x]} \tan \left[\frac{1}{2} (e + f x) \right] \\
& \quad \left. \left. \left(-\cos \left[\frac{1}{2} (e + f x) \right] \text{Sec}[e + f x] \sin \left[\frac{1}{2} (e + f x) \right] + \cos \left[\frac{1}{2} (e + f x) \right]^2 \text{Sec}[e + f x] \tan[e + f x] \right) \right) \Bigg) \Bigg) / \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 2 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1 + 2n) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]^n}{(a + a \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 6, 67 leaves, 4 steps):

$$\frac{\text{AppellF1} \left[\frac{1}{2}, 1 - n, 2, \frac{3}{2}, 1 - \text{Sec}[e + f x], \frac{1}{2} (1 - \text{Sec}[e + f x]) \right] \tan[e + f x]}{2 a f \sqrt{a + a \text{Sec}[e + f x]}}$$

$$\begin{aligned}
& 3 \left(-\frac{1}{3} (1-n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3} \left(-\frac{3}{2}+n \right) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \\
& \quad \left(2 (-1+n) \left(-\frac{3}{5} (2-n) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{3}{2}+n, 3-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} \left(-\frac{3}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + \right. \\
& \quad \left. (-3+2n) \left(-\frac{3}{5} (1-n) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} \left(-\frac{1}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) / \\
& \quad \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \left(2 (-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (-3+2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 + \\
& \quad \left(6 \left(\frac{3}{2}+n \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Cos} [e+fx] \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^n \right. \right. \\
& \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^{\frac{1}{2}+n} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 \right. \right. \\
& \quad \left. \left. - \operatorname{Cos} \left[\frac{1}{2} (e+fx) \right] \operatorname{Sec} [e+fx] \operatorname{Sin} \left[\frac{1}{2} (e+fx) \right] + \operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \operatorname{Tan} [e+fx] \right) \right) \right) / \\
& \quad \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left(2 (-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-3+2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right)
\end{aligned}$$

■ **Problem 312: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\operatorname{Sec} [e+fx])^n (a+a \operatorname{Sec} [e+fx])^{3/2} dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\left(2 a^2 (1+4 n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1-\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^n \operatorname{Sec}[e+f x]^{1-n} \operatorname{Sin}[e+f x] \right) / \left(f (1+2 n) \sqrt{a+a \operatorname{Sec}[e+f x]} \right) + \frac{2 a^2 (-\operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f (1+2 n) \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 5, 6383 leaves): Display of huge result suppressed!

■ **Problem 313: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\operatorname{Sec}[e+f x])^n \sqrt{a+a \operatorname{Sec}[e+f x]} dx$$

Optimal (type 5, 70 leaves, 3 steps):

$$\frac{2 a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1-\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^n \operatorname{Sec}[e+f x]^{1-n} \operatorname{Sin}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 5, 215 leaves):

$$-\frac{1}{f n (1+n)} i 2^{-\frac{1}{2}+n} e^{\frac{1}{2} i (e+f x)} \left(1+e^{2 i (e+f x)}\right)^{-\frac{1}{2}+n} \left(e^{-i (e+f x)} \left(1+e^{2 i (e+f x)}\right)\right)^{\frac{1}{2}-n} \operatorname{Cos}[e+f x]^{\frac{1}{2}+n} \\ \left((1+n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right) \\ \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] (-\operatorname{Sec}[e+f x])^n \sqrt{a(1+\operatorname{Sec}[e+f x])}$$

■ **Problem 314: Result more than twice size of optimal antiderivative.**

$$\int \frac{(-\operatorname{Sec}[e+f x])^n}{\sqrt{a+a \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 6, 75 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 1, 1+n, \operatorname{Sec}[e+f x], -\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f n \sqrt{1-\operatorname{Sec}[e+f x]} \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 6, 2977 leaves):

$$\left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n \right. \\ \left. (-\operatorname{Sec}[e+f x])^n \operatorname{Sec}[e+f x]^{-\frac{1}{2}-n+\frac{1}{2}} (-1+2 n) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \sqrt{1+\operatorname{Sec}[e+f x]} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right) / \\ \left(f \sqrt{a(1+\operatorname{Sec}[e+f x])} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \\ \left. \left. \left(2 (-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right.$$

$$\left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+n} \sqrt{1+\sec[e+fx]} \tan\left[\frac{1}{2}(e+fx)\right] \\ \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \Bigg) / \\ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \Bigg)$$

■ **Problem 315: Result more than twice size of optimal antiderivative.**

$$\int \frac{(-\sec[e+fx])^n}{(a+a\sec[e+fx])^{3/2}} dx$$

Optimal (type 6, 78 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 2, 1+n, \sec[e+fx], -\sec[e+fx]\right] (-\sec[e+fx])^n \tan[e+fx]}{a f n \sqrt{1-\sec[e+fx]} \sqrt{a+a\sec[e+fx]}}$$

Result (type 6, 3005 leaves):

$$\left(6 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ \left. (-\sec[e+fx])^n \sec[e+fx]^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^{\frac{3}{2}+n} \tan\left[\frac{1}{2}(e+fx)\right] \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) \Bigg) / \\ \left(f (a(1+\sec[e+fx]))^{3/2} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right. \\ \left. \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos[e+fx] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \right. \right. \\ \left. \left. \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]\right)^{\frac{3}{2}+n} \tan\left[\frac{1}{2}(e+fx)\right]^2 \left(-1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \right) \right) \Bigg) /$$

$$\begin{aligned}
& \left(6 \left(\frac{3}{2} + n \right) \text{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \cos [e + f x] \left(\sec \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \right. \\
& \quad \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x] \right)^{\frac{1}{2} + n} \tan \left[\frac{1}{2} (e + f x) \right] \left(-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 \\
& \quad \left. \left(-\cos \left[\frac{1}{2} (e + f x) \right] \sec [e + f x] \sin \left[\frac{1}{2} (e + f x) \right] + \cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x] \tan [e + f x] \right) \right) \Bigg/ \\
& \left(3 \text{AppellF1} \left[\frac{1}{2}, -\frac{3}{2} + n, 1 - n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left(2 (-1 + n) \text{AppellF1} \left[\frac{3}{2}, -\frac{3}{2} + n, 2 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (-3 + 2 n) \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

- **Problem 316: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\text{d Sec}[e + f x])^n (a + a \text{Sec}[e + f x])^{3/2} dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\begin{aligned}
& \left(2 a^2 (1 + 4 n) \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \text{Sec}[e + f x] \right] \sec [e + f x]^{1-n} (\text{d Sec}[e + f x])^n \sin [e + f x] \right) \Bigg/ \left(f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]} \right) + \\
& \frac{2 a^2 (\text{d Sec}[e + f x])^n \tan [e + f x]}{f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]}}
\end{aligned}$$

Result (type 5, 6383 leaves): Display of huge result suppressed!

- **Problem 317: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\text{d Sec}[e + f x])^n \sqrt{a + a \text{Sec}[e + f x]} dx$$

Optimal (type 5, 70 leaves, 3 steps):

$$\frac{2 a \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 - \text{Sec}[e + f x] \right] \sec [e + f x]^{1-n} (\text{d Sec}[e + f x])^n \sin [e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]}}$$

Result (type 5, 215 leaves):

$$\begin{aligned}
& - \frac{1}{f n (1+n)} i 2^{-\frac{1}{2}+n} e^{\frac{1}{2} i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)^{-\frac{1}{2}+n} \left(e^{-i (e+f x)} \left(1 + e^{2 i (e+f x)}\right)\right)^{\frac{1}{2}-n} \operatorname{Cos}[e+f x]^{\frac{1}{2}+n} \\
& \left((1+n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] (d \operatorname{Sec}[e+f x])^n \sqrt{a(1+\operatorname{Sec}[e+f x])}
\end{aligned}$$

■ **Problem 318: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sec}[e+f x])^n}{\sqrt{a+a \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 6, 75 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 1, 1+n, \operatorname{Sec}[e+f x], -\operatorname{Sec}[e+f x]\right] (d \operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f n \sqrt{1-\operatorname{Sec}[e+f x]} \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 6, 2977 leaves):

$$\begin{aligned}
& \left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n \right. \\
& \left. \operatorname{Sec}[e+f x]^{-\frac{1}{2}-n+\frac{1}{2}(-1+2n)} (d \operatorname{Sec}[e+f x])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \sqrt{1+\operatorname{Sec}[e+f x]} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right) / \\
& \left(f \sqrt{a(1+\operatorname{Sec}[e+f x])} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \\
& \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \\
& \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \left. \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Cos}[e+f x] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^{1+n} \right. \right. \right. \\
& \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \sqrt{1+\operatorname{Sec}[e+f x]}\right) / \left(\sqrt{2} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left(2(-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (-1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)\right) - \right. \\
& \left. \left(3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]\right)^n \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(2(-1+n) \left(-\frac{3}{5}(2-n) \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}+n, 3-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \right. \right. \\
& \quad \left. \frac{3}{5} \left(-\frac{1}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) + \\
& \quad (-1+2n) \left(-\frac{3}{5}(1-n) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2}+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] + \right. \\
& \quad \left. \frac{3}{5} \left(\frac{1}{2}+n \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2}+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) \right) \Big/ \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + (-1+2n) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right)^2 + \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \right)^n \right. \\
& \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^n \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \operatorname{Tan}[e+fx] \right) \Big/ \\
& \left(\sqrt{2} \sqrt{1+\operatorname{Sec}[e+fx]} \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1+2n) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right) \right) + \\
& \left(3\sqrt{2}n \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \operatorname{Cos}[e+fx] \left(\operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \right)^n \right. \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^{-1+n} \sqrt{1+\operatorname{Sec}[e+fx]} \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right. \\
& \quad \left. \left. \left(-\operatorname{Cos} \left[\frac{1}{2}(e+fx) \right] \operatorname{Sec}[e+fx] \operatorname{Sin} \left[\frac{1}{2}(e+fx) \right] + \operatorname{Cos} \left[\frac{1}{2}(e+fx) \right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) \right) \Big/ \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \\
& \quad \left(2(-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (-1+2n) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right]^2 \right) \right) \Big/
\end{aligned}$$

■ **Problem 319: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sec}[e + f x])^n}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 6, 78 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}, 2, 1+n, \operatorname{Sec}[e + f x], -\operatorname{Sec}[e + f x]\right] (d \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{a f n \sqrt{1 - \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 6, 3005 leaves):

$$\begin{aligned} & \left(6 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^n \right. \\ & \quad \left. \operatorname{Sec}[e+fx]^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)} (d \operatorname{Sec}[e+fx])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^{\frac{3}{2}+n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\ & \left(f (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left(2 (-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ & \quad \left. \left. (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \\ & \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}[e+fx] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \right. \\ & \quad \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^{\frac{3}{2}+n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \right) \right) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(2 (-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (-3+2n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}[e+fx] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{1+n} \right. \\ & \quad \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^{\frac{3}{2}+n} \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)^2 \right) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{3}{2}+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(2 (-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{3}{2}+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \end{aligned}$$

$$\left. \left. \left. (-3 + 2n) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} + n, 1 - n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right)$$

- **Problem 320: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\sec[e + f x])^n (a - a \sec[e + f x])^{5/2} dx$$

Optimal (type 5, 178 leaves, 4 steps):

$$\frac{2 a^3 (3 + 24 n + 16 n^2) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 + \sec[e + f x] \right] \tan[e + f x]}{f (1 + 2 n) (3 + 2 n) \sqrt{a - a \sec[e + f x]}} +$$

$$\frac{2 a^3 (7 + 4 n) (-\sec[e + f x])^n \tan[e + f x]}{f (1 + 2 n) (3 + 2 n) \sqrt{a - a \sec[e + f x]}} + \frac{2 a^2 (-\sec[e + f x])^n \sqrt{a - a \sec[e + f x]} \tan[e + f x]}{f (3 + 2 n)}$$

Result (type 5, 455 leaves):

$$\frac{1}{f} 2^{-\frac{5}{2} + n} e^{\frac{1}{2} i (e + f (1 - 2n) x)} \left(\frac{e^{i (e + f x)}}{1 + e^{2 i (e + f x)}} \right)^{-\frac{1}{2} + n} \left(1 + e^{2 i (e + f x)} \right)^{-\frac{1}{2} + n} \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^5$$

$$\left(\frac{e^{i f n x} \operatorname{Hypergeometric2F1} \left[\frac{n}{2}, \frac{5}{2} + n, \frac{2+n}{2}, -e^{2 i (e + f x)} \right]}{n} - \frac{5 e^{i (e + f (1+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{1+n}{2}, \frac{5}{2} + n, \frac{3+n}{2}, -e^{2 i (e + f x)} \right]}{1 + n} + \right.$$

$$\frac{10 e^{i (2e + f (2+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{2+n}{2}, \frac{5}{2} + n, \frac{4+n}{2}, -e^{2 i (e + f x)} \right]}{2 + n} - \frac{10 e^{i (3e + f (3+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{5}{2} + n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2 i (e + f x)} \right]}{3 + n} +$$

$$\left. \frac{5 e^{i (4e + f (4+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{5}{2} + n, \frac{4+n}{2}, \frac{6+n}{2}, -e^{2 i (e + f x)} \right]}{4 + n} - \frac{e^{i (5e + f (5+n) x)} \operatorname{Hypergeometric2F1} \left[\frac{5}{2} + n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2 i (e + f x)} \right]}{5 + n} \right)$$

$$(-\sec[e + f x])^n \sec[e + f x]^{-\frac{5}{2} - n} (a - a \sec[e + f x])^{5/2}$$

- **Problem 321: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\sec[e + f x])^n (a - a \sec[e + f x])^{3/2} dx$$

Optimal (type 5, 108 leaves, 4 steps):

$$\frac{2 a^2 (1 + 4 n) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 + \sec[e + f x] \right] \tan[e + f x]}{f (1 + 2 n) \sqrt{a - a \sec[e + f x]}} + \frac{2 a^2 (-\sec[e + f x])^n \tan[e + f x]}{f (1 + 2 n) \sqrt{a - a \sec[e + f x]}}$$

Result (type 5, 377 leaves):

$$\frac{1}{f n (1+n) (2+n) (3+n)} 2^{-\frac{3}{2}+n} e^{\frac{1}{2} i (e+f (1-2n) x)} \left(\frac{e^{i (e+f x)}}{1+e^{2 i (e+f x)}} \right)^{-\frac{1}{2}+n} (1+e^{2 i (e+f x)})^{-\frac{1}{2}+n}$$

$$\text{Csc} \left[\frac{1}{2} (e+f x) \right]^3 \left(-e^{i f n x} (6+11 n+6 n^2+n^3) \text{Hypergeometric2F1} \left[\frac{n}{2}, \frac{3}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)} \right] + \right.$$

$$3 e^{i (e+f (1+n) x)} n (6+5 n+n^2) \text{Hypergeometric2F1} \left[\frac{1+n}{2}, \frac{3}{2}+n, \frac{3+n}{2}, -e^{2 i (e+f x)} \right] +$$

$$e^{2 i e n} (1+n) \left(-3 e^{i f (2+n) x} (3+n) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{2+n}{2}, \frac{4+n}{2}, -e^{2 i (e+f x)} \right] + \right.$$

$$\left. \left. e^{i (e+f (3+n) x)} (2+n) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2 i (e+f x)} \right] \right) \right) (-\text{Sec}[e+f x])^n \text{Sec}[e+f x]^{-\frac{3}{2}-n} (a-a \text{Sec}[e+f x])^{3/2}$$

- **Problem 322: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (-\text{Sec}[e+f x])^n \sqrt{a-a \text{Sec}[e+f x]} dx$$

Optimal (type 5, 47 leaves, 2 steps):

$$\frac{2 a \text{Hypergeometric2F1} \left[\frac{1}{2}, 1-n, \frac{3}{2}, 1+\text{Sec}[e+f x] \right] \text{Tan}[e+f x]}{f \sqrt{a-a \text{Sec}[e+f x]}}$$

Result (type 5, 236 leaves):

$$\frac{1}{f n (1+n)} 2^{-\frac{1}{2}+n} e^{-\frac{1}{2} i (e+f (1+2n) x)} \left(\frac{e^{i (e+f x)}}{1+e^{2 i (e+f x)}} \right)^{\frac{1}{2}+n} (1+e^{2 i (e+f x)})^{\frac{1}{2}+n} \text{Csc} \left[\frac{e}{2} + \frac{f x}{2} \right]$$

$$\left(e^{i f n x} (1+n) \text{Hypergeometric2F1} \left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)} \right] - e^{i (e+f (1+n) x)} n \text{Hypergeometric2F1} \left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)} \right] \right)$$

$$(-\text{Sec}[e+f x])^n \text{Sec}[e+f x]^{-\frac{1}{2}-n} \sqrt{a-a \text{Sec}[e+f x]}$$

- **Problem 323: Unable to integrate problem.**

$$\int \frac{(-\text{Sec}[e+f x])^n}{\sqrt{a-a \text{Sec}[e+f x]}} dx$$

Optimal (type 6, 58 leaves, 4 steps):

$$\frac{\text{AppellF1} \left[\frac{1}{2}, 1-n, 1, \frac{3}{2}, 1+\text{Sec}[e+f x], \frac{1}{2} (1+\text{Sec}[e+f x]) \right] \text{Tan}[e+f x]}{f \sqrt{a-a \text{Sec}[e+f x]}}$$

Result (type 8, 28 leaves):

$$\int \frac{(-\text{Sec}[e+f x])^n}{\sqrt{a-a \text{Sec}[e+f x]}} dx$$

■ **Problem 324: Unable to integrate problem.**

$$\int \frac{(-\operatorname{Sec}[e + f x])^n}{(a - a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 6, 64 leaves, 4 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{1}{2}, 1 - n, 2, \frac{3}{2}, 1 + \operatorname{Sec}[e + f x], \frac{1}{2}(1 + \operatorname{Sec}[e + f x])\right] \operatorname{Tan}[e + f x]}{2 a f \sqrt{a - a \operatorname{Sec}[e + f x]}}$$

Result (type 8, 28 leaves):

$$\int \frac{(-\operatorname{Sec}[e + f x])^n}{(a - a \operatorname{Sec}[e + f x])^{3/2}} dx$$

■ **Problem 325: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^n (a - a \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{2 a^2 \operatorname{Sec}[e + f x]^{1+n} \operatorname{Sin}[e + f x]}{f (1 + 2 n) \sqrt{a - a \operatorname{Sec}[e + f x]}} + \left(2 a^2 (1 + 4 n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 - n, \frac{3}{2}, 1 + \operatorname{Sec}[e + f x]\right] (-\operatorname{Sec}[e + f x])^{-n} \operatorname{Sec}[e + f x]^{1+n} \operatorname{Sin}[e + f x] \right) / \left(f (1 + 2 n) \sqrt{a - a \operatorname{Sec}[e + f x]} \right)$$

Result (type 5, 366 leaves):

$$\frac{1}{f n (1 + n) (2 + n) (3 + n) \operatorname{Sec}[e + f x]^{3/2}} 2^{-\frac{3}{2}+n} e^{\frac{1}{2} i (e+f (1-2 n) x)} \left(\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}} \right)^{-\frac{1}{2}+n} \left((1 + e^{2 i (e+f x)})^{-\frac{1}{2}+n} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \left(-e^{i f n x} (6 + 11 n + 6 n^2 + n^3) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{3}{2} + n, \frac{2 + n}{2}, -e^{2 i (e+f x)}\right] + 3 e^{i (e+f (1+n) x)} n (6 + 5 n + n^2) \operatorname{Hypergeometric2F1}\left[\frac{1 + n}{2}, \frac{3}{2} + n, \frac{3 + n}{2}, -e^{2 i (e+f x)}\right] + e^{2 i e} n (1 + n) \left(-3 e^{i f (2+n) x} (3 + n) \operatorname{Hypergeometric2F1}\left[\frac{3}{2} + n, \frac{2 + n}{2}, \frac{4 + n}{2}, -e^{2 i (e+f x)}\right] + e^{i (e+f (3+n) x)} (2 + n) \operatorname{Hypergeometric2F1}\left[\frac{3}{2} + n, \frac{3 + n}{2}, \frac{5 + n}{2}, -e^{2 i (e+f x)}\right] \right) \right) (a - a \operatorname{Sec}[e + f x])^{3/2}$$

■ **Problem 326: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^n \sqrt{a - a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 5, 69 leaves, 3 steps):

$$\frac{2 a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1+\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^{-n} \operatorname{Sec}[e+f x]^{1+n} \operatorname{Sin}[e+f x]}{f \sqrt{a-a \operatorname{Sec}[e+f x]}}$$

Result (type 5, 222 leaves):

$$\frac{1}{f n (1+n) \sqrt{\operatorname{Sec}[e+f x]}} 2^{-\frac{1}{2}+n} e^{-\frac{1}{2} i (e+f (1+2 n) x)} \left(\frac{e^{i (e+f x)}}{1+e^{2 i (e+f x)}} \right)^{\frac{1}{2}+n} \left(1+e^{2 i (e+f x)} \right)^{\frac{1}{2}+n} \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]$$

$$\left(e^{i f n x} (1+n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] - e^{i (e+f (1+n) x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right)$$

$$\sqrt{a-a \operatorname{Sec}[e+f x]}$$

■ **Problem 327: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\operatorname{d} \operatorname{Sec}[e+f x])^n (a-a \operatorname{Sec}[e+f x])^{3/2} dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{2 a^2 (\operatorname{d} \operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f (1+2 n) \sqrt{a-a \operatorname{Sec}[e+f x]}} +$$

$$\left(2 a^2 (1+4 n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1+\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^{-n} (\operatorname{d} \operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x] \right) /$$

$$\left(f (1+2 n) \sqrt{a-a \operatorname{Sec}[e+f x]} \right)$$

Result (type 5, 377 leaves):

$$\frac{1}{f n (1+n) (2+n) (3+n)} 2^{-\frac{3}{2}+n} e^{\frac{1}{2} i (e+f (1-2 n) x)} \left(\frac{e^{i (e+f x)}}{1+e^{2 i (e+f x)}} \right)^{-\frac{1}{2}+n} \left(1+e^{2 i (e+f x)} \right)^{-\frac{1}{2}+n}$$

$$\operatorname{Csc}\left[\frac{1}{2} (e+f x)\right]^3 \left(-e^{i f n x} (6+11 n+6 n^2+n^3) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{3}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] +$$

$$3 e^{i (e+f (1+n) x)} n (6+5 n+n^2) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{3}{2}+n, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] +$$

$$e^{2 i e} n (1+n) \left(-3 e^{i f (2+n) x} (3+n) \operatorname{Hypergeometric2F1}\left[\frac{3}{2}+n, \frac{2+n}{2}, \frac{4+n}{2}, -e^{2 i (e+f x)}\right] +$$

$$e^{i (e+f (3+n) x)} (2+n) \operatorname{Hypergeometric2F1}\left[\frac{3}{2}+n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2 i (e+f x)}\right] \right) \right) \operatorname{Sec}[e+f x]^{-\frac{3}{2}-n} (\operatorname{d} \operatorname{Sec}[e+f x])^n (a-a \operatorname{Sec}[e+f x])^{3/2}$$

■ **Problem 328: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (\operatorname{d} \operatorname{Sec}[e+f x])^n \sqrt{a-a \operatorname{Sec}[e+f x]} dx$$

Optimal (type 5, 69 leaves, 3 steps):

$$\frac{2 a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1-n, \frac{3}{2}, 1+\operatorname{Sec}[e+f x]\right] (-\operatorname{Sec}[e+f x])^{-n} (d \operatorname{Sec}[e+f x])^n \operatorname{Tan}[e+f x]}{f \sqrt{a-a \operatorname{Sec}[e+f x]}}$$

Result (type 5, 236 leaves):

$$\frac{1}{f n (1+n)} 2^{-\frac{1}{2}+n} e^{-\frac{1}{2} i (e+f (1+2 n) x)} \left(\frac{e^{i (e+f x)}}{1+e^{2 i (e+f x)}} \right)^{\frac{1}{2}+n} (1+e^{2 i (e+f x)})^{\frac{1}{2}+n} \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]$$

$$\left(e^{i f n x} (1+n) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, \frac{1}{2}+n, \frac{2+n}{2}, -e^{2 i (e+f x)}\right] - e^{i (e+f (1+n) x)} n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1+n}{2}, \frac{3+n}{2}, -e^{2 i (e+f x)}\right] \right)$$

$$\operatorname{Sec}[e+f x]^{-\frac{1}{2}-n} (d \operatorname{Sec}[e+f x])^n \sqrt{a-a \operatorname{Sec}[e+f x]}$$

■ **Problem 329: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+f x]^n (1+\operatorname{Sec}[e+f x])^m dx$$

Optimal (type 6, 72 leaves, 2 steps):

$$\frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1}{2}, 1-n, \frac{1}{2}-m, \frac{3}{2}, 1-\operatorname{Sec}[e+f x], \frac{1}{2}(1-\operatorname{Sec}[e+f x])\right] \operatorname{Tan}[e+f x]}{f \sqrt{1+\operatorname{Sec}[e+f x]}}$$

Result (type 6, 2246 leaves):

$$\left(3 \times 2^{1+m} \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right.$$

$$\left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right)^{-1+n} \operatorname{Sec}[e+f x]^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^{m+n} (1+\operatorname{Sec}[e+f x])^m \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right) /$$

$$\left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right.$$

$$\left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + \right. \right.$$

$$\left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right)$$

$$\left(\left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right)^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^{m+n} \right) / \right.$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, \right. \right.$$

$$\left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] + (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \right) +$$

$$\left(3 \times 2^{1+m} (-1+n) \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \right)^{-1+n} \right.$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 + (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big)^2 + \\
& \left(3 \times 2^{1+m} (m+n) \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2\right)^{-1+n} \right. \\
& \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{-1+m+n} \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right) \Big) / \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 330: Result more than twice size of optimal antiderivative.**

$$\int (1 - \sec[e+fx])^m \sec[e+fx]^n dx$$

Optimal (type 6, 89 leaves, 2 steps):

$$\left(\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}+m, 1-n, \frac{1}{2}, \frac{3}{2}+m, 1-\sec[e+fx], \frac{1}{2}(1-\sec[e+fx])\right] (1-\sec[e+fx])^m \tan[e+fx] \right) / \left(f(1+2m) \sqrt{1+\sec[e+fx]} \right)$$

Result (type 6, 2751 leaves):

$$\begin{aligned}
& \left(2(3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-\frac{1}{2}+m+n} (1-\sec[e+fx])^m \sec[e+fx]^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+n} \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\sec\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \right) \Big) / \\
& \left(f(1+2m) \left((3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \\
& \left(\left(2 (3+2m) \left(-\frac{1}{2}+m+n \right) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \right. \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \right)^{m+n} \tan \left[\frac{1}{2} (e+fx) \right] \left(\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\sec \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{1+2m} \right) / \right. \\
& \quad \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(2 (3+2m) \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \right)^{m+n} \left(\frac{\tan \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\sec \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{1+2m} \left(-\frac{1}{\frac{3}{2}+m} \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2}+m \right) (1-n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{3}{2}+m} \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2}+m \right) (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) / \right. \\
& \quad \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Bigg) + \\
& \left(2(3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{-\frac{1}{2}+m+n} \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{2m} \left(\frac{1}{2} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} - \frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right) \right) / \\
& \left((3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(2(3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2\right)^{-\frac{1}{2}+m+n} \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \right) \\
& \left(2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+ \right. \right. \right. \\
& \left. \left. m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (3+2m) \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m\right) (1-n) \operatorname{AppellF1}\left[\right. \right. \\
& \left. \left. \frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}+m} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2} + m \right) (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 + m + n, 1 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + \\
& 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \left((-1+n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (2-n) \operatorname{AppellF1} \left[\frac{5}{2} + m, m+n, 3-n, \frac{7}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right. \right. \\
& \quad \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (m+n) \operatorname{AppellF1} \left[\frac{5}{2} + m, 1 + m + n, 2-n, \frac{7}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) + (m+n) \left(-\frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (1-n) \operatorname{AppellF1} \left[\frac{5}{2} + m, 1 + m + n, \right. \right. \\
& \quad \left. \left. 2-n, \frac{7}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \frac{1}{\frac{5}{2} + m} \left(\frac{3}{2} + m \right) (1 + m + n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{5}{2} + m, 2 + m + n, 1-n, \frac{7}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) \Bigg) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m+n, 1-n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\
& \quad 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m+n, 2-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + (m+n) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 + m + n, 1-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) + \\
& \left(2 (3+2m) (m+n) \operatorname{AppellF1} \left[\frac{1}{2} + m, m+n, 1-n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-\frac{1}{2} + m + n} \right. \\
& \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \right)^{-1 + m + n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{1 + 2m} \right) \right)
\end{aligned}$$

$$\left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \sec[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \tan[e+fx] \right) \right/$$

$$\left((1+2m) \left((3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.$$

$$\left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.$$

$$\left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)$$

■ **Problem 331: Result more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^n (a+a \sec[e+fx])^m dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$\frac{1}{f} 2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1}{2}, 1-n, \frac{1}{2}-m, \frac{3}{2}, 1-\sec[e+fx], \frac{1}{2}(1-\sec[e+fx])\right] (1+\sec[e+fx])^{-\frac{1}{2}-m} (a+a \sec[e+fx])^m \tan[e+fx]$$

Result (type 6, 2248 leaves):

$$\left(3 \times 2^{1+m} \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right.$$

$$\left. \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \sec[e+fx]^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+n} (a(1+\sec[e+fx]))^m \tan\left[\frac{1}{2}(e+fx)\right] \right) \right/$$

$$\left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.$$

$$\left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right.$$

$$\left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right)$$

$$\left(\left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+n} \right) \right/$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right.$$

$$\begin{aligned}
& \left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^{m+n} \right. \\
& \quad \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left(2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \quad \left. 3 \left(-\frac{1}{3} (1-n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3} (m+n) \right. \right. \\
& \quad \quad \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + 2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right. \\
& \quad \quad \left. \left((-1+n) \left(-\frac{3}{5} (2-n) \operatorname{AppellF1} \left[\frac{5}{2}, m+n, 3-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \right. \\
& \quad \quad \quad \left. \left. \frac{3}{5} (m+n) \operatorname{AppellF1} \left[\frac{5}{2}, 1+m+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) + \right. \\
& \quad \quad \left. (m+n) \left(-\frac{3}{5} (1-n) \operatorname{AppellF1} \left[\frac{5}{2}, 1+m+n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
& \quad \quad \quad \left. \left. \frac{3}{5} (1+m+n) \operatorname{AppellF1} \left[\frac{5}{2}, 2+m+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \right) \Bigg/ \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 + \\
& \left(3 \times 2^{1+m} (m+n) \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} \right. \\
& \quad \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^{-1+m+n} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \\
& \quad \left. \left(-\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right] \operatorname{Sec} [e+fx] \operatorname{Sin} \left[\frac{1}{2} (e+fx) \right] + \operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \operatorname{Tan} [e+fx] \right) \right) \Bigg/ \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int (1 - \operatorname{Sec}[e + f x])^m (-\operatorname{Sec}[e + f x])^n dx$$

Optimal (type 6, 70 leaves, 2 steps):

$$\frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1}{2}, 1-n, \frac{1}{2}-m, \frac{3}{2}, 1+\operatorname{Sec}[e+fx], \frac{1}{2}(1+\operatorname{Sec}[e+fx])\right] \operatorname{Tan}[e+fx]}{f \sqrt{1-\operatorname{Sec}[e+fx]}}$$

Result (type 6, 2753 leaves):

$$\left(2 (3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\ \left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-\frac{1}{2}+m+n} (1-\operatorname{Sec}[e+fx])^m (-\operatorname{Sec}[e+fx])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \right) \right) /$$

$$\left(f (1+2m) \left((3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ \left. \left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ \left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \\ \left(\left(2 (3+2m) \left(-\frac{1}{2}+m+n \right) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-\frac{1}{2}+m+n} \right. \right. \\ \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \right) \right) \right) /$$

$$\begin{aligned}
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left(2(3+2m) \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{1+2m} \left(-\frac{1}{\frac{3}{2}+m} \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2}+m \right) (1-n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{3}{2}+m} \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2}+m \right) (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& \quad 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) + \\
& \left(2(3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \right. \\
& \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{2m} \left(\frac{1}{2} \sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2} - \frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2}{2 \sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}} \right) \right) \right) / \\
& \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 - \\
& \left(2 (3+2 m) \operatorname{AppellF1} \left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-\frac{1}{2}+m+n} \right. \\
& \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Sec}[e+f x] \right)^{m+n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2}} \right)^{1+2 m} \right) \\
& \left(2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] + (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+ \right. \right. \right. \\
& \quad \left. \left. m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + (3+2 m) \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m \right) (1-n) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \frac{1}{\frac{3}{2}+m} \right. \right. \\
& \quad \left. \left. \left(\frac{1}{2}+m \right) (m+n) \operatorname{AppellF1} \left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) \right) + \\
& 2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \left((-1+n) \left(-\frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m \right) (2-n) \operatorname{AppellF1} \left[\frac{5}{2}+m, m+n, 3-n, \frac{7}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m \right) (m+n) \operatorname{AppellF1} \left[\frac{5}{2}+m, 1+m+n, 2-n, \frac{7}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) + (m+n) \left(-\frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m \right) (1-n) \operatorname{AppellF1} \left[\frac{5}{2}+m, 1+m+n, \right. \right. \\
& \quad \left. \left. 2-n, \frac{7}{2}+m, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] + \frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m \right) (1+m+n) \right) \right)
\end{aligned}$$

$$\int (-\operatorname{Sec}[e + f x])^n (a + a \operatorname{Sec}[e + f x])^m dx$$

Optimal (type 6, 87 leaves, 3 steps):

$$\left(\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2} + m, 1 - n, \frac{1}{2}, \frac{3}{2} + m, 1 + \operatorname{Sec}[e + f x], \frac{1}{2} (1 + \operatorname{Sec}[e + f x]) \right] (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x] \right) / \left(f (1 + 2m) \sqrt{1 - \operatorname{Sec}[e + f x]} \right)$$

Result (type 6, 2250 leaves):

$$\begin{aligned} & \left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right. \\ & \quad \left. \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-1+n} (-\operatorname{Sec}[e + f x])^n \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec}[e + f x] \right)^{m+n} (a (1 + \operatorname{Sec}[e + f x]))^m \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) / \\ & \left(f \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\ & \quad 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\ & \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \\ & \left(\left(3 \times 2^m \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^n \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec}[e + f x] \right)^{m+n} \right) / \right. \\ & \quad \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\ & \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) + \\ & \left(3 \times 2^{1+m} (-1+n) \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-1+n} \right. \\ & \quad \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec}[e + f x] \right)^{m+n} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) / \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\ & \quad 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\ & \quad \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) + \\ & \left(3 \times 2^{1+m} \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-1+n} \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec}[e + f x] \right)^{m+n} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right. \\ & \quad \left. \left(-\frac{1}{3} (1-n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \right. \right. \end{aligned}$$

$$2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\ \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \Bigg)$$

■ **Problem 336: Unable to integrate problem.**

$$\int (-\sec[e+fx])^n (a - a \sec[e+fx])^m dx$$

Optimal (type 6, 87 leaves, 3 steps):

$$\frac{1}{f} 2^{\frac{1}{2}+m} \operatorname{AppellF1} \left[\frac{1}{2}, 1-n, \frac{1}{2}-m, \frac{3}{2}, 1+\sec[e+fx], \frac{1}{2} (1+\sec[e+fx]) \right] (1-\sec[e+fx])^{-\frac{1}{2}-m} (a - a \sec[e+fx])^m \tan[e+fx]$$

Result (type 8, 26 leaves):

$$\int (-\sec[e+fx])^n (a - a \sec[e+fx])^m dx$$

■ **Problem 337: Result more than twice size of optimal antiderivative.**

$$\int (d \sec[e+fx])^n (1 + \sec[e+fx])^m dx$$

Optimal (type 6, 79 leaves, 2 steps):

$$\frac{\operatorname{AppellF1} \left[n, \frac{1}{2}, \frac{1}{2}-m, 1+n, \sec[e+fx], -\sec[e+fx] \right] (d \sec[e+fx])^n \tan[e+fx]}{f n \sqrt{1-\sec[e+fx]} \sqrt{1+\sec[e+fx]}}$$

Result (type 6, 2248 leaves):

$$\left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \\ \left. \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n} (d \sec[e+fx])^n \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^{m+n} (1 + \sec[e+fx])^m \tan \left[\frac{1}{2} (e+fx) \right] \right) / \\ \left(f \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\ \left. \left. 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \right. \\ \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \\ \left(\left(3 \times 2^m \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^n \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec[e+fx] \right)^{m+n} \right) / \right. \\ \left. \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \right.$$

$$\begin{aligned} & \frac{3}{5} (1+m+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m+n, 1-n, \frac{7}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]\right) \Bigg) \Bigg) \Bigg) \Bigg) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\ & \left(3 \times 2^{1+m} (m+n) \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \right. \\ & \quad \left. \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{-1+m+n} \tan\left[\frac{1}{2}(e+fx)\right] \right. \right. \\ & \quad \left. \left. \left(-\cos\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \tan[e+fx] \right) \right) \right) \Bigg) \Bigg) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+n, 1-n, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\ & \quad \left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}, m+n, 2-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (m+n) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+n, 1-n, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \end{aligned}$$

■ **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int (1 - \operatorname{Sec}[e+fx])^m (d \operatorname{Sec}[e+fx])^n dx$$

Optimal (type 6, 79 leaves, 2 steps):

$$\frac{\operatorname{AppellF1}\left[n, \frac{1}{2}-m, \frac{1}{2}, 1+n, \operatorname{Sec}[e+fx], -\operatorname{Sec}[e+fx]\right] (d \operatorname{Sec}[e+fx])^n \tan[e+fx]}{f n \sqrt{1 - \operatorname{Sec}[e+fx]} \sqrt{1 + \operatorname{Sec}[e+fx]}}$$

Result (type 6, 2753 leaves):

$$\left(2 (3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\ \left. \left(\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-\frac{1}{2}+m+n} (1 - \operatorname{Sec}[e+fx])^m (d \operatorname{Sec}[e+fx])^n \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \right) \right) /$$

$$\begin{aligned}
& \left(f (1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m+n, 1-n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m+n, 2-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1+m+n, 1-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \\
& \left(\left(2 (3+2m) \left(-\frac{1}{2} + m+n \right) \operatorname{AppellF1} \left[\frac{1}{2} + m, m+n, 1-n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \right. \right. \\
& \left. \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^{m+n} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{1+2m} \right) \right) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m+n, 1-n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \right. \\
& 2 \left((-1+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m+n, 2-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1+m+n, 1-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) + \\
& \left(2 (3+2m) \left(\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-\frac{1}{2}+m+n} \left(\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec} [e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}} \right)^{1+2m} \left(-\frac{1}{\frac{3}{2} + m} \right. \right. \\
& \left. \left. \left(\frac{1}{2} + m \right) (1-n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m+n, 2-n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{\frac{3}{2} + m} \right) \right)
\end{aligned}$$

$$\left. \left(\frac{1}{2} + m \right) (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 + m + n, 1 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \Bigg/$$

$$\left((1 + 2m) \left((3 + 2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m + n, 1 - n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right.$$

$$2 \left((-1 + n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m + n, 2 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right.$$

$$\left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 + m + n, 1 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \Bigg) +$$

$$\left(2 (3 + 2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m + n, 1 - n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-\frac{1}{2} + m + n} \right.$$

$$\left. \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \right)^{m+n} \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right)^{2m} \left(\frac{1}{2} \sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2} - \frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}} \right) \right) \Bigg/$$

$$\left((3 + 2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m + n, 1 - n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right.$$

$$2 \left((-1 + n) \operatorname{AppellF1} \left[\frac{3}{2} + m, m + n, 2 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right.$$

$$\left. \left. (m+n) \operatorname{AppellF1} \left[\frac{3}{2} + m, 1 + m + n, 1 - n, \frac{5}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) -$$

$$\left(2 (3 + 2m) \operatorname{AppellF1} \left[\frac{1}{2} + m, m + n, 1 - n, \frac{3}{2} + m, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right)^{-\frac{1}{2} + m + n} \right.$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{m+n} \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}} \right)^{1+2m} \\
& \left(2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (3+2m) \left(-\frac{1}{\frac{3}{2}+m} \left(\frac{1}{2}+m\right) (1-n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{3}{2}+m} \right) \right) \left(\frac{1}{2}+m\right) (m+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, 1+m+n, 1-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left((-1+n) \left(-\frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m\right) (2-n) \operatorname{AppellF1}\left[\frac{5}{2}+m, m+n, 3-n, \frac{7}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m\right) (m+n) \operatorname{AppellF1}\left[\frac{5}{2}+m, 1+m+n, 2-n, \frac{7}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) + (m+n) \left(-\frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m\right) (1-n) \operatorname{AppellF1}\left[\frac{5}{2}+m, 1+m+n, 2-n, \frac{7}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{\frac{5}{2}+m} \left(\frac{3}{2}+m\right) (1+m+n) \right) \operatorname{AppellF1}\left[\frac{5}{2}+m, 2+m+n, 1-n, \frac{7}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Bigg) / \\
& \left((1+2m) \left((3+2m) \operatorname{AppellF1}\left[\frac{1}{2}+m, m+n, 1-n, \frac{3}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \left((-1+n) \operatorname{AppellF1}\left[\frac{3}{2}+m, m+n, 2-n, \frac{5}{2}+m, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (m+n) \right) \right) \right)
\end{aligned}$$

$$\int (d \operatorname{Sec}[e + f x])^n (a - a \operatorname{Sec}[e + f x])^m dx$$

Optimal (type 6, 96 leaves, 3 steps):

$$-\frac{1}{f n \sqrt{1 + \operatorname{Sec}[e + f x]}} \operatorname{AppellF1}\left[n, \frac{1}{2} - m, \frac{1}{2}, 1 + n, \operatorname{Sec}[e + f x], -\operatorname{Sec}[e + f x]\right] (1 - \operatorname{Sec}[e + f x])^{-\frac{1}{2} - m} (d \operatorname{Sec}[e + f x])^n (a - a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]$$

Result (type 8, 26 leaves):

$$\int (d \operatorname{Sec}[e + f x])^n (a - a \operatorname{Sec}[e + f x])^m dx$$

■ **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[e + f x])^m dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 1, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Sec}[e + f x]), 1 + \operatorname{Sec}[e + f x]\right] (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]}{f (1 + 2m) \sqrt{1 - \operatorname{Sec}[e + f x]}}$$

Result (type 6, 1653 leaves):

$$\begin{aligned} & \left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x] \right)^m (a (1 + \operatorname{Sec}[e + f x]))^m \operatorname{Sin}[e + f x] \right) / \\ & \left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right) \\ & \left(\left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \operatorname{Cos}[e + f x] \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x] \right)^m \right) / \right. \\ & \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - \right. \right. \\ & \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right) + \\ & \left(3 \times 2^m \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x] \right)^m \operatorname{Sin}[e + f x] \left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \right. \right. \\ & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + \frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] \right) \right) / \\ & \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2\right] - \right. \right. \end{aligned}$$

$$\begin{aligned}
& m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^m \operatorname{Sin}[e+fx] \right. \\
& \left(-2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& 3 \left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. \frac{1}{3} m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) - \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, m, 3, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \left. \frac{3}{5} m \operatorname{AppellF1}\left[\frac{5}{2}, 1+m, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \left. m \left(-\frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1+m, 2, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \left. \left. \frac{3}{5} (1+m) \operatorname{AppellF1}\left[\frac{5}{2}, 2+m, 1, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big)^2 + \\
& \left(3 \times 2^m m \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]\right)^{-1+m} \operatorname{Sin}[e+fx] \right. \\
& \left(-\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \Big/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big)
\end{aligned}$$

■ **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+fx] (a + a \operatorname{Sec}[e+fx])^m dx$$

$$\begin{aligned} & \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] - m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) + \\ & \left(2 \operatorname{AppellF1} \left[\frac{1}{2}, m, 2, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) / \left(\operatorname{AppellF1} \left[\frac{1}{2}, m, 2, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \frac{2}{3} \left(-2 \operatorname{AppellF1} \left[\frac{3}{2}, m, 3, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \right. \\ & \quad \left. \left. \left. m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) \\ & \left(-\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] \operatorname{Sec} [e + f x] \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right] + \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \operatorname{Tan} [e + f x] \right) \end{aligned}$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{d} \operatorname{Sec} [e + f x])^{3/2} (a + a \operatorname{Sec} [e + f x])^m \operatorname{d} x$$

Optimal (type 6, 98 leaves, 3 steps):

$$\frac{1}{3 f \sqrt{1 - \operatorname{Sec} [e + f x]}} 2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, \frac{1}{2} - m, \frac{5}{2}, \operatorname{Sec} [e + f x], -\operatorname{Sec} [e + f x] \right] (\operatorname{d} \operatorname{Sec} [e + f x])^{3/2} (1 + \operatorname{Sec} [e + f x])^{-\frac{1}{2} - m} (a + a \operatorname{Sec} [e + f x])^m \operatorname{Tan} [e + f x]$$

Result (type 6, 2529 leaves):

$$\begin{aligned} & - \left(\left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right. \right. \\ & \quad \left. \left. \sqrt{\operatorname{Sec} [e + f x]} (\operatorname{d} \operatorname{Sec} [e + f x])^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \right)^m (a (1 + \operatorname{Sec} [e + f x]))^m \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\ & \left(f \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \\ & \quad \left(\operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2}, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \\ & \quad \left. \left. (3 + 2m) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2} + m, -\frac{1}{2}, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \\ & \left(\left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2} + m, -\frac{1}{2}, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \sqrt{\operatorname{Sec} [e + f x]} \right. \right. \\ & \quad \left. \left. \left(\operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} [e + f x] \right)^m \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) / \left(\left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right)^2 \right) \end{aligned}$$

$$-\frac{1}{f \sqrt{1 - \text{Sec}[e + f x]}}$$

$$2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \text{Sec}[e + f x], -\text{Sec}[e + f x]\right] \sqrt{d \text{Sec}[e + f x]} (1 + \text{Sec}[e + f x])^{-\frac{1}{2}-m} (a + a \text{Sec}[e + f x])^m \text{Tan}[e + f x]$$

Result (type 6, 2225 leaves):

$$\left(2^{1+m} \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right. \\ \left. \sqrt{d \text{Sec}[e + f x]} \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^{\frac{1}{2}+m} (a (1 + \text{Sec}[e + f x]))^m \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) /$$

$$\left(f \sqrt{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \left(\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - (1 + 2m) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \\ \left(\left(2^m \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \sqrt{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^{\frac{1}{2}+m} \right) / \right. \\ \left. \left(\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \right. \\ \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - (1 + 2m) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) - \\ \left(2^m \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^{\frac{1}{2}+m} \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \\ \left(\sqrt{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \left(\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - \right. \right. \\ \left. \left. \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - (1 + 2m) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2}, \frac{5}{2}, \right. \right. \right. \right. \\ \left. \left. \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) + \left(2^{1+m} \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^{\frac{1}{2}+m} \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \\ \left(-\frac{1}{6} \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{3} \left(\frac{1}{2} + m\right) \right)$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \Bigg) / \left(\sqrt{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \right. \\
& \left. \left(\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - (1+2m) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(2^{1+m} \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \text{Sec}[e+fx] \right)^{\frac{1}{2}+m} \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \left(-\frac{1}{6} \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \frac{1}{3} \left(\frac{1}{2}+m \right) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \\
& \quad \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \\
& \quad \left. (1+2m) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \\
& \quad \frac{1}{3} \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{9}{10} \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}+m, \frac{5}{2}, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{3}{5} \left(\frac{1}{2}+m \right) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] - \right. \\
& \quad \left. (1+2m) \left(-\frac{3}{10} \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}+m, \frac{3}{2}, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} \left(\frac{3}{2}+m \right) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}+m, \frac{1}{2}, \frac{7}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Bigg) / \\
& \left(\sqrt{\text{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \left(\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}+m, \frac{1}{2}, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \frac{1}{3} \left(\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}+m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - (1+2m) \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}+m, \frac{1}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) +
\end{aligned}$$

$$\left(2^{1+m} \left(\frac{1}{2} + m \right) \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec[e + f x] \right)^{-\frac{1}{2}+m} \right. \\ \left. \tan \left[\frac{1}{2} (e + f x) \right] \left(-\cos \left[\frac{1}{2} (e + f x) \right] \sec[e + f x] \sin \left[\frac{1}{2} (e + f x) \right] + \cos \left[\frac{1}{2} (e + f x) \right]^2 \sec[e + f x] \tan[e + f x] \right) \right) / \\ \left(\sqrt{\sec \left[\frac{1}{2} (e + f x) \right]^2} \left(\text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} + m, \frac{1}{2}, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] - \right. \right. \\ \left. \frac{1}{3} \left(\text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] - \right. \right. \\ \left. \left. \left. (1 + 2m) \text{AppellF1} \left[\frac{3}{2}, \frac{3}{2} + m, \frac{1}{2}, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) \right) \right)$$

■ **Problem 349: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[e + f x])^m}{\sqrt{d \sec[e + f x]}} dx$$

Optimal (type 6, 96 leaves, 3 steps):

$$\left(2 \text{AppellF1} \left[-\frac{1}{2}, \frac{1}{2}, \frac{1}{2} - m, \frac{1}{2}, \sec[e + f x], -\sec[e + f x] \right] (1 + \sec[e + f x])^{-\frac{1}{2}-m} (a + a \sec[e + f x])^m \tan[e + f x] \right) / \\ \left(f \sqrt{1 - \sec[e + f x]} \sqrt{d \sec[e + f x]} \right)$$

Result (type 6, 2424 leaves):

$$- \left(\left(3 \times 2^{1+m} \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} + m, \frac{3}{2}, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \sqrt{\sec[e + f x]} \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec[e + f x] \right)^{-\frac{1}{2}+m} \right. \right. \\ \left. (1 + \sec[e + f x])^{-m} (a (1 + \sec[e + f x]))^m \left(\cos[2(e + f x)] \left(\frac{(1 + \sec[e + f x])^m}{2 \sqrt{\sec[e + f x]}} - \frac{1}{2} i \sqrt{\sec[e + f x]} (1 + \sec[e + f x])^m \sin[e + f x] \right) + \right. \right. \\ \left. \left. \frac{1}{2} (1 + \sec[e + f x])^m + \frac{1}{2} i (1 + \sec[e + f x])^m \sin[2(e + f x)] \right) \right] \sqrt{\sec[e + f x]} + \\ \left. \sqrt{\sec[e + f x]} \sin[e + f x] \left(-\frac{1}{2} i (1 + \sec[e + f x])^m + \frac{1}{2} (1 + \sec[e + f x])^m \sin[2(e + f x)] \right) \right) \tan \left[\frac{1}{2} (e + f x) \right] \right) /$$

$$\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} + m, \frac{3}{2}, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)$$

- **Problem 350: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[e + f x])^m}{(d \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 6, 98 leaves, 3 steps):

$$\left(2 \text{AppellF1}\left[-\frac{3}{2}, \frac{1}{2}, \frac{1}{2} - m, -\frac{1}{2}, \text{Sec}[e + f x], -\text{Sec}[e + f x]\right] (1 + \text{Sec}[e + f x])^{-\frac{1}{2}-m} (a + a \text{Sec}[e + f x])^m \text{Tan}[e + f x] \right) / \left(3 f \sqrt{1 - \text{Sec}[e + f x]} (d \text{Sec}[e + f x])^{3/2} \right)$$

Result (type 6, 5127 leaves): Display of huge result suppressed!

- **Problem 351: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x]^{7/2} (a + a \text{Sec}[c + d x]) dx$$

Optimal (type 4, 111 leaves, 7 steps):

$$\frac{6 a \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{10 a \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{10 a \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{21 d} + \frac{2 a \text{Cos}[c + d x]^{3/2} \text{Sin}[c + d x]}{5 d} + \frac{2 a \text{Cos}[c + d x]^{5/2} \text{Sin}[c + d x]}{7 d}$$

Result (type 5, 490 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(-\frac{3 \cot[c]}{5d} + \frac{23 \cos[dx] \sin[c]}{84d} + \right. \right. \\
& \quad \left. \left. \frac{\cos[2dx] \sin[2c]}{10d} + \frac{\cos[3dx] \sin[3c]}{28d} + \frac{23 \cos[c] \sin[dx]}{84d} + \frac{\cos[2c] \sin[2dx]}{10d} + \frac{\cos[3c] \sin[3dx]}{28d} \right) - \right. \\
& \quad \left. \frac{1}{21d \sqrt{1 + \cot[c]^2}} 5 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \quad \left. \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}} \right) - \right. \\
& \quad \left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)
\end{aligned}$$

- **Problem 352: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{5/2} (a + a \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 87 leaves, 6 steps):

$$\frac{6 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5 d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 d} + \frac{2 a \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d} + \frac{2 a \cos[c+dx]^{3/2} \sin[c+dx]}{5 d}$$

Result (type 5, 458 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(-\frac{3 \cot[c]}{5d} + \frac{\cos[dx] \sin[c]}{3d} + \frac{\cos[2dx] \sin[2c]}{10d} + \frac{\cos[c] \sin[dx]}{3d} + \frac{\cos[2c] \sin[2dx]}{10d} \right) - \right. \\
& \frac{1}{3d \sqrt{1 + \cot[c]^2}} (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right\] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \right)
\end{aligned}$$

■ **Problem 353: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{2a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a \sqrt{\cos[c+dx]} \sin[c+dx]}{3d}$$

Result (type 5, 424 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(-\frac{\cot[c]}{d} + \frac{\cos[dx] \sin[c]}{3d} + \frac{\cos[c] \sin[dx]}{3d} \right) - \frac{1}{3d \sqrt{1 + \cot[c]^2}} \right. \\
& (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} (1 + \cos[c+dx]) \\
& \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]\right]^2 \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 354: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a + a \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 35 leaves, 4 steps):

$$\frac{2a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d}$$

Result (type 5, 155 leaves):

$$\frac{1}{2d} a \sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2$$

$$\left(-2 \sqrt{\cos[dx - \operatorname{ArcTan}[\cot[c]]]^2} \sqrt{\csc[c]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \right.$$

$$\left. \sin[c] + \tan[dx + \operatorname{ArcTan}[\tan[c]]] - \frac{\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \tan[dx + \operatorname{ArcTan}[\tan[c]]]}{\sqrt{\sin[dx + \operatorname{ArcTan}[\tan[c]]]^2}} \right)$$

- **Problem 355: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Sec}[c+dx]}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 57 leaves, 5 steps):

$$-\frac{2a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2a \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 5, 413 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\frac{\operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} \right) - \frac{1}{d \sqrt{1 + \operatorname{Cot}[c]^2}} \right. \\
& (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{2d} (1 + \cos[c+dx]) \right. \\
& \left. \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) \right) / \\
& \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left. \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 356: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Sec}[c+dx]}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$-\frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 d} + \frac{2 a \operatorname{Sin}[c+dx]}{3 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{2 a \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 444 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\frac{\csc[c] \sec[c]}{d} + \frac{\sec[c] \sec[c+dx]^2 \sin[dx]}{3d} + \frac{\sec[c] \sec[c+dx] (\sin[c] + 3 \sin[dx])}{3d} \right) - \right. \\
& \frac{1}{3d \sqrt{1 + \cot[c]^2}} (1 + \cos[c+dx]) \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{2d} (1 + \cos[c+dx]) \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \Bigg)
\end{aligned}$$

■ **Problem 357: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sec[c+dx]}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 4, 111 leaves, 7 steps):

$$-\frac{6a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2a \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{2a \sin[c+dx]}{3d \cos[c+dx]^{3/2}} + \frac{6a \sin[c+dx]}{5d \sqrt{\cos[c+dx]}}$$

Result (type 5, 477 leaves):

a

$$\left(\sqrt{\cos[c+dx]} (1+\cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \left(\frac{3\operatorname{Csc}[c]\operatorname{Sec}[c]}{5d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^3\sin[dx]}{5d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx]^2(3\sin[c]+5\sin[dx])}{15d} + \frac{\operatorname{Sec}[c]\operatorname{Sec}[c+dx](5\sin[c]+9\sin[dx])}{15d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}}(1+\cos[c+dx])\operatorname{Csc}[c] \right.$$

$$\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx-\operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \operatorname{Sec}[dx-\operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx-\operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1+\cot[c]^2}\sin[c]\sin[dx-\operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx-\operatorname{ArcTan}[\cot[c]]]} + \frac{1}{10d}3(1+\cos[c+dx])\operatorname{Csc}[c]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx+\operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx+\operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1-\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[dx+\operatorname{ArcTan}[\tan[c]]]\tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2\cos[dx+\operatorname{ArcTan}[\tan[c]]]\sqrt{1+\tan[c]^2}}{\cos[c]^2+\sin[c]^2}}{\sqrt{\cos[c]\cos[dx+\operatorname{ArcTan}[\tan[c]]]}\sqrt{1+\tan[c]^2}} \right)$$

- **Problem 358: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \operatorname{Sec}[c+dx]}{\cos[c+dx]^{7/2}} dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$-\frac{6a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{10a \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} + \frac{2a \sin[c+dx]}{7d \cos[c+dx]^{7/2}} + \frac{2a \sin[c+dx]}{5d \cos[c+dx]^{5/2}} + \frac{10a \sin[c+dx]}{21d \cos[c+dx]^{3/2}} + \frac{6a \sin[c+dx]}{5d \sqrt{\cos[c+dx]}}$$

Result (type 5, 505 leaves):

a

$$\begin{aligned}
& \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\frac{3 \operatorname{Csc}[c] \operatorname{Sec}[c]}{5d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 \sin[dx]}{7d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (5 \sin[c] + 7 \sin[dx])}{35d} \right) \right. \\
& \quad \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (21 \sin[c] + 25 \sin[dx])}{105d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (25 \sin[c] + 63 \sin[dx])}{105d} \right) - \\
& \quad \frac{1}{21d \sqrt{1 + \cot[c]^2}} 5 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} + \frac{1}{10d} 3 (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \quad \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \quad \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \quad \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 359: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{9/2} (a + a \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 4, 147 leaves, 10 steps):

$$\begin{aligned}
& \frac{32 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{15d} + \frac{20 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21d} + \frac{20 a^2 \sqrt{\cos[c+dx]} \sin[c+dx]}{21d} + \\
& \frac{32 a^2 \cos[c+dx]^{3/2} \sin[c+dx]}{45d} + \frac{4 a^2 \cos[c+dx]^{5/2} \sin[c+dx]}{7d} + \frac{2 a^2 \cos[c+dx]^{7/2} \sin[c+dx]}{9d}
\end{aligned}$$

Result (type 5, 548 leaves) :

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-\frac{8 \cot [c]}{15 d}+\frac{23 \cos [d x] \sin [c]}{84 d}+\frac{37 \cos [2 d x] \sin [2 c]}{360 d}+\frac{\cos [3 d x] \sin [3 c]}{28 d}\right. \\ & \quad \left.+\frac{\cos [4 d x] \sin [4 c]}{144 d}+\frac{23 \cos [c] \sin [d x]}{84 d}+\frac{37 \cos [2 c] \sin [2 d x]}{360 d}+\frac{\cos [3 c] \sin [3 d x]}{28 d}+\frac{\cos [4 c] \sin [4 d x]}{144 d}\right)- \\ & \quad \frac{1}{21 d \sqrt{1+\cot [c]^2}} 5 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\ & \quad (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \quad \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{15 d} 4 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \quad \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \quad \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}} \sqrt{1+\tan [c]^2}\right)- \\ & \quad \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}}\right) \end{aligned}$$

■ **Problem 360: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{7 / 2} (a+a \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 121 leaves, 9 steps) :

$$\begin{aligned} & \frac{12 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{7 d}+ \\ & \frac{8 a^2 \sqrt{\cos [c+d x]} \sin [c+d x]}{7 d}+\frac{4 a^2 \cos [c+d x]^{3 / 2} \sin [c+d x]}{5 d}+\frac{2 a^2 \cos [c+d x]^{5 / 2} \sin [c+d x]}{7 d} \end{aligned}$$

Result (type 5, 516 leaves) :

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-\frac{3 \cot [c]}{5 d}+\frac{17 \cos [d x] \sin [c]}{56 d}+\frac{\cos [2 d x] \sin [2 c]}{10 d}+\right. \\ & \quad \left.\frac{\cos [3 d x] \sin [3 c]}{56 d}+\frac{17 \cos [c] \sin [d x]}{56 d}+\frac{\cos [2 c] \sin [2 d x]}{10 d}+\frac{\cos [3 c] \sin [3 d x]}{56 d}\right)-\frac{1}{7 d \sqrt{1+\cot [c]^2}} \\ & 2 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{10 d} 3 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\ & \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}\right) \end{aligned}$$

- **Problem 361: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{5 / 2} (a+a \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 95 leaves, 8 steps):

$$\frac{16 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{4 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{4 a^2 \sqrt{\cos [c+d x]} \sin [c+d x]}{3 d}+\frac{2 a^2 \cos [c+d x]^{3 / 2} \sin [c+d x]}{5 d}$$

Result (type 5, 484 leaves):

$$\begin{aligned} & \cos[c + dx]^{5/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 \left(-\frac{4 \cot[c]}{5d} + \frac{\cos[dx] \sin[c]}{3d} + \frac{\cos[2dx] \sin[2c]}{20d} + \frac{\cos[c] \sin[dx]}{3d} + \frac{\cos[2c] \sin[2dx]}{20d} \right) - \\ & \frac{1}{3d \sqrt{1 + \cot[c]^2}} \cos[c + dx]^2 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\ & \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{5d} 2 \cos[c + dx]^2 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4} \\ & (a + a \sec[c + dx])^2 \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\ & \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\ & \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \end{aligned}$$

■ **Problem 362: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} (a + a \sec[c + dx])^2 dx$$

Optimal (type 4, 67 leaves, 7 steps):

$$\frac{4 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2 a^2 \sqrt{\cos[c + dx]} \sin[c + dx]}{3d}$$

Result (type 5, 450 leaves):

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-\frac{\cot [c]}{d}+\frac{\cos [d x] \sin [c]}{6 d}+\frac{\cos [c] \sin [d x]}{6 d}\right)-\frac{1}{3 d \sqrt{1+\cot [c]^2}} \\ & 2 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{2 d} \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\ & \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \end{aligned}$$

- **Problem 364: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^2}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 4, 91 leaves, 8 steps):

$$-\frac{4 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{2 a^2 \sin [c+d x]}{3 d \cos [c+d x]^{3 / 2}}+\frac{4 a^2 \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 5, 470 leaves):

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(\frac{\operatorname{Csc}[c] \operatorname{Sec}[c]}{d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \sin [d x]}{6 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](\sin [c]+6 \sin [d x])}{6 d}\right)- \\ & \frac{1}{3 d \sqrt{1+\cot [c]^2}} 2 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\ & \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{2 d} \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4} \\ & (a+a \operatorname{Sec}[c+d x])^2\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\ & \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}\right) \end{aligned}$$

■ **Problem 365: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^2}{\cos [c+d x]^{3 / 2}} d x$$

Optimal (type 4, 121 leaves, 9 steps):

$$-\frac{16 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{4 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+\frac{2 a^2 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}}+\frac{4 a^2 \sin [c+d x]}{3 d \cos [c+d x]^{3 / 2}}+\frac{16 a^2 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}$$

Result (type 5, 503 leaves):

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 \left(\frac{4 \csc [c] \sec [c]}{5 d} + \frac{\sec [c] \sec [c+d x]^3 \sin [d x]}{10 d} + \right. \\ & \quad \left. \frac{\sec [c] \sec [c+d x]^2 (3 \sin [c]+10 \sin [d x])}{30 d} + \frac{\sec [c] \sec [c+d x] (5 \sin [c]+12 \sin [d x])}{15 d} \right) - \frac{1}{3 d \sqrt{1+\cot [c]^2}} \\ & \cos [c+d x]^2 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2 \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 \\ & \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \frac{1}{5 d} 2 \cos [c+d x]^2 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 \\ & \left(\operatorname{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\ & \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \\ & \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \end{aligned}$$

- **Problem 366: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sec [c+d x])^2}{\cos [c+d x]^{5 / 2}} d x$$

Optimal (type 4, 147 leaves, 10 steps):

$$\begin{aligned} & -\frac{12 a^2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d} + \frac{8 a^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{7 d} + \\ & \frac{2 a^2 \sin [c+d x]}{7 d \cos [c+d x]^{7 / 2}} + \frac{4 a^2 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}} + \frac{8 a^2 \sin [c+d x]}{7 d \cos [c+d x]^{3 / 2}} + \frac{12 a^2 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}} \end{aligned}$$

Result (type 5, 531 leaves):

$$\begin{aligned} & \cos [c+d x]^{5 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \left(\frac{3 \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^4 \operatorname{Sin}[d x]}{14 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(7 \operatorname{Sin}[c]+10 \operatorname{Sin}[d x])}{35 d}+\right. \\ & \left.\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3(5 \operatorname{Sin}[c]+14 \operatorname{Sin}[d x])}{70 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](10 \operatorname{Sin}[c]+21 \operatorname{Sin}[d x])}{35 d}\right)-\frac{1}{7 d \sqrt{1+\operatorname{Cot}[c]^2}} \\ & 2 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ & \sqrt{1+\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}+\frac{1}{10 d} 3 \cos [c+d x]^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \\ & \left(\sqrt{1-\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2}\right)- \\ & \left.\frac{\frac{\operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}}\right) \end{aligned}$$

■ **Problem 367: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{9 / 2} (a+a \operatorname{Sec}[c+d x])^3 d x$$

Optimal (type 4, 147 leaves, 17 steps):

$$\begin{aligned} & \frac{68 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d}+\frac{44 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+\frac{44 a^3 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{21 d}+ \\ & \frac{68 a^3 \operatorname{Cos}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{45 d}+\frac{6 a^3 \operatorname{Cos}[c+d x]^{5 / 2} \operatorname{Sin}[c+d x]}{7 d}+\frac{2 a^3 \operatorname{Cos}[c+d x]^{7 / 2} \operatorname{Sin}[c+d x]}{9 d} \end{aligned}$$

Result (type 5, 548 leaves):

$$\begin{aligned} & \cos[c + dx]^{7/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \left(-\frac{17 \cot[c]}{30d} + \frac{97 \cos[dx] \sin[c]}{336d} + \frac{73 \cos[2dx] \sin[2c]}{720d} + \frac{3 \cos[3dx] \sin[3c]}{112d} + \right. \\ & \quad \left. \frac{\cos[4dx] \sin[4c]}{288d} + \frac{97 \cos[c] \sin[dx]}{336d} + \frac{73 \cos[2c] \sin[2dx]}{720d} + \frac{3 \cos[3c] \sin[3dx]}{112d} + \frac{\cos[4c] \sin[4dx]}{288d} \right) - \\ & \frac{1}{42d \sqrt{1 + \cot[c]^2}} 11 \cos[c + dx]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\ & (a + a \sec[c + dx])^3 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{60d} 17 \cos[c + dx]^3 \operatorname{Csc}[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\ & \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\ & \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \end{aligned}$$

- **Problem 368: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{7/2} (a + a \sec[c + dx])^3 dx$$

Optimal (type 4, 121 leaves, 15 steps):

$$\begin{aligned} & \frac{28 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \frac{52 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21d} + \\ & \frac{52 a^3 \sqrt{\cos[c + dx]} \sin[c + dx]}{21d} + \frac{6 a^3 \cos[c + dx]^{3/2} \sin[c + dx]}{5d} + \frac{2 a^3 \cos[c + dx]^{5/2} \sin[c + dx]}{7d} \end{aligned}$$

Result (type 5, 516 leaves):

$$\begin{aligned} & \cos [c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3\left(-\frac{7 \cot [c]}{10 d}+\frac{107 \cos [d x] \sin [c]}{336 d}+\frac{3 \cos [2 d x] \sin [2 c]}{40 d}+\right. \\ & \left.\frac{\cos [3 d x] \sin [3 c]}{112 d}+\frac{107 \cos [c] \sin [d x]}{336 d}+\frac{3 \cos [2 c] \sin [2 d x]}{40 d}+\frac{\cos [3 c] \sin [3 d x]}{112 d}\right)-\frac{1}{42 d \sqrt{1+\cot [c]^2}} \\ & 13 \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-\frac{1}{20 d} 7 \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\ & \left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}}\right) \end{aligned}$$

■ **Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{5 / 2} (a+a \operatorname{Sec}[c+d x])^3 d x$$

Optimal (type 4, 91 leaves, 13 steps):

$$\frac{36 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{4 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{2 a^3 \sqrt{\cos [c+d x]} \sin [c+d x]}{d}+\frac{2 a^3 \cos [c+d x]^{3 / 2} \sin [c+d x]}{5 d}$$

Result (type 5, 484 leaves):

$$\begin{aligned} & \cos[c + dx]^{7/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \left(-\frac{9 \cot[c]}{10d} + \frac{\cos[dx] \sin[c]}{4d} + \frac{\cos[2dx] \sin[2c]}{40d} + \frac{\cos[c] \sin[dx]}{4d} + \frac{\cos[2c] \sin[2dx]}{40d} \right) - \\ & \frac{1}{2d \sqrt{1 + \cot[c]^2}} \cos[c + dx]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\ & (a + a \sec[c + dx])^3 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\ & \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{20d} 9 \cos[c + dx]^3 \operatorname{Csc}[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\ & \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\ & \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \end{aligned}$$

■ **Problem 370: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} (a + a \sec[c + dx])^3 dx$$

Optimal (type 4, 91 leaves, 13 steps):

$$\frac{4 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{20 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2 a^3 \sin[c + dx]}{d \sqrt{\cos[c + dx]}} + \frac{2 a^3 \sqrt{\cos[c + dx]} \sin[c + dx]}{3d}$$

Result (type 5, 481 leaves):

$$\begin{aligned} & \text{Cos}[c + dx]^{7/2} \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 \\ & \left(-\frac{(1 + 3 \text{Cos}[2c]) \text{Csc}[c] \text{Sec}[c]}{8d} + \frac{\text{Cos}[dx] \text{Sin}[c]}{12d} + \frac{\text{Cos}[c] \text{Sin}[dx]}{12d} + \frac{\text{Sec}[c] \text{Sec}[c + dx] \text{Sin}[dx]}{4d} \right) - \frac{1}{6d \sqrt{1 + \text{Cot}[c]^2}} \\ & 5 \text{Cos}[c + dx]^3 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 \\ & \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\ & \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{4d} \text{Cos}[c + dx]^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 \\ & \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ & \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ & \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \end{aligned}$$

■ **Problem 371: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\text{Cos}[c + dx]} (a + a \text{Sec}[c + dx])^3 dx$$

Optimal (type 4, 91 leaves, 13 steps):

$$-\frac{4a^3 \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{20a^3 \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2a^3 \text{Sin}[c + dx]}{3d \text{Cos}[c + dx]^{3/2}} + \frac{6a^3 \text{Sin}[c + dx]}{d \sqrt{\text{Cos}[c + dx]}}$$

Result (type 5, 479 leaves):

$$\begin{aligned} & \cos [c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(-\frac{(-5+\cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{8 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \sin [d x]}{12 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](\sin [c]+9 \sin [d x])}{12 d}\right)-\frac{1}{6 d \sqrt{1+\cot [c]^2}} \\ & 5 \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\ & \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}+\frac{1}{4 d} \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \\ & \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)- \\ & \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \end{aligned}$$

■ **Problem 372: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^3}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 4, 117 leaves, 15 steps):

$$-\frac{36 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{4 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{d}+\frac{2 a^3 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}}+\frac{2 a^3 \sin [c+d x]}{d \cos [c+d x]^{3 / 2}}+\frac{36 a^3 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}$$

Result (type 5, 501 leaves):

$$\begin{aligned} & \cos [c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\frac{9 \operatorname{Csc}[c] \operatorname{Sec}[c]}{10 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \operatorname{Sin}[d x]}{20 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(\operatorname{Sin}[c]+5 \operatorname{Sin}[d x])}{20 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](5 \operatorname{Sin}[c]+18 \operatorname{Sin}[d x])}{20 d}\right) \\ & \frac{1}{2 d \sqrt{1+\operatorname{Cot}[c]^2}} \operatorname{Cos}[c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \\ & (a+a \operatorname{Sec}[c+d x])^3 \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ & \sqrt{1+\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}+\frac{1}{20 d} 9 \operatorname{Cos}[c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \\ & \left(\sqrt{1-\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2}\right) - \\ & \frac{\frac{\operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \end{aligned}$$

■ **Problem 373: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^3}{\operatorname{Cos}[c+d x]^{3 / 2}} d x$$

Optimal (type 4, 147 leaves, 17 steps):

$$\begin{aligned} & -\frac{28 a^3 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{52 a^3 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+ \\ & \frac{2 a^3 \operatorname{Sin}[c+d x]}{7 d \operatorname{Cos}[c+d x]^{7 / 2}}+\frac{6 a^3 \operatorname{Sin}[c+d x]}{5 d \operatorname{Cos}[c+d x]^{5 / 2}}+\frac{52 a^3 \operatorname{Sin}[c+d x]}{21 d \operatorname{Cos}[c+d x]^{3 / 2}}+\frac{28 a^3 \operatorname{Sin}[c+d x]}{5 d \sqrt{\operatorname{Cos}[c+d x]}} \end{aligned}$$

Result (type 5, 531 leaves):

$$\begin{aligned} & \cos [c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\frac{7 \operatorname{Csc}[c] \operatorname{Sec}[c]}{10 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^4 \operatorname{Sin}[d x]}{28 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3(5 \operatorname{Sin}[c]+21 \operatorname{Sin}[d x])}{140 d}+\right. \\ & \left.\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2(63 \operatorname{Sin}[c]+130 \operatorname{Sin}[d x])}{420 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](65 \operatorname{Sin}[c]+147 \operatorname{Sin}[d x])}{210 d}\right)-\frac{1}{42 d \sqrt{1+\operatorname{Cot}[c]^2}} \\ & 13 \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ & \sqrt{1+\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}+\frac{1}{20 d} 7 \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 \\ & \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \\ & \left(\sqrt{1-\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2}\right)- \\ & \left(\frac{\frac{\operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}}\right) \end{aligned}$$

■ **Problem 374: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^{5 / 2}}{a+a \operatorname{Sec}[c+d x]} d x$$

Optimal (type 4, 128 leaves, 9 steps):

$$\begin{aligned} & \frac{21 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 a d}-\frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 a d}- \\ & \frac{5 \sqrt{\operatorname{Cos}[c+d x]} \operatorname{Sin}[c+d x]}{3 a d}+\frac{7 \operatorname{Cos}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{5 a d}-\frac{\operatorname{Cos}[c+d x]^{3 / 2} \operatorname{Sin}[c+d x]}{d(a+a \operatorname{Sec}[c+d x])} \end{aligned}$$

Result (type 5, 314 leaves):

$$\frac{1}{15 a (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \left(\left(2 i \sqrt{2} e^{-i(c+dx)} \left(63 (1 + e^{2i(c+dx)}) + 63 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 25 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + d x] \right) / \left(d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} + 1 / \left(d \sqrt{\operatorname{Cos}[c + d x]} \right) \left(-96 \operatorname{Cot}[c] - 30 \operatorname{Csc}[c] - 20 \operatorname{Cos}[d x] \operatorname{Sin}[c] + 6 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c] - 30 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] - 20 \operatorname{Cos}[c] \operatorname{Sin}[d x] + 6 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x] \right) \right) \right)$$

- **Problem 375: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2}}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 100 leaves, 8 steps):

$$-\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a d} + \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a d} + \frac{5 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 a d} - \frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 5, 292 leaves):

$$\frac{1}{3 a (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \left(- \left(2 i \sqrt{2} e^{-i(c+dx)} \left(9 (1 + e^{2i(c+dx)}) + 9 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + d x] \right) / \left(d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} + \frac{12 \operatorname{Cot}[c] + 6 \operatorname{Csc}[c] + 4 \operatorname{Cos}[d x] \operatorname{Sin}[c] + 6 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + 4 \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d \sqrt{\operatorname{Cos}[c + d x]}} \right)$$

- **Problem 376: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + d x]}}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 72 leaves, 7 steps):

$$\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a d} - \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{a d} - \frac{\operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])}$$

Result (type 5, 270 leaves):

$$\frac{1}{a(1 + \operatorname{Sec}[c + dx])} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \left(\left(2i\sqrt{2} e^{-i(c+dx)} \left(3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + e^{i(c+dx)}(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + dx] \right) / \left(d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \right) - \frac{2(2\operatorname{Cot}[c] + \operatorname{Csc}[c] + \operatorname{Sec}\left[\frac{c}{2}\right]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d\sqrt{\operatorname{Cos}[c + dx]}} \right)$$

- **Problem 377: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 70 leaves, 7 steps):

$$-\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{\operatorname{Sin}[c + dx]}{d\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])}$$

Result (type 5, 262 leaves):

$$\frac{1}{a(1 + \operatorname{Sec}[c + dx])} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \left(- \left(2i\sqrt{2} e^{-i(c+dx)} \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + e^{i(c+dx)}(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + dx] \right) / \left(d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \right) + \frac{2(\operatorname{Csc}[c] + \operatorname{Sec}\left[\frac{c}{2}\right]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d\sqrt{\operatorname{Cos}[c + dx]}} \right)$$

- **Problem 378: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 70 leaves, 7 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{\operatorname{Sin}[c + dx]}{d\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])}$$

Result (type 5, 263 leaves):

$$\frac{1}{a(1 + \operatorname{Sec}[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left(\left(2i\sqrt{2} e^{-i(c+dx)} \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - e^{i(c+dx)} \right. \right. \right. \\ \left. \left. \left. (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}[c + dx] \right) / \left(d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} - \frac{2(\operatorname{Csc}[c] + \operatorname{Sec}\left[\frac{c}{2}\right]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d\sqrt{\operatorname{Cos}[c + dx]}} \right) \right)$$

- **Problem 379: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 96 leaves, 8 steps):

$$-\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{3 \operatorname{Sin}[c + dx]}{ad\sqrt{\operatorname{Cos}[c + dx]}} - \frac{\operatorname{Sin}[c + dx]}{d \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])}$$

Result (type 5, 303 leaves):

$$\frac{1}{a(1 + \operatorname{Sec}[c + dx])} \cos\left[\frac{1}{2}(c + dx)\right]^2 \left(\frac{\left(2 \operatorname{Cos}\left[\frac{1}{2}(c - dx)\right] + \operatorname{Cos}\left[\frac{1}{2}(3c + dx)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(c + 3dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]}{2d \operatorname{Cos}[c + dx]^{3/2}} - \right. \\ \left. \left(2i\sqrt{2} e^{-i(c+dx)} \left(3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - e^{i(c+dx)}(-1 + e^{2ic}) \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}[c + dx] \right) / \left(d(-1 + e^{2ic}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \right) \right)$$

- **Problem 380: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + dx]^{7/2} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 124 leaves, 9 steps):

$$\frac{3 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} + \frac{5 \operatorname{Sin}[c + dx]}{3ad \operatorname{Cos}[c + dx]^{3/2}} - \frac{3 \operatorname{Sin}[c + dx]}{ad\sqrt{\operatorname{Cos}[c + dx]}} - \frac{\operatorname{Sin}[c + dx]}{d \operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])}$$

Result (type 5, 338 leaves):

$$\frac{1}{3 a (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\left(-1 / (4 d \operatorname{Cos}[c + d x]^{5/2}) \left(10 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 8 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 4 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 9 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right]\right)\right.$$

$$\left.\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] + \left(2 i \sqrt{2} e^{-i(c+d x)} \left(9 \left(1 + e^{2 i(c+d x)}\right) + 9(-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 5 e^{i(c+d x)}(-1 + e^{2 i c})\right.\right.\right.$$

$$\left.\left.\sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \operatorname{Sec}[c + d x]\right) / \left(d(-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)}(1 + e^{2 i(c+d x)})}\right)$$

- **Problem 381: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{5/2}}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 160 leaves, 10 steps):

$$\frac{56 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 a^2 d} - \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} - \frac{5 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{a^2 d} +$$

$$\frac{56 \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{15 a^2 d} - \frac{3 \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{a^2 d (1 + \operatorname{Sec}[c + d x])} - \frac{\operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 5, 366 leaves):

$$\frac{1}{5 a^2 (1 + \operatorname{Sec}[c + d x])^2}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^4 \left(\left(4 i \sqrt{2} e^{-i(c+d x)} \left(56 \left(1 + e^{2 i(c+d x)} \right) + 56 \left(-1 + e^{2 i c} \right) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 25 e^{i(c+d x)} \right.\right. \right.$$

$$\left. \left. \left. \left(-1 + e^{2 i c} \right) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \operatorname{Sec}[c + d x]^2 \right) / \left(d \left(-1 + e^{2 i c} \right) \sqrt{e^{-i(c+d x)} \left(1 + e^{2 i(c+d x)} \right)} \right) +$$

$$\frac{1}{3 d \operatorname{Cos}[c + d x]^{3/2}} \left(-216 \operatorname{Cot}[c] - 120 \operatorname{Csc}[c] - 40 \operatorname{Cos}[d x] \operatorname{Sin}[c] + 6 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c] - 120 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] + \right.$$

$$\left. 5 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Sin}\left[\frac{d x}{2}\right] - 40 \operatorname{Cos}[c] \operatorname{Sin}[d x] + 6 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x] + 5 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right] \right)$$

- **Problem 382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2}}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 138 leaves, 9 steps):

$$-\frac{7 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{10 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} +$$

$$\frac{10 \sqrt{\cos[c+dx]} \sin[c+dx]}{3 a^2 d} - \frac{7 \sqrt{\cos[c+dx]} \sin[c+dx]}{3 a^2 d (1 + \sec[c+dx])} - \frac{\sqrt{\cos[c+dx]} \sin[c+dx]}{3 d (a + a \sec[c+dx])^2}$$

Result (type 5, 341 leaves):

$$\frac{1}{3 a^2 (1 + \sec[c+dx])^2}$$

$$\cos\left[\frac{1}{2}(c+dx)\right]^4 \left(- \left(4 i \sqrt{2} e^{-i(c+dx)} \left(21 (1 + e^{2i(c+dx)}) + 21 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 10 e^{i(c+dx)} \right. \right. \right.$$

$$\left. \left. (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \sec[c+dx]^2 \right) / \left(d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) +$$

$$\frac{1}{d \cos[c+dx]^{3/2}} \left(48 \cot[c] + 36 \csc[c] + 8 \cos[dx] \sin[c] + 36 \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{dx}{2}\right] - \right.$$

$$\left. 2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c+dx)\right]^3 \sin\left[\frac{dx}{2}\right] + 8 \cos[c] \sin[dx] - 2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{c}{2}\right] \right)$$

■ **Problem 383: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]}}{(a + a \sec[c+dx])^2} dx$$

Optimal (type 4, 112 leaves, 8 steps):

$$\frac{4 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{5 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{5 \sin[c+dx]}{3 a^2 d \sqrt{\cos[c+dx]} (1 + \sec[c+dx])} - \frac{\sin[c+dx]}{3 d \sqrt{\cos[c+dx]} (a + a \sec[c+dx])^2}$$

Result (type 5, 374 leaves):

$$\left(4 i \sqrt{2} e^{-i(c+dx)} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(12 (1 + e^{2i(c+dx)}) + 12 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \right.$$

$$\left. 5 e^{i(c+dx)} (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \sec[c+dx]^2 \right) /$$

$$\left(3 d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} (a + a \sec[c+dx])^2 \right) +$$

$$\frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(-\frac{8 \cot\left[\frac{c}{2}\right]}{d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{3 d} + \frac{2 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3 d} \right)}{\cos[c+dx]^{3/2} (a + a \sec[c+dx])^2}$$

- **Problem 384: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} (a+a \sec[c+dx])^2} dx$$

Optimal (type 4, 109 leaves, 8 steps):

$$-\frac{\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2 \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} + \frac{\sin[c+dx]}{a^2 d \sqrt{\cos[c+dx]} (1+\sec[c+dx])} - \frac{\sin[c+dx]}{3 d \cos[c+dx]^{3/2} (a+a \sec[c+dx])^2}$$

Result (type 5, 656 leaves):

$$\begin{aligned} & -\frac{1}{2(a+a \sec[c+dx])^2} i \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c+dx]^2 \\ & \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2(1+e^{2 i d x}) \cos[c] + 2 i (-1+e^{2 i d x}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i d x} \cos[2c] + i e^{2 i d x} \sin[2c]} \right) / (3 i d (1+e^{2 i d x}) \cos[c] - 3 d (-1+e^{2 i d x}) \sin[c]) - \right. \\ & \left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2(1+e^{2 i d x}) \cos[c] + 2 i (-1+e^{2 i d x}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2 i d x} \cos[2c] + i e^{2 i d x} \sin[2c]} \right) / (-i d (1+e^{2 i d x}) \cos[c] + d (-1+e^{2 i d x}) \sin[c]) \right) - \\ & \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c+dx]^2 \right. \\ & \quad \left. \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\ & \quad \left. \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \left(3 d \sqrt{1 + \text{Cot}[c]^2} (a+a \sec[c+dx])^2 \right) + \\ & \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(\frac{4 \text{Csc}[c]}{d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} - \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sin\left[\frac{dx}{2}\right]}{3 d} - \frac{2 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3 d} \right)}{\cos[c+dx]^{3/2} (a+a \sec[c+dx])^2} \end{aligned}$$

- **Problem 386: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{5/2} (a+a \sec[c+dx])^2} dx$$

Optimal (type 4, 109 leaves, 8 steps):

$$\frac{\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{2 \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{\text{Sin}[c+dx]}{a^2 d \sqrt{\text{Cos}[c+dx]} (1+\text{Sec}[c+dx])} - \frac{\text{Sin}[c+dx]}{3 d \text{Cos}[c+dx]^{3/2} (a+a \text{Sec}[c+dx])^2}$$

Result (type 5, 370 leaves):

$$\begin{aligned} & \left(4 i \sqrt{2} e^{-i(c+dx)} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(3 (1+e^{2i(c+dx)}) + 3 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right. \\ & \quad \left. \left. 2 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \text{Sec}[c+dx]^2 \right) / \\ & \left(3 d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} (a+a \text{Sec}[c+dx])^2 \right) + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(-\frac{4 \text{Csc}[c]}{d} - \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sin}\left[\frac{dx}{2}\right]}{d} - \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \text{Sin}\left[\frac{dx}{2}\right]}{3 d} - \frac{2 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3 d} \right)}{\text{Cos}[c+dx]^{3/2} (a+a \text{Sec}[c+dx])^2} \end{aligned}$$

■ **Problem 387: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\text{Cos}[c+dx]^{7/2} (a+a \text{Sec}[c+dx])^2} dx$$

Optimal (type 4, 136 leaves, 9 steps):

$$\begin{aligned} & -\frac{4 \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{5 \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} + \\ & \frac{4 \text{Sin}[c+dx]}{a^2 d \sqrt{\text{Cos}[c+dx]}} - \frac{5 \text{Sin}[c+dx]}{3 a^2 d \text{Cos}[c+dx]^{3/2} (1+\text{Sec}[c+dx])} - \frac{\text{Sin}[c+dx]}{3 d \text{Cos}[c+dx]^{5/2} (a+a \text{Sec}[c+dx])^2} \end{aligned}$$

Result (type 5, 393 leaves):

$$\begin{aligned} & -\left(4 i \sqrt{2} e^{-i(c+dx)} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(12 (1+e^{2i(c+dx)}) + 12 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - \right. \right. \\ & \quad \left. \left. 5 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \text{Sec}[c+dx]^2 \right) / \\ & \left(3 d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} (a+a \text{Sec}[c+dx])^2 \right) + \\ & \left(\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \left(\frac{8 \text{Cot}\left[\frac{c}{2}\right] \text{Sec}[c]}{d} + \frac{8 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sin}\left[\frac{dx}{2}\right]}{d} + \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \text{Sin}\left[\frac{dx}{2}\right]}{3 d} + \right. \right. \\ & \quad \left. \left. \frac{8 \text{Sec}[c] \text{Sec}[c+dx] \text{Sin}[dx]}{d} + \frac{2 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / (\text{Cos}[c+dx]^{3/2} (a+a \text{Sec}[c+dx])^2) \end{aligned}$$

- **Problem 388: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{9/2} (a+a \sec[c+dx])^2} dx$$

Optimal (type 4, 162 leaves, 10 steps):

$$\frac{7 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{10 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} + \frac{10 \sin[c+dx]}{3 a^2 d \cos[c+dx]^{3/2}} - \frac{7 \sin[c+dx]}{a^2 d \sqrt{\cos[c+dx]}} - \frac{7 \sin[c+dx]}{3 a^2 d \cos[c+dx]^{5/2} (1+\sec[c+dx])} - \frac{\sin[c+dx]}{3 d \cos[c+dx]^{7/2} (a+a \sec[c+dx])^2}$$

Result (type 5, 372 leaves):

$$\frac{1}{3 a^2 (1+\sec[c+dx])^2} \cos\left[\frac{1}{2}(c+dx)\right]^4 \left(-\frac{1}{8 d \cos[c+dx]^{7/2}} \left(82 \cos\left[\frac{1}{2}(c-dx)\right] + 65 \cos\left[\frac{1}{2}(3c+dx)\right] + 68 \cos\left[\frac{1}{2}(c+3dx)\right] + 37 \cos\left[\frac{1}{2}(5c+3dx)\right] + 53 \cos\left[\frac{1}{2}(3c+5dx)\right] + 10 \cos\left[\frac{1}{2}(7c+5dx)\right] + 21 \cos\left[\frac{1}{2}(5c+7dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 + \left(4 i \sqrt{2} e^{-i(c+dx)} \left(21 (1+e^{2i(c+dx)}) + 21 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - 10 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c+dx]^2 \right) / \left(d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \right) \right)$$

- **Problem 389: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[c+dx]^{5/2}}{(a+a \sec[c+dx])^3} dx$$

Optimal (type 4, 207 leaves, 11 steps):

$$\frac{231 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{21 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} - \frac{21 \sqrt{\cos[c+dx]} \sin[c+dx]}{2 a^3 d} + \frac{77 \cos[c+dx]^{3/2} \sin[c+dx]}{10 a^3 d} - \frac{\cos[c+dx]^{3/2} \sin[c+dx]}{5 d (a+a \sec[c+dx])^3} - \frac{4 \cos[c+dx]^{3/2} \sin[c+dx]}{5 a d (a+a \sec[c+dx])^2} - \frac{63 \cos[c+dx]^{3/2} \sin[c+dx]}{10 d (a^3+a^3 \sec[c+dx])}$$

Result (type 5, 391 leaves):

$$\frac{1}{5 a^3 d (1 + \operatorname{Sec}[c + d x])^3} 2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6$$

$$\left(\left(42 i \sqrt{2} e^{-i(c+dx)} \left(11 (1 + e^{2i(c+dx)}) + 11 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 5 e^{i(c+dx)} (-1 + e^{2ic}) \right. \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + d x]^3 \right) / \left((-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) + \frac{1}{\operatorname{Cos}[c + d x]^{5/2}}$$

$$\left(-264 \operatorname{Cot}[c] - 198 \operatorname{Csc}[c] + \frac{1}{16} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \left(-1210 \operatorname{Sin}\left[\frac{d x}{2}\right] + 770 \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 840 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 150 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - \right. \right.$$

$$\left. \left. 238 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 40 \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 5 \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] - 5 \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right) \right)$$

- **Problem 390: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2}}{(a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 181 leaves, 10 steps):

$$-\frac{119 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{11 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{2 a^3 d} + \frac{11 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{2 a^3 d} -$$

$$\frac{\sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} - \frac{2 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 a d (a + a \operatorname{Sec}[c + d x])^2} - \frac{119 \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{30 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 375 leaves):

$$\frac{1}{5 a^3 (1 + \operatorname{Sec}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6$$

$$\left(- \left(4 i \sqrt{2} e^{-i(c+dx)} \left(119 (1 + e^{2i(c+dx)}) + 119 (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 55 e^{i(c+dx)} (-1 + e^{2ic}) \right. \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c + d x]^3 \right) / \left(d (-1 + e^{2ic}) \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \right) +$$

$$\frac{1}{3 d \operatorname{Cos}[c + d x]^{5/2}} \left(720 \operatorname{Cot}[c] + 708 \operatorname{Csc}[c] + \frac{1}{4} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \left(1061 \operatorname{Sin}\left[\frac{d x}{2}\right] - 709 \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 715 \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - \right. \right.$$

$$\left. \left. 170 \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 202 \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 25 \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 5 \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + 5 \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] \right) \right)$$

- **Problem 391: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + d x]}}{(a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$\frac{49 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{13 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6 a^3 d} - \frac{\operatorname{Sin}[c+dx]}{5 d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^3} - \frac{8 \operatorname{Sin}[c+dx]}{15 a d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^2} - \frac{13 \operatorname{Sin}[c+dx]}{6 d \sqrt{\operatorname{Cos}[c+dx]} (a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 5, 357 leaves):

$$\frac{1}{15 a^3 (1+\operatorname{Sec}[c+dx])^3} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^6 \left(-\frac{1}{8 d \operatorname{Cos}[c+dx]^{5/2}} \left(806 \operatorname{Cos}\left[\frac{1}{2}(c-dx)\right] + 664 \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + 470 \operatorname{Cos}\left[\frac{1}{2}(c+3dx)\right] + 265 \operatorname{Cos}\left[\frac{1}{2}(5c+3dx)\right] + 117 \operatorname{Cos}\left[\frac{1}{2}(3c+5dx)\right] + 30 \operatorname{Cos}\left[\frac{1}{2}(7c+5dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 + \left(4 i \sqrt{2} e^{-i(c+dx)} \left(147 (1+e^{2i(c+dx)}) + 147 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 65 e^{i(c+dx)} (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c+dx]^3 \right) / \left(d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \right)$$

- **Problem 392: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$-\frac{9 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} - \frac{\operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^3} + \frac{2 \operatorname{Sin}[c+dx]}{5 a d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^2} + \frac{\operatorname{Sin}[c+dx]}{2 d \sqrt{\operatorname{Cos}[c+dx]} (a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 5, 721 leaves):

$$\begin{aligned}
& - \frac{1}{10 (a + a \operatorname{Sec}[c + dx])^3} 9 i \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \\
& \left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^3 \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left(d \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(\frac{36 \operatorname{Csc}[c]}{5 d} + \frac{36 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \frac{12 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} \right. \right. \\
& \quad \left. \left. \frac{12 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right) \right) / \left(\operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 393: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} + \frac{\operatorname{Sin}[c + dx]}{5 d \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} - \\
& \frac{\operatorname{Sin}[c + dx]}{15 a d \sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])^2} + \frac{\operatorname{Sin}[c + dx]}{6 d \sqrt{\operatorname{Cos}[c + dx]} (a^3 + a^3 \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 342 leaves):

$$\frac{1}{15 a^3 (1 + \operatorname{Sec}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6$$

$$\left(1 / (8 d \operatorname{Cos}[c + d x]^{5/2}) \left(14 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 16 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 20 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] - 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right]\right)\right)$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 -$$

$$\left(4 i \sqrt{2} e^{-i(c+d x)} \left(3(1 + e^{2 i(c+d x)}) + 3(-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 5 e^{i(c+d x)}(-1 + e^{2 i c})\right)\right)$$

$$\left(\sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \operatorname{Sec}[c + d x]^3 \Big/ \left(d(-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)}(1 + e^{2 i(c+d x)})}\right)$$

- **Problem 394: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$\frac{\operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{6 a^3 d} - \frac{\operatorname{Sin}[c + d x]}{5 d \operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^3} -$$

$$\frac{4 \operatorname{Sin}[c + d x]}{15 a d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2} + \frac{\operatorname{Sin}[c + d x]}{6 d \sqrt{\operatorname{Cos}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 342 leaves):

$$\frac{1}{15 a^3 (1 + \operatorname{Sec}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6$$

$$\left(-1 / (8 d \operatorname{Cos}[c + d x]^{5/2}) \left(4 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 26 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 10 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 5 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 3 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right]\right)\right)$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 +$$

$$\left(4 i \sqrt{2} e^{-i(c+d x)} \left(3(1 + e^{2 i(c+d x)}) + 3(-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 5 e^{i(c+d x)}(-1 + e^{2 i c})\right)\right)$$

$$\left(\sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right]\right) \operatorname{Sec}[c + d x]^3 \Big/ \left(d(-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)}(1 + e^{2 i(c+d x)})}\right)$$

- **Problem 395: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c + d x]^{7/2} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 155 leaves, 9 steps):

$$\frac{9 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} - \frac{\operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^3} - \frac{2 \operatorname{Sin}[c+dx]}{5 a d \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^2} - \frac{9 \operatorname{Sin}[c+dx]}{10 d \sqrt{\operatorname{Cos}[c+dx]} (a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 5, 721 leaves):

$$\frac{1}{10 (a+a \operatorname{Sec}[c+dx])^3} 9 i \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^3$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / \left(3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / \left(-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c] \right) \right) -$$

$$\left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^3 \right.$$

$$\left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right.$$

$$\left. \left. \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left(d \sqrt{1 + \operatorname{Cot}[c]^2} (a+a \operatorname{Sec}[c+dx])^3 \right) +$$

$$\left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \left(-\frac{36 \operatorname{Csc}[c]}{5 d} - \frac{36 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \operatorname{Sin}\left[\frac{dx}{2}\right]}{5 d} \right. \right.$$

$$\left. \left. \frac{8 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right) \right) / \left(\operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^3 \right)$$

■ **Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{9/2} (a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 181 leaves, 10 steps):

$$\begin{aligned}
& - \frac{49 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} - \frac{13 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6 a^3 d} + \frac{49 \operatorname{Sin}[c+dx]}{10 a^3 d \sqrt{\operatorname{Cos}[c+dx]}} - \\
& \frac{\operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^3} - \frac{8 \operatorname{Sin}[c+dx]}{15 a d \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^2} - \frac{13 \operatorname{Sin}[c+dx]}{6 d \operatorname{Cos}[c+dx]^{3/2} (a^3+a^3 \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 5, 372 leaves):

$$\begin{aligned}
& \frac{1}{15 a^3 (1+\operatorname{Sec}[c+dx])^3} \\
& \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^6 \left(\frac{1}{16 d \operatorname{Cos}[c+dx]^{7/2}} \left(1284 \operatorname{Cos}\left[\frac{1}{2}(c-dx)\right] + 921 \operatorname{Cos}\left[\frac{1}{2}(3c+dx)\right] + 1243 \operatorname{Cos}\left[\frac{1}{2}(c+3dx)\right] + 374 \operatorname{Cos}\left[\frac{1}{2}(5c+3dx)\right] + \right. \right. \\
& \quad \left. \left. 670 \operatorname{Cos}\left[\frac{1}{2}(3c+5dx)\right] + 65 \operatorname{Cos}\left[\frac{1}{2}(7c+5dx)\right] + 147 \operatorname{Cos}\left[\frac{1}{2}(5c+7dx)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 - \right. \\
& \quad \left. \left(4 i \sqrt{2} e^{-i(c+dx)} \left(147 (1+e^{2i(c+dx)}) + 147 (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] - 65 e^{i(c+dx)} \right. \right. \right. \\
& \quad \left. \left. \left. (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c+dx]^3 \right) / \left(d (-1+e^{2ic}) \sqrt{e^{-i(c+dx)} (1+e^{2i(c+dx)})} \right) \right)
\end{aligned}$$

■ **Problem 397: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{11/2} (a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 207 leaves, 11 steps):

$$\begin{aligned}
& \frac{119 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10 a^3 d} + \frac{11 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{2 a^3 d} + \frac{11 \operatorname{Sin}[c+dx]}{2 a^3 d \operatorname{Cos}[c+dx]^{3/2}} - \frac{119 \operatorname{Sin}[c+dx]}{10 a^3 d \sqrt{\operatorname{Cos}[c+dx]}} - \\
& \frac{\operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{9/2} (a+a \operatorname{Sec}[c+dx])^3} - \frac{2 \operatorname{Sin}[c+dx]}{3 a d \operatorname{Cos}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^2} - \frac{119 \operatorname{Sin}[c+dx]}{30 d \operatorname{Cos}[c+dx]^{5/2} (a^3+a^3 \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 5, 402 leaves):

$$\frac{1}{5 a^3 (1 + \operatorname{Sec}[c + d x])^3} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^6 \left(-\frac{1}{96 d \operatorname{Cos}[c + d x]^{9/2}} \left(5134 \operatorname{Cos}\left[\frac{1}{2}(c - d x)\right] + 4148 \operatorname{Cos}\left[\frac{1}{2}(3 c + d x)\right] + 4664 \operatorname{Cos}\left[\frac{1}{2}(c + 3 d x)\right] + 2476 \operatorname{Cos}\left[\frac{1}{2}(5 c + 3 d x)\right] + 3340 \operatorname{Cos}\left[\frac{1}{2}(3 c + 5 d x)\right] + 944 \operatorname{Cos}\left[\frac{1}{2}(7 c + 5 d x)\right] + 1620 \operatorname{Cos}\left[\frac{1}{2}(5 c + 7 d x)\right] + 165 \operatorname{Cos}\left[\frac{1}{2}(9 c + 7 d x)\right] + 357 \operatorname{Cos}\left[\frac{1}{2}(7 c + 9 d x)\right] \right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 + \left(4 i \sqrt{2} e^{-i(c+d x)} \left(119 (1 + e^{2 i(c+d x)}) + 119 (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] - 55 e^{i(c+d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+d x)}\right] \right) \operatorname{Sec}[c + d x]^3 \right) / \left(d (-1 + e^{2 i c}) \sqrt{e^{-i(c+d x)} (1 + e^{2 i(c+d x)})} \right)$$

■ **Problem 402: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]}}{\sqrt{\operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 57 leaves, 3 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d}$$

Result (type 5, 113 leaves):

$$-\frac{1}{3 d} 2 i e^{\frac{1}{2} i(c+d x)} \sqrt{\operatorname{Cos}[c+d x]} \left(3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right] + e^{i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1 + \operatorname{Sec}[c+d x])}$$

■ **Problem 403: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]}}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{\sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d} + \frac{a \operatorname{Sin}[c+d x]}{d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 5, 146 leaves):

$$\frac{1}{3d} \sqrt{\cos[c+dx]} \sqrt{a(1+\sec[c+dx])} \left(-3i e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec\left[\frac{1}{2}(c+dx)\right] - \right. \\ \left. i e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \sec\left[\frac{1}{2}(c+dx)\right] + 3 \sec[c+dx] \tan\left[\frac{1}{2}(c+dx)\right] \right)$$

- **Problem 404: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a \sec[c+dx]}}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{3\sqrt{a} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} + \frac{a \sin[c+dx]}{2d \cos[c+dx]^{5/2} \sqrt{a+a \sec[c+dx]}} + \frac{3a \sin[c+dx]}{4d \cos[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}}$$

Result (type 5, 145 leaves):

$$\frac{1}{4d} \sqrt{\cos[c+dx]} \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \left(-3i e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - \right. \\ \left. i e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + \sec[c+dx] (3+2 \sec[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

- **Problem 408: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\cos[c+dx]} (a+a \sec[c+dx])^{3/2} dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$\frac{2a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{2a^2 \sin[c+dx]}{d \sqrt{\cos[c+dx]} \sqrt{a+a \sec[c+dx]}}$$

Result (type 5, 132 leaves):

$$-\frac{1}{3d(1+e^{i(c+dx)})} 2i a \sqrt{\cos[c+dx]} \\ \left(-3+3e^{i(c+dx)}+6e^{i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] + 2e^{2i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \right) \\ \sqrt{a(1+\sec[c+dx])}$$

- **Problem 409: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec[c+dx])^{3/2}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 95 leaves, 5 steps):

$$\frac{3 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d} + \frac{a^2 \operatorname{Sin}[c+d x]}{d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 5, 171 leaves):

$$\frac{1}{2 d \sqrt{\operatorname{Cos}[c+d x]}} a (1 + \operatorname{Cos}[c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{a (1 + \operatorname{Sec}[c+d x])} \\ \left(-3 i e^{\frac{1}{2} i (c+d x)} \operatorname{Cos}[c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] - \right. \\ \left. i e^{\frac{3}{2} i (c+d x)} \operatorname{Cos}[c+d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 410: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+d x])^{3/2}}{\operatorname{Cos}[c+d x]^{3/2}} dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{7 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{4 d} + \frac{a^2 \operatorname{Sin}[c+d x]}{2 d \operatorname{Cos}[c+d x]^{5/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{7 a^2 \operatorname{Sin}[c+d x]}{4 d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 5, 167 leaves):

$$\frac{1}{24 d \operatorname{Cos}[c+d x]^{3/2}} \\ a (1 + \operatorname{Cos}[c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c+d x])} \left(-21 i e^{\frac{1}{2} i (c+d x)} \operatorname{Cos}[c+d x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \right. \\ \left. 7 i e^{\frac{3}{2} i (c+d x)} \operatorname{Cos}[c+d x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + 3 (2 + 7 \operatorname{Cos}[c+d x]) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 411: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+d x])^{3/2}}{\operatorname{Cos}[c+d x]^{5/2}} dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$\frac{11 a^{3/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{8 d} + \\ \frac{a^2 \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{7/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{11 a^2 \operatorname{Sin}[c+d x]}{12 d \operatorname{Cos}[c+d x]^{5/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{11 a^2 \operatorname{Sin}[c+d x]}{8 d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 5, 176 leaves) :

$$\frac{1}{48 d \operatorname{Cos}[c + d x]^{5/2}}$$

$$a (1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c + d x])} \left(-33 i e^{\frac{1}{2} i (c + d x)} \operatorname{Cos}[c + d x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - \right.$$

$$\left. 11 i e^{\frac{3}{2} i (c + d x)} \operatorname{Cos}[c + d x]^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + (8 + 22 \operatorname{Cos}[c + d x] + 33 \operatorname{Cos}[c + d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 415: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 3, 138 leaves, 5 steps) :

$$\frac{2 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{d} +$$

$$\frac{14 a^3 \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{2 a^2 \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d}$$

Result (type 5, 155 leaves) :

$$\frac{1}{12 d} a^2 \sqrt{\operatorname{Cos}[c + d x]} (1 + \operatorname{Cos}[c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 \sqrt{a (1 + \operatorname{Sec}[c + d x])} \left(-6 i e^{\frac{1}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - \right.$$

$$\left. 2 i e^{\frac{3}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + 15 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] \right)$$

■ **Problem 416: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 3, 132 leaves, 5 steps) :

$$\frac{5 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{a^3 \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{a^2 \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Cos}[c + d x]}}$$

Result (type 5, 167 leaves) :

$$\frac{1}{12 d \sqrt{\cos [c+d x]}}$$

$$a^2 (1 + \cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1 + \operatorname{Sec}[c+d x])} \left(-15 i e^{\frac{1}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \right.$$

$$\left. 5 i e^{\frac{3}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + 3 (1 + 2 \cos [c+d x]) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 417: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+d x])^{5/2}}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{19 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{4 d} + \frac{9 a^3 \sin [c+d x]}{4 d \cos [c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{2 d \cos [c+d x]^{3/2}}$$

Result (type 5, 171 leaves):

$$\frac{1}{48 d \cos [c+d x]^{3/2}}$$

$$a^2 (1 + \cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1 + \operatorname{Sec}[c+d x])} \left(-57 i e^{\frac{1}{2} i (c+d x)} \cos [c+d x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \right.$$

$$\left. 19 i e^{\frac{3}{2} i (c+d x)} \cos [c+d x]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + 3 (2 + 11 \cos [c+d x]) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 418: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+d x])^{5/2}}{\cos [c+d x]^{3/2}} dx$$

Optimal (type 3, 180 leaves, 6 steps):

$$\frac{25 a^{5/2} \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{8 d} +$$

$$\frac{13 a^3 \sin [c+d x]}{12 d \cos [c+d x]^{5/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{25 a^3 \sin [c+d x]}{8 d \cos [c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 d \cos [c+d x]^{5/2}}$$

Result (type 5, 180 leaves):

$$\frac{1}{96 d \cos [c+d x]^{5/2}} a^2 (1+\cos [c+d x])^2 \sec \left[\frac{1}{2} (c+d x) \right]^5 \sqrt{a(1+\sec [c+d x])} \left(-75 i e^{\frac{1}{2} i (c+d x)} \cos [c+d x]^3 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)} \right] - 25 i e^{\frac{3}{2} i (c+d x)} \cos [c+d x]^3 \operatorname{Hypergeometric2F1} \left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)} \right] + (8+34 \cos [c+d x]+75 \cos [c+d x]^2) \sin \left[\frac{1}{2} (c+d x) \right] \right)$$

■ **Problem 419: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{5/2}}{\cos [c+d x]^{5/2}} dx$$

Optimal (type 3, 220 leaves, 7 steps):

$$\frac{163 a^{5/2} \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}} \right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 d} + \frac{17 a^3 \sin [c+d x]}{24 d \cos [c+d x]^{7/2} \sqrt{a+a \sec [c+d x]}} + \frac{163 a^3 \sin [c+d x]}{96 d \cos [c+d x]^{5/2} \sqrt{a+a \sec [c+d x]}} + \frac{163 a^3 \sin [c+d x]}{64 d \cos [c+d x]^{3/2} \sqrt{a+a \sec [c+d x]}} + \frac{a^2 \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{4 d \cos [c+d x]^{7/2}}$$

Result (type 5, 190 leaves):

$$\frac{1}{768 d \cos [c+d x]^{7/2}} a^2 (1+\cos [c+d x])^2 \sec \left[\frac{1}{2} (c+d x) \right]^5 \sqrt{a(1+\sec [c+d x])} \left(-489 i e^{\frac{1}{2} i (c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)} \right] - 163 i e^{\frac{3}{2} i (c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1} \left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)} \right] + (48+184 \cos [c+d x]+326 \cos [c+d x]^2+489 \cos [c+d x]^3) \sin \left[\frac{1}{2} (c+d x) \right] \right)$$

■ **Problem 441: Unable to integrate problem.**

$$\int (d \cos [e+f x])^n (a+a \sec [e+f x])^3 dx$$

Optimal (type 5, 244 leaves, 8 steps):

$$\frac{a^3 (7-4 n) (d \cos [e+f x])^n \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos [e+f x]^2 \right] \sin [e+f x]}{f (2-n) n \sqrt{\sin [e+f x]^2}} - \frac{a^3 (1-4 n) \cos [e+f x] (d \cos [e+f x])^n \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [e+f x]^2 \right] \sin [e+f x]}{f (1-n) (1+n) \sqrt{\sin [e+f x]^2}} + \frac{a^3 (5-2 n) (d \cos [e+f x])^n \tan [e+f x]}{f (1-n) (2-n)} + \frac{(d \cos [e+f x])^n (a^3+a^3 \sec [e+f x]) \tan [e+f x]}{f (2-n)}$$

Result (type 8, 25 leaves) :

$$\int (d \cos [e + f x])^n (a + a \sec [e + f x])^3 dx$$

■ **Problem 442: Unable to integrate problem.**

$$\int (d \cos [e + f x])^n (a + a \sec [e + f x])^2 dx$$

Optimal (type 5, 179 leaves, 7 steps) :

$$\frac{2 a^2 (d \cos [e + f x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos [e + f x]^2\right] \sin [e + f x]}{f n \sqrt{\sin [e + f x]^2}} - \frac{a^2 (1 - 2 n) \cos [e + f x] (d \cos [e + f x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [e + f x]^2\right] \sin [e + f x]}{f (1 - n) (1 + n) \sqrt{\sin [e + f x]^2}} + \frac{a^2 (d \cos [e + f x])^n \tan [e + f x]}{f (1 - n)}$$

Result (type 8, 25 leaves) :

$$\int (d \cos [e + f x])^n (a + a \sec [e + f x])^2 dx$$

■ **Problem 443: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d \cos [e + f x])^n (a + a \sec [e + f x]) dx$$

Optimal (type 5, 132 leaves, 5 steps) :

$$\frac{a (d \cos [e + f x])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos [e + f x]^2\right] \sin [e + f x]}{f n \sqrt{\sin [e + f x]^2}} - \frac{a (d \cos [e + f x])^{1+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos [e + f x]^2\right] \sin [e + f x]}{d f (1 + n) \sqrt{\sin [e + f x]^2}}$$

Result (type 6, 3856 leaves) :

$$\left(a \cos [e + f x]^n (d \cos [e + f x])^n (1 + \sec [e + f x]) \tan \left[\frac{1}{2} (e + f x) \right] \right. \\ \left. \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, 1 - n, n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) / \left(\left(-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right. \right. \right. \\ \left. \left. \left(-3 \operatorname{AppellF1} \left[\frac{1}{2}, 1 - n, n, \frac{3}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + 2 \left(n \operatorname{AppellF1} \left[\frac{3}{2}, 1 - n, 1 + n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \right. \right. \\ \left. \left. \left. -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] + (-1 + n) \operatorname{AppellF1} \left[\frac{3}{2}, 2 - n, n, \frac{5}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) +$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, -n, 1+n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(n \operatorname{AppellF1}\left[\frac{3}{2}, 1-n, 1+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+n) \operatorname{AppellF1}\left[\frac{3}{2}, -n, 2+n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 \right)$$

■ **Problem 444: Unable to integrate problem.**

$$\int \frac{(d \cos[e+fx])^n}{a+a \sec[e+fx]} dx$$

Optimal (type 5, 178 leaves, 7 steps):

$$\frac{(d \cos[e+fx])^n \sin[e+fx]}{f(a+a \sec[e+fx])} - \frac{\cos[e+fx] (d \cos[e+fx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a f \sqrt{\sin[e+fx]^2}} + \frac{(1+n) \cos[e+fx]^2 (d \cos[e+fx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a f (2+n) \sqrt{\sin[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{(d \cos[e+fx])^n}{a+a \sec[e+fx]} dx$$

■ **Problem 445: Unable to integrate problem.**

$$\int \frac{(d \cos[e+fx])^n}{(a+a \sec[e+fx])^2} dx$$

Optimal (type 5, 215 leaves, 8 steps):

$$\frac{2(2+n)(d \cos[e+fx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{3 a^2 f \sqrt{\sin[e+fx]^2}} - \frac{(3+2n) \cos[e+fx] (d \cos[e+fx])^n \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{3 a^2 f \sqrt{\sin[e+fx]^2}} - \frac{2(2+n)(d \cos[e+fx])^n \tan[e+fx]}{3 a^2 f (1+\sec[e+fx])} - \frac{(d \cos[e+fx])^n \tan[e+fx]}{3 f (a+a \sec[e+fx])^2}$$

Result (type 8, 25 leaves):

$$\int \frac{(d \cos[e+fx])^n}{(a+a \sec[e+fx])^2} dx$$

■ **Problem 446: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^4 (a + b \text{Sec}[c + d x]) dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{3 b \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{a \text{Tan}[c + d x]}{d} + \frac{3 b \text{Sec}[c + d x] \text{Tan}[c + d x]}{8 d} + \frac{b \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{4 d} + \frac{a \text{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$\begin{aligned} & - \frac{3 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \\ & \frac{b}{16 d \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{b}{16 d \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b}{16 d \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \\ & \frac{3 b}{16 d \left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{2 a \text{Tan}[c + d x]}{3 d} + \frac{a \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3 d} \end{aligned}$$

■ **Problem 449: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x]) dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{a \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{b \text{Tan}[c + d x]}{d}$$

Result (type 3, 81 leaves):

$$- \frac{a \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b \text{Tan}[c + d x]}{d}$$

■ **Problem 450: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[c + d x]) dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$a x + \frac{b \text{ArcTanh}[\text{Sin}[c + d x]]}{d}$$

Result (type 3, 73 leaves):

$$a x - \frac{b \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d}$$

■ **Problem 456: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^4 (a+b \sec [c+d x])^2 d x$$

Optimal (type 3, 135 leaves, 7 steps):

$$\frac{3 a b \operatorname{ArcTanh}[\sin [c+d x]]}{4 d} + \frac{(5 a^2+4 b^2) \tan [c+d x]}{5 d} + \frac{3 a b \sec [c+d x] \tan [c+d x]}{4 d} +$$

$$\frac{a b \sec [c+d x]^3 \tan [c+d x]}{2 d} + \frac{b^2 \sec [c+d x]^4 \tan [c+d x]}{5 d} + \frac{(5 a^2+4 b^2) \tan [c+d x]^3}{15 d}$$

Result (type 3, 301 leaves):

$$-\frac{3 a b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{4 d} + \frac{3 a b \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{4 d} + \frac{a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} +$$

$$\frac{3 a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 a b}{8 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} +$$

$$\frac{2 a^2 \tan [c+d x]}{3 d} + \frac{8 b^2 \tan [c+d x]}{15 d} + \frac{a^2 \sec [c+d x]^2 \tan [c+d x]}{3 d} + \frac{4 b^2 \sec [c+d x]^2 \tan [c+d x]}{15 d} + \frac{b^2 \sec [c+d x]^4 \tan [c+d x]}{5 d}$$

■ **Problem 457: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^3 (a+b \sec [c+d x])^2 d x$$

Optimal (type 3, 110 leaves, 6 steps):

$$\frac{(4 a^2+3 b^2) \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} + \frac{2 a b \tan [c+d x]}{d} + \frac{(4 a^2+3 b^2) \sec [c+d x] \tan [c+d x]}{8 d} + \frac{b^2 \sec [c+d x]^3 \tan [c+d x]}{4 d} + \frac{2 a b \tan [c+d x]^3}{3 d}$$

Result (type 3, 375 leaves):

$$-\frac{a^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{3 b^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} +$$

$$\frac{a^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{3 b^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{b^2}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} +$$

$$\frac{a^2}{4 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{3 b^2}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{b^2}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} -$$

$$\frac{a^2}{4 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{3 b^2}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{4 a b \tan [c+d x]}{3 d} + \frac{2 a b \sec [c+d x]^2 \tan [c+d x]}{3 d}$$

■ **Problem 459: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$\frac{(2 a^2 + b^2) \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} + \frac{2 a b \text{Tan}[c + d x]}{d} + \frac{b^2 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 d}$$

Result (type 3, 218 leaves):

$$\begin{aligned} & \frac{1}{4 d} \text{Sec}[c + d x]^2 \left(-2 a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\ & (2 a^2 + b^2) \text{Cos}[2(c + d x)] \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\ & 2 a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 2 b^2 \text{Sin}[c + d x] + 4 a b \text{Sin}[2(c + d x)] \end{aligned}$$

■ **Problem 460: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$a^2 x + \frac{2 a b \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{b^2 \text{Tan}[c + d x]}{d}$$

Result (type 3, 77 leaves):

$$\frac{1}{d} \left(a \left(a c + a d x - 2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + b^2 \text{Tan}[c + d x] \right)$$

■ **Problem 461: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x] (a + b \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$2 a b x + \frac{b^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{a^2 \text{Sin}[c + d x]}{d}$$

Result (type 3, 105 leaves):

$$2 a b x - \frac{b^2 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b^2 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a^2 \text{Cos}[d x] \text{Sin}[c]}{d} + \frac{a^2 \text{Cos}[c] \text{Sin}[d x]}{d}$$

■ **Problem 466: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^3 (a + b \text{Sec}[c + d x])^3 dx$$

Optimal (type 3, 189 leaves, 8 steps) :

$$\frac{a(4a^2 + 9b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} - \frac{(3a^4 - 52a^2b^2 - 16b^4) \operatorname{Tan}[c + dx]}{30bd} - \frac{a(6a^2 - 71b^2) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{120d} -$$

$$\frac{(3a^2 - 16b^2)(a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]}{60bd} - \frac{a(a + b \operatorname{Sec}[c + dx])^3 \operatorname{Tan}[c + dx]}{20bd} + \frac{(a + b \operatorname{Sec}[c + dx])^4 \operatorname{Tan}[c + dx]}{5bd}$$

Result (type 3, 619 leaves) :

$$\frac{(-4a^3 - 9ab^2) \operatorname{Log}[\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)]]}{8d} + \frac{(4a^3 + 9ab^2) \operatorname{Log}[\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)]]}{8d} +$$

$$\frac{15ab^2 + 2b^3}{80d (\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)])^4} + \frac{60a^3 + 60a^2b + 135ab^2 + 19b^3}{240d (\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)])^2} + \frac{b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)]}{20d (\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)])^5} +$$

$$\frac{b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)]}{20d (\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)])^5} + \frac{-15ab^2 - 2b^3}{80d (\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)])^4} + \frac{-60a^3 - 60a^2b - 135ab^2 - 19b^3}{240d (\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)])^2} +$$

$$\frac{2(15a^2b \operatorname{Sin}[\frac{1}{2}(c + dx)] + 4b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)])}{15d (\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)])} + \frac{2(15a^2b \operatorname{Sin}[\frac{1}{2}(c + dx)] + 4b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)])}{15d (\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)])} +$$

$$\frac{60a^2b \operatorname{Sin}[\frac{1}{2}(c + dx)] + 19b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)]}{120d (\operatorname{Cos}[\frac{1}{2}(c + dx)] - \operatorname{Sin}[\frac{1}{2}(c + dx)])^3} + \frac{60a^2b \operatorname{Sin}[\frac{1}{2}(c + dx)] + 19b^3 \operatorname{Sin}[\frac{1}{2}(c + dx)]}{120d (\operatorname{Cos}[\frac{1}{2}(c + dx)] + \operatorname{Sin}[\frac{1}{2}(c + dx)])^3}$$

■ **Problem 467: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^2 (a + b \operatorname{Sec}[c + dx])^3 dx$$

Optimal (type 3, 130 leaves, 7 steps) :

$$\frac{3b(4a^2 + b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} + \frac{a(a^2 + 4b^2) \operatorname{Tan}[c + dx]}{2d} +$$

$$\frac{b(2a^2 + 3b^2) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{8d} + \frac{a(a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]}{4d} + \frac{(a + b \operatorname{Sec}[c + dx])^3 \operatorname{Tan}[c + dx]}{4d}$$

Result (type 3, 455 leaves) :

$$\begin{aligned}
& - \frac{3(4a^2b + b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{3(4a^2b + b^3) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \\
& \frac{b^3}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{12a^2b + 4ab^2 + 3b^3}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{2d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \frac{b^3}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{2d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{-12a^2b - 4ab^2 - 3b^3}{16d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 2ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 2ab^2 \sin\left[\frac{1}{2}(c + dx)\right]}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 468: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + b \sec[c + dx])^3 dx$$

Optimal (type 3, 99 leaves, 6 steps):

$$\frac{a(2a^2 + 3b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{2b(4a^2 + b^2) \tan[c + dx]}{3d} + \frac{5ab^2 \sec[c + dx] \tan[c + dx]}{6d} + \frac{b(a + b \sec[c + dx])^2 \tan[c + dx]}{3d}$$

Result (type 3, 206 leaves):

$$\begin{aligned}
& - \frac{1}{24d} \sec[c + dx]^3 \left(9a(2a^2 + 3b^2) \cos[c + dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\
& \quad \left. 3a(2a^2 + 3b^2) \cos[3(c + dx)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \right. \\
& \quad \left. 4b(9a^2 + 4b^2 + 9ab \cos[c + dx] + (9a^2 + 2b^2) \cos[2(c + dx)]) \sin[c + dx] \right)
\end{aligned}$$

■ **Problem 469: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[c + dx])^3 dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$a^3 x + \frac{b(6a^2 + b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{5ab^2 \tan[c + dx]}{2d} + \frac{b^2(a + b \sec[c + dx]) \tan[c + dx]}{2d}$$

Result (type 3, 256 leaves):

$$\frac{1}{4d} \operatorname{Sec}[c+dx]^2 \left(2a^3c + 2a^3dx - 6a^2b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$6a^2b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Cos}[2(c+dx)]$$

$$\left(2a^3(c+dx) - b(6a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b(6a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 2$$

$$b^3 \operatorname{Sin}[c+dx] + 6ab^2 \operatorname{Sin}[2(c+dx)]$$

■ **Problem 477: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^4 dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$\frac{ab(4a^2+3b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2d} + \frac{2(3a^4+28a^2b^2+4b^4) \operatorname{Tan}[c+dx]}{15d} + \frac{ab(6a^2+29b^2) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{30d} +$$

$$\frac{(3a^2+4b^2)(a+b \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]}{15d} + \frac{a(a+b \operatorname{Sec}[c+dx])^3 \operatorname{Tan}[c+dx]}{5d} + \frac{(a+b \operatorname{Sec}[c+dx])^4 \operatorname{Tan}[c+dx]}{5d}$$

Result (type 3, 663 leaves):

$$\frac{(-4a^3b-3ab^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{(4a^3b+3ab^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{2d} +$$

$$\frac{10ab^3+b^4}{40d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{240a^3b+120a^2b^2+180ab^3+19b^4}{240d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{20d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} +$$

$$\frac{b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{20d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \frac{-10ab^3-b^4}{40d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{-240a^3b-120a^2b^2-180ab^3-19b^4}{240d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{15a^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 60a^2b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 8b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \frac{15a^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 60a^2b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 8b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{15d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} +$$

$$\frac{120a^2b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 19b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{120d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{120a^2b^2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 19b^4 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{120d \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3}$$

■ **Problem 478: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx] (a+b \operatorname{Sec}[c+dx])^4 dx$$

Optimal (type 3, 146 leaves, 7 steps):

$$\frac{(8a^4 + 24a^2b^2 + 3b^4) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{ab(19a^2 + 16b^2) \tan[c + dx]}{6d} +$$

$$\frac{b^2(26a^2 + 9b^2) \sec[c + dx] \tan[c + dx]}{24d} + \frac{7ab(a + b \sec[c + dx])^2 \tan[c + dx]}{12d} + \frac{b(a + b \sec[c + dx])^3 \tan[c + dx]}{4d}$$

Result (type 3, 807 leaves):

$$\frac{(-8a^4 - 24a^2b^2 - 3b^4) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] (a + b \sec[c + dx])^4}{8d(b + a \cos[c + dx])^4} +$$

$$\frac{(8a^4 + 24a^2b^2 + 3b^4) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] (a + b \sec[c + dx])^4}{8d(b + a \cos[c + dx])^4} +$$

$$\frac{b^4 \cos[c + dx]^4 (a + b \sec[c + dx])^4}{16d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(72a^2b^2 + 16ab^3 + 9b^4) \cos[c + dx]^4 (a + b \sec[c + dx])^4}{48d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} +$$

$$\frac{2ab^3 \cos[c + dx]^4 (a + b \sec[c + dx])^4 \sin\left[\frac{1}{2}(c + dx)\right]}{3d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \frac{b^4 \cos[c + dx]^4 (a + b \sec[c + dx])^4}{16d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} +$$

$$\frac{2ab^3 \cos[c + dx]^4 (a + b \sec[c + dx])^4 \sin\left[\frac{1}{2}(c + dx)\right]}{3d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(-72a^2b^2 - 16ab^3 - 9b^4) \cos[c + dx]^4 (a + b \sec[c + dx])^4}{48d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} +$$

$$\frac{4 \cos[c + dx]^4 (a + b \sec[c + dx])^4 \left(3a^3b \sin\left[\frac{1}{2}(c + dx)\right] + 2ab^3 \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} +$$

$$\frac{4 \cos[c + dx]^4 (a + b \sec[c + dx])^4 \left(3a^3b \sin\left[\frac{1}{2}(c + dx)\right] + 2ab^3 \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3d(b + a \cos[c + dx])^4 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}$$

■ **Problem 479: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[c + dx])^4 dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$a^4 x + \frac{2ab(2a^2 + b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{b^2(17a^2 + 2b^2) \tan[c + dx]}{3d} + \frac{4ab^3 \sec[c + dx] \tan[c + dx]}{3d} + \frac{b^2(a + b \sec[c + dx])^2 \tan[c + dx]}{3d}$$

Result (type 3, 246 leaves):

$$\frac{1}{12d} \operatorname{Sec}[c+dx]^3 \left(9a \operatorname{Cos}[c+dx] \left(a^3(c+dx) - 2b(2a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 2b(2a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 3a \operatorname{Cos}[3(c+dx)] \left(a^3(c+dx) - 2b(2a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 2b(2a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 4b^2(9a^2+2b^2+6ab \operatorname{Cos}[c+dx] + (9a^2+b^2) \operatorname{Cos}[2(c+dx)]) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 480: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] (a+b \operatorname{Sec}[c+dx])^4 dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$4a^3bx + \frac{b^2(12a^2+b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2d} + \frac{a^2(2a^2-b^2) \operatorname{Sin}[c+dx]}{2d} + \frac{b^2(a+b \operatorname{Sec}[c+dx])^2 \operatorname{Sin}[c+dx]}{2d} + \frac{3ab^3 \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 280 leaves):

$$\frac{1}{4d} \operatorname{Sec}[c+dx]^2 \left(8a^3bc + 8a^3bdx - 12a^2b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - b^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 12a^2b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b \operatorname{Cos}[2(c+dx)] \right. \\ \left. \left(8a^3(c+dx) - b(12a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b(12a^2+b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + (a^4+2b^4) \operatorname{Sin}[c+dx] + 8ab^3 \operatorname{Sin}[2(c+dx)] + a^4 \operatorname{Sin}[3(c+dx)] \right)$$

■ **Problem 486: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+dx])^5 dx$$

Optimal (type 3, 158 leaves, 7 steps):

$$a^5x + \frac{b(40a^4+40a^2b^2+3b^4) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} + \frac{ab^2(53a^2+20b^2) \operatorname{Tan}[c+dx]}{6d} + \frac{b^3(58a^2+9b^2) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{24d} + \frac{11ab^2(a+b \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]}{12d} + \frac{b^2(a+b \operatorname{Sec}[c+dx])^3 \operatorname{Tan}[c+dx]}{4d}$$

Result (type 3, 855 leaves):

$$\begin{aligned}
& \frac{a^5 (c + dx) \cos [c + dx]^5 (a + b \sec [c + dx])^5}{d (b + a \cos [c + dx])^5} + \\
& \frac{(-40 a^4 b - 40 a^2 b^3 - 3 b^5) \cos [c + dx]^5 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right] (a + b \sec [c + dx])^5}{8 d (b + a \cos [c + dx])^5} + \\
& \frac{(40 a^4 b + 40 a^2 b^3 + 3 b^5) \cos [c + dx]^5 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right] (a + b \sec [c + dx])^5}{8 d (b + a \cos [c + dx])^5} + \\
& \frac{b^5 \cos [c + dx]^5 (a + b \sec [c + dx])^5}{16 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^4} + \frac{(120 a^2 b^3 + 20 a b^4 + 9 b^5) \cos [c + dx]^5 (a + b \sec [c + dx])^5}{48 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{5 a b^4 \cos [c + dx]^5 (a + b \sec [c + dx])^5 \sin \left[\frac{1}{2} (c + dx) \right]}{6 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} - \frac{b^5 \cos [c + dx]^5 (a + b \sec [c + dx])^5}{16 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{5 a b^4 \cos [c + dx]^5 (a + b \sec [c + dx])^5 \sin \left[\frac{1}{2} (c + dx) \right]}{6 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \frac{(-120 a^2 b^3 - 20 a b^4 - 9 b^5) \cos [c + dx]^5 (a + b \sec [c + dx])^5}{48 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{10 \cos [c + dx]^5 (a + b \sec [c + dx])^5 \left(3 a^3 b^2 \sin \left[\frac{1}{2} (c + dx) \right] + a b^4 \sin \left[\frac{1}{2} (c + dx) \right] \right)}{3 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{10 \cos [c + dx]^5 (a + b \sec [c + dx])^5 \left(3 a^3 b^2 \sin \left[\frac{1}{2} (c + dx) \right] + a b^4 \sin \left[\frac{1}{2} (c + dx) \right] \right)}{3 d (b + a \cos [c + dx])^5 \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)}
\end{aligned}$$

■ **Problem 505: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c + dx]^3}{(a + b \sec [c + dx])^2} dx$$

Optimal (type 3, 261 leaves, 8 steps):

$$\begin{aligned}
& -\frac{b (a^2 + 4 b^2) x}{a^5} + \frac{2 b^4 (5 a^2 - 4 b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{a+b}} \right]}{a^5 (a-b)^{3/2} (a+b)^{3/2} d} + \frac{(2 a^4 + 7 a^2 b^2 - 12 b^4) \sin [c + dx]}{3 a^4 (a^2 - b^2) d} - \\
& \frac{b (a^2 - 2 b^2) \cos [c + dx] \sin [c + dx]}{a^3 (a^2 - b^2) d} + \frac{(a^2 - 4 b^2) \cos [c + dx]^2 \sin [c + dx]}{3 a^2 (a^2 - b^2) d} + \frac{b^2 \cos [c + dx]^2 \sin [c + dx]}{a (a^2 - b^2) d (a + b \sec [c + dx])}
\end{aligned}$$

Result (type 3, 176 leaves):

$$\frac{1}{12 a^5 d} \left(-12 b (-i a + 2 b) (i a + 2 b) (c + d x) + \frac{24 b^4 (-5 a^2 + 4 b^2) \operatorname{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{3/2}} + \right.$$

$$\left. 9 a (a^2 + 4 b^2) \sin[c + d x] + \frac{12 a b^5 \sin[c + d x]}{(-a+b)(a+b)(b+a \cos[c + d x])} - 6 a^2 b \sin[2(c + d x)] + a^3 \sin[3(c + d x)] \right)$$

■ **Problem 529: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(5 + 3 \sec[c + d x])^3} dx$$

Optimal (type 3, 120 leaves, 6 steps):

$$\frac{x}{125} + \frac{8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right]}{256000 d} -$$

$$\frac{8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right]}{256000 d} + \frac{9 \tan[c + d x]}{160 d (5 + 3 \sec[c + d x])^2} + \frac{963 \tan[c + d x]}{12800 d (5 + 3 \sec[c + d x])}$$

Result (type 3, 241 leaves):

$$\frac{1}{512000 d (3 + 5 \cos[c + d x])^2} \left(88064 c + 88064 d x + 359523 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] + \right.$$

$$60 \cos[c + d x] \left(2048 (c + d x) + 8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] - 8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) +$$

$$25 \cos[2(c + d x)] \left(2048 (c + d x) + 8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] - 8361 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) -$$

$$\left. 359523 \operatorname{Log}\left[2 \cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] + 115560 \sin[c + d x] + 110700 \sin[2(c + d x)] \right)$$

■ **Problem 530: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(5 + 3 \sec[c + d x])^4} dx$$

Optimal (type 3, 145 leaves, 7 steps):

$$\frac{x}{625} + \frac{278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{20\,480\,000 d} - \frac{278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{20\,480\,000 d} +$$

$$\frac{3 \operatorname{Tan}[c+dx]}{80 d (5+3 \operatorname{Sec}[c+dx])^3} + \frac{519 \operatorname{Tan}[c+dx]}{12\,800 d (5+3 \operatorname{Sec}[c+dx])^2} + \frac{38\,733 \operatorname{Tan}[c+dx]}{1\,024\,000 d (5+3 \operatorname{Sec}[c+dx])}$$

Result (type 3, 344 leaves):

$$\frac{1}{81\,920\,000 d (3+5 \operatorname{Cos}[c+dx])^3} \left(18\,284\,544 c + 18\,284\,544 dx + 4\,096\,000 c \operatorname{Cos}[3(c+dx)] + 4\,096\,000 dx \operatorname{Cos}[3(c+dx)] + \right.$$

$$155\,208\,258 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 34\,768\,875 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] +$$

$$915 \operatorname{Cos}[c+dx] \left(32\,768 (c+dx) + 278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) +$$

$$450 \operatorname{Cos}[2(c+dx)] \left(32\,768 (c+dx) + 278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 278\,151 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) -$$

$$155\,208\,258 \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 34\,768\,875 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] +$$

$$\left. 52\,174\,260 \operatorname{Sin}[c+dx] + 51\,462\,000 \operatorname{Sin}[2(c+dx)] + 24\,286\,500 \operatorname{Sin}[3(c+dx)] \right)$$

■ **Problem 531: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+dx]^3 \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 292 leaves, 5 steps):

$$\frac{1}{15 b^3 d} 2 (a-b) \sqrt{a+b} (2 a^2 - 9 b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{15 b^2 d} 2 (a-b) \sqrt{a+b} (2 a+9 b) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{4 a \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{15 b d} + \frac{2 (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Tan}[c+dx]}{5 b d}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[c+dx]^3 \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

■ **Problem 532: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 241 leaves, 4 steps) :

$$-\frac{1}{3b^2d} 2a(a-b)\sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{2(a-b)\sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{3bd} + \frac{2\sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 8, 25 leaves) :

$$\int \operatorname{Sec}[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

■ **Problem 534: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 125 leaves, 1 step) :

$$-\frac{1}{\sqrt{a+b}d}$$

$$2 \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b \operatorname{Sec}[c+dx]}}\right], \frac{a-b}{a+b}\right] \sqrt{-\frac{b(1-\operatorname{Sec}[c+dx])}{a+b \operatorname{Sec}[c+dx]}} \sqrt{\frac{b(1+\operatorname{Sec}[c+dx])}{a+b \operatorname{Sec}[c+dx]}} (a+b \operatorname{Sec}[c+dx])$$

Result (type 8, 16 leaves) :

$$\int \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

■ **Problem 535: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 330 leaves, 6 steps) :

$$\begin{aligned}
& \frac{(a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{bd} + \\
& \frac{\sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d} - \\
& \frac{b \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{ad} + \frac{\sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d}
\end{aligned}$$

Result (type 4, 2713 leaves):

$$\begin{aligned}
& \left(\operatorname{Cos}[c+dx] \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \right. \\
& \left(i(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \\
& \left. 2ib \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \\
& \left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} (b+a \operatorname{Cos}[c+dx]) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4} \right. \\
& \left. \frac{1}{\sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(i (a - b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + \right. \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - \\
& \left. \sqrt{2} \sqrt{\frac{-a + b}{a + b}} \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} (b + a \operatorname{Cos}[c + d x]) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) + \\
& \frac{1}{2 \sqrt{\frac{-a + b}{a + b}} (b + a \operatorname{Cos}[c + d x])^{3/2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^4}} a \sqrt{\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec}[c + d x] \operatorname{Sin}[c + d x]} \\
& \left(i (a - b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + \right. \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - \\
& \left. \sqrt{2} \sqrt{\frac{-a + b}{a + b}} \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} (b + a \operatorname{Cos}[c + d x]) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) - \\
& \frac{1}{2 \sqrt{\frac{-a + b}{a + b}} \sqrt{b + a \operatorname{Cos}[c + d x]} \left(\operatorname{Cos}[c + d x] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^4 \right)^{3/2}} \sqrt{\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec}[c + d x]} \\
& \left(i (a - b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + \right. \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} -
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a\cos[c+dx]) \tan\left[\frac{1}{2}(c+dx)\right] \right) \\
& \left(-\sec\left[\frac{1}{2}(c+dx)\right]^4 \sin[c+dx] + 2\cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \frac{1}{\sqrt{\frac{-a+b}{a+b}} \sqrt{b+a\cos[c+dx]} \sqrt{\cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^4} \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}} \\
& \left(-\frac{\sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{2}} + \sqrt{2} a \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sin[c+dx] \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \left. \frac{\sqrt{\frac{-a+b}{a+b}} (b+a\cos[c+dx]) \left(\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{2} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \left(i(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(c+dx)\right] \right], \frac{a+b}{a-b} \right] \left(-\frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]}{a+b} + \frac{(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{a+b} \right) \right) \right) / \\
& \left(2 \sqrt{\frac{(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \left(i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] \right], \frac{a+b}{a-b} \right] \right. \right. \\
& \left. \left. \left(-\frac{a \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx]}{a+b} + \frac{(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{a+b} \right) \right) \right) / \\
& \left(\sqrt{\frac{(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \frac{b \sqrt{\frac{-a+b}{a+b}} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\left(1 - \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b} \right) \sqrt{1 + \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \sqrt{1 + \frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \right)} - \right.
\end{aligned}$$

$$\left. \frac{(a-b) \sqrt{\frac{-a+b}{a+b}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a-b}}}{2 \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}\right] +$$

$$\left(\left(i (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$\sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a \cos[c+dx]) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)$$

$$\left(-\cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) \Big/$$

$$\left(2 \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \cos[c+dx]} \sqrt{\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4} \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \right) \Big) \Big)$$

■ **Problem 536: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 396 leaves, 7 steps):

$$\frac{(a-b)\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4ad} +$$

$$\frac{\sqrt{a+b}(2a+b)\operatorname{Cot}[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4ad} - \frac{1}{4a^2d}$$

$$\frac{\sqrt{a+b}(4a^2-b^2)\operatorname{Cot}[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4ad} +$$

$$\frac{b\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{4ad} + \frac{\operatorname{Cos}[c+dx]\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1173 leaves):

$$\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[2(c+dx)]}{4d} +$$

$$\left(\sqrt{a+b}\operatorname{Sec}[c+dx]\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}}\left(a b\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]+b^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right)-\right.$$

$$2ab\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^3+ab\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5-b^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5-8ia^2\operatorname{EllipticPi}\left[-\frac{a+b}{a-b},\right.$$

$$i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+$$

$$2ib^2\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-8ia^2\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\tan\left[\frac{1}{2}(c+dx)\right]^2\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+$$

$$\begin{aligned}
& 2 i b^2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i(a-b) b \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i (2 a^2 - a b - b^2) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 a \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(a \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) \right)
\end{aligned}$$

■ **Problem 537: Unable to integrate problem.**

$$\int \sec[c+dx]^4 (a+b \sec[c+dx])^{3/2} dx$$

Optimal (type 4, 405 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (8 a^4 + 33 a^2 b^2 + 147 b^4) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (8 a^3 + 6 a^2 b + 39 a b^2 - 147 b^3) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{2 a (8 a^2 + 39 b^2) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{315 b^2 d} + \frac{2 (8 a^2 + 49 b^2) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{315 b^2 d} - \\
& \frac{8 a (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{63 b^2 d} + \frac{2 \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{9 b d}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[c+d x]^4 (a+b \operatorname{Sec}[c+d x])^{3/2} dx$$

■ **Problem 538: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+d x]^3 (a+b \operatorname{Sec}[c+d x])^{3/2} dx$$

Optimal (type 4, 342 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{105 b^3 d} 4 a (a-b) \sqrt{a+b} (3 a^2 - 41 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 b^2 d} 2 (a-b) \sqrt{a+b} (6 a^2 + 57 a b - 25 b^2) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{2 (6 a^2 - 25 b^2) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{105 b d} - \frac{4 a (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{35 b d} + \frac{2 (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{7 b d}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[c+d x]^3 (a+b \operatorname{Sec}[c+d x])^{3/2} dx$$

■ **Problem 539: Unable to integrate problem.**

$$\int \sec [c+d x]^2 (a+b \sec [c+d x])^{3 / 2} d x$$

Optimal (type 4, 282 leaves, 5 steps):

$$\begin{aligned} & -\frac{1}{5 b^2 d} 2(a-b) \sqrt{a+b} \left(a^2+3 b^2\right) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} \\ & -\frac{1}{5 b d} 2(a-3 b)(a-b) \sqrt{a+b} \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{2 a \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{5 d} + \frac{2(a+b \sec [c+d x])^{3 / 2} \tan [c+d x]}{5 d} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \sec [c+d x]^2 (a+b \sec [c+d x])^{3 / 2} d x$$

■ **Problem 540: Attempted integration timed out after 120 seconds.**

$$\int \sec [c+d x] (a+b \sec [c+d x])^{3 / 2} d x$$

Optimal (type 4, 249 leaves, 4 steps):

$$\begin{aligned} & -\frac{1}{3 b d} 8 a(a-b) \sqrt{a+b} \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{1}{3 b d} \\ & 2(a-b)(3 a-b) \sqrt{a+b} \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{2 b \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{3 d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 541: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \sec [c+d x])^{3 / 2} d x$$

Optimal (type 4, 309 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{d} (a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{d} \\
& 2(2a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{2a \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d}
\end{aligned}$$

Result (type 4, 882 leaves):

$$\begin{aligned}
& \frac{2 b \cos [c+d x] (a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{d (b+a \cos [c+d x])} + \\
& \left(2 (a+b \sec [c+d x])^{3 / 2} \left(a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] + b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] - 2 a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right]^3 + \right. \right. \\
& a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right]^5 - b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right]^5 + 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} + \\
& 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (c+d x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - i (a-b) b \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - \\
& i (a-b)^2 \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \left. \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} \right) \right) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d (b+a \cos [c+d x])^{3 / 2} \sec [c+d x]^{3 / 2} \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2}} \left(-1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \right. \\
& \left. \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right)^{3 / 2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c+d x) \right]^2}} \right)
\end{aligned}$$

■ **Problem 542: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c + dx] (a + b \sec[c + dx])^{3/2} dx$$

Optimal (type 4, 334 leaves, 6 steps):

$$\frac{a(a-b)\sqrt{a+b}\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{bd} +$$

$$\frac{\sqrt{a+b}(a+2b)\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d} -$$

$$\frac{3b\sqrt{a+b}\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d} + \frac{a\sqrt{a+b}\sec[c+dx]\sin[c+dx]}{d}$$

Result (type 4, 642 leaves):

$$\begin{aligned}
& \left((a + b \operatorname{Sec}[c + d x])^{3/2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right. \\
& \left. - i a (a - b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \right. \\
& \left. \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} + 2 i (a - b) b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] \right. \\
& \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - \right. \\
& \left. 6 i a b \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \right. \\
& \left. \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} + a \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right)^2\right) \right) \Bigg) / \\
& \left(\sqrt{\frac{-a + b}{a + b}} d (b + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2} \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right)^2\right) \right)
\end{aligned}$$

- **Problem 543: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Sec}[c + d x])^{3/2} dx$$

Optimal (type 4, 390 leaves, 7 steps):

$$\frac{5(a-b)\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4d} + \frac{1}{4d}$$

$$\frac{\sqrt{a+b}(2a+5b)\operatorname{Cot}[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4ad} - \frac{1}{4ad}$$

$$\frac{\sqrt{a+b}(4a^2+3b^2)\operatorname{Cot}[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}}}{4d} + \frac{5b\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{4d} + \frac{a\operatorname{Cos}[c+dx]\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1159 leaves):

$$\frac{a\operatorname{Cos}[c+dx](a+b\operatorname{Sec}[c+dx])^{3/2}\operatorname{Sin}[2(c+dx)]}{4d(b+a\operatorname{Cos}[c+dx])} -$$

$$\left((a+b\operatorname{Sec}[c+dx])^{3/2} \left(5ab\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 5b^2\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 10ab\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \right. \right.$$

$$5ab\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 5b^2\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 8i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right.$$

$$i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$6i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\left. \sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 8i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right.$$

$$\left. 6i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 5 i (a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i (2 a^2-a b-b^2) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 \sqrt{\frac{-a+b}{a+b}} d (b+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right. \\
& \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 544: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^4 (a+b \operatorname{Sec}[c+dx])^{5/2} dx$$

Optimal (type 4, 463 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{693 b^4 d} 2 a (a-b) \sqrt{a+b} (8 a^4 + 51 a^2 b^2 + 741 b^4) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{693 b^3 d} \\
& 2(a-b) \sqrt{a+b} (8 a^4 + 6 a^3 b + 57 a^2 b^2 - 606 a b^3 + 135 b^4) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2(8 a^4 + 57 a^2 b^2 + 135 b^4) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{693 b^2 d} + \\
& \frac{2 a (8 a^2 + 67 b^2) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{693 b^2 d} + \frac{2(8 a^2 + 81 b^2) (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{693 b^2 d} - \\
& \frac{8 a (a+b \operatorname{Sec}[c+d x])^{7/2} \operatorname{Tan}[c+d x]}{99 b^2 d} + \frac{2 \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{7/2} \operatorname{Tan}[c+d x]}{11 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 545: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+d x]^3 (a+b \operatorname{Sec}[c+d x])^{5/2} dx$$

Optimal (type 4, 399 leaves, 7 steps):

$$\begin{aligned}
& \frac{1}{315 b^3 d} 2(a-b) \sqrt{a+b} (10 a^4 - 279 a^2 b^2 - 147 b^4) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{315 b^2 d} \\
& 2(a-b) \sqrt{a+b} (10 a^3 + 165 a^2 b - 114 a b^2 + 147 b^3) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{4 a (5 a^2 - 57 b^2) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{315 b d} - \\
& \frac{2(10 a^2 - 49 b^2) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{315 b d} - \frac{4 a (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{63 b d} + \frac{2(a+b \operatorname{Sec}[c+d x])^{7/2} \operatorname{Tan}[c+d x]}{9 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 546: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x]^2 (a + b \text{Sec}[c + d x])^{5/2} dx$$

Optimal (type 4, 333 leaves, 6 steps):

$$-\frac{1}{21 b^2 d} 2 a (a - b) \sqrt{a + b} (3 a^2 + 29 b^2) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} -$$

$$\frac{1}{21 b d} 2 (a - b) \sqrt{a + b} (3 a^2 - 24 a b + 5 b^2) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 (3 a^2 + 5 b^2) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{21 d} +$$

$$\frac{2 a (a + b \text{Sec}[c + d x])^{3/2} \text{Tan}[c + d x]}{7 d} + \frac{2 (a + b \text{Sec}[c + d x])^{5/2} \text{Tan}[c + d x]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 547: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^{5/2} dx$$

Optimal (type 4, 296 leaves, 5 steps):

$$-\frac{1}{15 b d} 2 (a - b) \sqrt{a + b} (23 a^2 + 9 b^2) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{1}{15 b d} 2 (a - b) \sqrt{a + b} (15 a^2 - 8 a b + 9 b^2) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{16 a b \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{15 d} + \frac{2 b (a + b \text{Sec}[c + d x])^{3/2} \text{Tan}[c + d x]}{5 d}$$

Result (type 1, 1 leaves):

???

- **Problem 548: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 4, 352 leaves, 6 steps):

$$\begin{aligned} & -\frac{1}{3d} 14 a (a-b) \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3d} \\ & 2 \sqrt{a+b} (9a^2 - 7ab + b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\ & \frac{1}{d} 2a^2 \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \\ & \frac{2b^2 \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3d} \end{aligned}$$

Result (type 4, 713 leaves):

$$\begin{aligned}
& \left(2 (a + b \operatorname{Sec}[c + d x])^{5/2} \left(-7 i a (a - b) b \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \right. \right. \\
& \quad \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - \\
& \quad i (3 a^3 - 9 a^2 b + 7 a b^2 - b^3) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \\
& \quad \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 6 i a^3 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \quad \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + \\
& \quad \left. \left. \left. 7 a b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \left(b - b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) \right) \right) / \\
& \left(3 \sqrt{\frac{-a + b}{a + b}} d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2} \sqrt{\frac{1}{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right. \\
& \quad \left. \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^{3/2} \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \right) + \\
& \quad \frac{\operatorname{Cos}[c + d x]^2 (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{14}{3} a b \operatorname{Sin}[c + d x] + \frac{2}{3} b^2 \operatorname{Tan}[c + d x] \right)}{d (b + a \operatorname{Cos}[c + d x])^2}
\end{aligned}$$

■ **Problem 549: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 4, 353 leaves, 6 steps):

$$\frac{1}{bd} (a-b) \sqrt{a+b} (a^2 - 2b^2) \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{d}$$

$$\sqrt{a+b} (a^2 + 6ab - 2b^2) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{d} 5ab \sqrt{a+b} \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{a^2 \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{d}$$

Result (type 4, 784 leaves):

$$\frac{2 b^2 \cos [c+d x]^2 (a+b \sec [c+d x])^{5/2} \sin [c+d x]}{d (b+a \cos [c+d x])^2} +$$

$$\left((a+b \sec [c+d x])^{5/2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \left(a^3 \tan \left[\frac{1}{2}(c+d x)\right] + a^2 b \tan \left[\frac{1}{2}(c+d x)\right] - 2 a b^2 \tan \left[\frac{1}{2}(c+d x)\right] - 2 b^3 \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ \left. \left. 2 a^3 \tan \left[\frac{1}{2}(c+d x)\right]^3 + 4 a b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3 + a^3 \tan \left[\frac{1}{2}(c+d x)\right]^5 - a^2 b \tan \left[\frac{1}{2}(c+d x)\right]^5 - 2 a b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5 + \right. \right. \\ \left. \left. 2 b^3 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 10 a^2 b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 10 a^2 b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \right. \\ \left. \left. (a^3+a^2 b-2 a b^2-2 b^3) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 2 b(-3 a^2+3 a b+b^2) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \right) / \\ \left(d (b+a \cos [c+d x])^{5/2} \sec [c+d x]^{5/2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)$$

■ **Problem 550: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+b \sec [c+d x])^{5/2} dx$$

Optimal (type 4, 399 leaves, 7 steps):

$$\frac{9 a (a-b) \sqrt{a+b} \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{4 d} + \frac{1}{4 d}$$

$$\sqrt{a+b} (2 a^2+9 a b+8 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 d}$$

$$\sqrt{a+b} (4 a^2+15 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{9 a b \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{a^2 \operatorname{Cos}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 d}$$

Result (type 4, 4588 leaves):

$$\frac{a^2 \operatorname{Cos}[c+d x]^2 (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[2(c+d x)]}{4 d (b+a \operatorname{Cos}[c+d x])^2} +$$

$$\left(\left(\frac{a^3}{2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{3 a b^2}{\sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{11 a^2 b \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \right.$$

$$\left. \frac{b^3 \sqrt{\operatorname{Sec}[c+d x]}}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{9 a^2 b \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} \right) (a+b \operatorname{Sec}[c+d x])^{5/2}$$

$$\left(18 i a (a-b) b \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] - \right.$$

$$4 i (2 a^3 - a^2 b + 3 a b^2 - 4 b^3) \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] +$$

$$4 i a (4 a^2+15 b^2) \sqrt{\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{(a+b)(1+\operatorname{Cos}[c+d x])}} \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] -$$

$$9 a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Cos}[c+d x] (b+a \operatorname{Cos}[c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left. \right) \Bigg/$$

$$\begin{aligned}
& \left(4 \sqrt{\frac{-a+b}{a+b}} d (b+a \cos [c+d x])^3 \sqrt{\sec \left[\frac{1}{2}(c+d x)\right]^2} \sec [c+d x]^{5/2} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \right. \\
& \left. \left(-1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \right) \left(-\left(\sqrt{\sec \left[\frac{1}{2}(c+d x)\right]^2} \tan \left[\frac{1}{2}(c+d x)\right] \right. \right. \\
& \left. \left(18 i a (a-b) b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] - \right. \right. \\
& \left. \left. 4 i\left(2 a^3-a^2 b+3 a b^2-4 b^3\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \right. \right. \\
& \left. \left. \frac{a+b}{a-b}\right]+4 i a\left(4 a^2+15 b^2\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \right. \right. \\
& \left. \left. \frac{a+b}{a-b}\right]-9 a b \sqrt{\frac{-a+b}{a+b}} \cos [c+d x](b+a \cos [c+d x]) \sec \left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right] \right) \Bigg) / \\
& \left(4 \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \cos [c+d x]} \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \left(-1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \right) + \\
& \left(a \sin [c+d x] \left(18 i a (a-b) b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] - \right. \right. \\
& \left. \left. 4 i\left(2 a^3-a^2 b+3 a b^2-4 b^3\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \right. \right. \\
& \left. \left. \frac{a+b}{a-b}\right]+4 i a\left(4 a^2+15 b^2\right) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \right. \right. \\
& \left. \left. \frac{a+b}{a-b}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 2 i \left(2 a^3 - a^2 b + 3 a b^2 - 4 b^3 \right) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b}\right] \\
& \left(\frac{\operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{(1 + \operatorname{Cos}[c + d x])^2} - \frac{\operatorname{Sin}[c + d x]}{1 + \operatorname{Cos}[c + d x]} \right) + \frac{1}{\sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}}} 2 i a \left(4 a^2 + 15 b^2 \right) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])}} \\
& \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b} \left(\frac{\operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{(1 + \operatorname{Cos}[c + d x])^2} - \frac{\operatorname{Sin}[c + d x]}{1 + \operatorname{Cos}[c + d x]} \right) + \right. \\
& \left. \frac{1}{\sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])}}} 9 i a (a - b) b \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b}\right] \\
& \left(-\frac{a \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])} + \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])^2} \right) - \frac{1}{\sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])}}} \\
& 2 i \left(2 a^3 - a^2 b + 3 a b^2 - 4 b^3 \right) \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b}\right] \\
& \left(-\frac{a \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])} + \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])^2} \right) + \frac{1}{\sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])}}} 2 i a \left(4 a^2 + 15 b^2 \right) \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} \\
& \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b} \left(-\frac{a \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])} + \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{(a + b) (1 + \operatorname{Cos}[c + d x])^2} \right) + \right. \\
& \left. 9 a^2 b \sqrt{\frac{-a + b}{a + b}} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sin}[c + d x] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 9 a b \sqrt{\frac{-a + b}{a + b}} (b + a \operatorname{Cos}[c + d x]) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sin}[c + d x] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 9 a b \sqrt{\frac{-a + b}{a + b}} \operatorname{Cos}[c + d x] (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 \sqrt{\frac{-a+b}{a+b}} (2a^3 - a^2b + 3ab^2 - 4b^3) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \\
& \frac{2a \sqrt{\frac{-a+b}{a+b}} (4a^2 + 15b^2) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\left(1 - \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right) \sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \left(9a(a-b)b \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}\right. \\
& \left. \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}}\right) / \left(\sqrt{1 + \frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) \Bigg) \\
& \left(4 \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a\cos[c+dx]} \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2} \operatorname{Sec}[c+dx] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) - \\
& \left(\left(18ia(a-b)b \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] - \right. \right. \\
& \left. 4i(2a^3 - a^2b + 3ab^2 - 4b^3) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \right. \right. \\
& \left. \left. \frac{a+b}{a-b}\right] + 4ia(4a^2 + 15b^2) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \right. \\
& \left. \left. i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] - 9ab \sqrt{\frac{-a+b}{a+b}} \cos[c+dx] (b+a\cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left. \left(-\cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \tan[c+dx]\right) \Bigg) /
\end{aligned}$$

$$\left(8 \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \cos[c+dx]} \sqrt{\sec\left[\frac{1}{2}(c+dx)\right]^2} \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]\right)^{3/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right)$$

■ **Problem 551: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 (a+b \sec[c+dx])^{5/2} dx$$

Optimal (type 4, 460 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{24bd} (a-b) \sqrt{a+b} (16a^2 + 33b^2) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{24d} \sqrt{a+b} (16a^2 + 26ab + 33b^2) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{8ad} 5b \sqrt{a+b} (4a^2 + b^2) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\ & \frac{(16a^2 + 33b^2) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{24d} + \\ & \frac{13ab \cos[c+dx] \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{12d} + \frac{a^2 \cos[c+dx]^2 \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{3d} \end{aligned}$$

Result (type 4, 1026 leaves):

$$\frac{\cos[c+dx]^2 (a+b \sec[c+dx])^{5/2} \left(\frac{1}{12} a^2 \sin[c+dx] + \frac{13}{24} ab \sin[2(c+dx)] + \frac{1}{12} a^2 \sin[3(c+dx)]\right)}{d (b+a \cos[c+dx])^2} +$$

$$\left((a+b \sec[c+dx])^{5/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\ \left. \left(16a^3 \tan\left[\frac{1}{2}(c+dx)\right] + 16a^2 b \tan\left[\frac{1}{2}(c+dx)\right] + 33a^2 b^2 \tan\left[\frac{1}{2}(c+dx)\right] + 33b^3 \tan\left[\frac{1}{2}(c+dx)\right] - 32a^3 \tan\left[\frac{1}{2}(c+dx)\right]^3 - \right. \right. \\ \left. \left. 66a^2 b^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 + 16a^3 \tan\left[\frac{1}{2}(c+dx)\right]^5 - 16a^2 b \tan\left[\frac{1}{2}(c+dx)\right]^5 + 33a^2 b^2 \tan\left[\frac{1}{2}(c+dx)\right]^5 - 33b^3 \tan\left[\frac{1}{2}(c+dx)\right]^5 \right) \right)$$

$$\begin{aligned}
& 120 a^2 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a^2 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (16 a^3 + 16 a^2 b + 33 a b^2 + 33 b^3) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 b (38 a^2 - 13 a b + 24 b^2) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(24 d (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 552: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+b \operatorname{Sec}[c+dx])^{5/2} dx$$

Optimal (type 4, 530 leaves, 9 steps):

$$\begin{aligned}
& \frac{1}{192 a d} (a-b) \sqrt{a+b} (284 a^2 + 15 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{192 a d} \sqrt{a+b} (72 a^3 + 284 a^2 b + 118 a b^2 + 15 b^3) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{64 a^2 d} \sqrt{a+b} (48 a^4 + 120 a^2 b^2 - 5 b^4) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{b(284 a^2 + 15 b^2) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{192 a d} + \frac{(36 a^2 + 59 b^2) \operatorname{Cos}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{96 d} + \\
& \frac{17 a b \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d} + \frac{a^2 \operatorname{Cos}[c+d x]^3 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}
\end{aligned}$$

Result (type 4, 1688 leaves):

$$\begin{aligned}
& \frac{1}{d (b+a \operatorname{Cos}[c+d x])^2} \\
& \operatorname{Cos}[c+d x]^2 (a+b \operatorname{Sec}[c+d x])^{5/2} \left(\frac{17}{96} a b \operatorname{Sin}[c+d x] + \frac{1}{192} (48 a^2 + 59 b^2) \operatorname{Sin}[2(c+d x)] + \frac{17}{96} a b \operatorname{Sin}[3(c+d x)] + \frac{1}{32} a^2 \operatorname{Sin}[4(c+d x)] \right) + \\
& \left((a+b \operatorname{Sec}[c+d x])^{5/2} \left(-284 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 284 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 15 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \\
& 15 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 568 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 30 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\
& 284 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 284 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 15 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 15 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 288 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 720 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right)
\end{aligned}$$

$$\left(192 a \sqrt{\frac{-a+b}{a+b}} d (b+a \cos [c+d x])^{5/2} \sec [c+d x]^{5/2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \left(-1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \right. \\ \left. \left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^{3/2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)$$

■ **Problem 553: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \sec [c+d x])^{7/2} dx$$

Optimal (type 4, 403 leaves, 7 steps):

$$-\frac{1}{15 d} 2(a-b) \sqrt{a+b} (58 a^2+9 b^2) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ \frac{1}{15 d} 2 \sqrt{a+b} (60 a^3-58 a^2 b+22 a b^2-9 b^3) \cot [c+d x] \\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{d} \\ 2 a^3 \sqrt{a+b} \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ \frac{26 a b^2 \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{15 d} + \frac{2 b^2 (a+b \sec [c+d x])^{3/2} \tan [c+d x]}{5 d}$$

Result (type 4, 1150 leaves):

$$\left(2 (a+b \sec [c+d x])^{7/2} \right. \\ \left. \left(58 a^3 b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] + 58 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] + 9 a b^3 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] + 9 b^4 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ \left. \left. 116 a^3 b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3 - 18 a b^3 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3 + 58 a^3 b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^5 - \right. \right.$$

$$\begin{aligned}
& 58 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 9 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 9 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 30 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i b (-58 a^3 + 58 a^2 b - 9 a b^2 + 9 b^3) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i (15 a^4 - 60 a^3 b + 58 a^2 b^2 - 22 a b^3 + 9 b^4) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) \Bigg) / \\
& \left(15 \sqrt{\frac{-a+b}{a+b}} d (b + a \operatorname{Cos}[c+dx])^{7/2} \operatorname{Sec}[c+dx]^{7/2} \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right. \\
& \left. \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \frac{1}{d (b + a \operatorname{Cos}[c+dx])^3} \operatorname{Cos}[c+dx]^3 (a + b \operatorname{Sec}[c+dx])^{7/2}
\end{aligned}$$

$$\left(\frac{2}{15} b (58 a^2 + 9 b^2) \sin[c + dx] + \frac{32}{15} a b^2 \tan[c + dx] + \frac{2}{5} b^3 \sec[c + dx] \tan[c + dx] \right)$$

■ **Problem 554: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^5}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 359 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{105 b^5 d} 8 a (a - b) \sqrt{a + b} (12 a^2 + 11 b^2) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \\ & \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{105 b^4 d} 2 \sqrt{a + b} (48 a^3 - 12 a^2 b + 44 a b^2 + 25 b^3) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{2(24 a^2 + 25 b^2) \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{105 b^3 d} - \\ & \frac{12 a \sec[c + dx] \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{35 b^2 d} + \frac{2 \sec[c + dx]^2 \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{7 b d} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\sec[c + dx]^5}{\sqrt{a + b \sec[c + dx]}} dx$$

■ **Problem 555: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^4}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 301 leaves, 5 steps):

$$\begin{aligned} & -\frac{1}{15 b^4 d} 2 (a - b) \sqrt{a + b} (8 a^2 + 9 b^2) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{15 b^3 d} 2 \sqrt{a + b} (8 a^2 - 2 a b + 9 b^2) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \\ & \frac{8 a \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{15 b^2 d} + \frac{2 \sec[c + dx] \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{5 b d} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Sec}[c + d x]^4}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 556: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 244 leaves, 4 steps) :

$$\frac{4 a (a - b) \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}}{3 b^3 d} + \frac{1}{3 b^2 d}$$

$$2 \sqrt{a + b} (2 a + b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} +$$

$$\frac{2 \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{3 b d}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^3}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 557: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 204 leaves, 3 steps) :

$$-\frac{1}{b^2 d} 2 (a - b) \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}}{b d}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^2}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 559: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a d}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 560: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 338 leaves, 6 steps):

$$\frac{(a-b) \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b d} +$$

$$\frac{\sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a d} +$$

$$\frac{b \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 d} + \frac{\sqrt{a+b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a d}$$

Result (type 4, 795 leaves):

$$\begin{aligned}
& \left(\sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \left. \left(a \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. a \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \right. \right. \\
& \left. \left. 2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 i b \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i \right. \right. \\
& \left. \left. (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \right. \\
& \left. \left. 2 i b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left(a \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b \operatorname{Sec}[c+dx]} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

- **Problem 561: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2}{\sqrt{a+b \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 401 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{4a^2d} 3(a-b)\sqrt{a+b} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{4a^2d} \\
& (2a-3b)\sqrt{a+b} \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{\frac{b(1+\sec[c+dx])}{a-b}} - \frac{1}{4a^3d} \\
& \sqrt{a+b} (4a^2+3b^2) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{\frac{b(1+\sec[c+dx])}{a-b}} - \\
& \frac{3b\sqrt{a+b\sec[c+dx]} \sin[c+dx]}{4a^2d} + \frac{\cos[c+dx] \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{2ad}
\end{aligned}$$

Result (type 4, 1195 leaves):

$$\begin{aligned}
& \frac{(b+a\cos[c+dx])\sec[c+dx]\sin[2(c+dx)]}{4ad\sqrt{a+b\sec[c+dx]}} - \left(\sqrt{b+a\cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(3ab\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] + 3b^2\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] - 6ab\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 + 3ab\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 - \right. \right. \\
& \left. \left. 3b^2\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 + 8ia^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \\
& \left. \left. \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6ib^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \right. \\
& \left. \left. 8ia^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \\
& \left. \left. \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6ib^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right)
\end{aligned}$$

$$\begin{aligned} & \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\ & 3i(a-b)b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\ & \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2i(2a^2 - ab + 3b^2) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\ & \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) \Bigg) / \\ & \left(4a^2 \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b} \operatorname{Sec}[c+dx] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \right. \\ & \left. \left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right) \right) \end{aligned}$$

■ **Problem 562: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+dx]^5}{(a+b \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 4, 399 leaves, 6 steps):

$$-\frac{1}{5b^5 \sqrt{a+b} d} 2(16a^4 - 8a^2b^2 - 3b^4) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{5b^4 \sqrt{a+b} d}$$

$$2(4a+3b)(4a^2+b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{2a^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{b(a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \frac{2a(8a^2 - 3b^2) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{5b^3(a^2 - b^2) d} + \frac{2(6a^2 - b^2) \operatorname{Sec}[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{5b^2(a^2 - b^2) d}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^5}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 563: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]^4}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 325 leaves, 5 steps) :

$$\frac{2 a (8 a^2 - 5 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 b^4 \sqrt{a+b} d} + \frac{1}{3 b^3 \sqrt{a+b} d}$$

$$- \frac{2 (2 a + b) (4 a + b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 (4 a^2 - b^2) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c + d x]}{3 b^2 (a^2 - b^2) d}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^4}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 564: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 257 leaves, 4 steps) :

$$- \frac{2 (2 a^2 - b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b^3 \sqrt{a+b} d} -$$

$$\frac{2 (2 a + b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b^2 \sqrt{a+b} d} - \frac{2 a^2 \operatorname{Tan}[c + d x]}{b (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 565: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 237 leaves, 4 steps):

$$\frac{2 a \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b^2 \sqrt{a+b} d} +$$

$$\frac{2 \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b \sqrt{a+b} d} + \frac{2 a \operatorname{Tan}[c + d x]}{(a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 567: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 347 leaves, 6 steps):

$$\frac{2 \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a \sqrt{a+b} d} -$$

$$\frac{2 \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a \sqrt{a+b} d} -$$

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 d} + \frac{2 b^2 \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}$$

Result (type 4, 1249 leaves):

$$\frac{(b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \left(\frac{2 b \operatorname{Sin}[c+d x]}{a (-a^2+b^2)} + \frac{2 b^2 \operatorname{Sin}[c+d x]}{a (a^2-b^2) (b+a \operatorname{Cos}[c+d x])} \right)}{d (a + b \operatorname{Sec}[c + d x])^{3/2}} +$$

$$\begin{aligned}
& \left(2 (b + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right. \\
& \left(a b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] + b^2 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] - 2 a b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 + a b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 - \right. \\
& b^2 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 - 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 2 i b^2 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - \\
& 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \tan \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 2 i b^2 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \tan \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - \\
& i (a - b) b \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + i (a^2 + a b - 2 b^2) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \left. \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right) \Bigg) /
\end{aligned}$$

$$\left(a \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2} \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right. \\ \left. \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) \right)$$

■ **Problem 568: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 396 leaves, 7 steps):

$$\frac{(a^2 - 3b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b} \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}\right]}{a^2 b \sqrt{a+b} d} +$$

$$\frac{(a + 3b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b} \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}\right]}{a^2 \sqrt{a+b} d} +$$

$$\frac{3b \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b} \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}\right]}{a^3 d} +$$

$$\frac{\operatorname{Sin}[c + d x]}{a d \sqrt{a+b \operatorname{Sec}[c + d x]}} + \frac{b(a^2 - 3b^2) \operatorname{Tan}[c + d x]}{a^2 (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c + d x]}}$$

Result (type 4, 1077 leaves):

$$\frac{(b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \left(-\frac{2b^2 \operatorname{Sin}[c + d x]}{a^2 (-a^2 + b^2)} - \frac{2b^3 \operatorname{Sin}[c + d x]}{a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])} \right)}{d (a + b \operatorname{Sec}[c + d x])^{3/2}} -$$

$$\left((b + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2} \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right)$$

$$\begin{aligned}
& \left(a^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + a^2 b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 a b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 2 a^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \right. \\
& 6 a b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + a^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - a^2 b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 6 a^2 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 a^2 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a^3 + a^2 b - 3 a b^2 - 3 b^3) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 a b (a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(a^2 (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{3/2} \sqrt{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 569:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cos [c+d x]^2}{(a+b \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 4, 470 leaves, 8 steps):

$$\begin{aligned} & -\frac{(7 a^2-15 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sec}[c+d x])}{a-b}}}{4 a^3 \sqrt{a+b} d} + \frac{1}{4 a^3 \sqrt{a+b} d} \\ & (2 a^2-5 a b-15 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{4 a^4 d} \\ & \sqrt{a+b} (4 a^2+15 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{\frac{-b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\ & \frac{5 b \operatorname{Sin}[c+d x]}{4 a^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{\operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 a d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{b^2 (7 a^2-15 b^2) \operatorname{Tan}[c+d x]}{4 a^3 (a^2-b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}} \end{aligned}$$

Result (type 4, 1745 leaves):

$$\begin{aligned} & \frac{(b+a \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^2 \left(\frac{2 b^3 \operatorname{Sin}[c+d x]}{a^3 (-a^2+b^2)} + \frac{2 b^4 \operatorname{Sin}[c+d x]}{a^3 (a^2-b^2) (b+a \operatorname{Cos}[c+d x])} + \frac{\operatorname{Sin}[2(c+d x)]}{4 a^2} \right)}{d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \\ & \left((b+a \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sec}[c+d x]^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\ & \left(-7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 15 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \right. \\ & 15 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 14 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 30 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\ & 7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 15 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\ & \left. 15 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 8 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-22 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& 30 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-8 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 22 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+30 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& i b\left(7 a^3-7 a^2 b-15 a b^2+15 b^3\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+ \\
& 2 i\left(2 a^4-a^3 b+9 a^2 b^2+5 a b^3-15 b^4\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(4 a^3 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{3/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 570: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+dx]^5}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 427 leaves, 6 steps):

$$\frac{1}{3(a-b)b^5(a+b)^{3/2}d}$$

$$8a(4a^4 - 7a^2b^2 + 2b^4) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{3(a-b)b^4(a+b)^{3/2}d} 2(16a^4 + 12a^3b - 16a^2b^2 - 9ab^3 - b^4) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{2a^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3b(a^2 - b^2)d(a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{4a^3(3a^2 - 5b^2) \operatorname{Tan}[c+dx]}{3b^3(a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{2(2a^2 - b^2) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3b^3(a^2 - b^2)d}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Sec}[c+dx]^5}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

■ **Problem 571: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+dx]^4}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 362 leaves, 5 steps) :

$$\begin{aligned}
 & - \frac{1}{3 (a-b) b^4 (a+b)^{3/2} d} 2 (8 a^4 - 15 a^2 b^2 + 3 b^4) \operatorname{Cot}[c+d x] \\
 & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{3 (a-b) b^3 (a+b)^{3/2} d} \\
 & 2 (8 a^3 + 6 a^2 b - 9 a b^2 - 3 b^3) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
 & \frac{2 a^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} - \frac{8 a^2 (a^2 - 2 b^2) \operatorname{Tan}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}
 \end{aligned}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c+d x]^4}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

■ **Problem 572: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+d x]^3}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 4, 337 leaves, 5 steps) :

$$\begin{aligned}
 & \frac{4 a (a^2 - 3 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 (a-b) b^3 (a+b)^{3/2} d} + \frac{1}{3 (a-b) b^2 (a+b)^{3/2} d} \\
 & 2 (2 a^2 + 3 a b - 3 b^2) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
 & \frac{2 a^2 \operatorname{Tan}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{4 a (a^2 - 3 b^2) \operatorname{Tan}[c+d x]}{3 b (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}
 \end{aligned}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c+d x]^3}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

■ **Problem 573: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+d x]^2}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 4, 317 leaves, 5 steps) :

$$\frac{2 (a^2 + 3 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 (a-b) b^2 (a+b)^{3/2} d} +$$

$$\frac{2 (a-3 b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 (a-b) b (a+b)^{3/2} d} +$$

$$\frac{2 a \operatorname{Tan}[c + d x]}{3 (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 (a^2 + 3 b^2) \operatorname{Tan}[c + d x]}{3 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

■ **Problem 574: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 4, 304 leaves, 5 steps) :

$$-\frac{8 a \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 (a-b) b (a+b)^{3/2} d} +$$

$$\frac{2 (3 a - b) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 (a-b) b (a+b)^{3/2} d} -$$

$$\frac{2 b \operatorname{Tan}[c + d x]}{3 (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} - \frac{8 a b \operatorname{Tan}[c + d x]}{3 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 23 leaves) :

$$\int \frac{\operatorname{Sec}[c + d x]}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

■ **Problem 575: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 4, 448 leaves, 7 steps) :

$$\begin{aligned}
& \frac{2 (7 a^2 - 3 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{3 a^2 (a-b)(a+b)^{3/2} d} - \frac{1}{3 a^2 (a-b)(a+b)^{3/2} d} \\
& \frac{2 (6 a^2 - a b - 3 b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^3 d} - \\
& \frac{2 \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^3 d} + \\
& \frac{2 b^2 \operatorname{Tan}[c + d x]}{3 a (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 b^2 (7 a^2 - 3 b^2) \operatorname{Tan}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 4, 1798 leaves):

$$\begin{aligned}
& \frac{1}{d (a+b \operatorname{Sec}[c+d x])^{5/2}} \\
& (b+a \operatorname{Cos}[c+d x])^3 \operatorname{Sec}[c+d x]^3 \left(\frac{2 b (-7 a^2 + 3 b^2) \operatorname{Sin}[c+d x]}{3 a^2 (-a^2 + b^2)^2} - \frac{2 b^3 \operatorname{Sin}[c+d x]}{3 a^2 (a^2 - b^2) (b+a \operatorname{Cos}[c+d x])^2} - \frac{8 (-2 a^2 b^2 \operatorname{Sin}[c+d x] + b^4 \operatorname{Sin}[c+d x])}{3 a^2 (a^2 - b^2)^2 (b+a \operatorname{Cos}[c+d x])} \right) + \\
& \left(2 (b+a \operatorname{Cos}[c+d x])^{5/2} \operatorname{Sec}[c+d x]^{5/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left(7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& \left. 3 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 14 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 6 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \right. \\
& \left. 7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \right. \\
& \left. 3 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 6 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+12 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}- \\
& 6 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-6 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
& 12 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-6 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}- \\
& i b\left(7 a^3-7 a^2 b-3 a b^2+3 b^3\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
& i\left(3 a^4+6 a^3 b-13 a^2 b^2-2 a b^3+6 b^4\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(3 a^2 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+dx])^{5/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 576: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 510 leaves, 8 steps):

$$\frac{1}{3 a^3 (a-b) b (a+b)^{3/2} d} (3 a^4 - 26 a^2 b^2 + 15 b^4) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3 a^3 (a-b) (a+b)^{3/2} d}$$

$$(3 a^3 + 21 a^2 b - 5 a b^2 - 15 b^3) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{5 b \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a^4 d} +$$

$$\frac{\operatorname{Sin}[c+dx]}{a d (a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{b(3 a^2 - 5 b^2) \operatorname{Tan}[c+dx]}{3 a^2 (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{b(3 a^4 - 26 a^2 b^2 + 15 b^4) \operatorname{Tan}[c+dx]}{3 a^3 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+dx]}}$$

Result (type 4, 1493 leaves):

$$\frac{1}{d (a+b \operatorname{Sec}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3$$

$$\left(-\frac{4 b^2 (-5 a^2 + 3 b^2) \operatorname{Sin}[c+dx]}{3 a^3 (-a^2 + b^2)^2} + \frac{2 b^4 \operatorname{Sin}[c+dx]}{3 a^3 (a^2 - b^2) (b+a \operatorname{Cos}[c+dx])^2} + \frac{2 (-11 a^2 b^3 \operatorname{Sin}[c+dx] + 7 b^5 \operatorname{Sin}[c+dx])}{3 a^3 (a^2 - b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right) -$$

$$\begin{aligned}
& \left((b + a \cos[c + dx])^{5/2} \sec[c + dx]^{5/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
& \left(3 a^5 \tan\left[\frac{1}{2}(c + dx)\right] + 3 a^4 b \tan\left[\frac{1}{2}(c + dx)\right] - 26 a^3 b^2 \tan\left[\frac{1}{2}(c + dx)\right] - 26 a^2 b^3 \tan\left[\frac{1}{2}(c + dx)\right] + 15 a b^4 \tan\left[\frac{1}{2}(c + dx)\right] + \right. \\
& 15 b^5 \tan\left[\frac{1}{2}(c + dx)\right] - 6 a^5 \tan\left[\frac{1}{2}(c + dx)\right]^3 + 52 a^3 b^2 \tan\left[\frac{1}{2}(c + dx)\right]^3 - 30 a b^4 \tan\left[\frac{1}{2}(c + dx)\right]^3 + 3 a^5 \tan\left[\frac{1}{2}(c + dx)\right]^5 - \\
& 3 a^4 b \tan\left[\frac{1}{2}(c + dx)\right]^5 - 26 a^3 b^2 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 26 a^2 b^3 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 15 a b^4 \tan\left[\frac{1}{2}(c + dx)\right]^5 - 15 b^5 \tan\left[\frac{1}{2}(c + dx)\right]^5 + \\
& 30 a^4 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} - \\
& 60 a^2 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + \\
& 30 b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + \\
& 30 a^4 b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \\
& \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} - 60 a^2 b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \tan\left[\frac{1}{2}(c + dx)\right]^2 \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + 30 b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \\
& \tan\left[\frac{1}{2}(c + dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + \\
& \left. \left(3 a^5 + 3 a^4 b - 26 a^3 b^2 - 26 a^2 b^3 + 15 a b^4 + 15 b^5 \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)
\end{aligned}$$

$$\left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2ab(6a^3 + 9a^2b - 2ab^2 - 5b^3) \operatorname{EllipticF}\left[\right. \\ \left. \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \\ \left(3a(a^3 - ab^2)^2 d (a+b \operatorname{Sec}[c+dx])^{5/2} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) \right)$$

■ **Problem 577: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 562 leaves, 9 steps):

$$-\frac{1}{12a^4(a-b)(a+b)^{3/2}d} (33a^4 - 170a^2b^2 + 105b^4) \operatorname{Cot}[c+dx] \\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{12a^4(a-b)(a+b)^{3/2}d} \\ (a+3b)(6a^3 - 45a^2b + 35b^3) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} - \\ \frac{1}{4a^5d} \sqrt{a+b} (4a^2 + 35b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} - \\ \frac{7b \operatorname{Sin}[c+dx]}{4a^2d(a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2ad(a+b \operatorname{Sec}[c+dx])^{3/2}} - \frac{b^2(27a^2 - 35b^2) \operatorname{Tan}[c+dx]}{12a^3(a^2 - b^2)d(a+b \operatorname{Sec}[c+dx])^{3/2}} - \frac{b^2(33a^4 - 170a^2b^2 + 105b^4) \operatorname{Tan}[c+dx]}{12a^4(a^2 - b^2)^2d\sqrt{a+b \operatorname{Sec}[c+dx]}}$$

Result (type 4, 2285 leaves):

$$\frac{1}{d(a+b \operatorname{Sec}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \\ \left(\frac{2b^3(-13a^2 + 9b^2) \operatorname{Sin}[c+dx]}{3a^4(-a^2 + b^2)^2} - \frac{2b^5 \operatorname{Sin}[c+dx]}{3a^4(a^2 - b^2)(b+a \operatorname{Cos}[c+dx])^2} - \frac{4(-7a^2b^4 \operatorname{Sin}[c+dx] + 5b^6 \operatorname{Sin}[c+dx])}{3a^4(a^2 - b^2)^2(b+a \operatorname{Cos}[c+dx])} + \frac{\operatorname{Sin}[2(c+dx)]}{4a^3} \right) -$$

$$\begin{aligned}
& \left((b + a \cos[c + dx])^{5/2} \sec[c + dx]^{5/2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
& \left(33 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] + 33 a^4 b^2 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] - 170 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] - \right. \\
& 170 a^2 b^4 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] + 105 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] + 105 b^6 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right] - \\
& 66 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^3 + 340 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^3 - 210 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^3 + \\
& 33 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 - 33 a^4 b^2 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 - 170 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 + \\
& 170 a^2 b^4 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 + 105 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 - 105 b^6 \sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]^5 + \\
& 24 i a^6 \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \\
& \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + 162 i a^4 b^2 \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} - \\
& 396 i a^2 b^4 \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \\
& \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} + 210 i b^6 \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 i a^6 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 162 i a^4 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 396 i a^2 b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 210 i b^6 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i b (-33 a^5 + 33 a^4 b + 170 a^3 b^2 - 170 a^2 b^3 - 105 a b^4 + 105 b^5) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i (6 a^6 - 3 a^5 b + 57 a^4 b^2 + 54 a^3 b^3 - 184 a^2 b^4 - 35 a b^5 + 105 b^6) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg/
\end{aligned}$$

$$\left(12 a^4 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^{5/2} \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \\ \left. \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right)$$

■ **Problem 578: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x])^{7/2}} dx$$

Optimal (type 4, 535 leaves, 8 steps):

$$\frac{1}{15 a^3 (a - b)^2 (a + b)^{5/2} d} \\ 2 (58 a^4 - 41 a^2 b^2 + 15 b^4) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} - \\ \frac{1}{15 a^3 (a - b)^2 (a + b)^{5/2} d} 2 (45 a^4 - 13 a^3 b - 36 a^2 b^2 + 5 a b^3 + 15 b^4) \operatorname{Cot}[c + d x] \\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} - \\ \frac{2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}}{a^4 d} + \\ \frac{2 b^2 \operatorname{Tan}[c + d x]}{5 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{5/2}} + \frac{2 b^2 (13 a^2 - 5 b^2) \operatorname{Tan}[c + d x]}{15 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 b^2 (58 a^4 - 41 a^2 b^2 + 15 b^4) \operatorname{Tan}[c + d x]}{15 a^3 (a^2 - b^2)^3 d \sqrt{a + b \operatorname{Sec}[c + d x]}}$$

Result (type 4, 2346 leaves):

$$\frac{1}{d (a + b \operatorname{Sec}[c + d x])^{7/2}} (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^4 \left(\frac{2 b (58 a^4 - 41 a^2 b^2 + 15 b^4) \operatorname{Sin}[c + d x]}{15 a^3 (-a^2 + b^2)^3} + \frac{2 b^4 \operatorname{Sin}[c + d x]}{5 a^3 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])^3} + \right. \\ \left. \frac{2 (-19 a^2 b^3 \operatorname{Sin}[c + d x] + 11 b^5 \operatorname{Sin}[c + d x])}{15 a^3 (a^2 - b^2)^2 (b + a \operatorname{Cos}[c + d x])^2} + \frac{2 (74 a^4 b^2 \operatorname{Sin}[c + d x] - 65 a^2 b^4 \operatorname{Sin}[c + d x] + 23 b^6 \operatorname{Sin}[c + d x])}{15 a^3 (a^2 - b^2)^3 (b + a \operatorname{Cos}[c + d x])} \right) +$$

$$\left(2 (b + a \cos [c + d x])^{7/2} \sec [c + d x]^{7/2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right.$$

$$\left(58 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] + 58 a^4 b^2 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] - 41 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] - \right.$$

$$41 a^2 b^4 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] + 15 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] + 15 b^6 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] -$$

$$116 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 + 82 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 - 30 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 +$$

$$58 a^5 b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 - 58 a^4 b^2 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 - 41 a^3 b^3 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 +$$

$$41 a^2 b^4 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 + 15 a b^5 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 - 15 b^6 \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^5 -$$

$$30 i a^6 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}$$

$$\sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 90 i a^4 b^2 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right]$$

$$\sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} -$$

$$90 i a^2 b^4 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}$$

$$\sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 30 i b^6 \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right]$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30 i a^6 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 90 i a^4 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 90 i a^2 b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 i b^6 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i b (-58 a^5 + 58 a^4 b + 41 a^3 b^2 - 41 a^2 b^3 - 15 a b^4 + 15 b^5) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i (15 a^6 + 45 a^5 b - 103 a^4 b^2 - 23 a^3 b^3 + 86 a^2 b^4 + 10 a b^5 - 30 b^6) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg/
\end{aligned}$$

$$\left(15 a^3 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + d x])^{7/2} \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \\ \left. \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right)$$

■ **Problem 615: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{7/2}}{(a + b \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 279 leaves, 10 steps):

$$- \frac{(3 a^2 - 2 b^2) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{b^2 (a^2 - b^2) d} - \\ \frac{a \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - a (3 a^2 - 5 b^2) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{b (a^2 - b^2) d (a - b) b^2 (a + b)^2 d} + \\ \frac{(3 a^2 - 2 b^2) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{b^2 (a^2 - b^2) d} - \frac{a^2 \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{b (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 4, 632 leaves):

$$\frac{\sqrt{\sec[c+dx]} \left(\frac{(-3a^2+2b^2)\sin[c+dx]}{b^2(-a^2+b^2)} + \frac{a^2\sin[c+dx]}{b(-a^2+b^2)(b+a\cos[c+dx])} \right)}{d} - \frac{1}{4(a-b)b^2(a+b)d}$$

$$\left(- \left(2(8a^2b-4b^3)\cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (a+b\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right.$$

$$\left. (a(b+a\cos[c+dx])(1-\cos[c+dx]^2)) + \right.$$

$$\left. \left(2(9a^3-10ab^2)\cos[c+dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right) \right.$$

$$\left. (a+b\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / (b(b+a\cos[c+dx])(1-\cos[c+dx]^2)) -$$

$$\left(2(3a^3-2ab^2)\cos[2(c+dx)] (a+b\sec[c+dx]) \left(2ab-2ab\sec[c+dx]^2+2ab\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right.$$

$$\left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + a(a-2b)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right.$$

$$\left. a^2\operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right.$$

$$\left. 2b^2\operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \Big/$$

$$\left(a^2b(b+a\cos[c+dx])(1-\cos[c+dx]^2)\sqrt{\sec[c+dx]}(2-\sec[c+dx]^2) \right)$$

■ **Problem 616: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{5/2}}{(a+b\sec[c+dx])^2} dx$$

Optimal (type 4, 214 leaves, 9 steps):

$$\frac{a\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{b(a^2-b^2)d} + \frac{\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{(a^2-b^2)d} +$$

$$\frac{(a^2-3b^2)\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{(a-b)b(a+b)^2d} - \frac{a^2\sqrt{\sec[c+dx]}\sin[c+dx]}{b(a^2-b^2)d(a+b\sec[c+dx])}$$

Result (type 4, 587 leaves):

$$\begin{aligned}
& \frac{\sqrt{\sec[c+dx]} \left(\frac{a \sin[c+dx]}{b(-a^2+b^2)} + \frac{a \sin[c+dx]}{(a^2-b^2)(b+a \cos[c+dx])} \right)}{d} + \\
& \frac{1}{4(a-b)b(a+b)d} \left(- \left(8b \cos[c+dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] (a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) / \\
& \left((b+a \cos[c+dx]) (1-\cos[c+dx]^2) \right) + \\
& \left(2(3a^2-4b^2) \cos[c+dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \right) \right) \\
& \left((a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left((b+a \cos[c+dx]) (1-\cos[c+dx]^2) \right) - \\
& \left(2 \cos[2(c+dx)] (a+b \sec[c+dx]) \left(2ab - 2ab \sec[c+dx]^2 + 2ab \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \right. \right. \\
& \left. \left. \sqrt{1-\sec[c+dx]^2} + a(a-2b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \left. \left. 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \left((b+a \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right)
\end{aligned}$$

■ **Problem 617: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2}}{(a+b \sec[c+dx])^2} dx$$

Optimal (type 4, 208 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{(a^2-b^2)d} - \frac{b \sqrt{\cos[c+dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{a(a^2-b^2)d} + \\
& \frac{(a^2+b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi} \left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2 \right] \sqrt{\sec[c+dx]}}{a(a-b)(a+b)^2d} + \frac{a \sqrt{\sec[c+dx]} \sin[c+dx]}{(a^2-b^2)d(a+b \sec[c+dx])}
\end{aligned}$$

Result (type 4, 633 leaves):

$$\begin{aligned}
& \frac{(b + a \cos[c + dx])^2 \sec[c + dx]^{5/2} \left(-\frac{\sin[c + dx]}{-a^2 + b^2} + \frac{b \sin[c + dx]}{(-a^2 + b^2)(b + a \cos[c + dx])} \right)}{d (a + b \sec[c + dx])^2} + \\
& \frac{1}{4(-a + b)(a + b)d(a + b \sec[c + dx])^2} (b + a \cos[c + dx])^2 \sec[c + dx]^2 \left(-\left(8b \cos[c + dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \left(a(b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) - \right. \\
& \quad \left. \left(2a \cos[c + dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \right) \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \left(b(b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) - \right. \\
& \quad \left. \left(2 \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2ab - 2ab \sec[c + dx]^2 + 2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \right. \right. \right. \\
& \quad \left. \left. \sqrt{1 - \sec[c + dx]^2} + a(a - 2b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \sin[c + dx] \right) / \\
& \quad \left. \left(a b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \right)
\end{aligned}$$

■ **Problem 618: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]}}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 227 leaves, 9 steps):

$$\begin{aligned}
& \frac{b \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a(a^2 - b^2)d} + \frac{(2a^2 - b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2(a^2 - b^2)d} - \\
& \frac{b(3a^2 - b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2(a - b)(a + b)^2d} - \frac{b \sqrt{\sec[c + dx]} \sin[c + dx]}{(a^2 - b^2)d(a + b \sec[c + dx])}
\end{aligned}$$

Result (type 4, 630 leaves):

$$\frac{(b + a \cos[c + dx])^2 \sec[c + dx]^{5/2} \left(\frac{b \sin[c + dx]}{a(-a^2 + b^2)} + \frac{b^2 \sin[c + dx]}{a(a^2 - b^2)(b + a \cos[c + dx])} \right)}{d(a + b \sec[c + dx])^2} +$$

$$\frac{1}{4(a - b)(a + b)d(a + b \sec[c + dx])^2} (b + a \cos[c + dx])^2 \sec[c + dx]^2 \left(- \left(8 \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right.$$

$$\left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) \right) / \left((b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) -$$

$$\left(2 \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) \right.$$

$$\left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \left((b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) -$$

$$\left(2 \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2ab - 2ab \sec[c + dx]^2 + 2ab \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \right. \right.$$

$$\left. \sqrt{1 - \sec[c + dx]^2} + a(a - 2b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + \right.$$

$$\left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - \right.$$

$$\left. 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \sin[c + dx] \Big/$$

$$\left(a^2 (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right)$$

■ **Problem 619: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sec[c + dx]} (a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 244 leaves, 9 steps):

$$\frac{(2a^2 - 3b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticE} \left[\frac{1}{2}(c + dx), 2 \right] \sqrt{\sec[c + dx]} - b(4a^2 - 3b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c + dx), 2 \right] \sqrt{\sec[c + dx]}}{a^2(a^2 - b^2)d} +$$

$$\frac{b^2(5a^2 - 3b^2) \sqrt{\cos[c + dx]} \operatorname{EllipticPi} \left[\frac{2a}{a+b}, \frac{1}{2}(c + dx), 2 \right] \sqrt{\sec[c + dx]}}{a^3(a - b)(a + b)^2d} + \frac{b^2 \sqrt{\sec[c + dx]} \sin[c + dx]}{a(a^2 - b^2)d(a + b \sec[c + dx])}$$

Result (type 4, 608 leaves):

$$\frac{\sqrt{\sec[c+dx]} \left(-\frac{b^2 \sin[c+dx]}{a^2(-a^2+b^2)} - \frac{b^3 \sin[c+dx]}{a^2(a^2-b^2)(b+a \cos[c+dx])} \right)}{d} +$$

$$\frac{1}{4a(-a+b)(a+b)d} \left(-\left(8b \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \right.$$

$$\left. \left((b+a \cos[c+dx]) (1-\cos[c+dx]^2) \right) + \right.$$

$$\left. \left(2(-2a^2+b^2) \cos[c+dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right. \right.$$

$$\left. \left. (a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left(b(b+a \cos[c+dx]) (1-\cos[c+dx]^2) \right) - \right.$$

$$\left. \left(2(-2a^2+3b^2) \cos[2(c+dx)] (a+b \sec[c+dx]) \left(2ab - 2ab \sec[c+dx]^2 + 2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right. \right.$$

$$\left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + a(a-2b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right.$$

$$\left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right.$$

$$\left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \left. \right.$$

$$\left. \left(a^2 b (b+a \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) \right)$$

■ **Problem 620: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sec[c+dx]^{3/2} (a+b \sec[c+dx])^2} dx$$

Optimal (type 4, 304 leaves, 10 steps):

$$\frac{b(4a^2-5b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^3(a^2-b^2)d} +$$

$$\frac{(2a^4+16a^2b^2-15b^4) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3a^4(a^2-b^2)d} -$$

$$\frac{b^3(7a^2-5b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^4(a-b)(a+b)^2d} +$$

$$\frac{(2a^2-5b^2) \sin[c+dx]}{3a^2(a^2-b^2)d \sqrt{\sec[c+dx]}} + \frac{b^2 \sin[c+dx]}{a(a^2-b^2)d \sqrt{\sec[c+dx]} (a+b \sec[c+dx])}$$

Result (type 4, 639 leaves):

$$\frac{1}{12 a^2 (a-b)(a+b)d} \left(- \left(2 (4 a^3 + 8 a b^2) \cos [c+d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\ \left. (a(b+a \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\ \left. \left(2 (-8 a^2 b + 5 b^3) \cos [c+d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \right. \\ \left. \left. (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / (b(b+a \cos [c+d x]) (1-\cos [c+d x]^2)) - \right. \\ \left. \left(2 (-12 a^2 b + 15 b^3) \cos [2(c+d x)] (a+b \sec [c+d x]) \left(2 a b - 2 a b \sec [c+d x]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \right. \\ \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + a(a-2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\ \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\ \left. \left(a^2 b (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \right) + \\ \frac{\sqrt{\sec [c+d x]} \left(\frac{b^3 \sin [c+d x]}{a^3 (-a^2+b^2)} + \frac{b^4 \sin [c+d x]}{a^3 (a^2-b^2) (b+a \cos [c+d x])} + \frac{\sin [2(c+d x)]}{3 a^2} \right)}{d}$$

■ **Problem 622: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c+d x]^{7/2}}{(a+b \sec [c+d x])^3} dx$$

Optimal (type 4, 315 leaves, 10 steps):

$$\frac{3 a (a^2 - 3 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]} - (a^2 - 7 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 b^2 (a^2 - b^2)^2 d} + \frac{(a^2 - 7 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 b (a^2 - b^2)^2 d} + \\ \frac{3 (a^4 - 2 a^2 b^2 + 5 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{4 (a-b)^2 b^2 (a+b)^3 d} - \\ \frac{a^2 \sec [c+d x]^{3/2} \sin [c+d x]}{2 b (a^2 - b^2) d (a+b \sec [c+d x])^2} - \frac{3 a^2 (a^2 - 3 b^2) \sqrt{\sec [c+d x]} \sin [c+d x]}{4 b^2 (a^2 - b^2)^2 d (a+b \sec [c+d x])}$$

Result (type 4, 697 leaves):

$$\begin{aligned}
& \frac{1}{16 (a-b)^2 b^2 (a+b)^2 d} \\
& \left(- \left(2 (8 a^3 b - 32 a b^3) \operatorname{Cos}[c+d x]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / \right. \\
& \quad \left. (a (b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2)) + \right. \\
& \quad \left(2 (9 a^4 - 19 a^2 b^2 + 16 b^4) \operatorname{Cos}[c+d x]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / (b (b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2)) - \\
& \quad \left(2 (3 a^4 - 9 a^2 b^2) \operatorname{Cos}[2 (c+d x)] (a+b \operatorname{Sec}[c+d x]) \left(2 a b - 2 a b \operatorname{Sec}[c+d x]^2 + 2 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + a (a-2 b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \operatorname{Sin}[c+d x] \right) / \\
& \quad \left. \left(a^2 b (b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) \right) + \\
& \quad \frac{\sqrt{\operatorname{Sec}[c+d x]} \left(\frac{3 a (-a^2+3 b^2) \operatorname{Sin}[c+d x]}{4 b^2 (-a^2+b^2)^2} - \frac{a \operatorname{Sin}[c+d x]}{2 (-a^2+b^2) (b+a \operatorname{Cos}[c+d x])^2} + \frac{a^3 \operatorname{Sin}[c+d x]-7 a b^2 \operatorname{Sin}[c+d x]}{4 b (-a^2+b^2)^2 (b+a \operatorname{Cos}[c+d x])} \right)}{d}
\end{aligned}$$

■ **Problem 623: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2}}{(a+b \operatorname{Sec}[c+d x])^3} dx$$

Optimal (type 4, 313 leaves, 10 steps):

$$\begin{aligned}
& \frac{(a^2+5 b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]} + 3 (a^2+b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b (a^2-b^2)^2 d} + \\
& \frac{(a^4-10 a^2 b^2-3 b^4) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \frac{1}{2} (c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 a (a-b)^2 b (a+b)^3 d} - \\
& \frac{a^2 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 b (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^2} + \frac{a (a^2-7 b^2) \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b (a^2-b^2)^2 d (a+b \operatorname{Sec}[c+d x])}
\end{aligned}$$

Result (type 4, 733 leaves):

$$\begin{aligned}
& \frac{1}{16 (a-b)^2 b (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^3} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \\
& \left(- \left(2 (8a^2b + 16b^3) \operatorname{Cos}[c+dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1 \right] (a+b \operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] \right) / \right. \\
& \quad \left(a (b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2) \right) + \\
& \quad \left(2 (3a^3 - 9ab^2) \operatorname{Cos}[c+dx]^2 \left(\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1 \right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] \right) / \left(b (b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2) \right) - \\
& \quad \left(2 (a^3 + 5ab^2) \operatorname{Cos}[2(c+dx)] (a+b \operatorname{Sec}[c+dx]) \left(2ab - 2ab \operatorname{Sec}[c+dx]^2 + 2ab \operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + a(a-2b) \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1 \right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin}[\sqrt{\operatorname{Sec}[c+dx]}], -1 \right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} \right) \operatorname{Sin}[c+dx] \right) / \\
& \quad \left(a^2 b (b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2) \sqrt{\operatorname{Sec}[c+dx]} (2-\operatorname{Sec}[c+dx]^2) \right) \Big) + \frac{1}{d (a+b \operatorname{Sec}[c+dx])^3} \\
& (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^{7/2} \left(-\frac{(a^2+5b^2) \operatorname{Sin}[c+dx]}{4b(-a^2+b^2)^2} + \frac{b \operatorname{Sin}[c+dx]}{2(-a^2+b^2)(b+a \operatorname{Cos}[c+dx])^2} + \right. \\
& \quad \left. \frac{3(a^2 \operatorname{Sin}[c+dx] + b^2 \operatorname{Sin}[c+dx])}{4(-a^2+b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right)
\end{aligned}$$

■ **Problem 624: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{3/2}}{(a+b \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 306 leaves, 10 steps):

$$\begin{aligned}
& \frac{(5a^2+b^2) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticE}[\frac{1}{2}(c+dx), 2] \sqrt{\operatorname{Sec}[c+dx]} - b(7a^2-b^2) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}[\frac{1}{2}(c+dx), 2] \sqrt{\operatorname{Sec}[c+dx]}}{4a(a^2-b^2)^2 d} + \frac{b(7a^2-b^2) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}[\frac{1}{2}(c+dx), 2] \sqrt{\operatorname{Sec}[c+dx]}}{4a^2(a^2-b^2)^2 d} \\
& + \frac{(3a^4+10a^2b^2-b^4) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticPi}[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2] \sqrt{\operatorname{Sec}[c+dx]}}{4a^2(a-b)^2(a+b)^3 d} \\
& + \frac{a \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2(a^2-b^2)d(a+b \operatorname{Sec}[c+dx])^2} + \frac{3(a^2+b^2) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4(a^2-b^2)^2 d(a+b \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 4, 724 leaves) :

$$\begin{aligned}
 & - \frac{1}{16 (a-b)^2 (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^3} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \\
 & \left(- \left(48 b \operatorname{Cos}[c+dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] (a+b \operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] \right) / \right. \\
 & \quad \left. (b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2) \right) + \\
 & \left(2 (-a^2-5b^2) \operatorname{Cos}[c+dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] \right) \right. \\
 & \quad \left. (a+b \operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] \right) / (b(b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2)) - \\
 & \left(2 (5a^2+b^2) \operatorname{Cos}[2(c+dx)] (a+b \operatorname{Sec}[c+dx]) \left(2ab-2ab \operatorname{Sec}[c+dx]^2+2ab \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] \right. \right. \\
 & \quad \left. \left. \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + a(a-2b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + \right. \right. \\
 & \quad \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} - \right. \right. \\
 & \quad \left. \left. 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+dx]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} \right) \operatorname{Sin}[c+dx] \right) / \\
 & \quad \left. \left(a^2 b (b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2) \sqrt{\operatorname{Sec}[c+dx]} (2-\operatorname{Sec}[c+dx]^2) \right) \right) + \\
 & \frac{1}{d (a+b \operatorname{Sec}[c+dx])^3} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^{7/2} \\
 & \left(\frac{(5a^2+b^2) \operatorname{Sin}[c+dx]}{4a(a^2-b^2)^2} + \right. \\
 & \quad \frac{b^2 \operatorname{Sin}[c+dx]}{2a(a^2-b^2)(b+a \operatorname{Cos}[c+dx])^2} + \\
 & \quad \left. \frac{-7a^2 b \operatorname{Sin}[c+dx] + b^3 \operatorname{Sin}[c+dx]}{4a(a^2-b^2)^2(b+a \operatorname{Cos}[c+dx])} \right)
 \end{aligned}$$

■ **Problem 625: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]}}{(a+b \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 323 leaves, 10 steps) :

$$\frac{3 b (3 a^2 - b^2) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{4 a^2 (a^2 - b^2)^2 d} +$$

$$\frac{(8 a^4 - 5 a^2 b^2 + 3 b^4) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{4 a^3 (a^2 - b^2)^2 d} -$$

$$\frac{3 b (5 a^4 - 2 a^2 b^2 + b^4) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{4 a^3 (a - b)^2 (a + b)^3 d} -$$

$$\frac{b \sqrt{\sec[c + d x]} \sin[c + d x]}{2 (a^2 - b^2) d (a + b \sec[c + d x])^2} - \frac{b (7 a^2 - b^2) \sqrt{\sec[c + d x]} \sin[c + d x]}{4 a (a^2 - b^2)^2 d (a + b \sec[c + d x])}$$

Result (type 4, 749 leaves):

$$\frac{1}{16 a (a - b)^2 (a + b)^2 d (a + b \sec[c + d x])^3} (b + a \cos[c + d x])^3 \sec[c + d x]^3$$

$$\left(- \left(2 (16 a^3 + 8 a b^2) \cos[c + d x]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] (a + b \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \sin[c + d x] \right) / \right.$$

$$\left. (a (b + a \cos[c + d x]) (1 - \cos[c + d x]^2)) + \right.$$

$$\left(2 (-5 a^2 b - b^3) \cos[c + d x]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right) \right.$$

$$\left. (a + b \sec[c + d x]) \sqrt{1 - \sec[c + d x]^2} \sin[c + d x] \right) / (b (b + a \cos[c + d x]) (1 - \cos[c + d x]^2)) -$$

$$\left(2 (9 a^2 b - 3 b^3) \cos[2(c + d x)] (a + b \sec[c + d x]) \left(2 a b - 2 a b \sec[c + d x]^2 + 2 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \right. \right.$$

$$\left. \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + a (a - 2 b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} + \right.$$

$$\left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} - \right.$$

$$\left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + d x]}\right], -1\right] \sqrt{\sec[c + d x]} \sqrt{1 - \sec[c + d x]^2} \right) \sin[c + d x] \Big/$$

$$\left(a^2 b (b + a \cos[c + d x]) (1 - \cos[c + d x]^2) \sqrt{\sec[c + d x]} (2 - \sec[c + d x]^2) \right) + \frac{1}{d (a + b \sec[c + d x])^3}$$

$$(b + a \cos[c + d x])^3 \sec[c + d x]^{7/2} \left(\frac{3 b (-3 a^2 + b^2) \sin[c + d x]}{4 a^2 (-a^2 + b^2)^2} - \frac{b^3 \sin[c + d x]}{2 a^2 (a^2 - b^2) (b + a \cos[c + d x])^2} + \right.$$

$$\left. \frac{11 a^2 b^2 \sin[c + d x] - 5 b^4 \sin[c + d x]}{4 a^2 (a^2 - b^2)^2 (b + a \cos[c + d x])} \right)$$

■ **Problem 626: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sec[c+dx] (a+b\sec[c+dx])^3}} dx$$

Optimal (type 4, 342 leaves, 10 steps):

$$\begin{aligned} & \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a^3(a^2-b^2)^2d} - \\ & \frac{3b(8a^4 - 11a^2b^2 + 5b^4) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a^4(a^2-b^2)^2d} + \\ & \frac{b^2(35a^4 - 38a^2b^2 + 15b^4) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{4a^4(a-b)^2(a+b)^3d} + \\ & \frac{b^2 \sqrt{\sec[c+dx]} \sin[c+dx]}{2a(a^2-b^2)d(a+b\sec[c+dx])^2} + \frac{b^2(11a^2-5b^2) \sqrt{\sec[c+dx]} \sin[c+dx]}{4a^2(a^2-b^2)^2d(a+b\sec[c+dx])} \end{aligned}$$

Result (type 4, 712 leaves):

$$\frac{1}{16 a^2 (a-b)^2 (a+b)^2 d} \left(- \left(2 \left(-32 a^3 b + 8 a b^3 \right) \operatorname{Cos}[c+d x]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / \right. \\ \left. (a(b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2)) + \right. \\ \left. \left(2 \left(8 a^4 - 7 a^2 b^2 + 5 b^4 \right) \operatorname{Cos}[c+d x]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right) \right. \right. \\ \left. \left. (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \operatorname{Sin}[c+d x] \right) / (b(b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2)) - \right. \\ \left. \left(2 \left(8 a^4 - 29 a^2 b^2 + 15 b^4 \right) \operatorname{Cos}[2(c+d x)] (a+b \operatorname{Sec}[c+d x]) \left(2 a b - 2 a b \operatorname{Sec}[c+d x]^2 + 2 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \right. \right. \right. \\ \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + a(a-2 b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + \right. \right. \\ \left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\ \left. \left. 2 b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+d x]}\right], -1\right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \operatorname{Sin}[c+d x] \right) / \\ \left. \left(a^2 b (b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Cos}[c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) \right) + \\ \frac{\sqrt{\operatorname{Sec}[c+d x]} \left(-\frac{b^2(-13 a^2+7 b^2) \operatorname{Sin}[c+d x]}{4 a^3(-a^2+b^2)^2} + \frac{b^4 \operatorname{Sin}[c+d x]}{2 a^3(a^2-b^2)(b+a \operatorname{Cos}[c+d x])^2} + \frac{3(-5 a^2 b^3 \operatorname{Sin}[c+d x]+3 b^5 \operatorname{Sin}[c+d x])}{4 a^3(a^2-b^2)^2(b+a \operatorname{Cos}[c+d x])} \right)}{d}$$

■ **Problem 628: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} dx$$

Optimal (type 4, 237 leaves, 12 steps):

$$\frac{b \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} + a \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{\operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]} + \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 4, 321 leaves):

$$\frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}}$$

$$\sqrt{a+b \operatorname{Sec}[c+d x]} \left(\frac{2 a \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{(a+b) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} - \left(2 i \sqrt{\frac{a(-1+\operatorname{Cos}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Cos}[c+d x])}{a-b}} \operatorname{Csc}[c+d x] \left(-2 b(a+b) \right. \right. \right. \right.$$

$$\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right.$$

$$\left. \left. \left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right)\right)\right)\right) / \left(a \sqrt{\frac{1}{a-b}} b \sqrt{b+a \operatorname{Cos}[c+d x]} \right) + 4 \operatorname{Tan}[c+d x] \left. \right)$$

■ **Problem 634: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{3/2} dx$$

Optimal (type 4, 299 leaves, 13 steps):

$$\frac{7 a b \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{(3 a^2 + 4 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{5 a \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} +$$

$$\frac{5 a \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{b \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 d}$$

Result (type 4, 549 leaves):

$$\begin{aligned}
& \frac{1}{16 d (b + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2}} \\
& (a + b \sec [c + d x])^{3/2} \left(\frac{8 a b \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} - \frac{2(-a^2-8 b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} - \right. \\
& \left. \left(10 i a^2 \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \sin [c+d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\cos [c+d x]}^2 \sqrt{\frac{a^2-a^2 \cos [c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \cos [c+d x])+2(b+a \cos [c+d x])^2\right) \right) \right) + \\
& \frac{(a+b \sec [c+d x])^{3/2} \left(\frac{5}{4} a \tan [c+d x] + \frac{1}{2} b \sec [c+d x] \tan [c+d x]\right)}{d(b+a \cos [c+d x]) \sec [c+d x]^{3/2}}
\end{aligned}$$

■ **Problem 635: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec [c+d x]} (a+b \sec [c+d x])^{3/2} dx$$

Optimal (type 4, 249 leaves, 12 steps):

$$\begin{aligned}
& \frac{(2 a^2+b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]} + 3 a b \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{b \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} + b \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{b \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d}
\end{aligned}$$

Result (type 4, 394 leaves) :

$$\frac{1}{4 d \operatorname{Sec}[c+d x]^{3/2}} (a+b \operatorname{Sec}[c+d x])^{3/2} \left(\frac{8 a^2 \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{(b+a \operatorname{Cos}[c+d x])^2} + \frac{10 a b \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{(b+a \operatorname{Cos}[c+d x])^2} - \right. \\ \left. \left(2 i \sqrt{\frac{a(-1+\operatorname{Cos}[c+d x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Cos}[c+d x])}{a-b}} \operatorname{Csc}[c+d x] \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \right. \\ \left. \left. \left. a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \right. \right. \\ \left. \left. \left. a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \right) / \left(a \sqrt{\frac{1}{a-b}} (b+a \operatorname{Cos}[c+d x])^{3/2} + \frac{4 b \operatorname{Tan}[c+d x]}{b+a \operatorname{Cos}[c+d x]} \right)$$

■ **Problem 640: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{5/2} dx$$

Optimal (type 4, 369 leaves, 14 steps) :

$$\frac{b(59 a^2 + 16 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{24 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\ \frac{5 a(a^2 + 4 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{8 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\ \frac{(33 a^2 + 16 b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{24 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{(33 a^2 + 16 b^2) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d} + \\ \frac{13 a b \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 d} + \frac{b^2 \operatorname{Sec}[c+d x]^{5/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d}$$

Result (type 4, 602 leaves) :

$$\begin{aligned}
& - \frac{1}{96 d (b + a \cos[c + dx])^{5/2} \sec[c + dx]^{5/2}} a (a + b \sec[c + dx])^{5/2} \\
& \left(- \frac{104 a b \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + 2(3a^2 - 104b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos[c+dx]}} + \right. \\
& \left. \frac{2i(33a^2 + 16b^2) \sqrt{\frac{a-a \cos[c+dx]}{a+b}} \sqrt{\frac{a+a \cos[c+dx]}{a-b}} \cos[2(c+dx)]}{\sqrt{b+a \cos[c+dx]}} \right. \\
& \left. \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \right) \sin[c+dx] \Bigg/ \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \cos[c+dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b+a \cos[c+dx]) + 2(b+a \cos[c+dx])^2) \right) \right) + \\
& \left((a + b \sec[c + dx])^{5/2} \left(\frac{1}{24} \sec[c + dx] (33 a^2 \sin[c + dx] + 16 b^2 \sin[c + dx]) + \frac{13}{12} a b \sec[c + dx] \tan[c + dx] + \right. \right. \\
& \left. \left. \frac{1}{3} b^2 \sec[c + dx]^2 \tan[c + dx] \right) \right) \Bigg/ (d (b + a \cos[c + dx])^2 \sec[c + dx]^{5/2})
\end{aligned}$$

■ **Problem 641: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec[c + dx]} (a + b \sec[c + dx])^{5/2} dx$$

Optimal (type 4, 314 leaves, 13 steps):

$$\begin{aligned}
& \frac{a (8 a^2 + 11 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{b (15 a^2 + 4 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{9 a b \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{9 a b \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{b^2 \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 d}
\end{aligned}$$

Result (type 4, 560 leaves):

$$\begin{aligned}
& \frac{1}{16 d (b+a \operatorname{Cos}[c+d x])^{5/2} \operatorname{Sec}[c+d x]^{5/2}} (a+b \operatorname{Sec}[c+d x])^{5/2} \\
& \left(\frac{2 (16 a^3 + 4 a b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2 (21 a^2 b + 8 b^3) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} \right) - \\
& \left(18 i a^2 \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \left(-2 b (a+b) \right. \right. \\
& \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \\
& \left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c+d x] \right) / \\
& \left(\sqrt{\frac{1}{a-b}} \sqrt{1 - \operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+d x]) + 2 (b+a \operatorname{Cos}[c+d x])^2) \right) + \\
& \frac{(a+b \operatorname{Sec}[c+d x])^{5/2} \left(\frac{9}{4} a b \operatorname{Tan}[c+d x] + \frac{1}{2} b^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x] \right)}{d (b+a \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^{5/2}}
\end{aligned}$$

- **Problem 642: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2}}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 263 leaves, 12 steps):

$$\frac{b(4a^2 + b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]} + 5ab^2 \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{(2a^2 - b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]} + b^2 \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 4, 538 leaves):

$$\frac{b^2 (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{d (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{3/2}} + \frac{1}{4 d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} a (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{24 a b \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{2(2a^2 + 9b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \left(2i(2a^2 - b^2) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right. \right. \\ \left. \left. - 2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right. \right. \right. \\ \left. \left. \left. - \frac{-a+b}{a+b} \right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c+dx] \right) / \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b+a \operatorname{Cos}[c+dx]) + 2(b+a \operatorname{Cos}[c+dx])^2) \right)$$

- **Problem 643: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2}}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 262 leaves, 12 steps):

$$\frac{2 a (a^2 + 2 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} + 2 b^3 \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{14 a b \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]} + 2 a^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 a^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}$$

Result (type 4, 540 leaves):

$$\frac{2 a^2 (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{3 d (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}} + \frac{1}{6 d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2 (2 a^3 + 18 a b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2 (7 a^2 b + 6 b^3) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \left(14 i a^2 \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \right. \right. \\ \left. \left. - 2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] \right. \right. \right. \\ \left. \left. \left. + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \operatorname{Sin}[c+d x] \right) / \left(\sqrt{\frac{1}{a-b}} \sqrt{1 - \operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c+d x]) + 2 (b + a \operatorname{Cos}[c+d x])^2) \right)$$

■ **Problem 647: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]^{7/2}}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 312 leaves, 13 steps):

$$\begin{aligned} & a \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} \\ & - \frac{\quad}{4 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\ & \frac{(3 a^2 + 4 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{3 a \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 b^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} - \\ & \frac{3 a \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b^2 d} + \frac{\operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 b d} \end{aligned}$$

Result (type 4, 555 leaves):

$$\begin{aligned}
& \frac{1}{16 b^2 d \sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} \\
& \left(\frac{8 a b \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2\left(9 a^2+8 b^2\right) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \\
& \left. \left(6 i a^2 \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \right. \\
& \left. \left. a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \right. \\
& \left. \left. \left. a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \operatorname{Sin}[c+d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \operatorname{Cos}[c+d x])+2(b+a \operatorname{Cos}[c+d x])^2\right) \right) \right) + \\
& \frac{(b+a \operatorname{Cos}[c+d x]) \sqrt{\operatorname{Sec}[c+d x]} \left(-\frac{3 a \operatorname{Tan}[c+d x]}{4 b^2} + \frac{\operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 b}\right)}{d \sqrt{a+b \operatorname{Sec}[c+d x]}}
\end{aligned}$$

■ **Problem 648: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2}}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 246 leaves, 12 steps):

$$\frac{\sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]} - a \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]}}{d \sqrt{a+b \sec[c+dx]} - b d \sqrt{a+b \sec[c+dx]}} + \frac{\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + \sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{b d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \sqrt{\sec[c+dx]} - b d}$$

Result (type 4, 461 leaves):

$$\frac{(b+a \cos[c+dx]) \sec[c+dx]^{3/2} \sin[c+dx]}{b d \sqrt{a+b \sec[c+dx]}} - \frac{1}{4 b d \sqrt{a+b \sec[c+dx]}}$$

$$+ a \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \left(\frac{6 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos[c+dx]}} + \right.$$

$$\left. \left(2 i \sqrt{\frac{a-a \cos[c+dx]}{a+b}} \sqrt{\frac{a+a \cos[c+dx]}{a-b}} \cos[2(c+dx)] \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + \right. \right.$$

$$\left. \left. a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + \right. \right.$$

$$\left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \sin[c+dx] \right) /$$

$$\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \cos[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \cos[c+dx]) + 2 (b+a \cos[c+dx])^2) \right)$$

■ **Problem 654: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{7/2}}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 345 leaves, 13 steps):

$$\begin{aligned}
& \frac{\sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{b d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \\
& \frac{3 a \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{b^2 d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \frac{(3 a^2 - b^2) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]}}{b^2 (a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\operatorname{Sec}[c+dx]}} - \\
& \frac{2 a^2 \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{b (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{(3 a^2 - b^2) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{b^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 4, 592 leaves):

$$\begin{aligned}
& - \frac{1}{4 (a-b) b^2 (a+b) d (a+b \operatorname{Sec}[c+dx])^{3/2}} a (b+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \\
& \left(\frac{8 a b \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{2 (9 a^2 - 7 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \\
& \left. \left(2 i (3 a^2 - b^2) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \left(-2 b (a+b) \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + \right. \right. \right. \\
& \left. \left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \right) \operatorname{Sin}[c+dx] \right) / \right. \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+dx]) + 2 (b+a \operatorname{Cos}[c+dx])^2) \right) \right) + \\
& \frac{(b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^{3/2} \left(-\frac{2 a^3 \operatorname{Sin}[c+dx]}{b^2 (-a^2+b^2) (b+a \operatorname{Cos}[c+dx])} + \frac{\operatorname{Tan}[c+dx]}{b^2} \right)}{d (a+b \operatorname{Sec}[c+dx])^{3/2}}
\end{aligned}$$

- **Problem 655: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2}}{(a+b \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 4, 206 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} - \frac{2 a^2 \sqrt{\sec [c+d x]} \sin [c+d x]}{b\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 4, 557 leaves):

$$\begin{aligned}
& \frac{2 a^2 (b+a \cos [c+d x]) \sec [c+d x]^{3/2} \sin [c+d x]}{b\left(-a^2+b^2\right) d (a+b \sec [c+d x])^{3/2}} + \\
& \frac{1}{2(a-b) b(a+b) d (a+b \sec [c+d x])^{3/2}} (b+a \cos [c+d x])^{3/2} \sec [c+d x]^{3/2} \left(\frac{4 a b \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \right. \\
& \frac{2\left(3 a^2-2 b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \left. \left(2 i a^2 \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
& \left. \left. \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \sin [c+d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\cos [c+d x]}^2 \sqrt{\frac{a^2-a^2 \cos [c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \cos [c+d x])+2(b+a \cos [c+d x])^2\right) \right) \right)
\end{aligned}$$

■ **Problem 661: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^{9/2}}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 458 leaves, 14 steps):

$$\begin{aligned}
& \frac{(5 a^2 - 3 b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]} - 5 a \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 b^2\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}} - \frac{5 a \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b^3 d \sqrt{a+b \sec [c+d x]}} \\
& \frac{\left(15 a^4-26 a^2 b^2+3 b^4\right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 b^3\left(a^2-b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} - \frac{2 a^2 \sec [c+d x]^{5 / 2} \sin [c+d x]}{3 b\left(a^2-b^2\right) d(a+b \sec [c+d x])^{3 / 2}} \\
& \frac{2 a^2\left(5 a^2-9 b^2\right) \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{a+b \sec [c+d x]}} + \frac{\left(15 a^4-26 a^2 b^2+3 b^4\right) \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 b^3\left(a^2-b^2\right)^2 d}
\end{aligned}$$

Result (type 4, 677 leaves):

$$\begin{aligned}
& - \frac{1}{12 (a-b)^2 b^3 (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^{5/2}} a (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \\
& \left(\frac{2 (20 a^3 b - 36 a b^3) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), \frac{2a}{a+b}\right] + 2 (45 a^4 - 86 a^2 b^2 + 33 b^4) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{2 i (15 a^4 - 26 a^2 b^2 + 3 b^4) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2 (c+dx)]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} \right) \\
& \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right. \right. \\
& \left. \left. + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+dx] \Big/ \\
& \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+dx]) + 2 (b+a \operatorname{Cos}[c+dx])^2) \right) \Big/ + \\
& \frac{1}{d (a+b \operatorname{Sec}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^{5/2} \left(-\frac{2 a^3 \operatorname{Sin}[c+dx]}{3 b^2 (-a^2 + b^2) (b+a \operatorname{Cos}[c+dx])^2} - \right. \\
& \left. \frac{4 (-3 a^5 \operatorname{Sin}[c+dx] + 5 a^3 b^2 \operatorname{Sin}[c+dx])}{3 b^3 (-a^2 + b^2)^2 (b+a \operatorname{Cos}[c+dx])} + \frac{\operatorname{Tan}[c+dx]}{b^3} \right)
\end{aligned}$$

■ **Problem 662: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+dx]^{7/2}}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 370 leaves, 13 steps):

$$\begin{aligned}
& - \frac{2 a \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 b\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b^2 d \sqrt{a+b \sec [c+d x]}} + \frac{2 a\left(3 a^2-7 b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} - \\
& \frac{2 a^2 \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 b\left(a^2-b^2\right) d\left(a+b \sec [c+d x]\right)^{3 / 2}} - \frac{2 a^2\left(3 a^2-7 b^2\right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 4, 661 leaves):

$$\frac{1}{6 (a-b)^2 b^2 (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2}$$

$$\left(\frac{2 (4 a^3 b - 12 a b^3) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{2 (9 a^4 - 19 a^2 b^2 + 6 b^4) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right.$$

$$\left. \frac{2 i (3 a^4 - 7 a^2 b^2) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2 (c+dx)]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} \right)$$

$$\left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right], \right.$$

$$\left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c+dx] \Bigg/$$

$$\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]} \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+dx]) + 2 (b+a \operatorname{Cos}[c+dx])^2) \right) \Bigg) +$$

$$\frac{(b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^{5/2} \left(\frac{2 a^2 \operatorname{Sin}[c+dx]}{3 b (-a^2+b^2) (b+a \operatorname{Cos}[c+dx])^2} + \frac{2 (-3 a^4 \operatorname{Sin}[c+dx] + 7 a^2 b^2 \operatorname{Sin}[c+dx])}{3 b^2 (-a^2+b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right)}{d (a+b \operatorname{Sec}[c+dx])^{5/2}}$$

■ **Problem 685: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx] (a+b \operatorname{Sec}[c+dx])^{1/3} dx$$

Optimal (type 6, 105 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}\right] (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b}\right)^{1/3}}$$

Result (type 6, 11281 leaves): Display of huge result suppressed!

■ **Problem 687: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^4 (a+b \operatorname{Sec}[c+dx])^{2/3} dx$$

Optimal (type 6, 362 leaves, 10 steps):

$$\frac{3 (9 a^2 + 32 b^2) (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{220 b^2 d} - \frac{9 a (a + b \operatorname{Sec}[c + d x])^{5/3} \operatorname{Tan}[c + d x]}{44 b^2 d} + \frac{3 \operatorname{Sec}[c + d x] (a + b \operatorname{Sec}[c + d x])^{5/3} \operatorname{Tan}[c + d x]}{11 b d} +$$

$$\left(a (18 a^2 + 49 b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x] \right) /$$

$$\left(110 \sqrt{2} b^3 d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \right) -$$

$$\left((9 a^4 + 23 a^2 b^2 - 32 b^4) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{1/3} \operatorname{Tan}[c + d x] \right) /$$

$$\left(55 \sqrt{2} b^3 d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3} \right)$$

Result (type 6, 33386 leaves): Display of huge result suppressed!

■ **Problem 688: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^{2/3} dx$$

Optimal (type 6, 305 leaves, 9 steps):

$$- \frac{9 a (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{40 b d} + \frac{3 (a + b \operatorname{Sec}[c + d x])^{5/3} \operatorname{Tan}[c + d x]}{8 b d} -$$

$$\left((6 a^2 - 25 b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x] \right) /$$

$$\left(20 \sqrt{2} b^2 d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \right) +$$

$$\frac{3 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{1/3} \operatorname{Tan}[c + d x]}{10 \sqrt{2} b^2 d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}}$$

Result (type 6, 29862 leaves): Display of huge result suppressed!

■ **Problem 689: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (a + b \operatorname{Sec}[c + d x])^{2/3} dx$$

Optimal (type 6, 260 leaves, 8 steps):

$$\frac{3 (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d} + \frac{2 \sqrt{2} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3}} -$$

$$\frac{2 \sqrt{2} (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{1/3} \operatorname{Tan}[c + d x]}{5 b d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}}$$

Result (type 6, 9607 leaves) : Display of huge result suppressed!

■ **Problem 690: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^{2/3} dx$$

Optimal (type 6, 105 leaves, 3 steps) :

$$\frac{\sqrt{2} \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b(1 - \text{Sec}[c + d x])}{a + b}\right] (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x]}{d \sqrt{1 + \text{Sec}[c + d x]} \left(\frac{a + b \text{Sec}[c + d x]}{a + b}\right)^{2/3}}$$

Result (type 6, 11285 leaves) : Display of huge result suppressed!

■ **Problem 692: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^{4/3} dx$$

Optimal (type 6, 108 leaves, 3 steps) :

$$\left(\sqrt{2} (a + b) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b(1 - \text{Sec}[c + d x])}{a + b}\right] (a + b \text{Sec}[c + d x])^{1/3} \text{Tan}[c + d x] \right) / \left(d \sqrt{1 + \text{Sec}[c + d x]} \left(\frac{a + b \text{Sec}[c + d x]}{a + b}\right)^{1/3} \right)$$

Result (type 6, 11551 leaves) : Display of huge result suppressed!

■ **Problem 694: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^4 (a + b \text{Sec}[c + d x])^{5/3} dx$$

Optimal (type 6, 412 leaves, 11 steps) :

$$\frac{3 a (18 a^2 + 97 b^2) (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x]}{1232 b^2 d} + \frac{3 (18 a^2 + 121 b^2) (a + b \text{Sec}[c + d x])^{5/3} \text{Tan}[c + d x]}{1232 b^2 d} - \frac{9 a (a + b \text{Sec}[c + d x])^{8/3} \text{Tan}[c + d x]}{77 b^2 d} + \frac{3 \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^{8/3} \text{Tan}[c + d x]}{14 b d} + \left((36 a^4 + 164 a^2 b^2 + 605 b^4) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b(1 - \text{Sec}[c + d x])}{a + b}\right] (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x] \right) / \left(616 \sqrt{2} b^3 d \sqrt{1 + \text{Sec}[c + d x]} \left(\frac{a + b \text{Sec}[c + d x]}{a + b}\right)^{2/3} \right) - \left(a (18 a^4 + 79 a^2 b^2 - 97 b^4) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b(1 - \text{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \text{Sec}[c + d x]}{a + b}\right)^{1/3} \text{Tan}[c + d x] \right) / \left(308 \sqrt{2} b^3 d \sqrt{1 + \text{Sec}[c + d x]} (a + b \text{Sec}[c + d x])^{1/3} \right)$$

Result (type 6, 44504 leaves) : Display of huge result suppressed!

■ **Problem 695: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^3 (a + b \text{Sec}[c + d x])^{5/3} dx$$

Optimal (type 6, 356 leaves, 10 steps):

$$\begin{aligned} & - \frac{3 (15 a^2 - 64 b^2) (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x]}{440 b d} - \frac{9 a (a + b \text{Sec}[c + d x])^{5/3} \text{Tan}[c + d x]}{88 b d} + \frac{3 (a + b \text{Sec}[c + d x])^{8/3} \text{Tan}[c + d x]}{11 b d} \\ & \left(a (30 a^2 - 373 b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b (1 - \text{Sec}[c + d x])}{a + b}\right] (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x] \right) / \\ & \left(220 \sqrt{2} b^2 d \sqrt{1 + \text{Sec}[c + d x]} \left(\frac{a + b \text{Sec}[c + d x]}{a + b} \right)^{2/3} \right) + \\ & \left((15 a^4 - 79 a^2 b^2 + 64 b^4) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b (1 - \text{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \text{Sec}[c + d x]}{a + b} \right)^{1/3} \text{Tan}[c + d x] \right) / \\ & (110 \sqrt{2} b^2 d \sqrt{1 + \text{Sec}[c + d x]} (a + b \text{Sec}[c + d x])^{1/3}) \end{aligned}$$

Result (type 6, 33405 leaves): Display of huge result suppressed!

■ **Problem 696: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^2 (a + b \text{Sec}[c + d x])^{5/3} dx$$

Optimal (type 6, 299 leaves, 9 steps):

$$\begin{aligned} & \frac{3 a (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x]}{8 d} + \frac{3 (a + b \text{Sec}[c + d x])^{5/3} \text{Tan}[c + d x]}{8 d} + \\ & \left((2 a^2 + 5 b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b (1 - \text{Sec}[c + d x])}{a + b}\right] (a + b \text{Sec}[c + d x])^{2/3} \text{Tan}[c + d x] \right) / \\ & \left(4 \sqrt{2} b d \sqrt{1 + \text{Sec}[c + d x]} \left(\frac{a + b \text{Sec}[c + d x]}{a + b} \right)^{2/3} \right) - \\ & \frac{a (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + d x]), \frac{b (1 - \text{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \text{Sec}[c + d x]}{a + b} \right)^{1/3} \text{Tan}[c + d x]}{2 \sqrt{2} b d \sqrt{1 + \text{Sec}[c + d x]} (a + b \text{Sec}[c + d x])^{1/3}} \end{aligned}$$

Result (type 6, 29873 leaves): Display of huge result suppressed!

■ **Problem 697: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^{5/3} dx$$

Optimal (type 6, 108 leaves, 3 steps):

$$\left(\sqrt{2} (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx] \right) /$$

$$\left(d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \right)$$

Result (type 6, 11531 leaves) : Display of huge result suppressed!

■ **Problem 699: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^4}{(a+b \operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 6, 313 leaves, 9 steps) :

$$-\frac{9a(a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx]}{20b^2d} + \frac{3 \operatorname{Sec}[c+dx] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx]}{8bd} +$$

$$\left((18a^2 + 25b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx] \right) /$$

$$\left(20\sqrt{2} b^3 d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \right) -$$

$$\frac{a(9a^2 + 11b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \operatorname{Tan}[c+dx]}{10\sqrt{2} b^3 d \sqrt{1 + \operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{1/3}}$$

Result (type 6, 29880 leaves) : Display of huge result suppressed!

■ **Problem 700: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^3}{(a+b \operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 6, 265 leaves, 8 steps) :

$$\frac{3(a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx]}{5bd} - \frac{3\sqrt{2} a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx]}{5b^2 d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3}} +$$

$$\left(\sqrt{2} (3a^2 + 2b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \operatorname{Tan}[c+dx] \right) /$$

$$\left(5b^2 d \sqrt{1 + \operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{1/3} \right)$$

Result (type 6, 18777 leaves) : Display of huge result suppressed!

■ **Problem 701: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 6, 219 leaves, 7 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{2/3}} -$$

$$\frac{\sqrt{2} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{1/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}}$$

Result (type 6, 10665 leaves): Display of huge result suppressed!

■ **Problem 702: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + b \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 6, 105 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{1/3} \operatorname{Tan}[c + d x]}{d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}}$$

Result (type 6, 310 leaves):

$$\left(15 (a - b)^2 (a + b) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a + b \operatorname{Sec}[c + d x]}{a - b}, \frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right] \right. \\ \left. \operatorname{Cos}[c + d x] \operatorname{Cot}[c + d x]^3 (1 + \operatorname{Sec}[c + d x]) (b - b \operatorname{Sec}[c + d x]) (a + b \operatorname{Sec}[c + d x])^{2/3}\right) /$$

$$\left(b^2 (-a + b) d \left(3 (a - b) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, \frac{3}{2}, \frac{8}{3}, \frac{a + b \operatorname{Sec}[c + d x]}{a - b}, \frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right] (b + a \operatorname{Cos}[c + d x]) + \right. \right. \\ \left. (a + b) \left(10 (a - b) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a + b \operatorname{Sec}[c + d x]}{a - b}, \frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right] \operatorname{Cos}[c + d x] + \right. \right. \\ \left. \left. \left. 3 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, \frac{1}{2}, \frac{8}{3}, \frac{a + b \operatorname{Sec}[c + d x]}{a - b}, \frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right] (b + a \operatorname{Cos}[c + d x])\right)\right)\right)$$

■ **Problem 704: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + b \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 6, 105 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{2/3} \operatorname{Tan}[c + d x]}{d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{2/3}}$$

Result (type 6, 310 leaves):

$$\left(24 (a-b)^2 (a+b) \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}, \frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right] \right. \\ \left. \operatorname{Cos}[c+dx] \operatorname{Cot}[c+dx]^3 (1+\operatorname{Sec}[c+dx]) (b-b \operatorname{Sec}[c+dx]) (a+b \operatorname{Sec}[c+dx])^{1/3} \right] / \\ \left(b^2 (-a+b) d \left(3 (a-b) \operatorname{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, \frac{3}{2}, \frac{7}{3}, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}, \frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right] (b+a \operatorname{Cos}[c+dx]) + \right. \right. \\ \left. (a+b) \left(8 (a-b) \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}, \frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right] \operatorname{Cos}[c+dx] + \right. \right. \\ \left. \left. \left. 3 \operatorname{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, \frac{1}{2}, \frac{7}{3}, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}, \frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right] (b+a \operatorname{Cos}[c+dx]) \right) \right) \right) \right)$$

■ **Problem 706: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]}{(a+b \operatorname{Sec}[c+dx])^{4/3}} dx$$

Optimal (type 6, 110 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1-\operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \operatorname{Tan}[c+dx]}{(a+b) d \sqrt{1+\operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{1/3}}$$

Result (type 6, 12814 leaves): Display of huge result suppressed!

■ **Problem 708: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^4}{(a+b \operatorname{Sec}[c+dx])^{5/3}} dx$$

Optimal (type 6, 378 leaves, 9 steps):

$$-\frac{3 a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{2/3}} + \frac{3 (3 a^2 - b^2) (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{4 b^2 (a^2 - b^2) d} - \\ \left(a (9 a^2 - 7 b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b (1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx] \right) / \\ \left(2 \sqrt{2} b^3 (a^2 - b^2) d \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \right) + \\ \left((9 a^4 - 10 a^2 b^2 - b^4) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b (1 - \operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \operatorname{Tan}[c+dx] \right) / \\ \left(2 \sqrt{2} b^3 (a^2 - b^2) d \sqrt{1+\operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{2/3} \right)$$

Result (type 6, 33538 leaves): Display of huge result suppressed!

■ **Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a + b \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 6, 307 leaves, 8 steps):

$$\begin{aligned} & - \frac{3 a^2 \operatorname{Tan}[c + d x]}{2 b (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{2/3}} + \\ & \left((3 a^2 - 2 b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x] \right) / \\ & \left(\sqrt{2} b^2 (a^2 - b^2) d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{1/3} \right) - \\ & \frac{a (3 a^2 - 4 b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \operatorname{Tan}[c + d x]}{\sqrt{2} b^2 (a^2 - b^2) d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{2/3}} \end{aligned}$$

Result (type 6, 30028 leaves): Display of huge result suppressed!

■ **Problem 710: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 6, 289 leaves, 8 steps):

$$\begin{aligned} & \frac{3 a \operatorname{Tan}[c + d x]}{2 (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{2/3}} - \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{\sqrt{2} b (a^2 - b^2) d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{1/3}} + \\ & \frac{(a^2 - 2 b^2) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \operatorname{Tan}[c + d x]}{\sqrt{2} b (a^2 - b^2) d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{2/3}} \end{aligned}$$

Result (type 6, 18901 leaves): Display of huge result suppressed!

■ **Problem 711: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]}{(a + b \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 6, 110 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \operatorname{Tan}[c + d x]}{(a + b) d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{2/3}}$$

Result (type 6, 12838 leaves): Display of huge result suppressed!

■ **Problem 713: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{2/3}}{a + b \text{Sec}[c + d x]} dx$$

Optimal (type 6, 174 leaves, 6 steps):

$$\frac{a \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \text{Sin}[c + d x]^2, \frac{a^2 \text{Sin}[c + d x]^2}{a^2 - b^2}\right] \text{Sin}[c + d x]}{(a^2 - b^2) d (\text{Cos}[c + d x]^2)^{1/6} \text{Sec}[c + d x]^{1/3}} - \frac{b \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \text{Sin}[c + d x]^2, \frac{a^2 \text{Sin}[c + d x]^2}{a^2 - b^2}\right] (\text{Cos}[c + d x]^2)^{1/3} \text{Sec}[c + d x]^{2/3} \text{Sin}[c + d x]}{(a^2 - b^2) d}$$

Result (type 6, 4609 leaves):

$$\left(9 (a^2 - b^2) \text{Sec}[c + d x]^{5/3} \text{Sin}[c + d x] \left(\left(b \text{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \sqrt{1 + \text{Tan}[c + d x]^2} \right) / \right. \right. \\ \left. \left(9 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \\ \left. \left(6 b^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) \right. \\ \left. \text{Tan}[c + d x]^2 \right) + \left(a \text{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) / \\ \left. \left(-9 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] - 2 \left(3 b^2 \text{AppellF1}\left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \\ \left. \left. 2 (-a^2 + b^2) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + d x]^2 \right) \right) / \\ \left(d (a + b \text{Sec}[c + d x]) (1 + \text{Tan}[c + d x]^2)^{2/3} (-a^2 + b^2 (1 + \text{Tan}[c + d x]^2)) \left(-\frac{1}{(1 + \text{Tan}[c + d x]^2)^{2/3} (-a^2 + b^2 (1 + \text{Tan}[c + d x]^2))^2} \right. \right. \\ \left. \left. 18 b^2 (a^2 - b^2) \text{Sec}[c + d x]^2 \text{Tan}[c + d x]^2 \left(\left(b \text{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \sqrt{1 + \text{Tan}[c + d x]^2} \right) / \right. \right. \\ \left. \left(9 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] + \left(6 b^2 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right. \right. \\ \left. \left. \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2} \right) + (-a^2 + b^2) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) \text{Tan}[c + d x]^2 \right) + \\ \left. \left(a \text{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) / \left(-9 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\text{Tan}[c + d x]^2, \frac{b^2 \text{Tan}[c + d x]^2}{a^2 - b^2}\right] \right) \right)$$

$$\begin{aligned}
& \frac{b^2 \tan^2[c+dx]}{a^2-b^2} - 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + \right. \\
& \left. 2 (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx] \Big) - \\
& \frac{1}{(1+\tan^2[c+dx])^{5/3} (-a^2+b^2 (1+\tan^2[c+dx]))} 12 (a^2-b^2) \operatorname{Sec}[c+dx]^2 \tan^2[c+dx]^2 \\
& \left(\left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \sqrt{1+\tan^2[c+dx]} \right) / \right. \\
& \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + \left(6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \left. \left. \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx]^2 \Big) + \\
& \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) / \left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] - \right. \\
& \left. 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + 2 (-a^2+b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx]^2 \right) \Big) + \frac{1}{(1+\tan^2[c+dx])^{2/3} (-a^2+b^2 (1+\tan^2[c+dx]))} \\
& 9 (a^2-b^2) \operatorname{Sec}[c+dx]^2 \left(\left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \sqrt{1+\tan^2[c+dx]} \right) / \right. \\
& \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + \left(6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan^2[c+dx], \right. \right. \right. \\
& \left. \left. \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx]^2 \Big) + \\
& \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) / \left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] - \right. \\
& \left. 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] + 2 (-a^2+b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \right) \tan^2[c+dx]^2 \right) \Big) + \frac{1}{(1+\tan^2[c+dx])^{2/3} (-a^2+b^2 (1+\tan^2[c+dx]))} \\
& 9 (a^2-b^2) \tan^2[c+dx] \left(\left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan^2[c+dx], \frac{b^2 \tan^2[c+dx]}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan^2[c+dx] \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{1 + \tan[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left(6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left(b \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \frac{1}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \sqrt{1 + \tan[c + dx]^2} \right) / \\
& \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + \left(6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) + \\
& \left(a \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \frac{4}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) / \\
& \left(-9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] - 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + 2 (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \tan[c + dx]^2 \right) - \\
& \left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \sqrt{1 + \tan[c + dx]^2} \left(2 \left(6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \right) \operatorname{Sec}[c + dx]^2 \tan[c + dx] + \right. \\
& \quad \left. 9 (a^2 - b^2) \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3 (a^2 - b^2)} - \right. \right. \\
& \quad \left. \left. \frac{1}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2} \right] \operatorname{Sec}[c + dx]^2 \tan[c + dx] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \tan[c+dx]^2 \left(6b^2 \left(\frac{1}{5(a^2-b^2)} 12b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{6}, 3, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \frac{1}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) + \\
& \quad (-a^2+b^2) \left(\frac{6b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{5(a^2-b^2)} - \right. \\
& \quad \left. \frac{7}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{13}{6}, 1, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \Bigg) \Bigg) / \\
& \left(9(a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left(6b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \Bigg)^2 - \\
& \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \left(-4 \left(3b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. 2(-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \sec[c+dx]^2 \tan[c+dx] - \right. \\
& \quad \left. 9(a^2-b^2) \left(\frac{2b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{3(a^2-b^2)} - \right. \right. \\
& \quad \left. \left. \frac{4}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) \right) - \\
& 2 \tan[c+dx]^2 \left(3b^2 \left(\frac{1}{5(a^2-b^2)} 12b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{2}{3}, 3, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \right. \right. \\
& \quad \left. \tan[c+dx] - \frac{4}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx] \right) + \\
& \quad \left. 2(-a^2+b^2) \left(\frac{6b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{3}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \sec[c+dx]^2 \tan[c+dx]}{5(a^2-b^2)} - \right. \right.
\end{aligned}$$

$$\left(\left(\left(\left(\left(\left(2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{8}{3}, 1, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) \right) \right) \right) \right) /$$

$$\left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] - 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{2}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right. \right.$$

$$\left. \left. + 2 (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right)^2 \right) \right)$$

■ **Problem 714: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{1/3}}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 6, 174 leaves, 6 steps):

$$\frac{a \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sin}[c+dx]}{(a^2-b^2) d (\operatorname{Cos}[c+dx]^2)^{1/3} \operatorname{Sec}[c+dx]^{2/3}} -$$

$$\frac{b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2} \right] (\operatorname{Cos}[c+dx]^2)^{1/6} \operatorname{Sec}[c+dx]^{1/3} \operatorname{Sin}[c+dx]}{(a^2-b^2) d}$$

Result (type 6, 4610 leaves):

$$\left(9 (a^2-b^2) \operatorname{Sec}[c+dx]^{4/3} \operatorname{Sin}[c+dx] \left(\left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \right.$$

$$\left. \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right.$$

$$\left. 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \right.$$

$$\left. \operatorname{Tan}[c+dx]^2 \right) + \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) /$$

$$\left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left(-6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right.$$

$$\left. 5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \right) /$$

$$\left(d (a+b \operatorname{Sec}[c+dx]) (1+\operatorname{Tan}[c+dx]^2)^{5/6} (-a^2+b^2 (1+\operatorname{Tan}[c+dx]^2)) \left(-\frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{5/6} (-a^2+b^2 (1+\operatorname{Tan}[c+dx]^2))^2} \right) \right)$$

$$\begin{aligned}
& \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) / \left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \\
& \left. \left(-6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \frac{1}{(1+\operatorname{Tan}[c+dx]^2)^{5/6} (-a^2+b^2(1+\operatorname{Tan}[c+dx]^2))} \\
& 9 (a^2-b^2) \operatorname{Tan}[c+dx] \left(\left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) / \right. \\
& \left(\sqrt{1+\operatorname{Tan}[c+dx]^2} \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) \Bigg) + \\
& \left(b \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} - \frac{2}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \sqrt{1+\operatorname{Tan}[c+dx]^2} \right) / \\
& \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) + \\
& \left(a \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} - \frac{5}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, \right. \right. \right. \\
& \left. \left. \left. 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) / \\
& \left(-9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \left(-6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \sqrt{1+\operatorname{Tan}[c+dx]^2} \left(4 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] + \right. \\
& \quad \left. 9 (a^2-b^2) \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} - \right. \right. \\
& \quad \left. \left. \frac{2}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \right. \\
& \quad \left. 2 \operatorname{Tan}[c+dx]^2 \left(3 b^2 \left(\frac{1}{5 (a^2-b^2)} 12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{3}, 3, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \right. \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[c+dx] - \frac{2}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) + \right. \\
& \quad \left. (-a^2+b^2) \left(\frac{6 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{4}{3}, 2, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{5 (a^2-b^2)} - \right. \right. \\
& \quad \left. \left. \frac{8}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{3}, 1, \frac{7}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) \Bigg/ \\
& \left(9 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + 2 \left(3 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{3}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{4}{3}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Tan}[c+dx]^2 \right)^2 - \\
& \left(a \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \left(2 \left(-6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. 5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \right) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] - \right. \\
& \quad \left. 9 (a^2-b^2) \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 (a^2-b^2)} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{5}{9} \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \\
& \tan[c+dx]^2 \left(-6 b^2 \left(\frac{1}{5(a^2-b^2)} 12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{5}{6}, 3, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \right. \right. \\
& \quad \left. \left. \tan[c+dx] - \operatorname{AppellF1} \left[\frac{5}{2}, \frac{11}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) + \right. \\
& \quad \left. 5(a^2-b^2) \left(\frac{1}{5(a^2-b^2)} 6 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{11}{6}, 2, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] - \right. \right. \\
& \quad \left. \left. \frac{11}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{17}{6}, 1, \frac{7}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \operatorname{Sec}[c+dx]^2 \tan[c+dx] \right) \right) \right) \Bigg/ \\
& \left(-9(a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + \left(-6 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{6}, 2, \frac{5}{2}, -\tan[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + 5(a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{11}{6}, 1, \frac{5}{2}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \right) \tan[c+dx]^2 \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 715: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[c+dx]^{1/3} (a+b \operatorname{Sec}[c+dx])} dx$$

Optimal (type 6, 174 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] \sin[c+dx]}{(a^2-b^2) d (\cos[c+dx]^2)^{1/6} \operatorname{Sec}[c+dx]^{1/3}} + \\
& \frac{a \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{2}{3}, 1, \frac{3}{2}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] (\cos[c+dx]^2)^{1/3} \operatorname{Sec}[c+dx]^{2/3} \sin[c+dx]}{(a^2-b^2) d}
\end{aligned}$$

Result (type 6, 9174 leaves): Display of huge result suppressed!

■ **Problem 716: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[c+dx]^{2/3} (a+b \operatorname{Sec}[c+dx])} dx$$

Optimal (type 6, 174 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \sin^2[c+dx], \frac{a^2 \sin^2[c+dx]}{a^2-b^2}\right] \sin[c+dx]}{(a^2-b^2) d (\cos^2[c+dx])^{1/3} \operatorname{Sec}[c+dx]^{2/3}} + \\
& \frac{a \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{5}{6}, 1, \frac{3}{2}, \sin^2[c+dx], \frac{a^2 \sin^2[c+dx]}{a^2-b^2}\right] (\cos^2[c+dx])^{1/6} \operatorname{Sec}[c+dx]^{1/3} \sin[c+dx]}{(a^2-b^2) d}
\end{aligned}$$

Result (type 6, 9178 leaves): Display of huge result suppressed!

■ **Problem 718: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^{5/3} \sqrt{a+b \operatorname{Sec}[c+dx]} \, dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\operatorname{Sec}[c+dx]^{5/3} \sqrt{a+b \operatorname{Sec}[c+dx]}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 720: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^{2/3} \sqrt{a+b \operatorname{Sec}[c+dx]} \, dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\operatorname{Sec}[c+dx]^{2/3} \sqrt{a+b \operatorname{Sec}[c+dx]}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 726: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\operatorname{Sec}[c+dx]^{7/3}} \, dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\operatorname{Sec}[c+dx]^{7/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 727: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^{7/3} (a+b \operatorname{Sec}[c+dx])^{3/2} \, dx$$

Optimal (type 9, 27 leaves, 0 steps) :

`Unintegrable[Sec[c + d x]7/3 (a + b Sec[c + d x])3/2, x]`

Result (type 1, 1 leaves) :

???

■ **Problem 728: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x]^{5/3} (a + b \text{Sec}[c + d x])^{3/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

`Unintegrable[Sec[c + d x]5/3 (a + b Sec[c + d x])3/2, x]`

Result (type 1, 1 leaves) :

???

■ **Problem 729: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x]^{4/3} (a + b \text{Sec}[c + d x])^{3/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

`Unintegrable[Sec[c + d x]4/3 (a + b Sec[c + d x])3/2, x]`

Result (type 1, 1 leaves) :

???

■ **Problem 730: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x]^{2/3} (a + b \text{Sec}[c + d x])^{3/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

`Unintegrable[Sec[c + d x]2/3 (a + b Sec[c + d x])3/2, x]`

Result (type 1, 1 leaves) :

???

■ **Problem 732: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \text{Sec}[c + d x])^{3/2}}{\text{Sec}[c + d x]^{1/3}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

`Unintegrable[$\frac{(a + b \text{Sec}[c + d x])^{3/2}}{\text{Sec}[c + d x]^{1/3}}$, x]`

Result (type 1, 1 leaves) :

???

■ **Problem 736: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{3/2}}{\operatorname{Sec}[c + d x]^{7/3}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{Sec}[c + d x])^{3/2}}{\operatorname{Sec}[c + d x]^{7/3}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 737: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{5/2}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 738: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + d x]^{5/3} (a + b \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\operatorname{Sec}[c + d x]^{5/3} (a + b \operatorname{Sec}[c + d x])^{5/2}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 739: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{5/2}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 740: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x]^{2/3} (a + b \text{Sec}[c + d x])^{5/2} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}[\text{Sec}[c + d x]^{2/3} (a + b \text{Sec}[c + d x])^{5/2}, x]$$

Result (type 1, 1 leaves):

???

■ **Problem 742: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{1/3}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{1/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 744: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{4/3}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{4/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 746: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{7/3}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \text{Sec}[c + d x])^{5/2}}{\text{Sec}[c + d x]^{7/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 756: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{7/3} \sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{7/3} \sqrt{a + b \operatorname{Sec}[c + d x]}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 758: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/3}}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{\operatorname{Sec}[c + d x]^{5/3}}{(a + b \operatorname{Sec}[c + d x])^{3/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 760: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x]^{2/3}}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{\operatorname{Sec}[c + d x]^{2/3}}{(a + b \operatorname{Sec}[c + d x])^{3/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 762: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{1/3} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{1/3} (a + b \operatorname{Sec}[c + d x])^{3/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

- **Problem 764: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{3/2}}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 766: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{3/2}}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 768: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/3}}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Sec}[c + d x]^{5/3}}{(a + b \operatorname{Sec}[c + d x])^{5/2}}, x\right]$$

Result (type 1, 1 leaves):

???

- **Problem 770: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x]^{2/3}}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Sec}[c + d x]^{2/3}}{(a + b \operatorname{Sec}[c + d x])^{5/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 772: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{1/3} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{1/3} (a + b \operatorname{Sec}[c + d x])^{5/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 774: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{4/3} (a + b \operatorname{Sec}[c + d x])^{5/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 776: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 9, 27 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{1}{\operatorname{Sec}[c + d x]^{7/3} (a + b \operatorname{Sec}[c + d x])^{5/2}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 780: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sec}[e + f x])^n}{a + b \operatorname{Sec}[e + f x]} dx$$

Optimal (type 6, 192 leaves, 6 steps) :

$$\frac{1}{(a^2 - b^2) f} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+n), 1, \frac{3}{2}, \sin[e+fx]^2, \frac{a^2 \sin[e+fx]^2}{a^2 - b^2}\right] \cos[e+fx] (\cos[e+fx]^2)^{\frac{1}{2}(-1+n)} (d \operatorname{Sec}[e+fx])^n \sin[e+fx] -$$

$$\frac{b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{n}{2}, 1, \frac{3}{2}, \sin[e+fx]^2, \frac{a^2 \sin[e+fx]^2}{a^2 - b^2}\right] (\cos[e+fx]^2)^{n/2} (d \operatorname{Sec}[e+fx])^n \sin[e+fx]}{(a^2 - b^2) f}$$

Result (type 6, 5280 leaves): Display of huge result suppressed!

■ **Problem 781: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Sec}[e+fx])^n}{(a+b \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 6, 299 leaves, 9 steps):

$$\frac{1}{(a^2 - b^2)^2 f} a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-3+n), 2, \frac{3}{2}, \sin[e+fx]^2, \frac{a^2 \sin[e+fx]^2}{a^2 - b^2}\right] \cos[e+fx] (\cos[e+fx]^2)^{\frac{1}{2}(-1+n)} (d \operatorname{Sec}[e+fx])^n \sin[e+fx] +$$

$$\frac{1}{(a^2 - b^2)^2 f} b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+n), 2, \frac{3}{2}, \sin[e+fx]^2, \frac{a^2 \sin[e+fx]^2}{a^2 - b^2}\right] \cos[e+fx] (\cos[e+fx]^2)^{\frac{1}{2}(-1+n)} (d \operatorname{Sec}[e+fx])^n \sin[e+fx] -$$

$$\frac{1}{(a^2 - b^2)^2 f} 2 a b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-2+n), 2, \frac{3}{2}, \sin[e+fx]^2, \frac{a^2 \sin[e+fx]^2}{a^2 - b^2}\right] (\cos[e+fx]^2)^{n/2} (d \operatorname{Sec}[e+fx])^n \sin[e+fx]$$

Result (type 6, 10428 leaves): Display of huge result suppressed!

■ **Problem 788: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^3 (a+b \operatorname{Sec}[e+fx])^m dx$$

Optimal (type 6, 273 leaves, 8 steps):

$$\frac{(a+b \operatorname{Sec}[e+fx])^{1+m} \operatorname{Tan}[e+fx]}{b f (2+m)} -$$

$$\left(\sqrt{2} a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-m, \frac{3}{2}, \frac{1}{2}(1 - \operatorname{Sec}[e+fx]), \frac{b(1 - \operatorname{Sec}[e+fx])}{a+b}\right] (a+b \operatorname{Sec}[e+fx])^m \left(\frac{a+b \operatorname{Sec}[e+fx]}{a+b}\right)^{-m} \operatorname{Tan}[e+fx] \right) /$$

$$\left(b^2 f (2+m) \sqrt{1 + \operatorname{Sec}[e+fx]} \right) + \left(\sqrt{2} (a^2 + b^2 (1+m)) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1 - \operatorname{Sec}[e+fx]), \frac{b(1 - \operatorname{Sec}[e+fx])}{a+b}\right] \right.$$

$$\left. (a+b \operatorname{Sec}[e+fx])^m \left(\frac{a+b \operatorname{Sec}[e+fx]}{a+b}\right)^{-m} \operatorname{Tan}[e+fx] \right) / \left(b^2 f (2+m) \sqrt{1 + \operatorname{Sec}[e+fx]} \right)$$

Result (type 6, 8908 leaves): Display of huge result suppressed!

■ **Problem 789: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^2 (a+b \operatorname{Sec}[e+fx])^m dx$$

Optimal (type 6, 220 leaves, 7 steps) :

$$\frac{1}{b f \sqrt{1 + \operatorname{Sec}[e + f x]}} \sqrt{2} (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[e + f x]), \frac{b (1 - \operatorname{Sec}[e + f x])}{a + b}\right]$$

$$(a + b \operatorname{Sec}[e + f x])^m \left(\frac{a + b \operatorname{Sec}[e + f x]}{a + b}\right)^{-m} \operatorname{Tan}[e + f x] - \frac{1}{b f \sqrt{1 + \operatorname{Sec}[e + f x]}}$$

$$\sqrt{2} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[e + f x]), \frac{b (1 - \operatorname{Sec}[e + f x])}{a + b}\right] (a + b \operatorname{Sec}[e + f x])^m \left(\frac{a + b \operatorname{Sec}[e + f x]}{a + b}\right)^{-m} \operatorname{Tan}[e + f x]$$

Result (type 6, 5564 leaves) : Display of huge result suppressed!

■ **Problem 790: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x])^m dx$$

Optimal (type 6, 103 leaves, 3 steps) :

$$\frac{1}{f \sqrt{1 + \operatorname{Sec}[e + f x]}} \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[e + f x]), \frac{b (1 - \operatorname{Sec}[e + f x])}{a + b}\right] (a + b \operatorname{Sec}[e + f x])^m \left(\frac{a + b \operatorname{Sec}[e + f x]}{a + b}\right)^{-m} \operatorname{Tan}[e + f x]$$

Result (type 6, 2828 leaves) :

$$- \left(\left(6 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] (b + a \operatorname{Cos}[e + f x])^m \operatorname{Sec}[e + f x]^{1+m} (a + b \operatorname{Sec}[e + f x])^m \right. \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] \right) / \left(f \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right) \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] + \right. \right.$$

$$\left. 2 \left(- (a - b) m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m, 1 - m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] + \right. \right.$$

$$\left. \left. (a + b) (1 + m) \operatorname{AppellF1}\left[\frac{3}{2}, 2 + m, -m, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] \right) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right)$$

$$\left(\left(6 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] (b + a \operatorname{Cos}[e + f x])^m \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x]^m \right. \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right) / \left(\left(-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right)^2 \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m, -m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2, \frac{(a - b) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}\right] + \right. \right.$$

$$\begin{aligned}
& 2 \left(- (a-b) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) (1+m) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \Bigg) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] (b+a \operatorname{Cos}[e+f x])^m \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Sec}[e+f x]^m \right) / \\
& \left(\left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right) \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left(- (a-b) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) (1+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) + \\
& \left(6 a (a+b) m \operatorname{AppellF1} \left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] (b+a \operatorname{Cos}[e+f x])^{-1+m} \right. \\
& \quad \left. \operatorname{Sec}[e+f x]^m \operatorname{Sin}[e+f x] \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right] \right) / \\
& \left(\left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right) \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left(- (a-b) m \operatorname{AppellF1} \left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] + (a+b) (1+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2 \right) \right) - \\
& \left(6 (a+b) m \operatorname{AppellF1} \left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{a+b} \right] (b+a \operatorname{Cos}[e+f x])^m \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \text{Sec}[e + f x]^{1+m} \text{Sin}[e + f x] \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \\
& \left(\left(-1 + \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] + \right. \\
& 2 \left(- (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] + (a+b)(1+m) \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) - \\
& \left(6(a+b)(b+a \text{Cos}[e + f x])^m \text{Sec}[e + f x]^m \text{Tan}\left[\frac{1}{2}(e + f x)\right] \left(-\frac{1}{3(a+b)} (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right] + \right. \right. \\
& \left. \left. \left. \frac{1}{3}(1+m) \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) / \\
& \left(\left(-1 + \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right)^2 \right) \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] + \right. \\
& 2 \left(- (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] + (a+b)(1+m) \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{2}, 2+m, -m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) + \\
& \left(6(a+b) \text{AppellF1}\left[\frac{1}{2}, 1+m, -m, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] (b+a \text{Cos}[e + f x])^m \text{Sec}[e + f x]^m \right. \\
& \left. \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \left(2 \left(- (a-b) m \text{AppellF1}\left[\frac{3}{2}, 1+m, 1-m, \frac{5}{2}, \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a+b}\right] + (a+b)(1+m) \right) \right)
\end{aligned}$$

$$-\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{bd} - \frac{2a \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{b(a+b)d} + \frac{2 \operatorname{Sin}[c+dx]}{bd \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 197 leaves):

$$\frac{1}{2bd} \left(\frac{6a \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} - \frac{2b \left(2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2b \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{a} + \right. \\ \left. \frac{4 \operatorname{Sin}[c+dx]}{\sqrt{\operatorname{Cos}[c+dx]}} + 1 \right) / \left(ab \sqrt{\operatorname{Sin}[c+dx]^2} \right) 2 \left(2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] - \right. \\ \left. 2b(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + (a^2 - 2b^2) \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx]$$

■ **Problem 823: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[c+dx]^{7/2} (a+b \operatorname{Sec}[c+dx])} dx$$

Optimal (type 4, 128 leaves, 11 steps):

$$\frac{2a \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{b^2 d} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3bd} + \frac{2a^2 \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{b^2(a+b)d} + \frac{2 \operatorname{Sin}[c+dx]}{3bd \operatorname{Cos}[c+dx]^{3/2}} - \frac{2a \operatorname{Sin}[c+dx]}{b^2 d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 4, 257 leaves):

$$\frac{1}{6b^2 d} \left(\frac{2(9a^2 + 2b^2) \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} + \right. \\ \left. 8b \left(2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2b \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right) + \left(3 \operatorname{Cos}[2(c+dx)] \left(-4ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] + \right. \right. \right. \\ \left. \left. 4b(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] - 2(a^2 - 2b^2) \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Cos}[c+dx]}\right], -1\right] \right) \operatorname{Sin}[c+dx] \right) / \\ \left(b \sqrt{1 - \operatorname{Cos}[c+dx]^2} (-1 + 2 \operatorname{Cos}[c+dx]^2) \right) \left. + \frac{\sqrt{\operatorname{Cos}[c+dx]} \left(-\frac{2a \operatorname{Tan}[c+dx]}{b^2} + \frac{2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{3b} \right)}{d} \right)$$

■ **Problem 837: Unable to integrate problem.**

$$\int \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 244 leaves, 10 steps):

$$\frac{4 b (a^2 - b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 (9 a^2 - 2 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2 b \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a d} + \frac{2 \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]} dx$$

■ **Problem 838: Unable to integrate problem.**

$$\int \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} dx$$

Optimal (type 4, 192 leaves, 9 steps):

$$\frac{2 (a^2 - b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 b \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} dx$$

■ **Problem 839: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} dx$$

Optimal (type 4, 67 leaves, 4 steps):

$$\frac{2 \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 4, 198 leaves):

$$\left(\sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} \right. \\ \left. \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right] - i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right]\right], \frac{-a+b}{a+b} \right] + \right. \\ \left. \sqrt{\frac{1}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sin[c+dx] \right) / \left(d \sqrt{\frac{1}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right)$$

■ **Problem 840: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 138 leaves, 8 steps):

$$\frac{2a \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{2b \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{\cos[c+dx]}} dx$$

■ **Problem 841: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 237 leaves, 13 steps):

$$\frac{b \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{a \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} - \\ \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]}}{d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} + \frac{\sqrt{a+b \operatorname{Sec}[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 842: Unable to integrate problem.**

$$\int \cos [c+d x]^{7 / 2}(a+b \operatorname{Sec}[c+d x])^{3 / 2} d x$$

Optimal (type 4, 303 leaves, 11 steps):

$$\frac{2\left(25 a^4-31 a^2 b^2+6 b^4\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{105 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{4 b\left(41 a^2-3 b^2\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{105 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2\left(25 a^2+3 b^2\right) \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{105 a d} +$$

$$\frac{16 b \cos [c+d x]^{3 / 2} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{35 d} + \frac{2 a \cos [c+d x]^{5 / 2} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{7 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{7 / 2}(a+b \operatorname{Sec}[c+d x])^{3 / 2} d x$$

■ **Problem 843: Unable to integrate problem.**

$$\int \cos [c+d x]^{5 / 2}(a+b \operatorname{Sec}[c+d x])^{3 / 2} d x$$

Optimal (type 4, 240 leaves, 10 steps):

$$\frac{2 b\left(a^2-b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{5 a d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2\left(3 a^2+b^2\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{5 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{4 b \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{5 d} + \frac{2 a \cos [c+d x]^{3 / 2} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{5 / 2}(a+b \operatorname{Sec}[c+d x])^{3 / 2} d x$$

■ **Problem 844: Unable to integrate problem.**

$$\int \cos [c+d x]^{3 / 2}(a+b \operatorname{Sec}[c+d x])^{3 / 2} d x$$

Optimal (type 4, 187 leaves, 9 steps):

$$\frac{2(a^2 - b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{3d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{8b \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + 2a \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{3d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 27 leaves):

$$\int \cos[c+dx]^{3/2} (a+b \sec[c+dx])^{3/2} dx$$

■ **Problem 845: Unable to integrate problem.**

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{3/2} dx$$

Optimal (type 4, 209 leaves, 12 steps):

$$\frac{2ab \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{2b^2 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + 2a \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{3/2} dx$$

■ **Problem 846: Unable to integrate problem.**

$$\int \frac{(a+b \sec[c+dx])^{3/2}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 249 leaves, 13 steps):

$$\frac{(2a^2 + b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + 3ab \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{b \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + b \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 27 leaves):

$$\int \frac{(a+b \sec[c+dx])^{3/2}}{\sqrt{\cos[c+dx]}} dx$$

■ **Problem 847: Unable to integrate problem.**

$$\int \frac{(a+b \sec[c+dx])^{3/2}}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 299 leaves, 14 steps):

$$\frac{7ab \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (3a^2 + 4b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{5a \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + b \sqrt{a+b \sec[c+dx]} \sin[c+dx] + \frac{5a \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}}}{4d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 27 leaves):

$$\int \frac{(a+b \sec[c+dx])^{3/2}}{\cos[c+dx]^{3/2}} dx$$

■ **Problem 848: Unable to integrate problem.**

$$\int \cos[c+dx]^{9/2} (a+b \sec[c+dx])^{5/2} dx$$

Optimal (type 4, 363 leaves, 12 steps):

$$\begin{aligned}
& \frac{4 b (57 a^4 - 62 a^2 b^2 + 5 b^4) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{315 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 (147 a^4 + 279 a^2 b^2 - 10 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{315 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \\
& \frac{2 b (163 a^2 + 5 b^2) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{315 a d} + \frac{2 (49 a^2 + 75 b^2) \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{315 d} + \\
& \frac{38 a b \cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{63 d} + \frac{2 a^2 \cos [c+d x]^{7/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{9 d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{9/2} (a+b \sec [c+d x])^{5/2} dx$$

■ **Problem 849: Unable to integrate problem.**

$$\int \cos [c+d x]^{7/2} (a+b \sec [c+d x])^{5/2} dx$$

Optimal (type 4, 303 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 (5 a^4 - 2 a^2 b^2 - 3 b^4) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{21 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 b (29 a^2 + 3 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{21 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 (5 a^2 + 9 b^2) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{21 d} + \\
& \frac{6 a b \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{7 d} + \frac{2 a^2 \cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{7 d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{7/2} (a+b \sec [c+d x])^{5/2} dx$$

■ **Problem 850: Unable to integrate problem.**

$$\int \cos [c+d x]^{5/2} (a+b \sec [c+d x])^{5/2} dx$$

Optimal (type 4, 239 leaves, 10 steps):

$$\frac{16 b (a^2 - b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + 2 (9 a^2 + 23 b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{15 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}{15 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{22 a b \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 d} + \frac{2 a^2 \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{5/2} (a+b \sec [c+d x])^{5/2} dx$$

■ **Problem 851: Unable to integrate problem.**

$$\int \cos [c+d x]^{3/2} (a+b \sec [c+d x])^{5/2} dx$$

Optimal (type 4, 262 leaves, 13 steps):

$$\frac{2 a (a^2 + 2 b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + 2 b^3 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{14 a b \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} + 2 a^2 \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} + 3 d}$$

Result (type 8, 27 leaves):

$$\int \cos [c+d x]^{3/2} (a+b \sec [c+d x])^{5/2} dx$$

■ **Problem 852: Unable to integrate problem.**

$$\int \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{5/2} dx$$

Optimal (type 4, 263 leaves, 13 steps):

$$\frac{b (4 a^2 + b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + 5 a b^2 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(2 a^2 - b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} + b^2 \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} + d \sqrt{\cos [c+d x]}}$$

Result (type 8, 27 leaves) :

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{5/2} dx$$

■ **Problem 853: Unable to integrate problem.**

$$\int \frac{(a+b \sec[c+dx])^{5/2}}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 314 leaves, 14 steps) :

$$\frac{a(8a^2+11b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + b(15a^2+4b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{9ab \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{4d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} + \frac{b^2 \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{2d \cos[c+dx]^{3/2}} + \frac{9ab \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}}$$

Result (type 8, 27 leaves) :

$$\int \frac{(a+b \sec[c+dx])^{5/2}}{\sqrt{\cos[c+dx]}} dx$$

■ **Problem 854: Unable to integrate problem.**

$$\int \frac{(a+b \sec[c+dx])^{5/2}}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 369 leaves, 15 steps) :

$$\frac{b(59a^2+16b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + 5a(a^2+4b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{24d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{(33a^2+16b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{24d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} + \frac{b^2 \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{3d \cos[c+dx]^{5/2}} + \frac{13ab \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{12d \cos[c+dx]^{3/2}} + \frac{(33a^2+16b^2) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{24d \sqrt{\cos[c+dx]}}$$

Result (type 8, 27 leaves) :

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2}}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

■ **Problem 855: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[c + d x]^{5/2}}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 249 leaves, 10 steps):

$$\begin{aligned} & - \frac{2 b (7 a^2 + 8 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^3 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 (9 a^2 + 8 b^2) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{15 a^3 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} \\ & - \frac{8 b \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{15 a^2 d} + \frac{2 \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{5 a d} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Cos}[c + d x]^{5/2}}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 856: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2}}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 195 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 (a^2 + 2 b^2) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} \\ & - \frac{4 b \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 a^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \frac{2 \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 a d} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Cos}[c + d x]^{3/2}}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 857: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{\cos[c + dx]}}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 142 leaves, 8 steps):

$$-\frac{2b \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{ad \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{2 \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{ad \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 1, 1 leaves):

???

■ **Problem 858: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 67 leaves, 4 steps):

$$\frac{2 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}$$

Result (type 4, 102 leaves):

$$-\frac{2i \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{-a+b}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{\frac{1}{1+\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}}$$

■ **Problem 859: Unable to integrate problem.**

$$\int \frac{1}{\cos[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 68 leaves, 4 steps):

$$\frac{2 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos[c+dx]^{3/2} \sqrt{a+b \sec[c+dx]}} dx$$

■ **Problem 860: Unable to integrate problem.**

$$\int \frac{1}{\cos[c+dx]^{5/2} \sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 246 leaves, 13 steps):

$$\frac{\sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] - a \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} - b d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{b d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} + b d \sqrt{\cos[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos[c+dx]^{5/2} \sqrt{a+b \sec[c+dx]}} dx$$

■ **Problem 861: Unable to integrate problem.**

$$\int \frac{1}{\cos[c+dx]^{7/2} \sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 312 leaves, 14 steps):

$$-\frac{a \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (3a^2 + 4b^2) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4 b d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} + 4 b^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{3 a \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4 b^2 d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} + 2 b d \cos[c+dx]^{3/2}} - \frac{3 a \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4 b^2 d \sqrt{\cos[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos[c+dx]^{7/2} \sqrt{a+b \sec[c+dx]}} dx$$

■ **Problem 862: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{5 / 2}}{(a+b \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

Optimal (type 4, 360 leaves, 11 steps):

$$\begin{aligned} & \frac{8 b\left(a^2+4 b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{5 a^4 d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}+ \\ & \frac{2\left(3 a^4+8 a^2 b^2-16 b^4\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{5 a^4\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}+\frac{2 b^2 \cos [c+d x]^{3 / 2} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \operatorname{Sec}[c+d x]}}- \\ & \frac{2 b\left(3 a^2-8 b^2\right) \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{5 a^3\left(a^2-b^2\right) d}+\frac{2\left(a^2-6 b^2\right) \cos [c+d x]^{3 / 2} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{5 a^2\left(a^2-b^2\right) d} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [c+d x]^{5 / 2}}{(a+b \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

■ **Problem 863: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{3 / 2}}{(a+b \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

Optimal (type 4, 289 leaves, 10 steps):

$$\begin{aligned} & \frac{2\left(a^2+8 b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}-\frac{2 b\left(5 a^2-8 b^2\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 a^3\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}+ \\ & \frac{2 b^2 \sqrt{\cos [c+d x]} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \operatorname{Sec}[c+d x]}}+\frac{2\left(a^2-4 b^2\right) \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 a^2\left(a^2-b^2\right) d} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [c+d x]^{3 / 2}}{(a+b \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

■ **Problem 864: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos[c+dx]}}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 214 leaves, 9 steps):

$$\begin{aligned} & \frac{4b \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{a^2 d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \\ & \frac{2(a^2 - 2b^2) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{a^2 (a^2 - b^2) d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} + \frac{2b^2 \sin[c+dx]}{a (a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{\cos[c+dx]}}{(a+b \sec[c+dx])^{3/2}} dx$$

■ **Problem 865: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 200 leaves, 9 steps):

$$\begin{aligned} & \frac{2 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{a d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \\ & \frac{2b \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{a (a^2 - b^2) d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} - \frac{2b \sin[c+dx]}{(a^2 - b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{3/2}} dx$$

■ **Problem 866: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[c+dx]^{3/2} (a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$-\frac{2\sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{(a^2-b^2)d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} + \frac{2a \sin[c+dx]}{(a^2-b^2)d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}$$

Result (type 4, 1845 leaves):

$$\frac{2a(b+a \cos[c+dx]) \sin[c+dx]}{(a^2-b^2)d \cos[c+dx]^{3/2} (a+b \sec[c+dx])^{3/2}} -$$

$$\left((b+a \cos[c+dx])^{5/2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \sqrt{1+\sec[c+dx]} \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] - \right.$$

$$\left. i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] + \sqrt{\frac{1}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sin[c+dx] \right) /$$

$$\left((a^2-b^2)(-a^2+b^2)d \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} (a+b \sec[c+dx])^{3/2} \right.$$

$$\left. - \left(\sqrt{b+a \cos[c+dx]} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] - \right.$$

$$\left. i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] + \sqrt{\frac{1}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sin[c+dx] \right) /$$

$$\left(2(a^2-b^2) \cos[c+dx]^{3/2} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sqrt{1+\sec[c+dx]} \right) + \left(a \sqrt{\cos[c+dx]} \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1+\sec[c+dx]} \right.$$

$$\sin[c+dx] \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] - i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\tan\left[\frac{1}{2}(c+dx)\right] \right] \right], \frac{-a+b}{a+b} \right] +$$

$$\left. \sqrt{\frac{1}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sin[c+dx] \right) / \left(2(a^2-b^2) \sqrt{b+a \cos[c+dx]} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right) +$$

$$\begin{aligned}
& \left(\sqrt{b+a \cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1+\operatorname{Sec}[c+d x]} \sin [c+d x] \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) - \right. \\
& \quad \left. i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) + \sqrt{\frac{1}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \sin [c+d x] \Bigg) / \\
& \left(2\left(a^2-b^2\right) \sqrt{\cos [c+d x]} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \right) + \left(\sqrt{\cos [c+d x]} \sqrt{b+a \cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right. \\
& \quad \left. \sqrt{1+\operatorname{Sec}[c+d x]} \left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2} \right) \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) - \right. \\
& \quad \left. i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) + \sqrt{\frac{1}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \sin [c+d x] \Bigg) / \\
& \left(2\left(a^2-b^2\right) \left(\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])} \right)^{3/2} \right) - \left(\sqrt{\cos [c+d x]} \sqrt{b+a \cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1+\operatorname{Sec}[c+d x]} \right. \\
& \quad \left. \left(i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) - i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right]\right], \frac{-a+b}{a+b}\right) + \\
& \quad \left. \sqrt{\frac{1}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \sin [c+d x] \right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \Bigg) / \left(\left(a^2-b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \right) - \\
& \frac{1}{\left(a^2-b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \sqrt{\cos [c+d x]} \sqrt{b+a \cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1+\operatorname{Sec}[c+d x]} \\
& \left(\cos [c+d x] \sqrt{\frac{1}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} + \frac{1}{2} \left(\frac{1}{1+\cos [c+d x]} \right)^{3/2} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \sin [c+d x]^2 + \right.
\end{aligned}$$

$$\frac{\sqrt{\frac{1}{1+\cos[c+dx]} \sin[c+dx] \left(-\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a \cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right)}}{2 \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} + \left. \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2}{2 \sqrt{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1+\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right] - \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1+\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2 \sqrt{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right] \right. \right. \right)$$

■ **Problem 867: Unable to integrate problem.**

$$\int \frac{1}{\cos[c+dx]^{5/2} (a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 206 leaves, 10 steps):

$$\frac{2 \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{bd \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{2a \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{b(a^2-b^2) d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} - \frac{2a^2 \sin[c+dx]}{b(a^2-b^2) d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos[c+dx]^{5/2} (a+b \sec[c+dx])^{3/2}} dx$$

■ **Problem 868: Unable to integrate problem.**

$$\int \frac{1}{\cos[c+dx]^{7/2} (a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 345 leaves, 14 steps):

$$\frac{\sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] - 3 a \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} - b^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{(3 a^2 - b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b^2 (a^2 - b^2) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

$$\frac{2 a^2 \sin [c+d x]}{b (a^2 - b^2) d \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]}} + \frac{(3 a^2 - b^2) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{b^2 (a^2 - b^2) d \sqrt{\cos [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos [c+d x]^{7/2} (a+b \sec [c+d x])^{3/2}} dx$$

■ **Problem 869: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{3/2}}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 391 leaves, 11 steps):

$$\frac{2 (a^4 + 16 a^2 b^2 - 16 b^4) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^4 (a^2 - b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{8 b (2 a^4 - 7 a^2 b^2 + 4 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^4 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 b^2 \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a (a^2 - b^2) d (a+b \sec [c+d x])^{3/2}} + \frac{4 b^2 (5 a^2 - 3 b^2) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a+b \sec [c+d x]}} + \frac{2 (a^4 - 13 a^2 b^2 + 8 b^4) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a^3 (a^2 - b^2)^2 d}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos [c+d x]^{3/2}}{(a+b \sec [c+d x])^{5/2}} dx$$

■ **Problem 870: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]}}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 317 leaves, 10 steps):

$$\frac{2 b (9 a^2 - 8 b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 (a^2 - b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 (3 a^4 - 15 a^2 b^2 + 8 b^4) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^3 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2 b^2 \sin [c+d x]}{3 a (a^2 - b^2) d \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2}} + \frac{8 b^2 (2 a^2 - b^2) \sin [c+d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{\cos [c+d x]}}{(a+b \sec [c+d x])^{5/2}} dx$$

■ **Problem 871: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 302 leaves, 10 steps):

$$\frac{2 (3 a^2 - 2 b^2) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^2 (a^2 - b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{4 b (3 a^2 - b^2) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} -$$

$$\frac{2 b \sin [c+d x]}{3 (a^2 - b^2) d \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2}} - \frac{2 b (5 a^2 - b^2) \sin [c+d x]}{3 a (a^2 - b^2)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{5/2}} dx$$

■ **Problem 872: Unable to integrate problem.**

$$\int \frac{1}{\cos [c+d x]^{3/2} (a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 281 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 b \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a\left(a^2-b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{2\left(3 a^2+b^2\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a\left(a^2-b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \\
& \frac{2 a \sin [c+d x]}{3\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2}} + \frac{4\left(a^2+b^2\right) \sin [c+d x]}{3\left(a^2-b^2\right)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{5 / 2}} d x$$

■ **Problem 873: Unable to integrate problem.**

$$\int \frac{1}{\cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 277 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3\left(a^2-b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{8 b \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3\left(a^2-b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} - \\
& \frac{2 a^2 \sin [c+d x]}{3 b\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2}} + \frac{2 a\left(a^2-5 b^2\right) \sin [c+d x]}{3 b\left(a^2-b^2\right)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{5 / 2}} d x$$

■ **Problem 874: Unable to integrate problem.**

$$\int \frac{1}{\cos [c+d x]^{7 / 2}(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 370 leaves, 14 steps):

$$\begin{aligned}
& \frac{2 a \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 b\left(a^2-b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 a\left(3 a^2-7 b^2\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} - \\
& \frac{2 a^2 \sin [c+d x]}{3 b\left(a^2-b^2\right) d \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{3 / 2}} - \frac{2 a^2\left(3 a^2-7 b^2\right) \sin [c+d x]}{3 b^2\left(a^2-b^2\right)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\cos [c+d x]^{7 / 2}(a+b \sec [c+d x])^{5 / 2}} dx$$

■ **Problem 878: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \cos [e+f x])^n}{a+b \sec [e+f x]} dx$$

Optimal (type 6, 196 leaves, 7 steps):

$$\begin{aligned}
& \frac{1}{\left(a^2-b^2\right) f} a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1-n), 1, \frac{3}{2}, \sin [e+f x]^2, \frac{a^2 \sin [e+f x]^2}{a^2-b^2}\right] \cos [e+f x](d \cos [e+f x])^n\left(\cos [e+f x]^2\right)^{\frac{1}{2}(-1-n)} \sin [e+f x] - \\
& \frac{b \operatorname{AppellF1}\left[\frac{1}{2},-\frac{n}{2}, 1, \frac{3}{2}, \sin [e+f x]^2, \frac{a^2 \sin [e+f x]^2}{a^2-b^2}\right](d \cos [e+f x])^n\left(\cos [e+f x]^2\right)^{-n / 2} \sin [e+f x]}{\left(a^2-b^2\right) f}
\end{aligned}$$

Result (type 6, 5216 leaves): Display of huge result suppressed!

■ **Problem 879: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \cos [e+f x])^n}{(a+b \sec [e+f x])^2} dx$$

Optimal (type 6, 309 leaves, 10 steps):

$$\frac{1}{(a^2 - b^2)^2 f} a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-3 - n), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] (d \cos[e + f x])^n (\cos[e + f x]^2)^{\frac{1}{2}(-1-n)} \sin[e + f x] +$$

$$\frac{1}{(a^2 - b^2)^2 f} b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1 - n), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] (d \cos[e + f x])^n (\cos[e + f x]^2)^{\frac{1}{2}(-1-n)} \sin[e + f x] -$$

$$\frac{1}{(a^2 - b^2)^2 f} 2 a b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-2 - n), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2}\right] (d \cos[e + f x])^n (\cos[e + f x]^2)^{-n/2} \sin[e + f x]$$

Result (type 6, 10296 leaves): Display of huge result suppressed!

Test results for the 306 problems in "4.5.1.3 (d sin)^n (a+b sec)^m.m"

- **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x] (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 30 leaves, 6 steps):

$$\frac{a \operatorname{Log}[1 - \operatorname{Cos}[c + d x]]}{d} - \frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d}$$

Result (type 3, 65 leaves):

$$-\frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{a \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}[\operatorname{Sin}[c + d x]]}{d}$$

- **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^2 (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 37 leaves, 7 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} - \frac{a \operatorname{Cot}[c + d x]}{d} - \frac{a \operatorname{Csc}[c + d x]}{d}$$

Result (type 3, 106 leaves):

$$-\frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{2 d} - \frac{a \operatorname{Cot}[c + d x]}{d} - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{2 d}$$

- **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^4 (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 69 leaves, 8 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} - \frac{a \operatorname{Cot}[c + d x]}{d} - \frac{a \operatorname{Cot}[c + d x]^3}{3 d} - \frac{a \operatorname{Csc}[c + d x]}{d} - \frac{a \operatorname{Csc}[c + d x]^3}{3 d}$$

Result (type 3, 190 leaves):

$$\begin{aligned} & - \frac{7 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{12 d} - \frac{2 a \operatorname{Cot}[c + d x]}{3 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{24 d} - \\ & \frac{a \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2}{3 d} - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \\ & \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{7 a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{12 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{24 d} \end{aligned}$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^6 (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 101 leaves, 8 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} - \frac{a \operatorname{Cot}[c + d x]}{d} - \frac{2 a \operatorname{Cot}[c + d x]^3}{3 d} - \frac{a \operatorname{Cot}[c + d x]^5}{5 d} - \frac{a \operatorname{Csc}[c + d x]}{d} - \frac{a \operatorname{Csc}[c + d x]^3}{3 d} - \frac{a \operatorname{Csc}[c + d x]^5}{5 d}$$

Result (type 3, 272 leaves):

$$\begin{aligned} & - \frac{149 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{8 a \operatorname{Cot}[c + d x]}{15 d} - \frac{29 a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} - \\ & \frac{4 a \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^2}{15 d} - \frac{a \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]^4}{5 d} - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \\ & \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{149 a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{29 a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d} \end{aligned}$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^8 (a + a \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 131 leaves, 8 steps):

$$\begin{aligned} & \frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} - \frac{a \operatorname{Cot}[c + d x]}{d} - \frac{a \operatorname{Cot}[c + d x]^3}{d} - \frac{3 a \operatorname{Cot}[c + d x]^5}{5 d} - \\ & \frac{a \operatorname{Cot}[c + d x]^7}{7 d} - \frac{a \operatorname{Csc}[c + d x]}{d} - \frac{a \operatorname{Csc}[c + d x]^3}{3 d} - \frac{a \operatorname{Csc}[c + d x]^5}{5 d} - \frac{a \operatorname{Csc}[c + d x]^7}{7 d} \end{aligned}$$

Result (type 3, 354 leaves):

$$\frac{2161 a \cot\left[\frac{1}{2}(c+dx)\right]}{3360 d} - \frac{16 a \cot[c+dx]}{35 d} - \frac{481 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{6720 d} - \frac{3 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{280 d} -$$

$$\frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{896 d} - \frac{8 a \cot[c+dx] \csc[c+dx]^2}{35 d} - \frac{6 a \cot[c+dx] \csc[c+dx]^4}{35 d} - \frac{a \cot[c+dx] \csc[c+dx]^6}{7 d} -$$

$$\frac{a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{2161 a \tan\left[\frac{1}{2}(c+dx)\right]}{3360 d} -$$

$$\frac{481 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{6720 d} - \frac{3 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{280 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{896 d}$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]^{10} (a + a \sec[c+dx]) dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$\frac{a \operatorname{ArcTanh}[\sin[c+dx]]}{d} - \frac{a \cot[c+dx]}{d} - \frac{4 a \cot[c+dx]^3}{3 d} - \frac{6 a \cot[c+dx]^5}{5 d} - \frac{4 a \cot[c+dx]^7}{7 d} -$$

$$\frac{a \cot[c+dx]^9}{9 d} - \frac{a \csc[c+dx]}{d} - \frac{a \csc[c+dx]^3}{3 d} - \frac{a \csc[c+dx]^5}{5 d} - \frac{a \csc[c+dx]^7}{7 d} - \frac{a \csc[c+dx]^9}{9 d}$$

Result (type 3, 436 leaves):

$$\frac{53089 a \cot\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{128 a \cot[c+dx]}{315 d} - \frac{12769 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^2}{161280 d} - \frac{751 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^4}{53760 d} -$$

$$\frac{71 a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^6}{32256 d} - \frac{a \cot\left[\frac{1}{2}(c+dx)\right] \csc\left[\frac{1}{2}(c+dx)\right]^8}{4608 d} - \frac{64 a \cot[c+dx] \csc[c+dx]^2}{315 d} -$$

$$\frac{16 a \cot[c+dx] \csc[c+dx]^4}{105 d} - \frac{8 a \cot[c+dx] \csc[c+dx]^6}{63 d} - \frac{a \cot[c+dx] \csc[c+dx]^8}{9 d} - \frac{a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} +$$

$$\frac{a \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{53089 a \tan\left[\frac{1}{2}(c+dx)\right]}{80640 d} - \frac{12769 a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{161280 d} -$$

$$\frac{751 a \sec\left[\frac{1}{2}(c+dx)\right]^4 \tan\left[\frac{1}{2}(c+dx)\right]}{53760 d} - \frac{71 a \sec\left[\frac{1}{2}(c+dx)\right]^6 \tan\left[\frac{1}{2}(c+dx)\right]}{32256 d} - \frac{a \sec\left[\frac{1}{2}(c+dx)\right]^8 \tan\left[\frac{1}{2}(c+dx)\right]}{4608 d}$$

■ **Problem 26: Result more than twice size of optimal antiderivative.**

$$\int \csc[c+dx]^5 (a + a \sec[c+dx])^2 dx$$

Optimal (type 3, 115 leaves, 5 steps):

$$-\frac{a^4}{4d(a-a\cos[c+dx])^2} - \frac{5a^3}{4d(a-a\cos[c+dx])} + \frac{17a^2 \operatorname{Log}[1-\cos[c+dx]]}{8d} - \frac{2a^2 \operatorname{Log}[\cos[c+dx]]}{d} - \frac{a^2 \operatorname{Log}[1+\cos[c+dx]]}{8d} + \frac{a^2 \operatorname{Sec}[c+dx]}{d}$$

Result (type 3, 598 leaves):

$$\begin{aligned} & -\frac{5\cos[c+dx]^2 \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{32d} - \frac{\cos[c+dx]^2 \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{64d} \\ & - \frac{\cos[c+dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{16d} - \frac{\cos[c+dx]^2 \operatorname{Log}[\cos[c+dx]] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{2d} + \\ & \frac{17\cos[c+dx]^2 \operatorname{Log}\left[\sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{16d} + \frac{\cos[c+dx]^2 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2}{4d} + \\ & \frac{\cos[c+dx]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2 \sin\left[\frac{dx}{2}\right]}{4d(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right])(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} - \frac{\cos[c+dx]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2 \sin\left[\frac{dx}{2}\right]}{4d(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right])(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\ & x \cos[c+dx]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a\operatorname{Sec}[c+dx])^2 \left(-\frac{17}{32} \operatorname{Cot}\left[\frac{c}{2}\right] + \frac{1}{32} (8+9\cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] - \frac{1}{32} \operatorname{Tan}\left[\frac{c}{2}\right] - \frac{\operatorname{Tan}[c]}{2} \right) \end{aligned}$$

■ **Problem 27: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^7 (a+a\operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$\begin{aligned} & -\frac{a^5}{12d(a-a\cos[c+dx])^3} - \frac{3a^4}{8d(a-a\cos[c+dx])^2} - \frac{23a^3}{16d(a-a\cos[c+dx])} + \\ & \frac{a^3}{16d(a+a\cos[c+dx])} + \frac{9a^2 \operatorname{Log}[1-\cos[c+dx]]}{4d} - \frac{2a^2 \operatorname{Log}[\cos[c+dx]]}{d} - \frac{a^2 \operatorname{Log}[1+\cos[c+dx]]}{4d} + \frac{a^2 \operatorname{Sec}[c+dx]}{d} \end{aligned}$$

Result (type 3, 697 leaves):

$$\begin{aligned}
& \frac{23 \cos [c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{128 d}-\frac{3 \cos [c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{128 d} \\
& \frac{\cos [c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{384 d}-\frac{\cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{8 d} \\
& \frac{\cos [c+d x]^2 \operatorname{Log}[\cos [c+d x]] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{2 d}+\frac{9 \cos [c+d x]^2 \operatorname{Log}\left[\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{8 d} \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{4 d}+\frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^2}{128 d} \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{4 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}-\frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{4 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} \\
& +x \cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-\frac{9}{16} \cot \left[\frac{c}{2}\right]+\frac{1}{16}(4+5 \cos [c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]-\frac{1}{16} \tan \left[\frac{c}{2}\right]-\frac{\tan [c]}{2}\right)
\end{aligned}$$

■ **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+d x])^2 \sin [c+d x]^2 dx$$

Optimal (type 3, 73 leaves, 9 steps):

$$-\frac{a^2 x}{2}+\frac{2 a^2 \operatorname{ArcTanh}[\sin [c+d x]]}{d}-\frac{2 a^2 \sin [c+d x]}{d}-\frac{a^2 \cos [c+d x] \sin [c+d x]}{2 d}+\frac{a^2 \tan [c+d x]}{d}$$

Result (type 3, 243 leaves):

$$\begin{aligned}
& \frac{1}{16} a^2 (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \\
& \left(-2 x-\frac{8 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{d}+\frac{8 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{d}-\frac{8 \cos [d x] \sin [c]}{d}-\right. \\
& \left.\frac{\cos [2 d x] \sin [2 c]}{d}-\frac{8 \cos [c] \sin [d x]}{d}-\frac{\cos [2 c] \sin [2 d x]}{d}+\frac{4 \sin \left[\frac{d x}{2}\right]}{d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)}\right. \\
& \left.\frac{4 \sin \left[\frac{d x}{2}\right]}{d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)}\right)
\end{aligned}$$

■ **Problem 33: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^2 (a+a \operatorname{Sec}[c+d x])^2 dx$$

Optimal (type 3, 57 leaves, 11 steps):

$$\frac{2 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} - \frac{2 a^2 \operatorname{Cot}[c+d x]}{d} - \frac{2 a^2 \operatorname{Csc}[c+d x]}{d} + \frac{a^2 \operatorname{Tan}[c+d x]}{d}$$

Result (type 3, 401 leaves):

$$\begin{aligned} & - \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c+d x])^2}{2 d} + \\ & \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c+d x])^2}{2 d} + \\ & \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{2 d} + \\ & \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{4 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{4 d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} \end{aligned}$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^4 (a + a \operatorname{Sec}[c+d x])^2 dx$$

Optimal (type 3, 87 leaves, 8 steps):

$$\frac{2 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} + \frac{10 a^2 \operatorname{Tan}[c+d x]}{3 d} - \frac{2 a^2 \operatorname{Tan}[c+d x]}{d (1 - \operatorname{Cos}[c+d x])} - \frac{a^4 \operatorname{Tan}[c+d x]}{3 d (a - a \operatorname{Cos}[c+d x])^2}$$

Result (type 3, 228 leaves):

$$\begin{aligned} & \frac{1}{24 d} a^2 (1 + \operatorname{Cos}[c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \left(-\operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 - (-8 + 7 \operatorname{Cos}[c+d x]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^3 \operatorname{Sin}\left[\frac{d x}{2}\right] + \right. \\ & \left. 6 \left(-2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + \right. \\ & \left. \left. \operatorname{Sin}[d x] / \left(\left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right)\right) \right) \end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+d x]^6 (a + a \operatorname{Sec}[c+d x])^2 dx$$

Optimal (type 3, 129 leaves, 12 steps):

$$\begin{aligned} & \frac{2 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} - \frac{4 a^2 \operatorname{Cot}[c+d x]}{d} - \frac{5 a^2 \operatorname{Cot}[c+d x]^3}{3 d} - \\ & \frac{2 a^2 \operatorname{Cot}[c+d x]^5}{5 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]}{d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^3}{3 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^5}{5 d} + \frac{a^2 \operatorname{Tan}[c+d x]}{d} \end{aligned}$$

Result (type 3, 317 leaves) :

$$\frac{1}{7680 d} a^2 \cos [c+d x] \sec \left[\frac{1}{2} (c+d x) \right]^4 (1+\sec [c+d x])^2$$

$$\left(-3840 \cos [c+d x] \log \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + 3840 \cos [c+d x] \log \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) +$$

$$\text{Csc}[2 c] \text{Csc} \left[\frac{1}{2} (c+d x) \right]^4 \text{Csc}[c+d x] (320 \sin [2 c] - 596 \sin [d x] + 864 \sin [2 d x] + 216 \sin [c-d x] - 416 \sin [c+d x] +$$

$$624 \sin [2 (c+d x)] - 416 \sin [3 (c+d x)] + 104 \sin [4 (c+d x)] - 596 \sin [2 c+d x] - 680 \sin [3 c+d x] +$$

$$894 \sin [c+2 d x] + 224 \sin [2 (c+2 d x)] + 894 \sin [3 c+2 d x] + 480 \sin [4 c+2 d x] - 776 \sin [c+3 d x] -$$

$$596 \sin [2 c+3 d x] - 596 \sin [4 c+3 d x] - 120 \sin [5 c+3 d x] + 149 \sin [3 c+4 d x] + 149 \sin [5 c+4 d x])$$

■ **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c+d x]^8 (a+a \sec [c+d x])^2 dx$$

Optimal (type 3, 163 leaves, 12 steps) :

$$\frac{2 a^2 \text{ArcTanh}[\sin [c+d x]]}{d} - \frac{5 a^2 \text{Cot}[c+d x]}{d} - \frac{3 a^2 \text{Cot}[c+d x]^3}{d} - \frac{7 a^2 \text{Cot}[c+d x]^5}{5 d} -$$

$$\frac{2 a^2 \text{Cot}[c+d x]^7}{7 d} - \frac{2 a^2 \text{Csc}[c+d x]}{d} - \frac{2 a^2 \text{Csc}[c+d x]^3}{3 d} - \frac{2 a^2 \text{Csc}[c+d x]^5}{5 d} - \frac{2 a^2 \text{Csc}[c+d x]^7}{7 d} + \frac{a^2 \text{Tan}[c+d x]}{d}$$

Result (type 3, 428 leaves) :

$$\frac{1}{13762560 d} a^2 \cos [c+d x] \sec \left[\frac{1}{2} (c+d x) \right]^4 (1+\sec [c+d x])^2$$

$$\left(-6881280 \cos [c+d x] \log \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + 6881280 \cos [c+d x] \log \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) -$$

$$32 \text{Csc}[2 c] \text{Csc} \left[\frac{1}{2} (c+d x) \right]^4 \text{Csc}[c+d x]^3 (-9856 \sin [2 c] + 17288 \sin [d x] - 29056 \sin [2 d x] - 7264 \sin [c-d x] + 14208 \sin [c+d x] -$$

$$19536 \sin [2 (c+d x)] + 7104 \sin [3 (c+d x)] + 7104 \sin [4 (c+d x)] - 7104 \sin [5 (c+d x)] + 1776 \sin [6 (c+d x)] +$$

$$17288 \sin [2 c+d x] + 20384 \sin [3 c+d x] - 23771 \sin [c+2 d x] + 7104 \sin [2 (c+2 d x)] - 23771 \sin [3 c+2 d x] -$$

$$8960 \sin [4 c+2 d x] + 19984 \sin [c+3 d x] + 8644 \sin [2 c+3 d x] + 8644 \sin [4 c+3 d x] - 6160 \sin [5 c+3 d x] +$$

$$8644 \sin [3 c+4 d x] + 8644 \sin [5 c+4 d x] + 6720 \sin [6 c+4 d x] - 12144 \sin [3 c+5 d x] - 8644 \sin [4 c+5 d x] -$$

$$8644 \sin [6 c+5 d x] - 1680 \sin [7 c+5 d x] + 3456 \sin [4 c+6 d x] + 2161 \sin [5 c+6 d x] + 2161 \sin [7 c+6 d x])$$

■ **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c+d x]^{10} (a+a \sec [c+d x])^2 dx$$

Optimal (type 3, 201 leaves, 12 steps) :

$$\frac{2 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} - \frac{6 a^2 \operatorname{Cot}[c+d x]}{d} - \frac{14 a^2 \operatorname{Cot}[c+d x]^3}{3 d} - \frac{16 a^2 \operatorname{Cot}[c+d x]^5}{5 d} - \frac{9 a^2 \operatorname{Cot}[c+d x]^7}{7 d} - \frac{2 a^2 \operatorname{Cot}[c+d x]^9}{9 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]}{d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^3}{3 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^5}{5 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^7}{7 d} - \frac{2 a^2 \operatorname{Csc}[c+d x]^9}{9 d} + \frac{a^2 \operatorname{Tan}[c+d x]}{d}$$

Result (type 3, 1050 leaves):

$$\frac{6899 \operatorname{Cos}[c+d x]^2 \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{80640 d} - \frac{193 \operatorname{Cos}[c+d x]^2 \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{13440 d} - \frac{71 \operatorname{Cos}[c+d x]^2 \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{32256 d} - \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{4608 d} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]-\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{2 d} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2}+\frac{d x}{2}\right]+\operatorname{Sin}\left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{2 d} + \frac{123041 \operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{161280 d} + \frac{6899 \operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^3 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{80640 d} + \frac{193 \operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{13440 d} + \frac{71 \operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^7 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{32256 d} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x}{2}\right]^9 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{4608 d} + \frac{803 \operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{7680 d} + \frac{49 \operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^7 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{7680 d} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^9 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}\left[\frac{d x}{2}\right]}{2560 d} + \frac{\operatorname{Cos}[c+d x] \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Sin}[d x]}{4 d} + \frac{49 \operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{7680 d} + \frac{\operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{2560 d}$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + dx]^7 (a + a \operatorname{Sec}[c + dx])^3 dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$\frac{a^6}{6d(a - a \operatorname{Cos}[c + dx])^3} - \frac{7a^5}{8d(a - a \operatorname{Cos}[c + dx])^2} - \frac{31a^4}{8d(a - a \operatorname{Cos}[c + dx])} + \frac{111a^3 \operatorname{Log}[1 - \operatorname{Cos}[c + dx]]}{16d} - \frac{7a^3 \operatorname{Log}[\operatorname{Cos}[c + dx]]}{d} + \frac{a^3 \operatorname{Log}[1 + \operatorname{Cos}[c + dx]]}{16d} + \frac{3a^3 \operatorname{Sec}[c + dx]}{d} + \frac{a^3 \operatorname{Sec}[c + dx]^2}{2d}$$

Result (type 3, 799 leaves):

$$\frac{31 \operatorname{Cos}[c + dx]^3 \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{128d} - \frac{7 \operatorname{Cos}[c + dx]^3 \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{256d} - \frac{\operatorname{Cos}[c + dx]^3 \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{384d} + \frac{\operatorname{Cos}[c + dx]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{64d} - \frac{7 \operatorname{Cos}[c + dx]^3 \operatorname{Log}[\operatorname{Cos}[c + dx]] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{8d} + \frac{111 \operatorname{Cos}[c + dx]^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{64d} + \frac{3 \operatorname{Cos}[c + dx]^3 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{8d} + \frac{\operatorname{Cos}[c + dx]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \frac{3 \operatorname{Cos}[c + dx]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{8d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \frac{\operatorname{Cos}[c + dx]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3}{32d \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \frac{3 \operatorname{Cos}[c + dx]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 \operatorname{Sin}\left[\frac{dx}{2}\right]}{8d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + x \operatorname{Cos}[c + dx]^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 \left(-\frac{111}{128} \operatorname{Cot}\left[\frac{c}{2}\right] + \frac{1}{128} (56 + 55 \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] + \frac{1}{128} \operatorname{Tan}\left[\frac{c}{2}\right] - \frac{7 \operatorname{Tan}[c]}{8}\right)$$

■ **Problem 51: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx])^3 \operatorname{Sin}[c + dx]^2 dx$$

Optimal (type 3, 98 leaves, 11 steps):

$$-\frac{5a^3x}{2} + \frac{5a^3 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} - \frac{3a^3 \operatorname{Sin}[c + dx]}{d} - \frac{a^3 \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{2d} + \frac{3a^3 \operatorname{Tan}[c + dx]}{d} + \frac{a^3 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d}$$

Result (type 3, 300 leaves):

$$\frac{1}{32} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left(-10x - \frac{10 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{10 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} - \frac{12 \cos[dx] \sin[c]}{d} - \frac{\cos[2dx] \sin[2c]}{d} - \frac{12 \cos[c] \sin[dx]}{d} - \frac{\cos[2c] \sin[2dx]}{d} + \frac{1}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{12 \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \frac{1}{d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{12 \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} \right)$$

■ **Problem 52: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + dx]^2 (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 80 leaves, 9 steps):

$$\frac{9 a^3 \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} - \frac{4 a^3 \sin[c + dx]}{d (1 - \cos[c + dx])} + \frac{3 a^3 \tan[c + dx]}{d} + \frac{a^3 \sec[c + dx] \tan[c + dx]}{2 d}$$

Result (type 3, 244 leaves):

$$\frac{1}{32 d} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6 \left(-18 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 18 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 16 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \sin\left[\frac{dx}{2}\right] + \frac{1}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{1}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + (12 \sin[dx]) / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right)$$

■ **Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \csc[c + dx]^4 (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 110 leaves, 11 steps):

$$\frac{11 a^3 \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} - \frac{2 a^3 \sin[c + dx]}{3 d (1 - \cos[c + dx])^2} - \frac{17 a^3 \sin[c + dx]}{3 d (1 - \cos[c + dx])} + \frac{3 a^3 \tan[c + dx]}{d} + \frac{a^3 \sec[c + dx] \tan[c + dx]}{2 d}$$

Result (type 3, 678 leaves):

$$\begin{aligned}
& - \frac{\cos [c+d x]^3 \cot \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{24 d} - \\
& \frac{11 \cos [c+d x]^3 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{16 d} + \\
& \frac{11 \cos [c+d x]^3 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{16 d} + \\
& \frac{17 \cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{24 d} + \\
& \frac{\cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{24 d} + \\
& \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{32 d\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \frac{3 \cos [c+d x]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{8 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} - \\
& \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{32 d\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \frac{3 \cos [c+d x]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{8 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \csc [c+d x]^6 (a+a \sec [c+d x])^3 dx$$

Optimal (type 3, 165 leaves, 10 steps):

$$\begin{aligned}
& \frac{13 a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{152 a^3 \tan [c+d x]}{15 d} + \frac{13 a^3 \sec [c+d x] \tan [c+d x]}{2 d} - \\
& \frac{a^6 \sec [c+d x] \tan [c+d x]}{5 d(a-a \cos [c+d x])^3} - \frac{11 a^5 \sec [c+d x] \tan [c+d x]}{15 d(a-a \cos [c+d x])^2} - \frac{76 a^6 \sec [c+d x] \tan [c+d x]}{15 d\left(a^3-a^3 \cos [c+d x]\right)}
\end{aligned}$$

Result (type 3, 353 leaves):

$$\begin{aligned}
& - \frac{1}{30720d} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6 \sec[c + dx]^2 \\
& \left(24960 \cos[c + dx]^2 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) + \\
& \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^5 \sec[c] \left(-1235 \sin\left[\frac{dx}{2}\right] + 3805 \sin\left[\frac{3dx}{2}\right] + 4329 \sin\left[c - \frac{dx}{2}\right] - 1989 \sin\left[c + \frac{dx}{2}\right] - \right. \\
& 3575 \sin\left[2c + \frac{dx}{2}\right] + 475 \sin\left[c + \frac{3dx}{2}\right] + 2005 \sin\left[2c + \frac{3dx}{2}\right] + 2275 \sin\left[3c + \frac{3dx}{2}\right] - 2673 \sin\left[c + \frac{5dx}{2}\right] + \\
& 105 \sin\left[2c + \frac{5dx}{2}\right] - 1593 \sin\left[3c + \frac{5dx}{2}\right] - 975 \sin\left[4c + \frac{5dx}{2}\right] + 1325 \sin\left[2c + \frac{7dx}{2}\right] - 255 \sin\left[3c + \frac{7dx}{2}\right] + \\
& \left. 875 \sin\left[4c + \frac{7dx}{2}\right] + 195 \sin\left[5c + \frac{7dx}{2}\right] - 304 \sin\left[3c + \frac{9dx}{2}\right] + 90 \sin\left[4c + \frac{9dx}{2}\right] - 214 \sin\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + dx]^8 (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 192 leaves, 17 steps):

$$\begin{aligned}
& \frac{15 a^3 \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \frac{13 a^3 \operatorname{Cot}[c + dx]}{d} - \frac{7 a^3 \operatorname{Cot}[c + dx]^3}{d} - \frac{3 a^3 \operatorname{Cot}[c + dx]^5}{d} - \frac{4 a^3 \operatorname{Cot}[c + dx]^7}{7d} - \\
& \frac{15 a^3 \operatorname{Csc}[c + dx]}{2d} - \frac{5 a^3 \operatorname{Csc}[c + dx]^3}{2d} - \frac{3 a^3 \operatorname{Csc}[c + dx]^5}{2d} - \frac{15 a^3 \operatorname{Csc}[c + dx]^7}{14d} + \frac{a^3 \operatorname{Csc}[c + dx]^7 \sec[c + dx]^2}{2d} + \frac{3 a^3 \operatorname{Tan}[c + dx]}{d}
\end{aligned}$$

Result (type 3, 430 leaves):

$$\begin{aligned}
& \frac{1}{917504d} a^3 \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^6 (1 + \sec[c + dx])^3 \\
& \left(-860160 \cos[c + dx]^2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 860160 \cos[c + dx]^2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \\
& 8 \operatorname{Csc}[2c] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^6 \operatorname{Csc}[c + dx] (5264 \sin[2c] - 9580 \sin[dx] + 8480 \sin[2dx] + 2776 \sin[c - dx] - 6080 \sin[c + dx] + \\
& 8816 \sin[2(c + dx)] - 7904 \sin[3(c + dx)] + 4864 \sin[4(c + dx)] - 1824 \sin[5(c + dx)] + 304 \sin[6(c + dx)] - \\
& 9580 \sin[2c + dx] - 10024 \sin[3c + dx] + 13891 \sin[c + 2dx] + 7720 \sin[2(c + 2dx)] + 13891 \sin[3c + 2dx] + \\
& 10080 \sin[4c + 2dx] - 10060 \sin[c + 3dx] - 12454 \sin[2c + 3dx] - 12454 \sin[4c + 3dx] - 6580 \sin[5c + 3dx] + \\
& 7664 \sin[3c + 4dx] + 7664 \sin[5c + 4dx] + 2520 \sin[6c + 4dx] - 3420 \sin[3c + 5dx] - 2874 \sin[4c + 5dx] - \\
& \left. 2874 \sin[6c + 5dx] - 420 \sin[7c + 5dx] + 640 \sin[4c + 6dx] + 479 \sin[5c + 6dx] + 479 \sin[7c + 6dx] \right)
\end{aligned}$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + dx]^{10} (a + a \sec[c + dx])^3 dx$$

Optimal (type 3, 232 leaves, 17 steps):

$$\begin{aligned}
& \frac{17 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} - \frac{16 a^3 \operatorname{Cot}[c+d x]}{d} - \frac{34 a^3 \operatorname{Cot}[c+d x]^3}{3 d} - \frac{36 a^3 \operatorname{Cot}[c+d x]^5}{5 d} - \\
& \frac{19 a^3 \operatorname{Cot}[c+d x]^7}{7 d} - \frac{4 a^3 \operatorname{Cot}[c+d x]^9}{9 d} - \frac{17 a^3 \operatorname{Csc}[c+d x]}{2 d} - \frac{17 a^3 \operatorname{Csc}[c+d x]^3}{6 d} - \frac{17 a^3 \operatorname{Csc}[c+d x]^5}{10 d} - \\
& \frac{17 a^3 \operatorname{Csc}[c+d x]^7}{14 d} - \frac{17 a^3 \operatorname{Csc}[c+d x]^9}{18 d} + \frac{a^3 \operatorname{Csc}[c+d x]^9 \operatorname{Sec}[c+d x]^2}{2 d} + \frac{3 a^3 \operatorname{Tan}[c+d x]}{d}
\end{aligned}$$

Result (type 3, 1000 leaves):

$$\begin{aligned}
& \frac{9833 \cos [c+d x]^3 \cot \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{80640 d} \\
& \frac{979 \cos [c+d x]^3 \cot \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{53760 d} \\
& \frac{5 \cos [c+d x]^3 \cot \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^6 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{2016 d} - \frac{\cos [c+d x]^3 \cot \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^8 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{4608 d} \\
& \frac{17 \cos [c+d x]^3 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{16 d} + \\
& \frac{17 \cos [c+d x]^3 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3}{16 d} + \\
& \frac{197147 \cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{161280 d} + \\
& \frac{9833 \cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{80640 d} + \\
& \frac{979 \cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{53760 d} + \\
& \frac{5 \cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^7 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{2016 d} + \\
& \frac{\cos [c+d x]^3 \csc \left[\frac{c}{2}\right] \csc \left[\frac{c}{2}+\frac{d x}{2}\right]^9 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{4608 d} - \\
& \frac{35 \cos [c+d x]^3 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^7 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{1536 d} \\
& \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^9 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2}\right]}{1536 d} + \frac{\cos [c+d x] \sec [c] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 \sin [d x]}{16 d} + \\
& \frac{\cos [c+d x]^2 \sec [c] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (\sin [c]+6 \sin [d x])}{16 d} - \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \sec [c+d x])^3 \tan \left[\frac{c}{2}\right]}{1536 d}
\end{aligned}$$

■ **Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc [c+d x]^4}{a+a \sec [c+d x]} dx$$

Optimal (type 3, 55 leaves, 7 steps):

$$\frac{\text{Cot}[c + dx]^3}{3ad} + \frac{\text{Cot}[c + dx]^5}{5ad} - \frac{\text{Csc}[c + dx]^5}{5ad}$$

Result (type 3, 116 leaves):

$$-\frac{1}{960ad(1+\text{Sec}[c+dx])} \text{Csc}[c] \text{Csc}[c+dx]^3 \text{Sec}[c+dx] (240 \text{Sin}[c] - 96 \text{Sin}[dx] - 54 \text{Sin}[c+dx] - 18 \text{Sin}[2(c+dx)] + 18 \text{Sin}[3(c+dx)] + 9 \text{Sin}[4(c+dx)] - 32 \text{Sin}[c+2dx] + 32 \text{Sin}[2c+3dx] + 16 \text{Sin}[3c+4dx])$$

■ **Problem 71: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^6}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 3, 73 leaves, 7 steps):

$$\frac{\text{Cot}[c + dx]^3}{3ad} + \frac{2 \text{Cot}[c + dx]^5}{5ad} + \frac{\text{Cot}[c + dx]^7}{7ad} - \frac{\text{Csc}[c + dx]^7}{7ad}$$

Result (type 3, 158 leaves):

$$\frac{1}{53760ad(1+\text{Sec}[c+dx])} \text{Csc}[c] \text{Csc}[c+dx]^5 \text{Sec}[c+dx] (-8960 \text{Sin}[c] + 2560 \text{Sin}[dx] + 1500 \text{Sin}[c+dx] + 375 \text{Sin}[2(c+dx)] - 750 \text{Sin}[3(c+dx)] - 300 \text{Sin}[4(c+dx)] + 150 \text{Sin}[5(c+dx)] + 75 \text{Sin}[6(c+dx)] + 640 \text{Sin}[c+2dx] - 1280 \text{Sin}[2c+3dx] - 512 \text{Sin}[3c+4dx] + 256 \text{Sin}[4c+5dx] + 128 \text{Sin}[5c+6dx])$$

■ **Problem 72: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^8}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{\text{Cot}[c + dx]^3}{3ad} + \frac{3 \text{Cot}[c + dx]^5}{5ad} + \frac{3 \text{Cot}[c + dx]^7}{7ad} + \frac{\text{Cot}[c + dx]^9}{9ad} - \frac{\text{Csc}[c + dx]^9}{9ad}$$

Result (type 3, 200 leaves):

$$-\frac{1}{5160960ad(1+\text{Sec}[c+dx])} \text{Csc}[c] \text{Csc}[c+dx]^7 \text{Sec}[c+dx] (645120 \text{Sin}[c] - 143360 \text{Sin}[dx] - 85750 \text{Sin}[c+dx] - 17150 \text{Sin}[2(c+dx)] + 51450 \text{Sin}[3(c+dx)] + 17150 \text{Sin}[4(c+dx)] - 17150 \text{Sin}[5(c+dx)] - 7350 \text{Sin}[6(c+dx)] + 2450 \text{Sin}[7(c+dx)] + 1225 \text{Sin}[8(c+dx)] - 28672 \text{Sin}[c+2dx] + 86016 \text{Sin}[2c+3dx] + 28672 \text{Sin}[3c+4dx] - 28672 \text{Sin}[4c+5dx] - 12288 \text{Sin}[5c+6dx] + 4096 \text{Sin}[6c+7dx] + 2048 \text{Sin}[7c+8dx])$$

■ **Problem 73: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^{10}}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 3, 109 leaves, 7 steps):

$$\frac{\text{Cot}[c + dx]^3}{3ad} + \frac{4 \text{Cot}[c + dx]^5}{5ad} + \frac{6 \text{Cot}[c + dx]^7}{7ad} + \frac{4 \text{Cot}[c + dx]^9}{9ad} + \frac{\text{Cot}[c + dx]^{11}}{11ad} - \frac{\text{Csc}[c + dx]^{11}}{11ad}$$

Result (type 3, 242 leaves):

$$\frac{1}{454164480ad(1 + \text{Sec}[c + dx])} \text{Csc}[c] \text{Csc}[c + dx]^9 \text{Sec}[c + dx] \\ (-45416448 \text{Sin}[c] + 8257536 \text{Sin}[dx] + 5000940 \text{Sin}[c + dx] + 833490 \text{Sin}[2(c + dx)] - 3333960 \text{Sin}[3(c + dx)] - 952560 \text{Sin}[4(c + dx)] + \\ 1428840 \text{Sin}[5(c + dx)] + 535815 \text{Sin}[6(c + dx)] - 357210 \text{Sin}[7(c + dx)] - 158760 \text{Sin}[8(c + dx)] + 39690 \text{Sin}[9(c + dx)] + \\ 19845 \text{Sin}[10(c + dx)] + 1376256 \text{Sin}[c + 2dx] - 5505024 \text{Sin}[2c + 3dx] - 1572864 \text{Sin}[3c + 4dx] + 2359296 \text{Sin}[4c + 5dx] + \\ 884736 \text{Sin}[5c + 6dx] - 589824 \text{Sin}[6c + 7dx] - 262144 \text{Sin}[7c + 8dx] + 65536 \text{Sin}[8c + 9dx] + 32768 \text{Sin}[9c + 10dx])$$

■ **Problem 99: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[c + dx]^5}{(a + a \text{Sec}[c + dx])^3} dx$$

Optimal (type 3, 128 leaves, 6 steps):

$$\frac{3 \text{ArcTanh}[\text{Cos}[c + dx]]}{128a^3d} - \frac{1}{128ad(a - a \text{Cos}[c + dx])^2} - \frac{a^2}{40d(a + a \text{Cos}[c + dx])^5} + \\ \frac{1}{64d(a + a \text{Cos}[c + dx])^4} - \frac{1}{64ad(a + a \text{Cos}[c + dx])^2} - \frac{3}{128d(a^3 + a^3 \text{Cos}[c + dx])}$$

Result (type 3, 412 leaves):

$$-\frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[c + dx]^3}{32d(a + a \text{Sec}[c + dx])^3} - \frac{3 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^3}{32d(a + a \text{Sec}[c + dx])^3} - \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Cot}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^3}{64d(a + a \text{Sec}[c + dx])^3} + \\ \frac{3 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^3}{16d(a + a \text{Sec}[c + dx])^3} - \frac{3 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Log}\left[\text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^3}{16d(a + a \text{Sec}[c + dx])^3} + \frac{3 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[c + dx]^3}{128d(a + a \text{Sec}[c + dx])^3} - \\ \frac{\text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^3}{160d(a + a \text{Sec}[c + dx])^3} + \frac{x \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Sec}[c + dx]^3 \left(\frac{3}{32} \text{Cot}\left[\frac{c}{2}\right] - \frac{3}{32} \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] + \frac{3}{32} \text{Tan}\left[\frac{c}{2}\right]\right)}{(a + a \text{Sec}[c + dx])^3}$$

■ **Problem 119: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + a \text{Sec}[c + dx])^2}{(e \text{Sin}[c + dx])^{5/2}} dx$$

Optimal (type 4, 234 leaves, 16 steps):

$$\frac{2 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{e \sin [c+d x]}}{\sqrt{e}}\right]}{d e^{5 / 2}}+\frac{2 a^2 \operatorname{ArcTanh}\left[\frac{\sqrt{e \sin [c+d x]}}{\sqrt{e}}\right]}{d e^{5 / 2}}-\frac{4 a^2}{3 d e\left(e \sin [c+d x]\right)^{3 / 2}}-\frac{2 a^2 \cos [c+d x]}{3 d e\left(e \sin [c+d x]\right)^{3 / 2}}-\frac{2 a^2 \sec [c+d x]}{3 d e\left(e \sin [c+d x]\right)^{3 / 2}}+\frac{7 a^2 \operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{3 d e^2 \sqrt{e \sin [c+d x]}}+\frac{5 a^2 \sec [c+d x] \sqrt{e \sin [c+d x]}}{3 d e^3}$$

Result (type 1, 1 leaves):

???

- **Problem 121: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(e \sin [c+d x]\right)^{5 / 2}}{a+a \sec [c+d x]} d x$$

Optimal (type 4, 104 leaves, 7 steps):

$$-\frac{4 e^2 \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{5 a d \sqrt{\sin [c+d x]}}+\frac{2 e\left(e \sin [c+d x]\right)^{3 / 2}}{3 a d}-\frac{2 e \cos [c+d x]\left(e \sin [c+d x]\right)^{3 / 2}}{5 a d}$$

Result (type 5, 232 leaves):

$$\left(2 \cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]\left(e \sin [c+d x]\right)^{5 / 2}\right. \\ \left.\left(\left(2 e^{-i d x} \sqrt{2-2 e^{2 i(c+d x)}}\left(3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)}\right]+e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2 i(c+d x)}\right]\right) \sec [c]\right) / \right. \\ \left.\left(\sqrt{-i e^{-i(c+d x)}\left(-1+e^{2 i(c+d x)}\right)}\right)+\sqrt{\sin [c+d x]}\left(10 \cos [d x] \sin [c]-3 \cos [2 d x] \sin [2 c]+10 \cos [c] \sin [d x]-3 \cos [2 c] \sin [2 d x]-12 \tan [c]\right)\right) / \left(15 a d(1+\sec [c+d x]) \sin [c+d x]\right)^{5 / 2}$$

- **Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e \sin [c+d x]}}{a+a \sec [c+d x]} d x$$

Optimal (type 4, 95 leaves, 7 steps):

$$-\frac{2 e}{a d \sqrt{e \sin [c+d x]}}+\frac{2 e \cos [c+d x]}{a d \sqrt{e \sin [c+d x]}}+\frac{4 \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{a d \sqrt{\sin [c+d x]}}$$

Result (type 5, 249 leaves):

$$\left(2 \left(3 - 9 e^{2 i c} + 6 e^{i(c+d x)} - 9 e^{2 i(c+d x)} + 3 e^{2 i(2 c+d x)} + 6 e^{i(3 c+d x)} + \right. \right. \\ \left. \left. 12 e^{2 i c} \sqrt{1 - e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)} \right] + 4 e^{2 i(c+d x)} \sqrt{1 - e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2 i(c+d x)} \right] \right) \\ \left. \sqrt{e \operatorname{Sin}[c+d x]} \right) / \left(3 a d (1 + i e^{i c}) (i + e^{i c}) (-1 + e^{i(c+d x)}) (1 + e^{i(c+d x)}) \right)$$

■ **Problem 125: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c+d x]) (e \operatorname{Sin}[c+d x])^{3/2}} dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$-\frac{2 e}{5 a d (e \operatorname{Sin}[c+d x])^{5/2}} + \frac{2 e \operatorname{Cos}[c+d x]}{5 a d (e \operatorname{Sin}[c+d x])^{5/2}} - \frac{4 \operatorname{Cos}[c+d x]}{5 a d e \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{4 \operatorname{EllipticE} \left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2 \right] \sqrt{e \operatorname{Sin}[c+d x]}}{5 a d e^2 \sqrt{\operatorname{Sin}[c+d x]}}$$

Result (type 5, 175 leaves):

$$-\left(e^{-i(3 c+2 d x)} (1 + e^{2 i c}) \left(\sqrt{1 - e^{2 i(c+d x)}} (1 + 2 e^{i(c+d x)} + 2 e^{2 i(c+d x)}) + (-1 + e^{i(c+d x)}) (1 + e^{i(c+d x)})^3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)} \right] \right) \right. \\ \left. \operatorname{Sec}[c] \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right] \right) / \left(5 a d \sqrt{1 - e^{2 i(c+d x)}} (e \operatorname{Sin}[c+d x])^{3/2} \right)$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Sin}[c+d x])^{5/2}}{(a + a \operatorname{Sec}[c+d x])^2} dx$$

Optimal (type 4, 187 leaves, 14 steps):

$$\frac{4 e^3}{a^2 d \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{2 e^3 \operatorname{Cos}[c+d x]}{a^2 d \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{2 e^3 \operatorname{Cos}[c+d x]^3}{a^2 d \sqrt{e \operatorname{Sin}[c+d x]}} - \\ \frac{44 e^2 \operatorname{EllipticE} \left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2 \right] \sqrt{e \operatorname{Sin}[c+d x]}}{5 a^2 d \sqrt{\operatorname{Sin}[c+d x]}} + \frac{4 e (e \operatorname{Sin}[c+d x])^{3/2}}{3 a^2 d} - \frac{12 e \operatorname{Cos}[c+d x] (e \operatorname{Sin}[c+d x])^{3/2}}{5 a^2 d}$$

Result (type 5, 451 leaves):

$$\frac{1}{(a + a \operatorname{Sec}[c + dx])^2}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}[c + dx]^2 \operatorname{Sec}[c + dx]^2 \left(\frac{16 \operatorname{Cos}[dx] \operatorname{Sin}[c]}{3d} - \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \left(8 \operatorname{Sin}\left[\frac{c}{2}\right] + 3 \operatorname{Sin}\left[\frac{3c}{2}\right]\right)}{5d} - \frac{4 \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{5d} + \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} + \frac{16 \operatorname{Cos}[c] \operatorname{Sin}[dx]}{3d} - \frac{4 \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{5d} \right) (e \operatorname{Sin}[c + dx])^{5/2} + \left(44 i \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \right. \\ \left. - \frac{2 i e^{-i dx} \sqrt{2 - 2 e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2i(c+dx)}\right]}{d \sqrt{-i e^{-i(c+dx)}} (-1 + e^{2i(c+dx)})} - \frac{2 i e^{i dx} \sqrt{2 - 2 e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2i(c+dx)}\right]}{3d \sqrt{-i e^{-i(c+dx)}} (-1 + e^{2i(c+dx)})} \right) \\ \operatorname{Sec}[c + dx]^2 (e \operatorname{Sin}[c + dx])^{5/2} \Bigg/ \left(5 (a + a \operatorname{Sec}[c + dx])^2 \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \operatorname{Sin}[c + dx]^{5/2} \right)$$

■ **Problem 130: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{e \operatorname{Sin}[c + dx]}}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 188 leaves, 15 steps):

$$\frac{4 e^3}{5 a^2 d (e \operatorname{Sin}[c + dx])^{5/2}} - \frac{2 e^3 \operatorname{Cos}[c + dx]}{5 a^2 d (e \operatorname{Sin}[c + dx])^{5/2}} - \frac{2 e^3 \operatorname{Cos}[c + dx]^3}{5 a^2 d (e \operatorname{Sin}[c + dx])^{5/2}} - \frac{4 e}{a^2 d \sqrt{e \operatorname{Sin}[c + dx]}} + \frac{16 e \operatorname{Cos}[c + dx]}{5 a^2 d \sqrt{e \operatorname{Sin}[c + dx]}} + \frac{28 \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \operatorname{Sin}[c + dx]}}{5 a^2 d \sqrt{\operatorname{Sin}[c + dx]}}$$

Result (type 5, 222 leaves):

$$\frac{1}{15 a^2 d (1 + \operatorname{Sec}[c + dx])^2} 4 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sec}[c + dx]^2 \sqrt{e \operatorname{Sin}[c + dx]} \\ \left(\left(56 i e^{2ic} \left(3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2i(c+dx)}\right] + e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2i(c+dx)}\right] \right) \right) \Bigg/ \left((1 + e^{2ic}) \sqrt{1 - e^{2i(c+dx)}} \right) + \frac{3}{4} \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 \left(49 \operatorname{Sin}\left[\frac{1}{2}(c - dx)\right] + 35 \operatorname{Sin}\left[\frac{1}{2}(3c + dx)\right] - 23 \operatorname{Sin}\left[\frac{1}{2}(c + 3dx)\right] + 5 \operatorname{Sin}\left[\frac{1}{2}(5c + 3dx)\right] \right) \right)$$

■ **Problem 132: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c + dx])^2 (e \operatorname{Sin}[c + dx])^{3/2}} dx$$

Optimal (type 4, 224 leaves, 17 steps):

$$\frac{4 e^3}{9 a^2 d (e \sin [c+d x])^{9/2}} - \frac{2 e^3 \cos [c+d x]}{9 a^2 d (e \sin [c+d x])^{9/2}} - \frac{2 e^3 \cos [c+d x]^3}{9 a^2 d (e \sin [c+d x])^{9/2}} - \frac{4 e}{5 a^2 d (e \sin [c+d x])^{5/2}} +$$

$$\frac{16 e \cos [c+d x]}{45 a^2 d (e \sin [c+d x])^{5/2}} - \frac{4 \cos [c+d x]}{15 a^2 d e \sqrt{e \sin [c+d x]}} - \frac{4 \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{15 a^2 d e^2 \sqrt{\sin [c+d x]}}$$

Result (type 5, 222 leaves):

$$\left(\cos \left[\frac{1}{2} (c+d x) \right]^4 \left(\frac{96 i (1 - e^{2 i (c+d x)})^{3/2} \left(-\sqrt{1 - e^{2 i (c+d x)}} + (1 + e^{2 i c}) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i (c+d x)}\right] \right)}{(1 + e^{2 i c}) (1 + e^{2 i (c+d x)})^2} \right) \right.$$

$$\left. 2 (28 \cos [c] + 31 \cos [d x] + 16 \cos [2 c+d x] + 12 \cos [c+2 d x] + 3 \cos [2 c+3 d x]) \sec [c] \right.$$

$$\left. \sec \left[\frac{1}{2} (c+d x) \right]^2 \sec [c+d x]^2 \tan \left[\frac{1}{2} (c+d x) \right] \right) / (45 a^2 d (1 + \sec [c+d x])^2 (e \sin [c+d x])^{3/2})$$

■ **Problem 134: Unable to integrate problem.**

$$\int (a + a \sec [c+d x])^3 (e \sin [c+d x])^m dx$$

Optimal (type 5, 247 leaves, 9 steps):

$$\frac{a^3 \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] (e \sin [c+d x])^{1+m}}{d e (1+m) \sqrt{\cos [c+d x]^2}} +$$

$$\frac{3 a^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] (e \sin [c+d x])^{1+m}}{d e (1+m)} + \frac{a^3 \operatorname{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] (e \sin [c+d x])^{1+m}}{d e (1+m)} +$$

$$\frac{3 a^3 \sqrt{\cos [c+d x]^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] \sec [c+d x] (e \sin [c+d x])^{1+m}}{d e (1+m)}$$

Result (type 8, 25 leaves):

$$\int (a + a \sec [c+d x])^3 (e \sin [c+d x])^m dx$$

■ **Problem 135: Unable to integrate problem.**

$$\int (a + a \sec [c+d x])^2 (e \sin [c+d x])^m dx$$

Optimal (type 5, 195 leaves, 7 steps):

$$\frac{a^2 \operatorname{Cos}[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c + d x]^2\right] (e \operatorname{Sin}[c + d x])^{1+m}}{d e (1+m) \sqrt{\operatorname{Cos}[c + d x]^2}} +$$

$$\frac{2 a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c + d x]^2\right] (e \operatorname{Sin}[c + d x])^{1+m}}{d e (1+m)} +$$

$$\frac{a^2 \sqrt{\operatorname{Cos}[c + d x]^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c + d x]^2\right] \operatorname{Sec}[c + d x] (e \operatorname{Sin}[c + d x])^{1+m}}{d e (1+m)}$$

Result (type 8, 25 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^2 (e \operatorname{Sin}[c + d x])^m dx$$

■ **Problem 136: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x]) (e \operatorname{Sin}[c + d x])^m dx$$

Optimal (type 5, 119 leaves, 5 steps):

$$\frac{a \operatorname{Cos}[c + d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c + d x]^2\right] (e \operatorname{Sin}[c + d x])^{1+m}}{d e (1+m) \sqrt{\operatorname{Cos}[c + d x]^2}} +$$

$$\frac{a \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c + d x]^2\right] (e \operatorname{Sin}[c + d x])^{1+m}}{d e (1+m)}$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Sec}[c + d x]) (e \operatorname{Sin}[c + d x])^m dx$$

■ **Problem 137: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Sin}[c + d x])^m}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 5, 100 leaves, 5 steps):

$$-\frac{e (e \operatorname{Sin}[c + d x])^{-1+m}}{a d (1-m)} + \frac{e \operatorname{Cos}[c + d x] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} (-1+m), \frac{1+m}{2}, \operatorname{Sin}[c + d x]^2\right] (e \operatorname{Sin}[c + d x])^{-1+m}}{a d (1-m) \sqrt{\operatorname{Cos}[c + d x]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \operatorname{Sin}[c + d x])^m}{a + a \operatorname{Sec}[c + d x]} dx$$

■ **Problem 138: Unable to integrate problem.**

$$\int \frac{(e \sin[c + dx])^m}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 5, 207 leaves, 9 steps):

$$\frac{2 e^3 (e \sin[c + dx])^{-3+m}}{a^2 d (3 - m)} - \frac{e^3 \cos[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}(-3 + m), \frac{1}{2}(-1 + m), \sin^2[c + dx]\right] (e \sin[c + dx])^{-3+m}}{a^2 d (3 - m) \sqrt{\cos[c + dx]^2}} - \frac{e^3 \cos[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}(-3 + m), \frac{1}{2}(-1 + m), \sin^2[c + dx]\right] (e \sin[c + dx])^{-3+m}}{a^2 d (3 - m) \sqrt{\cos[c + dx]^2}} - \frac{2 e (e \sin[c + dx])^{-1+m}}{a^2 d (1 - m)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \sin[c + dx])^m}{(a + a \sec[c + dx])^2} dx$$

■ **Problem 139: Unable to integrate problem.**

$$\int \frac{(e \sin[c + dx])^m}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 5, 236 leaves, 12 steps):

$$-\frac{4 e^5 (e \sin[c + dx])^{-5+m}}{a^3 d (5 - m)} + \frac{e^5 \cos[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1}{2}(-5 + m), \frac{1}{2}(-3 + m), \sin^2[c + dx]\right] (e \sin[c + dx])^{-5+m}}{a^3 d (5 - m) \sqrt{\cos[c + dx]^2}} + \frac{3 e^5 \cos[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}(-5 + m), \frac{1}{2}(-3 + m), \sin^2[c + dx]\right] (e \sin[c + dx])^{-5+m}}{a^3 d (5 - m) \sqrt{\cos[c + dx]^2}} + \frac{7 e^3 (e \sin[c + dx])^{-3+m}}{a^3 d (3 - m)} - \frac{3 e (e \sin[c + dx])^{-1+m}}{a^3 d (1 - m)}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \sin[c + dx])^m}{(a + a \sec[c + dx])^3} dx$$

■ **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c + dx])^{3/2} (e \sin[c + dx])^m dx$$

Optimal (type 6, 106 leaves, 5 steps):

$$\frac{1}{d} 2 a e \operatorname{AppellF1}\left[-\frac{1}{2}, \frac{1-m}{2}, \frac{1}{2}(-2-m), \frac{1}{2}, \cos[c+dx], -\cos[c+dx]\right] \\ (1-\cos[c+dx])^{\frac{1-m}{2}} (1+\cos[c+dx])^{-m/2} \sqrt{a+a \operatorname{Sec}[c+dx]} (e \sin[c+dx])^{-1+m}$$

Result (type 6, 7867 leaves): Display of huge result suppressed!

- **Problem 141: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \operatorname{Sec}[c+dx]} (e \sin[c+dx])^m dx$$

Optimal (type 6, 107 leaves, 5 steps):

$$-\frac{1}{d} 2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1-m}{2}, -\frac{m}{2}, \frac{3}{2}, \cos[c+dx], -\cos[c+dx]\right] \\ (1-\cos[c+dx])^{\frac{1-m}{2}} \cos[c+dx] (1+\cos[c+dx])^{-m/2} \sqrt{a+a \operatorname{Sec}[c+dx]} (e \sin[c+dx])^{-1+m}$$

Result (type 6, 5279 leaves): Display of huge result suppressed!

- **Problem 142: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c+dx])^m}{\sqrt{a+a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 6, 115 leaves, 5 steps):

$$-\frac{1}{3 d \sqrt{a+a \operatorname{Sec}[c+dx]}} \\ 2 e \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1-m}{2}, \frac{2-m}{2}, \frac{5}{2}, \cos[c+dx], -\cos[c+dx]\right] (1-\cos[c+dx])^{\frac{1-m}{2}} \cos[c+dx] (1+\cos[c+dx])^{1-\frac{m}{2}} (e \sin[c+dx])^{-1+m}$$

Result (type 6, 2679 leaves):

$$\left(\sqrt{2} (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\ \left. \cos[c+dx] \sqrt{1+\operatorname{Sec}[c+dx]} \sin[c+dx]^m (e \sin[c+dx])^m \tan\left[\frac{1}{2}(c+dx)\right]\right) / \\ \left(d (1+m) \sqrt{a(1+\operatorname{Sec}[c+dx])}\right) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\ \left(2 (1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -\frac{1}{2}, 2+m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\ \left. \left. \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, 1+m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\ \left(\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1+\operatorname{Sec}[c+dx]}\right)\right)$$

$$\begin{aligned}
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left(2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, 2+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, 1+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\
& \left(\sqrt{2} (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Cos}[c+dx] \sqrt{1+\operatorname{Sec}[c+dx]} \right. \\
& \quad \left. \operatorname{Sin}[c+dx]^m \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(- \left(2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, 2+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, 1+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (3+m) \right. \\
& \quad \left(-\frac{1}{3+m} (1+m)^2 \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, -\frac{1}{2}, 2+m, 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \quad \frac{1}{2(3+m)} (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{1}{2}, 1+m, 1 + \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) - \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(-\frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{1}{2}, 2+m, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{1}{2(5+m)} (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{3}{2}, 1+m, 1 + \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + 2(1+m) \left(-\frac{1}{5+m} (2+m) (3+m) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \quad \left. \left. \left. 1 + \frac{3+m}{2}, -\frac{1}{2}, 3+m, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{2(5+m)} (3+m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{1}{2}, 2+m, 1 + \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) \right) / \\
& \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, 1+m, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left(2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, 2+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \operatorname{AppellF1} \left[\right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \frac{1}{2}, 1+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right) /
\end{aligned}$$

■ **Problem 143: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Sin}[c+dx])^m}{(a+a \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 6, 120 leaves, 5 steps) :

$$-\frac{1}{5 a d \sqrt{a+a \operatorname{Sec}[c+d x]}} 2 e \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1-m}{2}, \frac{4-m}{2}, \frac{7}{2}, \operatorname{Cos}[c+d x], -\operatorname{Cos}[c+d x]\right] (1-\operatorname{Cos}[c+d x])^{\frac{1-m}{2}} \operatorname{Cos}[c+d x]^2 (1+\operatorname{Cos}[c+d x])^{1-\frac{m}{2}} (e \operatorname{Sin}[c+d x])^{-1+m}$$

Result (type 6, 5702 leaves) : Display of huge result suppressed !

■ **Problem 144: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+d x])^n (e \operatorname{Sin}[c+d x])^m dx$$

Optimal (type 6, 130 leaves, 5 steps) :

$$-\frac{1}{d(1-n)} e \operatorname{AppellF1}\left[1-n, \frac{1-m}{2}, \frac{1}{2}(1-m-2n), 2-n, \operatorname{Cos}[c+d x], -\operatorname{Cos}[c+d x]\right] (1-\operatorname{Cos}[c+d x])^{\frac{1-m}{2}} \operatorname{Cos}[c+d x] (1+\operatorname{Cos}[c+d x])^{\frac{1}{2}(1-m-2n)} (a+a \operatorname{Sec}[c+d x])^n (e \operatorname{Sin}[c+d x])^{-1+m}$$

Result (type 6, 2135 leaves) :

$$\begin{aligned} & \left(2^n (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right. \\ & \quad \left. \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n (a(1+\operatorname{Sec}[c+d x]))^n \operatorname{Sin}[c+d x]^{1+m} (e \operatorname{Sin}[c+d x])^m\right) / \\ & \left(d(1+m) \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \right. \\ & \left. \left(\left(2^n (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Cos}[c+d x] \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n \right. \right. \right. \\ & \quad \left. \left. \operatorname{Sin}[c+d x]^m\right) / \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\ & \quad \left. \left. 2 \left((1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, n, 2+m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \right. \\ & \quad \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+n, 1+m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) + \right. \\ & \left. \left(2^n (3+m) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n \operatorname{Sin}[c+d x]^{1+m} \left(-1 / (3+m)(1+m)^2 \operatorname{AppellF1}\left[1+\frac{1+m}{2}, n, 2+m, 1+\frac{3+m}{2}, \right. \right. \right. \end{aligned}$$

$$\left(2^n (3+m) n \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \right)^{-1+n} \right. \\ \left. \operatorname{Sin}[c+dx]^{1+m} \left(-\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sec}[c+dx] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) \right) / \\ \left((1+m) \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, n, 1+m, \frac{3+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\ \left. \left. 2 \left((1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, n, 2+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\ \left. \left. \left. n \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+n, 1+m, \frac{5+m}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right)$$

■ **Problem 145: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]^7 dx$$

Optimal (type 5, 180 leaves, 4 steps):

$$\frac{(3-n)(8-n)(16-n) \operatorname{Hypergeometric2F1}[6, 4+n, 5+n, 1+\operatorname{Sec}[c+dx]] (a + a \operatorname{Sec}[c+dx])^{4+n}}{42 a^4 d (1-n) (4+n)} - \\ \frac{\operatorname{Cos}[c+dx]^7 (1-\operatorname{Sec}[c+dx])^2 (a + a \operatorname{Sec}[c+dx])^{4+n}}{a^4 d (1-n)} + \frac{\operatorname{Cos}[c+dx]^7 (a + a \operatorname{Sec}[c+dx])^{4+n} (6(8-n) - (108 - 25n + n^2) \operatorname{Sec}[c+dx])}{42 a^4 d (1-n)}$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]^7 dx$$

■ **Problem 148: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx] dx$$

Optimal (type 5, 42 leaves, 2 steps):

$$\frac{\operatorname{Hypergeometric2F1}[2, 1+n, 2+n, 1+\operatorname{Sec}[c+dx]] (a + a \operatorname{Sec}[c+dx])^{1+n}}{a d (1+n)}$$

Result (type 5, 95 leaves):

$$\frac{1}{d (1+n)} 2^{1+n} (-\operatorname{Cos}[c+dx])^{1+n} \operatorname{Hypergeometric2F1} \left[n, 1+n, 2+n, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \right] \\ \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \right)^{1+n} (1 + \operatorname{Sec}[c+dx])^{-n} (a (1 + \operatorname{Sec}[c+dx]))^n$$

- **Problem 152: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^4 dx$$

Optimal (type 6, 230 leaves, 11 steps):

$$-\left(\operatorname{AppellF1}\left[1 - n, -\frac{1}{2}, \frac{1}{2} - n, 2 - n, \operatorname{Cos}[c + d x], -\operatorname{Cos}[c + d x]\right] (1 + \operatorname{Cos}[c + d x])^{\frac{1}{2}-n} (n - n \operatorname{Cos}[c + d x]) \operatorname{Cot}[c + d x] (a + a \operatorname{Sec}[c + d x])^n \right) /$$

$$\left(d (1 - n) \sqrt{1 - \operatorname{Cos}[c + d x]} \right) - \frac{\operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d} + \frac{1}{d}$$

$$2^{\frac{1}{2}+n} \operatorname{AppellF1}\left[\frac{1}{2}, -4 + n, \frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Cos}[c + d x], \frac{1}{2} (1 - \operatorname{Cos}[c + d x])\right] \operatorname{Cos}[c + d x]^n (1 + \operatorname{Cos}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]$$

Result (type 6, 7115 leaves): Display of huge result suppressed!

- **Problem 153: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^2 dx$$

Optimal (type 6, 95 leaves, 6 steps):

$$-\frac{1}{d(1-n)} \operatorname{AppellF1}\left[1 - n, -\frac{1}{2}, -\frac{1}{2} - n, 2 - n, \operatorname{Cos}[c + d x], -\operatorname{Cos}[c + d x]\right] \sqrt{1 - \operatorname{Cos}[c + d x]} (1 + \operatorname{Cos}[c + d x])^{\frac{1}{2}-n} \operatorname{Cot}[c + d x] (a + a \operatorname{Sec}[c + d x])^n$$

Result (type 6, 4297 leaves):

$$\left(2^{3+n} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}[c + d x] \right)^n (1 + \operatorname{Sec}[c + d x])^{-n} (a (1 + \operatorname{Sec}[c + d x]))^n \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right.$$

$$\left(\operatorname{Cos}[2(c + d x)] \left(-\frac{1}{4} (1 + \operatorname{Sec}[c + d x])^n - \frac{1}{2} (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^2 - \frac{1}{4} (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^4 \right) + \right.$$

$$\frac{1}{4} i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[2(c + d x)] + \frac{1}{2} i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^2 \operatorname{Sin}[2(c + d x)] + \frac{1}{4} i (1 + \operatorname{Sec}[c + d x])^n$$

$$\operatorname{Sin}[c + d x]^4 \operatorname{Sin}[2(c + d x)] + \operatorname{Cos}[c + d x]^4 \left(-\frac{1}{4} \operatorname{Cos}[2(c + d x)] (1 + \operatorname{Sec}[c + d x])^n + \frac{1}{4} i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[2(c + d x)] \right) +$$

$$\operatorname{Cos}[c + d x]^3 (-i \operatorname{Cos}[2(c + d x)] (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] - (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \operatorname{Sin}[2(c + d x)]) +$$

$$\operatorname{Cos}[c + d x]^2 \left(\operatorname{Cos}[2(c + d x)] \left(\frac{1}{2} (1 + \operatorname{Sec}[c + d x])^n + \frac{3}{2} (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^2 \right) - \right.$$

$$\frac{1}{2} i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[2(c + d x)] - \frac{3}{2} i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^2 \operatorname{Sin}[2(c + d x)] \left. \right) +$$

$$\operatorname{Cos}[c + d x] \left(\operatorname{Cos}[2(c + d x)] \left(i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] + i (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^3 \right) + \right.$$

$$\left. \left. (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \operatorname{Sin}[2(c + d x)] + (1 + \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]^3 \operatorname{Sin}[2(c + d x)] \right) \right) /$$

$$\left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 2, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) /$$

$$\left(n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 3, \frac{5}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \left(-\cos \left[\frac{1}{2} (c+dx) \right] \sec [c+dx] \sin \left[\frac{1}{2} (c+dx) \right] + \cos \left[\frac{1}{2} (c+dx) \right]^2 \sec [c+dx] \tan [c+dx] \right) \right)$$

■ **Problem 156: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c+dx])^n \sin [c+dx]^{3/2} dx$$

Optimal (type 6, 105 leaves, 5 steps):

$$-\left(\operatorname{AppellF1} \left[1-n, -\frac{1}{4}, -\frac{1}{4}-n, 2-n, \cos [c+dx], -\cos [c+dx] \right] \cos [c+dx] (1+\cos [c+dx])^{-\frac{1}{4}-n} (a+a \sec [c+dx])^n \sqrt{\sin [c+dx]} \right) / \left(d(1-n)(1-\cos [c+dx])^{1/4} \right)$$

Result (type 6, 4151 leaves):

$$\left(5 \times 2^{1+n} \cot \left[\frac{1}{2} (c+dx) \right]^2 \left(\cos \left[\frac{1}{2} (c+dx) \right]^2 \sec [c+dx] \right)^n (1+\sec [c+dx])^{-n} \right. \\ \left. (a(1+\sec [c+dx]))^n \sin [c+dx]^{5/2} \left(-\frac{1}{2} \cos [2(c+dx)] (1+\sec [c+dx])^n \sin [c+dx]^{3/2} + \right. \right. \\ \left. \left. \sin [c+dx]^{3/2} \left(\frac{1}{2} (1+\sec [c+dx])^n - \frac{1}{2} i (1+\sec [c+dx])^n \sin [2(c+dx)] \right) + \cos [c+dx] \right. \right. \\ \left. \left. \left(-\frac{1}{2} i \cos [2(c+dx)] (1+\sec [c+dx])^n \sqrt{\sin [c+dx]} + \sqrt{\sin [c+dx]} \left(\frac{1}{2} i (1+\sec [c+dx])^n + \frac{1}{2} (1+\sec [c+dx])^n \sin [2(c+dx)] \right) \right) \right) \right) / \\ \left(\left(\operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{3}{2}, \frac{5}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \right) / \right. \\ \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{3}{2}, \frac{5}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{5}{2}, \frac{9}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right. \\ \left. \left. 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{3}{2}, \frac{9}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\ \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{5}{2}, \frac{5}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] / \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{5}{2}, \frac{5}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \\ \left. 2 \left(-5 \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{7}{2}, \frac{9}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right. \\ \left. \left. 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{5}{2}, \frac{9}{4}, \tan \left[\frac{1}{2} (c+dx) \right]^2, -\tan \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) / \\ \left(d \left(25 \times 2^n \cos [c+dx] \cot \left[\frac{1}{2} (c+dx) \right]^2 \left(\cos \left[\frac{1}{2} (c+dx) \right]^2 \sec [c+dx] \right)^n \sin [c+dx]^{3/2} \right. \right.$$

$$\begin{aligned}
& 5 \left(-\frac{1}{2} \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{7}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{1}{5} n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{5}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) + \\
& 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(-5 \left(-\frac{35}{18} \operatorname{AppellF1} \left[\frac{9}{4}, n, \frac{9}{2}, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \right. \\
& \quad \left. \frac{5}{9} n \operatorname{AppellF1} \left[\frac{9}{4}, 1+n, \frac{7}{2}, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) + \\
& \quad 2n \left(-\frac{25}{18} \operatorname{AppellF1} \left[\frac{9}{4}, 1+n, \frac{7}{2}, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \frac{5}{9} (1+n) \operatorname{AppellF1} \left[\frac{9}{4}, 2+n, \frac{5}{2}, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \Big) \Big) \Big) \Big) / \\
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{5}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(-5 \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{7}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{5}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Big) \Big) + \\
& 5 \times 2^{1+n} n \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right]^2 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \right)^{-1+n} \operatorname{Sin} [c+dx]^{5/2} \\
& \left(\left(\operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{3}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right) / \right. \\
& \quad \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{3}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(-3 \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{5}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{3}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\
& \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{5}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] / \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{5}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \\
& \quad 2 \left(-5 \operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{7}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \\
& \quad \left. \left. 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{5}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Big) \Big) \\
& \left(-\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sec} [c+dx] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \operatorname{Tan} [c+dx] \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 157: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec} [c+dx])^n \sqrt{\operatorname{Sin} [c+dx]} dx$$

Optimal (type 6, 105 leaves, 5 steps):

$$-\frac{1}{d(1-n)\sqrt{\sin[c+dx]}} \operatorname{AppellF1}\left[1-n, \frac{1}{4}, \frac{1}{4}-n, 2-n, \cos[c+dx], -\cos[c+dx]\right] (1-\cos[c+dx])^{1/4} \cos[c+dx] (1+\cos[c+dx])^{\frac{1}{4}-n} (a+a\sec[c+dx])^n$$

Result (type 6, 1758 leaves):

$$\begin{aligned} & \left(7 \times 2^{1+n} \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]\right)^n (a(1+\sec[c+dx]))^n \sin[c+dx]^2\right) / \\ & \left(d \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\ & \quad \left. \left. 2n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\ & \left(\left(21 \times 2^n \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \cos[c+dx] \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]\right)^n \sqrt{\sin[c+dx]}\right) / \right. \\ & \quad \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2 + \left(7 \times 2^{1+n} \right. \\ & \quad \left.\left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]\right)^n \sin[c+dx]^{3/2} \left(-\frac{9}{14} \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \\ & \quad \left. \left. \tan\left[\frac{1}{2}(c+dx)\right] + \frac{3}{7} n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) / \right. \\ & \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2 - \\ & \left(7 \times 2^{1+n} \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]\right)^n \sin[c+dx]^{3/2} \right. \\ & \quad \left(6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\ & \quad \left. 21 \left(-\frac{9}{14} \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \frac{3}{7} n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \\
& 6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-3 \left(-\frac{35}{22} \operatorname{AppellF1}\left[\frac{11}{4}, n, \frac{7}{2}, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \frac{7}{11} n \operatorname{AppellF1}\left[\frac{11}{4}, 1+n, \frac{5}{2}, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. 2 n \left(-\frac{21}{22} \operatorname{AppellF1}\left[\frac{11}{4}, 1+n, \frac{5}{2}, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{7}{11} (1+n) \operatorname{AppellF1}\left[\frac{11}{4}, 2+n, \frac{3}{2}, \frac{15}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)\right)\right) / \\
& \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + \\
& \left(7 \times 2^{1+n} n \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]\right)^{-1+n} \operatorname{Sin}[c+dx]^{3/2} \right. \\
& \quad \left. \left(-\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]\right)\right) / \\
& \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, n, \frac{3}{2}, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \quad \left. 6 \left(-3 \operatorname{AppellF1}\left[\frac{7}{4}, n, \frac{5}{2}, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2 n \operatorname{AppellF1}\left[\frac{7}{4}, 1+n, \frac{3}{2}, \frac{11}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)
\end{aligned}$$

■ **Problem 158: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^n}{\sqrt{\operatorname{Sin}[c+dx]}} dx$$

Optimal (type 6, 105 leaves, 5 steps):

$$-\frac{1}{d(1-n)\operatorname{Sin}[c+dx]^{3/2}} \operatorname{AppellF1}\left[1-n, \frac{3}{4}, \frac{3}{4}-n, 2-n, \operatorname{Cos}[c+dx], -\operatorname{Cos}[c+dx]\right] (1-\operatorname{Cos}[c+dx])^{3/4} \operatorname{Cos}[c+dx] (1+\operatorname{Cos}[c+dx])^{\frac{3}{4}-n} (a+a \operatorname{Sec}[c+dx])^n$$

Result (type 6, 1735 leaves):

$$\left(5 \times 2^{1+n} \operatorname{AppellF1}\left[\frac{1}{4}, n, \frac{1}{2}, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]\right)^n (a(1+\operatorname{Sec}[c+dx]))^n\right) /$$

$$\begin{aligned}
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{1}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{3}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{1}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2 + \\
& \left(5 \times 2^{1+n} n \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{1}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \right)^{-1+n} \sqrt{\operatorname{Sin} [c+dx]} \right. \\
& \quad \left. \left(-\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sec} [c+dx] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \operatorname{Tan} [c+dx] \right) \right) / \\
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, n, \frac{1}{2}, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{5}{4}, n, \frac{3}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2n \operatorname{AppellF1} \left[\frac{5}{4}, 1+n, \frac{1}{2}, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Big)
\end{aligned}$$

■ **Problem 159: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec} [c + dx])^n}{\operatorname{Sin} [c + dx]^{3/2}} dx$$

Optimal (type 6, 105 leaves, 5 steps):

$$\begin{aligned}
& - \frac{1}{d (1-n) \operatorname{Sin} [c + dx]^{5/2}} \\
& \operatorname{AppellF1} \left[1-n, \frac{5}{4}, \frac{5}{4} - n, 2-n, \operatorname{Cos} [c + dx], -\operatorname{Cos} [c + dx] \right] (1 - \operatorname{Cos} [c + dx])^{5/4} \operatorname{Cos} [c + dx] (1 + \operatorname{Cos} [c + dx])^{5-n} (a + a \operatorname{Sec} [c + dx])^n
\end{aligned}$$

Result (type 6, 1743 leaves):

$$\begin{aligned}
& - \left(\left(3 \times 2^{1+n} \operatorname{AppellF1} \left[-\frac{1}{4}, n, -\frac{1}{2}, \frac{3}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Csc} [c+dx]^2 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \right)^n (a (1 + \operatorname{Sec} [c+dx]))^n \right) \right) / \\
& \left(d \left(3 \operatorname{AppellF1} \left[-\frac{1}{4}, n, -\frac{1}{2}, \frac{3}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(\operatorname{AppellF1} \left[\frac{3}{4}, n, \frac{1}{2}, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. 2n \operatorname{AppellF1} \left[\frac{3}{4}, 1+n, -\frac{1}{2}, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \\
& \left(\left(3 \times 2^n \operatorname{AppellF1} \left[-\frac{1}{4}, n, -\frac{1}{2}, \frac{3}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Cos} [c+dx] \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec} [c+dx] \right)^n \right) \right) / \\
& \left(\operatorname{Sin} [c+dx]^{3/2} \left(3 \operatorname{AppellF1} \left[-\frac{1}{4}, n, -\frac{1}{2}, \frac{3}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left(\operatorname{AppellF1} \left[\frac{3}{4}, n, \frac{1}{2}, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2n \operatorname{AppellF1} \left[\frac{3}{4}, 1+n, -\frac{1}{2}, \frac{7}{4}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) -
\end{aligned}$$

$$-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + 2 n \operatorname{AppellF1}\left[\frac{3}{4}, 1+n, -\frac{1}{2}, \frac{7}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)$$

■ **Problem 164: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] (a+b \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 26 leaves, 5 steps):

$$-\frac{a \operatorname{ArcTanh}[\operatorname{Cos}[c+dx]]}{d} + \frac{b \operatorname{Log}[\operatorname{Tan}[c+dx]]}{d}$$

Result (type 3, 65 leaves):

$$-\frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} - \frac{b \operatorname{Log}[\operatorname{Cos}[c+dx]]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b \operatorname{Log}[\operatorname{Sin}[c+dx]]}{d}$$

■ **Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^2 (a+b \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 37 leaves, 7 steps):

$$\frac{b \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} - \frac{a \operatorname{Cot}[c+dx]}{d} - \frac{b \operatorname{Csc}[c+dx]}{d}$$

Result (type 3, 106 leaves):

$$-\frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{2d} - \frac{a \operatorname{Cot}[c+dx]}{d} - \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{2d}$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^4 (a+b \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 69 leaves, 8 steps):

$$\frac{b \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} - \frac{a \operatorname{Cot}[c+dx]}{d} - \frac{a \operatorname{Cot}[c+dx]^3}{3d} - \frac{b \operatorname{Csc}[c+dx]}{d} - \frac{b \operatorname{Csc}[c+dx]^3}{3d}$$

Result (type 3, 190 leaves):

$$-\frac{7b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{12d} - \frac{2a \operatorname{Cot}[c+dx]}{3d} - \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{24d} - \frac{a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{3d} - \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{d} - \frac{7b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{12d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24d}$$

■ **Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + d x]^6 (a + b \text{Sec}[c + d x]) dx$$

Optimal (type 3, 101 leaves, 8 steps):

$$\frac{b \text{ArcTanh}[\text{Sin}[c + d x]]}{d} - \frac{a \text{Cot}[c + d x]}{d} - \frac{2 a \text{Cot}[c + d x]^3}{3 d} - \frac{a \text{Cot}[c + d x]^5}{5 d} - \frac{b \text{Csc}[c + d x]}{d} - \frac{b \text{Csc}[c + d x]^3}{3 d} - \frac{b \text{Csc}[c + d x]^5}{5 d}$$

Result (type 3, 272 leaves):

$$\begin{aligned} & - \frac{149 b \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{8 a \text{Cot}[c + d x]}{15 d} - \frac{29 b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} - \frac{b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} \\ & - \frac{4 a \text{Cot}[c + d x] \text{Csc}[c + d x]^2}{15 d} - \frac{a \text{Cot}[c + d x] \text{Csc}[c + d x]^4}{5 d} - \frac{b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \\ & \frac{b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{149 b \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{29 b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d} \end{aligned}$$

■ **Problem 178: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + d x]^3 (a + b \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\begin{aligned} & - \frac{(2 a b + (a^2 + b^2) \text{Cos}[c + d x]) \text{Csc}[c + d x]^2}{2 d} + \frac{(a + b)(a + 3 b) \text{Log}[1 - \text{Cos}[c + d x]]}{4 d} \\ & - \frac{2 a b \text{Log}[\text{Cos}[c + d x]]}{d} - \frac{(a - 3 b)(a - b) \text{Log}[1 + \text{Cos}[c + d x]]}{4 d} + \frac{b^2 \text{Sec}[c + d x]}{d} \end{aligned}$$

Result (type 3, 329 leaves):

$$\begin{aligned}
& - \frac{1}{2d \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)} \\
& \operatorname{Csc}[c+dx]^4 \left(2a^2 - 2b^2 + 2(a^2 + 3b^2) \operatorname{Cos}[2(c+dx)] - a^2 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] + 4ab \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& 3b^2 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] - 4ab \operatorname{Cos}[3(c+dx)] \operatorname{Log}[\operatorname{Cos}[c+dx]] + \\
& a^2 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 4ab \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \\
& 3b^2 \operatorname{Cos}[3(c+dx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \operatorname{Cos}[c+dx] \left(8ab + (a^2 - 4ab + 3b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 4ab \operatorname{Log}[\operatorname{Cos}[c+dx]] - a^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 4ab \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 3b^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \Big)
\end{aligned}$$

■ **Problem 182: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 59 leaves, 8 steps):

$$\frac{2ab \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} - \frac{(a^2+b^2) \operatorname{Cot}[c+dx]}{d} - \frac{2ab \operatorname{Csc}[c+dx]}{d} + \frac{b^2 \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 138 leaves):

$$\begin{aligned}
& - \left(\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(4ab \operatorname{Cos}[c+dx] + (a^2 + 2b^2) \operatorname{Cos}[2(c+dx)] + \right. \right. \\
& \left. \left. a \left(a + 2b \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Sin}[2(c+dx)] \right) \right) \Big) / \left(4d \left(-1 + \right. \right. \\
& \left. \left. \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right)
\end{aligned}$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^4 (a+b \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 100 leaves, 9 steps):

$$\frac{2ab \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} - \frac{(a^2+2b^2) \operatorname{Cot}[c+dx]}{d} - \frac{(a^2+b^2) \operatorname{Cot}[c+dx]^3}{3d} - \frac{2ab \operatorname{Csc}[c+dx]}{d} - \frac{2ab \operatorname{Csc}[c+dx]^3}{3d} + \frac{b^2 \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 259 leaves):

$$\frac{1}{96 d \left(-1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)^2} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left(-3 a^2 - 14 a b \operatorname{Cos}[c+dx] - 2(a^2 + 4 b^2) \operatorname{Cos}[2(c+dx)] + 6 a b \operatorname{Cos}[3(c+dx)] + a^2 \operatorname{Cos}[4(c+dx)] + 4 b^2 \operatorname{Cos}[4(c+dx)] -\right.$$

$$6 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[2(c+dx)] + 6 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[2(c+dx)] +$$

$$\left.3 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[4(c+dx)] - 3 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[4(c+dx)]\right)$$

■ **Problem 184: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^6 (a+b \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 143 leaves, 9 steps):

$$\frac{2 a b \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} - \frac{(a^2 + 3 b^2) \operatorname{Cot}[c+dx]}{d} - \frac{(2 a^2 + 3 b^2) \operatorname{Cot}[c+dx]^3}{3 d}$$

$$\frac{(a^2 + b^2) \operatorname{Cot}[c+dx]^5}{5 d} - \frac{2 a b \operatorname{Csc}[c+dx]}{d} - \frac{2 a b \operatorname{Csc}[c+dx]^3}{3 d} - \frac{2 a b \operatorname{Csc}[c+dx]^5}{5 d} + \frac{b^2 \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 368 leaves):

$$\frac{1}{7680 d \left(-1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)^2}$$

$$\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^7 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \left(40 a^2 + 196 a b \operatorname{Cos}[c+dx] + 20(a^2 + 6 b^2) \operatorname{Cos}[2(c+dx)] - 130 a b \operatorname{Cos}[3(c+dx)] -\right.$$

$$16 a^2 \operatorname{Cos}[4(c+dx)] - 96 b^2 \operatorname{Cos}[4(c+dx)] + 30 a b \operatorname{Cos}[5(c+dx)] + 4 a^2 \operatorname{Cos}[6(c+dx)] + 24 b^2 \operatorname{Cos}[6(c+dx)] +$$

$$75 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[2(c+dx)] - 75 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[2(c+dx)] -$$

$$60 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[4(c+dx)] + 60 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[4(c+dx)] +$$

$$\left.15 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[6(c+dx)] - 15 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[6(c+dx)]\right)$$

■ **Problem 189: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^3 dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\begin{aligned}
& - \frac{a^2 \left(b \left(3 + \frac{b^2}{a^2} \right) + a \left(1 + \frac{3b^2}{a^2} \right) \cos[c + dx] \right) \operatorname{Csc}[c + dx]^2}{2d} + \frac{(a + b)^2 (a + 4b) \operatorname{Log}[1 - \cos[c + dx]]}{4d} - \\
& \frac{b (3a^2 + 2b^2) \operatorname{Log}[\cos[c + dx]]}{d} - \frac{(a - 4b) (a - b)^2 \operatorname{Log}[1 + \cos[c + dx]]}{4d} + \frac{3ab^2 \operatorname{Sec}[c + dx]}{d} + \frac{b^3 \operatorname{Sec}[c + dx]^2}{2d}
\end{aligned}$$

Result (type 3, 669 leaves):

$$\begin{aligned}
& \frac{3ab^2 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^3}{d (b + a \cos[c + dx])^3} + \frac{(-a^3 - 3a^2b - 3ab^2 - b^3) \cos[c + dx]^3 \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Sec}[c + dx])^3}{8d (b + a \cos[c + dx])^3} + \\
& \frac{(-a^3 + 6a^2b - 9ab^2 + 4b^3) \cos[c + dx]^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Sec}[c + dx])^3}{2d (b + a \cos[c + dx])^3} + \\
& \frac{(-3a^2b - 2b^3) \cos[c + dx]^3 \operatorname{Log}[\cos[c + dx]] (a + b \operatorname{Sec}[c + dx])^3}{d (b + a \cos[c + dx])^3} + \\
& \frac{(a^3 + 6a^2b + 9ab^2 + 4b^3) \cos[c + dx]^3 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Sec}[c + dx])^3}{2d (b + a \cos[c + dx])^3} + \\
& \frac{(a^3 - 3a^2b + 3ab^2 - b^3) \cos[c + dx]^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Sec}[c + dx])^3}{8d (b + a \cos[c + dx])^3} + \\
& \frac{b^3 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^3}{4d (b + a \cos[c + dx])^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{3ab^2 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^3 \sin\left[\frac{1}{2}(c + dx)\right]}{d (b + a \cos[c + dx])^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{b^3 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^3}{4d (b + a \cos[c + dx])^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{3ab^2 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^3 \sin\left[\frac{1}{2}(c + dx)\right]}{d (b + a \cos[c + dx])^3 \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 190: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx])^3 \sin[c + dx]^6 dx$$

Optimal (type 3, 299 leaves, 21 steps):

$$\begin{aligned}
& \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b \operatorname{ArcTanh}[\sin[c + dx]]}{d} - \frac{5b^3 \operatorname{ArcTanh}[\sin[c + dx]]}{2d} - \\
& \frac{3a^2b \sin[c + dx]}{d} + \frac{5b^3 \sin[c + dx]}{2d} - \frac{5a^3 \cos[c + dx] \sin[c + dx]}{16d} - \frac{a^2b \sin[c + dx]^3}{d} + \frac{5b^3 \sin[c + dx]^3}{6d} - \\
& \frac{5a^3 \cos[c + dx] \sin[c + dx]^3}{24d} - \frac{3a^2b \sin[c + dx]^5}{5d} - \frac{a^3 \cos[c + dx] \sin[c + dx]^5}{6d} + \frac{45a^2b^2 \tan[c + dx]}{8d} - \\
& \frac{15a^2b^2 \sin[c + dx]^2 \tan[c + dx]}{8d} - \frac{3a^2b^2 \sin[c + dx]^4 \tan[c + dx]}{4d} + \frac{b^3 \sin[c + dx]^3 \tan[c + dx]^2}{2d}
\end{aligned}$$

Result (type 3, 818 leaves) :

$$\begin{aligned}
 & \frac{5 a (a^2 - 18 b^2) (c + d x) \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{16 d (b + a \cos [c + d x])^3} + \\
 & \frac{(-6 a^2 b + 5 b^3) \cos [c + d x]^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3}{2 d (b + a \cos [c + d x])^3} + \\
 & \frac{(6 a^2 b - 5 b^3) \cos [c + d x]^3 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3}{2 d (b + a \cos [c + d x])^3} + \\
 & \frac{b^3 \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{4 d (b + a \cos [c + d x])^3 \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{3 a b^2 \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin\left[\frac{1}{2}(c + d x)\right]}{d (b + a \cos [c + d x])^3 \left(\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right)} - \\
 & \frac{b^3 \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{4 d (b + a \cos [c + d x])^3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{3 a b^2 \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin\left[\frac{1}{2}(c + d x)\right]}{d (b + a \cos [c + d x])^3 \left(\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right)} + \\
 & \frac{3 b (-11 a^2 + 6 b^2) \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [c + d x]}{8 d (b + a \cos [c + d x])^3} - \frac{3 a (5 a^2 - 32 b^2) \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [2 (c + d x)]}{64 d (b + a \cos [c + d x])^3} - \\
 & \frac{b (-21 a^2 + 4 b^2) \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [3 (c + d x)]}{48 d (b + a \cos [c + d x])^3} + \frac{3 a (a^2 - 2 b^2) \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [4 (c + d x)]}{64 d (b + a \cos [c + d x])^3} - \\
 & \frac{3 a^2 b \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [5 (c + d x)]}{80 d (b + a \cos [c + d x])^3} - \frac{a^3 \cos [c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \sin [6 (c + d x)]}{192 d (b + a \cos [c + d x])^3}
 \end{aligned}$$

■ **Problem 191: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 \sin [c + d x]^4 dx$$

Optimal (type 3, 236 leaves, 8 steps) :

$$\begin{aligned}
 & \frac{3}{8} a (a^2 - 12 b^2) x + \frac{3 b (2 a^2 - b^2) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} - \frac{b (17 a^2 - b^2) \sin [c + d x]}{2 d} - \\
 & \frac{a (21 a^2 - 2 b^2) \cos [c + d x] \sin [c + d x]}{8 d} - \frac{(6 a^2 - b^2) (b + a \cos [c + d x])^2 \sin [c + d x]}{4 b d} - \\
 & \frac{(4 a^2 - b^2) (b + a \cos [c + d x])^3 \sin [c + d x]}{4 b^2 d} + \frac{a (b + a \cos [c + d x])^4 \tan [c + d x]}{b^2 d} + \frac{(b + a \cos [c + d x])^4 \sec [c + d x] \tan [c + d x]}{2 b d}
 \end{aligned}$$

Result (type 3, 696 leaves) :

$$\begin{aligned}
& \frac{3 a (a^2 - 12 b^2) (c + d x) \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{8 d (b + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{3 (-2 a^2 b + b^3) \operatorname{Cos}[c + d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3}{2 d (b + a \operatorname{Cos}[c + d x])^3} - \\
& \frac{3 (-2 a^2 b + b^3) \operatorname{Cos}[c + d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3}{2 d (b + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{b^3 \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{4 d (b + a \operatorname{Cos}[c + d x])^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{3 a b^2 \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{d (b + a \operatorname{Cos}[c + d x])^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} - \\
& \frac{b^3 \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3}{4 d (b + a \operatorname{Cos}[c + d x])^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{3 a b^2 \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{d (b + a \operatorname{Cos}[c + d x])^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} + \\
& \frac{b (-15 a^2 + 4 b^2) \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{4 d (b + a \operatorname{Cos}[c + d x])^3} - \frac{a (a^2 - 3 b^2) \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[2(c + d x)]}{4 d (b + a \operatorname{Cos}[c + d x])^3} + \\
& \frac{a^2 b \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[3(c + d x)]}{4 d (b + a \operatorname{Cos}[c + d x])^3} + \frac{a^3 \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[4(c + d x)]}{32 d (b + a \operatorname{Cos}[c + d x])^3}
\end{aligned}$$

■ **Problem 192: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]^2 dx$$

Optimal (type 3, 138 leaves, 8 steps):

$$\begin{aligned}
& \frac{1}{2} a (a^2 - 6 b^2) x + \frac{b (6 a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} - \frac{15 a^2 b \operatorname{Sin}[c + d x]}{2 d} - \\
& \frac{5 a^3 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 d} + \frac{3 a (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Tan}[c + d x]}{2 d} + \frac{(b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}
\end{aligned}$$

Result (type 3, 327 leaves):

$$\begin{aligned}
& \frac{1}{4 d} \operatorname{Sec}[c + d x]^2 \left(a^3 c - 6 a b^2 c + a^3 d x - 6 a b^2 d x - 6 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 6 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Cos}[2(c + d x)] \\
& \left(a (a^2 - 6 b^2) (c + d x) + (-6 a^2 b + b^3) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - b (-6 a^2 + b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& \left(-3 a^2 b + 2 b^3 \right) \operatorname{Sin}[c + d x] - \frac{1}{2} a^3 \operatorname{Sin}[2(c + d x)] + 6 a b^2 \operatorname{Sin}[2(c + d x)] - 3 a^2 b \operatorname{Sin}[3(c + d x)] - \frac{1}{4} a^3 \operatorname{Sin}[4(c + d x)] \Big)
\end{aligned}$$

■ **Problem 193: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + d x]^2 (a + b \text{Sec}[c + d x])^3 dx$$

Optimal (type 3, 133 leaves, 15 steps):

$$\frac{3 a^2 b \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{3 b^3 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} - \frac{a^3 \text{Cot}[c + d x]}{d} - \frac{3 a b^2 \text{Cot}[c + d x]}{d} - \frac{3 a^2 b \text{Csc}[c + d x]}{d} - \frac{3 b^3 \text{Csc}[c + d x]}{2 d} + \frac{b^3 \text{Csc}[c + d x] \text{Sec}[c + d x]^2}{2 d} + \frac{3 a b^2 \text{Tan}[c + d x]}{d}$$

Result (type 3, 406 leaves):

$$-\frac{1}{16 d \left(-1 + \text{Cot}\left[\frac{1}{2}(c + d x)\right]\right)^2} \text{Csc}\left[\frac{1}{2}(c + d x)\right]^5 \text{Sec}\left[\frac{1}{2}(c + d x)\right] \\ \left(12 a^2 b + 2 b^3 + 6 a (a^2 + 2 b^2) \text{Cos}[c + d x] + 6 (2 a^2 b + b^3) \text{Cos}[2(c + d x)] + 2 a^3 \text{Cos}[3(c + d x)] + 12 a b^2 \text{Cos}[3(c + d x)] + \right. \\ \left. 6 a^2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] + 3 b^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] - \right. \\ \left. 6 a^2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] - 3 b^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[c + d x] + \right. \\ \left. 6 a^2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[3(c + d x)] + 3 b^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[3(c + d x)] - \right. \\ \left. 6 a^2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[3(c + d x)] - 3 b^3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \text{Sin}[3(c + d x)]\right)$$

■ **Problem 194: Result more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c + d x]^4 (a + b \text{Sec}[c + d x])^3 dx$$

Optimal (type 3, 205 leaves, 17 steps):

$$\frac{3 a^2 b \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{5 b^3 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} - \frac{a^3 \text{Cot}[c + d x]}{d} - \frac{6 a b^2 \text{Cot}[c + d x]}{d} - \frac{a^3 \text{Cot}[c + d x]^3}{3 d} - \frac{a b^2 \text{Cot}[c + d x]^3}{d} - \frac{3 a^2 b \text{Csc}[c + d x]}{d} - \frac{5 b^3 \text{Csc}[c + d x]}{2 d} - \frac{a^2 b \text{Csc}[c + d x]^3}{d} - \frac{5 b^3 \text{Csc}[c + d x]^3}{6 d} + \frac{b^3 \text{Csc}[c + d x]^3 \text{Sec}[c + d x]^2}{2 d} + \frac{3 a b^2 \text{Tan}[c + d x]}{d}$$

Result (type 3, 610 leaves):

$$\begin{aligned}
& - \frac{1}{768 d \left(-1 + \operatorname{Cot} \left[\frac{1}{2} (c + d x) \right] \right)^2} \\
& \operatorname{Csc} \left[\frac{1}{2} (c + d x) \right]^7 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^3 \left(84 a^2 b + 22 b^3 + 32 a (a^2 + 3 b^2) \operatorname{Cos}[c + d x] + 8 (6 a^2 b + 5 b^3) \operatorname{Cos}[2 (c + d x)] + 4 a^3 \operatorname{Cos}[3 (c + d x)] + \right. \\
& 48 a b^2 \operatorname{Cos}[3 (c + d x)] - 36 a^2 b \operatorname{Cos}[4 (c + d x)] - 30 b^3 \operatorname{Cos}[4 (c + d x)] - 4 a^3 \operatorname{Cos}[5 (c + d x)] - 48 a b^2 \operatorname{Cos}[5 (c + d x)] + \\
& 36 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[c + d x] + 30 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[c + d x] - \\
& 36 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[c + d x] - 30 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[c + d x] + \\
& 18 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[3 (c + d x)] + 15 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[3 (c + d x)] - \\
& 18 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[3 (c + d x)] - 15 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[3 (c + d x)] - \\
& 18 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[5 (c + d x)] - 15 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[5 (c + d x)] + \\
& \left. 18 a^2 b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[5 (c + d x)] + 15 b^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \operatorname{Sin}[5 (c + d x)] \right)
\end{aligned}$$

■ **Problem 195: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c + d x]^6 (a + b \operatorname{Sec}[c + d x])^3 dx$$

Optimal (type 3, 279 leaves, 17 steps):

$$\begin{aligned}
& \frac{3 a^2 b \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{7 b^3 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} - \frac{a^3 \operatorname{Cot}[c + d x]}{d} - \frac{9 a b^2 \operatorname{Cot}[c + d x]}{d} - \frac{2 a^3 \operatorname{Cot}[c + d x]^3}{3 d} - \\
& \frac{3 a b^2 \operatorname{Cot}[c + d x]^3}{d} - \frac{a^3 \operatorname{Cot}[c + d x]^5}{5 d} - \frac{3 a b^2 \operatorname{Cot}[c + d x]^5}{5 d} - \frac{3 a^2 b \operatorname{Csc}[c + d x]}{d} - \frac{7 b^3 \operatorname{Csc}[c + d x]}{2 d} - \frac{a^2 b \operatorname{Csc}[c + d x]^3}{d} - \\
& \frac{7 b^3 \operatorname{Csc}[c + d x]^3}{6 d} - \frac{3 a^2 b \operatorname{Csc}[c + d x]^5}{5 d} - \frac{7 b^3 \operatorname{Csc}[c + d x]^5}{10 d} + \frac{b^3 \operatorname{Csc}[c + d x]^5 \operatorname{Sec}[c + d x]^2}{2 d} + \frac{3 a b^2 \operatorname{Tan}[c + d x]}{d}
\end{aligned}$$

Result (type 3, 812 leaves):

$$\begin{aligned}
& - \frac{1}{61440 d \left(-1 + \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]\right)^2} \\
& \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^9 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \left(1176 a^2 b + 412 b^3 + 80 a (5 a^2 + 18 b^2) \operatorname{Cos}[c+dx] + 66 (6 a^2 b + 7 b^3) \operatorname{Cos}[2(c+dx)] + \right. \\
& 16 a^3 \operatorname{Cos}[3(c+dx)] + 288 a b^2 \operatorname{Cos}[3(c+dx)] - 600 a^2 b \operatorname{Cos}[4(c+dx)] - 700 b^3 \operatorname{Cos}[4(c+dx)] - 48 a^3 \operatorname{Cos}[5(c+dx)] - \\
& 864 a b^2 \operatorname{Cos}[5(c+dx)] + 180 a^2 b \operatorname{Cos}[6(c+dx)] + 210 b^3 \operatorname{Cos}[6(c+dx)] + 16 a^3 \operatorname{Cos}[7(c+dx)] + 288 a b^2 \operatorname{Cos}[7(c+dx)] + \\
& 450 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] + 525 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - \\
& 450 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] - 525 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[c+dx] + \\
& 90 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] + 105 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] - \\
& 90 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] - 105 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[3(c+dx)] - \\
& 270 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] - 315 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] + \\
& 270 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] + 315 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[5(c+dx)] + \\
& 90 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] + 105 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] - \\
& \left. 90 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] - 105 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sin}[7(c+dx)] \right)
\end{aligned}$$

■ **Problem 202: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c+dx]^5}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\begin{aligned}
& \frac{(4 a^2 b - a (3 a^2 + b^2) \operatorname{Cos}[c+dx]) \operatorname{Csc}[c+dx]^2}{8 (a^2 - b^2)^2 d} + \frac{(b - a \operatorname{Cos}[c+dx]) \operatorname{Csc}[c+dx]^4}{4 (a^2 - b^2) d} + \\
& \frac{a (3 a + b) \operatorname{Log}[1 - \operatorname{Cos}[c+dx]]}{16 (a+b)^3 d} - \frac{a (3 a - b) \operatorname{Log}[1 + \operatorname{Cos}[c+dx]]}{16 (a-b)^3 d} + \frac{a^4 b \operatorname{Log}[b + a \operatorname{Cos}[c+dx]]}{(a^2 - b^2)^3 d}
\end{aligned}$$

Result (type 3, 409 leaves):

$$\frac{(-3a - b)(b + a \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx]}{32(a + b)^2 d (a + b \operatorname{Sec}[c + dx])} -$$

$$\frac{(b + a \cos[c + dx]) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sec}[c + dx]}{64(a + b)d(a + b \operatorname{Sec}[c + dx])} + \frac{(3a^2 - ab)(b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]}{8(-a + b)^3 d (a + b \operatorname{Sec}[c + dx])} -$$

$$\frac{a^4 b (b + a \cos[c + dx]) \operatorname{Log}[b + a \cos[c + dx]] \operatorname{Sec}[c + dx]}{(-a^2 + b^2)^3 d (a + b \operatorname{Sec}[c + dx])} + \frac{(3a^2 + ab)(b + a \cos[c + dx]) \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]}{8(a + b)^3 d (a + b \operatorname{Sec}[c + dx])} +$$

$$\frac{(3a - b)(b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx]}{32(-a + b)^2 d (a + b \operatorname{Sec}[c + dx])} - \frac{(b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sec}[c + dx]}{64(-a + b)d(a + b \operatorname{Sec}[c + dx])}$$

■ **Problem 226: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Csc}[c + dx]^3}{(a + b \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 3, 229 leaves, 5 steps):

$$-\frac{b^3}{2(a^2 - b^2)^2 d (b + a \cos[c + dx])^2} + \frac{b^2(3a^2 + b^2)}{(a^2 - b^2)^3 d (b + a \cos[c + dx])} + \frac{(b(3a^2 + b^2) - a(a^2 + 3b^2) \cos[c + dx]) \operatorname{Csc}[c + dx]^2}{2(a^2 - b^2)^3 d} +$$

$$\frac{(a - 2b) \operatorname{Log}[1 - \cos[c + dx]]}{4(a + b)^4 d} - \frac{(a + 2b) \operatorname{Log}[1 + \cos[c + dx]]}{4(a - b)^4 d} + \frac{b(3a^4 + 8a^2 b^2 + b^4) \operatorname{Log}[b + a \cos[c + dx]]}{(a^2 - b^2)^4 d}$$

Result (type 3, 332 leaves):

$$-\frac{2i(3a^4 b + 8a^2 b^3 + b^5)(c + dx)}{(a - b)^4 (a + b)^4 d} - \frac{i(-a - 2b) \operatorname{ArcTan}[\operatorname{Tan}[c + dx]]}{2(-a + b)^4 d} - \frac{i(a - 2b) \operatorname{ArcTan}[\operatorname{Tan}[c + dx]]}{2(a + b)^4 d} -$$

$$\frac{b^3}{2(-a + b)^2 (a + b)^2 d (b + a \cos[c + dx])^2} - \frac{b^2(3a^2 + b^2)}{(-a + b)^3 (a + b)^3 d (b + a \cos[c + dx])} - \frac{\operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2}{8(a + b)^3 d} +$$

$$\frac{(-a - 2b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right]^2}{4(-a + b)^4 d} + \frac{(3a^4 b + 8a^2 b^3 + b^5) \operatorname{Log}[b + a \cos[c + dx]]}{(-a^2 + b^2)^4 d} + \frac{(a - 2b) \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right]^2}{4(a + b)^4 d} - \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2}{8(-a + b)^3 d}$$

■ **Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[c + dx]^5}{(a + b \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 3, 313 leaves, 7 steps):

$$\begin{aligned}
& - \frac{a^2 b^3}{2 (a^2 - b^2)^3 d (b + a \cos [c + d x])^2} + \frac{3 a^2 b^2 (a^2 + b^2)}{(a^2 - b^2)^4 d (b + a \cos [c + d x])} + \\
& \frac{(4 b (3 a^4 + 8 a^2 b^2 + b^4) - 3 a (a^4 + 10 a^2 b^2 + 5 b^4) \cos [c + d x]) \operatorname{Csc}[c + d x]^2}{8 (a^2 - b^2)^4 d} + \frac{(b (3 a^2 + b^2) - a (a^2 + 3 b^2) \cos [c + d x]) \operatorname{Csc}[c + d x]^4}{4 (a^2 - b^2)^3 d} + \\
& \frac{3 a (a - 3 b) \operatorname{Log}[1 - \cos [c + d x]]}{16 (a + b)^5 d} - \frac{3 a (a + 3 b) \operatorname{Log}[1 + \cos [c + d x]]}{16 (a - b)^5 d} + \frac{3 a^2 b (a^4 + 5 a^2 b^2 + 2 b^4) \operatorname{Log}[b + a \cos [c + d x]]}{(a^2 - b^2)^5 d}
\end{aligned}$$

Result (type 3, 780 leaves):

$$\begin{aligned}
& \frac{a^2 b^3 (b + a \cos [c + d x]) \operatorname{Sec}[c + d x]^3}{2 (-a + b)^3 (a + b)^3 d (a + b \operatorname{Sec}[c + d x])^3} + \frac{3 a^2 b^2 (-i a + b) (i a + b) (b + a \cos [c + d x])^2 \operatorname{Sec}[c + d x]^3}{(-a + b)^4 (a + b)^4 d (a + b \operatorname{Sec}[c + d x])^3} - \\
& \frac{6 i (a^6 b + 5 a^4 b^3 + 2 a^2 b^5) (c + d x) (b + a \cos [c + d x])^3 \operatorname{Sec}[c + d x]^3}{(a - b)^5 (a + b)^5 d (a + b \operatorname{Sec}[c + d x])^3} + \frac{3 i (-a^2 + 3 a b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Sec}[c + d x]^3}{8 (a + b)^5 d (a + b \operatorname{Sec}[c + d x])^3} - \\
& \frac{3 i (a^2 + 3 a b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Sec}[c + d x]^3}{8 (-a + b)^5 d (a + b \operatorname{Sec}[c + d x])^3} + \frac{3 (-a + b) (b + a \cos [c + d x])^3 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^3}{32 (a + b)^4 d (a + b \operatorname{Sec}[c + d x])^3} - \\
& \frac{(b + a \cos [c + d x])^3 \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^3}{64 (a + b)^3 d (a + b \operatorname{Sec}[c + d x])^3} + \frac{3 (a^2 + 3 a b) (b + a \cos [c + d x])^3 \operatorname{Log}\left[\cos\left[\frac{1}{2} (c + d x)\right]^2\right] \operatorname{Sec}[c + d x]^3}{16 (-a + b)^5 d (a + b \operatorname{Sec}[c + d x])^3} - \\
& \frac{3 (a^6 b + 5 a^4 b^3 + 2 a^2 b^5) (b + a \cos [c + d x])^3 \operatorname{Log}[b + a \cos [c + d x]] \operatorname{Sec}[c + d x]^3}{(-a^2 + b^2)^5 d (a + b \operatorname{Sec}[c + d x])^3} - \\
& \frac{3 (-a^2 + 3 a b) (b + a \cos [c + d x])^3 \operatorname{Log}\left[\sin\left[\frac{1}{2} (c + d x)\right]^2\right] \operatorname{Sec}[c + d x]^3}{16 (a + b)^5 d (a + b \operatorname{Sec}[c + d x])^3} + \\
& \frac{3 (a + b) (b + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^3}{32 (-a + b)^4 d (a + b \operatorname{Sec}[c + d x])^3} - \frac{(b + a \cos [c + d x])^3 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^3}{64 (-a + b)^3 d (a + b \operatorname{Sec}[c + d x])^3}
\end{aligned}$$

■ **Problem 228: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin [c + d x]^6}{(a + b \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 3, 539 leaves, 11 steps):

$$\begin{aligned}
& \frac{(5a^6 - 180a^4b^2 + 600a^2b^4 - 448b^6)x}{16a^9} - \frac{\sqrt{a-b}b\sqrt{a+b}(6a^4 - 47a^2b^2 + 56b^4)\text{ArcTanh}\left[\frac{\sqrt{a-b}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^9d} + \\
& \frac{b(213a^4 - 985a^2b^2 + 840b^4)\sin[c+dx]}{30a^8d} - \frac{(43a^4 - 244a^2b^2 + 224b^4)\cos[c+dx]\sin[c+dx]}{16a^7d} + \\
& \frac{(45a^4 - 291a^2b^2 + 280b^4)\cos[c+dx]^2\sin[c+dx]}{30a^6bd} - \frac{(24a^4 - 169a^2b^2 + 168b^4)\cos[c+dx]^3\sin[c+dx]}{24a^5b^2d} - \\
& \frac{\cos[c+dx]^4\sin[c+dx]}{4bd(b+a\cos[c+dx])^2} + \frac{a\cos[c+dx]^5\sin[c+dx]}{10b^2d(b+a\cos[c+dx])^2} + \frac{(9a^4 - 60a^2b^2 + 56b^4)\cos[c+dx]^5\sin[c+dx]}{60a^3b^2d(b+a\cos[c+dx])^2} + \\
& \frac{4b\cos[c+dx]^6\sin[c+dx]}{15a^2d(b+a\cos[c+dx])^2} - \frac{\cos[c+dx]^7\sin[c+dx]}{6ad(b+a\cos[c+dx])^2} + \frac{(15a^4 - 110a^2b^2 + 112b^4)\cos[c+dx]^4\sin[c+dx]}{20a^4b^2d(b+a\cos[c+dx])}
\end{aligned}$$

Result (type 3, 2091 leaves):

$$\begin{aligned}
& - \left((b+a\cos[c+dx])^3 \sec[c+dx]^3 \left(8(c+dx) + \frac{2b(15a^4 - 20a^2b^2 + 8b^4)\text{ArcTanh}\left[\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right. \right. \\
& \left. \left. \frac{ab(3a^2 - 4b^2)\sin[c+dx]}{(a-b)(a+b)(b+a\cos[c+dx])^2} - \frac{3a(2a^4 - 7a^2b^2 + 4b^4)\sin[c+dx]}{(a-b)^2(a+b)^2(b+a\cos[c+dx])} \right) \right) / (64a^3d(a+b\sec[c+dx])^3) + \\
& \frac{3(b+a\cos[c+dx])^3 \sec[c+dx]^3 \left(\frac{6ab\text{ArcTanh}\left[\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{(b(a^2+2b^2)+a(2a^2+b^2)\cos[c+dx])\sin[c+dx]}{(b+a\cos[c+dx])^2} \right)}{256(a-b)^2(a+b)^2d(a+b\sec[c+dx])^3} + \\
& \frac{1}{1024a^7d(a+b\sec[c+dx])^3} \\
& 3(b+a\cos[c+dx])^3 \sec[c+dx]^3 \left(\frac{12b(105a^8 - 840a^6b^2 + 2016a^4b^4 - 1920a^2b^6 + 640b^8)\text{ArcTanh}\left[\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right. \\
& \left. \frac{1}{(a^2-b^2)^2(b+a\cos[c+dx])^2} (48a^{10}c - 960a^8b^2c + 1776a^6b^4c + 2976a^4b^6c - 7680a^2b^8c + 3840b^{10}c + 48a^{10}dx - 960a^8b^2dx + \right.
\end{aligned}$$

$$\begin{aligned}
& 1776 a^6 b^4 d x + 2976 a^4 b^6 d x - 7680 a^2 b^8 d x + 3840 b^{10} d x + 192 a b (a^2 - b^2)^2 (a^4 - 20 a^2 b^2 + 40 b^4) (c + d x) \operatorname{Cos}[c + d x] + \\
& 48 (a^3 - a b^2)^2 (a^4 - 20 a^2 b^2 + 40 b^4) (c + d x) \operatorname{Cos}[2(c + d x)] + 114 a^9 b \operatorname{Sin}[c + d x] + 788 a^7 b^3 \operatorname{Sin}[c + d x] - 5696 a^5 b^5 \operatorname{Sin}[c + d x] + \\
& 8640 a^3 b^7 \operatorname{Sin}[c + d x] - 3840 a b^9 \operatorname{Sin}[c + d x] - 36 a^{10} \operatorname{Sin}[2(c + d x)] + 1221 a^8 b^2 \operatorname{Sin}[2(c + d x)] - 5182 a^6 b^4 \operatorname{Sin}[2(c + d x)] + \\
& 6880 a^4 b^6 \operatorname{Sin}[2(c + d x)] - 2880 a^2 b^8 \operatorname{Sin}[2(c + d x)] + 120 a^9 b \operatorname{Sin}[3(c + d x)] - 560 a^7 b^3 \operatorname{Sin}[3(c + d x)] + \\
& 760 a^5 b^5 \operatorname{Sin}[3(c + d x)] - 320 a^3 b^7 \operatorname{Sin}[3(c + d x)] - 8 a^{10} \operatorname{Sin}[4(c + d x)] + 56 a^8 b^2 \operatorname{Sin}[4(c + d x)] - 88 a^6 b^4 \operatorname{Sin}[4(c + d x)] + \\
& 40 a^4 b^6 \operatorname{Sin}[4(c + d x)] - 8 a^9 b \operatorname{Sin}[5(c + d x)] + 16 a^7 b^3 \operatorname{Sin}[5(c + d x)] - 8 a^5 b^5 \operatorname{Sin}[5(c + d x)] + 2 a^{10} \operatorname{Sin}[6(c + d x)] - \\
& 4 a^8 b^2 \operatorname{Sin}[6(c + d x)] + 2 a^6 b^4 \operatorname{Sin}[6(c + d x)] \Bigg) + \frac{1}{256 (a + b \operatorname{Sec}[c + d x])^3} (b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^3 \\
& \left(- \left(b (-693 a^{10} + 9240 a^8 b^2 - 36960 a^6 b^4 + 63360 a^4 b^6 - 49280 a^2 b^8 + 14336 b^{10}) \operatorname{ArcTanH} \left[\frac{(-a + b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) / \\
& \left(a^9 \sqrt{a^2 - b^2} (-a^2 + b^2)^2 d \right) - \frac{1}{60 a^9 (a^2 - b^2)^2 d (b + a \operatorname{Cos}[c + d x])^2} \\
& (-1200 a^{12} (c + d x) + 43200 a^{10} b^2 (c + d x) - 198000 a^8 b^4 (c + d x) + 83040 a^6 b^6 (c + d x) + 691200 a^4 b^8 (c + d x) - 1048320 a^2 b^{10} (c + d x) + \\
& 430080 b^{12} (c + d x) - 4800 a^{11} b (c + d x) \operatorname{Cos}[c + d x] + 182400 a^9 b^3 (c + d x) \operatorname{Cos}[c + d x] - 1156800 a^7 b^5 (c + d x) \operatorname{Cos}[c + d x] + \\
& 2645760 a^5 b^7 (c + d x) \operatorname{Cos}[c + d x] - 2526720 a^3 b^9 (c + d x) \operatorname{Cos}[c + d x] + 860160 a b^{11} (c + d x) \operatorname{Cos}[c + d x] - \\
& 1200 a^{12} (c + d x) \operatorname{Cos}[2(c + d x)] + 45600 a^{10} b^2 (c + d x) \operatorname{Cos}[2(c + d x)] - 289200 a^8 b^4 (c + d x) \operatorname{Cos}[2(c + d x)] + \\
& 661440 a^6 b^6 (c + d x) \operatorname{Cos}[2(c + d x)] - 631680 a^4 b^8 (c + d x) \operatorname{Cos}[2(c + d x)] + 215040 a^2 b^{10} (c + d x) \operatorname{Cos}[2(c + d x)] - \\
& 4530 a^{11} b \operatorname{Sin}[c + d x] - 11060 a^9 b^3 \operatorname{Sin}[c + d x] + 332800 a^7 b^5 \operatorname{Sin}[c + d x] - 1042880 a^5 b^7 \operatorname{Sin}[c + d x] + 1155840 a^3 b^9 \operatorname{Sin}[c + d x] - \\
& 430080 a b^{11} \operatorname{Sin}[c + d x] + 900 a^{12} \operatorname{Sin}[2(c + d x)] - 49125 a^{10} b^2 \operatorname{Sin}[2(c + d x)] + 362830 a^8 b^4 \operatorname{Sin}[2(c + d x)] - \\
& 903680 a^6 b^6 \operatorname{Sin}[2(c + d x)] + 911680 a^4 b^8 \operatorname{Sin}[2(c + d x)] - 322560 a^2 b^{10} \operatorname{Sin}[2(c + d x)] - 4344 a^{11} b \operatorname{Sin}[3(c + d x)] + \\
& 37808 a^9 b^3 \operatorname{Sin}[3(c + d x)] - 98424 a^7 b^5 \operatorname{Sin}[3(c + d x)] + 100800 a^5 b^7 \operatorname{Sin}[3(c + d x)] - 35840 a^3 b^9 \operatorname{Sin}[3(c + d x)] + \\
& 200 a^{12} \operatorname{Sin}[4(c + d x)] - 3256 a^{10} b^2 \operatorname{Sin}[4(c + d x)] + 10392 a^8 b^4 \operatorname{Sin}[4(c + d x)] - 11816 a^6 b^6 \operatorname{Sin}[4(c + d x)] + \\
& 4480 a^4 b^8 \operatorname{Sin}[4(c + d x)] + 392 a^{11} b \operatorname{Sin}[5(c + d x)] - 1680 a^9 b^3 \operatorname{Sin}[5(c + d x)] + 2184 a^7 b^5 \operatorname{Sin}[5(c + d x)] - 896 a^5 b^7 \operatorname{Sin}[5(c + d x)] - \\
& 50 a^{12} \operatorname{Sin}[6(c + d x)] + 324 a^{10} b^2 \operatorname{Sin}[6(c + d x)] - 498 a^8 b^4 \operatorname{Sin}[6(c + d x)] + 224 a^6 b^6 \operatorname{Sin}[6(c + d x)] - 64 a^{11} b \operatorname{Sin}[7(c + d x)] + \\
& 128 a^9 b^3 \operatorname{Sin}[7(c + d x)] - 64 a^7 b^5 \operatorname{Sin}[7(c + d x)] + 20 a^{12} \operatorname{Sin}[8(c + d x)] - 40 a^{10} b^2 \operatorname{Sin}[8(c + d x)] + 20 a^8 b^4 \operatorname{Sin}[8(c + d x)] \Bigg)
\end{aligned}$$

■ **Problem 229: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[c + d x]^4}{(a + b \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 3, 333 leaves, 9 steps):

$$\frac{3(a^4 - 24a^2b^2 + 40b^4)x}{8a^7} - \frac{3b(2a^4 - 11a^2b^2 + 10b^4) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^7 \sqrt{a-b} \sqrt{a+b} d} +$$

$$\frac{b(13a^2 - 30b^2) \operatorname{Sin}[c+dx]}{2a^6 d} - \frac{3(7a^2 - 20b^2) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{8a^5 d} + \frac{(3a^2 - 10b^2) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{2a^4 b d} -$$

$$\frac{(4a^2 - 15b^2) \operatorname{Cos}[c+dx]^3 \operatorname{Sin}[c+dx]}{4a^3 b^2 d} - \frac{(a^2 - b^2) \operatorname{Cos}[c+dx]^4 \operatorname{Sin}[c+dx]}{2a^2 b d (b+a \operatorname{Cos}[c+dx])^2} + \frac{(2a^2 - 7b^2) \operatorname{Cos}[c+dx]^4 \operatorname{Sin}[c+dx]}{2a^2 b^2 d (b+a \operatorname{Cos}[c+dx])}$$

Result (type 3, 1320 leaves):

$$- \left(3(b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \left(8(c+dx) + \frac{2b(15a^4 - 20a^2b^2 + 8b^4) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \right. \right.$$

$$\left. \left. \frac{ab(3a^2 - 4b^2) \operatorname{Sin}[c+dx]}{(a-b)(a+b)(b+a \operatorname{Cos}[c+dx])^2} - \frac{3a(2a^4 - 7a^2b^2 + 4b^4) \operatorname{Sin}[c+dx]}{(a-b)^2(a+b)^2(b+a \operatorname{Cos}[c+dx])} \right) \right) / (128a^3 d (a+b \operatorname{Sec}[c+dx])^3) +$$

$$\frac{3(b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \left(\frac{6ab \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + \frac{(b(a^2+2b^2)+a(2a^2+b^2) \operatorname{Cos}[c+dx]) \operatorname{Sin}[c+dx]}{(b+a \operatorname{Cos}[c+dx])^2} \right)}{128(a-b)^2(a+b)^2 d (a+b \operatorname{Sec}[c+dx])^3} -$$

$$\left((b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3 \left(-24(a^2 - 8b^2)(c+dx) + \frac{6b(-35a^6 + 140a^4b^2 - 168a^2b^4 + 64b^6) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} - \right. \right.$$

$$\left. \left. 96ab \operatorname{Sin}[c+dx] + \frac{ab(-5a^4 + 20a^2b^2 - 16b^4) \operatorname{Sin}[c+dx]}{(a-b)(a+b)(b+a \operatorname{Cos}[c+dx])^2} + \frac{a(10a^6 - 115a^4b^2 + 220a^2b^4 - 112b^6) \operatorname{Sin}[c+dx]}{(a-b)^2(a+b)^2(b+a \operatorname{Cos}[c+dx])} + 8a^2 \operatorname{Sin}[2(c+dx)] \right) \right) /$$

$$(128a^5 d (a+b \operatorname{Sec}[c+dx])^3) + \frac{1}{256a^7 d (a+b \operatorname{Sec}[c+dx])^3} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^3$$

$$\left(\frac{12 b (105 a^8 - 840 a^6 b^2 + 2016 a^4 b^4 - 1920 a^2 b^6 + 640 b^8) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2-b^2)^{5/2}} + \frac{1}{(a^2-b^2)^2 (b+a \operatorname{Cos}[c+dx])^2} \right.$$

$$\left. \begin{aligned} & (48 a^{10} c - 960 a^8 b^2 c + 1776 a^6 b^4 c + 2976 a^4 b^6 c - 7680 a^2 b^8 c + 3840 b^{10} c + 48 a^{10} dx - 960 a^8 b^2 dx + 1776 a^6 b^4 dx + 2976 a^4 b^6 dx - \\ & 7680 a^2 b^8 dx + 3840 b^{10} dx + 192 a b (a^2 - b^2)^2 (a^4 - 20 a^2 b^2 + 40 b^4) (c + dx) \operatorname{Cos}[c + dx] + 48 (a^3 - a b^2)^2 (a^4 - 20 a^2 b^2 + 40 b^4) \\ & (c + dx) \operatorname{Cos}[2(c + dx)] + 114 a^9 b \operatorname{Sin}[c + dx] + 788 a^7 b^3 \operatorname{Sin}[c + dx] - 5696 a^5 b^5 \operatorname{Sin}[c + dx] + 8640 a^3 b^7 \operatorname{Sin}[c + dx] - \\ & 3840 a b^9 \operatorname{Sin}[c + dx] - 36 a^{10} \operatorname{Sin}[2(c + dx)] + 1221 a^8 b^2 \operatorname{Sin}[2(c + dx)] - 5182 a^6 b^4 \operatorname{Sin}[2(c + dx)] + 6880 a^4 b^6 \operatorname{Sin}[2(c + dx)] - \\ & 2880 a^2 b^8 \operatorname{Sin}[2(c + dx)] + 120 a^9 b \operatorname{Sin}[3(c + dx)] - 560 a^7 b^3 \operatorname{Sin}[3(c + dx)] + 760 a^5 b^5 \operatorname{Sin}[3(c + dx)] - 320 a^3 b^7 \operatorname{Sin}[3(c + dx)] - \\ & 8 a^{10} \operatorname{Sin}[4(c + dx)] + 56 a^8 b^2 \operatorname{Sin}[4(c + dx)] - 88 a^6 b^4 \operatorname{Sin}[4(c + dx)] + 40 a^4 b^6 \operatorname{Sin}[4(c + dx)] - 8 a^9 b \operatorname{Sin}[5(c + dx)] + \\ & 16 a^7 b^3 \operatorname{Sin}[5(c + dx)] - 8 a^5 b^5 \operatorname{Sin}[5(c + dx)] + 2 a^{10} \operatorname{Sin}[6(c + dx)] - 4 a^8 b^2 \operatorname{Sin}[6(c + dx)] + 2 a^6 b^4 \operatorname{Sin}[6(c + dx)] \end{aligned} \right)$$

- **Problem 233: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Sin}[c + dx])^{7/2}}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 4, 516 leaves, 15 steps):

$$\begin{aligned} & \frac{b (a^2 - b^2)^{5/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{9/2} d} - \frac{b (a^2 - b^2)^{5/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{9/2} d} + \\ & \frac{2 (5 a^4 - 28 a^2 b^2 + 21 b^4) e^4 \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c + dx]}}{21 a^5 d \sqrt{e \operatorname{Sin}[c + dx]}} + \\ & \frac{b^2 (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c + dx]}}{a^5 (a^2 - b^2 - a \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + dx]}} + \\ & \frac{b^2 (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c + dx]}}{a^5 (a^2 - b^2 + a \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + dx]}} + \\ & \frac{2 e^3 (21 b (a^2 - b^2) - a (5 a^2 - 7 b^2) \operatorname{Cos}[c + dx]) \sqrt{e \operatorname{Sin}[c + dx]}}{21 a^4 d} + \frac{2 e (7 b - 5 a \operatorname{Cos}[c + dx]) (e \operatorname{Sin}[c + dx])^{5/2}}{35 a^2 d} \end{aligned}$$

Result (type 6, 2249 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \operatorname{Sec}[c + d x])} \\
& (b + a \operatorname{Cos}[c + d x]) \left(-\frac{(23 a^2 - 28 b^2) \operatorname{Cos}[c + d x]}{42 a^3} - \frac{b \operatorname{Cos}[2(c + d x)]}{5 a^2} + \frac{\operatorname{Cos}[3(c + d x)]}{14 a} \right) \operatorname{Csc}[c + d x]^3 \operatorname{Sec}[c + d x] (e \operatorname{Sin}[c + d x])^{7/2} - \\
& \frac{1}{420 a^3 d (a + b \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]^{7/2}} (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] (e \operatorname{Sin}[c + d x])^{7/2} \\
& \left(\frac{1}{(b + a \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x]^2)} 2 (-100 a^3 + 98 a b^2) \operatorname{Cos}[c + d x]^2 (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] \right) \right) - \\
& \left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \left(\left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right) + \\
& \frac{1}{(b + a \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} 2 (89 a^2 b - 70 b^3) \operatorname{Cos}[c + d x] (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \\
& \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] \right) \right) + \\
& \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \right) / \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{b (a^2 - b^2)^{3/4} e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{7/2} d} - \frac{b (a^2 - b^2)^{3/4} e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{7/2} d} - \\
& \frac{b^2 (a^2 - b^2) e^3 \operatorname{EllipticPi}\left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^4 \left(a - \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} - \frac{b^2 (a^2 - b^2) e^3 \operatorname{EllipticPi}\left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^4 \left(a + \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\
& \frac{2 (3 a^2 - 5 b^2) e^2 \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c+dx]}}{5 a^3 d \sqrt{\sin[c+dx]}} + \frac{2 e (5 b - 3 a \cos[c+dx]) (e \sin[c+dx])^{3/2}}{15 a^2 d}
\end{aligned}$$

Result (type 6, 1247 leaves):

$$\begin{aligned}
& - \frac{1}{5 a^2 d (a + b \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]^{5/2}} (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \\
& (e \operatorname{Sin}[c + d x])^{5/2} \left(\frac{1}{(b + a \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x]^2)} 2 (-3 a^2 + 5 b^2) \operatorname{Cos}[c + d x]^2 (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2 + b^2)^{1/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \sqrt{2} \sqrt{a} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] \right) - \right. \\
& \left. \left(7 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \left(3 \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \left. \right) + \\
& \frac{1}{6 (b + a \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} a b \operatorname{Cos}[c + d x] (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \left(\frac{1}{\sqrt{a} (a^2 - b^2)^{1/4}} \right. \\
& (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \sqrt{a} \right. \right. \\
& \left. \left. (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] \right) \left. \right) + \\
& \left(56 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \right) / \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \left. \right) \left. \right) + \\
& \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Csc}[c + d x]^2 \operatorname{Sec}[c + d x] (e \operatorname{Sin}[c + d x])^{5/2} \left(\frac{2 b \operatorname{Sin}[c + d x]}{3 a^2} - \frac{\operatorname{Sin}[2(c + d x)]}{5 a} \right)}{d (a + b \operatorname{Sec}[c + d x])}
\end{aligned}$$

■ **Problem 235: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{3/2}}{a + b \sec[c + dx]} dx$$

Optimal (type 4, 444 leaves, 14 steps):

$$\begin{aligned} & - \frac{b (a^2 - b^2)^{1/4} e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{5/2} d} - \frac{b (a^2 - b^2)^{1/4} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{5/2} d} + \\ & \frac{2 (a^2 - 3b^2) e^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{3 a^3 d \sqrt{e \sin[c+dx]}} + \frac{b^2 (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^3 \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\ & \frac{b^2 (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^3 \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \frac{2 e (3b - a \cos[c+dx]) \sqrt{e \sin[c+dx]}}{3 a^2 d} \end{aligned}$$

Result (type 6, 2159 leaves):

$$\begin{aligned} & - \frac{2 (b + a \cos[c + dx]) \operatorname{Csc}[c + dx] (e \sin[c + dx])^{3/2}}{3 a d (a + b \sec[c + dx])} + \frac{1}{6 a d (a + b \sec[c + dx]) \sin[c + dx]^{3/2}} \\ & (b + a \cos[c + dx]) \sec[c + dx] (e \sin[c + dx])^{3/2} \left(\frac{1}{(b + a \cos[c + dx]) (1 - \sin[c + dx]^2)} 4 a \cos[c + dx]^2 \left(b + a \sqrt{1 - \sin[c + dx]^2}\right) \right. \\ & \left. \left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \\ & \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + a \sin[c + dx]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + a \sin[c + dx]\right] \right) \right) - \\ & \left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2}\right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \\ & \left(\left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{a^2 \sin[c + dx]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2}\right] \right) \sin[c + dx]^2 \right) \end{aligned}$$

$$\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] + 2 \left(2 a^2 \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] + \right. \\ \left. (a^2-b^2) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] \text{Sin}[c+dx]^2 \right) (b^2 + a^2 (-1 + \text{Sin}[c+dx]^2)) \Bigg)$$

■ **Problem 236: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{e \text{Sin}[c+dx]}}{a+b \text{Sec}[c+dx]} dx$$

Optimal (type 4, 356 leaves, 13 steps):

$$\frac{b \sqrt{e} \text{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \text{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right] - b \sqrt{e} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \text{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right] - b^2 e \text{EllipticPi}\left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\text{Sin}[c+dx]}}{a^{3/2} (a^2-b^2)^{1/4} d} - \frac{b^2 e \text{EllipticPi}\left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\text{Sin}[c+dx]}}{a^2 (a - \sqrt{a^2-b^2}) d \sqrt{e \text{Sin}[c+dx]}} - \\ + \frac{2 \text{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{e \text{Sin}[c+dx]}}{a d \sqrt{\text{Sin}[c+dx]}}$$

Result (type 6, 548 leaves):

$$\frac{1}{d (b+a \text{Cos}[c+dx]) \sqrt{\text{Sin}[c+dx]}} 2 \left(b+a \sqrt{\text{Cos}[c+dx]^2} \right) \sqrt{e \text{Sin}[c+dx]} \\ \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2+b^2)^{1/4}} b \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \text{Log}\left[\sqrt{-a^2+b^2} - \right. \right. \\ \left. \left. \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + a \text{Sin}[c+dx]\right] - \text{Log}\left[\sqrt{-a^2+b^2} + \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + a \text{Sin}[c+dx]\right] \right) - \\ \left(7 a (a^2-b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] \sqrt{\text{Cos}[c+dx]^2} \text{Sin}[c+dx]^{3/2} \right) / \\ \left(3 (-a^2+b^2+a^2 \text{Sin}[c+dx]^2) \left(7 (a^2-b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] + 2 \left(2 a^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\ \left. \left. \left. \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] + (-a^2+b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Sin}[c+dx]^2, \frac{a^2 \text{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \text{Sin}[c+dx]^2 \right) \Bigg)$$

■ **Problem 237: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x]) \sqrt{e \operatorname{Sin}[c + d x]}} dx$$

Optimal (type 4, 370 leaves, 13 steps):

$$\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{a} (a^2 - b^2)^{3/4} d \sqrt{e}} - \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{a} (a^2 - b^2)^{3/4} d \sqrt{e}} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{a d \sqrt{e \operatorname{Sin}[c + d x]}} +$$

$$\frac{b^2 \operatorname{EllipticPi}\left[\frac{2a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{a \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \operatorname{Sin}[c + d x]}} + \frac{b^2 \operatorname{EllipticPi}\left[\frac{2a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{a \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \operatorname{Sin}[c + d x]}}$$

Result (type 6, 546 leaves):

$$\frac{1}{d (b + a \operatorname{Cos}[c + d x]) \sqrt{e \operatorname{Sin}[c + d x]}} 2 \left(b + a \sqrt{\operatorname{Cos}[c + d x]^2} \right) \sqrt{\operatorname{Sin}[c + d x]}$$

$$\left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right.$$

$$\left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] \right) -$$

$$\left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Cos}[c + d x]^2} \sqrt{\operatorname{Sin}[c + d x]} \right) /$$

$$\left((-a^2 + b^2 + a^2 \operatorname{Sin}[c + d x]^2) \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right.$$

$$\left. \left. \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) \right)$$

■ **Problem 238: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x]) (e \operatorname{Sin}[c + d x])^{3/2}} dx$$

Optimal (type 4, 430 leaves, 14 steps):

$$\begin{aligned}
& \frac{\sqrt{a} b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right)^{1 / 4} \sqrt{e}}\right]}{\left(a^2-b^2\right)^{5 / 4} d e^{3 / 2}} - \frac{\sqrt{a} b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right)^{1 / 4} \sqrt{e}}\right]}{\left(a^2-b^2\right)^{5 / 4} d e^{3 / 2}} + \\
& \frac{2(b-a \cos [c+d x])}{\left(a^2-b^2\right) d e \sqrt{e \sin [c+d x]}} - \frac{b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right) d e \sqrt{e \sin [c+d x]}} - \\
& \frac{b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right) d e \sqrt{e \sin [c+d x]}} - \frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right) d e^2 \sqrt{\sin [c+d x]}}
\end{aligned}$$

Result (type 6, 1229 leaves):

$$\begin{aligned}
& \frac{1}{(a-b)(a+b)d(a+b\operatorname{Sec}[c+dx])(e\operatorname{Sin}[c+dx])^{3/2}} \\
& a(b+a\operatorname{Cos}[c+dx])\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]^{3/2} \left(\frac{1}{(b+a\operatorname{Cos}[c+dx])(1-\operatorname{Sin}[c+dx]^2)} 2a\operatorname{Cos}[c+dx]^2 \left(b+a\sqrt{1-\operatorname{Sin}[c+dx]^2} \right) \right. \\
& \left. \left(\frac{1}{4\sqrt{2}a^{3/2}(-a^2+b^2)^{1/4}} b \left(-2\operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2\operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2}-\sqrt{2}\sqrt{a}\right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]}+a\operatorname{Sin}[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2}+\sqrt{2}\sqrt{a}(-a^2+b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]}+a\operatorname{Sin}[c+dx]\right] \right) \right. \\
& \left. \left(7a(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sin}[c+dx]^{3/2}\sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left(3\left(7(a^2-b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + 2\left(2a^2\operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sin}[c+dx]^2 \right) (b^2+a^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) + \\
& \frac{1}{6(b+a\operatorname{Cos}[c+dx])\sqrt{1-\operatorname{Sin}[c+dx]^2}} b\operatorname{Cos}[c+dx] \left(b+a\sqrt{1-\operatorname{Sin}[c+dx]^2} \right) \left(\frac{1}{\sqrt{a}(a^2-b^2)^{1/4}} \right. \\
& (3+3i) \left(2\operatorname{ArcTan}\left[1-\frac{(1+i)\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - 2\operatorname{ArcTan}\left[1+\frac{(1+i)\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2}-(1+i)\sqrt{a}\right. \right. \\
& \left. \left. (a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]}+ia\operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2}+(1+i)\sqrt{a}(a^2-b^2)^{1/4}\sqrt{\operatorname{Sin}[c+dx]}+ia\operatorname{Sin}[c+dx]\right] \right) \right) + \\
& \left(56b(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sin}[c+dx]^{3/2} \right) / \\
& \left(\sqrt{1-\operatorname{Sin}[c+dx]^2} \left(7(a^2-b^2)\operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + 2\left(2a^2\operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + (a^2-b^2)\operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2\operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sin}[c+dx]^2 \right) \right) \\
& \left. \left. \left. (b^2+a^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) - \frac{2(b-a\operatorname{Cos}[c+dx])(b+a\operatorname{Cos}[c+dx])\operatorname{Tan}[c+dx]}{(-a^2+b^2)d(a+b\operatorname{Sec}[c+dx])(e\operatorname{Sin}[c+dx])^{3/2}}
\end{aligned}$$

- **Problem 239: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + dx]) (e \operatorname{Sin}[c + dx])^{5/2}} dx$$

Optimal (type 4, 452 leaves, 14 steps):

$$\begin{aligned} & - \frac{a^{3/2} b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{(a^2-b^2)^{7/4} d e^{5/2}} - \frac{a^{3/2} b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{(a^2-b^2)^{7/4} d e^{5/2}} + \\ & \frac{2(b-a \operatorname{Cos}[c+dx])}{3(a^2-b^2) d e (e \operatorname{Sin}[c+dx])^{3/2}} + \frac{2 a \operatorname{EllipticF}\left[\frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{3(a^2-b^2) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{a b^2 \operatorname{EllipticPi}\left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)(a^2-b^2-a\sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{a b^2 \operatorname{EllipticPi}\left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2}(c-\frac{\pi}{2}+dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)(a^2-b^2+a\sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} \end{aligned}$$

Result (type 6, 1233 leaves):

$$\begin{aligned} & - \frac{1}{3(a-b)(a+b)d(a+b \operatorname{Sec}[c+dx])(e \operatorname{Sin}[c+dx])^{5/2}} \\ & a(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] \operatorname{Sin}[c+dx]^{5/2} \left(- \frac{1}{(b+a \operatorname{Cos}[c+dx])(1-\operatorname{Sin}[c+dx])^2} 2a \operatorname{Cos}[c+dx]^2 (b+a \sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\ & \left(\left(b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \right. \right. \right. \\ & \left. \left. \left. \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] \right) \right) / \\ & \left(4 \sqrt{2} \sqrt{a} (-a^2+b^2)^{3/4} - \left(5 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \sqrt{\operatorname{Sin}[c+dx]} \right. \right. \\ & \left. \left. \sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left(\left(5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \\ & \left. \left. 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\ & \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sin}[c+dx]^2 (b^2+a^2(-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) + \end{aligned}$$

$$\frac{1}{(b + a \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 4 b \cos[c + dx] \left(b + a \sqrt{1 - \sin[c + dx]^2} \right) \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \right. \\ \left. \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \right. \right. \right. \\ \left. \left. \left. \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] \right) \right) + \\ \left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c + dx]} \right) / \\ \left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\ \left. \left. \left. \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \right) \sin[c + dx]^2 \right) \\ \left. \left. \left. \left(b^2 + a^2 (-1 + \sin[c + dx]^2) \right) \right) \right) \right) - \frac{2 (b - a \cos[c + dx]) (b + a \cos[c + dx]) \tan[c + dx]}{3 (-a^2 + b^2) d (a + b \sec[c + dx]) (e \sin[c + dx])^{5/2}}$$

- **Problem 240: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[c + dx]) (e \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 511 leaves, 15 steps):

$$\frac{a^{5/2} b \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{e \sin[c + dx]}}{(a^2 - b^2)^{1/4} \sqrt{e}} \right]}{(a^2 - b^2)^{9/4} d e^{7/2}} - \frac{a^{5/2} b \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{e \sin[c + dx]}}{(a^2 - b^2)^{1/4} \sqrt{e}} \right]}{(a^2 - b^2)^{9/4} d e^{7/2}} + \frac{2 (b - a \cos[c + dx])}{5 (a^2 - b^2) d e (e \sin[c + dx])^{5/2}} + \\ \frac{2 (5 a^2 b - a (3 a^2 + 2 b^2) \cos[c + dx])}{5 (a^2 - b^2)^2 d e^3 \sqrt{e \sin[c + dx]}} - \frac{a^2 b^2 \operatorname{EllipticPi} \left[\frac{2 a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{(a^2 - b^2)^2 (a - \sqrt{a^2 - b^2}) d e^3 \sqrt{e \sin[c + dx]}} - \\ \frac{a^2 b^2 \operatorname{EllipticPi} \left[\frac{2 a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{\sin[c + dx]}}{(a^2 - b^2)^2 (a + \sqrt{a^2 - b^2}) d e^3 \sqrt{e \sin[c + dx]}} - \frac{2 a (3 a^2 + 2 b^2) \operatorname{EllipticE} \left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2 \right] \sqrt{e \sin[c + dx]}}{5 (a^2 - b^2)^2 d e^4 \sqrt{\sin[c + dx]}}$$

Result (type 6, 1324 leaves):

$$\frac{1}{5 (a - b)^2 (a + b)^2 d (a + b \sec[c + dx]) (e \sin[c + dx])^{7/2}}$$

$$\begin{aligned}
& a (b + a \cos [c + d x]) \sec [c + d x] \sin [c + d x]^{7/2} \left(\frac{1}{(b + a \cos [c + d x]) (1 - \sin [c + d x]^2)} 2 (3 a^3 + 2 a b^2) \cos [c + d x]^2 (b + a \sqrt{1 - \sin [c + d x]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2 + b^2)^{1/4}} b \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} - \sqrt{2} \sqrt{a} \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + a \sin [c + d x] \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + a \sin [c + d x] \right] \right) - \right. \\
& \left. \left(7 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \sin [c + d x]^{3/2} \sqrt{1 - \sin [c + d x]^2} \right) / \left(3 \left(7 (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \right) \sin [c + d x]^2 (b^2 + a^2 (-1 + \sin [c + d x]^2)) \right) \right) \right) + \\
& \frac{1}{12 (b + a \cos [c + d x]) \sqrt{1 - \sin [c + d x]^2}} (8 a^2 b + 2 b^3) \cos [c + d x] (b + a \sqrt{1 - \sin [c + d x]^2}) \\
& \left(\frac{1}{\sqrt{a} (a^2 - b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + i a \sin [c + d x] \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + i a \sin [c + d x] \right] \right) \right) + \\
& \left(56 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \sin [c + d x]^{3/2} \right) / \left(\sqrt{1 - \sin [c + d x]^2} \left(7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \right) \sin [c + d x]^2 (b^2 + a^2 (-1 + \sin [c + d x]^2)) \right) \right) \right) \right) + \\
& \left((b + a \cos [c + d x]) \left(-\frac{2 (-5 a^2 b + 3 a^3 \cos [c + d x] + 2 a b^2 \cos [c + d x]) \operatorname{Csc} [c + d x]}{5 (-a^2 + b^2)^2} - \frac{2 (b - a \cos [c + d x]) \operatorname{Csc} [c + d x]^3}{5 (-a^2 + b^2)} \right) \right) \\
& \sin [c + d x]^3 \\
& \tan [
\end{aligned}$$

$$\left. \begin{array}{l} c + d x \\ (a + b \operatorname{Sec}[c + d x]) (e \operatorname{Sin}[c + d x])^{7/2} \end{array} \right) / (d)$$

■ **Problem 241: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \operatorname{Sin}[c + d x])^{9/2}}{(a + b \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 1070 leaves, 35 steps):

$$\begin{aligned} & - \frac{7 b^3 (a^2 - b^2)^{3/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{13/2} d} + \frac{2 b (a^2 - b^2)^{7/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{13/2} d} + \frac{7 b^3 (a^2 - b^2)^{3/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{13/2} d} \\ & - \frac{2 b (a^2 - b^2)^{7/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{13/2} d} + \frac{7 b^4 (a^2 - b^2) e^5 \operatorname{EllipticPi}\left[\frac{2 a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{2 a^7 (a - \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}} \\ & - \frac{2 b^2 (a^2 - b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2 a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{a^7 (a - \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}} + \\ & - \frac{7 b^4 (a^2 - b^2) e^5 \operatorname{EllipticPi}\left[\frac{2 a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{2 a^7 (a + \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}} \\ & - \frac{2 b^2 (a^2 - b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2 a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{a^7 (a + \sqrt{a^2 - b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}} + \frac{14 e^4 \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{e \operatorname{Sin}[c + d x]}}{15 a^2 d \sqrt{\operatorname{Sin}[c + d x]}} \\ & - \frac{7 b^2 (3 a^2 - 5 b^2) e^4 \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{e \operatorname{Sin}[c + d x]}}{5 a^6 d \sqrt{\operatorname{Sin}[c + d x]}} - \frac{4 b^2 (8 a^2 - 5 b^2) e^4 \operatorname{EllipticE}\left[\frac{1}{2} (c - \frac{\pi}{2} + d x), 2\right] \sqrt{e \operatorname{Sin}[c + d x]}}{5 a^6 d \sqrt{\operatorname{Sin}[c + d x]}} \\ & + \frac{14 e^3 \operatorname{Cos}[c + d x] (e \operatorname{Sin}[c + d x])^{3/2}}{45 a^2 d} - \frac{7 b^2 e^3 (5 b - 3 a \operatorname{Cos}[c + d x]) (e \operatorname{Sin}[c + d x])^{3/2}}{15 a^5 d} \\ & + \frac{4 b e^3 (5 (a^2 - b^2) + 3 a b \operatorname{Cos}[c + d x]) (e \operatorname{Sin}[c + d x])^{3/2}}{15 a^5 d} + \frac{4 b e (e \operatorname{Sin}[c + d x])^{7/2}}{7 a^3 d} - \frac{2 e \operatorname{Cos}[c + d x] (e \operatorname{Sin}[c + d x])^{7/2}}{9 a^2 d} + \frac{b^2 e (e \operatorname{Sin}[c + d x])^{7/2}}{a^3 d (b + a \operatorname{Cos}[c + d x])} \end{aligned}$$

Result (type 6, 1368 leaves):

$$\begin{aligned}
& \frac{1}{30 a^5 d (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]^{9/2}} (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 (e \operatorname{Sin}[c + d x])^{9/2} \\
& \left(\frac{1}{(b + a \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x]^2)} 2 (14 a^4 - 159 a^2 b^2 + 165 b^4) \operatorname{Cos}[c + d x]^2 (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2 + b^2)^{1/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] \right) \right) - \\
& \left(7 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \left(3 \left(7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) + \\
& \frac{1}{12 (b + a \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} (-46 a^3 b + 66 a b^3) \operatorname{Cos}[c + d x] (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \\
& \left(\frac{1}{\sqrt{a} (a^2 - b^2)^{1/4}} (3 + 3 i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] \right) \right) + \\
& \left(56 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Sin}[c + d x]^{3/2} \right) / \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right) + \\
& \frac{1}{d (a + b \operatorname{Sec}[c + d x])^2} (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Csc}[c + d x]^4 \operatorname{Sec}[c + d x]^2 (e \operatorname{Sin}[c + d x])^{9/2} \\
& \left(-\frac{b (-37 a^2 + 56 b^2) \operatorname{Sin}[c + d x]}{21 a^5} + \right.
\end{aligned}$$

$$\frac{a^2 b^2 \sin[c + dx] - b^4 \sin[c + dx]}{a^5 (b + a \cos[c + dx])} -$$

$$\frac{(19 a^2 - 54 b^2) \sin[2 (c + dx)]}{90 a^4} -$$

$$\frac{b \sin[3 (c + dx)]}{7 a^3} +$$

$$\frac{\sin[4 (c + dx)]}{36 a^2} \Bigg)$$

- **Problem 242: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + dx])^{7/2}}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 1101 leaves, 35 steps):

$$\begin{aligned}
& \frac{5 b^3 (a^2 - b^2)^{1/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{11/2} d} - \frac{2 b (a^2 - b^2)^{5/4} e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{11/2} d} + \\
& \frac{5 b^3 (a^2 - b^2)^{1/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{11/2} d} - \frac{2 b (a^2 - b^2)^{5/4} e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{11/2} d} + \\
& \frac{10 e^4 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{21 a^2 d \sqrt{e \sin[c+dx]}} - \frac{5 b^2 (a^2 - 3 b^2) e^4 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{3 a^6 d \sqrt{e \sin[c+dx]}} - \\
& \frac{4 b^2 (4 a^2 - 3 b^2) e^4 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{3 a^6 d \sqrt{e \sin[c+dx]}} - \frac{5 b^4 (a^2 - b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 a^6 \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\
& \frac{2 b^2 (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2 a}{a - \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^6 \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} - \\
& \frac{5 b^4 (a^2 - b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 a^6 \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\
& \frac{2 b^2 (a^2 - b^2)^2 e^4 \operatorname{EllipticPi}\left[\frac{2 a}{a + \sqrt{a^2 - b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^6 \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} - \frac{10 e^3 \cos[c+dx] \sqrt{e \sin[c+dx]}}{21 a^2 d} - \\
& \frac{5 b^2 e^3 (3 b - a \cos[c+dx]) \sqrt{e \sin[c+dx]}}{3 a^5 d} + \frac{4 b e^3 (3 (a^2 - b^2) + a b \cos[c+dx]) \sqrt{e \sin[c+dx]}}{3 a^5 d} + \\
& \frac{4 b e (e \sin[c+dx])^{5/2}}{5 a^3 d} - \frac{2 e \cos[c+dx] (e \sin[c+dx])^{5/2}}{7 a^2 d} + \frac{b^2 e (e \sin[c+dx])^{5/2}}{a^3 d (b + a \cos[c+dx])}
\end{aligned}$$

Result (type 6, 2295 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \operatorname{Sec}[c + dx])^2} (b + a \cos[c + dx])^2 \left(-\frac{(23 a^2 - 84 b^2) \cos[c + dx]}{42 a^4} - \frac{b^2 (-a^2 + b^2)}{a^5 (b + a \cos[c + dx])} - \frac{2 b \cos[2 (c + dx)]}{5 a^3} + \frac{\cos[3 (c + dx)]}{14 a^2} \right) \\
& \operatorname{Csc}[c + dx]^3 \operatorname{Sec}[c + dx]^2 (e \sin[c + dx])^{7/2} + \frac{1}{210 a^5 d (a + b \operatorname{Sec}[c + dx])^2 \sin[c + dx]^{7/2}} (b + a \cos[c + dx])^2 \operatorname{Sec}[c + dx]^2
\end{aligned}$$

$$\begin{aligned}
& (e \sin[c + dx])^{7/2} \left(\frac{1}{(b + a \cos[c + dx]) (1 - \sin[c + dx])^2} 2 (50 a^4 - 273 a^2 b^2 + 105 b^4) \cos[c + dx]^2 (b + a \sqrt{1 - \sin[c + dx]^2}) \right. \\
& \left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + a \sin[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + a \sin[c + dx] \right] \right) - \\
& \left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left(\left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \right) \sin[c + dx]^2 (b^2 + a^2 (-1 + \sin[c + dx]^2)) \right) \right) \Bigg) + \\
& \frac{1}{(b + a \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2 (-139 a^3 b + 210 a b^3) \cos[c + dx] (b + a \sqrt{1 - \sin[c + dx]^2}) \\
& \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] \right) \right) + \\
& \left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c + dx]} \right) / \left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \right) \sin[c + dx]^2 (b^2 + a^2 (-1 + \sin[c + dx]^2)) \right) \right) \Bigg) + \\
& \frac{1}{(b + a \cos[c + dx]) (1 - 2 \sin[c + dx]^2) \sqrt{1 - \sin[c + dx]^2}} (231 a^3 b - 420 a b^3) \cos[c + dx] \cos[2(c + dx)]
\end{aligned}$$

$$\begin{aligned}
& \left(b + a \sqrt{1 - \sin[c + dx]^2} \right) \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (a^2 - 2b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right]}{a^{3/2} (a^2 - b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (a^2 - 2b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right]}{a^{3/2} (a^2 - b^2)^{3/4}} \right) + \\
& \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (a^2 - 2b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right]}{a^{3/2} (a^2 - b^2)^{3/4}} - \\
& \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (a^2 - 2b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right]}{a^{3/2} (a^2 - b^2)^{3/4}} + \frac{4 \sqrt{\sin[c+dx]}}{a} + \\
& \left(10 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c+dx]} \right) / \left(\sqrt{1 - \sin[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\right. \right. \right. \\
& \left. \left. \left. \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sin[c+dx]^2 \right) (b^2 + a^2 (-1 + \sin[c+dx]^2)) \right) \right) + \\
& \left(36 b (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sin[c+dx]^{5/2} \right) / \left(5 \sqrt{1 - \sin[c+dx]^2} \left(9 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sin[c+dx]^2 \right) (b^2 + a^2 (-1 + \sin[c+dx]^2)) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 243: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \sin[c + dx])^{5/2}}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 850 leaves, 32 steps):

$$\begin{aligned}
& - \frac{3 b^3 e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{9/2} (a^2-b^2)^{1/4} d} + \frac{2 b (a^2-b^2)^{3/4} e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{9/2} d} + \frac{3 b^3 e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{9/2} (a^2-b^2)^{1/4} d} - \\
& \frac{2 b (a^2-b^2)^{3/4} e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{9/2} d} + \frac{3 b^4 e^3 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{2 a^5 (a-\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} - \\
& \frac{2 b^2 (a^2-b^2) e^3 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{a^5 (a-\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} + \frac{3 b^4 e^3 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{2 a^5 (a+\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} - \\
& \frac{2 b^2 (a^2-b^2) e^3 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{\sin[c+dx]}}{a^5 (a+\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} + \frac{6 e^2 \operatorname{EllipticE}\left[\frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{e \sin[c+dx]}}{5 a^2 d \sqrt{\sin[c+dx]}} - \\
& \frac{7 b^2 e^2 \operatorname{EllipticE}\left[\frac{1}{2} (c-\frac{\pi}{2}+dx), 2\right] \sqrt{e \sin[c+dx]}}{a^4 d \sqrt{\sin[c+dx]}} + \frac{4 b e (e \sin[c+dx])^{3/2}}{3 a^3 d} - \frac{2 e \cos[c+dx] (e \sin[c+dx])^{3/2}}{5 a^2 d} + \frac{b^2 e (e \sin[c+dx])^{3/2}}{a^3 d (b+a \cos[c+dx])}
\end{aligned}$$

Result (type 6, 1280 leaves):

$$\begin{aligned}
& - \frac{1}{10 a^3 d (a+b \sec[c+dx])^2 \sin[c+dx]^{5/2}} (b+a \cos[c+dx])^2 \sec[c+dx]^2 \\
& (e \sin[c+dx])^{5/2} \left(\frac{1}{(b+a \cos[c+dx]) (1-\sin[c+dx])^2} 2 (-6 a^2+35 b^2) \cos[c+dx]^2 (b+a \sqrt{1-\sin[c+dx]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2+b^2)^{1/4}} b \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2}-\sqrt{2} \sqrt{a} \right. \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]}+a \sin[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2}+\sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]}+a \sin[c+dx]\right] \right) \right) - \\
& \left(7 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2}\right] \sin[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \left(3 \left(7 (a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2}\right] \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2}\right] \right) \sin[c+dx]^2 (b^2+a^2 (-1+\sin[c+dx]^2)) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{6 (b + a \cos [c + d x]) \sqrt{1 - \sin [c + d x]^2}} 7 a b \cos [c + d x] \left(b + a \sqrt{1 - \sin [c + d x]^2} \right) \left(\frac{1}{\sqrt{a} (a^2 - b^2)^{1/4}} \right. \\
& (3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1 + i) \sqrt{a} \right. \right. \\
& \left. \left. (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + i a \sin [c + d x] \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + i a \sin [c + d x] \right] \right) + \\
& \left(56 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \sin [c + d x]^{3/2} \right) / \left(\sqrt{1 - \sin [c + d x]^2} \left(7 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c + d x]^2, \frac{a^2 \sin [c + d x]^2}{a^2 - b^2} \right] \right) \sin [c + d x]^2 \right) \left(b^2 + a^2 (-1 + \sin [c + d x]^2) \right) \right) \left. \right) + \\
& \frac{1}{d (a + b \sec [c + d x])^2} (b + a \cos [c + d x])^2 \csc [c + d x]^2 \sec [c + d x]^2 (e \sin [c + d x])^{5/2} \\
& \left(\frac{4 b \sin [c + d x]}{3 a^3} + \frac{b^2 \sin [c + d x]}{a^3 (b + a \cos [c + d x])} - \frac{\sin [2 (c + d x)]}{5 a^2} \right)
\end{aligned}$$

■ **Problem 244: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin [c + d x])^{3/2}}{(a + b \sec [c + d x])^2} dx$$

Optimal (type 4, 882 leaves, 32 steps):

$$\begin{aligned}
& \frac{b^3 e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{7/2} (a^2-b^2)^{3/4} d} - \frac{2 b (a^2-b^2)^{1/4} e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{7/2} d} + \frac{b^3 e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{7/2} (a^2-b^2)^{3/4} d} - \\
& \frac{2 b (a^2-b^2)^{1/4} e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{7/2} d} + \frac{2 e^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{3 a^2 d \sqrt{e \sin[c+dx]}} - \\
& \frac{5 b^2 e^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^4 d \sqrt{e \sin[c+dx]}} - \frac{b^4 e^2 \operatorname{EllipticPi}\left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 a^4 \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\
& \frac{2 b^2 (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^4 \left(a^2 - b^2 - a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} - \\
& \frac{b^4 e^2 \operatorname{EllipticPi}\left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 a^4 \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \frac{2 b^2 (a^2 - b^2) e^2 \operatorname{EllipticPi}\left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{a^4 \left(a^2 - b^2 + a \sqrt{a^2 - b^2}\right) d \sqrt{e \sin[c+dx]}} + \\
& \frac{4 b e \sqrt{e \sin[c+dx]}}{a^3 d} - \frac{2 e \cos[c+dx] \sqrt{e \sin[c+dx]}}{3 a^2 d} + \frac{b^2 e \sqrt{e \sin[c+dx]}}{a^3 d (b + a \cos[c+dx])}
\end{aligned}$$

Result (type 6, 2212 leaves):

$$\begin{aligned}
& \frac{(b + a \cos[c+dx])^2 \left(-\frac{2 \cos[c+dx]}{3 a^2} + \frac{b^2}{a^3 (b + a \cos[c+dx])}\right) \csc[c+dx] \sec[c+dx]^2 (e \sin[c+dx])^{3/2}}{d (a + b \sec[c+dx])^2} - \\
& \frac{1}{6 a^3 d (a + b \sec[c+dx])^2 \sin[c+dx]^{3/2}} (b + a \cos[c+dx])^2 \sec[c+dx]^2 (e \sin[c+dx])^{3/2} \\
& \left(\frac{1}{(b + a \cos[c+dx]) (1 - \sin[c+dx])^2} 2 (-2 a^2 + 3 b^2) \cos[c+dx]^2 \left(b + a \sqrt{1 - \sin[c+dx]^2}\right) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + a \sin[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + a \sin[c+dx]\right] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c+dx]} \sqrt{1 - \sin[c+dx]^2} \right) / \\
& \left(\left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \right. \right. \right. \\
& \quad \left. \left. \left. \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right) + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \right) \sin[c+dx]^2 \right) \\
& \left. (b^2 + a^2 (-1 + \sin[c+dx]^2)) \right) \left. \right) + \frac{1}{(b+a \cos[c+dx]) \sqrt{1 - \sin[c+dx]^2}} 8 a b \cos[c+dx] \left(b + a \sqrt{1 - \sin[c+dx]^2} \right) \\
& \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1+i) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right] \right) \right) + \\
& \left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c+dx]} \right) / \left(\sqrt{1 - \sin[c+dx]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \right) \sin[c+dx]^2 \right) (b^2 + a^2 (-1 + \sin[c+dx]^2)) \right) \left. \right) - \\
& \frac{1}{(b+a \cos[c+dx]) (1 - 2 \sin[c+dx]^2) \sqrt{1 - \sin[c+dx]^2}} 6 a b \cos[c+dx] \cos[2(c+dx)] \left(b + a \sqrt{1 - \sin[c+dx]^2} \right) \\
& \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (a^2 - 2 b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}} \right]}{a^{3/2} (a^2 - b^2)^{3/4}} - \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (a^2 - 2 b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{a} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}} \right]}{a^{3/2} (a^2 - b^2)^{3/4}} \right) + \\
& \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (a^2 - 2 b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right]}{a^{3/2} (a^2 - b^2)^{3/4}} - \\
& \frac{\left(\frac{1}{4} - \frac{i}{4} \right) (a^2 - 2 b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1+i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + i a \sin[c+dx] \right]}{a^{3/2} (a^2 - b^2)^{3/4}} + \frac{4 \sqrt{\sin[c+dx]}}{a} + \\
& \left(10 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c+dx]} \right) / \left(\sqrt{1 - \sin[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \Big] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] + \right. \\ & \left. (a^2-b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] \sin[c+dx]^2 \right) (b^2+a^2(-1+\sin[c+dx]^2)) \Big) + \\ & \left(36 b (-a^2+b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] \sin[c+dx]^{5/2} \right) / \left(5 \sqrt{1-\sin[c+dx]^2} \left(9 (a^2-b^2) \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\ & \left. \left. \left. (a^2-b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c+dx]^2, \frac{a^2 \sin[c+dx]^2}{a^2-b^2} \right] \right) \sin[c+dx]^2 \right) (b^2+a^2(-1+\sin[c+dx]^2)) \right) \Big) \Big) \end{aligned}$$

■ **Problem 245: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{e \sin[c+dx]}}{(a+b \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 809 leaves, 27 steps):

$$\begin{aligned} & \frac{b^3 \sqrt{e} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}} \right]}{2 a^{5/2} (a^2-b^2)^{5/4} d} + \frac{2 b \sqrt{e} \operatorname{ArcTan} \left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}} \right]}{a^{5/2} (a^2-b^2)^{1/4} d} - \frac{b^3 \sqrt{e} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}} \right]}{2 a^{5/2} (a^2-b^2)^{5/4} d} - \frac{2 b \sqrt{e} \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{e \sin[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}} \right]}{a^{5/2} (a^2-b^2)^{1/4} d} \\ & - \frac{2 b^2 e \operatorname{EllipticPi} \left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{\sin[c+dx]}}{a^3 (a-\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} - \frac{b^4 e \operatorname{EllipticPi} \left[\frac{2a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{\sin[c+dx]}}{2 a^3 (a^2-b^2) (a-\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} \\ & - \frac{2 b^2 e \operatorname{EllipticPi} \left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{\sin[c+dx]}}{a^3 (a+\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} - \frac{b^4 e \operatorname{EllipticPi} \left[\frac{2a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{\sin[c+dx]}}{2 a^3 (a^2-b^2) (a+\sqrt{a^2-b^2}) d \sqrt{e \sin[c+dx]}} + \\ & - \frac{2 \operatorname{EllipticE} \left[\frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{e \sin[c+dx]}}{a^2 d \sqrt{\sin[c+dx]}} - \frac{b^2 \operatorname{EllipticE} \left[\frac{1}{2} (c-\frac{\pi}{2}+dx), 2 \right] \sqrt{e \sin[c+dx]}}{a^2 (a^2-b^2) d \sqrt{\sin[c+dx]}} + \frac{b^2 (e \sin[c+dx])^{3/2}}{a (a^2-b^2) d e (b+a \operatorname{Cos}[c+dx])} \end{aligned}$$

Result (type 6, 1248 leaves):

$$\begin{aligned}
& \frac{1}{2 a (-a+b) (a+b) d (a+b \operatorname{Sec}[c+d x])^2 \sqrt{\operatorname{Sin}[c+d x]}} \\
& \left((b+a \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^2 \sqrt{e \operatorname{Sin}[c+d x]} \left(\frac{1}{(b+a \operatorname{Cos}[c+d x]) (1-\operatorname{Sin}[c+d x]^2)} 2 (-2 a^2+3 b^2) \operatorname{Cos}[c+d x]^2 (b+a \sqrt{1-\operatorname{Sin}[c+d x]^2}) \right. \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2+b^2)^{1/4}} b \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2}-\right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + a \operatorname{Sin}[c+d x]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + a \operatorname{Sin}[c+d x]\right] \right) \right) - \\
& \left(7 a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sin}[c+d x]^{3/2} \sqrt{1-\operatorname{Sin}[c+d x]^2} \right) / \left(3 \left(7 (a^2-b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sin}[c+d x]^2 \right) (b^2+a^2 (-1+\operatorname{Sin}[c+d x]^2)) \right) \right) \right) + \\
& \frac{1}{6 (b+a \operatorname{Cos}[c+d x]) \sqrt{1-\operatorname{Sin}[c+d x]^2}} a b \operatorname{Cos}[c+d x] (b+a \sqrt{1-\operatorname{Sin}[c+d x]^2}) \left(\frac{1}{\sqrt{a} (a^2-b^2)^{1/4}} \right. \\
& \left. (3+3 i) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{a} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{a} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - (1+i) \sqrt{a} \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i a \operatorname{Sin}[c+d x]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + (1+i) \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + i a \operatorname{Sin}[c+d x]\right] \right) \right) + \\
& \left(56 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sin}[c+d x]^{3/2} \right) / \\
& \left(\sqrt{1-\operatorname{Sin}[c+d x]^2} \left(7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] + (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+d x]^2, \frac{a^2 \operatorname{Sin}[c+d x]^2}{a^2-b^2}\right] \operatorname{Sin}[c+d x]^2 \right) \right) \right) \\
& \left. \left. \left. (b^2+a^2 (-1+\operatorname{Sin}[c+d x]^2)) \right) \right) \right) + \frac{b^2 (b+a \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x] \sqrt{e \operatorname{Sin}[c+d x]} \operatorname{Tan}[c+d x]}{a (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^2}
\end{aligned}$$

■ **Problem 246: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + dx])^2 \sqrt{e \operatorname{Sin}[c + dx]}} dx$$

Optimal (type 4, 838 leaves, 27 steps):

$$\begin{aligned} & - \frac{3 b^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{3/2} (a^2-b^2)^{7/4} d \sqrt{e}} - \frac{2 b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{3/2} (a^2-b^2)^{3/4} d \sqrt{e}} \\ & - \frac{3 b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 a^{3/2} (a^2-b^2)^{7/4} d \sqrt{e}} - \frac{2 b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{a^{3/2} (a^2-b^2)^{3/4} d \sqrt{e}} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{a^2 d \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{b^2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{a^2 (a^2-b^2) d \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{a^2 \left(a^2-b^2-a \sqrt{a^2-b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{3 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 a^2 (a^2-b^2) \left(a^2-b^2-a \sqrt{a^2-b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{a^2 \left(a^2-b^2+a \sqrt{a^2-b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{3 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 a^2 (a^2-b^2) \left(a^2-b^2+a \sqrt{a^2-b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{b^2 \sqrt{e \operatorname{Sin}[c+dx]}}{a (a^2-b^2) d e (b+a \operatorname{Cos}[c+dx])} \end{aligned}$$

Result (type 6, 1246 leaves):

1

$$2 a (-a + b) (a + b) d (a + b \operatorname{Sec}[c + d x])^2 \sqrt{e \operatorname{Sin}[c + d x]}$$

$$\begin{aligned} & (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \sqrt{\operatorname{Sin}[c + d x]} \left(\frac{1}{(b + a \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x])^2} (-2 a^2 + b^2) \operatorname{Cos}[c + d x]^2 (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \right. \\ & \left. \left(\frac{1}{4 \sqrt{2} \sqrt{a} (-a^2 + b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \\ & \left. \left. \left. \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{a} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + a \operatorname{Sin}[c + d x]\right] \right) \right) - \\ & \left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \\ & \left(\left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) \\ & \left. (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \left. \right) + \frac{1}{(b + a \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} 4 a b \operatorname{Cos}[c + d x] (b + a \sqrt{1 - \operatorname{Sin}[c + d x]^2}) \\ & \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\ & \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i a \operatorname{Sin}[c + d x]\right] \right) \right) + \\ & \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \right) / \\ & \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \\ & \left. \left. 2 \left(2 a^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] + (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{a^2 \operatorname{Sin}[c + d x]^2}{a^2 - b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) (b^2 + a^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \left. \right) \left. \right) + \frac{b^2 (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^2 \sqrt{e \operatorname{Sin}[c + d x]}} \end{aligned}$$

■ **Problem 247: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x])^2 (e \operatorname{Sin}[c + d x])^{3/2}} dx$$

Optimal (type 4, 1054 leaves, 33 steps):

$$\begin{aligned} & \frac{5 b^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{a} (a^2-b^2)^{9/4} d e^{3/2}} + \frac{2 b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{a} (a^2-b^2)^{5/4} d e^{3/2}} - \frac{5 b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{a} (a^2-b^2)^{9/4} d e^{3/2}} - \frac{2 b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{\sqrt{a} (a^2-b^2)^{5/4} d e^{3/2}} \\ & + \frac{2 \operatorname{Cos}[c+d x]}{a^2 d e \sqrt{e \operatorname{Sin}[c+d x]}} + \frac{b^2}{a (a^2-b^2) d e (b+a \operatorname{Cos}[c+d x]) \sqrt{e \operatorname{Sin}[c+d x]}} + \frac{4 b (a-b \operatorname{Cos}[c+d x])}{a^2 (a^2-b^2) d e \sqrt{e \operatorname{Sin}[c+d x]}} + \\ & \frac{b^2 (5 a b - (3 a^2 + 2 b^2) \operatorname{Cos}[c+d x])}{a^2 (a^2-b^2)^2 d e \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{5 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{2 a (a^2-b^2)^2 (a-\sqrt{a^2-b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} - \\ & \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{a (a^2-b^2) (a-\sqrt{a^2-b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{5 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{2 a (a^2-b^2)^2 (a+\sqrt{a^2-b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} - \\ & \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{a (a^2-b^2) (a+\sqrt{a^2-b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} - \frac{2 \operatorname{EllipticE}\left[\frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{e \operatorname{Sin}[c+d x]}}{a^2 d e^2 \sqrt{\operatorname{Sin}[c+d x]}} - \\ & \frac{4 b^2 \operatorname{EllipticE}\left[\frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{e \operatorname{Sin}[c+d x]}}{a^2 (a^2-b^2) d e^2 \sqrt{\operatorname{Sin}[c+d x]}} - \frac{b^2 (3 a^2 + 2 b^2) \operatorname{EllipticE}\left[\frac{1}{2} (c-\frac{\pi}{2}+d x), 2\right] \sqrt{e \operatorname{Sin}[c+d x]}}{a^2 (a^2-b^2)^2 d e^2 \sqrt{\operatorname{Sin}[c+d x]}} \end{aligned}$$

Result (type 6, 1316 leaves):

$$\begin{aligned}
& \frac{1}{2 (a-b)^2 (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^2 (e \operatorname{Sin}[c+dx])^{3/2}} \\
& (b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^2 \operatorname{Sin}[c+dx]^{3/2} \left(\frac{1}{(b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Sin}[c+dx])^2} 2 (2a^3+3ab^2) \operatorname{Cos}[c+dx]^2 (b+a \sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} a^{3/2} (-a^2+b^2)^{1/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} - \sqrt{2} \sqrt{a} \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} + \sqrt{2} \sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] \right) - \right. \\
& \left. \left(7a (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sin}[c+dx]^{3/2} \sqrt{1-\operatorname{Sin}[c+dx]^2} \right) / \left(3 \left(7 (a^2-b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + 2 \left(2a^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sin}[c+dx]^2 \right) (b^2+a^2 (-1+\operatorname{Sin}[c+dx]^2)) \right) \right) + \\
& \frac{1}{12 (b+a \operatorname{Cos}[c+dx]) \sqrt{1-\operatorname{Sin}[c+dx]^2}} (6a^2b+4b^3) \operatorname{Cos}[c+dx] (b+a \sqrt{1-\operatorname{Sin}[c+dx]^2}) \\
& \left(\frac{1}{\sqrt{a} (a^2-b^2)^{1/4}} (3+3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{a} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{a^2-b^2} - (1+i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + i a \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{a^2-b^2} + (1+i) \sqrt{a} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + i a \operatorname{Sin}[c+dx]\right] \right) + \right. \\
& \left. \left(56b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \operatorname{Sin}[c+dx]^{3/2} \right) / \left(\sqrt{1-\operatorname{Sin}[c+dx]^2} \left(7 (a^2-b^2) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + 2 \left(2a^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{a^2 \operatorname{Sin}[c+dx]^2}{a^2-b^2}\right] \right) \operatorname{Sin}[c+dx]^2 \right) (b^2+a^2 (-1+\operatorname{Sin}[c+dx]^2)) \right) \right) \right) + \\
& \frac{(b+a \operatorname{Cos}[c+dx])^2 \left(-\frac{2(-2ab+a^2 \operatorname{Cos}[c+dx]+b^2 \operatorname{Cos}[c+dx]) \operatorname{Csc}[c+dx]}{(-a^2+b^2)^2} + \frac{a b^2 \operatorname{Sin}[c+dx]}{(-a^2+b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right) \operatorname{Tan}[c+dx]^2}{d (a+b \operatorname{Sec}[c+dx])^2 (e \operatorname{Sin}[c+dx])^{3/2}}
\end{aligned}$$

■ **Problem 248: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + dx])^2 (e \operatorname{Sin}[c + dx])^{5/2}} dx$$

Optimal (type 4, 1089 leaves, 33 steps):

$$\begin{aligned} & -\frac{7\sqrt{a} b^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2(a^2-b^2)^{11/4} d e^{5/2}} - \frac{2\sqrt{a} b \operatorname{ArcTan}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{(a^2-b^2)^{7/4} d e^{5/2}} - \frac{7\sqrt{a} b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{2(a^2-b^2)^{11/4} d e^{5/2}} \\ & - \frac{2\sqrt{a} b \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{e \operatorname{Sin}[c+dx]}}{(a^2-b^2)^{1/4} \sqrt{e}}\right]}{(a^2-b^2)^{7/4} d e^{5/2}} - \frac{2 \operatorname{Cos}[c+dx]}{3 a^2 d e (e \operatorname{Sin}[c+dx])^{3/2}} + \frac{b^2}{a (a^2-b^2) d e (b+a \operatorname{Cos}[c+dx]) (e \operatorname{Sin}[c+dx])^{3/2}} + \\ & \frac{4 b (a-b \operatorname{Cos}[c+dx])}{3 a^2 (a^2-b^2) d e (e \operatorname{Sin}[c+dx])^{3/2}} + \frac{b^2 (7 a b - (5 a^2 + 2 b^2) \operatorname{Cos}[c+dx])}{3 a^2 (a^2-b^2)^2 d e (e \operatorname{Sin}[c+dx])^{3/2}} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{3 a^2 d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{4 b^2 \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{3 a^2 (a^2-b^2) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{b^2 (5 a^2 + 2 b^2) \operatorname{EllipticF}\left[\frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{3 a^2 (a^2-b^2)^2 d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{7 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 (a^2-b^2)^2 (a^2-b^2-a \sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a-\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2) (a^2-b^2-a \sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \\ & \frac{7 b^4 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 (a^2-b^2)^2 (a^2-b^2+a \sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} + \frac{2 b^2 \operatorname{EllipticPi}\left[\frac{2 a}{a+\sqrt{a^2-b^2}}, \frac{1}{2} (c - \frac{\pi}{2} + dx), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{(a^2-b^2) (a^2-b^2+a \sqrt{a^2-b^2}) d e^2 \sqrt{e \operatorname{Sin}[c+dx]}} \end{aligned}$$

Result (type 6, 1320 leaves):

$$\begin{aligned} & -\frac{1}{6(a-b)^2(a+b)^2 d (a+b \operatorname{Sec}[c+dx])^2 (e \operatorname{Sin}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^2 \\ & \operatorname{Sin}[c+dx]^{5/2} \left(\frac{1}{(b+a \operatorname{Cos}[c+dx]) (1-\operatorname{Sin}[c+dx])^2} 2(-2a^3-5ab^2) \operatorname{Cos}[c+dx]^2 (b+a \sqrt{1-\operatorname{Sin}[c+dx]^2}) \right. \\ & \left(\frac{1}{4\sqrt{2}\sqrt{a}(-a^2+b^2)^{3/4}} b \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{a}\sqrt{\operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2+b^2} - \sqrt{2}\sqrt{a} \right. \right. \\ & \left. \left. (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] + \operatorname{Log}\left[\sqrt{-a^2+b^2} + \sqrt{2}\sqrt{a} (-a^2+b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + a \operatorname{Sin}[c+dx]\right] \right) \end{aligned}$$

$$\begin{aligned}
& \left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c + dx]} \sqrt{1 - \sin[c + dx]^2} \right) / \left(\left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \right) \sin[c + dx]^2 \right) (b^2 + a^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) + \\
& \frac{1}{(b + a \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} 2 (10 a^2 b + 4 b^3) \cos[c + dx] (b + a \sqrt{1 - \sin[c + dx]^2}) \\
& \left(-\frac{1}{(a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{a} \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{a} \sqrt{\sin[c + dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1 + i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{a} (a^2 - b^2)^{1/4} \sqrt{\sin[c + dx]} + i a \sin[c + dx] \right] \right) \right) + \\
& \left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \sqrt{\sin[c + dx]} \right) / \left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + 2 \left(2 a^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{a^2 \sin[c + dx]^2}{a^2 - b^2} \right] \right) \sin[c + dx]^2 \right) (b^2 + a^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) \Bigg) + \\
& \left((b + a \cos[c + dx])^2 \left(\frac{a b^2}{(-a^2 + b^2)^2 (b + a \cos[c + dx])} - \frac{2 (-2 a b + a^2 \cos[c + dx] + b^2 \cos[c + dx]) \operatorname{Csc}[c + dx]^2}{3 (-a^2 + b^2)^2} \right) \right. \\
& \left. \frac{\sin[c + dx]}{\tan[c + dx]^2} \right) / (d \\
& \frac{(a + b \sec[c + dx])^2}{(e \sin[c + dx])^{5/2}}
\end{aligned}$$

■ **Problem 249: Unable to integrate problem.**

$$\int \sqrt{a + b \sec[e + f x]} dx$$

Optimal (type 4, 125 leaves, 1 step):

$$-\frac{1}{\sqrt{a+b} f}$$

$$2 \cot[e + f x] \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b \sec[e + f x]}}\right], \frac{a-b}{a+b}\right] \sqrt{\frac{b(1-\sec[e + f x])}{a+b \sec[e + f x]}} \sqrt{\frac{b(1+\sec[e + f x])}{a+b \sec[e + f x]}} (a+b \sec[e + f x])$$

Result (type 8, 16 leaves):

$$\int \sqrt{a+b \sec[e + f x]} dx$$

■ **Problem 251: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \sec[e + f x])^{3/2} dx$$

Optimal (type 4, 309 leaves, 5 steps):

$$-\frac{1}{f} (a-b) \sqrt{a+b} \cot[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e + f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e + f x])}{a+b}} \sqrt{-\frac{b(1+\sec[e + f x])}{a-b}} + \frac{1}{f}$$

$$2(2a-b) \sqrt{a+b} \cot[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e + f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e + f x])}{a+b}} \sqrt{-\frac{b(1+\sec[e + f x])}{a-b}} -$$

$$\frac{2a \sqrt{a+b} \cot[e + f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e + f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e + f x])}{a+b}} \sqrt{-\frac{b(1+\sec[e + f x])}{a-b}}}{f}$$

Result (type 4, 882 leaves):

$$\begin{aligned}
& \frac{2 b \cos [e+f x] (a+b \sec [e+f x])^{3 / 2} \sin [e+f x]}{f (b+a \cos [e+f x])} + \\
& \left(2 (a+b \sec [e+f x])^{3 / 2} \left(a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] + b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] - 2 a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right]^3 + \right. \right. \\
& a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right]^5 - b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right]^5 + 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (e+f x) \right]^2 + b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} + \\
& 2 i a^2 \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (e+f x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (e+f x) \right]^2 + b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} - i (a-b) b \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (e+f x) \right]^2 + b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} - \\
& i (a-b)^2 \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2} \\
& \left. \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (e+f x) \right]^2 + b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} \right) \right] / \\
& \left(\sqrt{\frac{-a+b}{a+b}} f (b+a \cos [e+f x])^{3 / 2} \sec [e+f x]^{3 / 2} \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (e+f x) \right]^2}} \left(-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right. \\
& \left. \left(1 + \tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^{3 / 2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (e+f x) \right]^2 + b \tan \left[\frac{1}{2} (e+f x) \right]^2}{1 + \tan \left[\frac{1}{2} (e+f x) \right]^2}} \right)
\end{aligned}$$

■ **Problem 253: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{2 \sqrt{a + b} \operatorname{Cot}[e + f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a f}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

■ **Problem 255: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 4, 347 leaves, 6 steps):

$$\frac{2 \operatorname{Cot}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a \sqrt{a + b} f} - \frac{2 \operatorname{Cot}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a \sqrt{a + b} f} - \frac{2 \sqrt{a + b} \operatorname{Cot}[e + f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a^2 f} + \frac{2 b^2 \operatorname{Tan}[e + f x]}{a (a^2 - b^2) f \sqrt{a + b \operatorname{Sec}[e + f x]}}$$

Result (type 4, 1249 leaves):

$$\frac{(b + a \operatorname{Cos}[e + f x])^2 \operatorname{Sec}[e + f x]^2 \left(\frac{2 b \operatorname{Sin}[e+f x]}{a (-a^2+b^2)} + \frac{2 b^2 \operatorname{Sin}[e+f x]}{a (a^2-b^2) (b+a \operatorname{Cos}[e+f x])} \right)}{f (a + b \operatorname{Sec}[e + f x])^{3/2}} + \left(2 (b + a \operatorname{Cos}[e + f x])^{3/2} \operatorname{Sec}[e + f x]^{3/2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}} \right)$$

$$\begin{aligned}
& \left(a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] - 2 a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^3 + a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^5 - \right. \\
& b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^5 - 2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + 2 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} - \\
& 2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + 2 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} - \\
& i(a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + i\left(a^2+a b-2 b^2\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}\right) \Bigg) / \\
& \left(a \sqrt{\frac{-a+b}{a+b}} \left(a^2-b^2\right) f(a+b \operatorname{Sec}[e+f x])^{3/2} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}} \right.
\end{aligned}$$

$$\left(a \left(-1 + \tan\left[\frac{1}{2}(e + f x)\right] \right)^2 - b \left(1 + \tan\left[\frac{1}{2}(e + f x)\right] \right)^2 \right)$$

■ **Problem 256: Unable to integrate problem.**

$$\int \frac{\csc[e + f x]^2}{(a + b \sec[e + f x])^{3/2}} dx$$

Optimal (type 4, 318 leaves, 6 steps):

$$\frac{4 a \cot[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e+f x])}{a+b}} \sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}}{(a-b)(a+b)^{3/2} f} -$$

$$\frac{(3 a-b) \cot[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e+f x])}{a+b}} \sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}}{(a-b)(a+b)^{3/2} f} -$$

$$\frac{\cot[e + f x]}{f(a+b \sec[e + f x])^{3/2}} + \frac{b^2 \tan[e + f x]}{(a^2 - b^2) f(a+b \sec[e + f x])^{3/2}} + \frac{4 a b^2 \tan[e + f x]}{(a^2 - b^2)^2 f \sqrt{a+b \sec[e + f x]}}$$

Result (type 8, 25 leaves):

$$\int \frac{\csc[e + f x]^2}{(a + b \sec[e + f x])^{3/2}} dx$$

■ **Problem 260: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin[c + d x])^m}{a + b \sec[c + d x]} dx$$

Optimal (type 6, 232 leaves, 4 steps):

$$-\frac{1}{a^2 d(1-m)} b e \operatorname{AppellF1}\left[1-m, \frac{1-m}{2}, \frac{1-m}{2}, 2-m, -\frac{a-b}{b+a \cos[c+d x]}, \frac{a+b}{b+a \cos[c+d x]}\right] \left(-\frac{a(1-\cos[c+d x])}{b+a \cos[c+d x]}\right)^{\frac{1-m}{2}}$$

$$\left(\frac{a(1+\cos[c+d x])}{b+a \cos[c+d x]}\right)^{\frac{1-m}{2}} (e \sin[c+d x])^{-1+m} + \frac{\cos[c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2[c+d x]\right] (e \sin[c+d x])^{1+m}}{a d e(1+m) \sqrt{\cos[c+d x]^2}}$$

Result (type 6, 3387 leaves):

$$-\left(\left(2 \sin[c+d x]^m (e \sin[c+d x])^m \tan\left[\frac{1}{2}(c+d x)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \left(\sec\left[\frac{1}{2}(c+d x)\right]^2\right)^m -\right.\right.\right.$$

$$\begin{aligned}
& \left(b (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \right) / \\
& \left((b+a \operatorname{Cos}[c+dx]) \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad 2 \left((-a+b) \operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + (a+b) m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, \right. \right. \\
& \quad \quad \left. \left. 1, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left. \right) \right) / \left(a d (1+m) (a+b \operatorname{Sec}[c+dx]) \right) \\
& \left(-\frac{1}{a(1+m)} \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sin}[c+dx]^m \left(-\operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right)^m - \right. \right. \\
& \quad \left(b (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \right) / \\
& \quad \left((b+a \operatorname{Cos}[c+dx]) \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \quad 2 \left((-a+b) \operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + (a+b) m \right. \\
& \quad \quad \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \left. \right) \right) - \frac{1}{a(1+m)} \\
& 2 m \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]^{-1+m} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-\operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right)^m - \right. \\
& \quad \left(b (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \right) / \\
& \quad \left((b+a \operatorname{Cos}[c+dx]) \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((-a+b) \operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \quad \left. (a+b) m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \frac{1}{a(1+m)} \\
& 2 \operatorname{Sin}[c+dx]^m \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(-m \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right)^m \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \quad \left. \left(b(a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right) \right) / \\
& \quad \left((b+a \operatorname{Cos}[c+dx]) \left(- (a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left((-a+b) \operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (a+b) m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) - \\
& \quad \left(a b (a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sin}[c+dx] \right) / \\
& \quad \left((b+a \operatorname{Cos}[c+dx])^2 \left(- (a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left((-a+b) \operatorname{AppellF1} \left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (a+b) m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) - \\
& \quad \left(b(a+b)(3+m) \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \left(\frac{1}{(a+b)(3+m)} (a-b)(1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, m, 2, 1 + \frac{3+m}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\left]\sec\left[\frac{1}{2}(c+dx)\right]^2\tan\left[\frac{1}{2}(c+dx)\right]-\frac{1}{3+m}m(1+m)\right. \\
& \left.\text{AppellF1}\left[1+\frac{1+m}{2}, 1+m, 1, 1+\frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(c+dx)\right]^2\tan\left[\frac{1}{2}(c+dx)\right]\right]\right) / \\
& \left((b+a\cos[c+dx])\left(- (a+b)(3+m)\text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]+ \right.\right. \\
& \left. 2\left(- (a+b)\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]+ (a+b)m\text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \right.\right. \right. \\
& \left. \left. \left. \frac{5+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\right)\tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)-\frac{1}{2}(1+m)\csc\left[\frac{1}{2}(c+dx)\right]\sec\left[\frac{1}{2}(c+dx)\right] \\
& \left(\sec\left[\frac{1}{2}(c+dx)\right]^2\right)^m\left(-\text{Hypergeometric2F1}\left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]+ \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-1-m}\right)+ \\
& \left(b(a+b)(3+m)\text{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\cos\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \left(2\left(- (a+b)\text{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]+ (a+b)m \right.\right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\right)\sec\left[\frac{1}{2}(c+dx)\right]^2\tan\left[\frac{1}{2}(c+dx)\right]- \right.\right. \\
& (a+b)(3+m)\left(\frac{1}{(a+b)(3+m)}(a-b)(1+m)\text{AppellF1}\left[1+\frac{1+m}{2}, m, 2, 1+\frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \right.\right. \\
& \left. \left. \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(c+dx)\right]^2\tan\left[\frac{1}{2}(c+dx)\right]-\frac{1}{3+m}m(1+m)\text{AppellF1}\left[1+\frac{1+m}{2}, 1+m, \right.\right. \\
& \left. \left. \left. 1, 1+\frac{3+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right]\sec\left[\frac{1}{2}(c+dx)\right]^2\tan\left[\frac{1}{2}(c+dx)\right]\right)\right)+ \\
& \left. 2\tan\left[\frac{1}{2}(c+dx)\right]^2\left(- (a+b)\left(\frac{1}{(a+b)(5+m)}2(a-b)(3+m)\text{AppellF1}\left[1+\frac{3+m}{2}, m, 3, 1+\frac{5+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \right.\right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{5+m} m(3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. 2, 1 + \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \right) + \right. \\
& \left. (a+b) m \left(\frac{1}{(a+b)(5+m)} (a-b)(3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{5+m} (1+m)(3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. 2+m, 1, 1 + \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \right) \right) \right) \right) / \\
& \left((b+a \operatorname{Cos}[c+dx]) \left(- (a+b)(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) + \right. \\
& \left. 2 \left((-a+b) \operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) + \right. \\
& \left. \left. (a+b) m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 261: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Sin}[c+dx])^m}{(a+b \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 6, 405 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{a^3 d (1-m)} 2 b e \operatorname{AppellF1}\left[1-m, \frac{1-m}{2}, \frac{1-m}{2}, 2-m, -\frac{a-b}{b+a \cos [c+d x]}, \frac{a+b}{b+a \cos [c+d x]}\right] \\
& \left(-\frac{a(1-\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} \left(\frac{a(1+\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} (e \sin [c+d x])^{-1+m} + \\
& \left(b^2 e \operatorname{AppellF1}\left[2-m, \frac{1-m}{2}, \frac{1-m}{2}, 3-m, -\frac{a-b}{b+a \cos [c+d x]}, \frac{a+b}{b+a \cos [c+d x]}\right] \left(-\frac{a(1-\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}}\right. \\
& \left.\left(\frac{a(1+\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} (e \sin [c+d x])^{-1+m}\right) / \left(a^3 d (2-m) (b+a \cos [c+d x])\right) + \\
& \frac{\cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] (e \sin [c+d x])^{1+m}}{a^2 d e (1+m) \sqrt{\cos [c+d x]^2}}
\end{aligned}$$

Result (type 6, 9072 leaves) : Display of huge result suppressed!

■ **Problem 262: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e \sin [c+d x])^m}{(a+b \sec [c+d x])^3} dx$$

Optimal (type 6, 580 leaves, 7 steps) :

$$\begin{aligned}
& -\frac{1}{a^4 d (1-m)} 3 b e \operatorname{AppellF1}\left[1-m, \frac{1-m}{2}, \frac{1-m}{2}, 2-m, -\frac{a-b}{b+a \cos [c+d x]}, \frac{a+b}{b+a \cos [c+d x]}\right] \\
& \left(-\frac{a(1-\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} \left(\frac{a(1+\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} (e \sin [c+d x])^{-1+m} - \\
& \left(b^3 e \operatorname{AppellF1}\left[3-m, \frac{1-m}{2}, \frac{1-m}{2}, 4-m, -\frac{a-b}{b+a \cos [c+d x]}, \frac{a+b}{b+a \cos [c+d x]}\right] \left(-\frac{a(1-\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}}\right. \\
& \left.\left(\frac{a(1+\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} (e \sin [c+d x])^{-1+m}\right) / \left(a^4 d (3-m) (b+a \cos [c+d x])^2\right) + \\
& \left(3 b^2 e \operatorname{AppellF1}\left[2-m, \frac{1-m}{2}, \frac{1-m}{2}, 3-m, -\frac{a-b}{b+a \cos [c+d x]}, \frac{a+b}{b+a \cos [c+d x]}\right] \left(-\frac{a(1-\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}}\right. \\
& \left.\left(\frac{a(1+\cos [c+d x])}{b+a \cos [c+d x]}\right)^{\frac{1-m}{2}} (e \sin [c+d x])^{-1+m}\right) / \left(a^4 d (2-m) (b+a \cos [c+d x])\right) + \\
& \frac{\cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \sin [c+d x]^2\right] (e \sin [c+d x])^{1+m}}{a^3 d e (1+m) \sqrt{\cos [c+d x]^2}}
\end{aligned}$$

Result (type 6, 12336 leaves) :

$$\begin{aligned}
 & \left((e \operatorname{Sin}[c + d x])^m \left(\left((a + b) (3 + m) \operatorname{AppellF1} \left[\frac{1 + m}{2}, 1 + m, 3, \frac{3 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right)^{1+m} \right) / \right. \\
 & \left. \left((1 + m) \left(a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^3 \right. \right. \\
 & \left. \left(- (a + b) (3 + m) \operatorname{AppellF1} \left[\frac{1 + m}{2}, 1 + m, 3, \frac{3 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] + \right. \right. \\
 & \left. \left. 2 \left(-3 (a - b) \operatorname{AppellF1} \left[\frac{3 + m}{2}, 1 + m, 4, \frac{5 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] + (a + b) (1 + m) \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1} \left[\frac{3 + m}{2}, 2 + m, 3, \frac{5 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) - \\
 & \left(3 (a + b) (5 + m) \operatorname{AppellF1} \left[\frac{3 + m}{2}, 1 + m, 3, \frac{5 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right. \\
 & \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right)^{1+m} \right) / \left((3 + m) \left(a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^3 \right. \\
 & \left. \left(- (a + b) (5 + m) \operatorname{AppellF1} \left[\frac{3 + m}{2}, 1 + m, 3, \frac{5 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] + \right. \right. \\
 & \left. \left. 2 \left(-3 (a - b) \operatorname{AppellF1} \left[\frac{5 + m}{2}, 1 + m, 4, \frac{7 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] + \right. \right. \right. \\
 & \left. \left. \left. (a + b) (1 + m) \operatorname{AppellF1} \left[\frac{5 + m}{2}, 2 + m, 3, \frac{7 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \right) + \\
 & \left(3 (a + b) (7 + m) \operatorname{AppellF1} \left[\frac{5 + m}{2}, 1 + m, 3, \frac{7 + m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b} \right] \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^4 \right. \\
 & \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right)^{1+m} \right) / \left((5 + m) \left(a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^3 \right)
 \end{aligned}$$

$$\begin{aligned}
& \left(- (a+b) (7+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) - \\
& \left((a+b) (9+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^6 \right. \\
& \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) / \left((7+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \right. \\
& \left. - (a+b) (9+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) \Bigg) / \\
& \left(d (a+b \operatorname{Sec}[c+dx])^3 \right) \left(- \left(\left(3 (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right. \right. \right. \\
& \left. \left. \left(-a \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + b \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) \right) / \\
& \left((1+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^4 \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 4, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 2+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) + \\
& \left((a+b) (3+m) \left(\frac{1}{(a+b) (3+m)} 3 (a-b) (1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+m, 4, 1 + \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{3+m} (1+m)^2 \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 2+m, 3, 1 + \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) \right) / \\
& \left((1+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 4, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right) \right) + \\
& \quad \left. \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 2+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right) + \\
& \left(9 (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \left(-a \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + b \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) \right) / \\
& \left((3+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^4 \left(- (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 4, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right) \right) + \\
& \quad \left. \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 2+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) / \left((3+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \right. \\
& \quad \left(- (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 4, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 2+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \\
& \left(3 (a+b) (5+m) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(\frac{1}{(a+b) (5+m)} - 3 (a-b) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 4, 1 + \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 2+m, 3, 1 + \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right) / \\
& \left((3+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \left(- (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 4, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 2+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \\
& \left(9 (a+b) (7+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^4 \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \Big/ \\
& \left((5+m) \left(a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^4 \left(- (a+b) (7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) + \\
& \left(6 (a+b) (7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \right) \Big/ \left((5+m) \left(a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \right. \\
& \quad \left(- (a+b) (7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left(-3 (a-b) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) + \\
& \left(3 (a+b) (7+m) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 \left(\frac{1}{(a+b) (7+m)} - 3 (a-b) (5+m) \operatorname{AppellF1}\left[1 + \frac{5+m}{2}, 1+m, 4, 1 + \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{7+m} (1+m) (5+m) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1 + \frac{5+m}{2}, 2+m, 3, 1 + \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Big)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \Bigg/ \left((5+m) \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \right. \\
& \left. - (a+b) (7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \\
& \left. 2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) + \\
& \left(3(a+b) (9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^6 \right. \\
& \left. - a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \Bigg/ \\
& \left((7+m) \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^4 \left(- (a+b) (9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + 2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \right. \\
& \left. \left. (a+b) (1+m) \operatorname{AppellF1}\left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) - \\
& \left(3(a+b) (9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^5 \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \right) \Bigg/ \left((7+m) \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \right. \\
& \left. - (a+b) (9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \quad \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) - \\
& \left((a+b) (9+m) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^6 \left(\frac{1}{(a+b) (9+m)} 3 (a-b) (7+m) \operatorname{AppellF1} \left[1 + \frac{7+m}{2}, 1+m, 4, 1 + \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{9+m} (1+m) (7+m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[1 + \frac{7+m}{2}, 2+m, 3, 1 + \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \\
& \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \Bigg) / \left((7+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \right) \\
& \left(- (a+b) (9+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \quad \left. 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \Bigg) + \\
& \left((a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \right. \\
& \quad \left. \left(- \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \left(\left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \right) \\
& \left(- (a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 4, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \quad \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 2+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\
& \left(3 (a+b) (1+m) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((3+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \left(- (a+b) (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 4, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 2+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) + \\
& \left(3 (a+b) (1+m) (7+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^4 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \left((5+m) \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \left(- (a+b) (7+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3 (a-b) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (1+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left((a+b)(1+m)(9+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^6 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^m \left(-\frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} + \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2 \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \right) \right) / \\
& \quad \left((7+m) \left(a+b-a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3 \left(- (a+b)(9+m) \operatorname{AppellF1} \left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2 \left(-3(a-b) \operatorname{AppellF1} \left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1} \left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) - \\
& \quad \left((a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, 1+m, 3, \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \left(\frac{\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right)^{1+m} \right. \\
& \quad \left. \left(2 \left(-3(a-b) \operatorname{AppellF1} \left[\frac{3+m}{2}, 1+m, 4, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + (a+b)(1+m) \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, 2+m, 3, \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \quad \left. (a+b)(3+m) \left(\frac{1}{(a+b)(3+m)} 3(a-b)(1+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1+m, 4, 1 + \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{3+m} (1+m)^2 \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 2+m, 3, 1 + \frac{3+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \quad \left. \left. \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) + 2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left(-3(a-b) \left(\frac{1}{(a+b)(5+m)} \right) \right. \\
& \quad \left. 4(a-b)(3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1+m, 5, 1 + \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right. \\
& \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \frac{1}{5+m} (1+m)(3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 2+m, 4, 1 + \frac{5+m}{2}, -\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-3(a-b) \left(\frac{1}{(a+b)(7+m)} 4(a-b)(5+m) \operatorname{AppellF1}\left[1+\frac{5+m}{2}, 1+m, 5, 1+\frac{7+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{7+m}(1+m)(5+m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[1+\frac{5+m}{2}, 2+m, 4, 1+\frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& (a+b)(1+m) \left(\frac{1}{(a+b)(7+m)} 3(a-b)(5+m) \operatorname{AppellF1}\left[1+\frac{5+m}{2}, 2+m, 4, 1+\frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{7+m}(2+m)(5+m) \operatorname{AppellF1}\left[1+\frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. 3+m, 3, 1+\frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left((3+m) \left(a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \left(- (a+b)(5+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 3, \frac{5+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + 2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 4, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right) + \right. \\
& \quad \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 2+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) - \\
& \left(3(a+b)(7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 \right. \\
& \quad \left. \left(\frac{\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \left(2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \right) \right) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - (a+b)(7+m) \left(\frac{1}{(a+b)(7+m)} {}_3F_2\left(1+\frac{5+m}{2}, 1+m, 4, 1+\frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{7+m}(1+m)(5+m) \right. \\
& \left. \operatorname{AppellF1}\left[1+\frac{5+m}{2}, 2+m, 3, 1+\frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) + \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left(-3(a-b) \left(\frac{1}{(a+b)(9+m)} {}_4F_3\left(1+\frac{7+m}{2}, 1+m, 5, 1+\frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{9+m}(1+m)(7+m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1+\frac{7+m}{2}, 2+m, 4, 1+\frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) + \\
& (a+b)(1+m) \left(\frac{1}{(a+b)(9+m)} {}_3F_2\left(1+\frac{7+m}{2}, 2+m, 4, 1+\frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{9+m}(2+m)(7+m) \operatorname{AppellF1}\left[1+\frac{7+m}{2}, 3+m, 3, 1+\frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Bigg) \Bigg) / \\
& \left((5+m) \left(a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \left(- (a+b)(7+m) \operatorname{AppellF1}\left[\frac{5+m}{2}, 1+m, 3, \frac{7+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) + 2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 4, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right. \right. \\
& \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 2+m, 3, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) \Bigg) + \\
& \left((a+b)(9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^6 \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{1+m} \left(2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \right. \\
& \quad \left. \left. (a+b)(1+m) \operatorname{AppellF1}\left[\frac{9+m}{2}, 2+m, 3, \frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - (a+b)(9+m) \left(\frac{1}{(a+b)(9+m)} 3(a-b)(7+m) \operatorname{AppellF1}\left[1+\frac{7+m}{2}, 1+m, 4, 1+\frac{9+m}{2}, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \frac{1}{9+m}(1+m)(7+m) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+\frac{7+m}{2}, 2+m, 3, 1+\frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-3(a-b) \left(\frac{1}{(a+b)(11+m)} 4(a-b)(9+m) \operatorname{AppellF1}\left[1+\frac{9+m}{2}, 1+m, 5, 1+\frac{11+m}{2}, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \frac{1}{11+m}(1+m)(9+m) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+\frac{9+m}{2}, 2+m, 4, 1+\frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& (a+b)(1+m) \left(\frac{1}{(a+b)(11+m)} 3(a-b)(9+m) \operatorname{AppellF1}\left[1+\frac{9+m}{2}, 2+m, 4, 1+\frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \frac{1}{11+m}(2+m)(9+m) \operatorname{AppellF1}\left[1+\frac{9+m}{2}, \right. \\
& \quad \left. \left. 3+m, 3, 1+\frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Big/ \\
& \left((7+m) \left(a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^3 \left(- (a+b)(9+m) \operatorname{AppellF1}\left[\frac{7+m}{2}, 1+m, 3, \frac{9+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] + 2 \left(-3(a-b) \operatorname{AppellF1}\left[\frac{9+m}{2}, 1+m, 4, \frac{11+m}{2}, -\tan\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \right)^n \sin[c+dx] + \frac{4}{3} (a-b) (-2+n) (b+a \cos[c+dx])^n \left(\sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-2+n} \\
& \left(- \left(9 \operatorname{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) / \right. \\
& \quad \left(-2an \operatorname{AppellF1}\left[3, n, 1-n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + \right. \\
& \quad (a-b) \left(2n \operatorname{AppellF1}\left[3, 1+n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 3 \operatorname{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \left(4 \operatorname{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \right) / \right. \\
& \quad \left(-an \operatorname{AppellF1}\left[4, n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + (a-b) \left(n \operatorname{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 2 \operatorname{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \right)^n \tan\left[\frac{1}{2}(c+dx)\right] + \frac{4}{3} (a-b) (b+a \cos[c+dx])^n \left(\sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-2+n} \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \right)^n \\
& \left(- \left(9 \sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \left(1 / (3(a-b)) 4an \operatorname{AppellF1}\left[3, n, 1-n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\right. \right. \right. \\
& \quad \left. \left. \frac{1}{2}(c+dx) \right] - \frac{4}{3} n \operatorname{AppellF1}\left[3, 1+n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / \right. \\
& \quad \left(-2an \operatorname{AppellF1}\left[3, n, 1-n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + (a-b) \right. \\
& \quad \left. \left(2n \operatorname{AppellF1}\left[3, 1+n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 3 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right) + \\
& \left(4 \left(1 / (2(a-b)) 3 a n \text{AppellF1}\left[4, n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \left. \left. \frac{3}{2} n \text{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(-a n \text{AppellF1}\left[4, n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + (a-b) \left(n \text{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 2 \text{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)\right) - \\
& \left(9 \text{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) / \\
& \left(-2 a n \text{AppellF1}\left[3, n, 1-n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + \right. \\
& \left.(a-b) \left(2 n \text{AppellF1}\left[3, 1+n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + \right. \right. \\
& \left. \left. 3 \text{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)\right) + \\
& \left(9 \text{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(-2 a n \left(-\frac{1}{2(a-b)} 3 a (1-n) \right. \right. \right. \\
& \left. \left. \text{AppellF1}\left[4, n, 2-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right] - \frac{3}{2} n \text{AppellF1}\left[4, \right. \right. \right. \\
& \left. \left. \left. 1+n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right]\right)\right) + (a-b) \left(3 \text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{3(a-b)} 4 a n \operatorname{AppellF1} \left[3, n, 1-n, 4, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \quad \left. \frac{4}{3} n \operatorname{AppellF1} \left[3, 1+n, -n, 4, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] \right) + \\
& 2 n \left(\frac{1}{2(a-b)} 3 a n \operatorname{AppellF1} \left[4, 1+n, 1-n, 5, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \quad \left. \frac{3}{2} (1+n) \operatorname{AppellF1} \left[4, 2+n, -n, 5, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] \right) + \\
& \quad \left. \left. \left. \left. \left. 3 \operatorname{AppellF1} \left[2, n, -n, 3, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \tan \left[\frac{1}{2} (c+dx) \right] \right] \right) \right) \right) \right) / \\
& \left(-2 a n \operatorname{AppellF1} \left[3, n, 1-n, 4, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] + (a-b) \right. \\
& \quad \left(2 n \operatorname{AppellF1} \left[3, 1+n, -n, 4, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] + \right. \\
& \quad \left. \left. \left. \left. \left. 3 \operatorname{AppellF1} \left[2, n, -n, 3, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \sec \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \right) \right) \right) \right) - \\
& \left(4 \operatorname{AppellF1} \left[3, n, -n, 4, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \left(-a n \left(-\frac{1}{5(a-b)} 8 a (1-n) \operatorname{AppellF1} \left[5, n, 2-n, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 6, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] - \frac{8}{5} n \operatorname{AppellF1} \left[5, 1+n, 1-n, 6, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] \right) \right) + (a-b) \left(2 \sec \left[\frac{1}{2} (c+dx) \right]^2 \right) \right. \\
& \quad \left. \left(\frac{1}{2(a-b)} 3 a n \operatorname{AppellF1} \left[4, n, 1-n, 5, 2 \cos \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \cos \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \cos \left[\frac{1}{2} (c+dx) \right] \sin \left[\frac{1}{2} (c+dx) \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{3}{2} n \operatorname{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right]\right] + \\
& n \left(\frac{1}{5(a-b)} 8 a n \operatorname{AppellF1}\left[5, 1+n, 1-n, 6, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \left. \frac{8}{5} (1+n) \operatorname{AppellF1}\left[5, 2+n, -n, 6, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \cos\left[\frac{1}{2}(c+dx)\right] \sin\left[\frac{1}{2}(c+dx)\right]\right] + \\
& \left. 2 \operatorname{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) \Bigg) / \\
& \left(-a n \operatorname{AppellF1}\left[4, n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + (a-b) \left(n \operatorname{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \left. \left. \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 2 \operatorname{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \Bigg) + \\
& \frac{4}{3} (a-b) n (b+a \cos[c+dx])^n \left(\sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{-2+n} \left(- \left(9 \operatorname{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) / \left(-2 a n \operatorname{AppellF1}\left[3, n, 1-n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + \right. \right. \\
& \left. \left. (a-b) \left(2 n \operatorname{AppellF1}\left[3, 1+n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + 3 \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[2, n, -n, 3, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \Bigg) + \\
& \left(4 \operatorname{AppellF1}\left[3, n, -n, 4, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] \right) / \left(-a n \operatorname{AppellF1}\left[4, n, 1-n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \left. \left. \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + (a-b) \left(n \operatorname{AppellF1}\left[4, 1+n, -n, 5, 2 \cos\left[\frac{1}{2}(c+dx)\right]^2, \frac{2a \cos\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] + \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right) + \\
& \left(2 a(a-b) n \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] (b+a \operatorname{Cos}[c+d x])^{-1+n} \operatorname{Sec}[c+d x]^n \operatorname{Sin}[c+d x]\right) / \\
& \left(-a n \operatorname{AppellF1}\left[2, n, 1-n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] + (a-b)\left(n \operatorname{AppellF1}\left[2, 1+n, -n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \right.\right.\right. \\
& \left.\left.\left.\frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] + \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)\right) - \\
& \left(2(a-b) n \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] (b+a \operatorname{Cos}[c+d x])^n \operatorname{Sec}[c+d x]^{1+n} \operatorname{Sin}[c+d x]\right) / \\
& \left(-a n \operatorname{AppellF1}\left[2, n, 1-n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] + (a-b)\left(n \operatorname{AppellF1}\left[2, 1+n, -n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \right.\right.\right. \\
& \left.\left.\left.\frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] + \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)\right) + \\
& \left(2(a-b) \operatorname{AppellF1}\left[1, n, -n, 2, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] (b+a \operatorname{Cos}[c+d x])^n \operatorname{Sec}[c+d x]^n\right. \\
& \left(-a n\left(-\frac{1}{3(a-b)} 4 a(1-n) \operatorname{AppellF1}\left[3, n, 2-n, 4, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] - \frac{4}{3}\right.\right. \\
& \left.\left. n \operatorname{AppellF1}\left[3, 1+n, 1-n, 4, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right) + \\
& (a-b)\left(\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\left(\frac{1}{a-b} a n \operatorname{AppellF1}\left[2, n, 1-n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{2}(c+d x)\right.\right. \\
& \left.\left.- n \operatorname{AppellF1}\left[2, 1+n, -n, 3, 2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2, \frac{2 a \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2}{a-b}\right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right) +
\end{aligned}$$

$$\begin{aligned}
& n \left(\frac{1}{3(a-b)} 4 a n \operatorname{AppellF1} \left[3, 1+n, 1-n, 4, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \quad \left. \frac{4}{3} (1+n) \operatorname{AppellF1} \left[3, 2+n, -n, 4, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right) + \\
& \quad \operatorname{AppellF1} \left[1, n, -n, 2, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \Big) \Big) \Big) \Big) / \\
& \left(-a n \operatorname{AppellF1} \left[2, n, 1-n, 3, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] + (a-b) \left(n \operatorname{AppellF1} \left[2, 1+n, -n, 3, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] + \operatorname{AppellF1} \left[1, n, -n, 2, 2 \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2, \frac{2 a \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2}{a-b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 271: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[c+dx] (a+b \operatorname{Sec}[c+dx])^n dx$$

Optimal (type 5, 115 leaves, 4 steps):

$$\frac{\operatorname{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a-b} \right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{2(a-b)d(1+n)} - \frac{\operatorname{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{2(a+b)d(1+n)}$$

Result (type 6, 3438 leaves):

$$\begin{aligned}
& \left(b(-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{(-a+b) \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2b}, \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right. \\
& \quad \left. (b+a \operatorname{Cos}[c+dx])^n \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx]^{-1+n} (a+b \operatorname{Sec}[c+dx])^n \right) \Big) / \\
& \left(d(-1+n) \left(2b(-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{(-a+b) \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2b}, \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right] \operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 + \right. \right. \\
& \quad \left. \left(- (a-b) n \operatorname{AppellF1} \left[2-n, 1-n, 1, 3-n, \frac{(-a+b) \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{2b}, \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 b \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] \cos [c+d x] \Bigg) \\
& \left(-\left(\left(b(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right]\right.\right.\right. \\
& \quad \left.\left.\left.(b+a \cos [c+d x]\right)^n \cot \left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{-1+n}\right) / \left((-1+n)\right.\right. \\
& \quad \left.\left.\left(2 b(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] \cos \left[\frac{1}{2}(c+d x)\right]^2 +\right.\right.\right. \\
& \quad \left.\left.\left(- (a-b) n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] - 2 b\right.\right.\right. \\
& \quad \left.\left.\left.\operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] \cos [c+d x]\right)\right)\right) - \\
& \left(a b(-2+n) n \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right]\right. \\
& \quad \left.\left.(b+a \cos [c+d x]\right)^{-1+n} \cot \left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{-1+n} \sin [c+d x]\right) / \left((-1+n)\right. \\
& \quad \left.\left(2 b(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] \cos \left[\frac{1}{2}(c+d x)\right]^2 +\right.\right. \\
& \quad \left.\left.\left(- (a-b) n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] -\right.\right.\right. \\
& \quad \left.\left.\left.2 b \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] \cos [c+d x]\right)\right)\right) + \\
& \left(b(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{2 b}, \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right]\right.
\end{aligned}$$

$$\begin{aligned}
& \left. (b + a \cos [c + d x])^n \cot \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^n \sin [c + d x] \right) / \\
& \left(2 b (-2 + n) \operatorname{AppellF1} \left[1 - n, -n, 1, 2 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \cos \left[\frac{1}{2} (c + d x) \right]^2 + \right. \\
& \left. - (a - b) n \operatorname{AppellF1} \left[2 - n, 1 - n, 1, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \left. 2 b \operatorname{AppellF1} \left[2 - n, -n, 2, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \cos [c + d x] \left. + \left(b (-2 + n) \right. \right. \\
& \left. (b + a \cos [c + d x])^n \cot \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^{-1+n} \left(\frac{1}{2 - n} (1 - n) \operatorname{AppellF1} \left[2 - n, -n, 2, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \right. \right. \right. \\
& \left. \left. \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right) \left(-\sec \left[\frac{1}{2} (c + d x) \right]^2 \sin [c + d x] + \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \right) \right. \\
& \left. \frac{1}{2 - n} (1 - n) n \operatorname{AppellF1} \left[2 - n, 1 - n, 1, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \right. \\
& \left. \left. \left(-\frac{(-a + b) \sec \left[\frac{1}{2} (c + d x) \right]^2 \sin [c + d x]}{2 b} + \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]}{2 b} \right) \right) \right) / \left((-1 + n) \right. \\
& \left(2 b (-2 + n) \operatorname{AppellF1} \left[1 - n, -n, 1, 2 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \cos \left[\frac{1}{2} (c + d x) \right]^2 + \right. \\
& \left. - (a - b) n \operatorname{AppellF1} \left[2 - n, 1 - n, 1, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \left. 2 b \operatorname{AppellF1} \left[2 - n, -n, 2, 3 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \cos [c + d x] \left. \right) \left. - \right. \\
& \left(b (-2 + n) \operatorname{AppellF1} \left[1 - n, -n, 1, 2 - n, \frac{(-a + b) \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2}{2 b}, \cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right] \right. \\
& \left. (b + a \cos [c + d x])^n \cot \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^{-1+n} \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-2b(-2+n) \operatorname{AppellF1} \left[1-n, -n, 1, 2-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \cos \left[\frac{1}{2}(c+dx) \right] \sin \left[\frac{1}{2}(c+dx) \right] - \right. \\
& \left. \left(- (a-b)n \operatorname{AppellF1} \left[2-n, 1-n, 1, 3-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] - \right. \right. \\
& \left. \left. 2b \operatorname{AppellF1} \left[2-n, -n, 2, 3-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right) \sin[c+dx] + \right. \\
& \left. 2b(-2+n) \cos \left[\frac{1}{2}(c+dx) \right]^2 \left(\frac{1}{2-n} (1-n) \operatorname{AppellF1} \left[2-n, -n, 2, 3-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \right. \right. \right. \\
& \left. \left. \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \left(-\operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \tan \left[\frac{1}{2}(c+dx) \right] \right) - \frac{1}{2-n} \right. \right. \\
& \left. \left. (1-n)n \operatorname{AppellF1} \left[2-n, 1-n, 1, 3-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right. \right. \\
& \left. \left. \left(-\frac{(-a+b) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \sin[c+dx]}{2b} + \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \tan \left[\frac{1}{2}(c+dx) \right]}{2b} \right) \right) \right) + \\
& \cos[c+dx] \left(-2b \left(\frac{1}{3-n} 2(2-n) \operatorname{AppellF1} \left[3-n, -n, 3, 4-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \right. \right. \right. \right. \\
& \left. \left. \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \left(-\operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \tan \left[\frac{1}{2}(c+dx) \right] \right) - \frac{1}{3-n} \right. \right. \\
& \left. \left. (2-n)n \operatorname{AppellF1} \left[3-n, 1-n, 2, 4-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right. \right. \\
& \left. \left. \left(-\frac{(-a+b) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \sin[c+dx]}{2b} + \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \tan \left[\frac{1}{2}(c+dx) \right]}{2b} \right) \right) \right) - \\
& (a-b)n \left(\frac{1}{3-n} (2-n) \operatorname{AppellF1} \left[3-n, 1-n, 2, 4-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right. \\
& \left. \left(-\operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \tan \left[\frac{1}{2}(c+dx) \right] \right) + \frac{1}{3-n} \right. \\
& \left. \left. (1-n)(2-n) \operatorname{AppellF1} \left[3-n, 2-n, 1, 4-n, \frac{(-a+b) \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2}{2b}, \cos[c+dx] \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left((a+b) (b+a \cos [c+dx])^n \csc [c+dx]^2 \sec [c+dx]^n (a+b \sec [c+dx])^n \right. \\
& \left. \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \tan \left[\frac{1}{2} (c+dx) \right] \right) \right) / \right. \\
& \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + \right. \\
& \left. 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + \right. \\
& \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) - \\
& \operatorname{AppellF1} \left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] / \left((a+b) \operatorname{AppellF1} \left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \right. \right. \\
& \left. \left. \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \tan \left[\frac{1}{2} (c+dx) \right] + 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + \right. \\
& \left. (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^3 \right) \left. \right) / \\
& \left(2d \left(-\frac{1}{2} a (a+b) n (b+a \cos [c+dx])^{-1+n} \sec [c+dx]^n \sin [c+dx] \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right) \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2} (c+dx) \right] \right) / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + \right. \\
& \left. 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) + (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) - \operatorname{AppellF1} \left[-\frac{1}{2}, n, -n, \frac{1}{2}, \tan \left[\frac{1}{2} (c+dx) \right]^2, \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \Bigg/ \left((a+b) \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, \right. \right. \\
& \quad \left. \left. -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \Bigg) + \frac{1}{2} (a+b) n (b+a \operatorname{Cos}[c+dx])^n \\
& \operatorname{Sec}[c+dx]^{1+n} \operatorname{Sin}[c+dx] \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \Bigg/ \right. \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1}\left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 - \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \\
& \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right) \Bigg/ \left((a+b) \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \\
& \quad \left. (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \Bigg) + \\
& \frac{1}{2} (a+b) (b+a \operatorname{Cos}[c+dx])^n \operatorname{Sec}[c+dx]^n \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg/ \right. \\
& \left(2 \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) + \left(3 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \left. \left(-1 / (3(a+b))(a-b)n \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \right. \\
& \left. \left. \frac{1}{3} n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) / \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \\
& \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \Bigg) - \\
& \left(1 / (a+b)(a-b)n \operatorname{AppellF1} \left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] - \right. \\
& \left. n \operatorname{AppellF1} \left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) / \\
& \left((a+b) \operatorname{AppellF1} \left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^3 \Bigg) + \left(\operatorname{AppellF1} \left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b} \right] \left(\frac{1}{2} (a+b) \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\
& 3n \left((-a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, -n, \right. \right. \\
& \left. \left. \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + (a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \\
& \left(\frac{1}{a+b} (a-b) n \operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \left. n \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \\
& 2n \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \left((-a+b) \left(\frac{1}{3(a+b)} (a-b)(1-n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 2-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + (a+b) \left(-\frac{1}{3(a+b)} (a-b) n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1-n, \frac{5}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} (1+n) \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, 2+n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \right) \right) \Big/ \\
& \left((a+b) \operatorname{AppellF1}\left[-\frac{1}{2}, n, -n, \frac{1}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 2n \left((-a+b) \operatorname{AppellF1}\left[\frac{1}{2}, n, 1-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] + \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 1+n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \right)^2 -
\end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right. \\
& \left(2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + 3(a+b) \right. \\
& \left. \left(-\frac{1}{3(a+b)} (a-b)n \operatorname{AppellF1} \left[\frac{3}{2}, n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \right. \\
& \left. \left. \frac{1}{3} n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right. \\
& \left. 2n \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \left((-a+b) \left(\frac{1}{5(a+b)} 3(a-b)(1-n) \operatorname{AppellF1} \left[\frac{5}{2}, n, 2-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right. \right. \\
& \left. \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{3}{5} n \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, 1-n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \right. \right. \\
& \left. \left. \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + (a+b) \left(-\frac{1}{5(a+b)} 3(a-b)n \operatorname{AppellF1} \left[\frac{5}{2}, 1+n, 1-n, \frac{7}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \frac{3}{5} (1+n) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}, 2+n, -n, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \right) \right) \right) \right) / \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, n, -n, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + 2n \left((-a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. n, 1-n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] + \right. \right. \\
& \left. \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, -n, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2, \frac{(a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{a+b} \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 276: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c + d x]^4 (a + b \text{Sec}[c + d x])^n dx$$

Optimal (type 6, 424 leaves, ? steps):

$$\begin{aligned} & -\frac{1}{2\sqrt{2}d} \\ & 3 \text{AppellF1}\left[-\frac{1}{2}, \frac{5}{2}, -n, \frac{1}{2}, \frac{1}{2}(1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] \text{Cot}[c + dx] \sqrt{1 + \text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a + b}\right)^{-n} - \\ & \frac{1}{6\sqrt{2}d} \text{AppellF1}\left[-\frac{3}{2}, \frac{5}{2}, -n, -\frac{1}{2}, \frac{1}{2}(1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] \text{Cot}[c + dx]^3 \\ & (1 + \text{Sec}[c + dx])^{3/2} (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a + b}\right)^{-n} + \frac{1}{\sqrt{2}d\sqrt{1 + \text{Sec}[c + dx]}} \\ & \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a + b}\right)^{-n} \text{Tan}[c + dx] + \\ & \frac{1}{2\sqrt{2}d\sqrt{1 + \text{Sec}[c + dx]}} \\ & \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2}(1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a + b}\right)^{-n} \text{Tan}[c + dx] \end{aligned}$$

Result (type 6, 8963 leaves): Display of huge result suppressed!

■ **Problem 294: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \text{Csc}[c + dx])^{3/2}}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 4, 145 leaves, 8 steps):

$$\begin{aligned} & -\frac{4e \text{Cos}[c + dx] \sqrt{e \text{Csc}[c + dx]}}{5ad} + \frac{2e \text{Cot}[c + dx] \text{Csc}[c + dx] \sqrt{e \text{Csc}[c + dx]}}{5ad} - \\ & \frac{2e \text{Csc}[c + dx]^2 \sqrt{e \text{Csc}[c + dx]}}{5ad} - \frac{4e \sqrt{e \text{Csc}[c + dx]} \text{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + dx), 2\right] \sqrt{\text{Sin}[c + dx]}}{5ad} \end{aligned}$$

Result (type 5, 219 leaves):

$$\frac{1}{5 a (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 (e \operatorname{Csc}[c + d x])^{3/2}$$

$$\left(\left(8 \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{i e^{i(c+d x)}}{-1 + e^{2 i(c+d x)}}} \left(-1 + e^{2 i(c+d x)} + (1 + e^{2 i c}) \sqrt{1 - e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)}\right] \right) \operatorname{Sec}[c + d x] \right) / \right.$$

$$\left. \left(d (1 + e^{2 i c}) \operatorname{Csc}[c + d x]^{3/2} - \frac{2 \left(4 \operatorname{Cos}[d x] \operatorname{Sec}[c] + \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \right) \operatorname{Tan}[c + d x]}{d} \right) \right)$$

- **Problem 296: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{e \operatorname{Csc}[c + d x]} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 99 leaves, 7 steps):

$$\frac{2 \operatorname{Cot}[c + d x]}{a d \sqrt{e \operatorname{Csc}[c + d x]}} - \frac{2 \operatorname{Csc}[c + d x]}{a d \sqrt{e \operatorname{Csc}[c + d x]}} + \frac{4 \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + d x), 2\right]}{a d \sqrt{e \operatorname{Csc}[c + d x]} \sqrt{\operatorname{Sin}[c + d x]}}$$

Result (type 5, 82 leaves):

$$- \frac{2 \left(2 i - \operatorname{Cot}[c + d x] + \operatorname{Csc}[c + d x] - \frac{4 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)}\right]}{\sqrt{1 - e^{2 i(c+d x)}}} \right)}{a d \sqrt{e \operatorname{Csc}[c + d x]}}$$

- **Problem 298: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(e \operatorname{Csc}[c + d x])^{5/2} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 120 leaves, 7 steps):

$$- \frac{4 \operatorname{EllipticE}\left[\frac{1}{2}(c - \frac{\pi}{2} + d x), 2\right]}{5 a d e^2 \sqrt{e \operatorname{Csc}[c + d x]} \sqrt{\operatorname{Sin}[c + d x]}} + \frac{2 \operatorname{Sin}[c + d x]}{3 a d e^2 \sqrt{e \operatorname{Csc}[c + d x]}} - \frac{2 \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{5 a d e^2 \sqrt{e \operatorname{Csc}[c + d x]}}$$

Result (type 5, 91 leaves):

$$\frac{24 i - \frac{48 i \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)}\right]}{\sqrt{1 - e^{2 i(c+d x)}}} + 20 \operatorname{Sin}[c + d x] - 6 \operatorname{Sin}[2(c + d x)]}{30 a d e^2 \sqrt{e \operatorname{Csc}[c + d x]}}$$

- **Problem 301: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \operatorname{Csc}[c + d x])^{3/2}}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 250 leaves, 16 steps):

$$\begin{aligned} & - \frac{4 e \cos [c+d x] \sqrt{e \csc [c+d x]}}{15 a^2 d} + \frac{16 e \cot [c+d x] \csc [c+d x] \sqrt{e \csc [c+d x]}}{45 a^2 d} - \\ & \frac{2 e \cot [c+d x]^3 \csc [c+d x] \sqrt{e \csc [c+d x]}}{9 a^2 d} - \frac{4 e \csc [c+d x]^2 \sqrt{e \csc [c+d x]}}{5 a^2 d} - \frac{2 e \cot [c+d x] \csc [c+d x]^3 \sqrt{e \csc [c+d x]}}{9 a^2 d} + \\ & \frac{4 e \csc [c+d x]^4 \sqrt{e \csc [c+d x]}}{9 a^2 d} - \frac{4 e \sqrt{e \csc [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c-\frac{\pi}{2}+d x), 2\right] \sqrt{\sin [c+d x]}}{15 a^2 d} \end{aligned}$$

Result (type 5, 238 leaves):

$$\begin{aligned} & \frac{1}{15 a^2 (1+\sec [c+d x])^2} \cos \left[\frac{1}{2}(c+d x) \right]^4 (e \csc [c+d x])^{3/2} \sec [c+d x] \\ & \left(\left(16 \sqrt{2} e^{-i(c+d x)} \sqrt{\frac{i e^{i(c+d x)}}{-1+e^{2 i(c+d x)}}} \left(-1+e^{2 i(c+d x)}+(1+e^{2 i c}) \sqrt{1-e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 i(c+d x)}\right]\right) \sec [c+d x] \right) / \right. \\ & \left. (d(1+e^{2 i c}) \csc [c+d x]^{3/2}) - \frac{2(24 \cos [d x] \sec [c]+(8+13 \cos [c+d x]) \sec [\frac{1}{2}(c+d x)]^4) \tan [c+d x]}{3 d} \right) \end{aligned}$$

■ **Problem 303: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{e \csc [c+d x]} (a+a \sec [c+d x])^2} dx$$

Optimal (type 4, 199 leaves, 14 steps):

$$\begin{aligned} & \frac{16 \cot [c+d x]}{5 a^2 d \sqrt{e \csc [c+d x]}} - \frac{2 \cot [c+d x]^3}{5 a^2 d \sqrt{e \csc [c+d x]}} - \frac{4 \csc [c+d x]}{a^2 d \sqrt{e \csc [c+d x]}} - \\ & \frac{2 \cot [c+d x] \csc [c+d x]^2}{5 a^2 d \sqrt{e \csc [c+d x]}} + \frac{4 \csc [c+d x]^3}{5 a^2 d \sqrt{e \csc [c+d x]}} + \frac{28 \operatorname{EllipticE}\left[\frac{1}{2}(c-\frac{\pi}{2}+d x), 2\right]}{5 a^2 d \sqrt{e \csc [c+d x]} \sqrt{\sin [c+d x]}} \end{aligned}$$

Result (type 5, 241 leaves):

$$\left(4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{\operatorname{Csc}[c+dx]} \operatorname{Sec}[c+dx]^2 \right. \\ \left. \left(-1/(1+e^{2ic}) 28\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}} \left(-1+e^{2i(c+dx)} + (1+e^{2ic}) \sqrt{1-e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2i(c+dx)}\right]\right) \right. \right. \\ \left. \left. \sqrt{\operatorname{Csc}[c+dx]} \left(-(-23+5\operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Sec}[c] + 2 \left(-10 + \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + 5\operatorname{Sin}[c] \operatorname{Sin}[dx] \right) \right) \right) \right) \right) / \\ \left(5a^2 d \sqrt{e \operatorname{Csc}[c+dx]} (1 + \operatorname{Sec}[c+dx])^2 \right)$$

■ **Problem 305: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(e \operatorname{Csc}[c+dx])^{5/2} (a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 215 leaves, 13 steps):

$$-\frac{2 \operatorname{Cot}[c+dx]}{a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]}} - \frac{2 \operatorname{Cos}[c+dx]^2 \operatorname{Cot}[c+dx]}{a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]}} + \frac{4 \operatorname{Csc}[c+dx]}{a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]}} - \\ \frac{44 \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right]}{5a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]}} + \frac{4 \operatorname{Sin}[c+dx]}{3a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]}} - \frac{12 \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{5a^2 d e^2 \sqrt{e \operatorname{Csc}[c+dx]}}$$

Result (type 5, 351 leaves):

$$\left(176\sqrt{2} e^{-i(c+dx)} \sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}[c+dx]^{5/2} \right. \\ \left. \left(-1+e^{2i(c+dx)} + (1+e^{2ic}) \sqrt{1-e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2i(c+dx)}\right]\right) \operatorname{Sec}[c+dx]^2 \right) / \\ \left(5d(1+e^{2ic})(e \operatorname{Csc}[c+dx])^{5/2}(a+a \operatorname{Sec}[c+dx])^2 + \right. \\ \left. \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}[c+dx]^3 \operatorname{Sec}[c+dx]^2 \left(\frac{56}{3d} - \frac{8 \operatorname{Cos}[2c] \operatorname{Cos}[2dx]}{3d} + \frac{2 \operatorname{Cos}[3c] \operatorname{Cos}[3dx]}{5d} + \frac{(-129+47 \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Sec}[c]}{5d} - \right. \right. \right. \\ \left. \left. \left. \frac{94 \operatorname{Sin}[c] \operatorname{Sin}[dx]}{5d} + \frac{8 \operatorname{Sin}[2c] \operatorname{Sin}[2dx]}{3d} - \frac{2 \operatorname{Sin}[3c] \operatorname{Sin}[3dx]}{5d} \right) \right) \right) / \left((e \operatorname{Csc}[c+dx])^{5/2} (a+a \operatorname{Sec}[c+dx])^2 \right)$$

■ **Problem 306: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(e \operatorname{Csc}[c + d x])^{7/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 172 leaves, 13 steps):

$$-\frac{4}{a^2 d e^3 \sqrt{e \operatorname{Csc}[c + d x]}} + \frac{26 \operatorname{Cos}[c + d x]}{21 a^2 d e^3 \sqrt{e \operatorname{Csc}[c + d x]}} +$$

$$\frac{2 \operatorname{Cos}[c + d x]^3}{7 a^2 d e^3 \sqrt{e \operatorname{Csc}[c + d x]}} + \frac{52 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right]}{21 a^2 d e^3 \sqrt{e \operatorname{Csc}[c + d x]} \sqrt{\operatorname{Sin}[c + d x]}} + \frac{4 \operatorname{Sin}[c + d x]^2}{5 a^2 d e^3 \sqrt{e \operatorname{Csc}[c + d x]}}$$

Result (type 4, 365 leaves):

$$\left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}[c + d x]^4 \operatorname{Sec}[c + d x]^2 \left(\frac{58 \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{21 d} - \frac{2 \operatorname{Cos}[d x] \operatorname{Sec}[c] (-520 \operatorname{Sin}[c] + 357 \operatorname{Sin}[2 c])}{105 d} - \frac{4 \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{5 d} + \right. \right.$$

$$\left. \frac{\operatorname{Cos}[4 d x] \operatorname{Sin}[4 c]}{7 d} - \frac{4 (-260 + 357 \operatorname{Cos}[c]) \operatorname{Sin}[d x]}{105 d} + \frac{58 \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{21 d} - \frac{4 \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{5 d} + \frac{\operatorname{Cos}[4 c] \operatorname{Sin}[4 d x]}{7 d} \right) \Bigg) /$$

$$\left((e \operatorname{Csc}[c + d x])^{7/2} (a + a \operatorname{Sec}[c + d x])^2 \right) - \left(104 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}[c + d x]^{7/2} \operatorname{Sec}[c + d x]^2 \right.$$

$$\left. \left(\frac{2 \sqrt{\operatorname{Csc}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} \left(-c + \frac{\pi}{2} - d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{d} + \right. \right.$$

$$\left. \left. \frac{2 \operatorname{Cos}[c + d x]^2 \operatorname{Sec}[c]}{d \sqrt{\operatorname{Csc}[c + d x]} \sqrt{(-1 + \operatorname{Csc}[c + d x])^2} \operatorname{Sin}[c + d x]^2 \sqrt{1 - \operatorname{Sin}[c + d x]^2}} \right) \right) / \left(21 (e \operatorname{Csc}[c + d x])^{7/2} (a + a \operatorname{Sec}[c + d x])^2 \right)$$

Test results for the 365 problems in "4.5.1.4 (d tan)^n (a+b sec)^m.m"

■ **Problem 12: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^4 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a x + \frac{3 a \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} - \frac{(8 a + 3 a \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{8 d} + \frac{(4 a + 3 a \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^3}{12 d}$$

Result (type 3, 230 leaves):

$$\begin{aligned}
 & a x - \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
 & \frac{a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{5 a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\
 & \frac{5 a}{16 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{4 a \operatorname{Tan}[c+d x]}{3 d} + \frac{a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}
 \end{aligned}$$

■ **Problem 13: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+d x]) \operatorname{Tan}[c+d x]^2 dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$-a x - \frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} + \frac{(2 a + a \operatorname{Sec}[c+d x]) \operatorname{Tan}[c+d x]}{2 d}$$

Result (type 3, 142 leaves):

$$\begin{aligned}
 & -a x + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
 & \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{a}{4 d\left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{a \operatorname{Tan}[c+d x]}{d}
 \end{aligned}$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+d x]^4 (a + a \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$a x - \frac{\operatorname{Cot}[c+d x]^3 (a + a \operatorname{Sec}[c+d x])}{3 d} + \frac{\operatorname{Cot}[c+d x] (3 a + 2 a \operatorname{Sec}[c+d x])}{3 d}$$

Result (type 3, 136 leaves):

$$\begin{aligned}
 & a x + \frac{5 a \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]}{12 d} + \frac{4 a \operatorname{Cot}[c+d x]}{3 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{24 d} - \\
 & \frac{a \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2}{3 d} + \frac{5 a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{12 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 d}
 \end{aligned}$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+d x]^6 (a + a \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-a x - \frac{\text{Cot}[c + d x]^5 (a + a \text{Sec}[c + d x])}{5 d} + \frac{\text{Cot}[c + d x]^3 (5 a + 4 a \text{Sec}[c + d x])}{15 d} - \frac{\text{Cot}[c + d x] (15 a + 8 a \text{Sec}[c + d x])}{15 d}$$

Result (type 3, 219 leaves):

$$-a x - \frac{89 a \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{23 a \text{Cot}[c + d x]}{15 d} + \frac{31 a \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} -$$

$$\frac{a \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} + \frac{11 a \text{Cot}[c + d x] \text{Csc}[c + d x]^2}{15 d} - \frac{a \text{Cot}[c + d x] \text{Csc}[c + d x]^4}{5 d} -$$

$$\frac{89 a \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} + \frac{31 a \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{a \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d}$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^8 (a + a \text{Sec}[c + d x]) dx$$

Optimal (type 3, 111 leaves, 5 steps):

$$a x - \frac{\text{Cot}[c + d x]^7 (a + a \text{Sec}[c + d x])}{7 d} + \frac{\text{Cot}[c + d x]^5 (7 a + 6 a \text{Sec}[c + d x])}{35 d} +$$

$$\frac{\text{Cot}[c + d x] (35 a + 16 a \text{Sec}[c + d x])}{35 d} - \frac{\text{Cot}[c + d x]^3 (35 a + 24 a \text{Sec}[c + d x])}{105 d}$$

Result (type 3, 300 leaves):

$$a x + \frac{381 a \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{1120 d} + \frac{176 a \text{Cot}[c + d x]}{105 d} - \frac{179 a \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{2240 d} + \frac{a \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{70 d} -$$

$$\frac{a \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^6}{896 d} - \frac{122 a \text{Cot}[c + d x] \text{Csc}[c + d x]^2}{105 d} + \frac{22 a \text{Cot}[c + d x] \text{Csc}[c + d x]^4}{35 d} - \frac{a \text{Cot}[c + d x] \text{Csc}[c + d x]^6}{7 d} +$$

$$\frac{381 a \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{1120 d} - \frac{179 a \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{2240 d} + \frac{a \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{70 d} - \frac{a \text{Sec}\left[\frac{1}{2}(c + d x)\right]^6 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{896 d}$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^{10} (a + a \text{Sec}[c + d x]) dx$$

Optimal (type 3, 140 leaves, 6 steps):

$$-a x - \frac{\text{Cot}[c + d x]^9 (a + a \text{Sec}[c + d x])}{9 d} + \frac{\text{Cot}[c + d x]^7 (9 a + 8 a \text{Sec}[c + d x])}{63 d} -$$

$$\frac{\text{Cot}[c + d x]^5 (21 a + 16 a \text{Sec}[c + d x])}{105 d} + \frac{\text{Cot}[c + d x]^3 (105 a + 64 a \text{Sec}[c + d x])}{315 d} - \frac{\text{Cot}[c + d x] (315 a + 128 a \text{Sec}[c + d x])}{315 d}$$

Result (type 3, 383 leaves) :

$$\begin{aligned}
 & -a x - \frac{25\,609 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]}{80\,640 d} - \frac{563 a \operatorname{Cot}[c+dx]}{315 d} + \frac{14\,711 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{161\,280 d} - \frac{1231 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4}{53\,760 d} + \\
 & \frac{109 a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^6}{32\,256 d} - \frac{a \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^8}{4\,608 d} + \frac{506 a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^2}{315 d} - \frac{136 a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^4}{105 d} + \\
 & \frac{37 a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^6}{63 d} - \frac{a \operatorname{Cot}[c+dx] \operatorname{Csc}[c+dx]^8}{9 d} - \frac{25\,609 a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{80\,640 d} + \frac{14\,711 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{161\,280 d} - \\
 & \frac{1231 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{53\,760 d} + \frac{109 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^6 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{32\,256 d} - \frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^8 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4\,608 d}
 \end{aligned}$$

■ **Problem 29: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]^6 dx$$

Optimal (type 3, 161 leaves, 12 steps) :

$$\begin{aligned}
 & -a^2 x - \frac{5 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8 d} + \frac{a^2 \operatorname{Tan}[c+dx]}{d} + \frac{5 a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{8 d} - \\
 & \frac{a^2 \operatorname{Tan}[c+dx]^3}{3 d} - \frac{5 a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^3}{12 d} + \frac{a^2 \operatorname{Tan}[c+dx]^5}{5 d} + \frac{a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^5}{3 d} + \frac{a^2 \operatorname{Tan}[c+dx]^7}{7 d}
 \end{aligned}$$

Result (type 3, 337 leaves) :

$$\begin{aligned}
 & \frac{1}{215\,040 d} a^2 (1 + \operatorname{Cos}[c+dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Sec}[c+dx]^7 \\
 & \left(33\,600 \operatorname{Cos}[c+dx]^7 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\
 & \quad \operatorname{Sec}[c] (-14\,700 dx \operatorname{Cos}[dx] - 14\,700 dx \operatorname{Cos}[2c+dx] - 8820 dx \operatorname{Cos}[2c+3dx] - 8820 dx \operatorname{Cos}[4c+3dx] - 2940 dx \operatorname{Cos}[4c+5dx] - \\
 & \quad 2940 dx \operatorname{Cos}[6c+5dx] - 420 dx \operatorname{Cos}[6c+7dx] - 420 dx \operatorname{Cos}[8c+7dx] + 24\,640 \operatorname{Sin}[dx] - 16\,240 \operatorname{Sin}[2c+dx] + \\
 & \quad 2975 \operatorname{Sin}[c+2dx] + 2975 \operatorname{Sin}[3c+2dx] + 14\,448 \operatorname{Sin}[2c+3dx] - 10\,080 \operatorname{Sin}[4c+3dx] + 980 \operatorname{Sin}[3c+4dx] + 980 \operatorname{Sin}[5c+4dx] + \\
 & \quad \left. 6496 \operatorname{Sin}[4c+5dx] - 1680 \operatorname{Sin}[6c+5dx] + 1155 \operatorname{Sin}[5c+6dx] + 1155 \operatorname{Sin}[7c+6dx] + 1168 \operatorname{Sin}[6c+7dx] \right)
 \end{aligned}$$

■ **Problem 30: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]^4 dx$$

Optimal (type 3, 119 leaves, 10 steps) :

$$a^2 x + \frac{3 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{4 d} - \frac{a^2 \operatorname{Tan}[c+dx]}{d} - \frac{3 a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{4 d} + \frac{a^2 \operatorname{Tan}[c+dx]^3}{3 d} + \frac{a^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^3}{2 d} + \frac{a^2 \operatorname{Tan}[c+dx]^5}{5 d}$$

Result (type 3, 1173 leaves) :

$$\begin{aligned}
& \frac{1}{4} x \cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 - \frac{3 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{16 d} + \\
& \frac{3 \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2}{16 d} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{80 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^5} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(3 \cos \left[\frac{c}{2}\right]-2 \sin \left[\frac{c}{2}\right]\right)}{80 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^4} - \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{480 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-151 \cos \left[\frac{c}{2}\right]+149 \sin \left[\frac{c}{2}\right]\right)}{960 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} - \\
& \frac{17 \cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{60 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} + \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{80 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^5} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(-3 \cos \left[\frac{c}{2}\right]-2 \sin \left[\frac{c}{2}\right]\right)}{80 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^4} - \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{480 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2\left(151 \cos \left[\frac{c}{2}\right]+149 \sin \left[\frac{c}{2}\right]\right)}{960 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} - \frac{17 \cos [c+d x]^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 \sin \left[\frac{d x}{2}\right]}{60 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+d x])^2 \operatorname{Tan}[c+d x]^2 dx$$

Optimal (type 3, 72 leaves, 8 steps):

$$-a^2 x - \frac{a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} + \frac{a^2 \operatorname{Tan}[c+d x]}{d} + \frac{a^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{d} + \frac{a^2 \operatorname{Tan}[c+d x]^3}{3 d}$$

Result (type 3, 773 leaves):

$$\begin{aligned}
& -\frac{1}{4} x \cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 + \frac{\cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2}{4 d} - \\
& \frac{\cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2}{4 d} + \\
& \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 \sin \left[\frac{d x}{2}\right]}{24 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2\left(7 \cos \left[\frac{c}{2}\right]-5 \sin \left[\frac{c}{2}\right]\right)}{48 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \\
& \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 \sin \left[\frac{d x}{2}\right]}{6 d\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} + \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 \sin \left[\frac{d x}{2}\right]}{24 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2\left(-7 \cos \left[\frac{c}{2}\right]-5 \sin \left[\frac{c}{2}\right]\right)}{48 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \frac{\cos [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 \sin \left[\frac{d x}{2}\right]}{6 d\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^8 (a+a \sec [c+d x])^2 dx$$

Optimal (type 3, 139 leaves, 12 steps):

$$\begin{aligned}
& a^2 x + \frac{a^2 \cot [c+d x]}{d} - \frac{a^2 \cot [c+d x]^3}{3 d} + \frac{a^2 \cot [c+d x]^5}{5 d} - \\
& \frac{2 a^2 \cot [c+d x]^7}{7 d} + \frac{2 a^2 \csc [c+d x]}{d} - \frac{2 a^2 \csc [c+d x]^3}{d} + \frac{6 a^2 \csc [c+d x]^5}{5 d} - \frac{2 a^2 \csc [c+d x]^7}{7 d}
\end{aligned}$$

Result (type 3, 312 leaves):

$$\begin{aligned}
& \frac{1}{860160 d} a^2 \csc \left[\frac{c}{2}\right] \csc \left[\frac{1}{2}(c+d x)\right]^7 \sec \left[\frac{c}{2}\right] \sec \left[\frac{1}{2}(c+d x)\right]^3 \\
& (5880 d x \cos [d x] - 5880 d x \cos [2 c+d x] - 3360 d x \cos [c+2 d x] + 3360 d x \cos [3 c+2 d x] - 1260 d x \cos [2 c+3 d x] + \\
& 1260 d x \cos [4 c+3 d x] + 1680 d x \cos [3 c+4 d x] - 1680 d x \cos [5 c+4 d x] - 420 d x \cos [4 c+5 d x] + \\
& 420 d x \cos [6 c+5 d x] + 4032 \sin [c] - 9632 \sin [d x] - 16002 \sin [c+d x] + 9144 \sin [2(c+d x)] + 3429 \sin [3(c+d x)] - \\
& 4572 \sin [4(c+d x)] + 1143 \sin [5(c+d x)] - 11760 \sin [2 c+d x] + 8864 \sin [c+2 d x] + 3360 \sin [3 c+2 d x] + \\
& 2064 \sin [2 c+3 d x] + 2520 \sin [4 c+3 d x] - 4432 \sin [3 c+4 d x] - 1680 \sin [5 c+4 d x] + 1528 \sin [4 c+5 d x])
\end{aligned}$$

■ **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^{10} (a+a \sec [c+d x])^2 dx$$

Optimal (type 3, 179 leaves, 13 steps):

$$-a^2 x - \frac{a^2 \cot [c+d x]}{d} + \frac{a^2 \cot [c+d x]^3}{3 d} - \frac{a^2 \cot [c+d x]^5}{5 d} + \frac{a^2 \cot [c+d x]^7}{7 d} - \frac{2 a^2 \cot [c+d x]^9}{9 d} -$$

$$\frac{2 a^2 \csc [c+d x]}{d} + \frac{8 a^2 \csc [c+d x]^3}{3 d} - \frac{12 a^2 \csc [c+d x]^5}{5 d} + \frac{8 a^2 \csc [c+d x]^7}{7 d} - \frac{2 a^2 \csc [c+d x]^9}{9 d}$$

Result (type 3, 428 leaves):

$$-\frac{1}{330301440 d} a^2 \csc \left[\frac{c}{2} \right] \csc \left[\frac{1}{2} (c+d x) \right]^9 \sec \left[\frac{c}{2} \right] \sec \left[\frac{1}{2} (c+d x) \right]^5$$

$$(453600 d x \cos [d x] - 453600 d x \cos [2 c+d x] - 201600 d x \cos [c+2 d x] + 201600 d x \cos [3 c+2 d x] - 191520 d x \cos [2 c+3 d x] +$$

$$191520 d x \cos [4 c+3 d x] + 161280 d x \cos [3 c+4 d x] - 161280 d x \cos [5 c+4 d x] + 10080 d x \cos [4 c+5 d x] - 10080 d x \cos [6 c+5 d x] -$$

$$40320 d x \cos [5 c+6 d x] + 40320 d x \cos [7 c+6 d x] + 10080 d x \cos [6 c+7 d x] - 10080 d x \cos [8 c+7 d x] + 259584 \sin [c] -$$

$$897024 \sin [d x] - 1152405 \sin [c+d x] + 512180 \sin [2 (c+d x)] + 486571 \sin [3 (c+d x)] - 409744 \sin [4 (c+d x)] -$$

$$25609 \sin [5 (c+d x)] + 102436 \sin [6 (c+d x)] - 25609 \sin [7 (c+d x)] - 825216 \sin [2 c+d x] + 622976 \sin [c+2 d x] +$$

$$142464 \sin [3 c+2 d x] + 297088 \sin [2 c+3 d x] + 430080 \sin [4 c+3 d x] - 424192 \sin [3 c+4 d x] - 188160 \sin [5 c+4 d x] +$$

$$2048 \sin [4 c+5 d x] - 40320 \sin [6 c+5 d x] + 112768 \sin [5 c+6 d x] + 40320 \sin [7 c+6 d x] - 38272 \sin [6 c+7 d x])$$

■ **Problem 49: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c+d x])^3 \tan [c+d x]^2 dx$$

Optimal (type 3, 98 leaves, 11 steps):

$$-a^3 x - \frac{13 a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} + \frac{a^3 \tan [c+d x]}{d} + \frac{11 a^3 \sec [c+d x] \tan [c+d x]}{8 d} + \frac{a^3 \sec [c+d x]^3 \tan [c+d x]}{4 d} + \frac{a^3 \tan [c+d x]^3}{d}$$

Result (type 3, 230 leaves):

$$-\frac{1}{64 d} a^3 \sec [c+d x]^4 \left(24 d x - 39 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + 39 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] + \right.$$

$$4 \cos [2 (c+d x)] \left(8 d x - 13 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + 13 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) +$$

$$\cos [4 (c+d x)] \left(8 d x - 13 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] - \sin \left[\frac{1}{2} (c+d x) \right] \right] + 13 \log \left[\cos \left[\frac{1}{2} (c+d x) \right] + \sin \left[\frac{1}{2} (c+d x) \right] \right] \right) -$$

$$\left. 38 \sin [c+d x] - 32 \sin [2 (c+d x)] - 22 \sin [3 (c+d x)] \right)$$

■ **Problem 50: Result more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^2 (a + a \sec [c+d x])^3 dx$$

Optimal (type 3, 49 leaves, 11 steps):

$$-a^3 x + \frac{a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{d} - \frac{4 a^3 \cot [c+d x]}{d} - \frac{4 a^3 \csc [c+d x]}{d}$$

Result (type 3, 109 leaves):

$$-\frac{1}{8d} a^3 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left(dx + \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \sin\left[\frac{dx}{2}\right]\right)$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^{10} (a + a \operatorname{Sec}[c + dx])^3 dx$$

Optimal (type 3, 179 leaves, 16 steps):

$$-a^3 x - \frac{a^3 \cot[c + dx]}{d} + \frac{a^3 \cot[c + dx]^3}{3d} - \frac{a^3 \cot[c + dx]^5}{5d} + \frac{a^3 \cot[c + dx]^7}{7d} - \frac{4 a^3 \cot[c + dx]^9}{9d} -$$

$$\frac{3 a^3 \operatorname{Csc}[c + dx]}{d} + \frac{13 a^3 \operatorname{Csc}[c + dx]^3}{3d} - \frac{21 a^3 \operatorname{Csc}[c + dx]^5}{5d} + \frac{15 a^3 \operatorname{Csc}[c + dx]^7}{7d} - \frac{4 a^3 \operatorname{Csc}[c + dx]^9}{9d}$$

Result (type 3, 370 leaves):

$$\frac{1}{41287680d} a^3 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^9 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3$$

$$(-181440 dx \cos[dx] + 181440 dx \cos[2c + dx] + 136080 dx \cos[c + 2dx] - 136080 dx \cos[3c + 2dx] + 10080 dx \cos[2c + 3dx] -$$

$$10080 dx \cos[4c + 3dx] - 60480 dx \cos[3c + 4dx] + 60480 dx \cos[5c + 4dx] + 30240 dx \cos[4c + 5dx] -$$

$$30240 dx \cos[6c + 5dx] - 5040 dx \cos[5c + 6dx] + 5040 dx \cos[7c + 6dx] - 169344 \sin[c] + 338112 \sin[dx] + 675036 \sin[c + dx] -$$

$$506277 \sin[2(c + dx)] - 37502 \sin[3(c + dx)] + 225012 \sin[4(c + dx)] - 112506 \sin[5(c + dx)] + 18751 \sin[6(c + dx)] +$$

$$431424 \sin[2c + dx] - 375552 \sin[c + 2dx] - 201600 \sin[3c + 2dx] + 41248 \sin[2c + 3dx] - 84000 \sin[4c + 3dx] +$$

$$155712 \sin[3c + 4dx] + 100800 \sin[5c + 4dx] - 98016 \sin[4c + 5dx] - 30240 \sin[6c + 5dx] + 21376 \sin[5c + 6dx])$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx]^{12} (a + a \operatorname{Sec}[c + dx])^3 dx$$

Optimal (type 3, 213 leaves, 17 steps):

$$a^3 x + \frac{a^3 \cot[c + dx]}{d} - \frac{a^3 \cot[c + dx]^3}{3d} + \frac{a^3 \cot[c + dx]^5}{5d} - \frac{a^3 \cot[c + dx]^7}{7d} + \frac{a^3 \cot[c + dx]^9}{9d} - \frac{4 a^3 \cot[c + dx]^{11}}{11d} +$$

$$\frac{3 a^3 \operatorname{Csc}[c + dx]}{d} - \frac{16 a^3 \operatorname{Csc}[c + dx]^3}{3d} + \frac{34 a^3 \operatorname{Csc}[c + dx]^5}{5d} - \frac{36 a^3 \operatorname{Csc}[c + dx]^7}{7d} + \frac{19 a^3 \operatorname{Csc}[c + dx]^9}{9d} - \frac{4 a^3 \operatorname{Csc}[c + dx]^{11}}{11d}$$

Result (type 3, 1035 leaves):

$$\begin{aligned}
& \frac{1}{8} x \cos [c+d x]^3 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 - \frac{112229 \cos [c+d x]^3 \cot \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3}{1419264 d} + \\
& \frac{6155 \cos [c+d x]^3 \cot \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3}{236544 d} - \\
& \frac{1033 \cos [c+d x]^3 \cot \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3}{177408 d} + \\
& \frac{155 \cos [c+d x]^3 \cot \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3}{202752 d} - \frac{\cos [c+d x]^3 \cot \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^{10} \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3}{22528 d} - \\
& \frac{347267 \cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{1419264 d} + \\
& \frac{112229 \cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^3 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{1419264 d} - \\
& \frac{6155 \cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^5 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{236544 d} + \\
& \frac{1033 \cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^7 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{177408 d} - \\
& \frac{155 \cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^9 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{202752 d} + \\
& \frac{\cos [c+d x]^3 \csc \left[\frac{c}{2} \right] \csc \left[\frac{c}{2} + \frac{d x}{2} \right]^{11} \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{22528 d} - \\
& \frac{743 \cos [c+d x]^3 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^7 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{30720 d} + \\
& \frac{7 \cos [c+d x]^3 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^9 (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{3840 d} - \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^{11} (a+a \sec [c+d x])^3 \sin \left[\frac{d x}{2} \right]}{10240 d} + \\
& \frac{7 \cos [c+d x]^3 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a+a \sec [c+d x])^3 \tan \left[\frac{c}{2} \right]}{3840 d} - \frac{\cos [c+d x]^3 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^{10} (a+a \sec [c+d x])^3 \tan \left[\frac{c}{2} \right]}{10240 d}
\end{aligned}$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]^8}{a+a \sec [c+d x]} dx$$

Optimal (type 3, 105 leaves, 6 steps):

$$\frac{x}{a} - \frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{16 ad} - \frac{(16 - 5 \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]}{16 ad} + \frac{(8 - 5 \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]^3}{24 ad} - \frac{(6 - 5 \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]^5}{30 ad}$$

Result (type 3, 301 leaves):

$$\frac{1}{3840 ad (1 + \operatorname{Sec}[c + dx])} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx] \left(2400 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) + \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^6 \right. \\ \left. (2400 dx \operatorname{Cos}[c] + 1800 dx \operatorname{Cos}[c + 2dx] + 1800 dx \operatorname{Cos}[3c + 2dx] + 720 dx \operatorname{Cos}[3c + 4dx] + 720 dx \operatorname{Cos}[5c + 4dx] + 120 dx \operatorname{Cos}[5c + 6dx] + \right. \\ \left. 120 dx \operatorname{Cos}[7c + 6dx] + 3680 \operatorname{Sin}[c] + 450 \operatorname{Sin}[dx] + 450 \operatorname{Sin}[2c + dx] - 3360 \operatorname{Sin}[c + 2dx] + 2160 \operatorname{Sin}[3c + 2dx] - 25 \operatorname{Sin}[2c + 3dx] - \right. \\ \left. 25 \operatorname{Sin}[4c + 3dx] - 1488 \operatorname{Sin}[3c + 4dx] + 720 \operatorname{Sin}[5c + 4dx] + 165 \operatorname{Sin}[4c + 5dx] + 165 \operatorname{Sin}[6c + 5dx] - 368 \operatorname{Sin}[5c + 6dx]) \right)$$

■ **Problem 65: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + dx]^6}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$-\frac{x}{a} + \frac{3 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8 ad} + \frac{(8 - 3 \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]}{8 ad} - \frac{(4 - 3 \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]^3}{12 ad}$$

Result (type 3, 893 leaves):

$$\begin{aligned}
& - \frac{2 x \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] - 3 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + dx]}{a + a \operatorname{Sec}[c + dx]} + \frac{4 d (a + a \operatorname{Sec}[c + dx])}{4 d (a + a \operatorname{Sec}[c + dx])} + \\
& \frac{3 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + dx]}{4 d (a + a \operatorname{Sec}[c + dx])} + \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx]}{8 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} - \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \left(-19 \operatorname{Cos}\left[\frac{c}{2}\right] + 11 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{24 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{8 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx]}{8 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} - \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \left(19 \operatorname{Cos}\left[\frac{c}{2}\right] + 11 \operatorname{Sin}\left[\frac{c}{2}\right]\right)}{24 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{8 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \operatorname{Sec}[c + dx]) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + dx]^4}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{x}{a} - \frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a d} - \frac{(2 - \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]}{2 a d}$$

Result (type 3, 241 leaves):

$$\frac{1}{2 a (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Sec}[c + d x] \left(4 x + \frac{2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} - \frac{2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{1}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{1}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - (4 \operatorname{Sin}[d x]) \right) / \left(d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) \right)$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + d x]^2}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$-\frac{x}{a} + \frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d}$$

Result (type 3, 60 leaves):

$$-\frac{d x + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{a d}$$

■ **Problem 69: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^4}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 88 leaves, 5 steps):

$$\frac{x}{a} + \frac{\operatorname{Cot}[c + d x] (15 - 8 \operatorname{Sec}[c + d x])}{15 a d} - \frac{\operatorname{Cot}[c + d x]^3 (5 - 4 \operatorname{Sec}[c + d x])}{15 a d} + \frac{\operatorname{Cot}[c + d x]^5 (1 - \operatorname{Sec}[c + d x])}{5 a d}$$

Result (type 3, 254 leaves):

$$\frac{1}{1920 a d (1 + \operatorname{Sec}[c + d x])} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] (360 d x \operatorname{Cos}[d x] - 360 d x \operatorname{Cos}[2 c + d x] + 120 d x \operatorname{Cos}[c + 2 d x] - 120 d x \operatorname{Cos}[3 c + 2 d x] - 120 d x \operatorname{Cos}[2 c + 3 d x] + 120 d x \operatorname{Cos}[4 c + 3 d x] - 60 d x \operatorname{Cos}[3 c + 4 d x] + 60 d x \operatorname{Cos}[5 c + 4 d x] - 200 \operatorname{Sin}[c] - 584 \operatorname{Sin}[d x] + 534 \operatorname{Sin}[c + d x] + 178 \operatorname{Sin}[2(c + d x)] - 178 \operatorname{Sin}[3(c + d x)] - 89 \operatorname{Sin}[4(c + d x)] - 520 \operatorname{Sin}[2 c + d x] - 248 \operatorname{Sin}[c + 2 d x] - 120 \operatorname{Sin}[3 c + 2 d x] + 248 \operatorname{Sin}[2 c + 3 d x] + 120 \operatorname{Sin}[4 c + 3 d x] + 184 \operatorname{Sin}[3 c + 4 d x])$$

■ **Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^6}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 117 leaves, 6 steps) :

$$-\frac{x}{a} + \frac{\text{Cot}[c + dx]^3 (35 - 24 \text{Sec}[c + dx])}{105 a d} - \frac{\text{Cot}[c + dx] (35 - 16 \text{Sec}[c + dx])}{35 a d} - \frac{\text{Cot}[c + dx]^5 (7 - 6 \text{Sec}[c + dx])}{35 a d} + \frac{\text{Cot}[c + dx]^7 (1 - \text{Sec}[c + dx])}{7 a d}$$

Result (type 3, 359 leaves) :

$$\frac{1}{107520 a d (1 + \text{Sec}[c + dx])} \text{Csc}\left[\frac{c}{2}\right] \text{Csc}[c + dx]^5 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + dx] (-16800 dx \text{Cos}[dx] + 16800 dx \text{Cos}[2c + dx] - 4200 dx \text{Cos}[c + 2dx] + 4200 dx \text{Cos}[3c + 2dx] + 8400 dx \text{Cos}[2c + 3dx] - 8400 dx \text{Cos}[4c + 3dx] + 3360 dx \text{Cos}[3c + 4dx] - 3360 dx \text{Cos}[5c + 4dx] - 1680 dx \text{Cos}[4c + 5dx] + 1680 dx \text{Cos}[6c + 5dx] - 840 dx \text{Cos}[5c + 6dx] + 840 dx \text{Cos}[7c + 6dx] + 3136 \text{Sin}[c] + 30112 \text{Sin}[dx] - 22860 \text{Sin}[c + dx] - 5715 \text{Sin}[2(c + dx)] + 11430 \text{Sin}[3(c + dx)] + 4572 \text{Sin}[4(c + dx)] - 2286 \text{Sin}[5(c + dx)] - 1143 \text{Sin}[6(c + dx)] + 26208 \text{Sin}[2c + dx] + 14080 \text{Sin}[c + 2dx] - 16400 \text{Sin}[2c + 3dx] - 11760 \text{Sin}[4c + 3dx] - 7904 \text{Sin}[3c + 4dx] - 3360 \text{Sin}[5c + 4dx] + 3952 \text{Sin}[4c + 5dx] + 1680 \text{Sin}[6c + 5dx] + 2816 \text{Sin}[5c + 6dx])$$

■ **Problem 79: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + dx]^8}{(a + a \text{Sec}[c + dx])^2} dx$$

Optimal (type 3, 119 leaves, 11 steps) :

$$\frac{x}{a^2} - \frac{3 \text{ArcTanh}[\text{Sin}[c + dx]]}{4 a^2 d} - \frac{\text{Tan}[c + dx]}{a^2 d} + \frac{3 \text{Sec}[c + dx] \text{Tan}[c + dx]}{4 a^2 d} + \frac{\text{Tan}[c + dx]^3}{3 a^2 d} - \frac{\text{Sec}[c + dx] \text{Tan}[c + dx]^3}{2 a^2 d} + \frac{\text{Tan}[c + dx]^5}{5 a^2 d}$$

Result (type 3, 1167 leaves) :

$$\begin{aligned}
& \frac{4x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2}{(a+a\sec[c+dx])^2} + \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c+dx]^2}{d(a+a\sec[c+dx])^2} - \\
& \frac{3 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c+dx]^2}{d(a+a\sec[c+dx])^2} + \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{5d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^5} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \left(-2\cos\left[\frac{c}{2}\right] + 3\sin\left[\frac{c}{2}\right]\right)}{5d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} - \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{30d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \left(149\cos\left[\frac{c}{2}\right] - 151\sin\left[\frac{c}{2}\right]\right)}{60d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \\
& \frac{68\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{15d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{5d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^5} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \left(2\cos\left[\frac{c}{2}\right] + 3\sin\left[\frac{c}{2}\right]\right)}{5d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^4} - \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{30d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \left(-149\cos\left[\frac{c}{2}\right] - 151\sin\left[\frac{c}{2}\right]\right)}{60d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \\
& \frac{68\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^2 \sin\left[\frac{dx}{2}\right]}{15d(a+a\sec[c+dx])^2 \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]^6}{(a+a\sec[c+dx])^2} dx$$

Optimal (type 3, 72 leaves, 9 steps):

$$-\frac{x}{a^2} + \frac{\text{ArcTanh}[\text{Sin}[c + dx]]}{a^2 d} + \frac{\text{Tan}[c + dx]}{a^2 d} - \frac{\text{Sec}[c + dx] \text{Tan}[c + dx]}{a^2 d} + \frac{\text{Tan}[c + dx]^3}{3 a^2 d}$$

Result (type 3, 767 leaves):

$$\begin{aligned} & -\frac{4x \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2}{(a + a \text{Sec}[c + dx])^2} - \frac{4 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^2}{d (a + a \text{Sec}[c + dx])^2} + \\ & \frac{4 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^2}{d (a + a \text{Sec}[c + dx])^2} + \frac{2 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 (-5 \text{Cos}\left[\frac{c}{2}\right] + 7 \text{Sin}\left[\frac{c}{2}\right])}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\ & \frac{8 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] - \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\ & \frac{2 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\ & \frac{\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 (5 \text{Cos}\left[\frac{c}{2}\right] + 7 \text{Sin}\left[\frac{c}{2}\right])}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\ & \frac{8 \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + dx]^2 \text{Sin}\left[\frac{dx}{2}\right]}{3 d (a + a \text{Sec}[c + dx])^2 (\text{Cos}\left[\frac{c}{2}\right] + \text{Sin}\left[\frac{c}{2}\right]) (\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} \end{aligned}$$

■ **Problem 81: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + dx]^4}{(a + a \text{Sec}[c + dx])^2} dx$$

Optimal (type 3, 34 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{2 \text{ArcTanh}[\text{Sin}[c + dx]]}{a^2 d} + \frac{\text{Tan}[c + dx]}{a^2 d}$$

Result (type 3, 177 leaves):

$$\frac{1}{a^2 d (1 + \operatorname{Sec}[c + d x])^2}$$

$$4 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Sec}[c + d x]^2 \left(d x + 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) +$$

$$\operatorname{Sin}[d x] / \left(\left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right)$$

■ **Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^4}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 139 leaves, 13 steps):

$$\frac{x}{a^2} + \frac{\operatorname{Cot}[c + d x]}{a^2 d} - \frac{\operatorname{Cot}[c + d x]^3}{3 a^2 d} + \frac{\operatorname{Cot}[c + d x]^5}{5 a^2 d} - \frac{2 \operatorname{Cot}[c + d x]^7}{7 a^2 d} - \frac{2 \operatorname{Csc}[c + d x]}{a^2 d} + \frac{2 \operatorname{Csc}[c + d x]^3}{a^2 d} - \frac{6 \operatorname{Csc}[c + d x]^5}{5 a^2 d} + \frac{2 \operatorname{Csc}[c + d x]^7}{7 a^2 d}$$

Result (type 3, 314 leaves):

$$\frac{1}{26880 a^2 d (1 + \operatorname{Sec}[c + d x])^2}$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}[c + d x]^3 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (5880 d x \operatorname{Cos}[d x] - 5880 d x \operatorname{Cos}[2 c + d x] + 3360 d x \operatorname{Cos}[c + 2 d x] - 3360 d x \operatorname{Cos}[3 c + 2 d x] -$$

$$1260 d x \operatorname{Cos}[2 c + 3 d x] + 1260 d x \operatorname{Cos}[4 c + 3 d x] - 1680 d x \operatorname{Cos}[3 c + 4 d x] + 1680 d x \operatorname{Cos}[5 c + 4 d x] - 420 d x \operatorname{Cos}[4 c + 5 d x] +$$

$$420 d x \operatorname{Cos}[6 c + 5 d x] - 4032 \operatorname{Sin}[c] - 9632 \operatorname{Sin}[d x] + 16002 \operatorname{Sin}[c + d x] + 9144 \operatorname{Sin}[2(c + d x)] - 3429 \operatorname{Sin}[3(c + d x)] -$$

$$4572 \operatorname{Sin}[4(c + d x)] - 1143 \operatorname{Sin}[5(c + d x)] - 11760 \operatorname{Sin}[2 c + d x] - 8864 \operatorname{Sin}[c + 2 d x] - 3360 \operatorname{Sin}[3 c + 2 d x] +$$

$$2064 \operatorname{Sin}[2 c + 3 d x] + 2520 \operatorname{Sin}[4 c + 3 d x] + 4432 \operatorname{Sin}[3 c + 4 d x] + 1680 \operatorname{Sin}[5 c + 4 d x] + 1528 \operatorname{Sin}[4 c + 5 d x])$$

■ **Problem 85: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^6}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 179 leaves, 14 steps):

$$-\frac{x}{a^2} - \frac{\operatorname{Cot}[c + d x]}{a^2 d} + \frac{\operatorname{Cot}[c + d x]^3}{3 a^2 d} - \frac{\operatorname{Cot}[c + d x]^5}{5 a^2 d} + \frac{\operatorname{Cot}[c + d x]^7}{7 a^2 d} -$$

$$\frac{2 \operatorname{Cot}[c + d x]^9}{9 a^2 d} + \frac{2 \operatorname{Csc}[c + d x]}{a^2 d} - \frac{8 \operatorname{Csc}[c + d x]^3}{3 a^2 d} + \frac{12 \operatorname{Csc}[c + d x]^5}{5 a^2 d} - \frac{8 \operatorname{Csc}[c + d x]^7}{7 a^2 d} + \frac{2 \operatorname{Csc}[c + d x]^9}{9 a^2 d}$$

Result (type 3, 802 leaves):

$$\begin{aligned}
& - \frac{4 \times \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec^2[c+dx]}{(a + a \sec[c+dx])^2} + \frac{17 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cot\left[\frac{c}{2}\right] \cot\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec^2[c+dx]}{160 d (a + a \sec[c+dx])^2} - \frac{\cot\left[\frac{c}{2}\right] \cot\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec^2[c+dx]}{160 d (a + a \sec[c+dx])^2} + \\
& \frac{201 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \cot\left[\frac{c}{2} + \frac{dx}{2}\right] \csc\left[\frac{c}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{160 d (a + a \sec[c+dx])^2} - \frac{17 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cot\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \csc\left[\frac{c}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{160 d (a + a \sec[c+dx])^2} + \\
& \frac{\cot\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \csc\left[\frac{c}{2} + \frac{dx}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{160 d (a + a \sec[c+dx])^2} - \frac{7891 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{5040 d (a + a \sec[c+dx])^2} + \\
& \frac{63881 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sec\left[\frac{c}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{10080 d (a + a \sec[c+dx])^2} + \frac{313 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{840 d (a + a \sec[c+dx])^2} - \\
& \frac{109 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{2016 d (a + a \sec[c+dx])^2} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sec^2[c+dx] \sin\left[\frac{dx}{2}\right]}{288 d (a + a \sec[c+dx])^2} + \frac{313 \sec^2[c+dx] \tan\left[\frac{c}{2}\right]}{840 d (a + a \sec[c+dx])^2} - \\
& \frac{7891 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec^2[c+dx] \tan\left[\frac{c}{2}\right]}{5040 d (a + a \sec[c+dx])^2} - \frac{109 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec^2[c+dx] \tan\left[\frac{c}{2}\right]}{2016 d (a + a \sec[c+dx])^2} + \frac{\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec^2[c+dx] \tan\left[\frac{c}{2}\right]}{288 d (a + a \sec[c+dx])^2}
\end{aligned}$$

■ **Problem 97: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan^8[c+dx]}{(a + a \sec[c+dx])^3} dx$$

Optimal (type 3, 99 leaves, 12 steps):

$$\frac{x}{a^3} - \frac{13 \operatorname{ArcTanh}[\sin[c+dx]]}{8 a^3 d} - \frac{\tan[c+dx]}{a^3 d} + \frac{11 \sec[c+dx] \tan[c+dx]}{8 a^3 d} + \frac{\sec^3[c+dx] \tan[c+dx]}{4 a^3 d} - \frac{\tan^3[c+dx]}{a^3 d}$$

Result (type 3, 230 leaves):

$$\begin{aligned}
& \frac{1}{64 a^3 d} \sec^4[c+dx] \left(24 dx + 39 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 4 \cos[2(c+dx)] \left(8 dx + 13 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 13 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& \cos[4(c+dx)] \left(8 dx + 13 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - 13 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& 39 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] + 38 \sin[c+dx] - 32 \sin[2(c+dx)] + 22 \sin[3(c+dx)]
\end{aligned}$$

■ **Problem 98: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan^6[c+dx]}{(a + a \sec[c+dx])^3} dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$-\frac{x}{a^3} + \frac{7 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2 a^3 d} - \frac{5 \operatorname{Tan}[c + dx]}{2 a^3 d} - \frac{(1 - \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]}{2 a^3 d}$$

Result (type 3, 241 leaves):

$$\frac{1}{a^3 (1 + \operatorname{Sec}[c + dx])^3} + 2 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^6 \operatorname{Sec}[c + dx]^3 \left(-4x - \frac{14 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{14 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{1}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{1}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - (12 \operatorname{Sin}[dx]) / \left(d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \right)$$

■ **Problem 99: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + dx]^4}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 3, 46 leaves, 12 steps):

$$\frac{x}{a^3} + \frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^3 d} - \frac{4 \operatorname{Tan}[c + dx]}{a^2 d (a + a \operatorname{Sec}[c + dx])}$$

Result (type 3, 117 leaves):

$$\frac{1}{a^3 d (1 + \operatorname{Sec}[c + dx])^3} 8 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^3 \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \left(dx - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) - 4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right] \right)$$

■ **Problem 100: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + dx]^2}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 3, 60 leaves, 12 steps):

$$-\frac{x}{a^3} + \frac{2 \operatorname{Tan}[c + dx]}{a^2 d (a + a \operatorname{Sec}[c + dx])} - \frac{\operatorname{Tan}[c + dx]^3}{3 d (a + a \operatorname{Sec}[c + dx])^3}$$

Result (type 3, 125 leaves):

$$-\frac{1}{480 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \left(180 dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 180 dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 60 dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 60 dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] - 471 \operatorname{Sin}\left[\frac{dx}{2}\right] + 351 \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 277 \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 3 \operatorname{Sin}\left[2c + \frac{3dx}{2}\right]\right)$$

■ **Problem 102: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^4}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 177 leaves, 17 steps):

$$\frac{x}{a^3} + \frac{\operatorname{Cot}[c+dx]}{a^3 d} - \frac{\operatorname{Cot}[c+dx]^3}{3 a^3 d} + \frac{\operatorname{Cot}[c+dx]^5}{5 a^3 d} - \frac{\operatorname{Cot}[c+dx]^7}{7 a^3 d} + \frac{4 \operatorname{Cot}[c+dx]^9}{9 a^3 d} - \frac{3 \operatorname{Csc}[c+dx]}{a^3 d} + \frac{13 \operatorname{Csc}[c+dx]^3}{3 a^3 d} - \frac{21 \operatorname{Csc}[c+dx]^5}{5 a^3 d} + \frac{15 \operatorname{Csc}[c+dx]^7}{7 a^3 d} - \frac{4 \operatorname{Csc}[c+dx]^9}{9 a^3 d}$$

Result (type 3, 366 leaves):

$$\frac{1}{80640 a^3 d (1 + \operatorname{Sec}[c+dx])^3} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}[2(c+dx)]^3 \operatorname{Sec}\left[\frac{c}{2}\right] (181440 dx \operatorname{Cos}[dx] - 181440 dx \operatorname{Cos}[2c+dx] + 136080 dx \operatorname{Cos}[c+2dx] - 136080 dx \operatorname{Cos}[3c+2dx] - 10080 dx \operatorname{Cos}[2c+3dx] + 10080 dx \operatorname{Cos}[4c+3dx] - 60480 dx \operatorname{Cos}[3c+4dx] + 60480 dx \operatorname{Cos}[5c+4dx] - 30240 dx \operatorname{Cos}[4c+5dx] + 30240 dx \operatorname{Cos}[6c+5dx] - 5040 dx \operatorname{Cos}[5c+6dx] + 5040 dx \operatorname{Cos}[7c+6dx] - 169344 \operatorname{Sin}[c] - 338112 \operatorname{Sin}[dx] + 675036 \operatorname{Sin}[c+dx] + 506277 \operatorname{Sin}[2(c+dx)] - 37502 \operatorname{Sin}[3(c+dx)] - 225012 \operatorname{Sin}[4(c+dx)] - 112506 \operatorname{Sin}[5(c+dx)] - 18751 \operatorname{Sin}[6(c+dx)] - 431424 \operatorname{Sin}[2c+dx] - 375552 \operatorname{Sin}[c+2dx] - 201600 \operatorname{Sin}[3c+2dx] - 41248 \operatorname{Sin}[2c+3dx] + 84000 \operatorname{Sin}[4c+3dx] + 155712 \operatorname{Sin}[3c+4dx] + 100800 \operatorname{Sin}[5c+4dx] + 98016 \operatorname{Sin}[4c+5dx] + 30240 \operatorname{Sin}[6c+5dx] + 21376 \operatorname{Sin}[5c+6dx])$$

■ **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a+a \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2} dx$$

Optimal (type 4, 310 leaves, 17 steps):

$$\frac{a e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} + \frac{a e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} + \frac{6 a e^2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 d \sqrt{\operatorname{Sin}[2c+2dx]}} - \frac{6 a e \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 d} + \frac{2 e (5 a + 3 a \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{3/2}}{15 d}$$

Result (type 4, 332 leaves):

$$\begin{aligned}
& a \left(\frac{1}{d} \right. \\
& \left. \cos[c+dx] \cot[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 (1+\sec[c+dx]) (e \tan[c+dx])^{5/2} \left(-\frac{3}{5} \sin[c+dx] + \frac{1}{3} \tan[c+dx] + \frac{1}{5} \sec[c+dx] \tan[c+dx]\right) + \right. \\
& \left. \frac{1}{10 d \tan[c+dx]^{5/2}} \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^2 (1+\sec[c+dx]) (e \tan[c+dx])^{5/2} \right. \\
& \left. \left(-\frac{5}{2} \operatorname{Csc}[c+dx] \left(-\operatorname{ArcSin}[\cos[c+dx] - \sin[c+dx]] - \operatorname{Log}\left[\cos[c+dx] + \sin[c+dx] + \sqrt{\sin[2(c+dx)]}\right] \right) \sqrt{\sin[2(c+dx)]} \right. \right. \\
& \left. \left. \sqrt{\tan[c+dx]} + \frac{1}{\sqrt{1+\tan[c+dx]^2}} 6 \sec[c+dx] \left((-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c+dx]}\right], -1\right] - \right. \right. \right. \\
& \left. \left. \left. (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c+dx]}\right], -1\right] + \frac{\tan[c+dx]^{3/2}}{\sqrt{1+\tan[c+dx]^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 105: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \sec[c+dx]) (e \tan[c+dx])^{3/2} dx$$

Optimal (type 4, 282 leaves, 16 steps):

$$\begin{aligned}
& \frac{a e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} + \\
& \frac{a e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c+dx] - \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} d} - \frac{a e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c+dx] + \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} d} - \\
& \frac{a e^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \sec[c+dx] \sqrt{\sin[2c+2dx]}}{3 d \sqrt{e \tan[c+dx]}} + \frac{2 e (3 a + a \sec[c+dx]) \sqrt{e \tan[c+dx]}}{3 d}
\end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
& - \frac{1}{12 d (-1 + \tan[c + dx])^2} a e \cos[2(c + dx)] \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \\
& \sqrt{\sec[c + dx]^2} \sqrt{e \tan[c + dx]} \left(4 (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + dx]}\right], -1\right] \sqrt{\tan[c + dx]} + \right. \\
& \left. \sqrt{\sec[c + dx]^2} \left(12 \sin[c + dx] + 3 \operatorname{ArcSin}[\cos[c + dx] - \sin[c + dx]] \sqrt{\sin[2(c + dx)]} - \right. \right. \\
& \left. \left. 3 \operatorname{Log}\left[\cos[c + dx] + \sin[c + dx] + \sqrt{\sin[2(c + dx)]}\right] \sqrt{\sin[2(c + dx)]} + 4 \tan[c + dx] \right) \right)
\end{aligned}$$

■ **Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \sec[c + dx]) \sqrt{e \tan[c + dx]} dx$$

Optimal (type 4, 272 leaves, 16 steps):

$$\begin{aligned}
& - \frac{a \sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c + dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} + \frac{a \sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c + dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} + \\
& \frac{a \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e \tan[c + dx]} - \sqrt{2} \sqrt{e \tan[c + dx]}\right]}{2 \sqrt{2} d} - \frac{a \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e \tan[c + dx]} + \sqrt{2} \sqrt{e \tan[c + dx]}\right]}{2 \sqrt{2} d} - \\
& \frac{2 a \cos[c + dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \tan[c + dx]}}{d \sqrt{\sin[2c + 2dx]}} + \frac{2 a \cos[c + dx] (e \tan[c + dx])^{3/2}}{d e}
\end{aligned}$$

Result (type 4, 207 leaves):

$$\begin{aligned}
& - \frac{1}{4 d \sqrt{\sec[c + dx]^2}} a (1 + \cos[c + dx]) \operatorname{Csc}[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \\
& \left(\left(\operatorname{ArcSin}[\cos[c + dx] - \sin[c + dx]] + \operatorname{Log}\left[\cos[c + dx] + \sin[c + dx] + \sqrt{\sin[2(c + dx)]}\right] \right) \sqrt{\sec[c + dx]^2} \sqrt{\sin[2(c + dx)]} + \right. \\
& \left. 4 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + dx]}\right], -1\right] \sqrt{\tan[c + dx]} - \right. \\
& \left. 4 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + dx]}\right], -1\right] \sqrt{\tan[c + dx]} \right) \sqrt{e \tan[c + dx]}
\end{aligned}$$

■ **Problem 107: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{a + a \sec[c + dx]}{\sqrt{e \tan[c + dx]}} dx$$

Optimal (type 4, 244 leaves, 15 steps):

$$\begin{aligned}
& - \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d \sqrt{e}} + \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d \sqrt{e}} - \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d \sqrt{e}} + \\
& \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d \sqrt{e}} + \frac{a \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{d \sqrt{e \operatorname{Tan}[c+dx]}}
\end{aligned}$$

Result (type 6, 1511 leaves):

$$\begin{aligned}
& \left(45 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] (1 + \operatorname{Sec}[c+dx]) \operatorname{Sin}[c+dx] \right. \\
& \quad \left. \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - 4 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + 2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) / \\
& \quad \left(d \left(5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \left(-2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \quad \left(225 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] - \right. \\
& \quad 450 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - \\
& \quad 180 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \quad \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2 + 90 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \quad \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\
& \quad 360 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 180 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \\
& \quad \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad 360 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \quad \left. \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 180 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sin\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \\
& 72 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 72 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^3 + 18 \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 400 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 3, \frac{13}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^3 + 200 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 2, \frac{13}{4}, \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^3 - 150 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \\
& \left. \text{AppellF1}\left[\frac{9}{4}, \frac{5}{2}, 1, \frac{13}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^3\right) \sqrt{e \tan[c+dx]}
\end{aligned}$$

■ **Problem 108: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \operatorname{Sec}[c+dx]}{(e \tan[c+dx])^{3/2}} dx$$

Optimal (type 4, 305 leaves, 17 steps):

$$\begin{aligned}
& \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{3/2}} - \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{3/2}} - \\
& \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c+dx] - \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} d e^{3/2}} + \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c+dx] + \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} d e^{3/2}} - \\
& \frac{2(a + a \operatorname{Sec}[c+dx])}{d e \sqrt{e \tan[c+dx]}} - \frac{2 a \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \tan[c+dx]}}{d e^2 \sqrt{\sin[2c+2dx]}} + \frac{2 a \operatorname{Cos}[c+dx] (e \tan[c+dx])^{3/2}}{d e^3}
\end{aligned}$$

Result (type 4, 312 leaves):

$$\begin{aligned}
& a \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (1+\operatorname{Sec}[c+dx]) \operatorname{Sin}[c+dx] \left(-\operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}[c+dx]\right) \operatorname{Tan}[c+dx]}{d (e \operatorname{Tan}[c+dx])^{3/2}} + \right. \\
& \frac{1}{2 d (e \operatorname{Tan}[c+dx])^{3/2}} \operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (1+\operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]^{3/2} \\
& \left. \left(-\frac{1}{2} \operatorname{Csc}[c+dx] \left(-\operatorname{ArcSin}[\operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]] - \operatorname{Log}\left[\operatorname{Cos}[c+dx] + \operatorname{Sin}[c+dx] + \sqrt{\operatorname{Sin}[2(c+dx)]}\right] \right) \sqrt{\operatorname{Sin}[2(c+dx)]} \right. \right. \\
& \left. \left. \sqrt{\operatorname{Tan}[c+dx]} - \frac{1}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} 2 \operatorname{Sec}[c+dx] \left((-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} \right], -1 \right] - \right. \right. \right. \\
& \left. \left. \left. (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} \right], -1 \right] + \frac{\operatorname{Tan}[c+dx]^{3/2}}{\sqrt{1+\operatorname{Tan}[c+dx]^2}} \right) \right) \right)
\end{aligned}$$

■ **Problem 109: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \operatorname{Sec}[c+dx]}{(e \operatorname{Tan}[c+dx])^{5/2}} dx$$

Optimal (type 4, 282 leaves, 16 steps):

$$\begin{aligned}
& \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{5/2}} - \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{5/2}} + \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d e^{5/2}} \\
& - \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d e^{5/2}} - \frac{2(a + a \operatorname{Sec}[c+dx])}{3 d e (e \operatorname{Tan}[c+dx])^{3/2}} - \frac{a \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 d e^2 \sqrt{e \operatorname{Tan}[c+dx]}}
\end{aligned}$$

Result (type 4, 200 leaves):

$$\begin{aligned}
& - \frac{1}{6 d e^3 \sqrt{\operatorname{Sec}[c+dx]^2}} a \operatorname{Csc}[c+dx] \\
& \left(\sqrt{\operatorname{Sec}[c+dx]^2} \left(2 \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] + 2 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] - 3 \operatorname{ArcSin}[\operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]] \sqrt{\operatorname{Sin}[2(c+dx)]} + \right. \right. \\
& \left. \left. 3 \operatorname{Log}\left[\operatorname{Cos}[c+dx] + \operatorname{Sin}[c+dx] + \sqrt{\operatorname{Sin}[2(c+dx)]}\right] \sqrt{\operatorname{Sin}[2(c+dx)]} \right) - \right. \\
& \left. 4 (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} \right], -1 \right] \sqrt{\operatorname{Tan}[c+dx]} \right) \sqrt{e \operatorname{Tan}[c+dx]}
\end{aligned}$$

■ **Problem 110: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \operatorname{Sec}[c + d x]}{(e \operatorname{Tan}[c + d x])^{7/2}} dx$$

Optimal (type 4, 346 leaves, 18 steps):

$$\begin{aligned} & - \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{7/2}} + \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{7/2}} + \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} d e^{7/2}} - \\ & \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} d e^{7/2}} - \frac{2(a + a \operatorname{Sec}[c + d x])}{5 d e (e \operatorname{Tan}[c + d x])^{5/2}} + \frac{2(5 a + 3 a \operatorname{Sec}[c + d x])}{5 d e^3 \sqrt{e \operatorname{Tan}[c + d x]}} + \\ & \frac{6 a \operatorname{Cos}[c + d x] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right] \sqrt{e \operatorname{Tan}[c + d x]}}{5 d e^4 \sqrt{\operatorname{Sin}[2 c + 2 d x]}} - \frac{6 a \operatorname{Cos}[c + d x] (e \operatorname{Tan}[c + d x])^{3/2}}{5 d e^5} \end{aligned}$$

Result (type 4, 360 leaves):

$$\begin{aligned} & a \left(\frac{1}{d (e \operatorname{Tan}[c + d x])^{7/2}} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 (1 + \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x] \right. \\ & \left. \left(\frac{19}{20} \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right] - \frac{1}{20} \operatorname{Cot}\left[\frac{1}{2} (c + d x)\right] \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^2 - \frac{3}{5} \operatorname{Sin}[c + d x] - \frac{1}{4} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right) \operatorname{Tan}[c + d x]^3 + \right. \\ & \left. \frac{1}{10 d (e \operatorname{Tan}[c + d x])^{7/2}} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 (1 + \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^{7/2} \right. \\ & \left. \left(\frac{5}{2} \operatorname{Csc}[c + d x] \left(-\operatorname{ArcSin}[\operatorname{Cos}[c + d x] - \operatorname{Sin}[c + d x]] - \operatorname{Log}[\operatorname{Cos}[c + d x] + \operatorname{Sin}[c + d x] + \sqrt{\operatorname{Sin}[2 (c + d x)]}] \right) \sqrt{\operatorname{Sin}[2 (c + d x)]} \right. \right. \\ & \left. \left. \sqrt{\operatorname{Tan}[c + d x]} + \frac{1}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} 6 \operatorname{Sec}[c + d x] \left((-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c + d x]}\right], -1\right] - \right. \right. \right. \\ & \left. \left. \left. (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c + d x]}\right], -1\right] + \frac{\operatorname{Tan}[c + d x]^{3/2}}{\sqrt{1 + \operatorname{Tan}[c + d x]^2}} \right) \right) \right) \end{aligned}$$

■ **Problem 111: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[c + d x])^2 (e \operatorname{Tan}[c + d x])^{5/2} dx$$

Optimal (type 4, 366 leaves, 21 steps):

$$\frac{a^2 e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a^2 e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a^2 e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} +$$

$$\frac{a^2 e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} + \frac{12 a^2 e^2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 d \sqrt{\operatorname{Sin}[2c+2dx]}} +$$

$$\frac{2 a^2 e (e \operatorname{Tan}[c+dx])^{3/2}}{3 d} - \frac{12 a^2 e \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 d} + \frac{4 a^2 e \operatorname{Sec}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 d} + \frac{2 a^2 (e \operatorname{Tan}[c+dx])^{7/2}}{7 d e}$$

Result (type 4, 338 leaves):

$$\left(\left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{dx}{2} \right)\right] \right)^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^2 (a + a \operatorname{Sec}[c+dx])^2 (e \operatorname{Tan}[c+dx])^{5/2} \right.$$

$$\left. \left(\frac{1}{20 d} \left(48 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] - 48 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + \right.$$

$$\left. 5 \sqrt{2} \left(2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] + \right.$$

$$\left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \right) + \frac{2 \operatorname{Tan}[c+dx]^{3/2} \left(35 + 15 \operatorname{Tan}[c+dx]^2 + 42 \sqrt{1 + \operatorname{Tan}[c+dx]^2} \right)}{105 d} \right) \Bigg/$$

$$\left(4 \left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \operatorname{ArcTan}[\operatorname{Tan}[c+dx]]) \right)\right] \right) \right)^2 \operatorname{Tan}[c+dx]^{5/2} (1 + \operatorname{Tan}[c+dx]^2)^2$$

■ **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[c+dx])^2 (e \operatorname{Tan}[c+dx])^{3/2} dx$$

Optimal (type 4, 335 leaves, 20 steps):

$$\frac{a^2 e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} - \frac{a^2 e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d} + \frac{a^2 e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} -$$

$$\frac{a^2 e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d} - \frac{2 a^2 e^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 d \sqrt{e \operatorname{Tan}[c+dx]}} +$$

$$\frac{2 a^2 e \sqrt{e \operatorname{Tan}[c+dx]}}{d} + \frac{4 a^2 e \operatorname{Sec}[c+dx] \sqrt{e \operatorname{Tan}[c+dx]}}{3 d} + \frac{2 a^2 (e \operatorname{Tan}[c+dx])^{5/2}}{5 d e}$$

Result (type 4, 323 leaves):

$$\left(\left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{dx}{2} \right) \right] \right)^2 \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \sec [c + dx]^2 (a + a \sec [c + dx])^2 (e \tan [c + dx])^{3/2} \right. \\ \left. \left(\frac{1}{d} \left(\frac{2}{3} (-1)^{1/4} \text{EllipticF} \left[i \text{ArcSinh} \left[(-1)^{1/4} \sqrt{\tan [c + dx]} \right], -1 \right] + 1 / \left(4 \sqrt{2} \right) \left(2 \text{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [c + dx]} \right] - \right. \right. \right. \right. \\ \left. \left. \left. 2 \text{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [c + dx]} \right] + \text{Log} \left[1 - \sqrt{2} \sqrt{\tan [c + dx]} + \tan [c + dx] \right] - \text{Log} \left[1 + \sqrt{2} \sqrt{\tan [c + dx]} + \tan [c + dx] \right] \right) \right) \right) + \right. \\ \left. \left. \frac{2 \left(\sqrt{\tan [c + dx]} + \frac{1}{5} \tan [c + dx]^{5/2} + \frac{2}{3} \sqrt{\tan [c + dx]} \sqrt{1 + \tan [c + dx]^2} \right)}{d} \right) \right) / \\ \left(4 \left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \text{ArcTan} [\tan [c + dx]]) \right) \right] \right) \right)^2 \tan [c + dx]^{3/2} (1 + \tan [c + dx]^2)^2 \right)$$

■ **Problem 113: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \sec [c + dx])^2 \sqrt{e \tan [c + dx]} dx$$

Optimal (type 4, 309 leaves, 19 steps):

$$- \frac{a^2 \sqrt{e} \text{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{e \tan [c + dx]}}{\sqrt{e}} \right]}{\sqrt{2} d} + \frac{a^2 \sqrt{e} \text{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{e \tan [c + dx]}}{\sqrt{e}} \right]}{\sqrt{2} d} + \\ \frac{a^2 \sqrt{e} \text{Log} \left[\sqrt{e} + \sqrt{e} \tan [c + dx] - \sqrt{2} \sqrt{e \tan [c + dx]} \right]}{2 \sqrt{2} d} - \frac{a^2 \sqrt{e} \text{Log} \left[\sqrt{e} + \sqrt{e} \tan [c + dx] + \sqrt{2} \sqrt{e \tan [c + dx]} \right]}{2 \sqrt{2} d} - \\ \frac{4 a^2 \cos [c + dx] \text{EllipticE} \left[c - \frac{\pi}{4} + dx, 2 \right] \sqrt{e \tan [c + dx]}}{d \sqrt{\sin [2c + 2dx]}} + \frac{2 a^2 (e \tan [c + dx])^{3/2}}{3 d e} + \frac{4 a^2 \cos [c + dx] (e \tan [c + dx])^{3/2}}{d e}$$

Result (type 4, 249 leaves):

$$\frac{1}{12 d \sqrt{\tan[c + d x]}} a^2 \cos\left[\frac{1}{2}(c + d x)\right]^4 \sec\left[\frac{1}{2} \operatorname{ArcTan}[\tan[c + d x]]\right]^4$$

$$\sqrt{e \tan[c + d x]} \left(-6 \sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[c + d x]}\right] + 6 \sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[c + d x]}\right]\right) -$$

$$48 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + d x]}\right], -1\right] + 48 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + d x]}\right], -1\right] +$$

$$3 \sqrt{2} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[c + d x]} + \tan[c + d x]\right] - 3 \sqrt{2} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[c + d x]} + \tan[c + d x]\right] + 8 \tan[c + d x]^{3/2}$$

■ **Problem 114: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sec[c + d x])^2}{\sqrt{e \tan[c + d x]}} dx$$

Optimal (type 4, 278 leaves, 18 steps):

$$-\frac{a^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d \sqrt{e}} + \frac{a^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d \sqrt{e}} - \frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c + d x] - \sqrt{2} \sqrt{e \tan[c + d x]}\right]}{2 \sqrt{2} d \sqrt{e}} +$$

$$\frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c + d x] + \sqrt{2} \sqrt{e \tan[c + d x]}\right]}{2 \sqrt{2} d \sqrt{e}} + \frac{2 a^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + d x, 2\right] \sec[c + d x] \sqrt{\sin[2 c + 2 d x]}}{d \sqrt{e \tan[c + d x]}} + \frac{2 a^2 \sqrt{e \tan[c + d x]}}{d e}$$

Result (type 4, 218 leaves):

$$\frac{1}{4 d \sqrt{e \tan[c + d x]}} a^2 \cos\left[\frac{1}{2}(c + d x)\right]^4 \sec\left[\frac{1}{2} \operatorname{ArcTan}[\tan[c + d x]]\right]^4$$

$$\left(-2 \sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[c + d x]}\right] + 2 \sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[c + d x]}\right] - 16 (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan[c + d x]}\right], -1\right] -$$

$$\sqrt{2} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[c + d x]} + \tan[c + d x]\right] + \sqrt{2} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[c + d x]} + \tan[c + d x]\right] + 8 \sqrt{\tan[c + d x]}\right) \sqrt{\tan[c + d x]}$$

■ **Problem 115: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sec[c + d x])^2}{(e \tan[c + d x])^{3/2}} dx$$

Optimal (type 4, 310 leaves, 20 steps):

$$\frac{a^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{3/2}} - \frac{a^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{3/2}} -$$

$$\frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c + d x] - \sqrt{2} \sqrt{e \tan[c + d x]}\right]}{2 \sqrt{2} d e^{3/2}} + \frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan[c + d x] + \sqrt{2} \sqrt{e \tan[c + d x]}\right]}{2 \sqrt{2} d e^{3/2}} -$$

$$\frac{4 a^2}{d e \sqrt{e \tan[c + d x]}} - \frac{4 a^2 \cos[c + d x]}{d e \sqrt{e \tan[c + d x]}} - \frac{4 a^2 \cos[c + d x] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right] \sqrt{e \tan[c + d x]}}{d e^2 \sqrt{\sin[2 c + 2 d x]}}$$

Result (type 4, 304 leaves) :

$$\begin{aligned}
 & - \frac{1}{4 d e \sqrt{e \operatorname{Tan}[c+d x]}} \\
 & a^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}\left[\frac{1}{2} \operatorname{ArcTan}[\operatorname{Tan}[c+d x]]\right]^4 \left(16+16 \sqrt{\operatorname{Sec}[c+d x]^2}-2 \sqrt{2} \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}\right] \sqrt{\operatorname{Tan}[c+d x]}+\right. \\
 & \quad \left.2 \sqrt{2} \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}\right] \sqrt{\operatorname{Tan}[c+d x]}+16(-1)^{3 / 4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\operatorname{Tan}[c+d x]}\right],-1\right] \sqrt{\operatorname{Tan}[c+d x]}-\right. \\
 & \quad \left.16(-1)^{3 / 4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\operatorname{Tan}[c+d x]}\right],-1\right] \sqrt{\operatorname{Tan}[c+d x]}+\right. \\
 & \quad \left.\sqrt{2} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}+\operatorname{Tan}[c+d x]\right] \sqrt{\operatorname{Tan}[c+d x]}-\sqrt{2} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}+\operatorname{Tan}[c+d x]\right] \sqrt{\operatorname{Tan}[c+d x]}\right)
 \end{aligned}$$

■ **Problem 116: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^2}{(e \operatorname{Tan}[c+d x])^{5 / 2}} d x$$

Optimal (type 4, 316 leaves, 20 steps) :

$$\begin{aligned}
 & \frac{a^2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{5 / 2}}-\frac{a^2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{5 / 2}}+ \\
 & \frac{a^2 \operatorname{Log}\left[\sqrt{e}+\sqrt{e} \operatorname{Tan}[c+d x]-\sqrt{2} \sqrt{e \operatorname{Tan}[c+d x]}\right]}{2 \sqrt{2} d e^{5 / 2}}-\frac{a^2 \operatorname{Log}\left[\sqrt{e}+\sqrt{e} \operatorname{Tan}[c+d x]+\sqrt{2} \sqrt{e \operatorname{Tan}[c+d x]}\right]}{2 \sqrt{2} d e^{5 / 2}}- \\
 & \frac{4 a^2}{3 d e(e \operatorname{Tan}[c+d x])^{3 / 2}}-\frac{4 a^2 \operatorname{Sec}[c+d x]}{3 d e(e \operatorname{Tan}[c+d x])^{3 / 2}}-\frac{2 a^2 \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \operatorname{Sec}[c+d x] \sqrt{\operatorname{Sin}[2 c+2 d x]}}{3 d e^2 \sqrt{e \operatorname{Tan}[c+d x]}}
 \end{aligned}$$

Result (type 4, 281 leaves) :

$$\begin{aligned}
 & \frac{1}{24 d e^2 \sqrt{e \operatorname{Tan}[c+d x]}} a^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Cos}[c+d x] \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \\
 & \operatorname{Sec}\left[\frac{1}{2} \operatorname{ArcTan}[\operatorname{Tan}[c+d x]]\right]^4 \left(-16-16 \sqrt{\operatorname{Sec}[c+d x]^2}+6 \sqrt{2} \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}\right] \operatorname{Tan}[c+d x]^{3 / 2}-\right. \\
 & \quad \left.6 \sqrt{2} \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}\right] \operatorname{Tan}[c+d x]^{3 / 2}+16(-1)^{1 / 4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\operatorname{Tan}[c+d x]}\right],-1\right] \operatorname{Tan}[c+d x]^{3 / 2}+\right. \\
 & \quad \left.3 \sqrt{2} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}+\operatorname{Tan}[c+d x]\right] \operatorname{Tan}[c+d x]^{3 / 2}-3 \sqrt{2} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+d x]}+\operatorname{Tan}[c+d x]\right] \operatorname{Tan}[c+d x]^{3 / 2}\right)
 \end{aligned}$$

■ **Problem 117: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^2}{(e \operatorname{Tan}[c+d x])^{7 / 2}} d x$$

Optimal (type 4, 370 leaves, 22 steps) :

$$\begin{aligned}
& - \frac{a^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{7/2}} + \frac{a^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} d e^{7/2}} + \frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d e^{7/2}} \\
& \frac{a^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} d e^{7/2}} - \frac{4 a^2}{5 d e (e \operatorname{Tan}[c+dx])^{5/2}} - \frac{4 a^2 \operatorname{Sec}[c+dx]}{5 d e (e \operatorname{Tan}[c+dx])^{5/2}} + \\
& \frac{2 a^2}{d e^3 \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{12 a^2 \operatorname{Cos}[c+dx]}{5 d e^3 \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{12 a^2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 d e^4 \sqrt{\operatorname{Sin}[2c+2dx]}}
\end{aligned}$$

Result (type 4, 367 leaves):

$$\begin{aligned}
& \left(\left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{dx}{2}\right)\right] \right)^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^2 (a + a \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]^{7/2} \right. \\
& \left. \left(\frac{1}{20 d} \left(48 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] - 48 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + \right. \right. \right. \\
& \left. \left. 5 \sqrt{2} \left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - \right. \right. \right. \\
& \left. \left. \left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \right) \right) + \frac{2 \left(-\frac{2}{5 \operatorname{Tan}[c+dx]^{5/2}} + \frac{1}{\sqrt{\operatorname{Tan}[c+dx]}} + \left(-\frac{2}{5 \operatorname{Tan}[c+dx]^{5/2}} + \frac{6}{5 \sqrt{\operatorname{Tan}[c+dx]}} \right) \sqrt{1 + \operatorname{Tan}[c+dx]^2} \right)}{d} \right) \Bigg) / \\
& \left(4 \left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \operatorname{ArcTan}[\operatorname{Tan}[c+dx]])\right)\right] \right) \right)^2 (e \operatorname{Tan}[c+dx])^{7/2} (1 + \operatorname{Tan}[c+dx]^2)^2
\end{aligned}$$

■ **Problem 118: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{11/2}}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 330 leaves, 18 steps):

$$\frac{e^{11/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} - \frac{e^{11/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{e^{11/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} -$$

$$\frac{e^{11/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} + \frac{5 e^6 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{21 a d \sqrt{e \operatorname{Tan}[c+dx]}} +$$

$$\frac{2 e^5 (21 - 5 \operatorname{Sec}[c+dx]) \sqrt{e \operatorname{Tan}[c+dx]}}{21 a d} - \frac{2 e^3 (7 - 5 \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2}}{35 a d}$$

Result (type 4, 316 leaves):

$$\left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c+dx]^2 (e \operatorname{Tan}[c+dx])^{11/2} \left(1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right) \right.$$

$$\left. \left(\frac{1}{d} 2 \left(-\frac{5}{21} (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + 1 / \left(4 \sqrt{2}\right) \left(2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - \right. \right. \right.$$

$$\left. \left. 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \right) \right) +$$

$$\left. \left. \frac{2 \left(\sqrt{\operatorname{Tan}[c+dx]} - \frac{1}{5} \operatorname{Tan}[c+dx]^{5/2} + \sqrt{1 + \operatorname{Tan}[c+dx]^2} \left(-\frac{5}{21} \sqrt{\operatorname{Tan}[c+dx]} + \frac{1}{7} \operatorname{Tan}[c+dx]^{5/2} \right) \right)}{d} \right) \right) /$$

$$\left((1 + \operatorname{Cos}[c+dx]) (a + a \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]^{11/2} (1 + \operatorname{Tan}[c+dx]^2) \right)$$

■ **Problem 119: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{9/2}}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 326 leaves, 18 steps):

$$- \frac{e^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{e^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} -$$

$$\frac{e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} + \frac{6 e^4 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 a d \sqrt{\operatorname{Sin}[2c+2dx]}} -$$

$$\frac{6 e^3 \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 a d} - \frac{2 e^3 (5 - 3 \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{3/2}}{15 a d}$$

Result (type 4, 305 leaves) :

$$\left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c+dx]^2 (e \operatorname{Tan}[c+dx])^{9/2} \left(1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right) \right. \\ \left. \left(\frac{1}{20d} \left(24 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] - 24 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + \right. \right. \right. \\ \left. \left. \left. 5\sqrt{2} \left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - \right. \right. \right. \\ \left. \left. \left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right]\right) + \frac{2 \operatorname{Tan}[c+dx]^{3/2} \left(-5 + 3 \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right)}{15d} \right) \right) \Bigg/ \\ \left((1 + \operatorname{Cos}[c+dx]) (a + a \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]^{9/2} (1 + \operatorname{Tan}[c+dx]^2) \right)$$

■ **Problem 120: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{7/2}}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 295 leaves, 17 steps) :

$$-\frac{e^{7/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} ad} + \frac{e^{7/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} ad} - \\ \frac{e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2\sqrt{2} ad} + \frac{e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2\sqrt{2} ad} - \\ \frac{e^4 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3ad \sqrt{e \operatorname{Tan}[c+dx]}} - \frac{2e^3 (3 - \operatorname{Sec}[c+dx]) \sqrt{e \operatorname{Tan}[c+dx]}}{3ad}$$

Result (type 4, 262 leaves) :

$$\frac{1}{6ad (1 + \operatorname{Sec}[c+dx])^2 \sqrt{\operatorname{Tan}[c+dx]}} \\ e^3 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \left(1 + \sqrt{\operatorname{Sec}[c+dx]^2}\right) \left(-6\sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + 6\sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right]\right) + \\ 8(-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] - 3\sqrt{2} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] + \\ 3\sqrt{2} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - 24\sqrt{\operatorname{Tan}[c+dx]} + 8\sqrt{\operatorname{Sec}[c+dx]^2} \sqrt{\operatorname{Tan}[c+dx]} \Bigg) \sqrt{e \operatorname{Tan}[c+dx]}$$

■ **Problem 121: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^{5/2}}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 285 leaves, 17 steps):

$$\frac{e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} - \frac{e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} - \frac{e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} + \frac{e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} - \frac{2 e^2 \operatorname{Cos}[c + d x] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right] \sqrt{e \operatorname{Tan}[c + d x]}}{a d \sqrt{\operatorname{Sin}[2 c + 2 d x]}} + \frac{2 e \operatorname{Cos}[c + d x] (e \operatorname{Tan}[c + d x])^{3/2}}{a d}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^{5/2}}{a + a \operatorname{Sec}[c + d x]} dx$$

■ **Problem 122: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^{3/2}}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 257 leaves, 16 steps):

$$\frac{e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} - \frac{e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} - \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} + \frac{e^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + d x, 2\right] \operatorname{Sec}[c + d x] \sqrt{\operatorname{Sin}[2 c + 2 d x]}}{a d \sqrt{e \operatorname{Tan}[c + d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^{3/2}}{a + a \operatorname{Sec}[c + d x]} dx$$

■ **Problem 123: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{e \operatorname{Tan}[c + d x]}}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 315 leaves, 18 steps):

$$\begin{aligned}
& - \frac{\sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{\sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \\
& \frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} - \frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d} + \\
& \frac{2 e (1 - \operatorname{Sec}[c+dx])}{a d \sqrt{e \operatorname{Tan}[c+dx]}} - \frac{2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{a d \sqrt{\operatorname{Sin}[2c+2dx]}} + \frac{2 \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{a d e}
\end{aligned}$$

Result (type 4, 261 leaves):

$$\begin{aligned}
& \frac{1}{a d \sqrt{\operatorname{Tan}[c+dx]}} \operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right]^2 \left(1 + \sqrt{\operatorname{Sec}[c+dx]^2}\right) \\
& \left(-(-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + \right. \\
& \left. \frac{1}{4 \sqrt{2}} \left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - \right. \right. \\
& \left. \left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right]\right) + \frac{1}{\sqrt{\operatorname{Tan}[c+dx]}} - \frac{\sqrt{\operatorname{Sec}[c+dx]^2}}{\sqrt{\operatorname{Tan}[c+dx]}} \right) \sqrt{e \operatorname{Tan}[c+dx]}
\end{aligned}$$

■ **Problem 124: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c+dx]) \sqrt{e \operatorname{Tan}[c+dx]}} dx$$

Optimal (type 4, 290 leaves, 17 steps):

$$\begin{aligned}
& - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d \sqrt{e}} + \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d \sqrt{e}} - \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d \sqrt{e}} + \\
& \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d \sqrt{e}} + \frac{2 e (1 - \operatorname{Sec}[c+dx])}{3 a d (e \operatorname{Tan}[c+dx])^{3/2}} - \frac{\operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 a d \sqrt{e \operatorname{Tan}[c+dx]}}
\end{aligned}$$

Result (type 4, 225 leaves):

$$\frac{1}{24 a d \sqrt{e \tan [c+d x]}} \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\left(1+\sqrt{\operatorname{Sec}[c+d x]^2}\right)$$

$$\left(8(-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan [c+d x]}\right],-1\right]+3 \sqrt{2}\left(-2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]+2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]\right)-\right.$$

$$\left.\operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]+\operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]\right)-\frac{8\left(-1+\sqrt{\operatorname{Sec}[c+d x]^2}\right)}{\tan [c+d x]^{3/2}} \sqrt{\tan [c+d x]}$$

■ **Problem 125: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+a \operatorname{Sec}[c+d x])\left(e \tan [c+d x]\right)^{3/2}} dx$$

Optimal (type 4, 359 leaves, 19 steps):

$$\frac{\operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{e \tan [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d e^{3/2}}-\frac{\operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{e \tan [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d e^{3/2}}-\frac{\operatorname{Log}\left[\sqrt{e}+\sqrt{e} \tan [c+d x]-\sqrt{2} \sqrt{e \tan [c+d x]}\right]}{2 \sqrt{2} a d e^{3/2}}+$$

$$\frac{\operatorname{Log}\left[\sqrt{e}+\sqrt{e} \tan [c+d x]+\sqrt{2} \sqrt{e \tan [c+d x]}\right]}{2 \sqrt{2} a d e^{3/2}}+\frac{2 e(1-\operatorname{Sec}[c+d x])}{5 a d\left(e \tan [c+d x]\right)^{5/2}}-\frac{2(5-3 \operatorname{Sec}[c+d x])}{5 a d e \sqrt{e \tan [c+d x]}}+$$

$$\frac{6 \cos [c+d x] \operatorname{EllipticE}\left[c-\frac{\pi}{4}+d x, 2\right] \sqrt{e \tan [c+d x]}}{5 a d e^2 \sqrt{\sin [2 c+2 d x]}}-\frac{6 \cos [c+d x]\left(e \tan [c+d x]\right)^{3/2}}{5 a d e^3}$$

Result (type 4, 346 leaves):

$$\left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^{3/2} \left(1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right) \right. \\ \left. \left(\frac{1}{20d} \left(24 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] - 24 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + \right. \right. \right. \\ \left. \left. 5\sqrt{2} \left(2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] + \right. \right. \right. \\ \left. \left. \left. \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \right) \right) + \frac{2 \left(\frac{1}{5 \operatorname{Tan}[c+dx]^{5/2}} - \frac{1}{\sqrt{\operatorname{Tan}[c+dx]}} + \left(-\frac{1}{5 \operatorname{Tan}[c+dx]^{5/2}} + \frac{3}{5 \sqrt{\operatorname{Tan}[c+dx]}} \right) \sqrt{1 + \operatorname{Tan}[c+dx]^2} \right)}{d} \right) \Bigg) / \\ \left((1 + \operatorname{Cos}[c+dx]) (a + a \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{3/2} (1 + \operatorname{Tan}[c+dx]^2) \right)$$

■ **Problem 126: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2}} dx$$

Optimal (type 4, 328 leaves, 18 steps):

$$\frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d e^{5/2}} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a d e^{5/2}} + \\ \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d e^{5/2}} - \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a d e^{5/2}} + \\ \frac{2 e (1 - \operatorname{Sec}[c+dx])}{7 a d (e \operatorname{Tan}[c+dx])^{7/2}} - \frac{2 (7 - 5 \operatorname{Sec}[c+dx])}{21 a d e (e \operatorname{Tan}[c+dx])^{3/2}} + \frac{5 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c + 2dx]}}{21 a d e^2 \sqrt{e \operatorname{Tan}[c+dx]}}$$

Result (type 4, 304 leaves):

$$\left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]^{5/2} \left(1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right) \right. \\ \left. \left(1/d2 \left(-\frac{5}{21} (-1)^{1/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Tan}[c+dx]}\right], -1\right] + 1/\left(4\sqrt{2}\right) \left(2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] - \right. \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right]\right) \right) \right) + \\ \left. \left. \left. \left. \frac{2 \left(3 - 3 \sqrt{1 + \operatorname{Tan}[c+dx]^2} + \operatorname{Tan}[c+dx]^2 \left(-7 + 5 \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right)\right)}{21 d \operatorname{Tan}[c+dx]^{7/2}} \right) \right) \right) \right) / \\ \left((1 + \operatorname{Cos}[c+dx]) (a + a \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2} (1 + \operatorname{Tan}[c+dx]^2) \right)$$

■ **Problem 127: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{13/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 372 leaves, 22 steps):

$$\frac{e^{13/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{13/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{13/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} + \\ \frac{e^{13/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \frac{12 e^6 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 a^2 d \sqrt{\operatorname{Sin}[2c+2dx]}} + \\ \frac{2 e^5 (e \operatorname{Tan}[c+dx])^{3/2}}{3 a^2 d} + \frac{12 e^5 \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 a^2 d} - \frac{4 e^5 \operatorname{Sec}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{5 a^2 d} + \frac{2 e^3 (e \operatorname{Tan}[c+dx])^{7/2}}{7 a^2 d}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{13/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 128: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{11/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 339 leaves, 21 steps):

$$\frac{e^{11/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{11/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} + \frac{e^{11/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} -$$

$$\frac{e^{11/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} + \frac{2 e^6 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} +$$

$$\frac{2 e^5 \sqrt{e \operatorname{Tan}[c+dx]}}{a^2 d} - \frac{4 e^5 \operatorname{Sec}[c+dx] \sqrt{e \operatorname{Tan}[c+dx]}}{3 a^2 d} + \frac{2 e^3 (e \operatorname{Tan}[c+dx])^{5/2}}{5 a^2 d}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{11/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 129: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{9/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 312 leaves, 20 steps):

$$-\frac{e^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} + \frac{e^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} +$$

$$\frac{e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \frac{e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} +$$

$$\frac{4 e^4 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{a^2 d \sqrt{\operatorname{Sin}[2c+2dx]}} + \frac{2 e^3 (e \operatorname{Tan}[c+dx])^{3/2}}{3 a^2 d} - \frac{4 e^3 \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{a^2 d}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{9/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 130: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{7/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 281 leaves, 19 steps):

$$\begin{aligned}
& - \frac{e^{7/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} + \frac{e^{7/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} + \\
& \frac{e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \frac{2 e^4 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{2 e^3 \sqrt{e \operatorname{Tan}[c+dx]}}{a^2 d}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{7/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 131: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{5/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 310 leaves, 21 steps):

$$\begin{aligned}
& \frac{e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \\
& \frac{e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} + \frac{e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \\
& \frac{4 e^3}{a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{4 e^3 \operatorname{Cos}[c+dx]}{a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{4 e^2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{a^2 d \sqrt{\operatorname{Sin}[2c+2dx]}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{5/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 132: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c+dx])^{3/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 316 leaves, 21 steps):

$$\frac{e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} - \frac{e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} +$$

$$\frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} -$$

$$\frac{4 e^3}{3 a^2 d (e \operatorname{Tan}[c+dx])^{3/2}} + \frac{4 e^3 \operatorname{Sec}[c+dx]}{3 a^2 d (e \operatorname{Tan}[c+dx])^{3/2}} + \frac{2 e^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 a^2 d \sqrt{e \operatorname{Tan}[c+dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c+dx])^{3/2}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 133: Unable to integrate problem.**

$$\int \frac{\sqrt{e \operatorname{Tan}[c+dx]}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 363 leaves, 23 steps):

$$-\frac{\sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} + \frac{\sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d} + \frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} -$$

$$\frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a^2 d} - \frac{4 e^3}{5 a^2 d (e \operatorname{Tan}[c+dx])^{5/2}} + \frac{4 e^3 \operatorname{Sec}[c+dx]}{5 a^2 d (e \operatorname{Tan}[c+dx])^{5/2}} +$$

$$\frac{2 e}{a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} - \frac{12 e \operatorname{Cos}[c+dx]}{5 a^2 d \sqrt{e \operatorname{Tan}[c+dx]}} - \frac{12 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{5 a^2 d \sqrt{\operatorname{Sin}[2c+2dx]}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{e \operatorname{Tan}[c+dx]}}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

■ **Problem 134: Mathematica result simpler than optimal antiderivative, IF it can be verified!**

$$\int \frac{1}{(a + a \operatorname{Sec}[c+dx])^2 \sqrt{e \operatorname{Tan}[c+dx]}} dx$$

Optimal (type 4, 365 leaves, 23 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d \sqrt{e}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a^2 d \sqrt{e}} - \frac{\text{Log}\left[\sqrt{e} + \sqrt{e \tan[c+dx]} - \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} a^2 d \sqrt{e}} + \\
& \frac{\text{Log}\left[\sqrt{e} + \sqrt{e \tan[c+dx]} + \sqrt{2} \sqrt{e \tan[c+dx]}\right]}{2 \sqrt{2} a^2 d \sqrt{e}} - \frac{4 e^3}{7 a^2 d (e \tan[c+dx])^{7/2}} + \frac{4 e^3 \text{Sec}[c+dx]}{7 a^2 d (e \tan[c+dx])^{7/2}} + \\
& \frac{2 e}{3 a^2 d (e \tan[c+dx])^{3/2}} - \frac{20 e \text{Sec}[c+dx]}{21 a^2 d (e \tan[c+dx])^{3/2}} - \frac{10 \text{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \text{Sec}[c+dx] \sqrt{\sin[2c+2dx]}}{21 a^2 d \sqrt{e \tan[c+dx]}}
\end{aligned}$$

Result (type 3, 247 leaves):

$$\begin{aligned}
& - \left((60 - 126 \cos[c] + 40 \cos[2c] - 84 \cos[dx] + 26 \cos[c-dx] + 80 \cos[c+dx] + 20 \cos[2(c+dx)] - 84 \cos[2c+dx] + \right. \\
& \quad \left. 26 \cos[3c+dx] - 21 \cos[c+2dx] - 21 \cos[3c+2dx]) \text{Sec}[2c] \sin[c+dx] \right) / \left(42 a^2 d (1 + \cos[c+dx])^2 \sqrt{e \tan[c+dx]} \right) - \\
& \frac{1}{42 a^2 d \sqrt{e \tan[c+dx]}} \text{Sec}[2c] \text{Sec}[c+dx] \left(21 \text{ArcSin}[\cos[c+dx] - \sin[c+dx]] \cos[2c] - \right. \\
& \quad \left. 21 \cos[2c] \text{Log}\left[\cos[c+dx] + \sin[c+dx] + \sqrt{\sin[2(c+dx)]}\right] + 2(-10 + 21 \cos[c]) \sqrt{\sin[2(c+dx)]} \right) \sqrt{\sin[2(c+dx)]}
\end{aligned}$$

■ **Problem 135: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \text{Sec}[c+dx]} \tan[c+dx]^5 dx$$

Optimal (type 3, 147 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{a} \text{ArcTanh}\left[\frac{\sqrt{a+a \text{Sec}[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{2 \sqrt{a+a \text{Sec}[c+dx]}}{d} + \\
& \frac{2 (a + a \text{Sec}[c+dx])^{3/2}}{3 a d} + \frac{2 (a + a \text{Sec}[c+dx])^{5/2}}{5 a^2 d} - \frac{6 (a + a \text{Sec}[c+dx])^{7/2}}{7 a^3 d} + \frac{2 (a + a \text{Sec}[c+dx])^{9/2}}{9 a^4 d}
\end{aligned}$$

Result (type 3, 533 leaves):

$$\begin{aligned}
& \frac{1}{144 d} 5 (1 - 2 \operatorname{Cos}[c + d x] + 2 \operatorname{Cos}[2 (c + d x)] - 2 \operatorname{Cos}[3 (c + d x)] + 2 \operatorname{Cos}[4 (c + d x)]) \operatorname{Sec}[c + d x]^4 \sqrt{a (1 + \operatorname{Sec}[c + d x])} - \frac{1}{504 d} \\
& 5 (11 - 22 \operatorname{Cos}[c + d x] + 22 \operatorname{Cos}[2 (c + d x)] - 4 \operatorname{Cos}[3 (c + d x)] + 4 \operatorname{Cos}[4 (c + d x)]) \operatorname{Sec}[c + d x]^4 \sqrt{a (1 + \operatorname{Sec}[c + d x])} + \frac{1}{504 d} \\
& (107 - 88 \operatorname{Cos}[c + d x] + 88 \operatorname{Cos}[2 (c + d x)] - 16 \operatorname{Cos}[3 (c + d x)] + 16 \operatorname{Cos}[4 (c + d x)]) \operatorname{Sec}[c + d x]^4 \sqrt{a (1 + \operatorname{Sec}[c + d x])} - \frac{1}{1008 d} \\
& (109 + 34 \operatorname{Cos}[c + d x] + 176 \operatorname{Cos}[2 (c + d x)] - 32 \operatorname{Cos}[3 (c + d x)] + 32 \operatorname{Cos}[4 (c + d x)]) \operatorname{Sec}[c + d x]^4 \sqrt{a (1 + \operatorname{Sec}[c + d x])} + \frac{1}{5040 d} \\
& (557 + 902 \operatorname{Cos}[c + d x] + 778 \operatorname{Cos}[2 (c + d x)] + 374 \operatorname{Cos}[3 (c + d x)] + 256 \operatorname{Cos}[4 (c + d x)]) \operatorname{Sec}[c + d x]^4 \sqrt{a (1 + \operatorname{Sec}[c + d x])} + \\
& \frac{1}{5040 d} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \sqrt{a (1 + \operatorname{Sec}[c + d x])} \\
& \left(5040 \sqrt{2} \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right]^2 \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2\right] \right) \right. \\
& \left. \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} + \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] (9008 - 1984 \operatorname{Sec}[c + d x] - 1032 \operatorname{Sec}[c + d x]^2 + 230 \operatorname{Sec}[c + d x]^3 + 35 \operatorname{Sec}[c + d x]^4) \right)
\end{aligned}$$

■ **Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 3, 99 leaves, 6 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + a \operatorname{Sec}[c + d x]}}{\sqrt{a}}\right]}{d} - \frac{2 \sqrt{a + a \operatorname{Sec}[c + d x]}}{d} - \frac{2 (a + a \operatorname{Sec}[c + d x])^{3/2}}{3 a d} + \frac{2 (a + a \operatorname{Sec}[c + d x])^{5/2}}{5 a^2 d}$$

Result (type 3, 315 leaves):

$$\begin{aligned}
& - \frac{3(1 - 2 \cos[c + dx] + 2 \cos[2(c + dx)]) \sec[c + dx]^2 \sqrt{a(1 + \sec[c + dx])}}{20d} + \\
& \frac{(7 - 4 \cos[c + dx] + 4 \cos[2(c + dx)]) \sec[c + dx]^2 \sqrt{a(1 + \sec[c + dx])}}{20d} - \\
& \frac{(13 + 14 \cos[c + dx] + 16 \cos[2(c + dx)]) \sec[c + dx]^2 \sqrt{a(1 + \sec[c + dx])}}{60d} + \frac{1}{60d} \sec\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \sec[c + dx])} \\
& \left(60 \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \left(-\log\left[\sec\left[\frac{1}{4}(c + dx)\right]^2\right] + \log\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4} - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2\right] \right) \right. \\
& \left. \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4 + \cos\left[\frac{1}{2}(c + dx)\right] (-92 + 16 \sec[c + dx] + 3 \sec[c + dx]^2)} \right)
\end{aligned}$$

■ **Problem 137: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sec[c + dx]} \tan[c + dx] dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$- \frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{2\sqrt{a+a \sec[c+dx]}}{d}$$

Result (type 3, 144 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(2 \cos\left[\frac{1}{2}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \left(\log\left[\sec\left[\frac{1}{4}(c + dx)\right]^2\right] - \log\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4} - 2 \tan\left[\frac{1}{4}(c + dx)\right]^2\right] \right) \right. \\
& \left. \sqrt{\cos[c + dx] \sec\left[\frac{1}{4}(c + dx)\right]^4} \right) \sec\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \sec[c + dx])}
\end{aligned}$$

■ **Problem 138: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx] \sqrt{a + a \sec[c + dx]} dx$$

Optimal (type 3, 73 leaves, 6 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec[c+dx]}}{\sqrt{a}}\right]}{d} - \frac{\sqrt{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{d}$$

Result (type 3, 237 leaves):

$$\frac{1}{2d} \cos\left[\frac{1}{4}(c+dx)\right]^2 \left(-2\sqrt{2} \operatorname{Log}\left[\sec\left[\frac{1}{4}(c+dx)\right]^2\right] + \operatorname{Log}\left[\tan\left[\frac{1}{4}(c+dx)\right]^2\right] - \operatorname{Log}\left[1 + \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] + \right. \\ \left. 2\sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] + \operatorname{Log}\left[3 - \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] \right) \\ \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}}$$

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^3 \sqrt{a+a \sec[c+dx]} dx$$

Optimal (type 3, 131 leaves, 8 steps):

$$-\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{7\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec[c+dx]}}{\sqrt{2}\sqrt{a}}\right]}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a+a \sec[c+dx]}} + \frac{a}{2d(1-\sec[c+dx])\sqrt{a+a \sec[c+dx]}}$$

Result (type 3, 273 leaves):

$$\frac{1}{16d} \left(-2 \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 + \cos\left[\frac{1}{4}(c+dx)\right]^2 \right) \\ \left(16\sqrt{2} \operatorname{Log}\left[\sec\left[\frac{1}{4}(c+dx)\right]^2\right] - 7 \operatorname{Log}\left[\tan\left[\frac{1}{4}(c+dx)\right]^2\right] + 7 \operatorname{Log}\left[1 + \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 3 \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] - 16\sqrt{2} \right. \\ \left. \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - 2 \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] - 7 \operatorname{Log}\left[3 - \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 - \tan\left[\frac{1}{4}(c+dx)\right]^2}\right] \right) \\ \sqrt{\cos[c+dx] \sec\left[\frac{1}{4}(c+dx)\right]^4 \sec\left[\frac{1}{2}(c+dx)\right] - 4 \left(-3 + \sec\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{a(1+\sec[c+dx])}}$$

■ **Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \sec[c+dx]} \tan[c+dx]^6 dx$$

Optimal (type 3, 222 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2 a \operatorname{Tan}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{2 a^2 \operatorname{Tan}[c+dx]^3}{3 d (a+a \operatorname{Sec}[c+dx])^{3/2}} + \\
& \frac{2 a^3 \operatorname{Tan}[c+dx]^5}{5 d (a+a \operatorname{Sec}[c+dx])^{5/2}} + \frac{2 a^4 \operatorname{Tan}[c+dx]^7}{d (a+a \operatorname{Sec}[c+dx])^{7/2}} + \frac{10 a^5 \operatorname{Tan}[c+dx]^9}{9 d (a+a \operatorname{Sec}[c+dx])^{9/2}} + \frac{2 a^6 \operatorname{Tan}[c+dx]^{11}}{11 d (a+a \operatorname{Sec}[c+dx])^{11/2}}
\end{aligned}$$

Result (type 4, 959 leaves):

$$\begin{aligned}
& \frac{1}{64 d \sqrt{\operatorname{Sec}[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(\frac{1}{3465} 2 (14153 + 108232 \operatorname{Cos}[c+dx] + 19924 \operatorname{Cos}[2(c+dx)] + 56884 \operatorname{Cos}[3(c+dx)] + 6086 \operatorname{Cos}[4(c+dx)] + 13016 \operatorname{Cos}[5(c+dx)]) \right. \\
& \operatorname{Sec}[c+dx]^{11/2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 512 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \left. \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}} \right) + \\
& \frac{1}{176 d} 3 (1 + 2 \operatorname{Cos}[c+dx] + 2 \operatorname{Cos}[2(c+dx)] + 2 \operatorname{Cos}[3(c+dx)] + 2 \operatorname{Cos}[4(c+dx)] + 2 \operatorname{Cos}[5(c+dx)]) \\
& \operatorname{Sec}[c+dx]^5 \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{1056 d} \\
& 5 (13 + 26 \operatorname{Cos}[c+dx] + 26 \operatorname{Cos}[2(c+dx)] + 26 \operatorname{Cos}[3(c+dx)] + 4 \operatorname{Cos}[4(c+dx)] + 4 \operatorname{Cos}[5(c+dx)]) \\
& \operatorname{Sec}[c+dx]^5 \sqrt{a(1+\operatorname{Sec}[c+dx])} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5544 d} \\
& 5 (151 + 302 \operatorname{Cos}[c+dx] + 104 \operatorname{Cos}[2(c+dx)] + 104 \operatorname{Cos}[3(c+dx)] + 16 \operatorname{Cos}[4(c+dx)] + 16 \operatorname{Cos}[5(c+dx)]) \\
& \operatorname{Sec}[c+dx]^5 \sqrt{a(1+\operatorname{Sec}[c+dx])} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{2464 d}
\end{aligned}$$

$$\begin{aligned}
& (71 + 604 \operatorname{Cos}[c + d x] + 208 \operatorname{Cos}[2(c + d x)] + 208 \operatorname{Cos}[3(c + d x)] + 32 \operatorname{Cos}[4(c + d x)] + 32 \operatorname{Cos}[5(c + d x)]) \\
& \operatorname{Sec}[c + d x]^5 \sqrt{a(1 + \operatorname{Sec}[c + d x])} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + \frac{1}{18480 d} \\
& (-587 + 2522 \operatorname{Cos}[c + d x] - 646 \operatorname{Cos}[2(c + d x)] + 1664 \operatorname{Cos}[3(c + d x)] + 256 \operatorname{Cos}[4(c + d x)] + 256 \operatorname{Cos}[5(c + d x)]) \\
& \operatorname{Sec}[c + d x]^5 \sqrt{a(1 + \operatorname{Sec}[c + d x])} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \frac{1}{22176 d} \\
& (-1867 + 3658 \operatorname{Cos}[c + d x] - 2678 \operatorname{Cos}[2(c + d x)] + 1942 \operatorname{Cos}[3(c + d x)] - 874 \operatorname{Cos}[4(c + d x)] + 512 \operatorname{Cos}[5(c + d x)]) \\
& \operatorname{Sec}[c + d x]^5 \sqrt{a(1 + \operatorname{Sec}[c + d x])} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]
\end{aligned}$$

- **Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]^4 dx$$

Optimal (type 3, 160 leaves, 4 steps):

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{d} - \frac{2 a \operatorname{Tan}[c + d x]}{d \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{2 a^2 \operatorname{Tan}[c + d x]^3}{3 d (a + a \operatorname{Sec}[c + d x])^{3/2}} + \frac{6 a^3 \operatorname{Tan}[c + d x]^5}{5 d (a + a \operatorname{Sec}[c + d x])^{5/2}} + \frac{2 a^4 \operatorname{Tan}[c + d x]^7}{7 d (a + a \operatorname{Sec}[c + d x])^{7/2}}$$

Result (type 4, 681 leaves):

$$\begin{aligned}
& \frac{1}{16 d \sqrt{\sec [c+d x]}} \sec \left[\frac{1}{2} (c+d x) \right] \sqrt{a (1+\sec [c+d x])} \\
& \left(-\frac{2}{105} (127+954 \cos [c+d x]+142 \cos [2(c+d x)]+352 \cos [3(c+d x)]) \sec [c+d x]^{7/2} \sin \left[\frac{1}{2} (c+d x) \right] -128 (-3-2 \sqrt{2}) \cos \left[\frac{1}{4} (c+d x) \right]^4 \right. \\
& \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2} (c+d x) \right]}{1+\cos \left[\frac{1}{2} (c+d x) \right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2} (c+d x) \right]}{1+\cos \left[\frac{1}{2} (c+d x) \right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2} (c+d x) \right] \right) \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (c+d x) \right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right) \right. \\
& \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2} (c+d x) \right] \right) \sec \left[\frac{1}{4} (c+d x) \right]^2 \sec [c+d x]^{3/2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4} (c+d x) \right]^2}} \right) - \\
& \frac{1}{14 d} (1+2 \cos [c+d x]+2 \cos [2(c+d x)]+2 \cos [3(c+d x)]) \sec [c+d x]^3 \sqrt{a (1+\sec [c+d x])} \\
& \tan \left[\frac{1}{2} (c+d x) \right] + \frac{1}{140 d} \\
& 3 (9+18 \cos [c+d x]+4 \cos [2(c+d x)]+4 \cos [3(c+d x)]) \sec [c+d x]^3 \\
& \sqrt{a (1+\sec [c+d x])} \tan \left[\frac{1}{2} (c+d x) \right] - \frac{1}{210 d} \\
& (1+72 \cos [c+d x]+16 \cos [2(c+d x)]+16 \cos [3(c+d x)]) \sec [c+d x]^3 \sqrt{a (1+\sec [c+d x])} \tan \left[\frac{1}{2} (c+d x) \right] + \\
& \frac{1}{280 d} \\
& (-33+74 \cos [c+d x]-38 \cos [2(c+d x)]+32 \cos [3(c+d x)]) \\
& \sec [c+d x]^3 \sqrt{a (1+\sec [c+d x])} \tan \left[\frac{1}{2} (c+d x) \right]
\end{aligned}$$

- **Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+a \sec [c+d x]} \tan [c+d x]^2 dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$-\frac{2\sqrt{a}\operatorname{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{d} + \frac{2a\tan[c+dx]}{d\sqrt{a+a\sec[c+dx]}} + \frac{2a^2\tan[c+dx]^3}{3d(a+a\sec[c+dx])^{3/2}}$$

Result (type 4, 479 leaves):

$$\frac{1}{4d\sqrt{\sec[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\left(\frac{2}{3}(1+8\cos[c+dx])\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 32(-3-2\sqrt{2})\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}\right.$$

$$\left.\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\right.$$

$$\left.\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)\right.$$

$$\left.\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\operatorname{Sec}[c+dx]^{3/2}\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}\right) +$$

$$\frac{(1+2\cos[c+dx])\operatorname{Sec}[c+dx]\sqrt{a(1+\sec[c+dx])}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{3d} -$$

$$\frac{(-1+4\cos[c+dx])\operatorname{Sec}[c+dx]\sqrt{a(1+\sec[c+dx])}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{6d}$$

■ **Problem 147: Result more than twice size of optimal antiderivative.**

$$\int (a+a\sec[c+dx])^{3/2}\tan[c+dx]^5 dx$$

Optimal (type 3, 169 leaves, 9 steps):

$$-\frac{2a^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{a+a\sec[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{2a\sqrt{a+a\sec[c+dx]}}{d} + \frac{2(a+a\sec[c+dx])^{3/2}}{3d} +$$

$$\frac{2(a+a\sec[c+dx])^{5/2}}{5ad} + \frac{2(a+a\sec[c+dx])^{7/2}}{7a^2d} - \frac{2(a+a\sec[c+dx])^{9/2}}{3a^3d} + \frac{2(a+a\sec[c+dx])^{11/2}}{11a^4d}$$

Result (type 3, 752 leaves):

$$\begin{aligned}
& \frac{1}{352 d} (-1 + 2 \operatorname{Cos}[c + d x] - 2 \operatorname{Cos}[2 (c + d x)] + 2 \operatorname{Cos}[3 (c + d x)] - 2 \operatorname{Cos}[4 (c + d x)] + 2 \operatorname{Cos}[5 (c + d x)]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 \\
& (a (1 + \operatorname{Sec}[c + d x]))^{3/2} + \frac{1}{6336 d} 5 (-13 + 26 \operatorname{Cos}[c + d x] - 26 \operatorname{Cos}[2 (c + d x)] + 26 \operatorname{Cos}[3 (c + d x)] - 4 \operatorname{Cos}[4 (c + d x)] + 4 \operatorname{Cos}[5 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} - \frac{1}{11088 d} \\
& 5 (-151 + 302 \operatorname{Cos}[c + d x] - 104 \operatorname{Cos}[2 (c + d x)] + 104 \operatorname{Cos}[3 (c + d x)] - 16 \operatorname{Cos}[4 (c + d x)] + 16 \operatorname{Cos}[5 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} + \frac{1}{14784 d} \\
& 5 (-71 + 604 \operatorname{Cos}[c + d x] - 208 \operatorname{Cos}[2 (c + d x)] + 208 \operatorname{Cos}[3 (c + d x)] - 32 \operatorname{Cos}[4 (c + d x)] + 32 \operatorname{Cos}[5 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} - \frac{1}{15840 d} \\
& (587 + 2522 \operatorname{Cos}[c + d x] + 646 \operatorname{Cos}[2 (c + d x)] + 1664 \operatorname{Cos}[3 (c + d x)] - 256 \operatorname{Cos}[4 (c + d x)] + 256 \operatorname{Cos}[5 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} + \frac{1}{14784 d} \\
& (1867 + 3658 \operatorname{Cos}[c + d x] + 2678 \operatorname{Cos}[2 (c + d x)] + 1942 \operatorname{Cos}[3 (c + d x)] + 874 \operatorname{Cos}[4 (c + d x)] + 512 \operatorname{Cos}[5 (c + d x)]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \\
& \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} - \frac{1}{221760 d} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 (a (1 + \operatorname{Sec}[c + d x]))^{3/2} \left(-110880 \sqrt{2} \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right]^2 \right. \\
& \left. \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2\right] \right) \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} + \right. \\
& \left. \left. \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] (-208256 + 48688 \operatorname{Sec}[c + d x] + 32784 \operatorname{Sec}[c + d x]^2 - 8840 \operatorname{Sec}[c + d x]^3 - 2660 \operatorname{Sec}[c + d x]^4 + 315 \operatorname{Sec}[c + d x]^5) \right) \right)
\end{aligned}$$

■ **Problem 148: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} - \frac{2 a \sqrt{a+a \operatorname{Sec}[c+d x]}}{d} - \frac{2 (a+a \operatorname{Sec}[c+d x])^{3/2}}{3 d} - \frac{2 (a+a \operatorname{Sec}[c+d x])^{5/2}}{5 a d} + \frac{2 (a+a \operatorname{Sec}[c+d x])^{7/2}}{7 a^2 d}$$

Result (type 3, 399 leaves):

$$\begin{aligned}
& -\frac{1}{280d} 3 (-9 + 18 \cos [c + dx] - 4 \cos [2 (c + dx)] + 4 \cos [3 (c + dx)]) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec} [c + dx]^2 (a (1 + \operatorname{Sec} [c + dx]))^{3/2} + \\
& \frac{1}{210d} (-1 + 72 \cos [c + dx] - 16 \cos [2 (c + dx)] + 16 \cos [3 (c + dx)]) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec} [c + dx]^2 (a (1 + \operatorname{Sec} [c + dx]))^{3/2} - \\
& \frac{1}{560d} 3 (33 + 74 \cos [c + dx] + 38 \cos [2 (c + dx)] + 32 \cos [3 (c + dx)]) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec} [c + dx]^2 (a (1 + \operatorname{Sec} [c + dx]))^{3/2} - \\
& \frac{1}{1680d} \cos [c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec} [c + dx]))^{3/2} \\
& \left(840 \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right]^2 \left(\operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right] - \operatorname{Log} \left[2 + \sqrt{2} \sqrt{\cos [c + dx] \operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^4 - 2 \tan \left[\frac{1}{4} (c + dx) \right]^2} \right] \right) \right. \\
& \left. \sqrt{\cos [c + dx] \operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^4 + \cos \left[\frac{1}{2} (c + dx) \right] (1408 - 284 \operatorname{Sec} [c + dx] - 102 \operatorname{Sec} [c + dx]^2 + 15 \operatorname{Sec} [c + dx]^3)} \right)
\end{aligned}$$

■ **Problem 149: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec} [c + dx])^{3/2} \tan [c + dx] dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$-\frac{2 a^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + a \operatorname{Sec} [c + dx]}}{\sqrt{a}} \right]}{d} + \frac{2 a \sqrt{a + a \operatorname{Sec} [c + dx]}}{d} + \frac{2 (a + a \operatorname{Sec} [c + dx])^{3/2}}{3 d}$$

Result (type 3, 158 leaves):

$$\begin{aligned}
& \frac{1}{6d} \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \\
& \left(2 + \cos [c + dx] \left(8 + 3 \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right]^2 \left(\operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^2 \right] - \operatorname{Log} \left[2 + \sqrt{2} \sqrt{\cos [c + dx] \operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^4 - 2 \tan \left[\frac{1}{4} (c + dx) \right]^2} \right] \right) \right. \right. \\
& \left. \left. \sqrt{\cos [c + dx] \operatorname{Sec} \left[\frac{1}{4} (c + dx) \right]^4 + \cos \left[\frac{1}{2} (c + dx) \right] (1408 - 284 \operatorname{Sec} [c + dx] - 102 \operatorname{Sec} [c + dx]^2 + 15 \operatorname{Sec} [c + dx]^3)} \right) \right) (a (1 + \operatorname{Sec} [c + dx]))^{3/2}
\end{aligned}$$

■ **Problem 150: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + dx] (a + a \operatorname{Sec} [c + dx])^{3/2} dx$$

Optimal (type 3, 73 leaves, 6 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} - \frac{2 \sqrt{2} a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{d}$$

Result (type 3, 238 leaves):

$$\begin{aligned} & \frac{1}{2 d} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^6 \left(-\sqrt{2} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] + \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[1 + \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 3 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \right. \\ & \left. \sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \operatorname{Log}\left[3 - \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \\ & \left(\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4 \right)^{3/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \end{aligned}$$

■ **Problem 151: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c+d x]^3 (a+a \operatorname{Sec}[c+d x])^{3/2} dx$$

Optimal (type 3, 109 leaves, 7 steps):

$$-\frac{2 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} + \frac{5 a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} d} + \frac{a \sqrt{a+a \operatorname{Sec}[c+d x]}}{2 d (1-\operatorname{Sec}[c+d x])}$$

Result (type 3, 325 leaves):

$$\begin{aligned} & \frac{1}{d} \operatorname{Cos}[c+d x] \left(\frac{1}{4} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{8} \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} + \\ & \frac{1}{16 d} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}[c+d x] \left(8 \sqrt{2} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - 5 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \right. \\ & \left. 5 \operatorname{Log}\left[1 + \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 3 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - 8 \sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - \right. \\ & \left. 5 \operatorname{Log}\left[3 - \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \end{aligned}$$

■ **Problem 153: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]^6 dx$$

Optimal (type 3, 258 leaves, 4 steps) :

$$\begin{aligned}
 & - \frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} + \frac{2 a^2 \operatorname{Tan}[c+d x]}{d \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{2 a^3 \operatorname{Tan}[c+d x]^3}{3 d (a+a \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 a^4 \operatorname{Tan}[c+d x]^5}{5 d (a+a \operatorname{Sec}[c+d x])^{5/2}} + \\
 & \frac{30 a^5 \operatorname{Tan}[c+d x]^7}{7 d (a+a \operatorname{Sec}[c+d x])^{7/2}} + \frac{34 a^6 \operatorname{Tan}[c+d x]^9}{9 d (a+a \operatorname{Sec}[c+d x])^{9/2}} + \frac{14 a^7 \operatorname{Tan}[c+d x]^{11}}{11 d (a+a \operatorname{Sec}[c+d x])^{11/2}} + \frac{2 a^8 \operatorname{Tan}[c+d x]^{13}}{13 d (a+a \operatorname{Sec}[c+d x])^{13/2}}
 \end{aligned}$$

Result (type 4, 1214 leaves) :

$$\begin{aligned}
 & \frac{1}{256 d \operatorname{Sec}[c+d x]^{3/2}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \\
 & \left(\frac{1}{45045} 2(1410481 + 633920 \operatorname{Cos}[c+d x] + 2153438 \operatorname{Cos}[2(c+d x)] + 345060 \operatorname{Cos}[3(c+d x)] + 915630 \operatorname{Cos}[4(c+d x)] + \right. \\
 & \quad \left. 86048 \operatorname{Cos}[5(c+d x)] + 176138 \operatorname{Cos}[6(c+d x)]) \operatorname{Sec}[c+d x]^{13/2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 1024(-3-2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^4 \right. \\
 & \quad \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \\
 & \quad \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\
 & \quad \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{3/2} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}} \right) + \frac{1}{1664 d} \\
 & 3(1+2 \operatorname{Cos}[c+d x] + 2 \operatorname{Cos}[2(c+d x)] + 2 \operatorname{Cos}[3(c+d x)] + 2 \operatorname{Cos}[4(c+d x)] + 2 \operatorname{Cos}[5(c+d x)] + 2 \operatorname{Cos}[6(c+d x)]) \\
 & \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \\
 & \operatorname{Sec}[c+d x]^5 \\
 & (a(1+\operatorname{Sec}[c+d x]))^{3/2} \\
 & \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \frac{1}{18304 d} \\
 & 3(15+30 \operatorname{Cos}[c+d x] + 30 \operatorname{Cos}[2(c+d x)] + 30 \operatorname{Cos}[3(c+d x)] + 30 \operatorname{Cos}[4(c+d x)] + 4 \operatorname{Cos}[5(c+d x)] + 4 \operatorname{Cos}[6(c+d x)]) \\
 & \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^5 \\
 & (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{164736 d}
 \end{aligned}$$

$$25 (203 + 406 \cos [c + d x] + 406 \cos [2 (c + d x)] + 120 \cos [3 (c + d x)] + 120 \cos [4 (c + d x)] + 16 \cos [5 (c + d x)] + 16 \cos [6 (c + d x)]) \\ \sec \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^5 (a (1 + \sec [c + d x]))^{3/2} \tan \left[\frac{1}{2} (c + d x) \right] + \frac{1}{128 128 d}$$

$$15 (835 + 812 \cos [c + d x] + 812 \cos [2 (c + d x)] + 240 \cos [3 (c + d x)] + 240 \cos [4 (c + d x)] + 32 \cos [5 (c + d x)] + 32 \cos [6 (c + d x)]) \\ \sec \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^5 (a (1 + \sec [c + d x]))^{3/2} \tan \left[\frac{1}{2} (c + d x) \right] - \frac{1}{49 280 d}$$

$$(3677 + 490 \cos [c + d x] + 6496 \cos [2 (c + d x)] + 1920 \cos [3 (c + d x)] + 1920 \cos [4 (c + d x)] + 256 \cos [5 (c + d x)] + 256 \cos [6 (c + d x)]) \\ \sec \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^5 (a (1 + \sec [c + d x]))^{3/2} \tan \left[\frac{1}{2} (c + d x) \right] + \frac{1}{1 153 152 d}$$

$$17 (4351 - 5026 \cos [c + d x] + 6986 \cos [2 (c + d x)] - 2166 \cos [3 (c + d x)] + 3840 \cos [4 (c + d x)] + 512 \cos [5 (c + d x)] + 512 \cos [6 (c + d x)]) \\ \sec \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^5 (a (1 + \sec [c + d x]))^{3/2} \tan \left[\frac{1}{2} (c + d x) \right] - \frac{1}{128 128 d}$$

$$(14 401 - 26 110 \cos [c + d x] + 21 938 \cos [2 (c + d x)] - 14 670 \cos [3 (c + d x)] + 9354 \cos [4 (c + d x)] - \\ 3958 \cos [5 (c + d x)] + 2048 \cos [6 (c + d x)]) \sec \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x]^5 (a (1 + \sec [c + d x]))^{3/2} \tan \left[\frac{1}{2} (c + d x) \right]$$

- **Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^{3/2} \tan [c + d x]^4 dx$$

Optimal (type 3, 194 leaves, 4 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \sec [c + d x]}} \right]}{d} - \frac{2 a^2 \tan [c + d x]}{d \sqrt{a + a \sec [c + d x]}} + \frac{2 a^3 \tan [c + d x]^3}{3 d (a + a \sec [c + d x])^{3/2}} + \\ \frac{14 a^4 \tan [c + d x]^5}{5 d (a + a \sec [c + d x])^{5/2}} + \frac{10 a^5 \tan [c + d x]^7}{7 d (a + a \sec [c + d x])^{7/2}} + \frac{2 a^6 \tan [c + d x]^9}{9 d (a + a \sec [c + d x])^{9/2}}$$

Result (type 4, 872 leaves):

$$\begin{aligned}
& \frac{1}{64 d \operatorname{Sec}[c+d x]^{3/2}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \\
& \left(-\frac{2}{315} (2897 + 1258 \operatorname{Cos}[c+d x] + 3988 \operatorname{Cos}[2(c+d x)] + 496 \operatorname{Cos}[3(c+d x)] + 1126 \operatorname{Cos}[4(c+d x)]) \operatorname{Sec}[c+d x]^{9/2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] - \right. \\
& 256 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \left. \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{3/2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}} \right) - \\
& \frac{1}{288 d} (1 + 2 \operatorname{Cos}[c+d x] + 2 \operatorname{Cos}[2(c+d x)] + 2 \operatorname{Cos}[3(c+d x)] + 2 \operatorname{Cos}[4(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \operatorname{Sec}[c+d x]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{336 d} \\
& (11 + 22 \operatorname{Cos}[c+d x] + 22 \operatorname{Cos}[2(c+d x)] + 4 \operatorname{Cos}[3(c+d x)] + 4 \operatorname{Cos}[4(c+d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \\
& \frac{1}{720 d} (107 + 88 \operatorname{Cos}[c+d x] + 88 \operatorname{Cos}[2(c+d x)] + 16 \operatorname{Cos}[3(c+d x)] + 16 \operatorname{Cos}[4(c+d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{10080 d} \\
& 11 (109 - 34 \operatorname{Cos}[c+d x] + 176 \operatorname{Cos}[2(c+d x)] + 32 \operatorname{Cos}[3(c+d x)] + 32 \operatorname{Cos}[4(c+d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \frac{1}{3360 d} \\
& (557 - 902 \operatorname{Cos}[c+d x] + 778 \operatorname{Cos}[2(c+d x)] - 374 \operatorname{Cos}[3(c+d x)] + 256 \operatorname{Cos}[4(c+d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]
\end{aligned}$$

- **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]^2 dx$$

Optimal (type 3, 128 leaves, 4 steps):

$$-\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} + \frac{2 a^2 \operatorname{Tan}[c+d x]}{d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{2 a^3 \operatorname{Tan}[c+d x]^3}{d (a+a \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 a^4 \operatorname{Tan}[c+d x]^5}{5 d (a+a \operatorname{Sec}[c+d x])^{5/2}}$$

Result (type 4, 604 leaves):

$$\frac{1}{16 d \operatorname{Sec}[c+d x]^{3/2}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2}$$

$$\left(\frac{2}{15} (43 + 16 \operatorname{Cos}[c+d x] + 46 \operatorname{Cos}[2(c+d x)]) \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 64 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^4 \right.$$

$$\sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\left. \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{3/2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}} \right) -$$

$$\frac{1}{40 d} (1 + 2 \operatorname{Cos}[c+d x] + 2 \operatorname{Cos}[2(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] (a(1+\operatorname{Sec}[c+d x]))^{3/2}$$

$$\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \frac{1}{24 d}$$

$$(7 + 4 \operatorname{Cos}[c+d x] + 4 \operatorname{Cos}[2(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]$$

$$(a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{40 d}$$

$$(13 - 14 \operatorname{Cos}[c+d x] + 16 \operatorname{Cos}[2(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x] (a(1+\operatorname{Sec}[c+d x]))^{3/2} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]$$

- **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^2 (a + a \operatorname{Sec}[c + d x])^{3/2} dx$$

Optimal (type 3, 64 leaves, 3 steps):

$$-\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} - \frac{2 a \operatorname{Cot}[c+d x] \sqrt{a+a \operatorname{Sec}[c+d x]}}{d}$$

Result (type 4, 389 leaves):

$$\frac{\cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \left(-\frac{1}{2} \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{d} +$$

$$\frac{1}{d} 4(-3-2 \sqrt{2}) \cos \left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}}$$

- **Problem 159: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]^5 dx$$

Optimal (type 3, 193 leaves, 10 steps):

$$-\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} + \frac{2 a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{d} + \frac{2 a(a+a \operatorname{Sec}[c+d x])^{3/2}}{3 d} + \frac{2(a+a \operatorname{Sec}[c+d x])^{5/2}}{5 d} +$$

$$\frac{2(a+a \operatorname{Sec}[c+d x])^{7/2}}{7 a d} + \frac{2(a+a \operatorname{Sec}[c+d x])^{9/2}}{9 a^2 d} - \frac{6(a+a \operatorname{Sec}[c+d x])^{11/2}}{11 a^3 d} + \frac{2(a+a \operatorname{Sec}[c+d x])^{13/2}}{13 a^4 d}$$

Result (type 3, 925 leaves):

$$\begin{aligned}
& - \frac{1}{3328 d} 5 (1 - 2 \operatorname{Cos}[c + d x] + 2 \operatorname{Cos}[2 (c + d x)] - 2 \operatorname{Cos}[3 (c + d x)] + 2 \operatorname{Cos}[4 (c + d x)] - 2 \operatorname{Cos}[5 (c + d x)] + 2 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} + \frac{1}{36608 d} \\
& 35 (15 - 30 \operatorname{Cos}[c + d x] + 30 \operatorname{Cos}[2 (c + d x)] - 30 \operatorname{Cos}[3 (c + d x)] + 30 \operatorname{Cos}[4 (c + d x)] - 4 \operatorname{Cos}[5 (c + d x)] + 4 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} - \frac{1}{25344 d} \\
& 5 (203 - 406 \operatorname{Cos}[c + d x] + 406 \operatorname{Cos}[2 (c + d x)] - 120 \operatorname{Cos}[3 (c + d x)] + 120 \operatorname{Cos}[4 (c + d x)] - 16 \operatorname{Cos}[5 (c + d x)] + 16 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} + \frac{1}{256256 d} \\
& 15 (835 - 812 \operatorname{Cos}[c + d x] + 812 \operatorname{Cos}[2 (c + d x)] - 240 \operatorname{Cos}[3 (c + d x)] + 240 \operatorname{Cos}[4 (c + d x)] - 32 \operatorname{Cos}[5 (c + d x)] + 32 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} + \frac{1}{3843840 d} \\
& (3677 - 490 \operatorname{Cos}[c + d x] + 6496 \operatorname{Cos}[2 (c + d x)] - 1920 \operatorname{Cos}[3 (c + d x)] + 1920 \operatorname{Cos}[4 (c + d x)] - 256 \operatorname{Cos}[5 (c + d x)] + 256 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} - \frac{1}{768768 d} \\
& 5 (4351 + 5026 \operatorname{Cos}[c + d x] + 6986 \operatorname{Cos}[2 (c + d x)] + 2166 \operatorname{Cos}[3 (c + d x)] + 3840 \operatorname{Cos}[4 (c + d x)] - 512 \operatorname{Cos}[5 (c + d x)] + 512 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} + \frac{1}{768768 d} 5 (14401 + 26110 \operatorname{Cos}[c + d x] + \\
& 21938 \operatorname{Cos}[2 (c + d x)] + 14670 \operatorname{Cos}[3 (c + d x)] + 9354 \operatorname{Cos}[4 (c + d x)] + 3958 \operatorname{Cos}[5 (c + d x)] + 2048 \operatorname{Cos}[6 (c + d x)]) \\
& \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^4 \operatorname{Sec}[c + d x]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} - \frac{1}{11531520 d} \operatorname{Cos}[c + d x]^2 \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^5 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} \\
& \left(-2882880 \sqrt{2} \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right]^2 \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2}\right] \right) \right. \\
& \left. \sqrt{\operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^4} + \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] (-5636416 + 1376768 \operatorname{Sec}[c + d x] + \right. \\
& \left. 1129584 \operatorname{Sec}[c + d x]^2 - 340720 \operatorname{Sec}[c + d x]^3 - 152320 \operatorname{Sec}[c + d x]^4 + 28980 \operatorname{Sec}[c + d x]^5 + 3465 \operatorname{Sec}[c + d x]^6) \right)
\end{aligned}$$

■ **Problem 160: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{5/2} \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 3, 145 leaves, 8 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} - \frac{2 a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{d} - \frac{2 a (a+a \operatorname{Sec}[c+d x])^{3/2}}{3 d} - \frac{2 (a+a \operatorname{Sec}[c+d x])^{5/2}}{5 d} - \frac{2 (a+a \operatorname{Sec}[c+d x])^{7/2}}{7 a d} + \frac{2 (a+a \operatorname{Sec}[c+d x])^{9/2}}{9 a^2 d}$$

Result (type 3, 603 leaves):

$$\begin{aligned} & \frac{1}{576 d} 7 (1 - 2 \operatorname{Cos}[c+d x] + 2 \operatorname{Cos}[2(c+d x)] - 2 \operatorname{Cos}[3(c+d x)] + 2 \operatorname{Cos}[4(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]^2 (a(1+\operatorname{Sec}[c+d x]))^{5/2} - \\ & \frac{1}{2016 d} 11 (11 - 22 \operatorname{Cos}[c+d x] + 22 \operatorname{Cos}[2(c+d x)] - 4 \operatorname{Cos}[3(c+d x)] + 4 \operatorname{Cos}[4(c+d x)]) \\ & \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]^2 (a(1+\operatorname{Sec}[c+d x]))^{5/2} + \frac{1}{1440 d} \\ & (107 - 88 \operatorname{Cos}[c+d x] + 88 \operatorname{Cos}[2(c+d x)] - 16 \operatorname{Cos}[3(c+d x)] + 16 \operatorname{Cos}[4(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]^2 (a(1+\operatorname{Sec}[c+d x]))^{5/2} + \\ & \frac{1}{4032 d} (109 + 34 \operatorname{Cos}[c+d x] + 176 \operatorname{Cos}[2(c+d x)] - 32 \operatorname{Cos}[3(c+d x)] + 32 \operatorname{Cos}[4(c+d x)]) \\ & \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]^2 (a(1+\operatorname{Sec}[c+d x]))^{5/2} - \frac{1}{4032 d} \\ & (557 + 902 \operatorname{Cos}[c+d x] + 778 \operatorname{Cos}[2(c+d x)] + 374 \operatorname{Cos}[3(c+d x)] + 256 \operatorname{Cos}[4(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]^2 (a(1+\operatorname{Sec}[c+d x]))^{5/2} - \\ & \frac{1}{20160 d} \operatorname{Cos}[c+d x]^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} \\ & \left(5040 \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4 - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}\right] \right) \right. \\ & \left. + \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} + \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] (9008 - 1984 \operatorname{Sec}[c+d x] - 1032 \operatorname{Sec}[c+d x]^2 + 230 \operatorname{Sec}[c+d x]^3 + 35 \operatorname{Sec}[c+d x]^4) \right) \end{aligned}$$

■ **Problem 161: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x] dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$- \frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} + \frac{2 a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{d} + \frac{2 a (a+a \operatorname{Sec}[c+d x])^{3/2}}{3 d} + \frac{2 (a+a \operatorname{Sec}[c+d x])^{5/2}}{5 d}$$

Result (type 3, 337 leaves):

$$\begin{aligned}
& - \frac{9(1 - 2 \cos[c + dx] + 2 \cos[2(c + dx)]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{80d} + \\
& \frac{(7 - 4 \cos[c + dx] + 4 \cos[2(c + dx)]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{48d} + \\
& \frac{(13 + 14 \cos[c + dx] + 16 \cos[2(c + dx)]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{48d} - \\
& \frac{1}{240d} \cos[c + dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \\
& \left(60 \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right]^2 \left(-\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2\right] + \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] \right) \right. \\
& \left. \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} + \cos\left[\frac{1}{2}(c + dx)\right] (-92 + 16 \operatorname{Sec}[c + dx] + 3 \operatorname{Sec}[c + dx]^2) \right)
\end{aligned}$$

■ **Problem 162: Result more than twice size of optimal antiderivative.**

$$\int \cot[c + dx] (a + a \operatorname{Sec}[c + dx])^{5/2} dx$$

Optimal (type 3, 95 leaves, 7 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+dx]}}{\sqrt{a}}\right]}{d} - \frac{4 \sqrt{2} a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{d} + \frac{2 a^2 \sqrt{a+a \operatorname{Sec}[c+dx]}}{d}$$

Result (type 3, 268 leaves):

$$\begin{aligned}
& - \frac{1}{4d} a^2 (1 + \cos[c + dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(-2 + \right. \\
& \cos\left[\frac{1}{4}(c + dx)\right]^2 \left(\sqrt{2} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2\right] - 2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] + 2 \operatorname{Log}\left[1 + \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} - 3 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] - \right. \\
& \left. \sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] - 2 \operatorname{Log}\left[3 - \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2\right] \right) \\
& \left. \sqrt{\cos[c + dx] \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \right)
\end{aligned}$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^3 (a + a \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$-\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} + \frac{3 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{\sqrt{2} d} + \frac{a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{d(1-\operatorname{Sec}[c+d x])}$$

Result (type 3, 329 leaves):

$$\begin{aligned} & \frac{1}{d} \cos [c+d x]^2 \left(\frac{1}{4} \cos \left[\frac{1}{2} (c+d x) \right] - \frac{1}{8} \cot \left[\frac{1}{2} (c+d x) \right] \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} + \\ & \frac{1}{16 d} \cos \left[\frac{1}{4} (c+d x) \right]^2 \cos [c+d x]^2 \left(4 \sqrt{2} \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{4} (c+d x) \right]^2 \right] - 3 \operatorname{Log} \left[\operatorname{Tan} \left[\frac{1}{4} (c+d x) \right]^2 \right] + \right. \\ & \left. 3 \operatorname{Log} \left[1 + \sqrt{\cos [c+d x] \operatorname{Sec} \left[\frac{1}{4} (c+d x) \right]^4 - 3 \operatorname{Tan} \left[\frac{1}{4} (c+d x) \right]^2} \right] - 4 \sqrt{2} \operatorname{Log} \left[2 + \sqrt{2} \sqrt{\cos [c+d x] \operatorname{Sec} \left[\frac{1}{4} (c+d x) \right]^4 - 2 \operatorname{Tan} \left[\frac{1}{4} (c+d x) \right]^2} \right] - \right. \\ & \left. 3 \operatorname{Log} \left[3 - \sqrt{\cos [c+d x] \operatorname{Sec} \left[\frac{1}{4} (c+d x) \right]^4 - \operatorname{Tan} \left[\frac{1}{4} (c+d x) \right]^2} \right] \right) \sqrt{\cos [c+d x] \operatorname{Sec} \left[\frac{1}{4} (c+d x) \right]^4} \operatorname{Sec} \left[\frac{1}{2} (c+d x) \right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} \end{aligned}$$

■ **Problem 164: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^5 (a + a \operatorname{Sec}[c + d x])^{5/2} dx$$

Optimal (type 3, 147 leaves, 8 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{d} - \frac{43 a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{16 \sqrt{2} d} - \frac{a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{4 d(1-\operatorname{Sec}[c+d x])^2} - \frac{11 a^2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{16 d(1-\operatorname{Sec}[c+d x])}$$

Result (type 3, 355 leaves):

$$\frac{1}{d} \cos [c + d x]^2 \left(-\frac{15}{64} \cos \left[\frac{1}{2} (c + d x) \right] + \frac{19}{128} \cot \left[\frac{1}{2} (c + d x) \right] \csc \left[\frac{1}{2} (c + d x) \right] - \frac{1}{64} \cot \left[\frac{1}{2} (c + d x) \right] \csc \left[\frac{1}{2} (c + d x) \right]^3 \right)$$

$$\sec \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \sec [c + d x]))^{5/2} + \frac{1}{256 d} \cos \left[\frac{1}{4} (c + d x) \right]^2 \cos [c + d x]^2$$

$$\left(-64 \sqrt{2} \log \left[\sec \left[\frac{1}{4} (c + d x) \right]^2 \right] + 43 \log \left[\tan \left[\frac{1}{4} (c + d x) \right]^2 \right] - 43 \log \left[1 + \sqrt{\cos [c + d x] \sec \left[\frac{1}{4} (c + d x) \right]^4} - 3 \tan \left[\frac{1}{4} (c + d x) \right]^2 \right] + \right.$$

$$64 \sqrt{2} \log \left[2 + \sqrt{2} \sqrt{\cos [c + d x] \sec \left[\frac{1}{4} (c + d x) \right]^4} - 2 \tan \left[\frac{1}{4} (c + d x) \right]^2 \right] +$$

$$\left. 43 \log \left[3 - \sqrt{\cos [c + d x] \sec \left[\frac{1}{4} (c + d x) \right]^4} - \tan \left[\frac{1}{4} (c + d x) \right]^2 \right] \right) \sqrt{\cos [c + d x] \sec \left[\frac{1}{4} (c + d x) \right]^4} \sec \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \sec [c + d x]))^{5/2}$$

■ **Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^{5/2} \tan [c + d x]^6 dx$$

Optimal (type 3, 290 leaves, 4 steps):

$$-\frac{2 a^{5/2} \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \sec [c + d x]}} \right]}{d} + \frac{2 a^3 \tan [c + d x]}{d \sqrt{a + a \sec [c + d x]}} - \frac{2 a^4 \tan [c + d x]^3}{3 d (a + a \sec [c + d x])^{3/2}} + \frac{2 a^5 \tan [c + d x]^5}{5 d (a + a \sec [c + d x])^{5/2}} + \frac{62 a^6 \tan [c + d x]^7}{7 d (a + a \sec [c + d x])^{7/2}} +$$

$$\frac{98 a^7 \tan [c + d x]^9}{9 d (a + a \sec [c + d x])^{9/2}} + \frac{62 a^8 \tan [c + d x]^{11}}{11 d (a + a \sec [c + d x])^{11/2}} + \frac{18 a^9 \tan [c + d x]^{13}}{13 d (a + a \sec [c + d x])^{13/2}} + \frac{2 a^{10} \tan [c + d x]^{15}}{15 d (a + a \sec [c + d x])^{15/2}}$$

Result (type 4, 1415 leaves):

$$\frac{1}{1024 d \sec [c + d x]^{5/2}}$$

$$\sec \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \sec [c + d x]))^{5/2} \left(\frac{1}{45045} 2 (636923 + 4980406 \cos [c + d x] + 984986 \cos [2 (c + d x)] + 3075074 \cos [3 (c + d x)] + \right.$$

$$437114 \cos [4 (c + d x)] + 1097774 \cos [5 (c + d x)] + 92054 \cos [6 (c + d x)] + 182144 \cos [7 (c + d x)])$$

$$\left. \sec [c + d x]^{15/2} \sin \left[\frac{1}{2} (c + d x) \right] + 2048 (-3 - 2\sqrt{2}) \cos \left[\frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \right)$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} - \frac{1}{1920d}$$

$$(1 + 2 \cos[c + dx] + 2 \cos[2(c + dx)] + 2 \cos[3(c + dx)] + 2 \cos[4(c + dx)] + 2 \cos[5(c + dx)] + 2 \cos[6(c + dx)] + 2 \cos[7(c + dx)])$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^4$$

$$\sec[c + dx]^5$$

$$(a(1 + \sec[c + dx]))^{5/2}$$

$$\tan\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2496d}$$

$$(17 + 34 \cos[c + dx] + 34 \cos[2(c + dx)] + 34 \cos[3(c + dx)] + 34 \cos[4(c + dx)] + 34 \cos[5(c + dx)] + 4 \cos[6(c + dx)] + 4 \cos[7(c + dx)])$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^4 \sec[c + dx]^5$$

$$(a(1 + \sec[c + dx]))^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] - \frac{1}{54912d}$$

$$5(263 + 526 \cos[c + dx] + 526 \cos[2(c + dx)] + 526 \cos[3(c + dx)] + 136 \cos[4(c + dx)] + 136 \cos[5(c + dx)] + 16 \cos[6(c + dx)] + 16 \cos[7(c + dx)])$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^4 \sec[c + dx]^5 (a(1 + \sec[c + dx]))^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] + \frac{1}{29952d}$$

$$(1241 + 2482 \cos[c + dx] + 1052 \cos[2(c + dx)] + 1052 \cos[3(c + dx)] + 272 \cos[4(c + dx)] + 272 \cos[5(c + dx)] + 32 \cos[6(c + dx)] + 32 \cos[7(c + dx)]) \sec\left[\frac{1}{2}(c + dx)\right]^4 \sec[c + dx]^5 (a(1 + \sec[c + dx]))^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] -$$

$$\frac{1}{524160d} (3493 + 19856 \cos[c + dx] + 8416 \cos[2(c + dx)] + 8416 \cos[3(c + dx)] + 2176 \cos[4(c + dx)] + 2176 \cos[5(c + dx)] +$$

$$256 \cos[6(c + dx)] + 256 \cos[7(c + dx)]) \sec\left[\frac{1}{2}(c + dx)\right]^4 \sec[c + dx]^5 (a(1 + \sec[c + dx]))^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] -$$

$$\frac{1}{1441440d} (-2023 + 21694 \cos[c + dx] - 1186 \cos[2(c + dx)] + 16832 \cos[3(c + dx)] + 4352 \cos[4(c + dx)] +$$

$$4352 \cos[5(c + dx)] + 512 \cos[6(c + dx)] + 512 \cos[7(c + dx)])$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^4 \sec[c + dx]^5 (a(1 + \sec[c + dx]))^{5/2} \tan\left[\frac{1}{2}(c + dx)\right] + \frac{1}{1153152d}$$

$$(-23107 + 56746 \cos[c + dx] - 34774 \cos[2(c + dx)] + 37298 \cos[3(c + dx)] -$$

$$\begin{aligned}
& 12\,622 \operatorname{Cos}[4(c+dx)] + 17\,408 \operatorname{Cos}[5(c+dx)] + 2048 \operatorname{Cos}[6(c+dx)] + 2048 \operatorname{Cos}[7(c+dx)] \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Sec}[c+dx]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{658\,944d} \\
& (-52\,649 + 100\,622 \operatorname{Cos}[c+dx] - 82\,418 \operatorname{Cos}[2(c+dx)] + 61\,726 \operatorname{Cos}[3(c+dx)] - 38\,114 \operatorname{Cos}[4(c+dx)] + 21\,946 \operatorname{Cos}[5(c+dx)] - \\
& 8774 \operatorname{Cos}[6(c+dx)] + 4096 \operatorname{Cos}[7(c+dx)]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \operatorname{Sec}[c+dx]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]
\end{aligned}$$

■ **Problem 166: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx])^{5/2} \operatorname{Tan}[c + dx]^4 dx$$

Optimal (type 3, 224 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} - \frac{2 a^3 \operatorname{Tan}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2 a^4 \operatorname{Tan}[c+dx]^3}{3 d (a+a \operatorname{Sec}[c+dx])^{3/2}} + \\
& \frac{6 a^5 \operatorname{Tan}[c+dx]^5}{d (a+a \operatorname{Sec}[c+dx])^{5/2}} + \frac{34 a^6 \operatorname{Tan}[c+dx]^7}{7 d (a+a \operatorname{Sec}[c+dx])^{7/2}} + \frac{14 a^7 \operatorname{Tan}[c+dx]^9}{9 d (a+a \operatorname{Sec}[c+dx])^{9/2}} + \frac{2 a^8 \operatorname{Tan}[c+dx]^{11}}{11 d (a+a \operatorname{Sec}[c+dx])^{11/2}}
\end{aligned}$$

Result (type 4, 1033 leaves):

$$\begin{aligned}
& \frac{1}{256 d \operatorname{Sec}[c+dx]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \\
& \left(-\frac{1}{3465} 2(14\,153 + 108\,232 \operatorname{Cos}[c+dx] + 19\,924 \operatorname{Cos}[2(c+dx)] + 56\,884 \operatorname{Cos}[3(c+dx)] + 60\,86 \operatorname{Cos}[4(c+dx)] + 13\,016 \operatorname{Cos}[5(c+dx)]) \right. \\
& \left. \operatorname{Sec}[c+dx]^{11/2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 512(-3-2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \right. \\
& \left. \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \right. \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\
& \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}} \right) + \\
& \frac{1}{704 d} 3(1+2 \operatorname{Cos}[c+dx] + 2 \operatorname{Cos}[2(c+dx)] + 2 \operatorname{Cos}[3(c+dx)] + 2 \operatorname{Cos}[4(c+dx)] + 2 \operatorname{Cos}[5(c+dx)])
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \\
& \text{Sec}[c+dx]^3 \\
& (a(1+\text{Sec}[c+dx]))^{5/2} \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{12672d} \\
29 & (13 + 26 \text{Cos}[c+dx] + 26 \text{Cos}[2(c+dx)] + 26 \text{Cos}[3(c+dx)] + 4 \text{Cos}[4(c+dx)] + 4 \text{Cos}[5(c+dx)]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \text{Sec}[c+dx]^3 (a(1+\text{Sec}[c+dx]))^{5/2} \text{Tan}\left[\frac{1}{2}(c+dx)\right] + \\
& \frac{1}{2464d} (151 + 302 \text{Cos}[c+dx] + 104 \text{Cos}[2(c+dx)] + 104 \text{Cos}[3(c+dx)] + 16 \text{Cos}[4(c+dx)] + 16 \text{Cos}[5(c+dx)]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \text{Sec}[c+dx]^3 (a(1+\text{Sec}[c+dx]))^{5/2} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{49280d} \\
3 & (71 + 604 \text{Cos}[c+dx] + 208 \text{Cos}[2(c+dx)] + 208 \text{Cos}[3(c+dx)] + 32 \text{Cos}[4(c+dx)] + 32 \text{Cos}[5(c+dx)]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \text{Sec}[c+dx]^3 (a(1+\text{Sec}[c+dx]))^{5/2} \text{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{1}{44352d} \\
& (-587 + 2522 \text{Cos}[c+dx] - 646 \text{Cos}[2(c+dx)] + 1664 \text{Cos}[3(c+dx)] + 256 \text{Cos}[4(c+dx)] + 256 \text{Cos}[5(c+dx)]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \text{Sec}[c+dx]^3 (a(1+\text{Sec}[c+dx]))^{5/2} \text{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{88704d} \\
5 & (-1867 + 3658 \text{Cos}[c+dx] - 2678 \text{Cos}[2(c+dx)] + 1942 \text{Cos}[3(c+dx)] - 874 \text{Cos}[4(c+dx)] + 512 \text{Cos}[5(c+dx)]) \\
& \text{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \text{Sec}[c+dx]^3 (a(1+\text{Sec}[c+dx]))^{5/2} \text{Tan}\left[\frac{1}{2}(c+dx)\right]
\end{aligned}$$

- **Problem 167: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]^2 dx$$

Optimal (type 3, 160 leaves, 4 steps):

$$-\frac{2a^{5/2} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{d} + \frac{2a^3 \text{Tan}[c+dx]}{d \sqrt{a+a \text{Sec}[c+dx]}} + \frac{14a^4 \text{Tan}[c+dx]^3}{3d (a+a \text{Sec}[c+dx])^{3/2}} + \frac{2a^5 \text{Tan}[c+dx]^5}{d (a+a \text{Sec}[c+dx])^{5/2}} + \frac{2a^6 \text{Tan}[c+dx]^7}{7d (a+a \text{Sec}[c+dx])^{7/2}}$$

Result (type 4, 644 leaves):

$$\frac{1}{64 d \operatorname{Sec}[c+d x]^{5/2}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2}$$

$$\left(\frac{2}{105}(127+954 \cos [c+d x]+142 \cos [2(c+d x)]+352 \cos [3(c+d x)]) \operatorname{Sec}[c+d x]^{7/2} \sin \left[\frac{1}{2}(c+d x)\right]+128(-3-2 \sqrt{2}) \cos \left[\frac{1}{4}(c+d x)\right]^4\right.$$

$$\sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}[c+d x]^{3/2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}} -$$

$$\frac{1}{28 d}(1+2 \cos [c+d x]+2 \cos [2(c+d x)]+2 \cos [3(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4 \operatorname{Sec}[c+d x]$$

$$(a(1+\operatorname{Sec}[c+d x]))^{5/2} \tan \left[\frac{1}{2}(c+d x)\right]+\frac{1}{80 d}$$

$$(9+18 \cos [c+d x]+4 \cos [2(c+d x)]+4 \cos [3(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4$$

$$\operatorname{Sec}[c+d x](a(1+\operatorname{Sec}[c+d x]))^{5/2} \tan \left[\frac{1}{2}(c+d x)\right]-\frac{1}{224 d}$$

$$(-33+74 \cos [c+d x]-38 \cos [2(c+d x)]+32 \cos [3(c+d x)]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^4$$

$$\operatorname{Sec}[c+d x](a(1+\operatorname{Sec}[c+d x]))^{5/2} \tan \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 168: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^2 (a+a \operatorname{Sec}[c+d x])^{5/2} dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d}-\frac{4 a^2 \cot [c+d x] \sqrt{a+a \operatorname{Sec}[c+d x]}}{d}$$

Result (type 4, 397 leaves):

$$\frac{\cos [c+d x]^2 \sec \left[\frac{1}{2}(c+d x)\right]^5 (a(1+\sec [c+d x]))^{5/2} \left(-\frac{1}{2} \csc \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)}{d} +$$

$$\frac{1}{d} 2(-3-2 \sqrt{2}) \cos \left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \sec \left[\frac{1}{4}(c+d x)\right]^2 \sec \left[\frac{1}{2}(c+d x)\right]^5 (a(1+\sec [c+d x]))^{5/2} \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4}(c+d x)\right]^2}}$$

■ **Problem 169: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot [c+d x]^4 (a+a \sec [c+d x])^{5/2} dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right]}{d} + \frac{2 a^2 \cot [c+d x] \sqrt{a+a \sec [c+d x]}}{d} - \frac{2 a \cot [c+d x]^3 (a+a \sec [c+d x])^{3/2}}{3 d}$$

Result (type 4, 417 leaves):

$$\frac{1}{d} \cos [c+d x]^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} \left(\frac{5}{12} \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right] - \frac{1}{24} \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^3 - \frac{2}{3} \sin\left[\frac{1}{2}(c+d x)\right]\right) -$$

$$\frac{1}{d} 2(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]}{1+\cos\left[\frac{1}{2}(c+d x)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]}{1+\cos\left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+d x)\right]^2}}$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]^3}{\sqrt{a+a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} d} - \frac{2 \sqrt{a+a \operatorname{Sec}[c+d x]}}{a d} + \frac{2(a+a \operatorname{Sec}[c+d x])^{3/2}}{3 a^2 d}$$

Result (type 3, 165 leaves):

$$-\frac{1}{3 d \sqrt{a}(1+\operatorname{Sec}[c+d x])} 2 \cos\left[\frac{1}{2}(c+d x)\right] \left(\cos\left[\frac{1}{2}(c+d x)\right](-2+4 \cos [c+d x])+\right.$$

$$\left.3 \sqrt{2} \cos\left[\frac{1}{4}(c+d x)\right]^6 \left(\log \left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right]-\log \left[2+\sqrt{2} \sqrt{\cos [c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4}-2 \tan\left[\frac{1}{4}(c+d x)\right]^2}\right]\right)\right.$$

$$\left.\left(\cos [c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4\right)^{3/2}\right) \operatorname{Sec}[c+d x]^2$$

■ **Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]}{\sqrt{a+a \sec [c+d x]}} d x$$

Optimal (type 3, 31 leaves, 3 steps) :

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} d}$$

Result (type 3, 131 leaves) :

$$\left(2 \sqrt{2} \cos \left[\frac{1}{2}(c+d x)\right] \left(\log \left[\sec \left[\frac{1}{4}(c+d x)\right]^2\right] - \log \left[2+\sqrt{2} \sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} - 2 \tan \left[\frac{1}{4}(c+d x)\right]^2\right]\right) \sec \left[\frac{1}{4}(c+d x)\right]^2\right) /$$

$$\left(d \sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} \sqrt{a(1+\sec [c+d x])}\right)$$

■ **Problem 174: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot [c+d x]}{\sqrt{a+a \sec [c+d x]}} d x$$

Optimal (type 3, 92 leaves, 7 steps) :

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+a \sec [c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{\sqrt{2} \sqrt{a} d} - \frac{1}{d \sqrt{a+a \sec [c+d x]}}$$

Result (type 3, 263 leaves) :

$$\frac{1}{2 d \sqrt{a(1+\sec [c+d x])}}$$

$$\left(2-4 \cos \left[\frac{1}{2}(c+d x)\right]^2 - \cos \left[\frac{1}{4}(c+d x)\right]^2 \cos \left[\frac{1}{2}(c+d x)\right] \left(4 \sqrt{2} \log \left[\sec \left[\frac{1}{4}(c+d x)\right]^2\right] - \log \left[\tan \left[\frac{1}{4}(c+d x)\right]^2\right] + \log \left[1+\sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} - 3 \tan \left[\frac{1}{4}(c+d x)\right]^2 - 4 \sqrt{2} \log \left[2+\sqrt{2} \sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} - 2 \tan \left[\frac{1}{4}(c+d x)\right]^2}\right]\right) - \log \left[3-\sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} - \tan \left[\frac{1}{4}(c+d x)\right]^2}\right]\right) \sqrt{\cos [c+d x] \sec \left[\frac{1}{4}(c+d x)\right]^4} \sec [c+d x]$$

■ **Problem 175: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^3}{\sqrt{a + a \text{Sec}[c + dx]}} dx$$

Optimal (type 3, 152 leaves, 9 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a+a \text{Sec}[c+dx]}}{\sqrt{a}}\right]}{\sqrt{a} d} + \frac{9 \text{ArcTanh}\left[\frac{\sqrt{a+a \text{Sec}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{8 \sqrt{2} \sqrt{a} d} - \frac{a}{12 d (a + a \text{Sec}[c + dx])^{3/2}} + \frac{a}{2 d (1 - \text{Sec}[c + dx]) (a + a \text{Sec}[c + dx])^{3/2}} + \frac{7}{8 d \sqrt{a + a \text{Sec}[c + dx]}}$$

Result (type 3, 351 leaves):

$$\frac{1}{16 d \sqrt{a} (1 + \text{Sec}[c + dx])} \text{Cos}\left[\frac{1}{4} (c + dx)\right]^2 \text{Cos}\left[\frac{1}{2} (c + dx)\right] \left(32 \sqrt{2} \text{Log}\left[\text{Sec}\left[\frac{1}{4} (c + dx)\right]^2\right] - 9 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (c + dx)\right]^2\right] + 9 \text{Log}\left[1 + \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4 - 3 \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2}\right] - 32 \sqrt{2} \text{Log}\left[2 + \sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4 - 2 \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2}\right] - 9 \text{Log}\left[3 - \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4 - \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2}\right] \right) \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4} \text{Sec}[c + dx] + \frac{1}{d \sqrt{a} (1 + \text{Sec}[c + dx])} \text{Cos}\left[\frac{1}{2} (c + dx)\right] \left(\frac{31}{12} \text{Cos}\left[\frac{1}{2} (c + dx)\right] - \frac{1}{8} \text{Cot}\left[\frac{1}{2} (c + dx)\right] \text{Csc}\left[\frac{1}{2} (c + dx)\right] - \frac{4}{3} \text{Sec}\left[\frac{1}{2} (c + dx)\right] + \frac{1}{12} \text{Sec}\left[\frac{1}{2} (c + dx)\right]^3 \right) \text{Sec}[c + dx]$$

■ **Problem 177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + dx]^6}{\sqrt{a + a \text{Sec}[c + dx]}} dx$$

Optimal (type 3, 189 leaves, 4 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{\sqrt{a} d} + \frac{2 \text{Tan}[c + dx]}{d \sqrt{a + a \text{Sec}[c + dx]}} - \frac{2 a \text{Tan}[c + dx]^3}{3 d (a + a \text{Sec}[c + dx])^{3/2}} + \frac{2 a^2 \text{Tan}[c + dx]^5}{5 d (a + a \text{Sec}[c + dx])^{5/2}} + \frac{6 a^3 \text{Tan}[c + dx]^7}{7 d (a + a \text{Sec}[c + dx])^{7/2}} + \frac{2 a^4 \text{Tan}[c + dx]^9}{9 d (a + a \text{Sec}[c + dx])^{9/2}}$$

Result (type 4, 469 leaves):

$$\frac{1}{d \sqrt{a (1 + \operatorname{Sec}[c + d x])}}$$

$$\cos\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x] \left(\frac{1532}{315} \sin\left[\frac{1}{2}(c + d x)\right] + \frac{136}{315} \operatorname{Sec}[c + d x] \sin\left[\frac{1}{2}(c + d x)\right] - \frac{176}{105} \operatorname{Sec}[c + d x]^2 \sin\left[\frac{1}{2}(c + d x)\right] - \frac{4}{63} \operatorname{Sec}[c + d x]^3 \sin\left[\frac{1}{2}(c + d x)\right] + \frac{4}{9} \operatorname{Sec}[c + d x]^4 \sin\left[\frac{1}{2}(c + d x)\right] \right) +$$

$$\frac{1}{d \sqrt{a (1 + \operatorname{Sec}[c + d x])}} 16 (-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + d x)\right]^4 \cos\left[\frac{1}{2}(c + d x)\right] \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]}{1 + \cos\left[\frac{1}{2}(c + d x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + d x)\right]^2 \operatorname{Sec}[c + d x]^2 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + d x)\right]^2}}$$

- **Problem 178: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + d x]^4}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 125 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} - \frac{2 \tan[c + d x]}{d \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{2 a \tan[c + d x]^3}{3 d (a + a \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 a^2 \tan[c + d x]^5}{5 d (a + a \operatorname{Sec}[c + d x])^{5/2}}$$

Result (type 4, 425 leaves):

$$\frac{1}{d \sqrt{a} (1 + \operatorname{Sec}[c + dx])} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \left(-\frac{68}{15} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - \frac{4}{15} \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \frac{4}{5} \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) -$$

$$\frac{1}{d \sqrt{a} (1 + \operatorname{Sec}[c + dx])} 16 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right]^4 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]$$

$$\sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Sec}[c + dx]^2} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 179: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + dx]^2}{\sqrt{a + a \operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 63 leaves, 3 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{\sqrt{a} d} + \frac{2 \operatorname{Tan}[c + dx]}{d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 4, 379 leaves):

$$\frac{4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{d \sqrt{a(1+\operatorname{Sec}[c+dx])}} +$$

$$\frac{1}{d \sqrt{a(1+\operatorname{Sec}[c+dx])}} 16(-3-2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^2 \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}$$

■ **Problem 184: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^3}{(a+a \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+dx]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{2 \sqrt{a+a \operatorname{Sec}[c+dx]}}{a^2 d}$$

Result (type 3, 155 leaves):

$$\frac{1}{d(a(1+\operatorname{Sec}[c+dx]))^{3/2}} 4 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left(2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^2 \left(-\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2\right] + \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}\right]\right)\right)$$

$$\sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^4} \operatorname{Sec}[c+dx]^2$$

■ **Problem 185: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]}{(a+a \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 3, 54 leaves, 4 steps) :

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{2}{a d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 179 leaves) :

$$\left(4 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \left(-1+2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}\right]\right)\right.\right. \\ \left.\left.\sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4}\right] \operatorname{Sec}[c+d x]\right) / \left(a d (1+\operatorname{Cos}[c+d x]) \sqrt{a(1+\operatorname{Sec}[c+d x])}\right)$$

■ **Problem 186: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+d x]}{(a+a \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 120 leaves, 8 steps) :

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{1}{3 d (a+a \operatorname{Sec}[c+d x])^{3/2}} - \frac{3}{2 a d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 292 leaves) :

$$\frac{1}{6 a d (1+\operatorname{Cos}[c+d x]) \sqrt{a(1+\operatorname{Sec}[c+d x])}} \\ \left(-2+26 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 - 44 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^4 - 3 \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^3 \left(8 \sqrt{2} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \right.\right.\right. \\ \left.\left.\operatorname{Log}\left[1+\sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 3 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - 8 \sqrt{2} \operatorname{Log}\left[2+\sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - \right.\right.\right. \\ \left.\left.\left.2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[3-\sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right]\right) \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4}\right) \operatorname{Sec}[c+d x]$$

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^3}{(a + a \text{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 176 leaves, 10 steps):

$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a+a \text{Sec}[c+dx]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{11 \text{ArcTanh}\left[\frac{\sqrt{a+a \text{Sec}[c+dx]}}{\sqrt{2} \sqrt{a}}\right]}{16 \sqrt{2} a^{3/2} d} - \frac{3 a}{20 d (a + a \text{Sec}[c + dx])^{5/2}} + \frac{a}{2 d (1 - \text{Sec}[c + dx]) (a + a \text{Sec}[c + dx])^{5/2}} + \frac{5}{24 d (a + a \text{Sec}[c + dx])^{3/2}} + \frac{21}{16 a d \sqrt{a + a \text{Sec}[c + dx]}}$$

Result (type 3, 375 leaves):

$$\frac{1}{16 d (a (1 + \text{Sec}[c + dx]))^{3/2}} \text{Cos}\left[\frac{1}{4} (c + dx)\right]^2 \text{Cos}\left[\frac{1}{2} (c + dx)\right]^3$$

$$\left(64 \sqrt{2} \text{Log}\left[\text{Sec}\left[\frac{1}{4} (c + dx)\right]^2\right] - 11 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (c + dx)\right]^2\right] + 11 \text{Log}\left[1 + \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4} - 3 \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2\right] - 64 \sqrt{2} \right.$$

$$\left. \text{Log}\left[2 + \sqrt{2} \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4} - 2 \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2\right] - 11 \text{Log}\left[3 - \sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4} - \text{Tan}\left[\frac{1}{4} (c + dx)\right]^2\right] \right)$$

$$\sqrt{\text{Cos}[c + dx] \text{Sec}\left[\frac{1}{4} (c + dx)\right]^4} \text{Sec}[c + dx]^2 + \frac{1}{d (a (1 + \text{Sec}[c + dx]))^{3/2}} \text{Cos}\left[\frac{1}{2} (c + dx)\right]^3$$

$$\left(\frac{449}{60} \text{Cos}\left[\frac{1}{2} (c + dx)\right] - \frac{1}{8} \text{Cot}\left[\frac{1}{2} (c + dx)\right] \text{Csc}\left[\frac{1}{2} (c + dx)\right] - \frac{281}{60} \text{Sec}\left[\frac{1}{2} (c + dx)\right] + \frac{19}{30} \text{Sec}\left[\frac{1}{2} (c + dx)\right]^3 - \frac{1}{20} \text{Sec}\left[\frac{1}{2} (c + dx)\right]^5 \right) \text{Sec}[c + dx]^2$$

■ **Problem 189: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + dx]^6}{(a + a \text{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$-\frac{2 \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{a^{3/2} d} + \frac{2 \text{Tan}[c + dx]}{a d \sqrt{a + a \text{Sec}[c + dx]}} - \frac{2 \text{Tan}[c + dx]^3}{3 d (a + a \text{Sec}[c + dx])^{3/2}} + \frac{2 a \text{Tan}[c + dx]^5}{5 d (a + a \text{Sec}[c + dx])^{5/2}} + \frac{2 a^2 \text{Tan}[c + dx]^7}{7 d (a + a \text{Sec}[c + dx])^{7/2}}$$

Result (type 4, 453 leaves):

$$\frac{1}{d (a (1 + \operatorname{Sec}[c + d x]))^{3/2}} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \operatorname{Sec}[c + d x]^2$$

$$\left(\frac{1168}{105} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] - \frac{256}{105} \operatorname{Sec}[c + d x] \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] - \frac{64}{35} \operatorname{Sec}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + \frac{8}{7} \operatorname{Sec}[c + d x]^3 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) +$$

$$\frac{1}{d (a (1 + \operatorname{Sec}[c + d x]))^{3/2}} 32 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right]^4 \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4} (c + d x)\right]^2 \operatorname{Sec}[c + d x]^3 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4} (c + d x)\right]^2}}$$

■ **Problem 190: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c + d x]^4}{(a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{a^{3/2} d} - \frac{2 \operatorname{Tan}[c + d x]}{a d \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{2 \operatorname{Tan}[c + d x]^3}{3 d (a + a \operatorname{Sec}[c + d x])^{3/2}}$$

Result (type 4, 409 leaves):

$$\frac{\cos\left[\frac{1}{2}(c+dx)\right]^3 \sec[c+dx]^2 \left(-\frac{32}{3} \sin\left[\frac{1}{2}(c+dx)\right] + \frac{8}{3} \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right]\right)}{d(a(1+\sec[c+dx]))^{3/2}}$$

$$\frac{1}{d(a(1+\sec[c+dx]))^{3/2}} 32(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \cos\left[\frac{1}{2}(c+dx)\right]^3$$

$$\sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2 \sec[c+dx]^3 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

■ **Problem 195: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]^5}{(a+a\sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a+a\sec[c+dx]}}{\sqrt{a}}\right]}{a^{5/2}d} - \frac{6\sqrt{a+a\sec[c+dx]}}{a^3d} + \frac{2(a+a\sec[c+dx])^{3/2}}{3a^4d}$$

Result (type 3, 215 leaves):

$$\frac{1}{d(a(1+\sec[c+dx]))^{5/2}} 8\sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right]^2 \cos\left[\frac{1}{2}(c+dx)\right]^5$$

$$\left(\text{Log}\left[\sec\left[\frac{1}{4}(c+dx)\right]^2\right] - \text{Log}\left[2+\sqrt{2}\sqrt{\cos[c+dx]\sec\left[\frac{1}{4}(c+dx)\right]^4} - 2\tan\left[\frac{1}{4}(c+dx)\right]^2\right]\right) \sqrt{\cos[c+dx]\sec\left[\frac{1}{4}(c+dx)\right]^4 \sec[c+dx]^3 +$$

$$\frac{\cos\left[\frac{1}{2}(c+dx)\right]^5 \sec[c+dx]^3 \left(-\frac{128}{3} \cos\left[\frac{1}{2}(c+dx)\right] + \frac{16}{3} \cos\left[\frac{1}{2}(c+dx)\right] \sec[c+dx]\right)}{d(a(1+\sec[c+dx]))^{5/2}}$$

■ **Problem 196: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]^3}{(a+a\sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{5/2} d} - \frac{4}{a^2 d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 179 leaves):

$$\begin{aligned} & - \left(8 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^4 \left(-2 + 4 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 + \right. \right. \\ & \quad \left. \left. \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \right. \right. \\ & \quad \left. \left. \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} \right) \operatorname{Sec}[c+d x] \right) / \left(a^2 d (1 + \operatorname{Cos}[c+d x])^2 \sqrt{a(1 + \operatorname{Sec}[c+d x])} \right) \end{aligned}$$

■ **Problem 197: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+d x]}{(a+a \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{5/2} d} + \frac{2}{3 a d (a+a \operatorname{Sec}[c+d x])^{3/2}} + \frac{2}{a^2 d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 197 leaves):

$$\begin{aligned} & \left(4 \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 - 10 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^4 + 16 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^6 + 6 \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \right. \right. \\ & \quad \left. \left. \left(\operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right) \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} \right. \right. \\ & \quad \left. \left. \operatorname{Sec}[c+d x] \right) / \left(3 a^2 d (1 + \operatorname{Cos}[c+d x])^2 \sqrt{a(1 + \operatorname{Sec}[c+d x])} \right) \end{aligned}$$

■ **Problem 198: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+d x]}{(a+a \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 3, 144 leaves, 9 steps) :

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{5/2} d} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+a \operatorname{Sec}[c+d x]}}{\sqrt{2} \sqrt{a}}\right]}{4 \sqrt{2} a^{5/2} d} - \frac{1}{5 d (a+a \operatorname{Sec}[c+d x])^{5/2}} - \frac{1}{2 a d (a+a \operatorname{Sec}[c+d x])^{3/2}} - \frac{7}{4 a^2 d \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 347 leaves) :

$$\frac{1}{2 d (a (1 + \operatorname{Sec}[c+d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]^2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\left(-16 \sqrt{2} \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2\right] + \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] - \operatorname{Log}\left[1 + \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 3 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \right.$$

$$\left. 16 \sqrt{2} \operatorname{Log}\left[2 + \sqrt{2} \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - 2 \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] + \operatorname{Log}\left[3 - \sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} - \operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2\right] \right)$$

$$\sqrt{\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^4} \operatorname{Sec}[c+d x]^3 + \frac{1}{d (a (1 + \operatorname{Sec}[c+d x]))^{5/2}}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \left(-\frac{98}{5} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \frac{67}{5} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] - \frac{11}{5} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 + \frac{1}{5} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \right) \operatorname{Sec}[c+d x]^3$$

- **Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+d x]^6}{(a+a \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 3, 127 leaves, 4 steps) :

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{a^{5/2} d} + \frac{2 \operatorname{Tan}[c+d x]}{a^2 d \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{2 \operatorname{Tan}[c+d x]^3}{3 a d (a+a \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 \operatorname{Tan}[c+d x]^5}{5 d (a+a \operatorname{Sec}[c+d x])^{5/2}}$$

Result (type 4, 431 leaves) :

$$\frac{1}{d (a (1 + \operatorname{Sec}[c + d x]))^{5/2}} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5 \operatorname{Sec}[c + d x]^3 \left(\frac{368}{15} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - \frac{176}{15} \operatorname{Sec}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \frac{16}{5} \operatorname{Sec}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) +$$

$$\frac{1}{d (a (1 + \operatorname{Sec}[c + d x]))^{5/2}} 64 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right]^4 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^5$$

$$\sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + d x)\right]^2 \operatorname{Sec}[c + d x]^4 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + d x)\right]^2}}$$

■ **Problem 208: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^n (e \operatorname{Tan}[c + d x])^m dx$$

Optimal (type 6, 125 leaves, 1 step):

$$\frac{1}{d e (1 + m)} 2^{1+m+n} \operatorname{AppellF1}\left[\frac{1+m}{2}, m+n, 1, \frac{3+m}{2}, -\frac{a - a \operatorname{Sec}[c + d x]}{a + a \operatorname{Sec}[c + d x]}, \frac{a - a \operatorname{Sec}[c + d x]}{a + a \operatorname{Sec}[c + d x]}\right]$$

$$\left(\frac{1}{1 + \operatorname{Sec}[c + d x]}\right)^{1+m+n} (a + a \operatorname{Sec}[c + d x])^n (e \operatorname{Tan}[c + d x])^{1+m}$$

Result (type 8, 25 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^n (e \operatorname{Tan}[c + d x])^m dx$$

■ **Problem 209: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^3 (e \operatorname{Tan}[c + d x])^m dx$$

Optimal (type 5, 243 leaves, 8 steps):

$$\frac{3 a^3 (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)} + \frac{a^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\operatorname{Tan}[c+d x]^2\right] (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)} + \frac{1}{d e (1+m)}$$

$$3 a^3 (\operatorname{Cos}[c+d x]^2)^{\frac{2+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c+d x]^2\right] \operatorname{Sec}[c+d x] (e \operatorname{Tan}[c+d x])^{1+m} +$$

$$\frac{a^3 (\operatorname{Cos}[c+d x]^2)^{\frac{4+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{4+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c+d x]^2\right] \operatorname{Sec}[c+d x]^3 (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)}$$

Result (type 8, 25 leaves):

$$\int (a + a \operatorname{Sec}[c+d x])^3 (e \operatorname{Tan}[c+d x])^m dx$$

■ **Problem 210: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c+d x])^2 (e \operatorname{Tan}[c+d x])^m dx$$

Optimal (type 5, 161 leaves, 7 steps):

$$\frac{a^2 (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)} + \frac{a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\operatorname{Tan}[c+d x]^2\right] (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)} + \frac{1}{d e (1+m)}$$

$$2 a^2 (\operatorname{Cos}[c+d x]^2)^{\frac{2+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c+d x]^2\right] \operatorname{Sec}[c+d x] (e \operatorname{Tan}[c+d x])^{1+m}$$

Result (type 8, 25 leaves):

$$\int (a + a \operatorname{Sec}[c+d x])^2 (e \operatorname{Tan}[c+d x])^m dx$$

■ **Problem 211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+d x]) (e \operatorname{Tan}[c+d x])^m dx$$

Optimal (type 5, 129 leaves, 4 steps):

$$\frac{a \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\operatorname{Tan}[c+d x]^2\right] (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)} +$$

$$\frac{a (\operatorname{Cos}[c+d x]^2)^{\frac{2+m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \operatorname{Sin}[c+d x]^2\right] \operatorname{Sec}[c+d x] (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m)}$$

Result (type 6, 2548 leaves):

$$\left(a (1 + \operatorname{Sec}[c+d x]) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left(\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \left(\operatorname{Cos}[c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right)^m + \right. \right.$$

$$\left. \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) /$$

$$\begin{aligned}
& \left. 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \Big) \Big) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \frac{1}{2} (1+m) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right)^m \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, 1+m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-1-m} \right) - \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + (3+m) \left(-\frac{1}{3+m} \right. \right. \\
& \quad \left. \left. (1+m) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, m, 2, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3+m} \right. \right. \\
& \quad \left. \left. m (1+m) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, 1+m, 1, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{1}{5+m} 2 (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, m, 3, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m} m (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - m \left(-\frac{1}{5+m} (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1+m, 2, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{1}{5+m} (1+m) (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 2+m, 1, 1 + \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Big) \Big) \Big) / \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, m, 1, \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, m, 2, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1+m, 1, \frac{5+m}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \tan[c+dx]^m \Big) \Big)
\end{aligned}$$

■ **Problem 212: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{e \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+m), \frac{1+m}{2}, -\operatorname{Tan}[c + d x]^2\right] (e \operatorname{Tan}[c + d x])^{-1+m}}{a d (1-m)} - \frac{1}{a d (1-m)}$$

$$e (\operatorname{Cos}[c + d x]^2)^{m/2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1+m), \frac{m}{2}, \frac{1+m}{2}, \operatorname{Sin}[c + d x]^2\right] \operatorname{Sec}[c + d x] (e \operatorname{Tan}[c + d x])^{-1+m}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{a + a \operatorname{Sec}[c + d x]} dx$$

■ **Problem 213: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 5, 169 leaves, 8 steps):

$$-\frac{e^3 (e \operatorname{Tan}[c + d x])^{-3+m}}{a^2 d (3-m)} - \frac{e^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-3+m), \frac{1}{2}(-1+m), -\operatorname{Tan}[c + d x]^2\right] (e \operatorname{Tan}[c + d x])^{-3+m}}{a^2 d (3-m)} + \frac{1}{a^2 d (3-m)}$$

$$2 e^3 (\operatorname{Cos}[c + d x]^2)^{\frac{1}{2}(-2+m)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-3+m), \frac{1}{2}(-2+m), \frac{1}{2}(-1+m), \operatorname{Sin}[c + d x]^2\right] \operatorname{Sec}[c + d x] (e \operatorname{Tan}[c + d x])^{-3+m}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

■ **Problem 214: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 5, 252 leaves, 9 steps):

$$\frac{3 e^5 (e \operatorname{Tan}[c + d x])^{-5+m}}{a^3 d (5-m)} + \frac{e^5 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), -\operatorname{Tan}[c + d x]^2\right] (e \operatorname{Tan}[c + d x])^{-5+m}}{a^3 d (5-m)} - \frac{1}{a^3 d (5-m)}$$

$$3 e^5 (\operatorname{Cos}[c + d x]^2)^{\frac{1}{2}(-4+m)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-5+m), \frac{1}{2}(-4+m), \frac{1}{2}(-3+m), \operatorname{Sin}[c + d x]^2\right] \operatorname{Sec}[c + d x] (e \operatorname{Tan}[c + d x])^{-5+m} -$$

$$\frac{1}{a^3 d (5-m)} e^5 (\operatorname{Cos}[c + d x]^2)^{\frac{1}{2}(-2+m)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-5+m), \frac{1}{2}(-2+m), \frac{1}{2}(-3+m), \operatorname{Sin}[c + d x]^2\right] \operatorname{Sec}[c + d x]^3 (e \operatorname{Tan}[c + d x])^{-5+m}$$

Result (type 8, 25 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^3} dx$$

■ **Problem 215: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^{3/2} (e \operatorname{Tan}[c + d x])^m dx$$

Optimal (type 6, 131 leaves, 1 step):

$$\frac{1}{d e (1+m)} 2^{\frac{5}{2}+m} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{3}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}, \frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}\right] \\ \left(\frac{1}{1+\operatorname{Sec}[c+d x]}\right)^{\frac{5}{2}+m} (a+a \operatorname{Sec}[c+d x])^{3/2} (e \operatorname{Tan}[c+d x])^{1+m}$$

Result (type 8, 27 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^{3/2} (e \operatorname{Tan}[c + d x])^m dx$$

■ **Problem 216: Unable to integrate problem.**

$$\int \sqrt{a + a \operatorname{Sec}[c + d x]} (e \operatorname{Tan}[c + d x])^m dx$$

Optimal (type 6, 131 leaves, 1 step):

$$\frac{1}{d e (1+m)} 2^{\frac{3}{2}+m} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}, \frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}\right] \left(\frac{1}{1+\operatorname{Sec}[c+d x]}\right)^{\frac{3}{2}+m} \sqrt{a+a \operatorname{Sec}[c+d x]} (e \operatorname{Tan}[c+d x])^{1+m}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + a \operatorname{Sec}[c + d x]} (e \operatorname{Tan}[c + d x])^m dx$$

■ **Problem 217: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 6, 131 leaves, 1 step):

$$\frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}, \frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}\right] \left(\frac{1}{1+\operatorname{Sec}[c+d x]}\right)^{\frac{1}{2}+m} (e \operatorname{Tan}[c+d x])^{1+m}}{d e (1+m) \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 218: Unable to integrate problem.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 6, 131 leaves, 1 step):

$$\frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{3}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}, \frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}\right] \left(\frac{1}{1+\operatorname{Sec}[c+d x]}\right)^{-\frac{1}{2}+m} (e \operatorname{Tan}[c + d x])^{1+m}}{d e (1+m) (a + a \operatorname{Sec}[c + d x])^{3/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{(e \operatorname{Tan}[c + d x])^m}{(a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 219: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^7 dx$$

Optimal (type 5, 123 leaves, 4 steps):

$$\frac{7 (a + a \operatorname{Sec}[c + d x])^{4+n}}{a^4 d (4+n)} + \frac{\operatorname{Hypergeometric2F1}[1, 4+n, 5+n, 1+\operatorname{Sec}[c+d x]] (a + a \operatorname{Sec}[c + d x])^{4+n}}{a^4 d (4+n)} - \frac{5 (a + a \operatorname{Sec}[c + d x])^{5+n}}{a^5 d (5+n)} + \frac{(a + a \operatorname{Sec}[c + d x])^{6+n}}{a^6 d (6+n)}$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^7 dx$$

■ **Problem 220: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^5 dx$$

Optimal (type 5, 97 leaves, 4 steps):

$$-\frac{3 (a + a \operatorname{Sec}[c + d x])^{3+n}}{a^3 d (3+n)} - \frac{\operatorname{Hypergeometric2F1}[1, 3+n, 4+n, 1+\operatorname{Sec}[c+d x]] (a + a \operatorname{Sec}[c + d x])^{3+n}}{a^3 d (3+n)} + \frac{(a + a \operatorname{Sec}[c + d x])^{4+n}}{a^4 d (4+n)}$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^5 dx$$

■ **Problem 221: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 5, 69 leaves, 3 steps):

$$\frac{(a + a \operatorname{Sec}[c + d x])^{2+n}}{a^2 d (2+n)} + \frac{\operatorname{Hypergeometric2F1}[1, 2+n, 3+n, 1 + \operatorname{Sec}[c + d x]] (a + a \operatorname{Sec}[c + d x])^{2+n}}{a^2 d (2+n)}$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^3 dx$$

■ **Problem 223: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x] (a + a \operatorname{Sec}[c + d x])^n dx$$

Optimal (type 5, 74 leaves, 4 steps):

$$-\frac{\operatorname{Hypergeometric2F1}\left[1, n, 1+n, \frac{1}{2}(1 + \operatorname{Sec}[c + d x])\right] (a + a \operatorname{Sec}[c + d x])^n}{2 d n} + \frac{\operatorname{Hypergeometric2F1}[1, n, 1+n, 1 + \operatorname{Sec}[c + d x]] (a + a \operatorname{Sec}[c + d x])^n}{d n}$$

Result (type 6, 2553 leaves):

$$\begin{aligned} & \left(2^{-2+n} \operatorname{Cos}[c + d x] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 (a (1 + \operatorname{Sec}[c + d x]))^n \left(\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^n \right. \\ & \left(-\frac{1}{n} \left(1 - \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]^2 \right)^n \operatorname{Hypergeometric2F1}\left[n, n, 1+n, \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]^2\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \right) + \\ & \left(4 \operatorname{AppellF1}\left[1, n, 1, 2, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) / \\ & \left(-2 \operatorname{AppellF1}\left[1, n, 1, 2, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] + \left(\operatorname{AppellF1}\left[2, n, 2, 3, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \right) - \right. \\ & \left. n \operatorname{AppellF1}\left[2, 1+n, 1, 3, \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left. \right) / \\ & \left(d \left(\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 \right) \left(-\frac{1}{\left(\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 \right)^2} 2^{-1+n} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \left(\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \right)^n \right. \right. \\ & \left. \left. \left(\frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 + \frac{3}{2} \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \left(-\frac{1}{n} \left(1 - \operatorname{Cot}\left[\frac{1}{2}(c + d x)\right]^2 \right)^n \operatorname{Hypergeometric2F1}[n, n, 1+n, \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \cot\left[\frac{1}{2}(c+dx)\right]^2 \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \left(4 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \\
& \left(-2 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(\operatorname{AppellF1}\left[2, n, 2, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[2, 1+n, 1, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) + \\
& \left(2^{-2+n} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^n \left(-1/n \left(1 - \cot\left[\frac{1}{2}(c+dx)\right]^2\right)^n \operatorname{Hypergeometric2F1}\left[n, n, 1+n, \cot\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \left(4 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \right. \right. \\
& \quad \left. \left. \left(-2 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(\operatorname{AppellF1}\left[2, n, 2, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1}\left[2, 1+n, 1, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right) / \right. \\
& \left. \left(\tan\left[\frac{1}{2}(c+dx)\right] + \tan\left[\frac{1}{2}(c+dx)\right]^3\right) + \left(2^{-1+n} n \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^{1+n} \right. \right. \\
& \quad \left. \left(-1/n \left(1 - \cot\left[\frac{1}{2}(c+dx)\right]^2\right)^n \operatorname{Hypergeometric2F1}\left[n, n, 1+n, \cot\left[\frac{1}{2}(c+dx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \right. \right. \\
& \quad \left. \left(4 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) / \right. \right. \\
& \quad \left. \left(-2 \operatorname{AppellF1}\left[1, n, 1, 2, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(\operatorname{AppellF1}\left[2, n, 2, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[2, 1+n, 1, 3, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) / \right. \\
& \left. \left(\tan\left[\frac{1}{2}(c+dx)\right] + \tan\left[\frac{1}{2}(c+dx)\right]^3\right) + \frac{1}{\tan\left[\frac{1}{2}(c+dx)\right] + \tan\left[\frac{1}{2}(c+dx)\right]^3} 2^{-1+n} \tan\left[\frac{1}{2}(c+dx)\right] \left(\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}\right)^n \right. \\
& \quad \left. \left(-\frac{1}{n} \left(1 - \cot\left[\frac{1}{2}(c+dx)\right]^2\right)^n \operatorname{Hypergeometric2F1}\left[n, n, 1+n, \cot\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \quad \left. \left. \cot\left[\frac{1}{2}(c+dx)\right] \left(1 - \cot\left[\frac{1}{2}(c+dx)\right]^2\right)^{-1+n} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Hypergeometric2F1}\left[n, n, 1+n, \cot\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(c+dx)\right]^2 - n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \tan\left[\frac{1}{2}(c+dx)\right] - 3\left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \\
& 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, n, 3, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad \left. n\left(-\frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5}(1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right)\right) / \\
& \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 - \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^n \\
& \left(-\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-2-n}\right) + \right. \\
& \quad \left. \frac{1}{2} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-1-n}\right)\right)\right) + \\
& 2^{1+n} n \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]\right)^{-1+n} \tan\left[\frac{1}{2}(c+dx)\right] \left(-\left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2+n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^n + \right. \\
& \quad \left. \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \cos\left[\frac{1}{2}(c+dx)\right]^2\right) / \right. \\
& \quad \left. \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2\left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \right. \\
& \quad \left. \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right) \\
& \left. \left(-\cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \tan[c+dx]\right)\right)
\end{aligned}$$

■ **Problem 227: Result more than twice size of optimal antiderivative.**

$$\int \cot [c + d x]^2 (a + a \operatorname{Sec}[c + d x])^n dx$$

Optimal (type 6, 102 leaves, 1 step):

$$-\frac{1}{d} 2^{-1+n} \operatorname{AppellF1}\left[-\frac{1}{2}, -2+n, 1, \frac{1}{2}, -\frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}, \frac{a-a \operatorname{Sec}[c+d x]}{a+a \operatorname{Sec}[c+d x]}\right] \cot [c+d x] \left(\frac{1}{1+\operatorname{Sec}[c+d x]}\right)^{-1+n} (a+a \operatorname{Sec}[c+d x])^n$$

Result (type 6, 2492 leaves):

$$\begin{aligned} & \left(2^{-3+n} \cos [c+d x]^2 \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \left(\cos\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n\right. \\ & (a(1+\operatorname{Sec}[c+d x]))^n \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sin\left[\frac{1}{2}(c+d x)\right]^2\right) / \right. \\ & \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\ & \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+d x)\right]^2 + \left(\cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)^n\right. \\ & \left. \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, n, \frac{1}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2\right] + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right)\right) / \right. \\ & \left. \left(d \left(-2^{-2+n} \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 \left(\cos\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n\right.\right. \right. \\ & \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \sin\left[\frac{1}{2}(c+d x)\right]^2\right) / \right. \\ & \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\ & \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+d x)\right]^2 + \left(\cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right)^n\right. \\ & \left. \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, n, \frac{1}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2\right] + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2\right] \tan\left[\frac{1}{2}(c+d x)\right]^2\right)\right) + \right. \\ & \left. 2^{-1+n} \cot\left[\frac{1}{2}(c+d x)\right] \left(\cos\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Sec}[c+d x]\right)^n\right. \\ & \left(\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \cos\left[\frac{1}{2}(c+d x)\right] \sin\left[\frac{1}{2}(c+d x)\right]\right) / \right. \\ & \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] - \right. \right. \\ & \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+d x)\right]^2 + \right. \\ & \left. \left(12 \sin\left[\frac{1}{2}(c+d x)\right]^2 \left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+d x)\right]^2, -\tan\left[\frac{1}{2}(c+d x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \tan\left[\frac{1}{2}(c+d x)\right] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \Big/ \\
& \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \\
& n \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^{-1+n} \left(-\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] + \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, n, \frac{1}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - \\
& \left(12 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \sin\left[\frac{1}{2}(c+dx)\right]^2 \left(2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(c+dx)\right] - 3 \left(-\frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{3} n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \right. \\
& \quad \left. 2 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, n, 3, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} n \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\
& \quad \left. \left. n \left(-\frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1+n, 2, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} (1+n) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n, 1, \frac{7}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right)\right) \Big/ \\
& \left(-3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right]\right) \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 + \\
& \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right)^n \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \quad \left. \frac{1}{2} \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \left(\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, n, \frac{1}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] - \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-n}\right) + \right. \\
& \quad \left. \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, n, \frac{3}{2}, \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{-n}\right)\right)\right) \Big/ +
\end{aligned}$$

$$\begin{aligned}
& 2^{-1+n} n \cot \left[\frac{1}{2} (c + d x) \right] \left(\cos \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x] \right)^{-1+n} \\
& \left(\left(12 \operatorname{AppellF1} \left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \sin \left[\frac{1}{2} (c + d x) \right]^2 \right) / \right. \\
& \left(-3 \operatorname{AppellF1} \left[\frac{1}{2}, n, 1, \frac{3}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] + 2 \left(\operatorname{AppellF1} \left[\frac{3}{2}, n, 2, \frac{5}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1} \left[\frac{3}{2}, 1+n, 1, \frac{5}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2, -\tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 + \left(\cos [c + d x] \sec \left[\frac{1}{2} (c + d x) \right]^2 \right)^n \right. \\
& \left. \left(-\operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, n, \frac{1}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2 \right] + \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, n, \frac{3}{2}, \tan \left[\frac{1}{2} (c + d x) \right]^2 \right] \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \\
& \left. \left(-\cos \left[\frac{1}{2} (c + d x) \right] \sec [c + d x] \sin \left[\frac{1}{2} (c + d x) \right] + \cos \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x] \tan [c + d x] \right) \right) \right)
\end{aligned}$$

■ **Problem 228: Unable to integrate problem.**

$$\int \cot [c + d x]^4 (a + a \sec [c + d x])^n dx$$

Optimal (type 6, 106 leaves, 1 step):

$$-\frac{1}{3d} 2^{-3+n} \operatorname{AppellF1} \left[-\frac{3}{2}, -4+n, 1, -\frac{1}{2}, -\frac{a - a \sec [c + d x]}{a + a \sec [c + d x]}, \frac{a - a \sec [c + d x]}{a + a \sec [c + d x]} \right] \cot [c + d x]^3 \left(\frac{1}{1 + \sec [c + d x]} \right)^{-3+n} (a + a \sec [c + d x])^n$$

Result (type 8, 23 leaves):

$$\int \cot [c + d x]^4 (a + a \sec [c + d x])^n dx$$

■ **Problem 229: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^n \tan [c + d x]^{3/2} dx$$

Optimal (type 6, 114 leaves, 1 step):

$$\frac{1}{5d} 2^{\frac{7}{2}+n} \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}+n, 1, \frac{9}{4}, -\frac{a - a \sec [c + d x]}{a + a \sec [c + d x]}, \frac{a - a \sec [c + d x]}{a + a \sec [c + d x]} \right] \left(\frac{1}{1 + \sec [c + d x]} \right)^{\frac{5}{2}+n} (a + a \sec [c + d x])^n \tan [c + d x]^{5/2}$$

Result (type 6, 11753 leaves): Display of huge result suppressed!

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^n \sqrt{\tan [c + d x]} dx$$

Optimal (type 6, 114 leaves, 1 step):

$$\frac{1}{3d} 2^{\frac{5}{2}+n} \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, -\frac{a - a \sec [c + d x]}{a + a \sec [c + d x]}, \frac{a - a \sec [c + d x]}{a + a \sec [c + d x]} \right] \left(\frac{1}{1 + \sec [c + d x]} \right)^{\frac{3}{2}+n} (a + a \sec [c + d x])^n \tan [c + d x]^{3/2}$$

$$\begin{aligned}
& (1+2n) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}+n, 1, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \\
& 21 \left(-\frac{3}{7} \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}+n, 2, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{3}{7} \left(\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}+n, 1, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + \\
& 6 \tan\left[\frac{1}{2}(c+dx)\right]^2 \left(-2 \left(-\frac{14}{11} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}+n, 3, \frac{15}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \frac{7}{11} \left(\frac{1}{2}+n\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}+n, 2, \frac{15}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right) + (1+2n) \\
& \quad \left(-\frac{7}{11} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}+n, 2, \frac{15}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + \frac{7}{11} \left(\frac{3}{2}+n\right) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[\frac{11}{4}, \frac{5}{2}+n, 1, \frac{15}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]\right)\right) \sqrt{\tan[c+dx]} \Big/ \\
& \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + 6 \left(-2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}+n, 2, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + (1+2n) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}+n, 1, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \Big)^2 + \\
& \left(7 \times 2^{1+n} n \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]\right)^{-1+n} \sin[c+dx] \right. \\
& \quad \left. \sqrt{\tan[c+dx]} \left(-\cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \tan[c+dx]\right)\right) \Big/ \\
& \left(21 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& 6 \left(-2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}+n, 2, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \\
& \quad \left. \left. (1+2n) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}+n, 1, \frac{11}{4}, \tan\left[\frac{1}{2}(c+dx)\right]^2, -\tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 231: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^n}{\sqrt{\tan[c + dx]}} dx$$

Optimal (type 6, 111 leaves, 1 step):

$$\frac{1}{d} 2^{\frac{3}{2}+n} \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}+n, 1, \frac{5}{4}, -\frac{a - a \operatorname{Sec}[c + dx]}{a + a \operatorname{Sec}[c + dx]}, \frac{a - a \operatorname{Sec}[c + dx]}{a + a \operatorname{Sec}[c + dx]}\right] \left(\frac{1}{1 + \operatorname{Sec}[c + dx]}\right)^{\frac{1}{2}+n} (a + a \operatorname{Sec}[c + dx])^n \sqrt{\tan[c + dx]}$$

$$\begin{aligned}
& \left(-2 \left(2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} + n, 2, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 - 2 n) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} + n, 1, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + \right. \\
& \quad 5 \left(-\frac{1}{5} \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} + n, 2, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + \right. \\
& \quad \left. \frac{1}{5} \left(-\frac{1}{2} + n \right) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} + n, 1, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) - \\
& \quad 2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \left(2 \left(-\frac{10}{9} \operatorname{AppellF1} \left[\frac{9}{4}, -\frac{1}{2} + n, 3, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + \right. \right. \\
& \quad \left. \left. \frac{5}{9} \left(-\frac{1}{2} + n \right) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2} + n, 2, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) + \right. \\
& \quad \left. (1 - 2 n) \left(-\frac{5}{9} \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2} + n, 2, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + \frac{5}{9} \left(\frac{1}{2} + n \right) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2} + n, 1, \frac{13}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \right) \sqrt{\operatorname{Tan}[c + d x]} \Big/ \\
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} + n, 1, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - 2 \left(2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} + n, 2, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + (1 - 2 n) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} + n, 1, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \Big)^2 + \\
& \left(5 \times 2^{1+n} n \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} + n, 1, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \operatorname{Cos}[c + d x] \left(\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec}[c + d x] \right)^{-1+n} \right. \\
& \quad \left. \sqrt{\operatorname{Tan}[c + d x]} \left(-\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] \operatorname{Sec}[c + d x] \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x] \right) \right) \Big/ \\
& \left(5 \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2} + n, 1, \frac{5}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2} + n, 2, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 - 2 n) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2} + n, 1, \frac{9}{4}, \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \Big) \Big)
\end{aligned}$$

■ **Problem 232: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^n}{\operatorname{Tan}[c + d x]^{3/2}} dx$$

Optimal (type 6, 112 leaves, 1 step):

$$\frac{2^{\frac{1}{2}+n} \text{AppellF1}\left[-\frac{1}{4}, -\frac{3}{2}+n, 1, \frac{3}{4}, -\frac{a-a \operatorname{Sec}[c+dx]}{a+a \operatorname{Sec}[c+dx]}, \frac{a-a \operatorname{Sec}[c+dx]}{a+a \operatorname{Sec}[c+dx]}\right] \left(\frac{1}{1+\operatorname{Sec}[c+dx]}\right)^{-\frac{1}{2}+n} (a+a \operatorname{Sec}[c+dx])^n}{d \sqrt{\operatorname{Tan}[c+dx]}}$$

Result (type 6, 5312 leaves): Display of huge result suppressed!

■ **Problem 233: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \operatorname{Cot}[c+dx])^{5/2} (a+a \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 320 leaves, 17 steps):

$$\begin{aligned} & -\frac{2 (e \operatorname{Cot}[c+dx])^{5/2} (a+a \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]}{3 d} - \frac{a (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{EllipticF}\left[c-\frac{\pi}{4}+dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]} \operatorname{Tan}[c+dx]^2}{3 d} + \\ & \frac{a \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{Tan}[c+dx]^{5/2}}{\sqrt{2} d} - \frac{a \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{Tan}[c+dx]^{5/2}}{\sqrt{2} d} + \\ & \frac{a (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \operatorname{Tan}[c+dx]^{5/2}}{2 \sqrt{2} d} - \\ & \frac{a (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \operatorname{Tan}[c+dx]^{5/2}}{2 \sqrt{2} d} \end{aligned}$$

Result (type 4, 185 leaves):

$$\begin{aligned} & -\frac{1}{6 d \operatorname{Cot}[c+dx]^{5/2}} \\ & a (e \operatorname{Cot}[c+dx])^{5/2} \operatorname{Sec}[c+dx] \left(\sqrt{\operatorname{Cot}[c+dx]} \left(4 (1+\operatorname{Cos}[c+dx]) \operatorname{Cot}[c+dx] - 3 \operatorname{ArcSin}[\operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]] \sqrt{\operatorname{Sin}[2(c+dx)]} \right) + \right. \\ & \quad \left. 3 \operatorname{Log}\left[\operatorname{Cos}[c+dx] + \operatorname{Sin}[c+dx] + \sqrt{\operatorname{Sin}[2(c+dx)]}\right] \sqrt{\operatorname{Sin}[2(c+dx)]} \right) + \\ & \quad 2 (-1)^{1/4} \sqrt{\operatorname{Csc}[c+dx]^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Cot}[c+dx]}\right], -1\right] \operatorname{Sin}[2(c+dx)] \end{aligned}$$

■ **Problem 234: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \operatorname{Cot}[c+dx])^{3/2} (a+a \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 346 leaves, 18 steps):

$$\begin{aligned}
& - \frac{2 (e \cot [c+d x])^{3/2} (a+a \sec [c+d x]) \tan [c+d x]}{d} - \frac{2 a (e \cot [c+d x])^{3/2} \operatorname{EllipticE}\left[c-\frac{\pi}{4}+d x, 2\right] \sin [c+d x] \tan [c+d x]}{d \sqrt{\sin [2 c+2 d x]}} + \\
& \frac{a \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right] (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}}{\sqrt{2} d} - \frac{a \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right] (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}}{\sqrt{2} d} - \\
& \frac{a (e \cot [c+d x])^{3/2} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \tan [c+d x]^{3/2}}{2 \sqrt{2} d} + \\
& \frac{a (e \cot [c+d x])^{3/2} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \tan [c+d x]^{3/2}}{2 \sqrt{2} d} + \frac{2 a (e \cot [c+d x])^{3/2} \sin [c+d x] \tan [c+d x]^2}{d}
\end{aligned}$$

Result (type 4, 210 leaves):

$$\begin{aligned}
& \frac{1}{2 d \sqrt{\csc [c+d x]^2}} a e \sqrt{e \cot [c+d x]} \sec [c+d x] \left(4 (-1)^{3/4} \sqrt{\cot [c+d x]} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot [c+d x]}\right], -1\right] - \right. \\
& \left. 4 (-1)^{3/4} \sqrt{\cot [c+d x]} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot [c+d x]}\right], -1\right] + \sqrt{\csc [c+d x]^2} \left(-4 \cos [c+d x] + \right. \right. \\
& \left. \left. \operatorname{ArcSin}[\cos [c+d x]-\sin [c+d x]] \sqrt{\sin [2(c+d x)]} + \operatorname{Log}\left[\cos [c+d x]+\sin [c+d x]+\sqrt{\sin [2(c+d x)]}\right] \sqrt{\sin [2(c+d x)]}\right)\right)
\end{aligned}$$

■ **Problem 235: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{e \cot [c+d x]} (a+a \sec [c+d x]) dx$$

Optimal (type 4, 274 leaves, 16 steps):

$$\begin{aligned}
& \frac{a \sqrt{e \cot [c+d x]} \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \sec [c+d x] \sqrt{\sin [2 c+2 d x]}}{d} - \frac{a \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right] \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}{\sqrt{2} d} + \\
& \frac{a \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right] \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}{\sqrt{2} d} - \frac{a \sqrt{e \cot [c+d x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \sqrt{\tan [c+d x]}}{2 \sqrt{2} d} + \\
& \frac{a \sqrt{e \cot [c+d x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \sqrt{\tan [c+d x]}}{2 \sqrt{2} d}
\end{aligned}$$

Result (type 4, 169 leaves):

$$\frac{1}{4 d \sqrt{\text{Csc}[c + d x]^2}} a (1 + \text{Cos}[c + d x]) \sqrt{e \text{Cot}[c + d x]} \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2$$

$$\text{Sec}[c + d x] \left(4 (-1)^{1/4} \sqrt{\text{Cot}[c + d x]} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Cot}[c + d x]}\right], -1\right] + \sqrt{\text{Csc}[c + d x]^2} \left(-\text{ArcSin}[\text{Cos}[c + d x] - \text{Sin}[c + d x]] + \text{Log}\left[\text{Cos}[c + d x] + \text{Sin}[c + d x] + \sqrt{\text{Sin}[2(c + d x)]}\right] \right) \sqrt{\text{Sin}[2(c + d x)]} \right)$$

■ **Problem 236: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \text{Sec}[c + d x]}{\sqrt{e \text{Cot}[c + d x]}} dx$$

Optimal (type 4, 299 leaves, 17 steps):

$$\frac{2 a \text{Sin}[c + d x]}{d \sqrt{e \text{Cot}[c + d x]}} - \frac{2 a \text{Cos}[c + d x] \text{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right]}{d \sqrt{e \text{Cot}[c + d x]} \sqrt{\text{Sin}[2 c + 2 d x]}} - \frac{a \text{ArcTan}\left[1 - \sqrt{2} \sqrt{\text{Tan}[c + d x]}\right]}{\sqrt{2} d \sqrt{e \text{Cot}[c + d x]} \sqrt{\text{Tan}[c + d x]}} +$$

$$\frac{a \text{ArcTan}\left[1 + \sqrt{2} \sqrt{\text{Tan}[c + d x]}\right]}{\sqrt{2} d \sqrt{e \text{Cot}[c + d x]} \sqrt{\text{Tan}[c + d x]}} + \frac{a \text{Log}\left[1 - \sqrt{2} \sqrt{\text{Tan}[c + d x]} + \text{Tan}[c + d x]\right]}{2 \sqrt{2} d \sqrt{e \text{Cot}[c + d x]} \sqrt{\text{Tan}[c + d x]}} - \frac{a \text{Log}\left[1 + \sqrt{2} \sqrt{\text{Tan}[c + d x]} + \text{Tan}[c + d x]\right]}{2 \sqrt{2} d \sqrt{e \text{Cot}[c + d x]} \sqrt{\text{Tan}[c + d x]}}$$

Result (type 4, 200 leaves):

$$-\frac{1}{2 d \sqrt{e \text{Cot}[c + d x]}} a \text{Csc}[c + d x] \left(-4 - \frac{4 (-1)^{3/4} \sqrt{\text{Cot}[c + d x]} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Cot}[c + d x]}\right], -1\right]}{\sqrt{\text{Csc}[c + d x]^2}} + \frac{4 (-1)^{3/4} \sqrt{\text{Cot}[c + d x]} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Cot}[c + d x]}\right], -1\right]}{\sqrt{\text{Csc}[c + d x]^2}} + \frac{\text{ArcSin}[\text{Cos}[c + d x] - \text{Sin}[c + d x]] \sqrt{\text{Sin}[2(c + d x)]} + \text{Log}\left[\text{Cos}[c + d x] + \text{Sin}[c + d x] + \sqrt{\text{Sin}[2(c + d x)]}\right] \sqrt{\text{Sin}[2(c + d x)]}}{\sqrt{\text{Csc}[c + d x]^2}} \right)$$

■ **Problem 237: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + a \text{Sec}[c + d x]}{(e \text{Cot}[c + d x])^{3/2}} dx$$

Optimal (type 4, 320 leaves, 17 steps):

$$\frac{2 \cot [c+d x] (3 a+a \operatorname{Sec}[c+d x])}{3 d (e \cot [c+d x])^{3 / 2}}-\frac{a \cot [c+d x] \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \sqrt{\sin [2 c+2 d x]}}{3 d (e \cot [c+d x])^{3 / 2}}+\frac{a \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}-\frac{a \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}+\frac{a \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}-\frac{a \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}$$

Result (type 4, 224 leaves):

$$\frac{1}{12 d (e \cot [c+d x])^{3 / 2} (-1+\cot [c+d x])^2} a (1+\cos [c+d x]) \cos [2(c+d x)] \operatorname{Csc}[c+d x] \sqrt{\operatorname{Csc}[c+d x]^2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \left(-4(-1)^{1 / 4} \cot [c+d x]^{3 / 2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\cot [c+d x]}\right],-1\right]+3 \cot [c+d x] \operatorname{Log}\left[\cos [c+d x]+\sin [c+d x]+\sqrt{\sin [2(c+d x)]}\right] \sqrt{\sin [2(c+d x)]}\right)$$

■ **Problem 238: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \cot [c+d x])^{5 / 2} (a+a \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 357 leaves, 21 steps):

$$\frac{4 a^2 (e \cot [c+d x])^{5 / 2} \tan [c+d x]}{3 d}-\frac{4 a^2 (e \cot [c+d x])^{5 / 2} \operatorname{Sec}[c+d x] \tan [c+d x]}{3 d}+\frac{2 a^2 (e \cot [c+d x])^{5 / 2} \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \operatorname{Sec}[c+d x] \sqrt{\sin [2 c+2 d x]} \tan [c+d x]^2}{3 d}+\frac{a^2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right] (e \cot [c+d x])^{5 / 2} \tan [c+d x]^{5 / 2}}{\sqrt{2} d}-\frac{a^2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right] (e \cot [c+d x])^{5 / 2} \tan [c+d x]^{5 / 2}}{\sqrt{2} d}+\frac{a^2 (e \cot [c+d x])^{5 / 2} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \tan [c+d x]^{5 / 2}}{2 \sqrt{2} d}-\frac{a^2 (e \cot [c+d x])^{5 / 2} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \tan [c+d x]^{5 / 2}}{2 \sqrt{2} d}$$

Result (type 4, 332 leaves):

$$\left(\left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{dx}{2} \right) \right] \right)^2 (e \cot [c + dx])^{5/2} \operatorname{Csc}[c + dx]^2 \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \right. \\ \left. (a + a \operatorname{Sec}[c + dx])^2 \left(- \frac{4 \cot [c + dx]^{3/2} \left(1 + \sqrt{1 + \tan [c + dx]^2} \right)}{3d} - \frac{1}{d} \left(1 / (4\sqrt{2}) \left(2 \operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\cot [c + dx]} \right] - \right. \right. \right. \right. \right. \\ \left. \left. \left. 2 \operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\cot [c + dx]} \right] - \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\cot [c + dx]} + \cot [c + dx] \right] + \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\cot [c + dx]} + \cot [c + dx] \right] \right) \right) + \right. \\ \left. \left. \left. \left. \left. \frac{2 (-1)^{1/4} \cot [c + dx] \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c + dx]} \right], -1 \right] \sqrt{1 + \tan [c + dx]^2}}{3 \sqrt{1 + \cot [c + dx]^2}} \right) \right) \right) \right) \right) \Bigg/ \\ \left(4 \left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \operatorname{ArcCot} [\cot [c + dx]]) \right) \right] \right)^2 \sqrt{\cot [c + dx]} (1 + \cot [c + dx]^2)^2 \right)$$

■ **Problem 239: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e \cot [c + dx])^{3/2} (a + a \operatorname{Sec}[c + dx])^2 dx$$

Optimal (type 4, 343 leaves, 21 steps):

$$\frac{4 a^2 (e \cot [c + dx])^{3/2} \sin [c + dx]}{d} - \frac{4 a^2 (e \cot [c + dx])^{3/2} \tan [c + dx]}{d} - \\ \frac{4 a^2 (e \cot [c + dx])^{3/2} \operatorname{EllipticE} \left[c - \frac{\pi}{4} + dx, 2 \right] \sin [c + dx] \tan [c + dx]}{d \sqrt{\sin [2c + 2dx]}} + \\ \frac{a^2 \operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [c + dx]} \right] (e \cot [c + dx])^{3/2} \tan [c + dx]^{3/2}}{\sqrt{2} d} - \frac{a^2 \operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [c + dx]} \right] (e \cot [c + dx])^{3/2} \tan [c + dx]^{3/2}}{\sqrt{2} d} \\ \frac{a^2 (e \cot [c + dx])^{3/2} \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan [c + dx]} + \tan [c + dx] \right] \tan [c + dx]^{3/2}}{2 \sqrt{2} d} + \\ \frac{a^2 (e \cot [c + dx])^{3/2} \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan [c + dx]} + \tan [c + dx] \right] \tan [c + dx]^{3/2}}{2 \sqrt{2} d}$$

Result (type 4, 410 leaves):

$$\left(\left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{dx}{2} \right) \right] \right)^2 \sqrt{\cot [c+dx]} (e \cot [c+dx])^{3/2} \csc [c+dx]^2 \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a + a \sec [c+dx])^2 \right. \\ \left. - \frac{4 \sqrt{\cot [c+dx]}}{d} - \frac{1}{d} \left(- \frac{\operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2 \sqrt{\cot [c+dx]}}{\sqrt{2}} \right]}{2 \sqrt{2}} - \frac{\operatorname{ArcTan} \left[\frac{\sqrt{2} + 2 \sqrt{\cot [c+dx]}}{\sqrt{2}} \right]}{2 \sqrt{2}} + \frac{\operatorname{Log} \left[1 - \sqrt{2} \sqrt{\cot [c+dx]} + \cot [c+dx] \right]}{4 \sqrt{2}} - \right. \right. \\ \left. \frac{\operatorname{Log} \left[1 + \sqrt{2} \sqrt{\cot [c+dx]} + \cot [c+dx] \right]}{4 \sqrt{2}} - 1 / \left(1 + \cot [c+dx]^2 \right) 2 (-1)^{3/4} \sqrt{1 - i \cot [c+dx]} \sqrt{1 + i \cot [c+dx]} \right. \\ \left. \cot [c+dx] \left(\operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c+dx]} \right], -1 \right] - \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c+dx]} \right], -1 \right] \right) \right. \\ \left. \left. \left. \left. \left. \sqrt{(1 + \cot [c+dx]^2) \tan [c+dx]^2} \right) \right) \right) \right) \right) / \left(4 \left(1 + \cos \left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \operatorname{ArcCot} [\cot [c+dx]]) \right) \right] \right) \right)^2 (1 + \cot [c+dx]^2)^2 \right)$$

■ **Problem 240: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{e \cot [c+dx]} (a + a \sec [c+dx])^2 dx$$

Optimal (type 4, 311 leaves, 19 steps):

$$\frac{2 a^2 \sqrt{e \cot [c+dx]} \operatorname{EllipticF} \left[c - \frac{\pi}{4} + dx, 2 \right] \sec [c+dx] \sqrt{\sin [2c+2dx]} - a^2 \operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [c+dx]} \right] \sqrt{e \cot [c+dx]} \sqrt{\tan [c+dx]}}{d} + \frac{\sqrt{2} d}{\sqrt{2} d} \\ \frac{a^2 \operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [c+dx]} \right] \sqrt{e \cot [c+dx]} \sqrt{\tan [c+dx]} - a^2 \sqrt{e \cot [c+dx]} \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan [c+dx]} + \tan [c+dx] \right] \sqrt{\tan [c+dx]}}{\sqrt{2} d} - \frac{2 \sqrt{2} d}{2 \sqrt{2} d} \\ \frac{a^2 \sqrt{e \cot [c+dx]} \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan [c+dx]} + \tan [c+dx] \right] \sqrt{\tan [c+dx]}}{2 \sqrt{2} d} + \frac{2 a^2 \sqrt{e \cot [c+dx]} \tan [c+dx]}{d}$$

Result (type 4, 284 leaves):

$$\frac{1}{16 d \sqrt{e \cot [c+dx]} \sqrt{\csc [c+dx]^2}} a^2 e (1 + \cos [c+dx])^2 \\ \left(\sqrt{\csc [c+dx]^2} \left(8 + 2 \sqrt{2} \operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\cot [c+dx]} \right] \sqrt{\cot [c+dx]} - 2 \sqrt{2} \operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\cot [c+dx]} \right] \sqrt{\cot [c+dx]} - \right. \right. \\ \left. \left. \sqrt{2} \sqrt{\cot [c+dx]} \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\cot [c+dx]} + \cot [c+dx] \right] + \sqrt{2} \sqrt{\cot [c+dx]} \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\cot [c+dx]} + \cot [c+dx] \right] \right) + \right. \\ \left. 16 (-1)^{1/4} \cot [c+dx]^{3/2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c+dx]} \right], -1 \right] \sqrt{\sec [c+dx]^2} \right) \sec \left[\frac{1}{2} \operatorname{ArcCot} [\cot [c+dx]] \right]^4$$

■ **Problem 241: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2}{\sqrt{e \operatorname{Cot}[c + d x]}} dx$$

Optimal (type 4, 339 leaves, 20 steps):

$$\frac{4 a^2 \operatorname{Sin}[c + d x]}{d \sqrt{e \operatorname{Cot}[c + d x]}} - \frac{4 a^2 \operatorname{Cos}[c + d x] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right]}{d \sqrt{e \operatorname{Cot}[c + d x]} \sqrt{\operatorname{Sin}[2 c + 2 d x]}} - \frac{a^2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} d \sqrt{e \operatorname{Cot}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]}} + \frac{a^2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} d \sqrt{e \operatorname{Cot}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]}} +$$

$$\frac{a^2 \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} d \sqrt{e \operatorname{Cot}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]}} - \frac{a^2 \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} d \sqrt{e \operatorname{Cot}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]}} + \frac{2 a^2 \operatorname{Tan}[c + d x]}{3 d \sqrt{e \operatorname{Cot}[c + d x]}}$$

Result (type 4, 441 leaves):

$$\left(\left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{d x}{2} \right)\right] \right)^2 \operatorname{Cot}[c + d x]^{5/2} \operatorname{Csc}[c + d x]^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 \right.$$

$$\left. \left(-\frac{1}{d} \left(\frac{\operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2 \sqrt{\operatorname{Cot}[c + d x]}}{\sqrt{2}}\right]}{2 \sqrt{2}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} + 2 \sqrt{\operatorname{Cot}[c + d x]}}{\sqrt{2}}\right]}{2 \sqrt{2}} - \frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c + d x]} + \operatorname{Cot}[c + d x]\right]}{4 \sqrt{2}} + \right. \right.$$

$$\frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c + d x]} + \operatorname{Cot}[c + d x]\right]}{4 \sqrt{2}} - 1 / \left((1 + \operatorname{Cot}[c + d x])^2 (-1)^{3/4} \sqrt{1 - i \operatorname{Cot}[c + d x]} \sqrt{1 + i \operatorname{Cot}[c + d x]} \right.$$

$$\left. \left. \operatorname{Cot}[c + d x] \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Cot}[c + d x]}\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Cot}[c + d x]}\right], -1\right] \right) \right.$$

$$\left. \left. \sqrt{(1 + \operatorname{Cot}[c + d x])^2 \operatorname{Tan}[c + d x]^2} - \frac{2 \left(-\frac{1}{3 \operatorname{Cot}[c + d x]^{3/2}} - 2 \sqrt{\operatorname{Cot}[c + d x]} \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right)}{d} \right) \right) /$$

$$\left(4 \left(1 + \operatorname{Cos}\left[2 \left(\frac{c}{2} + \frac{1}{2} (-c + \operatorname{ArcCot}[\operatorname{Cot}[c + d x]]) \right)\right] \right)^2 \sqrt{e \operatorname{Cot}[c + d x]} (1 + \operatorname{Cot}[c + d x])^2 \right)$$

■ **Problem 242: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2}{(e \operatorname{Cot}[c + d x])^{3/2}} dx$$

Optimal (type 4, 375 leaves, 21 steps):

$$\frac{2 a^2 \cot [c+d x]}{d (e \cot [c+d x])^{3 / 2}}+\frac{4 a^2 \csc [c+d x]}{3 d (e \cot [c+d x])^{3 / 2}}-\frac{2 a^2 \cot [c+d x] \csc [c+d x] \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \sqrt{\sin [2 c+2 d x]}}{3 d (e \cot [c+d x])^{3 / 2}}+\frac{a^2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}-\frac{a^2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}+\frac{a^2 \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}-\frac{a^2 \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} d (e \cot [c+d x])^{3 / 2} \tan [c+d x]^{3 / 2}}+\frac{2 a^2 \tan [c+d x]}{5 d (e \cot [c+d x])^{3 / 2}}$$

Result (type 4, 346 leaves):

$$\left(\left(1+\cos \left[2\left(\frac{c}{2}+\frac{d x}{2}\right)\right]\right)^2 \cot [c+d x]^{7 / 2} \csc [c+d x]^2 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\right. \\ \left.(a+a \sec [c+d x])^2\left(-\frac{1}{d}\left(1 / \left(4 \sqrt{2}\right)\left(2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right]-2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right]\right)-\right. \right. \\ \left. \left.\operatorname{Log}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]+\operatorname{Log}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]\right)+\right. \\ \left.\frac{2(-1)^{1 / 4} \cot [c+d x] \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\cot [c+d x]}\right],-1\right] \sqrt{1+\tan [c+d x]^2}}{3 \sqrt{1+\cot [c+d x]^2}}\right)+ \\ \left.\frac{2\left(3+5 \cot [c+d x]^2\left(3+2 \sqrt{1+\tan [c+d x]^2}\right)\right)}{15 d \cot [c+d x]^{5 / 2}}\right) \left/\left(4\left(1+\cos \left[2\left(\frac{c}{2}+\frac{1}{2}(-c+\operatorname{ArcCot}[\cot [c+d x]])\right)\right]\right)\right)^2\right. \\ \left.\left.(e \cot [c+d x])^{3 / 2}\left(1+\cot [c+d x]^2\right)^2\right)\right)$$

■ **Problem 243: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(e \cot [c+d x])^{3 / 2}}{a+a \sec [c+d x]} d x$$

Optimal (type 4, 405 leaves, 20 steps):

$$\begin{aligned}
& \frac{2 \operatorname{Cot}[c+dx] (e \operatorname{Cot}[c+dx])^{3/2} (1 - \operatorname{Sec}[c+dx])}{5 a d} - \frac{2 (e \operatorname{Cot}[c+dx])^{3/2} (5 - 3 \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]}{5 a d} + \\
& \frac{6 (e \operatorname{Cot}[c+dx])^{3/2} \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sin}[c+dx] \operatorname{Tan}[c+dx]}{5 a d \sqrt{\operatorname{Sin}[2c+2dx]}} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] (e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Tan}[c+dx]^{3/2}}{\sqrt{2} a d} - \\
& \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]}\right] (e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Tan}[c+dx]^{3/2}}{\sqrt{2} a d} - \frac{(e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \operatorname{Tan}[c+dx]^{3/2}}{2 \sqrt{2} a d} + \\
& \frac{(e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c+dx]} + \operatorname{Tan}[c+dx]\right] \operatorname{Tan}[c+dx]^{3/2}}{2 \sqrt{2} a d} - \frac{6 (e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Sin}[c+dx] \operatorname{Tan}[c+dx]^2}{5 a d}
\end{aligned}$$

Result (type 4, 424 leaves):

$$\begin{aligned}
& \left(2 \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (e \operatorname{Cot}[c+dx])^{3/2} \operatorname{Csc}[c+dx] \operatorname{Sec}[c+dx] \left(1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right) \right. \\
& \left. - \frac{1}{d} \left(- \frac{\operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2\sqrt{\operatorname{Cot}[c+dx]}}{\sqrt{2}}\right]}{2\sqrt{2}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} + 2\sqrt{\operatorname{Cot}[c+dx]}}{\sqrt{2}}\right]}{2\sqrt{2}} + \frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Cot}[c+dx]} + \operatorname{Cot}[c+dx]\right]}{4\sqrt{2}} - \right. \right. \\
& \left. \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Cot}[c+dx]} + \operatorname{Cot}[c+dx]\right]}{4\sqrt{2}} + 1 / \left(5 \left(1 + \operatorname{Cot}[c+dx]^2\right)\right)^3 (-1)^{3/4} \sqrt{1 - i \operatorname{Cot}[c+dx]} \sqrt{1 + i \operatorname{Cot}[c+dx]} \right. \\
& \left. \left. \operatorname{Cot}[c+dx] \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Cot}[c+dx]}\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\operatorname{Cot}[c+dx]}\right], -1\right]\right) \right. \right. \\
& \left. \left. \sqrt{\left(1 + \operatorname{Cot}[c+dx]^2\right) \operatorname{Tan}[c+dx]^2} - \frac{2 \left(\sqrt{\operatorname{Cot}[c+dx]} + \frac{1}{5} \operatorname{Cot}[c+dx]^{5/2} \left(-1 + \sqrt{1 + \operatorname{Tan}[c+dx]^2}\right)\right)}{d} \right) \right) / \\
& \left((1 + \operatorname{Cos}[c+dx]) \sqrt{\operatorname{Cot}[c+dx]} (1 + \operatorname{Cot}[c+dx]^2) (a + a \operatorname{Sec}[c+dx]) \right)
\end{aligned}$$

■ **Problem 244: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{e \operatorname{Cot}[c+dx]}}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 325 leaves, 18 steps):

$$\frac{2 \cot [c+d x] \sqrt{e \cot [c+d x]} (1-\sec [c+d x])}{3 a d}-\frac{\sqrt{e \cot [c+d x]} \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \sec [c+d x] \sqrt{\sin [2 c+2 d x]}}{3 a d}-$$

$$\frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right] \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}{\sqrt{2} a d}+\frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right] \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}{\sqrt{2} a d}-$$

$$\frac{\sqrt{e \cot [c+d x]} \operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \sqrt{\tan [c+d x]}}{2 \sqrt{2} a d}+$$

$$\frac{\sqrt{e \cot [c+d x]} \operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right] \sqrt{\tan [c+d x]}}{2 \sqrt{2} a d}$$

Result (type 4, 313 leaves):

$$\frac{1}{(1+\cos [c+d x])\left(1+\cot [c+d x]^2\right)(a+a \sec [c+d x])}$$

$$2 \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^2 \sqrt{\cot [c+d x]} \sqrt{e \cot [c+d x]} \csc [c+d x] \sec [c+d x]\left(1+\sqrt{1+\tan [c+d x]^2}\right)\left(-\frac{2 \cot [c+d x]^{3 / 2}\left(-1+\sqrt{1+\tan [c+d x]^2}\right)}{3 d}-\frac{1}{d}\right.$$

$$2\left(\frac{1}{4 \sqrt{2}}\left(-2 \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right]+2 \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right]+\operatorname{Log}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]-\operatorname{Log}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]\right)+\frac{(-1)^{1 / 4} \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[(-1)^{1 / 4} \sqrt{\cot [c+d x]}\right],-1\right] \sqrt{1+\tan [c+d x]^2}}{3 \sqrt{1+\cot [c+d x]^2}}\left.\right)$$

■ **Problem 245: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{e \cot [c+d x]}(a+a \sec [c+d x])} d x$$

Optimal (type 4, 347 leaves, 19 steps):

$$\frac{2 \cot [c+d x](1-\sec [c+d x])}{a d \sqrt{e \cot [c+d x]}}+\frac{2 \sin [c+d x]}{a d \sqrt{e \cot [c+d x]}}-\frac{2 \cos [c+d x] \operatorname{EllipticE}\left[c-\frac{\pi}{4}+d x, 2\right]}{a d \sqrt{e \cot [c+d x]} \sqrt{\sin [2 c+2 d x]}}-\frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}+$$

$$\frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}+\frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}-\frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}$$

Result (type 4, 310 leaves):

$$\frac{1}{4 a d \sqrt{e \cot [c+d x]}}$$

$$\cot [c+d x]^{3/2} \left(1 + \sqrt{\sec [c+d x]^2} \right) \left(2 \sqrt{2} \operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\cot [c+d x]} \right] - 2 \sqrt{2} \operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\cot [c+d x]} \right] + 8 \sqrt{\cot [c+d x]} + \right.$$

$$\left. \sqrt{2} \operatorname{Log} \left[1 - \sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x] \right] - \sqrt{2} \operatorname{Log} \left[1 + \sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x] \right] + \right.$$

$$4 (-1)^{3/4} \sqrt{\csc [c+d x]^2} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]^2} \sin [2 (c+d x)] -$$

$$4 (-1)^{3/4} \sqrt{\csc [c+d x]^2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[(-1)^{1/4} \sqrt{\cot [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]^2} \sin [2 (c+d x)] \left. \right) \operatorname{Tan} \left[\frac{1}{2} (c+d x) \right]$$

■ **Problem 246: Unable to integrate problem.**

$$\int \frac{1}{(e \cot [c+d x])^{3/2} (a + a \sec [c+d x])} dx$$

Optimal (type 4, 290 leaves, 17 steps):

$$\frac{\cot [c+d x] \operatorname{Csc} [c+d x] \operatorname{EllipticF} \left[c - \frac{\pi}{4} + d x, 2 \right] \sqrt{\sin [2 c+2 d x]}}{a d (e \cot [c+d x])^{3/2}} + \frac{\operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [c+d x]} \right]}{\sqrt{2} a d (e \cot [c+d x])^{3/2} \operatorname{Tan} [c+d x]^{3/2}} -$$

$$\frac{\operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [c+d x]} \right]}{\sqrt{2} a d (e \cot [c+d x])^{3/2} \operatorname{Tan} [c+d x]^{3/2}} + \frac{\operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x] \right]}{2 \sqrt{2} a d (e \cot [c+d x])^{3/2} \operatorname{Tan} [c+d x]^{3/2}} - \frac{\operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x] \right]}{2 \sqrt{2} a d (e \cot [c+d x])^{3/2} \operatorname{Tan} [c+d x]^{3/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \cot [c+d x])^{3/2} (a + a \sec [c+d x])} dx$$

■ **Problem 247: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(e \cot [c+d x])^{5/2} (a + a \sec [c+d x])} dx$$

Optimal (type 4, 325 leaves, 18 steps):

$$\frac{2 \cos [c+d x] \cot [c+d x]}{a d (e \cot [c+d x])^{5/2}} - \frac{2 \cos [c+d x] \cot [c+d x]^2 \operatorname{EllipticE} \left[c - \frac{\pi}{4} + d x, 2 \right]}{a d (e \cot [c+d x])^{5/2} \sqrt{\sin [2 c+2 d x]}} + \frac{\operatorname{ArcTan} \left[1 - \sqrt{2} \sqrt{\tan [c+d x]} \right]}{\sqrt{2} a d (e \cot [c+d x])^{5/2} \operatorname{Tan} [c+d x]^{5/2}} -$$

$$\frac{\operatorname{ArcTan} \left[1 + \sqrt{2} \sqrt{\tan [c+d x]} \right]}{\sqrt{2} a d (e \cot [c+d x])^{5/2} \operatorname{Tan} [c+d x]^{5/2}} - \frac{\operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x] \right]}{2 \sqrt{2} a d (e \cot [c+d x])^{5/2} \operatorname{Tan} [c+d x]^{5/2}} + \frac{\operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x] \right]}{2 \sqrt{2} a d (e \cot [c+d x])^{5/2} \operatorname{Tan} [c+d x]^{5/2}}$$

Result (type 4, 324 leaves):

$$\frac{1}{2\sqrt{2}ade^3} \sqrt{\cot[c+dx]} \sqrt{e \cot[c+dx]} \left(1 + \sqrt{\sec[c+dx]^2}\right) \left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]}\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]}\right]\right) -$$

$$\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] + \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] + 4\sqrt{2} \sqrt{\cot[c+dx]} \sqrt{\sec[c+dx]^2} +$$

$$2(-1)^{3/4} \sqrt{2} \sqrt{\csc[c+dx]^2} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]^2} \sin[2(c+dx)] -$$

$$2(-1)^{3/4} \sqrt{2} \sqrt{\csc[c+dx]^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]^2} \sin[2(c+dx)] \left)\tan\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 248: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(e \cot[c+dx])^{7/2} (a + a \sec[c+dx])} dx$$

Optimal (type 4, 335 leaves, 18 steps):

$$\frac{2 \cot[c+dx]^3 (3 - \sec[c+dx])}{3ad(e \cot[c+dx])^{7/2}} - \frac{\cot[c+dx]^3 \csc[c+dx] \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{\sin[2c+2dx]}}{3ad(e \cot[c+dx])^{7/2}} -$$

$$\frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[c+dx]}\right]}{\sqrt{2} ad(e \cot[c+dx])^{7/2} \tan[c+dx]^{7/2}} + \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[c+dx]}\right]}{\sqrt{2} ad(e \cot[c+dx])^{7/2} \tan[c+dx]^{7/2}} -$$

$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]\right]}{2\sqrt{2} ad(e \cot[c+dx])^{7/2} \tan[c+dx]^{7/2}} + \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]\right]}{2\sqrt{2} ad(e \cot[c+dx])^{7/2} \tan[c+dx]^{7/2}}$$

Result (type 4, 313 leaves):

$$\left(2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cot[c+dx]^{9/2} \csc[c+dx] \sec[c+dx]\right.$$

$$\left.\left(\left(1 + \sqrt{1 + \tan[c+dx]^2}\right) \left(\frac{2\left(-3 + \sqrt{1 + \tan[c+dx]^2}\right)}{3d\sqrt{\cot[c+dx]}} - 1/d2 \left(1/\left(4\sqrt{2}\right)\left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]}\right] +\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]}\right] + \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\cot[c+dx]} + \cot[c+dx]\right]\right)\right) +\right.$$

$$\left.\left.\left.\left.\frac{(-1)^{1/4} \cot[c+dx] \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot[c+dx]}\right], -1\right] \sqrt{1 + \tan[c+dx]^2}}{3\sqrt{1 + \cot[c+dx]^2}}\right)\right)\right)\right)/$$

$$\left((1 + \cos[c+dx]) (e \cot[c+dx])^{7/2} (1 + \cot[c+dx]^2) (a + a \sec[c+dx])\right)$$

■ **Problem 249: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(e \cot [c+d x])^{9/2} (a+a \sec [c+d x])} dx$$

Optimal (type 4, 371 leaves, 19 steps):

$$\begin{aligned} & -\frac{6 \cos [c+d x] \cot [c+d x]^3}{5 a d (e \cot [c+d x])^{9/2}} - \frac{2 \cot [c+d x]^3 (5-3 \sec [c+d x])}{15 a d (e \cot [c+d x])^{9/2}} + \\ & \frac{6 \cos [c+d x] \cot [c+d x]^4 \operatorname{EllipticE}\left[c-\frac{\pi}{4}+d x, 2\right]}{5 a d (e \cot [c+d x])^{9/2} \sqrt{\sin [2 c+2 d x]}} - \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a d (e \cot [c+d x])^{9/2} \tan [c+d x]^{9/2}} + \\ & \frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a d (e \cot [c+d x])^{9/2} \tan [c+d x]^{9/2}} + \frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a d (e \cot [c+d x])^{9/2} \tan [c+d x]^{9/2}} - \frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a d (e \cot [c+d x])^{9/2} \tan [c+d x]^{9/2}} \end{aligned}$$

Result (type 4, 425 leaves):

$$\begin{aligned} & \left(2 \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cot [c+d x]^{11/2} \operatorname{Csc}[c+d x] \sec [c+d x] \left(1 + \sqrt{1 + \tan [c+d x]^2} \right) \right. \\ & \left. - \frac{1}{d} \left(\frac{\operatorname{ArcTan}\left[\frac{-\sqrt{2}+2 \sqrt{\cot [c+d x]}}{\sqrt{2}}\right]}{2 \sqrt{2}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2}+2 \sqrt{\cot [c+d x]}}{\sqrt{2}}\right]}{2 \sqrt{2}} - \frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]}{4 \sqrt{2}} \right. \right. \\ & \left. \left. + \frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]}{4 \sqrt{2}} + 1 / \left(5 \left(1 + \cot [c+d x]^2 \right) \right)^{3/4} \sqrt{1-i \cot [c+d x]} \sqrt{1+i \cot [c+d x]} \right. \right. \\ & \left. \left. \cot [c+d x] \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot [c+d x]}\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\cot [c+d x]}\right], -1\right] \right) \right. \right. \\ & \left. \left. \sqrt{\left(1 + \cot [c+d x]^2 \right) \tan [c+d x]^2} - \frac{2 \left(5 + 3 \left(-1 + 3 \cot [c+d x]^2 \right) \sqrt{1 + \tan [c+d x]^2} \right)}{15 d \cot [c+d x]^{3/2}} \right) \right) / \\ & \left((1 + \cos [c+d x]) (e \cot [c+d x])^{9/2} (1 + \cot [c+d x]^2) (a + a \sec [c+d x]) \right) \end{aligned}$$

■ **Problem 250: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{e \cot [c+d x]} (a+a \sec [c+d x])^2} dx$$

Optimal (type 4, 413 leaves, 24 steps):

$$\frac{2 \cot [c+d x]}{a^2 d \sqrt{e \cot [c+d x]}} - \frac{12 \cos [c+d x] \cot [c+d x]}{5 a^2 d \sqrt{e \cot [c+d x]}} - \frac{4 \cot [c+d x]^3}{5 a^2 d \sqrt{e \cot [c+d x]}} +$$

$$\frac{4 \cot [c+d x]^2 \csc [c+d x]}{5 a^2 d \sqrt{e \cot [c+d x]}} - \frac{12 \cos [c+d x] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right]}{5 a^2 d \sqrt{e \cot [c+d x]} \sqrt{\sin [2 c+2 d x]}} - \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}} +$$

$$\frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}} + \frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x]\right]}{2 \sqrt{2} a^2 d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x]\right]}{2 \sqrt{2} a^2 d \sqrt{e \cot [c+d x]} \sqrt{\tan [c+d x]}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{\sqrt{e \cot [c+d x]} (a + a \sec [c+d x])^2} dx$$

■ **Problem 251: Unable to integrate problem.**

$$\int \frac{1}{(e \cot [c+d x])^{3/2} (a + a \sec [c+d x])^2} dx$$

Optimal (type 4, 359 leaves, 22 steps):

$$-\frac{4 \cot [c+d x]^3}{3 a^2 d (e \cot [c+d x])^{3/2}} + \frac{4 \cot [c+d x]^2 \csc [c+d x]}{3 a^2 d (e \cot [c+d x])^{3/2}} + \frac{2 \cot [c+d x] \csc [c+d x] \operatorname{EllipticF}\left[c - \frac{\pi}{4} + d x, 2\right] \sqrt{\sin [2 c+2 d x]}}{3 a^2 d (e \cot [c+d x])^{3/2}} +$$

$$\frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}} - \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}} +$$

$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan [c+d x]} + \tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{3/2} \tan [c+d x]^{3/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \cot [c+d x])^{3/2} (a + a \sec [c+d x])^2} dx$$

■ **Problem 252: Unable to integrate problem.**

$$\int \frac{1}{(e \cot [c+d x])^{5/2} (a + a \sec [c+d x])^2} dx$$

Optimal (type 4, 355 leaves, 22 steps):

$$\begin{aligned}
& - \frac{4 \cot [c+d x]^3}{a^2 d (e \cot [c+d x])^{5/2}} + \frac{4 \cos [c+d x] \cot [c+d x]^3}{a^2 d (e \cot [c+d x])^{5/2}} + \frac{4 \cos [c+d x] \cot [c+d x]^2 \operatorname{EllipticE}\left[c-\frac{\pi}{4}+d x, 2\right]}{a^2 d (e \cot [c+d x])^{5/2} \sqrt{\sin [2 c+2 d x]}} + \\
& \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{5/2} \tan [c+d x]^{5/2}} - \frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{5/2} \tan [c+d x]^{5/2}} - \\
& \frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{5/2} \tan [c+d x]^{5/2}} + \frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{5/2} \tan [c+d x]^{5/2}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \cot [c+d x])^{5/2} (a+a \sec [c+d x])^2} dx$$

■ **Problem 253: Unable to integrate problem.**

$$\int \frac{1}{(e \cot [c+d x])^{7/2} (a+a \sec [c+d x])^2} dx$$

Optimal (type 4, 321 leaves, 20 steps):

$$\begin{aligned}
& \frac{2 \cot [c+d x]^3}{a^2 d (e \cot [c+d x])^{7/2}} - \frac{2 \cot [c+d x]^3 \operatorname{Csc}[c+d x] \operatorname{EllipticF}\left[c-\frac{\pi}{4}+d x, 2\right] \sqrt{\sin [2 c+2 d x]}}{a^2 d (e \cot [c+d x])^{7/2}} - \frac{\operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{7/2} \tan [c+d x]^{7/2}} + \\
& \frac{\operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}\right]}{\sqrt{2} a^2 d (e \cot [c+d x])^{7/2} \tan [c+d x]^{7/2}} - \frac{\operatorname{Log}\left[1-\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{7/2} \tan [c+d x]^{7/2}} + \frac{\operatorname{Log}\left[1+\sqrt{2} \sqrt{\tan [c+d x]}+\tan [c+d x]\right]}{2 \sqrt{2} a^2 d (e \cot [c+d x])^{7/2} \tan [c+d x]^{7/2}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \cot [c+d x])^{7/2} (a+a \sec [c+d x])^2} dx$$

■ **Problem 254: Unable to integrate problem.**

$$\int \frac{1}{(e \cot [c+d x])^{9/2} (a+a \sec [c+d x])^2} dx$$

Optimal (type 4, 357 leaves, 21 steps):

$$\frac{2 \operatorname{Cot}[c + d x]^3}{3 a^2 d (e \operatorname{Cot}[c + d x])^{9/2}} - \frac{4 \operatorname{Cos}[c + d x] \operatorname{Cot}[c + d x]^3}{a^2 d (e \operatorname{Cot}[c + d x])^{9/2}} + \frac{4 \operatorname{Cos}[c + d x] \operatorname{Cot}[c + d x]^4 \operatorname{EllipticE}\left[c - \frac{\pi}{4} + d x, 2\right]}{a^2 d (e \operatorname{Cot}[c + d x])^{9/2} \sqrt{\operatorname{Sin}[2 c + 2 d x]}} -$$

$$\frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{9/2} \operatorname{Tan}[c + d x]^{9/2}} + \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{9/2} \operatorname{Tan}[c + d x]^{9/2}} +$$

$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{9/2} \operatorname{Tan}[c + d x]^{9/2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{9/2} \operatorname{Tan}[c + d x]^{9/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \operatorname{Cot}[c + d x])^{9/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

■ **Problem 255: Unable to integrate problem.**

$$\int \frac{1}{(e \operatorname{Cot}[c + d x])^{11/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 389 leaves, 22 steps):

$$\frac{2 \operatorname{Cot}[c + d x]^3}{5 a^2 d (e \operatorname{Cot}[c + d x])^{11/2}} + \frac{2 \operatorname{Cot}[c + d x]^5}{a^2 d (e \operatorname{Cot}[c + d x])^{11/2}} - \frac{4 \operatorname{Cot}[c + d x]^4 \operatorname{Csc}[c + d x]}{3 a^2 d (e \operatorname{Cot}[c + d x])^{11/2}} +$$

$$\frac{2 \operatorname{Cot}[c + d x]^5 \operatorname{Csc}[c + d x] \operatorname{EllipticF}\left[c - \frac{\pi}{4} + d x, 2\right] \sqrt{\operatorname{Sin}[2 c + 2 d x]}}{3 a^2 d (e \operatorname{Cot}[c + d x])^{11/2}} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{11/2} \operatorname{Tan}[c + d x]^{11/2}} -$$

$$\frac{\operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right]}{\sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{11/2} \operatorname{Tan}[c + d x]^{11/2}} + \frac{\operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{11/2} \operatorname{Tan}[c + d x]^{11/2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2 d (e \operatorname{Cot}[c + d x])^{11/2} \operatorname{Tan}[c + d x]^{11/2}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(e \operatorname{Cot}[c + d x])^{11/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

■ **Problem 265: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^4 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a x + \frac{3 b \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} - \frac{(8 a + 3 b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{8 d} + \frac{(4 a + 3 b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^3}{12 d}$$

Result (type 3, 230 leaves):

$$\begin{aligned}
& a x - \frac{3 b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\
& \frac{b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{5 b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\
& \frac{5 b}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{4 a \operatorname{Tan}[c+d x]}{3 d} + \frac{a \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 d}
\end{aligned}$$

■ **Problem 266: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^2 dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$-a x - \frac{b \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{(2 a + b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 142 leaves):

$$\begin{aligned}
& -a x + \frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \\
& \frac{b}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right] - \sin\left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{b}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right] + \sin\left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{a \operatorname{Tan}[c + d x]}{d}
\end{aligned}$$

■ **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^4 (a + b \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$a x - \frac{\operatorname{Cot}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])}{3 d} + \frac{\operatorname{Cot}[c + d x] (3 a + 2 b \operatorname{Sec}[c + d x])}{3 d}$$

Result (type 3, 136 leaves):

$$\begin{aligned}
& a x + \frac{5 b \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right]}{12 d} + \frac{4 a \operatorname{Cot}[c+d x]}{3 d} - \frac{b \operatorname{Cot}\left[\frac{1}{2}(c+d x)\right] \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2}{24 d} - \\
& \frac{a \operatorname{Cot}[c+d x] \operatorname{Csc}[c+d x]^2}{3 d} + \frac{5 b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{12 d} - \frac{b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{24 d}
\end{aligned}$$

■ **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[c + d x]^6 (a + b \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$-a x - \frac{\text{Cot}[c + d x]^5 (a + b \text{Sec}[c + d x])}{5 d} + \frac{\text{Cot}[c + d x]^3 (5 a + 4 b \text{Sec}[c + d x])}{15 d} - \frac{\text{Cot}[c + d x] (15 a + 8 b \text{Sec}[c + d x])}{15 d}$$

Result (type 3, 219 leaves):

$$-a x - \frac{89 b \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{240 d} - \frac{23 a \text{Cot}[c + d x]}{15 d} + \frac{31 b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{480 d} -$$

$$\frac{b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{160 d} + \frac{11 a \text{Cot}[c + d x] \text{Csc}[c + d x]^2}{15 d} - \frac{a \text{Cot}[c + d x] \text{Csc}[c + d x]^4}{5 d} -$$

$$\frac{89 b \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{240 d} + \frac{31 b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{480 d} - \frac{b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{160 d}$$

■ **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^8 (a + b \text{Sec}[c + d x]) dx$$

Optimal (type 3, 111 leaves, 5 steps):

$$a x - \frac{\text{Cot}[c + d x]^7 (a + b \text{Sec}[c + d x])}{7 d} + \frac{\text{Cot}[c + d x]^5 (7 a + 6 b \text{Sec}[c + d x])}{35 d} +$$

$$\frac{\text{Cot}[c + d x] (35 a + 16 b \text{Sec}[c + d x])}{35 d} - \frac{\text{Cot}[c + d x]^3 (35 a + 24 b \text{Sec}[c + d x])}{105 d}$$

Result (type 3, 300 leaves):

$$a x + \frac{381 b \text{Cot}\left[\frac{1}{2}(c + d x)\right]}{1120 d} + \frac{176 a \text{Cot}[c + d x]}{105 d} - \frac{179 b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^2}{2240 d} + \frac{b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^4}{70 d} -$$

$$\frac{b \text{Cot}\left[\frac{1}{2}(c + d x)\right] \text{Csc}\left[\frac{1}{2}(c + d x)\right]^6}{896 d} - \frac{122 a \text{Cot}[c + d x] \text{Csc}[c + d x]^2}{105 d} + \frac{22 a \text{Cot}[c + d x] \text{Csc}[c + d x]^4}{35 d} - \frac{a \text{Cot}[c + d x] \text{Csc}[c + d x]^6}{7 d} +$$

$$\frac{381 b \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{1120 d} - \frac{179 b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{2240 d} + \frac{b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{70 d} - \frac{b \text{Sec}\left[\frac{1}{2}(c + d x)\right]^6 \text{Tan}\left[\frac{1}{2}(c + d x)\right]}{896 d}$$

■ **Problem 278: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + d x]^5 (a + b \text{Sec}[c + d x])^2 dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{a^2 \text{Log}[\text{Cos}[c + d x]]}{d} + \frac{a (4 a + 3 b) \text{Log}[1 - \text{Sec}[c + d x]]}{8 d} + \frac{a (4 a - 3 b) \text{Log}[1 + \text{Sec}[c + d x]]}{8 d} +$$

$$\frac{a \text{Cot}[c + d x]^2 (2 a + 3 b \text{Sec}[c + d x])}{4 d} - \frac{\text{Cot}[c + d x]^4 (a^2 + b^2 + 2 a b \text{Sec}[c + d x])}{4 d}$$

Result (type 3, 385 leaves) :

$$\begin{aligned} & \frac{(7a^2 + 10ab + 3b^2) \cos[c + dx]^2 \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Sec}[c + dx])^2}{32d (b + a \cos[c + dx])^2} + \frac{(-a^2 - 2ab - b^2) \cos[c + dx]^2 \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right]^4 (a + b \operatorname{Sec}[c + dx])^2}{64d (b + a \cos[c + dx])^2} + \\ & \frac{(4a^2 - 3ab) \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Sec}[c + dx])^2}{4d (b + a \cos[c + dx])^2} + \frac{(4a^2 + 3ab) \cos[c + dx]^2 \operatorname{Log}\left[\sin\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Sec}[c + dx])^2}{4d (b + a \cos[c + dx])^2} + \\ & \frac{(7a^2 - 10ab + 3b^2) \cos[c + dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a + b \operatorname{Sec}[c + dx])^2}{32d (b + a \cos[c + dx])^2} + \frac{(-a^2 + 2ab - b^2) \cos[c + dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a + b \operatorname{Sec}[c + dx])^2}{64d (b + a \cos[c + dx])^2} \end{aligned}$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]^4 dx$$

Optimal (type 3, 116 leaves, 10 steps) :

$$\begin{aligned} & a^2 x + \frac{3ab \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{4d} - \frac{a^2 \operatorname{Tan}[c + dx]}{d} - \\ & \frac{3ab \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{4d} + \frac{a^2 \operatorname{Tan}[c + dx]^3}{3d} + \frac{ab \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]^3}{2d} + \frac{b^2 \operatorname{Tan}[c + dx]^5}{5d} \end{aligned}$$

Result (type 3, 355 leaves) :

$$\begin{aligned} & \frac{1}{960d} \operatorname{Sec}[c + dx]^5 \left(60a^2 c \cos[5(c + dx)] + 60a^2 dx \cos[5(c + dx)] - \right. \\ & 45ab \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 45ab \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \\ & 150a \cos[c + dx] \left(4a(c + dx) - 3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \\ & 75a \cos[3(c + dx)] \left(4a(c + dx) - 3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \\ & 80a^2 \sin[c + dx] + 120b^2 \sin[c + dx] - 60ab \sin[2(c + dx)] - 160a^2 \sin[3(c + dx)] - \\ & \left. 60b^2 \sin[3(c + dx)] - 150ab \sin[4(c + dx)] - 80a^2 \sin[5(c + dx)] + 12b^2 \sin[5(c + dx)] \right) \end{aligned}$$

■ **Problem 281: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]^2 dx$$

Optimal (type 3, 70 leaves, 8 steps) :

$$-a^2 x - \frac{ab \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{d} + \frac{a^2 \operatorname{Tan}[c + dx]}{d} + \frac{ab \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{d} + \frac{b^2 \operatorname{Tan}[c + dx]^3}{3d}$$

Result (type 3, 201 leaves) :

$$\frac{1}{12d} \operatorname{Sec}[c+dx]^3 \left(-9a \operatorname{Cos}[c+dx] \left(a(c+dx) - b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c+dx) \right] \right] + b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c+dx) \right] \right] \right) - \right. \\ \left. 3a \operatorname{Cos}[3(c+dx)] \left(a(c+dx) - b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2}(c+dx) \right] \right] + b \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2}(c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2}(c+dx) \right] \right] \right) \right) + \\ \left. 2(3a^2 + b^2 + 6ab \operatorname{Cos}[c+dx] + (3a^2 - b^2) \operatorname{Cos}[2(c+dx)]) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 286: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^9}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 250 leaves, 3 steps):

$$-\frac{\operatorname{Log}[\operatorname{Cos}[c+dx]]}{ad} - \frac{(a^2-b^2)^4 \operatorname{Log}[a+b \operatorname{Sec}[c+dx]]}{ab^8d} + \frac{(a^6-4a^4b^2+6a^2b^4-4b^6) \operatorname{Sec}[c+dx]}{b^7d} - \frac{a(a^4-4a^2b^2+6b^4) \operatorname{Sec}[c+dx]^2}{2b^6d} + \\ \frac{(a^4-4a^2b^2+6b^4) \operatorname{Sec}[c+dx]^3}{3b^5d} - \frac{a(a^2-4b^2) \operatorname{Sec}[c+dx]^4}{4b^4d} + \frac{(a^2-4b^2) \operatorname{Sec}[c+dx]^5}{5b^3d} - \frac{a \operatorname{Sec}[c+dx]^6}{6b^2d} + \frac{\operatorname{Sec}[c+dx]^7}{7bd}$$

Result (type 3, 520 leaves):

$$\frac{(a^7-4a^5b^2+6a^3b^4-4ab^6)(b+a \operatorname{Cos}[c+dx]) \operatorname{Log}[\operatorname{Cos}[c+dx]] \operatorname{Sec}[c+dx]}{b^8d(a+b \operatorname{Sec}[c+dx])} + \\ \frac{(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8)(b+a \operatorname{Cos}[c+dx]) \operatorname{Log}[b+a \operatorname{Cos}[c+dx]] \operatorname{Sec}[c+dx]}{ab^8d(a+b \operatorname{Sec}[c+dx])} - \\ \frac{(-a^2+2b^2)(a^4-2a^2b^2+2b^4)(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^2}{b^7d(a+b \operatorname{Sec}[c+dx])} - \frac{a(a^4-4a^2b^2+6b^4)(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^3}{2b^6d(a+b \operatorname{Sec}[c+dx])} + \\ \frac{(a^4-4a^2b^2+6b^4)(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^4}{3b^5d(a+b \operatorname{Sec}[c+dx])} + \frac{a(-a+2b)(a+2b)(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^5}{4b^4d(a+b \operatorname{Sec}[c+dx])} - \\ \frac{(-a+2b)(a+2b)(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^6}{5b^3d(a+b \operatorname{Sec}[c+dx])} - \frac{a(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^7}{6b^2d(a+b \operatorname{Sec}[c+dx])} + \frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^8}{7bd(a+b \operatorname{Sec}[c+dx])}$$

■ **Problem 287: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[c+dx]^7}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 170 leaves, 3 steps):

$$\frac{\text{Log}[\text{Cos}[c + dx]]}{ad} - \frac{(a^2 - b^2)^3 \text{Log}[a + b \text{Sec}[c + dx]]}{ab^6d} + \frac{(a^4 - 3a^2b^2 + 3b^4) \text{Sec}[c + dx]}{b^5d} -$$

$$\frac{a(a^2 - 3b^2) \text{Sec}[c + dx]^2}{2b^4d} + \frac{(a^2 - 3b^2) \text{Sec}[c + dx]^3}{3b^3d} - \frac{a \text{Sec}[c + dx]^4}{4b^2d} + \frac{\text{Sec}[c + dx]^5}{5bd}$$

Result (type 3, 371 leaves):

$$\frac{(a^5 - 3a^3b^2 + 3ab^4)(b + a \text{Cos}[c + dx]) \text{Log}[\text{Cos}[c + dx]] \text{Sec}[c + dx]}{b^6d(a + b \text{Sec}[c + dx])} +$$

$$\frac{(-a^6 + 3a^4b^2 - 3a^2b^4 + b^6)(b + a \text{Cos}[c + dx]) \text{Log}[b + a \text{Cos}[c + dx]] \text{Sec}[c + dx]}{ab^6d(a + b \text{Sec}[c + dx])} +$$

$$\frac{(a^4 - 3a^2b^2 + 3b^4)(b + a \text{Cos}[c + dx]) \text{Sec}[c + dx]^2}{b^5d(a + b \text{Sec}[c + dx])} + \frac{a(-a^2 + 3b^2)(b + a \text{Cos}[c + dx]) \text{Sec}[c + dx]^3}{2b^4d(a + b \text{Sec}[c + dx])} +$$

$$\frac{(a^2 - 3b^2)(b + a \text{Cos}[c + dx]) \text{Sec}[c + dx]^4}{3b^3d(a + b \text{Sec}[c + dx])} - \frac{a(b + a \text{Cos}[c + dx]) \text{Sec}[c + dx]^5}{4b^2d(a + b \text{Sec}[c + dx])} + \frac{(b + a \text{Cos}[c + dx]) \text{Sec}[c + dx]^6}{5bd(a + b \text{Sec}[c + dx])}$$

■ **Problem 293: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + dx]^5}{a + b \text{Sec}[c + dx]} dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$\frac{\text{Log}[\text{Cos}[c + dx]]}{ad} + \frac{(8a^2 + 21ab + 15b^2) \text{Log}[1 - \text{Sec}[c + dx]]}{16(a + b)^3d} + \frac{(8a^2 - 21ab + 15b^2) \text{Log}[1 + \text{Sec}[c + dx]]}{16(a - b)^3d} - \frac{b^6 \text{Log}[a + b \text{Sec}[c + dx]]}{a(a^2 - b^2)^3d} -$$

$$\frac{1}{16(a + b)d(1 - \text{Sec}[c + dx])^2} - \frac{5a + 7b}{16(a + b)^2d(1 - \text{Sec}[c + dx])} - \frac{1}{16(a - b)d(1 + \text{Sec}[c + dx])^2} - \frac{5a - 7b}{16(a - b)^2d(1 + \text{Sec}[c + dx])}$$

Result (type 3, 625 leaves):

$$\begin{aligned}
& \frac{2i(a^5 - 3a^3b^2 + 3ab^4)(c+dx)(b+a\cos[c+dx])\sec[c+dx]}{(a-b)^3(a+b)^3d(a+b\sec[c+dx])} - \frac{i(-8a^2 + 21ab - 15b^2)\operatorname{ArcTan}[\tan[c+dx]](b+a\cos[c+dx])\sec[c+dx]}{8(-a+b)^3d(a+b\sec[c+dx])} - \\
& \frac{i(8a^2 + 21ab + 15b^2)\operatorname{ArcTan}[\tan[c+dx]](b+a\cos[c+dx])\sec[c+dx]}{8(a+b)^3d(a+b\sec[c+dx])} + \frac{(7a+9b)(b+a\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2\sec[c+dx]}{32(a+b)^2d(a+b\sec[c+dx])} - \\
& \frac{(b+a\cos[c+dx])\operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4\sec[c+dx]}{64(a+b)d(a+b\sec[c+dx])} + \frac{(-8a^2 + 21ab - 15b^2)(b+a\cos[c+dx])\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]^2\right]\sec[c+dx]}{16(-a+b)^3d(a+b\sec[c+dx])} + \\
& \frac{b^6(b+a\cos[c+dx])\operatorname{Log}[b+a\cos[c+dx]]\sec[c+dx]}{a(-a^2+b^2)^3d(a+b\sec[c+dx])} + \frac{(8a^2 + 21ab + 15b^2)(b+a\cos[c+dx])\operatorname{Log}\left[\sin\left[\frac{1}{2}(c+dx)\right]^2\right]\sec[c+dx]}{16(a+b)^3d(a+b\sec[c+dx])} + \\
& \frac{(7a-9b)(b+a\cos[c+dx])\sec\left[\frac{1}{2}(c+dx)\right]^2\sec[c+dx]}{32(-a+b)^2d(a+b\sec[c+dx])} + \frac{(b+a\cos[c+dx])\sec\left[\frac{1}{2}(c+dx)\right]^4\sec[c+dx]}{64(-a+b)d(a+b\sec[c+dx])}
\end{aligned}$$

■ **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c+dx]^6}{a+b\sec[c+dx]} dx$$

Optimal (type 3, 198 leaves, 15 steps):

$$\begin{aligned}
& -\frac{x}{a} + \frac{(8a^4 - 20a^2b^2 + 15b^4)\operatorname{ArcTanh}[\sin[c+dx]]}{8b^5d} - \frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{ArcTanh}\left[\frac{\sqrt{a-b}\tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{ab^5d} - \\
& \frac{a(a^2 - 2b^2)\tan[c+dx]}{b^4d} + \frac{(4a^2 - 7b^2)\sec[c+dx]\tan[c+dx]}{8b^3d} - \frac{a\tan[c+dx]^3}{3b^2d} + \frac{\sec[c+dx]\tan[c+dx]^3}{4bd}
\end{aligned}$$

Result (type 3, 907 leaves):

$$\begin{aligned}
& - \frac{(c + dx) (b + a \cos[c + dx]) \sec[c + dx]}{ad (a + b \sec[c + dx])} - \frac{2(-a^2 + b^2)^3 \operatorname{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right] (b + a \cos[c + dx]) \sec[c + dx]}{ab^5 \sqrt{a^2 - b^2} d (a + b \sec[c + dx])} + \\
& \frac{(-8a^4 + 20a^2b^2 - 15b^4) (b + a \cos[c + dx]) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec[c + dx]}{8b^5 d (a + b \sec[c + dx])} + \\
& \frac{(8a^4 - 20a^2b^2 + 15b^4) (b + a \cos[c + dx]) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \sec[c + dx]}{8b^5 d (a + b \sec[c + dx])} + \\
& \frac{(b + a \cos[c + dx]) \sec[c + dx]}{16bd (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{(12a^2 - 4ab - 27b^2) (b + a \cos[c + dx]) \sec[c + dx]}{48b^3 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \\
& \frac{a (b + a \cos[c + dx]) \sec[c + dx] \sin\left[\frac{1}{2}(c + dx)\right]}{6b^2 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \frac{(b + a \cos[c + dx]) \sec[c + dx]}{16bd (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} - \\
& \frac{a (b + a \cos[c + dx]) \sec[c + dx] \sin\left[\frac{1}{2}(c + dx)\right]}{6b^2 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(-12a^2 + 4ab + 27b^2) (b + a \cos[c + dx]) \sec[c + dx]}{48b^3 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{(b + a \cos[c + dx]) \sec[c + dx] \left(-3a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 7ab^2 \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3b^4 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{(b + a \cos[c + dx]) \sec[c + dx] \left(-3a^3 \sin\left[\frac{1}{2}(c + dx)\right] + 7ab^2 \sin\left[\frac{1}{2}(c + dx)\right]\right)}{3b^4 d (a + b \sec[c + dx]) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]^4}{a + b \sec[c + dx]} dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$\frac{x}{a} + \frac{(2a^2 - 3b^2) \operatorname{ArcTanh}[\sin[c + dx]]}{2b^3 d} - \frac{2(a - b)^{3/2} (a + b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{ab^3 d} - \frac{a \tan[c + dx]}{b^2 d} + \frac{\sec[c + dx] \tan[c + dx]}{2bd}$$

Result (type 3, 287 leaves):

$$\frac{1}{4 d (a + b \operatorname{Sec}[c + d x])}$$

$$(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \left(\frac{4 c}{a} + \frac{4 d x}{a} + \frac{8 (a^2 - b^2)^{3/2} \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{a b^3} - \frac{4 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{b^3} + \right.$$

$$\frac{6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{b} + \frac{4 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{b^3} - \frac{6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]}{b} +$$

$$\left. \frac{1}{b \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{1}{b \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{4 a \operatorname{Tan}[c + d x]}{b^2} \right)$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^4}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 177 leaves, 15 steps):

$$\frac{x}{a} - \frac{2 b^5 \operatorname{ArcTanh}\left[\frac{\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{a+b}\right]}{a (a^2 - b^2)^{5/2} d} + \frac{a (a^2 - 2 b^2) \operatorname{Cot}[c + d x]}{(a^2 - b^2)^2 d} - \frac{a \operatorname{Cot}[c + d x]^3}{3 (a^2 - b^2) d} - \frac{b (a^2 - 2 b^2) \operatorname{Csc}[c + d x]}{(a^2 - b^2)^2 d} + \frac{b \operatorname{Csc}[c + d x]^3}{3 (a^2 - b^2) d}$$

Result (type 3, 416 leaves):

$$\frac{(c + d x) (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]}{a d (a + b \operatorname{Sec}[c + d x])} + \frac{2 b^5 \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right] (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]}{a \sqrt{a^2 - b^2} (-a^2 + b^2)^2 d (a + b \operatorname{Sec}[c + d x])} +$$

$$\frac{(8 a \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 11 b \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]) (b + a \operatorname{Cos}[c + d x]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c + d x]}{12 (a + b)^2 d (a + b \operatorname{Sec}[c + d x])} -$$

$$\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right] \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c + d x]}{24 (a + b) d (a + b \operatorname{Sec}[c + d x])} +$$

$$\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c + d x] (-8 a \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 11 b \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right])}{12 (-a + b)^2 d (a + b \operatorname{Sec}[c + d x])} -$$

$$\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c + d x] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{24 (-a + b) d (a + b \operatorname{Sec}[c + d x])}$$

■ **Problem 299: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + d x]^9}{(a + b \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 255 leaves, 3 steps):

$$\begin{aligned} & - \frac{\text{Log}[\text{Cos}[c + d x]]}{a^2 d} + \frac{(a^2 - b^2)^3 (7 a^2 + b^2) \text{Log}[a + b \text{Sec}[c + d x]]}{a^2 b^8 d} - \frac{2 a (3 a^4 - 8 a^2 b^2 + 6 b^4) \text{Sec}[c + d x]}{b^7 d} + \frac{(5 a^4 - 12 a^2 b^2 + 6 b^4) \text{Sec}[c + d x]^2}{2 b^6 d} \\ & - \frac{4 a (a^2 - 2 b^2) \text{Sec}[c + d x]^3}{3 b^5 d} + \frac{(3 a^2 - 4 b^2) \text{Sec}[c + d x]^4}{4 b^4 d} - \frac{2 a \text{Sec}[c + d x]^5}{5 b^3 d} + \frac{\text{Sec}[c + d x]^6}{6 b^2 d} + \frac{(a^2 - b^2)^4}{a b^8 d (a + b \text{Sec}[c + d x])} \end{aligned}$$

Result (type 3, 528 leaves):

$$\begin{aligned} & - \frac{(-a + b)^4 (a + b)^4 (b + a \text{Cos}[c + d x]) \text{Sec}[c + d x]^2}{a^2 b^7 d (a + b \text{Sec}[c + d x])^2} + \frac{(-7 a^6 + 20 a^4 b^2 - 18 a^2 b^4 + 4 b^6) (b + a \text{Cos}[c + d x])^2 \text{Log}[\text{Cos}[c + d x]] \text{Sec}[c + d x]^2}{b^8 d (a + b \text{Sec}[c + d x])^2} + \\ & \frac{(7 a^8 - 20 a^6 b^2 + 18 a^4 b^4 - 4 a^2 b^6 - b^8) (b + a \text{Cos}[c + d x])^2 \text{Log}[b + a \text{Cos}[c + d x]] \text{Sec}[c + d x]^2}{a^2 b^8 d (a + b \text{Sec}[c + d x])^2} - \\ & \frac{2 a (3 a^4 - 8 a^2 b^2 + 6 b^4) (b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^3}{b^7 d (a + b \text{Sec}[c + d x])^2} + \frac{(5 a^4 - 12 a^2 b^2 + 6 b^4) (b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^4}{2 b^6 d (a + b \text{Sec}[c + d x])^2} + \\ & \frac{4 a (-a^2 + 2 b^2) (b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^5}{3 b^5 d (a + b \text{Sec}[c + d x])^2} + \frac{(3 a^2 - 4 b^2) (b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^6}{4 b^4 d (a + b \text{Sec}[c + d x])^2} - \\ & \frac{2 a (b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^7}{5 b^3 d (a + b \text{Sec}[c + d x])^2} + \frac{(b + a \text{Cos}[c + d x])^2 \text{Sec}[c + d x]^8}{6 b^2 d (a + b \text{Sec}[c + d x])^2} \end{aligned}$$

■ **Problem 300: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + d x]^7}{(a + b \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 179 leaves, 3 steps):

$$\begin{aligned} & \frac{\text{Log}[\text{Cos}[c + d x]]}{a^2 d} + \frac{(a^2 - b^2)^2 (5 a^2 + b^2) \text{Log}[a + b \text{Sec}[c + d x]]}{a^2 b^6 d} - \frac{2 a (2 a^2 - 3 b^2) \text{Sec}[c + d x]}{b^5 d} + \\ & \frac{3 (a^2 - b^2) \text{Sec}[c + d x]^2}{2 b^4 d} - \frac{2 a \text{Sec}[c + d x]^3}{3 b^3 d} + \frac{\text{Sec}[c + d x]^4}{4 b^2 d} + \frac{(a^2 - b^2)^3}{a b^6 d (a + b \text{Sec}[c + d x])} \end{aligned}$$

Result (type 3, 383 leaves):

$$\frac{(-a+b)^3 (a+b)^3 (b+a \cos[c+dx]) \sec[c+dx]^2}{a^2 b^5 d (a+b \sec[c+dx])^2} + \frac{(-5a^4 + 9a^2 b^2 - 3b^4) (b+a \cos[c+dx])^2 \log[\cos[c+dx]] \sec[c+dx]^2}{b^6 d (a+b \sec[c+dx])^2} +$$

$$\frac{(5a^6 - 9a^4 b^2 + 3a^2 b^4 + b^6) (b+a \cos[c+dx])^2 \log[b+a \cos[c+dx]] \sec[c+dx]^2}{a^2 b^6 d (a+b \sec[c+dx])^2} + \frac{2a(-2a^2 + 3b^2) (b+a \cos[c+dx])^2 \sec[c+dx]^3}{b^5 d (a+b \sec[c+dx])^2} -$$

$$\frac{3(-a+b)(a+b)(b+a \cos[c+dx])^2 \sec[c+dx]^4}{2b^4 d (a+b \sec[c+dx])^2} - \frac{2a(b+a \cos[c+dx])^2 \sec[c+dx]^5}{3b^3 d (a+b \sec[c+dx])^2} + \frac{(b+a \cos[c+dx])^2 \sec[c+dx]^6}{4b^2 d (a+b \sec[c+dx])^2}$$

■ **Problem 305: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[c+dx]^3}{(a+b \sec[c+dx])^2} dx$$

Optimal (type 3, 197 leaves, 3 steps):

$$-\frac{\log[\cos[c+dx]]}{a^2 d} - \frac{(a+2b) \log[1-\sec[c+dx]]}{2(a+b)^3 d} - \frac{(a-2b) \log[1+\sec[c+dx]]}{2(a-b)^3 d} - \frac{b^4 (5a^2 - b^2) \log[a+b \sec[c+dx]]}{a^2 (a^2 - b^2)^3 d} +$$

$$\frac{1}{4(a+b)^2 d (1-\sec[c+dx])} + \frac{1}{4(a-b)^2 d (1+\sec[c+dx])} + \frac{b^4}{a(a^2 - b^2)^2 d (a+b \sec[c+dx])}$$

Result (type 3, 351 leaves):

$$\frac{1}{8d(a+b \sec[c+dx])^2} (b+a \cos[c+dx])$$

$$\left(-\frac{8b^5}{a^2(a-b)^2(a+b)^2} - \frac{16i(a^4 - 3a^2b^2 - 2b^4)(c+dx)(b+a \cos[c+dx])}{(a-b)^3(a+b)^3} + \frac{8i(a-2b) \operatorname{ArcTan}[\tan[c+dx]](b+a \cos[c+dx])}{(a-b)^3} + \right.$$

$$\frac{8i(a+2b) \operatorname{ArcTan}[\tan[c+dx]](b+a \cos[c+dx])}{(a+b)^3} - \frac{(b+a \cos[c+dx]) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^2}{(a+b)^2} +$$

$$\frac{4(a-2b)(b+a \cos[c+dx]) \log\left[\cos\left[\frac{1}{2}(c+dx)\right]^2\right]}{(-a+b)^3} + \frac{8b^4(-5a^2 + b^2)(b+a \cos[c+dx]) \log[b+a \cos[c+dx]]}{a^2(a^2 - b^2)^3} -$$

$$\left. \frac{4(a+2b)(b+a \cos[c+dx]) \log\left[\sin\left[\frac{1}{2}(c+dx)\right]^2\right]}{(a+b)^3} - \frac{(b+a \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{(a-b)^2} \right) \sec[c+dx]^2$$

■ **Problem 306: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cot[c+dx]^5}{(a+b \sec[c+dx])^2} dx$$

Optimal (type 3, 278 leaves, 3 steps):

$$\frac{\text{Log}[\text{Cos}[c + d x]]}{a^2 d} + \frac{(4 a^2 + 13 a b + 12 b^2) \text{Log}[1 - \text{Sec}[c + d x]]}{8 (a + b)^4 d} + \frac{(4 a^2 - 13 a b + 12 b^2) \text{Log}[1 + \text{Sec}[c + d x]]}{8 (a - b)^4 d} -$$

$$\frac{b^6 (7 a^2 - b^2) \text{Log}[a + b \text{Sec}[c + d x]]}{a^2 (a^2 - b^2)^4 d} - \frac{1}{16 (a + b)^2 d (1 - \text{Sec}[c + d x])^2} - \frac{5 a + 9 b}{16 (a + b)^3 d (1 - \text{Sec}[c + d x])} -$$

$$\frac{1}{16 (a - b)^2 d (1 + \text{Sec}[c + d x])^2} - \frac{5 a - 9 b}{16 (a - b)^3 d (1 + \text{Sec}[c + d x])} + \frac{b^6}{a (a^2 - b^2)^3 d (a + b \text{Sec}[c + d x])}$$

Result (type 3, 473 leaves):

$$\frac{1}{64 d (a + b \text{Sec}[c + d x])^2} (b + a \text{Cos}[c + d x]) \left(\frac{64 b^7}{a^2 (-a + b)^3 (a + b)^3} + \right.$$

$$\frac{128 i (a^6 - 4 a^4 b^2 + 6 a^2 b^4 + 3 b^6) (c + d x) (b + a \text{Cos}[c + d x])}{(a - b)^4 (a + b)^4} - \frac{16 i (4 a^2 - 13 a b + 12 b^2) \text{ArcTan}[\text{Tan}[c + d x]] (b + a \text{Cos}[c + d x])}{(a - b)^4} -$$

$$\frac{16 i (4 a^2 + 13 a b + 12 b^2) \text{ArcTan}[\text{Tan}[c + d x]] (b + a \text{Cos}[c + d x])}{(a + b)^4} + \frac{2 (7 a + 11 b) (b + a \text{Cos}[c + d x]) \text{Csc}\left[\frac{1}{2} (c + d x)\right]^2}{(a + b)^3} -$$

$$\frac{(b + a \text{Cos}[c + d x]) \text{Csc}\left[\frac{1}{2} (c + d x)\right]^4}{(a + b)^2} + \frac{8 (4 a^2 - 13 a b + 12 b^2) (b + a \text{Cos}[c + d x]) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right]^2\right]}{(a - b)^4} +$$

$$\frac{64 (-7 a^2 b^6 + b^8) (b + a \text{Cos}[c + d x]) \text{Log}[b + a \text{Cos}[c + d x]]}{a^2 (a^2 - b^2)^4} + \frac{8 (4 a^2 + 13 a b + 12 b^2) (b + a \text{Cos}[c + d x]) \text{Log}\left[\text{Sin}\left[\frac{1}{2} (c + d x)\right]^2\right]}{(a + b)^4} +$$

$$\left. \frac{2 (7 a - 11 b) (b + a \text{Cos}[c + d x]) \text{Sec}\left[\frac{1}{2} (c + d x)\right]^2}{(a - b)^3} - \frac{(b + a \text{Cos}[c + d x]) \text{Sec}\left[\frac{1}{2} (c + d x)\right]^4}{(a - b)^2} \right) \text{Sec}[c + d x]^2$$

■ **Problem 307: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + d x]^6}{(a + b \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 200 leaves, 16 steps):

$$-\frac{x}{a^2} - \frac{a (4 a^2 - 5 b^2) \text{ArcTanh}[\text{Sin}[c + d x]]}{b^5 d} + \frac{2 (a - b)^{3/2} (a + b)^{3/2} (4 a^2 + b^2) \text{ArcTanh}\left[\frac{\sqrt{a-b} \text{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a+b}}\right]}{a^2 b^5 d} +$$

$$\frac{(a^2 - b^2)^2 \text{Sin}[c + d x]}{a b^4 d (b + a \text{Cos}[c + d x])} + \frac{(3 a^2 - 2 b^2) \text{Tan}[c + d x]}{b^4 d} - \frac{a \text{Sec}[c + d x] \text{Tan}[c + d x]}{b^3 d} + \frac{\text{Tan}[c + d x]^3}{3 b^2 d}$$

Result (type 3, 865 leaves):

$$\begin{aligned}
& - \frac{(c+d x)(b+a \cos [c+d x])^2 \sec [c+d x]^2}{a^2 d(a+b \sec [c+d x])^2} - \frac{2(-a^2+b^2)^2(4 a^2+b^2) \operatorname{ArcTanh}\left[\frac{(-a+b) \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2-b^2}}\right](b+a \cos [c+d x])^2 \sec [c+d x]^2}{a^2 b^5 \sqrt{a^2-b^2} d(a+b \sec [c+d x])^2} + \\
& \frac{(4 a^3-5 a b^2)(b+a \cos [c+d x])^2 \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] \sec [c+d x]^2}{b^5 d(a+b \sec [c+d x])^2} + \\
& \frac{(-4 a^3+5 a b^2)(b+a \cos [c+d x])^2 \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] \sec [c+d x]^2}{b^5 d(a+b \sec [c+d x])^2} + \\
& \frac{(-6 a+b)(b+a \cos [c+d x])^2 \sec [c+d x]^2}{12 b^3 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right]}{6 b^2 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^3} + \\
& \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right]}{6 b^2 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{(6 a-b)(b+a \cos [c+d x])^2 \sec [c+d x]^2}{12 b^3 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^2\left(9 a^2 \sin \left[\frac{1}{2}(c+d x)\right]-7 b^2 \sin \left[\frac{1}{2}(c+d x)\right]\right)}{3 b^4 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)} + \\
& \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^2\left(9 a^2 \sin \left[\frac{1}{2}(c+d x)\right]-7 b^2 \sin \left[\frac{1}{2}(c+d x)\right]\right)}{3 b^4 d(a+b \sec [c+d x])^2\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)} + \\
& \frac{(b+a \cos [c+d x]) \sec [c+d x]^2\left(a^4 \sin [c+d x]-2 a^2 b^2 \sin [c+d x]+b^4 \sin [c+d x]\right)}{a b^4 d(a+b \sec [c+d x])^2}
\end{aligned}$$

■ **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]^4}{(a+b \sec [c+d x])^2} dx$$

Optimal (type 3, 150 leaves, 6 steps):

$$\frac{x}{a^2} - \frac{2 a \operatorname{ArcTanh}[\sin [c+d x]]}{b^3 d} + \frac{2 \sqrt{a-b} \sqrt{a+b} (2 a^2+b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{a^2 b^3 d} + \frac{(2 a^2-b^2) \sin [c+d x]}{a b^2 d(b+a \cos [c+d x])} + \frac{\tan [c+d x]}{b d(b+a \cos [c+d x])}$$

Result (type 3, 327 leaves):

$$\frac{1}{d (a + b \operatorname{Sec}[c + d x])^2}$$

$$(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^2 \left(\frac{(c + d x) (b + a \operatorname{Cos}[c + d x])}{a^2} + \frac{2 (-2 a^4 + a^2 b^2 + b^4) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{a^2 b^3 \sqrt{a^2-b^2}} (b + a \operatorname{Cos}[c + d x]) \right. +$$

$$\frac{2 a (b + a \operatorname{Cos}[c + d x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{b^3} - \frac{2 a (b + a \operatorname{Cos}[c + d x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{b^3} +$$

$$\left. \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{b^2 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} + \frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{b^2 (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])} + \frac{(a^2 - b^2) \operatorname{Sin}[c + d x]}{a b^2} \right)$$

- **Problem 312: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^{5/2}}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 761 leaves, 38 steps):

$$\begin{aligned}
& \frac{a e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} b^2 d} - \frac{(a^2 - b^2) e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a b^2 d} - \\
& \frac{a e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} b^2 d} + \frac{(a^2 - b^2) e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a b^2 d} - \\
& \frac{a e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} b^2 d} + \frac{(a^2 - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a b^2 d} + \\
& \frac{a e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} b^2 d} - \frac{(a^2 - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a b^2 d} + \\
& \frac{2 \sqrt{2} \sqrt{a-b} \sqrt{a+b} e^2 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticPi}\left[-\frac{\sqrt{a-b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c+dx]}}{\sqrt{1+\operatorname{Cos}[c+dx]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c+dx]}}{a b d \sqrt{\operatorname{Sin}[c+dx]}} - \\
& \frac{2 \sqrt{2} \sqrt{a-b} \sqrt{a+b} e^2 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticPi}\left[\frac{\sqrt{a-b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c+dx]}}{\sqrt{1+\operatorname{Cos}[c+dx]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c+dx]}}{a b d \sqrt{\operatorname{Sin}[c+dx]}} - \\
& \frac{2 e^2 \operatorname{Cos}[c+dx] \operatorname{EllipticE}\left[c - \frac{\pi}{4} + dx, 2\right] \sqrt{e \operatorname{Tan}[c+dx]}}{b d \sqrt{\operatorname{Sin}[2c+2dx]}} + \frac{2 e \operatorname{Cos}[c+dx] (e \operatorname{Tan}[c+dx])^{3/2}}{b d}
\end{aligned}$$

Result (type 6, 2965 leaves):

$$\begin{aligned}
& \frac{2 (b + a \operatorname{Cos}[c+dx]) \operatorname{Cot}[c+dx] (e \operatorname{Tan}[c+dx])^{5/2}}{b d (a + b \operatorname{Sec}[c+dx])} - \frac{1}{b d (a + b \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]^{5/2}} \\
& (b + a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] (e \operatorname{Tan}[c+dx])^{5/2} \left(\frac{1}{(b + a \operatorname{Cos}[c+dx]) (1 + \operatorname{Tan}[c+dx])^{3/2}} 4 a \operatorname{Sec}[c+dx]^2 (a + b \sqrt{1 + \operatorname{Tan}[c+dx]^2}) \right. \\
& \left. \left(\frac{1}{4 \sqrt{2} \sqrt{b} (-a^2 + b^2)^{1/4}} \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Tan}[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Tan}[c+dx]}}{(-a^2 + b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} - \right. \right. \right. \\
& \left. \left. \left. \sqrt{2} \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} + b \operatorname{Tan}[c+dx]\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} + \sqrt{2} \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} + b \operatorname{Tan}[c+dx]\right] \right) \right) + \\
& \left(7 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \operatorname{Tan}[c+dx]^{3/2} \right) / \left(3 \sqrt{1 + \operatorname{Tan}[c+dx]^2} (-7 (a^2 - b^2)) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] - 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \\
& \left. (-a^2+b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] \text{Tan}[c+dx]^2 \right) (-a^2+b^2 (1+\text{Tan}[c+dx]^2)) \Bigg) + \\
& \frac{1}{4 (b+a \text{Cos}[c+dx]) (1+\text{Tan}[c+dx]^2)} b \text{Sec}[c+dx] \left(a+b \sqrt{1+\text{Tan}[c+dx]^2} \right) \left(\frac{1}{a} \left(-6 \sqrt{2} \text{ArcTan}\left[1-\sqrt{2} \sqrt{\text{Tan}[c+dx]}\right] + \right. \right. \\
& \frac{1}{(a^2-b^2)^{1/4}} 3 \left(2 \sqrt{2} (a^2-b^2)^{1/4} \text{ArcTan}\left[1+\sqrt{2} \sqrt{\text{Tan}[c+dx]}\right] + (2+2i) \sqrt{b} \text{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\text{Tan}[c+dx]}}{(a^2-b^2)^{1/4}}\right] - \right. \\
& (2+2i) \sqrt{b} \text{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\text{Tan}[c+dx]}}{(a^2-b^2)^{1/4}}\right] + \sqrt{2} (a^2-b^2)^{1/4} \text{Log}\left[1-\sqrt{2} \sqrt{\text{Tan}[c+dx]} + \text{Tan}[c+dx]\right] - \\
& \left. \sqrt{2} (a^2-b^2)^{1/4} \text{Log}\left[1+\sqrt{2} \sqrt{\text{Tan}[c+dx]} + \text{Tan}[c+dx]\right] - (1+i) \sqrt{b} \text{Log}\left[\sqrt{a^2-b^2} - (1+i) \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\text{Tan}[c+dx]} + \right. \right. \\
& \left. \left. i b \text{Tan}[c+dx]\right] + (1+i) \sqrt{b} \text{Log}\left[\sqrt{a^2-b^2} + (1+i) \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\text{Tan}[c+dx]} + i b \text{Tan}[c+dx]\right] \right) \Bigg) - \\
& \left(56 b (-a^2+b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] \text{Tan}[c+dx]^{3/2} \right) / \left(\sqrt{1+\text{Tan}[c+dx]^2} \left(7 (a^2-b^2) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] + 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2+b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\text{Tan}[c+dx]^2, \frac{b^2 \text{Tan}[c+dx]^2}{a^2-b^2}\right] \text{Tan}[c+dx]^2 \right) (-a^2+b^2 (1+\text{Tan}[c+dx]^2)) \right) \right) \Bigg) + \\
& \frac{1}{(b+a \text{Cos}[c+dx]) (-1+\text{Tan}[c+dx]^2) \sqrt{1+\text{Tan}[c+dx]^2}} 2 a \text{Cos}[2(c+dx)] \text{Sec}[c+dx]^2 \left(a+b \sqrt{1+\text{Tan}[c+dx]^2} \right) \\
& \left(\frac{b \text{ArcTan}\left[\frac{-\sqrt{2}+2\sqrt{\text{Tan}[c+dx]}}{\sqrt{2}}\right]}{\sqrt{2} a^2} + \frac{b \text{ArcTan}\left[\frac{\sqrt{2}+2\sqrt{\text{Tan}[c+dx]}}{\sqrt{2}}\right]}{\sqrt{2} a^2} + \frac{(-1)^{1/4} (a^2-b^2)^{3/4} (-a^2+2b^2) \text{ArcTan}\left[\frac{-\sqrt{2} (a^2-b^2)^{1/4}+2(-1)^{1/4} \sqrt{b} \sqrt{\text{Tan}[c+dx]}}{\sqrt{2} (a^2-b^2)^{1/4}}\right]}{2 \sqrt{2} a^2 \sqrt{b} (-a^2+b^2)} \right) + \\
& \frac{(-1)^{1/4} (a^2-b^2)^{3/4} (-a^2+2b^2) \text{ArcTan}\left[\frac{\sqrt{2} (a^2-b^2)^{1/4}+2(-1)^{1/4} \sqrt{b} \sqrt{\text{Tan}[c+dx]}}{\sqrt{2} (a^2-b^2)^{1/4}}\right]}{2 \sqrt{2} a^2 \sqrt{b} (-a^2+b^2)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2} - \frac{b \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right]}{2 \sqrt{2} a^2} + \frac{1}{4 \sqrt{2} a^2 \sqrt{b} (-a^2 + b^2)} \\
& (-1)^{1/4} (a^2 - b^2)^{3/4} (-a^2 + 2b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} - (-1)^{1/4} \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c + d x]} + i b \operatorname{Tan}[c + d x]\right] - \\
& \frac{1}{4 \sqrt{2} a^2 \sqrt{b} (-a^2 + b^2)} (-1)^{1/4} (a^2 - b^2)^{3/4} (-a^2 + 2b^2) \operatorname{Log}\left[\sqrt{a^2 - b^2} + (-1)^{1/4} \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c + d x]} + i b \operatorname{Tan}[c + d x]\right] - \\
& \frac{\operatorname{Tan}[c + d x]^{3/2}}{a \sqrt{1 + \operatorname{Tan}[c + d x]^2}} + \left(14 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Tan}[c + d x]^{3/2}\right) / \\
& \left(3 \sqrt{1 + \operatorname{Tan}[c + d x]^2} \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right]\right) \operatorname{Tan}[c + d x]^2\right) \\
& \left. (-a^2 + b^2 (1 + \operatorname{Tan}[c + d x]^2))\right) - \left(7 b^2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Tan}[c + d x]^{3/2}\right) / \\
& \left(a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right]\right) \operatorname{Tan}[c + d x]^2\right) \\
& \left. (-a^2 + b^2 (1 + \operatorname{Tan}[c + d x]^2))\right) - \left(11 b^2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Tan}[c + d x]^{7/2}\right) / \\
& \left(7 a \sqrt{1 + \operatorname{Tan}[c + d x]^2} \left(-11 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] - \right. \\
& \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right]\right) \operatorname{Tan}[c + d x]^2\right) (-a^2 + b^2 (1 + \operatorname{Tan}[c + d x]^2))\right) \Bigg)
\end{aligned}$$

■ **Problem 313: Result unnecessarily involves higher level functions.**

$$\int \frac{(e \operatorname{Tan}[c + d x])^{3/2}}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 740 leaves, 35 steps):

$$\begin{aligned}
& \frac{a e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} b^2 d} - \frac{(a^2 - b^2) e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a b^2 d} - \\
& \frac{a e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} b^2 d} + \frac{(a^2 - b^2) e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a b^2 d} + \\
& \frac{a e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} b^2 d} - \frac{(a^2 - b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a b^2 d} - \\
& \frac{a e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} b^2 d} + \frac{(a^2 - b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a b^2 d} - \\
& \frac{2 \sqrt{2} \sqrt{a^2 - b^2} e^2 \operatorname{EllipticPi}\left[\frac{b}{a - \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\operatorname{Cos}[c+dx]}}{\sqrt{1 + \operatorname{Sin}[c+dx]}}\right], -1\right] \sqrt{\operatorname{Sin}[c+dx]}}{a b d \sqrt{-\operatorname{Cos}[c+dx]} \sqrt{e \operatorname{Tan}[c+dx]}} + \\
& \frac{2 \sqrt{2} \sqrt{a^2 - b^2} e^2 \operatorname{EllipticPi}\left[\frac{b}{a + \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\operatorname{Cos}[c+dx]}}{\sqrt{1 + \operatorname{Sin}[c+dx]}}\right], -1\right] \sqrt{\operatorname{Sin}[c+dx]}}{a b d \sqrt{-\operatorname{Cos}[c+dx]} \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{e^2 \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{b d \sqrt{e \operatorname{Tan}[c+dx]}}
\end{aligned}$$

Result (type 6, 755 leaves):

$$\begin{aligned}
& \frac{1}{d (a + b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]^{3/2} (1 + \operatorname{Tan}[c + d x]^2)} \\
& 2 \operatorname{Sec}[c + d x]^2 (e \operatorname{Tan}[c + d x])^{3/2} \left(a + b \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \left(\frac{1}{8 a} \left(2 \sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right] + \right. \right. \\
& \left. \frac{1}{\sqrt{b}} \left(-2 \sqrt{2} \sqrt{b} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]}\right] - (2 - 2 i) (a^2 - b^2)^{1/4} \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Tan}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \\
& \left. (2 - 2 i) (a^2 - b^2)^{1/4} \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Tan}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \sqrt{2} \sqrt{b} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right] - \sqrt{2} \sqrt{b} \operatorname{Log}\left[\right. \right. \\
& \left. \left. 1 + \sqrt{2} \sqrt{\operatorname{Tan}[c + d x]} + \operatorname{Tan}[c + d x]\right] - (1 - i) (a^2 - b^2)^{1/4} \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c + d x]} + i b \operatorname{Tan}[c + d x]\right] + \right. \\
& \left. (1 - i) (a^2 - b^2)^{1/4} \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c + d x]} + i b \operatorname{Tan}[c + d x]\right] \right) \left. \right) - \\
& \left(9 b (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Tan}[c + d x]^{5/2} \right) / \left(5 \sqrt{1 + \operatorname{Tan}[c + d x]^2} \right) \\
& \left(9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] + 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\operatorname{Tan}[c + d x]^2, \frac{b^2 \operatorname{Tan}[c + d x]^2}{a^2 - b^2}\right] \operatorname{Tan}[c + d x]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[c + d x]^2)) \right) \left. \right)
\end{aligned}$$

■ **Problem 314: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{e \operatorname{Tan}[c + d x]}}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 415 leaves, 21 steps):

$$\begin{aligned}
& \frac{\sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{\sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d} + \frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} \\
& \frac{\sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Tan}[c + d x]}\right]}{2 \sqrt{2} a d} + \frac{2 \sqrt{2} b \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[-\frac{\sqrt{a-b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c + d x]}}{\sqrt{1 + \operatorname{Cos}[c + d x]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c + d x]}}{a \sqrt{a-b} \sqrt{a+b} d \sqrt{\operatorname{Sin}[c + d x]}} \\
& \frac{2 \sqrt{2} b \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{\sqrt{a-b}}{\sqrt{a+b}}, \operatorname{ArcSin}\left[\frac{\sqrt{\operatorname{Sin}[c + d x]}}{\sqrt{1 + \operatorname{Cos}[c + d x]}}\right], -1\right] \sqrt{e \operatorname{Tan}[c + d x]}}{a \sqrt{a-b} \sqrt{a+b} d \sqrt{\operatorname{Sin}[c + d x]}}
\end{aligned}$$

Result (type 4, 232 leaves):

$$\begin{aligned}
& - \left(4 (b + a \cos [c + d x]) \operatorname{Csc}[c + d x] \right. \\
& \left. \left(i \operatorname{EllipticPi}\left[-i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{1}{2}(c + d x)\right]}\right], -1\right] - i \operatorname{EllipticPi}\left[i, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{1}{2}(c + d x)\right]}\right], -1\right] + 1 / \left(\sqrt{a-b} \sqrt{a+b}\right) \right. \right. \\
& \left. \left. b \left(\operatorname{EllipticPi}\left[-\frac{\sqrt{a-b}}{\sqrt{a+b}}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{1}{2}(c + d x)\right]}\right], -1\right] - \operatorname{EllipticPi}\left[\frac{\sqrt{a-b}}{\sqrt{a+b}}, -\operatorname{ArcSin}\left[\sqrt{\tan\left[\frac{1}{2}(c + d x)\right]}\right], -1\right] \right) \right) \right) \\
& \left. \sqrt{\tan\left[\frac{1}{2}(c + d x)\right] \sqrt{e \tan [c + d x]}} / \left(a d \sqrt{\cos [c + d x] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 (a + b \operatorname{Sec}[c + d x])} \right) \right)
\end{aligned}$$

■ **Problem 315: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x]) \sqrt{e \tan [c + d x]}} dx$$

Optimal (type 4, 422 leaves, 19 steps):

$$\begin{aligned}
& - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \tan [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d \sqrt{e}} + \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \tan [c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} a d \sqrt{e}} - \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan [c + d x] - \sqrt{2} \sqrt{e \tan [c + d x]}\right]}{2 \sqrt{2} a d \sqrt{e}} + \\
& \frac{\operatorname{Log}\left[\sqrt{e} + \sqrt{e} \tan [c + d x] + \sqrt{2} \sqrt{e \tan [c + d x]}\right]}{2 \sqrt{2} a d \sqrt{e}} - \frac{2 \sqrt{2} b \operatorname{EllipticPi}\left[\frac{b}{a - \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\cos [c + d x]}}{\sqrt{1 + \sin [c + d x]}}\right], -1\right] \sqrt{\sin [c + d x]}}{a \sqrt{a^2 - b^2} d \sqrt{-\cos [c + d x]} \sqrt{e \tan [c + d x]}} + \\
& \frac{2 \sqrt{2} b \operatorname{EllipticPi}\left[\frac{b}{a + \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\cos [c + d x]}}{\sqrt{1 + \sin [c + d x]}}\right], -1\right] \sqrt{\sin [c + d x]}}{a \sqrt{a^2 - b^2} d \sqrt{-\cos [c + d x]} \sqrt{e \tan [c + d x]}}
\end{aligned}$$

Result (type 6, 1860 leaves):

$$\begin{aligned}
& \frac{1}{2 d (a + b \operatorname{Sec}[c + d x]) \sqrt{e \tan [c + d x]}} \\
& (b + a \cos [c + d x]) \operatorname{Sec}[c + d x] \sqrt{\tan [c + d x]} \left(\frac{1}{(b + a \cos [c + d x]) (1 + \tan [c + d x])^2} 2 \operatorname{Sec}[c + d x]^3 (a + b \sqrt{1 + \tan [c + d x]^2}) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{\sqrt{b} (a^2 - b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) a \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\tan[c+dx]}}{(a^2 - b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\tan[c+dx]}}{(a^2 - b^2)^{1/4}} \right] + \operatorname{Log} \left[\sqrt{a^2 - b^2} - (1+i) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c+dx]} + i b \tan[c+dx] \right] - \operatorname{Log} \left[\sqrt{a^2 - b^2} + (1+i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c+dx]} + i b \tan[c+dx] \right] \right) + \right. \\
& \quad \left(5 b (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sqrt{\tan[c+dx]} \sqrt{1 + \tan[c+dx]^2} \right) / \left(\left(5 (a^2 - b^2) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] + 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right) (a^2 - b^2 (1 + \tan[c+dx]^2)) \right) \Bigg) - \\
& \frac{1}{(b + a \cos[c+dx]) (1 - \tan[c+dx]^2) (1 + \tan[c+dx]^2)} 2 \cos[2(c+dx)] \sec[c+dx]^3 \left(a + b \sqrt{1 + \tan[c+dx]^2} \right) \\
& \left(\frac{\operatorname{ArcTan} \left[\frac{-\sqrt{2} + 2\sqrt{\tan[c+dx]}}{\sqrt{2}} \right]}{\sqrt{2} a} - \frac{\operatorname{ArcTan} \left[\frac{\sqrt{2} + 2\sqrt{\tan[c+dx]}}{\sqrt{2}} \right]}{\sqrt{2} a} - \frac{(-1)^{3/4} (a^2 - b^2)^{1/4} (-a^2 + 2b^2) \operatorname{ArcTan} \left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2(-1)^{1/4} \sqrt{b} \sqrt{\tan[c+dx]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2\sqrt{2} a \sqrt{b} (-a^2 + b^2)} \right) - \\
& \frac{(-1)^{3/4} (a^2 - b^2)^{1/4} (-a^2 + 2b^2) \operatorname{ArcTan} \left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2(-1)^{1/4} \sqrt{b} \sqrt{\tan[c+dx]}}{\sqrt{2} (a^2 - b^2)^{1/4}} \right]}{2\sqrt{2} a \sqrt{b} (-a^2 + b^2)} + \\
& \frac{\operatorname{Log} \left[1 - \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx] \right]}{2\sqrt{2} a} - \frac{\operatorname{Log} \left[1 + \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx] \right]}{2\sqrt{2} a} + \frac{1}{4\sqrt{2} a \sqrt{b} (-a^2 + b^2)} \\
& (-1)^{3/4} (a^2 - b^2)^{1/4} (-a^2 + 2b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} - (-1)^{1/4} \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c+dx]} + i b \tan[c+dx] \right] - \\
& \frac{1}{4\sqrt{2} a \sqrt{b} (-a^2 + b^2)} (-1)^{3/4} (a^2 - b^2)^{1/4} (-a^2 + 2b^2) \operatorname{Log} \left[\sqrt{a^2 - b^2} + (-1)^{1/4} \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c+dx]} + i b \tan[c+dx] \right] + \\
& \left(5 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] \sqrt{\tan[c+dx]} \right) / \\
& \left(\sqrt{1 + \tan[c+dx]^2} \left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2 - b^2} \right] \right) \tan[c+dx]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{(b + a \cos[c + dx]) \sec[c + dx] \left(-\frac{2(b - a \cos[c + dx]) \csc[c + dx]}{-a^2 + b^2} + \frac{2b \sin[c + dx]}{-a^2 + b^2} \right) \tan[c + dx]^2}{d(a + b \sec[c + dx]) (e \tan[c + dx])^{3/2}} + \\
& \frac{1}{(a - b)(a + b)d(a + b \sec[c + dx]) (e \tan[c + dx])^{3/2}} (b + a \cos[c + dx]) \sec[c + dx] \tan[c + dx]^{3/2} \\
& \left(\frac{1}{12(b + a \cos[c + dx]) (1 + \tan[c + dx]^2)} (-a^2 + 3b^2) \sec[c + dx] \left(a + b \sqrt{1 + \tan[c + dx]^2} \right) \left(\frac{1}{a} \left(-6\sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[c + dx]}\right] \right) + \right. \right. \\
& \frac{1}{(a^2 - b^2)^{1/4}} 3 \left(2\sqrt{2} (a^2 - b^2)^{1/4} \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[c + dx]}\right] + (2 + 2i) \sqrt{b} \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\tan[c + dx]}}{(a^2 - b^2)^{1/4}}\right] - \right. \\
& (2 + 2i) \sqrt{b} \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\tan[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[c + dx]} + \tan[c + dx]\right] - \\
& \sqrt{2} (a^2 - b^2)^{1/4} \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[c + dx]} + \tan[c + dx]\right] - (1 + i) \sqrt{b} \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1 + i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c + dx]} + \right. \\
& \left. \left. i b \tan[c + dx]\right] + (1 + i) \sqrt{b} \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1 + i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\tan[c + dx]} + i b \tan[c + dx]\right] \right) \left. \right) - \\
& \left(56b(-a^2 + b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2}\right] \tan[c + dx]^{3/2} \right) / \left(\sqrt{1 + \tan[c + dx]^2} \left(7(a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2}\right] + 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2}\right] + \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[c + dx]^2, \frac{b^2 \tan[c + dx]^2}{a^2 - b^2}\right] \right) \tan[c + dx]^2 \right) (-a^2 + b^2 (1 + \tan[c + dx]^2)) \right) \left. \right) + \\
& \frac{1}{(b + a \cos[c + dx]) (-1 + \tan[c + dx]^2) \sqrt{1 + \tan[c + dx]^2}} 2ab \cos[2(c + dx)] \sec[c + dx]^2 \left(a + b \sqrt{1 + \tan[c + dx]^2} \right) \\
& \left(\frac{b \operatorname{ArcTan}\left[\frac{-\sqrt{2} + 2\sqrt{\tan[c + dx]}}{\sqrt{2}}\right]}{\sqrt{2} a^2} + \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{2} + 2\sqrt{\tan[c + dx]}}{\sqrt{2}}\right]}{\sqrt{2} a^2} + \frac{(-1)^{1/4} (a^2 - b^2)^{3/4} (-a^2 + 2b^2) \operatorname{ArcTan}\left[\frac{-\sqrt{2} (a^2 - b^2)^{1/4} + 2(-1)^{1/4} \sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2\sqrt{2} a^2 \sqrt{b} (-a^2 + b^2)} \right) + \\
& \frac{(-1)^{1/4} (a^2 - b^2)^{3/4} (-a^2 + 2b^2) \operatorname{ArcTan}\left[\frac{\sqrt{2} (a^2 - b^2)^{1/4} + 2(-1)^{1/4} \sqrt{b} \sqrt{\tan[c + dx]}}{\sqrt{2} (a^2 - b^2)^{1/4}}\right]}{2\sqrt{2} a^2 \sqrt{b} (-a^2 + b^2)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]\right]}{2\sqrt{2}a^2} - \frac{b \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[c+dx]} + \tan[c+dx]\right]}{2\sqrt{2}a^2} + \frac{1}{4\sqrt{2}a^2\sqrt{b}(-a^2+b^2)} \\
& (-1)^{1/4}(a^2-b^2)^{3/4}(-a^2+2b^2) \operatorname{Log}\left[\sqrt{a^2-b^2} - (-1)^{1/4}\sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\tan[c+dx]} + i b \tan[c+dx]\right] - \\
& \frac{1}{4\sqrt{2}a^2\sqrt{b}(-a^2+b^2)} (-1)^{1/4}(a^2-b^2)^{3/4}(-a^2+2b^2) \operatorname{Log}\left[\sqrt{a^2-b^2} + (-1)^{1/4}\sqrt{2}\sqrt{b}(a^2-b^2)^{1/4}\sqrt{\tan[c+dx]} + i b \tan[c+dx]\right] - \\
& \frac{\tan[c+dx]^{3/2}}{a\sqrt{1+\tan[c+dx]^2}} + \left(14a(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] \tan[c+dx]^{3/2}\right) / \\
& \left(3\sqrt{1+\tan[c+dx]^2} \left(-7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] - 2\left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right]\right) \tan[c+dx]^2 \right) \\
& \left. (-a^2+b^2(1+\tan[c+dx]^2))\right) - \left(7b^2(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] \tan[c+dx]^{3/2}\right) / \\
& \left(a\sqrt{1+\tan[c+dx]^2} \left(-7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] - 2\left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right]\right) \tan[c+dx]^2 \right) \\
& \left. (-a^2+b^2(1+\tan[c+dx]^2))\right) - \left(11b^2(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] \tan[c+dx]^{7/2}\right) / \\
& \left(7a\sqrt{1+\tan[c+dx]^2} \left(-11(a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] - \right. \right. \\
& \quad \left. \left. 2\left(2b^2 \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2}\right]\right) \tan[c+dx]^2\right) (-a^2+b^2(1+\tan[c+dx]^2))\right) \right)
\end{aligned}$$

■ **Problem 317: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sec}[c+dx]) (e \tan[c+dx])^{5/2}} dx$$

Optimal (type 4, 836 leaves, 36 steps):

$$\begin{aligned}
& \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 - b^2) d e^{5/2}} - \frac{b^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a (a^2 - b^2) d e^{5/2}} - \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 - b^2) d e^{5/2}} + \frac{b^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} a (a^2 - b^2) d e^{5/2}} + \\
& \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} (a^2 - b^2) d e^{5/2}} - \frac{b^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] - \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a (a^2 - b^2) d e^{5/2}} - \\
& \frac{a \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} (a^2 - b^2) d e^{5/2}} + \frac{b^2 \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Tan}[c+dx] + \sqrt{2} \sqrt{e \operatorname{Tan}[c+dx]}\right]}{2 \sqrt{2} a (a^2 - b^2) d e^{5/2}} - \\
& \frac{2 (a - b \operatorname{Sec}[c+dx])}{3 (a^2 - b^2) d e (e \operatorname{Tan}[c+dx])^{3/2}} - \frac{2 \sqrt{2} b^3 \operatorname{EllipticPi}\left[\frac{b}{a - \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\operatorname{Cos}[c+dx]}}{\sqrt{1 + \operatorname{Sin}[c+dx]}}\right], -1\right] \sqrt{\operatorname{Sin}[c+dx]}}{a (a^2 - b^2)^{3/2} d e^2 \sqrt{-\operatorname{Cos}[c+dx]} \sqrt{e \operatorname{Tan}[c+dx]}} + \\
& \frac{2 \sqrt{2} b^3 \operatorname{EllipticPi}\left[\frac{b}{a + \sqrt{a^2 - b^2}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\operatorname{Cos}[c+dx]}}{\sqrt{1 + \operatorname{Sin}[c+dx]}}\right], -1\right] \sqrt{\operatorname{Sin}[c+dx]}}{a (a^2 - b^2)^{3/2} d e^2 \sqrt{-\operatorname{Cos}[c+dx]} \sqrt{e \operatorname{Tan}[c+dx]}} + \frac{b \operatorname{EllipticF}\left[c - \frac{\pi}{4} + dx, 2\right] \operatorname{Sec}[c+dx] \sqrt{\operatorname{Sin}[2c+2dx]}}{3 (a^2 - b^2) d e^2 \sqrt{e \operatorname{Tan}[c+dx]}}
\end{aligned}$$

Result (type 6, 2554 leaves):

$$\begin{aligned}
& \frac{(b + a \operatorname{Cos}[c+dx]) \left(\frac{2a}{3(a^2 - b^2)} - \frac{2(-a + b \operatorname{Cos}[c+dx]) \operatorname{Csc}[c+dx]^2}{3(-a^2 + b^2)} \right) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^3}{d (a + b \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2}} - \\
& \frac{1}{6 (a - b) (a + b) d (a + b \operatorname{Sec}[c+dx]) (e \operatorname{Tan}[c+dx])^{5/2}} (b + a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]^{5/2} \\
& \left(\frac{1}{(b + a \operatorname{Cos}[c+dx]) (1 + \operatorname{Tan}[c+dx])^2} 2 (3a^2 - 5b^2) \operatorname{Sec}[c+dx]^3 \left(a + b \sqrt{1 + \operatorname{Tan}[c+dx]} \right)^2 \right) \left(-\frac{1}{\sqrt{b} (a^2 - b^2)^{3/4}} \right. \\
& \left. \left(\frac{1}{8} - \frac{i}{8} \right) a \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Tan}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\operatorname{Tan}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + \operatorname{Log}\left[\sqrt{a^2 - b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
& \left. \left. (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} + i b \operatorname{Tan}[c+dx] \right] - \operatorname{Log}\left[\sqrt{a^2 - b^2} + (1+i) \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Tan}[c+dx]} + i b \operatorname{Tan}[c+dx] \right] \right) \left. \right) + \\
& \left(5b (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \sqrt{\operatorname{Tan}[c+dx]} \sqrt{1 + \operatorname{Tan}[c+dx]^2} \right) / \left(\left(5 (a^2 - b^2) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] + 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, -\operatorname{Tan}[c+dx]^2, \frac{b^2 \operatorname{Tan}[c+dx]^2}{a^2 - b^2}\right] \right) \right) +
\end{aligned}$$

$$\begin{aligned} & \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \Big] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \\ & \left. (-a^2+b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \tan[c+dx]^2 \right) (-a^2+b^2 (1+\tan[c+dx]^2)) \Big) - \\ & \left(9 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \tan[c+dx]^{5/2} \right) / \left(5 \sqrt{1+\tan[c+dx]^2} \left(-9 (a^2-b^2) \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] + \right. \right. \right. \\ & \left. \left. \left. (-a^2+b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\tan[c+dx]^2, \frac{b^2 \tan[c+dx]^2}{a^2-b^2} \right] \tan[c+dx]^2 \right) (-a^2+b^2 (1+\tan[c+dx]^2)) \right) \right) \Big) \Big) \end{aligned}$$

■ **Problem 320: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \operatorname{Sec}[c+dx]} \tan[c+dx] dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$-\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{2\sqrt{a+b \operatorname{Sec}[c+dx]}}{d}$$

Result (type 3, 137 leaves):

$$\frac{1}{d \sqrt{b+a \operatorname{Cos}[c+dx]}} \left(2 \sqrt{b+a \operatorname{Cos}[c+dx]} + \sqrt{a \operatorname{Cos}[c+dx]} \operatorname{Log}\left[1 - \frac{\sqrt{b+a \operatorname{Cos}[c+dx]}}{\sqrt{a \operatorname{Cos}[c+dx]}}\right] - \sqrt{a \operatorname{Cos}[c+dx]} \operatorname{Log}\left[1 + \frac{\sqrt{b+a \operatorname{Cos}[c+dx]}}{\sqrt{a \operatorname{Cos}[c+dx]}}\right] \right) \sqrt{a+b \operatorname{Sec}[c+dx]}$$

■ **Problem 321: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a}}\right]}{d} - \frac{\sqrt{a-b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a-b}}\right]}{d} - \frac{\sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right]}{d}$$

Result (type 3, 4527 leaves):

$$\begin{aligned}
& \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}+\sqrt{a-b}\left(2 \sqrt{a} \operatorname{Log}\left[2 i a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-\right.\right. \\
& \left.\left. i b\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)+2 \sqrt{a} \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right] / \right. \\
& \left.\left(4 a^{3 / 2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)-\sqrt{a+b} \operatorname{Log}\left[\frac{1}{(a+b)^{3 / 2}} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2\left(2 i b-2 i a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-\right.\right.\right. \\
& \left.\left.2 \sqrt{a+b} \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right)\right]\right) \\
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right) / \left(4 \sqrt{a-b} \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \\
& \left.\sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)+ \\
& \left(i\left((a-b) \operatorname{Log}\left[\frac{2 i\left(a-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\sqrt{a-b}}+2 \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right.\right.\right. \\
& \left.\left.\sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}+\sqrt{a-b}\left(2 \sqrt{a} \operatorname{Log}\left[2 i a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-\right.\right.\right. \\
& \left.\left.\left. i b\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)+2 \sqrt{a} \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right] / \right.\right. \\
& \left.\left.\left(4 a^{3 / 2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)\right)-\sqrt{a+b} \operatorname{Log}\left[\frac{1}{(a+b)^{3 / 2}} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2\left(2 i b-2 i a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-\right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. 2 \sqrt{a+b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] \right) \right) \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) / \\
& \left(4 \sqrt{a-b} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \right. \\
& \left. \left(i \left((a-b) \operatorname{Log}\left[\frac{2i(a - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{\sqrt{a-b}}\right] + 2 \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \right. \\
& \left. \left. \left. \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] + \right. \right. \\
& \left. \left. \sqrt{a-b} \left(2 \sqrt{a} \operatorname{Log}\left[2i a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - i b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + 2 \sqrt{a} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] / \left(4 a^{3/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right) \right) - \sqrt{a+b} \right. \\
& \left. \operatorname{Log}\left[\frac{1}{(a+b)^{3/2}} \cot\left[\frac{1}{2}(c+dx)\right]^2 \left(2i b - 2i a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - 2 \sqrt{a+b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right] \right) \right) \right) \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(4 \sqrt{a-b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left(i \left((a-b) \operatorname{Log}\left[\frac{2i(a - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{\sqrt{a-b}} + 2 \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \right. \\
& \quad \left. \left. \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] + \sqrt{a-b} \left(2\sqrt{a} \operatorname{Log}\left[\left(2ia \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - \right. \right. \right. \right. \\
& \quad \left. \left. \left. ib \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + 2\sqrt{a} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right] \right) \right) / \\
& \quad \left(4a^{3/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \left) \right) - \sqrt{a+b} \operatorname{Log}\left[\frac{1}{(a+b)^{3/2}} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2 \left(2ib - 2ia \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - \right. \right. \right. \\
& \quad \left. \left. \left. 2\sqrt{a+b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right] \right) \right) \left) \right) \\
& \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \quad \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \right) / \\
& \left(4 \sqrt{a-b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) - \\
& \left(i \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left((a-b) \left(\frac{2i \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right)}{\sqrt{a-b}} + \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \left(\sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) + \right. \\
& \quad \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \Bigg) / \\
& \left(\frac{2i \left(a - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\sqrt{a-b}} + 2 \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \quad \left. \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) + \\
& \sqrt{a-b} \left(\left(8a^2 \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \left(\left(2ia \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - ib \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{a} \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \right. \right. \\
& \quad \left. \left(\sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) + \left(\sqrt{a} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right. \right. \\
& \quad \left. \left. \sqrt{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \left(\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) / \\
& \left(4a^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) - \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(2ia \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) - \right. \right.
\end{aligned}$$

$$\left(2 \sqrt{a-b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right)$$

- **Problem 322: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[c+dx]^3 \sqrt{a+b \sec[c+dx]} dx$$

Optimal (type 3, 215 leaves, 13 steps):

$$\begin{aligned} & -\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a}}\right]}{d} + \frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a-b}}\right]}{\sqrt{a-b} d} - \frac{3b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a-b}}\right]}{4\sqrt{a-b} d} + \\ & \frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right]}{\sqrt{a+b} d} + \frac{3b \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right]}{4\sqrt{a+b} d} - \frac{\cot[c+dx]^2 \sqrt{a+b \sec[c+dx]}}{2d} \end{aligned}$$

Result (type 3, 4909 leaves):

$$\frac{\left(\frac{1}{2} - \frac{1}{2} \operatorname{Csc}[c+dx]^2\right) \sqrt{a+b \sec[c+dx]}}{d} +$$

$$\begin{aligned} & \left(i \left(\sqrt{a+b} (-4a+3b) \operatorname{Log}\left[\frac{2i(a-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{\sqrt{a-b}}\right] + 2 \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \\ & \left. \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} + \sqrt{a-b} \left(-8\sqrt{a} \sqrt{a+b} \operatorname{Log}\left[-2ia \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + \right. \right. \right. \\ & \left. \left. \left. i b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - 2\sqrt{a} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right] \right) / \\ & \left. \left(16a^{3/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right) + (4a+3b) \operatorname{Log}\left[\frac{1}{\sqrt{a+b}(4a+3b)} \cot\left[\frac{1}{2}(c+dx)\right]^2 \right] \right) \\ & \left. \left(-2ib + 2ia \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) + 2\sqrt{a+b} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) \right) \end{aligned}$$

$$\left(\frac{-\frac{3b \operatorname{Csc}[c+dx]}{4\sqrt{b+a\cos[c+dx]}} \sqrt{\operatorname{Sec}[c+dx]} - \frac{a \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2\sqrt{b+a\cos[c+dx]}} - \frac{a \cos[2(c+dx)] \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2\sqrt{b+a\cos[c+dx]}}}{\sqrt{a+b\operatorname{Sec}[c+dx]}} \right)$$

$$\sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}$$

$$\sqrt{\frac{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}$$

$$\sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \Big/$$

$$\left(8\sqrt{a-b} \sqrt{a+b} d \sqrt{b+a\cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)$$

$$\left(\left(\sqrt{a+b} (-4a+3b) \operatorname{Log}\left[\frac{2i(a - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2)}{\sqrt{a-b}}\right] + 2\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \right.$$

$$\left. \left. \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \sqrt{a-b} \left(-8\sqrt{a} \sqrt{a+b} \operatorname{Log}\left[\left(-2ia\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) + \right. \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \left. i b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) - 2\sqrt{a} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) \right) \right) \right) \Big/$$

$$\left(16a^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + (4a+3b) \operatorname{Log}\left[\frac{1}{\sqrt{a+b}(4a+3b)} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2 \right] \left(-2ib + \right.$$

$$\begin{aligned}
& \frac{1}{16 \sqrt{a-b} \sqrt{a+b}} \sqrt{\frac{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(\sqrt{a+b} (-4a+3b) \operatorname{Log}\left[\right. \right. \\
& \left. \left. \frac{2i \left(a - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\sqrt{a-b}} + 2 \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] + \right. \\
& \left. \sqrt{a-b} \left(-8 \sqrt{a} \sqrt{a+b} \operatorname{Log}\left[-2i a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + i b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - 2 \sqrt{a} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \right. \\
& \left. \left. \left. \sqrt{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] / \left(16 a^{3/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \right) + \\
& \left(4a+3b \right) \operatorname{Log}\left[\frac{1}{\sqrt{a+b} (4a+3b)} \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2 \left(-2i b + 2i a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + 2 \sqrt{a+b} \right. \right. \\
& \left. \left. \left. \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right] \right] \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \left. \sqrt{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(-\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2} + \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) - \\
& \frac{1}{16 \sqrt{a-b} \sqrt{a+b}} \left(\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} i \left(\sqrt{a+b} (-4a+3b) \operatorname{Log}\left[\frac{2i \left(a - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\sqrt{a-b}} + \right. \right. \\
& \left. \left. 2 \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2} \right] \right) +
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a-b} \left(-8 \sqrt{a} \sqrt{a+b} \operatorname{Log} \left[\left(-2 i a \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) + i b \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) - 2 \sqrt{a} \sqrt{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right) / \left(16 a^{3/2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right] \right) + \\
& (4a+3b) \operatorname{Log} \left[\frac{1}{\sqrt{a+b} (4a+3b)} \operatorname{Cot} \left[\frac{1}{2} (c+dx) \right]^2 \left(-2 i b + 2 i a \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) + 2 \sqrt{a+b} \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \sqrt{a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right) \right] \right] \right) \\
& \sqrt{\frac{1}{1 - \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}} \sqrt{\frac{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}} \sqrt{a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \\
& \left(\frac{-a \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + b \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} - \right. \\
& \quad \left. \frac{\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \left(a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^2} \right) + \\
& \frac{1}{8 \sqrt{a-b} \sqrt{a+b}} i \sqrt{\frac{1}{1 - \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}} \sqrt{\frac{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2}} \\
& \sqrt{a+b - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \\
& \left(\left(\sqrt{a+b} (-4a+3b) \right) \frac{2 i \left(-a \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + b \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right)}{\sqrt{a-b}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \\
& \left(\sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) + \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}{\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \\
& \left(\frac{2i \left(a - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\sqrt{a-b}} + 2 \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) + \\
& \sqrt{a-b} \left(- \left(\left(128 a^2 \sqrt{a+b} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \left(\left(-2i a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + i b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) - \right. \right. \right. \\
& \left. \left(\sqrt{a} \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \right. \\
& \left. \left(\sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) - \left(\sqrt{a} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right. \right. \\
& \left. \left. \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) / \left(\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) / \\
& \left(16 a^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) - \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-2i a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \right. \right. \\
& \left. \left. i b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) - 2 \sqrt{a} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3b^2d} 2a(a-b)\sqrt{a+b}\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{3bd} 2\sqrt{a+b}(a+2b)\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\
& \frac{2\sqrt{a+b}\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d} + \frac{2\sqrt{a+b\sec[c+dx]}\tan[c+dx]}{3d}
\end{aligned}$$

Result (type 4, 692 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{a+b \operatorname{Sec}[c+dx]} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \right. \\
& \left. \left(-i a(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right. \right. \\
& \left. \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 i(a-b) b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \right. \\
& \left. \left. 6 i a b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \right. \\
& \left. \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \right. \\
& \left. \left. a \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right) \right) / \\
& \left(3 b \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right) + \\
& \frac{\sqrt{a+b \operatorname{Sec}[c+dx]} \left(\frac{2 a \operatorname{Sin}[c+dx]}{3 b} + \frac{2}{3} \operatorname{Tan}[c+dx]\right)}{d}
\end{aligned}$$

■ **Problem 324: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 125 leaves, 1 step):

$$-\frac{1}{\sqrt{a+b} d}$$

$$2 \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b \sec [c+d x]}}\right], \frac{a-b}{a+b}\right] \sqrt{-\frac{b(1-\sec [c+d x])}{a+b \sec [c+d x]}} \sqrt{\frac{b(1+\sec [c+d x])}{a+b \sec [c+d x]}} (a+b \sec [c+d x])$$

Result (type 8, 16 leaves):

$$\int \sqrt{a+b \sec [c+d x]} dx$$

■ **Problem 327: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]^3}{\sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 3, 79 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} d} - \frac{2 a \sqrt{a+b \sec [c+d x]}}{b^2 d} + \frac{2 (a+b \sec [c+d x])^{3/2}}{3 b^2 d}$$

Result (type 3, 194 leaves):

$$\frac{(b+a \cos [c+d x]) \sec [c+d x] \left(-\frac{4 a}{3 b^2} + \frac{2 \sec [c+d x]}{3 b}\right)}{d \sqrt{a+b \sec [c+d x]}} +$$

$$\left(\sqrt{a \cos [c+d x]} \sqrt{b+a \cos [c+d x]} \left(-\log \left[1 - \frac{\sqrt{b+a \cos [c+d x]}}{\sqrt{a \cos [c+d x]}}\right] + \log \left[1 + \frac{\sqrt{b+a \cos [c+d x]}}{\sqrt{a \cos [c+d x]}}\right]\right) \sin [c+d x] \tan [c+d x]\right) /$$

$$\left(a d (1 - \cos [c+d x])^2 \sqrt{a+b \sec [c+d x]}\right)$$

■ **Problem 328: Result more than twice size of optimal antiderivative.**

$$\int \frac{\tan [c+d x]}{\sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} d}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{b+a \cos [c+d x]} \left(\log \left[1 - \frac{\sqrt{b+a \cos [c+d x]}}{\sqrt{a \cos [c+d x]}}\right] - \log \left[1 + \frac{\sqrt{b+a \cos [c+d x]}}{\sqrt{a \cos [c+d x]}}\right]\right)}{d \sqrt{a \cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

$$\left. \sqrt{a+b} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]^2 \right] \sqrt{\sec[c+dx] \sin[c+dx]} \right/$$

$$\left(4\sqrt{a-b} \sqrt{a+b} \sqrt{b+a\cos[c+dx]} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) - \left(i \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{b+a\cos[c+dx]} \right)$$

$$\left(\sqrt{a-b} \operatorname{Log} \left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \cot\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{a+b}} \right] - \right.$$

$$\left. \sqrt{a+b} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]^2 \right] \right)$$

$$\left. \sec[c+dx]^{3/2} \sin[c+dx] \right/ \left(4\sqrt{a-b} \sqrt{a+b} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) -$$

$$\left(i \sqrt{b+a\cos[c+dx]} \left(\sqrt{a-b} \operatorname{Log} \left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \cot\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{a+b}} \right] - \right. \right.$$

$$\left. \left. \sqrt{a+b} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]^2 \right] \right) \right)$$

$$\begin{aligned}
& \left. \sqrt{\sec[c+dx]} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \right/ \\
& \left(4\sqrt{a-b}\sqrt{a+b} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} + i \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{b+a\cos[c+dx]} \right. \\
& \left. \left[\sqrt{a-b} \operatorname{Log} \left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \operatorname{Cot} \left[\frac{1}{2}(c+dx) \right]^2}{\sqrt{a+b}} \right] - \right. \right. \\
& \left. \left. \sqrt{a+b} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2}(c+dx) \right]^2 \right] \right] \right) \\
& \left. \sqrt{\sec[c+dx]} \left(-\frac{a\sin[c+dx]}{1+\cos[c+dx]} + \frac{(b+a\cos[c+dx])\sin[c+dx]}{(1+\cos[c+dx])^2} \right) \right/ \left(4\sqrt{a-b}\sqrt{a+b} \left(\frac{b+a\cos[c+dx]}{1+\cos[c+dx]} \right)^{3/2} \right) - \\
& \frac{1}{2\sqrt{a-b}\sqrt{a+b} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}}} i \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{b+a\cos[c+dx]} \sqrt{\sec[c+dx]} \\
& \left(\left(\sqrt{a-b} \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \operatorname{Cot} \left[\frac{1}{2}(c+dx) \right] \operatorname{Csc} \left[\frac{1}{2}(c+dx) \right]^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \cot\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{2\sqrt{a+b} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx]\sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \\
 & \left. \frac{2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{a\sin[c+dx]}{1+\cos[c+dx]} + \frac{(b+a\cos[c+dx])\sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}}} \right) \Bigg/ \left(2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \right. \\
 & \left. \left. \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \cot\left[\frac{1}{2}(c+dx)\right]^2 \right) - \left(\sqrt{a+b} \left(\frac{2\sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx]\sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \right. \right. \\
 & \left. \left. \frac{2\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{a\sin[c+dx]}{1+\cos[c+dx]} + \frac{(b+a\cos[c+dx])\sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}}} - 2i\sqrt{a-b} \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg/ \\
 & \left(\frac{2ia}{\sqrt{a-b}} + 4\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \Bigg) \Bigg) - \\
 & \left(i\sqrt{-1-\cos[c+dx]} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \cos[2(c+dx)] \csc[\right. \\
 & \left. c + \right. \\
 & \left. dx \right]
 \end{aligned}$$

$$\left(\frac{\sqrt{a} \sqrt{a-b} \operatorname{Log} \left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \cot \left[\frac{1}{2} (c+dx) \right]^2}{\sqrt{a+b}} \right]}{\sqrt{a+b}} \right) -$$

$$\sqrt{a+b} \left(\sqrt{a} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right.$$

$$\left. \left. 4\sqrt{a-b} \operatorname{Log} \left[-\frac{1}{8\sqrt{a}} i \cos \left[\frac{1}{2} (c+dx) \right]^2 \left(-2a-b-4i\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} + (2a-b) \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right] \right] \right)$$

$$\operatorname{Sec}[c+dx] \left/ \left(4\sqrt{a} \sqrt{a-b} \sqrt{a+b} d \sqrt{b+a\cos[c+dx]} \right) \right.$$

$$\sqrt{a+b} \operatorname{Sec}[c+dx]$$

$$\left(-\frac{1}{4\sqrt{a-b} \sqrt{a+b} (b+a\cos[c+dx])^{3/2}} i\sqrt{a} \sqrt{-1-\cos[c+dx]} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right)$$

$$\left(\frac{\sqrt{a} \sqrt{a-b} \operatorname{Log} \left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} \right) \cot \left[\frac{1}{2} (c+dx) \right]^2}{\sqrt{a+b}} \right]}{\sqrt{a+b}} \right) -$$

$$\sqrt{a+b} \left(\sqrt{a} \operatorname{Log} \left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{1+\cos[c+dx]}} - 2i\sqrt{a-b} \tan \left[\frac{1}{2} (c+dx) \right]^2 \right] + 4\sqrt{a-b} \operatorname{Log} \left[\right. \right.$$

$$\left. \left. \left. \left. \left. -\frac{1}{8\sqrt{a}} i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \left(-2a-b-4i\sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (2a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right] \right] \right) \right) \right) \right)$$

$$\operatorname{Sin}[c+dx] - \frac{1}{4\sqrt{a}\sqrt{a-b}\sqrt{a+b}\sqrt{-1-\operatorname{Cos}[c+dx]}\sqrt{b+a\operatorname{Cos}[c+dx]}} i \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}}$$

$$\left(\sqrt{a}\sqrt{a-b} \operatorname{Log}\left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \right) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{a+b}} \right] - \right.$$

$$\left. \sqrt{a+b} \left(\sqrt{a} \operatorname{Log}\left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} - 2i\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right] + 4\sqrt{a-b} \right) \right)$$

$$\left. \left. \left. \left. \left. \operatorname{Log}\left[-\frac{1}{8\sqrt{a}} i \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \left(-2a-b-4i\sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (2a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right] \right] \right) \right) \right) \right) \right)$$

$$\operatorname{Sin}[c+dx] - \frac{1}{4\sqrt{a}\sqrt{a-b}\sqrt{a+b}\sqrt{b+a\operatorname{Cos}[c+dx]}\sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}}} i \sqrt{-1-\operatorname{Cos}[c+dx]}$$

$$\left(\sqrt{a}\sqrt{a-b} \operatorname{Log}\left[\frac{2ia + \left(-2ia - 2ib + 4\sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \right) \operatorname{Cot}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{a+b}} \right] - \right.$$

$$\left. \sqrt{a+b} \left(\sqrt{a} \operatorname{Log}\left[\frac{2ia}{\sqrt{a-b}} + 4 \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} - 2i\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right] + 4\sqrt{a-b} \right) \right)$$

$$\begin{aligned}
& \frac{(b + a \cos [c + d x]) \left(\frac{a}{2(a^2 - b^2)} + \frac{(a - b \cos [c + d x]) \operatorname{Csc}[c + d x]^2}{2(-a^2 + b^2)} \right) \operatorname{Sec}[c + d x]}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\
& \left((b + a \cos [c + d x]) \left(-\frac{8(a^2 - b^2) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2\right]}{\sqrt{a}} + \frac{(a - b)(4a + 5b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]}{\sqrt{a + b}} - \right. \right. \\
& \frac{(a - b)(4a + 5b) \operatorname{Log}\left[a + b + 2\sqrt{a + b} \sqrt{-\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{-b - a \cos [c + d x]}{1 + \cos [c + d x]}} - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]}{\sqrt{a + b}} + \\
& \frac{(4a^2 - ab - 5b^2) \operatorname{Log}\left[-a + 2\sqrt{a - b} \sqrt{-\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{-b - a \cos [c + d x]}{1 + \cos [c + d x]}} + (a - b) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]}{\sqrt{a - b}} + \\
& \left. \left. \frac{8(a^2 - b^2) \operatorname{Log}\left[2a + b + 4\sqrt{a} \sqrt{-\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{-b - a \cos [c + d x]}{1 + \cos [c + d x]}} + (-2a + b) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right]}{\sqrt{a}} \right) \right) \\
& \left(\frac{ab \operatorname{Csc}[c + d x]}{4(-a^2 + b^2) \sqrt{b + a \cos [c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{a^2 \operatorname{Csc}[c + d x] \sqrt{\operatorname{Sec}[c + d x]}}{2(-a^2 + b^2) \sqrt{b + a \cos [c + d x]}} - \frac{3b^2 \operatorname{Csc}[c + d x] \sqrt{\operatorname{Sec}[c + d x]}}{4(-a^2 + b^2) \sqrt{b + a \cos [c + d x]}} + \right. \\
& \left. \frac{a^2 \cos [2(c + d x)] \operatorname{Csc}[c + d x] \sqrt{\operatorname{Sec}[c + d x]}}{2(-a^2 + b^2) \sqrt{b + a \cos [c + d x]}} - \frac{b^2 \cos [2(c + d x)] \operatorname{Csc}[c + d x] \sqrt{\operatorname{Sec}[c + d x]}}{2(-a^2 + b^2) \sqrt{b + a \cos [c + d x]}} \right) \operatorname{Sec}[c + d x] \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \Big/ \\
& \left(8(a^2 - b^2) d \sqrt{-(b + a \cos [c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{a + b \operatorname{Sec}[c + d x]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{8 (a^2 - b^2) \operatorname{Log} \left[2a + b + 4\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (-2a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right]}{\sqrt{a}} \right) \sqrt{\sec[c+dx]} \\
& \left. \operatorname{Sin}[c+dx] \sqrt{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} \right) / \left(16 (a^2 - b^2) \sqrt{b+a\cos[c+dx]} \sqrt{-(b+a\cos[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2} \right) + \\
& \frac{1}{16 (a^2 - b^2) \sqrt{-(b+a\cos[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}} \sqrt{b+a\cos[c+dx]} \\
& \left(-\frac{8 (a^2 - b^2) \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right]}{\sqrt{a}} + \frac{(a-b) (4a+5b) \operatorname{Log} \left[\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right]}{\sqrt{a+b}} - \right. \\
& \left. \frac{(a-b) (4a+5b) \operatorname{Log} \left[a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right]}{\sqrt{a+b}} + \frac{1}{\sqrt{a-b}} \right. \\
& \left. (4a^2 - ab - 5b^2) \operatorname{Log} \left[-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \\
& \left. \frac{8 (a^2 - b^2) \operatorname{Log} \left[2a+b+4\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (-2a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right]}{\sqrt{a}} \right) \\
& \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx] \sqrt{-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2} - \frac{1}{16 (a^2 - b^2) \left(-(b+a\cos[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right)^{3/2}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{b+a \cos [c+d x]} \left(-\frac{8\left(a^2-b^2\right) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a}}+\frac{(a-b)(4 a+5 b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a+b}}-\right. \\
& \frac{(a-b)(4 a+5 b) \operatorname{Log}\left[a+b+2 \sqrt{a+b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a+b}}+\frac{1}{\sqrt{a-b}} \\
& \left.(4 a^2-a b-5 b^2) \operatorname{Log}\left[-a+2 \sqrt{a-b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}+(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \right. \\
& \left. \frac{8\left(a^2-b^2\right) \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}+(-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a}}\right) \sqrt{\operatorname{Sec}[c+d x]} \\
& \left(a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sin [c+d x]-(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right) \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}+ \\
& \frac{1}{8\left(a^2-b^2\right) \sqrt{-(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{b+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \left(\frac{(a-b)(4 a+5 b) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}-\frac{8\left(a^2-b^2\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a}}-\right. \\
& \left.(a-b)(4 a+5 b) \frac{\sqrt{a+b} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}\left(-\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2}+\frac{\sin [c+d x]}{1+\cos [c+d x]}\right)}{\sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}}}+ \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}}{\sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}} - a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \right) / \\
& \left(\sqrt{a+b} \left(a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(4a^2 - ab - 5b^2 \right) \left(\frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \\
& \left. \frac{\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}}{\sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}} + (a-b) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) / \\
& \left(\sqrt{a-b} \left(-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} + (a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(8(a^2 - b^2) \left(\frac{2\sqrt{a} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \right. \\
& \left. \left. \frac{2\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}}{\sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}} + (-2a+b) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right) \right) / \\
& \left(\sqrt{a} \left(2a+b+4\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} + (-2a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 331: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[c + d x]^4}{\sqrt{a + b \text{Sec}[c + d x]}} dx$$

Optimal (type 4, 404 leaves, 11 steps):

$$\frac{2 \sqrt{a+b} \text{Cot}[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{15 b^4 d}}{a d}$$

$$2(a-b) \sqrt{a+b} (8 a^2 - 21 b^2) \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{-\frac{b(-1+\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{15 b^3 d} 2 \sqrt{a+b} (-8 a^2 + 2 a b + 21 b^2) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{-\frac{b(-1+\text{Sec}[c+dx])}{a+b}}$$

$$\sqrt{\frac{b(1+\text{Sec}[c+dx])}{-a+b}} - \frac{8 a \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{15 b^2 d} + \frac{2 \text{Sec}[c+dx] \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{5 b d}$$

Result (type 4, 839 leaves):

- **Problem 332: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[c + dx]^2}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 310 leaves, 6 steps):

$$\begin{aligned} & -\frac{1}{b^2 d} 2(a-b)\sqrt{a+b} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{2\sqrt{a+b} \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{bd} + \\ & \frac{2\sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{ad} \end{aligned}$$

Result (type 4, 2752 leaves):

$$\begin{aligned} & \frac{2(b+a \cos[c+dx]) \tan[c+dx]}{bd \sqrt{a+b \sec[c+dx]}} - \left(4 \sqrt{b+a \cos[c+dx]} \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx]} \right. \\ & \left(-i(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \\ & \left. 2ib \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \\ & \left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a \cos[c+dx]) \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) / \\ & \left(b^2 \sqrt{\frac{-a+b}{a+b}} d \sqrt{\cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^4} \sqrt{a+b \sec[c+dx]} \right) \end{aligned}$$

$$\left(\frac{1}{b \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \cos[c+dx]} \sqrt{\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}} 2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right.$$

$$\left. -i(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right.$$

$$2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a \cos[c+dx]) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) +$$

$$\frac{1}{b \sqrt{\frac{-a+b}{a+b}} (b+a \cos[c+dx])^{3/2} \sqrt{\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}} a \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \operatorname{Sin}[c+dx]}$$

$$\left(-i(a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right.$$

$$2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} (b+a \cos[c+dx]) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) -$$

$$\frac{1}{b \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \cos[c+dx]} \left(\cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4\right)^{3/2}} \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]}$$

$$\begin{aligned}
& \left(-i (a-b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} - \right. \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} + \\
& \left. \sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} (b+a \operatorname{Cos}[c+dx]) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \\
& \left(-\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4 \operatorname{Sin}[c+dx] + 2 \operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) + \\
& \frac{1}{b \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4}} 2 \sqrt{\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sec}[c+dx] \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)} \\
& \left(\frac{\sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} (b+a \operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{\sqrt{2}} - \sqrt{2} a \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \operatorname{Sin}[c+dx] \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] + \right. \\
& \left. \frac{\sqrt{\frac{-a+b}{a+b}} (b+a \operatorname{Cos}[c+dx]) \left(\frac{\operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{(1+\operatorname{Cos}[c+dx])^2} - \frac{\operatorname{Sin}[c+dx]}{1+\operatorname{Cos}[c+dx]} \right) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{2} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}}} - \left(i (a-b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \left(-\frac{a \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Sin}[c+dx]}{a+b} + \frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{a+b} \right) \right) \right) / \\
& \left(2 \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} - \left(i b \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \right) \right)
\end{aligned}$$

$$\left(-\frac{a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sin}[c+dx]}{a+b} + \frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{a+b} \right) /$$

$$\left(\sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \frac{b \sqrt{\frac{-a+b}{a+b}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\left(1 - \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a-b}\right) \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}\right)} +$$

$$\frac{(a-b) \sqrt{\frac{-a+b}{a+b}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a-b}}}{2 \sqrt{1 + \frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}\right) +$$

$$\left(\left(-i (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right.$$

$$2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\sqrt{2} \sqrt{\frac{-a+b}{a+b}} \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} (b+a \operatorname{Cos}[c+dx]) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^2 \right)$$

$$\left(-\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) /$$

$$\left(b \sqrt{\frac{-a+b}{a+b}} \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Cos}[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4} \sqrt{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Sec}[c+dx]} \right)$$

■ **Problem 333: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a d}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

■ **Problem 334: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c + d x]^2}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 361 leaves, 9 steps):

$$\frac{\operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{\sqrt{a + b} d} -$$

$$\frac{\operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{\sqrt{a + b} d} +$$

$$\frac{2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a d} -$$

$$\frac{\operatorname{Cot}[c + d x]}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \frac{b^2 \operatorname{Tan}[c + d x]}{(a^2 - b^2) d \sqrt{a + b \operatorname{Sec}[c + d x]}}$$

Result (type 4, 1198 leaves):

$$\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \left(\frac{(-b + a \operatorname{Cos}[c + d x]) \operatorname{Csc}[c + d x]}{-a^2 + b^2} + \frac{b \operatorname{Sin}[c + d x]}{-a^2 + b^2} \right)}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} -$$

$$\left(\sqrt{b + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \left(a b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + b^2 \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 2 a b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + \right.$$

$$\begin{aligned}
& a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 4 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 4 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& i (a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - i (2 a^2 - a b - b^2) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right) \Bigg) \Bigg) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left. \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4\right) \right)
\end{aligned}$$

Problem 337: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[c + d x]}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 54 leaves, 4 steps) :

$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{2}{a d \sqrt{a+b \text{Sec}[c+d x]}}$$

Result (type 3, 128 leaves) :

$$\frac{1}{a^2 d \sqrt{a+b \text{Sec}[c+d x]}} \left(2 a \text{Cos}[c+d x] + \sqrt{a \text{Cos}[c+d x]} \sqrt{b+a \text{Cos}[c+d x]} \left(\text{Log}\left[1 - \frac{\sqrt{b+a \text{Cos}[c+d x]}}{\sqrt{a \text{Cos}[c+d x]}}\right] - \text{Log}\left[1 + \frac{\sqrt{b+a \text{Cos}[c+d x]}}{\sqrt{a \text{Cos}[c+d x]}}\right] \right) \right) \text{Sec}[c+d x]$$

■ **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[c + d x]}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 142 leaves, 7 steps) :

$$\frac{2 \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{(a-b)^{3/2} d} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{(a+b)^{3/2} d} + \frac{2 b^2}{a (a^2 - b^2) d \sqrt{a+b \text{Sec}[c+d x]}}$$

Result (type 3, 6484 leaves) :

$$\frac{(b + a \text{Cos}[c + d x])^2 \left(-\frac{2 b^2}{a^2 (-a^2 + b^2)} - \frac{2 b^3}{a^2 (a^2 - b^2) (b + a \text{Cos}[c + d x])} \right) \text{Sec}[c + d x]^2}{d (a + b \text{Sec}[c + d x])^{3/2}} -$$

$$\left(\sqrt{-\frac{\text{Cos}[c + d x]}{1 + \text{Cos}[c + d x]}} (b + a \text{Cos}[c + d x])^2 \left(-2 \sqrt{a - b} \sqrt{a + b} (a^2 - b^2) \text{Log}\left[\text{Sec}\left[\frac{1}{2} (c + d x)\right]^2\right] + a^{3/2} (a - b)^{3/2} \text{Log}\left[\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) - \right.$$

$$a^{5/2} \sqrt{a - b} \text{Log}\left[a + b + 2 \sqrt{a + b} \sqrt{-\frac{\text{Cos}[c + d x]}{1 + \text{Cos}[c + d x]}} \sqrt{\frac{-b - a \text{Cos}[c + d x]}{1 + \text{Cos}[c + d x]}} - a \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] +$$

$$a^{3/2} \sqrt{a - b} b \text{Log}\left[a + b + 2 \sqrt{a + b} \sqrt{-\frac{\text{Cos}[c + d x]}{1 + \text{Cos}[c + d x]}} \sqrt{\frac{-b - a \text{Cos}[c + d x]}{1 + \text{Cos}[c + d x]}} - a \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right] +$$

$$\begin{aligned}
& a^{5/2} \sqrt{a+b} \operatorname{Log} \left[-a + 2\sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \\
& a^{3/2} b \sqrt{a+b} \operatorname{Log} \left[-a + 2\sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \\
& 2a^2 \sqrt{a-b} \sqrt{a+b} \operatorname{Log} \left[2a+b+4\sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (-2a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \\
& 2\sqrt{a-b} b^2 \sqrt{a+b} \operatorname{Log} \left[2a+b+4\sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (-2a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \Big) \\
& \left(-\frac{b \operatorname{Csc}[c+dx]}{(a^2-b^2) \sqrt{b+a\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} + \frac{a \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2(a^2-b^2) \sqrt{b+a\operatorname{Cos}[c+dx]}} + \frac{b^2 \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2a(a^2-b^2) \sqrt{b+a\operatorname{Cos}[c+dx]}} + \right. \\
& \left. \frac{a \operatorname{Cos}[2(c+dx)] \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2(a^2-b^2) \sqrt{b+a\operatorname{Cos}[c+dx]}} - \frac{b^2 \operatorname{Cos}[2(c+dx)] \operatorname{Csc}[c+dx] \sqrt{\operatorname{Sec}[c+dx]}}{2a(a^2-b^2) \sqrt{b+a\operatorname{Cos}[c+dx]}} \right) \operatorname{Sec}[c+dx]^2 \Big) / \\
& \left(2a^{3/2} \sqrt{a-b} \sqrt{a+b} (a^2-b^2) d \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} (a+b \operatorname{Sec}[c+dx])^{3/2} \left(\frac{1}{4\sqrt{a} \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{b+a\operatorname{Cos}[c+dx]}} \right. \right. \\
& \left. \left. \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \left(-2\sqrt{a-b} \sqrt{a+b} (a^2-b^2) \operatorname{Log} \left[\operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \right] + a^{3/2} (a-b)^{3/2} \operatorname{Log} \left[\operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] - \right. \right. \right. \\
& a^{5/2} \sqrt{a-b} \operatorname{Log} \left[a+b+2\sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \\
& a^{3/2} \sqrt{a-b} b \operatorname{Log} \left[a+b+2\sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} - a \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \\
& a^{5/2} \sqrt{a+b} \operatorname{Log} \left[-a+2\sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{-b-a\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} + (a-b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] +
\end{aligned}$$

$$\begin{aligned}
& a^{3/2} b \sqrt{a+b} \operatorname{Log}\left[-a+2 \sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \\
& 2 a^2 \sqrt{a-b} \sqrt{a+b} \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]- \\
& 2 \sqrt{a-b} b^2 \sqrt{a+b} \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \Big) \\
& \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]-\frac{1}{4 a^{3/2} \sqrt{a-b} \sqrt{a+b}\left(a^2-b^2\right) \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{b+a \operatorname{Cos}[c+d x]} \\
& \left(-2 \sqrt{a-b} \sqrt{a+b}\left(a^2-b^2\right) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right]+a^{3/2}(a-b)^{3/2} \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]-\right. \\
& a^{5/2} \sqrt{a-b} \operatorname{Log}\left[a+b+2 \sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \\
& a^{3/2} \sqrt{a-b} b \operatorname{Log}\left[a+b+2 \sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \\
& a^{5/2} \sqrt{a+b} \operatorname{Log}\left[-a+2 \sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \\
& a^{3/2} b \sqrt{a+b} \operatorname{Log}\left[-a+2 \sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]+ \\
& 2 a^2 \sqrt{a-b} \sqrt{a+b} \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]- \\
& 2 \sqrt{a-b} b^2 \sqrt{a+b} \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}}+(-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \Big)
\end{aligned}$$

$$\begin{aligned}
& \text{Sec}[c+dx]^{3/2} \text{Sin}[c+dx] - \frac{1}{4 a^{3/2} \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}}} \\
& \sqrt{b+a \text{Cos}[c+dx]} \left(-2 \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \text{Log}\left[\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right] + a^{3/2} (a-b)^{3/2} \text{Log}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& a^{5/2} \sqrt{a-b} \text{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& a^{3/2} \sqrt{a-b} b \text{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& a^{5/2} \sqrt{a+b} \text{Log}\left[-a+2\sqrt{a-b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} + (a-b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& a^{3/2} b \sqrt{a+b} \text{Log}\left[-a+2\sqrt{a-b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} + (a-b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& 2 a^2 \sqrt{a-b} \sqrt{a+b} \text{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} + (-2 a+b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \\
& \left. 2 \sqrt{a-b} b^2 \sqrt{a+b} \text{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} + (-2 a+b) \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \\
& \sqrt{\text{Sec}[c+dx]} \left(-\frac{\text{Cos}[c+dx] \text{Sin}[c+dx]}{(1+\text{Cos}[c+dx])^2} + \frac{\text{Sin}[c+dx]}{1+\text{Cos}[c+dx]} \right) + \frac{1}{4 a^{3/2} \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \left(\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}\right)^{3/2}} \\
& \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{b+a \text{Cos}[c+dx]} \left(-2 \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \text{Log}\left[\text{Sec}\left[\frac{1}{2}(c+dx)\right]^2\right] + a^{3/2} (a-b)^{3/2} \text{Log}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] - \right. \\
& a^{5/2} \sqrt{a-b} \text{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& a^{3/2} \sqrt{a-b} b \text{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} \sqrt{\frac{-b-a \text{Cos}[c+dx]}{1+\text{Cos}[c+dx]}} - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] +
\end{aligned}$$

$$\begin{aligned}
& a^{5/2} \sqrt{a+b} \operatorname{Log} \left[-a + 2 \sqrt{a-b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (a-b) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right] + \\
& a^{3/2} b \sqrt{a+b} \operatorname{Log} \left[-a + 2 \sqrt{a-b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (a-b) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right] + \\
& 2 a^2 \sqrt{a-b} \sqrt{a+b} \operatorname{Log} \left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (-2 a+b) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right] - \\
& 2 \sqrt{a-b} b^2 \sqrt{a+b} \operatorname{Log} \left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (-2 a+b) \tan \left[\frac{1}{2} (c+d x) \right]^2 \right] \Bigg) \\
& \sqrt{\sec [c+d x]} \left(\frac{a \sin [c+d x]}{1+\cos [c+d x]} + \frac{(-b-a \cos [c+d x]) \sin [c+d x]}{(1+\cos [c+d x])^2} \right) - \frac{1}{2 a^{3/2} \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}} \\
& \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]} \left(a^{3/2} (a-b)^{3/2} \operatorname{Csc} \left[\frac{1}{2} (c+d x) \right] \sec \left[\frac{1}{2} (c+d x) \right] - \right. \\
& 2 \sqrt{a-b} \sqrt{a+b} (a^2-b^2) \tan \left[\frac{1}{2} (c+d x) \right] - \left. a^{5/2} \sqrt{a-b} \left(\frac{\sqrt{a+b} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} \left(-\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} + \frac{\sin [c+d x]}{1+\cos [c+d x]} \right)}{\sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \right. \right. \\
& \left. \left. \frac{\sqrt{a+b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \left(\frac{a \sin [c+d x]}{1+\cos [c+d x]} + \frac{(-b-a \cos [c+d x]) \sin [c+d x]}{(1+\cos [c+d x])^2} \right)}{\sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}} - a \sec \left[\frac{1}{2} (c+d x) \right]^2 \tan \left[\frac{1}{2} (c+d x) \right] \right) \right) / \\
& \left(a+b+2 \sqrt{a+b} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} - a \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) + \left(a^{3/2} \sqrt{a-b} b \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sqrt{a+b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) + \sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{\sqrt{a+b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} - a \right. \\
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Bigg/ \left(a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} - a \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left(a^{5/2} \sqrt{a+b} \left(\frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) + \sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \\
& \left. (a-b) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Bigg/ \left(-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} + \right. \\
& \left. (a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) + \left(a^{3/2} b \sqrt{a+b} \left(\frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) + \right. \right. \\
& \left. \left. \frac{\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + (a-b) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \Bigg/ \\
& \left(-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} + (a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) +
\end{aligned}$$

$$\left(2 a^2 \sqrt{a-b} \sqrt{a+b} \left(\frac{2 \sqrt{a} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} \left(-\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} + \frac{\sin [c+d x]}{1+\cos [c+d x]} \right)}{\sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}}} + \right. \right.$$

$$\left. \left. \frac{2 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \left(\frac{a \sin [c+d x]}{1+\cos [c+d x]} + \frac{(-b-a \cos [c+d x]) \sin [c+d x]}{(1+\cos [c+d x])^2} \right)}{\sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}} + (-2 a+b) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right) \right) /$$

$$\left(2 a+b+4 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) -$$

$$\left(2 \sqrt{a-b} b^2 \sqrt{a+b} \left(\frac{2 \sqrt{a} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} \left(-\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} + \frac{\sin [c+d x]}{1+\cos [c+d x]} \right)}{\sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}}} + \right. \right.$$

$$\left. \left. \frac{2 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \left(\frac{a \sin [c+d x]}{1+\cos [c+d x]} + \frac{(-b-a \cos [c+d x]) \sin [c+d x]}{(1+\cos [c+d x])^2} \right)}{\sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}}} + (-2 a+b) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \right) \right) /$$

$$\left(2 a+b+4 \sqrt{a} \sqrt{-\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{-b-a \cos [c+d x]}{1+\cos [c+d x]}} + (-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) \right) \right)$$

■ **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+d x]^3}{(a+b \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 236 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2} d} + \frac{(4 a-7 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a-b}}\right]}{4(a-b)^{5/2} d} + \frac{(4 a+7 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right]}{4(a+b)^{5/2} d} + \\
& \frac{2 b^4}{a\left(a^2-b^2\right)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{4(a+b)^2 d(1-\operatorname{Sec}[c+d x])} + \frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{4(a-b)^2 d(1+\operatorname{Sec}[c+d x])}
\end{aligned}$$

Result (type 3, 4191 leaves):

$$\frac{(b+a \operatorname{Cos}[c+d x])^2 \left(\frac{a^4+a^2 b^2+4 b^4}{2 a^2(-a^2+b^2)^2} - \frac{2 b^5}{a^2(a^2-b^2)^2(b+a \operatorname{Cos}[c+d x])} + \frac{(-a^2-b^2+2 a b \operatorname{Cos}[c+d x]) \operatorname{Csc}[c+d x]^2}{2(-a^2+b^2)^2} \right) \operatorname{Sec}[c+d x]^2}{d(a+b \operatorname{Sec}[c+d x])^{3/2}} +$$

$$\left((b+a \operatorname{Cos}[c+d x])^2 \left(-8(a^2-b^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right] + \frac{a^{3/2}(a-b)^2(4 a+7 b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a+b}} \right) - \right.$$

$$\left. \frac{a^{3/2}(a-b)^2(4 a+7 b) \operatorname{Log}\left[a+b+2 \sqrt{a+b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right]}{\sqrt{a+b}} + \frac{1}{\sqrt{a-b}} \right.$$

$$\left. a^{3/2}(4 a-7 b)(a+b)^2 \operatorname{Log}\left[-a+2 \sqrt{a-b} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} + (a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] + \right.$$

$$\left. 8(a^2-b^2)^2 \operatorname{Log}\left[2 a+b+4 \sqrt{a} \sqrt{-\frac{\operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} \sqrt{\frac{-b-a \operatorname{Cos}[c+d x]}{1+\operatorname{Cos}[c+d x]}} + (-2 a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right] \right)$$

$$\left(\frac{a^2 b \operatorname{Csc}[c+d x]}{4(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} - \frac{7 b^3 \operatorname{Csc}[c+d x]}{4(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} - \frac{a^3 \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{2(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \right.$$

$$\left. \frac{3 a b^2 \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{2(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{b^4 \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{2 a(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \frac{a^3 \operatorname{Cos}[2(c+d x)] \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{2(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \right.$$

$$\left. \frac{a b^2 \operatorname{Cos}[2(c+d x)] \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \frac{b^4 \operatorname{Cos}[2(c+d x)] \operatorname{Csc}[c+d x] \sqrt{\operatorname{Sec}[c+d x]}}{2 a(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} \right) \operatorname{Sec}[c+d x]^2 \sqrt{-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} /$$

$$\left(8 a^{3/2} (a^2 - b^2)^2 d \sqrt{-(b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2} (a + b \operatorname{Sec}[c + dx])^{3/2} \right.$$

$$\left. \left(\left(\sqrt{b + a \cos[c + dx]} \left(-8 (a^2 - b^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2\right] + \frac{a^{3/2} (a - b)^2 (4a + 7b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right]}{\sqrt{a + b}} - \frac{1}{\sqrt{a + b}} \right. \right. \right.$$

$$\left. \left. \left. a^{3/2} (a - b)^2 (4a + 7b) \operatorname{Log}\left[a + b + 2\sqrt{a + b} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} - a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \frac{1}{\sqrt{a - b}} \right. \right. \right.$$

$$\left. \left. \left. a^{3/2} (4a - 7b) (a + b)^2 \operatorname{Log}\left[-a + 2\sqrt{a - b} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} + (a - b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \right.$$

$$\left. \left. \left. 8 (a^2 - b^2)^2 \operatorname{Log}\left[2a + b + 4\sqrt{a} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} + (-2a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right.$$

$$\left. \left. \left. \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) \right) / \left(16 a^{3/2} (a^2 - b^2)^2 \sqrt{-(b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2} \right) -$$

$$\left(\left(-8 (a^2 - b^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2\right] + \frac{a^{3/2} (a - b)^2 (4a + 7b) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right]}{\sqrt{a + b}} - \frac{1}{\sqrt{a + b}} \right. \right.$$

$$\left. \left. \left. a^{3/2} (a - b)^2 (4a + 7b) \operatorname{Log}\left[a + b + 2\sqrt{a + b} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} - a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \frac{1}{\sqrt{a - b}} \right. \right. \right.$$

$$\left. \left. \left. a^{3/2} (4a - 7b) (a + b)^2 \operatorname{Log}\left[-a + 2\sqrt{a - b} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} + (a - b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \right.$$

$$\left. \left. \left. 8 (a^2 - b^2)^2 \operatorname{Log}\left[2a + b + 4\sqrt{a} \sqrt{-\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{-b - a \cos[c + dx]}{1 + \cos[c + dx]}} + (-2a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \sqrt{\operatorname{Sec}[c + dx]} \right.$$

$$\begin{aligned}
& \left. \frac{\sin[c+dx] \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}{16\sqrt{a} (a^2-b^2)^2 \sqrt{b+a\cos[c+dx]} \sqrt{-(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \frac{1}{16 a^{3/2} (a^2 - b^2)^2 \sqrt{-(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{b+a\cos[c+dx]} \\
& \left(-8 (a^2 - b^2)^2 \operatorname{Log}\left[\sec\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{a^{3/2} (a-b)^2 (4a+7b) \operatorname{Log}\left[\tan\left[\frac{1}{2}(c+dx)\right]^2\right]}{\sqrt{a+b}} - \frac{1}{\sqrt{a+b}} \right. \\
& a^{3/2} (a-b)^2 (4a+7b) \operatorname{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} - a \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{1}{\sqrt{a-b}} \\
& a^{3/2} (4a-7b) (a+b)^2 \operatorname{Log}\left[-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& \left. 8 (a^2 - b^2)^2 \operatorname{Log}\left[2a+b+4\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (-2a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \\
& \frac{\sec[c+dx]^{3/2} \sin[c+dx] \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}{16 a^{3/2} (a^2 - b^2)^2 \left(-(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{3/2}} - \frac{1}{16 a^{3/2} (a^2 - b^2)^2 \left(-(b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{3/2}} \\
& \sqrt{b+a\cos[c+dx]} \left(-8 (a^2 - b^2)^2 \operatorname{Log}\left[\sec\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{a^{3/2} (a-b)^2 (4a+7b) \operatorname{Log}\left[\tan\left[\frac{1}{2}(c+dx)\right]^2\right]}{\sqrt{a+b}} - \frac{1}{\sqrt{a+b}} \right. \\
& a^{3/2} (a-b)^2 (4a+7b) \operatorname{Log}\left[a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} - a \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \frac{1}{\sqrt{a-b}} \\
& a^{3/2} (4a-7b) (a+b)^2 \operatorname{Log}\left[-a+2\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2\right] + \\
& \left. 8 (a^2 - b^2)^2 \operatorname{Log}\left[2a+b+4\sqrt{a} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a\cos[c+dx]}{1+\cos[c+dx]}} + (-2a+b) \tan\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \sqrt{\sec[c+dx]}
\end{aligned}$$

$$\begin{aligned}
& \left(a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] - (b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} + \\
& \frac{1}{8 a^{3/2} (a^2 - b^2)^2 \sqrt{-(b+a \cos[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{b+a \cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(\frac{a^{3/2} (a-b)^2 (4a+7b) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}} - 8 (a^2 - b^2)^2 \tan\left[\frac{1}{2}(c+dx)\right] - \right. \\
& \left. \left(a^{3/2} (a-b)^2 (4a+7b) \left(\frac{\sqrt{a+b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \right. \\
& \left. \left. \frac{\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}} - a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left(\sqrt{a+b} \left(a+b+2\sqrt{a+b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} - a \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) + \\
& \left(a^{3/2} (4a-7b) (a+b)^2 \left(\frac{\sqrt{a-b} \sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}} \left(-\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} + \frac{\sin[c+dx]}{1+\cos[c+dx]} \right)}{\sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}}} + \right. \right. \\
& \left. \left. \frac{\sqrt{a-b} \sqrt{-\frac{\cos[c+dx]}{1+\cos[c+dx]}} \left(\frac{a \sin[c+dx]}{1+\cos[c+dx]} + \frac{(-b-a \cos[c+dx]) \sin[c+dx]}{(1+\cos[c+dx])^2} \right)}{\sqrt{\frac{-b-a \cos[c+dx]}{1+\cos[c+dx]}}} + (a-b) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \frac{(b + a \cos [c + d x])^2 \operatorname{Sec}[c + d x]^2 \left(-\frac{2 \sin [c + d x]}{a b} + \frac{2 \sin [c + d x]}{a (b + a \cos [c + d x])} \right)}{d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \\
& \left(2 (b + a \cos [c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2} \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \right. \\
& \left. a \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} + b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} - \right. \\
& a \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} + b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right]^3 \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} + \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right] + \\
& 2 i b \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \tan \left[\frac{1}{2} (c + d x) \right]^2 \right. \\
& \left. \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - i (a - b) \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \right. \\
& \left. \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - 2 i b \right. \\
& \left. \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right] \right) \right) / \\
& \left(a b \sqrt{\frac{-a + b}{a + b}} d (a + b \operatorname{Sec}[c + d x])^{3/2} \left(1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^{3/2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right)
\end{aligned}$$

■ **Problem 342: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 347 leaves, 6 steps) :

$$\frac{2 \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a \sqrt{a+b} d} -$$

$$\frac{2 \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a \sqrt{a+b} d} -$$

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a^2 d} + \frac{2 b^2 \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}$$

Result (type 4, 1249 leaves) :

$$\frac{(b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \left(\frac{2 b \operatorname{Sin}[c+dx]}{a(-a^2+b^2)} + \frac{2 b^2 \operatorname{Sin}[c+dx]}{a(a^2-b^2)(b+a \operatorname{Cos}[c+dx])} \right)}{d (a + b \operatorname{Sec}[c + d x])^{3/2}} +$$

$$\left(2 (b + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right.$$

$$\left(a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 2 a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - \right.$$

$$b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - 2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}$$

$$\sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} + 2 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} -$$

$$2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+2 i b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}- \\
& i(a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+i\left(a^2+a b-2 b^2\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left.\left.\left.\left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right)\right]\right)\right) / \\
& \left(a \sqrt{\frac{-a+b}{a+b}}\left(a^2-b^2\right) d(a+b \operatorname{Sec}[c+dx])^{3 / 2}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right. \\
& \left.\left.\left.\left.\left(a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)-b\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)\right)\right)\right)
\end{aligned}$$

■ **Problem 343: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^{3 / 2}} dx$$

Optimal (type 4, 449 leaves, 14 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a+b} \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a^2 d} + \\
& \frac{2(a^2+b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{-\frac{b(-1+\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a(a-b)(a+b)^{3/2} d} - \frac{1}{a(a-b)(a+b)^{3/2} d} \\
& (a^2-ab+2b^2) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{-\frac{b(-1+\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{\operatorname{Cot}[c+dx]}{d(a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{b^2 \operatorname{Tan}[c+dx]}{(a^2-b^2)d(a+b \operatorname{Sec}[c+dx])^{3/2}} + \frac{2b^2(a^2+b^2) \operatorname{Tan}[c+dx]}{a(a^2-b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 4, 4307 leaves):

$$\begin{aligned}
& \frac{1}{d(a+b \operatorname{Sec}[c+dx])^{3/2}} (b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^2 \\
& \left(\frac{(2ab-a^2 \operatorname{Cos}[c+dx]-b^2 \operatorname{Cos}[c+dx]) \operatorname{Csc}[c+dx]}{(-a^2+b^2)^2} - \frac{2b(a^2+b^2) \operatorname{Sin}[c+dx]}{a(a^2-b^2)^2} + \frac{2b^4 \operatorname{Sin}[c+dx]}{a(a^2-b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right) - \\
& \left(2 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^2 (b+a \operatorname{Cos}[c+dx]) \right. \\
& \left(-\frac{a^3}{(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} + \frac{3ab^2}{(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} + \frac{a^2 b \sqrt{\operatorname{Sec}[c+dx]}}{2(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]}} \right. \\
& \left. \frac{b^3 \sqrt{\operatorname{Sec}[c+dx]}}{2(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{a^2 b \operatorname{Cos}[2(c+dx)] \sqrt{\operatorname{Sec}[c+dx]}}{(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{b^3 \operatorname{Cos}[2(c+dx)] \sqrt{\operatorname{Sec}[c+dx]}}{(-a^2+b^2)^2 \sqrt{b+a \operatorname{Cos}[c+dx]}} \right) \operatorname{Sec}[c+dx]^2 \\
& \left(-2ib(-a^3+a^2b-ab^2+b^3) \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] + \right. \\
& \left. i(2a^4-a^3b-2a^2b^2-3ab^3+4b^4) \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{(a+b)(1+\operatorname{Cos}[c+dx])}} \right. \\
& \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 i (a^2 - b^2)^2 \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] - \\
& b \sqrt{\frac{-a + b}{a + b}} (a^2 + b^2) \cos [c + d x] (b + a \cos [c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \tan\left[\frac{1}{2}(c + d x)\right] \Bigg) \Bigg) / \\
& \left(a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^{3/2} \left(\frac{1}{a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - 2 \cos\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]} \sin\left[\frac{1}{2}(c + d x)\right] \right) \right. \\
& \left. \left(-2 i b (-a^3 + a^2 b - a b^2 + b^3) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] + \right. \right. \\
& \left. \left. i (2 a^4 - a^3 b - 2 a^2 b^2 - 3 a b^3 + 4 b^4) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] - 4 i (a^2 - b^2)^2 \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] - b \sqrt{\frac{-a + b}{a + b}} (a^2 + b^2) \cos [c + d x] (b + a \cos [c + d x]) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \tan\left[\frac{1}{2}(c + d x)\right] \right) \Bigg) - \\
& \frac{1}{\sqrt{\frac{-a + b}{a + b}} (a^2 - b^2)^2 (b + a \cos [c + d x])^{3/2}} \cos\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x] \left(-2 i b (-a^3 + a^2 b - a b^2 + b^3) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \right. \\
& \left. \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] + i (2 a^4 - a^3 b - 2 a^2 b^2 - 3 a b^3 + 4 b^4) \right. \\
& \left. \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a + b}{a - b}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 i (a^2 - b^2)^2 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] - \\
& b \sqrt{\frac{-a + b}{a + b}} (a^2 + b^2) \cos[c + dx] (b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] \Bigg) - \\
& \frac{1}{a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2)^2 \sqrt{b + a \cos[c + dx]}} \cos\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx] \left(-2 i b (-a^3 + a^2 b - a b^2 + b^3) \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \right. \\
& \left. \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] + i (2 a^4 - a^3 b - 2 a^2 b^2 - 3 a b^3 + 4 b^4) \right. \\
& \left. \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] - \right. \\
& \left. 4 i (a^2 - b^2)^2 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] - \right. \\
& \left. b \sqrt{\frac{-a + b}{a + b}} (a^2 + b^2) \cos[c + dx] (b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] \right) - \frac{1}{a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2)^2 \sqrt{b + a \cos[c + dx]}} \\
& 2 \cos\left[\frac{1}{2}(c + dx)\right]^2 \sqrt{\operatorname{Sec}[c + dx]} \left(-\frac{1}{2} b \sqrt{\frac{-a + b}{a + b}} (a^2 + b^2) \cos[c + dx] (b + a \cos[c + dx]) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 - \frac{1}{\sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}}} \right. \\
& \left. i b (-a^3 + a^2 b - a b^2 + b^3) \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a + b}{a - b}\right] \right. \\
& \left. \left(\frac{\cos[c + dx] \sin[c + dx]}{(1 + \cos[c + dx])^2} - \frac{\sin[c + dx]}{1 + \cos[c + dx]} \right) + \frac{1}{2 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}}} i (2 a^4 - a^3 b - 2 a^2 b^2 - 3 a b^3 + 4 b^4) \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) - \frac{1}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} \\
& 2i(a^2-b^2)^2 \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left(\frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]}\right) - \frac{1}{\sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} i b(-a^3+a^2b-ab^2+b^3) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \\
& \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(-\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a\cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) + \\
& \frac{1}{2\sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} i(2a^4-a^3b-2a^2b^2-3ab^3+4b^4) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left(-\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a\cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) - \frac{1}{\sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} 2i(a^2-b^2)^2 \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \\
& \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(-\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a\cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2}\right) + \\
& a b \sqrt{\frac{-a+b}{a+b}} (a^2+b^2) \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \tan\left[\frac{1}{2}(c+dx)\right] + b \sqrt{\frac{-a+b}{a+b}} (a^2+b^2) (b+a\cos[c+dx]) \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \sin[c+dx] \tan\left[\frac{1}{2}(c+dx)\right] - b \sqrt{\frac{-a+b}{a+b}} (a^2+b^2) \cos[c+dx] (b+a\cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 - \frac{\sqrt{\frac{-a+b}{a+b}} (2a^4-a^3b-2a^2b^2-3ab^3+4b^4) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sec\left[\frac{1}{2}(c+dx)\right]^2}{2\sqrt{1+\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a-b}} \sqrt{1+\frac{(-a+b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} +
\end{aligned}$$

$$\begin{aligned}
& (d \operatorname{Tan}[e + f x])^n \left(a^3 \operatorname{Tan}[e + f x]^n + 3 a^2 b \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]^n + 3 a b^2 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^n + b^3 \operatorname{Sec}[e + f x]^3 \operatorname{Tan}[e + f x]^n \right) / \\
& \left(f (1+n) (b + a \operatorname{Cos}[e + f x])^3 \left(-\frac{1}{1+n} 2 n \operatorname{Sec}[e + f x]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(-b \left((3 a^2 - 3 a b + b^2) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + 2 b \left((3 a - 2 b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) + \right. \right. \right. \\
& \quad \left. \left. \left. 2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \right) \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^n - \right. \\
& \quad \left(a^3 (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, \right. \right. \\
& \quad \left. \left. 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}[e + f x]^{-1+n} - \\
& \quad \frac{1}{1+n} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \left(-b \left((3 a^2 - 3 a b + b^2) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 b \left((3 a - 2 b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \right) \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^n - \right. \\
& \quad \left(a^3 (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \right) / \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, \right. \right. \\
& \quad \left. \left. n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}[e + f x]^n - \\
& \quad \frac{1}{1+n} 2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(-b n \left((3 a^2 - 3 a b + b^2) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 b \left((3 a - 2 b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \right) \right) \right. \\
& \quad \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \right)^{-1+n} \left(-\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sin}[e + f x] + \operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) + \\
& \quad \left(a^3 (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) /
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2)^2 - \\
& b \left(\cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^2\right)^n \left(\frac{1}{2}(3a^2 - 3ab + b^2)(1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \sec\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^{-1-n} \right) + \\
& \quad 2b \left(b(1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \sec\left[\frac{1}{2}(e+fx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. \left(1 - \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^{-3-n} \right) + \frac{1}{2}(3a - 2b)(1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \sec\left[\frac{1}{2}(e+fx)\right] \right. \right. \\
& \quad \left. \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(e+fx)\right]^2\right)^{-2-n} \right) \right) \right) \tan[e+fx]^n \Big) \Big)
\end{aligned}$$

■ **Problem 345: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + fx])^2 (d \tan[e + fx])^n dx$$

Optimal (type 5, 160 leaves, 7 steps):

$$\begin{aligned}
& \frac{b^2 (d \tan[e + fx])^{1+n}}{df(1+n)} + \frac{a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\tan[e + fx]^2\right] (d \tan[e + fx])^{1+n}}{df(1+n)} + \frac{1}{df(1+n)} \\
& 2ab (\cos[e + fx]^2)^{\frac{2+n}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{2+n}{2}, \frac{3+n}{2}, \sin[e + fx]^2\right] \sec[e + fx] (d \tan[e + fx])^{1+n}
\end{aligned}$$

Result (type 6, 2894 leaves):

$$\begin{aligned}
& \left(2 \cos[e + fx]^2 (a + b \sec[e + fx])^2 \tan\left[\frac{1}{2}(e + fx)\right] \right. \\
& \quad \left(b \left((2a - b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2\right] + 2b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \right. \\
& \quad \left. \left(\cos[e + fx] \sec\left[\frac{1}{2}(e + fx)\right]^2\right)^n + \left(a^2 (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \cos\left[\frac{1}{2}(e + fx)\right]^2 \right) \right) / \\
& \quad \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2, -\tan\left[\frac{1}{2}(e + fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e + fx)\right]^2 \right) \Big) \\
& \quad \left. (d \tan[e + fx])^n (a^2 \tan[e + fx]^n + 2ab \sec[e + fx] \tan[e + fx]^n + b^2 \sec[e + fx]^2 \tan[e + fx]^n) \right) / \\
& \left(f(1+n) (b + a \cos[e + fx])^2 \left(\frac{1}{1+n} 2n \sec[e + fx]^2 \tan\left[\frac{1}{2}(e + fx)\right] \left(b \left((2a - b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e + fx)\right]^2\right] + \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2\right]\left(\cos[e+f x] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n + \\
& \left(a^2(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \cos\left[\frac{1}{2}(e+f x)\right]^2\right) / \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - 2\left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \tan[e+f x]^{-1+n} + \frac{1}{1+n} \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \left(b\left((2 a-b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right.\right. \\
& \quad \left.2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right)\left(\cos[e+f x] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^n + \\
& \left(a^2(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \cos\left[\frac{1}{2}(e+f x)\right]^2\right) / \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - 2\left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \tan[e+f x]^n + \frac{1}{1+n} 2 \tan\left[\frac{1}{2}(e+f x)\right] \left(b n\left((2 a-b) \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2\right] + \right.\right. \\
& \quad \left.2 b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \\
& \left(\cos[e+f x] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right)^{-1+n} \left(-\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \sin[e+f x] + \cos[e+f x] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\right) - \\
& \left(a^2(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \cos\left[\frac{1}{2}(e+f x)\right] \sin\left[\frac{1}{2}(e+f x)\right]\right) / \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - 2\left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, \right.\right.\right. \\
& \quad \left.\left.\left.-\tan\left[\frac{1}{2}(e+f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right]\right) \tan\left[\frac{1}{2}(e+f x)\right]^2\right) + \\
& \left(a^2(3+n) \cos\left[\frac{1}{2}(e+f x)\right]^2\right) \left(-1 / (3+n)(1+n) \operatorname{AppellF1}\left[1+\frac{1+n}{2}, n, 2, 1+\frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \right. \\
& \quad \left.\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right] + 1 / (3+n)n(1+n) \operatorname{AppellF1}\left[1+\frac{1+n}{2}, 1+n, 1, \right.\right. \\
& \quad \left.1+\frac{3+n}{2}, \tan\left[\frac{1}{2}(e+f x)\right]^2, -\tan\left[\frac{1}{2}(e+f x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]\right) /
\end{aligned}$$

$$\begin{aligned}
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(a^2 (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \left(-2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + (3+n) \left(-\frac{1}{3+n} \right. \right. \\
& \quad \quad \left. \left. (1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+n} \right. \right. \\
& \quad \quad \left. \left. n (1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, 1+n, 1, 1 + \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) - \\
& \quad 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{5+n} 2 (3+n) \operatorname{AppellF1}\left[1 + \frac{3+n}{2}, n, 3, 1 + \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \quad \quad \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+n} n (3+n) \operatorname{AppellF1}\left[1 + \frac{3+n}{2}, 1+n, 2, 1 + \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - n \left(-\frac{1}{5+n} (3+n) \operatorname{AppellF1}\left[1 + \frac{3+n}{2}, 1+n, 2, 1 + \frac{5+n}{2}, \right. \right. \right. \\
& \quad \quad \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+n} (1+n) (3+n) \operatorname{AppellF1}\left[1 + \frac{3+n}{2}, \right. \right. \\
& \quad \quad \quad \left. \left. 2+n, 1, 1 + \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) \Big/ \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^2 + \\
& b \left(\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^n \left(b (1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-2-n} \right) + \frac{1}{2} (2a-b) (1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1-n} \right) \right) \operatorname{Tan}[e+fx]^n \Big)
\end{aligned}$$

■ **Problem 346: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]) (d \operatorname{Tan}[e + f x])^n dx$$

Optimal (type 5, 129 leaves, 4 steps):

$$\frac{a \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\operatorname{Tan}[e + f x]^2\right] (d \operatorname{Tan}[e + f x])^{1+n}}{d f (1+n)} +$$

$$\frac{b (\operatorname{Cos}[e + f x])^{\frac{2+n}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, \frac{2+n}{2}, \frac{3+n}{2}, \operatorname{Sin}[e + f x]^2\right] \operatorname{Sec}[e + f x] (d \operatorname{Tan}[e + f x])^{1+n}}{d f (1+n)}$$

Result (type 6, 2597 leaves):

$$\left(2 \operatorname{Cos}[e + f x] (a + b \operatorname{Sec}[e + f x]) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right.$$

$$\left(\left.b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2\right)^n +\right.\right.$$

$$\left.\left(a (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2\right) / \right.$$

$$\left.\left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2,\right.\right.\right.\right.$$

$$\left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)\right)$$

$$(d \operatorname{Tan}[e + f x])^n (a \operatorname{Tan}[e + f x]^n + b \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]^n) \Big/ (f (1+n) (b + a \operatorname{Cos}[e + f x]))$$

$$\left(\frac{1}{1+n} 2 n \operatorname{Sec}[e + f x]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2\right)^n +\right.\right.$$

$$\left.\left(a (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2\right) / \right.$$

$$\left.\left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2,\right.\right.\right.\right.$$

$$\left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)\right)$$

$$\operatorname{Tan}[e + f x]^{-1+n} + \frac{1}{1+n} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \left(b \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2\right)^n +\right.$$

$$\left.\left(a (3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2\right) / \right.$$

$$\left.\left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2,\right.\right.\right.\right.$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big) \Big) \\
& \tan[e+fx]^n + \frac{1}{1+n} 2 \tan\left[\frac{1}{2}(e+fx)\right] \left(b n \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \\
& \left. \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+n} \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx] + \cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) - \right. \\
& \left. \left(a(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right] \sin\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(a(3+n) \cos\left[\frac{1}{2}(e+fx)\right]^2 \left(-1 / (3+n)(1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + 1 / (3+n)n(1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, 1+n, 1, \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) / \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \frac{1}{2} b(1+n) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right)^n \\
& \left(-\operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 1+n, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \left(1 - \tan\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1-n} \right) - \\
& \left(a(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \cos\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left(-2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + (3+n) \left(-\frac{1}{3+n} \right. \right. \right. \\
& \left. \left. \left. (1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \frac{1}{3+n} \right. \right. \right. \\
& \left. \left. \left. n(1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, 1+n, 1, 1 + \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \left(-\frac{1}{5+n} 2(3+n) \operatorname{AppellF1}\left[1+\frac{3+n}{2}, n, 3, 1+\frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+n} n(3+n) \operatorname{AppellF1}\left[1+\frac{3+n}{2}, 1+n, 2, 1+\frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] - n \left(-\frac{1}{5+n} (3+n) \operatorname{AppellF1}\left[1+\frac{3+n}{2}, 1+n, 2, 1+\frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \frac{1}{5+n} (1+n)(3+n) \operatorname{AppellF1}\left[1+\frac{3+n}{2}, 2+n, 1, 1+\frac{5+n}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) \Big/ \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \operatorname{Tan}[e+fx]^n \Big)
\end{aligned}$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int \frac{(d \operatorname{Tan}[e+fx])^n}{a+b \operatorname{Sec}[e+fx]} dx$$

Optimal (type 6, 266 leaves, ? steps):

$$\begin{aligned}
& \frac{1}{af(1-n)} d \operatorname{AppellF1}\left[1-n, \frac{1-n}{2}, \frac{1-n}{2}, 2-n, \frac{a+b}{a+b \operatorname{Sec}[e+fx]}, \frac{a-b}{a+b \operatorname{Sec}[e+fx]}\right] \left(-\frac{b(1-\operatorname{Sec}[e+fx])}{a+b \operatorname{Sec}[e+fx]} \right)^{\frac{1-n}{2}} \left(\frac{b(1+\operatorname{Sec}[e+fx])}{a+b \operatorname{Sec}[e+fx]} \right)^{\frac{1-n}{2}} \\
& \quad (d \operatorname{Tan}[e+fx])^{-1+n} (-\operatorname{Tan}[e+fx]^2)^{\frac{1-n}{2}+\frac{1}{2}(-1+n)} - \frac{d \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\operatorname{Tan}[e+fx]^2\right] (d \operatorname{Tan}[e+fx])^{-1+n} (-\operatorname{Tan}[e+fx]^2)^{\frac{1-n}{2}+\frac{1+n}{2}}}{af(1+n)}
\end{aligned}$$

Result (type 6, 4911 leaves):

$$\begin{aligned}
& \left(2(3+n) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] \left(\operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \right) \Big/ \\
& \quad \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \quad \left(b(a+b) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left((b + a \cos[e + f x]) \left((a + b) (3 + n) \operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \right. \\
& \quad 2 \left((a-b) \operatorname{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \\
& \quad \left. \left. (a+b) n \operatorname{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) \\
& \tan[e + f x]^n (d \tan[e + f x])^n \Big/ \left(a f (1+n) (a + b \sec[e + f x]) \left(\frac{1}{a(1+n)} 2n(3+n) \cos \left[\frac{1}{2} (e + f x) \right] \sec[e + f x]^2 \right. \right. \\
& \quad \left. \left. \sin \left[\frac{1}{2} (e + f x) \right] \left(\operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \right. \right. \\
& \quad \left((3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] - n \operatorname{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) - \\
& \quad \left. \left. \left(b (a+b) \operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \right) \right) \Big/ \\
& \quad \left((b + a \cos[e + f x]) \left((a + b) (3 + n) \operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] + \right. \right. \\
& \quad 2 \left((a-b) \operatorname{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] + (a+b) n \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e + f x) \right]^2}{a+b} \right] \right) \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) \tan[e + f x]^{-1+n} + \\
& \quad \frac{1}{a(1+n)} (3+n) \cos \left[\frac{1}{2} (e + f x) \right]^2 \left(\operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \Big/ \\
& \quad \left((3+n) \operatorname{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, -\tan \left[\frac{1}{2} (e + f x) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan\left[\frac{1}{2}(e+fx)\right]^2 - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \tan\left[\frac{1}{2}(e+fx)\right]^2 - \\
& \left(b(a+b) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \\
& \left((b+a \cos[e+fx]) \left((a+b)(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& 2 \left((a-b) \operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. \left. (a+b)n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \tan[e+fx]^n - \\
& \frac{1}{a(1+n)} (3+n) \sin\left[\frac{1}{2}(e+fx)\right]^2 \left(\operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] / \right. \\
& \left((3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] - n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 - \right. \\
& \left. \left(b(a+b) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) / \right. \\
& \left((b+a \cos[e+fx]) \left((a+b)(3+n) \operatorname{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \right. \\
& 2 \left((a-b) \operatorname{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] + \right. \\
& \left. \left. (a+b)n \operatorname{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b)\tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) \right) \tan[e+fx]^n + \\
& \frac{1}{a(1+n)} 2(3+n) \cos\left[\frac{1}{2}(e+fx)\right] \sin\left[\frac{1}{2}(e+fx)\right] \left(\left(-1 / (3+n)(1+n) \operatorname{AppellF1}\left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 1 / (3+n)n(1+n) \right. \\
& \left. \text{AppellF1}\left[1 + \frac{1+n}{2}, 1+n, 1, 1 + \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) / \\
& \left((3+n) \text{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] - 2 \left(\text{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] - n \text{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) - \\
& \left(a b (a+b) \text{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \text{Sin}[e+fx] \right) / \\
& \left((b+a \text{Cos}[e+fx])^2 \left((a+b) (3+n) \text{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] + \right. \right. \\
& \left. \left. 2 \left((a-b) \text{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) + \right. \right. \\
& \left. \left. (a+b) n \text{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) - \\
& \left(b (a+b) \left(1 / ((a+b) (3+n)) (a-b) (1+n) \text{AppellF1}\left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\
& \left. \left. \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] + 1 / (3+n)n(1+n) \text{AppellF1}\left[1 + \frac{1+n}{2}, \right. \right. \\
& \left. \left. 1+n, 1, 1 + \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \text{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \text{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& \left((b+a \text{Cos}[e+fx]) \left((a+b) (3+n) \text{AppellF1}\left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] + \right. \right. \\
& \left. \left. 2 \left((a-b) \text{AppellF1}\left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) + \right. \right. \\
& \left. \left. (a+b) n \text{AppellF1}\left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \frac{(a-b) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b} \right] \right) \text{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\text{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(-2 \left(\text{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - n \text{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\right. \right. \\
& \quad \left. \left. \frac{1}{2} (e+fx) \right] + (3+n) \left(-\frac{1}{3+n} (1+n) \text{AppellF1} \left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \right. \\
& \quad \left. \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3+n} n (1+n) \text{AppellF1} \left[1 + \frac{1+n}{2}, 1+n, 1, 1 + \frac{3+n}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) - 2 \tan \left[\frac{1}{2} (e+fx) \right]^2 \\
& \quad \left(-\frac{1}{5+n} 2 (3+n) \text{AppellF1} \left[1 + \frac{3+n}{2}, n, 3, 1 + \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. \frac{1}{5+n} n (3+n) \text{AppellF1} \left[1 + \frac{3+n}{2}, 1+n, 2, 1 + \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \right. \\
& \quad \left. \tan \left[\frac{1}{2} (e+fx) \right] - n \left(-\frac{1}{5+n} (3+n) \text{AppellF1} \left[1 + \frac{3+n}{2}, 1+n, 2, 1 + \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \right. \\
& \quad \left. \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{5+n} (1+n) (3+n) \text{AppellF1} \left[1 + \frac{3+n}{2}, 2+n, 1, \right. \right. \\
& \quad \left. \left. 1 + \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) \Big/ \\
& \left((3+n) \text{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - 2 \left(\text{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] - n \text{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 + \\
& \left(b (a+b) \text{AppellF1} \left[\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right. \\
& \quad \left. \left(2 \left((a-b) \text{AppellF1} \left[\frac{3+n}{2}, n, 2, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) + \right. \right. \\
& \quad \left. \left. (a+b) n \text{AppellF1} \left[\frac{3+n}{2}, 1+n, 1, \frac{5+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \right) \text{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. (a+b) (3+n) \left(\frac{1}{(a+b) (3+n)} (a-b) (1+n) \text{AppellF1} \left[1 + \frac{1+n}{2}, n, 2, 1 + \frac{3+n}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \frac{(a-b) \tan \left[\frac{1}{2} (e+fx) \right]^2}{a+b} \right] \right) \right)
\end{aligned}$$

Result (type 6, 30540 leaves) : Display of huge result suppressed!

- **Problem 354: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]^3 dx$$

Optimal (type 5, 102 leaves, 4 steps) :

$$-\frac{a (a + b \operatorname{Sec}[c + d x])^{1+n}}{b^2 d (1+n)} + \frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b \operatorname{Sec}[c+d x]}{a}\right] (a + b \operatorname{Sec}[c + d x])^{1+n}}{a d (1+n)} + \frac{(a + b \operatorname{Sec}[c + d x])^{2+n}}{b^2 d (2+n)}$$

Result (type 6, 7524 leaves) : Display of huge result suppressed!

- **Problem 355: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x] dx$$

Optimal (type 5, 48 leaves, 2 steps) :

$$-\frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b \operatorname{Sec}[c+d x]}{a}\right] (a + b \operatorname{Sec}[c + d x])^{1+n}}{a d (1+n)}$$

Result (type 6, 5900 leaves) : Display of huge result suppressed!

- **Problem 356: Unable to integrate problem.**

$$\int \operatorname{Cot}[c + d x] (a + b \operatorname{Sec}[c + d x])^n dx$$

Optimal (type 5, 162 leaves, 8 steps) :

$$-\frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+d x]}{a-b}\right] (a + b \operatorname{Sec}[c + d x])^{1+n}}{2 (a-b) d (1+n)} - \frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+d x]}{a+b}\right] (a + b \operatorname{Sec}[c + d x])^{1+n}}{2 (a+b) d (1+n)} + \frac{\operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b \operatorname{Sec}[c+d x]}{a}\right] (a + b \operatorname{Sec}[c + d x])^{1+n}}{a d (1+n)}$$

Result (type 8, 21 leaves) :

$$\int \operatorname{Cot}[c + d x] (a + b \operatorname{Sec}[c + d x])^n dx$$

- **Problem 357: Unable to integrate problem.**

$$\int \operatorname{Cot}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^n dx$$

Optimal (type 5, 279 leaves, 10 steps) :

$$\frac{\text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}\right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{2(a-b)d(1+n)} + \frac{\text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a+b}\right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{2(a+b)d(1+n)}$$

$$\frac{\text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1+\frac{b \operatorname{Sec}[c+dx]}{a}\right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{ad(1+n)} -$$

$$\frac{b \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a-b}\right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{4(a-b)^2 d(1+n)} +$$

$$\frac{b \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{a+b \operatorname{Sec}[c+dx]}{a+b}\right] (a+b \operatorname{Sec}[c+dx])^{1+n}}{4(a+b)^2 d(1+n)}$$

Result (type 8, 23 leaves):

$$\int \cot[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^n dx$$

Test results for the 241 problems in "4.5.2.1 (a+b sec)^m (c+d sec)^n.m"

- **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[e+fx])^2 (c-c \operatorname{Sec}[e+fx]) dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$a^2 c x + \frac{a^2 c \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{2f} - \frac{c(2a^2 + a^2 \operatorname{Sec}[e+fx]) \operatorname{Tan}[e+fx]}{2f}$$

Result (type 3, 141 leaves):

$$-\frac{1}{16f} a^2 c (-1 + \operatorname{Cos}[e+fx]) (1 + \operatorname{Cos}[e+fx])^2 \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Sec}[e+fx]$$

$$\left(\operatorname{Cos}[e+fx] \left(2e + 2fx - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \right) - (1 + 2 \operatorname{Cos}[e+fx]) \operatorname{Tan}[e+fx] \right)$$

- **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[e+fx])^2}{c-c \operatorname{Sec}[e+fx]} dx$$

Optimal (type 3, 56 leaves, 8 steps):

$$\frac{a^2 x}{c} - \frac{a^2 \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{cf} - \frac{4a^2 \operatorname{Tan}[e+fx]}{cf(1-\operatorname{Sec}[e+fx])}$$

Result (type 3, 169 leaves):

$$\frac{1}{c f (-1 + \cos[e + f x])} a^2 \operatorname{Csc}\left[\frac{e}{2}\right] \left(-\cos\left[\frac{f x}{2}\right] \left(f x + \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) + \cos\left[e + \frac{f x}{2}\right] \left(f x + \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) + 8 \sin\left[\frac{f x}{2}\right] \right) \sin\left[\frac{1}{2}(e + f x)\right]$$

■ **Problem 16: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^3}{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 78 leaves, 15 steps):

$$\frac{a^3 x}{c} - \frac{4 a^3 \operatorname{ArcTanh}[\sin[e + f x]]}{c f} + \frac{8 a^3 \operatorname{Cot}[e + f x]}{c f} + \frac{8 a^3 \operatorname{Csc}[e + f x]}{c f} - \frac{a^3 \operatorname{Tan}[e + f x]}{c f}$$

Result (type 3, 240 leaves):

$$\frac{1}{4 f (c - c \operatorname{Sec}[e + f x])} a^3 \cos[e + f x]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^4 (1 + \operatorname{Sec}[e + f x])^3 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(8 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \sin\left[\frac{f x}{2}\right] + (-f x - 4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + \sin[f x]) \right) \left(\left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) \left(\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]$$

■ **Problem 17: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^3}{(c - c \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 88 leaves, 13 steps):

$$\frac{a^3 x}{c^2} + \frac{a^3 \operatorname{ArcTanh}[\sin[e + f x]]}{c^2 f} - \frac{8 a^3 \operatorname{Tan}[e + f x]}{3 c^2 f (1 - \operatorname{Sec}[e + f x])^2} + \frac{4 a^3 \operatorname{Tan}[e + f x]}{3 c^2 f (1 - \operatorname{Sec}[e + f x])}$$

Result (type 3, 177 leaves):

$$\frac{1}{6 c^2 f (-1 + \cos[e + f x])^2} a^3 (1 + \cos[e + f x])^3 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(4 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \sin\left[\frac{f x}{2}\right] - 4 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + 3 \left(f x - \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^3 \right)$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^5}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 136 leaves, 26 steps):

$$\frac{c^5 x}{a^2} - \frac{47 c^5 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} + \frac{13 c^5 \operatorname{Tan}[e + f x]}{2 a^2 f} + \frac{112 c^5 \operatorname{Tan}[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])} - \frac{32 c^5 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(c^5 - c^5 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{2 a^2 f}$$

Result (type 3, 384 leaves):

$$\frac{1}{96 a^2 (1 + \operatorname{Sec}[e + f x])^2} \left(\operatorname{Cos}[e + f x]^3 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^6 (c - c \operatorname{Sec}[e + f x])^5 \left(-\frac{320 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} - \frac{64 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} + 3 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^3 \left(-4 x - \frac{94 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \frac{94 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \frac{1}{f (\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right])^2} - \frac{1}{f (\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right])^2} - (28 \operatorname{Sin}[f x]) / \left(f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) \right) - \frac{64 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{e}{2}\right]}{f} \right)$$

■ **Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^4}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 102 leaves, 21 steps):

$$\frac{c^4 x}{a^2} - \frac{6 c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^2 f} - \frac{16 c^4 \operatorname{Cot}[e + f x]}{a^2 f} - \frac{32 c^4 \operatorname{Cot}[e + f x]^3}{3 a^2 f} + \frac{32 c^4 \operatorname{Csc}[e + f x]^3}{3 a^2 f} + \frac{c^4 \operatorname{Tan}[e + f x]}{a^2 f}$$

Result (type 3, 753 leaves):

$$\begin{aligned}
& \frac{x \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \operatorname{Sec}[e + f x])^4}{4 (a + a \operatorname{Sec}[e + f x])^2} + \\
& \frac{3 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \operatorname{Sec}[e + f x])^4}{2 f (a + a \operatorname{Sec}[e + f x])^2} - \\
& \frac{3 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \operatorname{Sec}[e + f x])^4}{2 f (a + a \operatorname{Sec}[e + f x])^2} + \\
& \frac{4 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \operatorname{Sec}\left[\frac{e}{2}\right] (c - c \operatorname{Sec}[e + f x])^4 \sin\left[\frac{f x}{2}\right]}{3 f (a + a \operatorname{Sec}[e + f x])^2} + \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^7 \operatorname{Sec}\left[\frac{e}{2}\right] (c - c \operatorname{Sec}[e + f x])^4 \sin\left[\frac{f x}{2}\right]}{3 f (a + a \operatorname{Sec}[e + f x])^2} + \\
& \frac{\cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \operatorname{Sec}[e + f x])^4 \sin\left[\frac{f x}{2}\right]}{4 f (a + a \operatorname{Sec}[e + f x])^2 (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right])} + \\
& \frac{\cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \operatorname{Sec}[e + f x])^4 \sin\left[\frac{f x}{2}\right]}{4 f (a + a \operatorname{Sec}[e + f x])^2 (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right])} + \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^4 \tan\left[\frac{e}{2}\right]}{3 f (a + a \operatorname{Sec}[e + f x])^2}
\end{aligned}$$

■ **Problem 23: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^3}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 85 leaves, 13 steps):

$$\frac{c^3 x}{a^2} - \frac{c^3 \operatorname{ArcTanh}[\sin[e + f x]]}{a^2 f} - \frac{8 c^3 \tan[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])^2} + \frac{4 c^3 \tan[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])}$$

Result (type 3, 216 leaves):

$$\begin{aligned}
& - \frac{1}{6 a^2 f (1 + \cos[e + f x])^2} c^3 (-1 + \cos[e + f x])^3 \cot\left[\frac{1}{2} (e + f x)\right] \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2 \\
& \left(3 \cot\left[\frac{1}{2} (e + f x)\right]^3 \left(f x + \operatorname{Log}\left[\cos\left[\frac{1}{2} (e + f x)\right] - \sin\left[\frac{1}{2} (e + f x)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2} (e + f x)\right] + \sin\left[\frac{1}{2} (e + f x)\right]\right] \right) - \\
& 4 \cot\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right] + 4 \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right] + 4 \cot\left[\frac{1}{2} (e + f x)\right] \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2 \tan\left[\frac{e}{2}\right]
\end{aligned}$$

■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\frac{x}{a^2 c^3} + \frac{\text{Cot}[e + f x]^5 (1 + \text{Sec}[e + f x])}{5 a^2 c^3 f} - \frac{\text{Cot}[e + f x]^3 (5 + 4 \text{Sec}[e + f x])}{15 a^2 c^3 f} + \frac{\text{Cot}[e + f x] (15 + 8 \text{Sec}[e + f x])}{15 a^2 c^3 f}$$

Result (type 3, 257 leaves):

$$\frac{1}{30720 a^2 c^3 f} \text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{1}{2}(e + f x)\right]^5 \text{Sec}\left[\frac{e}{2}\right] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^3 (360 f x \text{Cos}[f x] - 360 f x \text{Cos}[2e + f x] - 120 f x \text{Cos}[e + 2f x] + 120 f x \text{Cos}[3e + 2f x] - 120 f x \text{Cos}[2e + 3f x] + 120 f x \text{Cos}[4e + 3f x] + 60 f x \text{Cos}[3e + 4f x] - 60 f x \text{Cos}[5e + 4f x] + 200 \text{Sin}[e] - 584 \text{Sin}[f x] - 534 \text{Sin}[e + f x] + 178 \text{Sin}[2(e + f x)] + 178 \text{Sin}[3(e + f x)] - 89 \text{Sin}[4(e + f x)] - 520 \text{Sin}[2e + f x] + 248 \text{Sin}[e + 2f x] + 120 \text{Sin}[3e + 2f x] + 248 \text{Sin}[2e + 3f x] + 120 \text{Sin}[4e + 3f x] - 184 \text{Sin}[3e + 4f x])$$

■ **Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \text{Sec}[e + f x])^5}{(a + a \text{Sec}[e + f x])^3} dx$$

Optimal (type 3, 162 leaves, 29 steps):

$$\frac{c^5 x}{a^3} + \frac{8 c^5 \text{ArcTanh}[\text{Sin}[e + f x]]}{a^3 f} + \frac{32 c^5 \text{Cot}[e + f x]}{a^3 f} + \frac{128 c^5 \text{Cot}[e + f x]^3}{3 a^3 f} + \frac{128 c^5 \text{Cot}[e + f x]^5}{5 a^3 f} - \frac{16 c^5 \text{Csc}[e + f x]}{a^3 f} + \frac{64 c^5 \text{Csc}[e + f x]^3}{3 a^3 f} - \frac{128 c^5 \text{Csc}[e + f x]^5}{5 a^3 f} - \frac{c^5 \text{Tan}[e + f x]}{a^3 f}$$

Result (type 3, 908 leaves):

$$\begin{aligned}
& - \frac{x \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \sec[e + f x])^5}{4 (a + a \sec[e + f x])^3} + \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \log\left[\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \sec[e + f x])^5}{f (a + a \sec[e + f x])^3} - \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \log\left[\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \sec[e + f x])^5}{f (a + a \sec[e + f x])^3} + \\
& \frac{56 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \sec\left[\frac{e}{2}\right] (c - c \sec[e + f x])^5 \sin\left[\frac{f x}{2}\right]}{15 f (a + a \sec[e + f x])^3} - \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^7 \sec\left[\frac{e}{2}\right] (c - c \sec[e + f x])^5 \sin\left[\frac{f x}{2}\right]}{15 f (a + a \sec[e + f x])^3} + \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^9 \sec\left[\frac{e}{2}\right] (c - c \sec[e + f x])^5 \sin\left[\frac{f x}{2}\right]}{5 f (a + a \sec[e + f x])^3} + \\
& \frac{\cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \sec[e + f x])^5 \sin\left[\frac{f x}{2}\right]}{4 f (a + a \sec[e + f x])^3 \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]\right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} + \\
& \frac{\cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \sec[e + f x])^5 \sin\left[\frac{f x}{2}\right]}{4 f (a + a \sec[e + f x])^3 \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]\right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} - \\
& \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \sec[e + f x])^5 \tan\left[\frac{e}{2}\right]}{15 f (a + a \sec[e + f x])^3} + \frac{2 \cos[e + f x]^2 \cot\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^8 (c - c \sec[e + f x])^5 \tan\left[\frac{e}{2}\right]}{5 f (a + a \sec[e + f x])^3}
\end{aligned}$$

■ **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \sec[e + f x])^3 (c - c \sec[e + f x])^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{x}{a^3 c^2} + \frac{\cot[e + f x] (15 - 8 \sec[e + f x])}{15 a^3 c^2 f} - \frac{\cot[e + f x]^3 (5 - 4 \sec[e + f x])}{15 a^3 c^2 f} + \frac{\cot[e + f x]^5 (1 - \sec[e + f x])}{5 a^3 c^2 f}$$

Result (type 3, 257 leaves):

$$\frac{1}{30720 a^3 c^2 f} \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^5 (360 fx \operatorname{Cos}[fx] - 360 fx \operatorname{Cos}[2e+fx] + 120 fx \operatorname{Cos}[e+2fx] - 120 fx \operatorname{Cos}[3e+2fx] - 120 fx \operatorname{Cos}[2e+3fx] + 120 fx \operatorname{Cos}[4e+3fx] - 60 fx \operatorname{Cos}[3e+4fx] + 60 fx \operatorname{Cos}[5e+4fx] - 200 \operatorname{Sin}[e] - 584 \operatorname{Sin}[fx] + 534 \operatorname{Sin}[e+fx] + 178 \operatorname{Sin}[2(e+fx)] - 178 \operatorname{Sin}[3(e+fx)] - 89 \operatorname{Sin}[4(e+fx)] - 520 \operatorname{Sin}[2e+fx] - 248 \operatorname{Sin}[e+2fx] - 120 \operatorname{Sin}[3e+2fx] + 248 \operatorname{Sin}[2e+3fx] + 120 \operatorname{Sin}[4e+3fx] + 184 \operatorname{Sin}[3e+4fx])$$

■ **Problem 39: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a \operatorname{Sec}[e+fx])^3 (c-c \operatorname{Sec}[e+fx])^4} dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$\frac{x}{a^3 c^4} - \frac{\operatorname{Cot}[e+fx]^7 (1+\operatorname{Sec}[e+fx])}{7 a^3 c^4 f} + \frac{\operatorname{Cot}[e+fx]^5 (7+6 \operatorname{Sec}[e+fx])}{35 a^3 c^4 f} + \frac{\operatorname{Cot}[e+fx] (35+16 \operatorname{Sec}[e+fx])}{35 a^3 c^4 f} - \frac{\operatorname{Cot}[e+fx]^3 (35+24 \operatorname{Sec}[e+fx])}{105 a^3 c^4 f}$$

Result (type 3, 362 leaves):

$$\frac{1}{6881280 a^3 c^4 f} \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^7 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^5 (16800 fx \operatorname{Cos}[fx] - 16800 fx \operatorname{Cos}[2e+fx] - 4200 fx \operatorname{Cos}[e+2fx] + 4200 fx \operatorname{Cos}[3e+2fx] - 8400 fx \operatorname{Cos}[2e+3fx] + 8400 fx \operatorname{Cos}[4e+3fx] + 3360 fx \operatorname{Cos}[3e+4fx] - 3360 fx \operatorname{Cos}[5e+4fx] + 1680 fx \operatorname{Cos}[4e+5fx] - 1680 fx \operatorname{Cos}[6e+5fx] - 840 fx \operatorname{Cos}[5e+6fx] + 840 fx \operatorname{Cos}[7e+6fx] + 3136 \operatorname{Sin}[e] - 30112 \operatorname{Sin}[fx] - 22860 \operatorname{Sin}[e+fx] + 5715 \operatorname{Sin}[2(e+fx)] + 11430 \operatorname{Sin}[3(e+fx)] - 4572 \operatorname{Sin}[4(e+fx)] - 2286 \operatorname{Sin}[5(e+fx)] + 1143 \operatorname{Sin}[6(e+fx)] - 26208 \operatorname{Sin}[2e+fx] + 14080 \operatorname{Sin}[e+2fx] + 16400 \operatorname{Sin}[2e+3fx] + 11760 \operatorname{Sin}[4e+3fx] - 7904 \operatorname{Sin}[3e+4fx] - 3360 \operatorname{Sin}[5e+4fx] - 3952 \operatorname{Sin}[4e+5fx] - 1680 \operatorname{Sin}[6e+5fx] + 2816 \operatorname{Sin}[5e+6fx])$$

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+a \operatorname{Sec}[e+fx])^3 (c-c \operatorname{Sec}[e+fx])^5} dx$$

Optimal (type 3, 210 leaves, 14 steps):

$$\frac{x}{a^3 c^5} + \frac{\operatorname{Cot}[e+fx]}{a^3 c^5 f} - \frac{\operatorname{Cot}[e+fx]^3}{3 a^3 c^5 f} + \frac{\operatorname{Cot}[e+fx]^5}{5 a^3 c^5 f} - \frac{\operatorname{Cot}[e+fx]^7}{7 a^3 c^5 f} + \frac{2 \operatorname{Cot}[e+fx]^9}{9 a^3 c^5 f} + \frac{2 \operatorname{Csc}[e+fx]}{a^3 c^5 f} - \frac{8 \operatorname{Csc}[e+fx]^3}{3 a^3 c^5 f} + \frac{12 \operatorname{Csc}[e+fx]^5}{5 a^3 c^5 f} - \frac{8 \operatorname{Csc}[e+fx]^7}{7 a^3 c^5 f} + \frac{2 \operatorname{Csc}[e+fx]^9}{9 a^3 c^5 f}$$

Result (type 3, 441 leaves):

1

$$2580480 a^3 c^5 f (-1 + \text{Sec}[e + f x])^5 (1 + \text{Sec}[e + f x])^3$$

$$\begin{aligned} & \text{Csc}\left[\frac{e}{2}\right] \text{Sec}\left[\frac{e}{2}\right] \text{Sec}[e + f x]^7 (453600 f x \text{Cos}[f x] - 453600 f x \text{Cos}[2e + f x] - 201600 f x \text{Cos}[e + 2f x] + 201600 f x \text{Cos}[3e + 2f x] - \\ & 191520 f x \text{Cos}[2e + 3f x] + 191520 f x \text{Cos}[4e + 3f x] + 161280 f x \text{Cos}[3e + 4f x] - 161280 f x \text{Cos}[5e + 4f x] + 10080 f x \text{Cos}[4e + 5f x] - \\ & 10080 f x \text{Cos}[6e + 5f x] - 40320 f x \text{Cos}[5e + 6f x] + 40320 f x \text{Cos}[7e + 6f x] + 10080 f x \text{Cos}[6e + 7f x] - 10080 f x \text{Cos}[8e + 7f x] + \\ & 259584 \text{Sin}[e] - 897024 \text{Sin}[f x] - 1152405 \text{Sin}[e + f x] + 512180 \text{Sin}[2(e + f x)] + 486571 \text{Sin}[3(e + f x)] - 409744 \text{Sin}[4(e + f x)] - \\ & 25609 \text{Sin}[5(e + f x)] + 102436 \text{Sin}[6(e + f x)] - 25609 \text{Sin}[7(e + f x)] - 825216 \text{Sin}[2e + f x] + 622976 \text{Sin}[e + 2f x] + \\ & 142464 \text{Sin}[3e + 2f x] + 297088 \text{Sin}[2e + 3f x] + 430080 \text{Sin}[4e + 3f x] - 424192 \text{Sin}[3e + 4f x] - 188160 \text{Sin}[5e + 4f x] + \\ & 2048 \text{Sin}[4e + 5f x] - 40320 \text{Sin}[6e + 5f x] + 112768 \text{Sin}[5e + 6f x] + 40320 \text{Sin}[7e + 6f x] - 38272 \text{Sin}[6e + 7f x]) \text{Tan}[e + f x] \end{aligned}$$

- **Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \text{Sec}[e + f x]}}{(c - c \text{Sec}[e + f x])^2} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$\frac{2\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + a \text{Sec}[e + f x]}}\right]}{c^2 f} + \frac{2 \text{Cot}[e + f x] \sqrt{a + a \text{Sec}[e + f x]}}{c^2 f} - \frac{2 \text{Cot}[e + f x]^3 (a + a \text{Sec}[e + f x])^{3/2}}{3 a c^2 f}$$

Result (type 4, 471 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x]^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(\frac{20}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{2}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{32}{3} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2} 32 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]}$$

$$\operatorname{Sec}[e + f x]^3 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 48: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e + f x]}}{(c - c \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^3 f} + \frac{2 \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^3 f} - \frac{2 \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 a c^3 f} + \frac{2 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 a^2 c^3 f}$$

Result (type 4, 487 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x]^3 \sqrt{a(1 + \operatorname{Sec}[e + f x])}$$

$$\sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \left(-\frac{272}{15} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] + \frac{56}{15} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{2}{5} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 + \frac{368}{15} \sin\left[\frac{1}{2}(e + f x)\right]\right) +$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} 64 (-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]}$$

$$\operatorname{Sec}[e + f x]^4 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e + f x]}}{(c - c \operatorname{Sec}[e + f x])^4} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^4 f} + \frac{2 \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^4 f} -$$

$$\frac{2 \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 a c^4 f} + \frac{2 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 a^2 c^4 f} - \frac{2 \operatorname{Cot}[e + f x]^7 (a + a \operatorname{Sec}[e + f x])^{7/2}}{7 a^3 c^4 f}$$

Result (type 4, 503 leaves):

$$\begin{aligned}
& \frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x]^4 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8 \\
& \left(\frac{4768}{105} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{1504}{105} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 + \frac{108}{35} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 - \frac{2}{7} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^7 - \frac{5632}{105} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) - \\
& \frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} 128 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]} \\
& \operatorname{Sec}[e + f x]^5 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}
\end{aligned}$$

- **Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 102 leaves, 5 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^2 f} + \frac{2 a \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^2 f} - \frac{4 \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^2 f}$$

Result (type 4, 473 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}[e + f x] (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(\frac{14}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{2}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{20}{3} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2} 16 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3}$$

$$\operatorname{Sec}[e + f x]^2 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^3 f} + \frac{2 a \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^3 f} - \frac{2 \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^3 f} + \frac{4 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 a c^3 f}$$

Result (type 4, 491 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}[e + f x]^2 (a (1 + \operatorname{Sec}[e + f x]))^{3/2}$$

$$\sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \left(-\frac{172}{15} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] + \frac{46}{15} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{2}{5} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 + \frac{208}{15} \sin\left[\frac{1}{2}(e + f x)\right]\right) +$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} 32 (-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3}$$

$$\operatorname{Sec}[e + f x]^3 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^4} dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\frac{2 a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^4 f} + \frac{2 a \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^4 f} -$$

$$\frac{2 \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^4 f} + \frac{2 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 a c^4 f} - \frac{4 \operatorname{Cot}[e + f x]^7 (a + a \operatorname{Sec}[e + f x])^{7/2}}{7 a^2 c^4 f}$$

Result (type 4, 507 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}[e + f x]^3 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8$$

$$\left(\frac{2864}{105} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{1112}{105} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 + \frac{94}{35} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 - \frac{2}{7} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^7 - \frac{3056}{105} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) -$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} 64 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3}$$

$$\operatorname{Sec}[e + f x]^4 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 61: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{(c - c \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^2 f} - \frac{8 a \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^2 f}$$

Result (type 4, 465 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(\frac{8}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{2}{3} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{8}{3} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^2} 8 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5}$$

$$\operatorname{Sec}[e + f x] (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 62: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{(c - c \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^3 f} + \frac{2 a^2 \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^3 f} + \frac{8 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 c^3 f}$$

Result (type 4, 489 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \operatorname{Sec}[e + f x] (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \left(-\frac{34}{5} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] + \frac{12}{5} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{2}{5} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 + \frac{36}{5} \sin\left[\frac{1}{2}(e + f x)\right]\right) + \frac{1}{f (c - c \operatorname{Sec}[e + f x])^3} 16 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right) \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5} \operatorname{Sec}[e + f x]^2 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 63: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{(c - c \operatorname{Sec}[e + f x])^4} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^4 f} + \frac{2 a^2 \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^4 f} - \frac{2 a \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^4 f} - \frac{8 \operatorname{Cot}[e + f x]^7 (a + a \operatorname{Sec}[e + f x])^{7/2}}{7 a c^4 f}$$

Result (type 4, 507 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \operatorname{Sec}[e + f x]^2 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8$$

$$\left(\frac{332}{21} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] - \frac{158}{21} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 + \frac{16}{7} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 - \frac{2}{7} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^7 - \frac{320}{21} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^4} 32 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5}$$

$$\operatorname{Sec}[e + f x]^3 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^8 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{(c - c \operatorname{Sec}[e + f x])^5} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\frac{2 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{c^5 f} + \frac{2 a^2 \operatorname{Cot}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{c^5 f} -$$

$$\frac{2 a \operatorname{Cot}[e + f x]^3 (a + a \operatorname{Sec}[e + f x])^{3/2}}{3 c^5 f} + \frac{2 \operatorname{Cot}[e + f x]^5 (a + a \operatorname{Sec}[e + f x])^{5/2}}{5 c^5 f} + \frac{8 \operatorname{Cot}[e + f x]^9 (a + a \operatorname{Sec}[e + f x])^{9/2}}{9 a^2 c^5 f}$$

Result (type 4, 523 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^5} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \operatorname{Sec}[e + f x]^3 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^{10}$$

$$\left(-\frac{1616}{45} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] + \frac{968}{45} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 - \frac{418}{45} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^5 + \frac{20}{9} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^7 - \frac{2}{9} \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^9 + \frac{1424}{45} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) +$$

$$\frac{1}{f (c - c \operatorname{Sec}[e + f x])^5} 64 (-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5}$$

$$\operatorname{Sec}[e + f x]^4 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^{10} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 73: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^3}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\frac{2 c^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{3/2} f} + \frac{2 \sqrt{2} c^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{3/2} f} - \frac{4 c^3 \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{c^3 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sin}[e + f x] \operatorname{Tan}[e + f x]^2}{f (a + a \operatorname{Sec}[e + f x])^{3/2}}$$

Result (type 3, 564 leaves):

$$\frac{1}{8 (a (1 + \operatorname{Sec}[e + f x]))^{3/2}} \operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (1 + \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^3$$

$$\left(\frac{3 \sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + f x]}}\right] \operatorname{Cos}[e + f x]^2 \sqrt{-1 + \operatorname{Sec}[e + f x]} (1 + \operatorname{Sec}[e + f x])^{3/2} \operatorname{Sin}[e + f x]}{f (1 + \operatorname{Cos}[e + f x]) \sqrt{1 - \operatorname{Cos}[e + f x]}^2 \sqrt{\operatorname{Cos}[e + f x]^2 (-1 + \operatorname{Sec}[e + f x]) (1 + \operatorname{Sec}[e + f x])}} - \right.$$

$$\left. \left(\left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + f x]}}\right] + \operatorname{ArcTan}\left[\frac{-2 + \sqrt{1 + \operatorname{Sec}[e + f x]}}{\sqrt{-1 + \operatorname{Sec}[e + f x]}}\right] - \operatorname{ArcTan}\left[\frac{2 + \sqrt{1 + \operatorname{Sec}[e + f x]}}{\sqrt{-1 + \operatorname{Sec}[e + f x]}}\right] \right) \operatorname{Cos}[e + f x]^2 \sqrt{-1 + \operatorname{Sec}[e + f x]} \right. \right.$$

$$\left. \left. (1 + \operatorname{Sec}[e + f x])^{3/2} \operatorname{Sin}[e + f x] \right) \right) / \left(f (1 + \operatorname{Cos}[e + f x]) \sqrt{1 - \operatorname{Cos}[e + f x]}^2 \sqrt{\operatorname{Cos}[e + f x]^2 (-1 + \operatorname{Sec}[e + f x]) (1 + \operatorname{Sec}[e + f x])} \right) \right) +$$

$$\frac{1}{(a (1 + \operatorname{Sec}[e + f x]))^{3/2}} \operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sqrt{(1 + \operatorname{Cos}[e + f x]) \operatorname{Sec}[e + f x]} (1 + \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^3$$

$$\left(\frac{3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]}{4 f} - \frac{\operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{4 f} + \frac{3 \operatorname{Tan}\left[\frac{e}{2}\right]}{4 f} - \frac{\operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2}\right]}{4 f} \right)$$

■ **Problem 79: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^5}{(a + a \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 260 leaves, 9 steps):

$$\frac{2 c^5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{5/2} f} - \frac{23 \sqrt{2} c^5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{a^{5/2} f} + \frac{21 c^5 \operatorname{Tan}[e + f x]}{a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

$$\frac{19 c^5 \operatorname{Tan}[e + f x]^3}{6 a f (a + a \operatorname{Sec}[e + f x])^{3/2}} + \frac{3 c^5 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sin}[e + f x] \operatorname{Tan}[e + f x]^4}{4 f (a + a \operatorname{Sec}[e + f x])^{5/2}} + \frac{a c^5 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^4 \operatorname{Sin}[e + f x]^2 \operatorname{Tan}[e + f x]^5}{4 f (a + a \operatorname{Sec}[e + f x])^{7/2}}$$

Result (type 3, 667 leaves):

$$\left(\begin{aligned} & \cos[e + fx]^5 \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^{10} (1 + \operatorname{Sec}[e + fx])^{5/2} (c - c \operatorname{Sec}[e + fx])^5 \\ & - \frac{22\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] \cos[e + fx]^2 \sqrt{-1 + \operatorname{Sec}[e + fx]} (1 + \operatorname{Sec}[e + fx])^{3/2} \sin[e + fx]}{f(1 + \cos[e + fx]) \sqrt{1 - \cos[e + fx]^2} \sqrt{\cos[e + fx]^2 (-1 + \operatorname{Sec}[e + fx])} (1 + \operatorname{Sec}[e + fx])} \\ & \left(\left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] + \operatorname{ArcTan}\left[\frac{-2 + \sqrt{1 + \operatorname{Sec}[e + fx]}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] - \operatorname{ArcTan}\left[\frac{2 + \sqrt{1 + \operatorname{Sec}[e + fx]}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] \right) \cos[e + fx]^2 \sqrt{-1 + \operatorname{Sec}[e + fx]} \right. \\ & \left. (1 + \operatorname{Sec}[e + fx])^{3/2} \sin[e + fx] \right) / \left(f(1 + \cos[e + fx]) \sqrt{1 - \cos[e + fx]^2} \sqrt{\cos[e + fx]^2 (-1 + \operatorname{Sec}[e + fx])} (1 + \operatorname{Sec}[e + fx]) \right) \Bigg) / \\ & (32(a(1 + \operatorname{Sec}[e + fx]))^{5/2}) + \frac{1}{(a(1 + \operatorname{Sec}[e + fx]))^{5/2}} \cos[e + fx]^5 \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^{10} \sqrt{(1 + \cos[e + fx]) \operatorname{Sec}[e + fx]} \\ & (1 + \operatorname{Sec}[e + fx])^{5/2} \\ & (c - c \operatorname{Sec}[e + fx])^5 \\ & \left(-\frac{(-1 + 37 \cos[e]) \sin\left[\frac{e}{2}\right]}{24f(\cos\left[\frac{e}{2}\right] + \cos\left[\frac{3e}{2}\right])} - \frac{19 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] \sin\left[\frac{fx}{2}\right]}{24f} + \frac{\operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \sin\left[\frac{fx}{2}\right]}{32f} + \right. \\ & \left. \frac{\operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \sin\left[\frac{fx}{2}\right]}{16f} + \frac{\operatorname{Sec}[e] \operatorname{Sec}[e + fx] \sin[fx]}{48f} + \frac{\operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \tan\left[\frac{e}{2}\right]}{32f} + \frac{\operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \tan\left[\frac{e}{2}\right]}{16f} \right) \end{aligned} \right)$$

■ **Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + fx])^4}{(a + a \operatorname{Sec}[e + fx])^{5/2}} dx$$

Optimal (type 3, 229 leaves, 8 steps):

$$\frac{2c^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + fx]}{\sqrt{a + a \operatorname{Sec}[e + fx]}}\right]}{a^{5/2} f} - \frac{11c^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + fx]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + fx]}}\right]}{\sqrt{2} a^{5/2} f} + \frac{7c^4 \operatorname{Tan}[e + fx]}{2a^2 f \sqrt{a + a \operatorname{Sec}[e + fx]}} - \frac{c^4 \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \sin[e + fx] \operatorname{Tan}[e + fx]^2}{4af(a + a \operatorname{Sec}[e + fx])^{3/2}} - \frac{c^4 \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^4 \sin[e + fx]^2 \operatorname{Tan}[e + fx]^3}{4f(a + a \operatorname{Sec}[e + fx])^{5/2}}$$

Result (type 3, 627 leaves):

$$\left(\begin{aligned} & \cos[e + fx]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^8 (1 + \operatorname{Sec}[e + fx])^{5/2} (c - c \operatorname{Sec}[e + fx])^4 \\ & \frac{\left(9\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] \cos[e + fx]^2 \sqrt{-1 + \operatorname{Sec}[e + fx]} (1 + \operatorname{Sec}[e + fx])^{3/2} \sin[e + fx] \right.}{f(1 + \cos[e + fx]) \sqrt{1 - \cos[e + fx]^2} \sqrt{\cos[e + fx]^2 (-1 + \operatorname{Sec}[e + fx])} (1 + \operatorname{Sec}[e + fx])} + \\ & \left. 2 \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] + \operatorname{ArcTan}\left[\frac{-2 + \sqrt{1 + \operatorname{Sec}[e + fx]}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] - \operatorname{ArcTan}\left[\frac{2 + \sqrt{1 + \operatorname{Sec}[e + fx]}}{\sqrt{-1 + \operatorname{Sec}[e + fx]}}\right] \right) \cos[e + fx]^2 \sqrt{-1 + \operatorname{Sec}[e + fx]} \right.}{(1 + \operatorname{Sec}[e + fx])^{3/2} \sin[e + fx]} \Bigg) / \left(f(1 + \cos[e + fx]) \sqrt{1 - \cos[e + fx]^2} \sqrt{\cos[e + fx]^2 (-1 + \operatorname{Sec}[e + fx])} (1 + \operatorname{Sec}[e + fx]) \right) \Bigg) / \\ & (32(a(1 + \operatorname{Sec}[e + fx]))^{5/2}) + \frac{1}{(a(1 + \operatorname{Sec}[e + fx]))^{5/2}} \cos[e + fx]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^8 \sqrt{(1 + \cos[e + fx]) \operatorname{Sec}[e + fx]} \\ & (1 + \operatorname{Sec}[e + fx])^{5/2} \\ & (c - c \operatorname{Sec}[e + fx])^4 \\ & \left(\frac{3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] \sin\left[\frac{fx}{2}\right]}{16f} + \frac{3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \sin\left[\frac{fx}{2}\right]}{32f} - \right. \\ & \left. \frac{\operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \sin\left[\frac{fx}{2}\right]}{16f} + \frac{3 \operatorname{Tan}\left[\frac{e}{2}\right]}{16f} + \frac{3 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Tan}\left[\frac{e}{2}\right]}{32f} - \frac{\operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Tan}\left[\frac{e}{2}\right]}{16f} \right) \end{aligned} \right)$$

■ **Problem 86: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a + a \operatorname{Sec}[e + fx]} (c - c \operatorname{Sec}[e + fx])^{7/2} dx$$

Optimal (type 3, 185 leaves, 5 steps):

$$\frac{a c^4 \operatorname{Log}[\cos[e + fx]] \operatorname{Tan}[e + fx]}{f \sqrt{a + a \operatorname{Sec}[e + fx]} \sqrt{c - c \operatorname{Sec}[e + fx]}} - \frac{a c^3 \sqrt{c - c \operatorname{Sec}[e + fx]} \operatorname{Tan}[e + fx]}{f \sqrt{a + a \operatorname{Sec}[e + fx]}} - \frac{a c^2 (c - c \operatorname{Sec}[e + fx])^{3/2} \operatorname{Tan}[e + fx]}{2 f \sqrt{a + a \operatorname{Sec}[e + fx]}} - \frac{a c (c - c \operatorname{Sec}[e + fx])^{5/2} \operatorname{Tan}[e + fx]}{3 f \sqrt{a + a \operatorname{Sec}[e + fx]}}$$

Result (type 3, 149 leaves):

$$\frac{1}{24 f} c^3 \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right] \\ \left(-22-18 \operatorname{Cos}[2(e+f x)]+3 i f x \operatorname{Cos}[3(e+f x)]+9 \operatorname{Cos}[e+f x]\left(2+i f x-\operatorname{Log}\left[1+e^{2 i(e+f x)}\right]\right)-3 \operatorname{Cos}[3(e+f x)] \operatorname{Log}\left[1+e^{2 i(e+f x)}\right]\right) \\ \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \operatorname{Sec}[e+f x]^2 \sqrt{a(1+\operatorname{Sec}[e+f x])} \sqrt{c-c \operatorname{Sec}[e+f x]}$$

■ **Problem 87: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+a \operatorname{Sec}[e+f x]}(c-c \operatorname{Sec}[e+f x])^{5/2} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{a c^3 \operatorname{Log}[\operatorname{Cos}[e+f x]] \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]}} - \frac{a c^2 \sqrt{c-c \operatorname{Sec}[e+f x]} \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]}} - \frac{a c(c-c \operatorname{Sec}[e+f x])^{3/2} \operatorname{Tan}[e+f x]}{2 f \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 3, 162 leaves):

$$-\frac{1}{16\left(1+e^{i(e+f x)}\right) f} \\ c^2 e^{-3 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^3\left(i+\operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]\right)\left(-1-i f x+4 \operatorname{Cos}[e+f x]+\operatorname{Log}\left[1+e^{2 i(e+f x)}\right]+\operatorname{Cos}[2(e+f x)]\left(-i f x+\operatorname{Log}\left[1+e^{2 i(e+f x)}\right]\right)\right) \\ \operatorname{Sec}[e+f x]^4 \sqrt{a(1+\operatorname{Sec}[e+f x])} \sqrt{c-c \operatorname{Sec}[e+f x]}$$

■ **Problem 88: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+a \operatorname{Sec}[e+f x]}(c-c \operatorname{Sec}[e+f x])^{3/2} dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$\frac{a c^2 \operatorname{Log}[\operatorname{Cos}[e+f x]] \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]}} - \frac{a c \sqrt{c-c \operatorname{Sec}[e+f x]} \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]}}$$

Result (type 3, 99 leaves):

$$\frac{1}{\left(1+e^{i(e+f x)}\right) f} i c\left(i+\operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]\right)\left(i+\operatorname{Cos}[e+f x]\left(f x+i \operatorname{Log}\left[1+e^{2 i(e+f x)}\right]\right)\right) \sqrt{a(1+\operatorname{Sec}[e+f x])} \sqrt{c-c \operatorname{Sec}[e+f x]}$$

■ **Problem 89: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{a c \operatorname{Log}[\operatorname{Cos}[e+f x]] \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]}}$$

Result (type 3, 84 leaves):

$$-\frac{(1 + e^{2i(e+fx)}) (fx + i \operatorname{Log}[1 + e^{2i(e+fx)}]) \sqrt{a(1 + \operatorname{Sec}[e+fx])} \sqrt{c - c \operatorname{Sec}[e+fx]}}{(-1 + e^{2i(e+fx)}) f}$$

■ **Problem 90: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e+fx]}}{\sqrt{c - c \operatorname{Sec}[e+fx]}} dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{a \operatorname{Log}[1 - \operatorname{Cos}[e+fx]] \operatorname{Tan}[e+fx]}{f \sqrt{a + a \operatorname{Sec}[e+fx]} \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 86 leaves):

$$-\frac{(-1 + e^{i(e+fx)}) (fx + 2i \operatorname{Log}[1 - e^{i(e+fx)}]) \sqrt{a(1 + \operatorname{Sec}[e+fx])}}{(1 + e^{i(e+fx)}) f \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

■ **Problem 91: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e+fx]}}{(c - c \operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$-\frac{a \operatorname{Tan}[e+fx]}{f \sqrt{a + a \operatorname{Sec}[e+fx]} (c - c \operatorname{Sec}[e+fx])^{3/2}} + \frac{a \operatorname{Log}[1 - \operatorname{Cos}[e+fx]] \operatorname{Tan}[e+fx]}{c f \sqrt{a + a \operatorname{Sec}[e+fx]} \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 107 leaves):

$$\frac{1}{f (c - c \operatorname{Sec}[e+fx])^{3/2}} (-1 + i f x - 2 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e+fx] (-i f x + 2 \operatorname{Log}[1 - e^{i(e+fx)}])) \operatorname{Sec}[e+fx] \sqrt{a(1 + \operatorname{Sec}[e+fx])} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]$$

■ **Problem 92: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e+fx]}}{(c - c \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 4 steps):

$$-\frac{a \operatorname{Tan}[e+fx]}{2 f \sqrt{a + a \operatorname{Sec}[e+fx]} (c - c \operatorname{Sec}[e+fx])^{5/2}} - \frac{a \operatorname{Tan}[e+fx]}{c f \sqrt{a + a \operatorname{Sec}[e+fx]} (c - c \operatorname{Sec}[e+fx])^{3/2}} + \frac{a \operatorname{Log}[1 - \operatorname{Cos}[e+fx]] \operatorname{Tan}[e+fx]}{c^2 f \sqrt{a + a \operatorname{Sec}[e+fx]} \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 152 leaves):

$$\left((3 - 3 i f x + \text{Cos}[e + f x] (-4 + 4 i f x - 8 \text{Log}[1 - e^{i(e+fx)}])) + 6 \text{Log}[1 - e^{i(e+fx)}] + \text{Cos}[2(e + f x)] (-i f x + 2 \text{Log}[1 - e^{i(e+fx)}]) \right) \sqrt{a(1 + \text{Sec}[e + f x])} \text{Tan}\left[\frac{1}{2}(e + f x)\right] \Big/ \left(2 c^2 f (-1 + \text{Cos}[e + f x])^2 \sqrt{c - c \text{Sec}[e + f x]}\right)$$

■ **Problem 93: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \text{Sec}[e + f x]}}{(c - c \text{Sec}[e + f x])^{7/2}} dx$$

Optimal (type 3, 188 leaves, 5 steps):

$$-\frac{a \text{Tan}[e + f x]}{3 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{7/2}} - \frac{a \text{Tan}[e + f x]}{2 c f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{5/2}} - \frac{a \text{Tan}[e + f x]}{c^2 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{3/2}} + \frac{a \text{Log}[1 - \text{Cos}[e + f x]] \text{Tan}[e + f x]}{c^3 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 198 leaves):

$$\left((-40 + 30 i f x - 3 i f x \text{Cos}[3(e + f x)] + 18 i \text{Cos}[2(e + f x)] (i + f x + 2 i \text{Log}[1 - e^{i(e+fx)}]) - 60 \text{Log}[1 - e^{i(e+fx)}] + 6 \text{Cos}[3(e + f x)] \text{Log}[1 - e^{i(e+fx)}] + 9 \text{Cos}[e + f x] (6 - 5 i f x + 10 \text{Log}[1 - e^{i(e+fx)}]) \right) \sqrt{a(1 + \text{Sec}[e + f x])} \text{Tan}\left[\frac{1}{2}(e + f x)\right] \Big/ \left(12 c^3 f (-1 + \text{Cos}[e + f x])^3 \sqrt{c - c \text{Sec}[e + f x]}\right)$$

■ **Problem 94: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \text{Sec}[e + f x])^{3/2} (c - c \text{Sec}[e + f x])^{5/2} dx$$

Optimal (type 3, 190 leaves, 5 steps):

$$\frac{a^2 c^3 \text{Log}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} - \frac{a^2 c^2 \sqrt{c - c \text{Sec}[e + f x]} \text{Tan}[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]}} - \frac{a^2 c (c - c \text{Sec}[e + f x])^{3/2} \text{Tan}[e + f x]}{2 f \sqrt{a + a \text{Sec}[e + f x]}} + \frac{a^2 (c - c \text{Sec}[e + f x])^{5/2} \text{Tan}[e + f x]}{3 f \sqrt{a + a \text{Sec}[e + f x]}}$$

Result (type 3, 157 leaves):

$$\frac{1}{24 f} i a c^2 \text{Csc}\left[\frac{1}{2}(e + f x)\right] \left(2 i + 6 i \text{Cos}[2(e + f x)] + 3 f x \text{Cos}[3(e + f x)] + \text{Cos}[e + f x] (6 i + 9 f x + 9 i \text{Log}[1 + e^{2i(e+fx)}]) + 3 i \text{Cos}[3(e + f x)] \text{Log}[1 + e^{2i(e+fx)}]\right) \text{Sec}\left[\frac{1}{2}(e + f x)\right] \text{Sec}[e + f x]^2 \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]}$$

■ **Problem 95: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$\frac{a^2 c^2 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{a^2 c^2 \operatorname{Tan}[e + f x]^3}{2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 159 leaves):

$$\frac{1}{8 (1 + e^{i(e+fx)}) f} i a c e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2 \left(i + \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] \right) \\ (i + fx + \operatorname{Cos}[2(e+fx)]) (fx + i \operatorname{Log}[1 + e^{2i(e+fx)}]) + i \operatorname{Log}[1 + e^{2i(e+fx)}] \operatorname{Sec}[e + fx]^3 \sqrt{a(1 + \operatorname{Sec}[e + fx])} \sqrt{c - c \operatorname{Sec}[e + fx]}$$

■ **Problem 96: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$\frac{a^2 c \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a c \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 128 leaves):

$$\frac{1}{2 (1 + e^{i(e+fx)}) f} a e^{-i(e+fx)} (1 + e^{2i(e+fx)}) \left(i + \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right] \right) \\ (1 + \operatorname{Cos}[e + fx]) (i f x - \operatorname{Log}[1 + e^{2i(e+fx)}]) \operatorname{Sec}[e + fx] \sqrt{a(1 + \operatorname{Sec}[e + fx])} \sqrt{c - c \operatorname{Sec}[e + fx]}$$

■ **Problem 97: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{a^2 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{2 a^2 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 105 leaves):

$$- \frac{a (-1 + e^{i(e+fx)}) (fx + 4 i \operatorname{Log}[1 - e^{i(e+fx)}] - i \operatorname{Log}[1 + e^{2i(e+fx)}]) \sqrt{a(1 + \operatorname{Sec}[e + fx])}}{(1 + e^{i(e+fx)}) f \sqrt{c - c \operatorname{Sec}[e + fx]}}$$

■ **Problem 98: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 100 leaves, 3 steps):

$$-\frac{2 a^2 \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{a^2 \operatorname{Log}[1 - \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 115 leaves):

$$\left(a \left(-2 + i f x - 2 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e + f x] \left(-i f x + 2 \operatorname{Log}[1 - e^{i(e+fx)}] \right) \right) \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \left(c f (-1 + \operatorname{Cos}[e + f x]) \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 99: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$-\frac{a^2 \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2}} - \frac{a^2 \operatorname{Tan}[e + f x]}{c f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{a^2 \operatorname{Log}[1 - \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 153 leaves):

$$\left(a \left(4 - 3 i f x + \operatorname{Cos}[e + f x] \left(-6 + 4 i f x - 8 \operatorname{Log}[1 - e^{i(e+fx)}] \right) + 6 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[2(e + f x)] \left(-i f x + 2 \operatorname{Log}[1 - e^{i(e+fx)}] \right) \right) \sqrt{a(1 + \operatorname{Sec}[e + f x])} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \left(2 c^2 f (-1 + \operatorname{Cos}[e + f x])^2 \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^{7/2}} dx$$

Optimal (type 3, 196 leaves, 5 steps):

$$-\frac{2 a^2 \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{7/2}} - \frac{a^2 \operatorname{Tan}[e + f x]}{2 c f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2}} - \frac{a^2 \operatorname{Tan}[e + f x]}{c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{a^2 \operatorname{Log}[1 - \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{c^3 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 489 leaves):

$$\left(8 i \sqrt{2} e^{\frac{1}{2} i (e+fx)} \sqrt{\frac{(1 + e^{i (e+fx)})^2}{1 + e^{2 i (e+fx)}}} (fx + 2 i \operatorname{Log}[1 - e^{i (e+fx)}]) \operatorname{Sec}[e + fx]^{7/2} (a (1 + \operatorname{Sec}[e + fx]))^{3/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^7 \right) /$$

$$\left((1 + e^{i (e+fx)}) \sqrt{\frac{e^{i (e+fx)}}{1 + e^{2 i (e+fx)}}} f (1 + \operatorname{Sec}[e + fx])^{3/2} (c - c \operatorname{Sec}[e + fx])^{7/2} \right) +$$

$$\left(\operatorname{Sec}[e + fx]^4 \sqrt{(1 + \operatorname{Cos}[e + fx]) \operatorname{Sec}[e + fx]} (a (1 + \operatorname{Sec}[e + fx]))^{3/2} \right.$$

$$\left(-\frac{61 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{3 f} + \frac{17 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{3 f} - \frac{2 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^5}{3 f} + \frac{35 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]}{3 f} + \frac{61 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{3 f} -$$

$$\frac{17 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sin}\left[\frac{fx}{2}\right]}{3 f} + \frac{2 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^6 \operatorname{Sin}\left[\frac{fx}{2}\right]}{3 f} \right) \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^7 \Big/ \left((1 + \operatorname{Sec}[e + fx])^{3/2} (c - c \operatorname{Sec}[e + fx])^{7/2} \right)$$

- **Problem 101: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[e + fx])^{5/2} (c - c \operatorname{Sec}[e + fx])^{5/2} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{a^3 c^3 \operatorname{Log}[\operatorname{Cos}[e + fx]] \operatorname{Tan}[e + fx]}{f \sqrt{a + a \operatorname{Sec}[e + fx]} \sqrt{c - c \operatorname{Sec}[e + fx]}} + \frac{a^3 c^3 \operatorname{Tan}[e + fx]^3}{2 f \sqrt{a + a \operatorname{Sec}[e + fx]} \sqrt{c - c \operatorname{Sec}[e + fx]}} - \frac{a^3 c^3 \operatorname{Tan}[e + fx]^5}{4 f \sqrt{a + a \operatorname{Sec}[e + fx]} \sqrt{c - c \operatorname{Sec}[e + fx]}}$$

Result (type 3, 164 leaves):

$$\frac{1}{16 f} i a^2 c^2 \operatorname{Csc}\left[\frac{1}{2} (e + fx)\right]$$

$$\left(2 i + 3 f x + \operatorname{Cos}[4 (e + fx)] (fx + i \operatorname{Log}[1 + e^{2 i (e+fx)}]) + 4 \operatorname{Cos}[2 (e + fx)] (i + fx + i \operatorname{Log}[1 + e^{2 i (e+fx)}]) + 3 i \operatorname{Log}[1 + e^{2 i (e+fx)}] \right)$$

$$\operatorname{Sec}\left[\frac{1}{2} (e + fx)\right] \operatorname{Sec}[e + fx]^3 \sqrt{a (1 + \operatorname{Sec}[e + fx])} \sqrt{c - c \operatorname{Sec}[e + fx]}$$

- **Problem 102: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[e + fx])^{5/2} (c - c \operatorname{Sec}[e + fx])^{3/2} dx$$

Optimal (type 3, 190 leaves, 5 steps):

$$\frac{a^3 c^2 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a^2 c^2 \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a c^2 (a + a \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{2 f \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^2 (a + a \operatorname{Sec}[e + f x])^{5/2} \operatorname{Tan}[e + f x]}{3 f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 149 leaves):

$$\frac{1}{24 f} a^2 c \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \left(2 + 6 \operatorname{Cos}[2(e + f x)] + 3 i f x \operatorname{Cos}[3(e + f x)] + \operatorname{Cos}[e + f x] \left(-6 + 9 i f x - 9 \operatorname{Log}\left[1 + e^{2 i (e + f x)}\right]\right) - 3 \operatorname{Cos}[3(e + f x)] \operatorname{Log}\left[1 + e^{2 i (e + f x)}\right]\right) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x]^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}$$

■ **Problem 103: Result unnecessarily involves imaginary or complex numbers.**

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{a^3 c \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a^2 c \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a c (a + a \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{2 f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 164 leaves):

$$\frac{1}{4(1 + e^{i(e + f x)}) f} a^2 e^{-i(e + f x)} (1 + e^{2 i (e + f x)}) \left(i + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]\right) \left((1 + i f x + 4 \operatorname{Cos}[e + f x] + \operatorname{Cos}[2(e + f x)]) (i f x - \operatorname{Log}[1 + e^{2 i (e + f x)}]) - \operatorname{Log}[1 + e^{2 i (e + f x)}]\right) \operatorname{Sec}[e + f x]^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}$$

■ **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 152 leaves, 3 steps):

$$\frac{a^3 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{4 a^3 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{a^3 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 198 leaves):

$$\left((1 + \cos[e + f x]) (-i f x + 8 \operatorname{Log}[1 - e^{i(e+fx)}] - 3 \operatorname{Log}[1 + e^{2i(e+fx)}]) \sqrt{\sec\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] (a(1 + \sec[e+fx]))^{5/2}} \right. \\ \left. \left(\cos\left[\frac{1}{2}(e+fx)\right] + i \sin\left[\frac{1}{2}(e+fx)\right] \right) \sin\left[\frac{1}{2}(e+fx)\right] \right) / \left((1 + e^{i(e+fx)}) \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} f (1 + \sec[e+fx])^{3/2} \sqrt{c - c \sec[e+fx]} \right)$$

- **Problem 105: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sec[e + f x])^{5/2}}{(c - c \sec[e + f x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$-\frac{4 a^3 \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2}} + \frac{a^3 \operatorname{Log}[\cos[e + f x]] \tan[e + f x]}{c f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 111 leaves):

$$\left(a^2 (-4 + i f x - \operatorname{Log}[1 + e^{2i(e+fx)}] + \cos[e + f x] (-i f x + \operatorname{Log}[1 + e^{2i(e+fx)}])) \sqrt{a(1 + \sec[e + f x])} \tan\left[\frac{1}{2}(e + f x)\right] \right) / \\ (c f (-1 + \cos[e + f x]) \sqrt{c - c \sec[e + f x]})$$

- **Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \sec[e + f x])^{5/2}}{(c - c \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 100 leaves, 3 steps):

$$-\frac{2 a^3 \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{5/2}} + \frac{a^3 \operatorname{Log}[1 - \cos[e + f x]] \tan[e + f x]}{c^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 155 leaves):

$$\left(a^2 (4 - 3 i f x + \cos[e + f x] (-8 + 4 i f x - 8 \operatorname{Log}[1 - e^{i(e+fx)}]) + 6 \operatorname{Log}[1 - e^{i(e+fx)}] + \cos[2(e + f x)] (-i f x + 2 \operatorname{Log}[1 - e^{i(e+fx)}])) \right) \\ \sqrt{a(1 + \sec[e + f x])} \tan\left[\frac{1}{2}(e + f x)\right] / \left(2 c^2 f (-1 + \cos[e + f x])^2 \sqrt{c - c \sec[e + f x]} \right)$$

- **Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[e + f x])^{5/2}}{(c - c \sec[e + f x])^{7/2}} dx$$

Optimal (type 3, 148 leaves, 4 steps):

$$-\frac{4 a^3 \operatorname{Tan}[e+f x]}{3 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c-c \operatorname{Sec}[e+f x])^{7/2}} - \frac{a^3 \operatorname{Tan}[e+f x]}{c^2 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c-c \operatorname{Sec}[e+f x])^{3/2}} + \frac{a^3 \operatorname{Log}[1-\operatorname{Cos}[e+f x]] \operatorname{Tan}[e+f x]}{c^3 f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]}}$$

Result (type 3, 489 leaves):

$$\left(8 i \sqrt{2} e^{\frac{1}{2} i (e+f x)} \sqrt{\frac{(1+e^{i(e+f x)})^2}{1+e^{2 i (e+f x)}}} (f x+2 i \operatorname{Log}[1-e^{i(e+f x)}]) \operatorname{Sec}[e+f x]^{7/2} (a(1+\operatorname{Sec}[e+f x]))^{5/2} \operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right]^7 \right) /$$

$$\left((1+e^{i(e+f x)}) \sqrt{\frac{e^{i(e+f x)}}{1+e^{2 i (e+f x)}}} f (1+\operatorname{Sec}[e+f x])^{5/2} (c-c \operatorname{Sec}[e+f x])^{7/2} \right) +$$

$$\left(\operatorname{Sec}[e+f x]^4 \sqrt{(1+\operatorname{Cos}[e+f x]) \operatorname{Sec}[e+f x]} (a(1+\operatorname{Sec}[e+f x]))^{5/2} \right.$$

$$\left(-\frac{80 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]}{3 f} + \frac{28 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]^3}{3 f} - \frac{4 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]^5}{3 f} + \frac{40 \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]}{3 f} + \frac{80 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]^2 \operatorname{Sin}\left[\frac{f x}{2}\right]}{3 f} -$$

$$\frac{28 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]^4 \operatorname{Sin}\left[\frac{f x}{2}\right]}{3 f} + \frac{4 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 \operatorname{Sin}\left[\frac{f x}{2}\right]}{3 f} \right) \operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right]^7 / \left((1+\operatorname{Sec}[e+f x])^{5/2} (c-c \operatorname{Sec}[e+f x])^{7/2} \right)$$

■ **Problem 108: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a+a \operatorname{Sec}[e+f x])^{5/2}}{(c-c \operatorname{Sec}[e+f x])^{9/2}} dx$$

Optimal (type 3, 194 leaves, 5 steps):

$$-\frac{a^3 \operatorname{Tan}[e+f x]}{f \sqrt{a+a \operatorname{Sec}[e+f x]} (c-c \operatorname{Sec}[e+f x])^{9/2}} - \frac{a^3 \operatorname{Tan}[e+f x]}{2 c^2 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c-c \operatorname{Sec}[e+f x])^{5/2}} -$$

$$\frac{a^3 \operatorname{Tan}[e+f x]}{c^3 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c-c \operatorname{Sec}[e+f x])^{3/2}} + \frac{a^3 \operatorname{Log}[1-\operatorname{Cos}[e+f x]] \operatorname{Tan}[e+f x]}{c^4 f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c-c \operatorname{Sec}[e+f x]}}$$

Result (type 3, 285 leaves):

$$\left(\frac{\text{Sec}[e + f x]^{9/2} (a (1 + \text{Sec}[e + f x]))^{5/2} \left(\frac{16 \sqrt{2} e^{\frac{1}{2} i (e+fx)} \sqrt{\frac{(1+e^i (e+fx))^2}{1+e^{2i (e+fx)}}} (-i f x + 2 \text{Log}[1 - e^i (e+fx)])}{(1 + e^i (e+fx)) \sqrt{\frac{e^i (e+fx)}{1+e^{2i (e+fx)}}} f} \right)}{1 / (8 f) (-54 + 89 \text{Cos}[e + f x] - 60 \text{Cos}[2 (e + f x)] + 23 \text{Cos}[3 (e + f x)] - 6 \text{Cos}[4 (e + f x)]) \text{Csc}\left[\frac{1}{2} (e + f x)\right]^8} \right. \\ \left. \text{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \right) \text{Sin}\left[\frac{1}{2} (e + f x)\right]^9 \Big/ \left((1 + \text{Sec}[e + f x])^{5/2} (c - c \text{Sec}[e + f x])^{9/2} \right)$$

■ **Problem 109: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \text{Sec}[e + f x])^{5/2}}{(c - c \text{Sec}[e + f x])^{11/2}} dx$$

Optimal (type 3, 244 leaves, 6 steps):

$$\frac{4 a^3 \text{Tan}[e + f x]}{5 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{11/2}} - \frac{a^3 \text{Tan}[e + f x]}{3 c^2 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{7/2}} - \frac{a^3 \text{Tan}[e + f x]}{2 c^3 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{5/2}} - \frac{a^3 \text{Tan}[e + f x]}{c^4 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{3/2}} + \frac{a^3 \text{Log}[1 - \text{Cos}[e + f x]] \text{Tan}[e + f x]}{c^5 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 299 leaves):

$$\left(\frac{\text{Sec}[e + f x]^{11/2} (a (1 + \text{Sec}[e + f x]))^{5/2} \left(\frac{32 i \sqrt{2} e^{\frac{1}{2} i (e+fx)} \sqrt{\frac{(1+e^i (e+fx))^2}{1+e^{2i (e+fx)}}} (f x + 2 i \text{Log}[1 - e^i (e+fx)])}{(1 + e^i (e+fx)) \sqrt{\frac{e^i (e+fx)}{1+e^{2i (e+fx)}}} f} \right)}{1 / (240 f) (5612 \text{Cos}[e + f x] - 5 (625 + 736 \text{Cos}[2 (e + f x)] - 367 \text{Cos}[3 (e + f x)] + 111 \text{Cos}[4 (e + f x)] - 21 \text{Cos}[5 (e + f x)])} \right) \\ \text{Csc}\left[\frac{1}{2} (e + f x)\right]^{10} \text{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \right) \text{Sin}\left[\frac{1}{2} (e + f x)\right]^{11} \Big/ \left((1 + \text{Sec}[e + f x])^{5/2} (c - c \text{Sec}[e + f x])^{11/2} \right)$$

■ **Problem 110: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \text{Sec}[e + f x])^{7/2}}{\sqrt{a + a \text{Sec}[e + f x]}} dx$$

Optimal (type 3, 204 leaves, 3 steps) :

$$\frac{c^4 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{8 c^4 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} -$$

$$\frac{4 c^4 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^4 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 153 leaves) :

$$\frac{1}{2 f \sqrt{a (1 + \operatorname{Sec}[e + f x])}} c^3 \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right]$$

$$\frac{(-1 + i f x + 8 \operatorname{Cos}[e + f x] - 16 \operatorname{Log}[1 + e^{i (e + f x)}] + 7 \operatorname{Log}[1 + e^{2 i (e + f x)}] + \operatorname{Cos}[2 (e + f x)]) (i f x - 16 \operatorname{Log}[1 + e^{i (e + f x)}] + 7 \operatorname{Log}[1 + e^{2 i (e + f x)}])}{\operatorname{Sec}[e + f x]^2 \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

■ **Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{5/2}}{\sqrt{a + a \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 151 leaves, 3 steps) :

$$\frac{c^3 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{4 c^3 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{c^3 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 315 leaves) :

$$\left(e^{\frac{1}{2} i (e + f x)} \sqrt{\frac{(1 + e^{i (e + f x)})^2}{1 + e^{2 i (e + f x)}}} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 (i f x - 8 \operatorname{Log}[1 + e^{i (e + f x)}] + 3 \operatorname{Log}[1 + e^{2 i (e + f x)}]) \sqrt{1 + \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2} \right) /$$

$$\left(4 \sqrt{2} (1 + e^{i (e + f x)}) \sqrt{\frac{e^{i (e + f x)}}{1 + e^{2 i (e + f x)}}} f \operatorname{Sec}[e + f x]^{5/2} \sqrt{a (1 + \operatorname{Sec}[e + f x])} \right) + \frac{1}{8 f \sqrt{a (1 + \operatorname{Sec}[e + f x])}}$$

$$\operatorname{Cos}[e + f x]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right] \sqrt{(1 + \operatorname{Cos}[e + f x]) \operatorname{Sec}[e + f x]} \sqrt{1 + \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2}}$$

■ **Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + a \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 102 leaves, 3 steps) :

$$\frac{c^2 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{2 c^2 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 103 leaves) :

$$\frac{c \left(1 + e^{i(e+fx)} \right) \left(fx + 4i \operatorname{Log} \left[1 + e^{i(e+fx)} \right] - i \operatorname{Log} \left[1 + e^{2i(e+fx)} \right] \right) \sqrt{c - c \operatorname{Sec} [e + fx]}}{\left(-1 + e^{i(e+fx)} \right) f \sqrt{a} \left(1 + \operatorname{Sec} [e + fx] \right)}$$

- **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c - c \operatorname{Sec} [e + fx]}}{\sqrt{a + a \operatorname{Sec} [e + fx]}} dx$$

Optimal (type 3, 49 leaves, 2 steps) :

$$\frac{c \operatorname{Log} [1 + \operatorname{Cos} [e + fx]] \operatorname{Tan} [e + fx]}{f \sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}}$$

Result (type 3, 105 leaves) :

$$\frac{\left(1 + e^{i(e+fx)} \right) \sqrt{\frac{c \left(-1 + e^{i(e+fx)} \right)^2}{1 + e^{2i(e+fx)}}} \left(fx + 2i \operatorname{Log} \left[1 + e^{i(e+fx)} \right] \right)}{\left(-1 + e^{i(e+fx)} \right) f \sqrt{a} \left(1 + \operatorname{Sec} [e + fx] \right)}$$

- **Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}} dx$$

Optimal (type 3, 46 leaves, 2 steps) :

$$\frac{\operatorname{Log} [\operatorname{Sin} [e + fx]] \operatorname{Tan} [e + fx]}{f \sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}}$$

Result (type 3, 122 leaves) :

$$\frac{2 \left(-1 + e^{i(e+fx)} \right) \operatorname{Cos} \left[\frac{1}{2} (e + fx) \right]^2 \left(fx + i \operatorname{Log} \left[1 - e^{i(e+fx)} \right] + i \operatorname{Log} \left[1 + e^{i(e+fx)} \right] \right) \operatorname{Sec} [e + fx]}{\left(1 + e^{i(e+fx)} \right) f \sqrt{a} \left(1 + \operatorname{Sec} [e + fx] \right) \sqrt{c - c \operatorname{Sec} [e + fx]}}$$

- **Problem 115: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \operatorname{Sec} [e + fx]} \left(c - c \operatorname{Sec} [e + fx] \right)^{3/2}} dx$$

Optimal (type 3, 168 leaves, 3 steps) :

$$\frac{\operatorname{Tan} [e + fx]}{2cf \left(1 - \operatorname{Cos} [e + fx] \right) \sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}} + \frac{3 \operatorname{Log} [1 - \operatorname{Cos} [e + fx]] \operatorname{Tan} [e + fx]}{4cf \sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}} + \frac{\operatorname{Log} [1 + \operatorname{Cos} [e + fx]] \operatorname{Tan} [e + fx]}{4cf \sqrt{a + a \operatorname{Sec} [e + fx]} \sqrt{c - c \operatorname{Sec} [e + fx]}}$$

Result (type 3, 143 leaves) :

$$\left((-1 + 2i f x - 3 \operatorname{Log}[1 - e^{i(e+fx)}] - \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[e + f x] (-2i f x + 3 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Log}[1 + e^{i(e+fx)}])) \operatorname{Tan}[e + f x] \right) / \left(2 c f (-1 + \operatorname{Cos}[e + f x]) \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 116: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 274 leaves, 3 steps) :

$$\frac{\frac{\operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{7 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{8 c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{8 c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{3 \operatorname{Tan}[e + f x]}}{4 c^2 f (1 - \operatorname{Sec}[e + f x])^2 \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{4 c^2 f (1 - \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 194 leaves) :

$$\left((8 - 12i f x + 21 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e + f x] (-10 + 16i f x - 28 \operatorname{Log}[1 - e^{i(e+fx)}] - 4 \operatorname{Log}[1 + e^{i(e+fx)}])) + 3 \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[2(e + f x)] (-4i f x + 7 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Log}[1 + e^{i(e+fx)}])) \operatorname{Tan}[e + f x] \right) / \left(8 c^2 f (-1 + \operatorname{Cos}[e + f x])^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 117: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{7/2}}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 215 leaves, 3 steps) :

$$\frac{\frac{c^4 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{4 c^4 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^4 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{8 c^4 \operatorname{Tan}[e + f x]}{a f (1 + \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 204 leaves) :

$$\left(c^3 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (-2 + i f x + 8 \operatorname{Log}[1 + e^{i(e+fx)}] + 2 \operatorname{Cos}[e + f x] (-9 + i f x + 8 \operatorname{Log}[1 + e^{i(e+fx)}] - 5 \operatorname{Log}[1 + e^{2i(e+fx)}])) + \operatorname{Cos}[2(e + f x)] (i f x + 8 \operatorname{Log}[1 + e^{i(e+fx)}] - 5 \operatorname{Log}[1 + e^{2i(e+fx)}]) - 5 \operatorname{Log}[1 + e^{2i(e+fx)}]) \operatorname{Sec}[e + f x] \sqrt{c - c \operatorname{Sec}[e + f x]} \right) / \left(2 a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a(1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 118: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{5/2}}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps) :

$$-\frac{4 c^3 \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^3 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 116 leaves) :

$$\left(i c^2 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (4 i + f x + \operatorname{Cos}[e + f x]) (f x + i \operatorname{Log}[1 + e^{2 i (e + f x)}]) + i \operatorname{Log}[1 + e^{2 i (e + f x)}]) \sqrt{c - c \operatorname{Sec}[e + f x]} \right) / \left(a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 119: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{3/2}}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 98 leaves, 3 steps) :

$$-\frac{2 c^2 \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^2 \operatorname{Log}[1 + \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 114 leaves) :

$$\left(i c \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (2 i + f x + \operatorname{Cos}[e + f x]) (f x + 2 i \operatorname{Log}[1 + e^{i (e + f x)}]) + 2 i \operatorname{Log}[1 + e^{i (e + f x)}]) \sqrt{c - c \operatorname{Sec}[e + f x]} \right) / \left(a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 120: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{c - c \operatorname{Sec}[e + f x]}}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 94 leaves, 3 steps) :

$$-\frac{c \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c \operatorname{Log}[1 + \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 106 leaves) :

$$\frac{1}{f (a (1 + \operatorname{Sec}[e + f x]))^{3/2}} i \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (i + f x + \operatorname{Cos}[e + f x]) (f x + 2 i \operatorname{Log}[1 + e^{i (e + f x)}]) + 2 i \operatorname{Log}[1 + e^{i (e + f x)}]) \operatorname{Sec}[e + f x] \sqrt{c - c \operatorname{Sec}[e + f x]}$$

■ **Problem 121: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 215 leaves, 3 steps):

$$\frac{\operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{4 a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} +$$

$$\frac{3 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{4 a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{2 a f (1 + \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 141 leaves):

$$\left((1 - 2 i f x + \operatorname{Log}[1 - e^{i(e+fx)}] + 3 \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[e + f x] (-2 i f x + \operatorname{Log}[1 - e^{i(e+fx)}] + 3 \operatorname{Log}[1 + e^{i(e+fx)}])) \operatorname{Tan}[e + f x] \right) /$$

$$\left(2 a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 122: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 101 leaves, 3 steps):

$$\frac{\operatorname{Cot}[e + f x]}{2 a c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Log}[\operatorname{Sin}[e + f x]] \operatorname{Tan}[e + f x]}{a c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 151 leaves):

$$\left((1 - i f x + \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[2(e + f x)] (i f x - \operatorname{Log}[1 - e^{i(e+fx)}] - \operatorname{Log}[1 + e^{i(e+fx)}]) + \operatorname{Log}[1 + e^{i(e+fx)}]) \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) /$$

$$\left(2 c f (-1 + \operatorname{Sec}[e + f x]) (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 123: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 347 leaves, 3 steps):

$$\frac{\operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{11 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{16 a c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} +$$

$$\frac{5 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{16 a c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{8 a c^2 f (1 - \operatorname{Sec}[e + f x])^2 \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} -$$

$$\frac{\operatorname{Tan}[e + f x]}{2 a c^2 f (1 - \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{8 a c^2 f (1 + \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 275 leaves) :

$$\left((14 - 16 i f x - 8 i f x \cos[3(e + f x)] + 22 \log[1 - e^{i(e + f x)}] + 11 \cos[3(e + f x)] \log[1 - e^{i(e + f x)}] + \cos[e + f x] (-12 + 8 i f x - 11 \log[1 - e^{i(e + f x)}] - 5 \log[1 + e^{i(e + f x)}]) + 2 \cos[2(e + f x)] (-5 + 8 i f x - 11 \log[1 - e^{i(e + f x)}] - 5 \log[1 + e^{i(e + f x)}]) + 10 \log[1 + e^{i(e + f x)}] + 5 \cos[3(e + f x)] \log[1 + e^{i(e + f x)}]) \tan[e + f x] \right) / \left(32 a^2 c^2 f (-1 + \cos[e + f x])^2 (1 + \cos[e + f x]) \sqrt{a(1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right)$$

■ **Problem 124: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \sec[e + f x])^{7/2}}{(a + a \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 220 leaves, 3 steps) :

$$\frac{c^4 \log[\cos[e + f x]] \tan[e + f x]}{a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{2 c^4 \log[1 + \sec[e + f x]] \tan[e + f x]}{a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{4 c^4 \tan[e + f x]}{a^2 f (1 + \sec[e + f x])^2 \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{4 c^4 \tan[e + f x]}{a^2 f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 157 leaves) :

$$\left(c^3 \cot\left[\frac{1}{2}(e + f x)\right] (4 \cos[e + f x] (-2 + i f x - 4 \log[1 + e^{i(e + f x)}] + \log[1 + e^{2i(e + f x)}]) + (3 + \cos[2(e + f x)]) (i f x - 4 \log[1 + e^{i(e + f x)}] + \log[1 + e^{2i(e + f x)}])) \sqrt{c - c \sec[e + f x]} \right) / \left(2 a^2 f (1 + \cos[e + f x])^2 \sqrt{a(1 + \sec[e + f x])} \right)$$

■ **Problem 125: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \sec[e + f x])^{5/2}}{(a + a \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 98 leaves, 3 steps) :

$$- \frac{2 c^3 \tan[e + f x]}{f (a + a \sec[e + f x])^{5/2} \sqrt{c - c \sec[e + f x]}} + \frac{c^3 \log[1 + \cos[e + f x]] \tan[e + f x]}{a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 154 leaves) :

$$\left(i c^2 \cot\left[\frac{1}{2}(e + f x)\right] (4 i + 3 f x + \cos[2(e + f x)] (f x + 2 i \log[1 + e^{i(e + f x)}]) + 4 \cos[e + f x] (2 i + f x + 2 i \log[1 + e^{i(e + f x)}]) + 6 i \log[1 + e^{i(e + f x)}]) \sqrt{c - c \sec[e + f x]} \right) / \left(2 a^2 f (1 + \cos[e + f x])^2 \sqrt{a(1 + \sec[e + f x])} \right)$$

■ **Problem 126: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^{3/2}}{(a + a \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$-\frac{c^2 \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{c^2 \operatorname{Tan}[e + f x]}{a f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^2 \operatorname{Log}[1 + \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 152 leaves):

$$\left(i c \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(4 i + 3 f x + \operatorname{Cos}[2(e + f x)](f x + 2 i \operatorname{Log}[1 + e^{i(e + f x)}]) + \operatorname{Cos}[e + f x](6 i + 4 f x + 8 i \operatorname{Log}[1 + e^{i(e + f x)}]) + 6 i \operatorname{Log}[1 + e^{i(e + f x)}]) \right) \right) / \left(2 a^2 f (1 + \operatorname{Cos}[e + f x])^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 127: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{c - c \operatorname{Sec}[e + f x]}}{(a + a \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{c \operatorname{Tan}[e + f x]}{2 f (a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{c \operatorname{Tan}[e + f x]}{a f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c \operatorname{Log}[1 + \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 151 leaves):

$$\left(i \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(3 i + 3 f x + \operatorname{Cos}[2(e + f x)](f x + 2 i \operatorname{Log}[1 + e^{i(e + f x)}]) + 4 \operatorname{Cos}[e + f x](i + f x + 2 i \operatorname{Log}[1 + e^{i(e + f x)}]) + 6 i \operatorname{Log}[1 + e^{i(e + f x)}]) \right) \right) / \left(2 a^2 f (1 + \operatorname{Cos}[e + f x])^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 128: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 270 leaves, 3 steps):

$$\frac{\operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{8 a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{7 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{8 a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Tan}[e + f x]}{4 a^2 f (1 + \operatorname{Sec}[e + f x])^2 \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{3 \operatorname{Tan}[e + f x]}{4 a^2 f (1 + \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 195 leaves):

$$\left((8 - 12 i f x + 3 \operatorname{Log}[1 - e^{i(e+fx)}] + 21 \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[2(e+fx)]) (-4 i f x + \operatorname{Log}[1 - e^{i(e+fx)}] + 7 \operatorname{Log}[1 + e^{i(e+fx)}]) + 2 \operatorname{Cos}[e+fx] (5 - 8 i f x + 2 \operatorname{Log}[1 - e^{i(e+fx)}] + 14 \operatorname{Log}[1 + e^{i(e+fx)}]) \right) \operatorname{Tan}[e+fx] \Big/ \left(8 a^2 f (1 + \operatorname{Cos}[e+fx])^2 \sqrt{a(1 + \operatorname{Sec}[e+fx])} \sqrt{c - c \operatorname{Sec}[e+fx]} \right)$$

- **Problem 129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e+fx])^{5/2} (c - c \operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 345 leaves, 3 steps):

$$\frac{\operatorname{Log}[\operatorname{Cos}[e+fx]] \operatorname{Tan}[e+fx]}{a^2 c f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} + \frac{5 \operatorname{Log}[1 - \operatorname{Sec}[e+fx]] \operatorname{Tan}[e+fx]}{16 a^2 c f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} + \frac{11 \operatorname{Log}[1 + \operatorname{Sec}[e+fx]] \operatorname{Tan}[e+fx]}{16 a^2 c f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} - \frac{\operatorname{Tan}[e+fx]}{8 a^2 c f (1 - \operatorname{Sec}[e+fx]) \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} - \frac{\operatorname{Tan}[e+fx]}{2 a^2 c f (1 + \operatorname{Sec}[e+fx]) \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 275 leaves):

$$\left((-14 + 16 i f x - 8 i f x \operatorname{Cos}[3(e+fx)] - 10 \operatorname{Log}[1 - e^{i(e+fx)}] + 5 \operatorname{Cos}[3(e+fx)] \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e+fx] (-12 + 8 i f x - 5 \operatorname{Log}[1 - e^{i(e+fx)}] - 11 \operatorname{Log}[1 + e^{i(e+fx)}]) - 22 \operatorname{Log}[1 + e^{i(e+fx)}] + 11 \operatorname{Cos}[3(e+fx)] \operatorname{Log}[1 + e^{i(e+fx)}] + 2 \operatorname{Cos}[2(e+fx)] (5 - 8 i f x + 5 \operatorname{Log}[1 - e^{i(e+fx)}] + 11 \operatorname{Log}[1 + e^{i(e+fx)}]) \right) \operatorname{Tan}[e+fx] \Big/ \left(32 a^2 c f (-1 + \operatorname{Cos}[e+fx]) (1 + \operatorname{Cos}[e+fx])^2 \sqrt{a(1 + \operatorname{Sec}[e+fx])} \sqrt{c - c \operatorname{Sec}[e+fx]} \right)$$

- **Problem 130: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e+fx])^{5/2} (c - c \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 151 leaves, 4 steps):

$$\frac{\operatorname{Cot}[e+fx]}{2 a^2 c^2 f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} - \frac{\operatorname{Cot}[e+fx]^3}{4 a^2 c^2 f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}} + \frac{\operatorname{Log}[\operatorname{Sin}[e+fx]] \operatorname{Tan}[e+fx]}{a^2 c^2 f \sqrt{a+a \operatorname{Sec}[e+fx]} \sqrt{c-c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 195 leaves):

$$\left(\operatorname{Csc}[e+fx]^3 (2 - 3 i f x + 3 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[2(e+fx)] (-4 + 4 i f x - 4 \operatorname{Log}[1 - e^{i(e+fx)}] - 4 \operatorname{Log}[1 + e^{i(e+fx)}]) + 3 \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[4(e+fx)] (-i f x + \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Log}[1 + e^{i(e+fx)}]) \right) \operatorname{Sec}[e+fx] \Big/ \left(8 a^2 c^2 f \sqrt{a(1 + \operatorname{Sec}[e+fx])} \sqrt{c - c \operatorname{Sec}[e+fx]} \right)$$

- **Problem 131: Unable to integrate problem.**

$$\int (1 + \operatorname{Sec}[e+fx])^m (c - c \operatorname{Sec}[e+fx])^n dx$$

Optimal (type 6, 92 leaves, 2 steps):

$$\left(2^{\frac{1}{2}+m} \text{AppellF1}\left[\frac{1}{2}+n, \frac{1}{2}-m, 1, \frac{3}{2}+n, \frac{1}{2}(1-\text{Sec}[e+fx]), 1-\text{Sec}[e+fx]\right] (c-c\text{Sec}[e+fx])^n \text{Tan}[e+fx] \right) / \left(f(1+2n) \sqrt{1+\text{Sec}[e+fx]} \right)$$

Result (type 8, 26 leaves):

$$\int (1 + \text{Sec}[e + f x])^m (c - c \text{Sec}[e + f x])^n dx$$

■ **Problem 132: Unable to integrate problem.**

$$\int (a + a \text{Sec}[e + f x])^m (c - c \text{Sec}[e + f x])^n dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$\frac{1}{f(1+2m)} 2^{\frac{1}{2}+n} c \text{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}-n, 1, \frac{3}{2}+m, \frac{1}{2}(1+\text{Sec}[e+fx]), 1+\text{Sec}[e+fx]\right] \\ (1 - \text{Sec}[e + f x])^{\frac{1}{2}-n} (a + a \text{Sec}[e + f x])^m (c - c \text{Sec}[e + f x])^{-1+n} \text{Tan}[e + f x]$$

Result (type 8, 28 leaves):

$$\int (a + a \text{Sec}[e + f x])^m (c - c \text{Sec}[e + f x])^n dx$$

■ **Problem 133: Unable to integrate problem.**

$$\int (a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^n dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$\frac{1}{7f} 2^{\frac{1}{2}+n} c \text{AppellF1}\left[\frac{7}{2}, \frac{1}{2}-n, 1, \frac{9}{2}, \frac{1}{2}(1+\text{Sec}[e+fx]), 1+\text{Sec}[e+fx]\right] \\ (1 - \text{Sec}[e + f x])^{\frac{1}{2}-n} (a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^{-1+n} \text{Tan}[e + f x]$$

Result (type 8, 28 leaves):

$$\int (a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^n dx$$

■ **Problem 134: Unable to integrate problem.**

$$\int (a + a \text{Sec}[e + f x])^2 (c - c \text{Sec}[e + f x])^n dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$\frac{1}{5f} 2^{\frac{1}{2}+n} c \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-n, 1, \frac{7}{2}, \frac{1}{2}(1+\text{Sec}[e+fx]), 1+\text{Sec}[e+fx]\right] \\ (1 - \text{Sec}[e + f x])^{\frac{1}{2}-n} (a + a \text{Sec}[e + f x])^2 (c - c \text{Sec}[e + f x])^{-1+n} \text{Tan}[e + f x]$$

Result (type 8, 28 leaves):

$$\int (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^n dx$$

■ **Problem 135: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^n dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{1}{3f} 2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - n, 1, \frac{5}{2}, \frac{1}{2} (1 + \operatorname{Sec}[e + f x]), 1 + \operatorname{Sec}[e + f x]\right] \\ (1 - \operatorname{Sec}[e + f x])^{\frac{1}{2}-n} (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^{-1+n} \operatorname{Tan}[e + f x]$$

Result (type 8, 26 leaves):

$$\int (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^n dx$$

■ **Problem 136: Unable to integrate problem.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^n}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$-\frac{1}{f (a + a \operatorname{Sec}[e + f x])} \\ 2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[-\frac{1}{2}, \frac{1}{2} - n, 1, \frac{1}{2}, \frac{1}{2} (1 + \operatorname{Sec}[e + f x]), 1 + \operatorname{Sec}[e + f x]\right] (1 - \operatorname{Sec}[e + f x])^{\frac{1}{2}-n} (c - c \operatorname{Sec}[e + f x])^{-1+n} \operatorname{Tan}[e + f x]$$

Result (type 8, 28 leaves):

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^n}{a + a \operatorname{Sec}[e + f x]} dx$$

■ **Problem 137: Unable to integrate problem.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^n}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$-\frac{1}{3f (a + a \operatorname{Sec}[e + f x])^2} \\ 2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[-\frac{3}{2}, \frac{1}{2} - n, 1, -\frac{1}{2}, \frac{1}{2} (1 + \operatorname{Sec}[e + f x]), 1 + \operatorname{Sec}[e + f x]\right] (1 - \operatorname{Sec}[e + f x])^{\frac{1}{2}-n} (c - c \operatorname{Sec}[e + f x])^{-1+n} \operatorname{Tan}[e + f x]$$

Result (type 8, 28 leaves):

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^n}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

■ **Problem 138: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^n dx$$

Optimal (type 5, 172 leaves, 4 steps):

$$\frac{6 a^3 (c - c \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \operatorname{Sec}[e + f x]\right] (c - c \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{2 a^3 (c - c \operatorname{Sec}[e + f x])^{1+n} \operatorname{Tan}[e + f x]}{c f (3 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 8, 30 leaves):

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^n dx$$

■ **Problem 139: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^n dx$$

Optimal (type 5, 119 leaves, 3 steps):

$$\frac{2 a^2 (c - c \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \operatorname{Sec}[e + f x]\right] (c - c \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 8, 30 leaves):

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^n dx$$

■ **Problem 140: Unable to integrate problem.**

$$\int \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^n dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{2 a \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \operatorname{Sec}[e + f x]\right] (c - c \operatorname{Sec}[e + f x])^n \operatorname{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 8, 30 leaves):

$$\int \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^n dx$$

■ **Problem 141: Unable to integrate problem.**

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^n}{\sqrt{a + a \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 5, 139 leaves, 4 steps) :

$$\frac{\text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, \frac{1}{2} (1 - \text{Sec}[e + f x])\right] (c - c \text{Sec}[e + f x])^n \text{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]}} + \frac{2 \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \text{Sec}[e + f x]\right] (c - c \text{Sec}[e + f x])^n \text{Tan}[e + f x]}{f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]}}$$

Result (type 8, 30 leaves) :

$$\int \frac{(c - c \text{Sec}[e + f x])^n}{\sqrt{a + a \text{Sec}[e + f x]}} dx$$

■ **Problem 142: Unable to integrate problem.**

$$\int \frac{(c - c \text{Sec}[e + f x])^n}{(a + a \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 5, 205 leaves, 5 steps) :

$$\frac{(5 - 2 n) \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, \frac{1}{2} (1 - \text{Sec}[e + f x])\right] (c - c \text{Sec}[e + f x])^n \text{Tan}[e + f x]}{4 a f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]}} + \frac{2 \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \text{Sec}[e + f x]\right] (c - c \text{Sec}[e + f x])^n \text{Tan}[e + f x]}{a f (1 + 2 n) \sqrt{a + a \text{Sec}[e + f x]}} - \frac{(c - c \text{Sec}[e + f x])^n \text{Tan}[e + f x]}{2 a f (1 + \text{Sec}[e + f x]) \sqrt{a + a \text{Sec}[e + f x]}}$$

Result (type 8, 30 leaves) :

$$\int \frac{(c - c \text{Sec}[e + f x])^n}{(a + a \text{Sec}[e + f x])^{3/2}} dx$$

■ **Problem 143: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a + a \text{Sec}[e + f x]}}{c + c \text{Sec}[e + f x]} dx$$

Optimal (type 3, 91 leaves, 6 steps) :

$$\frac{2 \sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + a \text{Sec}[e + f x]}}\right]}{c f} - \frac{\sqrt{2} \sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{2} \sqrt{a + a \text{Sec}[e + f x]}}\right]}{c f}$$

Result (type 3, 168 leaves) :

$$\frac{1}{c (1 + e^{i (e+fx)}) f} \sqrt{1 + e^{2i (e+fx)}} \left(f x - i \operatorname{ArcSinh}[e^{i (e+fx)}] + i \sqrt{2} \operatorname{Log}[1 + e^{i (e+fx)}] + i \operatorname{Log}[1 + \sqrt{1 + e^{2i (e+fx)}}] - i \sqrt{2} \operatorname{Log}[1 - e^{i (e+fx)} + \sqrt{2} \sqrt{1 + e^{2i (e+fx)}}] \right) \sqrt{a (1 + \operatorname{Sec}[e + f x])}$$

■ **Problem 146: Unable to integrate problem.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x]) \sqrt{c + d \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 319 leaves, 5 steps):

$$\frac{2 \sqrt{c+d} \operatorname{Cot}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d \operatorname{Sec}[e+fx]}}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \sqrt{\frac{d(1-\operatorname{Sec}[e+fx])}{c+d}} \sqrt{-\frac{d(1+\operatorname{Sec}[e+fx])}{c-d}}}{a(c-d)f} - \frac{2 \sqrt{c+d} \operatorname{Cot}[e + f x] \operatorname{EllipticPi}\left[\frac{c+d}{c}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d \operatorname{Sec}[e+fx]}}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \sqrt{\frac{d(1-\operatorname{Sec}[e+fx])}{c+d}} \sqrt{-\frac{d(1+\operatorname{Sec}[e+fx])}{c-d}}}{a c f} - \frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}[e+fx]}{1+\operatorname{Sec}[e+fx]}\right], \frac{c-d}{c+d}\right] \sqrt{\frac{1}{1+\operatorname{Sec}[e+fx]}} \sqrt{c+d \operatorname{Sec}[e+fx]}}{a(c-d)f \sqrt{\frac{c+d \operatorname{Sec}[e+fx]}{(c+d)(1+\operatorname{Sec}[e+fx])}}}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x]) \sqrt{c + d \operatorname{Sec}[e + f x]}} dx$$

■ **Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])^4 dx$$

Optimal (type 3, 271 leaves, 5 steps):

$$\frac{2 a d (2 c + d) (2 c^2 + 2 c d + d^2) \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^{3/2} c^4 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e+fx]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{2 d^2 (6 c^2 + 8 c d + 3 d^2) (a - a \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 d^3 (4 c + 3 d) (a - a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 a f \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{2 d^4 (a - a \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{7 a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 4, 589 leaves):

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^4}$$

$$\operatorname{Cos}[e + f x]^4 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right] \sqrt{a(1 + \operatorname{Sec}[e + f x])} (c + d \operatorname{Sec}[e + f x])^4 \left(\frac{8}{105} d (105 c^3 + 105 c^2 d + 56 c d^2 + 12 d^3) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \right.$$

$$\left. \frac{2}{7} d^4 \operatorname{Sec}[e + f x]^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{4}{35} \operatorname{Sec}[e + f x]^2 \left(14 c d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 3 d^4 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) +$$

$$\left. \frac{4}{105} \operatorname{Sec}[e + f x] \left(105 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 56 c d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 12 d^4 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) -$$

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^4} 8 (-3 - 2\sqrt{2}) c^4 \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Cos}[e + f x]^3$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]}$$

$$\sqrt{a(1 + \operatorname{Sec}[e + f x])} (c + d \operatorname{Sec}[e + f x])^4 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])^3 dx$$

Optimal (type 3, 205 leaves, 5 steps):

$$\frac{2 a d (3 c^2 + 3 c d + d^2) \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^{3/2} c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} -$$

$$\frac{2 d^2 (3 c + 2 d) (a - a \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 d^3 (a - a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 a f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 4, 519 leaves):

$$\frac{1}{f (d + c \cos[e + f x])^3} \cos[e + f x]^3 \sec\left[\frac{1}{2} (e + f x)\right] \sqrt{a (1 + \sec[e + f x])} (c + d \sec[e + f x])^3 \left(\frac{2}{15} d (45 c^2 + 30 c d + 8 d^2) \sin\left[\frac{1}{2} (e + f x)\right] + \frac{2}{5} d^3 \sec[e + f x]^2 \sin\left[\frac{1}{2} (e + f x)\right] + \frac{2}{15} \sec[e + f x] \left(15 c d^2 \sin\left[\frac{1}{2} (e + f x)\right] + 4 d^3 \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) - \frac{1}{f (d + c \cos[e + f x])^3} 8 (-3 - 2\sqrt{2}) c^3 \cos\left[\frac{1}{4} (e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right]}{1 + \cos\left[\frac{1}{2} (e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right]}{1 + \cos\left[\frac{1}{2} (e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right] \right) \cos[e + f x]^2 \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right] \right) \sec\left[\frac{1}{4} (e + f x)\right]^2 \sec\left[\frac{1}{2} (e + f x)\right]} \sqrt{a (1 + \sec[e + f x])} (c + d \sec[e + f x])^3 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (e + f x)\right]^2}$$

- **Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sec[e + f x]} (c + d \sec[e + f x])^2 dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{2 a d (2 c + d) \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} + \frac{2 a^{3/2} c^2 \text{ArcTanh}\left[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{a}}\right] \tan[e + f x]}{f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} - \frac{2 d^2 (a - a \sec[e + f x]) \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]}}$$

Result (type 4, 463 leaves):

$$\frac{1}{f (d + c \cos[e + f x])^2}$$

$$\cos[e + f x]^2 \sec\left[\frac{1}{2}(e + f x)\right] \sqrt{a(1 + \sec[e + f x])} (c + d \sec[e + f x])^2 \left(\frac{4}{3} d (3c + d) \sin\left[\frac{1}{2}(e + f x)\right] + \frac{2}{3} d^2 \sec[e + f x] \sin\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (d + c \cos[e + f x])^2} 8 (-3 - 2\sqrt{2}) c^2 \cos\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right) \cos[e + f x]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right) \sec\left[\frac{1}{4}(e + f x)\right]^2 \sec\left[\frac{1}{2}(e + f x)\right]}$$

$$\sqrt{a(1 + \sec[e + f x])} (c + d \sec[e + f x])^2 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 150: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sec[e + f x]} (c + d \sec[e + f x]) dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2\sqrt{a} c \text{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \sec[e + f x]}}\right]}{f} + \frac{2 a d \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}}$$

Result (type 4, 407 leaves):

$$\begin{aligned}
& - \frac{1}{f (d + c \operatorname{Cos}[e + f x])} 8 (-3 - 2\sqrt{2}) c \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]} \\
& \sqrt{a(1 + \operatorname{Sec}[e + f x])} (c + d \operatorname{Sec}[e + f x]) \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2} + \\
& \frac{2 d \operatorname{Cos}[e + f x] \sqrt{a(1 + \operatorname{Sec}[e + f x])} (c + d \operatorname{Sec}[e + f x]) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{f (d + c \operatorname{Cos}[e + f x])}
\end{aligned}$$

- **Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} (c + d \operatorname{Sec}[e + f x])^3 dx$$

Optimal (type 3, 241 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 (6 c + 13 d) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{35 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \\
& \frac{2 a^2 (c + d \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{7 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 (2 (36 c^3 + 243 c^2 d + 189 c d^2 + 52 d^3) + d (24 c^2 + 111 c d + 52 d^2) \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{105 f \sqrt{a + a \operatorname{Sec}[e + f x]}}
\end{aligned}$$

Result (type 4, 590 leaves):

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^3}$$

$$\operatorname{Cos}[e + f x]^4 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} (c + d \operatorname{Sec}[e + f x])^3 \left(\frac{1}{105} (105 c^3 + 525 c^2 d + 378 c d^2 + 104 d^3) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \right.$$

$$\left. \frac{1}{7} d^3 \operatorname{Sec}[e + f x]^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{35} \operatorname{Sec}[e + f x]^2 \left(21 c d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 13 d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) + \right.$$

$$\left. \frac{1}{105} \operatorname{Sec}[e + f x] \left(105 c^2 d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 189 c d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 52 d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) -$$

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^3} 4 (-3 - 2\sqrt{2}) c^3 \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Cos}[e + f x]^3$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3}$$

$$(a (1 + \operatorname{Sec}[e + f x]))^{3/2} (c + d \operatorname{Sec}[e + f x])^3 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

- **Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[e + f x])^{3/2} (c + d \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 3, 176 leaves, 5 steps):

$$\frac{2 a^{5/2} c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} +$$

$$\frac{2 a^2 (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 (2 (6 c^2 + 25 c d + 9 d^2) + d (4 c + 9 d) \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{15 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 4, 520 leaves):

$$\frac{1}{f (d + c \cos[e + f x])^2} \cos[e + f x]^3 \sec\left[\frac{1}{2} (e + f x)\right]^3 (a (1 + \sec[e + f x]))^{3/2} (c + d \sec[e + f x])^2 \left(\frac{1}{15} (15 c^2 + 50 c d + 18 d^2) \sin\left[\frac{1}{2} (e + f x)\right] + \frac{1}{5} d^2 \sec[e + f x]^2 \sin\left[\frac{1}{2} (e + f x)\right] + \frac{1}{15} \sec[e + f x] \left(10 c d \sin\left[\frac{1}{2} (e + f x)\right] + 9 d^2 \sin\left[\frac{1}{2} (e + f x)\right] \right) \right) - \frac{1}{f (d + c \cos[e + f x])^2} 4 (-3 - 2\sqrt{2}) c^2 \cos\left[\frac{1}{4} (e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right]}{1 + \cos\left[\frac{1}{2} (e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right]}{1 + \cos\left[\frac{1}{2} (e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right] \right) \cos[e + f x]^2 \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4} (e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2} (e + f x)\right] \right) \sec\left[\frac{1}{4} (e + f x)\right]^2 \sec\left[\frac{1}{2} (e + f x)\right]^3} (a (1 + \sec[e + f x]))^{3/2} (c + d \sec[e + f x])^2 \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4} (e + f x)\right]^2}$$

- **Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \sec[e + f x])^{3/2} (c + d \sec[e + f x]) dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\frac{2 a^{3/2} c \text{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \sec[e + f x]}}\right]}{f} + \frac{2 a^2 (3 c + 4 d) \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]}} + \frac{2 a d \sqrt{a + a \sec[e + f x]} \tan[e + f x]}{3 f}$$

Result (type 4, 460 leaves):

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])} \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 (a (1 + \operatorname{Sec}[e + f x]))^{3/2} (c + d \operatorname{Sec}[e + f x]) \left(\frac{1}{3} (3 c + 5 d) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{3} d \operatorname{Sec}[e + f x] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) -$$

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])} 4 (-3 - 2\sqrt{2}) c \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Cos}[e + f x]$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3}$$

$$(a (1 + \operatorname{Sec}[e + f x]))^{3/2} (c + d \operatorname{Sec}[e + f x]) \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 160: Result unnecessarily involves higher level functions.**

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c + d \operatorname{Sec}[e + f x])^3 dx$$

Optimal (type 3, 336 leaves, 5 steps):

$$\frac{2 a^3 (3 c^3 + 12 c^2 d + 12 c d^2 + 4 d^3) \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^{7/2} c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} +$$

$$\frac{2 a d (3 c^2 + 15 c d + 13 d^2) (a - a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 f \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{6 d^2 (c + 2 d) (a - a \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{7 f \sqrt{a + a \operatorname{Sec}[e + f x]}} +$$

$$\frac{2 d^3 (a - a \operatorname{Sec}[e + f x])^4 \operatorname{Tan}[e + f x]}{9 a f \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{2 (c^3 + 12 c^2 d + 24 c d^2 + 12 d^3) (a^3 - a^3 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 4, 665 leaves):

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^3}$$

$$\operatorname{Cos}[e + f x]^5 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} (c + d \operatorname{Sec}[e + f x])^3 \left(\frac{1}{630} (840 c^3 + 2709 c^2 d + 2070 c d^2 + 584 d^3) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \right.$$

$$\frac{1}{18} d^3 \operatorname{Sec}[e + f x]^4 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{126} \operatorname{Sec}[e + f x]^3 \left(27 c d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 26 d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) +$$

$$\frac{1}{210} \operatorname{Sec}[e + f x]^2 \left(63 c^2 d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 180 c d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 73 d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) +$$

$$\left. \frac{1}{630} \operatorname{Sec}[e + f x] \left(105 c^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 882 c^2 d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 1035 c d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 292 d^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) -$$

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^3} 2 (-3 - 2\sqrt{2}) c^3 \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Cos}[e + f x]^4$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5}$$

$$(a (1 + \operatorname{Sec}[e + f x]))^{5/2} (c + d \operatorname{Sec}[e + f x])^3 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c + d \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$\frac{2 a^3 (c + 2 d) (3 c + 2 d) \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^{7/2} c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a d (2 c + 5 d) (a - a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

$$\frac{2 d^2 (a - a \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{7 f \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{2 (c^2 + 8 c d + 8 d^2) (a^3 - a^3 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 4, 577 leaves) :

$$\frac{1}{f (d + c \operatorname{Cos}[e + f x])^2} \operatorname{Cos}[e + f x]^4 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 (a (1 + \operatorname{Sec}[e + f x]))^{5/2} (c + d \operatorname{Sec}[e + f x])^2 \left(\frac{1}{105} (140 c^2 + 301 c d + 115 d^2) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{14} d^2 \operatorname{Sec}[e + f x]^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + \frac{1}{35} \operatorname{Sec}[e + f x]^2 \left(7 c d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 10 d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) + \frac{1}{210} \operatorname{Sec}[e + f x] \left(35 c^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 196 c d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 115 d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \right) - \frac{1}{f (d + c \operatorname{Cos}[e + f x])^2} 2 (-3 - 2\sqrt{2}) c^2 \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Cos}[e + f x]^3 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sec}\left[\frac{1}{4}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5} (a (1 + \operatorname{Sec}[e + f x]))^{5/2} (c + d \operatorname{Sec}[e + f x])^2 \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c + d \operatorname{Sec}[e + f x]) dx$$

Optimal (type 3, 142 leaves, 6 steps) :

$$\frac{2 a^{5/2} c \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{f} + \frac{2 a^3 (35 c + 32 d) \operatorname{Tan}[e + f x]}{15 f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 a^2 (5 c + 8 d) \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{15 f} + \frac{2 a d (a + a \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{5 f}$$

Result (type 4, 501 leaves) :

$$\frac{1}{f (d + c \cos[e + f x])} \cos[e + f x]^3 \sec\left[\frac{1}{2}(e + f x)\right]^5 (a (1 + \sec[e + f x]))^{5/2} (c + d \sec[e + f x])$$

$$\left(\frac{1}{30} (40c + 43d) \sin\left[\frac{1}{2}(e + f x)\right] + \frac{1}{10} d \sec[e + f x]^2 \sin\left[\frac{1}{2}(e + f x)\right] + \frac{1}{30} \sec[e + f x] \left(5c \sin\left[\frac{1}{2}(e + f x)\right] + 14d \sin\left[\frac{1}{2}(e + f x)\right] \right) \right) -$$

$$\frac{1}{f (d + c \cos[e + f x])} 2 (-3 - 2\sqrt{2}) c \cos\left[\frac{1}{4}(e + f x)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right] \right) \cos[e + f x]^2$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]\right) \sec\left[\frac{1}{4}(e + f x)\right]^2 \sec\left[\frac{1}{2}(e + f x)\right]^5}$$

$$(a (1 + \sec[e + f x]))^{5/2} (c + d \sec[e + f x]) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(e + f x)\right]^2}$$

■ **Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + a \sec[e + f x]} (c + d \sec[e + f x])^2} dx$$

Optimal (type 3, 416 leaves, 12 steps):

$$\frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{a}}\right] \tan[e + f x]}{c^2 f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} -$$

$$\frac{\sqrt{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e + f x]}{(c - d)^2 f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} + \frac{\sqrt{a} d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \sec[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \tan[e + f x]}{c (c - d) (c + d)^{3/2} f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} +$$

$$\frac{2\sqrt{a} (2c - d) d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \sec[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \tan[e + f x]}{c^2 (c - d)^2 \sqrt{c + d} f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} + \frac{d^2 \tan[e + f x]}{c (c^2 - d^2) f \sqrt{a + a \sec[e + f x]} (c + d \sec[e + f x])}$$

Result (type 3, 2477 leaves):

$$\begin{aligned}
& \frac{\cos\left[\frac{1}{2}(e+fx)\right] (d+c\cos[e+fx])^2 \sec[e+fx]^3 \left(-\frac{2d^2 \sin\left[\frac{1}{2}(e+fx)\right]}{c^2(-c+d)(c+d)} + \frac{2d^3 \sin\left[\frac{1}{2}(e+fx)\right]}{c^2(-c+d)(c+d)(d+c\cos[e+fx])}\right)}{f\sqrt{a(1+\sec[e+fx])}(c+d\sec[e+fx])^2} - \\
& \left(\cos\left[\frac{1}{2}(e+fx)\right] (d+c\cos[e+fx])^2 \left(\frac{2\sqrt{2}d^{3/2}(5c^2+cd-2d^2) \operatorname{ArcTan}\left[\frac{\sqrt{d}\tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d}\sqrt{-\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right]}{\sqrt{-c-d}(c-d)} - \right. \right. \\
& \left. \sqrt{2}(c^2-d^2) \operatorname{Log}\left[\sec\left[\frac{1}{2}(e+fx)\right]\right]^2 \left(-1+2\cos[e+fx]-2\sqrt{-\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sin[e+fx]\right) + \right. \\
& \left. \sqrt{2}(c^2-d^2) \operatorname{Log}\left[\sec\left[\frac{1}{2}(e+fx)\right]\right]^2 \left(-1+2\cos[e+fx]+2\sqrt{-\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sin[e+fx]\right) + \right. \\
& \left. \frac{4c^2(c+d) \operatorname{Log}\left[\tan\left[\frac{1}{2}(e+fx)\right] + \sqrt{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2}\right]}{c-d} \right) \\
& \left(\frac{d\sec\left[\frac{1}{2}(e+fx)\right]}{(-c+d)(c+d)(d+c\cos[e+fx])\sqrt{\sec[e+fx]}} + \frac{d^2\sec\left[\frac{1}{2}(e+fx)\right]}{2c(-c+d)(c+d)(d+c\cos[e+fx])\sqrt{\sec[e+fx]}} - \right. \\
& \frac{c\sec\left[\frac{1}{2}(e+fx)\right]\sqrt{\sec[e+fx]}}{2(-c+d)(c+d)(d+c\cos[e+fx])} - \frac{c\cos[2(e+fx)]\sec\left[\frac{1}{2}(e+fx)\right]\sqrt{\sec[e+fx]}}{2(-c+d)(c+d)(d+c\cos[e+fx])} + \\
& \left. \frac{d^2\cos[2(e+fx)]\sec\left[\frac{1}{2}(e+fx)\right]\sqrt{\sec[e+fx]}}{2c(-c+d)(c+d)(d+c\cos[e+fx])} \right) \sec[e+fx]^{5/2} \sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]} \sqrt{-1+\tan\left[\frac{1}{2}(e+fx)\right]^2} /
\end{aligned}$$

$$\left(\frac{2 c^2 (c-d) (c+d) f \sqrt{a (1 + \operatorname{Sec}[e + f x])} (c + d \operatorname{Sec}[e + f x])^2}{4 c^2 (c-d) (c+d) \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}} \right) - \frac{1}{\sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}}$$

$$\left(\frac{2 \sqrt{2} d^{3/2} (5 c^2 + c d - 2 d^2) \operatorname{ArcTan}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{-c-d} \sqrt{-\frac{\operatorname{Cos}[e + f x]}{1 + \operatorname{Cos}[e + f x]}}}\right]}{\sqrt{-c-d} (c-d)} - \sqrt{2} (c^2 - d^2) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]\right]^2 \left(-1 + 2 \operatorname{Cos}[e + f x] - 2 \sqrt{-\frac{\operatorname{Cos}[e + f x]}{1 + \operatorname{Cos}[e + f x]}}}\right) \right.$$

$$\left. \operatorname{Sin}[e + f x] \right) + \sqrt{2} (c^2 - d^2) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]\right]^2 \left(-1 + 2 \operatorname{Cos}[e + f x] + 2 \sqrt{-\frac{\operatorname{Cos}[e + f x]}{1 + \operatorname{Cos}[e + f x]}} \operatorname{Sin}[e + f x]\right) +$$

$$\left. \frac{4 c^2 (c+d) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}\right]}{c-d} \right) \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \sqrt{\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x]} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] -$$

$$\frac{1}{2 c^2 (c-d) (c+d)} \sqrt{\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x]} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2} \left(- \sqrt{2} (c^2 - d^2) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \right)$$

$$\left(\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2 \left(-2 \operatorname{Cos}[e + f x] \sqrt{-\frac{\operatorname{Cos}[e + f x]}{1 + \operatorname{Cos}[e + f x]}} - 2 \operatorname{Sin}[e + f x] - \frac{\operatorname{Sin}[e + f x] \left(-\frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{(1 + \operatorname{Cos}[e + f x])^2} + \frac{\operatorname{Sin}[e + f x]}{1 + \operatorname{Cos}[e + f x]} \right)}{\sqrt{-\frac{\operatorname{Cos}[e + f x]}{1 + \operatorname{Cos}[e + f x]}}} \right) \right) +$$

$$\begin{aligned}
& \frac{2\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a\operatorname{Sec}[e+fx]}}{\sqrt{a}}\right] \operatorname{Tan}[e+fx]}{c^3 f \sqrt{a-a\operatorname{Sec}[e+fx]} \sqrt{a+a\operatorname{Sec}[e+fx]}} - \frac{\sqrt{2}\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a\operatorname{Sec}[e+fx]}}{\sqrt{2}\sqrt{a}}\right] \operatorname{Tan}[e+fx]}{(c-d)^3 f \sqrt{a-a\operatorname{Sec}[e+fx]} \sqrt{a+a\operatorname{Sec}[e+fx]}} + \\
& \frac{3\sqrt{a} d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d}\sqrt{a-a\operatorname{Sec}[e+fx]}}{\sqrt{a}\sqrt{c+d}}\right] \operatorname{Tan}[e+fx]}{4c(c-d)(c+d)^{5/2} f \sqrt{a-a\operatorname{Sec}[e+fx]} \sqrt{a+a\operatorname{Sec}[e+fx]}} + \frac{\sqrt{a}(2c-d)d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d}\sqrt{a-a\operatorname{Sec}[e+fx]}}{\sqrt{a}\sqrt{c+d}}\right] \operatorname{Tan}[e+fx]}{c^2(c-d)^2(c+d)^{3/2} f \sqrt{a-a\operatorname{Sec}[e+fx]} \sqrt{a+a\operatorname{Sec}[e+fx]}} + \\
& \frac{2\sqrt{a} d^{3/2} (3c^2 - 3cd + d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d}\sqrt{a-a\operatorname{Sec}[e+fx]}}{\sqrt{a}\sqrt{c+d}}\right] \operatorname{Tan}[e+fx]}{c^3(c-d)^3 \sqrt{c+d} f \sqrt{a-a\operatorname{Sec}[e+fx]} \sqrt{a+a\operatorname{Sec}[e+fx]}} + \frac{d^2 \operatorname{Tan}[e+fx]}{2c(c^2-d^2) f \sqrt{a+a\operatorname{Sec}[e+fx]} (c+d \operatorname{Sec}[e+fx])^2} + \\
& \frac{3d^2 \operatorname{Tan}[e+fx]}{4c(c-d)(c+d)^2 f \sqrt{a+a\operatorname{Sec}[e+fx]} (c+d \operatorname{Sec}[e+fx])} + \frac{(2c-d)d^2 \operatorname{Tan}[e+fx]}{c^2(c-d)^2(c+d) f \sqrt{a+a\operatorname{Sec}[e+fx]} (c+d \operatorname{Sec}[e+fx])}
\end{aligned}$$

Result (type 3, 2940 leaves):

$$\begin{aligned}
& \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] (d+c \operatorname{Cos}[e+fx])^3 \operatorname{Sec}[e+fx]^4 \left(-\frac{d^2(-13c^2 - cd + 6d^2) \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}{2c^3(-c+d)^2(c+d)^2} - \frac{d^4 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}{c^3(-c+d)(c+d)(d+c \operatorname{Cos}[e+fx])^2} + \right. \right. \\
& \left. \left. \frac{-15c^2 d^3 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] - cd^4 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + 8d^5 \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}{2c^3(-c+d)^2(c+d)^2(d+c \operatorname{Cos}[e+fx])} \right) \right) / \left(f \sqrt{a(1+\operatorname{Sec}[e+fx])} (c+d \operatorname{Sec}[e+fx])^3 \right) - \\
& \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] (d+c \operatorname{Cos}[e+fx])^3 \left(\frac{\sqrt{2} d^{3/2} (35c^4 + 14c^3 d - 21c^2 d^2 - 4cd^3 + 8d^4) \operatorname{ArcTan}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}\right]}{\sqrt{-c-d} (c-d)} - \right. \right. \\
& \left. \left. 2\sqrt{2} (c^2 - d^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]\right]^2 \left(-1 + 2 \operatorname{Cos}[e+fx] - 2 \sqrt{-\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}} \operatorname{Sin}[e+fx] \right) \right) + \right. \\
& \left. 2\sqrt{2} (c^2 - d^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]\right]^2 \left(-1 + 2 \operatorname{Cos}[e+fx] + 2 \sqrt{-\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}} \operatorname{Sin}[e+fx] \right) \right) + \right. \\
& \left. \frac{8c^3(c+d)^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\right]}{c-d} \right) \left(-\frac{2cd \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{(-c+d)^2(c+d)^2(d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{13 d^2 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]}{8(-c+d)^2(c+d)^2(d+c \cos [e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} + \frac{d^3 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]}{8 c(-c+d)^2(c+d)^2(d+c \cos [e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} + \\
& \frac{d^4 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]}{2 c^2(-c+d)^2(c+d)^2(d+c \cos [e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} + \frac{c^2 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{2(-c+d)^2(c+d)^2(d+c \cos [e+f x])} + \\
& \frac{3 d^2 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{8(-c+d)^2(c+d)^2(d+c \cos [e+f x])} + \frac{d^3 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{8 c(-c+d)^2(c+d)^2(d+c \cos [e+f x])} + \frac{c^2 \cos [2(e+f x)] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{2(-c+d)^2(c+d)^2(d+c \cos [e+f x])} - \\
& \left. \frac{d^2 \cos [2(e+f x)] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^2(c+d)^2(d+c \cos [e+f x])} + \frac{d^4 \cos [2(e+f x)] \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right] \sqrt{\operatorname{Sec}[e+f x]}}{2 c^2(-c+d)^2(c+d)^2(d+c \cos [e+f x])} \right) \operatorname{Sec}[e+f x]^{7/2} \\
& \left. \sqrt{\cos \left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]} \sqrt{-1+\tan \left[\frac{1}{2}(e+f x)\right]^2} \right) / \left(4 c^3(c-d)^2(c+d)^2 f \sqrt{a(1+\operatorname{Sec}[e+f x])} (c+d \operatorname{Sec}[e+f x])^3 \right. \\
& \left. \left(-\frac{1}{8 c^3(c-d)^2(c+d)^2 \sqrt{-1+\tan \left[\frac{1}{2}(e+f x)\right]^2}} \left(\frac{\sqrt{2} d^{3/2} (35 c^4+14 c^3 d-21 c^2 d^2-4 c d^3+8 d^4) \operatorname{ArcTan}\left[\frac{\sqrt{d} \tan \left[\frac{1}{2}(e+f x)\right]}{\sqrt{-c-d} \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}}}\right]}{\sqrt{-c-d}(c-d)} \right) \right. \right. \\
& \left. \left. 2 \sqrt{2} (c^2-d^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\left(-1+2 \cos [e+f x]-2 \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}} \sin [e+f x]\right)\right] \right) + \right. \\
& \left. \left. 2 \sqrt{2} (c^2-d^2)^2 \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\left(-1+2 \cos [e+f x]+2 \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}} \sin [e+f x]\right)\right] \right) \right) +
\end{aligned}$$

$$\left. \frac{8 c^3 (c+d)^2 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] + \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}\right]}{c-d}\right) \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]} -$$

$$\frac{1}{4 c^3 (c-d)^2 (c+d)^2} \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \left(- \left(2 \sqrt{2} (c^2 - d^2)^2 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \right. \right.$$

$$\left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \left(-2 \operatorname{Cos}[e+f x] \sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}} - 2 \operatorname{Sin}[e+f x] - \frac{\operatorname{Sin}[e+f x] \left(-\frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{(1 + \operatorname{Cos}[e+f x])^2} + \frac{\operatorname{Sin}[e+f x]}{1 + \operatorname{Cos}[e+f x]} \right)}{\sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}}} \right) + \right. \right.$$

$$\left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \left(-1 + 2 \operatorname{Cos}[e+f x] - 2 \sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}} \operatorname{Sin}[e+f x] \right) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right] \right) \right) /$$

$$\left(-1 + 2 \operatorname{Cos}[e+f x] - 2 \sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}} \operatorname{Sin}[e+f x] \right) + \left(2 \sqrt{2} (c^2 - d^2)^2 \operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]^2 \right.$$

$$\left. \left. \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \left(2 \operatorname{Cos}[e+f x] \sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}} - 2 \operatorname{Sin}[e+f x] + \frac{\operatorname{Sin}[e+f x] \left(-\frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{(1 + \operatorname{Cos}[e+f x])^2} + \frac{\operatorname{Sin}[e+f x]}{1 + \operatorname{Cos}[e+f x]} \right)}{\sqrt{-\frac{\operatorname{Cos}[e+f x]}{1 + \operatorname{Cos}[e+f x]}}} \right) + \right. \right.$$

$$\begin{aligned}
& \cos\left[\frac{1}{2}(e+fx)\right]^3 \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} (d+c\cos[e+fx]) \\
& \left(\frac{c \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{2(-c+d)(d+c\cos[e+fx])\sqrt{\operatorname{Sec}[e+fx]}} - \frac{2d \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{(-c+d)(d+c\cos[e+fx])\sqrt{\operatorname{Sec}[e+fx]}} - \frac{c \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)(d+c\cos[e+fx])} + \right. \\
& \left. \frac{3d \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{2(-c+d)(d+c\cos[e+fx])} - \frac{c \cos[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)(d+c\cos[e+fx])} + \frac{d \cos[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)(d+c\cos[e+fx])} \right) \\
& \left. \operatorname{Sec}[e+fx]^{5/2} \sqrt{1+\operatorname{Sec}[e+fx]} \right) / \left(c \sqrt{-c-d} (c-d)^2 f (a(1+\operatorname{Sec}[e+fx]))^{3/2} (c+d \operatorname{Sec}[e+fx]) \right) \\
& \left(\left(\left(\sqrt{-c-d} \left(-c(5c-9d) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) + 4\sqrt{2} (c-d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) + \right. \\
& \left. 4\sqrt{2} d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) \sqrt{1+\operatorname{Sec}[e+fx]} \left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \right) / \\
& \left(2c \sqrt{-c-d} (c-d)^2 \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} + \frac{1}{c \sqrt{-c-d} (c-d)^2} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sqrt{1+\operatorname{Sec}[e+fx]} \right)
\end{aligned}$$

$$\left(\frac{4\sqrt{2} d^{5/2} \left(\frac{\sqrt{d} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} - \frac{\sqrt{d} \left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\sqrt{-c-d} \left(\frac{\cos[e+fx]}{1+\cos[e+fx]} \right)^{3/2}} \right)}{1 - \frac{d(1+\cos[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{-c-d}} + \right.$$

$$\left. \sqrt{-c-d} \left(-\frac{c(5c-9d) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} + \frac{4\sqrt{2}(c-d)^2 \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} - \frac{\left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2 \left(\frac{\cos[e+fx]}{1+\cos[e+fx]} \right)^{3/2}} \right)}{1 + (1+\cos[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right) \right) +$$

$$\left(\left(\sqrt{-c-d} \left(-c(5c-9d) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] + 4\sqrt{2}(c-d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) + 4\sqrt{2} d^{5/2} \right. \right.$$

$$\left. \left. \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) / \left(2c\sqrt{-c-d} (c-d)^2 \sqrt{1+\operatorname{Sec}[e+fx]} \right) \right)$$

■ **Problem 176: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e+fx])^{3/2} (c + d \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 3, 560 leaves, 15 steps):

$$\begin{aligned}
& - \frac{\text{Tan}[e + f x]}{2 a (c - d)^2 f (1 + \text{Sec}[e + f x]) \sqrt{a + a \text{Sec}[e + f x]}} + \\
& \frac{2 \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a}}\right] \text{Tan}[e + f x]}{\sqrt{a} c^2 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \frac{\sqrt{2} (c - 3 d) \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \text{Tan}[e + f x]}{\sqrt{a} (c - d)^3 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \\
& \frac{\text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \text{Tan}[e + f x]}{2 \sqrt{2} \sqrt{a} (c - d)^2 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \frac{d^{5/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \text{Tan}[e + f x]}{\sqrt{a} c (c - d)^2 (c + d)^{3/2} f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \\
& \frac{2 (3 c - d) d^{5/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \text{Tan}[e + f x]}{\sqrt{a} c^2 (c - d)^3 \sqrt{c + d} f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \frac{d^3 \text{Tan}[e + f x]}{a c (c - d)^2 (c + d) f \sqrt{a + a \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])}
\end{aligned}$$

Result (type 3, 2118 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2} (e + f x)\right]^3 (d + c \cos[e + f x])^2 \text{Sec}[e + f x]^4 \right. \\
& \left. \left(- \frac{2 (c^3 + c^2 d + 2 d^3) \sin\left[\frac{1}{2} (e + f x)\right]}{c^2 (-c + d)^2 (c + d)} + \frac{4 d^4 \sin\left[\frac{1}{2} (e + f x)\right]}{c^2 (-c + d)^2 (c + d) (d + c \cos[e + f x])} + \frac{\text{Sec}\left[\frac{1}{2} (e + f x)\right] \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{(-c + d)^2} \right) \right) / \\
& (f (a (1 + \text{Sec}[e + f x]))^{3/2} (c + d \text{Sec}[e + f x])^2) + \\
& \left(\left((-c - d)^{3/2} \left(-c^2 (5 c - 13 d) \text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) + 4 \sqrt{2} (c - d)^3 \text{ArcTan}\left[\frac{\text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] \right) + \right. \\
& \left. 2 \sqrt{2} d^{5/2} (-7 c^2 - 3 c d + 2 d^2) \text{ArcTanh}\left[\frac{\sqrt{d} \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{-c - d} \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] \right) \cos\left[\frac{1}{2} (e + f x)\right]^3 (d + c \cos[e + f x])^2 \sqrt{\cos[e + f x] \text{Sec}\left[\frac{1}{2} (e + f x)\right]^2} \\
& \left(- \frac{c^2 \text{Sec}\left[\frac{1}{2} (e + f x)\right]}{2 (-c + d)^2 (c + d) (d + c \cos[e + f x]) \sqrt{\text{Sec}[e + f x]}} + \frac{3 c d \text{Sec}\left[\frac{1}{2} (e + f x)\right]}{2 (-c + d)^2 (c + d) (d + c \cos[e + f x]) \sqrt{\text{Sec}[e + f x]}} + \right. \\
& \left. \frac{4 d^2 \text{Sec}\left[\frac{1}{2} (e + f x)\right]}{(-c + d)^2 (c + d) (d + c \cos[e + f x]) \sqrt{\text{Sec}[e + f x]}} + \frac{d^3 \text{Sec}\left[\frac{1}{2} (e + f x)\right]}{c (-c + d)^2 (c + d) (d + c \cos[e + f x]) \sqrt{\text{Sec}[e + f x]}} + \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{c^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} - \frac{3cd \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{2(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} - \frac{3d^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{2(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} + \\
& \frac{c^2 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} - \frac{cd \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} - \\
& \left. \frac{d^2 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} + \frac{d^3 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{c(-c+d)^2(c+d)(d+c \operatorname{Cos}[e+fx])} \right) \\
& \operatorname{Sec}[e+fx]^{7/2} \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]} \Bigg/ \left(c^2 (-c-d)^{3/2} (c-d)^3 f (a(1+\operatorname{Sec}[e+fx]))^{3/2} (c+d \operatorname{Sec}[e+fx])^2 \right. \\
& \left. \left(\frac{1}{2c^2(-c-d)^{3/2}(c-d)^3} \left((-c-d)^{3/2} \left(-c^2(5c-13d) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] + 4\sqrt{2}(c-d)^3 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}\right]} \right) \right) + \right. \\
& \left. 2\sqrt{2}d^{5/2}(-7c^2-3cd+2d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}\right] \right) \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \\
& \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{3/2} \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sin}[e+fx] + \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) + \\
& \frac{1}{c^2(-c-d)^{3/2}(c-d)^3} \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]}
\end{aligned}$$

$$\left(\left(2 \sqrt{2} d^{5/2} (-7c^2 - 3cd + 2d^2) \left(\frac{\sqrt{d} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} - \frac{\sqrt{d} \left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\sqrt{-c-d} \left(\frac{\cos[e+fx]}{1+\cos[e+fx]} \right)^{3/2}} \right) \right) / \right. \\
\left. \left(1 - \frac{d(1+\cos[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{-c-d} \right) + \right. \\
\left. (-c-d)^{3/2} \left(-\frac{c^2(5c-13d) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} + \frac{4\sqrt{2}(c-d)^3 \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} - \frac{\left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2 \left(\frac{\cos[e+fx]}{1+\cos[e+fx]} \right)^{3/2}} \right)}{1+(1+\cos[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right) \right) + \right. \\
\left. \frac{1}{2c^2(-c-d)^{3/2}(c-d)^3 \sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]}} \left((-c-d)^{3/2} \left(-c^2(5c-13d) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] \right) + \right. \right. \\
\left. \left. 4\sqrt{2}(c-d)^3 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) + 2\sqrt{2}d^{5/2}(-7c^2-3cd+2d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] \right) \right) \\
\left. \left. \left. \sqrt{\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \left(-\cos\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) \right) \right)$$

■ **Problem 177: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{3/2} (c + d \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 802 leaves, 19 steps):

$$\begin{aligned} & - \frac{\operatorname{Tan}[e + f x]}{2 a (c - d)^3 f (1 + \operatorname{Sec}[e + f x]) \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a}}\right] \operatorname{Tan}[e + f x]}{\sqrt{a} c^3 f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \\ & \frac{\sqrt{2} (c - 4 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \operatorname{Tan}[e + f x]}{\sqrt{a} (c - d)^4 f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \operatorname{Tan}[e + f x]}{2 \sqrt{2} \sqrt{a} (c - d)^3 f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \\ & \frac{3 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \operatorname{Tan}[e + f x]}{4 \sqrt{a} c (c - d)^2 (c + d)^{5/2} f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{(3 c - d) d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \operatorname{Tan}[e + f x]}{\sqrt{a} c^2 (c - d)^3 (c + d)^{3/2} f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \\ & \frac{2 d^{5/2} (6 c^2 - 4 c d + d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \operatorname{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \operatorname{Tan}[e + f x]}{\sqrt{a} c^3 (c - d)^4 \sqrt{c + d} f \sqrt{a - a \operatorname{Sec}[e + f x]} \sqrt{a + a \operatorname{Sec}[e + f x]}} - \frac{d^3 \operatorname{Tan}[e + f x]}{2 a c (c - d)^2 (c + d) f \sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])^2} - \\ & \frac{(3 c - d) d^3 \operatorname{Tan}[e + f x]}{a c^2 (c - d)^3 (c + d) f \sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])} - \frac{3 d^3 \operatorname{Tan}[e + f x]}{4 a c (c^2 - d^2)^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])} \end{aligned}$$

Result (type 3, 2632 leaves):

$$\begin{aligned} & \left(\cos\left[\frac{1}{2} (e + f x)\right]^3 (d + c \operatorname{Cos}[e + f x])^3 \operatorname{Sec}[e + f x]^5 \right. \\ & \left. \left(- \frac{(-2 c^5 - 4 c^4 d - 2 c^3 d^2 - 17 c^2 d^3 - 5 c d^4 + 6 d^5) \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]}{c^3 (-c + d)^3 (c + d)^2} - \frac{2 d^5 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]}{c^3 (-c + d)^2 (c + d) (d + c \operatorname{Cos}[e + f x])^2} + \right. \right. \\ & \left. \left. \frac{-19 c^2 d^4 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] - 5 c d^5 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] + 8 d^6 \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]}{c^3 (-c + d)^3 (c + d)^2 (d + c \operatorname{Cos}[e + f x])} - \frac{\operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{(-c + d)^3} \right) \right) / \\ & \left(f (a (1 + \operatorname{Sec}[e + f x]))^{3/2} (c + d \operatorname{Sec}[e + f x])^3 - \left(2 c^3 (5 c - 17 d) (c + d)^2 \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - \right. \right. \end{aligned}$$

$$\left. \begin{aligned}
& 8 \sqrt{2} (c-d)^4 (c+d)^2 \operatorname{ArcTan} \left[\frac{\operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} \right] - \frac{\sqrt{2} d^{5/2} (63 c^4 + 54 c^3 d - 17 c^2 d^2 - 12 c d^3 + 8 d^4) \operatorname{ArcTanh} \left[\frac{\sqrt{d} \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} \right]}{\sqrt{-c-d}} \right] \\
& \operatorname{Cos} \left[\frac{1}{2} (e+f x) \right]^3 (d+c \operatorname{Cos}[e+f x])^3 \sqrt{\operatorname{Cos}[e+f x] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2} \left(\frac{c^3 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{2 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} - \right. \\
& \frac{c^2 d \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} - \frac{19 c d^2 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{2 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} - \\
& \frac{33 d^3 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{4 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} - \frac{3 d^4 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{4 c (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} + \\
& \frac{d^5 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]}{c^2 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x]) \sqrt{\operatorname{Sec}[e+f x]}} - \frac{c^3 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \frac{3 c^2 d \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{2 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \\
& \frac{3 c d^2 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \frac{9 d^3 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{4 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \frac{d^4 \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{4 c (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} - \\
& \frac{c^3 \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \frac{c^2 d \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \\
& \frac{2 c d^2 \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} - \frac{2 d^3 \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} - \\
& \left. \frac{d^4 \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{c (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} + \frac{d^5 \operatorname{Cos}[2(e+f x)] \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right] \sqrt{\operatorname{Sec}[e+f x]}}{c^2 (-c+d)^3 (c+d)^2 (d+c \operatorname{Cos}[e+f x])} \right) \\
& \left. \operatorname{Sec}[e+f x]^{9/2} \sqrt{\operatorname{Cos} \left[\frac{1}{2} (e+f x) \right]^2 \operatorname{Sec}[e+f x]} \right) / \left(2 c^3 (c-d)^4 (c+d)^2 f (a (1+\operatorname{Sec}[e+f x]))^{3/2} (c+d \operatorname{Sec}[e+f x])^3 \right)
\end{aligned} \right)$$

$$\left(-\frac{1}{4c^3(c-d)^4(c+d)^2} \left(2c^3(5c-17d)(c+d)^2 \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - 8\sqrt{2}(c-d)^4(c+d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right] - \right. \right.$$

$$\left. \frac{\sqrt{2}d^{5/2}(63c^4+54c^3d-17c^2d^2-12cd^3+8d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d}\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d}\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right]}{\sqrt{-c-d}} \right) \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}$$

$$\left(\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \right)^{3/2} \left(-\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sin}[e+fx] + \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) -$$

$$\frac{1}{2c^3(c-d)^4(c+d)^2} \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]}$$

$$\left(\frac{c^3(5c-17d)(c+d)^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} - \frac{8\sqrt{2}(c-d)^4(c+d)^2 \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}} - \frac{\left(\frac{\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{(1+\operatorname{Cos}[e+fx])^2} - \frac{\operatorname{Sin}[e+fx]}{1+\operatorname{Cos}[e+fx]}\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\left(\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}\right)^{3/2}} \right)}{1+(1+\operatorname{Cos}[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right) -$$

$$\left(\sqrt{2}d^{5/2}(63c^4+54c^3d-17c^2d^2-12cd^3+8d^4) \left(\frac{\sqrt{d} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2\sqrt{-c-d}\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}} - \frac{\sqrt{d} \left(\frac{\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{(1+\operatorname{Cos}[e+fx])^2} - \frac{\operatorname{Sin}[e+fx]}{1+\operatorname{Cos}[e+fx]}\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2\sqrt{-c-d}\left(\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}\right)^{3/2}} \right) \right) \Bigg/$$

$$\left(\sqrt{-c-d} \left(1 - \frac{d(1+\cos[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{-c-d} \right) \right) -$$

$$\frac{1}{4c^3(c-d)^4(c+d)^2 \sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]}} \left(2c^3(5c-17d)(c+d)^2 \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] -$$

$$8\sqrt{2}(c-d)^4(c+d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right] - \frac{\sqrt{2}d^{5/2}(63c^4+54c^3d-17c^2d^2-12cd^3+8d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}}\right]}{\sqrt{-c-d}}$$

$$\left. \left. \left. \sqrt{\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \left(-\cos\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \sin\left[\frac{1}{2}(e+fx)\right] + \cos\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) \right) \right)$$

■ **Problem 180: Result more than twice size of optimal antiderivative.**

$$\int \frac{c+d \operatorname{Sec}[e+fx]}{(a+a \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 164 leaves, 7 steps):

$$\frac{2c \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{a^{5/2} f} - \frac{(43c-3d) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{16\sqrt{2} a^{5/2} f} - \frac{(c-d) \operatorname{Tan}[e+fx]}{4f(a+a \operatorname{Sec}[e+fx])^{5/2}} - \frac{(11c-3d) \operatorname{Tan}[e+fx]}{16af(a+a \operatorname{Sec}[e+fx])^{3/2}}$$

Result (type 3, 343 leaves):

$$\left(\left((-43c + 3d) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]\right]\right) + 32\sqrt{2}c \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]}{\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right] \right) \right. \\ \left. \operatorname{Cos}\left[\frac{1}{2}(e + fx)\right]^4 \sqrt{\frac{\operatorname{Cos}[e + fx]}{1 + \operatorname{Cos}[e + fx]}} \operatorname{Sec}[e + fx]^{3/2} \sqrt{1 + \operatorname{Sec}[e + fx]} (c + d \operatorname{Sec}[e + fx]) \right) / \\ \left(4f(d + c \operatorname{Cos}[e + fx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2 (a(1 + \operatorname{Sec}[e + fx]))^{5/2}} \right) + \left(\operatorname{Cos}\left[\frac{1}{2}(e + fx)\right]^5 \operatorname{Sec}[e + fx]^2 (c + d \operatorname{Sec}[e + fx]) \right) \\ \left(\frac{1}{2}(-15c + 7d) \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2 \left(19c \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] - 11d \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] \right) \right) + \\ \left. \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^4 \left(-c \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] + d \operatorname{Sin}\left[\frac{1}{2}(e + fx)\right] \right) \right) / \left(f(d + c \operatorname{Cos}[e + fx]) (a(1 + \operatorname{Sec}[e + fx]))^{5/2} \right)$$

■ **Problem 181: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + fx])^{5/2} (c + d \operatorname{Sec}[e + fx])} dx$$

Optimal (type 3, 592 leaves, 16 steps):

$$\frac{\operatorname{Tan}[e + fx]}{4a^2(c-d)f(1 + \operatorname{Sec}[e + fx])^2 \sqrt{a + a \operatorname{Sec}[e + fx]}} - \frac{(c-2d)\operatorname{Tan}[e + fx]}{2a^2(c-d)^2f(1 + \operatorname{Sec}[e + fx]) \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\ \frac{3 \operatorname{Tan}[e + fx]}{16a^2(c-d)f(1 + \operatorname{Sec}[e + fx]) \sqrt{a + a \operatorname{Sec}[e + fx]}} + \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \operatorname{Sec}[e+fx]}}{\sqrt{a}}\right] \operatorname{Tan}[e + fx]}{a^{3/2}cf \sqrt{a-a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\ \frac{(c-2d) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \operatorname{Sec}[e+fx]}}{\sqrt{2}\sqrt{a}}\right] \operatorname{Tan}[e + fx]}{2\sqrt{2}a^{3/2}(c-d)^2f \sqrt{a-a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \operatorname{Sec}[e+fx]}}{\sqrt{2}\sqrt{a}}\right] \operatorname{Tan}[e + fx]}{16\sqrt{2}a^{3/2}(c-d)f \sqrt{a-a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\ \frac{\sqrt{2}(c^2 - 3cd + 3d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \operatorname{Sec}[e+fx]}}{\sqrt{2}\sqrt{a}}\right] \operatorname{Tan}[e + fx]}{a^{3/2}(c-d)^3f \sqrt{a-a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} + \frac{2d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d}\sqrt{a-a \operatorname{Sec}[e+fx]}}{\sqrt{a}\sqrt{c+d}}\right] \operatorname{Tan}[e + fx]}{a^{3/2}c(c-d)^3\sqrt{c+d}f \sqrt{a-a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}}$$

Result (type 3, 1826 leaves):

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} (e + f x) \right]^5 (d + c \cos [e + f x]) \sec [e + f x]^4 \right. \\
& \left. \left(\frac{(-15c + 23d) \sin \left[\frac{1}{2} (e + f x) \right]}{2(-c + d)^2} + \frac{\sec \left[\frac{1}{2} (e + f x) \right]^2 (19c \sin \left[\frac{1}{2} (e + f x) \right] - 27d \sin \left[\frac{1}{2} (e + f x) \right])}{4(-c + d)^2} + \frac{\sec \left[\frac{1}{2} (e + f x) \right]^3 \tan \left[\frac{1}{2} (e + f x) \right]}{2(-c + d)} \right) \right) / \\
& (f (a (1 + \sec [e + f x]))^{5/2} (c + d \sec [e + f x])) - \\
& \left(\left(\sqrt{-c - d} \left(c (43c^2 - 126cd + 115d^2) \operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (e + f x) \right] \right] - 32\sqrt{2} (c - d)^3 \operatorname{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] \right) \right. \right. \\
& \left. \left. 32\sqrt{2} d^{7/2} \operatorname{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] \right) \cos \left[\frac{1}{2} (e + f x) \right]^5 \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \right. \\
& (d + c \cos [e + f x]) \left(-\frac{11c^2 \sec \left[\frac{1}{2} (e + f x) \right]}{8(-c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \frac{51cd \sec \left[\frac{1}{2} (e + f x) \right]}{8(-c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} - \right. \\
& \frac{8d^2 \sec \left[\frac{1}{2} (e + f x) \right]}{(-c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \frac{2c^2 \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (d + c \cos [e + f x])} - \frac{43cd \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{8(-c + d)^2 (d + c \cos [e + f x])} + \\
& \frac{35d^2 \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{8(-c + d)^2 (d + c \cos [e + f x])} + \frac{2c^2 \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (d + c \cos [e + f x])} - \\
& \left. \frac{4cd \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (d + c \cos [e + f x])} + \frac{2d^2 \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (d + c \cos [e + f x])} \right) \\
& \left. \sec [e + f x]^{7/2} \sqrt{1 + \sec [e + f x]} \right) / \left(4c \sqrt{-c - d} (c - d)^3 f (a (1 + \sec [e + f x]))^{5/2} (c + d \sec [e + f x]) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\left(\sqrt{-c-d} \left(c (43 c^2 - 126 c d + 115 d^2) \operatorname{ArcSin} \left[\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right] - 32 \sqrt{2} (c-d)^3 \operatorname{ArcTan} \left[\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} \right] \right) \right) \right) + \right. \\
& \left. \left. \left. 32 \sqrt{2} d^{7/2} \operatorname{ArcTanh} \left[\frac{\sqrt{d} \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} \right] \sqrt{1+\operatorname{Sec}[e+f x]} \left(\frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{(1+\operatorname{Cos}[e+f x])^2} - \frac{\operatorname{Sin}[e+f x]}{1+\operatorname{Cos}[e+f x]} \right) \right) \right) \right) / \\
& \left(8 c \sqrt{-c-d} (c-d)^3 \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}} - \frac{1}{4 c \sqrt{-c-d} (c-d)^3} \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}} \sqrt{1+\operatorname{Sec}[e+f x]} \right) \\
& \left(\frac{32 \sqrt{2} d^{7/2} \left(\frac{\sqrt{d} \operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2}{2 \sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} - \frac{\sqrt{d} \left(\frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{(1+\operatorname{Cos}[e+f x])^2} - \frac{\operatorname{Sin}[e+f x]}{1+\operatorname{Cos}[e+f x]} \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{2 \sqrt{-c-d} \left(\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]} \right)^{3/2}} \right)}{1 - \frac{d (1+\operatorname{Cos}[e+f x]) \operatorname{Sec}[e+f x] \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2}{-c-d}} \right) + \right. \\
& \left. \left. \left. \sqrt{-c-d} \left(\frac{c (43 c^2 - 126 c d + 115 d^2) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2}} - \frac{32 \sqrt{2} (c-d)^3 \left(\frac{\operatorname{Sec} \left[\frac{1}{2} (e+f x) \right]^2}{2 \sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} - \frac{\left(\frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{(1+\operatorname{Cos}[e+f x])^2} - \frac{\operatorname{Sin}[e+f x]}{1+\operatorname{Cos}[e+f x]} \right) \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]}{2 \left(\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]} \right)^{3/2}} \right)}{1 + (1 + \operatorname{Cos}[e+f x]) \operatorname{Sec}[e+f x] \operatorname{Tan} \left[\frac{1}{2} (e+f x) \right]^2} \right) \right) \right) - \right. \\
& \left. \left(\left(\left(\sqrt{-c-d} \left(c (43 c^2 - 126 c d + 115 d^2) \operatorname{ArcSin} \left[\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right] - 32 \sqrt{2} (c-d)^3 \operatorname{ArcTan} \left[\frac{\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\operatorname{Cos}[e+f x]}{1+\operatorname{Cos}[e+f x]}}} \right] \right) \right) \right) + 32 \sqrt{2} d^{7/2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& (f (a (1 + \operatorname{Sec}[e + f x]))^{5/2} (c + d \operatorname{Sec}[e + f x])^2) - \left(c^2 (43 c^3 - 123 c^2 d + 53 c d^2 + 219 d^3) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right] - \right. \\
& \left. 32 \sqrt{2} (c - d)^4 (c + d) \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] + \frac{16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{-c - d} \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right]}{\sqrt{-c - d}} \right) \\
& \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^5 (d + c \operatorname{Cos}[e + f x])^2 \sqrt{\operatorname{Cos}[e + f x] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^2} \\
& \left(\frac{11 c^3 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{8 (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} - \frac{6 c^2 d \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} + \right. \\
& \frac{37 c d^2 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{8 (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} + \frac{16 d^3 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} + \\
& \frac{2 d^4 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{c (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} - \frac{2 c^3 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} + \\
& \frac{43 c^2 d \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} - \frac{2 c d^2 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} - \frac{59 d^3 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} - \\
& \frac{2 c^3 \operatorname{Cos}[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} + \frac{4 c^2 d \operatorname{Cos}[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} - \\
& \left. \frac{4 d^3 \operatorname{Cos}[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} + \frac{2 d^4 \operatorname{Cos}[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{c (-c + d)^3 (c + d) (d + c \operatorname{Cos}[e + f x])} \right)
\end{aligned}$$

$$\left(16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2) \left(\frac{\sqrt{d} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{2 \sqrt{-c-d} \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}} - \frac{\sqrt{d} \left(\frac{\cos[e + f x] \sin[e + f x]}{(1 + \cos[e + f x])^2} - \frac{\sin[e + f x]}{1 + \cos[e + f x]} \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{2 \sqrt{-c-d} \left(\frac{\cos[e + f x]}{1 + \cos[e + f x]} \right)^{3/2}} \right) \right) \sqrt{\dots}$$

$$\left(\sqrt{-c-d} \left(1 - \frac{d(1 + \cos[e + f x]) \operatorname{Sec}[e + f x] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{-c-d} \right) \right) - \dots$$

$$\frac{1}{8 c^2 (c-d)^4 (c+d) \sqrt{\cos\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sec}[e + f x]}} \left(c^2 (43 c^3 - 123 c^2 d + 53 c d^2 + 219 d^3) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right] \right) - \dots$$

$$32 \sqrt{2} (c-d)^4 (c+d) \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] + \frac{16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right]}{\sqrt{-c-d}}$$

$$\sqrt{\cos[e + f x] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \left(-\cos\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x] \sin\left[\frac{1}{2}(e + f x)\right] + \cos\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x] \right)}$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{5/2} (c + d \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 999 leaves, 23 steps):

$$\begin{aligned}
& - \frac{\text{Tan}[e + f x]}{4 a^2 (c - d)^3 f (1 + \text{Sec}[e + f x])^2 \sqrt{a + a \text{Sec}[e + f x]}} - \frac{3 \text{Tan}[e + f x]}{(c - 4 d) \text{Tan}[e + f x]} + \\
& \frac{2 a^2 (c - d)^4 f (1 + \text{Sec}[e + f x]) \sqrt{a + a \text{Sec}[e + f x]}}{16 a^2 (c - d)^3 f (1 + \text{Sec}[e + f x]) \sqrt{a + a \text{Sec}[e + f x]}} + \\
& \frac{2 \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a}}\right] \text{Tan}[e + f x]}{(c - 4 d) \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \text{Tan}[e + f x]} - \\
& \frac{a^{3/2} c^3 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}{2 \sqrt{2} a^{3/2} (c - d)^4 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} - \\
& \frac{3 \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \text{Tan}[e + f x]}{\sqrt{2} (c^2 - 5 c d + 10 d^2) \text{ArcTanh}\left[\frac{\sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{2} \sqrt{a}}\right] \text{Tan}[e + f x]} + \\
& \frac{16 \sqrt{2} a^{3/2} (c - d)^3 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}{a^{3/2} (c - d)^5 f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} + \\
& \frac{3 d^{7/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \text{Tan}[e + f x]}{(4 c - d) d^{7/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \text{Tan}[e + f x]} + \\
& \frac{4 a^{3/2} c (c - d)^3 (c + d)^{5/2} f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}{a^{3/2} c^2 (c - d)^4 (c + d)^{3/2} f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}} + \\
& \frac{2 d^{7/2} (10 c^2 - 5 c d + d^2) \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \text{Sec}[e + f x]}}{\sqrt{a} \sqrt{c + d}}\right] \text{Tan}[e + f x]}{d^4 \text{Tan}[e + f x]} + \\
& \frac{a^{3/2} c^3 (c - d)^5 \sqrt{c + d} f \sqrt{a - a \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}{2 a^2 c (c - d)^3 (c + d) f \sqrt{a + a \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])^2} + \\
& \frac{3 d^4 \text{Tan}[e + f x]}{(4 c - d) d^4 \text{Tan}[e + f x]} + \\
& \frac{4 a^2 c (c - d)^3 (c + d)^2 f \sqrt{a + a \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])}{a^2 c^2 (c - d)^4 (c + d) f \sqrt{a + a \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])}
\end{aligned}$$

Result (type 3, 2904 leaves):

$$\begin{aligned}
& \left(\text{Cos}\left[\frac{1}{2} (e + f x)\right]^5 (d + c \text{Cos}[e + f x])^3 \text{Sec}[e + f x]^6 \left(- \frac{3 (5 c^6 - 3 c^5 d - 21 c^4 d^2 - 13 c^3 d^3 - 28 c^2 d^4 - 12 c d^5 + 8 d^6) \text{Sin}\left[\frac{1}{2} (e + f x)\right]}{2 c^3 (-c + d)^4 (c + d)^2} - \right. \right. \\
& \frac{4 d^6 \text{Sin}\left[\frac{1}{2} (e + f x)\right]}{c^3 (-c + d)^3 (c + d) (d + c \text{Cos}[e + f x])^2} + \frac{\text{Sec}\left[\frac{1}{2} (e + f x)\right]^2 (19 c \text{Sin}\left[\frac{1}{2} (e + f x)\right] - 43 d \text{Sin}\left[\frac{1}{2} (e + f x)\right])}{4 (-c + d)^4} + \\
& \left. \frac{2 (-23 c^2 d^5 \text{Sin}\left[\frac{1}{2} (e + f x)\right] - 9 c d^6 \text{Sin}\left[\frac{1}{2} (e + f x)\right] + 8 d^7 \text{Sin}\left[\frac{1}{2} (e + f x)\right])}{c^3 (-c + d)^4 (c + d)^2 (d + c \text{Cos}[e + f x])} + \frac{\text{Sec}\left[\frac{1}{2} (e + f x)\right]^3 \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{2 (-c + d)^3} \right) \Bigg/ \\
& \left((f (a (1 + \text{Sec}[e + f x]))^{5/2} (c + d \text{Sec}[e + f x])^3) - \left(c^3 (c + d)^2 (43 c^2 - 206 c d + 355 d^2) \text{ArcSin}\left[\text{Tan}\left[\frac{1}{2} (e + f x)\right]\right] \right) - \right.
\end{aligned}$$

$$\left. \begin{aligned}
& 32 \sqrt{2} (c-d)^5 (c+d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right] + \frac{4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right]}{\sqrt{-c-d}} \\
& \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^5 (d+c \operatorname{Cos}[e+fx])^3 \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \\
& \left(-\frac{11 c^4 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} + \frac{45 c^3 d \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} - \right. \\
& \frac{5 c^2 d^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} - \frac{317 c d^3 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} - \\
& \frac{69 d^4 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{2 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} - \frac{7 d^5 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{2 c (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} + \\
& \frac{2 d^6 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]}{c^2 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx]) \sqrt{\operatorname{Sec}[e+fx]}} + \frac{2 c^4 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} - \frac{43 c^3 d \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} \\
& \frac{3 c^2 d^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \frac{123 c d^3 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \frac{95 d^4 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{8 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \\
& \frac{d^5 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{2 c (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \frac{2 c^4 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} - \\
& \frac{4 c^3 d \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} - \frac{2 c^2 d^2 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \\
& \frac{8 c d^3 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} - \frac{2 d^4 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{(-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} - \\
& \left. \frac{4 d^5 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{c (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} + \frac{2 d^6 \operatorname{Cos}[2(e+fx)] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{\operatorname{Sec}[e+fx]}}{c^2 (-c+d)^4 (c+d)^2 (d+c \operatorname{Cos}[e+fx])} \right)
\end{aligned} \right)$$

$$\begin{aligned}
& \left. \left(\text{Sec}[e + f x]^{11/2} \sqrt{\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x]} \right) / \left(4 c^3 (c - d)^5 (c + d)^2 f (a (1 + \text{Sec}[e + f x]))^{5/2} (c + d \text{Sec}[e + f x])^3 \right. \\
& \left. \left(-\frac{1}{8 c^3 (c - d)^5 (c + d)^2} \left(c^3 (c + d)^2 (43 c^2 - 206 c d + 355 d^2) \text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(e + f x)\right]\right] - 32 \sqrt{2} (c - d)^5 (c + d)^2 \text{ArcTan}\left[\frac{\text{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]}}}\right] + \right. \right. \right. \\
& \left. \left. \frac{4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \text{ArcTanh}\left[\frac{\sqrt{d} \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{-c - d} \sqrt{\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]}}}\right]}{\sqrt{-c - d}} \right) \sqrt{\text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \right) \\
& \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] \right)^{3/2} \left(-\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sin}[e + f x] + \text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) - \\
& \frac{1}{4 c^3 (c - d)^5 (c + d)^2} \sqrt{\text{Cos}[e + f x] \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \sqrt{\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x]} \\
& \left(\frac{c^3 (c + d)^2 (43 c^2 - 206 c d + 355 d^2) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} - \frac{32 \sqrt{2} (c - d)^5 (c + d)^2 \left(\frac{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2}{2 \sqrt{\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]}}} - \frac{\left(\frac{\text{Cos}[e + f x] \text{Sin}[e + f x]}{(1 + \text{Cos}[e + f x])^2} - \frac{\text{Sin}[e + f x]}{1 + \text{Cos}[e + f x]} \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{2 \left(\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]} \right)^{3/2}} \right)}{1 + (1 + \text{Cos}[e + f x]) \text{Sec}[e + f x] \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \right) +
\end{aligned}$$

$$\left(4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \left(\frac{\sqrt{d} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2}{2 \sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}} - \frac{\sqrt{d} \left(\frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{(1+\operatorname{Cos}[e+fx])^2} - \frac{\operatorname{Sin}[e+fx]}{1+\operatorname{Cos}[e+fx]} \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{2 \sqrt{-c-d} \left(\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]} \right)^{3/2}} \right) / \left(\sqrt{-c-d} \left(1 - \frac{d(1+\operatorname{Cos}[e+fx]) \operatorname{Sec}[e+fx] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{-c-d} \right) \right) \right) - \frac{1}{8 c^3 (c-d)^5 (c+d)^2 \sqrt{\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]}} \left(c^3 (c+d)^2 (43 c^2 - 206 c d + 355 d^2) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right] - 4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{-c-d} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}\right] \right) + 32 \sqrt{2} (c-d)^5 (c+d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}}}}\right] + \frac{\sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2} \left(-\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \operatorname{Sec}[e+fx] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right)}{\sqrt{-c-d}} \right)$$

■ **Problem 193: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Sec}[e+fx])^2}{(c+d \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$\frac{a^2 x}{c^3} - \frac{(3 b^2 c^4 d - 2 a b c^3 (2 c^2 + d^2) + a^2 (6 c^4 d - 5 c^2 d^3 + 2 d^5)) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{c^3 (c-d)^{5/2} (c+d)^{5/2} f} - \frac{d (bc - ad)^2 \operatorname{Sin}[e+fx]}{2 c^2 (c^2 - d^2) f (d + c \operatorname{Cos}[e+fx])^2} - \frac{(bc - ad) (3 ad (2 c^2 - d^2) - bc (2 c^2 + d^2)) \operatorname{Sin}[e+fx]}{2 c^2 (c^2 - d^2)^2 f (d + c \operatorname{Cos}[e+fx])}$$

Result (type 3, 493 leaves) :

$$\frac{1}{4 c^3 f (b + a \operatorname{Cos}[e+fx])^2 (c + d \operatorname{Sec}[e+fx])^3} (d + c \operatorname{Cos}[e+fx]) \operatorname{Sec}[e+fx] (a + b \operatorname{Sec}[e+fx])^2$$

$$\left(\frac{1}{(c^2 - d^2)^{5/2}} 4 (3 b^2 c^4 d - 2 a b c^3 (2 c^2 + d^2) + a^2 (6 c^4 d - 5 c^2 d^3 + 2 d^5)) \operatorname{ArcTanh}\left[\frac{(-c+d) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c^2 - d^2}}\right] (d + c \operatorname{Cos}[e+fx])^2 + \right.$$

$$\frac{1}{(c^2 - d^2)^2} (2 a^2 c^6 e - 6 a^2 c^2 d^4 e + 4 a^2 d^6 e + 2 a^2 c^6 f x - 6 a^2 c^2 d^4 f x + 4 a^2 d^6 f x + 8 a^2 c d (c^2 - d^2)^2 (e+fx) \operatorname{Cos}[e+fx] +$$

$$2 a^2 c^2 (c^2 - d^2)^2 (e+fx) \operatorname{Cos}[2(e+fx)] + 2 b^2 c^5 d \operatorname{Sin}[e+fx] - 12 a b c^4 d^2 \operatorname{Sin}[e+fx] + 10 a^2 c^3 d^3 \operatorname{Sin}[e+fx] +$$

$$4 b^2 c^3 d^3 \operatorname{Sin}[e+fx] - 4 a^2 c d^5 \operatorname{Sin}[e+fx] + 2 b^2 c^6 \operatorname{Sin}[2(e+fx)] - 8 a b c^5 d \operatorname{Sin}[2(e+fx)] +$$

$$\left. 6 a^2 c^4 d^2 \operatorname{Sin}[2(e+fx)] + b^2 c^4 d^2 \operatorname{Sin}[2(e+fx)] + 2 a b c^3 d^3 \operatorname{Sin}[2(e+fx)] - 3 a^2 c^2 d^4 \operatorname{Sin}[2(e+fx)] \right)$$

■ **Problem 195: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[e+fx])^3}{(c + d \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 254 leaves, 6 steps) :

$$\frac{a^3 x}{c^3} - \frac{(bc - ad) (2 a b c d (4 c^2 - d^2) - b^2 c^2 (c^2 + 2 d^2) - a^2 (6 c^4 - 5 c^2 d^2 + 2 d^4)) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{c^3 (c-d)^{5/2} (c+d)^{5/2} f} + \frac{(bc - ad)^2 (b + a \operatorname{Cos}[e+fx]) \operatorname{Sin}[e+fx]}{2 c (c^2 - d^2) f (d + c \operatorname{Cos}[e+fx])^2} + \frac{(bc - ad)^2 (5 a c^2 - 3 b c d - 2 a d^2) \operatorname{Sin}[e+fx]}{2 c^2 (c^2 - d^2)^2 f (d + c \operatorname{Cos}[e+fx])}$$

Result (type 3, 517 leaves) :

$$\frac{1}{4 c^3 f} \left(-\frac{1}{(c^2 - d^2)^{5/2}} 4 (-9 a b^2 c^4 d + 3 a^2 b c^3 (2 c^2 + d^2) + b^3 c^3 (c^2 + 2 d^2) + a^3 (-6 c^4 d + 5 c^2 d^3 - 2 d^5)) \operatorname{ArcTanh}\left[\frac{(-c + d) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c^2 - d^2}}\right] + \right. \\ \left. \frac{1}{(c^2 - d^2)^2 (d + c \operatorname{Cos}[e + f x])^2} (2 a^3 c^6 e - 6 a^3 c^2 d^4 e + 4 a^3 d^6 e + 2 a^3 c^6 f x - 6 a^3 c^2 d^4 f x + 4 a^3 d^6 f x + 8 a^3 c d (c^2 - d^2)^2 (e + f x) \operatorname{Cos}[e + f x] + \right. \\ \left. 2 a^3 (c^3 - c d^2)^2 (e + f x) \operatorname{Cos}[2 (e + f x)] + 2 b^3 c^6 \operatorname{Sin}[e + f x] + 6 a b^2 c^5 d \operatorname{Sin}[e + f x] - 18 a^2 b c^4 d^2 \operatorname{Sin}[e + f x] - 8 b^3 c^4 d^2 \operatorname{Sin}[e + f x] + \right. \\ \left. 10 a^3 c^3 d^3 \operatorname{Sin}[e + f x] + 12 a b^2 c^3 d^3 \operatorname{Sin}[e + f x] - 4 a^3 c d^5 \operatorname{Sin}[e + f x] + 6 a b^2 c^6 \operatorname{Sin}[2 (e + f x)] - 12 a^2 b c^5 d \operatorname{Sin}[2 (e + f x)] - \right. \\ \left. 3 b^3 c^5 d \operatorname{Sin}[2 (e + f x)] + 6 a^3 c^4 d^2 \operatorname{Sin}[2 (e + f x)] + 3 a b^2 c^4 d^2 \operatorname{Sin}[2 (e + f x)] + 3 a^2 b c^3 d^3 \operatorname{Sin}[2 (e + f x)] - 3 a^3 c^2 d^4 \operatorname{Sin}[2 (e + f x)] \right) \Bigg)$$

■ **Problem 196: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^3}{(c + d \operatorname{Sec}[e + f x])^4} dx$$

Optimal (type 3, 412 leaves, 7 steps):

$$\frac{a^3 x}{c^4} - \left((3 a b^2 c^4 d (4 c^2 + d^2) - b^3 c^5 (c^2 + 4 d^2) - a^2 b (6 c^7 + 9 c^5 d^2) + a^3 (8 c^6 d - 8 c^4 d^3 + 7 c^2 d^5 - 2 d^7)) \operatorname{ArcTanh}\left[\frac{\sqrt{c - d} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{c + d}}\right] \right) / \\ \left(c^4 \sqrt{c - d} \sqrt{c + d} (c^2 - d^2)^3 f \right) - \frac{d (b c - a d) (b + a \operatorname{Cos}[e + f x])^2 \operatorname{Sin}[e + f x]}{3 c (c^2 - d^2) f (d + c \operatorname{Cos}[e + f x])^3} + \frac{(b c - a d)^2 (3 b c^3 - 8 a c^2 d + 2 b c d^2 + 3 a d^3) \operatorname{Sin}[e + f x]}{6 c^3 (c^2 - d^2)^2 f (d + c \operatorname{Cos}[e + f x])^2} - \\ \frac{(b c - a d) (b^2 c^2 d (13 c^2 + 2 d^2) - a b c (18 c^4 + 17 c^2 d^2 - 5 d^4) + a^2 (34 c^4 d - 28 c^2 d^3 + 9 d^5)) \operatorname{Sin}[e + f x]}{6 c^3 (c^2 - d^2)^3 f (d + c \operatorname{Cos}[e + f x])}$$

Result (type 3, 885 leaves):

$$\frac{a^3 (e + f x) (d + c \cos[e + f x])^4 \sec[e + f x] (a + b \sec[e + f x])^3}{c^4 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^4} +$$

$$\left((6 a^2 b c^7 + b^3 c^7 - 8 a^3 c^6 d - 12 a b^2 c^6 d + 9 a^2 b c^5 d^2 + 4 b^3 c^5 d^2 + 8 a^3 c^4 d^3 - 3 a b^2 c^4 d^3 - 7 a^3 c^2 d^5 + 2 a^3 d^7) \operatorname{ArcTanh}\left[\frac{(-c + d) \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c^2 - d^2}}\right] \right.$$

$$\left. (d + c \cos[e + f x])^4 \sec[e + f x] (a + b \sec[e + f x])^3 \right) / \left(c^4 \sqrt{c^2 - d^2} (-c^2 + d^2)^3 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^4 \right) +$$

$$\left((d + c \cos[e + f x]) \sec[e + f x] (a + b \sec[e + f x])^3 (-b^3 c^3 d \sin[e + f x] + 3 a b^2 c^2 d^2 \sin[e + f x] - 3 a^2 b c d^3 \sin[e + f x] + a^3 d^4 \sin[e + f x]) \right) /$$

$$\left(3 c^3 (c^2 - d^2) f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^4 \right) +$$

$$\left((d + c \cos[e + f x])^2 \sec[e + f x] (a + b \sec[e + f x])^3 (3 b^3 c^5 \sin[e + f x] - 18 a b^2 c^4 d \sin[e + f x] + 27 a^2 b c^3 d^2 \sin[e + f x] + \right.$$

$$\left. 2 b^3 c^3 d^2 \sin[e + f x] - 12 a^3 c^2 d^3 \sin[e + f x] + 3 a b^2 c^2 d^3 \sin[e + f x] - 12 a^2 b c d^4 \sin[e + f x] + 7 a^3 d^5 \sin[e + f x]) \right) /$$

$$\left(6 c^3 (c^2 - d^2)^2 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^4 \right) + \frac{1}{6 c^3 (c^2 - d^2)^3 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^4}$$

$$\left((d + c \cos[e + f x])^3 \sec[e + f x] (a + b \sec[e + f x])^3 \right.$$

$$\left. (18 a b^2 c^6 \sin[e + f x] - 54 a^2 b c^5 d \sin[e + f x] - 13 b^3 c^5 d \sin[e + f x] + 36 a^3 c^4 d^2 \sin[e + f x] + 30 a b^2 c^4 d^2 \sin[e + f x] + 15 a^2 b c^3 d^3 \right.$$

$$\left. \sin[e + f x] - 2 b^3 c^3 d^3 \sin[e + f x] - 32 a^3 c^2 d^4 \sin[e + f x] - 3 a b^2 c^2 d^4 \sin[e + f x] - 6 a^2 b c d^5 \sin[e + f x] + 11 a^3 d^6 \sin[e + f x]) \right)$$

■ **Problem 197: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sec[e + f x])^3}{(c + d \sec[e + f x])^5} dx$$

Optimal (type 3, 622 leaves, 8 steps):

$$\frac{a^3 x}{c^5} - \left((15 a b^2 c^6 d (4 c^2 + 3 d^2) - 3 a^2 b c^5 (8 c^4 + 24 c^2 d^2 + 3 d^4) - b^3 c^5 (4 c^4 + 27 c^2 d^2 + 4 d^4) + a^3 (40 c^8 d - 40 c^6 d^3 + 63 c^4 d^5 - 36 c^2 d^7 + 8 d^9)) \right.$$

$$\left. \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c+d}}\right] \right) / \left(4 c^5 \sqrt{c-d} \sqrt{c+d} (c^2 - d^2)^4 f \right) +$$

$$\frac{d^2 (b + a \cos[e + f x])^3 \sin[e + f x]}{4 c (c^2 - d^2) f (d + c \cos[e + f x])^4} - \frac{d (8 b c^3 - 11 a c^2 d - b c d^2 + 4 a d^3) (b + a \cos[e + f x])^2 \sin[e + f x]}{12 c^2 (c^2 - d^2)^2 f (d + c \cos[e + f x])^3} -$$

$$\left((b c - a d) (2 a b c d (32 c^4 + c^2 d^2 + 2 d^4) - a^2 d^2 (58 c^4 - 35 c^2 d^2 + 12 d^4) - b^2 (12 c^6 + 25 c^4 d^2 - 2 c^2 d^4)) \sin[e + f x] \right) /$$

$$\left(24 c^4 (c^2 - d^2)^3 f (d + c \cos[e + f x])^2 \right) -$$

$$\left((b^3 c^3 d (68 c^4 + 39 c^2 d^2 - 2 d^4) + a^2 b c d (272 c^6 + 10 c^4 d^2 + 49 c^2 d^4 - 16 d^6) - 3 a b^2 c^2 (24 c^6 + 84 c^4 d^2 - 5 c^2 d^4 + 2 d^6) - \right.$$

$$\left. a^3 (212 c^6 d^2 - 210 c^4 d^4 + 139 c^2 d^6 - 36 d^8)) \sin[e + f x] \right) / \left(24 c^4 (c^2 - d^2)^4 f (d + c \cos[e + f x]) \right)$$

Result (type 3, 1285 leaves):

$$\begin{aligned}
& \frac{a^3 (e + f x) (d + c \cos[e + f x])^5 \sec[e + f x]^2 (a + b \sec[e + f x])^3}{c^5 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5} + \\
& \left((-24 a^2 b c^9 - 4 b^3 c^9 + 40 a^3 c^8 d + 60 a b^2 c^8 d - 72 a^2 b c^7 d^2 - 27 b^3 c^7 d^2 - 40 a^3 c^6 d^3 + 45 a b^2 c^6 d^3 - 9 a^2 b c^5 d^4 - 4 b^3 c^5 d^4 + \right. \\
& \quad \left. 63 a^3 c^4 d^5 - 36 a^3 c^2 d^7 + 8 a^3 d^9) \operatorname{ArcTanh}\left[\frac{(-c + d) \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c^2 - d^2}}\right] (d + c \cos[e + f x])^5 \sec[e + f x]^2 (a + b \sec[e + f x])^3 \right) / \\
& \left(4 c^5 \sqrt{c^2 - d^2} (-c^2 + d^2)^4 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5 \right) + \\
& \left((d + c \cos[e + f x]) \sec[e + f x]^2 (a + b \sec[e + f x])^3 (b^3 c^3 d^2 \sin[e + f x] - 3 a b^2 c^2 d^3 \sin[e + f x] + 3 a^2 b c d^4 \sin[e + f x] - a^3 d^5 \sin[e + f x]) \right) / \\
& \left(4 c^4 (c^2 - d^2) f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5 \right) + \\
& \left((d + c \cos[e + f x])^2 \sec[e + f x]^2 (a + b \sec[e + f x])^3 (-8 b^3 c^5 d \sin[e + f x] + 36 a b^2 c^4 d^2 \sin[e + f x] - 48 a^2 b c^3 d^3 \sin[e + f x] + \right. \\
& \quad \left. b^3 c^3 d^3 \sin[e + f x] + 20 a^3 c^2 d^4 \sin[e + f x] - 15 a b^2 c^2 d^4 \sin[e + f x] + 27 a^2 b c d^5 \sin[e + f x] - 13 a^3 d^6 \sin[e + f x]) \right) / \\
& \left(12 c^4 (c^2 - d^2)^2 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5 \right) + \frac{1}{24 c^4 (c^2 - d^2)^3 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5} \\
& \left((d + c \cos[e + f x])^3 \sec[e + f x]^2 (a + b \sec[e + f x])^3 (12 b^3 c^7 \sin[e + f x] - 108 a b^2 c^6 d \sin[e + f x] + \right. \\
& \quad 216 a^2 b c^5 d^2 \sin[e + f x] + 25 b^3 c^5 d^2 \sin[e + f x] - 120 a^3 c^4 d^3 \sin[e + f x] + 9 a b^2 c^4 d^3 \sin[e + f x] - 165 a^2 b c^3 d^4 \sin[e + f x] - \\
& \quad \left. 2 b^3 c^3 d^4 \sin[e + f x] + 131 a^3 c^2 d^5 \sin[e + f x] - 6 a b^2 c^2 d^5 \sin[e + f x] + 54 a^2 b c d^6 \sin[e + f x] - 46 a^3 d^7 \sin[e + f x]) \right) + \\
& \frac{1}{24 c^4 (c^2 - d^2)^4 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5} (d + c \cos[e + f x])^4 \sec[e + f x]^2 (a + b \sec[e + f x])^3 \\
& \left(72 a b^2 c^8 \sin[e + f x] - 288 a^2 b c^7 d \sin[e + f x] - 68 b^3 c^7 d \sin[e + f x] + 240 a^3 c^6 d^2 \sin[e + f x] + 252 a b^2 c^6 d^2 \sin[e + f x] + \right. \\
& \quad 24 a^2 b c^5 d^3 \sin[e + f x] - 39 b^3 c^5 d^3 \sin[e + f x] - 280 a^3 c^4 d^4 \sin[e + f x] - 15 a b^2 c^4 d^4 \sin[e + f x] - 69 a^2 b c^3 d^5 \sin[e + f x] + \\
& \quad \left. 2 b^3 c^3 d^5 \sin[e + f x] + 195 a^3 c^2 d^6 \sin[e + f x] + 6 a b^2 c^2 d^6 \sin[e + f x] + 18 a^2 b c d^7 \sin[e + f x] - 50 a^3 d^8 \sin[e + f x] \right)
\end{aligned}$$

- **Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \sec[e + f x]} (c + d \sec[e + f x]) dx$$

Optimal (type 4, 320 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{bf} 2(a-b)\sqrt{a+b} d \cot[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}} + \frac{1}{bf} \\
& 2\sqrt{a+b} (b(c-d)+ad) \cot[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}} - \\
& 2\sqrt{a+b} c \cot[e+fx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}}
\end{aligned}$$

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Result (type 4, 913 leaves):

$$\begin{aligned}
& \frac{2 d \cos [e+f x] \sqrt{a+b \sec [e+f x]} (c+d \sec [e+f x]) \sin [e+f x]}{f (d+c \cos [e+f x])} + \\
& \left(2 \sqrt{a+b \sec [e+f x]} (c+d \sec [e+f x]) \left(a \sqrt{\frac{-a+b}{a+b}} d \tan \left[\frac{1}{2} (e+f x) \right] + b \sqrt{\frac{-a+b}{a+b}} d \tan \left[\frac{1}{2} (e+f x) \right] - 2 a \sqrt{\frac{-a+b}{a+b}} d \tan \left[\frac{1}{2} (e+f x) \right]^3 + \right. \right. \\
& a \sqrt{\frac{-a+b}{a+b}} d \tan \left[\frac{1}{2} (e+f x) \right]^5 - b \sqrt{\frac{-a+b}{a+b}} d \tan \left[\frac{1}{2} (e+f x) \right]^5 + 2 i a c \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1-\tan \left[\frac{1}{2} (e+f x) \right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (e+f x) \right]^2+b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} + \\
& 2 i a c \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (e+f x) \right]^2 \sqrt{1-\tan \left[\frac{1}{2} (e+f x) \right]^2} \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (e+f x) \right]^2+b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} - i (a-b) d \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1-\tan \left[\frac{1}{2} (e+f x) \right]^2} \left(1+\tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (e+f x) \right]^2+b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} - \\
& i (a-b) (c-d) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (e+f x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1-\tan \left[\frac{1}{2} (e+f x) \right]^2} \\
& \left. \left(1+\tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (e+f x) \right]^2+b \tan \left[\frac{1}{2} (e+f x) \right]^2}{a+b}} \right) \right] / \\
& \left(\sqrt{\frac{-a+b}{a+b}} f \sqrt{b+a \cos [e+f x]} (d+c \cos [e+f x]) \sec [e+f x]^{3/2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2} (e+f x) \right]^2}} \left(-1+\tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \right. \\
& \left. \left(1+\tan \left[\frac{1}{2} (e+f x) \right]^2 \right)^{3/2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (e+f x) \right]^2+b \tan \left[\frac{1}{2} (e+f x) \right]^2}{1+\tan \left[\frac{1}{2} (e+f x) \right]^2}} \right)
\end{aligned}$$

■ **Problem 199: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{c+d \operatorname{Sec}[e+f x]} dx$$

Optimal (type 4, 220 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{c f} + \frac{2(b c-a d) \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sec}[e+f x]}{a+b}} \operatorname{Tan}[e+f x]}{c(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{c+d \operatorname{Sec}[e+f x]} dx$$

■ **Problem 201: Unable to integrate problem.**

$$\int \frac{(a+b \operatorname{Sec}[e+f x])^{3/2}}{c+d \operatorname{Sec}[e+f x]} dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\frac{2 b \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{d f} - \frac{2 a \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{c f} - \frac{2(b c-a d)^2 \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sec}[e+f x]}{a+b}} \operatorname{Tan}[e+f x]}{c d(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{(a+b \operatorname{Sec}[e+f x])^{3/2}}{c+d \operatorname{Sec}[e+f x]} dx$$

■ **Problem 204: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+f x]} (c+d \operatorname{Sec}[e+f x])} dx$$

Optimal (type 4, 216 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a c f}$$

$$\frac{2 d \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sec}[e+f x]}{a+b}} \operatorname{Tan}[e+f x]}{c(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+f x]} (c+d \operatorname{Sec}[e+f x])} dx$$

■ **Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{c+d \operatorname{Sec}[e+f x]}{(a+b \operatorname{Sec}[e+f x])^{3/2}} dx$$

Optimal (type 4, 376 leaves, 6 steps):

$$\frac{2(b c-a d) \operatorname{Cot}[e+f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a b \sqrt{a+b} f}$$

$$\frac{2(b c-a d) \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a b \sqrt{a+b} f}$$

$$\frac{2 \sqrt{a+b} c \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{a^2 f} + \frac{2 b(b c-a d) \operatorname{Tan}[e+f x]}{a(a^2-b^2) f \sqrt{a+b \operatorname{Sec}[e+f x]}}$$

Result (type 4, 1491 leaves):

$$\frac{(b+a \operatorname{Cos}[e+f x])^2 \operatorname{Sec}[e+f x] (c+d \operatorname{Sec}[e+f x]) \left(\frac{2(-b c+a d) \operatorname{Sin}[e+f x]}{a(a^2-b^2)} - \frac{2(-b^2 c \operatorname{Sin}[e+f x]+a b d \operatorname{Sin}[e+f x])}{a(a^2-b^2)(b+a \operatorname{Cos}[e+f x])} \right)}{f(d+c \operatorname{Cos}[e+f x])(a+b \operatorname{Sec}[e+f x])^{3/2}} +$$

$$\begin{aligned}
& \left(2 (b + a \operatorname{Cos}[e + f x])^{3/2} \sqrt{\operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x]) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}} \right. \\
& \left(a b \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] + b^2 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - a^2 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - a b \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \right. \\
& 2 a b \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^3 + 2 a^2 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^3 + a b \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^5 - \\
& b^2 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^5 - a^2 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^5 + a b \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^5 - \\
& 2 i a^2 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \\
& \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} + 2 i b^2 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} - \\
& 2 i a^2 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \\
& \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} + 2 i b^2 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} + \\
& i (a - b) (-b c + a d) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)
\end{aligned}$$

$$\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}}+i(a-b)(2bc+a(c-d)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}}\right) /$$

$$\left(a \sqrt{\frac{-a+b}{a+b}}\left(a^2-b^2\right) f(d+c \operatorname{Cos}[e+fx])\left(a+b \operatorname{Sec}[e+fx]\right)^{3/2}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right) \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}}\right.$$

$$\left.\left(a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)-b\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right)\right)\right)$$

- **Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{c+d \operatorname{Sec}[e+fx]}{(a+b \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 4, 495 leaves, 7 steps):

$$\frac{1}{3 a^2(a-b) b(a+b)^{3/2} f}$$

$$2\left(7 a^2 b c-3 b^3 c-4 a^3 d\right) \operatorname{Cot}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}} -$$

$$\frac{1}{3 a^2(a-b) b(a+b)^{3/2} f} 2\left(6 a^2 b c-a b^2 c-3 b^3 c-3 a^3 d+a^2 b d\right) \operatorname{Cot}[e+fx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}} -$$

$$2 \sqrt{a+b} c \operatorname{Cot}[e+fx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}}$$

$$\frac{2 b(b c-a d) \operatorname{Tan}[e+fx]}{3 a\left(a^2-b^2\right) f(a+b \operatorname{Sec}[e+fx])^{3/2}}+\frac{2 b\left(7 a^2 b c-3 b^3 c-4 a^3 d\right) \operatorname{Tan}[e+fx]}{3 a^2\left(a^2-b^2\right)^2 f \sqrt{a+b \operatorname{Sec}[e+fx]}}$$

Result (type 4, 2083 leaves):

$$\begin{aligned}
& \left((b + a \cos[e + f x])^3 \sec[e + f x]^2 (c + d \sec[e + f x]) \left(\frac{2(-7a^2bc + 3b^3c + 4a^3d) \sin[e + f x]}{3a^2(a^2 - b^2)^2} - \frac{2(b^3c \sin[e + f x] - ab^2d \sin[e + f x])}{3a^2(a^2 - b^2)(b + a \cos[e + f x])^2} - \right. \right. \\
& \left. \left. \frac{2(-8a^2b^2c \sin[e + f x] + 4b^4c \sin[e + f x] + 5a^3bd \sin[e + f x] - ab^3d \sin[e + f x])}{3a^2(a^2 - b^2)^2(b + a \cos[e + f x])} \right) \right) / (f(d + c \cos[e + f x])(a + b \sec[e + f x])^{5/2}) + \\
& \left(2(b + a \cos[e + f x])^{5/2} \sec[e + f x]^{3/2} (c + d \sec[e + f x]) \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(e + f x)\right]^2 + b \tan\left[\frac{1}{2}(e + f x)\right]^2}{1 + \tan\left[\frac{1}{2}(e + f x)\right]^2}} \right. \\
& \left(7a^3b \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right] + 7a^2b^2 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right] - 3ab^3 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right] - \right. \\
& 3b^4 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right] - 4a^4 \sqrt{\frac{-a + b}{a + b}} d \tan\left[\frac{1}{2}(e + f x)\right] - 4a^3b \sqrt{\frac{-a + b}{a + b}} d \tan\left[\frac{1}{2}(e + f x)\right] - \\
& 14a^3b \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^3 + 6ab^3 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^3 + 8a^4 \sqrt{\frac{-a + b}{a + b}} d \tan\left[\frac{1}{2}(e + f x)\right]^3 + \\
& 7a^3b \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^5 - 7a^2b^2 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^5 - 3ab^3 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^5 + \\
& 3b^4 \sqrt{\frac{-a + b}{a + b}} c \tan\left[\frac{1}{2}(e + f x)\right]^5 - 4a^4 \sqrt{\frac{-a + b}{a + b}} d \tan\left[\frac{1}{2}(e + f x)\right]^5 + 4a^3b \sqrt{\frac{-a + b}{a + b}} d \tan\left[\frac{1}{2}(e + f x)\right]^5 - \\
& 6ia^4c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(e + f x)\right]^2} \\
& \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(e + f x)\right]^2 + b \tan\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} + 12ia^2b^2c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(e + f x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(e + f x)\right]^2 + b \tan\left[\frac{1}{2}(e + f x)\right]^2}{a + b}} - \\
& 6ib^4c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left[\frac{1}{2}(e + f x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(e + f x)\right]^2}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}-6 i a^4 c \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}+ \\
& 12 i a^2 b^2 c \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}-6 i b^4 c \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}+ \\
& i(a-b)\left(-7 a^2 b c+3 b^3 c+4 a^3 d\right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}+ \\
& i(a-b)\left(-4 a b^2 c-6 b^3 c+3 a^3(c-d)+a^2 b(9 c+d)\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{a+b}}\right) / \\
& \left(3 a^2 \sqrt{\frac{-a+b}{a+b}}\left(a^2-b^2\right)^2 f(d+c \operatorname{Cos}[e+f x])\left(a+b \operatorname{Sec}[e+f x]\right)^{5 / 2}\left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2}}\right. \\
& \left.\left(a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)-b\left(1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right)\right)\right)
\end{aligned}$$

Problem 207: Unable to integrate problem.

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]} \sqrt{c + d \operatorname{Sec}[e + f x]} dx$$

Optimal (type 4, 389 leaves, 3 steps):

$$-\frac{1}{\sqrt{a+b} f} 2 \sqrt{c+d} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\sqrt{-\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x]) +$$

$$\frac{1}{\sqrt{\frac{a+b}{c+d} f}} 2 \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{a+b \operatorname{Sec}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\sqrt{-\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x])$$

Result (type 8, 31 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]} \sqrt{c + d \operatorname{Sec}[e + f x]} dx$$

■ **Problem 208: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \operatorname{Sec}[e + f x]}}{\sqrt{c + d \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$-\frac{1}{\sqrt{a+b} c f} 2 \sqrt{c+d} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\sqrt{-\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x])$$

Result (type 4, 554 leaves):

$$\frac{1}{f \sqrt{b+a \cos [e+f x]} \sqrt{c+d \sec [e+f x]}} 4 (b c-a d) \sqrt{d+c \cos [e+f x]} \sqrt{a+b \sec [e+f x]}$$

$$\left(\left(\sqrt{\frac{(c+d) \cot \left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \sqrt{\frac{(a+b)(d+c \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}}{\sqrt{2}}}\right], \frac{2(b c-a d)}{(a+b)(c-d)}\right] \sin \left[\frac{1}{2}(e+f x)\right]^4 \right) / \right.$$

$$\left. \left((c+d) \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right) - \left(a \sqrt{\frac{(c+d) \cot \left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b)(d+c \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \operatorname{Csc}[e+f x] \operatorname{EllipticPi}\left[\frac{b c-a d}{(a+b) c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos [e+f x]) \csc \left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}}{\sqrt{2}}}\right] \right), \right.$$

$$\left. \left. \frac{2(b c-a d)}{(a+b)(c-d)} \right] \sin \left[\frac{1}{2}(e+f x)\right]^4 \right) / \left((a+b) c \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right)$$

■ **Problem 209: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \sec [e+f x]}}{(c+d \sec [e+f x])^{3/2}} dx$$

Optimal (type 4, 598 leaves, 5 steps):

$$\begin{aligned}
& - \frac{1}{\sqrt{a+b} c^2 f} 2 \sqrt{c+d} \cot[e+f x] \operatorname{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]}{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x]) - \\
& \left(2 \sqrt{a+b} d \cot[e+f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] (1+\operatorname{Sec}[e+f x]) \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}}\right) / \\
& \left(c(c-d) \sqrt{c+d} f \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}}\right) - \\
& \left(2(a-b) \sqrt{a+b} d \cot[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \right. \\
& \left. \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])\right) / (c(c-d) \sqrt{c+d} (bc-ad) f)
\end{aligned}$$

Result (type 4, 1678 leaves):

$$\begin{aligned}
& \frac{1}{(c-d)(c+d) f \sqrt{b+a \operatorname{Cos}[e+f x]} (c+d \operatorname{Sec}[e+f x])^{3/2}} (d+c \operatorname{Cos}[e+f x])^{3/2} \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]} \\
& \left(\left(4bc(bc-ad) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \operatorname{Csc}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{bc-ad}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]^4 \right) / \right. \\
& \left. \left((a+b)(c+d) \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{d+c \operatorname{Cos}[e+f x]} \right) + 4(bc-ad)(a+c+bd) \right)
\end{aligned}$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right. \\
\left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \right. \\
\left. \left((a+b)(c+d) \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) - \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right. \\
\left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \right. \\
\left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)c \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) + \\
2ad \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos\left[\frac{1}{2}(e+fx)\right] \sqrt{d+c \cos[e+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{\frac{b+a \cos[e+fx]}{a+b}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)} \right) / \right. \\
\left. \left(ac \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}(e+fx)\right]^2}{b+a \cos[e+fx]}} \sqrt{b+a \cos[e+fx]} \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c \cos[e+fx])}{(c+d)(b+a \cos[e+fx])}} \right) - \right.$$

$$\begin{aligned}
& \frac{1}{ac} 2 (bc - ad) \left((bc + (a+b)d) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \\
& \sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \\
& \left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)(c+d) \sqrt{b+a\cos[e+fx]} \sqrt{d+c\cos[e+fx]} \right) - (bc+ad) \\
& \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \\
& \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \\
& \left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)c \sqrt{b+a\cos[e+fx]} \sqrt{d+c\cos[e+fx]} \right) + \\
& \left. \left. \frac{\sqrt{d+c\cos[e+fx]} \operatorname{Sin}[e+fx]}{c \sqrt{b+a\cos[e+fx]}} \right) \right) + \frac{2d(d+c\cos[e+fx]) \sqrt{a+b\sec[e+fx]} \operatorname{Tan}[e+fx]}{(-c^2+d^2)f(c+d\sec[e+fx])^{3/2}}
\end{aligned}$$

■ **Problem 210: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b\sec[e+fx]}}{(c+d\sec[e+fx])^{5/2}} dx$$

Optimal (type 4, 899 leaves, 7 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} d (6bc^3 - 7ac^2d - 2bcd^2 + 3ad^3) \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} \right. \\
& \quad \left. (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]} \right) / \\
& \quad \left(3c^2(c-d)^2(c+d)^{3/2}(bc-ad)^2 f \sqrt{b+a\cos[e+fx]} \sqrt{c+d\sec[e+fx]} \right) + \\
& \left(2\sqrt{a+b} (bc^2(3c^2+3cd-2d^2) - ad(9c^3-2c^2d-6cd^2+3d^3)) \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} \right. \\
& \quad \left. (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]} \right) / \\
& \quad \left(3c^3(c-d)^2(c+d)^{3/2}(bc-ad) f \sqrt{b+a\cos[e+fx]} \sqrt{c+d\sec[e+fx]} \right) - \\
& \left(2\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]} \right) / \\
& \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a\cos[e+fx]} \sqrt{c+d\sec[e+fx]} \right) + \frac{2d^2 \sqrt{a+b\sec[e+fx]} \operatorname{Sin}[e+fx]}{3c(c^2-d^2) f (d+c\cos[e+fx]) \sqrt{c+d\sec[e+fx]}}
\end{aligned}$$

Result (type 4, 1960 leaves):

$$\begin{aligned}
& \frac{1}{f(c+d\sec[e+fx])^{5/2}} (d+c\cos[e+fx])^3 \sec[e+fx]^2 \sqrt{a+b\sec[e+fx]} \\
& \left(\frac{2d^2 \operatorname{Sin}[e+fx]}{3c(c^2-d^2)(d+c\cos[e+fx])^2} - \frac{2(6bc^3d \operatorname{Sin}[e+fx] - 7ac^2d^2 \operatorname{Sin}[e+fx] - 2bcd^3 \operatorname{Sin}[e+fx] + 3ad^4 \operatorname{Sin}[e+fx])}{3c(bc-ad)(c^2-d^2)^2(d+c\cos[e+fx])} \right) + \\
& \left((d+c\cos[e+fx])^{5/2} \sec[e+fx]^2 \sqrt{a+b\sec[e+fx]} \left(4(bc-ad)(3b^2c^4 - 3abc^3d - a^2c^2d^2 + b^2c^2d^2 - abc^3d^3 + a^2d^4) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \\
& \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \\
& \left((a+b)(c+d) \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) + 4(bc-ad) (3abc^4 - 3a^2c^3d + 6b^2c^3d - 7abc^2d^2 - a^2cd^3 - 2b^2cd^3 + 4abd^4) \\
& \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \right. \right. \\
& \left. \left. \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)(c+d) \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) - \right. \\
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \\
& \left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) /
\end{aligned}$$

$$\left. \left((a+b) c \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) + 2 (6abc^3d - 7a^2c^2d^2 - 2abcd^3 + 3a^2d^4) \right.$$

$$\left. \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos\left[\frac{1}{2}(e+fx)\right] \sqrt{d+c \cos[e+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{b+a \cos[e+fx]}{a+b}}}\right]}, \frac{2(bc-ad)}{(-a+b)(c+d)}\right] \right) / \right.$$

$$\left. \left(ac \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}(e+fx)\right]^2}{b+a \cos[e+fx]}} \sqrt{b+a \cos[e+fx]} \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c \cos[e+fx])}{(c+d)(b+a \cos[e+fx])}} \right) - \right.$$

$$\frac{1}{ac} 2(bc-ad) \left((bc+(a+b)d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right.$$

$$\left. \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \frac{2(bc-ad)}{(a+b)(c-d)} \sin\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)(c+d) \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) - (bc+ad)$$

$$\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}$$

$$\operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)} \sin\left[\frac{1}{2}(e+fx)\right]^4 \right) /$$

$$\begin{aligned}
& \frac{1}{(c-d)(c+d)f(b+a\cos[ex+fx])^{3/2}(c+d\sec[ex+fx])^{3/2}}(d+c\cos[ex+fx])^{3/2}(a+b\sec[ex+fx])^{3/2} \\
& \left(\left(4(bc-ad)(abc-b^2d) \sqrt{\frac{(c+d)\cot\left[\frac{1}{2}(ex+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \right. \right. \\
& \left. \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \operatorname{Csc}[ex+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \right. \\
& \left. \left. \operatorname{Sin}\left[\frac{1}{2}(ex+fx)\right]^4 \right) / \left((a+b)(c+d)\sqrt{b+a\cos[ex+fx]}\sqrt{d+c\cos[ex+fx]} + 4(a^2c-b^2c)(bc-ad) \right. \right. \\
& \left. \left(\left(\sqrt{\frac{(c+d)\cot\left[\frac{1}{2}(ex+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \right. \right. \right. \\
& \left. \left. \operatorname{Csc}[ex+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(ex+fx)\right]^4 \right) / \right. \\
& \left. \left((a+b)(c+d)\sqrt{b+a\cos[ex+fx]}\sqrt{d+c\cos[ex+fx]} \right) - \left(\sqrt{\frac{(c+d)\cot\left[\frac{1}{2}(ex+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \operatorname{Csc}[ex+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\csc\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 (bc - ad)}{(a + b) (c - d)} \right] \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right]^4 \Bigg/ \left((a + b) c \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) \Bigg) + \\
& 2 (-a b c + a^2 d) \left(\left(\sqrt{\frac{-a + b}{a + b}} (a + b) \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right] \sqrt{d + c \operatorname{Cos}[e + f x]} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{-a + b}{a + b}} \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{b + a \operatorname{Cos}[e + f x]}{a + b}}} \right], \frac{2 (bc - ad)}{(-a + b) (c + d)} \right] \right) \Bigg/ \right. \\
& \left(a c \sqrt{\frac{(a + b) \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2}{b + a \operatorname{Cos}[e + f x]}} \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{\frac{b + a \operatorname{Cos}[e + f x]}{a + b}} \sqrt{\frac{(a + b) (d + c \operatorname{Cos}[e + f x])}{(c + d) (b + a \operatorname{Cos}[e + f x])}} \right) - \\
& \frac{1}{a c} 2 (bc - ad) \left((bc + (a + b) d) \sqrt{\frac{(c + d) \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}} \right. \\
& \left. \sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}} \operatorname{Csc}[e + f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}}}{\sqrt{2}}} \right], \right. \\
& \left. \frac{2 (bc - ad)}{(a + b) (c - d)} \right] \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right]^4 \Bigg/ \left((a + b) (c + d) \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) - (bc + ad) \\
& \sqrt{\frac{(c + d) \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}} \sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}} \\
& \left. \operatorname{Csc}[e + f x] \operatorname{EllipticPi} \left[\frac{bc - ad}{(a + b) c}, \operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{bc - ad}}}{\sqrt{2}}} \right], \frac{2 (bc - ad)}{(a + b) (c - d)} \right] \operatorname{Sin} \left[\frac{1}{2} (e + f x) \right]^4 \Bigg/ \right.
\end{aligned}$$

$$\left((a+b) c \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) + \frac{\sqrt{d+c \cos[e+fx]} \sin[e+fx]}{c \sqrt{b+a \cos[e+fx]}} \Bigg)$$

■ **Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \sec[e+fx])^{3/2}}{(c+d \sec[e+fx])^{5/2}} dx$$

Optimal (type 4, 919 leaves, 7 steps):

$$\begin{aligned} & - \left(2(a-b) \sqrt{a+b} (3bc^3 - 7ac^2d + bcd^2 + 3ad^3) \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c \cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c \cos[e+fx])}} \right. \\ & \quad \left. (d+c \cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+fx]}}{\sqrt{a+b} \sqrt{d+c \cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b \sec[e+fx]} \right) / \\ & \quad \left(3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f \sqrt{b+a \cos[e+fx]} \sqrt{c+d \sec[e+fx]} \right) - \\ & \quad \left(2\sqrt{a+b} (b^2c^3(3c+d) - 2abc^2(3c^2+2cd-d^2) + a^2d(9c^3-2c^2d-6cd^2+3d^3)) \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c \cos[e+fx])}} \right. \\ & \quad \left. \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c \cos[e+fx])}} (d+c \cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+fx]}}{\sqrt{a+b} \sqrt{d+c \cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \right. \\ & \quad \left. \sqrt{a+b \sec[e+fx]} \right) / \left(3c^3(c-d)^2(c+d)^{3/2}(bc-ad)f \sqrt{b+a \cos[e+fx]} \sqrt{c+d \sec[e+fx]} \right) - \\ & \quad \left(2a\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c \cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c \cos[e+fx])}} (d+c \cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \right. \\ & \quad \left. \operatorname{EllipticPi}\left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+fx]}}{\sqrt{a+b} \sqrt{d+c \cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b \sec[e+fx]} \right) / \\ & \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a \cos[e+fx]} \sqrt{c+d \sec[e+fx]} \right) - \frac{2d(bc-ad) \sqrt{a+b \sec[e+fx]} \sin[e+fx]}{3c(c^2-d^2)f(d+c \cos[e+fx]) \sqrt{c+d \sec[e+fx]}} \end{aligned}$$

Result (type 4, 1930 leaves):

$$\begin{aligned}
& \left((d + c \cos[ex + fx])^3 \sec[ex + fx] (a + b \sec[ex + fx])^{3/2} \right. \\
& \left. \left(\frac{2(-bcd \sin[ex + fx] + ad^2 \sin[ex + fx])}{3c(c^2 - d^2)(d + c \cos[ex + fx])^2} + \frac{2(3bc^3 \sin[ex + fx] - 7ac^2d \sin[ex + fx] + bcd^2 \sin[ex + fx] + 3ad^3 \sin[ex + fx])}{3c(c^2 - d^2)^2(d + c \cos[ex + fx])} \right) \right) / \\
& \frac{1}{(f(b + a \cos[ex + fx])(c + d \sec[ex + fx])^{5/2}) + \frac{1}{3c(c-d)^2(c+d)^2 f(b + a \cos[ex + fx])^{3/2}(c + d \sec[ex + fx])^{5/2}}} \\
& (d + c \cos[ex + fx])^{5/2} \sec[ex + fx] (a + b \sec[ex + fx])^{3/2} \\
& \left(\left(4(bc - ad)(3abc^3 + a^2c^2d - 4b^2c^2d + abcd^2 - a^2d^3) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(ex + fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b + a \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d + c \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}} \right. \text{Csc}[ex + fx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{-\frac{(a+b)(d + c \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}}{\sqrt{2}}\right], \frac{2(bc - ad)}{(a+b)(c-d)}\right] \right. \right. \\
& \left. \left. \sin\left[\frac{1}{2}(ex + fx)\right]^4 \right) / \left((a+b)(c+d) \sqrt{b + a \cos[ex + fx]} \sqrt{d + c \cos[ex + fx]} \right) + 4(bc - ad) \right) \\
& (3a^2c^3 - 3b^2c^3 + 4abc^2d + a^2cd^2 - b^2cd^2 - 4abd^3) \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(ex + fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b + a \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d + c \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}} \right. \text{Csc}[ex + fx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{-\frac{(a+b)(d + c \cos[ex + fx]) \csc\left[\frac{1}{2}(ex + fx)\right]^2}{bc - ad}}{\sqrt{2}}\right], \right. \right. \\
& \left. \left. \frac{2(bc - ad)}{(a+b)(c-d)} \right] \sin\left[\frac{1}{2}(ex + fx)\right]^4 \right) / \left((a+b)(c+d) \sqrt{b + a \cos[ex + fx]} \sqrt{d + c \cos[ex + fx]} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \\
& \left. \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \\
& \left. \left((a+b)c \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{d+c \operatorname{Cos}[e+fx]} \right) + 2(-3abc^3 + 7a^2c^2d - abc d^2 - 3a^2d^3) \right) \\
& \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \sqrt{d+c \operatorname{Cos}[e+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{\frac{b+a \operatorname{Cos}[e+fx]}{a+b}}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)}\right] \right) / \right. \\
& \left. \left(a c \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2}{b+a \operatorname{Cos}[e+fx]}} \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{\frac{b+a \operatorname{Cos}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c \operatorname{Cos}[e+fx])}{(c+d)(b+a \operatorname{Cos}[e+fx])}} \right) - \right. \\
& \left. \frac{1}{ac} 2(bc-ad) \left((bc+(a+b)d) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right) \right) ,
\end{aligned}$$

$$\left. \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]^4 \Bigg/ \left((a + b) (c + d) \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) - \left(b c + a d \right)$$

$$\sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2}{b c - a d}} \sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2}{b c - a d}}$$

$$\operatorname{Csc}[e + f x] \operatorname{EllipticPi}\left[\frac{b c - a d}{(a + b) c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) (d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]^4 \Bigg/$$

$$\left((a + b) c \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) + \frac{\sqrt{d + c \operatorname{Cos}[e + f x]} \operatorname{Sin}[e + f x]}{c \sqrt{b + a \operatorname{Cos}[e + f x]}} \Bigg)$$

■ **Problem 213: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{3/2}}{(c + d \operatorname{Sec}[e + f x])^{7/2}} dx$$

Optimal (type 4, 1122 leaves, 8 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} (2abcd (35c^4 - 8c^2d^2 + 5d^4) - a^2d^2 (58c^4 - 41c^2d^2 + 15d^4) - b^2 (15c^6 + 19c^4d^2 - 2c^2d^4)) \right. \\
& \quad \left. \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} (d+c\cos[e+fx])^{3/2} \right. \\
& \quad \left. \operatorname{Csc}[e+fx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]}\right) / \\
& \quad \left(15c^3(c-d)^3(c+d)^{5/2}(bc-ad)^2f\sqrt{b+a\cos[e+fx]}\sqrt{c+d\sec[e+fx]} \right) - \\
& \quad \left(2\sqrt{a+b} (b^2c^3(15c^3+10c^2d+9cd^2-2d^3) - 2abc^2(15c^4+20c^3d-4c^2d^2-4cd^3+5d^4) + \right. \\
& \quad \left. a^2d(60c^5-2c^4d-66c^3d^2+25c^2d^3+30cd^4-15d^5)) \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} \right. \\
& \quad \left. (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]}\right) / \\
& \quad \left(15c^4(c-d)^3(c+d)^{5/2}(bc-ad)f\sqrt{b+a\cos[e+fx]}\sqrt{c+d\sec[e+fx]} \right) - \\
& \quad \left(2a\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos[e+fx])}{(a+b)(d+c\cos[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b\sec[e+fx]}\right) / \\
& \quad \left(c^4\sqrt{c+d}f\sqrt{b+a\cos[e+fx]}\sqrt{c+d\sec[e+fx]} \right) + \frac{2d^2(b+a\cos[e+fx])\sqrt{a+b\sec[e+fx]}\sin[e+fx]}{5c(c^2-d^2)f(d+c\cos[e+fx])^2\sqrt{c+d\sec[e+fx]}} - \\
& \quad \frac{2d(10bc^3-13a^2c^2d-2bcd^2+5ad^3)\sqrt{a+b\sec[e+fx]}\sin[e+fx]}{15c^2(c^2-d^2)^2f(d+c\cos[e+fx])\sqrt{c+d\sec[e+fx]}}
\end{aligned}$$

Result (type 4, 2355 leaves):

$$\frac{1}{f(b+a\cos[e+fx])(c+d\sec[e+fx])^{7/2}} (d+c\cos[e+fx])^4 \sec[e+fx]^2 (a+b\sec[e+fx])^{3/2}$$

$$\begin{aligned}
& \left(-\frac{2(-bc d^2 \sin[ex+f] + ad^3 \sin[ex+f])}{5c^2(c^2-d^2)(d+c \cos[ex+f])^3} - \frac{4(5bc^3 d \sin[ex+f] - 8ac^2 d^2 \sin[ex+f] - bc d^3 \sin[ex+f] + 4ad^4 \sin[ex+f])}{15c^2(c^2-d^2)^2(d+c \cos[ex+f])^2} + \right. \\
& \left. (2(15b^2 c^6 \sin[ex+f] - 70abc^5 d \sin[ex+f] + 58a^2 c^4 d^2 \sin[ex+f] + 19b^2 c^4 d^2 \sin[ex+f] + 16abc^3 d^3 \sin[ex+f] - 41a^2 c^2 d^4 \sin[ex+f] \right. \\
& \left. - 2b^2 c^2 d^4 \sin[ex+f] - 10abcd^5 \sin[ex+f] + 15a^2 d^6 \sin[ex+f])) / (15c^2(bc-ad)(c^2-d^2)^3(d+c \cos[ex+f])) \right) + \\
& \left((d+c \cos[ex+f])^{7/2} \sec[ex+f]^2 (a+b \sec[ex+f])^{3/2} \left[\frac{1}{(a+b)(c+d) \sqrt{b+a \cos[ex+f]} \sqrt{d+c \cos[ex+f]}} 4(bc-ad) \right. \right. \\
& \left. \left. (-15ab^2 c^6 + 5a^2 bc^5 d + 25b^3 c^5 d + 13a^3 c^4 d^2 - 38ab^2 c^4 d^2 + 25a^2 bc^3 d^3 + 7b^3 c^3 d^3 - 18a^3 c^2 d^4 - 11ab^2 c^2 d^4 + 2a^2 bcd^5 + 5a^3 d^6) \right. \right. \\
& \left. \left. \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(ex+f)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \operatorname{Csc}[ex+f] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \sin\left[\frac{1}{2}(ex+f)\right]^4 + \right. \right. \\
& \left. \left. 4(bc-ad) (-15a^2 bc^6 + 15b^3 c^6 + 15a^3 c^5 d - 55ab^2 c^5 d + 33a^2 bc^4 d^2 + 19b^3 c^4 d^2 + 13a^3 c^3 d^3 + \right. \right. \\
& \left. \left. 35ab^2 c^3 d^3 - 70a^2 bc^2 d^4 - 2b^3 c^2 d^4 + 4a^3 cd^5 - 12ab^2 cd^5 + 20a^2 bd^6) \right. \right. \\
& \left. \left. \left(\left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(ex+f)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{Csc}[ex+f] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \sin\left[\frac{1}{2}(ex+f)\right]^4 \right) \right) \right) / \\
& \left((a+b)(c+d) \sqrt{b+a \cos[ex+f]} \sqrt{d+c \cos[ex+f]} \right) - \left(\sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(ex+f)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[ex+f]) \operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \operatorname{Csc}[ex+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \\
& \left. \frac{2(bc-ad)}{(a+b)(c-d)} \operatorname{Sin}\left[\frac{1}{2}(ex+fx)\right]^4 \right] / \left((a+b)c\sqrt{b+a\cos[ex+fx]}\sqrt{d+c\cos[ex+fx]} \right) + \\
& 2(15a^2b^2c^6 - 70a^2bc^5d + 58a^3c^4d^2 + 19a^2b^2c^4d^2 + 16a^2bc^3d^3 - 41a^3c^2d^4 - 2ab^2c^2d^4 - 10a^2bcd^5 + 15a^3d^6) \\
& \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b)\cos\left[\frac{1}{2}(ex+fx)\right] \sqrt{d+c\cos[ex+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}(ex+fx)\right]}{\sqrt{\frac{b+a\cos[ex+fx]}{a+b}}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)} \right] \right) / \right. \\
& \left. \left(ac\sqrt{\frac{(a+b)\cos\left[\frac{1}{2}(ex+fx)\right]^2}{b+a\cos[ex+fx]}} \sqrt{b+a\cos[ex+fx]} \sqrt{\frac{b+a\cos[ex+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c\cos[ex+fx])}{(c+d)(b+a\cos[ex+fx])}} \right) - \right. \\
& \left. \frac{1}{ac} 2(bc-ad) \left((bc+(a+b)d) \sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}(ex+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \operatorname{Csc}[ex+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \operatorname{Sin}\left[\frac{1}{2}(ex+fx)\right]^4 \right] / \left((a+b)(c+d)\sqrt{b+a\cos[ex+fx]}\sqrt{d+c\cos[ex+fx]} \right) - (bc+ad) \right. \\
& \left. \sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}(ex+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[ex+fx])\operatorname{Csc}\left[\frac{1}{2}(ex+fx)\right]^2}{bc-ad}} \right)
\end{aligned}$$

$$\begin{aligned}
& - \left(2 (a-b) \sqrt{a+b} (7 a c^2 - 4 b c d - 3 a d^2) \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} \right. \\
& \quad \left. (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \operatorname{Sec}[e + f x]}\right) / \\
& \quad \left(3 c^2 (c-d)^2 (c+d)^{3/2} f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) + \\
& \quad \left(2 \sqrt{a+b} (b^2 c^2 (c+3 d) - a b c (7 c^2 + 4 c d - 3 d^2) + a^2 (9 c^3 - 2 c^2 d - 6 c d^2 + 3 d^3)) \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \right. \\
& \quad \left. \sqrt{a+b \operatorname{Sec}[e + f x]}\right) / \left(3 c^3 (c-d)^2 (c+d)^{3/2} f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) - \\
& \quad \left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \right. \\
& \quad \left. \operatorname{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \operatorname{Sec}[e + f x]}\right) / \\
& \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) + \frac{2 (b c - a d)^2 \sqrt{a+b \operatorname{Sec}[e + f x]} \operatorname{Sin}[e + f x]}{3 c (c^2 - d^2) f (d + c \operatorname{Cos}[e + f x]) \sqrt{c+d \operatorname{Sec}[e + f x]}}
\end{aligned}$$

Result (type 4, 1996 leaves):

$$\begin{aligned}
& \left((d + c \operatorname{Cos}[e + f x])^3 (a + b \operatorname{Sec}[e + f x])^{5/2} \left(\frac{2 (b^2 c^2 \operatorname{Sin}[e + f x] - 2 a b c d \operatorname{Sin}[e + f x] + a^2 d^2 \operatorname{Sin}[e + f x])}{3 c (c^2 - d^2) (d + c \operatorname{Cos}[e + f x])^2} + \right. \right. \\
& \quad \left. \left. (2 (7 a b c^3 \operatorname{Sin}[e + f x] - 7 a^2 c^2 d \operatorname{Sin}[e + f x] - 4 b^2 c^2 d \operatorname{Sin}[e + f x] + a b c d^2 \operatorname{Sin}[e + f x] + 3 a^2 d^3 \operatorname{Sin}[e + f x])) / \right. \right. \\
& \quad \left. \left. (3 c (c^2 - d^2)^2 (d + c \operatorname{Cos}[e + f x])) \right) \right) / \left(f (b + a \operatorname{Cos}[e + f x])^2 (c + d \operatorname{Sec}[e + f x])^{5/2} \right) + \\
& \quad \frac{1}{3 c (c-d)^2 (c+d)^2 f (b+a \operatorname{Cos}[e + f x])^{5/2} (c+d \operatorname{Sec}[e + f x])^{5/2}} (d + c \operatorname{Cos}[e + f x])^{5/2} (a + b \operatorname{Sec}[e + f x])^{5/2}
\end{aligned}$$

$$\left(\left(4 (bc - ad) (2a^2bc^3 + b^3c^3 + a^3c^2d - 8ab^2c^2d + 2a^2bcd^2 + 3b^3cd^2 - a^3d^3) \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \right. \right.$$

$$\left. \sqrt{\frac{(c+d)(b+a\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right.$$

$$\left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) \right) /$$

$$\left((a+b)(c+d) \sqrt{b+a\cos[e+fx]} \sqrt{d+c\cos[e+fx]} \right) + 4(bc-ad) (3a^3c^3 - 7ab^2c^3 + 4b^3c^2d + a^3cd^2 + 3ab^2cd^2 - 4a^2bd^3)$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right.$$

$$\left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \right.$$

$$\left. \left. \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) \right) / \left((a+b)(c+d) \sqrt{b+a\cos[e+fx]} \sqrt{d+c\cos[e+fx]} \right) -$$

$$\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right)$$

$$\begin{aligned}
& \left. \left(\text{Csc}[e + f x] \text{EllipticPi} \left[\frac{b c - a d}{(a + b) c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \text{Csc} \left[\frac{1}{2} (e+f x) \right]^2}}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \text{Sin} \left[\frac{1}{2} (e + f x) \right]^4 \right) / \right. \\
& \left. \left((a + b) c \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]} \right) + 2 (-7 a^2 b c^3 + 7 a^3 c^2 d + 4 a b^2 c^2 d - a^2 b c d^2 - 3 a^3 d^3) \right. \\
& \left. \left(\left(\sqrt{\frac{-a + b}{a + b}} (a + b) \cos \left[\frac{1}{2} (e + f x) \right] \sqrt{d + c \cos[e + f x]} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a + b}{a + b}} \text{Sin} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{b + a \cos[e + f x]}{a + b}}}} \right], \frac{2 (b c - a d)}{(-a + b) (c + d)} \right] \right) / \right. \\
& \left. \left(a c \sqrt{\frac{(a + b) \cos \left[\frac{1}{2} (e + f x) \right]^2}{b + a \cos[e + f x]}} \sqrt{b + a \cos[e + f x]} \sqrt{\frac{b + a \cos[e + f x]}{a + b}} \sqrt{\frac{(a + b) (d + c \cos[e + f x])}{(c + d) (b + a \cos[e + f x])}} \right) - \right. \\
& \left. \frac{1}{a c} 2 (b c - a d) \left((b c + (a + b) d) \sqrt{\frac{(c + d) \cot \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \text{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \text{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \text{Csc}[e + f x] \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \text{Csc} \left[\frac{1}{2} (e+f x) \right]^2}}{b c - a d}}}{\sqrt{2}} \right], \right. \right. \\
& \left. \left. \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \text{Sin} \left[\frac{1}{2} (e + f x) \right]^4 \right) / \left((a + b) (c + d) \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]} \right) - (b c + a d) \\
& \left. \sqrt{\frac{(c + d) \cot \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \text{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \text{Csc} \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \right)
\end{aligned}$$

$$\left. \begin{aligned} & \text{Csc}[e + f x] \text{EllipticPi}\left[\frac{bc - ad}{(a + b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[e+fx])\text{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc - ad)}{(a + b)(c - d)}\right] \text{Sin}\left[\frac{1}{2}(e + fx)\right]^4 \right) / \\ & \left. \left. \left((a + b)c \sqrt{b + a \cos[e + fx]} \sqrt{d + c \cos[e + fx]} \right) + \frac{\sqrt{d + c \cos[e + fx]} \text{Sin}[e + fx]}{c \sqrt{b + a \cos[e + fx]}} \right) \right) \end{aligned} \right)$$

- **Problem 215: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{Sec}[e + f x])^{5/2}}{(c + d \text{Sec}[e + f x])^{7/2}} dx$$

Optimal (type 4, 1150 leaves, 8 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} (b^2 c^2 d (29 c^2 + 3 d^2) - a b c (35 c^4 + 34 c^2 d^2 - 5 d^4) + a^2 (58 c^4 d - 41 c^2 d^3 + 15 d^5)) \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \right. \\
& \left. \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \right. \\
& \left. \sqrt{a+b} \operatorname{Sec}[e + f x] \right) / \left(15 c^3 (c-d)^3 (c+d)^{5/2} (b c - a d) f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) + \\
& \left(2 \sqrt{a+b} (b^3 c^4 (5 c^2 + 24 c d + 3 d^2) - a b^2 c^3 (35 c^3 + 42 c^2 d + 21 c d^2 - 2 d^3) + a^2 b c^2 (45 c^4 + 48 c^3 d + c^2 d^2 - 8 c d^3 + 10 d^4) - \right. \\
& \left. a^3 d (60 c^5 - 2 c^4 d - 66 c^3 d^2 + 25 c^2 d^3 + 30 c d^4 - 15 d^5)) \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} \right. \\
& \left. (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b} \operatorname{Sec}[e + f x] \right) / \\
& \left(15 c^4 (c-d)^3 (c+d)^{5/2} (b c - a d) f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) - \\
& \left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \operatorname{Cos}[e + f x])}{(a+b) (d + c \operatorname{Cos}[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Cos}[e + f x])}{(a-b) (d + c \operatorname{Cos}[e + f x])}} (d + c \operatorname{Cos}[e + f x])^{3/2} \operatorname{Csc}[e + f x] \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e + f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e + f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b} \operatorname{Sec}[e + f x] \right) / \\
& \left(c^4 \sqrt{c+d} f \sqrt{b+a \operatorname{Cos}[e + f x]} \sqrt{c+d \operatorname{Sec}[e + f x]} \right) - \frac{2 d (b c - a d) (b + a \operatorname{Cos}[e + f x]) \sqrt{a+b} \operatorname{Sec}[e + f x] \operatorname{Sin}[e + f x]}{5 c (c^2 - d^2) f (d + c \operatorname{Cos}[e + f x])^2 \sqrt{c+d \operatorname{Sec}[e + f x]}} + \\
& \frac{2 (b c - a d) (5 b c^3 - 13 a c^2 d + 3 b c d^2 + 5 a d^3) \sqrt{a+b} \operatorname{Sec}[e + f x] \operatorname{Sin}[e + f x]}{15 c^2 (c^2 - d^2)^2 f (d + c \operatorname{Cos}[e + f x]) \sqrt{c+d \operatorname{Sec}[e + f x]}}
\end{aligned}$$

Result (type 4, 2314 leaves):

$$\frac{1}{f (b + a \operatorname{Cos}[e + f x])^2 (c + d \operatorname{Sec}[e + f x])^{7/2}}$$

$$\left(d + c \operatorname{Cos}[e + f x] \right)^4 \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x])^{5/2} \left(-\frac{2 (b^2 c^2 d \operatorname{Sin}[e + f x] - 2 a b c d^2 \operatorname{Sin}[e + f x] + a^2 d^3 \operatorname{Sin}[e + f x])}{5 c^2 (c^2 - d^2) (d + c \operatorname{Cos}[e + f x])^3} + \right.$$

$$\begin{aligned}
& \left(2 \left(5 b^2 c^4 \sin[e + f x] - 21 a b c^3 d \sin[e + f x] + 16 a^2 c^2 d^2 \sin[e + f x] + 3 b^2 c^2 d^2 \sin[e + f x] + \right. \right. \\
& \quad \left. \left. 5 a b c d^3 \sin[e + f x] - 8 a^2 d^4 \sin[e + f x] \right) \right) / \left(15 c^2 (c^2 - d^2)^2 (d + c \cos[e + f x])^2 \right) + \\
& \left(2 \left(35 a b c^5 \sin[e + f x] - 58 a^2 c^4 d \sin[e + f x] - 29 b^2 c^4 d \sin[e + f x] + 34 a b c^3 d^2 \sin[e + f x] + 41 a^2 c^2 d^3 \sin[e + f x] - \right. \right. \\
& \quad \left. \left. 3 b^2 c^2 d^3 \sin[e + f x] - 5 a b c d^4 \sin[e + f x] - 15 a^2 d^5 \sin[e + f x] \right) \right) / \left(15 c^2 (c^2 - d^2)^3 (d + c \cos[e + f x]) \right) \Bigg) + \\
& \frac{1}{15 c^2 (c - d)^3 (c + d)^3 f (b + a \cos[e + f x])^{5/2} (c + d \sec[e + f x])^{7/2}} (d + c \cos[e + f x])^{7/2} \sec[e + f x] \\
& (a + b \sec[e + f x])^{5/2} \left(\frac{1}{(a + b) (c + d) \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]}} \right. \\
& 4 (b c - a d) (10 a^2 b c^5 + 5 b^3 c^5 + 13 a^3 c^4 d - 48 a b^2 c^4 d + 15 a^2 b c^3 d^2 + 27 b^3 c^3 d^2 - 18 a^3 c^2 d^3 - 16 a b^2 c^2 d^3 + 7 a^2 b c d^4 + 5 a^3 d^5) \\
& \sqrt{\frac{(c + d) \cot\left[\frac{1}{2}(e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \\
& \operatorname{Csc}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}}}{\sqrt{2}}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)}\right] \sin\left[\frac{1}{2}(e + f x)\right]^4 + \\
& 4 (b c - a d) (15 a^3 c^5 - 35 a b^2 c^5 + 23 a^2 b c^4 d + 29 b^3 c^4 d + 13 a^3 c^3 d^2 - 5 a b^2 c^3 d^2 - 75 a^2 b c^2 d^3 + 3 b^3 c^2 d^3 + 4 a^3 c d^4 + 8 a b^2 c d^4 + 20 a^2 b d^5) \\
& \left(\left(\sqrt{\frac{(c + d) \cot\left[\frac{1}{2}(e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b) (d + c \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}}}{\sqrt{2}}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)}\right] \sin\left[\frac{1}{2}(e + f x)\right]^4 \right) \right) / \\
& \left((a + b) (c + d) \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]} \right) - \left(\sqrt{\frac{(c + d) \cot\left[\frac{1}{2}(e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{(a+b)(d+c\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \operatorname{Csc}[ex+f] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}}}{\sqrt{2}}}\right]\right], \\
& \left. \frac{2(bc-ad)}{(a+b)(c-d)} \left[\operatorname{Sin}\left[\frac{1}{2}(ex+f)\right]^4 \right] \right/ \left((a+b)c\sqrt{b+a\cos[ex+f]}\sqrt{d+c\cos[ex+f]} \right) \right) + \\
& 2(-35a^2bc^5 + 58a^3c^4d + 29ab^2c^4d - 34a^2bc^3d^2 - 41a^3c^2d^3 + 3ab^2c^2d^3 + 5a^2bcd^4 + 15a^3d^5) \\
& \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b)\cos\left[\frac{1}{2}(ex+f)\right] \sqrt{d+c\cos[ex+f]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}}\operatorname{Sin}\left[\frac{1}{2}(ex+f)\right]}{\sqrt{\frac{b+a\cos[ex+f]}{a+b}}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)} \right] \right) \right/ \\
& \left(a c \sqrt{\frac{(a+b)\cos\left[\frac{1}{2}(ex+f)\right]^2}{b+a\cos[ex+f]}} \sqrt{b+a\cos[ex+f]} \sqrt{\frac{b+a\cos[ex+f]}{a+b}} \sqrt{\frac{(a+b)(d+c\cos[ex+f])}{(c+d)(b+a\cos[ex+f])}} \right) - \\
& \frac{1}{ac} 2(bc-ad) \left((bc+(a+b)d) \sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}(ex+f)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \right. \\
& \left. \sqrt{-\frac{(a+b)(d+c\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \operatorname{Csc}[ex+f] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}}}{\sqrt{2}}}\right]\right], \right. \\
& \left. \frac{2(bc-ad)}{(a+b)(c-d)} \left[\operatorname{Sin}\left[\frac{1}{2}(ex+f)\right]^4 \right] \right) \right/ \left((a+b)(c+d)\sqrt{b+a\cos[ex+f]}\sqrt{d+c\cos[ex+f]} \right) - (bc+ad) \\
& \sqrt{\frac{(c+d)\operatorname{Cot}\left[\frac{1}{2}(ex+f)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c\cos[ex+f])\operatorname{Csc}\left[\frac{1}{2}(ex+f)\right]^2}{bc-ad}}
\end{aligned}$$

$$\left. \text{Csc}[e + f x] \text{EllipticPi}\left[\frac{bc - ad}{(a + b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c\cos[efx])\text{Csc}\left[\frac{1}{2}(efx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc - ad)}{(a + b)(c - d)}\right] \text{Sin}\left[\frac{1}{2}(efx)\right]^4 \right/$$

$$\left. \left. \left((a + b)c \sqrt{b + a \cos[efx]} \sqrt{d + c \cos[efx]} \right) + \frac{\sqrt{d + c \cos[efx]} \text{Sin}[efx]}{c \sqrt{b + a \cos[efx]}} \right) \right)$$

■ **Problem 216: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{Sec}[e + f x])^{5/2}}{(c + d \text{Sec}[e + f x])^{9/2}} dx$$

Optimal (type 4, 1428 leaves, 9 steps):

$$\left(2(a - b) \sqrt{a + b} (2b^3 c^3 d (133c^4 + 62c^2 d^2 - 3d^4) + 2a^2 b c d (406c^6 + 73c^4 d^2 + 132c^2 d^4 - 35d^6) - a b^2 c^2 (245c^6 + 852c^4 d^2 + 41c^2 d^4 + 14d^6) - \right.$$

$$a^3 (582c^6 d^2 - 485c^4 d^4 + 392c^2 d^6 - 105d^8) \left. \sqrt{-\frac{(bc - ad)(1 - \cos[efx])}{(a + b)(d + c \cos[efx])}} \sqrt{-\frac{(bc - ad)(1 + \cos[efx])}{(a - b)(d + c \cos[efx])}} \right.$$

$$\left. (d + c \cos[efx])^{3/2} \text{Csc}[efx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{b + a \cos[efx]}}{\sqrt{a + b} \sqrt{d + c \cos[efx]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \sqrt{a + b \text{Sec}[efx]} \right/$$

$$\left(105c^4 (c - d)^4 (c + d)^{7/2} (bc - ad)^2 f \sqrt{b + a \cos[efx]} \sqrt{c + d \text{Sec}[efx]} \right) +$$

$$\left(2 \sqrt{a + b} (b^3 c^4 (35c^4 + 231c^3 d + 67c^2 d^2 + 57c d^3 - 6d^4) - a b^2 c^3 (245c^5 + 413c^4 d + 439c^3 d^2 + 53c^2 d^3 - 12c d^4 + 14d^5) + \right.$$

$$a^2 b c^2 (315c^6 + 497c^5 d + 219c^4 d^2 - 73c^3 d^3 + 208c^2 d^4 + 56c d^5 - 70d^6) -$$

$$a^3 d (525c^7 + 57c^6 d - 699c^5 d^2 + 214c^4 d^3 + 672c^3 d^4 - 280c^2 d^5 - 210c d^6 + 105d^7) \left. \sqrt{-\frac{(bc - ad)(1 - \cos[efx])}{(a + b)(d + c \cos[efx])}} \sqrt{-\frac{(bc - ad)(1 + \cos[efx])}{(a - b)(d + c \cos[efx])}} (d + c \cos[efx])^{3/2} \text{Csc}[efx] \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{b + a \cos[efx]}}{\sqrt{a + b} \sqrt{d + c \cos[efx]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \sqrt{a + b \text{Sec}[efx]} \right/$$

$$\left(105c^5 (c - d)^4 (c + d)^{7/2} (bc - ad) f \sqrt{b + a \cos[efx]} \sqrt{c + d \text{Sec}[efx]} \right) -$$

$$\left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c-a d)(1-\operatorname{Cos}[e+f x])}{(a+b)(d+c \operatorname{Cos}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Cos}[e+f x])}{(a-b)(d+c \operatorname{Cos}[e+f x])}} (d+c \operatorname{Cos}[e+f x])^{3/2} \operatorname{Csc}[e+f x] \right. \\ \left. \operatorname{EllipticPi}\left[\frac{(a+b) c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \operatorname{Cos}[e+f x]}}{\sqrt{a+b} \sqrt{d+c \operatorname{Cos}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \sqrt{a+b \operatorname{Sec}[e+f x]}\right) / \\ \left(c^5 \sqrt{c+d} f \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{c+d \operatorname{Sec}[e+f x]} \right) + \frac{2 d^2 (b+a \operatorname{Cos}[e+f x])^2 \sqrt{a+b \operatorname{Sec}[e+f x]} \operatorname{Sin}[e+f x]}{7 c (c^2-d^2) f (d+c \operatorname{Cos}[e+f x])^3 \sqrt{c+d \operatorname{Sec}[e+f x]}} - \\ \frac{2 d (14 b c^3 - 19 a c^2 d - 2 b c d^2 + 7 a d^3) (b+a \operatorname{Cos}[e+f x]) \sqrt{a+b \operatorname{Sec}[e+f x]} \operatorname{Sin}[e+f x]}{35 c^2 (c^2-d^2)^2 f (d+c \operatorname{Cos}[e+f x])^2 \sqrt{c+d \operatorname{Sec}[e+f x]}} - \\ \left(2 (2 a b c d (91 c^4 - 2 c^2 d^2 + 7 d^4) - a^2 d^2 (162 c^4 - 101 c^2 d^2 + 35 d^4) - b^2 (35 c^6 + 67 c^4 d^2 - 6 c^2 d^4)) \sqrt{a+b \operatorname{Sec}[e+f x]} \operatorname{Sin}[e+f x] \right) / \\ \left(105 c^3 (c^2-d^2)^3 f (d+c \operatorname{Cos}[e+f x]) \sqrt{c+d \operatorname{Sec}[e+f x]} \right)$$

Result (type 4, 2949 leaves):

$$\frac{1}{f (b+a \operatorname{Cos}[e+f x])^2 (c+d \operatorname{Sec}[e+f x])^{9/2}} \\ (d+c \operatorname{Cos}[e+f x])^5 \operatorname{Sec}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^{5/2} \left(\frac{2 (b^2 c^2 d^2 \operatorname{Sin}[e+f x] - 2 a b c d^3 \operatorname{Sin}[e+f x] + a^2 d^4 \operatorname{Sin}[e+f x])}{7 c^3 (c^2-d^2) (d+c \operatorname{Cos}[e+f x])^4} + \right. \\ \left(2 (-14 b^2 c^4 d \operatorname{Sin}[e+f x] + 43 a b c^3 d^2 \operatorname{Sin}[e+f x] - 29 a^2 c^2 d^3 \operatorname{Sin}[e+f x] + 2 b^2 c^2 d^3 \operatorname{Sin}[e+f x] - \right. \\ \left. 19 a b c d^4 \operatorname{Sin}[e+f x] + 17 a^2 d^5 \operatorname{Sin}[e+f x]) \right) / (35 c^3 (c^2-d^2)^2 (d+c \operatorname{Cos}[e+f x])^3) + \\ \left(2 (35 b^2 c^6 \operatorname{Sin}[e+f x] - 224 a b c^5 d \operatorname{Sin}[e+f x] + 234 a^2 c^4 d^2 \operatorname{Sin}[e+f x] + 67 b^2 c^4 d^2 \operatorname{Sin}[e+f x] + 52 a b c^3 d^3 \operatorname{Sin}[e+f x] - \right. \\ \left. 209 a^2 c^2 d^4 \operatorname{Sin}[e+f x] - 6 b^2 c^2 d^4 \operatorname{Sin}[e+f x] - 20 a b c d^5 \operatorname{Sin}[e+f x] + 71 a^2 d^6 \operatorname{Sin}[e+f x]) \right) / \\ \left. \frac{1}{105 c^3 (b c-a d) (c^2-d^2)^4 (d+c \operatorname{Cos}[e+f x])} \right) + \\ \left. \frac{2 (245 a b^2 c^8 \operatorname{Sin}[e+f x] - 812 a^2 b c^7 d \operatorname{Sin}[e+f x] - 266 b^3 c^7 d \operatorname{Sin}[e+f x] + 582 a^3 c^6 d^2 \operatorname{Sin}[e+f x] + 852 a b^2 c^6 d^2 \operatorname{Sin}[e+f x] - \right. \\ \left. 146 a^2 b c^5 d^3 \operatorname{Sin}[e+f x] - 124 b^3 c^5 d^3 \operatorname{Sin}[e+f x] - 485 a^3 c^4 d^4 \operatorname{Sin}[e+f x] + 41 a b^2 c^4 d^4 \operatorname{Sin}[e+f x] - 264 a^2 b c^3 d^5 \operatorname{Sin}[e+f x] + \right. \\ \left. 6 b^3 c^3 d^5 \operatorname{Sin}[e+f x] + 392 a^3 c^2 d^6 \operatorname{Sin}[e+f x] + 14 a b^2 c^2 d^6 \operatorname{Sin}[e+f x] + 70 a^2 b c d^7 \operatorname{Sin}[e+f x] - 105 a^3 d^8 \operatorname{Sin}[e+f x]) \right) + \\ \left((d+c \operatorname{Cos}[e+f x])^{9/2} \operatorname{Sec}[e+f x]^2 (a+b \operatorname{Sec}[e+f x])^{5/2} \left(\frac{1}{(a+b)(c+d) \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{d+c \operatorname{Cos}[e+f x]}} \right. \right. \\ \left. \left. 4 (b c-a d) (-70 a^2 b^2 c^8 - 35 b^4 c^8 - 77 a^3 b c^7 d + 427 a b^3 c^7 d + 162 a^4 c^6 d^2 - 522 a^2 b^2 c^6 d^2 - 298 b^4 c^6 d^2 + 348 a^3 b c^5 d^3 + 666 a b^3 c^5 d^3 - \right. \right.$$

$$\begin{aligned}
& 263 a^4 c^4 d^4 - 586 a^2 b^2 c^4 d^4 - 51 b^4 c^4 d^4 + 127 a^3 b c^3 d^5 + 59 a b^3 c^3 d^5 + 136 a^4 c^2 d^6 + 26 a^2 b^2 c^2 d^6 - 14 a^3 b c d^7 - 35 a^4 d^8) \\
& \sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \\
& \operatorname{Csc}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}}{\sqrt{2}}}\right], \frac{2(b c-a d)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]^4 + 4(b c-a d) \\
& (-105 a^3 b c^8 + 245 a b^3 c^8 + 105 a^4 c^7 d - 567 a^2 b^2 c^7 d - 266 b^4 c^7 d + 190 a^3 b c^6 d^2 + 586 a b^3 c^6 d^2 + 162 a^4 c^5 d^3 + 706 a^2 b^2 c^5 d^3 - 124 b^4 c^5 d^3 - \\
& 1261 a^3 b c^4 d^4 - 83 a b^3 c^4 d^4 + 145 a^4 c^3 d^5 - 223 a^2 b^2 c^3 d^5 + 6 b^4 c^3 d^5 + 548 a^3 b c^2 d^6 + 20 a b^3 c^2 d^6 - 28 a^4 c d^7 + 84 a^2 b^2 c d^7 - 140 a^3 b d^8) \\
& \left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \right. \right. \\
& \left. \left. \operatorname{Csc}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}}{\sqrt{2}}}\right], \frac{2(b c-a d)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]^4 \right) / \right. \\
& \left. \left((a+b)(c+d) \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{d+c \operatorname{Cos}[e+f x]} \right) - \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \operatorname{Csc}[e+f x] \operatorname{EllipticPi}\left[\frac{b c-a d}{(a+b) c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}}{\sqrt{2}}}\right], \right. \right. \\
& \left. \left. \frac{2(b c-a d)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]^4 \right) / \left((a+b) c \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{d+c \operatorname{Cos}[e+f x]} \right) \right) + \\
& 2(245 a^2 b^2 c^8 - 812 a^3 b c^7 d - 266 a b^3 c^7 d + 582 a^4 c^6 d^2 + 852 a^2 b^2 c^6 d^2 - 146 a^3 b c^5 d^3 - 124 a b^3 c^5 d^3 - 485 a^4 c^4 d^4 + \\
& 41 a^2 b^2 c^4 d^4 - 264 a^3 b c^3 d^5 + 6 a b^3 c^3 d^5 + 392 a^4 c^2 d^6 + 14 a^2 b^2 c^2 d^6 + 70 a^3 b c d^7 - 105 a^4 d^8)
\end{aligned}$$

$$\left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos\left[\frac{1}{2}(e+fx)\right] \sqrt{d+c \cos[e+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin\left[\frac{1}{2}(e+fx)\right]}{\sqrt{\frac{b+a \cos[e+fx]}{a+b}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)}\right] \right) \right. \\
\left. \left(a c \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}(e+fx)\right]^2}{b+a \cos[e+fx]}} \sqrt{b+a \cos[e+fx]} \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c \cos[e+fx])}{(c+d)(b+a \cos[e+fx])}} \right) - \right. \\
\left. \frac{1}{ac} 2(bc-ad) \left((bc+(a+b)d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right. \\
\left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \right. \right. \\
\left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \right] \sin\left[\frac{1}{2}(e+fx)\right]^4 \right) \left/ \left((a+b)(c+d) \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) - (bc+ad) \right. \\
\left. \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \\
\left. \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)} \right] \sin\left[\frac{1}{2}(e+fx)\right]^4 \right) \left/ \right. \\
\left. \left((a+b)c \sqrt{b+a \cos[e+fx]} \sqrt{d+c \cos[e+fx]} \right) + \frac{\sqrt{d+c \cos[e+fx]} \sin[e+fx]}{c \sqrt{b+a \cos[e+fx]}} \right) \left. \right) \left. \right) \left/ \right.$$

$$(105 c^3 (c-d)^4 (c+d)^4 (-bc+ad) f (b+a \cos[e+fx])^{5/2} (c+d \sec[e+fx])^{9/2})$$

Problem 217: Unable to integrate problem.

$$\int \frac{(c + d \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 652 leaves, ? steps):

$$\begin{aligned}
& - \left(2 c (c + d) \operatorname{Cot}[e + f x] \operatorname{EllipticPi} \left[\frac{a (c + d)}{(a + b) c}, \operatorname{ArcSin} \left[\sqrt{\frac{(a + b) (c + d \operatorname{Sec}[e + f x])}{(c + d) (a + b \operatorname{Sec}[e + f x])}} \right], \frac{(a - b) (c + d)}{(a + b) (c - d)} \right] \sqrt{\frac{(b c - a d) (1 + \operatorname{Sec}[e + f x])}{(c - d) (a + b \operatorname{Sec}[e + f x])}} \right. \\
& \quad \left. (a + b \operatorname{Sec}[e + f x])^{3/2} \sqrt{\frac{(a + b) (b c - a d) (-1 + \operatorname{Sec}[e + f x]) (c + d \operatorname{Sec}[e + f x])}{(c + d)^2 (a + b \operatorname{Sec}[e + f x])^2}} \right) / \left(a (a + b) f \sqrt{c + d \operatorname{Sec}[e + f x]} \right) + \\
& \left(2 d (c + d) \operatorname{Cot}[e + f x] \operatorname{EllipticPi} \left[\frac{b (c + d)}{(a + b) d}, \operatorname{ArcSin} \left[\sqrt{\frac{(a + b) (c + d \operatorname{Sec}[e + f x])}{(c + d) (a + b \operatorname{Sec}[e + f x])}} \right], \frac{(a - b) (c + d)}{(a + b) (c - d)} \right] \sqrt{\frac{(b c - a d) (1 + \operatorname{Sec}[e + f x])}{(c - d) (a + b \operatorname{Sec}[e + f x])}} \right. \\
& \quad \left. (a + b \operatorname{Sec}[e + f x])^{3/2} \sqrt{-\frac{(a + b) (-b c + a d) (-1 + \operatorname{Sec}[e + f x]) (c + d \operatorname{Sec}[e + f x])}{(c + d)^2 (a + b \operatorname{Sec}[e + f x])^2}} \right) / \left(b (a + b) f \sqrt{c + d \operatorname{Sec}[e + f x]} \right) + \\
& \frac{1}{a b f \sqrt{\frac{(a + b) (c + d \operatorname{Sec}[e + f x])}{(c + d) (a + b \operatorname{Sec}[e + f x])}}} 2 (b c - a d) \operatorname{Cot}[e + f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{(a + b) (c + d \operatorname{Sec}[e + f x])}{(c + d) (a + b \operatorname{Sec}[e + f x])}} \right], \frac{(a - b) (c + d)}{(a + b) (c - d)} \right] \\
& \sqrt{\frac{(b c - a d) (-1 + \operatorname{Sec}[e + f x])}{(c + d) (a + b \operatorname{Sec}[e + f x])}} \sqrt{\frac{(b c - a d) (1 + \operatorname{Sec}[e + f x])}{(c - d) (a + b \operatorname{Sec}[e + f x])}} \sqrt{a + b \operatorname{Sec}[e + f x]} \sqrt{c + d \operatorname{Sec}[e + f x]}
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(c + d \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

- **Problem 218: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c + d \operatorname{Sec}[e + f x]}}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$-\frac{1}{a\sqrt{c+d}f} 2\sqrt{a+b} \operatorname{Cot}[e+fx] \operatorname{EllipticPi}\left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b\operatorname{Sec}[e+fx]}}{\sqrt{a+b}\sqrt{c+d\operatorname{Sec}[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+fx])}{(a+b)(c+d\operatorname{Sec}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(a-b)(c+d\operatorname{Sec}[e+fx])}} (c+d\operatorname{Sec}[e+fx])$$

Result (type 4, 554 leaves):

$$\frac{1}{f\sqrt{d+c\operatorname{Cos}[e+fx]}\sqrt{a+b\operatorname{Sec}[e+fx]}} 4(-bc+ad)\sqrt{b+a\operatorname{Cos}[e+fx]}\sqrt{c+d\operatorname{Sec}[e+fx]}$$

$$\left(\left(\sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{a-b}} \sqrt{-\frac{(c+d)(b+a\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}} \sqrt{\frac{(a+b)(d+c\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(c+d)(b+a\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}}}{\sqrt{2}}}\right], \frac{2(-bc+ad)}{(a-b)(c+d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) /$$

$$\left((a+b)\sqrt{b+a\operatorname{Cos}[e+fx]}\sqrt{d+c\operatorname{Cos}[e+fx]} \right) - \left(c \sqrt{\frac{(a+b)\operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{a-b}} \sqrt{-\frac{(c+d)(b+a\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}} \right.$$

$$\left. \sqrt{\frac{(a+b)(d+c\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{-bc+ad}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(c+d)(b+a\operatorname{Cos}[e+fx])\operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{-bc+ad}}}{\sqrt{2}}}\right], \right.$$

$$\left. \left. \frac{2(-bc+ad)}{(a-b)(c+d)} \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left(a(c+d)\sqrt{b+a\operatorname{Cos}[e+fx]}\sqrt{d+c\operatorname{Cos}[e+fx]} \right) \right)$$

■ **Problem 219: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{a+b\operatorname{Sec}[e+fx]}\sqrt{c+d\operatorname{Sec}[e+fx]}} dx$$

Optimal (type 4, 398 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{a\sqrt{a+b}cf} 2\sqrt{c+d} \operatorname{Cot}[e+fx] \operatorname{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{c+d}\operatorname{Sec}[e+fx]}{\sqrt{c+d}\sqrt{a+b}\operatorname{Sec}[e+fx]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(bc-ad)(1-\operatorname{Sec}[e+fx])}{(c+d)(a+b\operatorname{Sec}[e+fx])}} \sqrt{\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(c-d)(a+b\operatorname{Sec}[e+fx])}} (a+b\operatorname{Sec}[e+fx]) - \\
& \frac{1}{a\sqrt{c+d}(bc-ad)f} 2b\sqrt{a+b} \operatorname{Cot}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{a+b}\operatorname{Sec}[e+fx]}{\sqrt{a+b}\sqrt{c+d}\operatorname{Sec}[e+fx]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\
& \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+fx])}{(a+b)(c+d\operatorname{Sec}[e+fx])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+fx])}{(a-b)(c+d\operatorname{Sec}[e+fx])}} (c+d\operatorname{Sec}[e+fx])
\end{aligned}$$

Result (type 4, 249 leaves):

$$\begin{aligned}
& \left(4i \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\frac{b+a\operatorname{Cos}[e+fx]}{(a+b)(1+\operatorname{Cos}[e+fx])}} \right. \\
& \left. \sqrt{\frac{d+c\operatorname{Cos}[e+fx]}{(c+d)(1+\operatorname{Cos}[e+fx])}} \left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] - \right. \right. \\
& \left. \left. 2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \right) \right. \\
& \left. \operatorname{Sec}[e+fx] \right) / \left(\sqrt{\frac{-a+b}{a+b}} f \sqrt{a+b\operatorname{Sec}[e+fx]} \sqrt{c+d\operatorname{Sec}[e+fx]} \right)
\end{aligned}$$

■ **Problem 220: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a+b\operatorname{Sec}[e+fx]} (c+d\operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 4, 622 leaves, 6 steps):

$$\begin{aligned}
& - \left(2 (a-b) \sqrt{a+b} d^2 \operatorname{Cot}[e+f x] \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \right. \\
& \quad \left. \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])} \right) / \left(c(c-d) \sqrt{c+d} (bc-ad)^2 f \right) - \\
& \left(2 \sqrt{a+b} (2c-d) d \operatorname{Cot}[e+f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}} \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])} \right) / \left(c^2 (c-d) \sqrt{c+d} (bc-ad) f \right) - \\
& \frac{1}{a c^2 \sqrt{c+d} f} 2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticPi} \left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \\
& \quad \left. \sqrt{\frac{(bc-ad)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(bc-ad)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])} \right)
\end{aligned}$$

Result (type 4, 1731 leaves):

$$\begin{aligned}
& \frac{1}{(c-d)(c+d)(bc-ad)f \sqrt{a+b \operatorname{Sec}[e+f x]} (c+d \operatorname{Sec}[e+f x])^{3/2}} \sqrt{b+a \operatorname{Cos}[e+f x]} (d+c \operatorname{Cos}[e+f x])^{3/2} \\
& \operatorname{Sec}[e+f x]^2 \left(- \left(4 b c d (bc-ad) \sqrt{\frac{(c+d) \operatorname{Cot}[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}[\frac{1}{2}(e+f x)]^2}{bc-ad}} \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}[\frac{1}{2}(e+f x)]^2}{bc-ad}} \operatorname{Csc}[e+f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}[\frac{1}{2}(e+f x)]^2}{bc-ad}}}{\sqrt{2}}} \right], \frac{2(bc-ad)}{(a+b)(c-d)} \right] \right. \right. \\
& \quad \left. \left. \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) / \left((a+b)(c+d) \sqrt{b+a \operatorname{Cos}[e+f x]} \sqrt{d+c \operatorname{Cos}[e+f x]} \right) + 4(bc-ad)(bc^2 - acd - 2bd^2)
\end{aligned}$$

$$\left(\left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right.$$

$$\left. \left. \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right], \frac{2(bc-ad)}{(a+b)(c-d)}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \right.$$

$$\left. \left((a+b)(c+d) \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{d+c \operatorname{Cos}[e+fx]} \right) - \left(\sqrt{\frac{(c+d) \operatorname{Cot}\left[\frac{1}{2}(e+fx)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \right. \right.$$

$$\left. \left. \sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}} \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \operatorname{Cos}[e+fx]) \operatorname{Csc}\left[\frac{1}{2}(e+fx)\right]^2}{bc-ad}}}{\sqrt{2}}}\right] \right), \right. \right.$$

$$\left. \left. \frac{2(bc-ad)}{(a+b)(c-d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) / \left((a+b)c \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{d+c \operatorname{Cos}[e+fx]} \right) -$$

$$2ad^2 \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] \sqrt{d+c \operatorname{Cos}[e+fx]} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]}}{\sqrt{\frac{b+a \operatorname{Cos}[e+fx]}{a+b}}}\right], \frac{2(bc-ad)}{(-a+b)(c+d)} \right] \right) / \right.$$

$$\left. \left(ac \sqrt{\frac{(a+b) \operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]^2}{b+a \operatorname{Cos}[e+fx]}} \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{\frac{b+a \operatorname{Cos}[e+fx]}{a+b}} \sqrt{\frac{(a+b)(d+c \operatorname{Cos}[e+fx])}{(c+d)(b+a \operatorname{Cos}[e+fx])}} \right) - \right.$$

$$\begin{aligned}
& \frac{1}{a c} 2 (b c - a d) \left((b c + (a + b) d) \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d)(b + a \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \right. \\
& \sqrt{-\frac{(a + b)(d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \operatorname{Csc}[e + f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b)(d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}}}{\sqrt{2}}}\right], \right. \\
& \left. \left. \frac{2(b c - a d)}{(a + b)(c - d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^4 \right) / \left((a + b)(c + d) \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) - (b c + a d) \\
& \sqrt{\frac{(c + d) \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2}{c - d}} \sqrt{\frac{(c + d)(b + a \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \sqrt{-\frac{(a + b)(d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}} \\
& \operatorname{Csc}[e + f x] \operatorname{EllipticPi}\left[\frac{b c - a d}{(a + b) c}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a + b)(d + c \operatorname{Cos}[e + f x]) \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{b c - a d}}}{\sqrt{2}}}\right], \frac{2(b c - a d)}{(a + b)(c - d)} \right] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^4 \right) / \\
& \left((a + b) c \sqrt{b + a \operatorname{Cos}[e + f x]} \sqrt{d + c \operatorname{Cos}[e + f x]} \right) + \frac{\sqrt{d + c \operatorname{Cos}[e + f x]} \operatorname{Sin}[e + f x]}{c \sqrt{b + a \operatorname{Cos}[e + f x]}} \Bigg) + \\
& \frac{2 d^2 (b + a \operatorname{Cos}[e + f x]) (d + c \operatorname{Cos}[e + f x]) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{(-b c + a d) (-c^2 + d^2) f \sqrt{a + b \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])^{3/2}}
\end{aligned}$$

■ **Problem 222: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{1/3}}{(c + d \operatorname{Sec}[e + f x])^{4/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{Sec}[e + f x])^{1/3}}{(c + d \operatorname{Sec}[e + f x])^{4/3}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 223: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{1/3}}{(c + d \operatorname{Sec}[e + f x])^{7/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{Sec}[e + f x])^{1/3}}{(c + d \operatorname{Sec}[e + f x])^{7/3}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 225: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{2/3}}{(c + d \operatorname{Sec}[e + f x])^{5/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{Sec}[e + f x])^{2/3}}{(c + d \operatorname{Sec}[e + f x])^{5/3}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 226: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{2/3}}{(c + d \operatorname{Sec}[e + f x])^{8/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps) :

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{Sec}[e + f x])^{2/3}}{(c + d \operatorname{Sec}[e + f x])^{8/3}}, x\right]$$

Result (type 1, 1 leaves) :

???

■ **Problem 227: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{4/3}}{(c + d \operatorname{Sec}[e + f x])^{4/3}} dx$$

Optimal (type 9, 88 leaves, 1 step) :

$$\frac{(d + c \cos[e + f x])^{4/3} (a + b \sec[e + f x])^{4/3} \text{Unintegrable}\left[\frac{(b+a \cos[e+f x])^{4/3}}{(d+c \cos[e+f x])^{4/3}}, x\right]}{(b + a \cos[e + f x])^{4/3} (c + d \sec[e + f x])^{4/3}}$$

Result (type 1, 1 leaves):

???

■ **Problem 228: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \sec[e + f x])^{4/3}}{(c + d \sec[e + f x])^{7/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \sec[e + f x])^{4/3}}{(c + d \sec[e + f x])^{7/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 229: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \sec[e + f x])^{4/3}}{(c + d \sec[e + f x])^{10/3}} dx$$

Optimal (type 9, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \sec[e + f x])^{4/3}}{(c + d \sec[e + f x])^{10/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 230: Result more than twice size of optimal antiderivative.**

$$\int (c (d \sec[e + f x])^p)^n (a + a \sec[e + f x])^m dx$$

Optimal (type 6, 106 leaves, 4 steps):

$$-\frac{1}{f n p \sqrt{1 - \sec[e + f x]}} \text{AppellF1}\left[n p, \frac{1}{2}, \frac{1}{2} - m, 1 + n p, \sec[e + f x], -\sec[e + f x]\right] (c (d \sec[e + f x])^p)^n (1 + \sec[e + f x])^{-\frac{1}{2}-m} (a + a \sec[e + f x])^m \tan[e + f x]$$

Result (type 6, 2425 leaves):

$$\left(3 \times 2^{1+m} \text{AppellF1}\left[\frac{1}{2}, m + n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \left(\sec\left[\frac{1}{2}(e + f x)\right]^2\right)^{-1+n p}\right)$$

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+np} (c(d \sec[e+fx])^p)^n (a(1+\sec[e+fx]))^m \tan\left[\frac{1}{2}(e+fx)\right] \Big/ \\
& \left(f \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (m+np) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \left(\left(3 \times 2^m \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{np} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+np} \right) \Big/ \right. \\
& \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (m+np) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(3 \times 2^{1+m} (-1+np) \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+np} \right. \\
& \quad \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+np} \tan\left[\frac{1}{2}(e+fx)\right]^2 \Big/ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (m+np) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\
& \left(3 \times 2^{1+m} \left(\sec\left[\frac{1}{2}(e+fx)\right]^2 \right)^{-1+np} \left(\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx] \right)^{m+np} \tan\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left(-\frac{1}{3} (1-np) \operatorname{AppellF1}\left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{1}{3} (m+np) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right] \right) \Big/ \Big/ \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \\
& \quad \left. \left. (m+np) \operatorname{AppellF1}\left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan\left[\frac{1}{2}(e+fx)\right]^2, -\tan\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) -
\end{aligned}$$

$$2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, m + np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, -\tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\ \left. (m + np) \operatorname{AppellF1} \left[\frac{3}{2}, 1 + m + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, -\tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) \right)$$

■ **Problem 231: Unable to integrate problem.**

$$\int (c (d \operatorname{Sec}[e + fx])^p)^n (a + a \operatorname{Sec}[e + fx])^3 dx$$

Optimal (type 5, 275 leaves, 8 steps):

$$\frac{a^3 (7 + 4np) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{np}{2}, \frac{1}{2} (2 - np), \cos[e + fx]^2 \right] (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx]}{fn p (2 + np) \sqrt{\sin[e + fx]^2}} - \\ \left(a^3 (1 + 4np) \cos[e + fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 - np), \frac{1}{2} (3 - np), \cos[e + fx]^2 \right] (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx] \right) / \\ \left(f (1 - n^2 p^2) \sqrt{\sin[e + fx]^2} \right) + \frac{a^3 (5 + 2np) (c (d \operatorname{Sec}[e + fx])^p)^n \tan[e + fx]}{f (1 + np) (2 + np)} + \frac{(c (d \operatorname{Sec}[e + fx])^p)^n (a^3 + a^3 \operatorname{Sec}[e + fx]) \tan[e + fx]}{f (2 + np)}$$

Result (type 8, 29 leaves):

$$\int (c (d \operatorname{Sec}[e + fx])^p)^n (a + a \operatorname{Sec}[e + fx])^3 dx$$

■ **Problem 232: Unable to integrate problem.**

$$\int (c (d \operatorname{Sec}[e + fx])^p)^n (a + a \operatorname{Sec}[e + fx])^2 dx$$

Optimal (type 5, 205 leaves, 7 steps):

$$\frac{2 a^2 \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{np}{2}, \frac{1}{2} (2 - np), \cos[e + fx]^2 \right] (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx]}{fn p \sqrt{\sin[e + fx]^2}} - \\ \left(a^2 (1 + 2np) \cos[e + fx] \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 - np), \frac{1}{2} (3 - np), \cos[e + fx]^2 \right] (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx] \right) / \\ \left(f (1 - n^2 p^2) \sqrt{\sin[e + fx]^2} \right) + \frac{a^2 (c (d \operatorname{Sec}[e + fx])^p)^n \tan[e + fx]}{f (1 + np)}$$

Result (type 8, 29 leaves):

$$\int (c (d \operatorname{Sec}[e + fx])^p)^n (a + a \operatorname{Sec}[e + fx])^2 dx$$

■ **Problem 233: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (c (d \operatorname{Sec}[e + fx])^p)^n (a + a \operatorname{Sec}[e + fx]) dx$$

Optimal (type 5, 156 leaves, 6 steps):

$$\frac{a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{np}{2}, \frac{1}{2}(2-np), \cos[e+fx]^2\right] (c(d \operatorname{Sec}[e+fx])^p)^n \operatorname{Sin}[e+fx]}{f np \sqrt{\operatorname{Sin}[e+fx]^2}} - \left(a \operatorname{Cos}[e+fx] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(1-np), \frac{1}{2}(3-np), \cos[e+fx]^2\right] (c(d \operatorname{Sec}[e+fx])^p)^n \operatorname{Sin}[e+fx] \right) / \left(f(1-np) \sqrt{\operatorname{Sin}[e+fx]^2} \right)$$

Result (type 6, 4295 leaves):

$$\begin{aligned} & - \left(a \operatorname{Sec}[e+fx]^{np} (c(d \operatorname{Sec}[e+fx])^p)^n (1 + \operatorname{Sec}[e+fx]) \right. \\ & \quad \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, np, 1-np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}[e+fx] \right) / \right. \\ & \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, np, 1-np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, np, 2-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + np \operatorname{AppellF1}\left[\frac{3}{2}, 1+np, 1-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\ & \quad \operatorname{AppellF1}\left[\frac{1}{2}, 1+np, -np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] / \left(\operatorname{AppellF1}\left[\frac{1}{2}, 1+np, -np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\ & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \frac{2}{3} \left(np \operatorname{AppellF1}\left[\frac{3}{2}, 1+np, 1-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. (1+np) \operatorname{AppellF1}\left[\frac{3}{2}, 2+np, -np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\ & \left(f \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) \left(\frac{1}{(-1 + \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2)^2} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sec}[e+fx]^{np} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \right. \\ & \quad \left(\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, np, 1-np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \operatorname{Cos}[e+fx] \right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, np, 1-np, \frac{3}{2}, \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + 2 \left((-1+np) \operatorname{AppellF1}\left[\frac{3}{2}, np, 2-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\ & \quad \left. \left. np \operatorname{AppellF1}\left[\frac{3}{2}, 1+np, 1-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right) + \\ & \quad \operatorname{AppellF1}\left[\frac{1}{2}, 1+np, -np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] / \left(\operatorname{AppellF1}\left[\frac{1}{2}, 1+np, -np, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, \right. \right. \\ & \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + \frac{2}{3} \left(np \operatorname{AppellF1}\left[\frac{3}{2}, 1+np, 1-np, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2\right] + (1+np) \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{3}{2}, 2 + n p, -n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \tan\left[\frac{1}{2}(e + f x)\right]^2 \right) - \frac{1}{2(-1 + \tan\left[\frac{1}{2}(e + f x)\right]^2)} \\
& \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x]^{n p} \left(\left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \cos[e + f x] \right) / \right. \\
& \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + \\
& 2 \left((-1 + n p) \text{AppellF1}\left[\frac{3}{2}, n p, 2 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + \\
& n p \text{AppellF1}\left[\frac{3}{2}, 1 + n p, 1 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \tan\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\
& \text{AppellF1}\left[\frac{1}{2}, 1 + n p, -n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] / \left(\text{AppellF1}\left[\frac{1}{2}, 1 + n p, -n p, \frac{3}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + \frac{2}{3} \left(n p \text{AppellF1}\left[\frac{3}{2}, 1 + n p, 1 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \right) + \\
& \left. (1 + n p) \text{AppellF1}\left[\frac{3}{2}, 2 + n p, -n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \tan\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) - \\
& \frac{1}{-1 + \tan\left[\frac{1}{2}(e + f x)\right]^2} n p \text{Sec}[e + f x]^{1 + n p} \sin[e + f x] \tan\left[\frac{1}{2}(e + f x)\right]^2 \left(\left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \cos[e + f x] \right) / \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + \right. \\
& 2 \left((-1 + n p) \text{AppellF1}\left[\frac{3}{2}, n p, 2 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + \\
& n p \text{AppellF1}\left[\frac{3}{2}, 1 + n p, 1 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \tan\left[\frac{1}{2}(e + f x)\right]^2 \right) + \\
& \text{AppellF1}\left[\frac{1}{2}, 1 + n p, -n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] / \left(\text{AppellF1}\left[\frac{1}{2}, 1 + n p, -n p, \frac{3}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] + \frac{2}{3} \left(n p \text{AppellF1}\left[\frac{3}{2}, 1 + n p, 1 - n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) \right) + \\
& \left. (1 + n p) \text{AppellF1}\left[\frac{3}{2}, 2 + n p, -n p, \frac{5}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \tan\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) - \frac{1}{-1 + \tan\left[\frac{1}{2}(e + f x)\right]^2} \\
& \text{Sec}[e + f x]^{n p} \tan\left[\frac{1}{2}(e + f x)\right] \left(- \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \sin[e + f x] \right) / \right. \\
& \left. \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1 - n p, \frac{3}{2}, \tan\left[\frac{1}{2}(e + f x)\right]^2, -\tan\left[\frac{1}{2}(e + f x)\right]^2\right] \right) + 2 \right)
\end{aligned}$$

$$\int \frac{(c (d \operatorname{Sec}[e + f x])^p)^n}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 5, 208 leaves, 7 steps):

$$\frac{(c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Sin}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])} - \frac{1}{a f \sqrt{\operatorname{Sin}[e + f x]^2}}$$

$$\operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \operatorname{Cos}[e + f x]^2\right] (c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Sin}[e + f x] +$$

$$\left((1 - n p) \operatorname{Cos}[e + f x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (2 - n p), \frac{1}{2} (4 - n p), \operatorname{Cos}[e + f x]^2\right] (c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Sin}[e + f x] \right) /$$

$$(a f (2 - n p) \sqrt{\operatorname{Sin}[e + f x]^2})$$

Result (type 8, 29 leaves):

$$\int \frac{(c (d \operatorname{Sec}[e + f x])^p)^n}{a + a \operatorname{Sec}[e + f x]} dx$$

■ **Problem 235: Unable to integrate problem.**

$$\int \frac{(c (d \operatorname{Sec}[e + f x])^p)^n}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 5, 248 leaves, 8 steps):

$$\frac{2 (2 - n p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n p}{2}, \frac{1}{2} (2 - n p), \operatorname{Cos}[e + f x]^2\right] (c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Sin}[e + f x]}{3 a^2 f \sqrt{\operatorname{Sin}[e + f x]^2}} - \frac{1}{3 a^2 f \sqrt{\operatorname{Sin}[e + f x]^2}}$$

$$(3 - 2 n p) \operatorname{Cos}[e + f x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \operatorname{Cos}[e + f x]^2\right] (c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Sin}[e + f x] -$$

$$\frac{2 (2 - n p) (c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Tan}[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])} - \frac{(c (d \operatorname{Sec}[e + f x])^p)^n \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2}$$

Result (type 8, 29 leaves):

$$\int \frac{(c (d \operatorname{Sec}[e + f x])^p)^n}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

■ **Problem 240: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c (d \operatorname{Sec}[e + f x])^p)^n}{a + b \operatorname{Sec}[e + f x]} dx$$

Optimal (type 6, 206 leaves, 7 steps):

$$-\frac{1}{(a^2 - b^2) f} b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{np}{2}, 1, \frac{3}{2}, \sin[e + fx]^2, \frac{a^2 \sin[e + fx]^2}{a^2 - b^2}\right] (\cos[e + fx]^2)^{\frac{np}{2}} (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx] + \frac{1}{(a^2 - b^2) f}$$

$$a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1 + np), 1, \frac{3}{2}, \sin[e + fx]^2, \frac{a^2 \sin[e + fx]^2}{a^2 - b^2}\right] \cos[e + fx] (\cos[e + fx]^2)^{\frac{1}{2}(-1 + np)} (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx]$$

Result (type 6, 5411 leaves): Display of huge result suppressed!

■ **Problem 241: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c (d \operatorname{Sec}[e + fx])^p)^n}{(a + b \operatorname{Sec}[e + fx])^2} dx$$

Optimal (type 6, 322 leaves, 10 steps):

$$-\frac{1}{(a^2 - b^2)^2 f} 2 a b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-2 + np), 2, \frac{3}{2}, \sin[e + fx]^2, \frac{a^2 \sin[e + fx]^2}{a^2 - b^2}\right] (\cos[e + fx]^2)^{\frac{np}{2}} (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx] +$$

$$\frac{1}{(a^2 - b^2)^2 f} a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-3 + np), 2, \frac{3}{2}, \sin[e + fx]^2, \frac{a^2 \sin[e + fx]^2}{a^2 - b^2}\right]$$

$$\cos[e + fx] (\cos[e + fx]^2)^{\frac{1}{2}(-1 + np)} (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx] + \frac{1}{(a^2 - b^2)^2 f}$$

$$b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1 + np), 2, \frac{3}{2}, \sin[e + fx]^2, \frac{a^2 \sin[e + fx]^2}{a^2 - b^2}\right] \cos[e + fx] (\cos[e + fx]^2)^{\frac{1}{2}(-1 + np)} (c (d \operatorname{Sec}[e + fx])^p)^n \sin[e + fx]$$

Result (type 6, 10678 leaves):

$$\left((c (d \operatorname{Sec}[e + fx])^p)^n \right.$$

$$\left. \left(-\frac{2 b \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - \frac{np}{2}, \frac{3}{2}, -\tan[e + fx]^2\right] \tan[e + fx]}{a^3} + \frac{\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 - \frac{np}{2}, \frac{3}{2}, -\tan[e + fx]^2\right] \tan[e + fx]}{a^2} - \right.$$

$$\left. \left(6 b^3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\tan[e + fx]^2, \frac{b^2 \tan[e + fx]^2}{a^2 - b^2}\right] \tan[e + fx] (1 + \tan[e + fx]^2)^{\frac{1}{2}(1 + np)} \right) \right) /$$

$$\left(a \left(3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\tan[e + fx]^2, \frac{b^2 \tan[e + fx]^2}{a^2 - b^2}\right] + \right.$$

$$\left. \left(4 b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 3, \frac{5}{2}, -\tan[e + fx]^2, \frac{b^2 \tan[e + fx]^2}{a^2 - b^2}\right] + \right.$$

$$\left. \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\tan[e + fx]^2, \frac{b^2 \tan[e + fx]^2}{a^2 - b^2}\right] \right) \tan[e + fx]^2 \right) (a^2 - b^2 (1 + \tan[e + fx]^2))^2 \right) +$$

$$\begin{aligned}
& \left(6 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{np}{2}} \right) / \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (a^2 - b^2 (1 + \operatorname{Tan}[e + f x]^2))^2 \Big) + \\
& \left(6 b^3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1}{2}(1+np)} \right) / \\
& \left(a^3 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + f x]^2)) \Big) - \\
& \left(3 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x] (1 + \operatorname{Tan}[e + f x]^2)^{\frac{np}{2}} \right) / \\
& \left(a^2 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + f x]^2)) \Big) \Big) / \\
& \left(f (a + b \operatorname{Sec}[e + f x])^2 \left(-\frac{2 b \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - \frac{np}{2}, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2}{a^3} + \right. \right. \\
& \quad \left. \frac{\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 - \frac{np}{2}, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2 \right] \operatorname{Sec}[e + f x]^2}{a^2} - \right. \\
& \quad \left. \left(24 b^5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2 (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1}{2}(1+np)} \right) / \right. \\
& \quad \left. \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + f x]^2 \right) (a^2 - b^2 (1 + \operatorname{Tan}[e + f x]^2))^3 \Big) + \\
& \left(24 b^4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2 (1 + \operatorname{Tan}[e + f x]^2)^{\frac{np}{2}} \right) / \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (a^2 - b^2 (1 + \operatorname{Tan}[e + f x]^2))^3 \Big) - \\
& \left(6 b^3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1}{2}(1+np)} \right) / \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (a^2 - b^2 (1 + \operatorname{Tan}[e + f x]^2))^2 \Big) - \\
& \left(6 b^3 (a^2 - b^2) \operatorname{Tan}[e + f x] \left(\frac{4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 (a^2 - b^2)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \left(-\frac{1}{2} - \frac{np}{2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) (1 + \operatorname{Tan}[e + f x]^2)^{\frac{1}{2}(1+np)} \right) / \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 3, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \right) \operatorname{Tan}[e + f x]^2 \right) (a^2 - b^2 (1 + \operatorname{Tan}[e + f x]^2))^2 \Big) - \\
& \left(6 b^3 (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, \frac{b^2 \operatorname{Tan}[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& (1 + \tan[e + f x]^2)^{-1 + \frac{1}{2}(1+n p)} \Big/ \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (1 + n p) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \Big) + \\
& \left(6 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right) \Big/ \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right) \tan[e + f x]^2 (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \Big) + \\
& \left(6 b^2 (a^2 - b^2) \tan[e + f x] \left(\frac{4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]}{3 (a^2 - b^2)} + \right. \right. \\
& \left. \left. \frac{1}{3} n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right) \Big/ \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right) \tan[e + f x]^2 (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \Big) + \\
& \left(6 b^2 (a^2 - b^2) n p \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-1 + \frac{n p}{2}} \right) \Big/ \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right) \tan[e + f x]^2 (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + fx]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + fx]^2)) \Bigg) + \\
& \left(6 b^3 (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx]^2 \right. \\
& \left. (1 + \operatorname{Tan}[e + fx]^2)^{-1 + \frac{1}{2}(1 + np)} \right) / \left(a^3 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + fx]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + fx]^2)) \right) \Bigg) - \\
& \left(3 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + fx]^2 (1 + \operatorname{Tan}[e + fx]^2)^{\frac{np}{2}} \right) / \\
& \left(a^2 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + fx]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + fx]^2)) \right) \Bigg) - \\
& \left(3 b^2 (a^2 - b^2) \operatorname{Tan}[e + fx] \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx]}{3 (a^2 - b^2)} + \right. \right. \\
& \left. \left. \frac{1}{3} np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx] \right) (1 + \operatorname{Tan}[e + fx]^2)^{\frac{np}{2}} \right) / \\
& \left(a^2 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Tan}[e + fx]^2 \right) (-a^2 + b^2 (1 + \operatorname{Tan}[e + fx]^2)) \right) \Bigg) - \\
& \left(3 b^2 (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx]^2 (1 + \operatorname{Tan}[e + fx]^2)^{-1 + \frac{np}{2}} \right) / \\
& \left(a^2 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\operatorname{Tan}[e + fx]^2, \frac{b^2 \operatorname{Tan}[e + fx]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (1 + \tan[e + f x]^2)^{\frac{np}{2}} \left(2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \quad 3 (a^2 - b^2) \left(\frac{2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x]}{3 (a^2 - b^2)} + \right. \\
& \quad \left. \frac{1}{3} np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) + \\
& \quad \tan[e + f x]^2 \left(2 b^2 \left(\frac{12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{np}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x]}{5 (a^2 - b^2)} + \right. \right. \\
& \quad \left. \left. \frac{3}{5} np \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{np}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) + \right. \\
& \quad \left. (a^2 - b^2) np \left(\frac{6 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{np}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x]}{5 (a^2 - b^2)} - \right. \right. \\
& \quad \left. \left. \frac{6}{5} \left(1 - \frac{np}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, 2 - \frac{np}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) / \\
& \left(a^2 \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right)^2 \\
& \left. (-a^2 + b^2 (1 + \tan[e + f x]^2)) \right) - \left(6 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \tan[e + f x] \right. \\
& \quad \left. (1 + \tan[e + f x]^2)^{\frac{np}{2}} \left(2 \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2 - b^2) np \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \sec[e + f x]^2 \tan[e + f x] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a^2 - b^2) \left(\frac{4 b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]}{3 (a^2 - b^2)} + \right. \\
& \left. \frac{1}{3} np \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \\
& \tan[e + f x]^2 \left(4 b^2 \left(\frac{18 b^2 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{np}{2}, 4, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x]}{5 (a^2 - b^2)} + \right. \right. \\
& \left. \left. \frac{3}{5} np \operatorname{AppellF1}\left[\frac{5}{2}, 1 - \frac{np}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) + \right. \\
& \left. (a^2 - b^2) np \left(\frac{1}{5 (a^2 - b^2)} 12 b^2 \operatorname{AppellF1}\left[\frac{5}{2}, 1 - \frac{np}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{6}{5} \left(1 - \frac{np}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - \frac{np}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \operatorname{Sec}[e + f x]^2 \tan[e + f x] \right) \right) \right) / \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \left(4 b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) np \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, \right. \right. \\
& \left. \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \right) \tan[e + f x]^2 (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \right) \right)
\end{aligned}$$

Test results for the 286 problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n.m"

- Problem 1: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^4 dx$$

Optimal (type 3, 105 leaves, 12 steps):

$$\frac{7 a c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{8 f} - \frac{a c^4 \operatorname{Sec}[e + f x] \tan[e + f x]}{8 f} - \frac{3 a c^4 \operatorname{Sec}[e + f x]^3 \tan[e + f x]}{4 f} + \frac{4 a c^4 \tan[e + f x]^3}{3 f} + \frac{a c^4 \tan[e + f x]^5}{5 f}$$

Result (type 3, 499 leaves):

$$\begin{aligned}
& - \frac{1}{3840 f} a c^4 \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^5 \\
& \left(525 \operatorname{Cos}[2 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 525 \operatorname{Cos}[4 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\
& 105 \operatorname{Cos}[4 e + 5 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 105 \operatorname{Cos}[6 e + 5 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\
& 1050 \operatorname{Cos}[f x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) + \\
& 1050 \operatorname{Cos}[2 e + f x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) - \\
& 525 \operatorname{Cos}[2 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 525 \operatorname{Cos}[4 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\
& 105 \operatorname{Cos}[4 e + 5 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 105 \operatorname{Cos}[6 e + 5 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\
& 800 \operatorname{Sin}[f x] - 1920 \operatorname{Sin}[2 e + f x] + 780 \operatorname{Sin}[e + 2 f x] + 780 \operatorname{Sin}[3 e + 2 f x] + 640 \operatorname{Sin}[2 e + 3 f x] - \\
& \left. 720 \operatorname{Sin}[4 e + 3 f x] + 30 \operatorname{Sin}[3 e + 4 f x] + 30 \operatorname{Sin}[5 e + 4 f x] + 272 \operatorname{Sin}[4 e + 5 f x] \right)
\end{aligned}$$

■ **Problem 2: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^3 dx$$

Optimal (type 3, 86 leaves, 9 steps):

$$\frac{5 a c^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{8 f} - \frac{3 a c^3 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{8 f} - \frac{a c^3 \operatorname{Sec}[e + f x]^3 \operatorname{Tan}[e + f x]}{4 f} + \frac{2 a c^3 \operatorname{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 887 leaves):

$$\begin{aligned}
& a \left(\frac{5 \operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \operatorname{Sec}[e + f x])^3}{64 f} - \right. \\
& \frac{5 \operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \operatorname{Sec}[e + f x])^3}{64 f} + \\
& \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3}{128 f \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^4} - \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{24 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^3} + \\
& \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \left(\operatorname{Cos}\left[\frac{e}{2}\right] - 17 \operatorname{Sin}\left[\frac{e}{2}\right]\right)}{384 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^2} + \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{12 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} - \\
& \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3}{128 f \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^4} - \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{24 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^3} + \\
& \left. \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \left(-\operatorname{Cos}\left[\frac{e}{2}\right] - 17 \operatorname{Sin}\left[\frac{e}{2}\right]\right)}{384 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)^2} + \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{12 f \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} \right)
\end{aligned}$$

■ **Problem 3: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a c^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 f} - \frac{a c^2 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 f} + \frac{a c^2 \operatorname{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
& -\frac{1}{48 f} a c^2 \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^3 \\
& \left(3 \operatorname{Cos}[2 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 3 \operatorname{Cos}[4 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\
& 9 \operatorname{Cos}[f x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) + \\
& 9 \operatorname{Cos}[2 e + f x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) - \\
& 3 \operatorname{Cos}[2 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 3 \operatorname{Cos}[4 e + 3 f x] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) - \\
& \left. 12 \operatorname{Sin}[2 e + f x] + 6 \operatorname{Sin}[e + 2 f x] + 6 \operatorname{Sin}[3 e + 2 f x] + 4 \operatorname{Sin}[2 e + 3 f x] \right)
\end{aligned}$$

■ **Problem 14: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^2 (c - c \text{Sec}[e + f x]) dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a^2 c \text{ArcTanh}[\text{Sin}[e + f x]]}{2 f} - \frac{a^2 c \text{Sec}[e + f x] \text{Tan}[e + f x]}{2 f} - \frac{a^2 c \text{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 124 leaves):

$$\frac{1}{12 f} a^2 c \left(-6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 6 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \frac{3}{\left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{3}{\left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - 4 \text{Tan}[e + f x]^3 \right)$$

■ **Problem 15: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^2}{c - c \text{Sec}[e + f x]} dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{3 a^2 \text{ArcTanh}[\text{Sin}[e + f x]]}{c f} - \frac{3 a^2 \text{Tan}[e + f x]}{c f} - \frac{2 (a^2 + a^2 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{f (c - c \text{Sec}[e + f x])}$$

Result (type 3, 220 leaves):

$$\frac{1}{f (c - c \text{Sec}[e + f x])} 2 a^2 \text{Cos}\left[\frac{1}{2}(e + f x)\right] \text{Sec}[e + f x] \text{Sin}\left[\frac{1}{2}(e + f x)\right] \left(4 \text{Csc}\left[\frac{e}{2}\right] \text{Sec}\left[\frac{1}{2}(e + f x)\right] \text{Sin}\left[\frac{f x}{2}\right] + \left(-3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \text{Sin}[f x] \right) / \left(\left(\text{Cos}\left[\frac{e}{2}\right] - \text{Sin}\left[\frac{e}{2}\right]\right) \left(\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right]\right) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \right) \text{Tan}\left[\frac{1}{2}(e + f x)\right]$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^6 dx$$

Optimal (type 3, 227 leaves, 16 steps):

$$\frac{55 a^3 c^6 \text{ArcTanh}[\text{Sin}[e + f x]]}{128 f} - \frac{25 a^3 c^6 \text{Sec}[e + f x] \text{Tan}[e + f x]}{128 f} - \frac{15 a^3 c^6 \text{Sec}[e + f x]^3 \text{Tan}[e + f x]}{64 f} + \frac{5 a^3 c^6 \text{Sec}[e + f x] \text{Tan}[e + f x]^3}{24 f} + \frac{5 a^3 c^6 \text{Sec}[e + f x]^3 \text{Tan}[e + f x]^3}{16 f} - \frac{a^3 c^6 \text{Sec}[e + f x] \text{Tan}[e + f x]^5}{6 f} - \frac{3 a^3 c^6 \text{Sec}[e + f x]^3 \text{Tan}[e + f x]^5}{8 f} + \frac{4 a^3 c^6 \text{Tan}[e + f x]^7}{7 f} + \frac{a^3 c^6 \text{Tan}[e + f x]^9}{9 f}$$

Result (type 3, 1686 leaves) :

$$\begin{aligned}
& \frac{1}{33\,554\,432\,f} \, 9 \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 \\
& \left((c - c \operatorname{Sec}[e + f x])^6 \left(-1430 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 1430 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) - \right. \\
& \left. \frac{1}{32} \operatorname{Sec}[e + f x]^8 (4601 \sin[e + f x] + 3589 \sin[3(e + f x)] + 5441 \sin[5(e + f x)] - 715 \sin[7(e + f x)]) \right) - \\
& \frac{1}{16\,777\,216\,f} \, 11 \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6 \\
& \left(-210 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 210 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\
& \left. \frac{1}{32} \operatorname{Sec}[e + f x]^8 (5053 \sin[e + f x] + 2681 \sin[3(e + f x)] + 805 \sin[5(e + f x)] + 105 \sin[7(e + f x)]) \right) + \\
& \frac{1}{25\,165\,824\,f} \, 29 \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6 \\
& \left(-330 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 330 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\
& \left. \frac{1}{32} \operatorname{Sec}[e + f x]^8 (-6103 \sin[e + f x] + 4213 \sin[3(e + f x)] + 1265 \sin[5(e + f x)] + 165 \sin[7(e + f x)]) \right) - \\
& \frac{1}{8\,388\,608\,f} \, 5 \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6 \\
& \left(-858 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 858 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] + \right. \\
& \left. \frac{1}{32} \operatorname{Sec}[e + f x]^8 (3793 \sin[e + f x] - 8707 \sin[3(e + f x)] + 3289 \sin[5(e + f x)] + 429 \sin[7(e + f x)]) \right) + \\
& \frac{1}{33\,554\,432\,f} \, \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6 \\
& \left(-24\,310 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] + 24\,310 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) - \\
& \left. \frac{1}{32} \operatorname{Sec}[e + f x]^8 (45\,449 \sin[e + f x] + 93\,781 \sin[3(e + f x)] + 59\,729 \sin[5(e + f x)] + 20\,613 \sin[7(e + f x)]) \right) - \\
& \frac{1}{8192} \, 9 \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6 \\
& \left(\frac{32 \operatorname{Tan}[e + f x]}{63 f} + \frac{16 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{63 f} + \frac{4 \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{21 f} + \frac{10 \operatorname{Sec}[e + f x]^6 \operatorname{Tan}[e + f x]}{63 f} - \frac{\operatorname{Sec}[e + f x]^8 \operatorname{Tan}[e + f x]}{9 f} \right) + \\
& \frac{1}{8192} \, \cos[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6
\end{aligned}$$

$$\left(\frac{32 \operatorname{Tan}[e + f x]}{9 f} + \frac{16 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{9 f} - \frac{20 \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{3 f} + \frac{22 \operatorname{Sec}[e + f x]^6 \operatorname{Tan}[e + f x]}{9 f} - \frac{\operatorname{Sec}[e + f x]^8 \operatorname{Tan}[e + f x]}{9 f} \right) -$$

$$\frac{1}{65536} 3 \operatorname{Cos}[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6$$

$$\left(\frac{256 \operatorname{Tan}[e + f x]}{9 f} - \frac{448 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{9 f} + \frac{80 \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{3 f} - \frac{40 \operatorname{Sec}[e + f x]^6 \operatorname{Tan}[e + f x]}{9 f} + \frac{\operatorname{Sec}[e + f x]^8 \operatorname{Tan}[e + f x]}{9 f} \right) +$$

$$\frac{1}{16384} 3 \operatorname{Cos}[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6$$

$$\left(\frac{64 \operatorname{Tan}[e + f x]}{63 f} + \frac{32 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{63 f} + \frac{8 \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{21 f} - \frac{64 \operatorname{Sec}[e + f x]^6 \operatorname{Tan}[e + f x]}{63 f} + \frac{\operatorname{Sec}[e + f x]^8 \operatorname{Tan}[e + f x]}{9 f} \right) +$$

$$\frac{1}{65536} 55 \operatorname{Cos}[e + f x]^9 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{12} \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^6$$

$$\left(\frac{128 \operatorname{Tan}[e + f x]}{315 f} + \frac{64 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{315 f} + \frac{16 \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{105 f} + \frac{8 \operatorname{Sec}[e + f x]^6 \operatorname{Tan}[e + f x]}{63 f} + \frac{\operatorname{Sec}[e + f x]^8 \operatorname{Tan}[e + f x]}{9 f} \right)$$

■ **Problem 27: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^3}{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$-\frac{15 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 c f} - \frac{10 a^3 \operatorname{Tan}[e + f x]}{c f} - \frac{5 a^3 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 c f} - \frac{2 a (a + a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{f (c - c \operatorname{Sec}[e + f x])}$$

Result (type 3, 287 leaves):

$$\frac{1}{16 f (c - c \operatorname{Sec}[e + f x])} a^3 \operatorname{Cos}[e + f x]^2 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^4 (1 + \operatorname{Sec}[e + f x])^3 \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]$$

$$\left(32 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + \left(-30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + 30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) +$$

$$\frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} - \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} + (16 \operatorname{Sin}[f x]) /$$

$$\left(\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) \right) \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]$$

■ **Problem 28: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^3}{(c - c \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 119 leaves, 6 steps) :

$$\frac{5 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{c^2 f} + \frac{5 a^3 \operatorname{Tan}[e + f x]}{c^2 f} - \frac{2 a (a + a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 f (c - c \operatorname{Sec}[e + f x])^2} + \frac{10 (a^3 + a^3 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f (c^2 - c^2 \operatorname{Sec}[e + f x])}$$

Result (type 3, 402 leaves) :

$$\left(a^3 (1 + \operatorname{Cos}[e + f x])^3 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right. \\ \left. \left(\frac{1}{16} (-74 + 42 \operatorname{Cos}[e] - 76 \operatorname{Cos}[f x] + 120 \operatorname{Cos}[e + f x] - 46 \operatorname{Cos}[2(e + f x)] - 76 \operatorname{Cos}[2e + f x] + 23 \operatorname{Cos}[e + 2f x] + 23 \operatorname{Cos}[3e + 2f x]) \operatorname{Csc}\left[\frac{e}{2}\right]^3 \right. \right. \\ \left. \left. \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \operatorname{Sin}\left[\frac{f x}{2}\right] - 48 \operatorname{Csc}\left[\frac{e}{2}\right]^3 \operatorname{Csc}[e + f x]^4 \operatorname{Sin}[e] \operatorname{Sin}\left[\frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^7 + \operatorname{Cos}[e] \operatorname{Cos}[e + f x] \operatorname{Csc}\left[\frac{e}{2}\right]^2 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^4 \right. \right. \\ \left. \left. \left(4 \operatorname{Cot}\left[\frac{e}{2}\right] + 15 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - \right. \right. \\ \left. \left. 4 (-1 + 5 \operatorname{Cos}[e + f x]) \operatorname{Cot}\left[\frac{e}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sin}\left[\frac{f x}{2}\right] \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) \Big/ \\ \left(6 c^2 f (-1 + \operatorname{Cos}[e + f x])^2 \left(-1 + \operatorname{Cot}\left[\frac{e}{2}\right]\right) \left(1 + \operatorname{Cot}\left[\frac{e}{2}\right]\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right) \right)$$

■ **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^4}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 121 leaves, 10 steps) :

$$-\frac{35 c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a f} + \frac{28 c^4 \operatorname{Tan}[e + f x]}{a f} - \frac{21 c^4 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 a f} + \frac{2 c (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])} + \frac{7 c^4 \operatorname{Tan}[e + f x]^3}{3 a f}$$

Result (type 3, 1036 leaves) :

$$\begin{aligned}
& \frac{35 \cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6 \operatorname{Log}\left[\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right](c-c \sec [e+f x])^4}{16 f(a+a \sec [e+f x])} - \\
& \frac{35 \cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6 \operatorname{Log}\left[\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right](c-c \sec [e+f x])^4}{16 f(a+a \sec [e+f x])} + \\
& \frac{2 \cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right] \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^7 \sec \left[\frac{e}{2}\right](c-c \sec [e+f x])^4 \sin \left[\frac{f x}{2}\right]}{f(a+a \sec [e+f x])} + \\
& \frac{\cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4 \sin \left[\frac{f x}{2}\right]}{48 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^3} + \\
& \frac{\cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4\left(-7 \cos \left[\frac{e}{2}\right]+8 \sin \left[\frac{e}{2}\right]\right)}{48 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2} + \\
& \frac{35 \cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4 \sin \left[\frac{f x}{2}\right]}{24 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)} + \\
& \frac{\cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4 \sin \left[\frac{f x}{2}\right]}{48 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^3} + \\
& \frac{\cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4\left(7 \cos \left[\frac{e}{2}\right]+8 \sin \left[\frac{e}{2}\right]\right)}{48 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2} + \\
& \frac{35 \cos [e+f x]^3 \cot \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^6(c-c \sec [e+f x])^4 \sin \left[\frac{f x}{2}\right]}{24 f(a+a \sec [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [e+f x](c-c \sec [e+f x])^3}{a+a \sec [e+f x]} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$-\frac{15 c^3 \operatorname{ArcTanh}[\sin [e+f x]]}{2 a f} + \frac{10 c^3 \tan [e+f x]}{a f} - \frac{5 c^3 \sec [e+f x] \tan [e+f x]}{2 a f} + \frac{2 c(c-c \sec [e+f x])^2 \tan [e+f x]}{f(a+a \sec [e+f x])}$$

Result (type 3, 287 leaves):

$$\frac{1}{16 a f (1 + \operatorname{Sec}[e + f x])} \operatorname{Cos}[e + f x]^2 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^4 (c - c \operatorname{Sec}[e + f x])^3$$

$$\left(-32 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(-30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] +\right.\right.$$

$$\left.30 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} -$$

$$\left(16 \operatorname{Sin}[f x]\right) / \left(\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)\right)\right)$$

■ **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^2}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{3 c^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a f} + \frac{3 c^2 \operatorname{Tan}[e + f x]}{a f} + \frac{2 (c^2 - c^2 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])}$$

Result (type 3, 220 leaves):

$$\frac{1}{a f (1 + \operatorname{Sec}[e + f x])} 2 c^2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x] \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]$$

$$\left(4 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] +\right.\right.$$

$$\left.\operatorname{Sin}[f x] / \left(\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)\right)\right)$$

■ **Problem 42: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^5}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 164 leaves, 11 steps):

$$\frac{105 c^5 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} - \frac{84 c^5 \operatorname{Tan}[e + f x]}{a^2 f} + \frac{63 c^5 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 a^2 f} -$$

$$\frac{6 c^2 (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{f (a^2 + a^2 \operatorname{Sec}[e + f x])} + \frac{2 c (c - c \operatorname{Sec}[e + f x])^4 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} - \frac{7 c^5 \operatorname{Tan}[e + f x]^3}{a^2 f}$$

Result (type 3, 380 leaves):

$$\frac{1}{3072 a^2 f (1 + \operatorname{Sec}[e + f x])^2} \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^6 (c - c \operatorname{Sec}[e + f x])^5$$

$$\left(20160 \operatorname{Cos}[e + f x]^3 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^3 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + \right.$$

$$\operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e]$$

$$\left(-1323 \operatorname{Sin}\left[\frac{f x}{2}\right] + 3247 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 2901 \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 1197 \operatorname{Sin}\left[e + \frac{f x}{2}\right] - 3027 \operatorname{Sin}\left[2e + \frac{f x}{2}\right] - 273 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 1827 \operatorname{Sin}\left[2e + \frac{3 f x}{2}\right] - \right.$$

$$1693 \operatorname{Sin}\left[3e + \frac{3 f x}{2}\right] + 1995 \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] - 117 \operatorname{Sin}\left[2e + \frac{5 f x}{2}\right] + 1143 \operatorname{Sin}\left[3e + \frac{5 f x}{2}\right] - 969 \operatorname{Sin}\left[4e + \frac{5 f x}{2}\right] + 1173 \operatorname{Sin}\left[2e + \frac{7 f x}{2}\right] +$$

$$\left. 117 \operatorname{Sin}\left[3e + \frac{7 f x}{2}\right] + 747 \operatorname{Sin}\left[4e + \frac{7 f x}{2}\right] - 309 \operatorname{Sin}\left[5e + \frac{7 f x}{2}\right] + 494 \operatorname{Sin}\left[3e + \frac{9 f x}{2}\right] + 142 \operatorname{Sin}\left[4e + \frac{9 f x}{2}\right] + 352 \operatorname{Sin}\left[5e + \frac{9 f x}{2}\right]\right)$$

■ **Problem 43: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^4}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 150 leaves, 7 steps):

$$\frac{35 c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} - \frac{70 c^4 \operatorname{Tan}[e + f x]}{3 a^2 f} + \frac{35 c^4 \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{6 a^2 f} +$$

$$\frac{2 c (c - c \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} - \frac{14 (c^2 - c^2 \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 f (a^2 + a^2 \operatorname{Sec}[e + f x])}$$

Result (type 3, 349 leaves):

$$\frac{1}{3 a^2 f (1 + \operatorname{Sec}[e + f x])^2} c^4 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}[e + f x]^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^3$$

$$\left(-256 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] - 32 \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + \right.$$

$$3 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right]^3 \left(-70 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 70 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) +$$

$$\frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{1}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - (24 \operatorname{Sin}[f x]) / \left(\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right)\right)$$

$$\left.\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)\right) - 32 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Tan}\left[\frac{e}{2}\right]$$

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (c - c \text{Sec}[e + f x])^3}{(a + a \text{Sec}[e + f x])^2} dx$$

Optimal (type 3, 119 leaves, 6 steps):

$$\frac{5 c^3 \text{ArcTanh}[\text{Sin}[e + f x]]}{a^2 f} - \frac{5 c^3 \text{Tan}[e + f x]}{a^2 f} + \frac{2 c (c - c \text{Sec}[e + f x])^2 \text{Tan}[e + f x]}{3 f (a + a \text{Sec}[e + f x])^2} - \frac{10 (c^3 - c^3 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{3 f (a^2 + a^2 \text{Sec}[e + f x])}$$

Result (type 3, 671 leaves):

$$\begin{aligned} & \frac{5 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \text{Sec}[e + f x])^3}{2 f (a + a \text{Sec}[e + f x])^2} - \\ & \frac{5 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \text{Sec}[e + f x])^3}{2 f (a + a \text{Sec}[e + f x])^2} + \\ & \frac{10 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Sec}\left[\frac{e}{2}\right] (c - c \text{Sec}[e + f x])^3 \text{Sin}\left[\frac{f x}{2}\right]}{3 f (a + a \text{Sec}[e + f x])^2} + \\ & \frac{2 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \text{Sec}\left[\frac{e}{2}\right] (c - c \text{Sec}[e + f x])^3 \text{Sin}\left[\frac{f x}{2}\right]}{3 f (a + a \text{Sec}[e + f x])^2} + \\ & \frac{\text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (c - c \text{Sec}[e + f x])^3 \text{Sin}\left[\frac{f x}{2}\right]}{2 f (a + a \text{Sec}[e + f x])^2 \left(\text{Cos}\left[\frac{e}{2}\right] - \text{Sin}\left[\frac{e}{2}\right]\right) \left(\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} + \\ & \frac{\text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (c - c \text{Sec}[e + f x])^3 \text{Sin}\left[\frac{f x}{2}\right]}{2 f (a + a \text{Sec}[e + f x])^2 \left(\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right]\right) \left(\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right)} + \frac{2 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \text{Sec}[e + f x])^3 \text{Tan}\left[\frac{e}{2}\right]}{3 f (a + a \text{Sec}[e + f x])^2} \end{aligned}$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^2 (c - c \text{Sec}[e + f x])^2} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\text{Csc}[e + f x]}{a^2 c^2 f} - \frac{\text{Csc}[e + f x]^3}{3 a^2 c^2 f}$$

Result (type 3, 100 leaves):

$$\frac{5 \text{Cot}\left[\frac{1}{2}(e + f x)\right]}{12 f} - \frac{\text{Cot}\left[\frac{1}{2}(e + f x)\right] \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{24 f} + \frac{5 \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{12 f} - \frac{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{24 f}$$

$$a^2 c^2$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (c - c \text{Sec}[e + f x])^4}{(a + a \text{Sec}[e + f x])^3} dx$$

Optimal (type 3, 164 leaves, 7 steps):

$$-\frac{7 c^4 \text{ArcTanh}[\text{Sin}[e + f x]]}{a^3 f} + \frac{7 c^4 \text{Tan}[e + f x]}{a^3 f} + \frac{2 c (c - c \text{Sec}[e + f x])^3 \text{Tan}[e + f x]}{5 f (a + a \text{Sec}[e + f x])^3} -$$

$$\frac{14 (c^2 - c^2 \text{Sec}[e + f x])^2 \text{Tan}[e + f x]}{15 a f (a + a \text{Sec}[e + f x])^2} + \frac{14 (c^4 - c^4 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{3 f (a^3 + a^3 \text{Sec}[e + f x])}$$

Result (type 3, 826 leaves):

$$\frac{7 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \text{Sec}[e + f x])^4}{2 f (a + a \text{Sec}[e + f x])^3} -$$

$$\frac{7 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]\right] (c - c \text{Sec}[e + f x])^4}{2 f (a + a \text{Sec}[e + f x])^3} +$$

$$\frac{76 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Sec}\left[\frac{e}{2}\right] (c - c \text{Sec}[e + f x])^4 \text{Sin}\left[\frac{f x}{2}\right]}{15 f (a + a \text{Sec}[e + f x])^3} +$$

$$\frac{8 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \text{Sec}\left[\frac{e}{2}\right] (c - c \text{Sec}[e + f x])^4 \text{Sin}\left[\frac{f x}{2}\right]}{15 f (a + a \text{Sec}[e + f x])^3} +$$

$$\frac{2 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^7 \text{Sec}\left[\frac{e}{2}\right] (c - c \text{Sec}[e + f x])^4 \text{Sin}\left[\frac{f x}{2}\right]}{5 f (a + a \text{Sec}[e + f x])^3} +$$

$$\frac{\text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (c - c \text{Sec}[e + f x])^4 \text{Sin}\left[\frac{f x}{2}\right]}{2 f (a + a \text{Sec}[e + f x])^3 (\text{Cos}\left[\frac{e}{2}\right] - \text{Sin}\left[\frac{e}{2}\right]) (\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right])} +$$

$$\frac{\text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (c - c \text{Sec}[e + f x])^4 \text{Sin}\left[\frac{f x}{2}\right]}{2 f (a + a \text{Sec}[e + f x])^3 (\text{Cos}\left[\frac{e}{2}\right] + \text{Sin}\left[\frac{e}{2}\right]) (\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right])} +$$

$$\frac{8 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (c - c \text{Sec}[e + f x])^4 \text{Tan}\left[\frac{e}{2}\right]}{15 f (a + a \text{Sec}[e + f x])^3} + \frac{2 \text{Cos}[e + f x] \text{Cot}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (c - c \text{Sec}[e + f x])^4 \text{Tan}\left[\frac{e}{2}\right]}{5 f (a + a \text{Sec}[e + f x])^3}$$

■ **Problem 60: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^3} dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$\frac{\text{Csc}[e + f x]}{a^3 c^3 f} - \frac{2 \text{Csc}[e + f x]^3}{3 a^3 c^3 f} + \frac{\text{Csc}[e + f x]^5}{5 a^3 c^3 f}$$

Result (type 3, 159 leaves):

$$-\frac{1}{a^3 c^3} \left(-\frac{89 \text{Cot}\left[\frac{1}{2}(e + f x)\right]}{240 f} + \frac{31 \text{Cot}\left[\frac{1}{2}(e + f x)\right] \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2}{480 f} - \frac{\text{Cot}\left[\frac{1}{2}(e + f x)\right] \text{Csc}\left[\frac{1}{2}(e + f x)\right]^4}{160 f} - \frac{89 \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{240 f} + \frac{31 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{480 f} - \frac{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^4 \text{Tan}\left[\frac{1}{2}(e + f x)\right]}{160 f} \right)$$

■ **Problem 61: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^4} dx$$

Optimal (type 3, 99 leaves, 7 steps):

$$-\frac{\text{Cot}[e + f x]^7}{7 a^3 c^4 f} + \frac{\text{Csc}[e + f x]}{a^3 c^4 f} - \frac{\text{Csc}[e + f x]^3}{a^3 c^4 f} + \frac{3 \text{Csc}[e + f x]^5}{5 a^3 c^4 f} - \frac{\text{Csc}[e + f x]^7}{7 a^3 c^4 f}$$

Result (type 3, 211 leaves):

$$\frac{1}{35840 a^3 c^4 f} \text{Csc}[e] \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \text{Csc}[e + f x]^5 - (2912 \text{Sin}[e] + 416 \text{Sin}[f x] - 7620 \text{Sin}[e + f x] + 1905 \text{Sin}[2(e + f x)] + 3810 \text{Sin}[3(e + f x)] - 1524 \text{Sin}[4(e + f x)] - 762 \text{Sin}[5(e + f x)] + 381 \text{Sin}[6(e + f x)] - 2016 \text{Sin}[2e + f x] + 2080 \text{Sin}[e + 2f x] - 1680 \text{Sin}[3e + 2f x] + 240 \text{Sin}[2e + 3f x] + 560 \text{Sin}[4e + 3f x] - 880 \text{Sin}[3e + 4f x] + 560 \text{Sin}[5e + 4f x] + 400 \text{Sin}[4e + 5f x] - 560 \text{Sin}[6e + 5f x] + 80 \text{Sin}[5e + 6f x])$$

■ **Problem 62: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^3 (c - c \text{Sec}[e + f x])^5} dx$$

Optimal (type 3, 120 leaves, 10 steps):

$$\frac{2 \text{Cot}[e + f x]^9}{9 a^3 c^5 f} + \frac{\text{Csc}[e + f x]}{a^3 c^5 f} - \frac{5 \text{Csc}[e + f x]^3}{3 a^3 c^5 f} + \frac{9 \text{Csc}[e + f x]^5}{5 a^3 c^5 f} - \frac{\text{Csc}[e + f x]^7}{a^3 c^5 f} + \frac{2 \text{Csc}[e + f x]^9}{9 a^3 c^5 f}$$

Result (type 3, 257 leaves):

$$-\frac{1}{184320 a^3 c^5 f (-1 + \text{Sec}[e + f x])^5 (1 + \text{Sec}[e + f x])^3} \text{Csc}[e] \text{Sec}[e + f x]^7 (-33024 \text{Sin}[e] + 6144 \text{Sin}[f x] + 76455 \text{Sin}[e + f x] - 33980 \text{Sin}[2(e + f x)] - 32281 \text{Sin}[3(e + f x)] + 27184 \text{Sin}[4(e + f x)] + 1699 \text{Sin}[5(e + f x)] - 6796 \text{Sin}[6(e + f x)] + 1699 \text{Sin}[7(e + f x)] + 22656 \text{Sin}[2e + f x] - 17216 \text{Sin}[e + 2f x] + 4416 \text{Sin}[3e + 2f x] + 3200 \text{Sin}[2e + 3f x] - 15360 \text{Sin}[4e + 3f x] + 12160 \text{Sin}[3e + 4f x] - 1920 \text{Sin}[5e + 4f x] - 5120 \text{Sin}[4e + 5f x] + 5760 \text{Sin}[6e + 5f x] + 320 \text{Sin}[5e + 6f x] - 2880 \text{Sin}[7e + 6f x] + 640 \text{Sin}[6e + 7f x]) \text{Tan}[e + f x]$$

- **Problem 68: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])}{\sqrt{c - c \text{Sec}[e + f x]}} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{2\sqrt{2} a \text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{\sqrt{c} f} + \frac{2 a \text{Tan}[e + f x]}{f \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 167 leaves):

$$-\left(i \sqrt{2} a (-1 + e^{i(e+fx)}) \left(\sqrt{2} (1 + e^{i(e+fx)}) + 2 \sqrt{1 + e^{2i(e+fx)}} \text{Log}[1 - e^{i(e+fx)}] - 2 \sqrt{1 + e^{2i(e+fx)}} \text{Log}[1 + e^{i(e+fx)} + \sqrt{2} \sqrt{1 + e^{2i(e+fx)}}] \right) \right) / \left((1 + e^{2i(e+fx)}) f \sqrt{c - c \text{Sec}[e + f x]} \right)$$

- **Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])}{(c - c \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{a \text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{\sqrt{2} c^{3/2} f} - \frac{a \text{Tan}[e + f x]}{f (c - c \text{Sec}[e + f x])^{3/2}}$$

Result (type 3, 298 leaves):

$$a \left(\frac{1}{f (c - c \text{Sec}[e + f x])^{3/2}} 2 e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} \sqrt{1 + e^{2i(e+fx)}} \right. \\ \left. \left(\text{Log}[1 - e^{i(e+fx)}] - \text{Log}[1 + e^{i(e+fx)} + \sqrt{2} \sqrt{1 + e^{2i(e+fx)}}] \right) \text{Sec}[e + f x]^{3/2} \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 + \frac{1}{(c - c \text{Sec}[e + f x])^{3/2}} \right. \\ \left. \text{Sec}[e + f x]^2 \left(\frac{4 \text{Cos}\left[\frac{e}{2}\right] \text{Cos}\left[\frac{fx}{2}\right]}{f} - \frac{2 \text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} + \frac{2 \text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \text{Sin}\left[\frac{fx}{2}\right]}{f} - \frac{4 \text{Sin}\left[\frac{e}{2}\right] \text{Sin}\left[\frac{fx}{2}\right]}{f} \right) \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right)$$

- **Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])}{(c - c \text{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 113 leaves, 4 steps):

$$\frac{a \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{8 \sqrt{2} c^{5/2} f} - \frac{a \operatorname{Tan}[e+fx]}{2 f (c-c \operatorname{Sec}[e+fx])^{5/2}} + \frac{a \operatorname{Tan}[e+fx]}{8 c f (c-c \operatorname{Sec}[e+fx])^{3/2}}$$

Result (type 3, 362 leaves):

$$a \left(\frac{1}{2 f (c-c \operatorname{Sec}[e+fx])^{5/2}} \right. \\ \left. e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i (e+fx)}}{1+e^{2 i (e+fx)}}} \sqrt{1+e^{2 i (e+fx)}} \left(-\operatorname{Log}[1-e^{i (e+fx)}] + \operatorname{Log}\left[1+e^{i (e+fx)} + \sqrt{2} \sqrt{1+e^{2 i (e+fx)}}\right]\right) \operatorname{Sec}[e+fx]^{5/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 + \right. \\ \left. \frac{1}{(c-c \operatorname{Sec}[e+fx])^{5/2}} \operatorname{Sec}[e+fx]^3 \left(-\frac{3 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{fx}{2}\right]}{f} + \frac{7 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{2 f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{f} - \frac{7 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{2 f} + \right. \right. \\ \left. \left. \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} + \frac{3 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} \right) \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \right)$$

■ **Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^2}{\sqrt{c-c \operatorname{Sec}[e+fx]}} dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{4 \sqrt{2} a^2 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{\sqrt{c} f} + \frac{16 a^2 \operatorname{Tan}[e+fx]}{3 f \sqrt{c-c \operatorname{Sec}[e+fx]}} - \frac{2 a^2 \sqrt{c-c \operatorname{Sec}[e+fx]} \operatorname{Tan}[e+fx]}{3 c f}$$

Result (type 3, 292 leaves):

$$\frac{1}{3 f \sqrt{c-c \operatorname{Sec}[e+fx]}} 4 a^2 e^{-\frac{1}{2} i (e+fx)} \operatorname{Sec}[e+fx] \left(\operatorname{Cos}\left[\frac{1}{2} (e+fx)\right] + i \operatorname{Sin}\left[\frac{1}{2} (e+fx)\right] \right) \operatorname{Sin}\left[\frac{1}{2} (e+fx)\right] \\ \left(\operatorname{Cos}\left[\frac{1}{2} (e+fx)\right] \left(7 + 3 \sqrt{2} \sqrt{1+e^{2 i (e+fx)}} \operatorname{Log}[1-e^{i (e+fx)}] - 3 \sqrt{2} \sqrt{1+e^{2 i (e+fx)}} \operatorname{Log}\left[1+e^{i (e+fx)} + \sqrt{2} \sqrt{1+e^{2 i (e+fx)}}\right] + \operatorname{Sec}[e+fx] \right) - \right. \\ \left. 3 i \sqrt{2} \sqrt{1+e^{2 i (e+fx)}} \left(\operatorname{Log}[1-e^{i (e+fx)}] - \operatorname{Log}\left[1+e^{i (e+fx)} + \sqrt{2} \sqrt{1+e^{2 i (e+fx)}}\right] \right) \operatorname{Sin}\left[\frac{1}{2} (e+fx)\right] \right)$$

■ **Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^2}{(c-c \operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 113 leaves, 4 steps):

$$\frac{3\sqrt{2} a^2 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{c^{3/2} f} - \frac{2 a^2 \operatorname{Tan}[e+fx]}{f (c-c \operatorname{Sec}[e+fx])^{3/2}} - \frac{2 a^2 \operatorname{Tan}[e+fx]}{c f \sqrt{c-c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 337 leaves):

$$\left(3 e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \sqrt{1+e^{2i(e+fx)}} \left(\operatorname{Log}[1-e^{i(e+fx)}] - \operatorname{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] (a+a \operatorname{Sec}[e+fx])^2 \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right) / \left(f \sqrt{\operatorname{Sec}[e+fx]} (c-c \operatorname{Sec}[e+fx])^{3/2} \right) + \frac{1}{(c-c \operatorname{Sec}[e+fx])^{3/2}} \\ \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] (a+a \operatorname{Sec}[e+fx])^2 \left(\frac{4 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{fx}{2}\right]}{f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} + \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} - \frac{4 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} \right) \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^3$$

■ **Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^2}{(c-c \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{3 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{4 \sqrt{2} c^{5/2} f} - \frac{a^2 \operatorname{Tan}[e+fx]}{f (c-c \operatorname{Sec}[e+fx])^{5/2}} + \frac{5 a^2 \operatorname{Tan}[e+fx]}{4 c f (c-c \operatorname{Sec}[e+fx])^{3/2}}$$

Result (type 3, 378 leaves):

$$-\frac{1}{4 c^2 f (-1+\operatorname{Sec}[e+fx])^2 \sqrt{c-c \operatorname{Sec}[e+fx]}} \\ a^2 e^{-\frac{1}{2} i (e+fx)} \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^3 \sqrt{\operatorname{Sec}[e+fx]} (1+\operatorname{Sec}[e+fx])^2 \left(\frac{1}{16 \sqrt{\operatorname{Sec}[e+fx]}} e^{-\frac{3ie}{2}} (-1+e^{ie}) \left(\operatorname{Cos}\left[\frac{fx}{2}\right] + i \operatorname{Sin}\left[\frac{fx}{2}\right] \right) \right. \\ \left. \left(-9 i e^{ie} (1+e^{ie}) \operatorname{Cos}\left[\frac{fx}{2}\right] + i (1+e^{3ie}) \operatorname{Cos}\left[\frac{3fx}{2}\right] - 9 e^{ie} \operatorname{Sin}\left[\frac{fx}{2}\right] + 9 e^{2ie} \operatorname{Sin}\left[\frac{fx}{2}\right] + \operatorname{Sin}\left[\frac{3fx}{2}\right] - e^{3ie} \operatorname{Sin}\left[\frac{3fx}{2}\right] \right) + \right. \\ \left. 3 \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \sqrt{1+e^{2i(e+fx)}} \left(-\operatorname{Log}[1-e^{i(e+fx)}] + \operatorname{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]^4 \right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]$$

- **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^2}{(c - c \text{Sec}[e + f x])^{7/2}} dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$-\frac{a^2 \text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{16 \sqrt{2} c^{7/2} f} - \frac{(a^2 + a^2 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{3 f (c - c \text{Sec}[e + f x])^{7/2}} + \frac{a^2 \text{Tan}[e + f x]}{4 c f (c - c \text{Sec}[e + f x])^{5/2}} - \frac{a^2 \text{Tan}[e + f x]}{16 c^2 f (c - c \text{Sec}[e + f x])^{3/2}}$$

Result (type 3, 486 leaves):

$$\frac{1}{8 f (c - c \text{Sec}[e + f x])^{7/2}} e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i (e+fx)}}{1 + e^{2 i (e+fx)}}} \sqrt{1 + e^{2 i (e+fx)}} \left(-\text{Log}[1 - e^{i (e+fx)}] + \text{Log}[1 + e^{i (e+fx)} + \sqrt{2} \sqrt{1 + e^{2 i (e+fx)}}] \right)$$

$$\text{Sec}[e + f x]^{3/2} (a + a \text{Sec}[e + f x])^2 \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 + \frac{1}{(c - c \text{Sec}[e + f x])^{7/2}} \text{Sec}[e + f x]^2 (a + a \text{Sec}[e + f x])^2$$

$$\left(\frac{7 \text{Cos}\left[\frac{e}{2}\right] \text{Cos}\left[\frac{f x}{2}\right]}{12 f} - \frac{43 \text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]}{24 f} + \frac{17 \text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{12 f} - \frac{\text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5}{3 f} + \frac{43 \text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sin}\left[\frac{f x}{2}\right]}{24 f} - \right.$$

$$\left. \frac{17 \text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \text{Sin}\left[\frac{f x}{2}\right]}{12 f} + \frac{\text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \text{Sin}\left[\frac{f x}{2}\right]}{3 f} - \frac{7 \text{Sin}\left[\frac{e}{2}\right] \text{Sin}\left[\frac{f x}{2}\right]}{12 f} \right) \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \text{Tan}\left[\frac{e}{2} + \frac{f x}{2}\right]^4$$

- **Problem 83: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^3}{\sqrt{c - c \text{Sec}[e + f x]}} dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$-\frac{8 \sqrt{2} a^3 \text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{\sqrt{c} f} + \frac{8 a^3 \text{Tan}[e + f x]}{f \sqrt{c - c \text{Sec}[e + f x]}} + \frac{2 a (a + a \text{Sec}[e + f x])^2 \text{Tan}[e + f x]}{5 f \sqrt{c - c \text{Sec}[e + f x]}} + \frac{4 (a^3 + a^3 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{3 f \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 223 leaves):

$$-\left(2 i a^3 (-1 + e^{i (e+fx)}) \left(73 + 105 e^{i (e+fx)} + 190 e^{2 i (e+fx)} + 190 e^{3 i (e+fx)} + 105 e^{4 i (e+fx)} + 73 e^{5 i (e+fx)} + 60 \sqrt{2} (1 + e^{2 i (e+fx)})^{5/2} \text{Log}[1 - e^{i (e+fx)}] - \right. \right.$$

$$\left. \left. 60 \sqrt{2} (1 + e^{2 i (e+fx)})^{5/2} \text{Log}[1 + e^{i (e+fx)} + \sqrt{2} \sqrt{1 + e^{2 i (e+fx)}}] \right) \right) / \left(15 (1 + e^{2 i (e+fx)})^3 f \sqrt{c - c \text{Sec}[e + f x]} \right)$$

- **Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^3}{(c - c \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 168 leaves, 5 steps) :

$$\frac{10 \sqrt{2} a^3 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{c^{3/2} f} - \frac{a (a + a \operatorname{Sec}[e+fx])^2 \operatorname{Tan}[e+fx]}{f (c - c \operatorname{Sec}[e+fx])^{3/2}} - \frac{10 a^3 \operatorname{Tan}[e+fx]}{c f \sqrt{c - c \operatorname{Sec}[e+fx]}} - \frac{5 (a^3 + a^3 \operatorname{Sec}[e+fx]) \operatorname{Tan}[e+fx]}{3 c f \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 377 leaves) :

$$\left(5 e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i (e+fx)}}{1 + e^{2 i (e+fx)}}} \sqrt{1 + e^{2 i (e+fx)}} \right. \\ \left. \left(\operatorname{Log}\left[1 - e^{i (e+fx)}\right] - \operatorname{Log}\left[1 + e^{i (e+fx)} + \sqrt{2} \sqrt{1 + e^{2 i (e+fx)}}\right]\right) \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 (a + a \operatorname{Sec}[e+fx])^3 \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right) / \\ \left(f \operatorname{Sec}[e+fx]^{3/2} (c - c \operatorname{Sec}[e+fx])^{3/2} \right) + \frac{1}{(c - c \operatorname{Sec}[e+fx])^{3/2}} \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 (a + a \operatorname{Sec}[e+fx])^3 \\ \left(\frac{19 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{fx}{2}\right]}{3 f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} + \frac{\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] \operatorname{Sec}[e+fx]}{3 f} + \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} - \frac{19 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{3 f} \right) \operatorname{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^3$$

- **Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx] (a + a \operatorname{Sec}[e+fx])^3}{(c - c \operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 5 steps) :

$$- \frac{15 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{2 \sqrt{2} c^{5/2} f} - \frac{a (a + a \operatorname{Sec}[e+fx])^2 \operatorname{Tan}[e+fx]}{2 f (c - c \operatorname{Sec}[e+fx])^{5/2}} + \frac{5 (a^3 + a^3 \operatorname{Sec}[e+fx]) \operatorname{Tan}[e+fx]}{4 c f (c - c \operatorname{Sec}[e+fx])^{3/2}} + \frac{15 a^3 \operatorname{Tan}[e+fx]}{4 c^2 f \sqrt{c - c \operatorname{Sec}[e+fx]}}$$

Result (type 3, 411 leaves) :

$$\left(15 e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \sqrt{1+e^{2i(e+fx)}} \left(\text{Log}[1-e^{i(e+fx)}] - \text{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \right. \\ \left. \text{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] (a+a \text{Sec}[e+fx])^3 \text{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \right) / \left(4f \sqrt{\text{Sec}[e+fx]} (c-c \text{Sec}[e+fx])^{5/2} \right) + \\ \frac{1}{(c-c \text{Sec}[e+fx])^{5/2}} \text{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right] (a+a \text{Sec}[e+fx])^3 \left(\frac{9 \text{Cos}\left[\frac{e}{2}\right] \text{Cos}\left[\frac{fx}{2}\right]}{2f} - \frac{\text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{4f} - \frac{\text{Cot}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{2f} + \right. \\ \left. \frac{\text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \text{Sin}\left[\frac{fx}{2}\right]}{4f} + \frac{\text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \text{Sin}\left[\frac{fx}{2}\right]}{2f} - \frac{9 \text{Sin}\left[\frac{e}{2}\right] \text{Sin}\left[\frac{fx}{2}\right]}{2f} \right) \text{Tan}\left[\frac{e}{2} + \frac{fx}{2}\right]^5$$

- **Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e+fx]}{(a+a \text{Sec}[e+fx]) \sqrt{c-c \text{Sec}[e+fx]}} dx$$

Optimal (type 3, 89 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{\sqrt{2} a \sqrt{c} f} + \frac{\text{Tan}[e+fx]}{f (a+a \text{Sec}[e+fx]) \sqrt{c-c \text{Sec}[e+fx]}}$$

Result (type 3, 204 leaves):

$$-\left(i (-1+e^{2i(e+fx)}) \left(\sqrt{2} (1+e^{2i(e+fx)}) + (1+e^{i(e+fx)}) \sqrt{1+e^{2i(e+fx)}} \text{Log}[1-e^{i(e+fx)}] - (1+e^{i(e+fx)}) \sqrt{1+e^{2i(e+fx)}} \text{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \right) / \left(\sqrt{2} a (1+e^{2i(e+fx)})^2 f (1+\text{Sec}[e+fx]) \sqrt{c-c \text{Sec}[e+fx]} \right)$$

- **Problem 91: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e+fx]}{(a+a \text{Sec}[e+fx]) (c-c \text{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 122 leaves, 4 steps):

$$-\frac{3 \text{ArcTan}\left[\frac{\sqrt{c} \text{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \text{Sec}[e+fx]}}\right]}{4 \sqrt{2} a c^{3/2} f} - \frac{3 \text{Tan}[e+fx]}{4 a f (c-c \text{Sec}[e+fx])^{3/2}} + \frac{\text{Tan}[e+fx]}{f (a+a \text{Sec}[e+fx]) (c-c \text{Sec}[e+fx])^{3/2}}$$

Result (type 3, 220 leaves):

$$- \left(e^{-2i(e+fx)} \operatorname{Csc}[2(e+fx)] \left(3 - 8e^{i(e+fx)} - 4e^{3i(e+fx)} + e^{4i(e+fx)} - \right. \right. \\ \left. \left. 2e^{\frac{3}{2}i(e+fx)} \left(-4 + 3\sqrt{2}\sqrt{1+e^{2i(e+fx)}} \operatorname{Log}[1-e^{i(e+fx)}] - 3\sqrt{2}\sqrt{1+e^{2i(e+fx)}} \operatorname{Log}\left[1+e^{i(e+fx)} + \sqrt{2}\sqrt{1+e^{2i(e+fx)}}\right]\right) \right. \right. \\ \left. \left. \sin\left[\frac{1}{2}(e+fx)\right] \sin[e+fx] \right) \right) / \left(8acf\sqrt{c-c\operatorname{Sec}[e+fx]} \right)$$

■ **Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]}{(a+a\operatorname{Sec}[e+fx])(c-c\operatorname{Sec}[e+fx])^{5/2}} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$- \frac{15 \operatorname{ArcTan}\left[\frac{\sqrt{c}\operatorname{Tan}[e+fx]}{\sqrt{2}\sqrt{c-c\operatorname{Sec}[e+fx]}}\right]}{32\sqrt{2}ac^{5/2}f} - \frac{5 \operatorname{Tan}[e+fx]}{8af(c-c\operatorname{Sec}[e+fx])^{5/2}} + \frac{\operatorname{Tan}[e+fx]}{f(a+a\operatorname{Sec}[e+fx])(c-c\operatorname{Sec}[e+fx])^{5/2}} - \frac{15 \operatorname{Tan}[e+fx]}{32acf(c-c\operatorname{Sec}[e+fx])^{3/2}}$$

Result (type 3, 441 leaves):

$$\left(15e^{-\frac{1}{2}i(e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \sqrt{1+e^{2i(e+fx)}} \operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \left(\operatorname{Log}[1-e^{i(e+fx)}] - \operatorname{Log}\left[1+e^{i(e+fx)} + \sqrt{2}\sqrt{1+e^{2i(e+fx)}}\right] \right) \right. \\ \left. \operatorname{Sec}[e+fx]^{7/2} \sin\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \right) / \left(4f(a+a\operatorname{Sec}[e+fx])(c-c\operatorname{Sec}[e+fx])^{5/2} \right) + \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sec}[e+fx]^4 \right. \\ \left. - \frac{3 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{fx}{2}\right]}{2f} + \frac{15 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{4f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{2f} - \frac{2 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} - \frac{15 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{4f} + \right. \\ \left. \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sin}\left[\frac{fx}{2}\right]}{2f} + \frac{3 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{2f} \right) \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^5 \Big/ \left((a+a\operatorname{Sec}[e+fx])(c-c\operatorname{Sec}[e+fx])^{5/2} \right)$$

■ **Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]}{(a+a\operatorname{Sec}[e+fx])^2 \sqrt{c-c\operatorname{Sec}[e+fx]}} dx$$

Optimal (type 3, 138 leaves, 4 steps):

$$- \frac{\operatorname{ArcTan}\left[\frac{\sqrt{c}\operatorname{Tan}[e+fx]}{\sqrt{2}\sqrt{c-c\operatorname{Sec}[e+fx]}}\right]}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\operatorname{Tan}[e+fx]}{3f(a+a\operatorname{Sec}[e+fx])^2\sqrt{c-c\operatorname{Sec}[e+fx]}} + \frac{\operatorname{Tan}[e+fx]}{2f(a^2+a^2\operatorname{Sec}[e+fx])\sqrt{c-c\operatorname{Sec}[e+fx]}}$$

Result (type 3, 296 leaves):

$$\left(2 e^{-\frac{1}{2} i (e+fx)} \operatorname{Cos}\left[\frac{1}{2} (e+fx)\right] \left(\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \operatorname{Cos}\left[\frac{1}{2} (e+fx)\right]^3 \right. \right. \\ \left. \left. \left(5 \sqrt{2} (1+e^{i(e+fx)}) + 3 \sqrt{1+e^{2i(e+fx)}} \operatorname{Log}[1-e^{i(e+fx)}] - 3 \sqrt{1+e^{2i(e+fx)}} \operatorname{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \right. \right. \\ \left. \left. e^{\frac{1}{2} i (e+fx)} \sqrt{\operatorname{Sec}[e+fx]} - 7 e^{\frac{1}{2} i (e+fx)} \operatorname{Cos}\left[\frac{1}{2} (e+fx)\right]^2 \sqrt{\operatorname{Sec}[e+fx]} \right) \right) \\ \left. \operatorname{Sec}[e+fx]^{5/2} \operatorname{Sin}\left[\frac{1}{2} (e+fx)\right] \right) / \left(3 a^2 f (1+\operatorname{Sec}[e+fx])^2 \sqrt{c-c \operatorname{Sec}[e+fx]} \right)$$

■ **Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]}{(a+a \operatorname{Sec}[e+fx])^2 (c-c \operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$- \frac{5 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+fx]}}\right]}{8 \sqrt{2} a^2 c^{3/2} f} - \frac{5 \operatorname{Tan}[e+fx]}{8 a^2 f (c-c \operatorname{Sec}[e+fx])^{3/2}} + \\ \frac{\operatorname{Tan}[e+fx]}{3 f (a+a \operatorname{Sec}[e+fx])^2 (c-c \operatorname{Sec}[e+fx])^{3/2}} + \frac{5 \operatorname{Tan}[e+fx]}{6 f (a^2+a^2 \operatorname{Sec}[e+fx]) (c-c \operatorname{Sec}[e+fx])^{3/2}}$$

Result (type 3, 395 leaves):

$$- \left(5 e^{-\frac{1}{2} i (e+fx)} \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} \sqrt{1+e^{2i(e+fx)}} \operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \right. \\ \left. \left(\operatorname{Log}[1-e^{i(e+fx)}] - \operatorname{Log}[1+e^{i(e+fx)} + \sqrt{2} \sqrt{1+e^{2i(e+fx)}}] \right) \operatorname{Sec}[e+fx]^{7/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right) / \left(f (a+a \operatorname{Sec}[e+fx])^2 (c-c \operatorname{Sec}[e+fx])^{3/2} \right) + \\ \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \operatorname{Sec}[e+fx]^4 \left(-\frac{26 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{fx}{2}\right]}{3 f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} + \frac{20 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]}{3 f} - \frac{2 \operatorname{Sec}\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{3 f} \right. \right. \\ \left. \left. \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \operatorname{Sin}\left[\frac{fx}{2}\right]}{f} + \frac{26 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{fx}{2}\right]}{3 f} \right) \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right) / \left((a+a \operatorname{Sec}[e+fx])^2 (c-c \operatorname{Sec}[e+fx])^{3/2} \right)$$

- **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 203 leaves, 6 steps):

$$\begin{aligned} & - \frac{35 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+f x]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+f x]}}\right]}{64 \sqrt{2} a^2 c^{5/2} f} - \frac{35 \operatorname{Tan}[e+f x]}{48 a^2 f (c - c \operatorname{Sec}[e+f x])^{5/2}} + \frac{\operatorname{Tan}[e+f x]}{3 f (a + a \operatorname{Sec}[e+f x])^2 (c - c \operatorname{Sec}[e+f x])^{5/2}} + \\ & \frac{7 \operatorname{Tan}[e+f x]}{6 f (a^2 + a^2 \operatorname{Sec}[e+f x]) (c - c \operatorname{Sec}[e+f x])^{5/2}} - \frac{35 \operatorname{Tan}[e+f x]}{64 a^2 c f (c - c \operatorname{Sec}[e+f x])^{3/2}} \end{aligned}$$

Result (type 3, 465 leaves):

$$\begin{aligned} & \left(35 e^{-\frac{1}{2} i (e+f x)} \sqrt{\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}}} \sqrt{1 + e^{2 i (e+f x)}} \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(\operatorname{Log}[1 - e^{i (e+f x)}] - \operatorname{Log}\left[1 + e^{i (e+f x)} + \sqrt{2} \sqrt{1 + e^{2 i (e+f x)}}\right] \right) \right) \\ & \operatorname{Sec}[e + f x]^{9/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \Big/ \left(4 f (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^{5/2} \right) + \\ & \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}[e + f x]^5 \left(\frac{43 \cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right]}{6 f} + \frac{19 \cot\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]}{4 f} - \frac{\cot\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{2 f} - \frac{26 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 f} + \right. \right. \\ & \left. \left. \frac{2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{3 f} - \frac{19 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sin\left[\frac{f x}{2}\right]}{4 f} + \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sin\left[\frac{f x}{2}\right]}{2 f} - \frac{43 \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]}{6 f} \right) \right) \\ & \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \Big/ \left((a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^{5/2} \right) \end{aligned}$$

- **Problem 104: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^3 \sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned} & - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+f x]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+f x]}}\right]}{4 \sqrt{2} a^3 \sqrt{c} f} + \frac{\operatorname{Tan}[e+f x]}{5 f (a + a \operatorname{Sec}[e + f x])^3 \sqrt{c - c \operatorname{Sec}[e + f x]}} + \\ & \frac{\operatorname{Tan}[e+f x]}{6 a f (a + a \operatorname{Sec}[e + f x])^2 \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Tan}[e+f x]}{4 f (a^3 + a^3 \operatorname{Sec}[e + f x]) \sqrt{c - c \operatorname{Sec}[e + f x]}} \end{aligned}$$

Result (type 3, 334 leaves) :

$$\frac{1}{15 a^3 f (1 + \operatorname{Sec}[e + f x])^3 \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

$$2 e^{-\frac{1}{2} i (e+f x)} \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] \left(\sqrt{\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}}} \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^5 \left(37 \sqrt{2} (1 + e^{i (e+f x)}) + 15 \sqrt{1 + e^{2 i (e+f x)}} \operatorname{Log}[1 - e^{i (e+f x)}] - \right. \right.$$

$$15 \sqrt{1 + e^{2 i (e+f x)}} \operatorname{Log}\left[1 + e^{i (e+f x)} + \sqrt{2} \sqrt{1 + e^{2 i (e+f x)}}\right] \left. \right) - 3 e^{\frac{1}{2} i (e+f x)} \sqrt{\operatorname{Sec}[e + f x]} +$$

$$23 e^{\frac{1}{2} i (e+f x)} \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^2 \sqrt{\operatorname{Sec}[e + f x]} - 71 e^{\frac{1}{2} i (e+f x)} \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^4 \sqrt{\operatorname{Sec}[e + f x]} \right) \operatorname{Sec}[e + f x]^{7/2} \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]$$

■ **Problem 105: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 212 leaves, 6 steps) :

$$-\frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e+f x]}{\sqrt{2} \sqrt{c-c \operatorname{Sec}[e+f x]}}\right]}{16 \sqrt{2} a^3 c^{3/2} f} - \frac{7 \operatorname{Tan}[e + f x]}{16 a^3 f (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{\operatorname{Tan}[e + f x]}{5 f (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{3/2}} +$$

$$\frac{7 \operatorname{Tan}[e + f x]}{30 a f (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{7 \operatorname{Tan}[e + f x]}{12 f (a^3 + a^3 \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^{3/2}}$$

Result (type 3, 417 leaves) :

$$-\left(7 e^{-\frac{1}{2} i (e+f x)} \sqrt{\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}}} \sqrt{1 + e^{2 i (e+f x)}} \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \right.$$

$$\left. \left(\operatorname{Log}[1 - e^{i (e+f x)}] - \operatorname{Log}\left[1 + e^{i (e+f x)} + \sqrt{2} \sqrt{1 + e^{2 i (e+f x)}}\right] \right) \operatorname{Sec}[e + f x]^{9/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \right) / \left(f (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{3/2} \right) +$$

$$\left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \operatorname{Sec}[e + f x]^5 \left(-\frac{278 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right]}{15 f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]}{f} + \frac{242 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]}{15 f} - \frac{56 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{15 f} + \frac{2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^5}{5 f} + \right.$$

$$\left. \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Sin}\left[\frac{f x}{2}\right]}{f} + \frac{278 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]}{15 f} \right) \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^3 \right) / \left((a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{3/2} \right)$$

■ **Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 246 leaves, 7 steps):

$$\begin{aligned} & - \frac{63 \operatorname{ArcTan}\left[\frac{\sqrt{c} \operatorname{Tan}[e + f x]}{\sqrt{2} \sqrt{c - c \operatorname{Sec}[e + f x]}}\right]}{128 \sqrt{2} a^3 c^{5/2} f} - \frac{21 \operatorname{Tan}[e + f x]}{32 a^3 f (c - c \operatorname{Sec}[e + f x])^{5/2}} + \frac{\operatorname{Tan}[e + f x]}{5 f (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{5/2}} + \\ & \frac{3 \operatorname{Tan}[e + f x]}{10 a f (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x])^{5/2}} + \frac{21 \operatorname{Tan}[e + f x]}{20 f (a^3 + a^3 \operatorname{Sec}[e + f x]) (c - c \operatorname{Sec}[e + f x])^{5/2}} - \frac{63 \operatorname{Tan}[e + f x]}{128 a^3 c f (c - c \operatorname{Sec}[e + f x])^{3/2}} \end{aligned}$$

Result (type 3, 487 leaves):

$$\begin{aligned} & \left(63 e^{-\frac{1}{2} i (e + f x)} \sqrt{\frac{e^{i (e + f x)}}{1 + e^{2 i (e + f x)}}} \sqrt{1 + e^{2 i (e + f x)}} \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \left(\operatorname{Log}[1 - e^{i (e + f x)}] - \operatorname{Log}[1 + e^{i (e + f x)} + \sqrt{2} \sqrt{1 + e^{2 i (e + f x)}}] \right) \right) \\ & \operatorname{Sec}[e + f x]^{11/2} \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \Big/ \left(4 f (a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{5/2} \right) + \\ & \left(\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \operatorname{Sec}[e + f x]^6 \left(\frac{257 \operatorname{Cos}\left[\frac{e}{2}\right] \operatorname{Cos}\left[\frac{f x}{2}\right]}{10 f} + \frac{23 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]}{4 f} - \frac{\operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{2 f} - \frac{124 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]}{5 f} + \right. \right. \\ & \left. \frac{22 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{5 f} - \frac{2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^5}{5 f} - \frac{23 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Sin}\left[\frac{f x}{2}\right]}{4 f} + \frac{\operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sin}\left[\frac{f x}{2}\right]}{2 f} - \frac{257 \operatorname{Sin}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]}{10 f} \right) \\ & \left. \operatorname{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \right) \Big/ \left((a + a \operatorname{Sec}[e + f x])^3 (c - c \operatorname{Sec}[e + f x])^{5/2} \right) \end{aligned}$$

■ **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{5/2} dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{a (c - c \operatorname{Sec}[e + f x])^{5/2} \operatorname{Tan}[e + f x]}{3 f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 3, 87 leaves):

$$\frac{1}{12 f} c^2 (5 - 6 \operatorname{Cos}[e + f x] + 3 \operatorname{Cos}[2 (e + f x)]) \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}[e + f x]^2 \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}$$

■ **Problem 110: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 51 leaves, 1 step):

$$\frac{a \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 99 leaves):

$$\frac{i (-1 + e^{i(e+fx)}) (2 \operatorname{Log}[1 - e^{i(e+fx)}] - \operatorname{Log}[1 + e^{2i(e+fx)}]) \sqrt{a(1 + \operatorname{Sec}[e + f x])}}{(1 + e^{i(e+fx)}) f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

■ **Problem 117: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 95 leaves, 2 steps):

$$\frac{2 a^2 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{a \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 174 leaves):

$$\left(\sqrt{2} a (1 + \operatorname{Cos}[e + f x]) (4 \operatorname{Log}[1 - e^{i(e+fx)}] - 2 \operatorname{Log}[1 + e^{2i(e+fx)}]) \operatorname{Sec}[e + f x]^{3/2} \sqrt{a(1 + \operatorname{Sec}[e + f x])} \right. \\ \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + i \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] \right) / \left((1 + e^{i(e+fx)}) \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} f \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 118: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^{3/2}}{(c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 99 leaves, 2 steps):

$$-\frac{a \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f (c - c \operatorname{Sec}[e + f x])^{3/2}} - \frac{a^2 \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 134 leaves):

$$- \left(a \left(2 - 2 \operatorname{Log} \left[1 - e^{i(e+fx)} \right] + \operatorname{Cos} [e+fx] \left(2 \operatorname{Log} \left[1 - e^{i(e+fx)} \right] - \operatorname{Log} \left[1 + e^{2i(e+fx)} \right] \right) + \operatorname{Log} \left[1 + e^{2i(e+fx)} \right] \right) \sqrt{a(1+\operatorname{Sec}[e+fx])} \operatorname{Tan} \left[\frac{1}{2}(e+fx) \right] \right) /$$

$$\left(cf(-1+\operatorname{Cos}[e+fx]) \sqrt{c-c\operatorname{Sec}[e+fx]} \right)$$

■ **Problem 126: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx] (a+a\operatorname{Sec}[e+fx])^{5/2} \sqrt{c-c\operatorname{Sec}[e+fx]} dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{c(a+a\operatorname{Sec}[e+fx])^{5/2} \operatorname{Tan}[e+fx]}{3f\sqrt{c-c\operatorname{Sec}[e+fx]}}$$

Result (type 3, 88 leaves):

$$\frac{1}{6f} a^2 \operatorname{Cot} \left[\frac{1}{2}(e+fx) \right] \left(2 + 4 \operatorname{Cos}[e+fx] + \operatorname{Cos}[e+fx]^2 \operatorname{Sec} \left[\frac{1}{2}(e+fx) \right]^2 \right) \operatorname{Sec}[e+fx]^2 \sqrt{a(1+\operatorname{Sec}[e+fx])} \sqrt{c-c\operatorname{Sec}[e+fx]}$$

■ **Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a\operatorname{Sec}[e+fx])^{5/2}}{\sqrt{c-c\operatorname{Sec}[e+fx]}} dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{4a^3 \operatorname{Log}[1-\operatorname{Sec}[e+fx]] \operatorname{Tan}[e+fx]}{f\sqrt{a+a\operatorname{Sec}[e+fx]} \sqrt{c-c\operatorname{Sec}[e+fx]}} + \frac{2a^2 \sqrt{a+a\operatorname{Sec}[e+fx]} \operatorname{Tan}[e+fx]}{f\sqrt{c-c\operatorname{Sec}[e+fx]}} + \frac{a(a+a\operatorname{Sec}[e+fx])^{3/2} \operatorname{Tan}[e+fx]}{2f\sqrt{c-c\operatorname{Sec}[e+fx]}}$$

Result (type 3, 328 leaves):

$$\left(4\sqrt{2} e^{\frac{1}{2}i(e+fx)} \sqrt{\frac{(1+e^{i(e+fx)})^2}{1+e^{2i(e+fx)}}} \left(2 \operatorname{Log} [1 - e^{i(e+fx)}] - \operatorname{Log} [1 + e^{2i(e+fx)}] \right) \sqrt{\operatorname{Sec}[e+fx]} (a(1+\operatorname{Sec}[e+fx]))^{5/2} \operatorname{Sin} \left[\frac{e}{2} + \frac{fx}{2} \right] \right) /$$

$$\left((1+e^{i(e+fx)}) \sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}} f(1+\operatorname{Sec}[e+fx])^{5/2} \sqrt{c-c\operatorname{Sec}[e+fx]} \right) +$$

$$\left(\operatorname{Sec}[e+fx] \sqrt{(1+\operatorname{Cos}[e+fx]) \operatorname{Sec}[e+fx]} (a(1+\operatorname{Sec}[e+fx]))^{5/2} \left(\frac{5 \operatorname{Sec} \left[\frac{e}{2} + \frac{fx}{2} \right]}{2f} + \frac{\operatorname{Cos} \left[\frac{e}{2} + \frac{fx}{2} \right] \operatorname{Sec}[e+fx]}{f} \right) \operatorname{Sin} \left[\frac{e}{2} + \frac{fx}{2} \right] \right) /$$

$$\left((1+\operatorname{Sec}[e+fx])^{5/2} \sqrt{c-c\operatorname{Sec}[e+fx]} \right)$$

■ **Problem 128: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a\operatorname{Sec}[e+fx])^{5/2}}{(c-c\operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 145 leaves, 3 steps) :

$$-\frac{a(a+a\sec[e+fx])^{3/2}\tan[e+fx]}{f(c-c\sec[e+fx])^{3/2}} - \frac{4a^3\log[1-\sec[e+fx]]\tan[e+fx]}{cf\sqrt{a+a\sec[e+fx]}\sqrt{c-c\sec[e+fx]}} - \frac{2a^2\sqrt{a+a\sec[e+fx]}\tan[e+fx]}{cf\sqrt{c-c\sec[e+fx]}}$$

Result (type 3, 188 leaves) :

$$\left(a^2 (1 - 4 \log[1 - e^{i(e+fx)}] + \cos[e+fx] (-5 + 8 \log[1 - e^{i(e+fx)}] - 4 \log[1 + e^{2i(e+fx)}]) + \right. \\ \left. 2 \log[1 + e^{2i(e+fx)}] + \cos[2(e+fx)] (-4 \log[1 - e^{i(e+fx)}] + 2 \log[1 + e^{2i(e+fx)}]) \right) \\ \sec[e+fx] \sqrt{a(1+\sec[e+fx])} \tan\left[\frac{1}{2}(e+fx)\right] \Big/ \left(cf(-1+\cos[e+fx])\sqrt{c-c\sec[e+fx]} \right)$$

■ **Problem 129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[e+fx](a+a\sec[e+fx])^{5/2}}{(c-c\sec[e+fx])^{5/2}} dx$$

Optimal (type 3, 145 leaves, 3 steps) :

$$-\frac{a(a+a\sec[e+fx])^{3/2}\tan[e+fx]}{2f(c-c\sec[e+fx])^{5/2}} + \frac{a^2\sqrt{a+a\sec[e+fx]}\tan[e+fx]}{cf(c-c\sec[e+fx])^{3/2}} + \frac{a^3\log[1-\sec[e+fx]]\tan[e+fx]}{c^2f\sqrt{a+a\sec[e+fx]}\sqrt{c-c\sec[e+fx]}}$$

Result (type 3, 182 leaves) :

$$-\left(a^2 (4 - 6 \log[1 - e^{i(e+fx)}] + \cos[e+fx] (8 \log[1 - e^{i(e+fx)}] - 4 \log[1 + e^{2i(e+fx)}]) + \right. \\ \left. 3 \log[1 + e^{2i(e+fx)}] + \cos[2(e+fx)] (-2 \log[1 - e^{i(e+fx)}] + \log[1 + e^{2i(e+fx)}]) \right) \\ \sqrt{a(1+\sec[e+fx])} \tan\left[\frac{1}{2}(e+fx)\right] \Big/ \left(2c^2f(-1+\cos[e+fx])^2\sqrt{c-c\sec[e+fx]} \right)$$

■ **Problem 133: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[e+fx](c-c\sec[e+fx])^{5/2}}{\sqrt{a+a\sec[e+fx]}} dx$$

Optimal (type 3, 139 leaves, 3 steps) :

$$-\frac{4c^3\log[1+\sec[e+fx]]\tan[e+fx]}{f\sqrt{a+a\sec[e+fx]}\sqrt{c-c\sec[e+fx]}} - \frac{2c^2\sqrt{c-c\sec[e+fx]}\tan[e+fx]}{f\sqrt{a+a\sec[e+fx]}} - \frac{c(c-c\sec[e+fx])^{3/2}\tan[e+fx]}{2f\sqrt{a+a\sec[e+fx]}}$$

Result (type 3, 141 leaves) :

$$\frac{1}{2 f \sqrt{a (1 + \operatorname{Sec}[e + f x])}}$$

$$c^2 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(1 - 6 \operatorname{Cos}[e + f x] + 8 \operatorname{Log}\left[1 + e^{i(e+f x)}\right] + \operatorname{Cos}\left[2(e + f x)\right] \left(8 \operatorname{Log}\left[1 + e^{i(e+f x)}\right] - 4 \operatorname{Log}\left[1 + e^{2i(e+f x)}\right]\right) - 4 \operatorname{Log}\left[1 + e^{2i(e+f x)}\right]\right)$$

$$\operatorname{Sec}[e + f x]^2 \sqrt{c - c \operatorname{Sec}[e + f x]}$$

- **Problem 134: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + a \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 94 leaves, 2 steps):

$$-\frac{2 c^2 \operatorname{Log}\left[1 + \operatorname{Sec}[e + f x]\right] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{c \sqrt{c - c \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]}}$$

Result (type 3, 173 leaves):

$$\left(c e^{-2i(e+f x)} \left(1 + e^{2i(e+f x)}\right)^2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] \left(-1 + \operatorname{Cos}[e + f x] \left(4 \operatorname{Log}\left[1 + e^{i(e+f x)}\right] - 2 \operatorname{Log}\left[1 + e^{2i(e+f x)}\right]\right)\right)\right)$$

$$\operatorname{Sec}[e + f x]^3 \sqrt{c - c \operatorname{Sec}[e + f x]} \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + i \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \Big/ \left(2 \left(1 + e^{i(e+f x)}\right) f \sqrt{a (1 + \operatorname{Sec}[e + f x])}\right)$$

- **Problem 135: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] \sqrt{c - c \operatorname{Sec}[e + f x]}}{\sqrt{a + a \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 50 leaves, 1 step):

$$-\frac{c \operatorname{Log}\left[1 + \operatorname{Sec}[e + f x]\right] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 118 leaves):

$$i \left(1 + e^{i(e+f x)}\right) \sqrt{\frac{c \left(-1 + e^{i(e+f x)}\right)^2}{1 + e^{2i(e+f x)}} \left(2 \operatorname{Log}\left[1 + e^{i(e+f x)}\right] - \operatorname{Log}\left[1 + e^{2i(e+f x)}\right]\right)}$$

$$\frac{\quad}{\left(-1 + e^{i(e+f x)}\right) f \sqrt{a (1 + \operatorname{Sec}[e + f x])}}$$

- **Problem 136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 115 leaves):

$$-\frac{2 i \left(-1 + e^{i(e+fx)}\right) \text{Cos}\left[\frac{1}{2}(e+fx)\right]^2 \left(\text{Log}\left[1 - e^{i(e+fx)}\right] - \text{Log}\left[1 + e^{i(e+fx)}\right]\right) \text{Sec}[e + f x]}{\left(1 + e^{i(e+fx)}\right) f \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]}}$$

■ **Problem 137: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{\sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 95 leaves, 3 steps):

$$-\frac{\text{Tan}[e + f x]}{2 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{3/2}} - \frac{\text{ArcTanh}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{2 c f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 129 leaves):

$$\left((-1 - \text{Log}[1 - e^{i(e+fx)}]) + \text{Cos}[e + f x] \left(\text{Log}[1 - e^{i(e+fx)}] - \text{Log}[1 + e^{i(e+fx)}] \right) + \text{Log}[1 + e^{i(e+fx)}] \right) \text{Tan}[e + f x] / \left(2 c f (-1 + \text{Cos}[e + f x]) \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]} \right)$$

■ **Problem 138: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{\sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{\text{Tan}[e + f x]}{4 f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{5/2}} - \frac{\text{Tan}[e + f x]}{4 c f \sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])^{3/2}} - \frac{\text{ArcTanh}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{4 c^2 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 176 leaves):

$$\left((4 + 3 \text{Log}[1 - e^{i(e+fx)}]) + \text{Cos}[2(e+fx)] \left(\text{Log}[1 - e^{i(e+fx)}] - \text{Log}[1 + e^{i(e+fx)}] \right) - 3 \text{Log}[1 + e^{i(e+fx)}] + \text{Cos}[e + f x] \left(-6 - 4 \text{Log}[1 - e^{i(e+fx)}] + 4 \text{Log}[1 + e^{i(e+fx)}] \right) \right) \text{Tan}[e + f x] / \left(8 c^2 f (-1 + \text{Cos}[e + f x])^2 \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]} \right)$$

■ **Problem 139: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x] (c - c \text{Sec}[e + f x])^{5/2}}{(a + a \text{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 142 leaves, 3 steps):

$$\frac{4 c^3 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{2 c^2 \sqrt{c - c \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]}} + \frac{c (c - c \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{3/2}}$$

Result (type 3, 183 leaves):

$$-\left(c^2 \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (-1 + 4 \operatorname{Log}[1 + e^{i(e + f x)}]) + \operatorname{Cos}[e + f x] (-5 + 8 \operatorname{Log}[1 + e^{i(e + f x)}]) - 4 \operatorname{Log}[1 + e^{2i(e + f x)}]) \right) + \left(\operatorname{Cos}[2(e + f x)] (4 \operatorname{Log}[1 + e^{i(e + f x)}]) - 2 \operatorname{Log}[1 + e^{2i(e + f x)}]) - 2 \operatorname{Log}[1 + e^{2i(e + f x)}]) \right) \operatorname{Sec}[e + f x] \sqrt{c - c \operatorname{Sec}[e + f x]} \Big/ \left(a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 140: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x] (c - c \operatorname{Sec}[e + f x])^{3/2}}{(a + a \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 95 leaves, 2 steps):

$$\frac{c^2 \operatorname{Log}[1 + \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c \sqrt{c - c \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])^{3/2}}$$

Result (type 3, 132 leaves):

$$-\left(c \operatorname{Cot}\left[\frac{1}{2}(e + f x)\right] (-2 + 2 \operatorname{Log}[1 + e^{i(e + f x)}]) + \operatorname{Cos}[e + f x] (2 \operatorname{Log}[1 + e^{i(e + f x)}]) - \operatorname{Log}[1 + e^{2i(e + f x)}]) - \operatorname{Log}[1 + e^{2i(e + f x)}]) \sqrt{c - c \operatorname{Sec}[e + f x]} \right) \Big/ \left(a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \right)$$

■ **Problem 142: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 95 leaves, 3 steps):

$$\frac{\operatorname{Tan}[e + f x]}{2 f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{2 a f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 129 leaves):

$$\left((-1 + \operatorname{Log}[1 - e^{i(e + f x)}]) + \operatorname{Cos}[e + f x] (\operatorname{Log}[1 - e^{i(e + f x)}]) - \operatorname{Log}[1 + e^{i(e + f x)}]) - \operatorname{Log}[1 + e^{i(e + f x)}]) \operatorname{Tan}[e + f x] \right) \Big/ \left(2 a f (1 + \operatorname{Cos}[e + f x]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 143: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{\text{Csc}[e + f x]}{2 a c f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} - \frac{\text{ArcTanh}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{2 a c f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 89 leaves):

$$\frac{\text{Csc}[e + f x] + \left(\text{Log}\left[1 - e^{i(e+fx)}\right] - \text{Log}\left[1 + e^{i(e+fx)}\right] \right) \text{Tan}[e + f x]}{2 a c f \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]}}$$

■ **Problem 144: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^{3/2} (c - c \text{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{3 \text{Csc}[e + f x]}{8 a c^2 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} - \frac{\text{Tan}[e + f x]}{4 f (a + a \text{Sec}[e + f x])^{3/2} (c - c \text{Sec}[e + f x])^{5/2}} - \frac{3 \text{ArcTanh}[\text{Cos}[e + f x]] \text{Tan}[e + f x]}{8 a c^2 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}$$

Result (type 3, 243 leaves):

$$\left((-2 + 6 \text{Log}[1 - e^{i(e+fx)}] + 3 \text{Cos}[3(e+fx)] \text{Log}[1 - e^{i(e+fx)}] - 2 \text{Cos}[2(e+fx)] (5 + 3 \text{Log}[1 - e^{i(e+fx)}] - 3 \text{Log}[1 + e^{i(e+fx)}]) - 6 \text{Log}[1 + e^{i(e+fx)}] - 3 \text{Cos}[3(e+fx)] \text{Log}[1 + e^{i(e+fx)}] + \text{Cos}[e + f x] (4 - 3 \text{Log}[1 - e^{i(e+fx)}] + 3 \text{Log}[1 + e^{i(e+fx)}])) \text{Tan}[e + f x] \right) / \left(32 a c^2 f (-1 + \text{Cos}[e + f x])^2 (1 + \text{Cos}[e + f x]) \sqrt{a(1 + \text{Sec}[e + f x])} \sqrt{c - c \text{Sec}[e + f x]} \right)$$

■ **Problem 145: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x] (c - c \text{Sec}[e + f x])^{5/2}}{(a + a \text{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 145 leaves, 3 steps):

$$-\frac{c^3 \text{Log}[1 + \text{Sec}[e + f x]] \text{Tan}[e + f x]}{a^2 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} - \frac{c^2 \sqrt{c - c \text{Sec}[e + f x]} \text{Tan}[e + f x]}{a f (a + a \text{Sec}[e + f x])^{3/2}} + \frac{c (c - c \text{Sec}[e + f x])^{3/2} \text{Tan}[e + f x]}{2 f (a + a \text{Sec}[e + f x])^{5/2}}$$

Result (type 3, 178 leaves):

$$\left(c^2 \text{Cot}\left[\frac{1}{2}(e + f x)\right] (-4 + 6 \text{Log}[1 + e^{i(e+fx)}] + \text{Cos}[e + f x] (8 \text{Log}[1 + e^{i(e+fx)}] - 4 \text{Log}[1 + e^{2i(e+fx)}]) + \text{Cos}[2(e + f x)] (2 \text{Log}[1 + e^{i(e+fx)}] - \text{Log}[1 + e^{2i(e+fx)}]) - 3 \text{Log}[1 + e^{2i(e+fx)}]) \sqrt{c - c \text{Sec}[e + f x]} \right) / \left(2 a^2 f (1 + \text{Cos}[e + f x])^2 \sqrt{a(1 + \text{Sec}[e + f x])} \right)$$

■ **Problem 148: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x])^{5/2} \sqrt{c - c \text{Sec}[e + f x]}} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$\frac{\operatorname{Tan}[e + f x]}{4 f (a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Tan}[e + f x]}{4 a f (a + a \operatorname{Sec}[e + f x])^{3/2} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{4 a^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 176 leaves):

$$\left((-4 + 3 \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e + f x] (-6 + 4 \operatorname{Log}[1 - e^{i(e+fx)}] - 4 \operatorname{Log}[1 + e^{i(e+fx)}]) + \operatorname{Cos}[2(e + f x)] (\operatorname{Log}[1 - e^{i(e+fx)}] - \operatorname{Log}[1 + e^{i(e+fx)}]) - 3 \operatorname{Log}[1 + e^{i(e+fx)}]) \operatorname{Tan}[e + f x] \right) / \left(8 a^2 f (1 + \operatorname{Cos}[e + f x])^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 149: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{3 \operatorname{Csc}[e + f x]}{8 a^2 c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{\operatorname{Tan}[e + f x]}{4 f (a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^{3/2}} - \frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{8 a^2 c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 242 leaves):

$$- \left((2 + 6 \operatorname{Log}[1 - e^{i(e+fx)}] - 3 \operatorname{Cos}[3(e + f x)] \operatorname{Log}[1 - e^{i(e+fx)}] + \operatorname{Cos}[e + f x] (4 + 3 \operatorname{Log}[1 - e^{i(e+fx)}] - 3 \operatorname{Log}[1 + e^{i(e+fx)}]) - 6 \operatorname{Log}[1 + e^{i(e+fx)}] + 3 \operatorname{Cos}[3(e + f x)] \operatorname{Log}[1 + e^{i(e+fx)}] + \operatorname{Cos}[2(e + f x)] (10 - 6 \operatorname{Log}[1 - e^{i(e+fx)}] + 6 \operatorname{Log}[1 + e^{i(e+fx)}]) \right) \operatorname{Tan}[e + f x] / \left(32 a^2 c f (-1 + \operatorname{Cos}[e + f x]) (1 + \operatorname{Cos}[e + f x])^2 \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right)$$

■ **Problem 150: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 160 leaves, 4 steps):

$$\frac{3 \operatorname{Csc}[e + f x]}{8 a^2 c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{\operatorname{Cot}[e + f x]^2 \operatorname{Csc}[e + f x]}{4 a^2 c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{8 a^2 c^2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 105 leaves):

$$\frac{(1 - 5 \operatorname{Cos}[2(e + f x)]) \operatorname{Csc}[e + f x]^3 + 6 (\operatorname{Log}[1 - e^{i(e+fx)}] - \operatorname{Log}[1 + e^{i(e+fx)}]) \operatorname{Tan}[e + f x]}{16 a^2 c^2 f \sqrt{a(1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

■ **Problem 151: Unable to integrate problem.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m (c - c \operatorname{Sec}[e + f x])^n dx$$

Optimal (type 5, 101 leaves, 3 steps):

$$-\frac{1}{f(1+2m)} 2^{\frac{1}{2}+n} c \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+m, \frac{1}{2}-n, \frac{3}{2}+m, \frac{1}{2}(1+\operatorname{Sec}[e+fx])\right] (1-\operatorname{Sec}[e+fx])^{\frac{1}{2}-n} (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{-1+n} \operatorname{Tan}[e+fx]$$

Result (type 8, 34 leaves):

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^n dx$$

■ **Problem 154: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m}{c-c \operatorname{Sec}[e+fx]} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$-\frac{1}{f(c-c \operatorname{Sec}[e+fx])} 2^{\frac{1}{2}+m} a \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{1}{2}(1-\operatorname{Sec}[e+fx])\right] (1+\operatorname{Sec}[e+fx])^{\frac{1}{2}-m} (a+a \operatorname{Sec}[e+fx])^{-1+m} \operatorname{Tan}[e+fx]$$

Result (type 8, 34 leaves):

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m}{c-c \operatorname{Sec}[e+fx]} dx$$

■ **Problem 155: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m}{(c-c \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 5, 92 leaves, 3 steps):

$$-\frac{1}{3f(c-c \operatorname{Sec}[e+fx])^2} 2^{\frac{1}{2}+m} a \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{1}{2}(1-\operatorname{Sec}[e+fx])\right] (1+\operatorname{Sec}[e+fx])^{\frac{1}{2}-m} (a+a \operatorname{Sec}[e+fx])^{-1+m} \operatorname{Tan}[e+fx]$$

Result (type 8, 34 leaves):

$$\int \frac{\operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m}{(c-c \operatorname{Sec}[e+fx])^2} dx$$

■ **Problem 156: Unable to integrate problem.**

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{5/2} dx$$

Optimal (type 3, 160 leaves, 3 steps):

$$-\frac{64 c^3 (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]}{f (5 + 2 m) (3 + 8 m + 4 m^2) \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{16 c^2 (a + a \operatorname{Sec}[e + f x])^m \sqrt{c - c \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f (15 + 16 m + 4 m^2)} - \frac{2 c (a + a \operatorname{Sec}[e + f x])^m (c - c \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{f (5 + 2 m)}$$

Result (type 8, 36 leaves):

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m (c - c \operatorname{Sec}[e + f x])^{5/2} dx$$

■ **Problem 157: Unable to integrate problem.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m (c - c \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 3, 100 leaves, 2 steps):

$$-\frac{8 c^2 (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]}{f (3 + 8 m + 4 m^2) \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{2 c (a + a \operatorname{Sec}[e + f x])^m \sqrt{c - c \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f (3 + 2 m)}$$

Result (type 8, 36 leaves):

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m (c - c \operatorname{Sec}[e + f x])^{3/2} dx$$

■ **Problem 158: Unable to integrate problem.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 46 leaves, 1 step):

$$-\frac{2 c (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]}{f (1 + 2 m) \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 8, 36 leaves):

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

■ **Problem 159: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^m}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 5, 69 leaves, 2 steps):

$$-\frac{\operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Sec}[e + f x])\right] (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]}{f (1 + 2 m) \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 8, 36 leaves):

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^m}{\sqrt{c - c \sec[e + f x]}} dx$$

■ **Problem 160: Unable to integrate problem.**

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^m}{(c - c \sec[e + f x])^{3/2}} dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[2, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sec[e + f x])\right] (a + a \sec[e + f x])^m \tan[e + f x]}{2 c f (1 + 2 m) \sqrt{c - c \sec[e + f x]}}$$

Result (type 8, 36 leaves):

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^m}{(c - c \sec[e + f x])^{3/2}} dx$$

■ **Problem 161: Unable to integrate problem.**

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^m}{(c - c \sec[e + f x])^{5/2}} dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{\text{Hypergeometric2F1}\left[3, \frac{1}{2} + m, \frac{3}{2} + m, \frac{1}{2} (1 + \sec[e + f x])\right] (a + a \sec[e + f x])^m \tan[e + f x]}{4 c^2 f (1 + 2 m) \sqrt{c - c \sec[e + f x]}}$$

Result (type 8, 36 leaves):

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^m}{(c - c \sec[e + f x])^{5/2}} dx$$

■ **Problem 162: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[e + f x] (a + a \sec[e + f x])^m (c - c \sec[e + f x])^{-3-m} dx$$

Optimal (type 3, 169 leaves, 3 steps):

$$\frac{(a + a \sec[e + f x])^m (c - c \sec[e + f x])^{-3-m} \tan[e + f x]}{f (1 + 2 m)} + \frac{2 (a + a \sec[e + f x])^{1+m} (c - c \sec[e + f x])^{-3-m} \tan[e + f x]}{a f (3 + 8 m + 4 m^2)} - \frac{2 (a + a \sec[e + f x])^{2+m} (c - c \sec[e + f x])^{-3-m} \tan[e + f x]}{a^2 f (1 + 2 m) (15 + 16 m + 4 m^2)}$$

Result (type 3, 321 leaves):

$$\begin{aligned}
& - \frac{1}{(-1 + e^{i(e+fx)})^5 f (1+2m) (3+2m) (5+2m)} i 2^{3+m} \left(-i e^{-\frac{1}{2}i(e+fx)} (-1 + e^{i(e+fx)}) \right)^{-2m} (1 + e^{i(e+fx)}) \left(\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}} \right)^{-m} \\
& \left(\frac{(1 + e^{i(e+fx)})^2}{1 + e^{2i(e+fx)}} \right)^m (7 + 12m + 4m^2 - 4e^{i(e+fx)}(3+2m) - 4e^{3i(e+fx)}(3+2m) + e^{4i(e+fx)}(7 + 12m + 4m^2) + e^{2i(e+fx)}(22 + 24m + 8m^2)) \\
& \text{Sec}[e+fx]^{3+m} (1 + \text{Sec}[e+fx])^{-m} (a(1 + \text{Sec}[e+fx]))^m (c - c \text{Sec}[e+fx])^{-3-m} \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]^{-2(-3-m)}
\end{aligned}$$

- **Problem 163: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e+fx] (a + a \text{Sec}[e+fx])^m (c - c \text{Sec}[e+fx])^{-2-m} dx$$

Optimal (type 3, 104 leaves, 2 steps):

$$- \frac{(a + a \text{Sec}[e+fx])^m (c - c \text{Sec}[e+fx])^{-2-m} \text{Tan}[e+fx]}{f(1+2m)} + \frac{(a + a \text{Sec}[e+fx])^{1+m} (c - c \text{Sec}[e+fx])^{-2-m} \text{Tan}[e+fx]}{af(3+8m+4m^2)}$$

Result (type 3, 250 leaves):

$$\begin{aligned}
& \left(i 2^{3+m} \left(-i e^{-\frac{1}{2}i(e+fx)} (-1 + e^{i(e+fx)}) \right)^{-2m} (1 + e^{i(e+fx)}) \left(\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}} \right)^{-m} \left(\frac{(1 + e^{i(e+fx)})^2}{1 + e^{2i(e+fx)}} \right)^m (1 - e^{i(e+fx)} + m + e^{2i(e+fx)}(1+m)) \text{Sec}[e+fx]^{2+m} \right. \\
& \left. (1 + \text{Sec}[e+fx])^{-m} (a(1 + \text{Sec}[e+fx]))^m (c - c \text{Sec}[e+fx])^{-2-m} \text{Sin}\left[\frac{1}{2}(e+fx)\right]^{2(2+m)} \right) / \left((-1 + e^{i(e+fx)})^3 f(1+2m)(3+2m) \right)
\end{aligned}$$

- **Problem 164: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e+fx] (a + a \text{Sec}[e+fx])^m (c - c \text{Sec}[e+fx])^{-1-m} dx$$

Optimal (type 3, 47 leaves, 1 step):

$$- \frac{(a + a \text{Sec}[e+fx])^m (c - c \text{Sec}[e+fx])^{-1-m} \text{Tan}[e+fx]}{f(1+2m)}$$

Result (type 3, 208 leaves):

$$\begin{aligned}
& - \frac{1}{f+2fm} 2^{1+m} e^{-\frac{1}{2}i(e+fx)} \left(-i e^{-\frac{1}{2}i(e+fx)} (-1 + e^{i(e+fx)}) \right)^{-1-2m} (1 + e^{i(e+fx)}) \left(\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}} \right)^{-m} \\
& \left(\frac{(1 + e^{i(e+fx)})^2}{1 + e^{2i(e+fx)}} \right)^m \text{Sec}[e+fx]^{1+m} (1 + \text{Sec}[e+fx])^{-m} (a(1 + \text{Sec}[e+fx]))^m (c - c \text{Sec}[e+fx])^{-1-m} \text{Sin}\left[\frac{1}{2}(e+fx)\right]^{2(1+m)}
\end{aligned}$$

- **Problem 165: Unable to integrate problem.**

$$\int \text{Sec}[e+fx] (a + a \text{Sec}[e+fx])^m (c - c \text{Sec}[e+fx])^{-m} dx$$

Optimal (type 5, 101 leaves, 3 steps) :

$$-\frac{1}{f(1+2m)} 2^{\frac{1}{2}-m} c \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+m, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\operatorname{Sec}[e+fx])\right] (1-\operatorname{Sec}[e+fx])^{\frac{1}{2}+m} (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{-1-m} \operatorname{Tan}[e+fx]$$

Result (type 8, 36 leaves) :

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{-m} dx$$

■ **Problem 166: Unable to integrate problem.**

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{1-m} dx$$

Optimal (type 5, 99 leaves, 3 steps) :

$$-\frac{1}{f(1+2m)} 2^{\frac{3}{2}-m} c \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}+m, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\operatorname{Sec}[e+fx])\right] (1-\operatorname{Sec}[e+fx])^{-\frac{1}{2}+m} (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{-m} \operatorname{Tan}[e+fx]$$

Result (type 8, 38 leaves) :

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{1-m} dx$$

■ **Problem 167: Unable to integrate problem.**

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{2-m} dx$$

Optimal (type 5, 101 leaves, 3 steps) :

$$-\frac{1}{f(1+2m)} 2^{\frac{5}{2}-m} c^2 \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}+m, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\operatorname{Sec}[e+fx])\right] (1-\operatorname{Sec}[e+fx])^{-\frac{1}{2}+m} (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{-m} \operatorname{Tan}[e+fx]$$

Result (type 8, 38 leaves) :

$$\int \operatorname{Sec}[e+fx] (a+a \operatorname{Sec}[e+fx])^m (c-c \operatorname{Sec}[e+fx])^{2-m} dx$$

■ **Problem 168: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^2 (a+a \operatorname{Sec}[e+fx])^3 (c-c \operatorname{Sec}[e+fx]) dx$$

Optimal (type 3, 105 leaves, 10 steps) :

$$\frac{a^3 c \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{4 f} + \frac{a^3 c \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{4 f} - \frac{a^3 c \operatorname{Sec}[e + f x]^3 \operatorname{Tan}[e + f x]}{2 f} - \frac{2 a^3 c \operatorname{Tan}[e + f x]^3}{3 f} - \frac{a^3 c \operatorname{Tan}[e + f x]^5}{5 f}$$

Result (type 3, 276 leaves):

$$\begin{aligned} & - \frac{a^3 c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{4 f} + \frac{a^3 c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{4 f} - \\ & \frac{a^3 c}{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} + \frac{a^3 c}{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{a^3 c}{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{a^3 c}{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} + \frac{7 a^3 c \operatorname{Tan}[e + f x]}{15 f} - \frac{4 a^3 c \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{15 f} - \frac{a^3 c \operatorname{Sec}[e + f x]^4 \operatorname{Tan}[e + f x]}{5 f} \end{aligned}$$

■ **Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^2 (c - c \operatorname{Sec}[e + f x])}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{2 c \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a f} - \frac{c \operatorname{Tan}[e + f x]}{a f} - \frac{2 c \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])}$$

Result (type 3, 154 leaves):

$$\begin{aligned} & - \frac{1}{a} c \left(\frac{2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} - \frac{2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{f} + \right. \\ & \left. \frac{\operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]}{f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{\operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]}{f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} + \frac{2 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{f} \right) \end{aligned}$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^2 (c - c \operatorname{Sec}[e + f x])}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$- \frac{c \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^2 f} + \frac{7 c \operatorname{Tan}[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])} - \frac{2 c \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2}$$

Result (type 3, 335 leaves):

$$\frac{1}{6 a^2 f (1 + \operatorname{Sec}[e + f x])^2} c \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e + f x]^2$$

$$\left(3 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 3 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) +$$

$$9 \operatorname{Cos}\left[\frac{f x}{2}\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) +$$

$$9 \operatorname{Cos}\left[e + \frac{f x}{2}\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) -$$

$$3 \operatorname{Cos}\left[e + \frac{3 f x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] -$$

$$3 \operatorname{Cos}\left[2 e + \frac{3 f x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 24 \operatorname{Sin}\left[\frac{f x}{2}\right] - 6 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 10 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right]$$

■ **Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (g \operatorname{Sec}[e + f x])^p (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]) dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$\frac{a^2 c (\operatorname{Cos}[e + f x]^2)^{\frac{3+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{3+p}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2\right] (g \operatorname{Sec}[e + f x])^p \operatorname{Tan}[e + f x]^3}{3 f}$$

$$\frac{a^2 c (\operatorname{Cos}[e + f x]^2)^{\frac{4+p}{2}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{4+p}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2\right] (g \operatorname{Sec}[e + f x])^{1+p} \operatorname{Tan}[e + f x]^3}{3 f g}$$

Result (type 6, 13496 leaves):

$$\frac{1}{32 f} \operatorname{Cos}[e + f x]^4 (\operatorname{Cos}[e + f x]^2)^{\frac{1}{2}(-1+p)} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+p}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2\right]$$

$$\operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (g \operatorname{Sec}[e + f x])^p (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]) \operatorname{Sin}[e + f x] + \frac{1}{16 f}$$

$$\operatorname{Cos}[e + f x]^3 (\operatorname{Cos}[e + f x]^2)^{p/2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+p}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (g \operatorname{Sec}[e + f x])^p$$

$$(a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]) \operatorname{Sin}[e + f x] - \left(3 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}[e + f x]^{-3-p} (g \operatorname{Sec}[e + f x])^p\right.$$

$$\left. (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]) \left(-\operatorname{Sec}[e + f x]^{2+p} + 2 \operatorname{Cos}[2(e + f x)] \operatorname{Sec}[e + f x]^{2+p}\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2}\right)^p\right.$$

$$\left. \left(\left(4 \operatorname{AppellF1}\left[\frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)^2\right)\right) / \left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)\right)$$

$$\begin{aligned}
& \left(2 \operatorname{AppellF1} \left[\frac{1}{2}, 2+p, -p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(\frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, 2+p, 1-p, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3} (2+p) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, 3+p, -p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{2}{3} \right. \right. \\
& \quad \left. \left. \left(p \operatorname{AppellF1} \left[\frac{3}{2}, 2+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (2+p) \operatorname{AppellF1} \left[\frac{3}{2}, 3+p, -p, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \frac{2}{3} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right. \right. \\
& \quad \left. \left. \left(p \left(-\frac{3}{5} (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 2+p, 2-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{5} (2+p) \operatorname{AppellF1} \left[\frac{5}{2}, 3+p, 1-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) + \right. \\
& \quad \left. (2+p) \left(\frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 3+p, 1-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} (3+p) \operatorname{AppellF1} \left[\frac{5}{2}, 4+p, -p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \right) \right) / \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, 2+p, -p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + \frac{2}{3} \left(p \operatorname{AppellF1} \left[\frac{3}{2}, 2+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] + (2+p) \operatorname{AppellF1} \left[\frac{3}{2}, 3+p, -p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) \right) \right) \right) + \\
& \left(\operatorname{Cos} [2 (e+fx)] \operatorname{Csc} \left[\frac{e+fx}{2} \right]^2 \operatorname{Sec} \left[\frac{e+fx}{2} \right]^4 (g \operatorname{Sec} [e+fx])^p (a+a \operatorname{Sec} [e+fx])^2 \right. \\
& \quad \left. \operatorname{Sec} [\right. \\
& \quad \left. e+fx) \right] \operatorname{Tan} \left[\frac{1}{2} (e+ \right. \\
& \quad \left. f x) \right] \left(\frac{1+\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2}{1-\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2} \right)^p \\
& \quad \left. \left(- \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, 1+p, -p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right] \left(-1+\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, p, -p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + 2 p \left(\operatorname{AppellF1} \left[\frac{3}{2}, p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \operatorname{AppellF1} \left[\frac{3}{2}, 1+p, -p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 + \right. \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Cos} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \quad \left. \left(2 \left((-1+p) \operatorname{AppellF1} \left[\frac{3}{2}, p, 2-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. p \operatorname{AppellF1} \left[\frac{3}{2}, 1+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + 3 \left(-\frac{1}{3} (1-p) \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, p, 2-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \frac{1}{3} p \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 1+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) + 2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \\
& \quad \left((-1+p) \left(-\frac{3}{5} (2-p) \operatorname{AppellF1} \left[\frac{5}{2}, p, 3-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} p \operatorname{AppellF1} \left[\frac{5}{2}, 1+p, 2-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) + \\
& \quad p \left(-\frac{3}{5} (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 1+p, 2-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] + \right. \\
& \quad \left. \left. \frac{3}{5} (1+p) \operatorname{AppellF1} \left[\frac{5}{2}, 2+p, 1-p, \frac{7}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) \right) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + 2 \left((-1+p) \operatorname{AppellF1} \left[\frac{3}{2}, p, 2-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] + p \operatorname{AppellF1} \left[\frac{3}{2}, 1+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (e + f x) \right]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 177: Unable to integrate problem.**

$$\int \frac{(g \operatorname{Sec}[e + f x])^p (c - c \operatorname{Sec}[e + f x])}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 5, 226 leaves, 7 steps):

$$\begin{aligned}
& - \frac{c g (3 - 4 p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-p}{2}, \frac{3-p}{2}, \cos[e + f x]^2\right] (g \operatorname{Sec}[e + f x])^{-1+p} \sin[e + f x]}{3 a^2 f \sqrt{\sin[e + f x]^2}} + \\
& \frac{c (5 - 4 p) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{p}{2}, \frac{2-p}{2}, \cos[e + f x]^2\right] (g \operatorname{Sec}[e + f x])^p \sin[e + f x]}{3 a^2 f \sqrt{\sin[e + f x]^2}} - \\
& \frac{c (5 - 4 p) (g \operatorname{Sec}[e + f x])^p \tan[e + f x]}{3 a^2 f (1 + \operatorname{Sec}[e + f x])} - \frac{2 c (g \operatorname{Sec}[e + f x])^p \tan[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2}
\end{aligned}$$

Result (type 8, 36 leaves):

$$\int \frac{(g \operatorname{Sec}[e + f x])^p (c - c \operatorname{Sec}[e + f x])}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

■ **Problem 180: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^{5/2}}{\sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])} dx$$

Optimal (type 3, 140 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{\sqrt{a} c f} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[e + f x]} \sin[e + f x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[e + f x]}}\right]}{\sqrt{2} \sqrt{a} c f} + \frac{\operatorname{Csc}[e + f x] \sqrt{a + a \operatorname{Sec}[e + f x]}}{a c f \sqrt{\operatorname{Sec}[e + f x]}}
\end{aligned}$$

Result (type 3, 724 leaves):

$$\begin{aligned}
& \left(\text{Sec}[e + f x]^{3/2} \sqrt{(1 + \text{Cos}[e + f x]) \text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \right. \\
& \left. \left(-\frac{2 \text{Cot}[e]}{f} + \frac{\text{Csc}\left[\frac{e}{2}\right] \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right] \text{Sin}\left[\frac{f x}{2}\right]}{f} + \frac{\text{Sec}\left[\frac{e}{2}\right] \text{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right] \text{Sin}\left[\frac{f x}{2}\right]}{f} \right) \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \right) / \left(\sqrt{a(1 + \text{Sec}[e + f x])} (c - c \text{Sec}[e + f x]) \right) + \\
& \left(\text{Cos}[e + f x] \left(\text{Log}\left[1 - 2 \text{Sec}[e + f x] - 3 \text{Sec}[e + f x]^2 - 2 \sqrt{2} \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \sqrt{-1 + \text{Sec}[e + f x]^2}\right] - \right. \right. \\
& \quad \left. \left. \text{Log}\left[1 - 2 \text{Sec}[e + f x] - 3 \text{Sec}[e + f x]^2 + 2 \sqrt{2} \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \sqrt{-1 + \text{Sec}[e + f x]^2}\right] \right) \right) - \\
& \left((1 + \text{Sec}[e + f x])^{3/2} \sqrt{-1 + \text{Sec}[e + f x]^2} \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sin}[e + f x] \right) / \\
& \left(2 f (1 + \text{Cos}[e + f x]) \sqrt{2 - 2 \text{Cos}[e + f x]^2} \sqrt{1 - \text{Cos}[e + f x]^2} \sqrt{a(1 + \text{Sec}[e + f x])} (c - c \text{Sec}[e + f x]) \right) + \\
& \left(\text{Cos}[e + f x] \left(-8 \text{Log}[1 + \text{Sec}[e + f x]] + 8 \text{Log}\left[\sqrt{\text{Sec}[e + f x]} + \text{Sec}[e + f x]^{3/2} + \sqrt{1 + \text{Sec}[e + f x]} \sqrt{-1 + \text{Sec}[e + f x]^2}\right] \right) + \right. \\
& \quad \left. \sqrt{2} \left(-\text{Log}\left[1 - 2 \text{Sec}[e + f x] - 3 \text{Sec}[e + f x]^2 - 2 \sqrt{2} \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \sqrt{-1 + \text{Sec}[e + f x]^2}\right] + \right. \right. \\
& \quad \left. \left. \text{Log}\left[1 - 2 \text{Sec}[e + f x] - 3 \text{Sec}[e + f x]^2 + 2 \sqrt{2} \sqrt{\text{Sec}[e + f x]} \sqrt{1 + \text{Sec}[e + f x]} \sqrt{-1 + \text{Sec}[e + f x]^2}\right] \right) \right) (1 + \text{Sec}[e + f x])^{3/2} \\
& \left. \sqrt{-1 + \text{Sec}[e + f x]^2} \text{Sin}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sin}[e + f x] \right) / \left(2 f (1 + \text{Cos}[e + f x]) (1 - \text{Cos}[e + f x]^2) \sqrt{a(1 + \text{Sec}[e + f x])} (c - c \text{Sec}[e + f x]) \right)
\end{aligned}$$

■ **Problem 181: Result more than twice size of optimal antiderivative.**

$$\int \frac{(g \text{Sec}[e + f x])^{3/2}}{\sqrt{a + a \text{Sec}[e + f x]} (c - c \text{Sec}[e + f x])} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{g^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{g} \text{Tan}[e + f x]}{\sqrt{2} \sqrt{g \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}\right]}{\sqrt{2} \sqrt{a} c f} + \frac{g \text{Cot}[e + f x] \sqrt{g \text{Sec}[e + f x]} \sqrt{a + a \text{Sec}[e + f x]}}{a c f}$$

Result (type 3, 236 leaves):

$$\begin{aligned}
& - \left(a \cos\left[\frac{1}{2}(e+fx)\right] (g \sec[e+fx])^{5/2} \sin\left[\frac{1}{2}(e+fx)\right]^3 \left(-4 - 4 \sec[e+fx] + \right. \right. \\
& \quad \left. \left. 1 / \left(\sqrt{\sec\left[\frac{1}{2}(e+fx)\right]^2} \right) \left(\log\left[1 - 2 \sec[e+fx] - 3 \sec[e+fx]^2 - 2\sqrt{2} \sqrt{\sec[e+fx]} \sqrt{1 + \sec[e+fx]} \sqrt{\tan[e+fx]^2} \right] - \right. \right. \right. \\
& \quad \left. \left. \log\left[1 - 2 \sec[e+fx] - 3 \sec[e+fx]^2 + 2\sqrt{2} \sqrt{\sec[e+fx]} \sqrt{1 + \sec[e+fx]} \sqrt{\tan[e+fx]^2} \right] \right) \right. \\
& \quad \left. \left. \sqrt{\tan[e+fx]^2} \right) \right) / (c f g (-1 + \sec[e+fx])^2 (a (1 + \sec[e+fx]))^{3/2})
\end{aligned}$$

- **Problem 183: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^2}{\sqrt{a+a \sec[e+fx]} \sqrt{c-c \sec[e+fx]}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\log[\tan[e+fx]] \tan[e+fx]}{f \sqrt{a+a \sec[e+fx]} \sqrt{c-c \sec[e+fx]}}$$

Result (type 3, 129 leaves):

$$\begin{aligned}
& - \left(2i (-1 + e^{i(e+fx)}) \cos\left[\frac{1}{2}(e+fx)\right]^2 \left(\log[1 - e^{i(e+fx)}] + \log[1 + e^{i(e+fx)}] - \log[1 + e^{2i(e+fx)}] \right) \sec[e+fx] \right) / \\
& \quad \left((1 + e^{i(e+fx)}) f \sqrt{a(1 + \sec[e+fx])} \sqrt{c - c \sec[e+fx]} \right)
\end{aligned}$$

- **Problem 186: Result more than twice size of optimal antiderivative.**

$$\int \sec[e+fx] (a + a \sec[e+fx]) (c + d \sec[e+fx])^3 dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\begin{aligned}
& \frac{a(8c^3 + 12c^2d + 12cd^2 + 3d^3) \operatorname{ArcTanh}[\sin[e+fx]]}{8f} + \frac{a(3c^3 + 16c^2d + 12cd^2 + 4d^3) \tan[e+fx]}{6f} + \\
& \frac{ad(6c^2 + 20cd + 9d^2) \sec[e+fx] \tan[e+fx]}{24f} + \frac{a(3c + 4d)(c + d \sec[e+fx])^2 \tan[e+fx]}{12f} + \frac{a(c + d \sec[e+fx])^3 \tan[e+fx]}{4f}
\end{aligned}$$

Result (type 3, 1107 leaves):

■ **Problem 187: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^2 dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$\frac{a (2 c^2 + 2 c d + d^2) \text{ArcTanh}[\text{Sin}[e + f x]]}{2 f} + \frac{2 a (c^2 + 3 c d + d^2) \text{Tan}[e + f x]}{3 f} + \frac{a d (2 c + 3 d) \text{Sec}[e + f x] \text{Tan}[e + f x]}{6 f} + \frac{a (c + d \text{Sec}[e + f x])^2 \text{Tan}[e + f x]}{3 f}$$

Result (type 3, 240 leaves):

$$\frac{1}{24 f \left(-1 + \text{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \right)^3} + a \text{Sec}\left[\frac{1}{2} (e + f x)\right]^6 \left(9 (2 c^2 + 2 c d + d^2) \text{Cos}[e + f x] \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) + 3 (2 c^2 + 2 c d + d^2) \text{Cos}[3 (e + f x)] \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) - 4 (3 c^2 + 6 c d + 4 d^2 + 3 d (2 c + d) \text{Cos}[e + f x] + (3 c^2 + 6 c d + 2 d^2) \text{Cos}[2 (e + f x)]) \text{Sin}[e + f x] \right)$$

■ **Problem 188: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x]) dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{a (2 c + d) \text{ArcTanh}[\text{Sin}[e + f x]]}{2 f} + \frac{a (c + d) \text{Tan}[e + f x]}{f} + \frac{a d \text{Sec}[e + f x] \text{Tan}[e + f x]}{2 f}$$

Result (type 3, 154 leaves):

$$\frac{1}{4 f} a \left(-2 (2 c + d) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + 4 c \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + 2 d \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + \frac{d}{\left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} - \frac{d}{\left(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} + 4 (c + d) \text{Tan}[e + f x] \right)$$

■ **Problem 195: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^2 (c + d \text{Sec}[e + f x])^2 dx$$

Optimal (type 3, 176 leaves, 8 steps) :

$$\frac{a^2 (12 c^2 + 16 c d + 7 d^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{8 f} - \frac{a^2 (c^3 - 8 c^2 d - 20 c d^2 - 8 d^3) \operatorname{Tan}[e + f x]}{6 d f} - \frac{a^2 (2 c (c - 8 d) - 21 d^2) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{24 f} - \frac{a^2 (c - 8 d) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{12 d f} + \frac{a^2 (c + d \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{4 d f}$$

Result (type 3, 479 leaves) :

$$\begin{aligned} & -\frac{1}{192 f} a^2 \operatorname{Sec}[e + f x]^4 \left(108 c^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + \\ & 144 c d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 63 d^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\ & 12 (12 c^2 + 16 c d + 7 d^2) \operatorname{Cos}[2(e + f x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) + \\ & 3 (12 c^2 + 16 c d + 7 d^2) \operatorname{Cos}[4(e + f x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) - \\ & 108 c^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 144 c d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\ & 63 d^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 24 c^2 \operatorname{Sin}[e + f x] - 96 c d \operatorname{Sin}[e + f x] - 90 d^2 \operatorname{Sin}[e + f x] - \\ & 96 c^2 \operatorname{Sin}[2(e + f x)] - 224 c d \operatorname{Sin}[2(e + f x)] - 128 d^2 \operatorname{Sin}[2(e + f x)] - 24 c^2 \operatorname{Sin}[3(e + f x)] - \\ & 96 c d \operatorname{Sin}[3(e + f x)] - 42 d^2 \operatorname{Sin}[3(e + f x)] - 48 c^2 \operatorname{Sin}[4(e + f x)] - 80 c d \operatorname{Sin}[4(e + f x)] - 32 d^2 \operatorname{Sin}[4(e + f x)] \end{aligned}$$

■ **Problem 196: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^2 (c + d \operatorname{Sec}[e + f x]) dx$$

Optimal (type 3, 103 leaves, 6 steps) :

$$\frac{a^2 (3 c + 2 d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 f} + \frac{2 a^2 (3 c + 2 d) \operatorname{Tan}[e + f x]}{3 f} + \frac{a^2 (3 c + 2 d) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{6 f} + \frac{d (a + a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 f}$$

Result (type 3, 993 leaves) :

$$\begin{aligned}
& \frac{1}{8 f (d+c \cos [e+f x])} (-3 c-2 d) \cos [e+f x]^3 \operatorname{Log}\left[\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right] \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x]) + \\
& \frac{1}{8 f (d+c \cos [e+f x])} (3 c+2 d) \cos [e+f x]^3 \operatorname{Log}\left[\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right] \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x]) + \\
& \frac{d \cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x]) \sin \left[\frac{f x}{2}\right]}{24 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^3} + \\
& \left(\cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x])\left(3 c \cos \left[\frac{e}{2}\right]+7 d \cos \left[\frac{e}{2}\right]-3 c \sin \left[\frac{e}{2}\right]-5 d \sin \left[\frac{e}{2}\right]\right)\right) / \\
& \left(48 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2\right) + \\
& \frac{\cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x])\left(6 c \sin \left[\frac{f x}{2}\right]+5 d \sin \left[\frac{f x}{2}\right]\right)}{12 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]-\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)} + \\
& \frac{d \cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x]) \sin \left[\frac{f x}{2}\right]}{24 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^3} + \\
& \left(\cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x])\left(-3 c \cos \left[\frac{e}{2}\right]-7 d \cos \left[\frac{e}{2}\right]-3 c \sin \left[\frac{e}{2}\right]-5 d \sin \left[\frac{e}{2}\right]\right)\right) / \\
& \left(48 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)^2\right) + \\
& \frac{\cos [e+f x]^3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 (a+a \sec [e+f x])^2 (c+d \sec [e+f x])\left(6 c \sin \left[\frac{f x}{2}\right]+5 d \sin \left[\frac{f x}{2}\right]\right)}{12 f (d+c \cos [e+f x])\left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right)\left(\cos \left[\frac{e}{2}+\frac{f x}{2}\right]+\sin \left[\frac{e}{2}+\frac{f x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 197: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [e+f x] (a+a \sec [e+f x])^2}{c+d \sec [e+f x]} dx$$

Optimal (type 3, 95 leaves, 8 steps):

$$-\frac{a^2 (c-2 d) \operatorname{ArcTanh}[\sin [e+f x]]}{d^2 f} + \frac{2 a^2 (c-d)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \tan \left[\frac{1}{2}(e+f x)\right]}{\sqrt{c+d}}\right]}{d^2 \sqrt{c+d} f} + \frac{a^2 \tan [e+f x]}{d f}$$

Result (type 3, 329 leaves):

$$\frac{1}{4 d^2 f (c + d \operatorname{Sec}[e + f x])}$$

$$a^2 \operatorname{Cos}[e + f x] (d + c \operatorname{Cos}[e + f x]) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^4 (1 + \operatorname{Sec}[e + f x])^2 \left((c - 2d) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \right.$$

$$(c - 2d) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \frac{2 i (c - d)^2 \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e] + \operatorname{Sin}[e]) (c \operatorname{Sin}[e] + (-d + c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}}\right] (\operatorname{Cos}[e] - i \operatorname{Sin}[e])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}} +$$

$$\left. \frac{d \operatorname{Sin}\left[\frac{f x}{2}\right]}{(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right])} + \frac{d \operatorname{Sin}\left[\frac{f x}{2}\right]}{(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right])} \right)$$

- **Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^2}{(c + d \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 117 leaves, 8 steps):

$$\frac{a^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{d^2 f} - \frac{2 a^2 \sqrt{c - d} (c + 2d) \operatorname{ArcTanh}\left[\frac{\sqrt{c - d} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c + d}}\right]}{d^2 (c + d)^{3/2} f} - \frac{a^2 (c - d) \operatorname{Tan}[e + f x]}{d (c + d) f (c + d \operatorname{Sec}[e + f x])}$$

Result (type 3, 312 leaves):

$$\frac{1}{4 d^2 f (c + d \operatorname{Sec}[e + f x])^2} a^2 (d + c \operatorname{Cos}[e + f x]) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^4 (1 + \operatorname{Sec}[e + f x])^2$$

$$\left(- (d + c \operatorname{Cos}[e + f x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + (d + c \operatorname{Cos}[e + f x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \right.$$

$$\left. 2 (c^2 + c d - 2 d^2) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e] + \operatorname{Sin}[e]) (c \operatorname{Sin}[e] + (-d + c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}}\right] (d + c \operatorname{Cos}[e + f x]) (i \operatorname{Cos}[e] + \operatorname{Sin}[e]) \right) /$$

$$\left((c + d) \sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} + \frac{(c - d) d (d \operatorname{Sin}[e] - c \operatorname{Sin}[f x])}{c (c + d) (\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]) (\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right])} \right)$$

■ **Problem 199: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^2}{(c + d \text{Sec}[e + f x])^3} dx$$

Optimal (type 3, 130 leaves, 5 steps):

$$\frac{3 a^2 \text{ArcTanh}\left[\frac{\sqrt{c-d} \text{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{\sqrt{c-d} (c+d)^{5/2} f} + \frac{(a^2 + a^2 \text{Sec}[e + f x]) \text{Tan}[e + f x]}{2 (c+d) f (c+d \text{Sec}[e + f x])^2} + \frac{3 a^2 \text{Tan}[e + f x]}{2 (c+d)^2 f (c+d \text{Sec}[e + f x])}$$

Result (type 3, 249 leaves):

$$\left(\begin{aligned} & a^2 (d + c \text{Cos}[e + f x]) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^4 \text{Sec}[e + f x] (1 + \text{Sec}[e + f x])^2 \\ & - \frac{6 i \text{ArcTan}\left[\frac{(i \text{Cos}[e] + \text{Sin}[e]) (c \text{Sin}[e] + (-d + c \text{Cos}[e]) \text{Tan}\left[\frac{fx}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}}\right]}{\sqrt{c^2 - d^2} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2}} (d + c \text{Cos}[e + f x])^2 (\text{Cos}[e] - i \text{Sin}[e]) + \right. \\ & \left. \frac{(c - d) (c + d) \text{Sec}[e] (-d \text{Sin}[e] + c \text{Sin}[fx])}{c^2} + \right. \\ & \left. \frac{(d + c \text{Cos}[e + f x]) \text{Sec}[e] ((c^2 - 4 c d - 2 d^2) \text{Sin}[e] + c (4 c + d) \text{Sin}[fx])}{c^2} \right) / (8 (c + d)^2 f (c + d \text{Sec}[e + f x])^3) \end{aligned}$$

■ **Problem 204: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x] (a + a \text{Sec}[e + f x])^3 (c + d \text{Sec}[e + f x]) dx$$

Optimal (type 3, 125 leaves, 10 steps):

$$\frac{5 a^3 (4 c + 3 d) \text{ArcTanh}[\text{Sin}[e + f x]]}{8 f} + \frac{a^3 (4 c + 3 d) \text{Tan}[e + f x]}{f} + \frac{3 a^3 (4 c + 3 d) \text{Sec}[e + f x] \text{Tan}[e + f x]}{8 f} + \frac{d (a + a \text{Sec}[e + f x])^3 \text{Tan}[e + f x]}{4 f} + \frac{a^3 (4 c + 3 d) \text{Tan}[e + f x]^3}{12 f}$$

Result (type 3, 273 leaves):

$$\begin{aligned}
& - \frac{1}{1536 f} a^3 (1 + \cos[e + f x])^3 \sec\left[\frac{1}{2}(e + f x)\right]^6 \sec[e + f x]^4 \\
& \left(120 (4 c + 3 d) \cos[e + f x]^4 \left(\log\left[\cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right]\right] - \log\left[\cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right]\right] \right) - \right. \\
& \quad \sec[e] (-24 (11 c + 9 d) \sin[e] + (36 c + 69 d) \sin[f x] + 36 c \sin[2 e + f x] + 69 d \sin[2 e + f x] + \\
& \quad \quad 280 c \sin[e + 2 f x] + 264 d \sin[e + 2 f x] - 72 c \sin[3 e + 2 f x] - 24 d \sin[3 e + 2 f x] + 36 c \sin[2 e + 3 f x] + \\
& \quad \quad \left. 45 d \sin[2 e + 3 f x] + 36 c \sin[4 e + 3 f x] + 45 d \sin[4 e + 3 f x] + 88 c \sin[3 e + 4 f x] + 72 d \sin[3 e + 4 f x] \right)
\end{aligned}$$

■ **Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + f x] (a + a \sec[e + f x])^3}{c + d \sec[e + f x]} dx$$

Optimal (type 3, 153 leaves, 9 steps):

$$\begin{aligned}
& \frac{a^3 \operatorname{ArcTanh}[\sin[e + f x]]}{2 d f} + \frac{a^3 (c^2 - 3 c d + 3 d^2) \operatorname{ArcTanh}[\sin[e + f x]]}{d^3 f} - \\
& \frac{2 a^3 (c - d)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c+d}}\right]}{d^3 \sqrt{c+d} f} - \frac{a^3 (c - 3 d) \tan[e + f x]}{d^2 f} + \frac{a^3 \sec[e + f x] \tan^3[e + f x]}{2 d f}
\end{aligned}$$

Result (type 3, 419 leaves):

$$\frac{1}{32 d^3 f (c + d \operatorname{Sec}[e + f x])} a^3 \operatorname{Cos}[e + f x]^2 (d + c \operatorname{Cos}[e + f x]) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^6 (1 + \operatorname{Sec}[e + f x])^3$$

$$\left(-2 (2 c^2 - 6 c d + 7 d^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 2 (2 c^2 - 6 c d + 7 d^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) +$$

$$\frac{8 (c - d)^3 \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e] + \operatorname{Sin}[e]) (c \operatorname{Sin}[e] + (-d + c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}\right] (i \operatorname{Cos}[e] + \operatorname{Sin}[e])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}} +$$

$$\frac{d^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{4 (c - 3 d) d \operatorname{Sin}\left[\frac{f x}{2}\right]}{\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} -$$

$$\left. \frac{d^2}{\left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{4 (c - 3 d) d \operatorname{Sin}\left[\frac{f x}{2}\right]}{\left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)} \right)$$

■ **Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^3}{(c + d \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 161 leaves, 9 steps):

$$-\frac{a^3 (2 c - 3 d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{d^3 f} + \frac{2 a^3 (c - d)^{3/2} (2 c + 3 d) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{c+d}}\right]}{d^3 (c + d)^{3/2} f} +$$

$$\frac{2 a^3 c \operatorname{Tan}[e + f x]}{d^2 (c + d) f} - \frac{(c - d) (a^3 + a^3 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{d (c + d) f (c + d \operatorname{Sec}[e + f x])}$$

Result (type 3, 979 leaves):

$$\begin{aligned}
& \frac{1}{8 d^3 f (c+d \operatorname{Sec}[e+f x])^2} (2 c-3 d) \operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}+\frac{f x}{2}\right]-\operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right]\right] \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 + \\
& \frac{1}{8 d^3 f (c+d \operatorname{Sec}[e+f x])^2} (-2 c+3 d) \operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2}+\frac{f x}{2}\right]+\operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right]\right] \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 + \\
& \left((-c+d)^2 (2 c+3 d) \operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x])^2 \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 \right. \\
& \left. - \left(i \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]}} \right) \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right]+i c \operatorname{Sin}\left[e+\frac{f x}{2}\right] \right) \right) \operatorname{Cos}[e] \right) / \\
& \left(4 d^3 \sqrt{c^2-d^2} f \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]} \right) - \left(\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]}} \right) \right. \\
& \left. \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right]+i c \operatorname{Sin}\left[e+\frac{f x}{2}\right] \right) \right) \operatorname{Sin}[e] \right) / \left(4 d^3 \sqrt{c^2-d^2} f \sqrt{\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]} \right) \Bigg) / \\
& \left((c+d) (c+d \operatorname{Sec}[e+f x])^2 \right) + \left(\operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x]) \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 \right. \\
& \left. \left(-c^2 d \operatorname{Sin}[e]+2 c d^2 \operatorname{Sin}[e]-d^3 \operatorname{Sin}[e]+c^3 \operatorname{Sin}[f x]-2 c^2 d \operatorname{Sin}[f x]+c d^2 \operatorname{Sin}[f x] \right) \right) / \\
& \left(8 c d^2 (c+d) f (c+d \operatorname{Sec}[e+f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]-\operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}\right]+\operatorname{Sin}\left[\frac{e}{2}\right] \right) \right) + \\
& \frac{\operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x])^2 \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{8 d^2 f (c+d \operatorname{Sec}[e+f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]-\operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}+\frac{f x}{2}\right]-\operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right] \right)} + \\
& \frac{\operatorname{Cos}[e+f x] (d+c \operatorname{Cos}[e+f x])^2 \operatorname{Sec}\left[\frac{e}{2}+\frac{f x}{2}\right]^6 (a+a \operatorname{Sec}[e+f x])^3 \operatorname{Sin}\left[\frac{f x}{2}\right]}{8 d^2 f (c+d \operatorname{Sec}[e+f x])^2 \left(\operatorname{Cos}\left[\frac{e}{2}\right]+\operatorname{Sin}\left[\frac{e}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{e}{2}+\frac{f x}{2}\right]+\operatorname{Sin}\left[\frac{e}{2}+\frac{f x}{2}\right] \right)}
\end{aligned}$$

■ **Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x] (a+a \operatorname{Sec}[e+f x])^3}{(c+d \operatorname{Sec}[e+f x])^3} dx$$

Optimal (type 3, 188 leaves, 9 steps):

$$\frac{a^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{d^3 f} - \frac{a^3 \sqrt{c-d} (2c^2 + 6cd + 7d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{d^3 (c+d)^{5/2} f} -$$

$$\frac{(c-d) (a^3 + a^3 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{2d (c+d) f (c+d \operatorname{Sec}[e + f x])^2} - \frac{a^3 (c-d) (2c+5d) \operatorname{Tan}[e + f x]}{2d^2 (c+d)^2 f (c+d \operatorname{Sec}[e + f x])}$$

Result (type 3, 393 leaves):

$$\frac{1}{32 d^3 f (c+d \operatorname{Sec}[e + f x])^3} a^3 (d + c \operatorname{Cos}[e + f x]) \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^6 (1 + \operatorname{Sec}[e + f x])^3$$

$$\left(-4 (d + c \operatorname{Cos}[e + f x])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 4 (d + c \operatorname{Cos}[e + f x])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \right.$$

$$\left. 4 (2c^3 + 4c^2d + cd^2 - 7d^3) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e] + \operatorname{Sin}[e]) (c \operatorname{Sin}[e] + (-d + c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{fx}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}\right] (d + c \operatorname{Cos}[e + f x])^2 (i \operatorname{Cos}[e] + \operatorname{Sin}[e]) \right) /$$

$$\left((c+d)^2 \sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) + \frac{1}{c^2 (c+d)^2} (c-d) d \operatorname{Sec}[e] \left((2c^4 + 6c^3d + 5c^2d^2 + 12cd^3 + 2d^4) \operatorname{Sin}[e] - \right.$$

$$\left. c (d (7c^2 + 18cd + 2d^2) \operatorname{Sin}[fx] - d (c^2 + 6cd + 2d^2) \operatorname{Sin}[2e + fx] + c (2c^2 + 6cd + d^2) \operatorname{Sin}[e + 2fx]) \right)$$

■ **Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (a + a \operatorname{Sec}[e + f x])^3}{(c + d \operatorname{Sec}[e + f x])^4} dx$$

Optimal (type 3, 178 leaves, 6 steps):

$$\frac{5 a^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{\sqrt{c-d} (c+d)^{7/2} f} + \frac{a (a + a \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 (c+d) f (c+d \operatorname{Sec}[e + f x])^3} - \frac{5 a^3 (c-d) \operatorname{Tan}[e + f x]}{6 d (c+d)^2 f (c+d \operatorname{Sec}[e + f x])^2} + \frac{5 a^3 (c+4d) \operatorname{Tan}[e + f x]}{6 d (c+d)^3 f (c+d \operatorname{Sec}[e + f x])}$$

Result (type 3, 398 leaves):

$$\frac{1}{192 (c+d)^3 f (c+d \operatorname{Sec}[e+f x])^4} a^3 (d+c \operatorname{Cos}[e+f x]) \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^6 \operatorname{Sec}[e+f x]$$

$$(1+\operatorname{Sec}[e+f x])^3 \left(-\frac{120 i \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e]+\operatorname{Sin}[e]) (c \operatorname{Sin}[e]+(-d+c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{f x}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}}}\right]}{\sqrt{c^2-d^2} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}}}\right) (d+c \operatorname{Cos}[e+f x])^3 (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) +$$

$$\frac{1}{c^3} (c \operatorname{Sec}[e] (6 (8 c^4+6 c^3 d+30 c^2 d^2+9 c d^3+2 d^4) \operatorname{Sin}[f x]-3 (6 c^4-3 c^3 d+30 c^2 d^2+18 c d^3+4 d^4) \operatorname{Sin}[2 e+f x]+$$

$$c (3 (3 c^3+38 c^2 d+12 c d^2+2 d^3) \operatorname{Sin}[e+2 f x]+3 (3 c^3-6 c^2 d-6 c d^2-2 d^3) \operatorname{Sin}[3 e+2 f x]+$$

$$c (22 c^2+9 c d+2 d^2) \operatorname{Sin}[2 e+3 f x])) -2 d (66 c^4+27 c^3 d+50 c^2 d^2+18 c d^3+4 d^4) \operatorname{Tan}[e])$$

■ **Problem 210: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x] (c+d \operatorname{Sec}[e+f x])^4}{a+a \operatorname{Sec}[e+f x]} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\frac{d (8 c^3-12 c^2 d+12 c d^2-3 d^3) \operatorname{ArcTanh}[\operatorname{Sin}[e+f x]]}{2 a f} - \frac{(3 c-4 d) d (c+d \operatorname{Sec}[e+f x])^2 \operatorname{Tan}[e+f x]}{3 a f} +$$

$$\frac{(c-d) (c+d \operatorname{Sec}[e+f x])^3 \operatorname{Tan}[e+f x]}{f (a+a \operatorname{Sec}[e+f x])} - \frac{d (4 (3 c^3-16 c^2 d+12 c d^2-4 d^3)+d (6 c^2-20 c d+9 d^2) \operatorname{Sec}[e+f x]) \operatorname{Tan}[e+f x]}{6 a f}$$

Result (type 3, 1243 leaves):

$$\begin{aligned}
& \left((-8c^3d + 12c^2d^2 - 12cd^3 + 3d^4) \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \cos[e + fx]^3 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right] (c + d \operatorname{Sec}[e + fx])^4 \right) / \\
& \left(f (d + c \cos[e + fx])^4 (a + a \operatorname{Sec}[e + fx]) \right) + \\
& \left((8c^3d - 12c^2d^2 + 12cd^3 - 3d^4) \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \cos[e + fx]^3 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right] (c + d \operatorname{Sec}[e + fx])^4 \right) / \\
& \left(f (d + c \cos[e + fx])^4 (a + a \operatorname{Sec}[e + fx]) \right) + \frac{1}{48f(d + c \cos[e + fx])^4(a + a \operatorname{Sec}[e + fx])} \\
& \cos\left[\frac{e}{2} + \frac{fx}{2}\right] \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}[e] (c + d \operatorname{Sec}[e + fx])^4 \left(-18c^4 \sin\left[\frac{fx}{2}\right] + 72c^3d \sin\left[\frac{fx}{2}\right] - 36c^2d^2 \sin\left[\frac{fx}{2}\right] + 24cd^3 \sin\left[\frac{fx}{2}\right] + 6d^4 \sin\left[\frac{fx}{2}\right] + \right. \\
& 18c^4 \sin\left[\frac{3fx}{2}\right] - 72c^3d \sin\left[\frac{3fx}{2}\right] + 180c^2d^2 \sin\left[\frac{3fx}{2}\right] - 108cd^3 \sin\left[\frac{3fx}{2}\right] + 39d^4 \sin\left[\frac{3fx}{2}\right] - 72c^2d^2 \sin\left[e - \frac{fx}{2}\right] + \\
& 48cd^3 \sin\left[e - \frac{fx}{2}\right] - 24d^4 \sin\left[e - \frac{fx}{2}\right] - 36c^2d^2 \sin\left[e + \frac{fx}{2}\right] + 24cd^3 \sin\left[e + \frac{fx}{2}\right] - 6d^4 \sin\left[e + \frac{fx}{2}\right] - 18c^4 \sin\left[2e + \frac{fx}{2}\right] + \\
& 72c^3d \sin\left[2e + \frac{fx}{2}\right] - 144c^2d^2 \sin\left[2e + \frac{fx}{2}\right] + 96cd^3 \sin\left[2e + \frac{fx}{2}\right] - 24d^4 \sin\left[2e + \frac{fx}{2}\right] + 72c^2d^2 \sin\left[e + \frac{3fx}{2}\right] - 36cd^3 \sin\left[e + \frac{3fx}{2}\right] + \\
& 21d^4 \sin\left[e + \frac{3fx}{2}\right] + 18c^4 \sin\left[2e + \frac{3fx}{2}\right] - 72c^3d \sin\left[2e + \frac{3fx}{2}\right] + 72c^2d^2 \sin\left[2e + \frac{3fx}{2}\right] - 36cd^3 \sin\left[2e + \frac{3fx}{2}\right] + \\
& 9d^4 \sin\left[2e + \frac{3fx}{2}\right] - 36c^2d^2 \sin\left[3e + \frac{3fx}{2}\right] + 36cd^3 \sin\left[3e + \frac{3fx}{2}\right] - 9d^4 \sin\left[3e + \frac{3fx}{2}\right] + 36c^2d^2 \sin\left[e + \frac{5fx}{2}\right] - \\
& 12cd^3 \sin\left[e + \frac{5fx}{2}\right] + 7d^4 \sin\left[e + \frac{5fx}{2}\right] - 6c^4 \sin\left[2e + \frac{5fx}{2}\right] + 24c^3d \sin\left[2e + \frac{5fx}{2}\right] + 12cd^3 \sin\left[2e + \frac{5fx}{2}\right] + d^4 \sin\left[2e + \frac{5fx}{2}\right] + \\
& 12cd^3 \sin\left[3e + \frac{5fx}{2}\right] - 3d^4 \sin\left[3e + \frac{5fx}{2}\right] - 6c^4 \sin\left[4e + \frac{5fx}{2}\right] + 24c^3d \sin\left[4e + \frac{5fx}{2}\right] - 36c^2d^2 \sin\left[4e + \frac{5fx}{2}\right] + \\
& 36cd^3 \sin\left[4e + \frac{5fx}{2}\right] - 9d^4 \sin\left[4e + \frac{5fx}{2}\right] + 6c^4 \sin\left[2e + \frac{7fx}{2}\right] - 24c^3d \sin\left[2e + \frac{7fx}{2}\right] + 72c^2d^2 \sin\left[2e + \frac{7fx}{2}\right] - \\
& 48cd^3 \sin\left[2e + \frac{7fx}{2}\right] + 16d^4 \sin\left[2e + \frac{7fx}{2}\right] + 36c^2d^2 \sin\left[3e + \frac{7fx}{2}\right] - 24cd^3 \sin\left[3e + \frac{7fx}{2}\right] + 10d^4 \sin\left[3e + \frac{7fx}{2}\right] + \\
& \left. 6c^4 \sin\left[4e + \frac{7fx}{2}\right] - 24c^3d \sin\left[4e + \frac{7fx}{2}\right] + 36c^2d^2 \sin\left[4e + \frac{7fx}{2}\right] - 24cd^3 \sin\left[4e + \frac{7fx}{2}\right] + 6d^4 \sin\left[4e + \frac{7fx}{2}\right] \right)
\end{aligned}$$

■ **Problem 211: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + fx] (c + d \operatorname{Sec}[e + fx])^3}{a + a \operatorname{Sec}[e + fx]} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{3d(2c^2 - 2cd + d^2) \operatorname{ArcTanh}[\sin[e + fx]]}{2af} + \frac{(c - d)(c + d \operatorname{Sec}[e + fx])^2 \tan[e + fx]}{f(a + a \operatorname{Sec}[e + fx])} - \frac{d(4(c^2 - 3cd + d^2) + (2c - 3d)d \operatorname{Sec}[e + fx]) \tan[e + fx]}{2af}$$

Result (type 3, 275 leaves):

$$\frac{1}{a f (1 + \operatorname{Cos}[e + f x])} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^6 \operatorname{Sec}[e + f x]^2 \left(16 d^3 \operatorname{Csc}[e + f x]^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^4 + \right. \\ \left. \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right)^2\right) \left(3 d (2 c^2 - 2 c d + d^2) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) - \right. \\ \left. 2 (c^3 - 3 c^2 d + 9 c d^2 - 3 d^3) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] - 3 d (2 c^2 - 2 c d + d^2) \right. \\ \left. \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 + 2 (c - d)^3 \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^3 \right) \right)$$

■ **Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^2}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{(2c - d) d \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a f} + \frac{d^2 \operatorname{Tan}[e + f x]}{a f} + \frac{(c - d)^2 \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])}$$

Result (type 3, 237 leaves):

$$\left(2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Cos}[e + f x] (c + d \operatorname{Sec}[e + f x])^2 \left((c - d)^2 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + \right. \right. \\ \left. \left. d \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \left(- (2c - d) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + \right. \right. \\ \left. \left. (d \operatorname{Sin}[f x]) \right) / \left(\left(\operatorname{Cos}\left[\frac{e}{2}\right] - \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{e}{2}\right] + \operatorname{Sin}\left[\frac{e}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right) \right. \right. \\ \left. \left. \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)\right)\right) / (a f (d + c \operatorname{Cos}[e + f x])^2 (1 + \operatorname{Sec}[e + f x]))$$

■ **Problem 213: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])}{a + a \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{d \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a f} + \frac{(c - d) \operatorname{Tan}[e + f x]}{f (a + a \operatorname{Sec}[e + f x])}$$

Result (type 3, 109 leaves):

$$\frac{1}{a f (1 + \operatorname{Cos}[e + f x])} 2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \\ \left(d \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + (c - d) \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right]\right)$$

■ **Problem 214: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])} dx$$

Optimal (type 3, 83 leaves, 4 steps):

$$-\frac{2 d \text{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{a (c-d)^{3/2} \sqrt{c+d} f} + \frac{\text{Tan}[e + f x]}{(c-d) f (a + a \text{Sec}[e + f x])}$$

Result (type 3, 160 leaves):

$$\left(2 \cos\left[\frac{1}{2}(e+fx)\right] \left(\frac{2 d \text{ArcTan}\left[\frac{(i \cos[e] + \sin[e]) (c \sin[e] + (-d+c \cos[e]) \tan\left[\frac{fx}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2}}\right]}{\sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2}} \right) \cos\left[\frac{1}{2}(e+fx)\right] (i \cos[e] + \sin[e]) + \text{Sec}\left[\frac{e}{2}\right] \sin\left[\frac{fx}{2}\right] \right) \right) / (a (c-d) f (1 + \cos[e + f x]))$$

■ **Problem 215: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^2} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$-\frac{2 d (2 c + d) \text{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{a (c-d)^{5/2} (c+d)^{3/2} f} + \frac{(c+2d) \text{Tan}[e + f x]}{(c-d)^2 (c+d) f (a + a \text{Sec}[e + f x])} - \frac{d \text{Tan}[e + f x]}{(c^2-d^2) f (a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])}$$

Result (type 3, 286 leaves):

$$\left(2 \cos\left[\frac{1}{2}(e+fx)\right] (d + c \cos[e + f x]) \text{Sec}[e + f x]^3 \right. \\ \left. \left(\left(2 d (2 c + d) \text{ArcTan}\left[\frac{(i \cos[e] + \sin[e]) (c \sin[e] + (-d+c \cos[e]) \tan\left[\frac{fx}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2}}\right]} \right) \cos\left[\frac{1}{2}(e+fx)\right] (d + c \cos[e + f x]) (i \cos[e] + \sin[e]) \right) \right) / \\ \left((c+d) \sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2} + (d+c \cos[e + f x]) \text{Sec}\left[\frac{e}{2}\right] \sin\left[\frac{fx}{2}\right] + \right. \\ \left. \frac{d^2 \cos\left[\frac{1}{2}(e+fx)\right] (-d \sin[e] + c \sin[fx])}{c (c+d) (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right])} \right) \right) / (a (c-d)^2 f (1 + \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^2)$$

■ **Problem 216: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]}{(a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^3} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$-\frac{3 d (2 c^2 + 2 c d + d^2) \text{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c+d}}\right]}{a (c-d)^{7/2} (c+d)^{5/2} f} + \frac{d (2 c + 3 d) \tan[e + f x]}{2 a (c-d)^2 (c+d) f (c+d \text{Sec}[e + f x])^2} + \frac{\tan[e + f x]}{(c-d) f (a + a \text{Sec}[e + f x]) (c+d \text{Sec}[e + f x])^2} + \frac{d (2 c + d) (c + 4 d) \tan[e + f x]}{2 a (c-d)^3 (c+d)^2 f (c+d \text{Sec}[e + f x])}$$

Result (type 3, 1422 leaves):

$$\left((2 c^2 + 2 c d + d^2) \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (d + c \cos[e + f x])^3 \text{Sec}[e + f x]^4 \right. \\ \left. - \left(6 i d \text{ArcTan}\left[\text{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2 e] - i \sin[2 e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2 e] - i \sin[2 e]}} \right) \left(-i d \sin\left[\frac{f x}{2}\right] + i c \sin\left[e + \frac{f x}{2}\right] \right) \right) \right. \\ \left. \cos[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2 e] - i \sin[2 e]} \right) - \\ \left(6 d \text{ArcTan}\left[\text{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2 e] - i \sin[2 e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2 e] - i \sin[2 e]}} \right) \left(-i d \sin\left[\frac{f x}{2}\right] + i c \sin\left[e + \frac{f x}{2}\right] \right) \right) \\ \sin[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2 e] - i \sin[2 e]} \right) \Bigg) / \\ \left((-c + d)^3 (c + d)^2 (a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^3 \right) + \frac{1}{8 c^2 (-c + d)^3 (c + d)^2 f (a + a \text{Sec}[e + f x]) (c + d \text{Sec}[e + f x])^3}$$

$$\cos\left[\frac{e}{2} + \frac{f x}{2}\right]$$

$$(d + c \cos[e + f x]) \text{Sec}\left[\frac{e}{2}\right]$$

$$\text{Sec}[e] \text{Sec}[e + f x]^4$$

$$\left(8 c^5 d \sin\left[\frac{f x}{2}\right] + 10 c^4 d^2 \sin\left[\frac{f x}{2}\right] - 11 c^3 d^3 \sin\left[\frac{f x}{2}\right] - 17 c^2 d^4 \sin\left[\frac{f x}{2}\right] - 2 c d^5 \sin\left[\frac{f x}{2}\right] + 2 d^6 \sin\left[\frac{f x}{2}\right] - \right.$$

$$\left. 8 c^5 d \sin\left[\frac{3 f x}{2}\right] - 22 c^4 d^2 \sin\left[\frac{3 f x}{2}\right] - 27 c^3 d^3 \sin\left[\frac{3 f x}{2}\right] - 5 c^2 d^4 \sin\left[\frac{3 f x}{2}\right] + 2 c d^5 \sin\left[\frac{3 f x}{2}\right] + 4 c^6 \sin\left[e - \frac{f x}{2}\right] + \right.$$

$$\begin{aligned}
& 8 c^5 d \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 18 c^4 d^2 \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 35 c^3 d^3 \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 25 c^2 d^4 \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 2 c d^5 \operatorname{Sin}\left[e - \frac{f x}{2}\right] - 2 d^6 \operatorname{Sin}\left[e - \frac{f x}{2}\right] - \\
& 4 c^6 \operatorname{Sin}\left[e + \frac{f x}{2}\right] - 8 c^5 d \operatorname{Sin}\left[e + \frac{f x}{2}\right] - 6 c^4 d^2 \operatorname{Sin}\left[e + \frac{f x}{2}\right] - 7 c^3 d^3 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 5 c^2 d^4 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 2 c d^5 \operatorname{Sin}\left[e + \frac{f x}{2}\right] - \\
& 2 d^6 \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 8 c^5 d \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] + 22 c^4 d^2 \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] + 17 c^3 d^3 \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] + 13 c^2 d^4 \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] + \\
& 2 c d^5 \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] - 2 d^6 \operatorname{Sin}\left[2 e + \frac{f x}{2}\right] + 2 c^6 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 4 c^5 d \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 4 c^4 d^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - \\
& 19 c^3 d^3 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 5 c^2 d^4 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 2 c d^5 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 8 c^5 d \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - 16 c^4 d^2 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - \\
& c^3 d^3 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] + 2 c^2 d^4 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - 2 c d^5 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] + 2 c^6 \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] + 4 c^5 d \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] + \\
& 2 c^4 d^2 \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] + 7 c^3 d^3 \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] + 2 c^2 d^4 \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] - 2 c d^5 \operatorname{Sin}\left[3 e + \frac{3 f x}{2}\right] - 2 c^6 \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] - \\
& 4 c^5 d \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] - 8 c^4 d^2 \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] - 2 c^3 d^3 \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] + c^2 d^4 \operatorname{Sin}\left[e + \frac{5 f x}{2}\right] - 6 c^4 d^2 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] - \\
& 2 c^3 d^3 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] + c^2 d^4 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] - 2 c^6 \operatorname{Sin}\left[3 e + \frac{5 f x}{2}\right] - 4 c^5 d \operatorname{Sin}\left[3 e + \frac{5 f x}{2}\right] - 2 c^4 d^2 \operatorname{Sin}\left[3 e + \frac{5 f x}{2}\right]
\end{aligned}$$

■ **Problem 217: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^5}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 258 leaves, 8 steps):

$$\begin{aligned}
& \frac{5 (2 c - d) d^2 (2 c^2 - 3 c d + 2 d^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^2 f} - \frac{d (c^2 + 10 c d - 12 d^2) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 a^2 f} + \\
& \frac{(c - d) (c + 10 d) (c + d \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{3 f (a^2 + a^2 \operatorname{Sec}[e + f x])} + \frac{(c - d) (c + d \operatorname{Sec}[e + f x])^4 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} - \\
& \frac{d (4 (c^4 + 10 c^3 d - 44 c^2 d^2 + 40 c d^3 - 12 d^4) + d (2 c^3 + 20 c^2 d - 57 c d^2 + 30 d^3) \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{6 a^2 f}
\end{aligned}$$

Result (type 3, 743 leaves):

$$\begin{aligned}
& \left(10 (-4 c^3 d^2 + 8 c^2 d^3 - 7 c d^4 + 2 d^5) \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Cos}[e + f x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] (c + d \operatorname{Sec}[e + f x])^5 \right) / \\
& \left(f (d + c \operatorname{Cos}[e + f x])^5 (a + a \operatorname{Sec}[e + f x])^2 \right) - \\
& \left(10 (-4 c^3 d^2 + 8 c^2 d^3 - 7 c d^4 + 2 d^5) \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Cos}[e + f x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] (c + d \operatorname{Sec}[e + f x])^5 \right) / \\
& \left(f (d + c \operatorname{Cos}[e + f x])^5 (a + a \operatorname{Sec}[e + f x])^2 \right) + \frac{1}{24 f (d + c \operatorname{Cos}[e + f x])^5 (a + a \operatorname{Sec}[e + f x])^2} \\
& \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^3 (c + d \operatorname{Sec}[e + f x])^5 \left(6 c^5 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] - 30 c^4 d \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 60 c^3 d^2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] - \right. \\
& 15 c d^4 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] + 18 d^5 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right] - 2 c^5 \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] + 40 c^4 d \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] - 140 c^3 d^2 \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] + \\
& 320 c^2 d^3 \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] - 205 c d^4 \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] + 70 d^5 \operatorname{Sin}\left[\frac{3}{2}(e + f x)\right] + 6 c^5 \operatorname{Sin}\left[\frac{5}{2}(e + f x)\right] - 60 c^3 d^2 \operatorname{Sin}\left[\frac{5}{2}(e + f x)\right] + \\
& 240 c^2 d^3 \operatorname{Sin}\left[\frac{5}{2}(e + f x)\right] - 165 c d^4 \operatorname{Sin}\left[\frac{5}{2}(e + f x)\right] + 54 d^5 \operatorname{Sin}\left[\frac{5}{2}(e + f x)\right] + 15 c^4 d \operatorname{Sin}\left[\frac{7}{2}(e + f x)\right] - \\
& 60 c^3 d^2 \operatorname{Sin}\left[\frac{7}{2}(e + f x)\right] + 180 c^2 d^3 \operatorname{Sin}\left[\frac{7}{2}(e + f x)\right] - 135 c d^4 \operatorname{Sin}\left[\frac{7}{2}(e + f x)\right] + 42 d^5 \operatorname{Sin}\left[\frac{7}{2}(e + f x)\right] + 2 c^5 \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] + \\
& \left. 5 c^4 d \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] - 40 c^3 d^2 \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] + 100 c^2 d^3 \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] - 80 c d^4 \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] + 24 d^5 \operatorname{Sin}\left[\frac{9}{2}(e + f x)\right] \right)
\end{aligned}$$

■ **Problem 219: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^3}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{(3c - 2d) d^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^2 f} + \frac{(c - d) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(c^3 + 4 c^2 d - 12 c d^2 + 10 d^3 - (c - 4 d) d^2 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{3 f (a^2 + a^2 \operatorname{Sec}[e + f x])}$$

Result (type 3, 294 leaves):

$$\frac{1}{3 a^2 f (1 + \operatorname{Cos}[e + f x])^2}$$

$$2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^6 \operatorname{Sec}[e + f x] \left(6 d^2 (-3 c + 2 d) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) - \right.$$

$$8 (c - d)^3 \operatorname{Csc}[e + f x]^3 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^4 + 32 (c - d)^3 \operatorname{Csc}[e + f x]^5 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]^8 + 2 (2 c^3 + 3 c^2 d - 12 c d^2 + 13 d^3) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] +$$

$$6 (3 c - 2 d) d^2 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 -$$

$$\left. 2 (c - d)^2 (2 c + 7 d) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^3\right)$$

■ **Problem 220: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^2}{(a + a \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$\frac{d^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^2 f} + \frac{(c - d)^2 \operatorname{Tan}[e + f x]}{3 f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(c - d) (c + 5 d) \operatorname{Tan}[e + f x]}{3 f (a^2 + a^2 \operatorname{Sec}[e + f x])}$$

Result (type 3, 181 leaves):

$$-\frac{1}{3 a^2 f (1 + \operatorname{Cos}[e + f x])^2}$$

$$2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \left(6 d^2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^3 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]\right) + \right.$$

$$\left. (c - d)^2 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] - 4 (c^2 + c d - 2 d^2) \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sin}\left[\frac{f x}{2}\right] + (c - d)^2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \operatorname{Tan}\left[\frac{e}{2}\right]\right)$$

■ **Problem 222: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^2 (c + d \operatorname{Sec}[e + f x])} dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$\frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c+d}}\right]}{a^2 (c-d)^{5/2} \sqrt{c+d} f} + \frac{\operatorname{Tan}[e + f x]}{3 (c-d) f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(c - 4 d) \operatorname{Tan}[e + f x]}{3 (c-d)^2 f (a^2 + a^2 \operatorname{Sec}[e + f x])}$$

Result (type 3, 209 leaves):

$$\left(\cos\left[\frac{1}{2}(e+fx)\right] \left(-\frac{24 i d^2 \operatorname{ArcTan}\left[\frac{(i \cos[e]+\sin[e])(c \sin[e]+(-d+c \cos[e]) \tan\left[\frac{fx}{2}\right])}{\sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2}}\right]}{\sqrt{c^2-d^2} \sqrt{(\cos[e]-i \sin[e])^2}}\right) \cos\left[\frac{1}{2}(e+fx)\right]^3 (\cos[e]-i \sin[e]) \right. \right. \\ \left. \left. \sec\left[\frac{e}{2}\right] \left(3(c-3d) \sin\left[\frac{fx}{2}\right] - 3(c-2d) \sin\left[e+\frac{fx}{2}\right] + (2c-5d) \sin\left[e+\frac{3fx}{2}\right] \right) \right) \right) / (3 a^2 (c-d)^2 f (1+\cos[e+fx])^2)$$

- **Problem 223: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]}{(a+a \sec[e+fx])^2 (c+d \sec[e+fx])^2} dx$$

Optimal (type 3, 211 leaves, 7 steps):

$$\frac{2 d^2 (3 c+2 d) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{a^2 (c-d)^{7/2} (c+d)^{3/2} f} + \frac{d (c^2-6 c d-10 d^2) \tan[e+fx]}{3 a^2 (c-d)^3 (c+d) f (c+d \sec[e+fx])} + \\ \frac{(c-6 d) \tan[e+fx]}{3 a^2 (c-d)^2 f (1+\sec[e+fx]) (c+d \sec[e+fx])} + \frac{\tan[e+fx]}{3 (c-d) f (a+a \sec[e+fx])^2 (c+d \sec[e+fx])}$$

Result (type 3, 764 leaves):

$$\begin{aligned}
& \left((3c + 2d) \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 (d + c \cos[e + fx])^2 \sec[e + fx]^4 \right. \\
& \left. \left(\left(8i d^2 \operatorname{ArcTan}\left[\sec\left[\frac{fx}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} \right) \left(-i d \sin\left[\frac{fx}{2}\right] + i c \sin\left[e + \frac{fx}{2}\right] \right) \right) \right. \right. \\
& \left. \left. \cos[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \right. \\
& \left. \left(8d^2 \operatorname{ArcTan}\left[\sec\left[\frac{fx}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} \right) \left(-i d \sin\left[\frac{fx}{2}\right] + i c \sin\left[e + \frac{fx}{2}\right] \right) \right) \right. \\
& \left. \left. \sin[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) \Bigg) / \\
& \left((-c + d)^3 (c + d) (a + a \sec[e + fx])^2 (c + d \sec[e + fx])^2 \right) - \frac{2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right] (d + c \cos[e + fx])^2 \sec\left[\frac{e}{2}\right] \sec[e + fx]^4 \sin\left[\frac{fx}{2}\right]}{3 (-c + d)^2 f (a + a \sec[e + fx])^2 (c + d \sec[e + fx])^2} + \\
& \frac{8 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^3 (d + c \cos[e + fx])^2 \sec\left[\frac{e}{2}\right] \sec[e + fx]^4 (-c \sin\left[\frac{fx}{2}\right] + 4d \sin\left[\frac{fx}{2}\right])}{3 (-c + d)^3 f (a + a \sec[e + fx])^2 (c + d \sec[e + fx])^2} - \\
& \frac{4 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 (d + c \cos[e + fx]) \sec[e + fx]^4 (d^4 \sin[e] - c d^3 \sin[fx])}{c (-c + d)^3 (c + d) f (a + a \sec[e + fx])^2 (c + d \sec[e + fx])^2 (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right])} - \\
& \frac{2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (d + c \cos[e + fx])^2 \sec[e + fx]^4 \tan\left[\frac{e}{2}\right]}{3 (-c + d)^2 f (a + a \sec[e + fx])^2 (c + d \sec[e + fx])^2}
\end{aligned}$$

■ **Problem 224: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]}{(a + a \sec[e + fx])^2 (c + d \sec[e + fx])^3} dx$$

Optimal (type 3, 284 leaves, 8 steps):

$$\frac{d^2 (12 c^2 + 16 c d + 7 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{c+d}}\right]}{a^2 (c-d)^{9/2} (c+d)^{5/2} f} +$$

$$\frac{d (2 c^2 - 16 c d - 21 d^2) \operatorname{Tan}[e+fx]}{6 a^2 (c-d)^3 (c+d) f (c+d \operatorname{Sec}[e+fx])^2} + \frac{(c-8d) \operatorname{Tan}[e+fx]}{3 a^2 (c-d)^2 f (1+\operatorname{Sec}[e+fx]) (c+d \operatorname{Sec}[e+fx])^2} +$$

$$\frac{\operatorname{Tan}[e+fx]}{3 (c-d) f (a+a \operatorname{Sec}[e+fx])^2 (c+d \operatorname{Sec}[e+fx])^2} + \frac{d (2 c^3 - 16 c^2 d - 59 c d^2 - 32 d^3) \operatorname{Tan}[e+fx]}{6 a^2 (c-d)^4 (c+d)^2 f (c+d \operatorname{Sec}[e+fx])}$$

Result (type 3, 2220 leaves):

$$\left((12 c^2 + 16 c d + 7 d^2) \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (d + c \operatorname{Cos}[e+fx])^3 \operatorname{Sec}[e+fx]^5 \right.$$

$$\left. - \left(4 i d^2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}} \right) \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right] + i c \operatorname{Sin}\left[e + \frac{f x}{2}\right] \right) \right) \right.$$

$$\left. \operatorname{Cos}[e] \right) / \left(\sqrt{c^2-d^2} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) -$$

$$\left(4 d^2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2-d^2} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}} \right) \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right] + i c \operatorname{Sin}\left[e + \frac{f x}{2}\right] \right) \right)$$

$$\operatorname{Sin}[e] \right) / \left(\sqrt{c^2-d^2} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \left. \right) /$$

$$\left((-c+d)^4 (c+d)^2 (a+a \operatorname{Sec}[e+fx])^2 (c+d \operatorname{Sec}[e+fx])^3 \right) + \frac{1}{48 c^2 (-c+d)^4 (c+d)^2 f (a+a \operatorname{Sec}[e+fx])^2 (c+d \operatorname{Sec}[e+fx])^3}$$

$$\operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]$$

$$(d + c \operatorname{Cos}[e+fx]) \operatorname{Sec}\left[\frac{e}{2}\right]$$

$$\operatorname{Sec}[e] \operatorname{Sec}[e+fx]^5$$

$$\left(-16 c^7 \operatorname{Sin}\left[\frac{f x}{2}\right] + 14 c^6 d \operatorname{Sin}\left[\frac{f x}{2}\right] + 220 c^5 d^2 \operatorname{Sin}\left[\frac{f x}{2}\right] + 334 c^4 d^3 \operatorname{Sin}\left[\frac{f x}{2}\right] + 54 c^3 d^4 \operatorname{Sin}\left[\frac{f x}{2}\right] - 156 c^2 d^5 \operatorname{Sin}\left[\frac{f x}{2}\right] - \right.$$

$$48 c d^6 \operatorname{Sin}\left[\frac{f x}{2}\right] + 18 d^7 \operatorname{Sin}\left[\frac{f x}{2}\right] + 14 c^7 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 16 c^6 d \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 226 c^5 d^2 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 532 c^4 d^3 \operatorname{Sin}\left[\frac{3 f x}{2}\right] -$$

$$583 c^3 d^4 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 232 c^2 d^5 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 6 c d^6 \operatorname{Sin}\left[\frac{3 f x}{2}\right] + 6 d^7 \operatorname{Sin}\left[\frac{3 f x}{2}\right] - 12 c^7 \operatorname{Sin}\left[e - \frac{f x}{2}\right] + 20 c^6 d \operatorname{Sin}\left[e - \frac{f x}{2}\right] +$$

$$\begin{aligned}
& 236 c^5 d^2 \sin\left[e - \frac{f x}{2}\right] + 628 c^4 d^3 \sin\left[e - \frac{f x}{2}\right] + 778 c^3 d^4 \sin\left[e - \frac{f x}{2}\right] + 420 c^2 d^5 \sin\left[e - \frac{f x}{2}\right] + 48 c d^6 \sin\left[e - \frac{f x}{2}\right] - \\
& 18 d^7 \sin\left[e - \frac{f x}{2}\right] + 12 c^7 \sin\left[e + \frac{f x}{2}\right] - 20 c^6 d \sin\left[e + \frac{f x}{2}\right] - 236 c^5 d^2 \sin\left[e + \frac{f x}{2}\right] - 460 c^4 d^3 \sin\left[e + \frac{f x}{2}\right] - 310 c^3 d^4 \sin\left[e + \frac{f x}{2}\right] + \\
& 39 c^2 d^5 \sin\left[e + \frac{f x}{2}\right] + 48 c d^6 \sin\left[e + \frac{f x}{2}\right] - 18 d^7 \sin\left[e + \frac{f x}{2}\right] - 16 c^7 \sin\left[2 e + \frac{f x}{2}\right] + 14 c^6 d \sin\left[2 e + \frac{f x}{2}\right] + 220 c^5 d^2 \sin\left[2 e + \frac{f x}{2}\right] + \\
& 502 c^4 d^3 \sin\left[2 e + \frac{f x}{2}\right] + 522 c^3 d^4 \sin\left[2 e + \frac{f x}{2}\right] + 303 c^2 d^5 \sin\left[2 e + \frac{f x}{2}\right] + 48 c d^6 \sin\left[2 e + \frac{f x}{2}\right] - 18 d^7 \sin\left[2 e + \frac{f x}{2}\right] - \\
& 6 c^7 \sin\left[e + \frac{3 f x}{2}\right] + 6 c^6 d \sin\left[e + \frac{3 f x}{2}\right] + 126 c^5 d^2 \sin\left[e + \frac{3 f x}{2}\right] + 114 c^4 d^3 \sin\left[e + \frac{3 f x}{2}\right] - 159 c^3 d^4 \sin\left[e + \frac{3 f x}{2}\right] - \\
& 144 c^2 d^5 \sin\left[e + \frac{3 f x}{2}\right] - 6 c d^6 \sin\left[e + \frac{3 f x}{2}\right] + 6 d^7 \sin\left[e + \frac{3 f x}{2}\right] + 14 c^7 \sin\left[2 e + \frac{3 f x}{2}\right] - 16 c^6 d \sin\left[2 e + \frac{3 f x}{2}\right] - \\
& 226 c^5 d^2 \sin\left[2 e + \frac{3 f x}{2}\right] - 412 c^4 d^3 \sin\left[2 e + \frac{3 f x}{2}\right] - 235 c^3 d^4 \sin\left[2 e + \frac{3 f x}{2}\right] - 7 c^2 d^5 \sin\left[2 e + \frac{3 f x}{2}\right] + 6 c d^6 \sin\left[2 e + \frac{3 f x}{2}\right] - \\
& 6 d^7 \sin\left[2 e + \frac{3 f x}{2}\right] - 6 c^7 \sin\left[3 e + \frac{3 f x}{2}\right] + 6 c^6 d \sin\left[3 e + \frac{3 f x}{2}\right] + 126 c^5 d^2 \sin\left[3 e + \frac{3 f x}{2}\right] + 234 c^4 d^3 \sin\left[3 e + \frac{3 f x}{2}\right] + \\
& 189 c^3 d^4 \sin\left[3 e + \frac{3 f x}{2}\right] + 81 c^2 d^5 \sin\left[3 e + \frac{3 f x}{2}\right] + 6 c d^6 \sin\left[3 e + \frac{3 f x}{2}\right] - 6 d^7 \sin\left[3 e + \frac{3 f x}{2}\right] + 6 c^7 \sin\left[e + \frac{5 f x}{2}\right] - \\
& 14 c^6 d \sin\left[e + \frac{5 f x}{2}\right] - 134 c^5 d^2 \sin\left[e + \frac{5 f x}{2}\right] - 274 c^4 d^3 \sin\left[e + \frac{5 f x}{2}\right] - 193 c^3 d^4 \sin\left[e + \frac{5 f x}{2}\right] - 27 c^2 d^5 \sin\left[e + \frac{5 f x}{2}\right] + \\
& 6 c d^6 \sin\left[e + \frac{5 f x}{2}\right] - 6 c^7 \sin\left[2 e + \frac{5 f x}{2}\right] + 12 c^6 d \sin\left[2 e + \frac{5 f x}{2}\right] + 42 c^5 d^2 \sin\left[2 e + \frac{5 f x}{2}\right] - 48 c^4 d^3 \sin\left[2 e + \frac{5 f x}{2}\right] - \\
& 105 c^3 d^4 \sin\left[2 e + \frac{5 f x}{2}\right] - 27 c^2 d^5 \sin\left[2 e + \frac{5 f x}{2}\right] + 6 c d^6 \sin\left[2 e + \frac{5 f x}{2}\right] + 6 c^7 \sin\left[3 e + \frac{5 f x}{2}\right] - 14 c^6 d \sin\left[3 e + \frac{5 f x}{2}\right] - \\
& 134 c^5 d^2 \sin\left[3 e + \frac{5 f x}{2}\right] - 202 c^4 d^3 \sin\left[3 e + \frac{5 f x}{2}\right] - 61 c^3 d^4 \sin\left[3 e + \frac{5 f x}{2}\right] + 12 c^2 d^5 \sin\left[3 e + \frac{5 f x}{2}\right] - 6 c d^6 \sin\left[3 e + \frac{5 f x}{2}\right] - \\
& 6 c^7 \sin\left[4 e + \frac{5 f x}{2}\right] + 12 c^6 d \sin\left[4 e + \frac{5 f x}{2}\right] + 42 c^5 d^2 \sin\left[4 e + \frac{5 f x}{2}\right] + 24 c^4 d^3 \sin\left[4 e + \frac{5 f x}{2}\right] + 27 c^3 d^4 \sin\left[4 e + \frac{5 f x}{2}\right] + \\
& 12 c^2 d^5 \sin\left[4 e + \frac{5 f x}{2}\right] - 6 c d^6 \sin\left[4 e + \frac{5 f x}{2}\right] + 4 c^7 \sin\left[2 e + \frac{7 f x}{2}\right] - 14 c^6 d \sin\left[2 e + \frac{7 f x}{2}\right] - 40 c^5 d^2 \sin\left[2 e + \frac{7 f x}{2}\right] - \\
& 46 c^4 d^3 \sin\left[2 e + \frac{7 f x}{2}\right] - 12 c^3 d^4 \sin\left[2 e + \frac{7 f x}{2}\right] + 3 c^2 d^5 \sin\left[2 e + \frac{7 f x}{2}\right] - 24 c^4 d^3 \sin\left[3 e + \frac{7 f x}{2}\right] - 12 c^3 d^4 \sin\left[3 e + \frac{7 f x}{2}\right] + \\
& 3 c^2 d^5 \sin\left[3 e + \frac{7 f x}{2}\right] + 4 c^7 \sin\left[4 e + \frac{7 f x}{2}\right] - 14 c^6 d \sin\left[4 e + \frac{7 f x}{2}\right] - 40 c^5 d^2 \sin\left[4 e + \frac{7 f x}{2}\right] - 22 c^4 d^3 \sin\left[4 e + \frac{7 f x}{2}\right]
\end{aligned}$$

■ **Problem 225: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + f x] (c + d \sec[e + f x])^6}{(a + a \sec[e + f x])^3} dx$$

Optimal (type 3, 363 leaves, 9 steps) :

$$\frac{d^3 (40 c^3 - 90 c^2 d + 78 c d^2 - 23 d^3) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 a^3 f} - \frac{2 d (2 c^5 + 18 c^4 d + 107 c^3 d^2 - 472 c^2 d^3 + 456 c d^4 - 136 d^5) \operatorname{Tan}[e + f x]}{15 a^3 f} -$$

$$\frac{d^2 (4 c^4 + 36 c^3 d + 216 c^2 d^2 - 626 c d^3 + 345 d^4) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{30 a^3 f} -$$

$$\frac{d (2 c^3 + 18 c^2 d + 111 c d^2 - 136 d^3) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{15 a^3 f} + \frac{(c - d) (2 c^2 + 18 c d + 115 d^2) (c + d \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{15 f (a^3 + a^3 \operatorname{Sec}[e + f x])} +$$

$$\frac{(c - d) (2 c + 13 d) (c + d \operatorname{Sec}[e + f x])^4 \operatorname{Tan}[e + f x]}{15 a f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(c - d) (c + d \operatorname{Sec}[e + f x])^5 \operatorname{Tan}[e + f x]}{5 f (a + a \operatorname{Sec}[e + f x])^3}$$

Result (type 3, 1338 leaves) :

$$\begin{aligned}
& \left(4 \left(-40 c^3 d^3 + 90 c^2 d^4 - 78 c d^5 + 23 d^6 \right) \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \cos [e + f x]^3 \operatorname{Log} \left[\cos \left[\frac{e}{2} + \frac{f x}{2} \right] - \sin \left[\frac{e}{2} + \frac{f x}{2} \right] \right] (c + d \operatorname{Sec}[e + f x])^6 \right) / \\
& \left(f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) - \\
& \left(4 \left(-40 c^3 d^3 + 90 c^2 d^4 - 78 c d^5 + 23 d^6 \right) \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \cos [e + f x]^3 \operatorname{Log} \left[\cos \left[\frac{e}{2} + \frac{f x}{2} \right] + \sin \left[\frac{e}{2} + \frac{f x}{2} \right] \right] (c + d \operatorname{Sec}[e + f x])^6 \right) / \\
& \left(f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \left(2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^2 \cos [e + f x]^3 \operatorname{Sec} \left[\frac{e}{2} \right] (c + d \operatorname{Sec}[e + f x])^6 \right. \\
& \left. \left(c^6 \sin \left[\frac{e}{2} \right] - 6 c^5 d \sin \left[\frac{e}{2} \right] + 15 c^4 d^2 \sin \left[\frac{e}{2} \right] - 20 c^3 d^3 \sin \left[\frac{e}{2} \right] + 15 c^2 d^4 \sin \left[\frac{e}{2} \right] - 6 c d^5 \sin \left[\frac{e}{2} \right] + d^6 \sin \left[\frac{e}{2} \right] \right) \right) / \\
& \left(5 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \left(8 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \cos [e + f x]^3 \operatorname{Sec} \left[\frac{e}{2} \right] (c + d \operatorname{Sec}[e + f x])^6 \right. \\
& \left. \left(-4 c^6 \sin \left[\frac{e}{2} \right] + 9 c^5 d \sin \left[\frac{e}{2} \right] + 15 c^4 d^2 \sin \left[\frac{e}{2} \right] - 70 c^3 d^3 \sin \left[\frac{e}{2} \right] + 90 c^2 d^4 \sin \left[\frac{e}{2} \right] - 51 c d^5 \sin \left[\frac{e}{2} \right] + 11 d^6 \sin \left[\frac{e}{2} \right] \right) \right) / \\
& \left(15 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \left(2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right] \cos [e + f x]^3 \operatorname{Sec} \left[\frac{e}{2} \right] (c + d \operatorname{Sec}[e + f x])^6 \right. \\
& \left. \left(c^6 \sin \left[\frac{f x}{2} \right] - 6 c^5 d \sin \left[\frac{f x}{2} \right] + 15 c^4 d^2 \sin \left[\frac{f x}{2} \right] - 20 c^3 d^3 \sin \left[\frac{f x}{2} \right] + 15 c^2 d^4 \sin \left[\frac{f x}{2} \right] - 6 c d^5 \sin \left[\frac{f x}{2} \right] + d^6 \sin \left[\frac{f x}{2} \right] \right) \right) / \\
& \left(5 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \left(8 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^3 \cos [e + f x]^3 \operatorname{Sec} \left[\frac{e}{2} \right] (c + d \operatorname{Sec}[e + f x])^6 \right. \\
& \left. \left(-4 c^6 \sin \left[\frac{f x}{2} \right] + 9 c^5 d \sin \left[\frac{f x}{2} \right] + 15 c^4 d^2 \sin \left[\frac{f x}{2} \right] - 70 c^3 d^3 \sin \left[\frac{f x}{2} \right] + 90 c^2 d^4 \sin \left[\frac{f x}{2} \right] - 51 c d^5 \sin \left[\frac{f x}{2} \right] + 11 d^6 \sin \left[\frac{f x}{2} \right] \right) \right) / \\
& \left(15 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \left(8 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^5 \cos [e + f x]^3 \operatorname{Sec} \left[\frac{e}{2} \right] (c + d \operatorname{Sec}[e + f x])^6 \right. \\
& \left. \left(7 c^6 \sin \left[\frac{f x}{2} \right] + 18 c^5 d \sin \left[\frac{f x}{2} \right] + 30 c^4 d^2 \sin \left[\frac{f x}{2} \right] - 440 c^3 d^3 \sin \left[\frac{f x}{2} \right] + 855 c^2 d^4 \sin \left[\frac{f x}{2} \right] - 642 c d^5 \sin \left[\frac{f x}{2} \right] + 172 d^6 \sin \left[\frac{f x}{2} \right] \right) \right) / \\
& \left(15 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \frac{8 d^6 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \operatorname{Sec}[e] (c + d \operatorname{Sec}[e + f x])^6 \sin [f x]}{3 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3} - \\
& \left(4 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \cos [e + f x]^2 \operatorname{Sec}[e] (c + d \operatorname{Sec}[e + f x])^6 \left(-18 c d^5 \sin [e] + 9 d^6 \sin [e] - 90 c^2 d^4 \sin [f x] + 108 c d^5 \sin [f x] - 40 d^6 \sin [f x] \right) \right) / \\
& \left(3 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3 \right) + \\
& \frac{4 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \cos [e + f x] \operatorname{Sec}[e] (c + d \operatorname{Sec}[e + f x])^6 \left(2 d^6 \sin [e] + 18 c d^5 \sin [f x] - 9 d^6 \sin [f x] \right)}{3 f (d + c \cos [e + f x])^6 (a + a \operatorname{Sec}[e + f x])^3}
\end{aligned}$$

■ **Problem 228: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^3}{(a + a \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 133 leaves, 6 steps) :

$$\frac{d^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{a^3 f} + \frac{(c - d) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{5 f (a + a \operatorname{Sec}[e + f x])^3} + \frac{(c - d) (2 (2 c^2 + 8 c d + 11 d^2) + (2 c^2 + 11 c d + 29 d^2) \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{15 a f (a + a \operatorname{Sec}[e + f x])^2}$$

Result (type 3, 295 leaves) :

$$\frac{1}{30 a^3 f (1 + \operatorname{Cos}[e + f x])^3} \left(-240 d^3 \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]^6 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) + (c - d) \operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}\left[\frac{e}{2}\right] \left(5 (8 c^2 + 17 c d + 29 d^2) \operatorname{Sin}\left[\frac{f x}{2}\right] - 15 (2 c^2 + 5 c d + 5 d^2) \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 20 c^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 65 c d \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 95 d^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 15 c^2 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - 15 c d \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - 15 d^2 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] + 7 c^2 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] + 16 c d \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] + 22 d^2 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] \right) \right)$$

■ **Problem 231: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^3 (c + d \operatorname{Sec}[e + f x])} dx$$

Optimal (type 3, 181 leaves, 7 steps) :

$$-\frac{2 d^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{c+d}}\right]}{a^3 (c-d)^{7/2} \sqrt{c+d} f} + \frac{\operatorname{Tan}[e + f x]}{5 (c-d) f (a + a \operatorname{Sec}[e + f x])^3} + \frac{(2 c - 7 d) \operatorname{Tan}[e + f x]}{15 a (c-d)^2 f (a + a \operatorname{Sec}[e + f x])^2} + \frac{(2 c^2 - 9 c d + 22 d^2) \operatorname{Tan}[e + f x]}{15 (c-d)^3 f (a^3 + a^3 \operatorname{Sec}[e + f x])}$$

Result (type 3, 345 leaves) :

$$\frac{1}{30 a^3 (c-d)^3 f (1 + \operatorname{Cos}[e + f x])^3}$$

$$\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \left(\frac{480 d^3 \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[e] + \operatorname{Sin}[e]) (c \operatorname{Sin}[e] + (-d + c \operatorname{Cos}[e]) \operatorname{Tan}\left[\frac{f x}{2}\right])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}}}\right] \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^5 (i \operatorname{Cos}[e] + \operatorname{Sin}[e])}{\sqrt{c^2 - d^2} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}} + \operatorname{Sec}\left[\frac{e}{2}\right]} \right.$$

$$\left. \left(5 (8 c^2 - 27 c d + 37 d^2) \operatorname{Sin}\left[\frac{f x}{2}\right] - 15 (2 c^2 - 7 c d + 9 d^2) \operatorname{Sin}\left[e + \frac{f x}{2}\right] + 20 c^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 75 c d \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] + 115 d^2 \operatorname{Sin}\left[e + \frac{3 f x}{2}\right] - 15 \right.$$

$$\left. \left. c^2 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] + 45 c d \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] - 45 d^2 \operatorname{Sin}\left[2 e + \frac{3 f x}{2}\right] + 7 c^2 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] - 24 c d \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] + 32 d^2 \operatorname{Sin}\left[2 e + \frac{5 f x}{2}\right] \right) \right)$$

■ **Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]}{(a + a \operatorname{Sec}[e + f x])^3 (c + d \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$-\frac{2 d^3 (4 c + 3 d) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c+d}}\right]}{a^3 (c-d)^{9/2} (c+d)^{3/2} f} + \frac{d (2 c^3 - 12 c^2 d + 43 c d^2 + 72 d^3) \operatorname{Tan}[e + f x]}{15 a^3 (c-d)^4 (c+d) f (c+d \operatorname{Sec}[e + f x])} + \frac{\operatorname{Tan}[e + f x]}{5 (c-d) f (a + a \operatorname{Sec}[e + f x])^3 (c+d \operatorname{Sec}[e + f x])} +$$

$$\frac{(2 c - 9 d) \operatorname{Tan}[e + f x]}{15 a (c-d)^2 f (a + a \operatorname{Sec}[e + f x])^2 (c+d \operatorname{Sec}[e + f x])} + \frac{(2 c^2 - 12 c d + 45 d^2) \operatorname{Tan}[e + f x]}{15 (c-d)^3 f (a^3 + a^3 \operatorname{Sec}[e + f x]) (c+d \operatorname{Sec}[e + f x])}$$

Result (type 3, 1772 leaves):

$$\left((4 c + 3 d) \operatorname{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 (d + c \operatorname{Cos}[e + f x])^2 \operatorname{Sec}[e + f x]^5 \right.$$

$$\left. \left(\left(16 i d^3 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2 - d^2} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2 - d^2} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]}} \right) \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right] + i c \operatorname{Sin}\left[e + \frac{f x}{2}\right] \right) \right) \right.$$

$$\left. \operatorname{Cos}[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) +$$

$$\left(16 d^3 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{f x}{2}\right]\right] \left(\frac{\operatorname{Cos}[e]}{\sqrt{c^2 - d^2} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]}} - \frac{i \operatorname{Sin}[e]}{\sqrt{c^2 - d^2} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]}} \right) \left(-i d \operatorname{Sin}\left[\frac{f x}{2}\right] + i c \operatorname{Sin}\left[e + \frac{f x}{2}\right] \right) \right)$$

$$15 c d^4 \sin\left[3 e + \frac{7 f x}{2}\right] + 7 c^5 \sin\left[4 e + \frac{7 f x}{2}\right] - 27 c^4 d \sin\left[4 e + \frac{7 f x}{2}\right] + 38 c^3 d^2 \sin\left[4 e + \frac{7 f x}{2}\right] + 72 c^2 d^3 \sin\left[4 e + \frac{7 f x}{2}\right]$$

- **Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + f x]}{(a + a \sec[e + f x])^3 (c + d \sec[e + f x])^3} dx$$

Optimal (type 3, 368 leaves, 9 steps):

$$\begin{aligned} & - \frac{d^3 (20 c^2 + 30 c d + 13 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{c-d} \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c+d}}\right]}{a^3 (c-d)^{11/2} (c+d)^{5/2} f} + \frac{d (4 c^3 - 30 c^2 d + 146 c d^2 + 195 d^3) \operatorname{Tan}[e + f x]}{30 a^3 (c-d)^4 (c+d) f (c+d \sec[e + f x])^2} + \\ & \frac{\operatorname{Tan}[e + f x]}{5 (c-d) f (a + a \sec[e + f x])^3 (c+d \sec[e + f x])^2} + \frac{(2 c - 11 d) \operatorname{Tan}[e + f x]}{15 a (c-d)^2 f (a + a \sec[e + f x])^2 (c+d \sec[e + f x])^2} + \\ & \frac{(2 c^2 - 15 c d + 76 d^2) \operatorname{Tan}[e + f x]}{15 (c-d)^3 f (a^3 + a^3 \sec[e + f x]) (c+d \sec[e + f x])^2} + \frac{d (4 c^4 - 30 c^3 d + 142 c^2 d^2 + 525 c d^3 + 304 d^4) \operatorname{Tan}[e + f x]}{30 a^3 (c-d)^5 (c+d)^2 f (c+d \sec[e + f x])} \end{aligned}$$

Result (type 3, 1096 leaves):

$$\begin{aligned}
& \frac{4 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^4 (d + c \cos[e + fx])^3 \sec\left[\frac{e}{2}\right] \sec[e + fx]^6 (-8c \sin\left[\frac{e}{2}\right] + 23d \sin\left[\frac{e}{2}\right])}{15 (-c + d)^4 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3} + \\
& \left(\frac{(20c^2 + 30cd + 13d^2) \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^6 (d + c \cos[e + fx])^3 \sec[e + fx]^6}{\left(- \left(8i d^3 \operatorname{ArcTan}\left[\sec\left[\frac{fx}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} \right) \left(-i d \sin\left[\frac{fx}{2}\right] + i c \sin\left[e + \frac{fx}{2}\right] \right) \right)} \right. \\
& \left. \cos[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
& \left(8d^3 \operatorname{ArcTan}\left[\sec\left[\frac{fx}{2}\right]\right] \left(\frac{\cos[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} - \frac{i \sin[e]}{\sqrt{c^2 - d^2} \sqrt{\cos[2e] - i \sin[2e]}} \right) \left(-i d \sin\left[\frac{fx}{2}\right] + i c \sin\left[e + \frac{fx}{2}\right] \right) \right) \\
& \left. \sin[e] \right) / \left(\sqrt{c^2 - d^2} f \sqrt{\cos[2e] - i \sin[2e]} \right) \Big) / \\
& \left((-c + d)^5 (c + d)^2 (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3 \right) - \frac{2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right] (d + c \cos[e + fx])^3 \sec\left[\frac{e}{2}\right] \sec[e + fx]^6 \sin\left[\frac{fx}{2}\right]}{5 (-c + d)^3 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3} + \\
& \frac{4 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^3 (d + c \cos[e + fx])^3 \sec\left[\frac{e}{2}\right] \sec[e + fx]^6 (-8c \sin\left[\frac{fx}{2}\right] + 23d \sin\left[\frac{fx}{2}\right])}{15 (-c + d)^4 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3} - \\
& \left(8 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^5 (d + c \cos[e + fx])^3 \sec\left[\frac{e}{2}\right] \sec[e + fx]^6 \left(7c^2 \sin\left[\frac{fx}{2}\right] - 44cd \sin\left[\frac{fx}{2}\right] + 127d^2 \sin\left[\frac{fx}{2}\right] \right) \right) / \\
& \left(15 (-c + d)^5 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3 \right) + \\
& \frac{4 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^6 (d + c \cos[e + fx]) \sec[e] \sec[e + fx]^6 (d^6 \sin[e] - cd^5 \sin[fx])}{c^2 (-c + d)^4 (c + d) f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3} - \\
& \left(4 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^6 (d + c \cos[e + fx])^2 \sec[e] \sec[e + fx]^6 \right. \\
& \left. (-11c^2 d^5 \sin[e] - 6cd^6 \sin[e] + 2d^7 \sin[e] + 10c^3 d^4 \sin[fx] + 6c^2 d^5 \sin[fx] - cd^6 \sin[fx]) \right) / \\
& \left(c^2 (-c + d)^5 (c + d)^2 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3 \right) - \\
& \frac{2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (d + c \cos[e + fx])^3 \sec[e + fx]^6 \tan\left[\frac{e}{2}\right]}{5 (-c + d)^3 f (a + a \sec[e + fx])^3 (c + d \sec[e + fx])^3}
\end{aligned}$$

- **Problem 239: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(g \operatorname{Sec}[e + f x])^{3/2} \sqrt{a + a \operatorname{Sec}[e + f x]}}{c + d \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 149 leaves, 5 steps):

$$\frac{2 \sqrt{a} g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{g} \operatorname{Tan}[e+fx]}{\sqrt{g \operatorname{Sec}[e+fx]} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{d f} - \frac{2 \sqrt{a} \sqrt{c} g^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \operatorname{Tan}[e+fx]}{\sqrt{c+d} \sqrt{g \operatorname{Sec}[e+fx]} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{d \sqrt{c+d} f}$$

Result (type 3, 427 leaves):

$$\frac{1}{4 (i + \sqrt{2}) d \sqrt{c+d} f \sqrt{g \operatorname{Sec}[e+fx]}} (-2 i + \sqrt{2}) g^2$$

$$\left(2 \sqrt{c+d} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(e+fx)\right] - (-1 + \sqrt{2}) \sin\left[\frac{1}{4}(e+fx)\right]}{(1 + \sqrt{2}) \cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]}\right] + 2 \sqrt{c+d} \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(e+fx)\right] - (1 + \sqrt{2}) \sin\left[\frac{1}{4}(e+fx)\right]}{(-1 + \sqrt{2}) \cos\left[\frac{1}{4}(e+fx)\right] - \sin\left[\frac{1}{4}(e+fx)\right]}\right] \right) +$$

$$i \left(2 \sqrt{c+d} \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(e+fx)\right]\right] - \sqrt{c+d} \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(e+fx)\right]\right] - \right.$$

$$\left. \sqrt{c+d} \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(e+fx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(e+fx)\right]\right] + 2 \sqrt{c} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} - 2 \sqrt{c} \sin\left[\frac{1}{2}(e+fx)\right]\right] - \right.$$

$$\left. 2 \sqrt{c} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} + 2 \sqrt{c} \sin\left[\frac{1}{2}(e+fx)\right]\right] \right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{a (1 + \operatorname{Sec}[e+fx])}$$

- **Problem 243: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(g \operatorname{Sec}[e + f x])^{5/2}}{\sqrt{a + a \operatorname{Sec}[e + f x]} (c + d \operatorname{Sec}[e + f x])} dx$$

Optimal (type 3, 231 leaves, 8 steps):

$$\frac{2 g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{g} \operatorname{Tan}[e+fx]}{\sqrt{g \operatorname{Sec}[e+fx]} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{\sqrt{a} d f} + \frac{\sqrt{2} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{g} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{g \operatorname{Sec}[e+fx]} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{\sqrt{a} (c-d) f} - \frac{2 c^{3/2} g^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c} \sqrt{g} \operatorname{Tan}[e+fx]}{\sqrt{c+d} \sqrt{g \operatorname{Sec}[e+fx]} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{\sqrt{a} (c-d) d \sqrt{c+d} f}$$

Result (type 3, 1097 leaves):

$$\begin{aligned}
& \frac{1}{2 \left(i + \sqrt{2} \right) d \left(-c + d \right) \sqrt{c+d} f \sqrt{a} \left(1 + \operatorname{Sec}[e + f x] \right)} \\
& g^2 \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] \left(-2 \left(-2 i + \sqrt{2} \right) (c - d) \sqrt{c+d} \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \left(-1 + \sqrt{2} \right) \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]}{\left(1 + \sqrt{2} \right) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]} \right] - \right. \\
& \quad \left. 2 \left(-2 i + \sqrt{2} \right) (c - d) \sqrt{c+d} \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \left(1 + \sqrt{2} \right) \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]}{\left(-1 + \sqrt{2} \right) \operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]} \right] + 4 i d \sqrt{c+d} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]\right] \right) + \\
& \quad 4 \sqrt{2} d \sqrt{c+d} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]\right] - 4 i d \sqrt{c+d} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]\right] - \\
& \quad 4 \sqrt{2} d \sqrt{c+d} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{4}(e + f x)\right]\right] - 4 c \sqrt{c+d} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\
& \quad 2 i \sqrt{2} c \sqrt{c+d} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 4 d \sqrt{c+d} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 2 i \sqrt{2} d \sqrt{c+d} \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\
& \quad 2 c \sqrt{c+d} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + i \sqrt{2} c \sqrt{c+d} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\
& \quad 2 d \sqrt{c+d} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - i \sqrt{2} d \sqrt{c+d} \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\
& \quad 2 c \sqrt{c+d} \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + i \sqrt{2} c \sqrt{c+d} \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\
& \quad 2 d \sqrt{c+d} \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - i \sqrt{2} d \sqrt{c+d} \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - \\
& \quad 4 c^{3/2} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} - 2 \sqrt{c} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] - 2 i \sqrt{2} c^{3/2} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} - 2 \sqrt{c} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + \\
& \quad \left. 4 c^{3/2} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} + 2 \sqrt{c} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] + 2 i \sqrt{2} c^{3/2} \operatorname{Log}\left[\sqrt{2} \sqrt{c+d} + 2 \sqrt{c} \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \right) \sqrt{g \operatorname{Sec}[e + f x]}
\end{aligned}$$

■ **Problem 245: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x]) (c + d \operatorname{Sec}[e + f x])^3 dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$\begin{aligned}
& \frac{(8 a c^3 + 12 b c^2 d + 12 a c d^2 + 3 b d^3) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{8 f} + \frac{(4 a d (4 c^2 + d^2) + 3 b (c^3 + 4 c d^2)) \operatorname{Tan}[e + f x]}{6 f} + \\
& \frac{d (6 b c^2 + 20 a c d + 9 b d^2) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{24 f} + \frac{(3 b c + 4 a d) (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{12 f} + \frac{b (c + d \operatorname{Sec}[e + f x])^3 \operatorname{Tan}[e + f x]}{4 f}
\end{aligned}$$

Result (type 3, 1179 leaves):

$$\begin{aligned}
& \left((-8ac^3 - 12bc^2d - 12acd^2 - 3bd^3) \cos[ex + fx]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(ex + fx)\right] - \sin\left[\frac{1}{2}(ex + fx)\right]\right] (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3 \right) / \\
& (8f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3) + \\
& \left((8ac^3 + 12bc^2d + 12acd^2 + 3bd^3) \cos[ex + fx]^4 \operatorname{Log}\left[\cos\left[\frac{1}{2}(ex + fx)\right] + \sin\left[\frac{1}{2}(ex + fx)\right]\right] (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3 \right) / \\
& (8f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3) + \frac{bd^3 \cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3}{16f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] - \sin[\frac{1}{2}(ex + fx)])^4} + \\
& \frac{(36bc^2d + 36acd^2 + 12bcd^2 + 4ad^3 + 9bd^3) \cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3}{48f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] - \sin[\frac{1}{2}(ex + fx)])^2} - \\
& \frac{bd^3 \cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3}{16f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] + \sin[\frac{1}{2}(ex + fx)])^4} + \\
& \frac{(-36bc^2d - 36acd^2 - 12bcd^2 - 4ad^3 - 9bd^3) \cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3}{48f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] + \sin[\frac{1}{2}(ex + fx)])^2} + \\
& \frac{\cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3 (3bc^2d \sin[\frac{1}{2}(ex + fx)] + ad^3 \sin[\frac{1}{2}(ex + fx)])}{6f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] - \sin[\frac{1}{2}(ex + fx)])^3} + \\
& \frac{\cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^3 (3bc^2d \sin[\frac{1}{2}(ex + fx)] + ad^3 \sin[\frac{1}{2}(ex + fx)])}{6f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 (\cos[\frac{1}{2}(ex + fx)] + \sin[\frac{1}{2}(ex + fx)])^3} + \left(\cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) \right. \\
& \left. (c + d \operatorname{Sec}[ex + fx])^3 \left(3bc^3 \sin\left[\frac{1}{2}(ex + fx)\right] + 9ac^2d \sin\left[\frac{1}{2}(ex + fx)\right] + 6bcd^2 \sin\left[\frac{1}{2}(ex + fx)\right] + 2ad^3 \sin\left[\frac{1}{2}(ex + fx)\right] \right) \right) / \\
& \left(3f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 \left(\cos\left[\frac{1}{2}(ex + fx)\right] - \sin\left[\frac{1}{2}(ex + fx)\right] \right) \right) + \left(\cos[ex + fx]^4 (a + b \operatorname{Sec}[ex + fx]) \right. \\
& \left. (c + d \operatorname{Sec}[ex + fx])^3 \left(3bc^3 \sin\left[\frac{1}{2}(ex + fx)\right] + 9ac^2d \sin\left[\frac{1}{2}(ex + fx)\right] + 6bcd^2 \sin\left[\frac{1}{2}(ex + fx)\right] + 2ad^3 \sin\left[\frac{1}{2}(ex + fx)\right] \right) \right) / \\
& \left(3f(b + a \cos[ex + fx]) (d + c \cos[ex + fx])^3 \left(\cos\left[\frac{1}{2}(ex + fx)\right] + \sin\left[\frac{1}{2}(ex + fx)\right] \right) \right)
\end{aligned}$$

■ **Problem 246: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[ex + fx] (a + b \operatorname{Sec}[ex + fx]) (c + d \operatorname{Sec}[ex + fx])^2 dx$$

Optimal (type 3, 115 leaves, 6 steps):

$$\frac{(2 b c d + a (2 c^2 + d^2)) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 f} + \frac{2 (3 a c d + b (c^2 + d^2)) \operatorname{Tan}[e + f x]}{3 f} +$$

$$\frac{d (2 b c + 3 a d) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{6 f} + \frac{b (c + d \operatorname{Sec}[e + f x])^2 \operatorname{Tan}[e + f x]}{3 f}$$

Result (type 3, 239 leaves):

$$\frac{1}{6 f} \operatorname{Sec}[e + f x]^3 \left(-\frac{9}{4} (2 b c d + a (2 c^2 + d^2)) \operatorname{Cos}[e + f x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) - \right.$$

$$\left. \frac{3}{4} (2 b c d + a (2 c^2 + d^2)) \operatorname{Cos}[3 (e + f x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) + \right.$$

$$\left. (3 b c^2 + 6 a c d + 4 b d^2 + 3 d (2 b c + a d) \operatorname{Cos}[e + f x] + (3 b c^2 + 6 a c d + 2 b d^2) \operatorname{Cos}[2 (e + f x)]) \operatorname{Sin}[e + f x] \right)$$

■ **Problem 247: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x] (a + b \operatorname{Sec}[e + f x]) (c + d \operatorname{Sec}[e + f x]) dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{(2 a c + b d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 f} + \frac{(b c + a d) \operatorname{Tan}[e + f x]}{f} + \frac{b d \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 f}$$

Result (type 3, 164 leaves):

$$\frac{1}{4 f} \left(-2 (2 a c + b d) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + \right.$$

$$\left. 4 a c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + 2 b d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + \right.$$

$$\left. \frac{b d}{\left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} - \frac{b d}{\left(\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right)^2} + 4 (b c + a d) \operatorname{Tan}[e + f x] \right)$$

■ **Problem 252: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x] (c + d \operatorname{Sec}[e + f x])^4}{a + b \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 247 leaves, 12 steps):

$$\frac{d^3 (4 b c - a d) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 b^2 f} + \frac{d (2 b c - a d) (2 b^2 c^2 - 2 a b c d + a^2 d^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{b^4 f} + \frac{2 (b c - a d)^4 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^4 \sqrt{a+b} f} +$$

$$\frac{d^4 \operatorname{Tan}[e + f x]}{b f} + \frac{d^2 (6 b^2 c^2 - 4 a b c d + a^2 d^2) \operatorname{Tan}[e + f x]}{b^3 f} + \frac{d^3 (4 b c - a d) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{2 b^2 f} + \frac{d^4 \operatorname{Tan}[e + f x]^3}{3 b f}$$

Result (type 3, 1150 leaves) :

$$\begin{aligned}
& \frac{2 (b c - a d)^4 \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]}{\sqrt{a^2-b^2}}\right] \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4}{b^4 \sqrt{a^2-b^2} f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x])} + \\
& \left((-8 b^3 c^3 d + 12 a b^2 c^2 d^2 - 8 a^2 b c d^3 - 4 b^3 c d^3 + 2 a^3 d^4 + a b^2 d^4) \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) \right. \\
& \quad \left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4\right) / \left(2 b^4 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x])\right) + \\
& \left((8 b^3 c^3 d - 12 a b^2 c^2 d^2 + 8 a^2 b c d^3 + 4 b^3 c d^3 - 2 a^3 d^4 - a b^2 d^4) \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) \right. \\
& \quad \left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] (c+d \operatorname{Sec}[e+f x])^4\right) / \left(2 b^4 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x])\right) + \\
& \quad \frac{(12 b c d^3 - 3 a d^4 + b d^4) \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4}{12 b^2 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2} + \\
& \quad \frac{d^4 \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 b f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3} + \\
& \quad \frac{d^4 \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]}{6 b f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^3} + \\
& \quad \frac{(-12 b c d^3 + 3 a d^4 - b d^4) \operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4}{12 b^2 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right)^2} + \\
& \left(\operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) (c+d \operatorname{Sec}[e+f x])^4 \right. \\
& \quad \left. \left(18 b^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right) / \\
& \left(3 b^3 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \right) + \left(\operatorname{Cos}[e+f x]^3 (b+a \operatorname{Cos}[e+f x]) \right. \\
& \quad \left. (c+d \operatorname{Sec}[e+f x])^4 \left(18 b^2 c^2 d^2 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] - 12 a b c d^3 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 3 a^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] + 2 b^2 d^4 \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right] \right) \right) / \\
& \left(3 b^3 f (d+c \operatorname{Cos}[e+f x])^4 (a+b \operatorname{Sec}[e+f x]) \left(\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right) \right)
\end{aligned}$$

■ **Problem 253: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (c + d \text{Sec}[e + f x])^3}{a + b \text{Sec}[e + f x]} dx$$

Optimal (type 3, 170 leaves, 10 steps):

$$\frac{d^3 \text{ArcTanh}[\text{Sin}[e + f x]]}{2 b f} + \frac{d (3 b^2 c^2 - 3 a b c d + a^2 d^2) \text{ArcTanh}[\text{Sin}[e + f x]]}{b^3 f} +$$

$$\frac{2 (b c - a d)^3 \text{ArcTanh}\left[\frac{\sqrt{a-b} \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^3 \sqrt{a+b} f} + \frac{d^2 (3 b c - a d) \text{Tan}[e + f x]}{b^2 f} + \frac{d^3 \text{Sec}[e + f x] \text{Tan}[e + f x]}{2 b f}$$

Result (type 3, 389 leaves):

$$\frac{1}{4 b^3 f (d + c \text{Cos}[e + f x])^3 (a + b \text{Sec}[e + f x])} \text{Cos}[e + f x]^2 (b + a \text{Cos}[e + f x]) (c + d \text{Sec}[e + f x])^3$$

$$\left(\frac{8 (-b c + a d)^3 \text{ArcTanh}\left[\frac{(-a+b) \text{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{a^2 - b^2}}\right]}{\sqrt{a^2 - b^2}} - 2 d (-6 a b c d + 2 a^2 d^2 + b^2 (6 c^2 + d^2)) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \right) +$$

$$2 d (-6 a b c d + 2 a^2 d^2 + b^2 (6 c^2 + d^2)) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]\right] + \frac{b^2 d^3}{(\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right])^2} +$$

$$\left(\frac{4 b d^2 (3 b c - a d) \text{Sin}\left[\frac{1}{2} (e + f x)\right]}{\text{Cos}\left[\frac{1}{2} (e + f x)\right] - \text{Sin}\left[\frac{1}{2} (e + f x)\right]} - \frac{b^2 d^3}{(\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right])^2} + \frac{4 b d^2 (3 b c - a d) \text{Sin}\left[\frac{1}{2} (e + f x)\right]}{\text{Cos}\left[\frac{1}{2} (e + f x)\right] + \text{Sin}\left[\frac{1}{2} (e + f x)\right]} \right)$$

■ **Problem 258: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (c + d \text{Sec}[e + f x])^5}{(a + b \text{Sec}[e + f x])^2} dx$$

Optimal (type 3, 379 leaves, 16 steps):

$$\frac{d^4 (5bc - 2ad) \operatorname{ArcTanh}[\operatorname{Sin}[e + fx]]}{2b^3 f} + \frac{d^2 (10b^3 c^3 - 20ab^2 c^2 d + 15a^2 bc d^2 - 4a^3 d^3) \operatorname{ArcTanh}[\operatorname{Sin}[e + fx]]}{b^5 f} +$$

$$\frac{2(bc - ad)^5 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a+b}}\right]}{a(a-b)^{3/2} b^3 (a+b)^{3/2} f} + \frac{2(bc - ad)^4 (bc + 4ad) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a+b}}\right]}{a\sqrt{a-b} b^5 \sqrt{a+b} f} - \frac{(bc - ad)^5 \operatorname{Sin}[e + fx]}{b^4 (a^2 - b^2) f (b + a \operatorname{Cos}[e + fx])} +$$

$$\frac{d^5 \operatorname{Tan}[e + fx]}{b^2 f} + \frac{d^3 (10b^2 c^2 - 10ab^2 cd + 3a^2 d^2) \operatorname{Tan}[e + fx]}{b^4 f} + \frac{d^4 (5bc - 2ad) \operatorname{Sec}[e + fx] \operatorname{Tan}[e + fx]}{2b^3 f} + \frac{d^5 \operatorname{Tan}[e + fx]^3}{3b^2 f}$$

Result (type 3, 1137 leaves):

$$- \left(2(bc - ad)^4 (-abc - 4a^2 d + 5b^2 d) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{\sqrt{a^2 - b^2}}\right] \operatorname{Cos}[e + fx]^3 (b + a \operatorname{Cos}[e + fx])^2 (c + d \operatorname{Sec}[e + fx])^5 \right) /$$

$$\left(b^5 \sqrt{a^2 - b^2} (-a^2 + b^2) f (d + c \operatorname{Cos}[e + fx])^5 (a + b \operatorname{Sec}[e + fx])^2 \right) +$$

$$\left((-20b^3 c^3 d^2 + 40ab^2 c^2 d^3 - 30a^2 bc d^4 - 5b^3 c d^4 + 8a^3 d^5 + 2ab^2 d^5) \operatorname{Cos}[e + fx]^3 (b + a \operatorname{Cos}[e + fx])^2 \right.$$

$$\left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] - \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] (c + d \operatorname{Sec}[e + fx])^5 \right) / \left(2b^5 f (d + c \operatorname{Cos}[e + fx])^5 (a + b \operatorname{Sec}[e + fx])^2 \right) +$$

$$\left((20b^3 c^3 d^2 - 40ab^2 c^2 d^3 + 30a^2 bc d^4 + 5b^3 c d^4 - 8a^3 d^5 - 2ab^2 d^5) \operatorname{Cos}[e + fx]^3 (b + a \operatorname{Cos}[e + fx])^2 \right.$$

$$\left. \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] (c + d \operatorname{Sec}[e + fx])^5 \right) / \left(2b^5 f (d + c \operatorname{Cos}[e + fx])^5 (a + b \operatorname{Sec}[e + fx])^2 \right) +$$

$$\frac{1}{24b^4 (-a^2 + b^2) f (d + c \operatorname{Cos}[e + fx])^5 (a + b \operatorname{Sec}[e + fx])^2} (b + a \operatorname{Cos}[e + fx]) (c + d \operatorname{Sec}[e + fx])^5$$

$$(-60a^2 b^3 c^2 d^3 \operatorname{Sin}[e + fx] + 60b^5 c^2 d^3 \operatorname{Sin}[e + fx] + 45a^3 b^2 c d^4 \operatorname{Sin}[e + fx] - 45ab^4 c d^4 \operatorname{Sin}[e + fx] - 12a^4 b d^5 \operatorname{Sin}[e + fx] +$$

$$12b^5 d^5 \operatorname{Sin}[e + fx] + 6b^5 c^5 \operatorname{Sin}[2(e + fx)] - 30ab^4 c^4 d \operatorname{Sin}[2(e + fx)] + 60a^2 b^3 c^3 d^2 \operatorname{Sin}[2(e + fx)] -$$

$$120a^3 b^2 c^2 d^3 \operatorname{Sin}[2(e + fx)] + 60ab^4 c^2 d^3 \operatorname{Sin}[2(e + fx)] + 90a^4 b c d^4 \operatorname{Sin}[2(e + fx)] - 90a^2 b^3 c d^4 \operatorname{Sin}[2(e + fx)] +$$

$$30b^5 c d^4 \operatorname{Sin}[2(e + fx)] - 24a^5 d^5 \operatorname{Sin}[2(e + fx)] + 22a^3 b^2 d^5 \operatorname{Sin}[2(e + fx)] - 4ab^4 d^5 \operatorname{Sin}[2(e + fx)] -$$

$$60a^2 b^3 c^2 d^3 \operatorname{Sin}[3(e + fx)] + 60b^5 c^2 d^3 \operatorname{Sin}[3(e + fx)] + 45a^3 b^2 c d^4 \operatorname{Sin}[3(e + fx)] - 45ab^4 c d^4 \operatorname{Sin}[3(e + fx)] -$$

$$12a^4 b d^5 \operatorname{Sin}[3(e + fx)] + 8a^2 b^3 d^5 \operatorname{Sin}[3(e + fx)] + 4b^5 d^5 \operatorname{Sin}[3(e + fx)] + 3b^5 c^5 \operatorname{Sin}[4(e + fx)] - 15ab^4 c^4 d \operatorname{Sin}[4(e + fx)] +$$

$$30a^2 b^3 c^3 d^2 \operatorname{Sin}[4(e + fx)] - 60a^3 b^2 c^2 d^3 \operatorname{Sin}[4(e + fx)] + 30ab^4 c^2 d^3 \operatorname{Sin}[4(e + fx)] + 45a^4 b c d^4 \operatorname{Sin}[4(e + fx)] -$$

$$30a^2 b^3 c d^4 \operatorname{Sin}[4(e + fx)] - 12a^5 d^5 \operatorname{Sin}[4(e + fx)] + 7a^3 b^2 d^5 \operatorname{Sin}[4(e + fx)] + 2ab^4 d^5 \operatorname{Sin}[4(e + fx)])$$

■ **Problem 264: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e + fx] \sqrt{a + b \operatorname{Sec}[e + fx]}}{c + d \operatorname{Sec}[e + fx]} dx$$

Optimal (type 4, 213 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{d f}$$

$$\frac{2(b c-a d) \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sec}[e+f x]}{a+b}} \operatorname{Tan}[e+f x]}{d(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 35 leaves):

$$\int \frac{\operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]}}{c+d \operatorname{Sec}[e+f x]} dx$$

■ **Problem 265: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 4, 196 leaves, 1 step):

$$\frac{1}{d \sqrt{\frac{a+b}{c+d}} f} 2 \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{a+b \operatorname{Sec}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\sqrt{-\frac{(b c-a d)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(b c-a d)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x])$$

Result (type 8, 37 leaves):

$$\int \frac{\operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

■ **Problem 269: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+f x]^2}{\sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

Optimal (type 4, 396 leaves, 3 steps):

$$\frac{1}{b d \sqrt{\frac{a+b}{c+d}} f} 2 \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{b(c+d)}{(a+b) d}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d \operatorname{Sec}[e+f x]}}{\sqrt{a+b \operatorname{Sec}[e+f x]}}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right]$$

$$\sqrt{-\frac{(b c-a d)(1-\operatorname{Sec}[e+f x])}{(c+d)(a+b \operatorname{Sec}[e+f x])}} \sqrt{\frac{(b c-a d)(1+\operatorname{Sec}[e+f x])}{(c-d)(a+b \operatorname{Sec}[e+f x])}} (a+b \operatorname{Sec}[e+f x]) -$$

$$\frac{1}{b \sqrt{c+d} (b c-a d) f} 2 a \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b} \sqrt{c+d \operatorname{Sec}[e+f x]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]$$

$$\sqrt{\frac{(b c-a d)(1-\operatorname{Sec}[e+f x])}{(a+b)(c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(b c-a d)(1+\operatorname{Sec}[e+f x])}{(a-b)(c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])$$

Result (type 8, 39 leaves):

$$\int \frac{\operatorname{Sec}[e+f x]^2}{\sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{c+d \operatorname{Sec}[e+f x]}} dx$$

■ **Problem 270: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(g \operatorname{Sec}[e+f x])^{3/2} \sqrt{c+d \operatorname{Sec}[e+f x]}}{a+b \operatorname{Sec}[e+f x]} dx$$

Optimal (type 4, 170 leaves, 7 steps):

$$\frac{2 d g \sqrt{\frac{d+c \operatorname{Cos}[e+f x]}{c+d}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(e+f x), \frac{2c}{c+d}\right] \sqrt{g \operatorname{Sec}[e+f x]}}{b f \sqrt{c+d \operatorname{Sec}[e+f x]}} +$$

$$\frac{2(b c-a d) g \sqrt{\frac{d+c \operatorname{Cos}[e+f x]}{c+d}} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(e+f x), \frac{2c}{c+d}\right] \sqrt{g \operatorname{Sec}[e+f x]}}{b(a+b) f \sqrt{c+d \operatorname{Sec}[e+f x]}}$$

Result (type 4, 223 leaves):

$$\begin{aligned}
& - \left(2 i g \sqrt{-\frac{c(-1+\cos[e+fx])}{c+d}} \sqrt{\frac{c(1+\cos[e+fx])}{c-d}} \cot[e+fx] \right. \\
& \left. \left(\text{EllipticPi}\left[1-\frac{c}{d}, i \text{ArcSinh}\left[\sqrt{\frac{1}{c-d}} \sqrt{d+c \cos[e+fx]}\right], \frac{-c+d}{c+d}\right] - \text{EllipticPi}\left[\frac{a(-c+d)}{-bc+ad}, \right. \right. \right. \\
& \left. \left. \left. i \text{ArcSinh}\left[\sqrt{\frac{1}{c-d}} \sqrt{d+c \cos[e+fx]}\right], \frac{-c+d}{c+d}\right] \right) \sqrt{g \sec[e+fx]} \sqrt{c+d \sec[e+fx]} \right) / \left(b \sqrt{\frac{1}{c-d}} f \sqrt{d+c \cos[e+fx]} \right)
\end{aligned}$$

■ **Problem 272: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{g \sec[e+fx]} \sqrt{c+d \sec[e+fx]}}{a+b \cos[e+fx]} dx$$

Optimal (type 4, 168 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 d \sqrt{\frac{d+c \cos[e+fx]}{c+d}} \text{EllipticPi}\left[2, \frac{1}{2}(e+fx), \frac{2c}{c+d}\right] \sqrt{g \sec[e+fx]}}{a f \sqrt{c+d \sec[e+fx]}} + \\
& \frac{2(a c-b d) \sqrt{\frac{d+c \cos[e+fx]}{c+d}} \text{EllipticPi}\left[\frac{2b}{a+b}, \frac{1}{2}(e+fx), \frac{2c}{c+d}\right] \sqrt{g \sec[e+fx]}}{a(a+b) f \sqrt{c+d \sec[e+fx]}}
\end{aligned}$$

Result (type 4, 222 leaves):

$$\begin{aligned}
& - \left(2 i \sqrt{-\frac{c(-1+\cos[e+fx])}{c+d}} \sqrt{\frac{c(1+\cos[e+fx])}{c-d}} \cot[e+fx] \right. \\
& \left. \left(\text{EllipticPi}\left[1-\frac{c}{d}, i \text{ArcSinh}\left[\sqrt{\frac{1}{c-d}} \sqrt{d+c \cos[e+fx]}\right], \frac{-c+d}{c+d}\right] - \text{EllipticPi}\left[\frac{b(-c+d)}{-ac+bd}, \right. \right. \right. \\
& \left. \left. \left. i \text{ArcSinh}\left[\sqrt{\frac{1}{c-d}} \sqrt{d+c \cos[e+fx]}\right], \frac{-c+d}{c+d}\right] \right) \sqrt{g \sec[e+fx]} \sqrt{c+d \sec[e+fx]} \right) / \left(a \sqrt{\frac{1}{c-d}} f \sqrt{d+c \cos[e+fx]} \right)
\end{aligned}$$

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]}}{c + c \text{Sec}[e + f x]} dx$$

Optimal (type 4, 95 leaves, 1 step):

$$\frac{\text{EllipticE}\left[\text{ArcSin}\left[\frac{\text{Tan}[e + f x]}{1 + \text{Sec}[e + f x]}\right], \frac{a - b}{a + b}\right] \sqrt{\frac{1}{1 + \text{Sec}[e + f x]}} \sqrt{a + b \text{Sec}[e + f x]}}{c f \sqrt{\frac{a + b \text{Sec}[e + f x]}{(a + b)(1 + \text{Sec}[e + f x])}}}$$

Result (type 4, 1999 leaves):

$$\frac{\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]} \left(-2 \text{Sin}[e + f x] + 2 \text{Tan}\left[\frac{1}{2}(e + f x)\right]\right)}{f (c + c \text{Sec}[e + f x])} + \left(\text{Cos}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^5\right.$$

$$\left.\left(\frac{b}{\sqrt{b + a \text{Cos}[e + f x]} \sqrt{\text{Sec}[e + f x]}} + \frac{a \sqrt{\text{Sec}[e + f x]}}{\sqrt{b + a \text{Cos}[e + f x]}} + \frac{b \sqrt{\text{Sec}[e + f x]}}{\sqrt{b + a \text{Cos}[e + f x]}} + \frac{a \text{Cos}[2(e + f x)] \sqrt{\text{Sec}[e + f x]}}{\sqrt{b + a \text{Cos}[e + f x]}}\right) \sqrt{\text{Sec}[e + f x]}\right.$$

$$\left.\sqrt{1 + \text{Sec}[e + f x]} \sqrt{a + b \text{Sec}[e + f x]} \left(2 \text{Cos}\left[\frac{1}{2}(e + f x)\right] \sqrt{\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]}} \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a - b}{a + b}\right] + \right.$$

$$\left.\left.\sqrt{\frac{b + a \text{Cos}[e + f x]}{(a + b)(1 + \text{Cos}[e + f x])}} \left(-\text{Sin}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{3}{2}(e + f x)\right]\right)\right)\right) /$$

$$\left(4 f \left(\frac{1}{1 + \text{Cos}[e + f x]}\right)^{3/2} \sqrt{\frac{b + a \text{Cos}[e + f x]}{(a + b)(1 + \text{Cos}[e + f x])}} (c + c \text{Sec}[e + f x])\right.$$

$$\left.- \left(a \text{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \sqrt{1 + \text{Sec}[e + f x]} \text{Sin}[e + f x] \left(2 \text{Cos}\left[\frac{1}{2}(e + f x)\right] \sqrt{\frac{\text{Cos}[e + f x]}{1 + \text{Cos}[e + f x]}} \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a - b}{a + b}\right] + \right.$$

$$\left.\left.\sqrt{\frac{b + a \text{Cos}[e + f x]}{(a + b)(1 + \text{Cos}[e + f x])}} \left(-\text{Sin}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{3}{2}(e + f x)\right]\right)\right)\right) /$$

$$\begin{aligned}
& \left(8 \left(\frac{1}{1 + \cos[e + f x]} \right)^{3/2} \sqrt{b + a \cos[e + f x]} \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \right) - \left(3 \sqrt{b + a \cos[e + f x]} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \right. \\
& \left. \sqrt{1 + \operatorname{Sec}[e + f x]} \sin[e + f x] \left(2 \cos\left[\frac{1}{2}(e + f x)\right] \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \left. \left. \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \left(-\sin\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{3}{2}(e + f x)\right] \right) \right) \right) / \left(8 \sqrt{\frac{1}{1 + \cos[e + f x]}} \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \right) - \\
& \left(\sqrt{b + a \cos[e + f x]} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \sqrt{1 + \operatorname{Sec}[e + f x]} \left(-\frac{a \sin[e + f x]}{(a + b)(1 + \cos[e + f x])} + \frac{(b + a \cos[e + f x]) \sin[e + f x]}{(a + b)(1 + \cos[e + f x])^2} \right) \right. \\
& \left(2 \cos\left[\frac{1}{2}(e + f x)\right] \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a - b}{a + b}\right] + \right. \\
& \left. \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \left(-\sin\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{3}{2}(e + f x)\right] \right) \right) / \left(8 \left(\frac{1}{1 + \cos[e + f x]} \right)^{3/2} \left(\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])} \right)^{3/2} \right) + \\
& \left(5 \sqrt{b + a \cos[e + f x]} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \sqrt{1 + \operatorname{Sec}[e + f x]} \left(2 \cos\left[\frac{1}{2}(e + f x)\right] \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \operatorname{EllipticE}\left[\right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a - b}{a + b}\right] + \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \left(-\sin\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{3}{2}(e + f x)\right] \right) \right) \right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \right) / \\
& \left(8 \left(\frac{1}{1 + \cos[e + f x]} \right)^{3/2} \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \right) + \frac{1}{4 \left(\frac{1}{1 + \cos[e + f x]} \right)^{3/2} \sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}}} \\
& \sqrt{b + a \cos[e + f x]} \operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^5 \sqrt{1 + \operatorname{Sec}[e + f x]} \left(\sqrt{\frac{b + a \cos[e + f x]}{(a + b)(1 + \cos[e + f x])}} \left(-\frac{1}{2} \cos\left[\frac{1}{2}(e + f x)\right] + \frac{3}{2} \cos\left[\frac{3}{2}(e + f x)\right] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] \sin\left[\frac{1}{2}(e+fx)\right] + \\
& \frac{\cos\left[\frac{1}{2}(e+fx)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] \left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]}\right)}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} + \\
& \frac{\left(-\frac{a \sin[e+fx]}{(a+b)(1+\cos[e+fx])} + \frac{(b+a \cos[e+fx]) \sin[e+fx]}{(a+b)(1+\cos[e+fx])^2}\right) \left(-\sin\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{3}{2}(e+fx)\right]\right)}{2 \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}}} + \\
& \left. \frac{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}}}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \right) + \\
& \left(\sqrt{b+a \cos[e+fx]} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^5 \operatorname{Sec}[e+fx] \left(2 \cos\left[\frac{1}{2}(e+fx)\right] \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \left(-\sin\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{3}{2}(e+fx)\right]\right) \right) \operatorname{Tan}[e+fx] \right) / \\
& \left. \left(8 \left(\frac{1}{1+\cos[e+fx]} \right)^{3/2} \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \sqrt{1+\operatorname{Sec}[e+fx]} \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 274: Unable to integrate problem.**

$$\int \frac{(g \operatorname{Sec}[e+fx])^{3/2} \sqrt{a+b \operatorname{Sec}[e+fx]}}{c+c \operatorname{Sec}[e+fx]} dx$$

Optimal (type 4, 295 leaves, 11 steps):

$$\frac{g(b+a \cos[e+fx]) \operatorname{EllipticE}\left[\frac{1}{2}(e+fx), \frac{2a}{a+b}\right] \sqrt{g \sec[e+fx]}}{c f \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \sqrt{a+b \sec[e+fx]}} + \frac{(a-b) g \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(e+fx), \frac{2a}{a+b}\right] \sqrt{g \sec[e+fx]}}{c f \sqrt{a+b \sec[e+fx]}} +$$

$$\frac{2 b g \sqrt{\frac{b+a \cos[e+fx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(e+fx), \frac{2a}{a+b}\right] \sqrt{g \sec[e+fx]}}{c f \sqrt{a+b \sec[e+fx]}} - \frac{g(b+a \cos[e+fx]) \sqrt{g \sec[e+fx]} \sin[e+fx]}{f(c+c \cos[e+fx]) \sqrt{a+b \sec[e+fx]}}$$

Result (type 8, 41 leaves):

$$\int \frac{(g \sec[e+fx])^{3/2} \sqrt{a+b \sec[e+fx]}}{c+c \sec[e+fx]} dx$$

■ **Problem 275: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]}{\sqrt{a+b \sec[e+fx]} (c+c \sec[e+fx])} dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \cot[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e+fx])}{a+b}} \sqrt{-\frac{b(1+\sec[e+fx])}{a-b}}}{(a-b) c f} +$$

$$\frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\tan[e+fx]}{1+\sec[e+fx]}\right], \frac{a-b}{a+b}\right] \sqrt{\frac{1}{1+\sec[e+fx]}} \sqrt{a+b \sec[e+fx]}}{(a-b) c f \sqrt{\frac{a+b \sec[e+fx]}{(a+b)(1+\sec[e+fx])}}}$$

Result (type 4, 2173 leaves):

$$\frac{\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (b+a \cos[e+fx]) \sec[e+fx]^2 \left(\frac{2 \sin[e+fx]}{-a+b} - \frac{2 \tan\left[\frac{1}{2}(e+fx)\right]}{-a+b}\right)}{f \sqrt{a+b \sec[e+fx]} (c+c \sec[e+fx])} -$$

$$\left(2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \left(-\frac{b}{(-a+b) \sqrt{b+a \cos[e+fx]} \sqrt{\sec[e+fx]}} - \frac{a \sqrt{\sec[e+fx]}}{(-a+b) \sqrt{b+a \cos[e+fx]}} + \right. \right.$$

$$\left. \left. \frac{b \sqrt{\sec[e+fx]}}{(-a+b) \sqrt{b+a \cos[e+fx]}} - \frac{a \cos[2(e+fx)] \sqrt{\sec[e+fx]}}{(-a+b) \sqrt{b+a \cos[e+fx]}} \right) \sec[e+fx]^{3/2} \sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]}$$

$$\begin{aligned}
& \left((a-b) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \operatorname{Cos}[e+fx]) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{a+b}} + \right. \\
& \left. \sqrt{2} \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}} (b+a \operatorname{Cos}[e+fx]) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg) / \\
& \left(\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) f \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^4} \sqrt{a+b \operatorname{Sec}[e+fx]} (c+c \operatorname{Sec}[e+fx]) \right. \\
& \left. - \left(2 \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2 \sqrt{\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx]} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \left((a-b) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right], \frac{a+b}{a-b} \right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(b+a \operatorname{Cos}[e+fx]) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{a+b}} + \sqrt{2} \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}} (b+a \operatorname{Cos}[e+fx]) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) / \\
& \left(\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) \sqrt{b+a \operatorname{Cos}[e+fx]} \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^4} \right) - \left(a \sqrt{\operatorname{Cos} \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx]} \right. \\
& \left. \operatorname{Sin}[e+fx] \left((a-b) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \operatorname{Cos}[e+fx]) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{a+b}} + \right. \right. \\
& \left. \left. \sqrt{2} \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\operatorname{Cos}[e+fx]}{1+\operatorname{Cos}[e+fx]}} (b+a \operatorname{Cos}[e+fx]) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right) / \\
& \left(\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) (b+a \operatorname{Cos}[e+fx])^{3/2} \sqrt{\operatorname{Cos}[e+fx] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^4} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]} \left((a-b) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{(b+a\cos[e+fx]) \sec\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} + \right. \right. \\
& \quad \left. \left. \sqrt{2} \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} (b+a\cos[e+fx]) \tan\left[\frac{1}{2}(e+fx)\right] \right) \right. \\
& \quad \left. \left(-\sec\left[\frac{1}{2}(e+fx)\right]^4 \sin[e+fx] + 2\cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^4 \tan\left[\frac{1}{2}(e+fx)\right] \right) \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \right) / \\
& \quad \left(\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) \sqrt{b+a\cos[e+fx]} \left(\cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^4 \right)^{3/2} \right) - \\
& \quad \frac{1}{\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) \sqrt{b+a\cos[e+fx]} \sqrt{\cos[e+fx] \sec\left[\frac{1}{2}(e+fx)\right]^4}} 2 \sqrt{\cos\left[\frac{1}{2}(e+fx)\right]^2 \sec[e+fx]} \left(-1 + \tan\left[\frac{1}{2}(e+fx)\right]^2 \right) \\
& \quad \left(\frac{\sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} (b+a\cos[e+fx]) \sec\left[\frac{1}{2}(e+fx)\right]^2}{\sqrt{2}} - \sqrt{2} a \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sin[e+fx] \tan\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{\sqrt{\frac{a-b}{a+b}} (b+a\cos[e+fx]) \left(\frac{\cos[e+fx] \sin[e+fx]}{(1+\cos[e+fx])^2} - \frac{\sin[e+fx]}{1+\cos[e+fx]} \right) \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{2} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} + \left((a-b) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a-b}{a+b}} \tan\left[\frac{1}{2}(e+fx)\right]\right], \right. \right. \right. \\
& \quad \left. \left. \frac{a+b}{a-b} \right) \left(-\frac{a \sec\left[\frac{1}{2}(e+fx)\right]^2 \sin[e+fx]}{a+b} + \frac{(b+a\cos[e+fx]) \sec\left[\frac{1}{2}(e+fx)\right]^2 \tan\left[\frac{1}{2}(e+fx)\right]}{a+b} \right) \right) / \\
& \quad \left(2 \sqrt{\frac{(b+a\cos[e+fx]) \sec\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} + \frac{(a-b) \sqrt{\frac{a-b}{a+b}} \sec\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\frac{(b+a\cos[e+fx]) \sec\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} \sqrt{1 - \tan\left[\frac{1}{2}(e+fx)\right]^2}}{2 \sqrt{1 - \frac{(a-b) \tan\left[\frac{1}{2}(e+fx)\right]^2}{a+b}}} \right) -
\end{aligned}$$

$$\left(\left((a-b) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a-b}{a+b}} \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{\frac{(b+a \cos[e+fx]) \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^2}{a+b}} + \right. \right. \\ \left. \left. \sqrt{2} \sqrt{\frac{a-b}{a+b}} \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} (b+a \cos[e+fx]) \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right) \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (e+fx) \right]^2 \right) \right. \\ \left. \left(-\cos \left[\frac{1}{2} (e+fx) \right] \operatorname{Sec}[e+fx] \sin \left[\frac{1}{2} (e+fx) \right] + \cos \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx] \right) \right) / \\ \left(\left(\frac{a-b}{a+b} \right)^{3/2} (a+b) \sqrt{b+a \cos[e+fx]} \sqrt{\cos[e+fx] \operatorname{Sec} \left[\frac{1}{2} (e+fx) \right]^4} \sqrt{\cos \left[\frac{1}{2} (e+fx) \right]^2 \operatorname{Sec}[e+fx]} \right) \right)$$

■ **Problem 276: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^2}{\sqrt{a+b \operatorname{Sec}[e+fx]} (c+c \operatorname{Sec}[e+fx])} dx$$

Optimal (type 4, 214 leaves, 3 steps):

$$\frac{2 a \sqrt{a+b} \operatorname{Cot}[e+fx] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\operatorname{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+fx])}{a-b}}}{(a-b) b c f} - \\ \frac{\operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\operatorname{Tan}[e+fx]}{1+\operatorname{Sec}[e+fx]} \right], \frac{a-b}{a+b} \right] \sqrt{\frac{1}{1+\operatorname{Sec}[e+fx]}} \sqrt{a+b \operatorname{Sec}[e+fx]}}{(a-b) c f \sqrt{\frac{a+b \operatorname{Sec}[e+fx]}{(a+b)(1+\operatorname{Sec}[e+fx])}}}$$

Result (type 4, 1482 leaves):

$$\left(8 a \cos \left[\frac{e}{2} + \frac{fx}{2} \right]^2 \cos \left[\frac{1}{2} (e+fx) \right]^2 \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\operatorname{Tan} \left[\frac{1}{2} (e+fx) \right] \right], \frac{a-b}{a+b} \right] \right)$$

$$\begin{aligned}
& \left. \sqrt{\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Sec}[e+fx]^{3/2} \sqrt{1+\operatorname{Sec}[e+fx]}} \right) / \left((-a+b) f \sqrt{a+b \operatorname{Sec}[e+fx]} (c+c \operatorname{Sec}[e+fx]) \right) - \\
& \left(4 b \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \cos\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \left. \sqrt{\cos[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Sec}[e+fx]^{3/2} \sqrt{1+\operatorname{Sec}[e+fx]}} \right) / \left((-a+b) f \sqrt{a+b \operatorname{Sec}[e+fx]} (c+c \operatorname{Sec}[e+fx]) \right) + \\
& \frac{\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (b+a \cos[e+fx]) \operatorname{Sec}[e+fx]^2 \left(-\frac{2 \operatorname{Sin}[e+fx]}{-a+b} + \frac{2 \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]}{-a+b} \right)}{f \sqrt{a+b \operatorname{Sec}[e+fx]} (c+c \operatorname{Sec}[e+fx])} - \\
& \left(2 \cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \sqrt{b+a \cos[e+fx]} \operatorname{Sec}[e+fx]^{3/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} \right. \\
& \left(-a \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} + \right. \\
& \left. a \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} - b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2} + \right. \\
& \left. 4 i a \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} - \right. \\
& \left. 2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} + \right. \\
& \left. 4 i a \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}} - 2 i b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a+b}{a-b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\text{Csc}[e] \left(b - a \sqrt{1 + \text{Cot}[e]^2} \sin[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \right)}{a \sqrt{1 + \text{Cot}[e]^2} \left(1 + \frac{b \text{Csc}[e]}{a \sqrt{1 + \text{Cot}[e]^2}} \right)}, \right. \\
& \left. \frac{\text{Csc}[e] \left(b - a \sqrt{1 + \text{Cot}[e]^2} \sin[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]] \right)}{a \sqrt{1 + \text{Cot}[e]^2} \left(-1 + \frac{b \text{Csc}[e]}{a \sqrt{1 + \text{Cot}[e]^2}} \right)} \right] \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sqrt{b + a \cos[e + f x]} \text{Csc}\left[\frac{e}{2}\right] \\
& \text{Sec}\left[\frac{e}{2}\right] (g \text{Sec}[e + f x])^{3/2} \text{Sec}[f x - \text{ArcTan}[\text{Cot}[e]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[e]^2} - a \sqrt{1 + \text{Cot}[e]^2} \sin[f x - \text{ArcTan}[\text{Cot}[e]]]}{a \sqrt{1 + \text{Cot}[e]^2} - b \text{Csc}[e]}} \\
& \sqrt{\frac{a \sqrt{1 + \text{Cot}[e]^2} + a \sqrt{1 + \text{Cot}[e]^2} \sin[f x - \text{ArcTan}[\text{Cot}[e]]]}{a \sqrt{1 + \text{Cot}[e]^2} + b \text{Csc}[e]}} \sqrt{b - a \sqrt{1 + \text{Cot}[e]^2} \sin[e] \sin[f x - \text{ArcTan}[\text{Cot}[e]]]} + \\
& \left(a \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sqrt{b + a \cos[e + f x]} \text{Csc}\left[\frac{e}{2}\right] \text{Sec}\left[\frac{e}{2}\right] (g \text{Sec}[e + f x])^{3/2} \right. \\
& \left. \left(\left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[e] \left(b + a \cos[e] \cos[f x + \text{ArcTan}[\text{Tan}[e]] \right) \sqrt{1 + \text{Tan}[e]^2}}{a \sqrt{1 + \text{Tan}[e]^2} \left(1 - \frac{b \text{Sec}[e]}{a \sqrt{1 + \text{Tan}[e]^2}} \right)}, \right. \right. \right. \\
& \left. \left. \frac{\text{Sec}[e] \left(b + a \cos[e] \cos[f x + \text{ArcTan}[\text{Tan}[e]] \right) \sqrt{1 + \text{Tan}[e]^2}}{a \sqrt{1 + \text{Tan}[e]^2} \left(-1 - \frac{b \text{Sec}[e]}{a \sqrt{1 + \text{Tan}[e]^2}} \right)} \right) \sin[f x + \text{ArcTan}[\text{Tan}[e]]] \text{Tan}[e] \right) / \\
& \left(\sqrt{1 + \text{Tan}[e]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[e]^2} - a \cos[f x + \text{ArcTan}[\text{Tan}[e]]] \sqrt{1 + \text{Tan}[e]^2}}{b \text{Sec}[e] + a \sqrt{1 + \text{Tan}[e]^2}}} \right)
\end{aligned}$$

$$\left. \frac{\sqrt{\frac{a \sqrt{1 + \tan[e]^2} + a \cos[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}{-b \sec[e] + a \sqrt{1 + \tan[e]^2}} \sqrt{b + a \cos[e] \cos[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}}}{\frac{\frac{\sin[f x + \text{ArcTan}[\tan[e]]] \tan[e]}{\sqrt{1 + \tan[e]^2}} + \frac{2 a \cos[e] (b + a \cos[e] \cos[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2})}{a^2 \cos[e]^2 + a^2 \sin[e]^2}}{\sqrt{b + a \cos[e] \cos[f x + \text{ArcTan}[\tan[e]]] \sqrt{1 + \tan[e]^2}}}}}{\left(2 (-a + b) f \sqrt{a + b \sec[e + f x]} (c + c \sec[e + f x]) \right)} \right)$$

■ **Problem 278: Unable to integrate problem.**

$$\int \frac{(g \sec[e + f x])^{5/2}}{\sqrt{a + b \sec[e + f x]} (c + c \sec[e + f x])} dx$$

Optimal (type 4, 312 leaves, 11 steps):

$$\frac{g^2 (b + a \cos[e + f x]) \text{EllipticE}\left[\frac{1}{2} (e + f x), \frac{2a}{a+b}\right] \sqrt{g \sec[e + f x]} - g^2 \sqrt{\frac{b+a \cos[e+f x]}{a+b}} \text{EllipticF}\left[\frac{1}{2} (e + f x), \frac{2a}{a+b}\right] \sqrt{g \sec[e + f x]}}{(a - b) c f \sqrt{\frac{b+a \cos[e+f x]}{a+b}} \sqrt{a + b \sec[e + f x]} - c f \sqrt{a + b \sec[e + f x]}} + \frac{2 g^2 \sqrt{\frac{b+a \cos[e+f x]}{a+b}} \text{EllipticPi}\left[2, \frac{1}{2} (e + f x), \frac{2a}{a+b}\right] \sqrt{g \sec[e + f x]}}{c f \sqrt{a + b \sec[e + f x]}} + \frac{g^2 (b + a \cos[e + f x]) \sqrt{g \sec[e + f x]} \sin[e + f x]}{(a - b) f (c + c \cos[e + f x]) \sqrt{a + b \sec[e + f x]}}$$

Result (type 8, 41 leaves):

$$\int \frac{(g \sec[e + f x])^{5/2}}{\sqrt{a + b \sec[e + f x]} (c + c \sec[e + f x])} dx$$

■ **Problem 279: Unable to integrate problem.**

$$\int \frac{\sec[e + f x] \sqrt{a + b \sec[e + f x]}}{c + d \sec[e + f x]} dx$$

Optimal (type 4, 213 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}}{d f}$$

$$\frac{2(b c-a d) \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\operatorname{Sec}[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \operatorname{Sec}[e+f x]}{a+b}} \operatorname{Tan}[e+f x]}{d(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2}}$$

Result (type 8, 35 leaves):

$$\int \frac{\operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]}}{c+d \operatorname{Sec}[e+f x]} dx$$

■ **Problem 280: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(g \operatorname{Sec}[e+f x])^{3/2} \sqrt{a+b \operatorname{Sec}[e+f x]}}{c+d \operatorname{Sec}[e+f x]} dx$$

Optimal (type 4, 170 leaves, 7 steps):

$$\frac{2 b g \sqrt{\frac{b+a \operatorname{Cos}[e+f x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(e+f x), \frac{2 a}{a+b}\right] \sqrt{g \operatorname{Sec}[e+f x]}}{d f \sqrt{a+b \operatorname{Sec}[e+f x]}}$$

$$\frac{2(b c-a d) g \sqrt{\frac{b+a \operatorname{Cos}[e+f x]}{a+b}} \operatorname{EllipticPi}\left[\frac{2 c}{c+d}, \frac{1}{2}(e+f x), \frac{2 a}{a+b}\right] \sqrt{g \operatorname{Sec}[e+f x]}}{d(c+d) f \sqrt{a+b \operatorname{Sec}[e+f x]}}$$

Result (type 4, 223 leaves):

$$- \left(2 i g \sqrt{-\frac{a(-1+\operatorname{Cos}[e+f x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Cos}[e+f x])}{a-b}} \operatorname{Cot}[e+f x] \right.$$

$$\left. \left(\operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[e+f x]}\right], \frac{-a+b}{a+b}\right] - \operatorname{EllipticPi}\left[\frac{(a-b) c}{-b c+a d}, \right.$$

$$\left. i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[e+f x]}\right], \frac{-a+b}{a+b}\right] \sqrt{g \operatorname{Sec}[e+f x]} \sqrt{a+b \operatorname{Sec}[e+f x]} \right) / \left(\sqrt{\frac{1}{a-b}} d f \sqrt{b+a \operatorname{Cos}[e+f x]} \right)$$

■ **Problem 281: Unable to integrate problem.**

$$\int \frac{\text{Sec}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])} dx$$

Optimal (type 4, 102 leaves, 1 step):

$$\frac{2 \text{EllipticPi}\left[\frac{2d}{c+d}, \text{ArcSin}\left[\frac{\sqrt{1-\text{Sec}[e+fx]}}{\sqrt{2}}\right], \frac{2b}{a+b}\right] \sqrt{\frac{a+b \text{Sec}[e+fx]}{a+b}} \text{Tan}[e + f x]}{(c+d) f \sqrt{a + b \text{Sec}[e + f x]} \sqrt{-\text{Tan}[e + f x]^2}}$$

Result (type 8, 35 leaves):

$$\int \frac{\text{Sec}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])} dx$$

■ **Problem 282: Unable to integrate problem.**

$$\int \frac{\text{Sec}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])} dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$\frac{2 \sqrt{a+b} \text{Cot}[e + f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[e+fx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[e+fx])}{a-b}}}{b d f}$$

$$\frac{2 c \text{EllipticPi}\left[\frac{2d}{c+d}, \text{ArcSin}\left[\frac{\sqrt{1-\text{Sec}[e+fx]}}{\sqrt{2}}\right], \frac{2b}{a+b}\right] \sqrt{\frac{a+b \text{Sec}[e+fx]}{a+b}} \text{Tan}[e + f x]}{d (c+d) f \sqrt{a + b \text{Sec}[e + f x]} \sqrt{-\text{Tan}[e + f x]^2}}$$

Result (type 8, 37 leaves):

$$\int \frac{\text{Sec}[e + f x]^2}{\sqrt{a + b \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])} dx$$

■ **Problem 284: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(g \text{Sec}[e + f x])^{5/2}}{\sqrt{a + b \text{Sec}[e + f x]} (c + d \text{Sec}[e + f x])} dx$$

Optimal (type 4, 166 leaves, 7 steps):

$$\frac{2 g^2 \sqrt{\frac{b+a \text{Cos}[e+fx]}{a+b}} \text{EllipticPi}\left[2, \frac{1}{2} (e + f x), \frac{2a}{a+b}\right] \sqrt{g \text{Sec}[e + f x]} - 2 c g^2 \sqrt{\frac{b+a \text{Cos}[e+fx]}{a+b}} \text{EllipticPi}\left[\frac{2c}{c+d}, \frac{1}{2} (e + f x), \frac{2a}{a+b}\right] \sqrt{g \text{Sec}[e + f x]}}{d f \sqrt{a + b \text{Sec}[e + f x]} - d (c+d) f \sqrt{a + b \text{Sec}[e + f x]}}$$

Result (type 4, 246 leaves) :

$$\begin{aligned}
 & - \left(2 i g \sqrt{-\frac{a(-1 + \cos[e + f x])}{a + b}} \sqrt{\frac{a(1 + \cos[e + f x])}{a - b}} \sqrt{b + a \cos[e + f x]} \right. \\
 & \quad \left. \cot[e + f x] \left((-b c + a d) \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos[e + f x]}\right], \frac{-a + b}{a + b}\right] + \right. \right. \\
 & \quad \left. \left. b c \operatorname{EllipticPi}\left[\frac{(a - b) c}{-b c + a d}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos[e + f x]}\right], \frac{-a + b}{a + b}\right] \right) \right) \\
 & \quad \left(g \sec[e + f x] \right)^{3/2} \left/ \left(\sqrt{\frac{1}{a - b}} b d (-b c + a d) f \sqrt{a + b \sec[e + f x]} \right) \right)
 \end{aligned}$$

■ **Problem 285: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + f x] \tan[e + f x]^4}{(c - c \sec[e + f x])^7} dx$$

Optimal (type 3, 67 leaves, 4 steps) :

$$\frac{\cot\left[\frac{1}{2}(e + f x)\right]^5}{20 c^7 f} - \frac{\cot\left[\frac{1}{2}(e + f x)\right]^7}{14 c^7 f} + \frac{\cot\left[\frac{1}{2}(e + f x)\right]^9}{36 c^7 f}$$

Result (type 3, 151 leaves) :

$$\begin{aligned}
 & \frac{1}{23063040 c^7 f} \\
 & \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(e + f x)\right]^9 \left(-971082 \sin\left[\frac{f x}{2}\right] - 718830 \sin\left[e + \frac{f x}{2}\right] + 467208 \sin\left[e + \frac{3 f x}{2}\right] + 659400 \sin\left[2e + \frac{3 f x}{2}\right] - 303192 \sin\left[2e + \frac{5 f x}{2}\right] - \right. \\
 & \quad \left. 179640 \sin\left[3e + \frac{5 f x}{2}\right] + 30753 \sin\left[3e + \frac{7 f x}{2}\right] + 89955 \sin\left[4e + \frac{7 f x}{2}\right] - 13427 \sin\left[4e + \frac{9 f x}{2}\right] + 15 \sin\left[5e + \frac{9 f x}{2}\right] \right)
 \end{aligned}$$

Test results for the 634 problems in "4.5.3.1 (a+b sec)^m (d sec)^n (A+B sec).m"

■ **Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[c + d x]^m (b \sec[c + d x])^{4/3} (A + B \sec[c + d x]) dx$$

Optimal (type 5, 167 leaves, 6 steps) :

$$\left(3 A b \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (-1-3m), \frac{1}{6} (5-3m), \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^m (b \text{Sec}[c+dx])^{1/3} \text{Sin}[c+dx] \right) /$$

$$\left(d (1+3m) \sqrt{\text{Sin}[c+dx]^2} \right) +$$

$$\left(3 b B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (-4-3m), \frac{1}{6} (2-3m), \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^{1+m} (b \text{Sec}[c+dx])^{1/3} \text{Sin}[c+dx] \right) /$$

$$\left(d (4+3m) \sqrt{\text{Sin}[c+dx]^2} \right)$$

Result (type 6, 4860 leaves):

$$- \left(\left(18 \text{Sec}[c+dx]^{-2+m} (b \text{Sec}[c+dx])^{4/3} (A+B \text{Sec}[c+dx]) \right. \right.$$

$$\left. \left(B \text{Sec}[c+dx]^{\frac{1}{3}+m} + \text{Cos}[2(c+dx)] \left(\frac{1}{2} A \text{Sec}[c+dx]^{\frac{4}{3}+m} - \frac{1}{2} i A \text{Sec}[c+dx]^{\frac{7}{3}+m} \text{Sin}[c+dx] \right) \right) + \right.$$

$$\left. \text{Sec}[c+dx] \left(\frac{1}{2} A \text{Sec}[c+dx]^{\frac{1}{3}+m} + \frac{1}{2} i A \text{Sec}[c+dx]^{\frac{1}{3}+m} \text{Sin}[2(c+dx)] \right) \right) +$$

$$\left. \text{Sec}[c+dx]^2 \left(B \text{Sec}[c+dx]^{\frac{1}{3}+m} \text{Sin}[c+dx]^2 + \text{Sin}[c+dx] \left(-\frac{1}{2} i A \text{Sec}[c+dx]^{\frac{1}{3}+m} + \frac{1}{2} A \text{Sec}[c+dx]^{\frac{1}{3}+m} \text{Sin}[2(c+dx)] \right) \right) \right) \text{Tan} \left[\frac{1}{2} (c+dx) \right]$$

$$\left(\left((A-B) \text{AppellF1} \left[\frac{1}{2}, \frac{4}{3}+m, -\frac{1}{3}-m, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(-1 + \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) / \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{4}{3}+m, -\frac{1}{3}-m, \right. \right. \right.$$

$$\left. \left. \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left((1+3m) \text{AppellF1} \left[\frac{3}{2}, \frac{4}{3}+m, \frac{2}{3}-m, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right.$$

$$\left. (4+3m) \text{AppellF1} \left[\frac{3}{2}, \frac{7}{3}+m, -\frac{1}{3}-m, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 -$$

$$\left(2 B \text{AppellF1} \left[\frac{1}{2}, \frac{7}{3}+m, -\frac{1}{3}-m, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) / \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{7}{3}+m, -\frac{1}{3}-m, \frac{3}{2}, \right. \right.$$

$$\left. \left. \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + 2 \left((1+3m) \text{AppellF1} \left[\frac{3}{2}, \frac{7}{3}+m, \frac{2}{3}-m, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] + \right. \right.$$

$$\left. (7+3m) \text{AppellF1} \left[\frac{3}{2}, \frac{10}{3}+m, -\frac{1}{3}-m, \frac{5}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \right) \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) /$$

$$\left(d (B+A \text{Cos}[c+dx]) \left(-1 + \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right)^2 \left(\frac{1}{\left(-1 + \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right)^3} 36 \text{Sec} \left[\frac{1}{2} (c+dx) \right]^2 \text{Sec}[c+dx]^{\frac{1}{3}+m} \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right.$$

$$\left. \left(\left((A-B) \text{AppellF1} \left[\frac{1}{2}, \frac{4}{3}+m, -\frac{1}{3}-m, \frac{3}{2}, \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2, -\text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right] \left(-1 + \text{Tan} \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) / \left(9 \text{AppellF1} \left[\frac{1}{2}, \frac{4}{3}+m, \right. \right. \right.$$

Result (type 6, 5573 leaves) : Display of huge result suppressed!

■ **Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^m (A + B \text{Sec}[c + d x])}{(b \text{Sec}[c + d x])^{4/3}} dx$$

Optimal (type 5, 173 leaves, 6 steps) :

$$\frac{3 A \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6} (7 - 3 m), \frac{1}{6} (13 - 3 m), \text{Cos}[c + d x]^2\right] \text{Sec}[c + d x]^{-2+m} \text{Sin}[c + d x]}{b d (7 - 3 m) (b \text{Sec}[c + d x])^{1/3} \sqrt{\text{Sin}[c + d x]^2}} - \frac{3 B \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6} (4 - 3 m), \frac{1}{6} (10 - 3 m), \text{Cos}[c + d x]^2\right] \text{Sec}[c + d x]^{-1+m} \text{Sin}[c + d x]}{b d (4 - 3 m) (b \text{Sec}[c + d x])^{1/3} \sqrt{\text{Sin}[c + d x]^2}}$$

Result (type 6, 5557 leaves) : Display of huge result suppressed!

■ **Problem 39: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^{3/2} (b \text{Sec}[c + d x])^n (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 5, 163 leaves, 6 steps) :

$$\left(2 A \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} (-1 - 2 n), \frac{1}{4} (3 - 2 n), \text{Cos}[c + d x]^2\right] \sqrt{\text{Sec}[c + d x]} (b \text{Sec}[c + d x])^n \text{Sin}[c + d x] \right) / \left(d (1 + 2 n) \sqrt{\text{Sin}[c + d x]^2} \right) + \left(2 B \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} (-3 - 2 n), \frac{1}{4} (1 - 2 n), \text{Cos}[c + d x]^2\right] \text{Sec}[c + d x]^{3/2} (b \text{Sec}[c + d x])^n \text{Sin}[c + d x] \right) / \left(d (3 + 2 n) \sqrt{\text{Sin}[c + d x]^2} \right)$$

Result (type 6, 4819 leaves) :

$$\begin{aligned} & - \left(6 \sqrt{\text{Sec}[c + d x]} (b \text{Sec}[c + d x])^n \left(B \text{Sec}[c + d x]^{\frac{1}{2}+n} + \text{Cos}[2 (c + d x)] \left(\frac{1}{2} A \text{Sec}[c + d x]^{\frac{3}{2}+n} - \frac{1}{2} i A \text{Sec}[c + d x]^{\frac{5}{2}+n} \text{Sin}[c + d x] \right) \right) + \right. \\ & \quad \left. \text{Sec}[c + d x] \left(\frac{1}{2} A \text{Sec}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} i A \text{Sec}[c + d x]^{\frac{1}{2}+n} \text{Sin}[2 (c + d x)] \right) + \right. \\ & \quad \left. \text{Sec}[c + d x]^2 \left(B \text{Sec}[c + d x]^{\frac{1}{2}+n} \text{Sin}[c + d x]^2 + \text{Sin}[c + d x] \left(-\frac{1}{2} i A \text{Sec}[c + d x]^{\frac{1}{2}+n} + \frac{1}{2} A \text{Sec}[c + d x]^{\frac{1}{2}+n} \text{Sin}[2 (c + d x)] \right) \right) \right) \\ & \quad \text{Tan}\left[\frac{1}{2} (c + d x)\right] \left(\left((A - B) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + n, -\frac{1}{2} - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \left(-1 + \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) / \right. \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2} + n, -\frac{1}{2} - n, \frac{3}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + \left((1 + 2 n) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2} + n, \frac{1}{2} - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] + (3 + 2 n) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + n, -\frac{1}{2} - n, \frac{5}{2}, \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2, -\text{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) - \end{aligned}$$

$$\begin{aligned}
& \left. \frac{7}{2} + n, -\frac{1}{2} - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \\
& 3 \left(-\frac{1}{3} \left(-\frac{1}{2} - n \right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + n, \frac{1}{2} - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \left. \frac{1}{3} \left(\frac{5}{2} + n \right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} + n, -\frac{1}{2} - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \left((1+2n) \left(-\frac{3}{5} \left(\frac{1}{2} - n \right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2} + n, \frac{3}{2} - n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left(\frac{5}{2} + n \right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + n, \frac{1}{2} - n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) + (5+2n) \left(-\frac{3}{5} \left(-\frac{1}{2} - n \right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2} + n, \frac{1}{2} - n, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, \right. \right. \\
& \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \frac{3}{5} \left(\frac{7}{2} + n \right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2} + n, -\frac{1}{2} - n, \frac{7}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \Big/ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{5}{2} + n, -\frac{1}{2} - n, \frac{3}{2}, \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \left((1+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{5}{2} + n, \frac{1}{2} - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + \right. \right. \\
& \left. \left. (5+2n) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2} + n, -\frac{1}{2} - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \Big) \Big)
\end{aligned}$$

■ **Problem 40: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} (b \operatorname{Sec}[c+dx])^n (A + B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 5, 163 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 A \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1-2n), \frac{1}{4}(5-2n), \operatorname{Cos}[c+dx]^2\right] (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d(1-2n) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]^2}} + \\
& \frac{\left(2 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-1-2n), \frac{1}{4}(3-2n), \operatorname{Cos}[c+dx]^2\right] \sqrt{\operatorname{Sec}[c+dx]} (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx] \right)}{d(1+2n) \sqrt{\operatorname{Sin}[c+dx]^2}}
\end{aligned}$$

Result (type 6, 5543 leaves): Display of huge result suppressed!

■ **Problem 42: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(b \operatorname{Sec}[c+dx])^n (A + B \operatorname{Sec}[c+dx])}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 5, 163 leaves, 6 steps):

$$\frac{2 A \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(5-2n), \frac{1}{4}(9-2n), \cos[c+dx]^2\right] (b \sec[c+dx])^n \sin[c+dx]}{d(5-2n) \sec[c+dx]^{5/2} \sqrt{\sin[c+dx]^2}}$$

$$\frac{2 B \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3-2n), \frac{1}{4}(7-2n), \cos[c+dx]^2\right] (b \sec[c+dx])^n \sin[c+dx]}{d(3-2n) \sec[c+dx]^{3/2} \sqrt{\sin[c+dx]^2}}$$

Result (type 6, 5527 leaves): Display of huge result suppressed!

■ **Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^3 (a + a \sec[c+dx]) (A + B \sec[c+dx]) dx$$

Optimal (type 3, 106 leaves, 6 steps):

$$\frac{a(4A+3B) \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{a(A+B) \tan[c+dx]}{d} +$$

$$\frac{a(4A+3B) \sec[c+dx] \tan[c+dx]}{8d} + \frac{aB \sec[c+dx]^3 \tan[c+dx]}{4d} + \frac{a(A+B) \tan[c+dx]^3}{3d}$$

Result (type 3, 403 leaves):

$$\frac{aA \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{3aB \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{aA \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} +$$

$$\frac{3aB \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{aB}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{aA}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{3aB}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{aB}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{aA}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} -$$

$$\frac{3aB}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2aA \tan[c+dx]}{3d} + \frac{2aB \tan[c+dx]}{3d} + \frac{aA \sec[c+dx]^2 \tan[c+dx]}{3d} + \frac{aB \sec[c+dx]^2 \tan[c+dx]}{3d}$$

■ **Problem 46: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx] (a + a \sec[c+dx]) (A + B \sec[c+dx]) dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{a(2A+B) \operatorname{ArcTanh}[\sin[c+dx]]}{2d} + \frac{a(A+B) \tan[c+dx]}{d} + \frac{aB \sec[c+dx] \tan[c+dx]}{2d}$$

Result (type 3, 154 leaves):

$$\frac{1}{4d} a \left(-2 (2A+B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right] + \right. \\ \left. 4A \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right] + 2B \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right] + \right. \\ \left. \frac{B}{\left(\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right)^2} - \frac{B}{\left(\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right)^2} + 4(A+B) \tan [c+dx] \right)$$

■ **Problem 47: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + dx]) (A + B \sec [c + dx]) dx$$

Optimal (type 3, 32 leaves, 4 steps):

$$aAx + \frac{a(A+B) \operatorname{ArcTanh}[\sin [c+dx]]}{d} + \frac{aB \tan [c+dx]}{d}$$

Result (type 3, 159 leaves):

$$aAx - \frac{aA \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} - \frac{aB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \\ \frac{aA \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \tan [c+dx]}{d}$$

■ **Problem 48: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + dx] (a + a \sec [c + dx]) (A + B \sec [c + dx]) dx$$

Optimal (type 3, 32 leaves, 3 steps):

$$a(A+B)x + \frac{aB \operatorname{ArcTanh}[\sin [c+dx]]}{d} + \frac{aA \sin [c+dx]}{d}$$

Result (type 3, 104 leaves):

$$aAx + aBx - \frac{aB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aA \cos [dx] \sin [c]}{d} + \frac{aA \cos [c] \sin [dx]}{d}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + dx] (a + a \sec [c + dx])^2 (A + B \sec [c + dx]) dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{a^2 (3A+2B) \operatorname{ArcTanh}[\sin [c+dx]]}{2d} + \frac{2a^2 (3A+2B) \tan [c+dx]}{3d} + \frac{a^2 (3A+2B) \sec [c+dx] \tan [c+dx]}{6d} + \frac{B(a+a \sec [c+dx])^2 \tan [c+dx]}{3d}$$

Result (type 3, 993 leaves) :

$$\frac{1}{8 d (B + A \cos [c + d x])} (-3 A - 2 B) \cos [c + d x]^3 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) +$$

$$\frac{1}{8 d (B + A \cos [c + d x])} (3 A + 2 B) \cos [c + d x]^3 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) +$$

$$\frac{B \cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \sin\left[\frac{d x}{2}\right]}{24 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} +$$

$$\left(\cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \left(3 A \cos\left[\frac{c}{2}\right] + 7 B \cos\left[\frac{c}{2}\right] - 3 A \sin\left[\frac{c}{2}\right] - 5 B \sin\left[\frac{c}{2}\right]\right)\right) /$$

$$\left(48 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2\right) +$$

$$\frac{\cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \left(6 A \sin\left[\frac{d x}{2}\right] + 5 B \sin\left[\frac{d x}{2}\right]\right)}{12 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] - \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)} +$$

$$\frac{B \cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \sin\left[\frac{d x}{2}\right]}{24 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^3} +$$

$$\left(\cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \left(-3 A \cos\left[\frac{c}{2}\right] - 7 B \cos\left[\frac{c}{2}\right] - 3 A \sin\left[\frac{c}{2}\right] - 5 B \sin\left[\frac{c}{2}\right]\right)\right) /$$

$$\left(48 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2\right) +$$

$$\frac{\cos [c + d x]^3 \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \left(6 A \sin\left[\frac{d x}{2}\right] + 5 B \sin\left[\frac{d x}{2}\right]\right)}{12 d (B + A \cos [c + d x]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right] + \sin\left[\frac{c}{2} + \frac{d x}{2}\right]\right)}$$

■ **Problem 56: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) dx$$

Optimal (type 3, 82 leaves, 5 steps) :

$$a^2 A x + \frac{a^2 (4 A + 3 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a^2 (2 A + 3 B) \tan [c + d x]}{2 d} + \frac{B (a^2 + a^2 \sec [c + d x]) \tan [c + d x]}{2 d}$$

Result (type 3, 307 leaves) :

$$\frac{1}{16 (B + A \cos [c + d x])} a^2 \cos [c + d x]^3 \sec \left[\frac{1}{2} (c + d x) \right]^4 (1 + \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(4 A x - \frac{2 (4 A + 3 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \frac{2 (4 A + 3 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \right.$$

$$\left. \frac{B}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (A + 2 B) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)} - \right.$$

$$\left. \frac{B}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (A + 2 B) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a^2 (2 A + B) x + \frac{a^2 (A + 2 B) \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a^2 (A - B) \sin [c + d x]}{d} + \frac{B (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{d}$$

Result (type 3, 258 leaves):

$$\frac{1}{4 (B + A \cos [c + d x])} a^2 \cos [c + d x]^3 \sec \left[\frac{1}{2} (c + d x) \right]^4 (1 + \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left((2 A + B) x - \frac{(A + 2 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \frac{(A + 2 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \frac{A \cos [d x] \sin [c]}{d} + \right.$$

$$\left. \frac{A \cos [c] \sin [d x]}{d} + \frac{B \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)} + \frac{B \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 64: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x] (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) dx$$

Optimal (type 3, 125 leaves, 10 steps):

$$\frac{5 a^3 (4 A + 3 B) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a^3 (4 A + 3 B) \tan [c + d x]}{d} +$$

$$\frac{3 a^3 (4 A + 3 B) \sec [c + d x] \tan [c + d x]}{8 d} + \frac{B (a + a \sec [c + d x])^3 \tan [c + d x]}{4 d} + \frac{a^3 (4 A + 3 B) \tan [c + d x]^3}{12 d}$$

Result (type 3, 273 leaves):

$$\begin{aligned}
& - \frac{1}{1536 d} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6 \sec[c + dx]^4 \\
& \left(120 (4A + 3B) \cos[c + dx]^4 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \right. \\
& \quad \sec[c] (-24 (11A + 9B) \sin[c] + (36A + 69B) \sin[dx] + 36A \sin[2c + dx] + 69B \sin[2c + dx] + \\
& \quad 280A \sin[c + 2dx] + 264B \sin[c + 2dx] - 72A \sin[3c + 2dx] - 24B \sin[3c + 2dx] + 36A \sin[2c + 3dx] + \\
& \quad \left. 45B \sin[2c + 3dx] + 36A \sin[4c + 3dx] + 45B \sin[4c + 3dx] + 88A \sin[3c + 4dx] + 72B \sin[3c + 4dx] \right)
\end{aligned}$$

■ **Problem 65: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$\begin{aligned}
& a^3 A x + \frac{a^3 (7A + 5B) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{5a^3 (A + B) \tan[c + dx]}{2d} + \\
& \frac{aB (a + a \sec[c + dx])^2 \tan[c + dx]}{3d} + \frac{(3A + 5B) (a^3 + a^3 \sec[c + dx]) \tan[c + dx]}{6d}
\end{aligned}$$

Result (type 3, 1056 leaves):

$$\begin{aligned}
& \frac{A x \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])}{8(B+A \cos [c+d x])}+\frac{1}{16 d(B+A \cos [c+d x])} \\
& \frac{(-7 A-5 B) \cos [c+d x]^4 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])}{16 d(B+A \cos [c+d x])}+ \\
& \frac{1}{16 d(B+A \cos [c+d x])}(7 A+5 B) \cos [c+d x]^4 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x]) \\
& +\frac{B \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x]) \sin \left[\frac{d x}{2}\right]}{48 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3}+ \\
& \left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])\left(3 A \cos \left[\frac{c}{2}\right]+10 B \cos \left[\frac{c}{2}\right]-3 A \sin \left[\frac{c}{2}\right]-8 B \sin \left[\frac{c}{2}\right]\right)\right) / \\
& \left(96 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2\right)+ \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])\left(9 A \sin \left[\frac{d x}{2}\right]+11 B \sin \left[\frac{d x}{2}\right]\right)}{24 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}+ \\
& \frac{B \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x]) \sin \left[\frac{d x}{2}\right]}{48 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3}+ \\
& \left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])\left(-3 A \cos \left[\frac{c}{2}\right]-10 B \cos \left[\frac{c}{2}\right]-3 A \sin \left[\frac{c}{2}\right]-8 B \sin \left[\frac{c}{2}\right]\right)\right) / \\
& \left(96 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2\right)+ \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x])\left(9 A \sin \left[\frac{d x}{2}\right]+11 B \sin \left[\frac{d x}{2}\right]\right)}{24 d(B+A \cos [c+d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]\left(a+a \sec [c+d x]\right)^3(A+B \sec [c+d x]) d x$$

Optimal (type 3, 108 leaves, 5 steps):

$$\begin{aligned}
& a^3(3 A+B) x+\frac{a^3(6 A+7 B) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d}-\frac{5 a^3 B \sin [c+d x]}{2 d}+ \\
& \frac{a B\left(a+a \sec [c+d x]\right)^2 \sin [c+d x]}{2 d}+\frac{(A+2 B)\left(a^3+a^3 \sec [c+d x]\right) \sin [c+d x]}{d}
\end{aligned}$$

Result (type 3, 335 leaves):

1

32 (B + A Cos [c + d x])

$$a^3 \cos [c + d x]^4 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^6 (1 + \operatorname{Sec} [c + d x])^3 (A + B \operatorname{Sec} [c + d x]) \left(4 (3 A + B) x - \frac{2 (6 A + 7 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \right.$$

$$\left. \frac{2 (6 A + 7 B) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \frac{4 A \cos [d x] \sin [c]}{d} + \frac{4 A \cos [c] \sin [d x]}{d} + \right.$$

$$\left. \frac{B}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (A + 3 B) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)} - \right.$$

$$\left. \frac{B}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (A + 3 B) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 67: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^2 (a + a \operatorname{Sec} [c + d x])^3 (A + B \operatorname{Sec} [c + d x]) dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$\frac{1}{2} a^3 (7 A + 6 B) x + \frac{a^3 (A + 3 B) \operatorname{ArcTanh} [\sin [c + d x]]}{d} + \frac{5 a^3 A \sin [c + d x]}{2 d} +$$

$$\frac{a A \cos [c + d x] (a + a \operatorname{Sec} [c + d x])^2 \sin [c + d x]}{2 d} - \frac{(A - 2 B) (a^3 + a^3 \operatorname{Sec} [c + d x]) \sin [c + d x]}{2 d}$$

Result (type 3, 802 leaves):

$$\begin{aligned}
& \frac{(7A + 6B) x \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx])}{16(B + A \cos[c + dx])} + \frac{1}{8d(B + A \cos[c + dx])} \\
& \frac{(-A - 3B) \cos[c + dx]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx])}{8d(B + A \cos[c + dx])} + \\
& \frac{1}{8d(B + A \cos[c + dx])} (A + 3B) \cos[c + dx]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) + \\
& \frac{(3A + B) \cos[dx] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin[c]}{8d(B + A \cos[c + dx])} + \\
& \frac{A \cos[2dx] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin[2c]}{32d(B + A \cos[c + dx])} + \\
& \frac{(3A + B) \cos[c] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin[dx]}{8d(B + A \cos[c + dx])} + \\
& \frac{A \cos[2c] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin[2dx]}{32d(B + A \cos[c + dx])} + \\
& \frac{B \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin\left[\frac{dx}{2}\right]}{8d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{B \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx]) \sin\left[\frac{dx}{2}\right]}{8d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 74: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) dx$$

Optimal (type 3, 151 leaves, 7 steps):

$$\begin{aligned}
& a^4 A x + \frac{a^4 (48A + 35B) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{5a^4 (8A + 7B) \tan[c + dx]}{8d} + \frac{aB (a + a \sec[c + dx])^3 \tan[c + dx]}{4d} + \\
& \frac{(4A + 7B) (a^2 + a^2 \sec[c + dx])^2 \tan[c + dx]}{12d} + \frac{(32A + 35B) (a^4 + a^4 \sec[c + dx]) \tan[c + dx]}{24d}
\end{aligned}$$

Result (type 3, 326 leaves):

$$\frac{1}{3072 d} a^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^8 (1+\operatorname{Sec}[c+d x])^4$$

$$\left(-24(48 A+35 B) \operatorname{Cos}[c+d x]^4 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right)+\right.$$

$$\operatorname{Sec}[c] (72 A d x \operatorname{Cos}[c]+48 A d x \operatorname{Cos}[c+2 d x]+48 A d x \operatorname{Cos}[3 c+2 d x]+12 A d x \operatorname{Cos}[3 c+4 d x]+12 A d x \operatorname{Cos}[5 c+4 d x]-$$

$$480 A \operatorname{Sin}[c]-480 B \operatorname{Sin}[c]+48 A \operatorname{Sin}[d x]+105 B \operatorname{Sin}[d x]+48 A \operatorname{Sin}[2 c+d x]+105 B \operatorname{Sin}[2 c+d x]+$$

$$496 A \operatorname{Sin}[c+2 d x]+544 B \operatorname{Sin}[c+2 d x]-144 A \operatorname{Sin}[3 c+2 d x]-96 B \operatorname{Sin}[3 c+2 d x]+48 A \operatorname{Sin}[2 c+3 d x]+$$

$$\left.81 B \operatorname{Sin}[2 c+3 d x]+48 A \operatorname{Sin}[4 c+3 d x]+81 B \operatorname{Sin}[4 c+3 d x]+160 A \operatorname{Sin}[3 c+4 d x]+160 B \operatorname{Sin}[3 c+4 d x])\right)$$

■ **Problem 75: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x] (a+a \operatorname{Sec}[c+d x])^4 (A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 151 leaves, 6 steps):

$$a^4 (4 A+B) x + \frac{a^4 (13 A+12 B) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{2 d} - \frac{5 a^4 (A+2 B) \operatorname{Sin}[c+d x]}{2 d} +$$

$$\frac{a B (a+a \operatorname{Sec}[c+d x])^3 \operatorname{Sin}[c+d x]}{3 d} + \frac{(A+2 B) (a^2+a^2 \operatorname{Sec}[c+d x])^2 \operatorname{Sin}[c+d x]}{2 d} + \frac{(9 A+11 B) (a^4+a^4 \operatorname{Sec}[c+d x]) \operatorname{Sin}[c+d x]}{3 d}$$

Result (type 3, 1202 leaves):

$$\begin{aligned}
& \frac{(4A + B) \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx])}{16(B + A \cos[c + dx])} + \frac{1}{32d(B + A \cos[c + dx])} \\
& \frac{(-13A - 12B) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) +}{32d(B + A \cos[c + dx])} \\
& \frac{1}{32d(B + A \cos[c + dx])} (13A + 12B) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) + \\
& \frac{A \cos[dx] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \sin[c]}{16d(B + A \cos[c + dx])} + \\
& \frac{A \cos[c] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \sin[dx]}{16d(B + A \cos[c + dx])} + \\
& \frac{B \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \sin\left[\frac{dx}{2}\right]}{96d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \left(\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \left(3A \cos\left[\frac{c}{2}\right] + 13B \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 11B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(192d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2 \right) + \\
& \frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \left(3A \sin\left[\frac{dx}{2}\right] + 5B \sin\left[\frac{dx}{2}\right]\right)}{12d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{B \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \sin\left[\frac{dx}{2}\right]}{96d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \left(\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \left(-3A \cos\left[\frac{c}{2}\right] - 13B \cos\left[\frac{c}{2}\right] - 3A \sin\left[\frac{c}{2}\right] - 11B \sin\left[\frac{c}{2}\right]\right) \right) / \\
& \left(192d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2 \right) + \\
& \frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) \left(3A \sin\left[\frac{dx}{2}\right] + 5B \sin\left[\frac{dx}{2}\right]\right)}{12d(B + A \cos[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 76: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \sec[c + dx])^4 (A + B \sec[c + dx]) dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{1}{2} a^4 (13 A + 8 B) x + \frac{a^4 (8 A + 13 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{5 a^4 (A - B) \operatorname{Sin}[c + d x]}{2 d} +$$

$$\frac{a A \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{2 d} - \frac{(A - B) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 d} + \frac{(A + 6 B) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{2 d}$$

Result (type 3, 1018 leaves):

$$\frac{(13 A + 8 B) x \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{32 (B + A \operatorname{Cos}[c + d x])} + \frac{1}{32 d (B + A \operatorname{Cos}[c + d x])}$$

$$(-8 A - 13 B) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) +$$

$$\frac{1}{32 d (B + A \operatorname{Cos}[c + d x])} (8 A + 13 B) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) +$$

$$\frac{(4 A + B) \operatorname{Cos}[d x] \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}[c]}{16 d (B + A \operatorname{Cos}[c + d x])} +$$

$$\frac{A \operatorname{Cos}[2 d x] \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}[2 c]}{64 d (B + A \operatorname{Cos}[c + d x])} +$$

$$\frac{(4 A + B) \operatorname{Cos}[c] \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}[d x]}{16 d (B + A \operatorname{Cos}[c + d x])} +$$

$$\frac{A \operatorname{Cos}[2 c] \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}[2 d x]}{64 d (B + A \operatorname{Cos}[c + d x])} +$$

$$\frac{B \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{64 d (B + A \operatorname{Cos}[c + d x])} \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2 +$$

$$\frac{\operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) (A \operatorname{Sin}\left[\frac{d x}{2}\right] + 4 B \operatorname{Sin}\left[\frac{d x}{2}\right])}{16 d (B + A \operatorname{Cos}[c + d x])} \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right) -$$

$$\frac{B \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{64 d (B + A \operatorname{Cos}[c + d x])} \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)^2 +$$

$$\frac{\operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) (A \operatorname{Sin}\left[\frac{d x}{2}\right] + 4 B \operatorname{Sin}\left[\frac{d x}{2}\right])}{16 d (B + A \operatorname{Cos}[c + d x])} \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right)$$

■ **Problem 77: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^3 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 165 leaves, 6 steps) :

$$\frac{1}{2} a^4 (12 A + 13 B) x + \frac{a^4 (A + 4 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^4 (2 A + B) \operatorname{Sin}[c + d x]}{2 d} + \frac{a A \operatorname{Cos}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{3 d} + \frac{(2 A + B) \operatorname{Cos}[c + d x] (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 d} - \frac{(8 A - 3 B) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{6 d}$$

Result (type 3, 342 leaves) :

$$\frac{1}{192 (B + A \operatorname{Cos}[c + d x])} a^4 \operatorname{Cos}[c + d x]^5 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^8 (1 + \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \left(72 A x + 78 B x - \frac{12 (A + 4 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{12 (A + 4 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{3 (27 A + 16 B) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{d} + \frac{3 (4 A + B) \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{d} + \frac{A \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{d} + \frac{3 (27 A + 16 B) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d} + \frac{3 (4 A + B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{d} + \frac{A \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{d} + \frac{12 B \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} + \frac{12 B \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} \right)$$

■ **Problem 82: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4 (A + B \operatorname{Sec}[c + d x])}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 131 leaves, 6 steps) :

$$\frac{3 (A - B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a d} - \frac{(3 A - 4 B) \operatorname{Tan}[c + d x]}{a d} + \frac{3 (A - B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d} + \frac{(A - B) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])} - \frac{(3 A - 4 B) \operatorname{Tan}[c + d x]^3}{3 a d}$$

Result (type 3, 635 leaves) :

$$\frac{3(-A+B)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+B\sec[c+dx])}{d(B+A\cos[c+dx])(a+a\sec[c+dx])} -$$

$$\frac{3(-A+B)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+B\sec[c+dx])}{d(B+A\cos[c+dx])(a+a\sec[c+dx])} +$$

$$\frac{1}{48d(B+A\cos[c+dx])(a+a\sec[c+dx])} \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c+dx]^3 (A+B\sec[c+dx])$$

$$\left(6A\sin\left[\frac{dx}{2}\right]+6B\sin\left[\frac{dx}{2}\right]-27A\sin\left[\frac{3dx}{2}\right]+39B\sin\left[\frac{3dx}{2}\right]+12A\sin\left[c-\frac{dx}{2}\right]-24B\sin\left[c-\frac{dx}{2}\right]+6A\sin\left[c+\frac{dx}{2}\right]-\right.$$

$$6B\sin\left[c+\frac{dx}{2}\right]+24A\sin\left[2c+\frac{dx}{2}\right]-24B\sin\left[2c+\frac{dx}{2}\right]-9A\sin\left[c+\frac{3dx}{2}\right]+21B\sin\left[c+\frac{3dx}{2}\right]-9A\sin\left[2c+\frac{3dx}{2}\right]+$$

$$9B\sin\left[2c+\frac{3dx}{2}\right]+9A\sin\left[3c+\frac{3dx}{2}\right]-9B\sin\left[3c+\frac{3dx}{2}\right]-3A\sin\left[c+\frac{5dx}{2}\right]+7B\sin\left[c+\frac{5dx}{2}\right]+3A\sin\left[2c+\frac{5dx}{2}\right]+$$

$$B\sin\left[2c+\frac{5dx}{2}\right]+3A\sin\left[3c+\frac{5dx}{2}\right]-3B\sin\left[3c+\frac{5dx}{2}\right]+9A\sin\left[4c+\frac{5dx}{2}\right]-9B\sin\left[4c+\frac{5dx}{2}\right]-12A\sin\left[2c+\frac{7dx}{2}\right]+$$

$$\left.16B\sin\left[2c+\frac{7dx}{2}\right]-6A\sin\left[3c+\frac{7dx}{2}\right]+10B\sin\left[3c+\frac{7dx}{2}\right]-6A\sin\left[4c+\frac{7dx}{2}\right]+6B\sin\left[4c+\frac{7dx}{2}\right]\right)$$

■ **Problem 83: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^3 (A+B\sec[c+dx])}{a+a\sec[c+dx]} dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{(2A-3B)\operatorname{ArcTanh}[\sin[c+dx]]}{2ad} + \frac{2(A-B)\tan[c+dx]}{ad} - \frac{(2A-3B)\sec[c+dx]\tan[c+dx]}{2ad} + \frac{(A-B)\sec[c+dx]^2\tan[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 3, 671 leaves):

$$\begin{aligned}
& \frac{(2A - 3B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx])}{d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])} + \\
& \frac{(-2A + 3B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx])}{d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])} - \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \left(-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right]\right)}{d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])} + \frac{B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c + dx])}{2d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c + dx]) \left(-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right]\right)}{d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c + dx])}{2d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} - \\
& \frac{2 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c + dx]) \left(-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right]\right)}{d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx])}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$\frac{(A - B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{ad} + \frac{B \operatorname{Tan}[c + dx]}{ad} - \frac{(A - B) \operatorname{Tan}[c + dx]}{d (a + a \operatorname{Sec}[c + dx])}$$

Result (type 3, 224 leaves):

$$\begin{aligned}
& \left(2 \cos\left[\frac{1}{2}(c + dx)\right] (A + B \operatorname{Sec}[c + dx]) \right. \\
& \left. \left((-A + B) \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \cos\left[\frac{1}{2}(c + dx)\right] \left(-(A - B) \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) \right) \right. \\
& \left. \left(B \sin[dx] \right) / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right. \\
& \left. \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) / (ad (B + A \cos[c + dx]) (1 + \operatorname{Sec}[c + dx]))
\end{aligned}$$

■ **Problem 85: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{B \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d} + \frac{(A - B) \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 3, 109 leaves):

$$\frac{1}{a d (1 + \operatorname{Cos}[c + d x])} 2 \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \left(B \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \left(-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + (A - B) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right] \right)$$

■ **Problem 86: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$\frac{A x}{a} - \frac{(A - B) \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 3, 72 leaves):

$$\frac{\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(A d x \operatorname{Cos}\left[\frac{d x}{2}\right] + A d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 2 (-A + B) \operatorname{Sin}\left[\frac{d x}{2}\right] \right)}{a d (1 + \operatorname{Cos}[c + d x])}$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x]^2 (A + B \operatorname{Sec}[c + d x])}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\frac{(3 A - 2 B) x}{2 a} - \frac{2 (A - B) \operatorname{Sin}[c + d x]}{a d} + \frac{(3 A - 2 B) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 a d} - \frac{(A - B) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 3, 197 leaves):

$$\frac{1}{8 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(4 (3 A - 2 B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 4 (3 A - 2 B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 20 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 20 B \operatorname{Sin}\left[\frac{d x}{2}\right] - 4 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 4 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 3 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 4 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 3 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 4 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] \right)$$

■ **Problem 89: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \sec[c + dx])}{a + a \sec[c + dx]} dx$$

Optimal (type 3, 122 leaves, 6 steps):

$$-\frac{3(A-B)x}{2a} + \frac{(4A-3B)\sin[c+dx]}{ad} - \frac{3(A-B)\cos[c+dx]\sin[c+dx]}{2ad} - \frac{(A-B)\cos[c+dx]^2\sin[c+dx]}{d(a+a\sec[c+dx])} - \frac{(4A-3B)\sin[c+dx]^3}{3ad}$$

Result (type 3, 249 leaves):

$$\frac{1}{24ad(1+\cos[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right] \sec\left[\frac{c}{2}\right] \left(-36(A-B)dx \cos\left[\frac{dx}{2}\right] - 36(A-B)dx \cos\left[c + \frac{dx}{2}\right] + 69A \sin\left[\frac{dx}{2}\right] - 60B \sin\left[\frac{dx}{2}\right] + 21A \sin\left[c + \frac{dx}{2}\right] - 12B \sin\left[c + \frac{dx}{2}\right] + 18A \sin\left[c + \frac{3dx}{2}\right] - 9B \sin\left[c + \frac{3dx}{2}\right] + 18A \sin\left[2c + \frac{3dx}{2}\right] - 9B \sin\left[2c + \frac{3dx}{2}\right] - 2A \sin\left[2c + \frac{5dx}{2}\right] + 3B \sin\left[2c + \frac{5dx}{2}\right] - 2A \sin\left[3c + \frac{5dx}{2}\right] + 3B \sin\left[3c + \frac{5dx}{2}\right] + A \sin\left[3c + \frac{7dx}{2}\right] + A \sin\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 90: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^5 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{(7A-10B)\operatorname{ArcTanh}[\sin[c+dx]]}{2a^2d} - \frac{4(2A-3B)\tan[c+dx]}{a^2d} + \frac{(7A-10B)\sec[c+dx]\tan[c+dx]}{2a^2d} + \frac{(7A-10B)\sec[c+dx]^3\tan[c+dx]}{3a^2d(1+\sec[c+dx])} + \frac{(A-B)\sec[c+dx]^4\tan[c+dx]}{3d(a+a\sec[c+dx])^2} - \frac{4(2A-3B)\tan[c+dx]^3}{3a^2d}$$

Result (type 3, 764 leaves):

$$\begin{aligned}
& \frac{2(-7A + 10B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx] (A + B \sec[c + dx])}{d(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} - \\
& \frac{2(-7A + 10B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx] (A + B \sec[c + dx])}{d(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} + \\
& \frac{1}{96d(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^4 (A + B \sec[c + dx]) \\
& \left(45A \sin\left[\frac{dx}{2}\right] - 6B \sin\left[\frac{dx}{2}\right] - 201A \sin\left[\frac{3dx}{2}\right] + 310B \sin\left[\frac{3dx}{2}\right] + 195A \sin\left[c - \frac{dx}{2}\right] - 306B \sin\left[c - \frac{dx}{2}\right] - 51A \sin\left[c + \frac{dx}{2}\right] + \right. \\
& 42B \sin\left[c + \frac{dx}{2}\right] + 189A \sin\left[2c + \frac{dx}{2}\right] - 270B \sin\left[2c + \frac{dx}{2}\right] - A \sin\left[c + \frac{3dx}{2}\right] + 50B \sin\left[c + \frac{3dx}{2}\right] - 81A \sin\left[2c + \frac{3dx}{2}\right] + \\
& 90B \sin\left[2c + \frac{3dx}{2}\right] + 119A \sin\left[3c + \frac{3dx}{2}\right] - 170B \sin\left[3c + \frac{3dx}{2}\right] - 129A \sin\left[c + \frac{5dx}{2}\right] + 198B \sin\left[c + \frac{5dx}{2}\right] - \\
& 9A \sin\left[2c + \frac{5dx}{2}\right] + 42B \sin\left[2c + \frac{5dx}{2}\right] - 57A \sin\left[3c + \frac{5dx}{2}\right] + 66B \sin\left[3c + \frac{5dx}{2}\right] + 63A \sin\left[4c + \frac{5dx}{2}\right] - \\
& 90B \sin\left[4c + \frac{5dx}{2}\right] - 75A \sin\left[2c + \frac{7dx}{2}\right] + 114B \sin\left[2c + \frac{7dx}{2}\right] - 15A \sin\left[3c + \frac{7dx}{2}\right] + 36B \sin\left[3c + \frac{7dx}{2}\right] - \\
& 39A \sin\left[4c + \frac{7dx}{2}\right] + 48B \sin\left[4c + \frac{7dx}{2}\right] + 21A \sin\left[5c + \frac{7dx}{2}\right] - 30B \sin\left[5c + \frac{7dx}{2}\right] - 32A \sin\left[3c + \frac{9dx}{2}\right] + \\
& \left. 48B \sin\left[3c + \frac{9dx}{2}\right] - 12A \sin\left[4c + \frac{9dx}{2}\right] + 22B \sin\left[4c + \frac{9dx}{2}\right] - 20A \sin\left[5c + \frac{9dx}{2}\right] + 26B \sin\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 91: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 156 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(4A - 7B) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2d} + \frac{2(5A - 8B) \tan[c + dx]}{3a^2d} - \\
& \frac{(4A - 7B) \sec[c + dx] \tan[c + dx]}{2a^2d} + \frac{(5A - 8B) \sec[c + dx]^2 \tan[c + dx]}{3a^2d(1 + \sec[c + dx])} + \frac{(A - B) \sec[c + dx]^3 \tan[c + dx]}{3d(a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 3, 652 leaves):

$$\begin{aligned}
& \frac{2(-4A + 7B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])}{d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} + \\
& \frac{2(-4A + 7B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])}{d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} + \\
& \frac{1}{48d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sec}[c + dx]) \\
& \left(-14A \sin\left[\frac{dx}{2}\right] + 14B \sin\left[\frac{dx}{2}\right] + 64A \sin\left[\frac{3dx}{2}\right] - 97B \sin\left[\frac{3dx}{2}\right] - 84A \sin\left[c - \frac{dx}{2}\right] + 126B \sin\left[c - \frac{dx}{2}\right] + 42A \sin\left[c + \frac{dx}{2}\right] - \right. \\
& 42B \sin\left[c + \frac{dx}{2}\right] - 56A \sin\left[2c + \frac{dx}{2}\right] + 98B \sin\left[2c + \frac{dx}{2}\right] - 6A \sin\left[c + \frac{3dx}{2}\right] + 3B \sin\left[c + \frac{3dx}{2}\right] + 34A \sin\left[2c + \frac{3dx}{2}\right] - \\
& 37B \sin\left[2c + \frac{3dx}{2}\right] - 36A \sin\left[3c + \frac{3dx}{2}\right] + 63B \sin\left[3c + \frac{3dx}{2}\right] + 48A \sin\left[c + \frac{5dx}{2}\right] - 75B \sin\left[c + \frac{5dx}{2}\right] + 6A \sin\left[2c + \frac{5dx}{2}\right] - \\
& 15B \sin\left[2c + \frac{5dx}{2}\right] + 30A \sin\left[3c + \frac{5dx}{2}\right] - 39B \sin\left[3c + \frac{5dx}{2}\right] - 12A \sin\left[4c + \frac{5dx}{2}\right] + 21B \sin\left[4c + \frac{5dx}{2}\right] + \\
& \left. 20A \sin\left[2c + \frac{7dx}{2}\right] - 32B \sin\left[2c + \frac{7dx}{2}\right] + 6A \sin\left[3c + \frac{7dx}{2}\right] - 12B \sin\left[3c + \frac{7dx}{2}\right] + 14A \sin\left[4c + \frac{7dx}{2}\right] - 20B \sin\left[4c + \frac{7dx}{2}\right]\right)
\end{aligned}$$

■ **Problem 92: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sec}[c + dx])}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$\frac{(A - 2B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} - \frac{(A - 4B) \tan[c + dx]}{3a^2 d} - \frac{(A - 2B) \tan[c + dx]}{a^2 d (1 + \operatorname{Sec}[c + dx])} + \frac{(A - B) \operatorname{Sec}[c + dx]^2 \tan[c + dx]}{3d (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 3, 292 leaves):

$$\begin{aligned}
& \frac{1}{3a^2 d (B + A \cos[c + dx]) (1 + \operatorname{Sec}[c + dx])^2} 2 \cos\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \\
& (A + B \operatorname{Sec}[c + dx]) \left((-A + B) \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] - 2(4A - 7B) \cos\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + \cos\left[\frac{1}{2}(c + dx)\right]^3 \right. \\
& \left. \left(-6(A - 2B) \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]\right) + (6B \sin[dx]) / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \right. \right. \right. \\
& \left. \left. \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right) \right) \right) - (A - B) \cos\left[\frac{1}{2}(c + dx)\right] \tan\left[\frac{c}{2}\right]
\end{aligned}$$

■ **Problem 93: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 79 leaves, 4 steps) :

$$\frac{B \text{ArcTanh}[\text{Sin}[c + d x]]}{a^2 d} + \frac{(2A - 5B) \text{Tan}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} - \frac{(A - B) \text{Tan}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 169 leaves) :

$$-\frac{1}{3 a^2 d (1 + \text{Cos}[c + d x])^2} + 2 \text{Cos}\left[\frac{1}{2}(c + d x)\right] \left(6 B \text{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + (-A + B) \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] - 2(A - 4B) \text{Cos}\left[\frac{1}{2}(c + d x)\right]^2 \text{Sec}\left[\frac{c}{2}\right] \text{Sin}\left[\frac{d x}{2}\right] - (A - B) \text{Cos}\left[\frac{1}{2}(c + d x)\right] \text{Tan}\left[\frac{c}{2}\right] \right)$$

■ **Problem 95: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \text{Sec}[c + d x]}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 70 leaves, 3 steps) :

$$\frac{A x}{a^2} - \frac{(4A - B) \text{Tan}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} - \frac{(A - B) \text{Tan}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 153 leaves) :

$$\frac{1}{24 a^2 d} \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \left(9 A d x \text{Cos}\left[\frac{d x}{2}\right] + 9 A d x \text{Cos}\left[c + \frac{d x}{2}\right] + 3 A d x \text{Cos}\left[c + \frac{3 d x}{2}\right] + 3 A d x \text{Cos}\left[2c + \frac{3 d x}{2}\right] - 18 A \text{Sin}\left[\frac{d x}{2}\right] + 6 B \text{Sin}\left[\frac{d x}{2}\right] + 12 A \text{Sin}\left[c + \frac{d x}{2}\right] - 6 B \text{Sin}\left[c + \frac{d x}{2}\right] - 10 A \text{Sin}\left[c + \frac{3 d x}{2}\right] + 4 B \text{Sin}\left[c + \frac{3 d x}{2}\right] \right)$$

■ **Problem 96: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c + d x] (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 3, 98 leaves, 5 steps) :

$$-\frac{(2A - B) x}{a^2} + \frac{2(5A - 2B) \text{Sin}[c + d x]}{3 a^2 d} - \frac{(2A - B) \text{Sin}[c + d x]}{a^2 d (1 + \text{Sec}[c + d x])} - \frac{(A - B) \text{Sin}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 3, 245 leaves) :

$$\frac{1}{12 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-18 (2 A - B) d x \cos \left[\frac{d x}{2} \right] - 18 (2 A - B) d x \cos \left[c + \frac{d x}{2} \right] - 12 A d x \cos \left[c + \frac{3 d x}{2} \right] + 6 B d x \cos \left[c + \frac{3 d x}{2} \right] - 12 A d x \cos \left[2 c + \frac{3 d x}{2} \right] + 6 B d x \cos \left[2 c + \frac{3 d x}{2} \right] + 66 A \sin \left[\frac{d x}{2} \right] - 36 B \sin \left[\frac{d x}{2} \right] - 30 A \sin \left[c + \frac{d x}{2} \right] + 24 B \sin \left[c + \frac{d x}{2} \right] + 41 A \sin \left[c + \frac{3 d x}{2} \right] - 20 B \sin \left[c + \frac{3 d x}{2} \right] + 9 A \sin \left[2 c + \frac{3 d x}{2} \right] + 3 A \sin \left[2 c + \frac{5 d x}{2} \right] + 3 A \sin \left[3 c + \frac{5 d x}{2} \right] \right)$$

■ **Problem 97: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^2 (A + B \sec [c + d x])}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 143 leaves, 6 steps):

$$\frac{(7 A - 4 B) x}{2 a^2} - \frac{2 (8 A - 5 B) \sin [c + d x]}{3 a^2 d} + \frac{(7 A - 4 B) \cos [c + d x] \sin [c + d x]}{2 a^2 d} - \frac{(8 A - 5 B) \cos [c + d x] \sin [c + d x]}{3 a^2 d (1 + \sec [c + d x])} - \frac{(A - B) \cos [c + d x] \sin [c + d x]}{3 d (a + a \sec [c + d x])^2}$$

Result (type 3, 315 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(36 (7 A - 4 B) d x \cos \left[\frac{d x}{2} \right] + 36 (7 A - 4 B) d x \cos \left[c + \frac{d x}{2} \right] + 84 A d x \cos \left[c + \frac{3 d x}{2} \right] - 48 B d x \cos \left[c + \frac{3 d x}{2} \right] + 84 A d x \cos \left[2 c + \frac{3 d x}{2} \right] - 48 B d x \cos \left[2 c + \frac{3 d x}{2} \right] - 381 A \sin \left[\frac{d x}{2} \right] + 264 B \sin \left[\frac{d x}{2} \right] + 147 A \sin \left[c + \frac{d x}{2} \right] - 120 B \sin \left[c + \frac{d x}{2} \right] - 239 A \sin \left[c + \frac{3 d x}{2} \right] + 164 B \sin \left[c + \frac{3 d x}{2} \right] - 63 A \sin \left[2 c + \frac{3 d x}{2} \right] + 36 B \sin \left[2 c + \frac{3 d x}{2} \right] - 15 A \sin \left[2 c + \frac{5 d x}{2} \right] + 12 B \sin \left[2 c + \frac{5 d x}{2} \right] - 15 A \sin \left[3 c + \frac{5 d x}{2} \right] + 12 B \sin \left[3 c + \frac{5 d x}{2} \right] + 3 A \sin \left[3 c + \frac{7 d x}{2} \right] + 3 A \sin \left[4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 98: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + B \sec [c + d x])}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 170 leaves, 7 steps):

$$-\frac{(10 A - 7 B) x}{2 a^2} + \frac{4 (3 A - 2 B) \sin [c + d x]}{a^2 d} - \frac{(10 A - 7 B) \cos [c + d x] \sin [c + d x]}{2 a^2 d} - \frac{(10 A - 7 B) \cos [c + d x]^2 \sin [c + d x]}{3 a^2 d (1 + \sec [c + d x])} - \frac{(A - B) \cos [c + d x]^2 \sin [c + d x]}{3 d (a + a \sec [c + d x])^2} - \frac{4 (3 A - 2 B) \sin [c + d x]^3}{3 a^2 d}$$

Result (type 3, 369 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-36 (10 A - 7 B) d x \cos \left[\frac{d x}{2} \right] - 36 (10 A - 7 B) d x \cos \left[c + \frac{d x}{2} \right] - 120 A d x \cos \left[c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[c + \frac{3 d x}{2} \right] - 120 A d x \cos \left[2 c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[2 c + \frac{3 d x}{2} \right] + 516 A \sin \left[\frac{d x}{2} \right] - 381 B \sin \left[\frac{d x}{2} \right] - 156 A \sin \left[c + \frac{d x}{2} \right] + 147 B \sin \left[c + \frac{d x}{2} \right] + 342 A \sin \left[c + \frac{3 d x}{2} \right] - 239 B \sin \left[c + \frac{3 d x}{2} \right] + 118 A \sin \left[2 c + \frac{3 d x}{2} \right] - 63 B \sin \left[2 c + \frac{3 d x}{2} \right] + 30 A \sin \left[2 c + \frac{5 d x}{2} \right] - 15 B \sin \left[2 c + \frac{5 d x}{2} \right] + 30 A \sin \left[3 c + \frac{5 d x}{2} \right] - 15 B \sin \left[3 c + \frac{5 d x}{2} \right] - 3 A \sin \left[3 c + \frac{7 d x}{2} \right] + 3 B \sin \left[3 c + \frac{7 d x}{2} \right] - 3 A \sin \left[4 c + \frac{7 d x}{2} \right] + 3 B \sin \left[4 c + \frac{7 d x}{2} \right] + A \sin \left[4 c + \frac{9 d x}{2} \right] + A \sin \left[5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 99: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^5 (A + B \sec [c + d x])}{(a + a \sec [c + d x])^3} dx$$

Optimal (type 3, 202 leaves, 8 steps):

$$-\frac{(6 A - 13 B) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^3 d} + \frac{8 (9 A - 19 B) \tan [c + d x]}{15 a^3 d} - \frac{(6 A - 13 B) \sec [c + d x] \tan [c + d x]}{2 a^3 d} + \frac{(A - B) \sec [c + d x]^4 \tan [c + d x]}{5 d (a + a \sec [c + d x])^3} + \frac{(6 A - 11 B) \sec [c + d x]^3 \tan [c + d x]}{15 a d (a + a \sec [c + d x])^2} + \frac{4 (9 A - 19 B) \sec [c + d x]^2 \tan [c + d x]}{15 d (a^3 + a^3 \sec [c + d x])}$$

Result (type 3, 768 leaves):

$$\begin{aligned}
& \frac{4(-6A + 13B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + B \sec[c + dx])}{d(B + A \cos[c + dx]) (a + a \sec[c + dx])^3} + \\
& \frac{4(-6A + 13B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + B \sec[c + dx])}{d(B + A \cos[c + dx]) (a + a \sec[c + dx])^3} + \\
& \frac{1}{480 d (B + A \cos[c + dx]) (a + a \sec[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^4 (A + B \sec[c + dx]) \\
& \left(-870 A \sin\left[\frac{dx}{2}\right] + 1235 B \sin\left[\frac{dx}{2}\right] + 1830 A \sin\left[\frac{3 dx}{2}\right] - 3805 B \sin\left[\frac{3 dx}{2}\right] - 2094 A \sin\left[c - \frac{dx}{2}\right] + 4329 B \sin\left[c - \frac{dx}{2}\right] + \right. \\
& 1314 A \sin\left[c + \frac{dx}{2}\right] - 1989 B \sin\left[c + \frac{dx}{2}\right] - 1650 A \sin\left[2c + \frac{dx}{2}\right] + 3575 B \sin\left[2c + \frac{dx}{2}\right] - 450 A \sin\left[c + \frac{3 dx}{2}\right] + 475 B \sin\left[c + \frac{3 dx}{2}\right] + \\
& 1230 A \sin\left[2c + \frac{3 dx}{2}\right] - 2005 B \sin\left[2c + \frac{3 dx}{2}\right] - 1050 A \sin\left[3c + \frac{3 dx}{2}\right] + 2275 B \sin\left[3c + \frac{3 dx}{2}\right] + 1278 A \sin\left[c + \frac{5 dx}{2}\right] - \\
& 2673 B \sin\left[c + \frac{5 dx}{2}\right] - 90 A \sin\left[2c + \frac{5 dx}{2}\right] - 105 B \sin\left[2c + \frac{5 dx}{2}\right] + 918 A \sin\left[3c + \frac{5 dx}{2}\right] - 1593 B \sin\left[3c + \frac{5 dx}{2}\right] - \\
& 450 A \sin\left[4c + \frac{5 dx}{2}\right] + 975 B \sin\left[4c + \frac{5 dx}{2}\right] + 630 A \sin\left[2c + \frac{7 dx}{2}\right] - 1325 B \sin\left[2c + \frac{7 dx}{2}\right] + 60 A \sin\left[3c + \frac{7 dx}{2}\right] - \\
& 255 B \sin\left[3c + \frac{7 dx}{2}\right] + 480 A \sin\left[4c + \frac{7 dx}{2}\right] - 875 B \sin\left[4c + \frac{7 dx}{2}\right] - 90 A \sin\left[5c + \frac{7 dx}{2}\right] + 195 B \sin\left[5c + \frac{7 dx}{2}\right] + \\
& \left. 144 A \sin\left[3c + \frac{9 dx}{2}\right] - 304 B \sin\left[3c + \frac{9 dx}{2}\right] + 30 A \sin\left[4c + \frac{9 dx}{2}\right] - 90 B \sin\left[4c + \frac{9 dx}{2}\right] + 114 A \sin\left[5c + \frac{9 dx}{2}\right] - 214 B \sin\left[5c + \frac{9 dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 100: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 156 leaves, 7 steps):

$$\begin{aligned}
& \frac{(A - 3B) \operatorname{ArcTanh}[\sin[c + dx]]}{a^3 d} - \frac{(7A - 27B) \tan[c + dx]}{15 a^3 d} + \\
& \frac{(A - B) \sec[c + dx]^3 \tan[c + dx]}{5 d (a + a \sec[c + dx])^3} + \frac{(4A - 9B) \sec[c + dx]^2 \tan[c + dx]}{15 a d (a + a \sec[c + dx])^2} - \frac{(A - 3B) \tan[c + dx]}{d (a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 3, 642 leaves):

$$\frac{8(-A+3B)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right] \operatorname{Sec}[c+dx]^2 (A+B\operatorname{Sec}[c+dx])}{d(B+A\cos[c+dx])(a+a\operatorname{Sec}[c+dx])^3} -$$

$$\frac{8(-A+3B)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right] \operatorname{Sec}[c+dx]^2 (A+B\operatorname{Sec}[c+dx])}{d(B+A\cos[c+dx])(a+a\operatorname{Sec}[c+dx])^3} +$$

$$\frac{1}{120d(B+A\cos[c+dx])(a+a\operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2}+\frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (A+B\operatorname{Sec}[c+dx])$$

$$\left(160A\sin\left[\frac{dx}{2}\right]-255B\sin\left[\frac{dx}{2}\right]-167A\sin\left[\frac{3dx}{2}\right]+567B\sin\left[\frac{3dx}{2}\right]+170A\sin\left[c-\frac{dx}{2}\right]-600B\sin\left[c-\frac{dx}{2}\right]-170A\sin\left[c+\frac{dx}{2}\right]+375B\sin\left[c+\frac{dx}{2}\right]+160A\sin\left[2c+\frac{dx}{2}\right]-480B\sin\left[2c+\frac{dx}{2}\right]+75A\sin\left[c+\frac{3dx}{2}\right]-60B\sin\left[c+\frac{3dx}{2}\right]-167A\sin\left[2c+\frac{3dx}{2}\right]+402B\sin\left[2c+\frac{3dx}{2}\right]+75A\sin\left[3c+\frac{3dx}{2}\right]-225B\sin\left[3c+\frac{3dx}{2}\right]-95A\sin\left[c+\frac{5dx}{2}\right]+315B\sin\left[c+\frac{5dx}{2}\right]+15A\sin\left[2c+\frac{5dx}{2}\right]+30B\sin\left[2c+\frac{5dx}{2}\right]-95A\sin\left[3c+\frac{5dx}{2}\right]+240B\sin\left[3c+\frac{5dx}{2}\right]+15A\sin\left[4c+\frac{5dx}{2}\right]-45B\sin\left[4c+\frac{5dx}{2}\right]-22A\sin\left[2c+\frac{7dx}{2}\right]+72B\sin\left[2c+\frac{7dx}{2}\right]+15B\sin\left[3c+\frac{7dx}{2}\right]-22A\sin\left[4c+\frac{7dx}{2}\right]+57B\sin\left[4c+\frac{7dx}{2}\right]\right)$$

■ **Problem 104: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B\operatorname{Sec}[c+dx]}{(a+a\operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{Ax}{a^3} - \frac{(A-B)\operatorname{Tan}[c+dx]}{5d(a+a\operatorname{Sec}[c+dx])^3} - \frac{(7A-2B)\operatorname{Tan}[c+dx]}{15ad(a+a\operatorname{Sec}[c+dx])^2} - \frac{2(11A-B)\operatorname{Tan}[c+dx]}{15d(a^3+a^3\operatorname{Sec}[c+dx])}$$

Result (type 3, 241 leaves):

$$\frac{1}{480a^3d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \left(150Adx\cos\left[\frac{dx}{2}\right]+150Adx\cos\left[c+\frac{dx}{2}\right]+75Adx\cos\left[c+\frac{3dx}{2}\right]+75Adx\cos\left[2c+\frac{3dx}{2}\right]+15Adx\cos\left[2c+\frac{5dx}{2}\right]+15Adx\cos\left[3c+\frac{5dx}{2}\right]-370A\sin\left[\frac{dx}{2}\right]+80B\sin\left[\frac{dx}{2}\right]+270A\sin\left[c+\frac{dx}{2}\right]-60B\sin\left[c+\frac{dx}{2}\right]-230A\sin\left[c+\frac{3dx}{2}\right]+40B\sin\left[c+\frac{3dx}{2}\right]+90A\sin\left[2c+\frac{3dx}{2}\right]-30B\sin\left[2c+\frac{3dx}{2}\right]-64A\sin\left[2c+\frac{5dx}{2}\right]+14B\sin\left[2c+\frac{5dx}{2}\right]\right)$$

■ **Problem 105: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx](A+B\operatorname{Sec}[c+dx])}{(a+a\operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 136 leaves, 6 steps):

$$-\frac{(3A-B)x}{a^3} + \frac{2(36A-11B)\sin[c+dx]}{15a^3d} - \frac{(A-B)\sin[c+dx]}{5d(a+a\sec[c+dx])^3} - \frac{(9A-4B)\sin[c+dx]}{15ad(a+a\sec[c+dx])^2} - \frac{(3A-B)\sin[c+dx]}{d(a^3+a^3\sec[c+dx])}$$

Result (type 3, 365 leaves):

$$\frac{1}{120a^3d(1+\cos[c+dx])^3} \cos\left[\frac{1}{2}(c+dx)\right] \sec\left[\frac{c}{2}\right] \left(-300(3A-B)dx \cos\left[\frac{dx}{2}\right] - 300(3A-B)dx \cos\left[c+\frac{dx}{2}\right] - 450Adx \cos\left[c+\frac{3dx}{2}\right] + 150Bdx \cos\left[c+\frac{3dx}{2}\right] - 450Adx \cos\left[2c+\frac{3dx}{2}\right] + 150Bdx \cos\left[2c+\frac{3dx}{2}\right] - 90Adx \cos\left[2c+\frac{5dx}{2}\right] + 30Bdx \cos\left[2c+\frac{5dx}{2}\right] - 90Adx \cos\left[3c+\frac{5dx}{2}\right] + 30Bdx \cos\left[3c+\frac{5dx}{2}\right] + 1755A \sin\left[\frac{dx}{2}\right] - 740B \sin\left[\frac{dx}{2}\right] - 1125A \sin\left[c+\frac{dx}{2}\right] + 540B \sin\left[c+\frac{dx}{2}\right] + 1215A \sin\left[c+\frac{3dx}{2}\right] - 460B \sin\left[c+\frac{3dx}{2}\right] - 225A \sin\left[2c+\frac{3dx}{2}\right] + 180B \sin\left[2c+\frac{3dx}{2}\right] + 363A \sin\left[2c+\frac{5dx}{2}\right] - 128B \sin\left[2c+\frac{5dx}{2}\right] + 75A \sin\left[3c+\frac{5dx}{2}\right] + 15A \sin\left[3c+\frac{7dx}{2}\right] + 15A \sin\left[4c+\frac{7dx}{2}\right] \right)$$

■ **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (A+B\sec[c+dx])}{(a+a\sec[c+dx])^3} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\frac{(13A-6B)x}{2a^3} - \frac{8(19A-9B)\sin[c+dx]}{15a^3d} + \frac{(13A-6B)\cos[c+dx]\sin[c+dx]}{2a^3d} - \frac{(A-B)\cos[c+dx]\sin[c+dx]}{5d(a+a\sec[c+dx])^3} - \frac{(11A-6B)\cos[c+dx]\sin[c+dx]}{15ad(a+a\sec[c+dx])^2} - \frac{4(19A-9B)\cos[c+dx]\sin[c+dx]}{15d(a^3+a^3\sec[c+dx])}$$

Result (type 3, 435 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + d x])^3} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right]$$

$$\left(600 (13 A - 6 B) d x \cos \left[\frac{d x}{2} \right] + 600 (13 A - 6 B) d x \cos \left[c + \frac{d x}{2} \right] + 3900 A d x \cos \left[c + \frac{3 d x}{2} \right] - 1800 B d x \cos \left[c + \frac{3 d x}{2} \right] + 3900 A d x \cos \left[2 c + \frac{3 d x}{2} \right] - \right.$$

$$1800 B d x \cos \left[2 c + \frac{3 d x}{2} \right] + 780 A d x \cos \left[2 c + \frac{5 d x}{2} \right] - 360 B d x \cos \left[2 c + \frac{5 d x}{2} \right] + 780 A d x \cos \left[3 c + \frac{5 d x}{2} \right] - 360 B d x \cos \left[3 c + \frac{5 d x}{2} \right] -$$

$$12760 A \sin \left[\frac{d x}{2} \right] + 7020 B \sin \left[\frac{d x}{2} \right] + 7560 A \sin \left[c + \frac{d x}{2} \right] - 4500 B \sin \left[c + \frac{d x}{2} \right] - 9230 A \sin \left[c + \frac{3 d x}{2} \right] + 4860 B \sin \left[c + \frac{3 d x}{2} \right] +$$

$$930 A \sin \left[2 c + \frac{3 d x}{2} \right] - 900 B \sin \left[2 c + \frac{3 d x}{2} \right] - 2782 A \sin \left[2 c + \frac{5 d x}{2} \right] + 1452 B \sin \left[2 c + \frac{5 d x}{2} \right] - 750 A \sin \left[3 c + \frac{5 d x}{2} \right] + 300 B \sin \left[3 c + \frac{5 d x}{2} \right] -$$

$$\left. 105 A \sin \left[3 c + \frac{7 d x}{2} \right] + 60 B \sin \left[3 c + \frac{7 d x}{2} \right] - 105 A \sin \left[4 c + \frac{7 d x}{2} \right] + 60 B \sin \left[4 c + \frac{7 d x}{2} \right] + 15 A \sin \left[4 c + \frac{9 d x}{2} \right] + 15 A \sin \left[5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + B \sec [c + d x])}{(a + a \sec [c + d x])^3} dx$$

Optimal (type 3, 218 leaves, 8 steps):

$$-\frac{(23 A - 13 B) x}{2 a^3} + \frac{4 (34 A - 19 B) \sin [c + d x]}{5 a^3 d} - \frac{(23 A - 13 B) \cos [c + d x] \sin [c + d x]}{2 a^3 d} - \frac{(A - B) \cos [c + d x]^2 \sin [c + d x]}{5 d (a + a \sec [c + d x])^3}$$

$$\frac{(13 A - 8 B) \cos [c + d x]^2 \sin [c + d x]}{15 a d (a + a \sec [c + d x])^2} - \frac{(23 A - 13 B) \cos [c + d x]^2 \sin [c + d x]}{3 d (a^3 + a^3 \sec [c + d x])} - \frac{4 (34 A - 19 B) \sin [c + d x]^3}{15 a^3 d}$$

Result (type 3, 491 leaves):

$$\frac{1}{480 a^3 d (1 + \cos [c + d x])^3}$$

$$\cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-600 (23 A - 13 B) d x \cos \left[\frac{d x}{2} \right] - 600 (23 A - 13 B) d x \cos \left[c + \frac{d x}{2} \right] - 6900 A d x \cos \left[c + \frac{3 d x}{2} \right] + 3900 B d x \cos \left[c + \frac{3 d x}{2} \right] - \right.$$

$$6900 A d x \cos \left[2 c + \frac{3 d x}{2} \right] + 3900 B d x \cos \left[2 c + \frac{3 d x}{2} \right] - 1380 A d x \cos \left[2 c + \frac{5 d x}{2} \right] + 780 B d x \cos \left[2 c + \frac{5 d x}{2} \right] - 1380 A d x \cos \left[3 c + \frac{5 d x}{2} \right] +$$

$$780 B d x \cos \left[3 c + \frac{5 d x}{2} \right] + 20410 A \sin \left[\frac{d x}{2} \right] - 12760 B \sin \left[\frac{d x}{2} \right] - 11110 A \sin \left[c + \frac{d x}{2} \right] + 7560 B \sin \left[c + \frac{d x}{2} \right] + 15380 A \sin \left[c + \frac{3 d x}{2} \right] -$$

$$9230 B \sin \left[c + \frac{3 d x}{2} \right] - 380 A \sin \left[2 c + \frac{3 d x}{2} \right] + 930 B \sin \left[2 c + \frac{3 d x}{2} \right] + 4777 A \sin \left[2 c + \frac{5 d x}{2} \right] - 2782 B \sin \left[2 c + \frac{5 d x}{2} \right] +$$

$$1625 A \sin \left[3 c + \frac{5 d x}{2} \right] - 750 B \sin \left[3 c + \frac{5 d x}{2} \right] + 230 A \sin \left[3 c + \frac{7 d x}{2} \right] - 105 B \sin \left[3 c + \frac{7 d x}{2} \right] + 230 A \sin \left[4 c + \frac{7 d x}{2} \right] - 105 B \sin \left[4 c + \frac{7 d x}{2} \right] -$$

$$\left. 20 A \sin \left[4 c + \frac{9 d x}{2} \right] + 15 B \sin \left[4 c + \frac{9 d x}{2} \right] - 20 A \sin \left[5 c + \frac{9 d x}{2} \right] + 15 B \sin \left[5 c + \frac{9 d x}{2} \right] + 5 A \sin \left[5 c + \frac{11 d x}{2} \right] + 5 A \sin \left[6 c + \frac{11 d x}{2} \right] \right)$$

■ **Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + dx]^6 (A + B \text{Sec}[c + dx])}{(a + a \text{Sec}[c + dx])^4} dx$$

Optimal (type 3, 238 leaves, 9 steps):

$$\begin{aligned} & - \frac{(8A - 21B) \text{ArcTanh}[\text{Sin}[c + dx]]}{2a^4 d} + \frac{8(83A - 216B) \text{Tan}[c + dx]}{105a^4 d} - \frac{(8A - 21B) \text{Sec}[c + dx] \text{Tan}[c + dx]}{2a^4 d} + \frac{(52A - 129B) \text{Sec}[c + dx]^3 \text{Tan}[c + dx]}{105a^4 d (1 + \text{Sec}[c + dx])^2} \\ & + \frac{4(83A - 216B) \text{Sec}[c + dx]^2 \text{Tan}[c + dx]}{105a^4 d (1 + \text{Sec}[c + dx])} + \frac{(A - B) \text{Sec}[c + dx]^5 \text{Tan}[c + dx]}{7d (a + a \text{Sec}[c + dx])^4} + \frac{(A - 2B) \text{Sec}[c + dx]^4 \text{Tan}[c + dx]}{5ad (a + a \text{Sec}[c + dx])^3} \end{aligned}$$

Result (type 3, 880 leaves):

$$\begin{aligned} & - \frac{8(-8A + 21B) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^3 (A + B \text{Sec}[c + dx])}{d (B + A \text{Cos}[c + dx]) (a + a \text{Sec}[c + dx])^4} + \\ & \frac{8(-8A + 21B) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + dx]^3 (A + B \text{Sec}[c + dx])}{d (B + A \text{Cos}[c + dx]) (a + a \text{Sec}[c + dx])^4} + \\ & \frac{1}{6720 d (B + A \text{Cos}[c + dx]) (a + a \text{Sec}[c + dx])^4} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c] \text{Sec}[c + dx]^5 (A + B \text{Sec}[c + dx]) \\ & \left(-38668A \text{Sin}\left[\frac{dx}{2}\right] + 73206B \text{Sin}\left[\frac{dx}{2}\right] + 64384A \text{Sin}\left[\frac{3dx}{2}\right] - 166668B \text{Sin}\left[\frac{3dx}{2}\right] - 70896A \text{Sin}\left[c - \frac{dx}{2}\right] + 183162B \text{Sin}\left[c - \frac{dx}{2}\right] + \right. \\ & 50316A \text{Sin}\left[c + \frac{dx}{2}\right] - 100842B \text{Sin}\left[c + \frac{dx}{2}\right] - 59248A \text{Sin}\left[2c + \frac{dx}{2}\right] + 155526B \text{Sin}\left[2c + \frac{dx}{2}\right] - 22820A \text{Sin}\left[c + \frac{3dx}{2}\right] + \\ & 37380B \text{Sin}\left[c + \frac{3dx}{2}\right] + 48004A \text{Sin}\left[2c + \frac{3dx}{2}\right] - 101148B \text{Sin}\left[2c + \frac{3dx}{2}\right] - 39200A \text{Sin}\left[3c + \frac{3dx}{2}\right] + 102900B \text{Sin}\left[3c + \frac{3dx}{2}\right] + \\ & 46032A \text{Sin}\left[c + \frac{5dx}{2}\right] - 119364B \text{Sin}\left[c + \frac{5dx}{2}\right] - 8750A \text{Sin}\left[2c + \frac{5dx}{2}\right] + 8820B \text{Sin}\left[2c + \frac{5dx}{2}\right] + 35742A \text{Sin}\left[3c + \frac{5dx}{2}\right] - \\ & 78204B \text{Sin}\left[3c + \frac{5dx}{2}\right] - 19040A \text{Sin}\left[4c + \frac{5dx}{2}\right] + 49980B \text{Sin}\left[4c + \frac{5dx}{2}\right] + 24664A \text{Sin}\left[2c + \frac{7dx}{2}\right] - 64053B \text{Sin}\left[2c + \frac{7dx}{2}\right] - \\ & 1050A \text{Sin}\left[3c + \frac{7dx}{2}\right] - 3885B \text{Sin}\left[3c + \frac{7dx}{2}\right] + 19834A \text{Sin}\left[4c + \frac{7dx}{2}\right] - 44733B \text{Sin}\left[4c + \frac{7dx}{2}\right] - 5880A \text{Sin}\left[5c + \frac{7dx}{2}\right] + \\ & 15435B \text{Sin}\left[5c + \frac{7dx}{2}\right] + 8456A \text{Sin}\left[3c + \frac{9dx}{2}\right] - 21987B \text{Sin}\left[3c + \frac{9dx}{2}\right] + 630A \text{Sin}\left[4c + \frac{9dx}{2}\right] - 3675B \text{Sin}\left[4c + \frac{9dx}{2}\right] + \\ & 6986A \text{Sin}\left[5c + \frac{9dx}{2}\right] - 16107B \text{Sin}\left[5c + \frac{9dx}{2}\right] - 840A \text{Sin}\left[6c + \frac{9dx}{2}\right] + 2205B \text{Sin}\left[6c + \frac{9dx}{2}\right] + 1328A \text{Sin}\left[4c + \frac{11dx}{2}\right] - \\ & \left. 3456B \text{Sin}\left[4c + \frac{11dx}{2}\right] + 210A \text{Sin}\left[5c + \frac{11dx}{2}\right] - 840B \text{Sin}\left[5c + \frac{11dx}{2}\right] + 1118A \text{Sin}\left[6c + \frac{11dx}{2}\right] - 2616B \text{Sin}\left[6c + \frac{11dx}{2}\right] \right) \end{aligned}$$

■ **Problem 109: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^5 (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^4} dx$$

Optimal (type 3, 194 leaves, 8 steps):

$$\frac{(A - 4 B) \text{ArcTanh}[\text{Sin}[c + d x]]}{a^4 d} - \frac{(55 A - 244 B) \text{Tan}[c + d x]}{105 a^4 d} + \frac{(25 A - 88 B) \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{105 a^4 d (1 + \text{Sec}[c + d x])^2} - \frac{(A - 4 B) \text{Tan}[c + d x]}{a^4 d (1 + \text{Sec}[c + d x])} + \frac{(A - B) \text{Sec}[c + d x]^4 \text{Tan}[c + d x]}{7 d (a + a \text{Sec}[c + d x])^4} + \frac{(5 A - 12 B) \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{35 a d (a + a \text{Sec}[c + d x])^3}$$

Result (type 3, 754 leaves):

$$\frac{16 (-A + 4 B) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + d x]^3 (A + B \text{Sec}[c + d x])}{d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^4} - \frac{16 (-A + 4 B) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \text{Sec}[c + d x]^3 (A + B \text{Sec}[c + d x])}{d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^4} + \frac{1}{1680 d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^4} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c] \text{Sec}[c + d x]^4 (A + B \text{Sec}[c + d x]) \left(4165 A \text{Sin}\left[\frac{dx}{2}\right] - 10780 B \text{Sin}\left[\frac{dx}{2}\right] - 4445 A \text{Sin}\left[\frac{3 dx}{2}\right] + 18788 B \text{Sin}\left[\frac{3 dx}{2}\right] + 4795 A \text{Sin}\left[c - \frac{dx}{2}\right] - 20524 B \text{Sin}\left[c - \frac{dx}{2}\right] - 4795 A \text{Sin}\left[c + \frac{dx}{2}\right] + 14644 B \text{Sin}\left[c + \frac{dx}{2}\right] + 4165 A \text{Sin}\left[2c + \frac{dx}{2}\right] - 16660 B \text{Sin}\left[2c + \frac{dx}{2}\right] + 2275 A \text{Sin}\left[c + \frac{3 dx}{2}\right] - 4690 B \text{Sin}\left[c + \frac{3 dx}{2}\right] - 4445 A \text{Sin}\left[2c + \frac{3 dx}{2}\right] + 14378 B \text{Sin}\left[2c + \frac{3 dx}{2}\right] + 2275 A \text{Sin}\left[3c + \frac{3 dx}{2}\right] - 9100 B \text{Sin}\left[3c + \frac{3 dx}{2}\right] - 2785 A \text{Sin}\left[c + \frac{5 dx}{2}\right] + 11668 B \text{Sin}\left[c + \frac{5 dx}{2}\right] + 735 A \text{Sin}\left[2c + \frac{5 dx}{2}\right] - 630 B \text{Sin}\left[2c + \frac{5 dx}{2}\right] - 2785 A \text{Sin}\left[3c + \frac{5 dx}{2}\right] + 9358 B \text{Sin}\left[3c + \frac{5 dx}{2}\right] + 735 A \text{Sin}\left[4c + \frac{5 dx}{2}\right] - 2940 B \text{Sin}\left[4c + \frac{5 dx}{2}\right] - 1015 A \text{Sin}\left[2c + \frac{7 dx}{2}\right] + 4228 B \text{Sin}\left[2c + \frac{7 dx}{2}\right] + 105 A \text{Sin}\left[3c + \frac{7 dx}{2}\right] + 315 B \text{Sin}\left[3c + \frac{7 dx}{2}\right] - 1015 A \text{Sin}\left[4c + \frac{7 dx}{2}\right] + 3493 B \text{Sin}\left[4c + \frac{7 dx}{2}\right] + 105 A \text{Sin}\left[5c + \frac{7 dx}{2}\right] - 420 B \text{Sin}\left[5c + \frac{7 dx}{2}\right] - 160 A \text{Sin}\left[3c + \frac{9 dx}{2}\right] + 664 B \text{Sin}\left[3c + \frac{9 dx}{2}\right] + 105 B \text{Sin}\left[4c + \frac{9 dx}{2}\right] - 160 A \text{Sin}\left[5c + \frac{9 dx}{2}\right] + 559 B \text{Sin}\left[5c + \frac{9 dx}{2}\right] \right)$$

■ **Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \text{Sec}[c + d x]}{(a + a \text{Sec}[c + d x])^4} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{A x}{a^4} - \frac{(55 A - 6 B) \tan[c + d x]}{105 a^4 d (1 + \sec[c + d x])^2} - \frac{2 (80 A - 3 B) \tan[c + d x]}{105 a^4 d (1 + \sec[c + d x])} - \frac{(A - B) \tan[c + d x]}{7 d (a + a \sec[c + d x])^4} - \frac{(10 A - 3 B) \tan[c + d x]}{35 a d (a + a \sec[c + d x])^3}$$

Result (type 3, 329 leaves):

$$\begin{aligned} & \frac{1}{13440 a^4 d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + d x)\right]^7 \\ & \left(3675 A d x \cos\left[\frac{d x}{2}\right] + 3675 A d x \cos\left[c + \frac{d x}{2}\right] + 2205 A d x \cos\left[c + \frac{3 d x}{2}\right] + 2205 A d x \cos\left[2 c + \frac{3 d x}{2}\right] + 735 A d x \cos\left[2 c + \frac{5 d x}{2}\right] + \right. \\ & 735 A d x \cos\left[3 c + \frac{5 d x}{2}\right] + 105 A d x \cos\left[3 c + \frac{7 d x}{2}\right] + 105 A d x \cos\left[4 c + \frac{7 d x}{2}\right] - 9940 A \sin\left[\frac{d x}{2}\right] + 1260 B \sin\left[\frac{d x}{2}\right] + \\ & 8260 A \sin\left[c + \frac{d x}{2}\right] - 1260 B \sin\left[c + \frac{d x}{2}\right] - 7140 A \sin\left[c + \frac{3 d x}{2}\right] + 882 B \sin\left[c + \frac{3 d x}{2}\right] + 3780 A \sin\left[2 c + \frac{3 d x}{2}\right] - 630 B \sin\left[2 c + \frac{3 d x}{2}\right] - \\ & \left. 2800 A \sin\left[2 c + \frac{5 d x}{2}\right] + 294 B \sin\left[2 c + \frac{5 d x}{2}\right] + 840 A \sin\left[3 c + \frac{5 d x}{2}\right] - 210 B \sin\left[3 c + \frac{5 d x}{2}\right] - 520 A \sin\left[3 c + \frac{7 d x}{2}\right] + 72 B \sin\left[3 c + \frac{7 d x}{2}\right] \right) \end{aligned}$$

■ **Problem 115: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x] (A + B \sec[c + d x])}{(a + a \sec[c + d x])^4} dx$$

Optimal (type 3, 166 leaves, 7 steps):

$$\begin{aligned} & -\frac{(4 A - B) x}{a^4} + \frac{8 (83 A - 20 B) \sin[c + d x]}{105 a^4 d} - \frac{(88 A - 25 B) \sin[c + d x]}{105 a^4 d (1 + \sec[c + d x])^2} - \\ & \frac{(4 A - B) \sin[c + d x]}{a^4 d (1 + \sec[c + d x])} - \frac{(A - B) \sin[c + d x]}{7 d (a + a \sec[c + d x])^4} - \frac{(12 A - 5 B) \sin[c + d x]}{35 a d (a + a \sec[c + d x])^3} \end{aligned}$$

Result (type 3, 485 leaves):

1

$$1680 a^4 d (1 + \cos[c + dx])^4$$

$$\begin{aligned} & \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left(-7350(4A - B) dx \cos\left[\frac{dx}{2}\right] - 7350(4A - B) dx \cos\left[c + \frac{dx}{2}\right] - 17640A dx \cos\left[c + \frac{3dx}{2}\right] + 4410B dx \cos\left[c + \frac{3dx}{2}\right] - \right. \\ & 17640A dx \cos\left[2c + \frac{3dx}{2}\right] + 4410B dx \cos\left[2c + \frac{3dx}{2}\right] - 5880A dx \cos\left[2c + \frac{5dx}{2}\right] + 1470B dx \cos\left[2c + \frac{5dx}{2}\right] - \\ & 5880A dx \cos\left[3c + \frac{5dx}{2}\right] + 1470B dx \cos\left[3c + \frac{5dx}{2}\right] - 840A dx \cos\left[3c + \frac{7dx}{2}\right] + 210B dx \cos\left[3c + \frac{7dx}{2}\right] - \\ & 840A dx \cos\left[4c + \frac{7dx}{2}\right] + 210B dx \cos\left[4c + \frac{7dx}{2}\right] + 60830A \sin\left[\frac{dx}{2}\right] - 19880B \sin\left[\frac{dx}{2}\right] - 46130A \sin\left[c + \frac{dx}{2}\right] + \\ & 16520B \sin\left[c + \frac{dx}{2}\right] + 46116A \sin\left[c + \frac{3dx}{2}\right] - 14280B \sin\left[c + \frac{3dx}{2}\right] - 18060A \sin\left[2c + \frac{3dx}{2}\right] + 7560B \sin\left[2c + \frac{3dx}{2}\right] + \\ & 19292A \sin\left[2c + \frac{5dx}{2}\right] - 5600B \sin\left[2c + \frac{5dx}{2}\right] - 2100A \sin\left[3c + \frac{5dx}{2}\right] + 1680B \sin\left[3c + \frac{5dx}{2}\right] + \\ & \left. 3791A \sin\left[3c + \frac{7dx}{2}\right] - 1040B \sin\left[3c + \frac{7dx}{2}\right] + 735A \sin\left[4c + \frac{7dx}{2}\right] + 105A \sin\left[4c + \frac{9dx}{2}\right] + 105A \sin\left[5c + \frac{9dx}{2}\right] \right) \end{aligned}$$

■ **Problem 116: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$\begin{aligned} & \frac{(21A - 8B)x}{2a^4} - \frac{8(216A - 83B) \sin[c + dx]}{105a^4 d} + \frac{(21A - 8B) \cos[c + dx] \sin[c + dx]}{2a^4 d} - \frac{(129A - 52B) \cos[c + dx] \sin[c + dx]}{105a^4 d (1 + \sec[c + dx])^2} - \\ & \frac{4(216A - 83B) \cos[c + dx] \sin[c + dx]}{105a^4 d (1 + \sec[c + dx])} - \frac{(A - B) \cos[c + dx] \sin[c + dx]}{7d (a + a \sec[c + dx])^4} - \frac{(2A - B) \cos[c + dx] \sin[c + dx]}{5ad (a + a \sec[c + dx])^3} \end{aligned}$$

Result (type 3, 555 leaves):

$$\frac{1}{6720 a^4 d (1 + \cos[c + dx])^4} \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left(14700 (21A - 8B) dx \cos\left[\frac{dx}{2}\right] + 14700 (21A - 8B) dx \cos\left[c + \frac{dx}{2}\right] + 185220 A dx \cos\left[c + \frac{3dx}{2}\right] - 70560 B dx \cos\left[c + \frac{3dx}{2}\right] + 185220 A dx \cos\left[2c + \frac{3dx}{2}\right] - 70560 B dx \cos\left[2c + \frac{3dx}{2}\right] + 61740 A dx \cos\left[2c + \frac{5dx}{2}\right] - 23520 B dx \cos\left[2c + \frac{5dx}{2}\right] + 61740 A dx \cos\left[3c + \frac{5dx}{2}\right] - 23520 B dx \cos\left[3c + \frac{5dx}{2}\right] + 8820 A dx \cos\left[3c + \frac{7dx}{2}\right] - 3360 B dx \cos\left[3c + \frac{7dx}{2}\right] + 8820 A dx \cos\left[4c + \frac{7dx}{2}\right] - 3360 B dx \cos\left[4c + \frac{7dx}{2}\right] - 539490 A \sin\left[\frac{dx}{2}\right] + 243320 B \sin\left[\frac{dx}{2}\right] + 386190 A \sin\left[c + \frac{dx}{2}\right] - 184520 B \sin\left[c + \frac{dx}{2}\right] - 422478 A \sin\left[c + \frac{3dx}{2}\right] + 184464 B \sin\left[c + \frac{3dx}{2}\right] + 132930 A \sin\left[2c + \frac{3dx}{2}\right] - 72240 B \sin\left[2c + \frac{3dx}{2}\right] - 181461 A \sin\left[2c + \frac{5dx}{2}\right] + 77168 B \sin\left[2c + \frac{5dx}{2}\right] + 3675 A \sin\left[3c + \frac{5dx}{2}\right] - 8400 B \sin\left[3c + \frac{5dx}{2}\right] - 36003 A \sin\left[3c + \frac{7dx}{2}\right] + 15164 B \sin\left[3c + \frac{7dx}{2}\right] - 9555 A \sin\left[4c + \frac{7dx}{2}\right] + 2940 B \sin\left[4c + \frac{7dx}{2}\right] - 945 A \sin\left[4c + \frac{9dx}{2}\right] + 420 B \sin\left[4c + \frac{9dx}{2}\right] - 945 A \sin\left[5c + \frac{9dx}{2}\right] + 420 B \sin\left[5c + \frac{9dx}{2}\right] + 105 A \sin\left[5c + \frac{11dx}{2}\right] + 105 A \sin\left[6c + \frac{11dx}{2}\right] \right)$$

■ **Problem 117: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^3 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 256 leaves, 9 steps):

$$-\frac{(44A - 21B)x}{2a^4} + \frac{8(227A - 108B)\sin[c + dx]}{35a^4d} - \frac{(44A - 21B)\cos[c + dx]\sin[c + dx]}{2a^4d} - \frac{(178A - 87B)\cos[c + dx]^2\sin[c + dx]}{105a^4d(1 + \sec[c + dx])^2} - \frac{(44A - 21B)\cos[c + dx]^2\sin[c + dx]}{3a^4d(1 + \sec[c + dx])} - \frac{(A - B)\cos[c + dx]^2\sin[c + dx]}{7d(a + a\sec[c + dx])^4} - \frac{(16A - 9B)\cos[c + dx]^2\sin[c + dx]}{35ad(a + a\sec[c + dx])^3} - \frac{8(227A - 108B)\sin[c + dx]^3}{105a^4d}$$

Result (type 3, 611 leaves):

1

$$6720 a^4 d (1 + \operatorname{Cos}[c + d x])^4$$

$$\begin{aligned} & \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(-14700(44A - 21B) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 14700(44A - 21B) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 388080 A d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + \right. \\ & 185220 B d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 388080 A d x \operatorname{Cos}\left[2c + \frac{3 d x}{2}\right] + 185220 B d x \operatorname{Cos}\left[2c + \frac{3 d x}{2}\right] - 129360 A d x \operatorname{Cos}\left[2c + \frac{5 d x}{2}\right] + \\ & 61740 B d x \operatorname{Cos}\left[2c + \frac{5 d x}{2}\right] - 129360 A d x \operatorname{Cos}\left[3c + \frac{5 d x}{2}\right] + 61740 B d x \operatorname{Cos}\left[3c + \frac{5 d x}{2}\right] - 18480 A d x \operatorname{Cos}\left[3c + \frac{7 d x}{2}\right] + \\ & 8820 B d x \operatorname{Cos}\left[3c + \frac{7 d x}{2}\right] - 18480 A d x \operatorname{Cos}\left[4c + \frac{7 d x}{2}\right] + 8820 B d x \operatorname{Cos}\left[4c + \frac{7 d x}{2}\right] + 1010660 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 539490 B \operatorname{Sin}\left[\frac{d x}{2}\right] - \\ & 687260 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 386190 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 814107 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 422478 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 204645 A \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] + \\ & 132930 B \operatorname{Sin}\left[2c + \frac{3 d x}{2}\right] + 357609 A \operatorname{Sin}\left[2c + \frac{5 d x}{2}\right] - 181461 B \operatorname{Sin}\left[2c + \frac{5 d x}{2}\right] + 18025 A \operatorname{Sin}\left[3c + \frac{5 d x}{2}\right] + \\ & 3675 B \operatorname{Sin}\left[3c + \frac{5 d x}{2}\right] + 72522 A \operatorname{Sin}\left[3c + \frac{7 d x}{2}\right] - 36003 B \operatorname{Sin}\left[3c + \frac{7 d x}{2}\right] + 24010 A \operatorname{Sin}\left[4c + \frac{7 d x}{2}\right] - 9555 B \operatorname{Sin}\left[4c + \frac{7 d x}{2}\right] + \\ & 2310 A \operatorname{Sin}\left[4c + \frac{9 d x}{2}\right] - 945 B \operatorname{Sin}\left[4c + \frac{9 d x}{2}\right] + 2310 A \operatorname{Sin}\left[5c + \frac{9 d x}{2}\right] - 945 B \operatorname{Sin}\left[5c + \frac{9 d x}{2}\right] - 175 A \operatorname{Sin}\left[5c + \frac{11 d x}{2}\right] + \\ & \left. 105 B \operatorname{Sin}\left[5c + \frac{11 d x}{2}\right] - 175 A \operatorname{Sin}\left[6c + \frac{11 d x}{2}\right] + 105 B \operatorname{Sin}\left[6c + \frac{11 d x}{2}\right] + 35 A \operatorname{Sin}\left[6c + \frac{13 d x}{2}\right] + 35 A \operatorname{Sin}\left[7c + \frac{13 d x}{2}\right] \right) \end{aligned}$$

- **Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2 \sqrt{a} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{d} + \frac{2 a B \operatorname{Tan}[c + d x]}{d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 4, 407 leaves):

$$\begin{aligned}
& - \frac{1}{d (B + A \cos [c + d x])} 8 (-3 - 2 \sqrt{2}) A \cos \left[\frac{1}{4} (c + d x) \right]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \\
& \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\text{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \text{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \text{Sec} \left[\frac{1}{2} (c + d x) \right]} \\
& \sqrt{a (1 + \text{Sec} [c + d x])} (A + B \text{Sec} [c + d x]) \sqrt{3 - 2 \sqrt{2} - \text{Tan} \left[\frac{1}{4} (c + d x) \right]^2} + \\
& \frac{2 B \cos [c + d x] \sqrt{a (1 + \text{Sec} [c + d x])} (A + B \text{Sec} [c + d x]) \text{Tan} \left[\frac{1}{2} (c + d x) \right]}{d (B + A \cos [c + d x])}
\end{aligned}$$

- **Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] \sqrt{a + a \text{Sec} [c + d x]} (A + B \text{Sec} [c + d x]) dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{\sqrt{a} (A + 2 B) \text{ArcTan} \left[\frac{\sqrt{a} \text{Tan} [c + d x]}{\sqrt{a + a \text{Sec} [c + d x]}} \right]}{d} + \frac{a A \text{Sin} [c + d x]}{d \sqrt{a + a \text{Sec} [c + d x]}}$$

Result (type 4, 396 leaves):

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{2}A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}A \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{d} -$$

$$\frac{1}{d} 4(-3-2\sqrt{2})(A+2B) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$\frac{\sqrt{a} (3A+4B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a(3A+4B) \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aA \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 418 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{8}(A+4B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}(A+2B) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}A \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (3A+4B) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

- **Problem 125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$\frac{\sqrt{a} (5A+6B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a(5A+6B) \operatorname{Sin}[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a(5A+6B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aA \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 443 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{48}(11A+6B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{12}(4A+3B) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16}(A+2B) \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{24}A \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right) + \frac{1}{d} \left(1 + \frac{3}{2\sqrt{2}} \right) (5A+6B) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \right) \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 203 leaves, 6 steps):

$$\frac{5\sqrt{a}(7A+8B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64d} + \frac{5a(7A+8B) \operatorname{Sin}[c+dx]}{64d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{5a(7A+8B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{96d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a(7A+8B) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{24d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aA \operatorname{Cos}[c+dx]^3 \operatorname{Sin}[c+dx]}{4d\sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 465 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{384}(41A+88B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{1}{48}(11A+16B) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{128}(15A+8B) \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{48}(A+2B) \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + \frac{1}{64}A \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right]\right) + \\
& \quad \frac{1}{(-64+48\sqrt{2})d} 5(7A+8B) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \quad \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right) \\
& \quad \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

- **Problem 130: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\frac{2a^{3/2}A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2a^2(3A+4B) \operatorname{Tan}[c+dx]}{3d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2aB\sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 460 leaves):

$$\frac{1}{d (B + A \cos [c + d x])} \cos [c + d x]^2 \sec \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \sec [c + d x]))^{3/2} (A + B \sec [c + d x]) \left(\frac{1}{3} (3 A + 5 B) \sin \left[\frac{1}{2} (c + d x) \right] + \frac{1}{3} B \sec [c + d x] \sin \left[\frac{1}{2} (c + d x) \right] \right) -$$

$$\frac{1}{d (B + A \cos [c + d x])} 4 (-3 - 2 \sqrt{2}) A \cos \left[\frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]$$

$$\left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \sec \left[\frac{1}{4} (c + d x) \right]^2 \sec \left[\frac{1}{2} (c + d x) \right]^3}$$

$$(a (1 + \sec [c + d x]))^{3/2} (A + B \sec [c + d x]) \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (c + d x) \right]^2}$$

- **Problem 131: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + a \sec [c + d x])^{3/2} (A + B \sec [c + d x]) dx$$

Optimal (type 3, 103 leaves, 4 steps):

$$\frac{a^{3/2} (3 A + 2 B) \text{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \sec [c + d x]}} \right]}{d} + \frac{a^2 (A - 2 B) \sin [c + d x]}{d \sqrt{a + a \sec [c + d x]}} + \frac{2 a B \sqrt{a + a \sec [c + d x]} \sin [c + d x]}{d}$$

Result (type 4, 408 leaves):

$$\frac{1}{d} \cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\frac{1}{4}(-A + 4B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4}A \sin\left[\frac{3}{2}(c + dx)\right] \right) -$$

$$\frac{1}{d} 2(-3 - 2\sqrt{2})(3A + 2B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}}$$

- **Problem 132: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$\frac{a^{3/2} (7A + 12B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{4d} + \frac{a^2 (5A + 4B) \sin[c + dx]}{4d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{a A \cos[c + dx] \sqrt{a + a \operatorname{Sec}[c + dx]} \sin[c + dx]}{2d}$$

Result (type 4, 428 leaves):

$$\frac{1}{d} \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \left(-\frac{1}{16}(5A + 4B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{8}(3A + 2B) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{16}A \sin\left[\frac{5}{2}(c + dx)\right]\right) +$$

$$\frac{1}{d} \left(1 + \frac{3}{2\sqrt{2}}\right) (7A + 12B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

- **Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^{3/2} (A + B \sec[c + dx]) dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\frac{a^{3/2} (11A + 14B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{8d} + \frac{a^2 (11A + 14B) \sin[c + dx]}{8d \sqrt{a + a \sec[c + dx]}} +$$

$$\frac{a^2 (7A + 6B) \cos[c + dx] \sin[c + dx]}{12d \sqrt{a + a \sec[c + dx]}} + \frac{aA \cos[c + dx]^2 \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{3d}$$

Result (type 4, 450 leaves):

$$\frac{1}{d} \cos[c + dx] \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \left(-\frac{1}{96} (17A + 30B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{24} (7A + 9B) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{32} (3A + 2B) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{48} A \sin\left[\frac{7}{2}(c + dx)\right] \right) + \frac{1}{8d} (4 + 3\sqrt{2}) (11A + 14B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 (a + a \sec[c + dx])^{3/2} (A + B \sec[c + dx]) dx$$

Optimal (type 3, 209 leaves, 6 steps):

$$\frac{a^{3/2} (75A + 88B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{64d} + \frac{a^2 (75A + 88B) \sin[c + dx]}{64d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 (75A + 88B) \cos[c + dx] \sin[c + dx]}{96d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 (9A + 8B) \cos[c + dx]^2 \sin[c + dx]}{24d \sqrt{a + a \sec[c + dx]}} + \frac{aA \cos[c + dx]^3 \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{4d}$$

Result (type 4, 471 leaves):

$$\begin{aligned}
& \frac{1}{d} \cos[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(-\frac{1}{768} (129A + 136B) \sin\left[\frac{1}{2}(c + dx)\right] + \right. \\
& \quad \left. \frac{1}{96} (27A + 28B) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{256} (23A + 24B) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{96} (3A + 2B) \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{128} A \sin\left[\frac{9}{2}(c + dx)\right] \right) + \\
& \frac{1}{64d} (4 + 3\sqrt{2}) (75A + 88B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}
\end{aligned}$$

- **Problem 138: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 142 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{d} + \frac{2 a^3 (35 A + 32 B) \tan[c + dx]}{15 d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \\
& \frac{2 a^2 (5 A + 8 B) \sqrt{a + a \operatorname{Sec}[c + dx]} \tan[c + dx]}{15 d} + \frac{2 a B (a + a \operatorname{Sec}[c + dx])^{3/2} \tan[c + dx]}{5 d}
\end{aligned}$$

Result (type 4, 501 leaves):

$$\frac{1}{d (B + A \cos [c + d x])} \cos [c + d x]^3 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x])$$

$$\left(\frac{1}{30} (40 A + 43 B) \sin \left[\frac{1}{2} (c + d x) \right] + \frac{1}{10} B \operatorname{Sec} [c + d x]^2 \sin \left[\frac{1}{2} (c + d x) \right] + \frac{1}{30} \operatorname{Sec} [c + d x] \left(5 A \sin \left[\frac{1}{2} (c + d x) \right] + 14 B \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) -$$

$$\frac{1}{d (B + A \cos [c + d x])} 2 (-3 - 2 \sqrt{2}) A \cos \left[\frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]^2$$

$$\left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5}$$

$$(a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x]) \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (c + d x) \right]^2}$$

- **Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + a \operatorname{Sec} [c + d x])^{5/2} (A + B \operatorname{Sec} [c + d x]) dx$$

Optimal (type 3, 143 leaves, 5 steps):

$$\frac{a^{5/2} (5 A + 2 B) \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \operatorname{Sec} [c + d x]}} \right]}{d} - \frac{a^3 (3 A + 14 B) \sin [c + d x]}{3 d \sqrt{a + a \operatorname{Sec} [c + d x]}} +$$

$$\frac{2 a^2 (A + 2 B) \sqrt{a + a \operatorname{Sec} [c + d x]} \sin [c + d x]}{d} + \frac{2 a B (a + a \operatorname{Sec} [c + d x])^{3/2} \sin [c + d x]}{3 d}$$

Result (type 4, 434 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(\frac{1}{24} (9A + 32B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{6} B \sec[c + dx] \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{8} A \sin\left[\frac{3}{2}(c + dx)\right] \right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (5A + 2B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \cos[c + dx]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

Optimal (type 3, 154 leaves, 5 steps):

$$\frac{a^{5/2} (19A + 20B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{4d} + \frac{a^3 (9A - 4B) \sin[c + dx]}{4d \sqrt{a + a \sec[c + dx]}} -$$

$$\frac{a^2 (A - 4B) \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{2d} + \frac{aA \cos[c + dx] (a + a \sec[c + dx])^{3/2} \sin[c + dx]}{2d}$$

Result (type 4, 437 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(\frac{3}{32}(-3A + 4B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{16}(5A + 2B) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{32}A \sin\left[\frac{5}{2}(c + dx)\right] \right) +$$

$$\frac{1}{8d} (4 + 3\sqrt{2}) (19A + 20B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\cos[c + dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\frac{a^{5/2} (25A + 38B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{8d} + \frac{a^3 (49A + 54B) \sin[c + dx]}{24d \sqrt{a + a \sec[c + dx]}} +$$

$$\frac{a^2 (3A + 2B) \cos[c + dx] \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{4d} + \frac{a A \cos[c + dx]^2 (a + a \sec[c + dx])^{3/2} \sin[c + dx]}{3d}$$

Result (type 4, 458 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(-\frac{1}{192} (47A + 54B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{48} (16A + 15B) \sin\left[\frac{3}{2}(c + dx)\right] + \frac{1}{64} (5A + 2B) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{96} A \sin\left[\frac{7}{2}(c + dx)\right] \right) + \frac{1}{8d} \left(2 + \frac{3}{\sqrt{2}} \right) (25A + 38B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \cos[c + dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 (a + a \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

Optimal (type 3, 209 leaves, 6 steps):

$$\frac{a^{5/2} (163A + 200B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{64d} + \frac{a^3 (163A + 200B) \sin[c + dx]}{64d \sqrt{a + a \sec[c + dx]}} + \frac{a^3 (95A + 104B) \cos[c + dx] \sin[c + dx]}{96d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 (11A + 8B) \cos[c + dx]^2 \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{24d} + \frac{aA \cos[c + dx]^3 (a + a \sec[c + dx])^{3/2} \sin[c + dx]}{4d}$$

Result (type 4, 479 leaves):

$$\frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \left(-\frac{(265A+376B)\sin\left[\frac{1}{2}(c+dx)\right]}{1536} + \right. \\ \left. \frac{1}{192}(55A+64B)\sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{512}(47A+40B)\sin\left[\frac{5}{2}(c+dx)\right] + \frac{1}{192}(5A+2B)\sin\left[\frac{7}{2}(c+dx)\right] + \frac{1}{256}A\sin\left[\frac{9}{2}(c+dx)\right] \right) + \\ \frac{1}{64d} \left(2 + \frac{3}{\sqrt{2}} \right) (163A+200B) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right] \right) \\ \cos[c+dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\ \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)\sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)\sec\left[\frac{1}{4}(c+dx)\right]^2} \\ \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 143: Result unnecessarily involves higher level functions.**

$$\int \cos[c+dx]^5 (a+a\sec[c+dx])^{5/2} (A+B\sec[c+dx]) dx$$

Optimal (type 3, 254 leaves, 7 steps):

$$\frac{a^{5/2}(283A+326B)\text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{128d} + \frac{a^3(283A+326B)\sin[c+dx]}{128d\sqrt{a+a\sec[c+dx]}} + \\ \frac{a^3(283A+326B)\cos[c+dx]\sin[c+dx]}{192d\sqrt{a+a\sec[c+dx]}} + \frac{a^3(157A+170B)\cos[c+dx]^2\sin[c+dx]}{240d\sqrt{a+a\sec[c+dx]}} + \\ \frac{a^2(13A+10B)\cos[c+dx]^3\sqrt{a+a\sec[c+dx]}\sin[c+dx]}{40d} + \frac{aA\cos[c+dx]^4(a+a\sec[c+dx])^{3/2}\sin[c+dx]}{5d}$$

Result (type 4, 500 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2}$$

$$\left(-\frac{(2309A + 2650B) \sin\left[\frac{1}{2}(c + dx)\right]}{15360} + \frac{(509A + 550B) \sin\left[\frac{3}{2}(c + dx)\right]}{1920} + \frac{(95A + 94B) \sin\left[\frac{5}{2}(c + dx)\right]}{1024} + \right.$$

$$\left. \frac{1}{960} (32A + 25B) \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{512} (5A + 2B) \sin\left[\frac{9}{2}(c + dx)\right] + \frac{1}{640} A \sin\left[\frac{11}{2}(c + dx)\right] \right) +$$

$$\frac{1}{256d} (4 + 3\sqrt{2}) (283A + 326B) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\cos[c + dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 162: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^2 (A + B \sec[c + dx])}{(a + a \sec[c + dx])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$\frac{(5A + 19B) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A - B) \tan[c + dx]}{4d(a + a \sec[c + dx])^{5/2}} + \frac{(5A - 13B) \tan[c + dx]}{16ad(a + a \sec[c + dx])^{3/2}}$$

Result (type 3, 256 leaves):

$$\frac{(5A + 19B) \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \cos\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}[c + dx]^{5/2} \sqrt{1 + \operatorname{Sec}[c + dx]} + 4d \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}}{d(a(1 + \operatorname{Sec}[c + dx]))^{5/2}} \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^3 \left(-\frac{1}{2}(-A + 9B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(-A \sin\left[\frac{1}{2}(c + dx)\right] + B \sin\left[\frac{1}{2}(c + dx)\right]\right) + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(3A \sin\left[\frac{1}{2}(c + dx)\right] + 5B \sin\left[\frac{1}{2}(c + dx)\right]\right)\right)$$

■ **Problem 163: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])}{(a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 4 steps):

$$\frac{(3A + 5B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} + \frac{(A - B) \tan[c + dx]}{4d(a + a \operatorname{Sec}[c + dx])^{5/2}} + \frac{(3A + 5B) \tan[c + dx]}{16ad(a + a \operatorname{Sec}[c + dx])^{3/2}}$$

Result (type 3, 298 leaves):

$$\left((3A + 5B) \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \cos\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{Sec}[c + dx]^{3/2} \sqrt{1 + \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \right) / \left(4d(B + A \cos[c + dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}} \right) + \left(\cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx]) \left(\frac{1}{2}(7A + B) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right] \right) + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(-11A \sin\left[\frac{1}{2}(c + dx)\right] + 3B \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) / (d(B + A \cos[c + dx]) (a(1 + \operatorname{Sec}[c + dx]))^{5/2})$$

■ **Problem 164: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{(a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 164 leaves, 7 steps):

$$\frac{2A \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} - \frac{(43A - 3B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A - B) \tan[c + dx]}{4d(a + a \operatorname{Sec}[c + dx])^{5/2}} - \frac{(11A - 3B) \tan[c + dx]}{16ad(a + a \operatorname{Sec}[c + dx])^{3/2}}$$

Result (type 3, 343 leaves) :

$$\left(\left((-43 A + 3 B) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right] + 32 \sqrt{2} A \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{\frac{\operatorname{Cos}[c + dx]}{1 + \operatorname{Cos}[c + dx]}}}}\right] \right) \right. \\ \left. \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\operatorname{Cos}[c + dx]}{1 + \operatorname{Cos}[c + dx]}} \operatorname{Sec}[c + dx]^{3/2} \sqrt{1 + \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \right) / \\ \left(4 d (B + A \operatorname{Cos}[c + dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx]) \right) \right. \\ \left. \left(\frac{1}{2} (-15 A + 7 B) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(19 A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - 11 B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) + \right. \\ \left. \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(-A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) / (d (B + A \operatorname{Cos}[c + dx]) (a (1 + \operatorname{Sec}[c + dx]))^{5/2}) \right)$$

■ **Problem 167: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + A \operatorname{Sec}[c + dx]}{\sqrt{a - a \operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 89 leaves, 5 steps) :

$$\frac{2 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a - a \operatorname{Sec}[c + dx]}}\right]}{\sqrt{a} d} - \frac{2 \sqrt{2} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{2} \sqrt{a - a \operatorname{Sec}[c + dx]}}\right]}{\sqrt{a} d}$$

Result (type 3, 176 leaves) :

$$- \left(A (-1 + e^{i(c + dx)}) \right. \\ \left(\sqrt{2} dx + i \sqrt{2} \operatorname{ArcSinh}[e^{i(c + dx)}] + 4 i \operatorname{Log}[1 - e^{i(c + dx)}] + i \sqrt{2} \operatorname{Log}[1 + \sqrt{1 + e^{2i(c + dx)}}] - 4 i \operatorname{Log}[1 + e^{i(c + dx)} + \sqrt{2} \sqrt{1 + e^{2i(c + dx)}}] \right) \left. \right) / \\ \left(\sqrt{2} d \sqrt{1 + e^{2i(c + dx)}} \sqrt{a - a \operatorname{Sec}[c + dx]} \right)$$

■ **Problem 168: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx] (A + A \operatorname{Sec}[c + dx])}{\sqrt{a - a \operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 115 leaves, 6 steps) :

$$\frac{3 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} - \frac{2 \sqrt{2} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} + \frac{A \operatorname{Sin}[c+d x]}{d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 382 leaves) :

$$A \left(\left(e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1+e^{2 i (c+d x)}}} \sqrt{1+e^{2 i (c+d x)}} (1+\operatorname{Cos}[c+d x]) \right. \right. \\ \left. \left. \left(-3 i d x + 3 \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] + 4 \sqrt{2} \operatorname{Log}\left[1-e^{i (c+d x)}\right] + 3 \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+d x)}}\right] - 4 \sqrt{2} \operatorname{Log}\left[1+e^{i (c+d x)}+\sqrt{2} \sqrt{1+e^{2 i (c+d x)}}\right] \right) \right. \right. \\ \left. \left. \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right] \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Tan}\left[\frac{c}{2}+\frac{d x}{2}\right] \right) / \left(2 \sqrt{2} d \sqrt{a-a \operatorname{Sec}[c+d x]} \right) + \frac{1}{\sqrt{a-a \operatorname{Sec}[c+d x]}} \right. \\ \left. (1+\operatorname{Cos}[c+d x]) \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right] \operatorname{Sec}[c+d x] \left(\frac{\operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Cos}\left[\frac{d x}{2}\right]}{2 d} + \frac{\operatorname{Cos}\left[\frac{3 c}{2}\right] \operatorname{Cos}\left[\frac{3 d x}{2}\right]}{2 d} - \frac{\operatorname{Sin}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{d x}{2}\right]}{2 d} - \frac{\operatorname{Sin}\left[\frac{3 c}{2}\right] \operatorname{Sin}\left[\frac{3 d x}{2}\right]}{2 d} \right) \operatorname{Tan}\left[\frac{c}{2}+\frac{d x}{2}\right] \right)$$

■ **Problem 169: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^2 (A+A \operatorname{Sec}[c+d x])}{\sqrt{a-a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 155 leaves, 7 steps) :

$$\frac{11 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{4 \sqrt{a} d} - \frac{2 \sqrt{2} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} + \frac{5 A \operatorname{Sin}[c+d x]}{4 d \sqrt{a-a \operatorname{Sec}[c+d x]}} + \frac{A \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 332 leaves) :

$$\frac{1}{8 d \sqrt{a-a \operatorname{Sec}[c+d x]}} \\ A e^{-i (c+d x)} \left(7 + 6 e^{-i (c+d x)} + 7 e^{i (c+d x)} + e^{-2 i (c+d x)} + 6 e^{2 i (c+d x)} + e^{3 i (c+d x)} - 11 i d \sqrt{1+e^{2 i (c+d x)}} x + 11 \sqrt{1+e^{2 i (c+d x)}} \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] + \right. \\ \left. 16 \sqrt{2} \sqrt{1+e^{2 i (c+d x)}} \operatorname{Log}\left[1-e^{i (c+d x)}\right] + 11 \sqrt{1+e^{2 i (c+d x)}} \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+d x)}}\right] - \right. \\ \left. 16 \sqrt{2} \sqrt{1+e^{2 i (c+d x)}} \operatorname{Log}\left[1+e^{i (c+d x)}+\sqrt{2} \sqrt{1+e^{2 i (c+d x)}}\right] \right) \operatorname{Sec}[c+d x] \left(\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right] + i \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right] \right) \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]$$

■ **Problem 170: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^3 (A+A \operatorname{Sec}[c+d x])}{\sqrt{a-a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 192 leaves, 8 steps):

$$\frac{23 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{8 \sqrt{a} d} - \frac{2 \sqrt{2} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} +$$

$$\frac{9 A \sin [c+d x]}{8 d \sqrt{a-a \operatorname{Sec}[c+d x]}} + \frac{7 A \cos [c+d x] \sin [c+d x]}{12 d \sqrt{a-a \operatorname{Sec}[c+d x]}} + \frac{A \cos [c+d x]^2 \sin [c+d x]}{3 d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 362 leaves):

$$\frac{1}{48 d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

$$A e^{-i(c+d x)} \left(47 + 40 e^{-i(c+d x)} + 47 e^{i(c+d x)} + 9 e^{-2 i(c+d x)} + 40 e^{2 i(c+d x)} + 2 e^{-3 i(c+d x)} + 9 e^{3 i(c+d x)} + 2 e^{4 i(c+d x)} - 69 i d \sqrt{1+e^{2 i(c+d x)}} x + \right.$$

$$69 \sqrt{1+e^{2 i(c+d x)}} \operatorname{ArcSinh}\left[e^{i(c+d x)}\right] + 96 \sqrt{2} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Log}\left[1-e^{i(c+d x)}\right] + 69 \sqrt{1+e^{2 i(c+d x)}} \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right] -$$

$$\left. 96 \sqrt{2} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right) \operatorname{Sec}[c+d x] \left(\cos \left[\frac{1}{2}(c+d x)\right] + i \sin \left[\frac{1}{2}(c+d x)\right] \right) \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 171: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+A \operatorname{Sec}[c+d x]}{(a-a \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 116 leaves, 6 steps):

$$\frac{2 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{a^{3/2} d} - \frac{3 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{2} a^{3/2} d} - \frac{A \operatorname{Tan}[c+d x]}{d(a-a \operatorname{Sec}[c+d x])^{3/2}}$$

Result (type 3, 265 leaves):

$$\frac{1}{d(a-a \operatorname{Sec}[c+d x])^{3/2}} A \left(\sqrt{2} e^{-\frac{1}{2} i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \sqrt{1+e^{2 i(c+d x)}} \right.$$

$$\left. \left(2 i d x - 2 \operatorname{ArcSinh}\left[e^{i(c+d x)}\right] - 3 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right] - 2 \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right] + 3 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right] \right) - \right.$$

$$\left. \left(\cos \left[\frac{1}{2}(c+d x)\right] + \cos \left[\frac{3}{2}(c+d x)\right] \right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{\operatorname{Sec}[c+d x]} \right) \operatorname{Sec}[c+d x]^{3/2} \sin \left[\frac{1}{2}(c+d x)\right]^3$$

■ **Problem 172: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x] (A+A \operatorname{Sec}[c+d x])}{(a-a \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

Optimal (type 3, 146 leaves, 7 steps):

$$\frac{5 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{a^{3 / 2} d}-\frac{7 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{2} a^{3 / 2} d}-\frac{A \operatorname{Sin}[c+d x]}{d(a-a \operatorname{Sec}[c+d x])^{3 / 2}}+\frac{2 A \operatorname{Sin}[c+d x]}{a d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 281 leaves):

$$\frac{1}{d(a-a \operatorname{Sec}[c+d x])^{3 / 2}} A\left(\sqrt{2} e^{-\frac{1}{2} i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}}\sqrt{1+e^{2 i(c+d x)}}\right. \\ \left.+\left(5 i d x-5 \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]-7 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]-5 \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]+7 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \\ +\frac{1}{2}\left(-2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-3 \operatorname{Cos}\left[\frac{3}{2}(c+d x)\right]+\operatorname{Cos}\left[\frac{5}{2}(c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sec}[c+d x]^{3 / 2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^3$$

■ **Problem 173: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^2 (A+A \operatorname{Sec}[c+d x])}{(a-a \operatorname{Sec}[c+d x])^{3 / 2}} d x$$

Optimal (type 3, 194 leaves, 8 steps):

$$\frac{31 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{4 a^{3 / 2} d}-\frac{11 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{2} a^{3 / 2} d}-\frac{A \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{d(a-a \operatorname{Sec}[c+d x])^{3 / 2}}+\frac{13 A \operatorname{Sin}[c+d x]}{4 a d \sqrt{a-a \operatorname{Sec}[c+d x]}}+\frac{3 A \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 a d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 296 leaves):

$$\frac{1}{4 d(a-a \operatorname{Sec}[c+d x])^{3 / 2}} A\left(\sqrt{2} e^{-\frac{1}{2} i(c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}}\sqrt{1+e^{2 i(c+d x)}}\right. \\ \left.+\left(31 i d x-31 \operatorname{ArcSinh}\left[e^{i(c+d x)}\right]-44 \sqrt{2} \operatorname{Log}\left[1-e^{i(c+d x)}\right]-31 \operatorname{Log}\left[1+\sqrt{1+e^{2 i(c+d x)}}\right]+44 \sqrt{2} \operatorname{Log}\left[1+e^{i(c+d x)}+\sqrt{2} \sqrt{1+e^{2 i(c+d x)}}\right]\right)\right) \\ +\frac{1}{2}\left(-9 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-16 \operatorname{Cos}\left[\frac{3}{2}(c+d x)\right]+8 \operatorname{Cos}\left[\frac{5}{2}(c+d x)\right]+\operatorname{Cos}\left[\frac{7}{2}(c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{\operatorname{Sec}[c+d x]} \\ \operatorname{Sec}[c+d x]^{3 / 2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]^3$$

■ **Problem 174: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x]^3 (A+A \operatorname{Sec}[c+d x])}{(a-a \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 236 leaves, 9 steps):

$$\frac{85 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{8 a^{3/2} d} - \frac{15 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{2} a^{3/2} d} - \frac{A \cos [c+d x]^2 \sin [c+d x]}{d (a-a \operatorname{Sec}[c+d x])^{3/2}} +$$

$$\frac{35 A \sin [c+d x]}{8 a d \sqrt{a-a \operatorname{Sec}[c+d x]}} + \frac{25 A \cos [c+d x] \sin [c+d x]}{12 a d \sqrt{a-a \operatorname{Sec}[c+d x]}} + \frac{4 A \cos [c+d x]^2 \sin [c+d x]}{3 a d \sqrt{a-a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 314 leaves):

$$\frac{1}{8 (a-a \operatorname{Sec}[c+d x])^{3/2}} A \left(-1 / d 5 \sqrt{2} e^{-\frac{1}{2} i (c+d x)} \sqrt{\frac{e^{i (c+d x)}}{1+e^{2 i (c+d x)}}} \sqrt{1+e^{2 i (c+d x)}} \right.$$

$$\left. \left(-17 i d x + 17 \operatorname{ArcSinh}\left[e^{i (c+d x)}\right] + 24 \sqrt{2} \operatorname{Log}\left[1-e^{i (c+d x)}\right] + 17 \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+d x)}}\right] - 24 \sqrt{2} \operatorname{Log}\left[1+e^{i (c+d x)}+\sqrt{2} \sqrt{1+e^{2 i (c+d x)}}\right] \right) +$$

$$1 / (6 d) \left(-61 \cos \left[\frac{1}{2} (c+d x)\right] - 120 \cos \left[\frac{3}{2} (c+d x)\right] + 72 \cos \left[\frac{5}{2} (c+d x)\right] + 11 \cos \left[\frac{7}{2} (c+d x)\right] + 2 \cos \left[\frac{9}{2} (c+d x)\right] \right)$$

$$\left. \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2 \sqrt{\operatorname{Sec}[c+d x]} \right) \operatorname{Sec}[c+d x]^{3/2} \sin \left[\frac{1}{2} (c+d x)\right]^3$$

■ **Problem 175: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+A \operatorname{Sec}[c+d x]}{(a-a \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{2 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{a^{5/2} d} - \frac{23 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+d x]}}\right]}{8 \sqrt{2} a^{5/2} d} - \frac{A \operatorname{Tan}[c+d x]}{2 d (a-a \operatorname{Sec}[c+d x])^{5/2}} - \frac{7 A \operatorname{Tan}[c+d x]}{8 a d (a-a \operatorname{Sec}[c+d x])^{3/2}}$$

Result (type 3, 421 leaves):

$$\begin{aligned}
& A \left(\left(e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left(-16i dx + 16 \operatorname{ArcSinh}[e^{i(c+dx)}] + 23\sqrt{2} \operatorname{Log}[1-e^{i(c+dx)}] + 16 \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] \right) - \right. \\
& \quad \left. 23\sqrt{2} \operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Bigg) / \left(2\sqrt{2} d (a - a \operatorname{Sec}[c+dx])^{5/2} \right) + \\
& \quad \frac{1}{(a - a \operatorname{Sec}[c+dx])^{5/2}} \operatorname{Sec}[c+dx]^3 \left(-\frac{11 \operatorname{Cos}\left[\frac{c}{2}\right] \operatorname{Cos}\left[\frac{dx}{2}\right]}{d} + \frac{15 \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]}{2d} - \frac{\operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^3}{d} - \right. \\
& \quad \left. \frac{15 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sin}\left[\frac{dx}{2}\right]}{2d} + \frac{\operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} + \frac{11 \operatorname{Sin}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right]}{d} \right) \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \Bigg)
\end{aligned}$$

■ **Problem 176: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+dx] (A + A \operatorname{Sec}[c+dx])}{(a - a \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 184 leaves, 8 steps):

$$\frac{7 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} - \frac{79 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{8 \sqrt{2} a^{5/2} d} - \frac{A \operatorname{Sin}[c+dx]}{2 d (a - a \operatorname{Sec}[c+dx])^{5/2}} - \frac{11 A \operatorname{Sin}[c+dx]}{8 a d (a - a \operatorname{Sec}[c+dx])^{3/2}} + \frac{23 A \operatorname{Sin}[c+dx]}{8 a^2 d \sqrt{a - a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 298 leaves):

$$\begin{aligned}
& \frac{1}{4 d (a - a \operatorname{Sec}[c+dx])^{5/2}} A \left(\sqrt{2} e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \right. \\
& \quad \left. \left(-56i dx + 56 \operatorname{ArcSinh}[e^{i(c+dx)}] + 79\sqrt{2} \operatorname{Log}[1-e^{i(c+dx)}] + 56 \operatorname{Log}[1+\sqrt{1+e^{2i(c+dx)}}] - 79\sqrt{2} \operatorname{Log}[1+e^{i(c+dx)} + \sqrt{2}\sqrt{1+e^{2i(c+dx)}}] \right) \right) + \\
& \quad \frac{1}{4} \left(-12 \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + 23 \operatorname{Cos}\left[\frac{3}{2}(c+dx)\right] - 31 \operatorname{Cos}\left[\frac{5}{2}(c+dx)\right] + 4 \operatorname{Cos}\left[\frac{7}{2}(c+dx)\right] \right) \operatorname{Csc}\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{\operatorname{Sec}[c+dx]} \Bigg) \\
& \quad \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]^5
\end{aligned}$$

■ **Problem 177: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 (A + A \operatorname{Sec}[c+dx])}{(a - a \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 236 leaves, 9 steps):

$$\frac{59 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{4 a^{5/2} d} - \frac{167 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{8 \sqrt{2} a^{5/2} d} - \frac{A \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2 d (a-a \operatorname{Sec}[c+dx])^{5/2}} -$$

$$\frac{15 A \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{8 a d (a-a \operatorname{Sec}[c+dx])^{3/2}} + \frac{49 A \operatorname{Sin}[c+dx]}{8 a^2 d \sqrt{a-a \operatorname{Sec}[c+dx]}} + \frac{23 A \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{8 a^2 d \sqrt{a-a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 308 leaves):

$$\frac{1}{4 d (a-a \operatorname{Sec}[c+dx])^{5/2}}$$

$$A \left(\sqrt{2} e^{-\frac{1}{2} i (c+dx)} \sqrt{\frac{e^{i (c+dx)}}{1+e^{2 i (c+dx)}}} \sqrt{1+e^{2 i (c+dx)}} \left(-118 i dx + 118 \operatorname{ArcSinh}\left[e^{i (c+dx)}\right] + 167 \sqrt{2} \operatorname{Log}\left[1-e^{i (c+dx)}\right] + 118 \operatorname{Log}\left[1+\sqrt{1+e^{2 i (c+dx)}}\right] \right) - \right.$$

$$\left. 167 \sqrt{2} \operatorname{Log}\left[1+e^{i (c+dx)}+\sqrt{2} \sqrt{1+e^{2 i (c+dx)}}\right] \right) +$$

$$\frac{1}{4} \left(-19 \operatorname{Cos}\left[\frac{1}{2} (c+dx)\right] + 54 \operatorname{Cos}\left[\frac{3}{2} (c+dx)\right] - 62 \operatorname{Cos}\left[\frac{5}{2} (c+dx)\right] + 10 \operatorname{Cos}\left[\frac{7}{2} (c+dx)\right] + \operatorname{Cos}\left[\frac{9}{2} (c+dx)\right] \right)$$

$$\left. \operatorname{Csc}\left[\frac{1}{2} (c+dx)\right]^4 \sqrt{\operatorname{Sec}[c+dx]} \right) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}\left[\frac{1}{2} (c+dx)\right]^5$$

■ **Problem 178: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c+dx]^3 (A+A \operatorname{Sec}[c+dx])}{(a-a \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 280 leaves, 10 steps):

$$\frac{203 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{8 a^{5/2} d} - \frac{287 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a-a \operatorname{Sec}[c+dx]}}\right]}{8 \sqrt{2} a^{5/2} d} - \frac{A \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{2 d (a-a \operatorname{Sec}[c+dx])^{5/2}} -$$

$$\frac{19 A \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{8 a d (a-a \operatorname{Sec}[c+dx])^{3/2}} + \frac{21 A \operatorname{Sin}[c+dx]}{2 a^2 d \sqrt{a-a \operatorname{Sec}[c+dx]}} + \frac{119 A \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{24 a^2 d \sqrt{a-a \operatorname{Sec}[c+dx]}} + \frac{77 A \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{24 a^2 d \sqrt{a-a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 323 leaves):

$$\frac{1}{12 d (a - a \operatorname{Sec}[c + d x])^{5/2}} A \left(21 \sqrt{2} e^{-\frac{1}{2} i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \sqrt{1 + e^{2 i (c + d x)}} \right. \\ \left. \left(-29 i d x + 29 \operatorname{ArcSinh}\left[e^{i (c + d x)}\right] + 41 \sqrt{2} \operatorname{Log}\left[1 - e^{i (c + d x)}\right] + 29 \operatorname{Log}\left[1 + \sqrt{1 + e^{2 i (c + d x)}}\right] - 41 \sqrt{2} \operatorname{Log}\left[1 + e^{i (c + d x)} + \sqrt{2} \sqrt{1 + e^{2 i (c + d x)}}\right] \right) + \right. \\ \left. \frac{1}{8} \left(-173 \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + 575 \operatorname{Cos}\left[\frac{3}{2} (c + d x)\right] - 625 \operatorname{Cos}\left[\frac{5}{2} (c + d x)\right] + 112 \operatorname{Cos}\left[\frac{7}{2} (c + d x)\right] + 13 \operatorname{Cos}\left[\frac{9}{2} (c + d x)\right] + 2 \operatorname{Cos}\left[\frac{11}{2} (c + d x)\right] \right) \right. \\ \left. \operatorname{Csc}\left[\frac{1}{2} (c + d x)\right]^4 \sqrt{\operatorname{Sec}[c + d x]} \right) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]^5$$

- **Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 234 leaves, 9 steps):

$$\frac{4 a^2 (4 A + 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ \frac{4 a^2 (7 A + 6 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{4 a^2 (4 A + 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{5 d} + \\ \frac{4 a^2 (7 A + 6 B) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{21 d} + \frac{2 a^2 (7 A + 9 B) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{35 d} + \frac{2 B \operatorname{Sec}[c + d x]^{5/2} (a^2 + a^2 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{7 d}$$

Result (type 5, 731 leaves):

$$\begin{aligned}
& - \frac{1}{5 d (B + A \cos [c + d x])} \\
& 2 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos [c + d x]^3 \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) - \\
& \left(3 B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos [c + d x]^3 \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) \right) / \left(5 \sqrt{2} d (B + A \cos [c + d x]) \right) + \\
& \frac{A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{3 d (B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2}} + \\
& \frac{2 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{7 d (B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2}} + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) \right. \\
& \left. \left(\frac{(4 A + 3 B) \cos [d x] \operatorname{Csc}[c]}{5 d} + \frac{B \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \sin [d x]}{14 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (5 B \sin [c] + 7 A \sin [d x] + 14 B \sin [d x])}{70 d} \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (21 A \sin [c] + 42 B \sin [c] + 70 A \sin [d x] + 60 B \sin [d x])}{210 d} + \frac{(7 A + 6 B) \tan [c]}{21 d} \right) \right) / \left((B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2} \right)
\end{aligned}$$

- **Problem 187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 199 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 a^2 (5 A + 4 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \frac{4 a^2 (2 A + B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \\
& \frac{4 a^2 (5 A + 4 B) \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x]}{5 d} + \frac{2 a^2 (5 A + 7 B) \operatorname{Sec}[c + d x]^{3/2} \sin [c + d x]}{15 d} + \frac{2 B \operatorname{Sec}[c + d x]^{3/2} (a^2 + a^2 \operatorname{Sec}[c + d x]) \sin [c + d x]}{5 d}
\end{aligned}$$

Result (type 5, 685 leaves):

$$\begin{aligned}
& - \frac{1}{\sqrt{2} d (B + A \cos [c + d x])} \\
& A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos [c + d x]^3 \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) - \frac{1}{5 d (B + A \cos [c + d x])} \\
& 2 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos [c + d x]^3 \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) + \\
& \frac{2 A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{3 d (B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2}} + \\
& \frac{B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{3 d (B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2}} + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) \left(\frac{(5 A + 4 B) \cos [d x] \operatorname{Csc}[c]}{5 d} + \frac{B \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \sin [d x]}{10 d} + \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (3 B \sin [c] + 5 A \sin [d x] + 10 B \sin [d x])}{30 d} + \frac{(A + 2 B) \tan [c]}{6 d} \right) \right) / ((B + A \cos [c + d x]) \operatorname{Sec}[c + d x]^{5/2})
\end{aligned}$$

■ **Problem 188: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 160 leaves, 7 steps):

$$\begin{aligned}
& - \frac{4 a^2 B \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{4 a^2 (3 A + 2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \\
& \frac{2 a^2 (3 A + 5 B) \sqrt{\operatorname{Sec}[c + d x]} \sin [c + d x]}{3 d} + \frac{2 B \sqrt{\operatorname{Sec}[c + d x]} (a^2 + a^2 \operatorname{Sec}[c + d x]) \sin [c + d x]}{3 d}
\end{aligned}$$

Result (type 5, 313 leaves):

$$\frac{1}{12 d (B + A \cos [c + d x])} a^2 \sec \left[\frac{1}{2} (c + d x) \right]^4 (1 + \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(-\frac{1}{-1 + e^{2 i c}} 4 i \sqrt{2} e^{-i (c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \cos [c + d x]^3 \left(3 B (1 + e^{2 i (c + d x)}) + 3 B (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[\right. \right. \right.$$

$$\left. \left. \left. -\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] + (3 A + 2 B) e^{i (c + d x)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right] \right) + \right.$$

$$\left. \frac{-3 (-A - 4 B + A \cos [2 c]) \cos [d x] \csc [c] + 6 A \cos [c] \sin [d x] + 2 B \tan [c + d x]}{\sec [c + d x]^{5/2}} \right)$$

■ **Problem 189: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c + d x])^2 (A + B \sec [c + d x])}{\sec [c + d x]^{3/2}} dx$$

Optimal (type 4, 158 leaves, 7 steps):

$$\frac{4 a^2 A \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]} + 4 a^2 (2 A + 3 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{d} -$$

$$\frac{2 a^2 (A - 3 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{3 d} + \frac{2 A (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 183 leaves):

$$\frac{1}{3 d \sqrt{\sec [c + d x]}} a^2 \left(\cos \left[\frac{c}{2} \right] - i \sin \left[\frac{c}{2} \right] \right) \left(-i \cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(12 A - \frac{24 A \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right]}{\sqrt{1 + e^{2 i (c + d x)}}} + \right.$$

$$\left. \frac{8 (2 A + 3 B) e^{i (c + d x)} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i (c + d x)} \right]}{\sqrt{1 + e^{2 i (c + d x)}}} + 2 i A \sin [c + d x] + 6 i B \tan [c + d x] \right)$$

■ **Problem 190: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c + d x])^2 (A + B \sec [c + d x])}{\sec [c + d x]^{5/2}} dx$$

Optimal (type 4, 166 leaves, 7 steps):

$$\frac{4 a^2 (4 A + 5 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} +$$

$$\frac{4 a^2 (A + 2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a^2 (7 A + 5 B) \sin [c + d x]}{15 d \sqrt{\sec [c + d x]}} + \frac{2 A (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{5 d \sec [c + d x]^{3/2}}$$

Result (type 5, 155 leaves):

$$\frac{1}{30 d} a^2 \sqrt{\sec [c + d x]} \left(40 (A + 2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 24 i (4 A + 5 B) e^{-i(c+d x)} \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + \right.$$

$$\left. 2 \cos [c + d x] (-12 i (4 A + 5 B) + 10 (2 A + B) \sin [c + d x] + 3 A \sin [2(c + d x)]) \right)$$

■ **Problem 191: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c + d x])^2 (A + B \sec [c + d x])}{\sec [c + d x]^{7/2}} dx$$

Optimal (type 4, 201 leaves, 8 steps):

$$\frac{4 a^2 (3 A + 4 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{4 a^2 (6 A + 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} +$$

$$\frac{2 a^2 (9 A + 7 B) \sin [c + d x]}{35 d \sec [c + d x]^{3/2}} + \frac{4 a^2 (6 A + 7 B) \sin [c + d x]}{21 d \sqrt{\sec [c + d x]}} + \frac{2 A (a^2 + a^2 \sec [c + d x]) \sin [c + d x]}{7 d \sec [c + d x]^{5/2}}$$

Result (type 5, 207 leaves):

$$\frac{1}{420 d} a^2 e^{-i(2c+d x)} \sqrt{\sec [c + d x]} \left(80 (6 A + 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right.$$

$$\left. 336 i (3 A + 4 B) e^{-i(c+d x)} \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 2 \cos [c + d x] \right.$$

$$\left. (-504 i A - 672 i B + 5 (51 A + 56 B) \sin [c + d x] + 42 (2 A + B) \sin [2(c + d x)] + 15 A \sin [3(c + d x)]) \right) (\cos [2 c + d x] + i \sin [2 c + d x])$$

■ **Problem 192: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c + d x])^2 (A + B \sec [c + d x])}{\sec [c + d x]^{9/2}} dx$$

Optimal (type 4, 234 leaves, 9 steps):

$$\frac{4 a^2 (8 A+9 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} + \frac{4 a^2 (5 A+6 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} +$$

$$\frac{2 a^2 (11 A+9 B) \sin [c+d x]}{63 d \sec [c+d x]^{5 / 2}} + \frac{4 a^2 (8 A+9 B) \sin [c+d x]}{45 d \sec [c+d x]^{3 / 2}} + \frac{4 a^2 (5 A+6 B) \sin [c+d x]}{21 d \sqrt{\sec [c+d x]}} + \frac{2 A\left(a^2+a^2 \sec [c+d x]\right) \sin [c+d x]}{9 d \sec [c+d x]^{7 / 2}}$$

Result (type 5, 231 leaves):

$$\frac{1}{2520 d} a^2 e^{-i(2 c+d x)} \sqrt{\sec [c+d x]} \left(480 (5 A+6 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right.$$

$$672 i (8 A+9 B) e^{-i(c+d x)} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] +$$

$$2 \cos [c+d x] (-2688 i A-3024 i B+30(46 A+51 B) \sin [c+d x]+14(37 A+36 B) \sin [2(c+d x)] +$$

$$180 A \sin [3(c+d x)]+90 B \sin [3(c+d x)]+35 A \sin [4(c+d x)]) \left. \right) (\cos [2 c+d x]+i \sin [2 c+d x])$$

■ **Problem 193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^{3 / 2} (a+a \sec [c+d x])^3 (A+B \sec [c+d x]) dx$$

Optimal (type 4, 277 leaves, 10 steps):

$$\frac{4 a^3 (21 A+17 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} +$$

$$\frac{4 a^3 (13 A+11 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \frac{4 a^3 (21 A+17 B) \sqrt{\sec [c+d x]} \sin [c+d x]}{15 d} +$$

$$\frac{4 a^3 (13 A+11 B) \sec [c+d x]^{3 / 2} \sin [c+d x]}{21 d} + \frac{4 a^3 (24 A+23 B) \sec [c+d x]^{5 / 2} \sin [c+d x]}{105 d} +$$

$$\frac{2 a B \sec [c+d x]^{5 / 2} (a+a \sec [c+d x])^2 \sin [c+d x]}{9 d} + \frac{2 (9 A+13 B) \sec [c+d x]^{5 / 2} \left(a^3+a^3 \sec [c+d x]\right) \sin [c+d x]}{63 d}$$

Result (type 5, 773 leaves):

$$\begin{aligned}
& - \left(7 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \csc[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx]) \right) / \left(10 \sqrt{2} d (B + A \cos[c+dx]) \right) - \\
& \left(17 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \csc[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx]) \right) / \left(30 \sqrt{2} d (B + A \cos[c+dx]) \right) + \\
& \frac{13 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx])}{42 d (B + A \cos[c+dx]) \sec[c+dx]^{7/2}} + \\
& \frac{11 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx])}{42 d (B + A \cos[c+dx]) \sec[c+dx]^{7/2}} + \\
& \left(\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx]) \right. \\
& \quad \left(\frac{(21 A + 17 B) \cos[dx] \csc[c]}{30 d} + \frac{B \sec[c] \sec[c+dx]^4 \sin[dx]}{36 d} + \frac{\sec[c] \sec[c+dx]^3 (7 B \sin[c] + 9 A \sin[dx] + 27 B \sin[dx])}{252 d} \right. \\
& \quad \frac{\sec[c] \sec[c+dx]^2 (45 A \sin[c] + 135 B \sin[c] + 189 A \sin[dx] + 238 B \sin[dx])}{1260 d} + \\
& \quad \frac{\sec[c] \sec[c+dx] (189 A \sin[c] + 238 B \sin[c] + 390 A \sin[dx] + 330 B \sin[dx])}{1260 d} + \\
& \quad \left. \left. \frac{(13 A + 11 B) \tan[c]}{42 d} \right) \right) / \left((B + A \cos[c+dx]) \sec[c+dx]^{7/2} \right)
\end{aligned}$$

■ **Problem 194: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^3 (A + B \sec[c+dx]) dx$$

Optimal (type 4, 244 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^3 (9 A + 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \\
& \frac{4 a^3 (21 A + 13 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \\
& \frac{4 a^3 (9 A + 7 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \frac{4 a^3 (42 A + 41 B) \sec [c + d x]^{3/2} \sin [c + d x]}{105 d} + \\
& \frac{2 a B \sec [c + d x]^{3/2} (a + a \sec [c + d x])^2 \sin [c + d x]}{7 d} + \frac{2 (7 A + 11 B) \sec [c + d x]^{3/2} (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{35 d}
\end{aligned}$$

Result (type 5, 731 leaves):

$$\begin{aligned}
& - \left(9 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^4 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \right) / \left(10 \sqrt{2} d (B + A \cos [c + d x]) \right) - \\
& \left(7 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^4 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \right) / \left(10 \sqrt{2} d (B + A \cos [c + d x]) \right) + \\
& \frac{A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x])}{2 d (B + A \cos [c + d x]) \sec [c + d x]^{7/2}} + \\
& \frac{13 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x])}{42 d (B + A \cos [c + d x]) \sec [c + d x]^{7/2}} + \\
& \left(\sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \right. \\
& \quad \left(\frac{(9 A + 7 B) \cos [d x] \operatorname{Csc}[c]}{10 d} + \frac{B \sec [c] \sec [c + d x]^3 \sin [d x]}{28 d} + \frac{\sec [c] \sec [c + d x]^2 (5 B \sin [c] + 7 A \sin [d x] + 21 B \sin [d x])}{140 d} \right. \\
& \quad \left. \frac{\sec [c] \sec [c + d x] (21 A \sin [c] + 63 B \sin [c] + 105 A \sin [d x] + 130 B \sin [d x])}{420 d} \right. \\
& \quad \left. \left. \frac{(21 A + 26 B) \tan [c]}{84 d} \right) \right) / \left((B + A \cos [c + d x]) \sec [c + d x]^{7/2} \right)
\end{aligned}$$

■ **Problem 195: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$\begin{aligned} & - \frac{4 a^3 (5 A + 9 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (5 A + 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{4 a^3 (20 A + 21 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \\ & \frac{2 a B \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{5 d} + \frac{2 (5 A + 9 B) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 257 leaves):

$$\begin{aligned} & \frac{1}{30 d} \\ & a^3 e^{-i(2c+dx)} \operatorname{Sec}[c + d x]^{5/2} \left(90 i A \operatorname{Cos}[c + d x] + 162 i B \operatorname{Cos}[c + d x] + 30 i A \operatorname{Cos}[3(c + d x)] + 54 i B \operatorname{Cos}[3(c + d x)] + 40 (5 A + 3 B) \operatorname{Cos}[c + d x]^{5/2} \right. \\ & \left. \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] - 6 i (5 A + 9 B) e^{-3 i(c+dx)} (1 + e^{2 i(c+dx)})^{5/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] + 45 A \operatorname{Sin}[c + d x] + \right. \\ & \left. 66 B \operatorname{Sin}[c + d x] + 10 A \operatorname{Sin}[2(c + d x)] + 30 B \operatorname{Sin}[2(c + d x)] + 45 A \operatorname{Sin}[3(c + d x)] + 54 B \operatorname{Sin}[3(c + d x)] \right) (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx]) \end{aligned}$$

■ **Problem 196: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 199 leaves, 8 steps):

$$\begin{aligned} & \frac{4 a^3 (A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{20 a^3 (A + B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \\ & \frac{4 a^3 (A + 4 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d} + \frac{2 a A (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{2 (A - B) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{3 d} \end{aligned}$$

Result (type 5, 226 leaves):

$$\frac{1}{6d}$$

$$a^3 e^{-i(2c+dx)} \operatorname{Sec}[c+dx]^{3/2} \left(-12iA + 12iB - 12iA \cos[2(c+dx)] + 12iB \cos[2(c+dx)] + 40(A+B) \cos[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 12i(A-B) e^{-2i(c+dx)} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + A \sin[c+dx] + \right. \\ \left. 4B \sin[c+dx] + 6A \sin[2(c+dx)] + 18B \sin[2(c+dx)] + A \sin[3(c+dx)] \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 197: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx])}{\operatorname{Sec}[c+dx]^{5/2}} dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$\frac{4a^3(9A+5B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{4a^3(3A+5B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{3d} - \\ \frac{4a^3(6A-5B)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{15d} + \frac{2aA(a+a\operatorname{Sec}[c+dx])^2\sin[c+dx]}{5d\operatorname{Sec}[c+dx]^{3/2}} + \frac{2(9A+5B)(a^3+a^3\operatorname{Sec}[c+dx])\sin[c+dx]}{15d\sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 5, 220 leaves):

$$\frac{1}{30d} a^3 e^{-i(2c+dx)} \sqrt{\operatorname{Sec}[c+dx]} \left(-216iA \cos[c+dx] - 120iB \cos[c+dx] + 40(3A+5B) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 24i(9A+5B) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 3A \sin[c+dx] + \right. \\ \left. 60B \sin[c+dx] + 30A \sin[2(c+dx)] + 10B \sin[2(c+dx)] + 3A \sin[3(c+dx)] \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 198: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx])}{\operatorname{Sec}[c+dx]^{7/2}} dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$\frac{4a^3(7A+9B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{5d} + \\ \frac{4a^3(13A+21B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{21d} + \frac{4a^3(41A+42B)\sin[c+dx]}{105d\sqrt{\operatorname{Sec}[c+dx]}} + \\ \frac{2aA(a+a\operatorname{Sec}[c+dx])^2\sin[c+dx]}{7d\operatorname{Sec}[c+dx]^{5/2}} + \frac{2(11A+7B)(a^3+a^3\operatorname{Sec}[c+dx])\sin[c+dx]}{35d\operatorname{Sec}[c+dx]^{3/2}}$$

Result (type 5, 208 leaves):

$$\frac{1}{420 d} a^3 e^{-i(2c+dx)} \sqrt{\text{Sec}[c+dx]} \left(80 (13 A + 21 B) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 336 i (7 A + 9 B) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 \text{Cos}[c+dx] \right. \\ \left. (-168 i (7 A + 9 B) + 5 (107 A + 84 B) \text{Sin}[c+dx] + 42 (3 A + B) \text{Sin}[2(c+dx)] + 15 A \text{Sin}[3(c+dx)]) \right) (\text{Cos}[2c+dx] + i \text{Sin}[2c+dx])$$

■ **Problem 199: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \text{Sec}[c+dx])^3 (A + B \text{Sec}[c+dx])}{\text{Sec}[c+dx]^{9/2}} dx$$

Optimal (type 4, 244 leaves, 9 steps):

$$\frac{4 a^3 (17 A + 21 B) \sqrt{\text{Cos}[c+dx]} \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{15 d} + \\ \frac{4 a^3 (11 A + 13 B) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{21 d} + \frac{4 a^3 (23 A + 24 B) \text{Sin}[c+dx]}{105 d \text{Sec}[c+dx]^{3/2}} + \\ \frac{4 a^3 (11 A + 13 B) \text{Sin}[c+dx]}{21 d \sqrt{\text{Sec}[c+dx]}} + \frac{2 a A (a + a \text{Sec}[c+dx])^2 \text{Sin}[c+dx]}{9 d \text{Sec}[c+dx]^{7/2}} + \frac{2 (13 A + 9 B) (a^3 + a^3 \text{Sec}[c+dx]) \text{Sin}[c+dx]}{63 d \text{Sec}[c+dx]^{5/2}}$$

Result (type 5, 197 leaves):

$$\frac{1}{2520 d} a^3 \sqrt{\text{Sec}[c+dx]} \left(480 (11 A + 13 B) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 672 i (17 A + 21 B) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 \text{Cos}[c+dx] (-5712 i A - 7056 i B + \right. \\ \left. 30 (97 A + 107 B) \text{Sin}[c+dx] + 14 (73 A + 54 B) \text{Sin}[2(c+dx)] + 270 A \text{Sin}[3(c+dx)] + 90 B \text{Sin}[3(c+dx)] + 35 A \text{Sin}[4(c+dx)]) \right)$$

■ **Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c+dx])^3 (A + B \text{Sec}[c+dx])}{\text{Sec}[c+dx]^{11/2}} dx$$

Optimal (type 4, 277 leaves, 10 steps):

$$\frac{4 a^3 (15 A + 17 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{15 d} +$$

$$\frac{4 a^3 (105 A + 121 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{20 a^3 (21 A + 22 B) \sin [c + d x]}{693 d \sec [c + d x]^{5/2}} + \frac{4 a^3 (15 A + 17 B) \sin [c + d x]}{45 d \sec [c + d x]^{3/2}} +$$

$$\frac{4 a^3 (105 A + 121 B) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}} + \frac{2 a A (a + a \sec [c + d x])^2 \sin [c + d x]}{11 d \sec [c + d x]^{9/2}} + \frac{2 (15 A + 11 B) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{99 d \sec [c + d x]^{7/2}}$$

Result (type 5, 864 leaves):

$$\left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^4 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \right) / \left(2 \sqrt{2} d (B + A \cos [c + d x]) \right) +$$

$$\left(17 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^4 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \right) / \left(30 \sqrt{2} d (B + A \cos [c + d x]) \right) +$$

$$\frac{5 A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x])}{22 d (B + A \cos [c + d x]) \sec [c + d x]^{7/2}} +$$

$$\frac{11 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x])}{42 d (B + A \cos [c + d x]) \sec [c + d x]^{7/2}} +$$

$$\frac{1}{(B + A \cos [c + d x]) \sec [c + d x]^{7/2}}$$

$$\sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x]) \left(-\frac{(645 A + 743 B + 795 A \cos [2 c] + 889 B \cos [2 c]) \cos [d x] \operatorname{Csc}[c]}{2880 d} + \right.$$

$$\frac{(4473 A + 4664 B) \cos [2 d x] \sin [2 c]}{29568 d} + \frac{(165 A + 151 B) \cos [3 d x] \sin [3 c]}{2880 d} + \frac{(49 A + 33 B) \cos [4 d x] \sin [4 c]}{2464 d} +$$

$$\frac{(3 A + B) \cos [5 d x] \sin [5 c]}{576 d} + \frac{A \cos [6 d x] \sin [6 c]}{1408 d} + \frac{(795 A + 889 B) \cos [c] \sin [d x]}{1440 d} + \frac{(4473 A + 4664 B) \cos [2 c] \sin [2 d x]}{29568 d} +$$

$$\left. \frac{(165 A + 151 B) \cos [3 c] \sin [3 d x]}{2880 d} + \frac{(49 A + 33 B) \cos [4 c] \sin [4 d x]}{2464 d} + \frac{(3 A + B) \cos [5 c] \sin [5 d x]}{576 d} + \frac{A \cos [6 c] \sin [6 d x]}{1408 d} \right)$$

- **Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{7/2} (A + B \text{Sec}[c + d x])}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 229 leaves, 9 steps):

$$\begin{aligned} & \frac{3 (5 A - 7 B) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{5 a d} + \\ & \frac{5 (A - B) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{3 a d} - \frac{3 (5 A - 7 B) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{5 a d} + \\ & \frac{5 (A - B) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 a d} - \frac{(5 A - 7 B) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{5 a d} + \frac{(A - B) \text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 794 leaves):

$$\begin{aligned}
& \left(3 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(\sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right) - \\
& \left(21 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(5 \sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right) + \\
& \left(5 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& (3 d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) - \\
& \left(5 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& (3 d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \left(\frac{3(-5A + 7B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \right. \right. \\
& \left. \frac{(-A + B) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] (-\sin\left[\frac{c}{2}\right] + 5 \sin\left[\frac{3c}{2}\right])}{3d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{d} + \frac{4 B \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \sin[dx]}{5d} + \right. \\
& \left. \left. \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (3 B \sin[c] + 5 A \sin[dx] - 5 B \sin[dx])}{15d} \right) \right) / \left((B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right)
\end{aligned}$$

■ **Problem 202: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx])}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 4, 192 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3(A - B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{ad} - \frac{(3A - 5B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3ad} + \\
& \frac{3(A - B) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{ad} - \frac{(3A - 5B) \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{3ad} + \frac{(A - B) \operatorname{Sec}[c + dx]^{5/2} \sin[c + dx]}{d(a + a \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 371 leaves) :

$$\begin{aligned}
 & - \frac{1}{3 a d \left(1 + e^{2 i (c+d x)}\right) (B + A \operatorname{Cos}[c + d x]) (1 + \operatorname{Sec}[c + d x])} \\
 & e^{-\frac{3}{2} i (c+d x)} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \left((3 A - 5 B) e^{i (c+d x)} \left(1 + e^{i (c+d x)} + e^{2 i (c+d x)} + e^{3 i (c+d x)}\right) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] - \right. \\
 & \quad \left. i \left(9 A - 9 B + 6 A e^{i (c+d x)} - 4 B e^{i (c+d x)} + 12 A e^{2 i (c+d x)} - 10 B e^{2 i (c+d x)} + 6 A e^{3 i (c+d x)} - 8 B e^{3 i (c+d x)} + 3 A e^{4 i (c+d x)} - 5 B e^{4 i (c+d x)} - 9 (A - B) \right. \right. \\
 & \quad \left. \left. \sqrt{1 + e^{2 i (c+d x)}} \left(1 + e^{i (c+d x)} + e^{2 i (c+d x)} + e^{3 i (c+d x)}\right) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right]\right) \right) \sqrt{\operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x])
 \end{aligned}$$

- **Problem 203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x])}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 153 leaves, 7 steps) :

$$\begin{aligned}
 & \frac{(A - 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} + \\
 & \frac{(A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} - \frac{(A - 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{a d} + \frac{(A - B) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}
 \end{aligned}$$

Result (type 5, 705 leaves) :

$$\begin{aligned}
& \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(\sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right) - \\
& \left(3 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(\sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right) + \\
& \left(A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& (d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) - \\
& \left(B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& (d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \left(\frac{(-A + 3B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \right. \right. \\
& \left. \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{d} - \frac{2(-A + B) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \left((B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right)
\end{aligned}$$

■ **Problem 204: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx])}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 4, 123 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(A - B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a d} + \\
& \frac{(A + B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a d} + \frac{(A - B) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{d (a + a \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 207 leaves):

$$\left(\cos\left[\frac{1}{2}(c+dx)\right]^2 \left(\frac{4(A+B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{d} + 1/\left(d(1+e^{i(c+dx)})\right) 4i(A-B) \right. \right. \\ \left. \left. e^{-i(c+dx)} \left(1+e^{2i(c+dx)} - (1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \sqrt{\sec[c+dx]} \right) \right. \\ \left. (A+B\sec[c+dx]) \right) / \left(2a(B+A\cos[c+dx])(1+\sec[c+dx]) \right)$$

■ **Problem 205: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\sec[c+dx]}{\sqrt{\sec[c+dx]}(a+a\sec[c+dx])} dx$$

Optimal (type 4, 128 leaves, 6 steps):

$$\frac{(3A-B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} - \\ \frac{(A-B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} - \frac{(A-B)\sqrt{\sec[c+dx]}\sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 425 leaves):

$$\frac{1}{2ad(B+A\cos[c+dx])(1+\sec[c+dx])} \cos\left[\frac{1}{2}(c+dx)\right]^2 \\ \left(6\sqrt{2}Ae^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) - \right. \\ \left. 2\sqrt{2}Be^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) - \right. \\ \left. \frac{2((2A-B)\cos\left[\frac{1}{2}(c-dx)\right] + A\cos\left[\frac{1}{2}(3c+dx)\right]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{\sec[c+dx]}} \right) \\ \left. 4A\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]} + 4B\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]} \right) (A+B\sec[c+dx])$$

- **Problem 206: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 164 leaves, 7 steps):

$$\begin{aligned} & - \frac{3 (A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} + \\ & \frac{(5 A - 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a d} + \frac{(5 A - 3 B) \operatorname{Sin}[c + d x]}{3 a d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{(A - B) \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 479 leaves):

$$\begin{aligned} & \frac{1}{6 a d (B + A \operatorname{Cos}[c + d x]) (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right]^2 (A + B \operatorname{Sec}[c + d x]) \\ & \left(-18 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) + \right. \\ & 18 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) + \\ & 20 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - 12 B \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} + \\ & 2 \sqrt{\operatorname{Sec}[c + d x]} \left(3 (A - B) (2 + \operatorname{Cos}[2 c]) \operatorname{Cos}[d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] + 2 A \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c] - \right. \\ & \left. 6 (A - B) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \operatorname{Sin}\left[\frac{d x}{2}\right] - 12 (A - B) \operatorname{Cos}[c] \operatorname{Sin}[d x] + 2 A \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x] - 6 (A - B) \operatorname{Tan}\left[\frac{c}{2}\right] \right) \end{aligned}$$

- **Problem 207: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Sec}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 197 leaves, 8 steps):

$$\begin{aligned} & \frac{3 (7 A - 5 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 a d} - \frac{5 (A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 a d} + \\ & \frac{(7 A - 5 B) \operatorname{Sin}[c + d x]}{5 a d \operatorname{Sec}[c + d x]^{3/2}} - \frac{5 (A - B) \operatorname{Sin}[c + d x]}{3 a d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{(A - B) \operatorname{Sin}[c + d x]}{d \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 520 leaves) :

$$\frac{1}{60 a d (B + A \cos [c + d x]) (1 + \sec [c + d x])} \cos \left[\frac{1}{2} (c + d x) \right]^2 (A + B \sec [c + d x])$$

$$\left(252 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) - \right.$$

$$180 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \operatorname{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)} \right] \right) -$$

$$200 A \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]} + 200 B \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]} +$$

$$\sqrt{\sec [c + d x]} \left(-3 (51 A - 40 B + (33 A - 20 B) \cos [2 c]) \cos [d x] \operatorname{Csc} \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] - \right.$$

$$40 (A - B) \cos [2 d x] \sin [2 c] + 12 A \cos [3 d x] \sin [3 c] + 120 (A - B) \sec \left[\frac{c}{2} \right] \sec \left[\frac{1}{2} (c + d x) \right] \sin \left[\frac{d x}{2} \right] +$$

$$\left. 12 (33 A - 20 B) \cos [c] \sin [d x] - 40 (A - B) \cos [2 c] \sin [2 d x] + 12 A \cos [3 c] \sin [3 d x] + 120 (A - B) \tan \left[\frac{c}{2} \right] \right)$$

■ **Problem 208: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x]}{\sec [c + d x]^{7/2} (a + a \sec [c + d x])} dx$$

Optimal (type 4, 230 leaves, 9 steps) :

$$\frac{21 (A - B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]} - 5 (9 A - 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{5 a d} + \frac{21 a d}{7 a d \sec [c + d x]^{5/2}} - \frac{7 (A - B) \sin [c + d x]}{5 a d \sec [c + d x]^{3/2}} + \frac{5 (9 A - 7 B) \sin [c + d x]}{21 a d \sqrt{\sec [c + d x]}} - \frac{(A - B) \sin [c + d x]}{d \sec [c + d x]^{5/2} (a + a \sec [c + d x])}$$

Result (type 5, 864 leaves) :

$$\begin{aligned}
& - \left(21 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(5\sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])\right) + \\
& \left(21 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \right) / \left(5\sqrt{2} d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])\right) + \\
& \left(15 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& \quad (7 d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) - \\
& \left(5 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& \quad (3 d (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])) + \\
& \frac{1}{(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) \left(-\frac{3(-A + B)(17 + 11 \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{20 d} - \right. \\
& \quad \frac{(-27A + 14B) \cos[2dx] \sin[2c]}{21 d} + \frac{(-A + B) \cos[3dx] \sin[3c]}{5 d} + \frac{A \cos[4dx] \sin[4c]}{14 d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{d} \Big) + \\
& \quad \left. \frac{33(-A + B) \cos[c] \sin[dx]}{5 d} - \frac{(-27A + 14B) \cos[2c] \sin[2dx]}{21 d} + \frac{(-A + B) \cos[3c] \sin[3dx]}{5 d} + \frac{A \cos[4c] \sin[4dx]}{14 d} + \frac{2(-A + B) \tan\left[\frac{c}{2}\right]}{d} \right)
\end{aligned}$$

■ **Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{7/2} (A + B \operatorname{Sec}[c + dx])}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(4A - 7B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} - \\
& \frac{5(A - 2B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} + \frac{(4A - 7B) \sqrt{\sec[c + dx]} \sin[c + dx]}{a^2 d} - \\
& \frac{5(A - 2B) \sec[c + dx]^{3/2} \sin[c + dx]}{3a^2 d} + \frac{(4A - 7B) \sec[c + dx]^{5/2} \sin[c + dx]}{3a^2 d (1 + \sec[c + dx])} + \frac{(A - B) \sec[c + dx]^{7/2} \sin[c + dx]}{3d (a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 5, 841 leaves):

$$\begin{aligned}
& - \left(4 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) + \left(7 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / (d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) - \\
& \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3 d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(20 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3 d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \left(-\frac{2(-4A+7B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} \right) \right. \\
& \quad \left. + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-5A \sin\left[\frac{dx}{2}\right] + 8B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8B \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{3d} \right. \\
& \quad \left. + \frac{4(2B-5A \cos[c] + 10B \cos[c]) \operatorname{Sec}[c] \tan\left[\frac{c}{2}\right]}{3d} + \frac{2(-A+B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) / ((B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2)
\end{aligned}$$

- **Problem 210: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx])}{(a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 204 leaves, 8 steps):

$$\frac{(A-4B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{a^2d} + \frac{(2A-5B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{3a^2d}$$

$$\frac{(A-4B)\sqrt{\sec[c+dx]}\sin[c+dx]}{a^2d} + \frac{(2A-5B)\sec[c+dx]^{3/2}\sin[c+dx]}{3a^2d(1+\sec[c+dx])} + \frac{(A-B)\sec[c+dx]^{5/2}\sin[c+dx]}{3d(a+a\sec[c+dx])^2}$$

Result (type 5, 811 leaves):

$$\left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c+dx] (A + B \sec[c+dx]) \right) /$$

$$(d(B + A \cos[c+dx]) (a + a \sec[c+dx])^2) - \left(4 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ \left. \sec\left[\frac{c}{2}\right] \sec[c+dx] (A + B \sec[c+dx]) \right) / (d(B + A \cos[c+dx]) (a + a \sec[c+dx])^2) +$$

$$\left(4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{3/2} (A + B \sec[c+dx]) \sin[c] \right) / \\ (3d(B + A \cos[c+dx]) (a + a \sec[c+dx])^2) -$$

$$\left(10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{3/2} (A + B \sec[c+dx]) \sin[c] \right) / \\ (3d(B + A \cos[c+dx]) (a + a \sec[c+dx])^2) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c+dx]^{3/2} (A + B \sec[c+dx]) \left(\frac{2(-A+4B)\cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{d} - \right. \right. \\ \left. \frac{2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (-2A \sin\left[\frac{dx}{2}\right] + 5B \sin\left[\frac{dx}{2}\right])}{3d} - \right. \\ \left. \left. \frac{4(-2A+5B)\tan\left[\frac{c}{2}\right]}{3d} - \frac{2(-A+B)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / ((B + A \cos[c+dx]) (a + a \sec[c+dx])^2)$$

- **Problem 211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 4, 161 leaves, 7 steps):

$$\frac{B \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a^2 d} + \frac{(A + 2 B) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{3 a^2 d} - \frac{B \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{a^2 d (1 + \text{Sec}[c + d x])} + \frac{(A - B) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2}$$

Result (type 5, 617 leaves):

$$\left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\ \left. \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x] (A + B \text{Sec}[c + d x]) \right) / \left((d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^2) + \right. \\ \left(2 A \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\text{Cos}[c + d x]} \text{Csc}\left[\frac{c}{2}\right] \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x]) \text{Sin}[c] \right) / \\ (3 d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^2) + \\ \left(4 B \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\text{Cos}[c + d x]} \text{Csc}\left[\frac{c}{2}\right] \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x]) \text{Sin}[c] \right) / \\ (3 d (B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^2) + \left(\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x]) \right. \\ \left. \left(-\frac{2 B \text{Cos}[d x] \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right]}{d} + \frac{2 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \text{Sin}\left[\frac{dx}{2}\right] + B \text{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{4 \text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \text{Sin}\left[\frac{dx}{2}\right] + 2 B \text{Sin}\left[\frac{dx}{2}\right])}{3 d} \right. \right. \\ \left. \left. + \frac{4 (A + 2 B) \text{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{2 (-A + B) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((B + A \text{Cos}[c + d x]) (a + a \text{Sec}[c + d x])^2 \right)$$

- **Problem 212: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\text{Sec}[c + d x]} (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 4, 168 leaves, 7 steps):

$$\begin{aligned}
& - \frac{A \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \\
& \frac{(2A+B) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3a^2 d} + \frac{(2A+B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3a^2 d (1+\sec[c+dx])} + \frac{(A-B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3d (a+a \sec[c+dx])^2}
\end{aligned}$$

Result (type 5, 618 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx]) \right) / \left(d (B+A \cos[c+dx]) (a+a \operatorname{Sec}[c+dx])^2 \right) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& \left(3d (B+A \cos[c+dx]) (a+a \operatorname{Sec}[c+dx])^2 \right) + \\
& \left(2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& \left(3d (B+A \cos[c+dx]) (a+a \operatorname{Sec}[c+dx])^2 \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx]) \right. \\
& \left. \left(\frac{2A \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-4A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} \right. \right. \\
& \left. \left. \frac{4(-4A+B) \tan\left[\frac{c}{2}\right]}{3d} - \frac{2(-A+B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / \left((B+A \cos[c+dx]) (a+a \operatorname{Sec}[c+dx])^2 \right)
\end{aligned}$$

■ **Problem 213: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+dx]}{\sqrt{\sec[c+dx]} (a+a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 177 leaves, 7 steps):

$$\begin{aligned}
& \frac{(4A-B) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} - \\
& \frac{(5A-2B) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3a^2 d} - \frac{(5A-2B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3a^2 d (1+\sec[c+dx])} - \frac{(A-B) \sqrt{\sec[c+dx]} \sin[c+dx]}{3d (a+a \operatorname{Sec}[c+dx])^2}
\end{aligned}$$

Result (type 5, 830 leaves) :

$$\begin{aligned}
 & \left(4 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
 & \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / \left(d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2 \right) - \\
 & \left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
 & \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / \left(d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2 \right) - \\
 & \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
 & \left(3 d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2 \right) + \\
 & \left(4 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
 & \left(3 d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2 \right) + \\
 & \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \left(-\frac{2(3A - B + A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \right. \right. \\
 & \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-7 A \sin\left[\frac{dx}{2}\right] + 4 B \sin\left[\frac{dx}{2}\right])}{3 d} + \right. \\
 & \left. \left. \frac{8 A \cos[c] \sin[dx]}{d} - \frac{4(-7A + 4B) \tan\left[\frac{c}{2}\right]}{3 d} + \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2 \right)
 \end{aligned}$$

- **Problem 214: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c+dx]}{\operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 211 leaves, 8 steps) :

$$\begin{aligned}
& - \frac{(7A - 4B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} + 5(2A - B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \frac{5(2A - B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} \\
& - \frac{5(2A - B) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]}} - \frac{(7A - 4B) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]} (1 + \sec[c + dx])} - \frac{(A - B) \sin[c + dx]}{3d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 5, 875 leaves):

$$\begin{aligned}
& - \left(7\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx]) \right) / \\
& \quad (d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(4\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx]) \right) / (d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \quad \left(20A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& \quad (3d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \quad \left(10B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx]) \sin[c] \right) / \\
& \quad (3d(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \quad \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx]) \left(-\frac{2(-5A + 3B - 2A \cos[2c] + B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4A \cos[2dx] \sin[2c]}{3d} \right. \right. \\
& \quad \left. \left. + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-10A \sin\left[\frac{dx}{2}\right] + 7B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8(-2A + B) \cos[c] \sin[dx]}{d} \right. \right. \\
& \quad \left. \left. + \frac{4A \cos[2c] \sin[2dx]}{3d} + \frac{4(-10A + 7B) \tan\left[\frac{c}{2}\right]}{3d} - \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / ((B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2)
\end{aligned}$$

- **Problem 215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Sec}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 244 leaves, 9 steps):

$$\frac{7(8A - 5B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]} - 5(3A - 2B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{5a^2 d} + \frac{7(8A - 5B) \sin[c + dx]}{15a^2 d \operatorname{Sec}[c + dx]^{3/2}} - \frac{5(3A - 2B) \sin[c + dx]}{3a^2 d \sqrt{\operatorname{Sec}[c + dx]}} - \frac{(3A - 2B) \sin[c + dx]}{a^2 d \operatorname{Sec}[c + dx]^{3/2} (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B) \sin[c + dx]}{3d \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 5, 924 leaves):

$$\begin{aligned}
& \left(56 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) - \left(7 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx]) \right) / (d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) - \\
& \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(20 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2) + \frac{1}{(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^2} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx]) \left(\frac{(-151A + 100B - 73A \cos[2c] + 40B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{10d} + \right. \\
& \quad \frac{4(-2A+B) \cos[2dx] \sin[2c]}{3d} + \frac{2A \cos[3dx] \sin[3c]}{5d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} - \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-13A \sin\left[\frac{dx}{2}\right] + 10B \sin\left[\frac{dx}{2}\right])}{3d} - \frac{2(-73A + 40B) \cos[c] \sin[dx]}{5d} + \\
& \quad \left. \frac{4(-2A+B) \cos[2c] \sin[2dx]}{3d} + \frac{2A \cos[3c] \sin[3dx]}{5d} - \frac{4(-13A + 10B) \tan\left[\frac{c}{2}\right]}{3d} + \frac{2(-A+B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 216: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{9/2} (A + B \operatorname{Sec}[c+dx])}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 292 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{7 (7 A - 17 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{10 a^3 d} - \frac{(13 A - 33 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{6 a^3 d} + \\
 & \frac{7 (7 A - 17 B) \sqrt{\sec [c + d x]} \sin [c + d x]}{10 a^3 d} - \frac{(13 A - 33 B) \sec [c + d x]^{3/2} \sin [c + d x]}{6 a^3 d} + \\
 & \frac{(A - B) \sec [c + d x]^{9/2} \sin [c + d x]}{5 d (a + a \sec [c + d x])^3} + \frac{(A - 2 B) \sec [c + d x]^{7/2} \sin [c + d x]}{3 a d (a + a \sec [c + d x])^2} + \frac{7 (7 A - 17 B) \sec [c + d x]^{5/2} \sin [c + d x]}{30 d (a^3 + a^3 \sec [c + d x])}
 \end{aligned}$$

Result (type 5, 933 leaves):

$$\begin{aligned}
& - \left(49 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d(B + A \cos[c+dx])(a + a \operatorname{Sec}[c+dx])^3) + \left(119 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / (5d(B + A \cos[c+dx])(a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(26 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3d(B + A \cos[c+dx])(a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(22 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d(B + A \cos[c+dx])(a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(B + A \cos[c+dx])(a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \left(-\frac{14(-7A + 17B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \right. \\
& \quad \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-8A \sin\left[\frac{dx}{2}\right] + 13B \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-13A \sin\left[\frac{dx}{2}\right] + 29B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{16B \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{3d} + \\
& \quad \left. \frac{4(4B - 13A \cos[c] + 33B \cos[c]) \operatorname{Sec}[c] \tan\left[\frac{c}{2}\right]}{3d} + \frac{4(-8A + 13B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 217: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{7/2} (A + B \operatorname{Sec}[c+dx])}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 261 leaves, 9 steps):

$$\begin{aligned}
& \frac{(9A - 49B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10a^3d} + \\
& \frac{(3A - 13B) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6a^3d} - \frac{(9A - 49B) \sqrt{\sec[c + dx]} \sin[c + dx]}{10a^3d} + \\
& \frac{(A - B) \sec[c + dx]^{7/2} \sin[c + dx]}{5d(a + a \sec[c + dx])^3} + \frac{(3A - 8B) \sec[c + dx]^{5/2} \sin[c + dx]}{15ad(a + a \sec[c + dx])^2} + \frac{(3A - 13B) \sec[c + dx]^{3/2} \sin[c + dx]}{6d(a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 904 leaves):

$$\begin{aligned}
& \left(9 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(49 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(26B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \left(\frac{2(-9A + 49B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} \right. \right. \\
& \quad \left. \left. - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-3A \sin\left[\frac{dx}{2}\right] + 8B \sin\left[\frac{dx}{2}\right])}{15d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-3A \sin\left[\frac{dx}{2}\right] + 13B \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4(-3A + 13B) \tan\left[\frac{c}{2}\right]}{3d} \right. \right. \\
& \quad \left. \left. - \frac{4(-3A + 8B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) \right) / ((B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3)
\end{aligned}$$

- **Problem 218: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx])}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 220 leaves, 8 steps):

$$\frac{(A+9B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{10a^3d} + \frac{(A+3B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{6a^3d} +$$

$$\frac{(A-B)\sec[c+dx]^{5/2}\sin[c+dx]}{5d(a+a\sec[c+dx])^3} + \frac{(A-6B)\sec[c+dx]^{3/2}\sin[c+dx]}{15ad(a+a\sec[c+dx])^2} - \frac{(A+9B)\sqrt{\sec[c+dx]}\sin[c+dx]}{10d(a^3+a^3\sec[c+dx])}$$

Result (type 5, 899 leaves):

$$\left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c+dx]^2 (A+B\sec[c+dx]) \right) /$$

$$(5d(B+A\cos[c+dx])(a+a\sec[c+dx])^3) + \left(9\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \sec\left[\frac{c}{2}\right] \sec[c+dx]^2 (A+B\sec[c+dx]) \right) / (5d(B+A\cos[c+dx])(a+a\sec[c+dx])^3) +$$

$$\left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{5/2} (A+B\sec[c+dx]) \sin[c] \right) /$$

$$(3d(B+A\cos[c+dx])(a+a\sec[c+dx])^3) +$$

$$\left(2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{5/2} (A+B\sec[c+dx]) \sin[c] \right) /$$

$$(d(B+A\cos[c+dx])(a+a\sec[c+dx])^3) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c+dx]^{5/2} (A+B\sec[c+dx]) \left(-\frac{2(A+9B)\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}\right]}{5d} + \frac{2\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A\sin\left[\frac{dx}{2}\right] + B\sin\left[\frac{dx}{2}\right])}{5d} \right. \right.$$

$$\left. \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A\sin\left[\frac{dx}{2}\right] + 3B\sin\left[\frac{dx}{2}\right])}{3d} + \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (2A\sin\left[\frac{dx}{2}\right] + 3B\sin\left[\frac{dx}{2}\right])}{15d} + \frac{4(A+3B)\tan\left[\frac{c}{2}\right]}{3d} + \right.$$

$$\left. \frac{4(2A+3B)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{2(-A+B)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) / ((B+A\cos[c+dx])(a+a\sec[c+dx])^3)$$

- **Problem 219: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 4, 216 leaves, 8 steps):

$$\begin{aligned} & - \frac{(A - B) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{10 a^3 d} + \frac{(A + B) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{6 a^3 d} + \\ & \frac{(A - B) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{5 d (a + a \text{Sec}[c + d x])^3} - \frac{(A + 4 B) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{15 a d (a + a \text{Sec}[c + d x])^2} + \frac{(A + B) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{6 d (a^3 + a^3 \text{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 898 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \left(5d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3\right) + \\
& \left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \left(5d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3\right) + \\
& \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c]\right) / \\
& \quad (3d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c]\right) / \\
& \quad (3d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \left(-\frac{2(-A+B) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d}\right) + \right. \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-7A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right])}{15d} + \frac{4(A+B) \tan\left[\frac{c}{2}\right]}{3d} + \\
& \quad \left. \frac{4(-7A+2B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{2(-A+B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) / \left((B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3\right)
\end{aligned}$$

- **Problem 220: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx])}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 222 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(9A+B)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{10a^3d} + \frac{(3A+B)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{6a^3d} \\
& + \frac{(A-B)\sqrt{\sec[c+dx]}\sin[c+dx]}{5d(a+a\sec[c+dx])^3} + \frac{(3A+2B)\sqrt{\sec[c+dx]}\sin[c+dx]}{15ad(a+a\sec[c+dx])^2} + \frac{(3A+B)\sqrt{\sec[c+dx]}\sin[c+dx]}{6d(a^3+a^3\sec[c+dx])}
\end{aligned}$$

Result (type 5, 899 leaves):

$$\begin{aligned}
& - \left(9\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \right. \\
& \quad \operatorname{Csc}\left[\frac{c}{2}\right]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}[c+dx]^2(A+B\operatorname{Sec}[c+dx])\right) / \left(5d(B+A\cos[c+dx])(a+a\sec[c+dx])^3\right) - \\
& \left(\sqrt{2}Be^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}[c+dx]^2(A+B\operatorname{Sec}[c+dx])\right) / \left(5d(B+A\cos[c+dx])(a+a\sec[c+dx])^3\right) + \\
& \left(2A\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}[c+dx]^{5/2}(A+B\operatorname{Sec}[c+dx])\sin[c] \right) / \\
& \quad \left(d(B+A\cos[c+dx])(a+a\sec[c+dx])^3 \right) + \\
& \left(2B\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}[c+dx]^{5/2}(A+B\operatorname{Sec}[c+dx])\sin[c] \right) / \\
& \quad \left(3d(B+A\cos[c+dx])(a+a\sec[c+dx])^3 \right) + \\
& \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^6\operatorname{Sec}[c+dx]^{5/2}(A+B\operatorname{Sec}[c+dx]) \left(\frac{2(9A+B)\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \frac{4\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right](-9A\sin\left[\frac{dx}{2}\right]+B\sin\left[\frac{dx}{2}\right])}{3d} \right) \right. \\
& \quad \left. \frac{2\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^5(-A\sin\left[\frac{dx}{2}\right]+B\sin\left[\frac{dx}{2}\right])}{5d} - \frac{4\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^3(-12A\sin\left[\frac{dx}{2}\right]+7B\sin\left[\frac{dx}{2}\right])}{15d} + \frac{4(-9A+B)\tan\left[\frac{c}{2}\right]}{3d} \right. \\
& \quad \left. \left. \frac{4(-12A+7B)\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2\tan\left[\frac{c}{2}\right]}{15d} + \frac{2(-A+B)\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4\tan\left[\frac{c}{2}\right]}{5d} \right) \right) / \left((B+A\cos[c+dx])(a+a\sec[c+dx])^3 \right)
\end{aligned}$$

- **Problem 221: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 228 leaves, 8 steps):

$$\frac{(49 A - 9 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - (13 A - 3 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} - \frac{(A - B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} - \frac{(8 A - 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} - \frac{(13 A - 3 B) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 923 leaves):

$$\begin{aligned}
& \left(49 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(9 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(26 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \\
& \left(-\frac{2(39A - 9B + 10A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} - \right. \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-23A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-17A \sin\left[\frac{dx}{2}\right] + 12B \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \left. \frac{16A \cos[c] \sin[dx]}{d} - \frac{4(-23A + 9B) \tan\left[\frac{c}{2}\right]}{3d} + \frac{4(-17A + 12B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

- **Problem 222: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 261 leaves, 9 steps):

$$\begin{aligned}
& - \frac{7 (17 A - 7 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{10 a^3 d} + \\
& \frac{(33 A - 13 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{6 a^3 d} + \frac{(33 A - 13 B) \sin [c + d x]}{6 a^3 d \sqrt{\sec [c + d x]}} - \\
& \frac{(A - B) \sin [c + d x]}{5 d \sqrt{\sec [c + d x]} (a + a \sec [c + d x])^3} - \frac{(2 A - B) \sin [c + d x]}{3 a d \sqrt{\sec [c + d x]} (a + a \sec [c + d x])^2} - \frac{7 (17 A - 7 B) \sin [c + d x]}{30 d \sqrt{\sec [c + d x]} (a^3 + a^3 \sec [c + d x])}
\end{aligned}$$

Result (type 5, 968 leaves):

$$\begin{aligned}
& - \left(119 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(49 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / (5d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(22 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(26 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (3d(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \frac{1}{(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3} \\
& \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \left(-\frac{2(-89A + 39B - 30A \cos[2c] + 10B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \right. \\
& \quad \frac{8A \cos[2dx] \sin[2c]}{3d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-22A \sin\left[\frac{dx}{2}\right] + 17B \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-43A \sin\left[\frac{dx}{2}\right] + 23B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{16(-3A + B) \cos[c] \sin[dx]}{d} + \frac{8A \cos[2c] \sin[2dx]}{3d} + \\
& \quad \left. \frac{4(-43A + 23B) \tan\left[\frac{c}{2}\right]}{3d} - \frac{4(-22A + 17B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 223: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c+dx]}{\operatorname{Sec}[c+dx]^{5/2} (a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 294 leaves, 10 steps):

$$\frac{7 (33 A - 17 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{10 a^3 d} -$$

$$\frac{(21 A - 11 B) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{2 a^3 d} + \frac{7 (33 A - 17 B) \sin [c + d x]}{30 a^3 d \sec [c + d x]^{3/2}} - \frac{(21 A - 11 B) \sin [c + d x]}{2 a^3 d \sqrt{\sec [c + d x]}} -$$

$$\frac{(A - B) \sin [c + d x]}{5 d \sec [c + d x]^{3/2} (a + a \sec [c + d x])^3} - \frac{(12 A - 7 B) \sin [c + d x]}{15 a d \sec [c + d x]^{3/2} (a + a \sec [c + d x])^2} - \frac{3 (21 A - 11 B) \sin [c + d x]}{10 d \sec [c + d x]^{3/2} (a^3 + a^3 \sec [c + d x])}$$

Result (type 5, 1012 leaves):

$$\begin{aligned}
& \left(231 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / \\
& (5d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(119 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A + B \operatorname{Sec}[c+dx]) \right) / (5d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(42 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(22 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \sin[c] \right) / \\
& (d (B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(B + A \cos[c+dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{5/2} (A + B \operatorname{Sec}[c+dx]) \\
& \left(\frac{(-329A + 178B - 133A \cos[2c] + 60B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \frac{8(-3A + B) \cos[2dx] \sin[2c]}{3d} + \frac{4A \cos[3dx] \sin[3c]}{5d} - \right. \\
& \quad \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-27A \sin\left[\frac{dx}{2}\right] + 22B \sin\left[\frac{dx}{2}\right])}{15d} - \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-69A \sin\left[\frac{dx}{2}\right] + 43B \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4(-133A + 60B) \cos[c] \sin[dx]}{5d} + \frac{8(-3A + B) \cos[2c] \sin[2dx]}{3d} + \\
& \quad \left. \frac{4A \cos[3c] \sin[3dx]}{5d} - \frac{4(-69A + 43B) \tan\left[\frac{c}{2}\right]}{3d} + \frac{4(-27A + 22B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 224: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{5/2} \sqrt{a + a \operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 176 leaves, 5 steps):

$$\frac{\sqrt{a} (6A + 5B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a (6A + 5B) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a (6A + 5B) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 1184 leaves):

$$-\left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((-1+i) + \sqrt{2} \right) \left((18+6i)A + 6\sqrt{2}A + (15+5i)B + 5\sqrt{2}B \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \right.$$

$$\left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \sqrt{\operatorname{Sec}[c+dx]} \right) -$$

$$\left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((1+i) + \sqrt{2} \right) \left((-18+6i)A + 6\sqrt{2}A - (15-5i)B + 5\sqrt{2}B \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \right.$$

$$\left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \sqrt{\operatorname{Sec}[c+dx]} \right) +$$

$$\frac{(12A + 6i\sqrt{2}A + 10B + 5i\sqrt{2}B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{32 (i + \sqrt{2}) d \sqrt{\operatorname{Sec}[c+dx]}} -$$

$$\left(\left(\frac{1}{128} - \frac{i}{128} \right) \left((-1+i) + \sqrt{2} \right) \left((18+6i)A + 6\sqrt{2}A + (15+5i)B + 5\sqrt{2}B \right) \right.$$

$$\left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \sqrt{\operatorname{Sec}[c+dx]} \right) +$$

$$\left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((1+i) + \sqrt{2} \right) \left((-18+6i)A + 6\sqrt{2}A - (15-5i)B + 5\sqrt{2}B \right) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right.$$

$$\left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \sqrt{\operatorname{Sec}[c+dx]} \right) +$$

$$\frac{B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{12d \sqrt{\operatorname{Sec}[c+dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3} + \frac{(6A + 5B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{16d \sqrt{\operatorname{Sec}[c+dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)}$$

$$+ \frac{B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{12d \sqrt{\operatorname{Sec}[c+dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^3} +$$

$$\frac{(-6A - 5B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])}}{16d \sqrt{\operatorname{Sec}[c + dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \left(2A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{8d \sqrt{\operatorname{Sec}[c + dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \left(2A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{8d \sqrt{\operatorname{Sec}[c + dx]} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2}$$

- **Problem 225: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{\sqrt{a} (4A + 3B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{4d} + \frac{a (4A + 3B) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{4d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{aB \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{2d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 1002 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((-1+i) + \sqrt{2} \right) \left((12+4i)A + 4\sqrt{2}A + (9+3i)B + 3\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2}(i+\sqrt{2})d\sqrt{\operatorname{Sec}[c+dx]} \right) - \\
& \left(\left(\frac{1}{32} - \frac{i}{32} \right) \left((1+i) + \sqrt{2} \right) \left((-12+4i)A + 4\sqrt{2}A - (9-3i)B + 3\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2}(i+\sqrt{2})d\sqrt{\operatorname{Sec}[c+dx]} \right) + \\
& \frac{(8A+4i\sqrt{2}A+6B+3i\sqrt{2}B) \operatorname{Log}\left[\sqrt{2}+2\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{16(i+\sqrt{2})d\sqrt{\operatorname{Sec}[c+dx]}} - \\
& \left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((-1+i) + \sqrt{2} \right) \left((12+4i)A + 4\sqrt{2}A + (9+3i)B + 3\sqrt{2}B \right) \right. \\
& \quad \left. \operatorname{Log}\left[2-\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2}(i+\sqrt{2})d\sqrt{\operatorname{Sec}[c+dx]} \right) + \\
& \left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((1+i) + \sqrt{2} \right) \left((-12+4i)A + 4\sqrt{2}A - (9-3i)B + 3\sqrt{2}B \right) \operatorname{Log}\left[2+\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) / \left(\sqrt{2}(i+\sqrt{2})d\sqrt{\operatorname{Sec}[c+dx]} \right) + \\
& \frac{(4A+3B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{8d\sqrt{\operatorname{Sec}[c+dx]} (\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right])} + \frac{(-4A-3B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}}{8d\sqrt{\operatorname{Sec}[c+dx]} (\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])} + \\
& \frac{B\sqrt{a(1+\operatorname{Sec}[c+dx])} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4d\sqrt{\operatorname{Sec}[c+dx]} (\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right])^2} + \\
& \frac{B\sqrt{a(1+\operatorname{Sec}[c+dx])} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4d\sqrt{\operatorname{Sec}[c+dx]} (\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right])^2}
\end{aligned}$$

- **Problem 226: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a\operatorname{Sec}[c+dx]} (A+B\operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$\frac{\sqrt{a} (2A + B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a B \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 522 leaves):

$$\frac{1}{d \sqrt{\operatorname{Sec}[c+dx]}}$$

$$\left(\left(\frac{1}{32} + \frac{i}{32} \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right) \left(\frac{2i\sqrt{2} \left((-3+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) (2A+B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right]}{i + \sqrt{2}} \right) - \right.$$

$$\frac{2\sqrt{2} \left((-1+i) + \sqrt{2} \right) \left((3+i) + \sqrt{2} \right) (2A+B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right]}{i + \sqrt{2}} +$$

$$\frac{(4+4i) \left(-2i + \sqrt{2} \right) (2A+B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right]}{i + \sqrt{2}} +$$

$$\frac{i\sqrt{2} \left((-1+i) + \sqrt{2} \right) \left((3+i) + \sqrt{2} \right) (2A+B) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right]}{i + \sqrt{2}} +$$

$$\frac{\sqrt{2} \left((-3+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) (2A+B) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right]}{i + \sqrt{2}} +$$

$$\left. \frac{(8-8i)B}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} - \frac{(8-8i)B}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right)$$

- **Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx])}{\sqrt{\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{2\sqrt{a} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2aA \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 321 leaves) :

$$\frac{1}{4 d \sqrt{\operatorname{Sec}[c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1+\operatorname{Sec}[c+d x])}$$

$$\left(-2 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-(-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right]-2 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\left(1+\sqrt{2}\right) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right]\right)+$$

$$2 \sqrt{2} B \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-\sqrt{2} B \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]-$$

$$\left.\sqrt{2} B \operatorname{Log}\left[2+\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]+8 A \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 231: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 227 leaves, 6 steps) :

$$\frac{a^{3/2} (88 A+75 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{64 d} + \frac{a^2 (88 A+75 B) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{64 d \sqrt{a+a \operatorname{Sec}[c+d x]}} +$$

$$\frac{a^2 (88 A+75 B) \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{96 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 (8 A+9 B) \operatorname{Sec}[c+d x]^{7/2} \operatorname{Sin}[c+d x]}{24 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a B \operatorname{Sec}[c+d x]^{7/2} \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}$$

Result (type 3, 1376 leaves) :

$$-\left(\left(\frac{1}{1024}+\frac{i}{1024}\right)\left((-1+i)+\sqrt{2}\right)\left((264+88 i) A+88 \sqrt{2} A+(225+75 i) B+75 \sqrt{2} B\right)\right.$$

$$\left.\operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]+\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2}\right] /$$

$$\left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+d x]^{3/2}\right)-\left(\left(\frac{1}{1024}-\frac{i}{1024}\right)\left((1+i)+\sqrt{2}\right)\left((-264+88 i) A+88 \sqrt{2} A-(225-75 i) B+75 \sqrt{2} B\right) \operatorname{ArcTan}\left[\right.$$

$$\left.\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]-\sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]+\sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2}\right] / \left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+d x]^{3/2}\right)+$$

$$\left(\left(176 A+88 i \sqrt{2} A+150 B+75 i \sqrt{2} B\right) \operatorname{Log}\left[\sqrt{2}+2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2}\right) /$$

$$\begin{aligned}
& (512 (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{3/2}) - \\
& \left(\left(\frac{1}{2048} - \frac{i}{2048} \right) \left((-1 + i) + \sqrt{2} \right) \left((264 + 88i) A + 88\sqrt{2} A + (225 + 75i) B + 75\sqrt{2} B \right) \right. \\
& \quad \left. \operatorname{Log} \left[2 - \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{3/2} \right) + \\
& \left(\left(\frac{1}{2048} + \frac{i}{2048} \right) \left((1 + i) + \sqrt{2} \right) \left((-264 + 88i) A + 88\sqrt{2} A - (225 - 75i) B + 75\sqrt{2} B \right) \operatorname{Log} \left[2 + \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{3/2} \right) + \\
& \frac{(8A + 15B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2}}{192 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^3} + \frac{(88A + 75B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2}}{256 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{(-8A - 15B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2}}{192 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(-88A - 75B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2}}{256 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(24A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 19B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{128 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(24A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 19B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{128 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{32 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{32 d \operatorname{Sec}[c + dx]^{3/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^4}
\end{aligned}$$

■ **Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\frac{a^{3/2} (14 A + 11 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8 d} + \frac{a^2 (14 A + 11 B) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{8 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (6 A + 7 B) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{12 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a B \operatorname{Sec}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d}$$

Result (type 3, 1208 leaves):

$$-\left(\frac{1}{128} + \frac{i}{128}\right) \left((-1+i) + \sqrt{2}\right) \left((42+14i) A + 14\sqrt{2} A + (33+11i) B + 11\sqrt{2} B\right)$$

$$\operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2}\right] /$$

$$\left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2}\right) - \left(\frac{1}{128} - \frac{i}{128}\right) \left((1+i) + \sqrt{2}\right) \left((-42+14i) A + 14\sqrt{2} A - (33-11i) B + 11\sqrt{2} B\right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2}\right] / \left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2}\right) +$$

$$\left((28A + 14i\sqrt{2} A + 22B + 11i\sqrt{2} B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} /$$

$$\left(64(i+\sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2}\right) -$$

$$\left(\frac{1}{256} - \frac{i}{256}\right) \left((-1+i) + \sqrt{2}\right) \left((42+14i) A + 14\sqrt{2} A + (33+11i) B + 11\sqrt{2} B\right) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} / \left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2}\right) +$$

$$\left(\frac{1}{256} + \frac{i}{256}\right) \left((1+i) + \sqrt{2}\right) \left((-42+14i) A + 14\sqrt{2} A - (33-11i) B + 11\sqrt{2} B\right) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right]$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} / \left(\sqrt{2}(i+\sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2}\right) +$$

$$\frac{B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2}}{24 d \operatorname{Sec}[c+dx]^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{(14A + 11B) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2}}{32 d \operatorname{Sec}[c+dx]^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)}$$

$$+ \frac{B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2}}{24 d \operatorname{Sec}[c+dx]^{3/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} +$$

$$\frac{(-14A - 11B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2}}{32d \operatorname{Sec}[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(2A \sin\left[\frac{1}{2}(c + dx)\right] + 3B \sin\left[\frac{1}{2}(c + dx)\right]\right)}{16d \operatorname{Sec}[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} +$$

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(2A \sin\left[\frac{1}{2}(c + dx)\right] + 3B \sin\left[\frac{1}{2}(c + dx)\right]\right)}{16d \operatorname{Sec}[c + dx]^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2}$$

- **Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$\frac{a^{3/2} (12A + 7B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{4d} + \frac{a^2 (4A + 5B) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{4d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{aB \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{2d}$$

Result (type 3, 1040 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((-1 + i) + \sqrt{2} \right) \left((36 + 12i)A + 12\sqrt{2}A + (21 + 7i)B + 7\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \Big/ \left(\sqrt{2}(i + \sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2} \right) - \\
& \left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((1 + i) + \sqrt{2} \right) \left((-36 + 12i)A + 12\sqrt{2}A - (21 - 7i)B + 7\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \Big/ \left(\sqrt{2}(i + \sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2} \right) + \\
& \left((24A + 12i\sqrt{2}A + 14B + 7i\sqrt{2}B) \operatorname{Log} \left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right] \right] \right) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \Big/ \left(32(i + \sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2} \right) - \\
& \left(\left(\frac{1}{128} - \frac{i}{128} \right) \left((-1 + i) + \sqrt{2} \right) \left((36 + 12i)A + 12\sqrt{2}A + (21 + 7i)B + 7\sqrt{2}B \right) \right. \\
& \quad \left. \operatorname{Log} \left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] \right) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \Big/ \left(\sqrt{2}(i + \sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2} \right) + \\
& \left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((1 + i) + \sqrt{2} \right) \left((-36 + 12i)A + 12\sqrt{2}A - (21 - 7i)B + 7\sqrt{2}B \right) \operatorname{Log} \left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] \right) \\
& \quad \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \Big/ \left(\sqrt{2}(i + \sqrt{2}) d \operatorname{Sec}[c+dx]^{3/2} \right) + \\
& \frac{(4A + 7B) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2}}{16 d \operatorname{Sec}[c+dx]^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \frac{(-4A - 7B) \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^3 (a(1 + \operatorname{Sec}[c+dx]))^{3/2}}{16 d \operatorname{Sec}[c+dx]^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)} + \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \operatorname{Tan} \left[\frac{1}{2}(c+dx) \right]}{8 d \operatorname{Sec}[c+dx]^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2} + \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2}(c+dx) \right]^2 (a(1 + \operatorname{Sec}[c+dx]))^{3/2} \operatorname{Tan} \left[\frac{1}{2}(c+dx) \right]}{8 d \operatorname{Sec}[c+dx]^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2}
\end{aligned}$$

■ **Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^{3/2} (A + B \operatorname{Sec}[c+dx])}{\sqrt{\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$\frac{a^{3/2} (2A + 3B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a^2 (2A - B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a B \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d}$$

Result (type 3, 603 leaves):

$$\frac{1}{d \operatorname{Sec}[c+dx]^{3/2}} \left(\frac{1}{64} + \frac{i}{64} \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a(1+\operatorname{Sec}[c+dx]))^{3/2}$$

$$\left(\frac{1}{i+\sqrt{2}} 2i\sqrt{2} \left((-3+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) (2A+3B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] - \right.$$

$$\left. \frac{1}{i+\sqrt{2}} 2\sqrt{2} \left((-1+i) + \sqrt{2} \right) \left((3+i) + \sqrt{2} \right) (2A+3B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$\left. \frac{(4+4i) \left(-2i + \sqrt{2} \right) (2A+3B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]}{i+\sqrt{2}} + \frac{1}{i+\sqrt{2}} \right.$$

$$i\sqrt{2} \left((-1+i) + \sqrt{2} \right) \left((3+i) + \sqrt{2} \right) (2A+3B) \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] +$$

$$\left. \frac{1}{i+\sqrt{2}} \sqrt{2} \left((-3+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) (2A+3B) \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] - \right.$$

$$\left. \frac{(8-8i) B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} + (32-32i) A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \frac{(8-8i) B \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx])}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{2 a^{3/2} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2 a^2 (4A+3B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2 a A \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 348 leaves):

$$\frac{1}{12 d \sqrt{\sec[c+dx]}} a \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\left(-6 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] - 6 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \right) +$$

$$6 \sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] - 3 \sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] -$$

$$3 \sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] + 36 A \sin\left[\frac{1}{2}(c+dx)\right] + 24 B \sin\left[\frac{1}{2}(c+dx)\right] + 4 A \sin\left[\frac{3}{2}(c+dx)\right]$$

- **Problem 239: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^{5/2} (a + a \sec[c+dx])^{5/2} (A + B \sec[c+dx]) dx$$

Optimal (type 3, 274 leaves, 7 steps):

$$\frac{a^{5/2} (326 A + 283 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{128 d} + \frac{a^3 (326 A + 283 B) \sec[c+dx]^{3/2} \sin[c+dx]}{128 d \sqrt{a+a \sec[c+dx]}} +$$

$$\frac{a^3 (326 A + 283 B) \sec[c+dx]^{5/2} \sin[c+dx]}{192 d \sqrt{a+a \sec[c+dx]}} + \frac{a^3 (170 A + 157 B) \sec[c+dx]^{7/2} \sin[c+dx]}{240 d \sqrt{a+a \sec[c+dx]}} +$$

$$\frac{a^2 (10 A + 13 B) \sec[c+dx]^{7/2} \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{40 d} + \frac{a B \sec[c+dx]^{7/2} (a+a \sec[c+dx])^{3/2} \sin[c+dx]}{5 d}$$

Result (type 3, 1544 leaves):

$$-\left(\left(\frac{1}{4096} + \frac{i}{4096} \right) \left((-1+i) + \sqrt{2} \right) \left((978+326i) A + 326\sqrt{2} A + (849+283i) B + 283\sqrt{2} B \right) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \right) / \left(\sqrt{2} (i+\sqrt{2}) d \sec[c+dx]^{5/2} \right) -$$

$$\left(\left(\frac{1}{4096} - \frac{i}{4096} \right) \left((1+i) + \sqrt{2} \right) \left((-978+326i) A + 326\sqrt{2} A - (849-283i) B + 283\sqrt{2} B \right) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \right) / \left(\sqrt{2} (i+\sqrt{2}) d \sec[c+dx]^{5/2} \right) +$$

$$\left((652 A + 326 i \sqrt{2} A + 566 B + 283 i \sqrt{2} B) \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} /$$

$$\begin{aligned}
& (2048 (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2}) - \\
& \left(\left(\frac{1}{8192} - \frac{i}{8192} \right) \left((-1 + i) + \sqrt{2} \right) \left((978 + 326 i) A + 326 \sqrt{2} A + (849 + 283 i) B + 283 \sqrt{2} B \right) \right. \\
& \quad \left. \operatorname{Log} \left[2 - \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left(\left(\frac{1}{8192} + \frac{i}{8192} \right) \left((1 + i) + \sqrt{2} \right) \left((-978 + 326 i) A + 326 \sqrt{2} A - (849 - 283 i) B + 283 \sqrt{2} B \right) \operatorname{Log} \left[2 + \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{160 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^5} + \frac{(46 A + 59 B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{768 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(326 A + 283 B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{1024 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)} - \\
& \frac{B \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{160 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^5} + \\
& \frac{(-46 A - 59 B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{768 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(-326 A - 283 B) \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}{1024 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(2 A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 5 B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{128 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(2 A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 5 B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{128 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(86 A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 75 B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{512 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(86 A \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] + 75 B \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)}{512 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right)^2}
\end{aligned}$$

Problem 240: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Sec}[c + dx]^{3/2} (a + a \text{Sec}[c + dx])^{5/2} (A + B \text{Sec}[c + dx]) dx$$

Optimal (type 3, 227 leaves, 6 steps):

$$\frac{a^{5/2} (200 A + 163 B) \text{ArcSinh}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{64 d} + \frac{a^3 (200 A + 163 B) \text{Sec}[c + dx]^{3/2} \text{Sin}[c + dx]}{64 d \sqrt{a + a \text{Sec}[c + dx]}} + \frac{a^3 (104 A + 95 B) \text{Sec}[c + dx]^{5/2} \text{Sin}[c + dx]}{96 d \sqrt{a + a \text{Sec}[c + dx]}} + \frac{a^2 (8 A + 11 B) \text{Sec}[c + dx]^{5/2} \sqrt{a + a \text{Sec}[c + dx]} \text{Sin}[c + dx]}{24 d} + \frac{a B \text{Sec}[c + dx]^{5/2} (a + a \text{Sec}[c + dx])^{3/2} \text{Sin}[c + dx]}{4 d}$$

Result (type 3, 1376 leaves):

$$\begin{aligned} & - \left(\left(\frac{1}{2048} + \frac{i}{2048} \right) \left((-1 + i) + \sqrt{2} \right) \left((600 + 200 i) A + 200 \sqrt{2} A + (489 + 163 i) B + 163 \sqrt{2} B \right) \text{ArcTan} \left[\frac{\text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{4}(c + dx)\right]}{-\text{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) - \\ & \left(\left(\frac{1}{2048} - \frac{i}{2048} \right) \left((1 + i) + \sqrt{2} \right) \left((-600 + 200 i) A + 200 \sqrt{2} A - (489 - 163 i) B + 163 \sqrt{2} B \right) \text{ArcTan} \left[\frac{\text{Cos}\left[\frac{1}{4}(c + dx)\right] + \text{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{4}(c + dx)\right]}{\text{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \text{Cos}\left[\frac{1}{4}(c + dx)\right] - \text{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\ & \left((400 A + 200 i \sqrt{2} A + 326 B + 163 i \sqrt{2} B) \text{Log}\left[\sqrt{2} + 2 \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2} \right) / \left(1024 (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) - \\ & \left(\left(\frac{1}{4096} - \frac{i}{4096} \right) \left((-1 + i) + \sqrt{2} \right) \left((600 + 200 i) A + 200 \sqrt{2} A + (489 + 163 i) B + 163 \sqrt{2} B \right) \text{Log}\left[2 - \sqrt{2} \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\ & \left(\left(\frac{1}{4096} + \frac{i}{4096} \right) \left((1 + i) + \sqrt{2} \right) \left((-600 + 200 i) A + 200 \sqrt{2} A - (489 - 163 i) B + 163 \sqrt{2} B \right) \text{Log}\left[2 + \sqrt{2} \text{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\ & \frac{(8 A + 23 B) \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2}}{384 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{(200 A + 163 B) \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a (1 + \text{Sec}[c + dx]))^{5/2}}{512 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} \end{aligned}$$

$$\begin{aligned}
& \frac{(-8A - 23B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{384d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-200A - 163B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{512d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \left(40A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 43B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{256d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \left(40A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 43B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)}{256d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{B \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{64d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\
& \frac{B \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{64d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4}
\end{aligned}$$

■ **Problem 241: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{5/2} (38A + 25B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{8d} + \frac{a^3 (54A + 49B) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{24d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \\
& \frac{a^2 (2A + 3B) \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{4d} + \frac{aB \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{3d}
\end{aligned}$$

Result (type 3, 1208 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{256} + \frac{i}{256} \right) \left((-1 + i) + \sqrt{2} \right) \left((114 + 38i)A + 38\sqrt{2}A + (75 + 25i)B + 25\sqrt{2}B \right) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) - \left(\left(\frac{1}{256} - \frac{i}{256} \right) \left((1 + i) + \sqrt{2} \right) \left((-114 + 38i) A + 38\sqrt{2} A - (75 - 25i) B + 25\sqrt{2} B \right) \operatorname{ArcTan} \left[\right. \right. \\
& \left. \left. \frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right] \Big/ \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left((76A + 38i\sqrt{2}A + 50B + 25i\sqrt{2}B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right) \Big/ \\
& \left(128 (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) - \\
& \left(\left(\frac{1}{512} - \frac{i}{512} \right) \left((-1 + i) + \sqrt{2} \right) \left((114 + 38i) A + 38\sqrt{2} A + (75 + 25i) B + 25\sqrt{2} B \right) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \Big/ \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left(\left(\frac{1}{512} + \frac{i}{512} \right) \left((1 + i) + \sqrt{2} \right) \left((-114 + 38i) A + 38\sqrt{2} A - (75 - 25i) B + 25\sqrt{2} B \right) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \Big/ \left(\sqrt{2} (i + \sqrt{2}) d \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \frac{B \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{48 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \frac{(22A + 25B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{64 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} - \\
& \frac{B \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{48 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{(-22A - 25B) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}{64 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \left(2A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 5B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)}{32 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2} + \\
& \frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \left(2A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 5B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)}{32 d \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2}
\end{aligned}$$

■ **Problem 242: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx])}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\frac{a^{5/2} (20 A + 19 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4 d} + \frac{a^3 (4 A - 9 B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (4 A + 7 B) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d} + \frac{a B \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2 d}$$

Result (type 3, 1094 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((-1 + i) + \sqrt{2} \right) \left((60 + 20i)A + 20\sqrt{2}A + (57 + 19i)B + 19\sqrt{2}B \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\text{Cos} \left[\frac{1}{4}(c + dx) \right] - \text{Sin} \left[\frac{1}{4}(c + dx) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4}(c + dx) \right]}{-\text{Cos} \left[\frac{1}{4}(c + dx) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4}(c + dx) \right] - \text{Sin} \left[\frac{1}{4}(c + dx) \right]} \right] \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} \right] / \right. \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) - \left(\left(\frac{1}{128} - \frac{i}{128} \right) \left((1 + i) + \sqrt{2} \right) \left((-60 + 20i)A + 20\sqrt{2}A - (57 - 19i)B + 19\sqrt{2}B \right) \text{ArcTan} \left[\right. \right. \\
& \quad \left. \left. \frac{\text{Cos} \left[\frac{1}{4}(c + dx) \right] + \text{Sin} \left[\frac{1}{4}(c + dx) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4}(c + dx) \right]}{\text{Cos} \left[\frac{1}{4}(c + dx) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4}(c + dx) \right] - \text{Sin} \left[\frac{1}{4}(c + dx) \right]} \right] \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} \right] / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\
& \quad \left((40A + 20i\sqrt{2}A + 38B + 19i\sqrt{2}B) \text{Log} \left[\sqrt{2} + 2 \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right] \right) \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} / \\
& \quad \left(64 (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) - \\
& \quad \left(\left(\frac{1}{256} - \frac{i}{256} \right) \left((-1 + i) + \sqrt{2} \right) \left((60 + 20i)A + 20\sqrt{2}A + (57 + 19i)B + 19\sqrt{2}B \right) \text{Log} \left[2 - \sqrt{2} \text{Cos} \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right] \right) \\
& \quad \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\
& \quad \left(\left(\frac{1}{256} + \frac{i}{256} \right) \left((1 + i) + \sqrt{2} \right) \left((-60 + 20i)A + 20\sqrt{2}A - (57 - 19i)B + 19\sqrt{2}B \right) \text{Log} \left[2 + \sqrt{2} \text{Cos} \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right] \right) \\
& \quad \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} / \left(\sqrt{2} (i + \sqrt{2}) d \text{Sec}[c + dx]^{5/2} \right) + \\
& \quad \frac{(4A + 11B) \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2}}{32 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos} \left[\frac{1}{2}(c + dx) \right] - \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right)} + \frac{(-4A - 11B) \text{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2}}{32 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos} \left[\frac{1}{2}(c + dx) \right] + \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right)} + \\
& \quad \frac{A \text{Sec} \left[\frac{1}{2}(c + dx) \right]^4 (a(1 + \text{Sec}[c + dx]))^{5/2} \text{Tan} \left[\frac{1}{2}(c + dx) \right]}{2 d \text{Sec}[c + dx]^{5/2}} + \\
& \quad \frac{B \text{Sec} \left[\frac{1}{2}(c + dx) \right]^4 (a(1 + \text{Sec}[c + dx]))^{5/2} \text{Tan} \left[\frac{1}{2}(c + dx) \right]}{16 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos} \left[\frac{1}{2}(c + dx) \right] - \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2} + \\
& \quad \frac{B \text{Sec} \left[\frac{1}{2}(c + dx) \right]^4 (a(1 + \text{Sec}[c + dx]))^{5/2} \text{Tan} \left[\frac{1}{2}(c + dx) \right]}{16 d \text{Sec}[c + dx]^{5/2} \left(\text{Cos} \left[\frac{1}{2}(c + dx) \right] + \text{Sin} \left[\frac{1}{2}(c + dx) \right] \right)^2}
\end{aligned}$$

- **Problem 243: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 3, 177 leaves, 5 steps):

$$\frac{a^{5/2} (2A + 5B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a^3 (14A + 3B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{a^2 (2A - 3B) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d} + \frac{2aA (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 635 leaves):

$$\frac{1}{d \operatorname{Sec}[c + d x]^{5/2}} \left(\frac{1}{384} + \frac{i}{384} \right) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 (a (1 + \operatorname{Sec}[c + d x]))^{5/2}$$

$$\left(\frac{1}{i + \sqrt{2}} 6i\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2A + 5B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + d x)\right] - (-1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + d x)\right]}{\left(1 + \sqrt{2}\right) \cos\left[\frac{1}{4}(c + d x)\right] - \sin\left[\frac{1}{4}(c + d x)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] - \frac{1}{i + \sqrt{2}} 6\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2A + 5B) \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + d x)\right] - (1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + d x)\right]}{(-1 + \sqrt{2}) \cos\left[\frac{1}{4}(c + d x)\right] - \sin\left[\frac{1}{4}(c + d x)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] + \frac{(12 + 12i) \left(-2i + \sqrt{2} \right) (2A + 5B) \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}} 3i\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2A + 5B) \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] + \frac{1}{i + \sqrt{2}} 3\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2A + 5B) \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] + (16 - 16i) A \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] - \frac{(24 - 24i) B \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2}{-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]} + (48 - 48i) (5A + 2B) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \frac{(24 - 24i) B \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]} \right)$$

- **Problem 244: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\frac{2 a^{5/2} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2 a^3 (32 A + 35 B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{2 a^2 (8 A + 5 B) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15 d \sqrt{\operatorname{Sec}[c+dx]}} + \frac{2 a A (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{5 d \operatorname{Sec}[c+dx]^{3/2}}$$

Result (type 3, 376 leaves) :

$$\frac{1}{60 d \sqrt{\operatorname{Sec}[c+dx]}} a^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

$$\left(-30 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] - 30 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \right) +$$

$$30 \sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 15 \sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] -$$

$$15 \sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 300 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] +$$

$$300 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 50 A \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 20 B \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + 6 A \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \Bigg)$$

■ **Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2} (A+B \operatorname{Sec}[c+dx])}{\sqrt{a+a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 190 leaves, 7 steps) :

$$-\frac{(4 A - 7 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4 \sqrt{a} d} + \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a} d} +$$

$$\frac{(4 A - B) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{4 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{B \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{2 d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 684 leaves) :

$$\frac{1}{d \sqrt{a (1 + \operatorname{Sec}[c + d x])}}$$

$$\left(\frac{1}{64} + \frac{i}{64} \right) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]} \left(\frac{2 \sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (4 A - 7 B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]} \right]}{-1 + i \sqrt{2}} \right) + \right.$$

$$\frac{2 \sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (4 A - 7 B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]} \right]}{i + \sqrt{2}} -$$

$$(64 - 64 i) (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] + (64 - 64 i) (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] -$$

$$\frac{(4 + 4 i) \left(-2 i + \sqrt{2} \right) (4 A - 7 B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{i + \sqrt{2}} +$$

$$\frac{\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (4 A - 7 B) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{-1 + i \sqrt{2}} -$$

$$\frac{\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (4 A - 7 B) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{i + \sqrt{2}} + \frac{(8 - 8 i) (4 A - B)}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} +$$

$$\left. \frac{(16 - 16 i) B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{(16 - 16 i) B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{(8 - 8 i) (-4 A + B)}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} \right)$$

■ **Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x])}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(2 A - B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} + \frac{B \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 596 leaves):

$$\frac{1}{d \sqrt{a (1 + \operatorname{Sec}[c + d x])}}$$

$$\left(\frac{1}{16} + \frac{i}{16} \right) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]} \left(\frac{2 i \sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2 A - B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right]}{i + \sqrt{2}} \right) - \right.$$

$$\frac{2 \sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2 A - B) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right]}{i + \sqrt{2}} +$$

$$(16 - 16 i) (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] - (16 - 16 i) (A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] +$$

$$\frac{(4 + 4 i) \left(-2 i + \sqrt{2} \right) (2 A - B) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{i + \sqrt{2}} +$$

$$\frac{i \sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2 A - B) \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{i + \sqrt{2}} +$$

$$\frac{\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2 A - B) \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{i + \sqrt{2}} +$$

$$\left. \frac{(8 - 8 i) B}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} - \frac{(8 - 8 i) B}{\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]} \right)$$

- **Problem 250: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x])}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{2 B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d}$$

Result (type 3, 412 leaves):

$$\frac{1}{2 d \sqrt{a} (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]$$

$$\left(-2 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right] - 2 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right] - \right.$$

$$4 A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] + 4 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] +$$

$$4 A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] - 4 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]\right] + 2 \sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] -$$

$$\left. \sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \sqrt{\operatorname{Sec}[c + d x]}$$

■ **Problem 255: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{7/2} (A + B \operatorname{Sec}[c + d x])}{(a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 247 leaves, 8 steps):

$$-\frac{(12 A - 19 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{4 a^{3/2} d} + \frac{(9 A - 13 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} +$$

$$\frac{(A - B) \operatorname{Sec}[c + d x]^{7/2} \operatorname{Sin}[c + d x]}{2 d (a + a \operatorname{Sec}[c + d x])^{3/2}} + \frac{(6 A - 7 B) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{4 a d \sqrt{a + a \operatorname{Sec}[c + d x]}} - \frac{(A - 2 B) \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{2 a d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 1318 leaves):

$$-\left(\left(\frac{1}{8} + \frac{i}{8} \right) \left((-1 + i) + \sqrt{2} \right) \left((-36 - 12 i) A - 12 \sqrt{2} A + (57 + 19 i) B + 19 \sqrt{2} B \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right] \right.$$

$$\left. \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Sec}[c + d x]^{3/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d (a (1 + \operatorname{Sec}[c + d x]))^{3/2} \right) -$$

$$\left(\left(\frac{1}{8} - \frac{i}{8} \right) \left((1 + i) + \sqrt{2} \right) \left((36 - 12 i) A - 12 \sqrt{2} A - (57 - 19 i) B + 19 \sqrt{2} B \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}\right] \right.$$

$$\left. \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^3 \operatorname{Sec}[c + d x]^{3/2} \right) / \left(\sqrt{2} (i + \sqrt{2}) d (a (1 + \operatorname{Sec}[c + d x]))^{3/2} \right) +$$

$$\begin{aligned}
& \frac{(-9A + 13B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{3/2}}{d(a(1 + \operatorname{Sec}[c + dx]))^{3/2}} + \\
& \frac{(9A - 13B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{3/2}}{d(a(1 + \operatorname{Sec}[c + dx]))^{3/2}} + \\
& \frac{(-24A - 12i\sqrt{2}A + 38B + 19i\sqrt{2}B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{3/2}}{4(i + \sqrt{2})d(a(1 + \operatorname{Sec}[c + dx]))^{3/2}} - \\
& \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((-1 + i) + \sqrt{2} \right) \left((-36 - 12i)A - 12\sqrt{2}A + (57 + 19i)B + 19\sqrt{2}B \right) \cos\left[\frac{1}{2}(c + dx)\right]^3 \right. \\
& \quad \left. \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{3/2} \right) / \left(\sqrt{2}(i + \sqrt{2})d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \left(\left(\frac{1}{16} + \frac{i}{16} \right) \left((1 + i) + \sqrt{2} \right) \left((36 - 12i)A - 12\sqrt{2}A - (57 - 19i)B + 19\sqrt{2}B \right) \cos\left[\frac{1}{2}(c + dx)\right]^3 \right. \\
& \quad \left. \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{3/2} \right) / \left(\sqrt{2}(i + \sqrt{2})d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \frac{(A - B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2}}{2d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^2} + \frac{(-A + B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2}}{2d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^2} + \\
& \frac{(4A - 5B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2}}{2d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)} + \\
& \frac{B \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} + \\
& \frac{B \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2} \sin\left[\frac{1}{2}(c + dx)\right]}{d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2} + \\
& \frac{(-4A + 5B) \cos\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Sec}[c + dx]^{3/2}}{2d(a(1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)}
\end{aligned}$$

■ **Problem 256: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx])}{(a + a \operatorname{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{(2A - 3B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{3/2} d} - \frac{(5A - 9B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} +$$

$$\frac{(A - B) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{2d (a + a \operatorname{Sec}[c+dx])^{3/2}} - \frac{(A - 3B) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{2ad \sqrt{a + a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 1157 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{2} - \frac{i}{2} \right) \left((1+i) - i\sqrt{2} \right) \left((-6-2i)A - 2\sqrt{2}A + (9+3i)B + 3\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) \\
& \quad \cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2} \Big/ \left(\sqrt{2}(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \right) + \\
& \left(\left(\frac{1}{2} - \frac{i}{2} \right) \left((1+i) + \sqrt{2} \right) \left((6-2i)A - 2\sqrt{2}A - (9-3i)B + 3\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) \\
& \quad \cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2} \Big/ \left(\sqrt{2}(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \right) + \\
& \frac{(5A-9B)\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{3/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{3/2}} + \\
& \frac{(-5A+9B)\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{3/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{3/2}} + \\
& \frac{(4A+2i\sqrt{2}A-6B-3i\sqrt{2}B)\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Log}\left[\sqrt{2}+2\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{3/2}}{(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{3/2}} + \\
& \left(\left(\frac{1}{4} + \frac{i}{4} \right) \left((1+i) - i\sqrt{2} \right) \left((-6-2i)A - 2\sqrt{2}A + (9+3i)B + 3\sqrt{2}B \right) \cos\left[\frac{1}{2}(c+dx)\right]^3 \right. \\
& \quad \left. \operatorname{Log}\left[2-\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{3/2} \right) \Big/ \left(\sqrt{2}(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \right) - \\
& \left(\left(\frac{1}{4} + \frac{i}{4} \right) \left((1+i) + \sqrt{2} \right) \left((6-2i)A - 2\sqrt{2}A - (9-3i)B + 3\sqrt{2}B \right) \cos\left[\frac{1}{2}(c+dx)\right]^3 \right. \\
& \quad \left. \operatorname{Log}\left[2+\sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{3/2} \right) \Big/ \left(\sqrt{2}(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \right) + \\
& \frac{(-A+B)\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2}}{2d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right)^2} + \frac{(A-B)\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2}}{2d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right)^2} + \\
& \frac{2B\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} - \\
& \frac{2B\cos\left[\frac{1}{2}(c+dx)\right]^3 \operatorname{Sec}[c+dx]^{3/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)}
\end{aligned}$$

- **Problem 257: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$\frac{2 B \text{ArcSinh}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{a^{3/2} d} + \frac{(A - 5 B) \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \text{Sec}[c+dx]}}\right]}{2 \sqrt{2} a^{3/2} d} + \frac{(A - B) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{2 d (a + a \text{Sec}[c + d x])^{3/2}}$$

Result (type 3, 430 leaves):

$$\frac{1}{2 d (a (1 + \text{Sec}[c + d x]))^{3/2}} \text{Cos}\left[\frac{1}{2} (c + d x)\right]^3 \text{Sec}[c + d x]^{3/2} \left(-4 i \sqrt{2} B \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{1}{4} (c + d x)\right] - (-1 + \sqrt{2}) \text{Sin}\left[\frac{1}{4} (c + d x)\right]}{(1 + \sqrt{2}) \text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] - 4 i \sqrt{2} B \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{1}{4} (c + d x)\right] - (1 + \sqrt{2}) \text{Sin}\left[\frac{1}{4} (c + d x)\right]}{(-1 + \sqrt{2}) \text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] - 2 (A - 5 B) \text{Log}\left[\text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right]\right] + 2 (A - 5 B) \text{Log}\left[\text{Cos}\left[\frac{1}{4} (c + d x)\right] + \text{Sin}\left[\frac{1}{4} (c + d x)\right]\right] + 4 \sqrt{2} B \text{Log}\left[\sqrt{2} + 2 \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 2 \sqrt{2} B \text{Log}\left[2 - \sqrt{2} \text{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 2 \sqrt{2} B \text{Log}\left[2 + \sqrt{2} \text{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \frac{A - B}{(\text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right])^2} + \frac{-A + B}{(\text{Cos}\left[\frac{1}{4} (c + d x)\right] + \text{Sin}\left[\frac{1}{4} (c + d x)\right])^2} \right)$$

- **Problem 262: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{7/2} (A + B \text{Sec}[c + d x])}{(a + a \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 3, 246 leaves, 8 steps):

$$\frac{(2 A - 5 B) \text{ArcSinh}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{a^{5/2} d} - \frac{(43 A - 115 B) \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\text{Sec}[c+dx]} \text{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \text{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} + \frac{(A - B) \text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{4 d (a + a \text{Sec}[c + d x])^{5/2}} + \frac{(7 A - 15 B) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{16 a d (a + a \text{Sec}[c + d x])^{3/2}} - \frac{(11 A - 35 B) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{16 a^2 d \sqrt{a + a \text{Sec}[c + d x]}}$$

Result (type 3, 1304 leaves):

$$- \left((1 - i) \left((1 + i) - i \sqrt{2} \right) \left((-6 - 2 i) A - 2 \sqrt{2} A + (15 + 5 i) B + 5 \sqrt{2} B \right) \text{ArcTan}\left[\frac{\text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right] - \sqrt{2} \text{Sin}\left[\frac{1}{4} (c + d x)\right]}{-\text{Cos}\left[\frac{1}{4} (c + d x)\right] + \sqrt{2} \text{Cos}\left[\frac{1}{4} (c + d x)\right] - \text{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] \right)$$

$$\begin{aligned}
& \left. \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2} \right/ \left(\sqrt{2} (i+\sqrt{2}) d (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right) + \\
& \left((1-i) \left((1+i) + \sqrt{2} \right) \left((6-2i)A - 2\sqrt{2}A - (15-5i)B + 5\sqrt{2}B \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \right) \\
& \left. \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2} \right/ \left(\sqrt{2} (i+\sqrt{2}) d (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right) + \\
& \frac{(43A - 115B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{4d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{(-43A + 115B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{4d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \frac{2(4A + 2i\sqrt{2}A - 10B - 5i\sqrt{2}B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2}}{(i+\sqrt{2})d(a(1+\operatorname{Sec}[c+dx]))^{5/2}} + \\
& \left(\left(\frac{1}{2} + \frac{i}{2} \right) \left((1+i) - i\sqrt{2} \right) \left((-6-2i)A - 2\sqrt{2}A + (15+5i)B + 5\sqrt{2}B \right) \cos\left[\frac{1}{2}(c+dx)\right]^5 \right. \\
& \left. \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2} \right) \left/ \left(\sqrt{2} (i+\sqrt{2}) d (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right) - \right. \\
& \left(\left(\frac{1}{2} + \frac{i}{2} \right) \left((1+i) + \sqrt{2} \right) \left((6-2i)A - 2\sqrt{2}A - (15-5i)B + 5\sqrt{2}B \right) \cos\left[\frac{1}{2}(c+dx)\right]^5 \right. \\
& \left. \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}[c+dx]^{5/2} \right) \left/ \left(\sqrt{2} (i+\sqrt{2}) d (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right) + \right. \\
& \frac{(-A+B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8d(a(1+\operatorname{Sec}[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^4} + \frac{(-11A+19B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8d(a(1+\operatorname{Sec}[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^2} + \\
& \frac{(A-B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8d(a(1+\operatorname{Sec}[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^4} + \\
& \frac{(11A-19B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{8d(a(1+\operatorname{Sec}[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^2} + \\
& \frac{4B \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx]^{5/2}}{d(a(1+\operatorname{Sec}[c+dx]))^{5/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)} -
\end{aligned}$$

$$\frac{4 B \cos\left[\frac{1}{2}(c+dx)\right]^5 \sec[c+dx]^{5/2}}{d(a(1+\sec[c+dx]))^{5/2}\left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)}$$

- **Problem 263: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{5/2}(A+B\sec[c+dx])}{(a+a\sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$\frac{2 B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{a^{5/2} d} + \frac{(3 A - 43 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} +$$

$$\frac{(A - B) \sec[c+dx]^{5/2} \sin[c+dx]}{4 d (a + a \sec[c+dx])^{5/2}} + \frac{(3 A - 11 B) \sec[c+dx]^{3/2} \sin[c+dx]}{16 a d (a + a \sec[c+dx])^{3/2}}$$

Result (type 3, 988 leaves):

$$\begin{aligned}
& - \frac{1}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} 2 \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) B \\
& \operatorname{ArcTan} \left[\frac{\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right]}{-\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right]} \right] \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2} - \frac{1}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} \\
& 2 \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) B \operatorname{ArcTan} \left[\frac{\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right]}{\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right]} \right] \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2} + \\
& \frac{(-3A + 43B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right] \operatorname{Sec}[c + dx]^{5/2}}{4d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{(3A - 43B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right] \operatorname{Sec}[c + dx]^{5/2}}{4d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{4\sqrt{2} B \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Log} \left[\sqrt{2} + 2 \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec}[c + dx]^{5/2}}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \frac{1}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} \\
& i \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) B \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Log} \left[2 - \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec}[c + dx]^{5/2} + \\
& \frac{1}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} \\
& i \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) B \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Log} \left[2 + \sqrt{2} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec}[c + dx]^{5/2} + \\
& \frac{(A - B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right)^4} + \frac{(3A - 11B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] - \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right)^2} + \\
& \frac{(-A + B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right)^4} + \frac{(-3A + 11B) \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos} \left[\frac{1}{4} (c + dx) \right] + \operatorname{Sin} \left[\frac{1}{4} (c + dx) \right] \right)^2}
\end{aligned}$$

■ **Problem 266: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 203 leaves, 5 steps):

$$\frac{(75 A - 19 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d (a + a \operatorname{Sec}[c+dx])^{5/2}} - \frac{(13 A - 5 B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{16 a d (a + a \operatorname{Sec}[c+dx])^{3/2}} + \frac{(49 A - 9 B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{16 a^2 d \sqrt{a + a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 491 leaves):

$$\frac{(75 A - 19 B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \frac{(-75 A + 19 B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \frac{(-A + B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^4} + \frac{(21 A - 13 B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^2} + \frac{(A - B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^4} + \frac{(-21 A + 13 B) \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\operatorname{Cos}\left[\frac{1}{4} (c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + dx)\right]\right)^2} + \frac{16 A \operatorname{Cos}\left[\frac{1}{2} (c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}\left[\frac{1}{2} (c + dx)\right]}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}$$

■ **Problem 267: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 250 leaves, 6 steps):

$$\frac{(163 A - 75 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \operatorname{Sin}[c + dx]}{4 d \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{5/2}} - \frac{(17 A - 9 B) \operatorname{Sin}[c + dx]}{16 a d \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{3/2}} + \frac{(95 A - 39 B) \operatorname{Sin}[c + dx]}{48 a^2 d \sqrt{\operatorname{Sec}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]}} - \frac{(299 A - 147 B) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{48 a^2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 551 leaves):

$$\begin{aligned}
& \frac{(-163 A + 75 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{(163 A - 75 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{(A - B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right)^4} + \frac{(-29 A + 21 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right)^2} + \\
& \frac{(-A + B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right)^4} + \frac{(29 A - 21 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right)^2} + \\
& \frac{8 (-5 A + 2 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \frac{8 A \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \sin\left[\frac{3}{2}(c + dx)\right]}{3 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}
\end{aligned}$$

■ **Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 297 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(283 A - 163 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B) \sin[c + dx]}{4 d \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{5/2}} - \frac{(21 A - 13 B) \sin[c + dx]}{16 a d \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{3/2}} + \\
& \frac{(157 A - 85 B) \sin[c + dx]}{80 a^2 d \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]}} - \frac{(787 A - 475 B) \sin[c + dx]}{240 a^2 d \sqrt{\operatorname{Sec}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{(2671 A - 1495 B) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{240 a^2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}
\end{aligned}$$

Result (type 3, 609 leaves):

$$\begin{aligned}
& \frac{(283 A - 163 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{(-283 A + 163 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^{5/2}}{4 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{(-A + B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right)^4} + \\
& \frac{(37 A - 29 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]\right)^2} + \frac{(A - B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right)^4} + \\
& \frac{(-37 A + 29 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2}}{8 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right]\right)^2} - \frac{40 (-2 A + B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \sin\left[\frac{1}{2}(c + dx)\right]}{d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \\
& \frac{4 (-5 A + 2 B) \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \sin\left[\frac{3}{2}(c + dx)\right]}{3 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}} + \frac{4 A \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^{5/2} \sin\left[\frac{5}{2}(c + dx)\right]}{5 d (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}
\end{aligned}$$

■ **Problem 269: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx])^{2/3} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 6, 406 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 \sqrt{2} A \operatorname{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, \frac{1}{2}(1 + \operatorname{Sec}[c + dx]), 1 + \operatorname{Sec}[c + dx]\right] (a + a \operatorname{Sec}[c + dx])^{2/3} \operatorname{Tan}[c + dx]}{7 d \sqrt{1 - \operatorname{Sec}[c + dx]}} + \\
& \frac{3 B (a + a \operatorname{Sec}[c + dx])^{2/3} \operatorname{Tan}[c + dx]}{2 d (1 + \operatorname{Sec}[c + dx])} - \left(3^{3/4} B \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + dx])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + dx])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\
& \left. (a + a \operatorname{Sec}[c + dx])^{2/3} (2^{1/3} - (1 + \operatorname{Sec}[c + dx])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + dx])^{1/3} + (1 + \operatorname{Sec}[c + dx])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + dx])^{1/3})^2}} \operatorname{Tan}[c + dx] \right) / \\
& \left(2 \times 2^{1/3} d (1 - \operatorname{Sec}[c + dx]) (1 + \operatorname{Sec}[c + dx]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + dx])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + dx])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + dx])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 7123 leaves): Display of huge result suppressed!

■ **Problem 270: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 6, 354 leaves, 8 steps):

$$\frac{3\sqrt{2} A \operatorname{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2}(1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{d\sqrt{1 - \operatorname{Sec}[c + d x]}(a + a \operatorname{Sec}[c + d x])^{1/3}} -$$

$$\left(3^{3/4} B \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3})(1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3})(1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4}(2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3}(1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3})(1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3})(1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 6, 7136 leaves): Display of huge result suppressed!

■ **Problem 271: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{(a + a \operatorname{Sec}[c + d x])^{4/3}} dx$$

Optimal (type 6, 415 leaves, 9 steps):

$$\frac{3 B \tan [c+d x]}{5 a d (1+\sec [c+d x]) (a+a \sec [c+d x])^{1/3}} - \frac{3 \sqrt{2} A \operatorname{AppellF1}\left[-\frac{5}{6}, \frac{1}{2}, 1, \frac{1}{6}, \frac{1}{2}(1+\sec [c+d x]), 1+\sec [c+d x]\right] \tan [c+d x]}{5 a d \sqrt{1-\sec [c+d x]} (1+\sec [c+d x]) (a+a \sec [c+d x])^{1/3}}$$

$$\left(3^{3/4} B \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \sec [c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \sec [c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \sec [c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \sec [c + d x])^{1/3} + (1 + \sec [c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \sec [c + d x])^{1/3})^2}} \tan [c + d x] \right) /$$

$$\left(5 \times 2^{1/3} a d (1 - \sec [c + d x]) (a + a \sec [c + d x])^{1/3} \sqrt{-\frac{(1 + \sec [c + d x])^{1/3} (2^{1/3} - (1 + \sec [c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \sec [c + d x])^{1/3})^2}} \right)$$

Result (type 6, 7385 leaves) : Display of huge result suppressed!

■ **Problem 272: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^{4/3} (A + B \sec [c + d x]) dx$$

Optimal (type 6, 787 leaves, 11 steps) :

$$\begin{aligned}
& \frac{3 a B (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d} + \frac{1}{11 d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& 3 \sqrt{2} a A \operatorname{AppellF1}\left[\frac{11}{6}, \frac{1}{2}, 1, \frac{17}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x] - \\
& \frac{15 (1 + \sqrt{3}) a B (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3})) (1 + \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(15 \times 3^{1/4} a B \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(2 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(5 \times 3^{3/4} (1 - \sqrt{3}) a B \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(4 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 5276 leaves): Display of huge result suppressed!

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{1/3} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 6, 739 leaves, 10 steps):

$$\begin{aligned}
& \frac{3\sqrt{2} \text{A AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, \frac{1}{2}(1 + \text{Sec}[c + dx]), 1 + \text{Sec}[c + dx]\right] (a + a \text{Sec}[c + dx])^{1/3} \text{Tan}[c + dx]}{5d\sqrt{1 - \text{Sec}[c + dx]}} - \\
& \frac{3(1 + \sqrt{3}) \text{B}(a + a \text{Sec}[c + dx])^{1/3} \text{Tan}[c + dx]}{d(1 + \text{Sec}[c + dx])^{2/3} \left(2^{1/3} - (1 + \sqrt{3})\right) (1 + \text{Sec}[c + dx])^{1/3}} + \\
& \left(3 \times 2^{1/3} 3^{1/4} \text{B EllipticE}\left[\text{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}}\right], \frac{1}{4}(2 + \sqrt{3})\right] (a + a \text{Sec}[c + dx])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \text{Sec}[c + dx])^{1/3} + (1 + \text{Sec}[c + dx])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2} \text{Tan}[c + dx]} \right) / \\
& \left(d(1 - \text{Sec}[c + dx]) (1 + \text{Sec}[c + dx])^{2/3} \sqrt{-\frac{(1 + \text{Sec}[c + dx])^{1/3} (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2}} \right) + \\
& \left(3^{3/4} (1 - \sqrt{3}) \text{B EllipticF}\left[\text{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3}}\right], \frac{1}{4}(2 + \sqrt{3})\right] (a + a \text{Sec}[c + dx])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \text{Sec}[c + dx])^{1/3} + (1 + \text{Sec}[c + dx])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2} \text{Tan}[c + dx]} \right) / \\
& \left(2^{2/3} d(1 - \text{Sec}[c + dx]) (1 + \text{Sec}[c + dx])^{2/3} \sqrt{-\frac{(1 + \text{Sec}[c + dx])^{1/3} (2^{1/3} - (1 + \text{Sec}[c + dx])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \text{Sec}[c + dx])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 4726 leaves):

$$\begin{aligned}
& (3 \text{B Cos}[c + dx] ((1 + \text{Cos}[c + dx]) \text{Sec}[c + dx])^{1/3} (a(1 + \text{Sec}[c + dx]))^{1/3} (A + \text{B Sec}[c + dx]) \text{Sin}[c + dx]) / \\
& (d(\text{B} + \text{A Cos}[c + dx]) (1 + \text{Sec}[c + dx])^{1/3}) + \left(2^{1/3} \text{Cos}[c + dx]^2 \left(\text{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \text{Sec}[c + dx]\right)^{1/3} (a(1 + \text{Sec}[c + dx]))^{1/3} \right. \\
& \left. (\text{A} + \text{B Sec}[c + dx]) (\text{A}(1 + \text{Sec}[c + dx])^{1/3} + \text{B}(1 + \text{Sec}[c + dx])^{1/3} - 3 \text{B Cos}[c + dx] (1 + \text{Sec}[c + dx])^{1/3}) \right. \\
& \left. \text{Tan}\left[\frac{1}{2}(c + dx)\right] \left(\left(9(2\text{A} - \text{B}) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \text{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\text{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right]\right)\right) /
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{f n \sqrt{1 - \operatorname{Sec}[e + f x]}} \operatorname{B AppellF1}\left[n, \frac{1}{2}, -\frac{1}{2} - m, 1 + n, \operatorname{Sec}[e + f x], -\operatorname{Sec}[e + f x]\right] \\
& \quad (c \operatorname{Sec}[e + f x])^n (1 + \operatorname{Sec}[e + f x])^{-\frac{1}{2}-m} (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x] - \frac{1}{f n \sqrt{1 - \operatorname{Sec}[e + f x]}} \\
& (A - B) \operatorname{AppellF1}\left[n, \frac{1}{2}, \frac{1}{2} - m, 1 + n, \operatorname{Sec}[e + f x], -\operatorname{Sec}[e + f x]\right] (c \operatorname{Sec}[e + f x])^n (1 + \operatorname{Sec}[e + f x])^{-\frac{1}{2}-m} (a + a \operatorname{Sec}[e + f x])^m \operatorname{Tan}[e + f x]
\end{aligned}$$

Result (type 6, 4897 leaves):

$$\begin{aligned}
& \left(2^{1+m} \left(\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2\right)^n \operatorname{Sec}[e + f x]^{-1-n} (c \operatorname{Sec}[e + f x])^n \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sec}[e + f x]\right)^{m+n} (1 + \operatorname{Sec}[e + f x])^{-m}\right. \\
& \quad \left.(a (1 + \operatorname{Sec}[e + f x])\right)^m (A + B \operatorname{Sec}[e + f x]) \left(A \operatorname{Sec}[e + f x]^n (1 + \operatorname{Sec}[e + f x])^m + B \operatorname{Sec}[e + f x]^{1+n} (1 + \operatorname{Sec}[e + f x])^m\right) \\
& \quad \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right] \left(-\left(3 A \operatorname{AppellF1}\left[\frac{1}{2}, m + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Cos}[e + f x]\right) / \right. \\
& \quad \left.\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 \left((-1 + n) \operatorname{AppellF1}\left[\frac{3}{2}, m + n, 2 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right.\right.\right. \right. \\
& \quad \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + (m + n) \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m + n, 1 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right) - \\
& \quad \left.\left(B \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m + n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) / \left(\operatorname{AppellF1}\left[\frac{1}{2}, 1 + m + n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right.\right.\right. \\
& \quad \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \frac{2}{3} \left(n \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m + n, 1 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right. \\
& \quad \left.\left.(1 + m + n) \operatorname{AppellF1}\left[\frac{3}{2}, 2 + m + n, -n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)\right) / \\
& \left(f (B + A \operatorname{Cos}[e + f x]) \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right) \left(-\frac{1}{\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right)^2} 2^{1+m} \left(\operatorname{Sec}\left[\frac{1}{2}(e + f x)\right]^2\right)^{1+n} \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \operatorname{Sec}[e + f x]\right)^{m+n}\right.\right. \\
& \quad \left.\left.\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \left(-\left(3 A \operatorname{AppellF1}\left[\frac{1}{2}, m + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] \operatorname{Cos}[e + f x]\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, \right.\right.\right. \right. \\
& \quad \left.\left.\left.m + n, 1 - n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 \left((-1 + n) \operatorname{AppellF1}\left[\frac{3}{2}, m + n, 2 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, \right.\right.\right. \right. \\
& \quad \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + (m + n) \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m + n, 1 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right) - \\
& \quad \left.\left.\left(B \operatorname{AppellF1}\left[\frac{1}{2}, 1 + m + n, -n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right]\right) / \left(\operatorname{AppellF1}\left[\frac{1}{2}, 1 + m + n, -n, \frac{3}{2}, \right.\right.\right. \right. \\
& \quad \left.\left.\left.\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \frac{2}{3} \left(n \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m + n, 1 - n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]^2\right] + \right.\right.\right.
\end{aligned}$$

Optimal (type 3, 61 leaves, 5 steps) :

$$\frac{(2 a A + b B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{(A b + a B) \operatorname{Tan}[c + d x]}{d} + \frac{b B \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 164 leaves) :

$$\frac{1}{4 d} \left(-2 (2 a A + b B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. 4 a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 2 b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. \frac{b B}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b B}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 4 (A b + a B) \operatorname{Tan}[c + d x] \right)$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 35 leaves, 4 steps) :

$$a A x + \frac{(A b + a B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{b B \operatorname{Tan}[c + d x]}{d}$$

Result (type 3, 159 leaves) :

$$a A x - \frac{A b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \\ \frac{A b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b B \operatorname{Tan}[c + d x]}{d}$$

■ **Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 35 leaves, 3 steps) :

$$(A b + a B) x + \frac{b B \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{a A \operatorname{Sin}[c + d x]}{d}$$

Result (type 3, 104 leaves) :

$$A b x + a B x - \frac{b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{d} + \frac{a A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d}$$

■ **Problem 286: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^2 (a + b \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{(8 a A b + 4 a^2 B + 3 b^2 B) \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{(4 a^2 A b + 4 A b^3 - a^3 B + 8 a b^2 B) \text{Tan}[c + d x]}{6 b d} +$$

$$\frac{(8 a A b - 2 a^2 B + 9 b^2 B) \text{Sec}[c + d x] \text{Tan}[c + d x]}{24 d} + \frac{(4 A b - a B) (a + b \text{Sec}[c + d x])^2 \text{Tan}[c + d x]}{12 b d} + \frac{B (a + b \text{Sec}[c + d x])^3 \text{Tan}[c + d x]}{4 b d}$$

Result (type 3, 457 leaves):

$$\frac{1}{48 d} \left(-6 (8 a A b + 4 a^2 B + 3 b^2 B) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 6 (8 a A b + 4 a^2 B + 3 b^2 B) \text{Log}\left[\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \right.$$

$$\frac{3 b^2 B}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^4} + \frac{12 a^2 B + 8 a b (3 A + B) + b^2 (4 A + 9 B)}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{8 b (A b + 2 a B) \text{Sin}\left[\frac{1}{2} (c + d x)\right]}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^3} +$$

$$\frac{16 (3 a^2 A + 2 A b^2 + 4 a b B) \text{Sin}\left[\frac{1}{2} (c + d x)\right]}{\text{Cos}\left[\frac{1}{2} (c + d x)\right] - \text{Sin}\left[\frac{1}{2} (c + d x)\right]} - \frac{3 b^2 B}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^4} +$$

$$\left. \frac{8 b (A b + 2 a B) \text{Sin}\left[\frac{1}{2} (c + d x)\right]}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^3} - \frac{12 a^2 B + 8 a b (3 A + B) + b^2 (4 A + 9 B)}{\left(\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]\right)^2} + \frac{16 (3 a^2 A + 2 A b^2 + 4 a b B) \text{Sin}\left[\frac{1}{2} (c + d x)\right]}{\text{Cos}\left[\frac{1}{2} (c + d x)\right] + \text{Sin}\left[\frac{1}{2} (c + d x)\right]} \right)$$

■ **Problem 287: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + b \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 3, 116 leaves, 6 steps):

$$\frac{(2 a^2 A + A b^2 + 2 a b B) \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} + \frac{2 (3 a A b + a^2 B + b^2 B) \text{Tan}[c + d x]}{3 d} +$$

$$\frac{b (3 A b + 2 a B) \text{Sec}[c + d x] \text{Tan}[c + d x]}{6 d} + \frac{B (a + b \text{Sec}[c + d x])^2 \text{Tan}[c + d x]}{3 d}$$

Result (type 3, 239 leaves):

$$\frac{1}{6d} \operatorname{Sec}[c+dx]^3 \left(-\frac{9}{4} (2a^2A + Ab^2 + 2abB) \operatorname{Cos}[c+dx] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) - \right. \\ \left. \frac{3}{4} (2a^2A + Ab^2 + 2abB) \operatorname{Cos}[3(c+dx)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\ \left. (6ab + 3a^2B + 4b^2B + 3b(Ab + 2aB) \operatorname{Cos}[c+dx] + (6ab + 3a^2B + 2b^2B) \operatorname{Cos}[2(c+dx)]) \operatorname{Sin}[c+dx] \right)$$

■ **Problem 288: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c+dx])^2 (A + B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$a^2 Ax + \frac{(4aAb + 2a^2B + b^2B) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2d} + \frac{b(2Ab + 3aB) \operatorname{Tan}[c+dx]}{2d} + \frac{bB(a + b \operatorname{Sec}[c+dx]) \operatorname{Tan}[c+dx]}{2d}$$

Result (type 3, 345 leaves):

$$\frac{1}{4d} \operatorname{Sec}[c+dx]^2 \left(2a^2Ac + 2a^2Adx - 4aAb \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - 2a^2B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right. \\ \left. b^2B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + 4aAb \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\ \left. 2a^2B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + b^2B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \\ \left. \operatorname{Cos}[2(c+dx)] \left(2a^2A(c+dx) - (4aAb + 2a^2B + b^2B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \\ \left. \left. (4aAb + 2a^2B + b^2B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right) + 2b^2B \operatorname{Sin}[c+dx] + 2Ab^2 \operatorname{Sin}[2(c+dx)] + 4abB \operatorname{Sin}[2(c+dx)] \right)$$

■ **Problem 295: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx] (a + b \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$\frac{(8a^3A + 12aAb^2 + 12a^2bB + 3b^3B) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8d} + \frac{(16a^2Ab + 4Ab^3 + 3a^3B + 12ab^2B) \operatorname{Tan}[c+dx]}{6d} + \\ \frac{b(20aAb + 6a^2B + 9b^2B) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{24d} + \frac{(4Ab + 3aB)(a + b \operatorname{Sec}[c+dx])^2 \operatorname{Tan}[c+dx]}{12d} + \frac{B(a + b \operatorname{Sec}[c+dx])^3 \operatorname{Tan}[c+dx]}{4d}$$

Result (type 3, 1179 leaves):

$$\begin{aligned}
& \left((-8 a^3 A - 12 a A b^2 - 12 a^2 b B - 3 b^3 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \right) / \\
& \quad (8 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x])) + \\
& \left((8 a^3 A + 12 a A b^2 + 12 a^2 b B + 3 b^3 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \right) / \\
& \quad (8 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x])) + \frac{b^3 B \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{16 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^4} + \\
& \frac{(36 a A b^2 + 4 A b^3 + 36 a^2 b B + 12 a b^2 B + 9 b^3 B) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{48 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} - \\
& \frac{b^3 B \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{16 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^4} + \\
& \frac{(-36 a A b^2 - 4 A b^3 - 36 a^2 b B - 12 a b^2 B - 9 b^3 B) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{48 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} + \\
& \frac{\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) (A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])}{6 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} + \\
& \frac{\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) (A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])}{6 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} + \left(\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x]) \right)^3 \\
& \quad (A + B \operatorname{Sec}[c + d x]) \left(9 a^2 A b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) + \left(\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x]) \right)^3 \\
& \quad (A + B \operatorname{Sec}[c + d x]) \left(9 a^2 A b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right)
\end{aligned}$$

■ **Problem 296: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$a^3 A x + \frac{(6 a^2 A b + A b^3 + 2 a^3 B + 3 a b^2 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} +$$

$$\frac{b (9 a A b + 8 a^2 B + 2 b^2 B) \operatorname{Tan}[c + d x]}{3 d} + \frac{b^2 (3 A b + 5 a B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} + \frac{b B (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 968 leaves):

$$a^3 A (c + d x) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])$$

$$\frac{d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x])}{+}$$

$$\left((-6 a^2 A b - A b^3 - 2 a^3 B - 3 a b^2 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \right) /$$

$$(2 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x])) +$$

$$\left((6 a^2 A b + A b^3 + 2 a^3 B + 3 a b^2 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \right) /$$

$$(2 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x])) + \frac{(3 A b^3 + 9 a b^2 B + b^3 B) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{12 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right])^2} +$$

$$\frac{b^3 B \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{6 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right])^3} +$$

$$\frac{b^3 B \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]}{6 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right])^3} +$$

$$\frac{(-3 A b^3 - 9 a b^2 B - b^3 B) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x])}{12 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right])^2} +$$

$$\left(\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \left(9 a A b^2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 9 a^2 b B \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 2 b^3 B \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) /$$

$$\left(3 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) +$$

$$\left(\operatorname{Cos}[c + d x]^4 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) \left(9 a A b^2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 9 a^2 b B \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] + 2 b^3 B \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right) /$$

$$\left(3 d (b + a \operatorname{Cos}[c + d x])^3 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right) \right)$$

■ **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 119 leaves, 6 steps):

$$a^2 (3 A b + a B) x + \frac{b (6 a A b + 6 a^2 B + b^2 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} +$$

$$\frac{a^2 (2 a A - b B) \operatorname{Sin}[c + d x]}{2 d} + \frac{b B (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 d} + \frac{b^2 (A b + 2 a B) \operatorname{Tan}[c + d x]}{d}$$

Result (type 3, 399 leaves):

$$\frac{1}{4 d} \operatorname{Sec}[c + d x]^2$$

$$\left(6 a^2 A b c + 2 a^3 B c + 6 a^2 A b d x + 2 a^3 B d x - 6 a A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 6 a^2 b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \right.$$

$$b^3 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 6 a A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] +$$

$$6 a^2 b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + b^3 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \operatorname{Cos}[2(c + d x)] \left(2 a^2 (3 A b + a B) (c + d x) - \right.$$

$$b (6 a A b + 6 a^2 B + b^2 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + b (6 a A b + 6 a^2 B + b^2 B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \left. \right) +$$

$$\left. (a^3 A + 2 b^3 B) \operatorname{Sin}[c + d x] + 2 A b^3 \operatorname{Sin}[2(c + d x)] + 6 a b^2 B \operatorname{Sin}[2(c + d x)] + a^3 A \operatorname{Sin}[3(c + d x)] \right)$$

■ **Problem 304: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 200 leaves, 7 steps):

$$a^4 A x + \frac{(32 a^3 A b + 16 a A b^3 + 8 a^4 B + 24 a^2 b^2 B + 3 b^4 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{b (34 a^2 A b + 4 A b^3 + 19 a^3 B + 16 a b^2 B) \operatorname{Tan}[c + d x]}{6 d} +$$

$$\frac{b^2 (32 a A b + 26 a^2 B + 9 b^2 B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{24 d} + \frac{b (4 A b + 7 a B) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} + \frac{b B (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 455 leaves):

1

$$96 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x])$$

$$\operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \left(36 a^4 A (c + d x) + 48 a^4 A (c + d x) \operatorname{Cos}[2 (c + d x)] + 12 a^4 A (c + d x) \operatorname{Cos}[4 (c + d x)] - \right.$$

$$12 (32 a^3 A b + 16 a A b^3 + 8 a^4 B + 24 a^2 b^2 B + 3 b^4 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] +$$

$$12 (32 a^3 A b + 16 a A b^3 + 8 a^4 B + 24 a^2 b^2 B + 3 b^4 B) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] +$$

$$48 a A b^3 \operatorname{Sin}[c + d x] + 72 a^2 b^2 B \operatorname{Sin}[c + d x] + 33 b^4 B \operatorname{Sin}[c + d x] + 144 a^2 A b^2 \operatorname{Sin}[2 (c + d x)] + 32 A b^4 \operatorname{Sin}[2 (c + d x)] +$$

$$96 a^3 b B \operatorname{Sin}[2 (c + d x)] + 128 a b^3 B \operatorname{Sin}[2 (c + d x)] + 48 a A b^3 \operatorname{Sin}[3 (c + d x)] + 72 a^2 b^2 B \operatorname{Sin}[3 (c + d x)] +$$

$$\left. 9 b^4 B \operatorname{Sin}[3 (c + d x)] + 72 a^2 A b^2 \operatorname{Sin}[4 (c + d x)] + 8 A b^4 \operatorname{Sin}[4 (c + d x)] + 48 a^3 b B \operatorname{Sin}[4 (c + d x)] + 32 a b^3 B \operatorname{Sin}[4 (c + d x)] \right)$$

■ **Problem 305: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 195 leaves, 7 steps):

$$a^3 (4 A b + a B) x + \frac{b (12 a^2 A b + A b^3 + 8 a^3 B + 4 a b^2 B) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} +$$

$$\frac{a A (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{d} - \frac{b (6 a^3 A - 12 a A b^2 - 17 a^2 b B - 2 b^3 B) \operatorname{Tan}[c + d x]}{3 d} -$$

$$\frac{b^2 (6 a^2 A - 3 A b^2 - 8 a b B) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} - \frac{b (3 a A - b B) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 1051 leaves):

$$\begin{aligned}
& \frac{a^3 (4 A b + a B) (c + d x) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x])} + \\
& \left(\frac{(-12 a^2 A b^2 - A b^4 - 8 a^3 b B - 4 a b^3 B) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{2 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x])} \right) / \\
& \left(\frac{(12 a^2 A b^2 + A b^4 + 8 a^3 b B + 4 a b^3 B) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{2 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x])} \right) / \\
& \left(\frac{(3 A b^4 + 12 a b^3 B + b^4 B) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{12 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} + \right. \\
& \left. \frac{b^4 B \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} + \right. \\
& \left. \frac{b^4 B \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^3} + \right. \\
& \left. \frac{(-3 A b^4 - 12 a b^3 B - b^4 B) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x])}{12 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) (\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right])^2} + \right. \\
& \left. \left(\frac{2 \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \left(6 a A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + b^4 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)}{3 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)} \right) + \right. \\
& \left. \left(\frac{2 \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \left(6 a A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + b^4 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)}{3 d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)} \right) + \right. \\
& \left. \frac{a^4 A \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{d (b + a \operatorname{Cos}[c + d x])^4 (B + A \operatorname{Cos}[c + d x])} \right)
\end{aligned}$$

■ **Problem 311: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4 (A + B \operatorname{Sec}[c + d x])}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 187 leaves, 8 steps):

$$\frac{(2a^2 + b^2)(Ab - aB) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^4 d} - \frac{2a^3(Ab - aB) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^4 \sqrt{a+b} d} -$$

$$\frac{(3aAb - 3a^2B - 2b^2B) \operatorname{Tan}[c + dx]}{3b^3 d} + \frac{(Ab - aB) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2b^2 d} + \frac{B \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3bd}$$

Result (type 3, 422 leaves):

$$\frac{1}{12b^4 d} \left(\frac{24a^3(Ab - aB) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + 6(2a^2 + b^2)(-Ab + aB) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$6(2a^2 + b^2)(-Ab + aB) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \frac{b^2(3Ab + (-3a+b)B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{2b^3B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{4b(-3aAb + 3a^2B + 2b^2B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} +$$

$$\left. \frac{2b^3B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{b^2(3Ab + (-3a+b)B)}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4b(-3aAb + 3a^2B + 2b^2B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 312: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sec}[c + dx])}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 143 leaves, 7 steps):

$$- \frac{(2aAb - 2a^2B - b^2B) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^3 d} + \frac{2a^2(Ab - aB) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^3 \sqrt{a+b} d} + \frac{(Ab - aB) \operatorname{Tan}[c + dx]}{b^2 d} + \frac{B \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2bd}$$

Result (type 3, 300 leaves):

$$\frac{1}{4 b^3 d} \left(\frac{8 a^2 (-A b + a B) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} - 2 (-2 a A b + 2 a^2 B + b^2 B) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \right) +$$

$$2 (-2 a A b + 2 a^2 B + b^2 B) \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] + \frac{b^2 B}{\left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)^2} +$$

$$\left. \frac{4 b (A b - a B) \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]}{\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]} - \frac{b^2 B}{\left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)^2} + \frac{4 b (A b - a B) \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]}{\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]} \right)$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{(a + b \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 3, 292 leaves, 7 steps):

$$\frac{A x}{a^4} - \frac{(8 a^6 A b - 8 a^4 A b^3 + 7 a^2 A b^5 - 2 A b^7 - 2 a^7 B - 3 a^5 b^2 B) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{a^4 (a-b)^{7/2} (a+b)^{7/2} d} + \frac{b (A b - a B) \operatorname{Tan}[c + dx]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + dx])^3} +$$

$$\frac{b (8 a^2 A b - 3 A b^3 - 5 a^3 B) \operatorname{Tan}[c + dx]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + dx])^2} + \frac{b (26 a^4 A b - 17 a^2 A b^3 + 6 A b^5 - 11 a^5 B - 4 a^3 b^2 B) \operatorname{Tan}[c + dx]}{6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + dx])}$$

Result (type 3, 769 leaves):

$$\frac{1}{24 a^4 d (B + A \cos [c + d x]) (a + b \sec [c + d x])^4} (b + a \cos [c + d x]) \sec [c + d x]^3 (A + B \sec [c + d x])$$

$$\left(-\frac{1}{(a^2 - b^2)^{7/2}} 24 (-8 a^6 A b + 8 a^4 A b^3 - 7 a^2 A b^5 + 2 A b^7 + 2 a^7 B + 3 a^5 b^2 B) \operatorname{ArcTanh}\left[\frac{(-a + b) \tan\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a^2 - b^2}}\right] (b + a \cos [c + d x])^3 + \right.$$

$$\frac{1}{(a^2 - b^2)^3} (36 a^8 A b c - 84 a^6 A b^3 c + 36 a^4 A b^5 c + 36 a^2 A b^7 c - 24 A b^9 c + 36 a^8 A b d x - 84 a^6 A b^3 d x + 36 a^4 A b^5 d x + 36 a^2 A b^7 d x - 24 A b^9 d x +$$

$$18 a A (a^2 - b^2)^3 (a^2 + 4 b^2) (c + d x) \cos [c + d x] + 36 a^2 A b (a^2 - b^2)^3 (c + d x) \cos [2 (c + d x)] + 6 a^9 A c \cos [3 (c + d x)] - 18 a^7 A b^2 c$$

$$\cos [3 (c + d x)] + 18 a^5 A b^4 c \cos [3 (c + d x)] - 6 a^3 A b^6 c \cos [3 (c + d x)] + 6 a^9 A d x \cos [3 (c + d x)] - 18 a^7 A b^2 d x \cos [3 (c + d x)] +$$

$$18 a^5 A b^4 d x \cos [3 (c + d x)] - 6 a^3 A b^6 d x \cos [3 (c + d x)] + 36 a^7 A b^2 \sin [c + d x] + 72 a^5 A b^4 \sin [c + d x] - 57 a^3 A b^6 \sin [c + d x] +$$

$$24 a A b^8 \sin [c + d x] - 18 a^8 b B \sin [c + d x] - 39 a^6 b^3 B \sin [c + d x] - 18 a^4 b^5 B \sin [c + d x] + 120 a^6 A b^3 \sin [2 (c + d x)] -$$

$$90 a^4 A b^5 \sin [2 (c + d x)] + 30 a^2 A b^7 \sin [2 (c + d x)] - 54 a^7 b^2 B \sin [2 (c + d x)] - 6 a^5 b^4 B \sin [2 (c + d x)] + 36 a^7 A b^2 \sin [3 (c + d x)] -$$

$$\left. 32 a^5 A b^4 \sin [3 (c + d x)] + 11 a^3 A b^6 \sin [3 (c + d x)] - 18 a^8 b B \sin [3 (c + d x)] + 5 a^6 b^3 B \sin [3 (c + d x)] - 2 a^4 b^5 B \sin [3 (c + d x)] \right)$$

■ **Problem 342: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (A + B \sec [c + d x])}{(a + b \sec [c + d x])^4} dx$$

Optimal (type 3, 411 leaves, 8 steps):

$$-\frac{(4 A b - a B) x}{a^5} + \frac{1}{a^5 (a - b)^{7/2} (a + b)^{7/2} d}$$

$$b (20 a^6 A b - 35 a^4 A b^3 + 28 a^2 A b^5 - 8 A b^7 - 8 a^7 B + 8 a^5 b^2 B - 7 a^3 b^4 B + 2 a b^6 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a - b} \tan\left[\frac{1}{2}(c + d x)\right]}{\sqrt{a + b}}\right] +$$

$$\frac{(6 a^6 A - 65 a^4 A b^2 + 68 a^2 A b^4 - 24 A b^6 + 26 a^5 b B - 17 a^3 b^3 B + 6 a b^5 B) \sin [c + d x]}{6 a^4 (a^2 - b^2)^3 d} + \frac{b (A b - a B) \sin [c + d x]}{3 a (a^2 - b^2) d (a + b \sec [c + d x])^3} +$$

$$\frac{b (9 a^2 A b - 4 A b^3 - 6 a^3 B + a b^2 B) \sin [c + d x]}{6 a^2 (a^2 - b^2)^2 d (a + b \sec [c + d x])^2} + \frac{b (12 a^4 A b - 11 a^2 A b^3 + 4 A b^5 - 6 a^5 B + 2 a^3 b^2 B - a b^4 B) \sin [c + d x]}{2 a^3 (a^2 - b^2)^3 d (a + b \sec [c + d x])}$$

Result (type 3, 1372 leaves):

$$\begin{aligned}
& - \left(b \left(-20 a^6 A b + 35 a^4 A b^3 - 28 a^2 A b^5 + 8 A b^7 + 8 a^7 B - 8 a^5 b^2 B + 7 a^3 b^4 B - 2 a b^6 B \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right] \right. \\
& \quad \left. (b+a \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^3 (A+B \operatorname{Sec}[c+dx]) \right) \Bigg/ \left(a^5 \sqrt{a^2-b^2} (-a^2+b^2)^3 d (B+A \operatorname{Cos}[c+dx]) (a+b \operatorname{Sec}[c+dx])^4 \right) + \\
& \frac{1}{24 a^5 (a^2-b^2)^3 d (B+A \operatorname{Cos}[c+dx]) (a+b \operatorname{Sec}[c+dx])^4} (b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^3 (A+B \operatorname{Sec}[c+dx]) \\
& \quad (-144 a^8 A b^2 (c+dx) + 336 a^6 A b^4 (c+dx) - 144 a^4 A b^6 (c+dx) - 144 a^2 A b^8 (c+dx) + 96 A b^{10} (c+dx) + 36 a^9 b B (c+dx) - 84 a^7 b^3 B (c+dx) + \\
& \quad 36 a^5 b^5 B (c+dx) + 36 a^3 b^7 B (c+dx) - 24 a b^9 B (c+dx) - 72 a^9 A b (c+dx) \operatorname{Cos}[c+dx] - 72 a^7 A b^3 (c+dx) \operatorname{Cos}[c+dx] + \\
& \quad 648 a^5 A b^5 (c+dx) \operatorname{Cos}[c+dx] - 792 a^3 A b^7 (c+dx) \operatorname{Cos}[c+dx] + 288 a A b^9 (c+dx) \operatorname{Cos}[c+dx] + 18 a^{10} B (c+dx) \operatorname{Cos}[c+dx] + \\
& \quad 18 a^8 b^2 B (c+dx) \operatorname{Cos}[c+dx] - 162 a^6 b^4 B (c+dx) \operatorname{Cos}[c+dx] + 198 a^4 b^6 B (c+dx) \operatorname{Cos}[c+dx] - 72 a^2 b^8 B (c+dx) \operatorname{Cos}[c+dx] - \\
& \quad 144 a^8 A b^2 (c+dx) \operatorname{Cos}[2(c+dx)] + 432 a^6 A b^4 (c+dx) \operatorname{Cos}[2(c+dx)] - 432 a^4 A b^6 (c+dx) \operatorname{Cos}[2(c+dx)] + \\
& \quad 144 a^2 A b^8 (c+dx) \operatorname{Cos}[2(c+dx)] + 36 a^9 b B (c+dx) \operatorname{Cos}[2(c+dx)] - 108 a^7 b^3 B (c+dx) \operatorname{Cos}[2(c+dx)] + \\
& \quad 108 a^5 b^5 B (c+dx) \operatorname{Cos}[2(c+dx)] - 36 a^3 b^7 B (c+dx) \operatorname{Cos}[2(c+dx)] - 24 a^9 A b (c+dx) \operatorname{Cos}[3(c+dx)] + \\
& \quad 72 a^7 A b^3 (c+dx) \operatorname{Cos}[3(c+dx)] - 72 a^5 A b^5 (c+dx) \operatorname{Cos}[3(c+dx)] + 24 a^3 A b^7 (c+dx) \operatorname{Cos}[3(c+dx)] + \\
& \quad 6 a^{10} B (c+dx) \operatorname{Cos}[3(c+dx)] - 18 a^8 b^2 B (c+dx) \operatorname{Cos}[3(c+dx)] + 18 a^6 b^4 B (c+dx) \operatorname{Cos}[3(c+dx)] - 6 a^4 b^6 B (c+dx) \operatorname{Cos}[3(c+dx)] + \\
& \quad 18 a^9 A b \operatorname{Sin}[c+dx] - 90 a^7 A b^3 \operatorname{Sin}[c+dx] - 135 a^5 A b^5 \operatorname{Sin}[c+dx] + 228 a^3 A b^7 \operatorname{Sin}[c+dx] - 96 a A b^9 \operatorname{Sin}[c+dx] + \\
& \quad 36 a^8 b^2 B \operatorname{Sin}[c+dx] + 72 a^6 b^4 B \operatorname{Sin}[c+dx] - 57 a^4 b^6 B \operatorname{Sin}[c+dx] + 24 a^2 b^8 B \operatorname{Sin}[c+dx] + 6 a^{10} A \operatorname{Sin}[2(c+dx)] + \\
& \quad 18 a^8 A b^2 \operatorname{Sin}[2(c+dx)] - 300 a^6 A b^4 \operatorname{Sin}[2(c+dx)] + 336 a^4 A b^6 \operatorname{Sin}[2(c+dx)] - 120 a^2 A b^8 \operatorname{Sin}[2(c+dx)] + \\
& \quad 120 a^7 b^3 B \operatorname{Sin}[2(c+dx)] - 90 a^5 b^5 B \operatorname{Sin}[2(c+dx)] + 30 a^3 b^7 B \operatorname{Sin}[2(c+dx)] + 18 a^9 A b \operatorname{Sin}[3(c+dx)] - 114 a^7 A b^3 \operatorname{Sin}[3(c+dx)] + \\
& \quad 125 a^5 A b^5 \operatorname{Sin}[3(c+dx)] - 44 a^3 A b^7 \operatorname{Sin}[3(c+dx)] + 36 a^8 b^2 B \operatorname{Sin}[3(c+dx)] - 32 a^6 b^4 B \operatorname{Sin}[3(c+dx)] + \\
& \quad 11 a^4 b^6 B \operatorname{Sin}[3(c+dx)] + 3 a^{10} A \operatorname{Sin}[4(c+dx)] - 9 a^8 A b^2 \operatorname{Sin}[4(c+dx)] + 9 a^6 A b^4 \operatorname{Sin}[4(c+dx)] - 3 a^4 A b^6 \operatorname{Sin}[4(c+dx)] \Big)
\end{aligned}$$

■ **Problem 343: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 (A+B \operatorname{Sec}[c+dx])}{(a+b \operatorname{Sec}[c+dx])^4} dx$$

Optimal (type 3, 538 leaves, 9 steps):

$$\begin{aligned}
& \frac{(a^2 A + 20 A b^2 - 8 a b B) x}{2 a^6} - \frac{1}{a^6 (a-b)^{7/2} (a+b)^{7/2} d} \\
& b^2 (40 a^6 A b - 84 a^4 A b^3 + 69 a^2 A b^5 - 20 A b^7 - 20 a^7 B + 35 a^5 b^2 B - 28 a^3 b^4 B + 8 a b^6 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right] - \\
& \frac{(24 a^6 A b - 146 a^4 A b^3 + 167 a^2 A b^5 - 60 A b^7 - 6 a^7 B + 65 a^5 b^2 B - 68 a^3 b^4 B + 24 a b^6 B) \operatorname{Sin}[c+dx]}{6 a^5 (a^2 - b^2)^3 d} + \\
& \frac{(a^6 A - 23 a^4 A b^2 + 27 a^2 A b^4 - 10 A b^6 + 12 a^5 b B - 11 a^3 b^3 B + 4 a b^5 B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2 a^4 (a^2 - b^2)^3 d} + \\
& \frac{b (A b - a B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{3 a (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^3} + \frac{b (10 a^2 A b - 5 A b^3 - 7 a^3 B + 2 a b^2 B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{6 a^2 (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+dx])^2} + \\
& \frac{b (48 a^4 A b - 53 a^2 A b^3 + 20 A b^5 - 27 a^5 B + 20 a^3 b^2 B - 8 a b^4 B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{6 a^3 (a^2 - b^2)^3 d (a+b \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 3, 1578 leaves):

$$\frac{1}{a^6 \sqrt{a^2 - b^2} (-a^2 + b^2)^3 d}$$

$$b^2 (-40 a^6 A b + 84 a^4 A b^3 - 69 a^2 A b^5 + 20 A b^7 + 20 a^7 B - 35 a^5 b^2 B + 28 a^3 b^4 B - 8 a b^6 B) \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right] +$$

$$\frac{1}{96 a^6 (a^2 - b^2)^3 d (b + a \operatorname{Cos}[c + dx])^3}$$

$$\begin{aligned} & (72 a^{10} A b (c + dx) + 1272 a^8 A b^3 (c + dx) - 3288 a^6 A b^5 (c + dx) + 1512 a^4 A b^7 (c + dx) + 1392 a^2 A b^9 (c + dx) - \\ & 960 A b^{11} (c + dx) - 576 a^9 b^2 B (c + dx) + 1344 a^7 b^4 B (c + dx) - 576 a^5 b^6 B (c + dx) - 576 a^3 b^8 B (c + dx) + 384 a b^{10} B (c + dx) + \\ & 36 a^{11} A (c + dx) \operatorname{Cos}[c + dx] + 756 a^9 A b^2 (c + dx) \operatorname{Cos}[c + dx] + 396 a^7 A b^4 (c + dx) \operatorname{Cos}[c + dx] - 6084 a^5 A b^6 (c + dx) \operatorname{Cos}[c + dx] + \\ & 7776 a^3 A b^8 (c + dx) \operatorname{Cos}[c + dx] - 2880 a A b^{10} (c + dx) \operatorname{Cos}[c + dx] - 288 a^{10} b B (c + dx) \operatorname{Cos}[c + dx] - 288 a^8 b^3 B (c + dx) \operatorname{Cos}[c + dx] + \\ & 2592 a^6 b^5 B (c + dx) \operatorname{Cos}[c + dx] - 3168 a^4 b^7 B (c + dx) \operatorname{Cos}[c + dx] + 1152 a^2 b^9 B (c + dx) \operatorname{Cos}[c + dx] + 72 a^{10} A b (c + dx) \operatorname{Cos}[2(c + dx)] + \\ & 1224 a^8 A b^3 (c + dx) \operatorname{Cos}[2(c + dx)] - 4104 a^6 A b^5 (c + dx) \operatorname{Cos}[2(c + dx)] + 4248 a^4 A b^7 (c + dx) \operatorname{Cos}[2(c + dx)] - \\ & 1440 a^2 A b^9 (c + dx) \operatorname{Cos}[2(c + dx)] - 576 a^9 b^2 B (c + dx) \operatorname{Cos}[2(c + dx)] + 1728 a^7 b^4 B (c + dx) \operatorname{Cos}[2(c + dx)] - \\ & 1728 a^5 b^6 B (c + dx) \operatorname{Cos}[2(c + dx)] + 576 a^3 b^8 B (c + dx) \operatorname{Cos}[2(c + dx)] + 12 a^{11} A (c + dx) \operatorname{Cos}[3(c + dx)] + \\ & 204 a^9 A b^2 (c + dx) \operatorname{Cos}[3(c + dx)] - 684 a^7 A b^4 (c + dx) \operatorname{Cos}[3(c + dx)] + 708 a^5 A b^6 (c + dx) \operatorname{Cos}[3(c + dx)] - \\ & 240 a^3 A b^8 (c + dx) \operatorname{Cos}[3(c + dx)] - 96 a^{10} b B (c + dx) \operatorname{Cos}[3(c + dx)] + 288 a^8 b^3 B (c + dx) \operatorname{Cos}[3(c + dx)] - \\ & 288 a^6 b^5 B (c + dx) \operatorname{Cos}[3(c + dx)] + 96 a^4 b^7 B (c + dx) \operatorname{Cos}[3(c + dx)] + 6 a^{11} A \operatorname{Sin}[c + dx] - 270 a^9 A b^2 \operatorname{Sin}[c + dx] + \\ & 750 a^7 A b^4 \operatorname{Sin}[c + dx] + 1086 a^5 A b^6 \operatorname{Sin}[c + dx] - 2232 a^3 A b^8 \operatorname{Sin}[c + dx] + 960 a A b^{10} \operatorname{Sin}[c + dx] + 72 a^{10} b B \operatorname{Sin}[c + dx] - \\ & 360 a^8 b^3 B \operatorname{Sin}[c + dx] - 540 a^6 b^5 B \operatorname{Sin}[c + dx] + 912 a^4 b^7 B \operatorname{Sin}[c + dx] - 384 a^2 b^9 B \operatorname{Sin}[c + dx] - 60 a^{10} A b \operatorname{Sin}[2(c + dx)] - \\ & 372 a^8 A b^3 \operatorname{Sin}[2(c + dx)] + 2772 a^6 A b^5 \operatorname{Sin}[2(c + dx)] - 3300 a^4 A b^7 \operatorname{Sin}[2(c + dx)] + 1200 a^2 A b^9 \operatorname{Sin}[2(c + dx)] + \\ & 24 a^{11} B \operatorname{Sin}[2(c + dx)] + 72 a^9 b^2 B \operatorname{Sin}[2(c + dx)] - 1200 a^7 b^4 B \operatorname{Sin}[2(c + dx)] + 1344 a^5 b^6 B \operatorname{Sin}[2(c + dx)] - \\ & 480 a^3 b^8 B \operatorname{Sin}[2(c + dx)] + 9 a^{11} A \operatorname{Sin}[3(c + dx)] - 279 a^9 A b^2 \operatorname{Sin}[3(c + dx)] + 1143 a^7 A b^4 \operatorname{Sin}[3(c + dx)] - \\ & 1253 a^5 A b^6 \operatorname{Sin}[3(c + dx)] + 440 a^3 A b^8 \operatorname{Sin}[3(c + dx)] + 72 a^{10} b B \operatorname{Sin}[3(c + dx)] - 456 a^8 b^3 B \operatorname{Sin}[3(c + dx)] + \\ & 500 a^6 b^5 B \operatorname{Sin}[3(c + dx)] - 176 a^4 b^7 B \operatorname{Sin}[3(c + dx)] - 30 a^{10} A b \operatorname{Sin}[4(c + dx)] + 90 a^8 A b^3 \operatorname{Sin}[4(c + dx)] - \\ & 90 a^6 A b^5 \operatorname{Sin}[4(c + dx)] + 30 a^4 A b^7 \operatorname{Sin}[4(c + dx)] + 12 a^{11} B \operatorname{Sin}[4(c + dx)] - 36 a^9 b^2 B \operatorname{Sin}[4(c + dx)] + 36 a^7 b^4 B \operatorname{Sin}[4(c + dx)] - \\ & 12 a^5 b^6 B \operatorname{Sin}[4(c + dx)] + 3 a^{11} A \operatorname{Sin}[5(c + dx)] - 9 a^9 A b^2 \operatorname{Sin}[5(c + dx)] + 9 a^7 A b^4 \operatorname{Sin}[5(c + dx)] - 3 a^5 A b^6 \operatorname{Sin}[5(c + dx)] \end{aligned}$$

■ **Problem 348: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + dx]^4 \sqrt{a + b \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 4, 485 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^5 d} 2 (a-b) \sqrt{a+b} (24 a^3 A b + 57 a A b^3 - 16 a^4 B - 24 a^2 b^2 B + 147 b^4 B) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 A - 49 B) + 18 a b^2 (A - 2 B) + 12 a^2 b (2 A - B) - 16 a^3 B) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{2 (12 a^2 A b - 75 A b^3 - 8 a^3 B - 13 a b^2 B) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{315 b^3 d} + \frac{2 (9 a A b - 6 a^2 B + 49 b^2 B) \text{Sec}[c+d x] \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{315 b^2 d} + \\
& \frac{2 (9 A b + a B) \text{Sec}[c+d x]^2 \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{63 b d} + \frac{2 B \text{Sec}[c+d x]^3 \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{9 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 349: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+d x]^3 \sqrt{a+b \text{Sec}[c+d x]} (A+B \text{Sec}[c+d x]) dx$$

Optimal (type 4, 397 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{105 b^4 d} 2 (a-b) \sqrt{a+b} (14 a^2 A b - 63 A b^3 - 8 a^3 B - 19 a b^2 B) \text{Cot}[c+d x] \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{1}{105 b^3 d} \\
& 2 (a-b) \sqrt{a+b} (b^2 (63 A - 25 B) + 2 a b (7 A - 3 B) - 8 a^2 B) \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{2 (7 a A b - 4 a^2 B + 25 b^2 B) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{105 b^2 d} + \\
& \frac{2 (7 A b + a B) \text{Sec}[c+d x] \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{35 b d} + \frac{2 B \text{Sec}[c+d x]^2 \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{7 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 350: Unable to integrate problem.**

$$\int \text{Sec}[c + d x]^2 \sqrt{a + b \text{Sec}[c + d x]} (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$-\frac{1}{15 b^3 d} 2 (a - b) \sqrt{a + b} (5 a A b - 2 a^2 B + 9 b^2 B) \text{Cot}[c + d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} - \frac{1}{15 b^2 d}$$

$$2 (a - b) \sqrt{a + b} (5 A b - 2 a B - 9 b B) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}}$$

$$\sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 (5 A b - 2 a B) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{15 b d} + \frac{2 B (a + b \text{Sec}[c + d x])^{3/2} \text{Tan}[c + d x]}{5 b d}$$

Result (type 8, 35 leaves):

$$\int \text{Sec}[c + d x]^2 \sqrt{a + b \text{Sec}[c + d x]} (A + B \text{Sec}[c + d x]) dx$$

■ **Problem 351: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + d x] \sqrt{a + b \text{Sec}[c + d x]} (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 4, 256 leaves, 4 steps):

$$-\frac{1}{3 b^2 d} 2 (a - b) \sqrt{a + b} (3 A b + a B) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{1}{3 b d} 2 (a - b) \sqrt{a + b} (3 A - B) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{2 B \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{3 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 352: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \text{Sec}[c + d x]} (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 4, 320 leaves, 5 steps) :

$$\begin{aligned}
 & -\frac{1}{bd} 2 (a-b) \sqrt{a+b} B \cot [c+dx] \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{1}{bd} \\
 & 2 \sqrt{a+b} (Ab + (a-b) B) \cot [c+dx] \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
 & \frac{2A \sqrt{a+b} \cot [c+dx] \text{EllipticPi} \left[\frac{a+b}{a}, \text{ArcSin} \left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}}}{d}
 \end{aligned}$$

Result (type 4, 913 leaves) :

$$\begin{aligned}
& \frac{2 B \cos [c+d x] \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) \sin [c+d x]}{d (B+A \cos [c+d x])} + \\
& \left(2 \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) \left(a \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2} (c+d x) \right] + b \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2} (c+d x) \right] - 2 a \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2} (c+d x) \right]^3 + \right. \right. \\
& a \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2} (c+d x) \right]^5 - b \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2} (c+d x) \right]^5 + 2 i a A \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} + \\
& 2 i a A \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (c+d x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - i (a-b) B \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - \\
& i (a-b) (A-B) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \left. \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} \right) \right) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos [c+d x]} (B+A \cos [c+d x]) \sec [c+d x]^{3/2} \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c+d x) \right]^2}} \left(-1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \right. \\
& \left. \left(1 + \tan \left[\frac{1}{2} (c+d x) \right]^2 \right)^{3/2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c+d x) \right]^2}} \right)
\end{aligned}$$

- **Problem 353: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] \sqrt{a + b \sec[c + dx]} (A + B \sec[c + dx]) dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\frac{A(a-b)\sqrt{a+b}\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{bd} +$$

$$\frac{\sqrt{a+b}(A+2B)\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d} - \frac{1}{ad}$$

$$\frac{\sqrt{a+b}(Ab+2aB)\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d} +$$

$$\frac{A\sqrt{a+b}\sec[c+dx]\sin[c+dx]}{d}$$

Result (type 4, 1107 leaves):

$$\left(\sqrt{a+b}\sec[c+dx] \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(aA\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] + Ab\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] - 2aA\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^3 + aA\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 - \right.$$

$$Ab\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 - 2iAb\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\left. \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 4iAb\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right.$$

$$\left. \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right.$$

$$\begin{aligned}
& 2 i A b \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 4 i a B \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i A (a-b) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 i (a-b) B \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b-b \tan\left[\frac{1}{2}(c+dx)\right]^4+a \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right)
\end{aligned}$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 \sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx]) dx$$

Optimal (type 4, 429 leaves, 7 steps):

$$\frac{1}{4abd} (a-b) \sqrt{a+b} (Ab+4aB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{4ad} \sqrt{a+b} (Ab+2a(A+2B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{4a^2d} \sqrt{a+b} (4a^2A-Ab^2+4abB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{(Ab+4aB) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4ad} + \frac{A \operatorname{Cos}[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1161 leaves):

$$\frac{A \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[2(c+dx)]}{4d} + \left(\sqrt{a+b \operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(aAb \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + Ab^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4a^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 2aAb \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - \right.$$

$$8a^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + aAb \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - Ab^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 4a^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$8a^2A \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$2Ab^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$8abB \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$8a^2A \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+2 A b^2 \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-8 a b B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
& (a+b)(A b+4 a B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-2 a(2 a A-A b+4 b B) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left.\left.\left.\left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right)\right]\right)\right) / \\
& \left(4 a d \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3 / 2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 355: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+b \operatorname{Sec}[c+dx]}(A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 509 leaves, 8 steps):

$$\frac{1}{24 a^2 b d} (a-b) \sqrt{a+b} (16 a^2 A - 3 A b^2 + 6 a b B) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{24 a^2 d} \sqrt{a+b} (2 a+b) (8 a A - 3 A b + 6 a B) \cot [c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{8 a^3 d}$$

$$\sqrt{a+b} (4 a^2 A b + A b^3 + 8 a^3 B - 2 a b^2 B) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{(16 a^2 A - 3 A b^2 + 6 a b B) \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{24 a^2 d} +$$

$$\frac{(A b + 6 a B) \cos [c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{12 a d} + \frac{A \cos [c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 d}$$

Result (type 4, 1565 leaves):

$$\frac{\sqrt{a+b \operatorname{Sec}[c+d x]} \left(\frac{1}{12} A \sin [c+d x] + \frac{(A b + 6 a B) \sin [2(c+d x)]}{24 a} + \frac{1}{12} A \sin [3(c+d x)] \right)}{d} +$$

$$\left(\sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right.$$

$$\left(-16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right] - 16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right] + 3 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right] + 3 A b^3 \tan \left[\frac{1}{2}(c+d x)\right] - 6 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right] - \right.$$

$$6 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right] + 32 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^3 - 6 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3 + 12 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^3 - 16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^5 +$$

$$16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right]^5 + 3 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 3 A b^3 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 6 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^5 + 6 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 +$$

$$\left. 24 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) +$$

$$\begin{aligned}
& 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 12 a b^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 a b^2 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& (a+b) (16 a^2 A - 3 A b^2 + 6 a b B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 a (-A b^2 + 2 a b (7 A - 3 B) + 12 a^2 B) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(24 a^2 d \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right)$$

■ **Problem 356: Attempted integration timed out after 120 seconds.**

$$\int \sec[c+dx]^3 (a+b \sec[c+dx])^{3/2} (A+B \sec[c+dx]) dx$$

Optimal (type 4, 475 leaves, 7 steps):

$$\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (18 a^3 A b - 246 a A b^3 - 8 a^4 B - 33 a^2 b^2 B - 147 b^4 B)$$

$$\cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 A - 49 B) - 3 a b^2 (57 A - 13 B) - 6 a^2 b (3 A - B) + 8 a^3 B) \cot[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{2 (18 a^2 A b - 75 A b^3 - 8 a^3 B - 39 a b^2 B) \sqrt{a+b \sec[c+dx]} \tan[c+dx]}{315 b^2 d} - \frac{2 (18 a A b - 8 a^2 B - 49 b^2 B) (a+b \sec[c+dx])^{3/2} \tan[c+dx]}{315 b^2 d} +$$

$$\frac{2 (9 A b - 4 a B) (a+b \sec[c+dx])^{5/2} \tan[c+dx]}{63 b^2 d} + \frac{2 B \sec[c+dx] (a+b \sec[c+dx])^{5/2} \tan[c+dx]}{9 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 357: Attempted integration timed out after 120 seconds.**

$$\int \sec[c+dx]^2 (a+b \sec[c+dx])^{3/2} (A+B \sec[c+dx]) dx$$

Optimal (type 4, 388 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 b^3 d} 2 (a-b) \sqrt{a+b} (21 a^2 A b + 63 A b^3 - 6 a^3 B + 82 a b^2 B) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 b^2 d} \\
& 2 (a-b) \sqrt{a+b} (b^2 (63 A - 25 B) + 6 a^2 B - a (21 A b - 57 b B)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2(21 a A b - 6 a^2 B + 25 b^2 B) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{105 b d} + \\
& \frac{2(7 A b - 2 a B) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{35 b d} + \frac{2 B (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{7 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 358: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 4, 312 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{15 b^2 d} 2 (a-b) \sqrt{a+b} (20 a A b + 3 a^2 B + 9 b^2 B) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{15 b d} \\
& 2 (a-b) \sqrt{a+b} (15 a A - 5 A b - 3 a B + 9 b B) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \\
& \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2(5 A b + 3 a B) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{15 d} + \frac{2 B (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{5 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 360: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x] (a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 4, 361 leaves, 6 steps):

$$\frac{1}{bd} (a-b) \sqrt{a+b} (aA - 2bB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{d}$$

$$\sqrt{a+b} (2b(A-B) + a(A+4B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{d} \sqrt{a+b} (3Ab + 2aB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{aA \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d}$$

Result (type 4, 979 leaves):

$$\frac{2 b B \cos [c+d x] (a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{d (b+a \cos [c+d x])} +$$

$$\left((a+b \sec [c+d x])^{3 / 2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \left(a^2 A \tan \left[\frac{1}{2}(c+d x)\right] + a A b \tan \left[\frac{1}{2}(c+d x)\right] - 2 a b B \tan \left[\frac{1}{2}(c+d x)\right] - 2 b^2 B \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ \left. \left. 2 a^2 A \tan \left[\frac{1}{2}(c+d x)\right]^3 + 4 a b B \tan \left[\frac{1}{2}(c+d x)\right]^3 + a^2 A \tan \left[\frac{1}{2}(c+d x)\right]^5 - a A b \tan \left[\frac{1}{2}(c+d x)\right]^5 - 2 a b B \tan \left[\frac{1}{2}(c+d x)\right]^5 + \right. \right. \\ \left. \left. 2 b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 - 6 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 a A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \\ \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \right. \\ \left. \left. (a+b)(a A-2 b B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2(2 a b(A-B)+a^2 B-b^2(A+B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) / \\ \left(d (b+a \cos [c+d x])^{3 / 2} \sec [c+d x]^{3 / 2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)^{3 / 2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)$$

■ **Problem 361: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 428 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{4 b d}(a-b) \sqrt{a+b}(5 A b+4 a B) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}+ \\ & \frac{1}{4 d} \sqrt{a+b}(2 a A+5 A b+4 a B+8 b B) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}-\frac{1}{4 a d} \sqrt{a+b}(4 a^2 A+3 A b^2+12 a b B) \cot [c+d x] \\ & \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}+ \\ & \frac{(5 A b+4 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 d}+\frac{a A \cos [c+d x] \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 d} \end{aligned}$$

Result (type 4, 1598 leaves):

$$\begin{aligned} & \frac{a A \cos [c+d x](a+b \sec [c+d x])^{3 / 2} \sin [2(c+d x)]}{4 d(b+a \cos [c+d x])}- \\ & \left((a+b \sec [c+d x])^{3 / 2}\left(5 a A b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]+5 A b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]+4 a^2 \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2}(c+d x)\right]+ \right. \right. \\ & 4 a b \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2}(c+d x)\right]-10 a A b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3-8 a^2 \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2}(c+d x)\right]^3+ \\ & 5 a A b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^5-5 A b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^5+4 a^2 \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2}(c+d x)\right]^5- \\ & \left. 4 a b \sqrt{\frac{-a+b}{a+b}} B \tan \left[\frac{1}{2}(c+d x)\right]^5-8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \\ & \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-6 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 24 i a b B \text{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \\
& i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\left. \right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 8 i a^2 A \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i A b^2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 24 i a b B \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i (a-b) (5 A b + 4 a B) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i (a-b) (2 a A + b (A + 4 B)) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left. \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) \right] \Bigg/ \\
& \left(4 \sqrt{\frac{-a+b}{a+b}} d (b + a \cos[c+dx])^{3/2} \sec[c+dx]^{3/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)
\end{aligned}$$

$$\left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)^{3/2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

■ **Problem 362: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + b \sec[c + dx])^{3/2} (A + B \sec[c + dx]) dx$$

Optimal (type 4, 520 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{24abd} (a-b) \sqrt{a+b} (16a^2A + 3Ab^2 + 30abB) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} + \frac{1}{24ad} \sqrt{a+b} (16a^2A + 14aAb + 3Ab^2 + 12a^2B + 30abB) \\ & \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} - \\ & \frac{1}{8a^2d} \sqrt{a+b} (12a^2Ab - Ab^3 + 8a^3B + 6ab^2B) \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} + \frac{(16a^2A + 3Ab^2 + 30abB) \sqrt{a+b \sec[c + dx]} \sin[c + dx]}{24ad} + \\ & \frac{(7Ab + 6aB) \cos[c + dx] \sqrt{a+b \sec[c + dx]} \sin[c + dx]}{12d} + \frac{aA \cos[c + dx]^2 \sqrt{a+b \sec[c + dx]} \sin[c + dx]}{3d} \end{aligned}$$

Result (type 4, 1551 leaves):

$$\begin{aligned} & \frac{1}{d(b + a \cos[c + dx])} \cos[c + dx] (a + b \sec[c + dx])^{3/2} \left(\frac{1}{12} aA \sin[c + dx] + \frac{1}{24} (7Ab + 6aB) \sin[2(c + dx)] + \frac{1}{12} aA \sin[3(c + dx)] \right) + \\ & \left((a + b \sec[c + dx])^{3/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\ & \left. \left(16a^3A \tan\left[\frac{1}{2}(c + dx)\right] + 16a^2Ab \tan\left[\frac{1}{2}(c + dx)\right] + 3aAb^2 \tan\left[\frac{1}{2}(c + dx)\right] + 3Ab^3 \tan\left[\frac{1}{2}(c + dx)\right] + 30a^2bB \tan\left[\frac{1}{2}(c + dx)\right] + \right. \right. \\ & \left. \left. 30a^2b^2B \tan\left[\frac{1}{2}(c + dx)\right] - 32a^3A \tan\left[\frac{1}{2}(c + dx)\right]^3 - 6aAb^2 \tan\left[\frac{1}{2}(c + dx)\right]^3 - 60a^2bB \tan\left[\frac{1}{2}(c + dx)\right]^3 + 16a^3A \tan\left[\frac{1}{2}(c + dx)\right]^5 - \right. \right. \end{aligned}$$

$$\begin{aligned}
& 16 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 30 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 30 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 72 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 48 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 36 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 72 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 6 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 48 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 36 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + (a+b) (16 a^2 A + 3 A b^2 + 30 a b B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} -
\end{aligned}$$

$$2 a (12 a^2 B + b^2 (-7 A + 24 B) + a (26 A b - 6 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right]$$

$$\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} \right) \Bigg| \Bigg|$$

$$\left(24 a d (b + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Sec}[c + d x]^{3/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right)^{3/2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}}\right)$$

■ **Problem 363: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + d x]^3 (a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 566 leaves, 8 steps):

$$\frac{1}{3465 b^4 d} 2 (a - b) \sqrt{a + b} (110 a^4 A b - 3069 a^2 A b^3 - 1617 A b^5 - 40 a^5 B - 255 a^3 b^2 B - 3705 a b^4 B)$$

$$\operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{3465 b^3 d}$$

$$2 (a - b) \sqrt{a + b} (6 a b^3 (209 A - 505 B) - 3 b^4 (539 A - 225 B) - 15 a^2 b^2 (121 A - 19 B) + 40 a^4 B - a^3 (110 A b - 30 b B))$$

$$\operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 (110 a^3 A b - 1254 a A b^3 - 40 a^4 B - 285 a^2 b^2 B - 675 b^4 B) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{3465 b^2 d} -$$

$$\frac{2 (110 a^2 A b - 539 A b^3 - 40 a^3 B - 335 a b^2 B) (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]}{3465 b^2 d} - \frac{2 (22 a A b - 8 a^2 B - 81 b^2 B) (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Tan}[c + d x]}{693 b^2 d} +$$

$$\frac{2 (11 A b - 4 a B) (a + b \operatorname{Sec}[c + d x])^{7/2} \operatorname{Tan}[c + d x]}{99 b^2 d} + \frac{2 B \operatorname{Sec}[c + d x] (a + b \operatorname{Sec}[c + d x])^{7/2} \operatorname{Tan}[c + d x]}{11 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 364: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c + d x]^2 (a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 469 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (45 a^3 A b + 435 a A b^3 - 10 a^4 B + 279 a^2 b^2 B + 147 b^4 B) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{315 b^2 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 A - 49 B) - 6 a b^2 (60 A - 19 B) + 15 a^2 b (3 A - 11 B) - 10 a^3 B) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{2 (45 a^2 A b + 75 A b^3 - 10 a^3 B + 114 a b^2 B) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{315 b d} + \frac{2 (45 a A b - 10 a^2 B + 49 b^2 B) (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{315 b d} + \\
& \frac{2 (9 A b - 2 a B) (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{63 b d} + \frac{2 B (a+b \text{Sec}[c+d x])^{7/2} \text{Tan}[c+d x]}{9 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 365: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+d x] (a+b \text{Sec}[c+d x])^{5/2} (A+B \text{Sec}[c+d x]) dx$$

Optimal (type 4, 384 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 b^2 d} 2 (a-b) \sqrt{a+b} (161 a^2 A b + 63 A b^3 + 15 a^3 B + 145 a b^2 B) \text{Cot}[c+d x] \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{1}{105 b d} \\
& 2 (a-b) \sqrt{a+b} (b^2 (63 A - 25 B) - 8 a b (7 A - 15 B) + 15 a^2 (7 A - B)) \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{2 (56 a A b + 15 a^2 B + 25 b^2 B) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{105 d} + \\
& \frac{2 (7 A b + 5 a B) (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{35 d} + \frac{2 B (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{7 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 367: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + b \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

Optimal (type 4, 433 leaves, 7 steps):

$$\frac{1}{3bd} (a-b) \sqrt{a+b} (3a^2A - 6Ab^2 - 14abB) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} + \frac{1}{3d} \sqrt{a+b} (2ab(9A - 7B) - 2b^2(3A - B) + 3a^2(A + 6B))$$

$$\cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} - \frac{1}{d}$$

$$a \sqrt{a+b} (5Ab + 2aB) \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{aA(a+b \sec[c + dx])^{3/2} \sin[c + dx]}{d} - \frac{b(3aA - 2bB) \sqrt{a+b \sec[c + dx]} \tan[c + dx]}{3d}$$

Result (type 4, 1146 leaves):

$$\left((a + b \sec[c + dx])^{5/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right.$$

$$\left. \left(3a^3A \tan\left[\frac{1}{2}(c + dx)\right] + 3a^2Ab \tan\left[\frac{1}{2}(c + dx)\right] - 6aAb^2 \tan\left[\frac{1}{2}(c + dx)\right] - 6Ab^3 \tan\left[\frac{1}{2}(c + dx)\right] - 14a^2bB \tan\left[\frac{1}{2}(c + dx)\right] - \right.$$

$$14ab^2B \tan\left[\frac{1}{2}(c + dx)\right] - 6a^3A \tan\left[\frac{1}{2}(c + dx)\right]^3 + 12aAb^2 \tan\left[\frac{1}{2}(c + dx)\right]^3 + 28a^2bB \tan\left[\frac{1}{2}(c + dx)\right]^3 + 3a^3A \tan\left[\frac{1}{2}(c + dx)\right]^5 -$$

$$3a^2Ab \tan\left[\frac{1}{2}(c + dx)\right]^5 - 6aAb^2 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 6Ab^3 \tan\left[\frac{1}{2}(c + dx)\right]^5 - 14a^2bB \tan\left[\frac{1}{2}(c + dx)\right]^5 + 14ab^2B \tan\left[\frac{1}{2}(c + dx)\right]^5 -$$

$$30a^2Ab \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} -$$

$$12a^3B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a+b}} -$$

$$\begin{aligned}
& 30 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 a^3 B \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (3 a^2 A - 6 A b^2 - 14 a b B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (9 a^2 b (A - B) + 3 a^3 B - b^3 (3 A + B) - a b^2 (9 A + 7 B)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \\
& \left(3 d (b + a \text{Cos}[c+dx])^{5/2} \text{Sec}[c+dx]^{5/2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right) + \\
& \frac{\text{Cos}[c+dx]^2 (a+b \text{Sec}[c+dx])^{5/2} \left(\frac{2}{3} b (3 A b + 7 a B) \text{Sin}[c+dx] + \frac{2}{3} b^2 B \text{Tan}[c+dx]\right)}{d (b + a \text{Cos}[c+dx])^2}
\end{aligned}$$

■ **Problem 368: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c+dx]^2 (a+b \text{Sec}[c+dx])^{5/2} (A+B \text{Sec}[c+dx]) dx$$

Optimal (type 4, 450 leaves, 7 steps):

$$\frac{1}{4bd} (a-b) \sqrt{a+b} (9aAb + 4a^2B - 8b^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4d} \sqrt{a+b} (8b^2(A-B) + 2a^2(A+2B) + 3ab(3A+8B))$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{4d}$$

$$\sqrt{a+b} (4a^2A + 15Ab^2 + 20abB) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{a(7Ab+4aB) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d} + \frac{aA \operatorname{Cos}[c+dx] (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1338 leaves):

$$\frac{\operatorname{Cos}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^{5/2} (2b^2B \operatorname{Sin}[c+dx] + \frac{1}{4}a^2A \operatorname{Sin}[2(c+dx)])}{d(b+a \operatorname{Cos}[c+dx])^2} +$$

$$\left((a+b \operatorname{Sec}[c+dx])^{5/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(9a^2Ab \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 9aAb^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$4a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 8ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 8b^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 18a^2Ab \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 -$$

$$8a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 16ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 9a^2Ab \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 9aAb^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$4a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 8ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 8b^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$8a^3A \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$30aAb^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$40a^2bB \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$\begin{aligned}
& 8 a^3 A \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 a A b^2 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 40 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (9 a A b + 4 a^2 B - 8 b^2 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 \left(2 a^3 A - a^2 b (A - 12 B) + 12 a b^2 (A - B) - 4 b^3 (A + B)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 d (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 369: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 4, 518 leaves, 8 steps):

$$\frac{1}{24bd} (a-b) \sqrt{a+b} (16a^2A + 33Ab^2 + 54abB) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{24d} \sqrt{a+b} (16a^2A + 26aAb + 33Ab^2 + 12a^2B + 54abB + 48b^2B)$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{8ad} \sqrt{a+b} (20a^2Ab + 5Ab^3 + 8a^3B + 30ab^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{(16a^2A + 33Ab^2 + 54abB) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24d} +$$

$$\frac{a(3Ab + 2aB) \operatorname{Cos}[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d} + \frac{aA \operatorname{Cos}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 4, 1567 leaves):

$$\frac{1}{d(b+a \operatorname{Cos}[c+dx])^2} \operatorname{Cos}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^{5/2} \left(\frac{1}{12} a^2 A \operatorname{Sin}[c+dx] + \frac{1}{24} a (13Ab + 6aB) \operatorname{Sin}[2(c+dx)] + \frac{1}{12} a^2 A \operatorname{Sin}[3(c+dx)] \right) +$$

$$\left((a+b \operatorname{Sec}[c+dx])^{5/2} \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(16a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 16a^2 Ab \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 33aAb^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 33Ab^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 54a^2 bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$54a^2 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 32a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 66aAb^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 108a^2 bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 16a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$16a^2 Ab \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 33aAb^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 33Ab^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 54a^2 bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 54ab^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$120a^2 Ab \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$30Ab^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$\begin{aligned}
& 48 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 180 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 48 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 180 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (16 a^2 A + 33 A b^2 + 54 a b B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (24 b^3 (A - B) + 12 a^3 B + a b^2 (-13 A + 72 B) + a^2 (38 A b - 6 b B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) /
\end{aligned}$$

$$\left(24 d (b + a \cos [c + d x])^{5/2} \sec [c + d x]^{5/2} \left(1 + \tan \left[\frac{1}{2} (c + d x) \right] \right)^{3/2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right)$$

■ **Problem 371: Unable to integrate problem.**

$$\int \frac{\sec [c + d x]^3 (A + B \sec [c + d x])}{\sqrt{a + b \sec [c + d x]}} dx$$

Optimal (type 4, 329 leaves, 5 steps):

$$\frac{1}{15 b^4 d} 2 (a - b) \sqrt{a + b} (10 a A b - 8 a^2 B - 9 b^2 B) \cot [c + d x] \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec [c + d x]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right]$$

$$\sqrt{\frac{b (1 - \sec [c + d x])}{a + b}} \sqrt{-\frac{b (1 + \sec [c + d x])}{a - b}} + \frac{1}{15 b^3 d} 2 \sqrt{a + b} (b^2 (5 A - 9 B) - 8 a^2 B + 2 a b (5 A + B))$$

$$\cot [c + d x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec [c + d x]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b (1 - \sec [c + d x])}{a + b}} \sqrt{-\frac{b (1 + \sec [c + d x])}{a - b}} +$$

$$\frac{2 (5 A b - 4 a B) \sqrt{a + b \sec [c + d x]} \tan [c + d x]}{15 b^2 d} + \frac{2 B \sec [c + d x] \sqrt{a + b \sec [c + d x]} \tan [c + d x]}{5 b d}$$

Result (type 8, 35 leaves):

$$\int \frac{\sec [c + d x]^3 (A + B \sec [c + d x])}{\sqrt{a + b \sec [c + d x]}} dx$$

■ **Problem 372: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec [c + d x]^2 (A + B \sec [c + d x])}{\sqrt{a + b \sec [c + d x]}} dx$$

Optimal (type 4, 261 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{3 b^3 d} \\
& 2 (a-b) \sqrt{a+b} (3 A b-2 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{3 b^2 d} 2 \sqrt{a+b} (3 A b-(2 a+b) B) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{2 B \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{3 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 373: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+d x] (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 210 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{b^2 d} 2 (a-b) \sqrt{a+b} B \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{2 \sqrt{a+b} (A-B) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b d}
\end{aligned}$$

Result (type 8, 33 leaves):

$$\int \frac{\operatorname{Sec}[c+d x] (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

■ **Problem 375: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x] (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 348 leaves, 6 steps):

$$\begin{aligned}
& \frac{A(a-b)\sqrt{a+b}\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{abd} + \\
& \frac{A\sqrt{a+b}\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{ad} + \frac{1}{a^2d} \\
& \sqrt{a+b}(Ab-2aB)\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{A\sqrt{a+b}\operatorname{Sec}[c+dx]\sin[c+dx]}{ad}
\end{aligned}$$

Result (type 4, 1027 leaves):

$$\begin{aligned}
& \left(\sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \left. \left(a A \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& a A \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \\
& 2 i A b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 4 i a B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i A b \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 4 i a B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i A (a-b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left. \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 i (A b - a B) \right. \\
& \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left(a \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b \operatorname{Sec}[c+dx]} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

- **Problem 376: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (A + B \sec[c + dx])}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 435 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{4a^2bd} \\ & (a-b)\sqrt{a+b} (3Ab-4aB) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{4a^2d} \sqrt{a+b} (3Ab-2a(A+2B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{4a^3d} \sqrt{a+b} (4a^2A+3Ab^2-4abB) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \\ & \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{(3Ab-4aB)\sqrt{a+b\sec[c+dx]}\sin[c+dx]}{4a^2d} + \frac{A\cos[c+dx]\sqrt{a+b\sec[c+dx]}\sin[c+dx]}{2ad} \end{aligned}$$

Result (type 4, 1639 leaves):

$$\begin{aligned} & \frac{A(b+a\cos[c+dx])\sec[c+dx]\sin[2(c+dx)]}{4ad\sqrt{a+b\sec[c+dx]}} + \\ & \left(\sqrt{b+a\cos[c+dx]}\sqrt{\sec[c+dx]}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-3aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \\ & 3Ab^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] + 4a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right] + 4ab\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right] + 6aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^3 - \\ & 8a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^3 - 3aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 + 3Ab^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 + \\ & \left. \left. 4a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^5 - 4ab\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^5 - 8ia^2A\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \right. \end{aligned}$$

$$\begin{aligned}
& i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \\
& \left.\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\right. \\
& \left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-\right. \\
& 8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \\
& \left.\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-6 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\right. \\
& \left.\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+\right. \\
& 8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \\
& \left.\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-i(a-b)(-3 A b+4 a B) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\right. \\
& \left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+\right. \\
& \left.2 i\left(2 a^2 A+3 A b^2-a b(A+4 B)\right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(4a^2 \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b \operatorname{Sec}[c+dx]} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 377: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^3 (A+B \operatorname{Sec}[c+dx])}{\sqrt{a+b \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 525 leaves, 8 steps):

$$\frac{1}{24a^3bd} (a-b) \sqrt{a+b} (16a^2A + 15Ab^2 - 18abB) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{24a^3d} \sqrt{a+b} (16a^2A - 10aAb + 15Ab^2 + 12a^2B - 18abB)$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{8a^4d} \sqrt{a+b} (4a^2Ab + 5Ab^3 - 8a^3B - 6ab^2B) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1 - \operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+dx])}{a-b}} + \frac{(16a^2A + 15Ab^2 - 18abB) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24a^3d}$$

$$\frac{(5Ab - 6aB) \operatorname{Cos}[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12a^2d} + \frac{A \operatorname{Cos}[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3ad}$$

Result (type 4, 1585 leaves):

$$\frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] \left(\frac{A \operatorname{Sin}[c+dx]}{12a} + \frac{(-5Ab+6aB) \operatorname{Sin}[2(c+dx)]}{24a^2} + \frac{A \operatorname{Sin}[3(c+dx)]}{12a} \right)}{d \sqrt{a+b \operatorname{Sec}[c+dx]}}$$

$$\begin{aligned}
& \left(\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left. \left(16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right] + 16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right] + 15 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right] + 15 A b^3 \tan \left[\frac{1}{2}(c+d x)\right] - 18 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\
& 18 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right] - 32 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^3 - 30 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3 + 36 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^3 + 16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^5 - \\
& 16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right]^5 + 15 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 15 A b^3 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 18 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^5 + 18 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 + \\
& 24 a^2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 30 A b^3 \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 48 a^3 B \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 36 a b^2 B \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 24 a^2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 30 A b^3 \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 48 a^3 B \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}
\end{aligned}$$

$$\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-36 a b^2 \operatorname{B EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right]$$

$$\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+$$

$$(a+b)\left(16 a^2 A+15 A b^2-18 a b B\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)$$

$$\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-2 a\left(5 A b^2+2 a b(A-3 B)+12 a^2 B\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right]$$

$$\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) /$$

$$\left(24 a^3 d \sqrt{a+b \operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-b\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)$$

■ **Problem 378: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c+dx]^3(A+B \operatorname{Sec}[c+dx])}{(a+b \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 4, 329 leaves, 5 steps):

$$-\frac{1}{3 b^4 \sqrt{a+b} d} 2\left(6 a^2 A b-3 A b^3-8 a^3 B+5 a b^2 B\right) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}-\frac{1}{3 b^3 \sqrt{a+b} d}$$

$$2(2 a+b)(3 A b-(4 a+b) B) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}-$$

$$\frac{2 a^2(A b-a B) \operatorname{Tan}[c+dx]}{b^2\left(a^2-b^2\right) d \sqrt{a+b \operatorname{Sec}[c+dx]}}+\frac{2 B \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3 b^2 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 379: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c + dx]^2 (A + B \sec[c + dx])}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$\frac{1}{b^3 \sqrt{a+b} d} 2 (a A b - 2 a^2 B + b^2 B) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{1}{b^2 \sqrt{a+b} d} 2 (A b - (2 a + b) B) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{2 a (A b - a B) \tan[c + dx]}{b (a^2 - b^2) d \sqrt{a+b \sec[c + dx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 380: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c + dx] (A + B \sec[c + dx])}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 254 leaves, 4 steps):

$$-\frac{2 (A b - a B) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}}}{b^2 \sqrt{a+b} d} +$$

$$\frac{2 (A + B) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}}}{b \sqrt{a+b} d} - \frac{2 (A b - a B) \tan[c + dx]}{(a^2 - b^2) d \sqrt{a+b \sec[c + dx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 381: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx]}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 376 leaves, 6 steps):

$$\frac{2 (A b - a B) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} -$$

$$\frac{2 (A b - a B) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} -$$

$$\frac{2 A \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 d} + \frac{2 b (A b - a B) \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}$$

Result (type 4, 1491 leaves):

$$\frac{(b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x]) \left(\frac{2 (-A b + a B) \operatorname{Sin}[c + d x]}{a (a^2 - b^2)} - \frac{2 (-A b^2 \operatorname{Sin}[c + d x] + a b B \operatorname{Sin}[c + d x])}{a (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])} \right)}{d (B + A \operatorname{Cos}[c + d x]) (a + b \operatorname{Sec}[c + d x])^{3/2}} +$$

$$\left(2 (b + a \operatorname{Cos}[c + d x])^{3/2} \sqrt{\operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x]) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right.$$

$$\left. \left(a A b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + A b^2 \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a^2 \sqrt{\frac{-a + b}{a + b}} B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a b \sqrt{\frac{-a + b}{a + b}} B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - \right.$$

$$2 a A b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 2 a^2 \sqrt{\frac{-a + b}{a + b}} B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + a A b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$A b^2 \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - a^2 \sqrt{\frac{-a + b}{a + b}} B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + a b \sqrt{\frac{-a + b}{a + b}} B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$2 i a^2 A \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}$$

$$\left. \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} + 2 i A b^2 \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a + b}{a - b}\right] \right)$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2i a^2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2i a b^2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i(a-b)(-Ab+aB) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i(a-b)(2Ab+a(A-B)) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg/ \\
& \left(a \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2) d (B + A \cos[c+dx]) (a+b \sec[c+dx])^{3/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)
\end{aligned}$$

■ **Problem 382: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] (A+B \sec[c+dx])}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 427 leaves, 7 steps):

$$\frac{1}{a^2 b \sqrt{a+b} d} (a^2 A - 3 A b^2 + 2 a b B) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{a^2 \sqrt{a+b} d} (3 A b + a(A-2 B)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{a^3 d} \sqrt{a+b} (3 A b - 2 a B) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{A \operatorname{Sin}[c+dx]}{a d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{b(a^2 A - 3 A b^2 + 2 a b B) \operatorname{Tan}[c+dx]}{a^2 (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+dx]}}$$

Result (type 4, 1613 leaves):

$$\frac{(b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^2 \left(-\frac{2 b(A b-a B) \operatorname{Sin}[c+dx]}{a^2 (-a^2+b^2)} + \frac{2(-A b^3 \operatorname{Sin}[c+dx]+a b^2 B \operatorname{Sin}[c+dx])}{a^2 (a^2-b^2)(b+a \operatorname{Cos}[c+dx])} \right)}{d (a+b \operatorname{Sec}[c+dx])^{3/2}}$$

$$\left((b+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 2 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$2 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 2 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 6 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 4 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 2 a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 2 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$6 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$6 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$4 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\int \frac{\cos [c+d x]^2 (A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 531 leaves, 8 steps):

$$-\frac{1}{4 a^3 b \sqrt{a+b} d} (7 a^2 A b-15 A b^3-4 a^3 B+12 a b^2 B) \cot [c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}$$

$$\frac{1}{4 a^3 \sqrt{a+b} d} (15 A b^2+a b(5 A-12 B)-2 a^2(A+2 B)) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} -\frac{1}{4 a^4 d} \sqrt{a+b} (4 a^2 A+15 A b^2-12 a b B) \cot [c+d x]$$

$$\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}$$

$$\frac{(5 A b-4 a B) \sin [c+d x]}{4 a^2 d \sqrt{a+b \sec [c+d x]}}+\frac{A \cos [c+d x] \sin [c+d x]}{2 a d \sqrt{a+b \sec [c+d x]}}-\frac{b(7 a^2 A b-15 A b^3-4 a^3 B+12 a b^2 B) \tan [c+d x]}{4 a^3\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}}$$

Result (type 4, 47132 leaves): Display of huge result suppressed!

■ **Problem 384: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^3 (A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 630 leaves, 9 steps):

$$\frac{1}{24 a^4 b \sqrt{a+b} d} (16 a^4 A + 41 a^2 A b^2 - 105 A b^4 - 42 a^3 b B + 90 a b^3 B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{24 a^4 \sqrt{a+b} d} (105 A b^3 + 5 a b^2 (7 A - 18 B) + 4 a^3 (4 A + 3 B) - 6 a^2 b (A + 5 B))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{8 a^5 d}$$

$$\sqrt{a+b} (12 a^2 A b + 35 A b^3 - 8 a^3 B - 30 a b^2 B) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{(16 a^2 A + 35 A b^2 - 30 a b B) \operatorname{Sin}[c+d x]}{24 a^3 d \sqrt{a+b} \operatorname{Sec}[c+d x]} -$$

$$\frac{(7 A b - 6 A B) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{12 a^2 d \sqrt{a+b} \operatorname{Sec}[c+d x]} + \frac{A \operatorname{Cos}[c+d x]^2 \operatorname{Sin}[c+d x]}{3 a d \sqrt{a+b} \operatorname{Sec}[c+d x]} + \frac{b(16 a^4 A + 41 a^2 A b^2 - 105 A b^4 - 42 a^3 b B + 90 a b^3 B) \operatorname{Tan}[c+d x]}{24 a^4 (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c+d x]}$$

Result (type 4, 2343 leaves):

$$\frac{1}{d (a+b \operatorname{Sec}[c+d x])^{3/2}} (b+a \operatorname{Cos}[c+d x])^2 \operatorname{Sec}[c+d x]^2 \left(-\frac{(a^4 A - a^2 A b^2 + 24 A b^4 - 24 a b^3 B) \operatorname{Sin}[c+d x]}{12 a^4 (-a^2 + b^2)} - \right.$$

$$\left. \frac{2 (A b^5 \operatorname{Sin}[c+d x] - a b^4 B \operatorname{Sin}[c+d x])}{a^4 (a^2 - b^2) (b+a \operatorname{Cos}[c+d x])} + \frac{(-11 A b + 6 a B) \operatorname{Sin}[2(c+d x)]}{24 a^3} + \frac{A \operatorname{Sin}[3(c+d x)]}{12 a^2} \right) -$$

$$\left((b+a \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sec}[c+d x]^{3/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right.$$

$$\left(16 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 16 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 41 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 41 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 105 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right.$$

$$105 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 42 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 42 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 90 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] +$$

$$90 a b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 32 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 82 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 210 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 +$$

$$84 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 180 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 16 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 16 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 +$$

$$\begin{aligned}
& 41 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 41 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 105 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 105 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 42 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 42 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 90 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 90 a b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 72 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 138 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 210 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 48 a^5 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 132 a^3 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 180 a b^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 72 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 138 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 210 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -
\end{aligned}$$

$$\begin{aligned}
& 48 a^5 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 132 a^3 b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 180 a b^4 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (16 a^4 A + 41 a^2 A b^2 - 105 A b^4 - 42 a^3 b B + 90 a b^3 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 a (a+b) (-35 A b^3 + 12 a^3 B - 2 a^2 b (5 A + 9 B) + 3 a b^2 (7 A + 10 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \\
& \left(24 a^4 (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{3/2} \sqrt{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 385: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c+dx]^4 (A+B \operatorname{Sec}[c+dx])}{(a+b \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 510 leaves, 6 steps):

$$\begin{aligned}
& - \frac{1}{3(a-b)b^5(a+b)^{3/2}d} \left(8a^4Ab - 15a^2Ab^3 + 3Ab^5 - 16a^5B + 28a^3b^2B - 8ab^4B \right) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{1}{3b^4\sqrt{a+b}(a^2-b^2)d} \left(9ab^3(A-B) + b^4(3A-B) + 16a^4B - 2a^2b^2(3A+8B) - a^3(8Ab-12bB) \right) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{2a(Ab-aB)\text{Sec}[c+dx]^2 \text{Tan}[c+dx]}{3b(a^2-b^2)d(a+b\text{Sec}[c+dx])^{3/2}} - \\
& \frac{2a^2(3a^2Ab-7Ab^3-6a^3B+10ab^2B)\text{Tan}[c+dx]}{3b^3(a^2-b^2)^2d\sqrt{a+b\text{Sec}[c+dx]}} - \frac{2(aAb-2a^2B+b^2B)\sqrt{a+b\text{Sec}[c+dx]}\text{Tan}[c+dx]}{3b^3(a^2-b^2)d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 386: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c+dx]^3(A+B\text{Sec}[c+dx])}{(a+b\text{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 4, 417 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{3(a-b)b^4(a+b)^{3/2}d} \left(2a^3Ab - 6aAb^3 - 8a^4B + 15a^2b^2B - 3b^4B \right) \text{Cot}[c+dx] \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{1}{3b^3\sqrt{a+b}(a^2-b^2)d} \\
& \quad 2(2a^2b(A-3B) - 3b^3(A-B) - 8a^3B + 3ab^2(A+3B)) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{2a^2(Ab-aB)\text{Tan}[c+dx]}{3b^2(a^2-b^2)d(a+b\text{Sec}[c+dx])^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B+9ab^2B)\text{Tan}[c+dx]}{3b^2(a^2-b^2)^2d\sqrt{a+b\text{Sec}[c+dx]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 387: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x])}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 387 leaves, 5 steps):

$$\frac{1}{3 (a - b) b^3 (a + b)^{3/2} d}$$

$$2 (a^2 A b + 3 A b^3 + 2 a^3 B - 6 a b^2 B) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{1}{3 b^2 \sqrt{a + b} (a^2 - b^2) d} 2 (2 a^2 B - 3 b^2 (A + B) + a b (A + 3 B)) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 a (A b - a B) \text{Tan}[c + d x]}{3 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} + \frac{2 (a^2 A b + 3 A b^3 + 2 a^3 B - 6 a b^2 B) \text{Tan}[c + d x]}{3 b (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 388: Unable to integrate problem.**

$$\int \frac{\text{Sec}[c + d x] (A + B \text{Sec}[c + d x])}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 353 leaves, 5 steps):

$$-\frac{1}{3 (a - b) b^2 (a + b)^{3/2} d} 2 (4 a A b - a^2 B - 3 b^2 B) \text{Cot}[c + d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{1}{3 (a - b) b (a + b)^{3/2} d}$$

$$2 (3 a A - A b + a B - 3 b B) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 (A b - a B) \text{Tan}[c + d x]}{3 (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} - \frac{2 (4 a A b - a^2 B - 3 b^2 B) \text{Tan}[c + d x]}{3 (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}}$$

Result (type 8, 33 leaves):

$$\int \frac{\text{Sec}[c + d x] (A + B \text{Sec}[c + d x])}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

- **Problem 389: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 495 leaves, 7 steps):

$$\frac{1}{3 a^2 (a-b) b (a+b)^{3/2} d} + \frac{2 (7 a^2 A b - 3 A b^3 - 4 a^3 B) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}}}{3 a^2 (a-b) b (a+b)^{3/2} d} + \frac{2 (6 a^2 A b - a A b^2 - 3 A b^3 - 3 a^3 B + a^2 b B) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}}}{3 a^2 (a-b) b (a+b)^{3/2} d} + \frac{2 A \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}}}{a^3 d} + \frac{2 b (A b - a B) \operatorname{Tan}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 b (7 a^2 A b - 3 A b^3 - 4 a^3 B) \operatorname{Tan}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \operatorname{Sec}[c + d x]}}$$

Result (type 4, 2083 leaves):

$$\left((b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x]) \left(\frac{2 (-7 a^2 A b + 3 A b^3 + 4 a^3 B) \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2} - \frac{2 (A b^3 \operatorname{Sin}[c + d x] - a b^2 B \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])^2} - \frac{2 (-8 a^2 A b^2 \operatorname{Sin}[c + d x] + 4 A b^4 \operatorname{Sin}[c + d x] + 5 a^3 b B \operatorname{Sin}[c + d x] - a b^3 B \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2)^2 (b + a \operatorname{Cos}[c + d x])} \right) \right) / (d (B + A \operatorname{Cos}[c + d x]) (a + b \operatorname{Sec}[c + d x])^{5/2}) + \left(2 (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x]) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right) + \left(7 a^3 A b \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 7 a^2 A b^2 \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 3 a A b^3 \sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - \right.$$

$$\begin{aligned}
& 3 A b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 a^4 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 a^3 b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \\
& 14 a^3 A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 6 a A b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 8 a^4 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \\
& 7 a^3 A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 7 a^2 A b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a A b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 3 A b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 4 a^4 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 4 a^3 b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 6 i a^4 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 12 i a^2 A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 i A b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 i a^4 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 12 i a^2 A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 i A b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$i(a-b) \left(-7a^2Ab + 3Ab^3 + 4a^3B\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$i(a-b) \left(-4aAb^2 - 6Ab^3 + 3a^3(A-B) + a^2b(9A+B)\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) /$$

$$\left(3a^2 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^2 d (B + A \cos[c+dx]) (a+b \sec[c+dx])^{5/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}}\right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)$$

■ **Problem 390: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] (A + B \sec[c+dx])}{(a+b \sec[c+dx])^{5/2}} dx$$

Optimal (type 4, 582 leaves, 8 steps):

$$\frac{1}{3 a^3 (a-b) b (a+b)^{3/2} d} (3 a^4 A - 26 a^2 A b^2 + 15 A b^4 + 14 a^3 b B - 6 a b^3 B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{3 a^3 \sqrt{a+b} (a^2-b^2) d} (15 A b^3 + a b^2 (5 A - 6 B) - 3 a^3 (A - 4 B) - a^2 b (21 A + 2 B))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{a^4 d}$$

$$\sqrt{a+b} (5 A b - 2 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{A \operatorname{Sin}[c+d x]}{a d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{b(3 a^2 A - 5 A b^2 + 2 a b B) \operatorname{Tan}[c+d x]}{3 a^2 (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{b(3 a^4 A - 26 a^2 A b^2 + 15 A b^4 + 14 a^3 b B - 6 a b^3 B) \operatorname{Tan}[c+d x]}{3 a^3 (a^2-b^2)^2 d \sqrt{a+b} \operatorname{Sec}[c+d x]}$$

Result (type 4, 2390 leaves):

$$\frac{1}{d (a+b \operatorname{Sec}[c+d x])^{5/2}}$$

$$(b+a \operatorname{Cos}[c+d x])^3 \operatorname{Sec}[c+d x]^3 \left(-\frac{2 b (-10 a^2 A b + 6 A b^3 + 7 a^3 B - 3 a b^2 B) \operatorname{Sin}[c+d x]}{3 a^3 (-a^2+b^2)^2} + \frac{2 (A b^4 \operatorname{Sin}[c+d x] - a b^3 B \operatorname{Sin}[c+d x])}{3 a^3 (a^2-b^2) (b+a \operatorname{Cos}[c+d x])^2} + \right.$$

$$\left. \frac{2 (-11 a^2 A b^3 \operatorname{Sin}[c+d x] + 7 A b^5 \operatorname{Sin}[c+d x] + 8 a^3 b^2 B \operatorname{Sin}[c+d x] - 4 a b^4 B \operatorname{Sin}[c+d x])}{3 a^3 (a^2-b^2)^2 (b+a \operatorname{Cos}[c+d x])} \right) -$$

$$\left((b+a \operatorname{Cos}[c+d x])^{5/2} \operatorname{Sec}[c+d x]^{5/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right.$$

$$\left(3 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 26 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 26 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 15 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \right.$$

$$15 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 14 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 14 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 6 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 6 a b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] -$$

$$6 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 52 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 30 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 28 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 12 a^2 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 +$$

$$3 a^5 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^4 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 26 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 26 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 15 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 -$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 24 a^3 b^2 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 a b^4 B \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b)\left(3 a^4 A-26 a^2 A b^2+15 A b^4+14 a^3 b B-6 a b^3 B\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 a(a+b)\left(5 A b^3+3 a^3 B+3 a^2 b(-2 A+B)-a b^2(3 A+2 B)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left.\left.\left.\left.\left.\left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right)\right]\right]\right]\right] / \\
& \left(3 a\left(a^3-a b^2\right)^2 d(a+b \operatorname{Sec}[c+dx])^{5 / 2} \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)-b\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 391: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2(A+B \operatorname{Sec}[c+dx])}{(a+b \operatorname{Sec}[c+dx])^{5 / 2}} dx$$

Optimal (type 4, 686 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{12 a^4 (a-b) b (a+b)^{3/2} d} (33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{1}{12 a^4 \sqrt{a+b} (a^2-b^2) d} (105 A b^4 + 5 a b^3 (7 A - 12 B) + 6 a^4 (A + 2 B) - 5 a^2 b^2 (27 A + 4 B) - a^3 (27 A b - 84 b B)) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{4 a^5 d} \sqrt{a+b} (4 a^2 A + 35 A b^2 - 20 a b B) \text{Cot}[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{(7 A b - 4 a B) \text{Sin}[c+dx]}{4 a^2 d (a+b \text{Sec}[c+dx])^{3/2}} + \frac{A \text{Cos}[c+dx] \text{Sin}[c+dx]}{2 a d (a+b \text{Sec}[c+dx])^{3/2}} - \\
& \frac{b(27 a^2 A b - 35 A b^3 - 12 a^3 B + 20 a b^2 B) \text{Tan}[c+dx]}{12 a^3 (a^2-b^2) d (a+b \text{Sec}[c+dx])^{3/2}} - \frac{b(33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \text{Tan}[c+dx]}{12 a^4 (a^2-b^2)^2 d \sqrt{a+b \text{Sec}[c+dx]}}
\end{aligned}$$

Result(type 4, 5217 leaves):

$$\begin{aligned}
& \frac{1}{d (a+b \text{Sec}[c+dx])^{5/2}} \\
& (b+a \text{Cos}[c+dx])^3 \text{Sec}[c+dx]^3 \left(\frac{2 b^2 (-13 a^2 A b + 9 A b^3 + 10 a^3 B - 6 a b^2 B) \text{Sin}[c+dx]}{3 a^4 (-a^2+b^2)^2} - \frac{2 (A b^5 \text{Sin}[c+dx] - a b^4 B \text{Sin}[c+dx])}{3 a^4 (a^2-b^2) (b+a \text{Cos}[c+dx])^2} - \right. \\
& \left. \frac{2 (-14 a^2 A b^4 \text{Sin}[c+dx] + 10 A b^6 \text{Sin}[c+dx] + 11 a^3 b^3 B \text{Sin}[c+dx] - 7 a b^5 B \text{Sin}[c+dx])}{3 a^4 (a^2-b^2)^2 (b+a \text{Cos}[c+dx])} + \frac{A \text{Sin}[2(c+dx)]}{4 a^3} \right) + \\
& \left((b+a \text{Cos}[c+dx])^{5/2} \left(\frac{a^2 A}{2 (a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} + \frac{2 A b^2}{(a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} - \right. \right. \\
& \frac{7 A b^4}{6 a^2 (a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} - \frac{2 a b B}{(a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} + \\
& \left. \frac{2 b^3 B}{3 a (a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]}} - \frac{9 a A b \sqrt{\text{Sec}[c+dx]}}{8 (a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]}} + \frac{31 A b^3 \sqrt{\text{Sec}[c+dx]}}{12 a (a^2-b^2)^2 \sqrt{b+a \text{Cos}[c+dx]}} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{35 A b^5 \sqrt{\text{Sec}[c+d x]}}{24 a^3 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} + \frac{a^2 B \sqrt{\text{Sec}[c+d x]}}{2 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} - \frac{4 b^2 B \sqrt{\text{Sec}[c+d x]}}{3 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} + \frac{5 b^4 B \sqrt{\text{Sec}[c+d x]}}{6 a^2 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} - \\
& \frac{11 a A b \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{8 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} + \frac{85 A b^3 \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{12 a (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} - \frac{35 A b^5 \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{8 a^3 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} + \\
& \left. \frac{a^2 B \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{2 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} - \frac{13 b^2 B \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{3 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} + \frac{5 b^4 B \text{Cos}[2(c+d x)] \sqrt{\text{Sec}[c+d x]}}{2 a^2 (a^2 - b^2)^2 \sqrt{b+a \text{Cos}[c+d x]}} \right) \text{Sec}[c+d x]^{5/2} \\
& \left(\left((-33 a^4 A b + 170 a^2 A b^3 - 105 A b^5 + 12 a^5 B - 104 a^3 b^2 B + 60 a b^4 B) \text{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) / \right. \\
& \left. \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1+\text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) - \right. \\
& \left. \left((a+b) \left((33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \right. \\
& \left. \left. 2 a (6 a^4 A - 35 A b^4 + 3 a^2 b^2 (11 A - 4 B) - 3 a^3 b (3 A + 8 B) + a b^3 (21 A + 20 B)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. 6 (a-b)^2 (a+b) (4 a^2 A + 35 A b^2 - 20 a b B) \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right) \right. \\
& \left. \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \right) / \\
& \left. \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1-\text{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(b - b \text{Tan}\left[\frac{1}{2}(c+d x)\right]^4 + a \left(-1 + \text{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right)^2 \right) \right) \right) / \\
& \left(d (a+b \text{Sec}[c+d x])^{5/2} \left(\left((-33 a^4 A b + 170 a^2 A b^3 - 105 A b^5 + 12 a^5 B - 104 a^3 b^2 B + 60 a b^4 B) \text{Sec}\left[\frac{1}{2}(c+d x)\right] \right)^2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) / \left(24 a^4 (a^2-b^2)^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) + \\
& \left((a+b) \left((33 a^4 A b-170 a^2 A b^3+105 A b^5-12 a^5 B+104 a^3 b^2 B-60 a b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad 2 a (6 a^4 A-35 A b^4+3 a^2 b^2 (11 A-4 B)-3 a^3 b (3 A+8 B)+a b^3 (21 A+20 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \\
& \quad \left. \left. 6(a-b)^2(a+b)(4 a^2 A+35 A b^2-20 a b B) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \\
& \left. \left(-2 b \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 2 a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) \right) / \\
& \left(12 a^4 (a^2-b^2)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4+a \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right)^2 \right)^2 \right) + \\
& \left((a+b) \left((33 a^4 A b-170 a^2 A b^3+105 A b^5-12 a^5 B+104 a^3 b^2 B-60 a b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad 2 a (6 a^4 A-35 A b^4+3 a^2 b^2 (11 A-4 B)-3 a^3 b (3 A+8 B)+a b^3 (21 A+20 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \\
& \quad \left. \left. 6(a-b)^2(a+b)(4 a^2 A+35 A b^2-20 a b B) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \right) \right) \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^4} \left(b - b \tan\left[\frac{1}{2}(c + dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^2 \right) \right) - \\
& \left(\left((33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) + \right. \\
& \quad 2 a (6 a^4 A - 35 A b^4 + 3 a^2 b^2 (11 A - 4 B) - 3 a^3 b (3 A + 8 B) + a b^3 (21 A + 20 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) + \\
& \quad \left. 6 (a - b)^2 (a + b) (4 a^2 A + 35 A b^2 - 20 a b B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) \\
& \quad \left(-a \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] \right) \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \\
& \quad \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^4} \right) / \left(24 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
& \quad \left. \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \left(b - b \tan\left[\frac{1}{2}(c + dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^2 \right) \right) + \\
& \left((a + b) \left((33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) + \right. \\
& \quad 2 a (6 a^4 A - 35 A b^4 + 3 a^2 b^2 (11 A - 4 B) - 3 a^3 b (3 A + 8 B) + a b^3 (21 A + 20 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) + \\
& \quad \left. 6 (a - b)^2 (a + b) (4 a^2 A + 35 A b^2 - 20 a b B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right]\right], \frac{a - b}{a + b}\right) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \tan\left[\frac{1}{2}(c + dx)\right] \\
& \quad \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \right) / \left(24 a^4 (a^2 - b^2)^2 \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) - \\
& \left((-33 a^4 A b + 170 a^2 A b^3 - 105 A b^5 + 12 a^5 B - 104 a^3 b^2 B + 60 a b^4 B) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \left(24 a^4 (a^2 - b^2)^2 \left(\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \left((-33 a^4 A b + 170 a^2 A b^3 - 105 A b^5 + 12 a^5 B - 104 a^3 b^2 B + 60 a b^4 B) \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right] \left(\frac{-a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a + b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \left(24 a^4 (a^2 - b^2)^2 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) - \\
& \left((a+b) \left((33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + 104 a^3 b^2 B - 60 a b^4 B) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. 2 a (6 a^4 A - 35 A b^4 + 3 a^2 b^2 (11 A - 4 B) - 3 a^3 b (3 A + 8 B) + a b^3 (21 A + 20 B)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. 6 (a-b)^2 (a+b) (4 a^2 A + 35 A b^2 - 20 a b B) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} +
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) / \left(24 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) - \\
& \left((a+b) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(\frac{a \left(6 a^4 A - 35 A b^4 + 3 a^2 b^2 (11 A - 4 B) - 3 a^3 b (3 A + 8 B) + a b^3 (21 A + 20 B)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right. \right. \\
& \left. \left. \frac{3(a-b)^2(a+b)(4a^2A + 35Ab^2 - 20abB) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right. \right. \\
& \left. \left. + \left(33 a^4 A b - 170 a^2 A b^3 + 105 A b^5 - 12 a^5 B + \right. \right. \right. \\
& \left. \left. \left. 104 a^3 b^2 B - 60 a b^4 B \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \left(2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right) \right) / \right. \\
& \left. \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) \right)
\end{aligned}$$

Problem 392: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[e + f x] (A + A \text{Sec}[e + f x])}{\sqrt{a + b \text{Sec}[e + f x]}} dx$$

Optimal (type 4, 105 leaves, 1 step):

$$-\frac{1}{b^2 f} 2 A (a - b) \sqrt{a + b} \text{Cot}[e + f x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[e + f x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \text{Sec}[e + f x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[e + f x])}{a - b}}$$

Result (type 4, 283 leaves):

$$A \left(\frac{(b + a \text{Cos}[e + f x]) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 (1 + \text{Sec}[e + f x]) \text{Sin}[e + f x]}{b f \sqrt{a + b \text{Sec}[e + f x]}} + \left(\sqrt{\text{Sec}\left[\frac{1}{2}(e + f x)\right]^2} \sqrt{\text{Cos}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] (1 + \text{Sec}[e + f x])} \right. \right. \\ \left. \left. \frac{\left(\sqrt{\frac{a-b}{a+b}} (a+b) \sqrt{\frac{b+a \text{Cos}[e+f x]}{(a+b)(1+\text{Cos}[e+f x])}} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{a-b}{a+b}} \text{Tan}\left[\frac{1}{2}(e + f x)\right]\right], \frac{a+b}{a-b}\right] \right)}{\sqrt{\frac{\text{Cos}[e+f x]}{1+\text{Cos}[e+f x]}}} + (b + a \text{Cos}[e + f x]) \text{Tan}\left[\frac{1}{2}(e + f x)\right] \right) \right. \\ \left. \left(-1 + \text{Tan}\left[\frac{1}{2}(e + f x)\right]^2 \right) \right) / (b f \sqrt{\text{Sec}[e + f x]} \sqrt{a + b \text{Sec}[e + f x]})$$

■ **Problem 393: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x] (A - A \text{Sec}[e + f x])}{\sqrt{a + b \text{Sec}[e + f x]}} dx$$

Optimal (type 4, 107 leaves, 1 step):

$$\frac{2 A \sqrt{a - b} (a + b) \text{Cot}[e + f x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[e + f x]}}{\sqrt{a - b}}\right], \frac{a - b}{a + b}\right] \sqrt{\frac{b(1 - \text{Sec}[e + f x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[e + f x])}{a - b}}}{b^2 f}$$

Result (type 4, 2069 leaves):

$$\frac{(b + a \text{Cos}[e + f x]) \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 (A - A \text{Sec}[e + f x]) \text{Sin}[e + f x]}{b f \sqrt{a + b \text{Sec}[e + f x]}} - \left((b + a \text{Cos}[e + f x]) \text{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \text{Sec}\left[\frac{1}{2}(e + f x)\right]^4 \right)$$

$$\begin{aligned}
& \left(-\frac{1}{2\sqrt{b+a\cos[ex+f]}\sqrt{\sec[ex+f]}} - \frac{\sqrt{\sec[ex+f]}}{2\sqrt{b+a\cos[ex+f]}} - \frac{a\sqrt{\sec[ex+f]}}{2b\sqrt{b+a\cos[ex+f]}} - \frac{a\cos[2(ex+f)]\sqrt{\sec[ex+f]}}{2b\sqrt{b+a\cos[ex+f]}} \right) \\
& \sqrt{1+\sec[ex+f]} (A - A\sec[ex+f]) \left(2\sqrt{\frac{\cos[ex+f]}{1+\cos[ex+f]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(ex+f)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \left. \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} \sec\left[\frac{1}{2}(ex+f)\right] \left(-\sin\left[\frac{1}{2}(ex+f)\right] + \sin\left[\frac{3}{2}(ex+f)\right]\right) \right) \Bigg) / \\
& \left(8bf \left(\frac{1}{1+\cos[ex+f]}\right)^{3/2} \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} \sqrt{\sec[ex+f]} \sqrt{a+b\sec[ex+f]} \right. \\
& \left. \left(a\sec\left[\frac{1}{2}(ex+f)\right]^4 \sqrt{1+\sec[ex+f]} \sin[ex+f] \left(2\sqrt{\frac{\cos[ex+f]}{1+\cos[ex+f]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(ex+f)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} \sec\left[\frac{1}{2}(ex+f)\right] \left(-\sin\left[\frac{1}{2}(ex+f)\right] + \sin\left[\frac{3}{2}(ex+f)\right]\right) \right) \right) \right) / \\
& \left(16b \left(\frac{1}{1+\cos[ex+f]}\right)^{3/2} \sqrt{b+a\cos[ex+f]} \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} + \left(3\sqrt{b+a\cos[ex+f]} \sec\left[\frac{1}{2}(ex+f)\right]^4 \right. \right. \\
& \left. \left. \sqrt{1+\sec[ex+f]} \sin[ex+f] \left(2\sqrt{\frac{\cos[ex+f]}{1+\cos[ex+f]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(ex+f)\right]\right], \frac{a-b}{a+b}\right] + \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} \right. \right. \right. \\
& \left. \left. \left. \sec\left[\frac{1}{2}(ex+f)\right] \left(-\sin\left[\frac{1}{2}(ex+f)\right] + \sin\left[\frac{3}{2}(ex+f)\right]\right) \right) \right) \right) / \left(16b \sqrt{\frac{1}{1+\cos[ex+f]}} \sqrt{\frac{b+a\cos[ex+f]}{(a+b)(1+\cos[ex+f])}} \right) + \\
& \left(\sqrt{b+a\cos[ex+f]} \sec\left[\frac{1}{2}(ex+f)\right]^4 \sqrt{1+\sec[ex+f]} \left(-\frac{a\sin[ex+f]}{(a+b)(1+\cos[ex+f])} + \frac{(b+a\cos[ex+f])\sin[ex+f]}{(a+b)(1+\cos[ex+f])^2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(2 \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] + \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \right. \\
& \quad \left. \left(-\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{3}{2}(e+fx)\right]\right) \right] \Bigg) / \left(16b \left(\frac{1}{1+\cos[e+fx]}\right)^{3/2} \left(\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}\right)^{3/2} \right) - \\
& \left(\sqrt{b+a\cos[e+fx]} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \sqrt{1+\operatorname{Sec}[e+fx]} \left(2 \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad \left. \left. \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(-\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{3}{2}(e+fx)\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] \right) \right) / \\
& \left(4b \left(\frac{1}{1+\cos[e+fx]}\right)^{3/2} \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}} - \frac{1}{8b \left(\frac{1}{1+\cos[e+fx]}\right)^{3/2} \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}}} \sqrt{b+a\cos[e+fx]} \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \sqrt{1+\operatorname{Sec}[e+fx]} \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \left(-\frac{1}{2}\cos\left[\frac{1}{2}(e+fx)\right] + \frac{3}{2}\cos\left[\frac{3}{2}(e+fx)\right]\right) \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] + \right. \\
& \quad \left. \frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right], \frac{a-b}{a+b}\right] \left(\frac{\cos[e+fx]\operatorname{Sin}[e+fx]}{(1+\cos[e+fx])^2} - \frac{\operatorname{Sin}[e+fx]}{1+\cos[e+fx]}\right)}{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}}} + \frac{1}{2\sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}}} \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(-\frac{a\operatorname{Sin}[e+fx]}{(a+b)(1+\cos[e+fx])} + \frac{(b+a\cos[e+fx])\operatorname{Sin}[e+fx]}{(a+b)(1+\cos[e+fx])^2}\right) \left(-\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{3}{2}(e+fx)\right]\right) \right) + \\
& \quad \frac{1}{2} \sqrt{\frac{b+a\cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(-\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{3}{2}(e+fx)\right]\right) \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right] + \\
& \quad \left. \frac{\sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \sqrt{1 - \frac{(a-b)\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}{a+b}}}}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]^2}} \right) -
\end{aligned}$$

$$\left(\sqrt{b+a \cos[e+fx]} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Sec}[e+fx] \left(2 \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(e+fx)\right]\right]\right], \frac{a-b}{a+b}\right) + \right. \\ \left. \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right] \left(-\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right] + \operatorname{Sin}\left[\frac{3}{2}(e+fx)\right] \right) \operatorname{Tan}[e+fx] \right) / \\ \left(16b \left(\frac{1}{1+\cos[e+fx]} \right)^{3/2} \sqrt{\frac{b+a \cos[e+fx]}{(a+b)(1+\cos[e+fx])}} \sqrt{1+\operatorname{Sec}[e+fx]} \right) \right)$$

■ **Problem 415: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{7/2} (A+B \operatorname{Sec}[c+dx])}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 277 leaves, 11 steps):

$$\frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5b^3d} + \\ \frac{2(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3b^2d} + \\ \frac{2a^2(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{b^3(a+b)d} - \\ \frac{2(5aAb - 5a^2B - 3b^2B) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{5b^3d} + \frac{2(Ab - aB) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{3b^2d} + \frac{2B \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{5bd}$$

Result (type 4, 669 leaves):

$$\begin{aligned}
& -\frac{1}{30 b^3 d} \\
& \left(-\left(2 \left(-40 a A b^2 + 40 a^2 b B + 18 b^3 B \right) \cos [c+d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) \right. \\
& \quad \left. \right) / \left((a(b+a \cos [c+d x]) (1-\cos [c+d x]^2)) + \left(2 \left(-45 a^2 A b - 10 A b^3 + 45 a^3 B + 19 a b^2 B \right) \right. \right. \\
& \quad \left. \left. \cos [c+d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right) \right) \right. \\
& \quad \left. (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left((b(b+a \cos [c+d x]) (1-\cos [c+d x]^2)) - \right. \\
& \quad \left(2 \left(-15 a^2 A b + 15 a^3 B + 9 a b^2 B \right) \cos [2(c+d x)] (a+b \sec [c+d x]) \left(2 a b - 2 a b \sec [c+d x]^2 + \right. \right. \\
& \quad \left. \left. 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + a(a-2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left(a^2 b (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) + \\
& \quad \left. \sqrt{\sec [c+d x]} \left(\frac{2(-5 a A b+5 a^2 B+3 b^2 B) \sin [c+d x]}{5 b^3} + \frac{2 \sec [c+d x] (A b \sin [c+d x]-a B \sin [c+d x])}{3 b^2} + \frac{2 B \sec [c+d x] \tan [c+d x]}{5 b} \right) \right) \\
& \quad \left. \right) / d
\end{aligned}$$

■ **Problem 420: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sec [c+d x]}{\sec [c+d x]^{3/2} (a+b \sec [c+d x])} dx$$

Optimal (type 4, 196 leaves, 9 steps):

$$\begin{aligned}
& -\frac{2(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^2 d} + \\
& \frac{2\left(a^2 A+3 A b^2-3 a b B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{3 a^3 d} - \\
& \frac{2 b^2(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^3(a+b) d} + \frac{2 A \sin [c+d x]}{3 a d \sqrt{\sec [c+d x]}}
\end{aligned}$$

Result (type 4, 545 leaves):

$$\begin{aligned}
& - \frac{1}{6 a d} \left(\left(4 A \cos [c+d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\
& \quad \left. (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left(2 (A b-3 a B) \cos [c+d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / (b(b+a \cos [c+d x]) (1-\cos [c+d x]^2)) - \\
& \quad \left(2 (3 A b-3 a B) \cos [2(c+d x)] (a+b \sec [c+d x]) \left(2 a b-2 a b \sec [c+d x]^2+2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+a(a-2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+ \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}- \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left(a^2 b(b+a \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \Big) + \frac{A \sqrt{\sec [c+d x]} \sin [2(c+d x)]}{3 a d}
\end{aligned}$$

■ **Problem 421: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sec [c+d x]}{\sec [c+d x]^{5/2} (a+b \sec [c+d x])} dx$$

Optimal (type 4, 242 leaves, 10 steps):

$$\begin{aligned}
& \frac{2(3 a^2 A+5 A b^2-5 a b B) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{5 a^3 d} - \\
& \frac{2(a^2+3 b^2)(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{3 a^4 d} + \\
& \frac{2 b^3(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^4(a+b) d} + \frac{2 A \sin [c+d x]}{5 a d \sec [c+d x]^{3/2}} - \frac{2(A b-a B) \sin [c+d x]}{3 a^2 d \sqrt{\sec [c+d x]}}
\end{aligned}$$

Result (type 4, 617 leaves):

$$\frac{1}{30 a^2 d} \left(- \left(2 (8 a A b + 10 a^2 B) \cos [c + d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] (a + b \operatorname{Sec} [c + d x]) \sqrt{1 - \operatorname{Sec} [c + d x]^2} \sin [c + d x] \right) \right) /$$

$$\left(a (b + a \cos [c + d x]) (1 - \cos [c + d x]^2) \right) +$$

$$\left(2 (9 a^2 A + 5 A b^2 - 5 a b B) \cos [c + d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] \right) \right)$$

$$\left(a + b \operatorname{Sec} [c + d x] \right) \sqrt{1 - \operatorname{Sec} [c + d x]^2} \sin [c + d x] \Big) / \left(b (b + a \cos [c + d x]) (1 - \cos [c + d x]^2) \right) -$$

$$\left(2 (9 a^2 A + 15 A b^2 - 15 a b B) \cos [2 (c + d x)] (a + b \operatorname{Sec} [c + d x]) \left(2 a b - 2 a b \operatorname{Sec} [c + d x]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] \right) \right)$$

$$\sqrt{\operatorname{Sec} [c + d x]} \sqrt{1 - \operatorname{Sec} [c + d x]^2} + a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] \sqrt{\operatorname{Sec} [c + d x]} \sqrt{1 - \operatorname{Sec} [c + d x]^2} +$$

$$a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] \sqrt{\operatorname{Sec} [c + d x]} \sqrt{1 - \operatorname{Sec} [c + d x]^2} -$$

$$2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec} [c + d x]} \right], -1 \right] \sqrt{\operatorname{Sec} [c + d x]} \sqrt{1 - \operatorname{Sec} [c + d x]^2} \Big) \sin [c + d x] \Big) /$$

$$\left(a^2 b (b + a \cos [c + d x]) (1 - \cos [c + d x]^2) \sqrt{\operatorname{Sec} [c + d x]} (2 - \operatorname{Sec} [c + d x]^2) \right) \Big) +$$

$$\frac{\sqrt{\operatorname{Sec} [c + d x]} \left(\frac{A \sin [c + d x]}{10 a} + \frac{(-A b + a B) \sin [2 (c + d x)]}{3 a^2} + \frac{A \sin [3 (c + d x)]}{10 a} \right)}{d}$$

■ **Problem 423: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec} [c + d x]^{5/2} (A + B \operatorname{Sec} [c + d x])}{(a + b \operatorname{Sec} [c + d x])^2} dx$$

Optimal (type 4, 315 leaves, 10 steps):

$$\frac{(a A b - 3 a^2 B + 2 b^2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{b^2 (a^2 - b^2) d} +$$

$$\frac{(A b - a B) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{b (a^2 - b^2) d} +$$

$$\frac{(a^2 A b - 3 A b^3 - 3 a^3 B + 5 a b^2 B) \sqrt{\cos [c + d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a + b}, \frac{1}{2} (c + d x), 2 \right] \sqrt{\operatorname{Sec} [c + d x]}}{(a - b) b^2 (a + b)^2 d} -$$

$$\frac{(a A b - 3 a^2 B + 2 b^2 B) \sqrt{\operatorname{Sec} [c + d x]} \sin [c + d x]}{b^2 (a^2 - b^2) d} + \frac{a (A b - a B) \operatorname{Sec} [c + d x]^{3/2} \sin [c + d x]}{b (a^2 - b^2) d (a + b \operatorname{Sec} [c + d x])}$$

Result (type 4, 685 leaves):

$$\begin{aligned}
& - \frac{1}{4(a-b)b^2(a+b)d} \\
& \left(- \left(2(-4aAb^2 + 8a^2bB - 4b^3B) \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] (a+b\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \right. \right. \\
& \quad \left. \left. \sin[c+dx] \right) / \left(a(b+a\cos[c+dx])(1-\cos[c+dx]^2) \right) + \left(2(-3a^2Ab + 4Ab^3 + 9a^3B - 10ab^2B) \right. \right. \\
& \quad \left. \left. \cos[c+dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right) \right) \right. \\
& \quad \left. (a+b\sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) / \left(b(b+a\cos[c+dx])(1-\cos[c+dx]^2) \right) - \\
& \left(2(-a^2Ab + 3a^3B - 2ab^2B) \cos[2(c+dx)] (a+b\sec[c+dx]) \left(2ab - 2ab\sec[c+dx]^2 + 2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + a(a-2b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c+dx]}\right], -1\right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \left(a^2b(b+a\cos[c+dx])(1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) + \\
& \frac{\sqrt{\sec[c+dx]} \left(\frac{(aAb - 3a^2B + 2b^2B) \sin[c+dx]}{b^2(-a^2+b^2)} + \frac{-aAb \sin[c+dx] + a^2B \sin[c+dx]}{b(-a^2+b^2)(b+a\cos[c+dx])} \right)}{d}
\end{aligned}$$

■ **Problem 424: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2} (A+B\sec[c+dx])}{(a+b\sec[c+dx])^2} dx$$

Optimal (type 4, 257 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{b(a^2 - b^2)d} - \frac{(Ab - aB) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a(a^2 - b^2)d} + \\
& \frac{(a^2Ab + Ab^3 + a^3B - 3ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a(a-b)b(a+b)^2d} + \frac{a(Ab - aB) \sqrt{\sec[c+dx]} \sin[c+dx]}{b(a^2 - b^2)d(a+b\sec[c+dx])}
\end{aligned}$$

Result (type 4, 643 leaves):

$$\begin{aligned}
& \frac{1}{4b(-a+b)(a+b)d} \\
& \left(- \left(2(4Ab^2 - 4aAbB) \cos[c+dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] (a+b\operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \sin[c+dx] \right) / \right. \\
& \quad \left. (a(b+a\cos[c+dx])(1-\cos[c+dx]^2)) + \right. \\
& \quad \left(2(-aAb - 3a^2B + 4b^2B) \cos[c+dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] \right) \right. \\
& \quad \left. (a+b\operatorname{Sec}[c+dx]) \sqrt{1-\operatorname{Sec}[c+dx]^2} \sin[c+dx] \right) / (b(b+a\cos[c+dx])(1-\cos[c+dx]^2)) - \\
& \quad \left(2(aAb - a^2B) \cos[2(c+dx)] (a+b\operatorname{Sec}[c+dx]) \left(2ab - 2ab\operatorname{Sec}[c+dx]^2 + 2ab\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + a(a-2b)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} + \right. \right. \\
& \quad \left. \left. a^2\operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} - \right. \right. \\
& \quad \left. \left. 2b^2\operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\operatorname{Sec}[c+dx]}\right], -1\right] \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1-\operatorname{Sec}[c+dx]^2} \right) \sin[c+dx] \right) / \\
& \quad \left. \left(a^2b(b+a\cos[c+dx])(1-\cos[c+dx]^2) \sqrt{\operatorname{Sec}[c+dx]} (2-\operatorname{Sec}[c+dx]^2) \right) \right) + \\
& \frac{\sqrt{\operatorname{Sec}[c+dx]} \left(-\frac{(Ab-aB)\sin[c+dx]}{b(-a^2+b^2)} + \frac{Ab\sin[c+dx]-aB\sin[c+dx]}{(-a^2+b^2)(b+a\cos[c+dx])} \right)}{d}
\end{aligned}$$

■ **Problem 425: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]} (A+B\operatorname{Sec}[c+dx])}{(a+b\operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 263 leaves, 9 steps):

$$\begin{aligned}
& \frac{(Ab-aB)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{a(a^2-b^2)d} + \frac{(2a^2A-Ab^2-abB)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{a^2(a^2-b^2)d} \\
& - \frac{(3a^2Ab-Ab^3-a^3B-ab^2B)\sqrt{\cos[c+dx]}\operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]\sqrt{\operatorname{Sec}[c+dx]}}{a^2(a-b)(a+b)^2d} - \frac{(Ab-aB)\sqrt{\operatorname{Sec}[c+dx]}\sin[c+dx]}{(a^2-b^2)d(a+b\operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 4, 727 leaves):

$$\begin{aligned}
& \frac{1}{4 (a-b) (a+b) d (B+A \cos [c+d x]) (a+b \sec [c+d x])^2} (b+a \cos [c+d x])^2 \sec [c+d x] (A+B \sec [c+d x]) \\
& \left(- \left(2 (4 a A - 4 b B) \cos [c+d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \right. \\
& \quad \left(a (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \right) + \\
& \quad \left(2 (-A b+a B) \cos [c+d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x] \right) / \left(b (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \right) - \\
& \quad \left(2 (A b-a B) \cos [2(c+d x)] (a+b \sec [c+d x]) \left(2 a b - 2 a b \sec [c+d x]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + a (a-2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec [c+d x]} \right], -1 \right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left(a^2 b (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\sec [c+d x]} (2-\sec [c+d x]^2) \right) \left. \right) + \\
& \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^{3/2} (A+B \sec [c+d x]) \left(\frac{(-A b+a B) \sin [c+d x]}{a (a^2-b^2)} + \frac{A b^2 \sin [c+d x]-a b B \sin [c+d x]}{a (a^2-b^2) (b+a \cos [c+d x])} \right)}{d (B+A \cos [c+d x]) (a+b \sec [c+d x])^2}
\end{aligned}$$

■ **Problem 426: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sec [c+d x]}{\sqrt{\sec [c+d x]} (a+b \sec [c+d x])^2} dx$$

Optimal (type 4, 283 leaves, 9 steps):

$$\begin{aligned}
& \frac{(2 a^2 A - 3 A b^2 + a b B) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^2 (a^2 - b^2) d} - \\
& \frac{(4 a^2 A b - 3 A b^3 - 2 a^3 B + a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^3 (a^2 - b^2) d} + \\
& \frac{b (5 a^2 A b - 3 A b^3 - 3 a^3 B + a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\sec [c+d x]}}{a^3 (a-b) (a+b)^2 d} + \frac{b (A b - a B) \sqrt{\sec [c+d x]} \sin [c+d x]}{a (a^2 - b^2) d (a+b \sec [c+d x])}
\end{aligned}$$

Result (type 4, 657 leaves):

$$\begin{aligned}
& \frac{1}{4 a (-a+b) (a+b) d} \\
& \left(- \left(2 (4 a A b - 4 a^2 B) \cos [c+d x]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x] \right) / \right. \\
& \quad \left. (a (b+a \cos [c+d x]) (1-\cos [c+d x]^2)) + \right. \\
& \quad \left(2 (-2 a^2 A + A b^2 + a b B) \cos [c+d x]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+d x]) \sqrt{1-\operatorname{Sec}[c+d x]^2} \sin [c+d x] \right) / (b (b+a \cos [c+d x]) (1-\cos [c+d x]^2)) - \\
& \quad \left(2 (-2 a^2 A + 3 A b^2 - a b B) \cos [2 (c+d x)] (a+b \operatorname{Sec}[c+d x]) \left(2 a b - 2 a b \operatorname{Sec}[c+d x]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + a (a-2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} - \right. \right. \\
& \quad \left. \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\operatorname{Sec}[c+d x]} \right], -1 \right] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{1-\operatorname{Sec}[c+d x]^2} \right) \sin [c+d x] \right) / \\
& \quad \left. \left(a^2 b (b+a \cos [c+d x]) (1-\cos [c+d x]^2) \sqrt{\operatorname{Sec}[c+d x]} (2-\operatorname{Sec}[c+d x]^2) \right) \right) + \\
& \quad \frac{\sqrt{\operatorname{Sec}[c+d x]} \left(-\frac{b (A b - a B) \sin [c+d x]}{a^2 (-a^2 + b^2)} + \frac{-A b^3 \sin [c+d x] + a b^2 B \sin [c+d x]}{a^2 (a^2 - b^2) (b + a \cos [c+d x])} \right)}{d}
\end{aligned}$$

■ **Problem 431: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2} (A+B \operatorname{Sec}[c+d x])}{(a+b \operatorname{Sec}[c+d x])^3} dx$$

Optimal (type 4, 402 leaves, 10 steps):

$$\begin{aligned}
& \frac{(5 a^2 A b + A b^3 - a^3 B - 5 a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{4 a b (a^2 - b^2)^2 d} \\
& \frac{(7 a^2 A b - A b^3 - 3 a^3 B - 3 a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{4 a^2 (a^2 - b^2)^2 d} + \frac{1}{4 a^2 (a-b)^2 b (a+b)^3 d} \\
& \frac{(3 a^4 A b + 10 a^2 A b^3 - A b^5 + a^5 B - 10 a^3 b^2 B - 3 a b^4 B) \sqrt{\cos [c+d x]} \operatorname{EllipticPi} \left[\frac{2 a}{a+b}, \frac{1}{2} (c+d x), 2 \right] \sqrt{\operatorname{Sec}[c+d x]}}{2 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^2} + \frac{(3 a^2 A b + 3 A b^3 + a^3 B - 7 a b^2 B) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{4 b (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+d x])}
\end{aligned}$$

Result (type 4, 887 leaves) :

$$\frac{1}{16 (a-b)^2 b (a+b)^2 d (B+A \cos[c+dx]) (a+b \sec[c+dx])^3} (b+a \cos[c+dx])^3 \sec[c+dx]^2$$

$$(A+B \sec[c+dx]) \left(- \left(2 (-24 a A b^2 + 8 a^2 b B + 16 b^3 B) \cos[c+dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \right. \right.$$

$$\left. \left. (a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) \right) / \left((a+b+a \cos[c+dx]) (1-\cos[c+dx]^2) \right) +$$

$$\left(2 (a^2 A b + 5 A b^3 + 3 a^3 B - 9 a b^2 B) \cos[c+dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] + \right. \right.$$

$$\left. \left. \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \right) (a+b \sec[c+dx]) \sqrt{1-\sec[c+dx]^2} \sin[c+dx] \right) /$$

$$(b (b+a \cos[c+dx]) (1-\cos[c+dx]^2)) - \left(2 (-5 a^2 A b - A b^3 + a^3 B + 5 a b^2 B) \cos[2(c+dx)] (a+b \sec[c+dx]) \right.$$

$$\left. \left(2 a b - 2 a b \sec[c+dx]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + a (a-2b) \right. \right.$$

$$\left. \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \right.$$

$$\left. \left. \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} - 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c+dx]} \right], -1 \right] \sqrt{\sec[c+dx]} \sqrt{1-\sec[c+dx]^2} \right) \right)$$

$$\sin[c+dx] \left) / \left(a^2 b (b+a \cos[c+dx]) (1-\cos[c+dx]^2) \sqrt{\sec[c+dx]} (2-\sec[c+dx]^2) \right) \right) +$$

$$\left((b+a \cos[c+dx])^3 \sec[c+dx]^{5/2} (A+B \sec[c+dx]) \left(\frac{(5 a^2 A b + A b^3 - a^3 B - 5 a b^2 B) \sin[c+dx]}{4 a b (-a^2 + b^2)^2} - \frac{-A b^2 \sin[c+dx] + a b B \sin[c+dx]}{2 a (a^2 - b^2) (b+a \cos[c+dx])^2} + \right. \right.$$

$$\left. \left. \frac{-7 a^2 A b \sin[c+dx] + A b^3 \sin[c+dx] + 3 a^3 B \sin[c+dx] + 3 a b^2 B \sin[c+dx]}{4 a (a^2 - b^2)^2 (b+a \cos[c+dx])} \right) \right) / \left(d (B+A \cos[c+dx]) (a+b \sec[c+dx])^3 \right)$$

■ **Problem 432: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]} (A+B \sec[c+dx])}{(a+b \sec[c+dx])^3} dx$$

Optimal (type 4, 402 leaves, 10 steps) :

$$\frac{(9 a^2 A b - 3 A b^3 - 5 a^3 B - a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{4 a^2\left(a^2-b^2\right)^2 d} +$$

$$\frac{\left(8 a^4 A - 5 a^2 A b^2 + 3 A b^4 - 7 a^3 b B + a b^3 B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{4 a^3\left(a^2-b^2\right)^2 d} - \frac{1}{4 a^3(a-b)^2(a+b)^3 d}$$

$$\frac{\left(15 a^4 A b - 6 a^2 A b^3 + 3 A b^5 - 3 a^5 B - 10 a^3 b^2 B + a b^4 B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{2\left(a^2-b^2\right) d(a+b \sec [c+d x])^2} -$$

$$\frac{\left(A b - a B\right) \sqrt{\sec [c+d x]} \sin [c+d x]}{4 a\left(a^2-b^2\right)^2 d(a+b \sec [c+d x])} - \frac{\left(7 a^2 A b - A b^3 - 3 a^3 B - 3 a b^2 B\right) \sqrt{\sec [c+d x]} \sin [c+d x]}{2\left(a^2-b^2\right) d(a+b \sec [c+d x])^2}$$

Result (type 4, 890 leaves):

$$\frac{1}{16 a(a-b)^2(a+b)^2 d(B+A \cos [c+d x])(a+b \sec [c+d x])^3}(b+a \cos [c+d x])^3 \sec [c+d x]^2$$

$$(A+B \sec [c+d x])\left(-\left(2\left(16 a^3 A+8 a A b^2-24 a^2 b B\right) \cos [c+d x]^2 \operatorname{EllipticPi}\left[-\frac{b}{a},-\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right]\right.\right.$$

$$\left.\left.(a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x]\right) / (a(b+a \cos [c+d x])\left(1-\cos [c+d x]^2\right))\right)+$$

$$\left(2\left(-5 a^2 A b-A b^3+a^3 B+5 a b^2 B\right) \cos [c+d x]^2\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right]+\right.\right.$$

$$\left.\left.\operatorname{EllipticPi}\left[-\frac{b}{a},-\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right]\right)(a+b \sec [c+d x]) \sqrt{1-\sec [c+d x]^2} \sin [c+d x]\right) /$$

$$(b(b+a \cos [c+d x])\left(1-\cos [c+d x]^2\right))-\left(2\left(9 a^2 A b-3 A b^3-5 a^3 B-a b^2 B\right) \cos [2(c+d x)](a+b \sec [c+d x])\right.$$

$$\left.\left(2 a b-2 a b \sec [c+d x]^2+2 a b \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+a(a-2 b)\right.\right.$$

$$\left.\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}+a^2 \operatorname{EllipticPi}\left[-\frac{b}{a},-\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right]\right.$$

$$\left.\left.\sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}-2 b^2 \operatorname{EllipticPi}\left[-\frac{b}{a},-\operatorname{ArcSin}\left[\sqrt{\sec [c+d x]}\right],-1\right] \sqrt{\sec [c+d x]} \sqrt{1-\sec [c+d x]^2}\right)\right.$$

$$\left.\left.\sin [c+d x]\right) / \left(a^2 b(b+a \cos [c+d x])\left(1-\cos [c+d x]^2\right) \sqrt{\sec [c+d x]}\left(2-\sec [c+d x]^2\right)\right)\right)+$$

$$\left((b+a \cos [c+d x])^3 \sec [c+d x]^{5 / 2}(A+B \sec [c+d x])\left(\frac{\left(-9 a^2 A b+3 A b^3+5 a^3 B+a b^2 B\right) \sin [c+d x]}{4 a^2\left(-a^2+b^2\right)^2}-\frac{A b^3 \sin [c+d x]-a b^2 B \sin [c+d x]}{2 a^2\left(a^2-b^2\right)(b+a \cos [c+d x])^2}+\right.\right.$$

$$\left.\left.\frac{11 a^2 A b^2 \sin [c+d x]-5 A b^4 \sin [c+d x]-7 a^3 b B \sin [c+d x]+a b^3 B \sin [c+d x]}{4 a^2\left(a^2-b^2\right)^2(b+a \cos [c+d x])}\right)\right) / (d(B+A \cos [c+d x])(a+b \sec [c+d x])^3)$$

■ **Problem 435: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sec}[c + d x]^{3/2} \sqrt{a + b \text{Sec}[c + d x]} (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 4, 336 leaves, 13 steps):

$$\frac{(4 A b + 3 a B) \sqrt{\frac{b+a \text{Cos}[c+dx]}{a+b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{\text{Sec}[c + d x]}}{4 d \sqrt{a + b \text{Sec}[c + d x]}} +$$

$$\frac{(4 a A b - a^2 B + 4 b^2 B) \sqrt{\frac{b+a \text{Cos}[c+dx]}{a+b}} \text{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{\text{Sec}[c + d x]}}{4 b d \sqrt{a + b \text{Sec}[c + d x]}} -$$

$$\frac{(4 A b + a B) \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{a + b \text{Sec}[c + d x]}}{4 b d \sqrt{\frac{b+a \text{Cos}[c+dx]}{a+b}} \sqrt{\text{Sec}[c + d x]}} +$$

$$\frac{(4 A b + a B) \sqrt{\text{Sec}[c + d x]} \sqrt{a + b \text{Sec}[c + d x]} \text{Sin}[c + d x]}{4 b d} + \frac{B \text{Sec}[c + d x]^{3/2} \sqrt{a + b \text{Sec}[c + d x]} \text{Sin}[c + d x]}{2 d}$$

Result (type 4, 578 leaves):

$$\begin{aligned}
& - \frac{1}{16 b d \sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} \sqrt{a+b \sec [c+d x]} \\
& \left(\frac{8 a b B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] - 2(-4 a A b+3 a^2 B-8 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \frac{2(-4 a A b+3 a^2 B-8 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} \right) + \\
& \left(2 i(4 a A b+a^2 B) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \sin [c+d x] \Big/ \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\cos [c+d x]^2} \sqrt{\frac{a^2-a^2 \cos [c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \cos [c+d x])+2(b+a \cos [c+d x])^2\right) \right) \right) + \\
& \frac{\sqrt{a+b \sec [c+d x]} \left(\frac{\sec [c+d x](4 A b \sin [c+d x]+a B \sin [c+d x])}{4 b} + \frac{1}{2} B \sec [c+d x] \tan [c+d x]\right)}{d \sqrt{\sec [c+d x]}}
\end{aligned}$$

■ **Problem 436: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) dx$$

Optimal (type 4, 253 leaves, 12 steps):

$$\frac{(2 a A + b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{(2 A b + a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{B \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{B \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d}$$

Result (type 4, 377 leaves):

$$\frac{1}{4 d \sqrt{\sec [c+d x]}} \sqrt{a+b \sec [c+d x]} \left(\frac{8 a A \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{(a+b) \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2(4 A b + a B) \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{(a+b) \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} - \right.$$

$$\left. \left(2 i B \sqrt{-\frac{a(-1+\cos [c+d x])}{a+b}} \sqrt{\frac{a(1+\cos [c+d x])}{a-b}} \operatorname{Csc}[c+d x] \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right.$$

$$\left. \left. a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + \right. \right.$$

$$\left. \left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \right) / \left(a \sqrt{\frac{1}{a-b}} b \sqrt{b+a \cos [c+d x]} + 4 B \tan [c+d x] \right)$$

■ **Problem 441: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec [c+d x]^{3/2} (a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x]) dx$$

Optimal (type 4, 421 leaves, 14 steps):

$$\begin{aligned}
& \frac{(42 a A b + 17 a^2 B + 16 b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{24 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(6 a^2 A b + 8 A b^3 - a^3 B + 12 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{8 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{(30 a A b + 3 a^2 B + 16 b^2 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{24 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{(30 a A b + 3 a^2 B + 16 b^2 B) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 b d} + \\
& \frac{(6 A b + 7 a B) \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 d} + \frac{b B \operatorname{Sec}[c+d x]^{5/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 4, 673 leaves):

$$\begin{aligned}
& - \frac{1}{96 b d (b + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2}} (a + b \sec [c + d x])^{3/2} \left(\frac{2 (-24 a A b^2 - 28 a^2 b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \right. \\
& \frac{2 (-6 a^2 A b - 48 A b^3 + 9 a^3 B - 56 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \\
& \left(2 i (30 a^2 A b + 3 a^3 B + 16 a b^2 B) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right], \right. \right. \\
& \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \left. \right) \sin [c+d x] \Bigg/ \\
& \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos [c+d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \cos [c+d x]) + 2 (b + a \cos [c+d x])^2) \right) \Bigg) + \\
& \left((a + b \sec [c + d x])^{3/2} \left(\frac{1}{12} \sec [c + d x]^2 (6 A b \sin [c + d x] + 7 a B \sin [c + d x]) + \right. \right. \\
& \left. \frac{\sec [c + d x] (30 a A b \sin [c + d x] + 3 a^2 B \sin [c + d x] + 16 b^2 B \sin [c + d x])}{24 b} + \right. \\
& \left. \left. \frac{1}{3} b B \sec [c + d x]^2 \tan [c + d x] \right) \right) \Bigg/ (d (b + a \cos [c + d x]) \sec [c + d x]^{3/2})
\end{aligned}$$

■ **Problem 442: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec [c + d x]} (a + b \sec [c + d x])^{3/2} (A + B \sec [c + d x]) dx$$

Optimal (type 4, 339 leaves, 13 steps):

$$\begin{aligned}
& \frac{(8 a^2 A + 4 A b^2 + 7 a b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{4 d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{(12 a A b + 3 a^2 B + 4 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{4 d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{(4 A b + 5 a B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{4 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \\
& \frac{(4 A b + 5 a B) \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 d} + \frac{b B \sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 d}
\end{aligned}$$

Result (type 4, 595 leaves):

$$\begin{aligned}
& \frac{1}{16 d (b + a \cos [c + d x])^{3/2} \sec [c + d x]^{3/2}} (a + b \sec [c + d x])^{3/2} \\
& \left(\frac{2 (16 a^2 A + 4 a b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \frac{2 (20 a A b + a^2 B + 8 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \right. \\
& \left. \frac{2 i (-4 a A b - 5 a^2 B) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)]}{\sqrt{b+a \cos [c+d x]}} \right. \\
& \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \sin [c+d x] \Bigg/ \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos [c+d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c+d x]^2}{a^2}} \left(-a^2 + 2 b^2 - 4 b (b + a \cos [c+d x]) + 2 (b + a \cos [c+d x])^2 \right) \right) \right) + \\
& \left((a + b \sec [c + d x])^{3/2} \left(\frac{1}{4} \sec [c + d x] (4 A b \sin [c + d x] + 5 a B \sin [c + d x]) + \frac{1}{2} b B \sec [c + d x] \tan [c + d x] \right) \right) \Bigg/ \\
& (d (b + a \cos [c + d x]) \sec [c + d x]^{3/2})
\end{aligned}$$

- **Problem 443: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sec [c + d x])^{3/2} (A + B \sec [c + d x])}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 272 leaves, 12 steps):

$$\frac{(2 a A b + 2 a^2 B + b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{b(2 A b + 3 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{(2 a A - b B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{b B \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d}$$

Result (type 4, 554 leaves):

$$\frac{b B (a+b \sec [c+d x])^{3/2} \sin [c+d x]}{d (b+a \cos [c+d x]) \sqrt{\sec [c+d x]}} +$$

$$\frac{1}{4 d (b+a \cos [c+d x])^{3/2} \sec [c+d x]^{3/2}} (a+b \sec [c+d x])^{3/2} \left(\frac{2 (8 a A b + 4 a^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \right.$$

$$\frac{2 (2 a^2 A + 4 A b^2 + 5 a b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \left. \left(2 i (2 a^2 A - a b B) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \right. \right.$$

$$\left. \left. \cos [2(c+d x)] \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right. \right. \right.$$

$$\left. \left. \left. \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \sin [c+d x] \Big/$$

$$\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos [c+d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \cos [c+d x]) + 2 (b+a \cos [c+d x])^2) \right)$$

- **Problem 444: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 276 leaves, 12 steps):

$$\frac{2 (a^2 A - A b^2 + 3 a b B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c + d x]} + 2 b^2 B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c + d x]} + \frac{2 (4 A b + 3 a B) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a+b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]} + 2 a A \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a A \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 4, 563 leaves):

$$\begin{aligned}
& \frac{2 a A (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d (b+a \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x]^{3/2}} + \\
& \frac{1}{6 d (b+a \operatorname{Cos}[c+d x])^{3/2} \operatorname{Sec}[c+d x]^{3/2}} (a+b \operatorname{Sec}[c+d x])^{3/2} \left(\frac{2 (2 a^2 A+6 A b^2+12 a b B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \\
& \frac{2 (4 a A b+3 a^2 B+6 b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \\
& \left. \left(2 i (4 a A b+3 a^2 B) \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \right. \right. \\
& \left. \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right)\right) \operatorname{Sin}[c+d x] \right) / \\
& \left. \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \operatorname{Cos}[c+d x])+2(b+a \operatorname{Cos}[c+d x])^2\right) \right) \right) \right)
\end{aligned}$$

■ **Problem 448: Result unnecessarily involves imaginary or complex numbers.**

$$\int \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 4, 513 leaves, 15 steps):

$$\begin{aligned}
& \frac{(472 a^2 A b + 128 A b^3 + 133 a^3 B + 356 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{192 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{1}{64 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} \\
& (40 a^3 A b + 160 a A b^3 - 5 a^4 B + 120 a^2 b^2 B + 48 b^4 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} - \\
& \frac{(264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{192 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{(264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{192 b d} + \\
& \frac{(104 a A b + 59 a^2 B + 36 b^2 B) \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{96 d} + \\
& \frac{b(8 A b + 11 a B) \operatorname{Sec}[c+d x]^{5/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d} + \frac{b B \operatorname{Sec}[c+d x]^{5/2} (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{4 d}
\end{aligned}$$

Result (type 4, 768 leaves):

$$\begin{aligned}
& - \frac{1}{768 b d (b + a \cos [c + d x])^{5/2} \sec [c + d x]^{5/2}} \\
& (a + b \sec [c + d x])^{5/2} \left(\frac{2 (-416 a^2 A b^2 - 236 a^3 b B - 144 a b^3 B) \sqrt{\frac{b + a \cos [c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right]}{\sqrt{b + a \cos [c + d x]}} + \frac{1}{\sqrt{b + a \cos [c + d x]}} \right. \\
& 2 (24 a^3 A b - 832 a A b^3 + 45 a^4 B - 436 a^2 b^2 B - 288 b^4 B) \sqrt{\frac{b + a \cos [c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] + \\
& \left. \left(2 i (264 a^3 A b + 128 a A b^3 + 15 a^4 B + 284 a^2 b^2 B) \sqrt{\frac{a - a \cos [c + d x]}{a + b}} \sqrt{\frac{a + a \cos [c + d x]}{a - b}} \cos [2 (c + d x)] \right. \right. \\
& \left. \left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \frac{-a + b}{a + b}\right]\right) \right) \sin [c + d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \cos [c + d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \cos [c + d x]) + 2 (b + a \cos [c + d x])^2) \right) \right) + \\
& \left((a + b \sec [c + d x])^{5/2} \left(\frac{1}{24} \sec [c + d x]^3 (8 A b^2 \sin [c + d x] + 17 a b B \sin [c + d x]) + \right. \right. \\
& \frac{1}{96} \sec [c + d x]^2 (104 a A b \sin [c + d x] + 59 a^2 B \sin [c + d x] + 36 b^2 B \sin [c + d x]) + \\
& 1 / (192 b) \sec [c + d x] (264 a^2 A b \sin [c + d x] + 128 A b^3 \sin [c + d x] + 15 a^3 B \sin [c + d x] + 284 a b^2 B \sin [c + d x]) + \\
& \left. \left. \frac{1}{4} b^2 B \sec [c + d x]^3 \tan [c + d x] \right) \right) / (d (b + a \cos [c + d x])^2 \sec [c + d x]^{5/2})
\end{aligned}$$

■ **Problem 449: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec [c + d x]} (a + b \sec [c + d x])^{5/2} (A + B \sec [c + d x]) dx$$

Optimal (type 4, 422 leaves, 14 steps):

$$\begin{aligned}
& \frac{(48 a^3 A + 66 a A b^2 + 59 a^2 b B + 16 b^3 B) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{24 d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \\
& \frac{(30 a^2 A b + 8 A b^3 + 5 a^3 B + 20 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{8 d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \\
& \frac{(54 a A b + 33 a^2 B + 16 b^2 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]}}{24 d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\operatorname{Sec}[c+dx]}} + \\
& \frac{(54 a A b + 33 a^2 B + 16 b^2 B) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24 d} + \\
& \frac{b(2 A b + 3 a B) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d} + \frac{b B \operatorname{Sec}[c+dx]^{3/2} (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3 d}
\end{aligned}$$

Result (type 4, 678 leaves):

$$\begin{aligned}
& \frac{1}{96 d (b + a \cos [c + d x])^{5/2} \sec [c + d x]^{5/2}} (a + b \sec [c + d x])^{5/2} \left(\frac{2 (96 a^3 A + 24 a A b^2 + 52 a^2 b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \right. \\
& \frac{2 (126 a^2 A b + 48 A b^3 - 3 a^3 B + 104 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \\
& \left(2 i (-54 a^2 A b - 33 a^3 B - 16 a b^2 B) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \\
& \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \sin [c+d x] \Bigg) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos [c+d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \cos [c+d x]) + 2 (b + a \cos [c+d x])^2) \right) \right) + \\
& \left((a + b \sec [c + d x])^{5/2} \left(\frac{1}{12} \sec [c + d x]^2 (6 A b^2 \sin [c + d x] + 13 a b B \sin [c + d x]) + \right. \right. \\
& \frac{1}{24} \sec [c + d x] (54 a A b \sin [c + d x] + 33 a^2 B \sin [c + d x] + 16 b^2 B \sin [c + d x]) + \\
& \left. \left. \frac{1}{3} b^2 B \sec [c + d x]^2 \tan [c + d x] \right) \right) / (d (b + a \cos [c + d x])^2 \sec [c + d x]^{5/2})
\end{aligned}$$

■ **Problem 450: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \sec [c + d x])^{5/2} (A + B \sec [c + d x])}{\sqrt{\sec [c + d x]}} dx$$

Optimal (type 4, 359 leaves, 13 steps):

$$\begin{aligned}
& \frac{(16 a^2 A b + 4 A b^3 + 8 a^3 B + 11 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{b(20 a A b + 15 a^2 B + 4 b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(8 a^2 A - 4 A b^2 - 9 a b B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{b(4 A b + 7 a B) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{b B \sqrt{\operatorname{Sec}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{2 d}
\end{aligned}$$

Result (type 4, 628 leaves):

$$\begin{aligned}
& \frac{1}{16 d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2 (48 a^2 A b + 16 a^3 B + 4 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \\
& \frac{2 (8 a^3 A + 36 a A b^2 + 21 a^2 b B + 8 b^3 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \\
& \left. \left(2 i (8 a^3 A - 4 a A b^2 - 9 a^2 b B) \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \right. \right. \\
& \left. \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c+d x]) + 2 (b + a \operatorname{Cos}[c+d x])^2) \right) \right) + \\
& \left. \left((a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{1}{4} \operatorname{Sec}[c + d x] (4 A b^2 \operatorname{Sin}[c + d x] + 9 a b B \operatorname{Sin}[c + d x]) + \frac{1}{2} b^2 B \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x] \right) \right) / \right. \\
& \left. (d (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}) \right)
\end{aligned}$$

■ **Problem 451: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 349 leaves, 13 steps):

$$\begin{aligned}
& \frac{(2 a^3 A + 4 a A b^2 + 12 a^2 b B + 3 b^3 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{b^2 (2 A b + 5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(14 a A b + 6 a^2 B - 3 b^2 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} - \\
& \frac{b (2 a A - 3 b B) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 d} + \frac{2 a A (a+b \operatorname{Sec}[c+d x])^{3 / 2} \sin [c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 4, 599 leaves):

$$\begin{aligned}
& \frac{1}{12 d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2 (4 a^3 A + 36 a A b^2 + 36 a^2 b B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a+b}\right]}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \right. \\
& \frac{2 (14 a^2 A b + 12 A b^3 + 6 a^3 B + 27 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a+b}\right]}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \\
& \left(2 i (14 a^2 A b + 6 a^3 B - 3 a b^2 B) \sqrt{\frac{a - a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + d x]}{a - b}} \operatorname{Cos}[2 (c + d x)] \right. \\
& \left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right], \right. \right. \\
& \left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] \right) \operatorname{Sin}[c + d x] \Bigg) / \\
& \left. \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \operatorname{Cos}[c + d x]^2} \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \right) + \\
& \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2}{3} a^2 A \operatorname{Sin}[c + d x] + b^2 B \operatorname{Tan}[c + d x] \right)}{d (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}}
\end{aligned}$$

■ **Problem 452: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 342 leaves, 13 steps):

$$\begin{aligned}
& \frac{2 \left(8 a^2 A b - 8 A b^3 + 5 a^3 B + 10 a b^2 B \right) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{15 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{2 b^3 B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{2 \left(9 a^2 A + 23 A b^2 + 35 a b B \right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{15 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{2 a \left(8 A b + 5 a B \right) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{15 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 a A \left(a+b \operatorname{Sec}[c+d x] \right)^{3/2} \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 4, 616 leaves):

$$\begin{aligned}
& \frac{1}{30 d (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2}} \\
& (a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2 (34 a^2 A b + 30 A b^3 + 10 a^3 B + 90 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \\
& \frac{2 (9 a^3 A + 23 a A b^2 + 35 a^2 b B + 30 b^3 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \\
& \left(2 i (9 a^3 A + 23 a A b^2 + 35 a^2 b B) \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)] \right. \\
& \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+d x] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c+d x]) + 2 (b + a \operatorname{Cos}[c+d x])^2) \right) \right) + \\
& \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} \left(\frac{2}{15} a (11 A b + 5 a B) \operatorname{Sin}[c + d x] + \frac{1}{5} a^2 A \operatorname{Sin}[2(c + d x)] \right)}{d (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^{5/2}}
\end{aligned}$$

■ **Problem 456: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2} (A + B \operatorname{Sec}[c + d x])}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 344 leaves, 13 steps):

$$\begin{aligned}
& \frac{(4 A b - a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{4 b d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{(4 a A b - 3 a^2 B - 4 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{4 b^2 d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{(4 A b - 3 a B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{4 b^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \\
& \frac{(4 A b - 3 a B) \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 b^2 d} + \frac{B \sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 b d}
\end{aligned}$$

Result (type 4, 593 leaves):

$$\begin{aligned}
& \frac{1}{16 b^2 d \sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} \\
& \left(\frac{8 a b B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2(-12 a A b+9 a^2 B+8 b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \\
& \left. \frac{2 i(-4 a A b+3 a^2 B) \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \operatorname{Cos}[2(c+d x)]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} \right. \\
& \left. \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right)\right) \operatorname{Sin}[c+d x] \Bigg/ \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \operatorname{Cos}[c+d x])+2(b+a \operatorname{Cos}[c+d x])^2\right) \right) \right) + \\
& \frac{(b+a \operatorname{Cos}[c+d x]) \sqrt{\operatorname{Sec}[c+d x]} \left(\frac{\operatorname{Sec}[c+d x](4 A b \operatorname{Sin}[c+d x]-3 a B \operatorname{Sin}[c+d x])}{4 b^2} + \frac{B \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 b}\right)}{d \sqrt{a+b \operatorname{Sec}[c+d x]}}
\end{aligned}$$

■ **Problem 457: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2} (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 256 leaves, 12 steps):

$$\frac{B \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]} + (2Ab - aB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]}}{d \sqrt{a+b \sec[c+dx]}} + \frac{(2Ab - aB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]}}{bd \sqrt{a+b \sec[c+dx]}}$$

$$\frac{B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + B \sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{bd \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \sqrt{\sec[c+dx]}} + \frac{B \sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{bd}$$

Result (type 4, 339 leaves):

$$\frac{1}{4bd \sqrt{a+b \sec[c+dx]}}$$

$$\sqrt{\sec[c+dx]} \left(2(4Ab - 3aB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] - \frac{1}{a \sqrt{\frac{1}{a-b} b}} 2iB \sqrt{\frac{a(-1+\cos[c+dx])}{a+b}} \right.$$

$$\left. \sqrt{\frac{a(1+\cos[c+dx])}{a-b}} \sqrt{b+a \cos[c+dx]} \operatorname{Csc}[c+dx] \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + \right.$$

$$\left. a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + \right.$$

$$\left. \left. a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \right) + 4B(b+a \cos[c+dx]) \operatorname{Tan}[c+dx] \right)$$

■ **Problem 462: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c+dx]^{5/2} (A+B \sec[c+dx])}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 371 leaves, 13 steps):

$$\begin{aligned}
& \frac{B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]} + (2 A b-3 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b d \sqrt{a+b \sec [c+d x]}} + \frac{(2 a A b-3 a^2 B+b^2 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b^2 d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{b^2\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}}{2 a(A b-a B) \sec [c+d x]^{3 / 2} \sin [c+d x]} - \frac{(2 a A b-3 a^2 B+b^2 B) \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{b\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}}
\end{aligned}$$

Result (type 4, 647 leaves):

$$\begin{aligned}
& - \frac{1}{4 (a-b) b^2 (a+b) d (a+b \operatorname{Sec}[c+dx])^{3/2}} \\
& (b+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \left(\frac{2(-4aAb^2+4a^2bB) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \\
& \frac{2(-6a^2Ab+4Ab^3+9a^3B-7ab^2B) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \\
& \left(2i(-2a^2Ab+3a^3B-ab^2B) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right. \\
& \left. \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+dx] \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b+a \operatorname{Cos}[c+dx]) + 2(b+a \operatorname{Cos}[c+dx])^2) \right) \right) + \\
& \frac{(b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx]^{3/2} \left(\frac{2(a^2Ab \operatorname{Sin}[c+dx] - a^3B \operatorname{Sin}[c+dx])}{b^2(-a^2+b^2)(b+a \operatorname{Cos}[c+dx])} + \frac{B \operatorname{Tan}[c+dx]}{b^2} \right)}{d (a+b \operatorname{Sec}[c+dx])^{3/2}}
\end{aligned}$$

- **Problem 463: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx])}{(a+b \operatorname{Sec}[c+dx])^{3/2}} dx$$

Optimal (type 4, 220 leaves, 9 steps):

$$\frac{2 B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2(A b-a B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{2 a(A b-a B) \sqrt{\sec [c+d x]} \sin [c+d x]}{b\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}}$$

Result (type 4, 595 leaves):

$$-\frac{2(b+a \cos [c+d x]) \sec [c+d x]^{3/2}(a A b \sin [c+d x]-a^2 B \sin [c+d x])}{b\left(-a^2+b^2\right) d(a+b \sec [c+d x])^{3/2}} +$$

$$\frac{1}{2 b(-a+b)(a+b) d(a+b \sec [c+d x])^{3/2}}(b+a \cos [c+d x])^{3/2} \sec [c+d x]^{3/2}$$

$$\left(\frac{2\left(2 A b^2-2 a b B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} + \frac{2\left(a A b-3 a^2 B+2 b^2 B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}}\right) +$$

$$\left(2 i\left(a A b-a^2 B\right) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)]\right.$$

$$\left.-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] + a\left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right],\right.$$

$$\left.\frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \sin [c+d x] \Bigg/$$

$$\left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\cos [c+d x]}^2 \sqrt{\frac{a^2-a^2 \cos [c+d x]^2}{a^2}}\left(-a^2+2 b^2-4 b(b+a \cos [c+d x])+2(b+a \cos [c+d x])^2\right)\right)$$

■ **Problem 468: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^{5/2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 399 leaves, 13 steps):

$$\begin{aligned}
 & \frac{2 (A b - a B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 b \left(a^2 - b^2\right) d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
 & \frac{2 B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{b^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 \left(4 A b^3 + 3 a^3 B - 7 a b^2 B\right) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 b^2 \left(a^2 - b^2\right)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
 & \frac{2 a (A b - a B) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 b \left(a^2 - b^2\right) d \left(a+b \operatorname{Sec}[c+d x]\right)^{3/2}} - \frac{2 a \left(4 A b^3 + 3 a^3 B - 7 a b^2 B\right) \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^2 \left(a^2 - b^2\right)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}
 \end{aligned}$$

Result (type 4, 726 leaves):

$$\begin{aligned}
& \frac{1}{6 (a-b)^2 b^2 (a+b)^2 d (a+b \operatorname{Sec}[c+dx])^{5/2}} \\
& (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \left(\frac{2 (2 a^2 A b^2 + 6 A b^4 + 4 a^3 b B - 12 a b^3 B) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \\
& \frac{2 (4 a A b^3 + 9 a^4 B - 19 a^2 b^2 B + 6 b^4 B) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \\
& \left(2 i (4 a A b^3 + 3 a^4 B - 7 a^2 b^2 B) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2 (c+dx)] \right. \\
& \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+dx] \Big/ \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+dx]) + 2 (b+a \operatorname{Cos}[c+dx])^2) \right) \right) + \\
& \frac{1}{d (a+b \operatorname{Sec}[c+dx])^{5/2}} (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx]^{5/2} \left(-\frac{2 (a A b \operatorname{Sin}[c+dx] - a^2 B \operatorname{Sin}[c+dx])}{3 b (-a^2 + b^2) (b+a \operatorname{Cos}[c+dx])^2} - \right. \\
& \left. \frac{2 (4 a A b^3 \operatorname{Sin}[c+dx] + 3 a^4 B \operatorname{Sin}[c+dx] - 7 a^2 b^2 B \operatorname{Sin}[c+dx])}{3 b^2 (-a^2 + b^2)^2 (b+a \operatorname{Cos}[c+dx])} \right)
\end{aligned}$$

■ **Problem 479: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+dx]^m (a+b \operatorname{Sec}[c+dx])^4 (A+B \operatorname{Sec}[c+dx]) dx$$

Optimal (type 5, 544 leaves, 9 steps):

$$\frac{1}{d(1+m)(3+m)(4+m)} b \left(A b^3 (8+6m+m^2) + 4 a b^2 B (8+6m+m^2) + 2 a^3 B (19+8m+m^2) + a^2 A b (68+37m+5m^2) \right) \text{Sec}[c+dx]^{1+m} \text{Sin}[c+dx] +$$

$$\frac{b^2 (b^2 B (3+m)^2 + 2 a A b (4+m)^2 + a^2 B (26+9m+m^2)) \text{Sec}[c+dx]^{2+m} \text{Sin}[c+dx]}{d(2+m)(3+m)(4+m)} +$$

$$\frac{b(Ab(4+m) + aB(7+m)) \text{Sec}[c+dx]^{1+m} (a+b \text{Sec}[c+dx])^2 \text{Sin}[c+dx]}{d(3+m)(4+m)} + \frac{bB \text{Sec}[c+dx]^{1+m} (a+b \text{Sec}[c+dx])^3 \text{Sin}[c+dx]}{d(4+m)} -$$

$$\left((A b^4 m (2+m) + 4 a b^3 B m (2+m) + 6 a^2 A b^2 m (3+m) + 4 a^3 b B m (3+m) + a^4 A (3+4m+m^2)) \right.$$

$$\left. \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^{-1+m} \text{Sin}[c+dx] \right) / \left(d(1-m)(1+m)(3+m) \sqrt{\text{Sin}[c+dx]^2} \right) +$$

$$\left((b^4 B (3+4m+m^2) + 4 a A b^3 (4+5m+m^2) + 6 a^2 b^2 B (4+5m+m^2) + 4 a^3 A b (8+6m+m^2) + a^4 B (8+6m+m^2)) \right.$$

$$\left. \text{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{m}{2}, \frac{2-m}{2}, \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^m \text{Sin}[c+dx] \right) / \left(d m (2+m)(4+m) \sqrt{\text{Sin}[c+dx]^2} \right)$$

Result (type 8, 33 leaves):

$$\int \text{Sec}[c+dx]^m (a+b \text{Sec}[c+dx])^4 (A+B \text{Sec}[c+dx]) dx$$

■ **Problem 480: Unable to integrate problem.**

$$\int \text{Sec}[c+dx]^m (a+b \text{Sec}[c+dx])^3 (A+B \text{Sec}[c+dx]) dx$$

Optimal (type 5, 366 leaves, 8 steps):

$$\frac{b(b^2 B (2+m) + 3 a A b (3+m) + 2 a^2 B (4+m)) \text{Sec}[c+dx]^{1+m} \text{Sin}[c+dx]}{d(1+m)(3+m)} +$$

$$\frac{b^2 (A b (3+m) + a B (5+m)) \text{Sec}[c+dx]^{2+m} \text{Sin}[c+dx]}{d(2+m)(3+m)} + \frac{b B \text{Sec}[c+dx]^{1+m} (a+b \text{Sec}[c+dx])^2 \text{Sin}[c+dx]}{d(3+m)} -$$

$$\left((b^3 B m (2+m) + 3 a A b^2 m (3+m) + 3 a^2 b B m (3+m) + a^3 A (3+4m+m^2)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \text{Cos}[c+dx]^2 \right] \right.$$

$$\left. \text{Sec}[c+dx]^{-1+m} \text{Sin}[c+dx] \right) / \left(d(3+m) (1-m^2) \sqrt{\text{Sin}[c+dx]^2} \right) +$$

$$\left((A b^3 (1+m) + 3 a b^2 B (1+m) + 3 a^2 A b (2+m) + a^3 B (2+m)) \text{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{m}{2}, \frac{2-m}{2}, \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^m \text{Sin}[c+dx] \right) /$$

$$\left(d m (2+m) \sqrt{\text{Sin}[c+dx]^2} \right)$$

Result (type 8, 33 leaves):

$$\int \operatorname{Sec}[c + d x]^m (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x]) dx$$

■ **Problem 483: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^{7/2} (a + a \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 132 leaves, 8 steps):

$$\frac{6 a (A + B) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{2 a (5 A + 7 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{2 a (5 A + 7 B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{21 d} + \frac{2 a (A + B) \operatorname{Cos}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{5 d} + \frac{2 a A \operatorname{Cos}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{7 d}$$

Result (type 5, 872 leaves):

$$a \left(\sqrt{\operatorname{Cos}[c + d x]} (1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(-\frac{3 (A + B) \operatorname{Cot}[c]}{5 d} + \frac{(23 A + 28 B) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{84 d} + \frac{(A + B) \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{10 d} + \right. \right.$$

$$\left. \frac{A \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{28 d} + \frac{(23 A + 28 B) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{84 d} + \frac{(A + B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{10 d} + \frac{A \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{28 d} \right) - \frac{1}{21 d \sqrt{1 + \operatorname{Cot}[c]^2}}$$

$$5 A (1 + \operatorname{Cos}[c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} B (1 + \operatorname{Cos}[c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{10 d} 3 A (1 + \operatorname{Cos}[c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) - \frac{1}{10 d} 3 B (1 + \cos [c + d x]) \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c] \right) / \right.$$

$$\left. \left(\frac{\sqrt{1 - \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2}}{\frac{\sin [d x + \text{ArcTan}[\text{Tan}[c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}} \right) \right)$$

■ **Problem 484: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{5/2} (a + a \sec [c + d x]) (A + B \sec [c + d x]) dx$$

Optimal (type 4, 101 leaves, 7 steps):

$$\frac{2 a (3 A + 5 B) \text{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{2 a (A + B) \text{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{3 d} +$$

$$\frac{2 a (A + B) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 a A \cos [c + d x]^{3/2} \sin [c + d x]}{5 d}$$

Result (type 5, 830 leaves):

$$a \left(\sqrt{\cos [c + d x]} (1 + \cos [c + d x]) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \right)$$

$$\left(-\frac{(3A+5B)\cot[c]}{5d} + \frac{(A+B)\cos[dx]\sin[c]}{3d} + \frac{A\cos[2dx]\sin[2c]}{10d} + \frac{(A+B)\cos[c]\sin[dx]}{3d} + \frac{A\cos[2c]\sin[2dx]}{10d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}}$$

$$A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3d\sqrt{1+\cot[c]^2}} B(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{10d} 3A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{2d} B(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

- **Problem 485: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \sec[c + d x]) (A + B \sec[c + d x]) dx$$

Optimal (type 4, 70 leaves, 6 steps):

$$\frac{2 a (A + B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (A + 3 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a A \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d}$$

Result (type 5, 784 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(-\frac{(A + B) \cot[c]}{d} + \frac{A \cos[d x] \sin[c]}{3 d} + \frac{A \cos[c] \sin[d x]}{3 d} \right) - \frac{1}{3 d \sqrt{1 + \cot[c]^2}} \right.$$

$$A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{d \sqrt{1 + \cot[c]^2}} B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{2 d} A (1 + \cos[c + d x]) \text{Csc}[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) - \right.$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} - \frac{1}{2d} B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)$$

- **Problem 486: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \text{Sec}[c + d x]) (A + B \text{Sec}[c + d x]) dx$$

Optimal (type 4, 66 leaves, 6 steps):

$$\frac{2 a (A - B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (A + B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a B \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 783 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(-\frac{(A - 2 B + A \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{2 d} + \frac{B \text{Sec}[c] \text{Sec}[c + d x] \sin[d x]}{d} \right) - \frac{1}{d \sqrt{1 + \cot[c]^2}} \right.$$

$$A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}} -$$

$$\begin{aligned}
& \frac{1}{d \sqrt{1 + \cot[c]^2}} B (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} A (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left(\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) + \frac{1}{2d} B (1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left(\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right)
\end{aligned}$$

■ **Problem 487: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx]) (A + B \operatorname{Sec}[c + dx])}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 95 leaves, 7 steps):

$$-\frac{2 a (A+B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d} + \frac{2 a (3 A+B) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} + \frac{2 a B \sin [c+d x]}{3 d \cos [c+d x]^{3 / 2}} + \frac{2 a (A+B) \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 5, 813 leaves):

$$a \left(\sqrt{\cos [c+d x]} (1+\cos [c+d x]) \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \right. \\ \left. \left(\frac{(A+B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{B \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \sin [d x]}{3 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x] (B \sin [c]+3 A \sin [d x]+3 B \sin [d x])}{3 d} \right) - \frac{1}{d \sqrt{1+\cot [c]^2}} \right. \\ \left. A (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \right. \\ \left. \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \right. \\ \left. \frac{1}{3 d \sqrt{1+\cot [c]^2}} B (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \right. \\ \left. \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\ \left. \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} + \frac{1}{2 d} A (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \right. \\ \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\ \left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right. \\ \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) + \frac{1}{2 d} B (1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

■ **Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + dx]) (A + B \text{Sec}[c + dx])}{\text{Cos}[c + dx]^{3/2}} dx$$

Optimal (type 4, 132 leaves, 8 steps):

$$- \frac{2a(5A + 3B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \frac{2a(A + B) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \\ \frac{2aB \text{Sin}[c + dx]}{5d \text{Cos}[c + dx]^{5/2}} + \frac{2a(A + B) \text{Sin}[c + dx]}{3d \text{Cos}[c + dx]^{3/2}} + \frac{2a(5A + 3B) \text{Sin}[c + dx]}{5d \sqrt{\text{Cos}[c + dx]}}$$

Result (type 5, 865 leaves):

$$a \left(\sqrt{\text{Cos}[c + dx]} (1 + \text{Cos}[c + dx]) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left(\frac{(5A + 3B) \text{Csc}[c] \text{Sec}[c]}{5d} + \frac{B \text{Sec}[c] \text{Sec}[c + dx]^3 \text{Sin}[dx]}{5d} + \frac{\text{Sec}[c] \text{Sec}[c + dx]^2 (3B \text{Sin}[c] + 5A \text{Sin}[dx] + 5B \text{Sin}[dx])}{15d} \right. \\ \left. \left. \frac{\text{Sec}[c] \text{Sec}[c + dx] (5A \text{Sin}[c] + 5B \text{Sin}[c] + 15A \text{Sin}[dx] + 9B \text{Sin}[dx])}{15d} \right) - \frac{1}{3d \sqrt{1 + \text{Cot}[c]^2}} \right) \\ A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\ \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} B (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \frac{1}{2 d} A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) + \frac{1}{10 d} 3 B (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) \right)$$

■ **Problem 489: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{9/2} (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 4, 194 leaves, 9 steps):

$$\frac{4 a^2 (8 A+9 B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \frac{4 a^2 (5 A+6 B) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d} + \frac{4 a^2 (5 A+6 B) \sqrt{\cos [c+d x]} \sin [c+d x]}{21 d} +$$

$$\frac{4 a^2 (8 A+9 B) \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d} + \frac{2 a^2 (11 A+9 B) \cos [c+d x]^{5 / 2} \sin [c+d x]}{63 d} + \frac{2 A \cos [c+d x]^{5 / 2} \left(a^2+a^2 \cos [c+d x]\right) \sin [c+d x]}{9 d}$$

Result (type 5, 1086 leaves):

$$\frac{1}{B+A \cos [c+d x]} \cos [c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x])$$

$$\left(-\frac{(8 A+9 B) \cot [c]}{15 d}+\frac{(46 A+51 B) \cos [d x] \sin [c]}{168 d}+\frac{(37 A+36 B) \cos [2 d x] \sin [2 c]}{360 d}+\frac{(2 A+B) \cos [3 d x] \sin [3 c]}{56 d}+\frac{A \cos [4 d x] \sin [4 c]}{144 d}+\right.$$

$$\left.\frac{(46 A+51 B) \cos [c] \sin [d x]}{168 d}+\frac{(37 A+36 B) \cos [2 c] \sin [2 d x]}{360 d}+\frac{(2 A+B) \cos [3 c] \sin [3 d x]}{56 d}+\frac{A \cos [4 c] \sin [4 d x]}{144 d}\right)-$$

$$\frac{1}{21 d(B+A \cos [c+d x]) \sqrt{1+\cot [c]^2}} 5 A \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-}$$

$$\frac{1}{7 d(B+A \cos [c+d x]) \sqrt{1+\cot [c]^2}} 2 B \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-}$$

$$\frac{1}{15 d(B+A \cos [c+d x])} 4 A \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x])$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) /$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}\right)-$$

$$\frac{\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{10 d (B + A \cos[c + d x])} - \frac{1}{10 d (B + A \cos[c + d x])}$$

$$3 B \cos[c + d x]^3 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x])$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}$$

■ **Problem 490: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{7/2} (a + a \sec[c + d x])^2 (A + B \sec[c + d x]) dx$$

Optimal (type 4, 161 leaves, 8 steps):

$$\frac{4 a^2 (3 A + 4 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^2 (6 A + 7 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \frac{4 a^2 (6 A + 7 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a^2 (9 A + 7 B) \cos[c + d x]^{3/2} \sin[c + d x]}{35 d} + \frac{2 A \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{7 d}$$

Result (type 5, 1040 leaves):

$$\frac{1}{B + A \cos[c + d x]} \cos[c + d x]^{7/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2$$

$$(A + B \sec[c + d x]) \left(-\frac{(3 A + 4 B) \cot[c]}{5 d} + \frac{(51 A + 56 B) \cos[d x] \sin[c]}{168 d} + \frac{(2 A + B) \cos[2 d x] \sin[2 c]}{20 d} + \frac{A \cos[3 d x] \sin[3 c]}{56 d} + \frac{(51 A + 56 B) \cos[c] \sin[d x]}{168 d} + \frac{(2 A + B) \cos[2 c] \sin[2 d x]}{20 d} + \frac{A \cos[3 c] \sin[3 d x]}{56 d} \right) -$$

$$\frac{1}{7 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2}} 2 A \cos [c + d x]^3 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{3 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2}} B \cos [c + d x]^3 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{10 d (B + A \cos [c + d x])} 3 A \cos [c + d x]^3 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}} - \frac{1}{5 d (B + A \cos [c + d x])}$$

$$2 B \cos [c + d x]^3 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} \right)$$

- **Problem 491: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{5/2} (a + a \sec[c + d x])^2 (A + B \sec[c + d x]) dx$$

Optimal (type 4, 126 leaves, 7 steps):

$$\frac{4 a^2 (4 A + 5 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^2 (A + 2 B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a^2 (7 A + 5 B) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 A \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{5 d}$$

Result (type 5, 994 leaves):

$$\frac{1}{B + A \cos[c + d x]} \cos[c + d x]^{7/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x])$$

$$\left(-\frac{(4 A + 5 B) \cot[c]}{5 d} + \frac{(2 A + B) \cos[d x] \sin[c]}{6 d} + \frac{A \cos[2 d x] \sin[2 c]}{20 d} + \frac{(2 A + B) \cos[c] \sin[d x]}{6 d} + \frac{A \cos[2 c] \sin[2 d x]}{20 d} \right) -$$

$$\frac{1}{3 d (B + A \cos[c + d x]) \sqrt{1 + \cot[c]^2}} A \cos[c + d x]^3 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x]) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3 d (B + A \cos[c + d x]) \sqrt{1 + \cot[c]^2}} 2 B \cos[c + d x]^3 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x]) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{5 d (B + A \cos [c + d x])} 2 A \cos [c + d x]^3 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \text{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \left) - \frac{1}{2 d (B + A \cos [c + d x])}$$

$$B \cos [c + d x]^3 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \text{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \text{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \left) -$$

■ **Problem 492: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{3/2} (a + a \sec [c + d x])^2 (A + B \sec [c + d x]) dx$$

Optimal (type 4, 116 leaves, 7 steps):

$$\frac{4 a^2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d} + \frac{4 a^2 (2 A+3 B) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} +$$

$$\frac{2 a^2 (A-3 B) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 d} + \frac{2 B\left(a^2+a^2 \cos [c+d x]\right) \sin [c+d x]}{d \sqrt{\cos [c+d x]}}$$

Result (type 5, 735 leaves):

$$\begin{aligned}
& \frac{1}{B + A \cos[c + dx]} \cos[c + dx]^{7/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \\
& \left(-\frac{(2A - B + 2A \cos[2c] + B \cos[2c]) \csc[c] \sec[c]}{4d} + \frac{A \cos[dx] \sin[c]}{6d} + \frac{A \cos[c] \sin[dx]}{6d} + \frac{B \sec[c] \sec[c + dx] \sin[dx]}{2d} \right) - \\
& \frac{1}{3d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2}} 2A \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2}} B \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{2d (B + A \cos[c + dx])} A \cos[c + dx]^3 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}}
\end{aligned}$$

■ **Problem 493: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + dx]} (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) dx$$

Optimal (type 4, 120 leaves, 7 steps):

$$-\frac{4 a^2 B \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{d} + \frac{4 a^2 (3 A+2 B) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} + \frac{2 a^2 (3 A+5 B) \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{2 B\left(a^2+a^2 \operatorname{Cos}[c+d x]\right) \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{3 / 2}}$$

Result (type 5, 736 leaves):

$$\frac{1}{B+A \operatorname{Cos}[c+d x]} \operatorname{Cos}[c+d x]^{7 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x])$$

$$\left(-\frac{(-A-4 B+A \operatorname{Cos}[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{4 d} + \frac{B \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \operatorname{Sin}[d x]}{6 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x] (B \operatorname{Sin}[c]+3 A \operatorname{Sin}[d x]+6 B \operatorname{Sin}[d x])}{6 d}\right) -$$

$$\frac{1}{d (B+A \operatorname{Cos}[c+d x]) \sqrt{1+\operatorname{Cot}[c]^2}} A \operatorname{Cos}[c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{3 d (B+A \operatorname{Cos}[c+d x]) \sqrt{1+\operatorname{Cot}[c]^2}} 2 B \operatorname{Cos}[c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\operatorname{Sin}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} +$$

$$\frac{1}{2 d (B+A \operatorname{Cos}[c+d x])} B \operatorname{Cos}[c+d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x])$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) /$$

$$\left(\sqrt{1-\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2}\right) -$$

$$\frac{\frac{\operatorname{Sin}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}}$$

- **Problem 494: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])}{\sqrt{\operatorname{Cos}[c + d x]}} dx$$

Optimal (type 4, 159 leaves, 8 steps):

$$-\frac{4 a^2 (5 A + 4 B) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (2 A + B) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} + \frac{2 a^2 (5 A + 7 B) \operatorname{Sin}[c + d x]}{15 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^2 (5 A + 4 B) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 B (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{5 d \operatorname{Cos}[c + d x]^{5/2}}$$

Result (type 5, 1025 leaves):

$$\frac{1}{B + A \operatorname{Cos}[c + d x]} \operatorname{Cos}[c + d x]^{7/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])$$

$$\left(\frac{(5 A + 4 B) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{B \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \operatorname{Sin}[d x]}{10 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (3 B \operatorname{Sin}[c] + 5 A \operatorname{Sin}[d x] + 10 B \operatorname{Sin}[d x])}{30 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (5 A \operatorname{Sin}[c] + 10 B \operatorname{Sin}[c] + 30 A \operatorname{Sin}[d x] + 24 B \operatorname{Sin}[d x])}{30 d} \right) -$$

$$\frac{1}{3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 2 A \operatorname{Cos}[c + d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} B \operatorname{Cos}[c + d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} +$$

$$\frac{1}{2 d (B + A \operatorname{Cos}[c + d x])} A \operatorname{Cos}[c + d x]^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x])$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\ \left. \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\ \left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) + \frac{1}{5 d (B + A \cos [c + d x])}$$

$$2 B \cos [c + d x]^3 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x])$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\ \left. \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\ \left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) -$$

■ **Problem 495: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec [c + d x])^2 (A + B \sec [c + d x])}{\cos [c + d x]^{3/2}} dx$$

Optimal (type 4, 194 leaves, 9 steps):

$$- \frac{4 a^2 (4 A + 3 B) \text{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^2 (7 A + 6 B) \text{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{21 d} + \\ \frac{2 a^2 (7 A + 9 B) \sin [c + d x]}{35 d \cos [c + d x]^{5/2}} + \frac{4 a^2 (7 A + 6 B) \sin [c + d x]}{21 d \cos [c + d x]^{3/2}} + \frac{4 a^2 (4 A + 3 B) \sin [c + d x]}{5 d \sqrt{\cos [c + d x]}} + \frac{2 B (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{7 d \cos [c + d x]^{7/2}}$$

Result (type 5, 1067 leaves):

$$\begin{aligned}
& \frac{1}{B + A \cos[c + dx]} \cos[c + dx]^{7/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \\
& \left(\frac{(4A + 3B) \csc[c] \sec[c]}{5d} + \frac{B \sec[c] \sec[c + dx]^4 \sin[dx]}{14d} + \frac{\sec[c] \sec[c + dx]^3 (5B \sin[c] + 7A \sin[dx] + 14B \sin[dx])}{70d} + \right. \\
& \quad \frac{\sec[c] \sec[c + dx]^2 (21A \sin[c] + 42B \sin[c] + 70A \sin[dx] + 60B \sin[dx])}{210d} + \\
& \quad \left. \frac{\sec[c] \sec[c + dx] (35A \sin[c] + 30B \sin[c] + 84A \sin[dx] + 63B \sin[dx])}{105d} \right) - \\
& \frac{1}{3d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2}} A \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{7d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2}} 2B \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} +} \\
& \frac{1}{5d (B + A \cos[c + dx])} 2A \cos[c + dx]^3 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx]) \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -
\end{aligned}$$

$$\frac{\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{10 d (B + A \cos[c + d x])} + \frac{1}{10 d (B + A \cos[c + d x])}$$

$$3 B \cos[c + d x]^3 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x])$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right. \\ \left. \left. \frac{\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) \right)$$

■ **Problem 496: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{5/2} (A + B \text{Sec}[c + d x])}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 157 leaves, 7 steps):

$$\frac{3 (7 A - 5 B) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 a d} - \frac{5 (A - B) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 a d} - \\ \frac{5 (A - B) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a d} + \frac{(7 A - 5 B) \cos[c + d x]^{3/2} \sin[c + d x]}{5 a d} - \frac{(A - B) \cos[c + d x]^{5/2} \sin[c + d x]}{d (a + a \cos[c + d x])}$$

Result (type 5, 1292 leaves):

$$\frac{1}{20 (B + A \cos[c + d x]) (a + a \text{Sec}[c + d x])} 21 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] (A + B \text{Sec}[c + d x]) \\ \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \frac{1}{4 (B + A \operatorname{Cos}[c + d x]) (a + a \operatorname{Sec}[c + d x])} 3 i B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec}[c + d x]) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left(\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\operatorname{Cos}[c + d x]} (A + B \operatorname{Sec}[c + d x]) \left(\frac{2 (-5 A + 5 B - 16 A \operatorname{Cos}[c] + 10 B \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{4 (-A + B) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \right. \right. \\
& \quad \left. \left. \frac{2 A \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (-A \operatorname{Sin} \left[\frac{d x}{2} \right] + B \operatorname{Sin} \left[\frac{d x}{2} \right])}{d} + \frac{4 (-A + B) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 d} + \frac{2 A \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{5 d} \right) \right) / \\
& ((B + A \operatorname{Cos}[c + d x]) (a + a \operatorname{Sec}[c + d x])) + \left(5 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \right. \\
& \quad \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right) - \\
& \left(5 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec}[c + d x]) \right. \\
& \quad \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \left(3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right)
\end{aligned}$$

Problem 497: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos[c + dx]^{3/2} (A + B \sec[c + dx])}{a + a \sec[c + dx]} dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$-\frac{3(A-B) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(5A-3B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3ad} + \frac{(5A-3B) \sqrt{\cos[c+dx]} \sin[c+dx]}{3ad} - \frac{(A-B) \cos[c+dx]^{3/2} \sin[c+dx]}{d(a+a \cos[c+dx])}$$

Result (type 5, 1239 leaves):

$$\begin{aligned} & -\frac{1}{4(B+A \cos[c+dx])(a+a \sec[c+dx])} 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+B \sec[c+dx]) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) + \\ & \frac{1}{4(B+A \cos[c+dx])(a+a \sec[c+dx])} 3iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+B \sec[c+dx]) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) + \\ & \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} (A+B \sec[c+dx]) \left(-\frac{2(-A+B)(1+2\cos[c]) \operatorname{Csc}[c]}{d} + \frac{4A \cos[dx] \sin[c]}{3d} - \right. \right. \\ & \quad \left. \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{d} + \frac{4A \cos[c] \sin[dx]}{3d} \right) \right) / ((B+A \cos[c+dx])(a+a \sec[c+dx])) - \end{aligned}$$

$$\left(5 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ \left. (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right) + \\ \left(B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\ \left. \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right)$$

■ **Problem 498: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \operatorname{Sec}[c + dx])}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 4, 88 leaves, 5 steps):

$$\frac{(3A - B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1208 leaves):

$$\frac{1}{4(B + A \cos[c + dx])(a + a \operatorname{Sec}[c + dx])} 3i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \\ \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\ \frac{1}{4(B + A \cos[c + dx])(a + a \operatorname{Sec}[c + dx])} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx])$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \quad \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} (A + B \sec [c + d x]) \left(-\frac{2 (A - B + 2 A \cos [c]) \csc [c]}{d} + \frac{2 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (-A \sin \left[\frac{d x}{2} \right] + B \sin \left[\frac{d x}{2} \right])}{d} \right) \right) / \\
& \quad ((B + A \cos [c + d x]) (a + a \sec [c + d x])) + \\
& \quad \left(A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \sec \left[\frac{c}{2} \right] (A + B \sec [c + d x]) \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \quad \left(d (B + A \cos [c + d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right) - \\
& \quad \left(B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \sec \left[\frac{c}{2} \right] (A + B \sec [c + d x]) \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \quad \left(d (B + A \cos [c + d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right)
\end{aligned}$$

- **Problem 499: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x]}{\sqrt{\cos [c + d x]} (a + a \sec [c + d x])} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{(A - B) \text{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{a d} + \frac{(A + B) \text{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{a d} + \frac{(A - B) \sqrt{\cos [c + d x]} \sin [c + d x]}{d (a + a \cos [c + d x])}$$

Result (type 5, 1204 leaves):

$$\begin{aligned}
& - \frac{1}{4 (B + A \cos[c + dx]) (a + a \sec[c + dx])} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \frac{1}{4 (B + A \cos[c + dx]) (a + a \sec[c + dx])} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} (A + B \sec[c + dx]) \left(-\frac{2 (-A+B) \operatorname{Csc}[c]}{d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{d} \right) \\
& \frac{1}{(B + A \cos[c + dx]) (a + a \sec[c + dx])} - \\
& \left(A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx]) \right) - \\
& \left(B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$\left(d (B + A \cos[c + d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + d x]) \right)$$

■ **Problem 500: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + d x]}{\cos[c + d x]^{3/2} (a + a \sec[c + d x])} dx$$

Optimal (type 4, 113 leaves, 6 steps):

$$\frac{(A - 3 B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a d} + \frac{(A - B) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{a d} - \frac{(A - 3 B) \sin[c + d x]}{a d \sqrt{\cos[c + d x]}} + \frac{(A - B) \sin[c + d x]}{d \sqrt{\cos[c + d x]} (a + a \cos[c + d x])}$$

Result (type 5, 1240 leaves):

$$\frac{1}{4 (B + A \cos[c + d x]) (a + a \sec[c + d x])} i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + d x])$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\frac{1}{4 (B + A \cos[c + d x]) (a + a \sec[c + d x])} 3 i B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + d x])$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{\cos[c + d x]} (A + B \sec[c + d x]) \left(\frac{(2 B - A \cos[c] + B \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c]}{d} + \right.$$

$$\begin{aligned}
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(-A \operatorname{Sin}\left[\frac{dx}{2}\right] + B \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{d} + \frac{4 B \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{d} \right) / \left((B + A \operatorname{Cos}[c + dx]) (a + a \operatorname{Sec}[c + dx]) \right) - \\
& \left(A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (B + A \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right) + \\
& \left(B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (B + A \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right)
\end{aligned}$$

■ **Problem 501: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 (A - B) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{a d} - \frac{(3 A - 5 B) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{3 a d} - \\
& \frac{(3 A - 5 B) \operatorname{Sin}[c + dx]}{3 a d \operatorname{Cos}[c + dx]^{3/2}} + \frac{3 (A - B) \operatorname{Sin}[c + dx]}{a d \sqrt{\operatorname{Cos}[c + dx]}} + \frac{(A - B) \operatorname{Sin}[c + dx]}{d \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Cos}[c + dx])}
\end{aligned}$$

Result (type 5, 1277 leaves):

$$\begin{aligned}
& - \frac{1}{4 (B + A \operatorname{Cos}[c + dx]) (a + a \operatorname{Sec}[c + dx])} 3 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx]) \\
& \quad \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) -
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \\
& \frac{1}{4 (B + A \cos [c + d x]) (a + a \sec [c + d x])} 3 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \sec [c + d x]) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \right. \\
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \sqrt{\cos [c + d x]} (A + B \sec [c + d x]) \left(-\frac{(-A + B) (2 + \cos [c]) \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c]}{d} - \right. \right. \\
& \quad \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (-A \sin \left[\frac{d x}{2} \right] + B \sin \left[\frac{d x}{2} \right])}{d} + \frac{4 B \sec [c] \operatorname{Sec} [c + d x]^2 \sin [d x]}{3 d} + \\
& \quad \left. \left. \frac{4 \operatorname{Sec} [c] \operatorname{Sec} [c + d x] (B \sin [c] + 3 A \sin [d x] - 3 B \sin [d x])}{3 d} \right) \right) / \left((B + A \cos [c + d x]) (a + a \sec [c + d x]) \right) + \\
& \left(A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]}} \right) / \\
& \left(d (B + A \cos [c + d x]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \sec [c + d x]) \right) - \\
& \left(5 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right)
\end{aligned}$$

$$\left(\frac{\sqrt{-\sqrt{1 + \cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]}}{\dots} \right) /$$

$$\left(3 d (B + A \cos[c + dx]) \sqrt{1 + \cot^2[c]} (a + a \sec[c + dx]) \right)$$

■ **Problem 502: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + B \sec[c + dx])}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 4, 204 leaves, 8 steps):

$$\frac{7(8A - 5B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5a^2 d} - \frac{5(3A - 2B) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} - \frac{5(3A - 2B) \sqrt{\cos[c + dx]} \sin[c + dx]}{3a^2 d} +$$

$$\frac{7(8A - 5B) \cos[c + dx]^{3/2} \sin[c + dx]}{15a^2 d} - \frac{(3A - 2B) \cos[c + dx]^{5/2} \sin[c + dx]}{a^2 d (1 + \cos[c + dx])} - \frac{(A - B) \cos[c + dx]^{7/2} \sin[c + dx]}{3d (a + a \cos[c + dx])^2}$$

Result (type 5, 1396 leaves):

$$\frac{1}{5(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} 28 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + dx] (A + B \sec[c + dx])$$

$$\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\frac{1}{2(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} 7 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + dx] (A + B \sec[c + dx])$$

$$\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\begin{aligned}
& \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) - \\
& \left(20 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx]) \left(\frac{4(-20A + 15B - 36A \cos[c] + 20B \cos[c]) \operatorname{Csc}[c]}{5d} + \frac{8(-2A + B) \cos[dx] \sin[c]}{3d} + \frac{4A \cos[2dx] \sin[2c]}{5d} \right. \right. \\
& \quad \left. \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-4A \sin\left[\frac{dx}{2}\right] + 3B \sin\left[\frac{dx}{2}\right])}{d} + \frac{8(-2A + B) \cos[c] \sin[dx]}{3d} \right. \right. \\
& \quad \left. \left. \frac{4A \cos[2c] \sin[2dx]}{5d} - \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \left(\sqrt{\cos[c + dx]} (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)
\end{aligned}$$

- **Problem 503: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx])}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 171 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(7A - 4B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{5(2A - B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \\
& \frac{5(2A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{3a^2 d} - \frac{(7A - 4B) \cos[c + dx]^{3/2} \sin[c + dx]}{3a^2 d (1 + \cos[c + dx])} - \frac{(A - B) \cos[c + dx]^{5/2} \sin[c + dx]}{3d (a + a \cos[c + dx])^2}
\end{aligned}$$

Result (type 5, 1352 leaves):

$$\begin{aligned}
& - \frac{1}{2 (B + A \cos[c + dx]) (a + a \sec[c + dx])^2} 7 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \frac{1}{(B + A \cos[c + dx]) (a + a \sec[c + dx])^2} 2 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\
& \left(20 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right. \\
& \quad \left. (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \left(10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[c + dx] (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /
\end{aligned}$$

$$\left(3 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2 (a + a \sec [c + d x])^2} + \right. \\ \left. \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (A + B \sec [c + d x]) \left(-\frac{4 (-3 A + 2 B - 4 A \cos [c] + 2 B \cos [c]) \csc [c]}{d} + \frac{8 A \cos [d x] \sin [c]}{3 d} + \right. \right. \\ \left. \frac{2 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (-A \sin \left[\frac{d x}{2} \right] + B \sin \left[\frac{d x}{2} \right])}{3 d} - \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (-3 A \sin \left[\frac{d x}{2} \right] + 2 B \sin \left[\frac{d x}{2} \right])}{d} + \right. \\ \left. \frac{8 A \cos [c] \sin [d x]}{3 d} + \frac{2 (-A + B) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[\frac{c}{2} \right]}{3 d} \right) \Big/ \left(\sqrt{\cos [c + d x]} (B + A \cos [c + d x]) (a + a \sec [c + d x])^2 \right)$$

■ **Problem 504: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \sec [c + d x])}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$\frac{(4 A - B) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} - \frac{(5 A - 2 B) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} - \\ \frac{(5 A - 2 B) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 a^2 d (1 + \cos [c + d x])} - \frac{(A - B) \cos [c + d x]^{3/2} \sin [c + d x]}{3 d (a + a \cos [c + d x])^2}$$

Result (type 5, 1318 leaves):

$$\frac{1}{(B + A \cos [c + d x]) (a + a \sec [c + d x])^2} 2 i A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \sec [c + d x] (A + B \sec [c + d x]) \\ \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \Big/ (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \Big/ (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\ \frac{1}{2 (B + A \cos [c + d x]) (a + a \sec [c + d x])^2} i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \sec [c + d x] (A + B \sec [c + d x]) \\ \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \Big/ (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right.$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left(10 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec}[c + d x] \\
& \quad (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) - \\
& \left(4 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad \operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \left(\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (A + B \operatorname{Sec}[c + d x]) \right. \\
& \quad \left(-\frac{4 (2 A - B + 2 A \operatorname{Cos}[c]) \operatorname{Csc}[c]}{d} + \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (-2 A \operatorname{Sin} \left[\frac{d x}{2} \right] + B \operatorname{Sin} \left[\frac{d x}{2} \right])}{d} - \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (-A \operatorname{Sin} \left[\frac{d x}{2} \right] + B \operatorname{Sin} \left[\frac{d x}{2} \right])}{3 d} \right. \\
& \quad \left. \left. \frac{2 (-A + B) \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{3 d} \right) \right) / \left(\sqrt{\operatorname{Cos}[c + d x]} (B + A \operatorname{Cos}[c + d x]) (a + a \operatorname{Sec}[c + d x])^2 \right)
\end{aligned}$$

■ **Problem 505: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$-\frac{A \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{a^2 d} + \frac{(2 A + B) \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{3 a^2 d} + \frac{A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{a^2 d (1 + \operatorname{Cos}[c + d x])} - \frac{(A - B) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 5, 921 leaves):

$$\begin{aligned}
& - \frac{1}{2 (B + A \cos[c + dx]) (a + a \sec[c + dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) - \\
& \left(4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right. \\
& \quad \left. (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) - \\
& \left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[c + dx] (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \sec[c + dx]) \right. \\
& \quad \left. \left(\frac{4 A \operatorname{Csc}[c]}{d} + \frac{4 A \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{2 (-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \\
& \left(\sqrt{\cos[c + dx]} (B + A \cos[c + dx]) (a + a \sec[c + dx])^2 \right)
\end{aligned}$$

■ **Problem 506: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx]}{\cos[c + dx]^{3/2} (a + a \sec[c + dx])^2} dx$$

Optimal (type 4, 121 leaves, 6 steps) :

$$\frac{B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{(A+2B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3 a^2 d} - \frac{B \sqrt{\cos[c+dx]} \sin[c+dx]}{a^2 d (1+\cos[c+dx])} + \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{3 d (a+a \cos[c+dx])^2}$$

Result (type 5, 921 leaves) :

$$\begin{aligned}
& \frac{1}{2 (B + A \cos [c + d x]) (a + a \sec [c + d x])^2} i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x] (A + B \sec [c + d x]) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \left(2 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x] \right. \\
& \quad \left. (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \left(3 d (B + A \cos [c + d x]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \sec [c + d x])^2 \right) - \\
& \left(4 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left. \operatorname{Sec} [c + d x] (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \left(3 d (B + A \cos [c + d x]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \sec [c + d x])^2 \right) + \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (A + B \sec [c + d x]) \right. \\
& \quad \left. \left(-\frac{4 B \operatorname{Csc} [c]}{d} - \frac{4 B \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] \sin \left[\frac{d x}{2} \right]}{d} - \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (-A \sin \left[\frac{d x}{2} \right] + B \sin \left[\frac{d x}{2} \right])}{3 d} - \frac{2 (-A + B) \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{3 d} \right) \right) / \\
& \left(\sqrt{\cos [c + d x]} (B + A \cos [c + d x]) (a + a \sec [c + d x])^2 \right)
\end{aligned}$$

■ **Problem 507: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x]}{\cos [c + d x]^{5/2} (a + a \sec [c + d x])^2} dx$$

Optimal (type 4, 164 leaves, 7 steps):

$$\frac{(A - 4B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(2A - 5B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} -$$

$$\frac{(A - 4B) \operatorname{Sin}[c + dx]}{a^2 d \sqrt{\operatorname{Cos}[c + dx]}} + \frac{(2A - 5B) \operatorname{Sin}[c + dx]}{3a^2 d \sqrt{\operatorname{Cos}[c + dx]} (1 + \operatorname{Cos}[c + dx])} + \frac{(A - B) \operatorname{Sin}[c + dx]}{3d \sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Cos}[c + dx])^2}$$

Result (type 5, 1351 leaves):

$$\frac{1}{2(B + A \operatorname{Cos}[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])$$

$$\left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) -$$

$$\frac{1}{(B + A \operatorname{Cos}[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} 2i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx])$$

$$\left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) -$$

$$\left(4A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]$$

$$(A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\left. \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right.$$

$$\left. (3d(B + A \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) +$$

$$\left(10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ \left. \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) + \\ \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx]) \left(\frac{2 (2 B - A \cos[c] + 2 B \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{3 d} \right. \right. \\ \left. \left. + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + 2 B \sin\left[\frac{dx}{2}\right])}{d} + \frac{8 B \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \frac{2 (-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \\ \left(\sqrt{\cos[c + dx]} (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)$$

■ **Problem 508: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\cos[c + dx]^{7/2} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 197 leaves, 8 steps):

$$-\frac{(4A - 7B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} - \frac{5(A - 2B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} - \frac{5(A - 2B) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2}} + \\ \frac{(4A - 7B) \sin[c + dx]}{a^2 d \sqrt{\cos[c + dx]}} + \frac{(4A - 7B) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2} (1 + \cos[c + dx])} + \frac{(A - B) \sin[c + dx]}{3d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^2}$$

Result (type 5, 1392 leaves):

$$-\frac{1}{(B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^2} 2i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx]) \\ \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])\right]^2 \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])\right]^2 \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\begin{aligned}
& \frac{1}{2 (B + A \cos [c + d x]) (a + a \sec [c + d x])^2} 7 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x] (A + B \sec [c + d x]) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left(10 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x] \right. \\
& \quad (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(3 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^2 \right) - \\
& \left(20 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec} [c + d x] (A + B \sec [c + d x]) \operatorname{Sec} [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(3 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^2 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (A + B \sec [c + d x]) \left(-\frac{2 (-2 A + 4 B - 2 A \cos [c] + 3 B \cos [c]) \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c]}{d} - \right. \right. \\
& \quad \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (-A \sin \left[\frac{d x}{2} \right] + B \sin \left[\frac{d x}{2} \right])}{3 d} - \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (-2 A \sin \left[\frac{d x}{2} \right] + 3 B \sin \left[\frac{d x}{2} \right])}{d} + \\
& \quad \left. \frac{8 B \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^2 \sin [d x]}{3 d} + \frac{8 \operatorname{Sec} [c] \operatorname{Sec} [c + d x] (B \sin [c] + 3 A \sin [d x] - 6 B \sin [d x])}{3 d} - \frac{2 (-A + B) \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{3 d} \right) \right) / \\
& \left(\sqrt{\cos [c + d x]} (B + A \cos [c + d x]) (a + a \sec [c + d x])^2 \right)
\end{aligned}$$

Problem 509: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos[c + dx]^{3/2} (A + B \sec[c + dx])}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 221 leaves, 8 steps):

$$-\frac{7(17A - 7B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3 d} + \frac{(33A - 13B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3 d} + \frac{(33A - 13B) \sqrt{\cos[c + dx]} \sin[c + dx]}{6a^3 d} - \frac{(A - B) \cos[c + dx]^{7/2} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(2A - B) \cos[c + dx]^{5/2} \sin[c + dx]}{3ad(a + a \cos[c + dx])^2} - \frac{7(17A - 7B) \cos[c + dx]^{3/2} \sin[c + dx]}{30d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1448 leaves):

$$-\frac{1}{10(B + A \cos[c + dx])(a + a \sec[c + dx])^3} 119iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \sec[c + dx])$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) +$$

$$\frac{1}{10(B + A \cos[c + dx])(a + a \sec[c + dx])^3} 49iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \sec[c + dx])$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1 + e^{2idx})\cos[c] - 3d(-1 + e^{2idx})\sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx})\cos[c] + 2i(-1 + e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (-id(1 + e^{2idx})\cos[c] + d(-1 + e^{2idx})\sin[c]) \right) -$$

$$\left(22A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \right.$$

$$\left. (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \left(26 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx]^2 (A + B \sec[c + dx]) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx]) \right. \\
& \quad \left(-\frac{4 (-59 A + 29 B - 60 A \cos[c] + 20 B \cos[c]) \csc[c]}{5 d} + \frac{16 A \cos[dx] \sin[c]}{3 d} - \frac{2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5 d} + \right. \\
& \quad \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-19 A \sin\left[\frac{dx}{2}\right] + 14 B \sin\left[\frac{dx}{2}\right])}{15 d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (-59 A \sin\left[\frac{dx}{2}\right] + 29 B \sin\left[\frac{dx}{2}\right])}{5 d} + \frac{16 A \cos[c] \sin[dx]}{3 d} + \\
& \quad \left. \frac{4 (-19 A + 14 B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15 d} - \frac{2 (-A + B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5 d} \right) \right) / \left(\cos[c + dx]^{3/2} (B + A \cos[c + dx]) (a + a \sec[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 510: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \sec[c + dx])}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 188 leaves, 7 steps):

$$\begin{aligned}
& \frac{(49 A - 9 B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} - \frac{(13 A - 3 B) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} - \\
& \frac{(A - B) \cos[c + dx]^{5/2} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} - \frac{(8 A - 3 B) \cos[c + dx]^{3/2} \sin[c + dx]}{15 a d (a + a \cos[c + dx])^2} - \frac{(13 A - 3 B) \sqrt{\cos[c + dx]} \sin[c + dx]}{6 d (a^3 + a^3 \cos[c + dx])}
\end{aligned}$$

Result (type 5, 1415 leaves):

$$\frac{1}{10 (B + A \cos[c + dx]) (a + a \sec[c + dx])^3} {}_4F_9 \left[i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx]) \right]$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) - \\
& \frac{1}{10 (B + A \cos [c + d x]) (a + a \sec [c + d x])^3} 9 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \sec [c + d x]^2 (A + B \sec [c + d x]) \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) + \\
& \left(26 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2 \right] \sec \left[\frac{c}{2} \right] \sec [c + d x]^2 \right. \\
& \quad (A + B \sec [c + d x]) \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) \Big) / \\
& \left(3 d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) - \\
& \left(2 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2 \right] \sec \left[\frac{c}{2} \right] \right. \\
& \quad \sec [c + d x]^2 (A + B \sec [c + d x]) \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) \Big) / \\
& \left(d (B + A \cos [c + d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) +
\end{aligned}$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx]) \left(-\frac{4(29A - 9B + 20A \cos[c]) \csc[c]}{5d} + \frac{2 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (-29A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right])}{5d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-14A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right])}{15d} - \frac{4(-14A + 9B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{2(-A + B) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) \right) / (\cos[c + dx]^{3/2} (B + A \cos[c + dx]) (a + a \sec[c + dx])^3)$$

■ **Problem 511: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx]}{\sqrt{\cos[c + dx]} (a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 182 leaves, 7 steps):

$$-\frac{(9A + B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(3A + B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \frac{(A - B) \cos[c + dx]^{3/2} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(6A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} + \frac{(9A + B) \sqrt{\cos[c + dx]} \sin[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1407 leaves):

$$-\frac{1}{10(B + A \cos[c + dx]) (a + a \sec[c + dx])^3} 9iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx]) \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \frac{1}{10(B + A \cos[c + dx]) (a + a \sec[c + dx])^3} iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx]) \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left(2 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec}[c + d x]^2 \\
& \quad (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^3 \right) - \\
& \left(2 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right) \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x]) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (B + A \operatorname{Cos}[c + d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^3 \right) + \\
& \left(\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (A + B \operatorname{Sec}[c + d x]) \left(\frac{4 (9 A + B) \operatorname{Csc}[c]}{5 d} - \frac{2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^5 (-A \operatorname{Sin} \left[\frac{d x}{2} \right] + B \operatorname{Sin} \left[\frac{d x}{2} \right])}{5 d} \right. \right. \\
& \quad \left. \left. + \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (9 A \operatorname{Sin} \left[\frac{d x}{2} \right] + B \operatorname{Sin} \left[\frac{d x}{2} \right])}{5 d} + \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (-9 A \operatorname{Sin} \left[\frac{d x}{2} \right] + 4 B \operatorname{Sin} \left[\frac{d x}{2} \right])}{15 d} \right. \right. \\
& \quad \left. \left. + \frac{4 (-9 A + 4 B) \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{15 d} - \frac{2 (-A + B) \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Tan} \left[\frac{c}{2} \right]}{5 d} \right) \right) / \left(\operatorname{Cos}[c + d x]^{3/2} (B + A \operatorname{Cos}[c + d x]) (a + a \operatorname{Sec}[c + d x])^3 \right)
\end{aligned}$$

- **Problem 512: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 178 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A-B) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10a^3d} + \frac{(A+B) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6a^3d} - \\
& \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{5d(a+a\cos[c+dx])^3} + \frac{(4A+B) \sqrt{\cos[c+dx]} \sin[c+dx]}{15ad(a+a\cos[c+dx])^2} + \frac{(A-B) \sqrt{\cos[c+dx]} \sin[c+dx]}{10d(a^3+a^3\cos[c+dx])}
\end{aligned}$$

Result (type 5, 1406 leaves):

$$\begin{aligned}
& - \frac{1}{10(B+A\cos[c+dx])(a+a\sec[c+dx])^3} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A+B\sec[c+dx]) \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \right. \\
& \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \right. \\
& \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \\
& \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (-id(1+e^{2idx})\cos[c]+d(-1+e^{2idx})\sin[c]) \right) + \\
& \frac{1}{10(B+A\cos[c+dx])(a+a\sec[c+dx])^3} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A+B\sec[c+dx]) \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \right. \\
& \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \right. \\
& \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \\
& \left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (-id(1+e^{2idx})\cos[c]+d(-1+e^{2idx})\sin[c]) \right) - \\
& \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 \right. \\
& \left. (A+B\sec[c+dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3d(B+A\cos[c+dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a+a\sec[c+dx])^3 \right) -
\end{aligned}$$

$$\left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ \left. \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\ \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \operatorname{Sec}[c + dx]) \left(-\frac{4(-A + B) \operatorname{Csc}[c]}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} + \right. \right. \\ \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (4A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{15d} + \right. \\ \left. \left. \frac{4(4A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{2(-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right) \right) / \left(\cos[c + dx]^{3/2} (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)$$

■ **Problem 513: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx]}{\cos[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 180 leaves, 7 steps):

$$\frac{(A + 9B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(A + 3B) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} + \\ \frac{(A - B) \sqrt{\cos[c + dx]} \sin[c + dx]}{5d (a + a \cos[c + dx])^3} + \frac{(A - 6B) \sqrt{\cos[c + dx]} \sin[c + dx]}{15 a d (a + a \cos[c + dx])^2} - \frac{(A + 9B) \sqrt{\cos[c + dx]} \sin[c + dx]}{10 d (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1407 leaves):

$$\frac{1}{10 (B + A \cos[c + dx]) (a + a \operatorname{Sec}[c + dx])^3} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx]) \\ \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} \right) + \\
& \frac{1}{10 (B + A \cos[c + dx]) (a + a \sec[c + dx])^3} 9 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \sec[c + dx]) \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(3 i d (1 + e^{2idx}) \cos[c] - 3 d (-1 + e^{2idx}) \sin[c])} - \right. \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2 i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} \right) - \right. \\
& \left. \left(2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \right. \right. \\
& \left. (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \right) / \\
& \left(3 d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^3 \right) - \\
& \left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[c + dx]^2 (A + B \sec[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \right) / \\
& \left(d (B + A \cos[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx]) \left(-\frac{4 (A + 9 B) \operatorname{Csc}[c]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5 d} \right) - \right. \\
& \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] + 6 B \sin\left[\frac{dx}{2}\right])}{15 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 9 B \sin\left[\frac{dx}{2}\right])}{5 d} \right)
\end{aligned}$$

$$\left. \frac{4(-A+6B)\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} - \frac{2(-A+B)\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right) / \left(\operatorname{Cos}[c+dx]^{3/2} (B+A\operatorname{Cos}[c+dx]) (a+a\operatorname{Sec}[c+dx])^3 \right)$$

■ **Problem 514: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\operatorname{Sec}[c+dx]}{\operatorname{Cos}[c+dx]^{7/2} (a+a\operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 221 leaves, 8 steps):

$$\frac{(9A-49B)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10a^3d} + \frac{(3A-13B)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6a^3d} - \frac{(9A-49B)\operatorname{Sin}[c+dx]}{10a^3d\sqrt{\operatorname{Cos}[c+dx]}} +$$

$$\frac{(A-B)\operatorname{Sin}[c+dx]}{5d\sqrt{\operatorname{Cos}[c+dx]}(a+a\operatorname{Cos}[c+dx])^3} + \frac{(3A-8B)\operatorname{Sin}[c+dx]}{15ad\sqrt{\operatorname{Cos}[c+dx]}(a+a\operatorname{Cos}[c+dx])^2} + \frac{(3A-13B)\operatorname{Sin}[c+dx]}{6d\sqrt{\operatorname{Cos}[c+dx]}(a^3+a^3\operatorname{Cos}[c+dx])}$$

Result (type 5, 1447 leaves):

$$\frac{1}{10(B+A\operatorname{Cos}[c+dx])(a+a\operatorname{Sec}[c+dx])^3} 9iA\operatorname{Cos}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A+B\operatorname{Sec}[c+dx])$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c]+i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c]+2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c]+ie^{2idx}\operatorname{Sin}[2c]} \right) / \left(3id(1+e^{2idx})\operatorname{Cos}[c]-3d(-1+e^{2idx})\operatorname{Sin}[c] \right) - \right.$$

$$\left(2\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c]+i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c]+2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c]+ie^{2idx}\operatorname{Sin}[2c]} \right) / \left(-id(1+e^{2idx})\operatorname{Cos}[c]+d(-1+e^{2idx})\operatorname{Sin}[c] \right) \right) -$$

$$\frac{1}{10(B+A\operatorname{Cos}[c+dx])(a+a\operatorname{Sec}[c+dx])^3} 49iB\operatorname{Cos}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2 (A+B\operatorname{Sec}[c+dx])$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\operatorname{Cos}[c]+i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c]+2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c]+ie^{2idx}\operatorname{Sin}[2c]} \right) / \left(3id(1+e^{2idx})\operatorname{Cos}[c]-3d(-1+e^{2idx})\operatorname{Sin}[c] \right) - \right.$$

$$\left(2\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\operatorname{Cos}[c]+i\operatorname{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\operatorname{Cos}[c]+2i(-1+e^{2idx})\operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\operatorname{Cos}[2c]+ie^{2idx}\operatorname{Sin}[2c]} \right) / \left(-id(1+e^{2idx})\operatorname{Cos}[c]+d(-1+e^{2idx})\operatorname{Sin}[c] \right) \right) -$$

$$\left(2A\operatorname{Cos}\left[\frac{c}{2}+\frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2$$

$$\begin{aligned}
& \left. \left((A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \right. \\
& \left. \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right. \\
& \left(d (B + A \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(26 B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx]) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \right. \\
& \left(3 d (B + A \operatorname{Cos}[c + dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \operatorname{Sec}[c + dx]) \left(\frac{2 (20 B - 9 A \operatorname{Cos}[c] + 29 B \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5 d} + \right. \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \operatorname{Sin}\left[\frac{dx}{2}\right] + B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-6 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 11 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} + \right. \\
& \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-9 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 29 B \operatorname{Sin}\left[\frac{dx}{2}\right])}{5 d} + \frac{16 B \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{d} + \frac{4 (-6 A + 11 B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \right. \\
& \left. \left. \frac{2 (-A + B) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right) \right) / \left(\operatorname{Cos}[c + dx]^{3/2} (B + A \operatorname{Cos}[c + dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 519: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx]) dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{2 \sqrt{a} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right] \sqrt{\operatorname{Cos}[c + dx]} \sqrt{\operatorname{Sec}[c + dx]}}{d} + \frac{2 a A \operatorname{Sin}[c + dx]}{d \sqrt{\operatorname{Cos}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 5, 135 leaves):

$$-\frac{1}{3d(1+e^{i(c+dx)})} \sqrt{\cos[c+dx]} - 2i \sqrt{\cos[c+dx]} \left(3A(-1+e^{i(c+dx)}) + 6Be^{i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] + 2Be^{2i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \right) \sqrt{a(1+\sec[c+dx])}$$

■ **Problem 520: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a\sec[c+dx]} (A+B\sec[c+dx])}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{\sqrt{a} (2A+B) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{aB\sin[c+dx]}{d\cos[c+dx]^{3/2}\sqrt{a+a\sec[c+dx]}}$$

Result (type 5, 157 leaves):

$$\frac{1}{3d} \sqrt{\cos[c+dx]} \sqrt{a(1+\sec[c+dx])} \left(-3i(2A+B) e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] \sec\left[\frac{1}{2}(c+dx)\right] - i(2A+B) e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \sec\left[\frac{1}{2}(c+dx)\right] + 3B\sec[c+dx] \tan\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 521: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a\sec[c+dx]} (A+B\sec[c+dx])}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\frac{\sqrt{a} (4A+3B) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} + \frac{aB\sin[c+dx]}{2d\cos[c+dx]^{5/2}\sqrt{a+a\sec[c+dx]}} + \frac{a(4A+3B)\sin[c+dx]}{4d\cos[c+dx]^{3/2}\sqrt{a+a\sec[c+dx]}}$$

Result (type 5, 166 leaves):

$$\frac{1}{12d} \sqrt{\cos[c+dx]} \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \left(-3i(4A+3B) e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i(4A+3B) e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3\sec[c+dx] (4A+3B+2B\sec[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 522: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x])}{\operatorname{Cos}[c + d x]^{5/2}} dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\frac{\sqrt{a} (6 A + 5 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{8 d} +$$

$$\frac{a B \operatorname{Sin}[c + d x]}{3 d \operatorname{Cos}[c + d x]^{7/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{a (6 A + 5 B) \operatorname{Sin}[c + d x]}{12 d \operatorname{Cos}[c + d x]^{5/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{a (6 A + 5 B) \operatorname{Sin}[c + d x]}{8 d \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 5, 185 leaves):

$$\frac{1}{24 d} \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \sqrt{a (1 + \operatorname{Sec}[c + d x])}$$

$$\left(-3 i (6 A + 5 B) e^{\frac{1}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - i (6 A + 5 B) e^{\frac{3}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + \right.$$

$$\left. \operatorname{Sec}[c + d x] (3 (6 A + 5 B) + 2 (6 A + 5 B) \operatorname{Sec}[c + d x] + 8 B \operatorname{Sec}[c + d x]^2) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 527: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$\frac{2 a^{3/2} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{d} +$$

$$\frac{2 a^2 (4 A + 3 B) \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{2 a A \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d}$$

Result (type 5, 157 leaves):

$$\frac{1}{3 d} a \sqrt{\operatorname{Cos}[c + d x]} (1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c + d x])} \left(-3 i B e^{\frac{1}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - \right.$$

$$\left. i B e^{\frac{3}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + (5 A + 3 B + A \operatorname{Cos}[c + d x]) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 528: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x]) dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{a^{3/2} (2A + 3B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{a^2 (2A - B) \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 178 leaves):

$$\frac{1}{6d \sqrt{\operatorname{Cos}[c+dx]}} a (1 + \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c+dx])} - \left(-3i (2A + 3B) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i (2A + 3B) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3 (B + 2A \operatorname{Cos}[c+dx]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 529: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^{3/2} (A + B \operatorname{Sec}[c+dx])}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{a^{3/2} (12A + 7B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4d} + \frac{a^2 (4A + 5B) \operatorname{Sin}[c+dx]}{4d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 5, 189 leaves):

$$\frac{1}{24d \operatorname{Cos}[c+dx]^{3/2}} a (1 + \operatorname{Cos}[c+dx]) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c+dx])} - \left(-3i (12A + 7B) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i (12A + 7B) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3 (2B + (4A + 7B) \operatorname{Cos}[c+dx]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 530: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^{3/2} (A + B \operatorname{Sec}[c+dx])}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{a^{3/2} (14A + 11B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{8d} + \frac{a^2 (6A + 7B) \operatorname{Sin}[c+dx]}{12d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (14A + 11B) \operatorname{Sin}[c+dx]}{8d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d \operatorname{Cos}[c+dx]^{5/2}}$$

Result (type 5, 205 leaves) :

$$\frac{1}{48 d \cos [c+d x]^{5/2}} a (1+\cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \sqrt{a(1+\operatorname{Sec}[c+d x])}$$

$$\left(-3 i(14 A+11 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-i(14 A+11 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^3\right.$$

$$\left.\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(8 B+2(6 A+11 B) \cos [c+d x]+(42 A+33 B) \cos [c+d x]^2) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 531: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^{3/2}(A+B \operatorname{Sec}[c+d x])}{\cos [c+d x]^{5/2}} dx$$

Optimal (type 3, 247 leaves, 7 steps) :

$$\frac{a^{3/2}(88 A+75 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{64 d} + \frac{a^2(8 A+9 B) \sin [c+d x]}{24 d \cos [c+d x]^{7/2} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

$$+ \frac{a^2(88 A+75 B) \sin [c+d x]}{96 d \cos [c+d x]^{5/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2(88 A+75 B) \sin [c+d x]}{64 d \cos [c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a B \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{4 d \cos [c+d x]^{7/2}}$$

Result (type 5, 223 leaves) :

$$\frac{1}{384 d \cos [c+d x]^{7/2}}$$

$$a(1+\cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 \sqrt{a(1+\operatorname{Sec}[c+d x])} \left(-3 i(88 A+75 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-\right.$$

$$\left.i(88 A+75 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(48 B+8(8 A+15 B) \cos [c+d x]+2(88 A+75 B) \cos [c+d x]^2+3(88 A+75 B) \cos [c+d x]^3) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 535: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{5/2}(a+a \operatorname{Sec}[c+d x])^{5/2}(A+B \operatorname{Sec}[c+d x]) dx$$

Optimal (type 3, 192 leaves, 6 steps) :

$$\frac{2 a^{5/2} B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d} + \frac{2 a^3(32 A+35 B) \sin [c+d x]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

$$+ \frac{2 a^2(8 A+5 B) \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{15 d} + \frac{2 a A \cos [c+d x]^{3/2}(a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{5 d}$$

Result (type 5, 179 leaves) :

$$\frac{1}{60 d} a^2 \sqrt{\cos [c+d x]} (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\operatorname{Sec}[c+d x])}$$

$$\left(-30 i B e^{\frac{1}{2} i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-10 i B e^{\frac{3}{2} i(c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+\right.$$

$$\left.(89 A+80 B+2(14 A+5 B) \cos [c+d x]+3 A \cos [2(c+d x)]) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 536: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{3 / 2}(a+a \operatorname{Sec}[c+d x])^{5 / 2}(A+B \operatorname{Sec}[c+d x]) d x$$

Optimal (type 3, 197 leaves, 6 steps) :

$$\frac{a^{5 / 2}(2 A+5 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d}+\frac{a^3(14 A+3 B) \sin [c+d x]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

$$-\frac{a^2(2 A-3 B) \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 d \sqrt{\cos [c+d x]}}+\frac{2 a A \sqrt{\cos [c+d x]}(a+a \operatorname{Sec}[c+d x])^{3 / 2} \sin [c+d x]}{3 d}$$

Result (type 5, 200 leaves) :

$$\frac{1}{12 d \sqrt{\cos [c+d x]}} a^2(1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\operatorname{Sec}[c+d x])}$$

$$\left(-3 i(2 A+5 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-i(2 A+5 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]\right.$$

$$\left.\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(A+3 B+2(8 A+3 B) \cos [c+d x]+A \cos [2(c+d x)]) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 537: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\cos [c+d x]}(a+a \operatorname{Sec}[c+d x])^{5 / 2}(A+B \operatorname{Sec}[c+d x]) d x$$

Optimal (type 3, 200 leaves, 6 steps) :

$$\frac{a^{5 / 2}(20 A+19 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{4 d}+$$

$$\frac{a^3(4 A-9 B) \sin [c+d x]}{4 d \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}}+\frac{a^2(4 A+7 B) \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{4 d \sqrt{\cos [c+d x]}}+\frac{a B(a+a \operatorname{Sec}[c+d x])^{3 / 2} \sin [c+d x]}{2 d \sqrt{\cos [c+d x]}}$$

Result (type 5, 204 leaves) :

$$\frac{1}{48 d \cos [c+d x]^{3/2}} a^2 (1+\cos [c+d x])^2 \sec \left[\frac{1}{2}(c+d x) \right]^5 \sqrt{a(1+\sec [c+d x])}$$

$$\left(-3 i (20 A+19 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^2 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)} \right] - i (20 A+19 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^2 \right.$$

$$\left. \operatorname{Hypergeometric2F1} \left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)} \right] + 3 (2 B+(4 A+11 B) \cos [c+d x]+8 A \cos [c+d x]^2) \sin \left[\frac{1}{2}(c+d x) \right] \right)$$

■ **Problem 538: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{5/2} (A+B \sec [c+d x])}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{a^{5/2} (38 A+25 B) \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}} \right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{8 d} +$$

$$\frac{a^3 (54 A+49 B) \sin [c+d x]}{24 d \cos [c+d x]^{3/2} \sqrt{a+a \sec [c+d x]}} + \frac{a^2 (2 A+3 B) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{4 d \cos [c+d x]^{3/2}} + \frac{a B (a+a \sec [c+d x])^{3/2} \sin [c+d x]}{3 d \cos [c+d x]^{3/2}}$$

Result (type 5, 209 leaves):

$$\frac{1}{96 d \cos [c+d x]^{5/2}} a^2 (1+\cos [c+d x])^2 \sec \left[\frac{1}{2}(c+d x) \right]^5 \sqrt{a(1+\sec [c+d x])}$$

$$\left(-3 i (38 A+25 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^3 \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)} \right] - i (38 A+25 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^3 \right.$$

$$\left. \operatorname{Hypergeometric2F1} \left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)} \right] + (8 B+2 (6 A+17 B) \cos [c+d x]+(66 A+75 B) \cos [c+d x]^2) \sin \left[\frac{1}{2}(c+d x) \right] \right)$$

■ **Problem 539: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{5/2} (A+B \sec [c+d x])}{\cos [c+d x]^{3/2}} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$\frac{a^{5/2} (200 A+163 B) \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}} \right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 d} + \frac{a^3 (104 A+95 B) \sin [c+d x]}{96 d \cos [c+d x]^{5/2} \sqrt{a+a \sec [c+d x]}} +$$

$$\frac{a^3 (200 A+163 B) \sin [c+d x]}{64 d \cos [c+d x]^{3/2} \sqrt{a+a \sec [c+d x]}} + \frac{a^2 (8 A+11 B) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{24 d \cos [c+d x]^{5/2}} + \frac{a B (a+a \sec [c+d x])^{3/2} \sin [c+d x]}{4 d \cos [c+d x]^{5/2}}$$

Result (type 5, 225 leaves):

$$\frac{1}{768 d \cos [c+d x]^{7/2}} a^2 (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\sqrt{a(1+\operatorname{Sec}[c+d x])} \left(-3 i (200 A+163 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right] - \right.$$

$$i (200 A+163 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right] +$$

$$\left. (48 B+8(8 A+23 B) \cos [c+d x]+(272 A+326 B) \cos [c+d x]^2+(600 A+489 B) \cos [c+d x]^3) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 540: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x])}{\cos [c+d x]^{5/2}} dx$$

Optimal (type 3, 294 leaves, 8 steps):

$$\frac{a^{5/2} (326 A+283 B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{128 d} +$$

$$\frac{a^3 (170 A+157 B) \sin [c+d x]}{240 d \cos [c+d x]^{7/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^3 (326 A+283 B) \sin [c+d x]}{192 d \cos [c+d x]^{5/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^3 (326 A+283 B) \sin [c+d x]}{128 d \cos [c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} +$$

$$\frac{a^2 (10 A+13 B) \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{40 d \cos [c+d x]^{7/2}} + \frac{a B (a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{5 d \cos [c+d x]^{7/2}}$$

Result (type 5, 244 leaves):

$$\frac{1}{7680 d \cos [c+d x]^{9/2}} a^2 (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\sqrt{a(1+\operatorname{Sec}[c+d x])} \left(-15 i (326 A+283 B) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right] - \right.$$

$$5 i (326 A+283 B) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right] + (384 B+48(10 A+29 B) \cos [c+d x]+$$

$$8(230 A+283 B) \cos [c+d x]^2+10(326 A+283 B) \cos [c+d x]^3+15(326 A+283 B) \cos [c+d x]^4) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 546: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+d x]}{\cos [c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\frac{(2A - B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a} d} -$$

$$\frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a} d} + \frac{B \operatorname{Sin}[c+dx]}{d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 402 leaves):

$$\frac{1}{4 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{\operatorname{Sin}[c+dx]^2}}$$

$$\operatorname{Sin}[c+dx] \left(-\operatorname{Cos}[c+dx] \sqrt{1+\operatorname{Cos}[c+dx]} \left(2\sqrt{2} B \operatorname{Log}[1+\operatorname{Cos}[c+dx]] + (8A - 4B) \operatorname{Log}\left[\sqrt{\operatorname{Cos}[c+dx]} (1+\operatorname{Cos}[c+dx])\right] \right) - \right.$$

$$2\sqrt{2} A \operatorname{Log}\left[(1+\operatorname{Cos}[c+dx])^2\right] + \sqrt{2} B \operatorname{Log}\left[(1+\operatorname{Cos}[c+dx])^2\right] - 2\sqrt{2} B \operatorname{Log}\left[2\sqrt{1+\operatorname{Cos}[c+dx]} + \sqrt{2-2\operatorname{Cos}[c+dx]^2}\right] -$$

$$8A \operatorname{Log}\left[1+\operatorname{Cos}[c+dx] + \sqrt{1+\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]^2}\right] + 4B \operatorname{Log}\left[1+\operatorname{Cos}[c+dx] + \sqrt{1+\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]^2}\right] +$$

$$2\sqrt{2} A \operatorname{Log}\left[3+2\operatorname{Cos}[c+dx] - \operatorname{Cos}[c+dx]^2 + 2\sqrt{2} \sqrt{1+\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]^2}\right] -$$

$$\left. \sqrt{2} B \operatorname{Log}\left[3+2\operatorname{Cos}[c+dx] - \operatorname{Cos}[c+dx]^2 + 2\sqrt{2} \sqrt{1+\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sin}[c+dx]^2}\right] \right) + 4B \sqrt{\operatorname{Sin}[c+dx]^2}$$

■ **Problem 547: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c+dx]}{\operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 3, 230 leaves, 8 steps):

$$-\frac{(4A - 7B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4\sqrt{a} d} + \frac{\sqrt{2} (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{a} d} +$$

$$\frac{B \operatorname{Sin}[c+dx]}{2d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{(4A - B) \operatorname{Sin}[c+dx]}{4d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 481 leaves):

$$\left(\cos\left[\frac{1}{2}(c+dx)\right] \left(B \sec[c+dx]^2 \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2} \sec[c+dx] \left(4A \sin\left[\frac{1}{2}(c+dx)\right] - B \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) /$$

$$\left(d \sqrt{\cos[c+dx]} \sqrt{a(1+\sec[c+dx])} \right) + \frac{1}{8d\sqrt{a(1+\sec[c+dx])}} \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]}$$

$$\left(-\frac{1}{\sqrt{1-\cos[c+dx]}^2} \sqrt{2} (4A-B) \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \left(\log[1+\cos[c+dx]] - \log\left[2\sqrt{1+\cos[c+dx]} + \sqrt{2-2\cos[c+dx]^2}\right] \right) \right.$$

$$\left. \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] - \frac{1}{2\sqrt{1-\cos[c+dx]}^2} (-4A+7B) \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \right.$$

$$\left(-\sqrt{2} \log[(1+\cos[c+dx])^2] + 4 \log\left[\sqrt{\cos[c+dx]} + \cos[c+dx]^{3/2}\right] - 4 \log\left[1+\cos[c+dx] + \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] + \right.$$

$$\left. \sqrt{2} \log\left[3+2\cos[c+dx] - \cos[c+dx]^2 + 2\sqrt{2} \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx]$$

■ **Problem 560: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sec[c+dx]}{\cos[c+dx]^{5/2} (a+a \sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 234 leaves, 8 steps):

$$\frac{2B \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + (3A-43B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2} d} + \frac{(A-B) \sin[c+dx]}{4d \cos[c+dx]^{5/2} (a+a \sec[c+dx])^{5/2}} + \frac{(3A-11B) \sin[c+dx]}{16ad \cos[c+dx]^{3/2} (a+a \sec[c+dx])^{3/2}}$$

Result (type 3, 505 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(c+dx)\right] \right)^5 \\
& \left(\frac{1}{4} \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(3A \sin\left[\frac{1}{2}(c+dx)\right] - 11B \sin\left[\frac{1}{2}(c+dx)\right] \right) + \frac{1}{2} \sec\left[\frac{1}{2}(c+dx)\right]^4 \left(A \sin\left[\frac{1}{2}(c+dx)\right] - B \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \Bigg) / \\
& \left(d \cos[c+dx]^{5/2} (a(1+\sec[c+dx]))^{5/2} + \frac{1}{8d(a(1+\sec[c+dx]))^{5/2}} \cos\left[\frac{1}{2}(c+dx)\right]^5 \sec[c+dx]^{5/2} \right. \\
& \left. \left(-\frac{1}{\sqrt{1-\cos[c+dx]^2}} \sqrt{2} (3A-11B) \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \left(\log[1+\cos[c+dx]] - \log\left[2\sqrt{1+\cos[c+dx]} + \sqrt{2-2\cos[c+dx]^2}\right] \right) \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] - \frac{1}{\sqrt{1-\cos[c+dx]^2}} 16B \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \right. \right. \\
& \left. \left. \left(-\sqrt{2} \log[(1+\cos[c+dx])^2] + 4 \log\left[\sqrt{\cos[c+dx]} + \cos[c+dx]^{3/2}\right] - 4 \log\left[1+\cos[c+dx] + \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] + \right. \right. \\
& \left. \left. \sqrt{2} \log\left[3+2\cos[c+dx] - \cos[c+dx]^2 + 2\sqrt{2} \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] \right) \Bigg)
\end{aligned}$$

■ **Problem 578: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \sec[c+dx]}{\cos[c+dx]^{3/2} (a+b \sec[c+dx])} dx$$

Optimal (type 4, 86 leaves, 6 steps):

$$-\frac{2B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{bd} + \frac{2(Ab-aB) \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{b(a+b)d} + \frac{2B \sin[c+dx]}{bd \sqrt{\cos[c+dx]}}$$

Result (type 4, 208 leaves):

$$\begin{aligned}
& \frac{1}{2bd} \left(\frac{2(2Ab-3aB) \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} - \frac{2bB \left(2 \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - \frac{2b \operatorname{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right]}{a+b} \right)}{a} \right) + \\
& \frac{4B \sin[c+dx]}{\sqrt{\cos[c+dx]}} + 1 / \left(ab \sqrt{\sin[c+dx]^2} \right) 2B \left(2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] - \right. \\
& \left. 2b(a+b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] + (a^2-2b^2) \operatorname{EllipticPi}\left[-\frac{a}{b}, -\operatorname{ArcSin}\left[\sqrt{\cos[c+dx]}\right], -1\right] \right) \sin[c+dx] \Bigg)
\end{aligned}$$

■ **Problem 594: Unable to integrate problem.**

$$\int \cos [c+d x]^{7 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

Optimal (type 4, 343 leaves, 11 steps):

$$\frac{2\left(a^2-b^2\right)\left(25 a^2 A+8 A b^2-14 a b B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{105 a^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2\left(19 a^2 A b+8 A b^3+63 a^3 B-14 a b^2 B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{105 a^3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2\left(25 a^2 A-4 A b^2+7 a b B\right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{105 a^2 d} +$$

$$\frac{2(A b+7 a B) \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{35 a d} + \frac{2 A \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{7 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{7 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

■ **Problem 595: Unable to integrate problem.**

$$\int \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

Optimal (type 4, 267 leaves, 10 steps):

$$-\frac{2\left(a^2-b^2\right)\left(2 A b-5 a B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2\left(9 a^2 A-2 A b^2+5 a b B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2(A b+5 a B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a d} + \frac{2 A \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

■ **Problem 596: Unable to integrate problem.**

$$\int \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

Optimal (type 4, 201 leaves, 9 steps):

$$\frac{2 A\left(a^2-b^2\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2(A b+3 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 A \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

■ **Problem 597: Unable to integrate problem.**

$$\int \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

Optimal (type 4, 208 leaves, 12 steps):

$$\frac{2 a B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 b B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 8, 37 leaves):

$$\int \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]) d x$$

■ **Problem 598: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x])}{\sqrt{\cos [c+d x]}} d x$$

Optimal (type 4, 253 leaves, 13 steps):

$$\frac{(2aA + bB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (2Ab + aB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{B \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + B \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 37 leaves):

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx])}{\sqrt{\cos[c+dx]}} dx$$

■ **Problem 599: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx])}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 336 leaves, 14 steps):

$$\frac{(4Ab + 3aB) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (4aAb - a^2B + 4b^2B) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{(4Ab + aB) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + 4bd \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}{4bd \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{B \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{2d \cos[c+dx]^{3/2}} + \frac{(4Ab + aB) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4bd \sqrt{\cos[c+dx]}}$$

Result (type 8, 37 leaves):

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx])}{\cos[c+dx]^{3/2}} dx$$

■ **Problem 600: Unable to integrate problem.**

$$\int \cos[c+dx]^{9/2} (a+b \sec[c+dx])^{3/2} (A+B \sec[c+dx]) dx$$

Optimal (type 4, 427 leaves, 12 steps):

$$\begin{aligned}
& \frac{2(a^2 - b^2)(39a^2Ab + 8Ab^3 + 75a^3B - 18ab^2B) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{315a^3d\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}} + \frac{1}{315a^3d\sqrt{\frac{b+a\cos[c+dx]}{a+b}}} \\
& \frac{2(147a^4A + 33a^2Ab^2 + 8Ab^4 + 246a^3bB - 18ab^3B) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b\sec[c+dx]}}{315a^2d} + \\
& \frac{2(88a^2Ab - 4Ab^3 + 75a^3B + 9ab^2B) \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{315ad} + \\
& \frac{2(49a^2A + 3Ab^2 + 72abB) \cos[c+dx]^{3/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{315ad} + \\
& \frac{2(10Ab + 9aB) \cos[c+dx]^{5/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{63d} + \frac{2aA \cos[c+dx]^{7/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{9d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \cos[c+dx]^{9/2} (a+b\sec[c+dx])^{3/2} (A+B\sec[c+dx]) dx$$

■ **Problem 601: Unable to integrate problem.**

$$\int \cos[c+dx]^{7/2} (a+b\sec[c+dx])^{3/2} (A+B\sec[c+dx]) dx$$

Optimal (type 4, 342 leaves, 11 steps):

$$\begin{aligned}
& \frac{2(a^2 - b^2)(25a^2A - 6Ab^2 + 21abB) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{105a^2d\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}} + \\
& \frac{2(82a^2Ab - 6Ab^3 + 63a^3B + 21ab^2B) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b\sec[c+dx]}}{105a^2d\sqrt{\frac{b+a\cos[c+dx]}{a+b}}} + \\
& \frac{2(25a^2A + 3Ab^2 + 42abB) \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{105ad} + \\
& \frac{2(8Ab + 7aB) \cos[c+dx]^{3/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{35d} + \frac{2aA \cos[c+dx]^{5/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \cos[c+dx]^{7/2} (a+b\sec[c+dx])^{3/2} (A+B\sec[c+dx]) dx$$

■ **Problem 602: Unable to integrate problem.**

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 266 leaves, 10 steps):

$$\frac{2\left(a^2-b^2\right)\left(3 A b+5 a B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2\left(9 a^2 A+3 A b^2+20 a b B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2\left(6 A b+5 a B\right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 d} + \frac{2 a A \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

■ **Problem 603: Unable to integrate problem.**

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 276 leaves, 13 steps):

$$\frac{2\left(a^2 A-A b^2+3 a b B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 b^2 B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2\left(4 A b+3 a B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 a A \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

■ **Problem 604: Unable to integrate problem.**

$$\int \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 272 leaves, 13 steps):

$$\frac{(2 a A b + 2 a^2 B + b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + b(2 A b + 3 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(2 a A - b B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} + b B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 8, 37 leaves):

$$\int \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x]) dx$$

■ **Problem 605: Unable to integrate problem.**

$$\int \frac{(a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x])}{\sqrt{\cos [c+d x]}} dx$$

Optimal (type 4, 339 leaves, 14 steps):

$$\frac{(8 a^2 A + 4 A b^2 + 7 a b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + (12 a A b + 3 a^2 B + 4 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(4 A b + 5 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} + b B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

$$\frac{b B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 d \cos [c+d x]^{3/2}} + \frac{(4 A b + 5 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 d \sqrt{\cos [c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{(a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x])}{\sqrt{\cos [c+d x]}} dx$$

■ **Problem 606: Unable to integrate problem.**

$$\int \frac{(a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x])}{\cos [c+d x]^{3/2}} dx$$

Optimal (type 4, 421 leaves, 15 steps):

$$\begin{aligned}
& \frac{(42 a A b + 17 a^2 B + 16 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{24 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{(6 a^2 A b + 8 A b^3 - a^3 B + 12 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{8 b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \\
& \frac{(30 a A b + 3 a^2 B + 16 b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{24 b d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{b B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d \cos [c+d x]^{5/2}} + \\
& \frac{(6 A b + 7 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{12 d \cos [c+d x]^{3/2}} + \frac{(30 a A b + 3 a^2 B + 16 b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{24 b d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(a+b \sec [c+d x])^{3/2} (A+B \sec [c+d x])}{\cos [c+d x]^{3/2}} dx$$

■ **Problem 607: Unable to integrate problem.**

$$\int \cos [c+d x]^{11/2} (a+b \sec [c+d x])^{5/2} (A+B \sec [c+d x]) dx$$

Optimal (type 4, 519 leaves, 13 steps):

$$\begin{aligned}
& \left(2 (a^2 - b^2) (675 a^4 A + 285 a^2 A b^2 + 40 A b^4 + 1254 a^3 b B - 110 a b^3 B) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2a}{a + b}\right] \right) / \\
& \left(3465 a^3 d \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \right) + \frac{1}{3465 a^3 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}}} \\
& 2 (3705 a^4 A b + 255 a^2 A b^3 + 40 A b^5 + 1617 a^5 B + 3069 a^3 b^2 B - 110 a b^4 B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2a}{a + b}\right] \sqrt{a + b \sec[c + dx]} + \\
& \frac{1}{3465 a^2 d} 2 (675 a^4 A + 1025 a^2 A b^2 - 20 A b^4 + 1793 a^3 b B + 55 a b^3 B) \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \sin[c + dx] + \\
& \frac{2 (1145 a^2 A b + 15 A b^3 + 539 a^3 B + 825 a b^2 B) \cos[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{3465 a d} + \\
& \frac{2 (81 a^2 A + 113 A b^2 + 209 a b B) \cos[c + dx]^{5/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{693 d} + \\
& \frac{2 a (14 A b + 11 a B) \cos[c + dx]^{7/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{99 d} + \frac{2 a A \cos[c + dx]^{9/2} (a + b \sec[c + dx])^{3/2} \sin[c + dx]}{11 d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \cos[c + dx]^{11/2} (a + b \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

■ **Problem 608: Unable to integrate problem.**

$$\int \cos[c + dx]^{9/2} (a + b \sec[c + dx])^{5/2} (A + B \sec[c + dx]) dx$$

Optimal (type 4, 425 leaves, 12 steps):

$$\begin{aligned}
& \frac{2 (a^2 - b^2) (114 a^2 A b - 10 A b^3 + 75 a^3 B + 45 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{315 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{1}{315 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} \\
& \frac{2 (147 a^4 A + 279 a^2 A b^2 - 10 A b^4 + 435 a^3 b B + 45 a b^3 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} +}{315 a d} \\
& \frac{2 (163 a^2 A b + 5 A b^3 + 75 a^3 B + 135 a b^2 B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{315 a d} + \\
& \frac{2 (49 a^2 A + 75 A b^2 + 135 a b B) \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{315 d} + \\
& \frac{2 a (4 A b + 3 a B) \cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{21 d} + \frac{2 a A \cos [c+d x]^{7/2} (a+b \sec [c+d x])^{3/2} \sin [c+d x]}{9 d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{9/2} (a+b \sec [c+d x])^{5/2} (A+B \sec [c+d x]) dx$$

■ **Problem 609: Unable to integrate problem.**

$$\int \cos [c+d x]^{7/2} (a+b \sec [c+d x])^{5/2} (A+B \sec [c+d x]) dx$$

Optimal (type 4, 340 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 (a^2 - b^2) (25 a^2 A + 15 A b^2 + 56 a b B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{105 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{1}{105 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} \\
& \frac{2 (145 a^2 A b + 15 A b^3 + 63 a^3 B + 161 a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} +}{105 d} \\
& \frac{2 (25 a^2 A + 45 A b^2 + 77 a b B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{35 d} + \\
& \frac{2 a (10 A b + 7 a B) \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{35 d} + \frac{2 a A \cos [c+d x]^{5/2} (a+b \sec [c+d x])^{3/2} \sin [c+d x]}{7 d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{7/2} (a+b \sec [c+d x])^{5/2} (A+B \sec [c+d x]) dx$$

■ **Problem 610: Unable to integrate problem.**

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 342 leaves, 14 steps):

$$\frac{2\left(8 a^2 A b-8 A b^3+5 a^3 B+10 a b^2 B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]+2 b^3 B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}+\frac{2\left(9 a^2 A+23 A b^2+35 a b B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

$$+\frac{2 a(8 A b+5 a B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 d}+\frac{2 a A \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{5 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]) d x$$

■ **Problem 611: Unable to integrate problem.**

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]) d x$$

Optimal (type 4, 349 leaves, 14 steps):

$$\frac{\left(2 a^3 A+4 a A b^2+12 a^2 b B+3 b^3 B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]+b^2(2 A b+5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}+\frac{\left(14 a A b+6 a^2 B-3 b^2 B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

$$-\frac{b(2 a A-3 b B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d \sqrt{\cos [c+d x]}}+\frac{2 a A \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{3 d}$$

Result (type 8, 37 leaves):

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]) d x$$

■ **Problem 612: Unable to integrate problem.**

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{5/2} (A+B \sec[c+dx]) dx$$

Optimal (type 4, 359 leaves, 14 steps):

$$\frac{(16 a^2 A b + 4 A b^3 + 8 a^3 B + 11 a b^2 B) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4 d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} +$$

$$\frac{b (20 a A b + 15 a^2 B + 4 b^2 B) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4 d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} +$$

$$\frac{(8 a^2 A - 4 A b^2 - 9 a b B) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{4 d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} +$$

$$\frac{b (4 A b + 7 a B) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4 d \sqrt{\cos[c+dx]}} + \frac{b B (a+b \sec[c+dx])^{3/2} \sin[c+dx]}{2 d \sqrt{\cos[c+dx]}}$$

Result (type 8, 37 leaves):

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^{5/2} (A+B \sec[c+dx]) dx$$

■ **Problem 613: Unable to integrate problem.**

$$\int \frac{(a+b \sec[c+dx])^{5/2} (A+B \sec[c+dx])}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 422 leaves, 15 steps):

$$\begin{aligned}
& \frac{(48 a^3 A + 66 a A b^2 + 59 a^2 b B + 16 b^3 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{24 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(30 a^2 A b + 8 A b^3 + 5 a^3 B + 20 a b^2 B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{8 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{(54 a A b + 33 a^2 B + 16 b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{24 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \frac{b(2 A b + 3 a B) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d \operatorname{Cos}[c+d x]^{3/2}} + \\
& \frac{(54 a A b + 33 a^2 B + 16 b^2 B) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{b B (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x])}{\sqrt{\operatorname{Cos}[c+d x]}} dx$$

■ **Problem 614: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x])}{\operatorname{Cos}[c+d x]^{3/2}} dx$$

Optimal (type 4, 513 leaves, 16 steps):

$$\begin{aligned}
& \frac{(472 a^2 A b + 128 A b^3 + 133 a^3 B + 356 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{192 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{(40 a^3 A b + 160 a A b^3 - 5 a^4 B + 120 a^2 b^2 B + 48 b^4 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{64 b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{1}{192 b d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} \\
& \frac{(264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} +}{b(8 A b + 11 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]} + \frac{(104 a A b + 59 a^2 B + 36 b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{96 d \cos [c+d x]^{3/2}} + \\
& \frac{(264 a^2 A b + 128 A b^3 + 15 a^3 B + 284 a b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{192 b d \sqrt{\cos [c+d x]}} + \frac{b B (a+b \sec [c+d x])^{3/2} \sin [c+d x]}{4 d \cos [c+d x]^{5/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 615: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{5/2} (A+B \sec [c+d x])}{\sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 280 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2(7 a^2 A b + 8 A b^3 - 5 a^3 B - 10 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2(9 a^2 A + 8 A b^2 - 10 a b B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 a^3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} - \\
& \frac{2(4 A b - 5 a B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a^2 d} + \frac{2 A \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 a d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\cos [c+d x]^{5/2} (A+B \sec [c+d x])}{\sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 616: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{3 / 2} (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} d x$$

Optimal (type 4, 212 leaves, 9 steps) :

$$\frac{2\left(a^2 A+2 A b^2-3 a b B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{2(2 A b-3 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 A \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{3 a d}$$

Result (type 8, 37 leaves) :

$$\int \frac{\cos [c+d x]^{3 / 2} (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} d x$$

■ **Problem 617: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} d x$$

Optimal (type 4, 150 leaves, 8 steps) :

$$-\frac{2(A b-a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a d \sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 8, 37 leaves) :

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \operatorname{Sec}[c+d x])}{\sqrt{a+b \operatorname{Sec}[c+d x]}} d x$$

■ **Problem 618: Unable to integrate problem.**

$$\int \frac{A+B \operatorname{Sec}[c+d x]}{\sqrt{\cos [c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} d x$$

Optimal (type 4, 138 leaves, 8 steps) :

$$\frac{2 A \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A+B \sec [c+d x]}{\sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 619: Unable to integrate problem.**

$$\int \frac{A+B \sec [c+d x]}{\cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 256 leaves, 13 steps):

$$\frac{B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(2 A b-a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{B \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{b d \sqrt{\cos [c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A+B \sec [c+d x]}{\cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 620: Unable to integrate problem.**

$$\int \frac{A+B \sec [c+d x]}{\cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$\frac{(4 A b - a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] - (4 a A b - 3 a^2 B - 4 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} - 4 b^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

$$\frac{(4 A b - 3 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{4 b^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{B \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 b d \cos [c+d x]^{3/2}} + \frac{(4 A b - 3 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 b^2 d \sqrt{\cos [c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \sec [c+d x]}{\cos [c+d x]^{5/2} \sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 621: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{5/2} (A + B \sec [c+d x])}{(a+b \sec [c+d x])^{3/2}} dx$$

Optimal (type 4, 423 leaves, 11 steps):

$$\frac{2(12 a^2 A b + 48 A b^3 - 5 a^3 B - 40 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^4 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\left(2(9 a^4 A + 24 a^2 A b^2 - 48 A b^4 - 25 a^3 b B + 40 a b^3 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}\right) /$$

$$\left(15 a^4 (a^2 - b^2) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}\right) + \frac{2 b (A b - a B) \cos [c+d x]^{3/2} \sin [c+d x]}{a (a^2 - b^2) d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2(9 a^2 A b - 24 A b^3 - 5 a^3 B + 20 a b^2 B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a^3 (a^2 - b^2) d} +$$

$$\frac{2(a^2 A - 6 A b^2 + 5 a b B) \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 a^2 (a^2 - b^2) d}$$

Result (type 8, 37 leaves):

$$\int \frac{\cos [c+d x]^{5/2} (A + B \sec [c+d x])}{(a+b \sec [c+d x])^{3/2}} dx$$

■ **Problem 622: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{3 / 2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 326 leaves, 10 steps):

$$\frac{2\left(a^2 A+8 A b^2-6 a b B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2\left(5 a^2 A b-8 A b^3-3 a^3 B+6 a b^2 B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^3\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} +$$

$$\frac{2 b(A b-a B) \sqrt{\cos [c+d x]} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}} + \frac{2\left(a^2 A-4 A b^2+3 a b B\right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a^2\left(a^2-b^2\right) d}$$

Result (type 8, 37 leaves):

$$\int \frac{\cos [c+d x]^{3 / 2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

■ **Problem 623: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 235 leaves, 9 steps):

$$-\frac{2(2 A b-a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2\left(a^2 A-2 A b^2+a b B\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{a^2\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 b(A b-a B) \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{\sqrt{\cos [c+d x]}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{3 / 2}} d x$$

■ **Problem 624: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 215 leaves, 9 steps):

$$\frac{2 A \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2(A b - a B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{a(a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} - \frac{2(A b - a B) \operatorname{Sin}[c+d x]}{(a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 625: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 220 leaves, 10 steps):

$$\frac{2 B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{2(A b - a B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{b(a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \frac{2 a(A b - a B) \operatorname{Sin}[c+d x]}{b(a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 626: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{5/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 371 leaves, 14 steps):

$$\begin{aligned}
& \frac{B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] - (2 A b-3 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(2 a A b-3 a^2 B+b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \frac{(2 a A b-3 a^2 B+b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b^2\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \\
& \frac{2 a(A b-a B) \sin [c+d x]}{b\left(a^2-b^2\right) d \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]}} - \frac{(2 a A b-3 a^2 B+b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{b^2\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{A+B \sec [c+d x]}{\cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{3 / 2}} d x$$

■ **Problem 627: Attempted integration timed out after 120 seconds.**

$$\int \frac{A+B \sec [c+d x]}{\cos [c+d x]^{7 / 2}(a+b \sec [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 487 leaves, 15 steps):

$$\begin{aligned}
& \frac{(4 A b-5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] - (12 a A b-15 a^2 B-4 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 b^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{(12 a^2 A b-4 A b^3-15 a^3 B+7 a b^2 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{4 b^3\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \\
& \frac{2 a(A b-a B) \sin [c+d x]}{b\left(a^2-b^2\right) d \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]}} - \frac{(4 a A b-5 a^2 B+b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 b^2\left(a^2-b^2\right) d \cos [c+d x]^{3 / 2}} + \\
& \frac{(12 a^2 A b-4 A b^3-15 a^3 B+7 a b^2 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 b^3\left(a^2-b^2\right) d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 628: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{5 / 2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 588 leaves, 12 steps):

$$\begin{aligned} & - \left(2 \left(17 a^4 A b + 116 a^2 A b^3 - 128 A b^5 - 5 a^5 B - 80 a^3 b^2 B + 80 a b^4 B \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \right) / \\ & \left(15 a^5 \left(a^2 - b^2 \right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \right) + \\ & \left(2 \left(9 a^6 A + 55 a^4 A b^2 - 212 a^2 A b^4 + 128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B \right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) / \\ & \left(15 a^5 \left(a^2 - b^2 \right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \frac{2 b(A b - a B) \cos [c+d x]^{3 / 2} \sin [c+d x]}{3 a \left(a^2 - b^2 \right) d (a+b \sec [c+d x])^{3 / 2}} + \\ & \frac{2 b \left(12 a^2 A b - 8 A b^3 - 9 a^3 B + 5 a b^2 B \right) \cos [c+d x]^{3 / 2} \sin [c+d x]}{3 a^2 \left(a^2 - b^2 \right)^2 d \sqrt{a+b \sec [c+d x]}} - \frac{1}{15 a^4 \left(a^2 - b^2 \right)^2 d} \\ & \frac{2 \left(14 a^4 A b - 98 a^2 A b^3 + 64 A b^5 - 5 a^5 B + 65 a^3 b^2 B - 40 a b^4 B \right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a^3 \left(a^2 - b^2 \right)^2 d} + \\ & \frac{2 \left(3 a^4 A - 71 a^2 A b^2 + 48 A b^4 + 50 a^3 b B - 30 a b^3 B \right) \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a^3 \left(a^2 - b^2 \right)^2 d} \end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\cos [c+d x]^{5 / 2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5 / 2}} d x$$

■ **Problem 629: Unable to integrate problem.**

$$\int \frac{\cos [c+d x]^{3 / 2}(A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 472 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 (a^4 A + 16 a^2 A b^2 - 16 A b^4 - 9 a^3 b B + 8 a b^3 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^4 (a^2-b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \\
& \left(\frac{2 (8 a^4 A b - 28 a^2 A b^3 + 16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{2} \right) / \\
& \left(\frac{3 a^4 (a^2-b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}{3 a (a^2-b^2) d (a+b \sec [c+d x])^{3/2}} + \frac{2 b (A b - a B) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a (a^2-b^2) d (a+b \sec [c+d x])^{3/2}} + \right. \\
& \left. \frac{2 b (10 a^2 A b - 6 A b^3 - 7 a^3 B + 3 a b^2 B) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a^2 (a^2-b^2)^2 d \sqrt{a+b \sec [c+d x]}} + \right. \\
& \left. \frac{2 (a^4 A - 13 a^2 A b^2 + 8 A b^4 + 8 a^3 b B - 4 a b^3 B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a^3 (a^2-b^2)^2 d} \right)
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\cos [c+d x]^{3/2} (A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5/2}} dx$$

■ **Problem 630: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 368 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 (9 a^2 A b - 8 A b^3 - 3 a^3 B + 2 a b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 (a^2-b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \left(\frac{2 (3 a^4 A - 15 a^2 A b^2 + 8 A b^4 + 6 a^3 b B - 2 a b^3 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{2} \right) / \\
& \left(\frac{3 a^3 (a^2-b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}{3 a (a^2-b^2) d \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2}} + \frac{2 b (A b - a B) \sin [c+d x]}{3 a (a^2-b^2) d \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2}} + \frac{2 b (8 a^2 A b - 4 A b^3 - 5 a^3 B + a b^2 B) \sin [c+d x]}{3 a^2 (a^2-b^2)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} \right)
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \sec [c+d x])}{(a+b \sec [c+d x])^{5/2}} dx$$

■ **Problem 631: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 346 leaves, 10 steps):

$$\frac{2 (3 a^2 A - 2 A b^2 - a b B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{3 a^2 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{2 (6 a^2 A b - 2 A b^3 - 3 a^3 B - a b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} -$$

$$\frac{2 (A b - a B) \operatorname{Sin}[c+d x]}{3 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{3/2}} - \frac{2 (5 a^2 A b - A b^3 - 2 a^3 B - 2 a b^2 B) \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

■ **Problem 632: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 329 leaves, 10 steps):

$$-\frac{2 (A b - a B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{3 a (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} -$$

$$\frac{2 (3 a^2 A + A b^2 - 4 a b B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 a (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} +$$

$$\frac{2 a (A b - a B) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 (2 a^2 A b + 2 A b^3 + a^3 B - 5 a b^2 B) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

■ **Problem 633: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{5/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 399 leaves, 14 steps):

$$\frac{2 (A b - a B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] + 2 B \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{3 b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{b^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 b (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \frac{2 (4 A b^3 + 3 a^3 B - 7 a b^2 B) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 a (A b - a B) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d \operatorname{Cos}[c+d x]^{3/2} (a + b \operatorname{Sec}[c+d x])^{3/2}} - \frac{2 a (4 A b^3 + 3 a^3 B - 7 a b^2 B) \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 37 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{5/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

■ **Problem 634: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x]}{\operatorname{Cos}[c + d x]^{7/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 526 leaves, 15 steps):

$$\begin{aligned}
& - \frac{(2 a A b - 5 a^2 B + 3 b^2 B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + (2 A b - 5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 b^2 (a^2 - b^2) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{(2 A b - 5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \\
& \left((6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) / \\
& \left(3 b^3 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \frac{2 a (A b - a B) \sin [c+d x]}{3 b (a^2 - b^2) d \cos [c+d x]^{5/2} (a+b \sec [c+d x])^{3/2}} + \\
& \frac{2 a (2 a^2 A b - 6 A b^3 - 5 a^3 B + 9 a b^2 B) \sin [c+d x]}{3 b^2 (a^2 - b^2)^2 d \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]}} - \frac{(6 a^3 A b - 14 a A b^3 - 15 a^4 B + 26 a^2 b^2 B - 3 b^4 B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{\cos [c+d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Test results for the 70 problems in "4.5.4.1 (a+b sec)^m (A+B sec+C sec^2).m"

- Problem 2: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^5 (A+C \sec [c+d x]^2) d x$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{(6 A+5 C) \operatorname{ArcTanh}[\sin [c+d x]]}{16 d} + \frac{(6 A+5 C) \sec [c+d x] \tan [c+d x]}{16 d} + \frac{(6 A+5 C) \sec [c+d x]^3 \tan [c+d x]}{24 d} + \frac{C \sec [c+d x]^5 \tan [c+d x]}{6 d}$$

Result (type 3, 445 leaves):

$$\begin{aligned}
& - \frac{3 A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} - \frac{5 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{16 d} + \\
& \frac{3 A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{5 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{16 d} + \frac{C}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} + \\
& \frac{A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{3 A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{5 C}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{C}{48 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^6} - \frac{A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{5 C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{3 A}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{5 C}{32 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}
\end{aligned}$$

■ **Problem 8: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] (A+C \sec[c+dx])^2 dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{C \operatorname{ArcTanh}[\sin[c+dx]]}{d} + \frac{A \sin[c+dx]}{d}$$

Result (type 3, 92 leaves):

$$\begin{aligned}
& - \frac{C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A \cos[dx] \sin[c]}{d} + \frac{A \cos[c] \sin[dx]}{d}
\end{aligned}$$

■ **Problem 17: Result unnecessarily involves higher level functions.**

$$\int (b \sec[c+dx])^{3/2} (A+C \sec[c+dx])^2 dx$$

Optimal (type 4, 110 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 b^2 (5 A + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5 d \sqrt{\cos[c+dx]} \sqrt{b \sec[c+dx]}} + \frac{2 b (5 A + 3 C) \sqrt{b \sec[c+dx]} \sin[c+dx]}{5 d} + \frac{2 C (b \sec[c+dx])^{3/2} \tan[c+dx]}{5 d}
\end{aligned}$$

Result (type 5, 180 leaves):

$$\begin{aligned}
& - \left(4 i e^{-i(c+dx)} \cos[c+dx]^3 \right. \\
& \left. \left(-5 A (1 + e^{2i(c+dx)})^2 - C (3 + 8 e^{2i(c+dx)} + e^{4i(c+dx)}) + (5 A + 3 C) (1 + e^{2i(c+dx)})^{5/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \left. (b \sec[c+dx])^{3/2} (A+C \sec[c+dx])^2 \right) / \left(5 d (1 + e^{2i(c+dx)})^2 (A + 2 C + A \cos[2(c+dx)]) \right)
\end{aligned}$$

■ **Problem 19: Result unnecessarily involves higher level functions.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{\sqrt{b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 68 leaves, 3 steps):

$$\frac{2(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{b \operatorname{Sec}[c + d x]}} + \frac{2 C \operatorname{Tan}[c + d x]}{d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 5, 99 leaves):

$$\frac{2 i \left(A - 2 C + A e^{2 i (c + d x)} + 2 (-A + C) \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right)}{d \left(1 + e^{2 i (c + d x)} \right) \sqrt{b \operatorname{Sec}[c + d x]}}$$

■ **Problem 21: Result unnecessarily involves higher level functions.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 77 leaves, 3 steps):

$$\frac{2(3A + 5C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 b^2 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{b \operatorname{Sec}[c + d x]}} + \frac{2 A \operatorname{Tan}[c + d x]}{5 d (b \operatorname{Sec}[c + d x])^{5/2}}$$

Result (type 5, 135 leaves):

$$\frac{1}{10 d (b \operatorname{Sec}[c + d x])^{5/2}} e^{-i(2c + d x)} \operatorname{Sec}[c + d x]^2 \left(-4 i (3A + 5C) + \frac{8 i (3A + 5C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right]}{\sqrt{1 + e^{2 i (c + d x)}}} + 2 A \operatorname{Sin}[2(c + d x)] \right) (\operatorname{Cos}[2c + d x] + i \operatorname{Sin}[2c + d x])$$

■ **Problem 23: Result unnecessarily involves higher level functions.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(b \operatorname{Sec}[c + d x])^{9/2}} dx$$

Optimal (type 4, 112 leaves, 4 steps):

$$\frac{2(7A + 9C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 b^4 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{b \operatorname{Sec}[c + d x]}} + \frac{2(7A + 9C) \operatorname{Sin}[c + d x]}{45 b^3 d (b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 A \operatorname{Tan}[c + d x]}{9 d (b \operatorname{Sec}[c + d x])^{9/2}}$$

Result (type 5, 145 leaves):

$$\frac{1}{360 b^4 d \sqrt{b \operatorname{Sec}[c + d x]}}$$

$$e^{-i(2c+dx)} \left(-336 i A - 432 i C + \frac{96 i (7A + 9C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1 + e^{2i(c+dx)}}} + (76A + 72C) \operatorname{Sin}[2(c+dx)] + 10A \operatorname{Sin}[4(c+dx)] \right)$$

$$(\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx])$$

- **Problem 32: Result more than twice size of optimal antiderivative.**

$$\int -\operatorname{Cos}[e + f x] dx$$

Optimal (type 3, 11 leaves, 1 step):

$$-\frac{\operatorname{Sin}[e + f x]}{f}$$

Result (type 3, 23 leaves):

$$-\frac{\operatorname{Cos}[f x] \operatorname{Sin}[e]}{f} - \frac{\operatorname{Cos}[e] \operatorname{Sin}[f x]}{f}$$

- **Problem 34: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^3 (-3 + 2 \operatorname{Sec}[e + f x]^2) dx$$

Optimal (type 3, 19 leaves, 1 step):

$$-\frac{\operatorname{Cos}[e + f x]^2 \operatorname{Sin}[e + f x]}{f}$$

Result (type 3, 51 leaves):

$$\frac{2 \operatorname{Cos}[f x] \operatorname{Sin}[e]}{f} + \frac{2 \operatorname{Cos}[e] \operatorname{Sin}[f x]}{f} - \frac{9 \operatorname{Sin}[e + f x]}{4 f} - \frac{\operatorname{Sin}[3(e + f x)]}{4 f}$$

- **Problem 36: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^5 (-5 + 4 \operatorname{Sec}[e + f x]^2) dx$$

Optimal (type 3, 19 leaves, 1 step):

$$-\frac{\operatorname{Cos}[e + f x]^4 \operatorname{Sin}[e + f x]}{f}$$

Result (type 3, 44 leaves):

$$-\frac{\operatorname{Sin}[e + f x]}{8 f} - \frac{3 \operatorname{Sin}[3(e + f x)]}{16 f} - \frac{\operatorname{Sin}[5(e + f x)]}{16 f}$$

■ **Problem 37: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^3 (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{3 C \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{B \text{Tan}[c + d x]}{d} + \frac{3 C \text{Sec}[c + d x] \text{Tan}[c + d x]}{8 d} + \frac{C \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{4 d} + \frac{B \text{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 227 leaves):

$$\begin{aligned} & - \frac{3 C \text{Log}[\text{Cos}[\frac{1}{2}(c + d x)] - \text{Sin}[\frac{1}{2}(c + d x)]]}{8 d} + \frac{3 C \text{Log}[\text{Cos}[\frac{1}{2}(c + d x)] + \text{Sin}[\frac{1}{2}(c + d x)]]}{8 d} + \\ & \frac{C}{16 d (\text{Cos}[\frac{1}{2}(c + d x)] - \text{Sin}[\frac{1}{2}(c + d x)])^4} + \frac{C}{16 d (\text{Cos}[\frac{1}{2}(c + d x)] - \text{Sin}[\frac{1}{2}(c + d x)])^2} - \frac{C}{16 d (\text{Cos}[\frac{1}{2}(c + d x)] + \text{Sin}[\frac{1}{2}(c + d x)])^4} - \\ & \frac{3 C}{16 d (\text{Cos}[\frac{1}{2}(c + d x)] + \text{Sin}[\frac{1}{2}(c + d x)])^2} + \frac{2 B \text{Tan}[c + d x]}{3 d} + \frac{B \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3 d} \end{aligned}$$

■ **Problem 40: Result more than twice size of optimal antiderivative.**

$$\int (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{B \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{C \text{Tan}[c + d x]}{d}$$

Result (type 3, 81 leaves):

$$- \frac{B \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] - \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d} + \frac{B \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] + \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d} + \frac{C \text{Tan}[c + d x]}{d}$$

■ **Problem 41: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x] (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 16 leaves, 4 steps):

$$B x + \frac{C \text{ArcTanh}[\text{Sin}[c + d x]]}{d}$$

Result (type 3, 73 leaves):

$$B x - \frac{C \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] - \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d} + \frac{C \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] + \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d}$$

■ **Problem 54: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 122 leaves, 7 steps):

$$\frac{3 B \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} + \frac{(5 A+4 C) \tan [c+d x]}{5 d} + \frac{3 B \sec [c+d x] \tan [c+d x]}{8 d} +$$

$$\frac{B \sec [c+d x]^3 \tan [c+d x]}{4 d} + \frac{C \sec [c+d x]^4 \tan [c+d x]}{5 d} + \frac{(5 A+4 C) \tan [c+d x]^3}{15 d}$$

Result (type 3, 285 leaves):

$$-\frac{3 B \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 B \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{B}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} +$$

$$\frac{3 B}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{B}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 B}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} +$$

$$\frac{2 A \tan [c+d x]}{3 d} + \frac{8 C \tan [c+d x]}{15 d} + \frac{A \sec [c+d x]^2 \tan [c+d x]}{3 d} + \frac{4 C \sec [c+d x]^2 \tan [c+d x]}{15 d} + \frac{C \sec [c+d x]^4 \tan [c+d x]}{5 d}$$

■ **Problem 55: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^3 (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 97 leaves, 6 steps):

$$\frac{(4 A+3 C) \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} + \frac{B \tan [c+d x]}{d} + \frac{(4 A+3 C) \sec [c+d x] \tan [c+d x]}{8 d} + \frac{C \sec [c+d x]^3 \tan [c+d x]}{4 d} + \frac{B \tan [c+d x]^3}{3 d}$$

Result (type 3, 353 leaves):

$$-\frac{A \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} - \frac{3 C \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} +$$

$$\frac{A \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{2 d} + \frac{3 C \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{C}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} +$$

$$\frac{A}{4 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{3 C}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{C}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} -$$

$$\frac{A}{4 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \frac{3 C}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{2 B \tan [c+d x]}{3 d} + \frac{B \sec [c+d x]^2 \tan [c+d x]}{3 d}$$

■ **Problem 57: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{(2A + C) \text{ArcTanh}[\text{Sin}[c + d x]]}{2d} + \frac{B \text{Tan}[c + d x]}{d} + \frac{C \text{Sec}[c + d x] \text{Tan}[c + d x]}{2d}$$

Result (type 3, 151 leaves):

$$\frac{1}{4d} \left(-2(2A + C) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 4A \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \right. \\ \left. 2C \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \frac{C}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{C}{\left(\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + 4B \text{Tan}[c + d x] \right)$$

■ **Problem 58: Result more than twice size of optimal antiderivative.**

$$\int (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$A x + \frac{B \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{C \text{Tan}[c + d x]}{d}$$

Result (type 3, 84 leaves):

$$A x - \frac{B \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{B \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \text{Tan}[c + d x]}{d}$$

■ **Problem 59: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$B x + \frac{C \text{ArcTanh}[\text{Sin}[c + d x]]}{d} + \frac{A \text{Sin}[c + d x]}{d}$$

Result (type 3, 95 leaves):

$$B x - \frac{C \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{C \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{A \text{Cos}[d x] \text{Sin}[c]}{d} + \frac{A \text{Cos}[c] \text{Sin}[d x]}{d}$$

■ **Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (b \text{Sec}[c + d x])^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 178 leaves, 8 steps) :

$$-\frac{2b^2(5A+3C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d\sqrt{\cos[c+dx]}\sqrt{b\sec[c+dx]}} + \frac{2bB\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{b\sec[c+dx]}}{3d} +$$

$$\frac{2b(5A+3C)\sqrt{b\sec[c+dx]}\sin[c+dx]}{5d} + \frac{2B(b\sec[c+dx])^{3/2}\sin[c+dx]}{3d} + \frac{2C(b\sec[c+dx])^{3/2}\tan[c+dx]}{5d}$$

Result (type 5, 618 leaves) :

$$\frac{4B\cos[c+dx]^{7/2}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right](b\sec[c+dx])^{3/2}(A+B\sec[c+dx]+C\sec[c+dx]^2)}{3d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])} -$$

$$\left(2\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\operatorname{Csc}[c]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\right)$$

$$(b\sec[c+dx])^{3/2}(A+B\sec[c+dx]+C\sec[c+dx]^2)\left/\left(d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sec[c+dx]^{7/2}\right) -$$

$$\left(6\sqrt{2}Ce^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\operatorname{Csc}[c]\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\right)$$

$$(b\sec[c+dx])^{3/2}(A+B\sec[c+dx]+C\sec[c+dx]^2)\left/\left(5d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sec[c+dx]^{7/2}\right) +$$

$$\left(\cos[c+dx]^3(b\sec[c+dx])^{3/2}(A+B\sec[c+dx]+C\sec[c+dx]^2)\left(\frac{4(5A+3C)\cos[dx]\operatorname{Csc}[c]}{5d} + \frac{4C\sec[c]\sec[c+dx]^2\sin[dx]}{5d} + \frac{4\sec[c]\sec[c+dx](3C\sin[c]+5B\sin[dx])}{15d} + \frac{4B\tan[c]}{3d}\right)\right)\left/\left(A+2C+2B\cos[c+dx]+A\cos[2c+2dx]\right)$$

■ **Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{b\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 4, 136 leaves, 7 steps) :

$$-\frac{2bB\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d\sqrt{\cos[c+dx]}\sqrt{b\sec[c+dx]}} + \frac{2(3A+C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{b\sec[c+dx]}}{3d} +$$

$$\frac{2B\sqrt{b\sec[c+dx]}\sin[c+dx]}{d} + \frac{2C\sqrt{b\sec[c+dx]}\tan[c+dx]}{3d}$$

Result (type 5, 302 leaves) :

$$\left(4 \sqrt{b \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-1 / (-1 + e^{2 i c}) i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2 i(c+dx)}}} \left(3 B (1 + e^{2 i(c+dx)}) + 3 B (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] \right) \right. \right. \\ \left. \left. + (3 A + C) e^{i(c+dx)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+dx)}\right] \right) \right) + \\ \left. \sqrt{\operatorname{Sec}[c + d x]} (3 B \operatorname{Cos}[d x] \operatorname{Csc}[c] + C \operatorname{Tan}[c + d x]) \right) \Bigg/ (3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}[c + d x]^{5/2})$$

■ **Problem 67: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 110 leaves, 6 steps) :

$$\frac{2(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{b \operatorname{Sec}[c + d x]}} + \frac{2 B \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{b \operatorname{Sec}[c + d x]}}{b d} + \frac{2 C \operatorname{Tan}[c + d x]}{d \sqrt{b \operatorname{Sec}[c + d x]}}$$

Result (type 5, 135 leaves) :

$$\frac{1}{b d} e^{-i(c+dx)} \left(2 B e^{i(c+dx)} \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] - \right. \\ \left. i \left(A - 2 C + A e^{2 i(c+dx)} + 2(-A + C) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] \right) \right) \sqrt{b \operatorname{Sec}[c + d x]}$$

■ **Problem 68: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 117 leaves, 6 steps) :

$$\frac{2 B \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{b d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{b \operatorname{Sec}[c + d x]}} + \frac{2(A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{b \operatorname{Sec}[c + d x]}}{3 b^2 d} + \frac{2 A \operatorname{Tan}[c + d x]}{3 d (b \operatorname{Sec}[c + d x])^{3/2}}$$

Result (type 5, 143 leaves) :

$$\left(2 \left(6 i B \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] - 2 i (A + 3 C) e^{i(c+dx)} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)} \right] + \sqrt{1 + e^{2i(c+dx)}} (-3 i B + A \sin[c + dx]) \right) \right) / \left(3 b d \sqrt{1 + e^{2i(c+dx)}} \sqrt{b \sec[c + dx]} \right)$$

■ **Problem 69: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{(b \sec[c + dx])^{5/2}} dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\frac{2(3A + 5C) \operatorname{EllipticE} \left[\frac{1}{2}(c + dx), 2 \right]}{5b^2 d \sqrt{\cos[c + dx]} \sqrt{b \sec[c + dx]}} + \frac{2B \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c + dx), 2 \right] \sqrt{b \sec[c + dx]}}{3b^3 d} + \frac{2B \sin[c + dx]}{3b^2 d \sqrt{b \sec[c + dx]}} + \frac{2A \tan[c + dx]}{5d (b \sec[c + dx])^{5/2}}$$

Result (type 5, 183 leaves):

$$\frac{1}{30b^3 d} e^{-i(2c+dx)} \sqrt{b \sec[c + dx]} \left(20B \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c + dx), 2 \right] + 12i(3A + 5C) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] + 2 \cos[c + dx] (-6i(3A + 5C) + 10B \sin[c + dx] + 3A \sin[2(c + dx)]) \right) (\cos[2c + dx] + i \sin[2c + dx])$$

■ **Problem 70: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{(b \sec[c + dx])^{7/2}} dx$$

Optimal (type 4, 185 leaves, 8 steps):

$$\frac{6B \operatorname{EllipticE} \left[\frac{1}{2}(c + dx), 2 \right]}{5b^3 d \sqrt{\cos[c + dx]} \sqrt{b \sec[c + dx]}} + \frac{2(5A + 7C) \sqrt{\cos[c + dx]} \operatorname{EllipticF} \left[\frac{1}{2}(c + dx), 2 \right] \sqrt{b \sec[c + dx]}}{21b^4 d} + \frac{2B \sin[c + dx]}{5b^2 d (b \sec[c + dx])^{3/2}} + \frac{2(5A + 7C) \sin[c + dx]}{21b^3 d \sqrt{b \sec[c + dx]}} + \frac{2A \tan[c + dx]}{7d (b \sec[c + dx])^{7/2}}$$

Result (type 5, 177 leaves):

$$\left(504 i B \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] - 40 i (5A + 7C) e^{i(c+dx)} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)} \right] + \sqrt{1 + e^{2i(c+dx)}} (5(23A + 28C) \sin[c + dx] + 3(-84 i B + 14B \sin[2(c + dx)] + 5A \sin[3(c + dx)])) \right) / \left(210 b^3 d \sqrt{1 + e^{2i(c+dx)}} \sqrt{b \sec[c + dx]} \right)$$

Test results for the 1373 problems in "4.5.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2).m"

- **Problem 1: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sec}[c + d x]^2 (b \text{Sec}[c + d x])^{1/3} (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 95 leaves, 4 steps):

$$\frac{3 (10 A + 7 C) \text{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{4/3} \text{Sin}[c + d x]}{40 b d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{7/3} \text{Tan}[c + d x]}{10 b^2 d}$$

Result (type 5, 189 leaves):

$$\left(3 (b \text{Sec}[c + d x])^{1/3} (A + C \text{Sec}[c + d x]^2) \left(-2 i 2^{1/3} (10 A + 7 C) \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{1/3} (1 + e^{2i(c+dx)})^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)}\right] + \right. \right. \\ \left. \left. (5 (2 A + 3 C) + (10 A + 7 C) \text{Cos}[2(c + d x)]) \text{Sec}[c + d x]^{10/3} \text{Sin}[c + d x] \right) \right) / \left(40 d (A + 2 C + A \text{Cos}[2(c + d x)]) \text{Sec}[c + d x]^{7/3} \right)$$

- **Problem 2: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sec}[c + d x] (b \text{Sec}[c + d x])^{1/3} (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 92 leaves, 4 steps):

$$\frac{3 (7 A + 4 C) \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{1/3} \text{Sin}[c + d x]}{7 d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{4/3} \text{Tan}[c + d x]}{7 b d}$$

Result (type 5, 183 leaves):

$$- \left(6 i e^{-i(c+dx)} \text{Cos}[c + d x]^3 \right. \\ \left. \left(-7 A (1 + e^{2i(c+dx)})^2 - 2 C (2 + 5 e^{2i(c+dx)} + e^{4i(c+dx)}) + (7 A + 4 C) (1 + e^{2i(c+dx)})^{7/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)}\right] \right) \right) \\ \left(b \text{Sec}[c + d x] \right)^{4/3} (A + C \text{Sec}[c + d x]^2) / \left(7 b d (1 + e^{2i(c+dx)})^2 (A + 2 C + A \text{Cos}[2(c + d x)]) \right)$$

- **Problem 3: Result unnecessarily involves imaginary or complex numbers.**

$$\int (b \text{Sec}[c + d x])^{1/3} (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 88 leaves, 3 steps):

$$\frac{3 b (4 A + C) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos [c + d x]^2\right] \sin [c + d x]}{8 d (b \operatorname{Sec}[c + d x])^{2/3} \sqrt{\sin [c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{1/3} \tan [c + d x]}{4 d}$$

Result (type 5, 162 leaves):

$$\left(3 (b \operatorname{Sec}[c + d x])^{1/3} (A + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-i 2^{1/3} (4 A + C) \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{1/3} (1 + e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)}\right] + C \operatorname{Sec}[c + d x]^{4/3} \sin [c + d x] \right) \right) / \\ (2 d (A + 2 C + A \cos [2(c + d x)]) \operatorname{Sec}[c + d x]^{7/3})$$

■ **Problem 6: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 95 leaves, 4 steps):

$$\frac{3 (13 A + 10 C) \operatorname{Hypergeometric2F1}\left[-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos [c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{7/3} \sin [c + d x]}{91 b d \sqrt{\sin [c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{10/3} \tan [c + d x]}{13 b^2 d}$$

Result (type 5, 235 leaves):

$$-\left(12 i e^{-i(c+dx)} \cos [c + d x]^3 \left(-13 A (1 + e^{2i(c+dx)})^2 (2 + 5 e^{2i(c+dx)} + e^{4i(c+dx)}) - 2 C (10 + 45 e^{2i(c+dx)} + 79 e^{4i(c+dx)} + 21 e^{6i(c+dx)} + 5 e^{8i(c+dx)}) \right) \right. \\ \left. 2 (13 A + 10 C) (1 + e^{2i(c+dx)})^{13/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)}\right] \right) \\ (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) \Big/ (91 d (1 + e^{2i(c+dx)})^4 (A + 2 C + A \cos [2(c + d x)]))$$

■ **Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x] (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 92 leaves, 4 steps):

$$\frac{3 (10 A + 7 C) \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos [c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{4/3} \sin [c + d x]}{40 d \sqrt{\sin [c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{7/3} \tan [c + d x]}{10 b d}$$

Result (type 5, 192 leaves):

$$\left(3 (b \operatorname{Sec}[c + d x])^{7/3} (A + C \operatorname{Sec}[c + d x]^2) \left(-2 i 2^{1/3} (10 A + 7 C) \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{1/3} (1 + e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)}\right] + (5(2A + 3C) + (10A + 7C) \cos[2(c+dx)]) \operatorname{Sec}[c + d x]^{10/3} \sin[c + d x] \right) \right) / \left(40 b d (A + 2C + A \cos[2(c+dx)]) \operatorname{Sec}[c + d x]^{13/3} \right)$$

- **Problem 8: Result unnecessarily involves imaginary or complex numbers.**

$$\int (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{3 b (7 A + 4 C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{1/3} \sin[c + d x]}{7 d \sqrt{\sin[c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{4/3} \tan[c + d x]}{7 d}$$

Result (type 5, 180 leaves):

$$-\left(6 i e^{-i(c+dx)} \cos[c + d x]^3 \right. \\ \left. (-7 A (1 + e^{2i(c+dx)})^2 - 2 C (2 + 5 e^{2i(c+dx)} + e^{4i(c+dx)}) + (7 A + 4 C) (1 + e^{2i(c+dx)})^{7/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)}\right]) \right) \\ (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) / \left(7 d (1 + e^{2i(c+dx)})^2 (A + 2 C + A \cos[2(c+dx)]) \right)$$

- **Problem 9: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos[c + d x] (b \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 91 leaves, 4 steps):

$$-\frac{3 b^2 (4 A + C) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos[c + d x]^2\right] \sin[c + d x]}{8 d (b \operatorname{Sec}[c + d x])^{2/3} \sqrt{\sin[c + d x]^2}} + \frac{3 b C (b \operatorname{Sec}[c + d x])^{1/3} \tan[c + d x]}{4 d}$$

Result (type 5, 163 leaves):

$$\left(3 b (b \operatorname{Sec}[c + d x])^{1/3} (A + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-i 2^{1/3} (4 A + C) \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{1/3} (1 + e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)}\right] + C \operatorname{Sec}[c + d x]^{4/3} \sin[c + d x] \right) \right) / \\ (2 d (A + 2 C + A \cos[2(c+dx)]) \operatorname{Sec}[c + d x]^{7/3})$$

- **Problem 11: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2)}{(b \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 5, 95 leaves, 4 steps):

$$\frac{3 (8 A + 5 C) \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos [c+d x]^2\right] (b \operatorname{Sec}[c+d x])^{2/3} \sin [c+d x]}{16 b d \sqrt{\sin [c+d x]^2}} + \frac{3 C (b \operatorname{Sec}[c+d x])^{5/3} \tan [c+d x]}{8 b^2 d}$$

Result (type 5, 207 leaves):

$$-\left(3 i\left(-8 A\left(1+e^{2 i(c+d x)}\right)^2-C\left(5+14 e^{2 i(c+d x)}+e^{4 i(c+d x)}\right)+\left(8 A+5 C\right)\left(1+e^{2 i(c+d x)}\right)^{8/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6},-e^{2 i(c+d x)}\right]\right) \\ \left(A+C \operatorname{Sec}[c+d x]^2\right) \Bigg/ \left(4 \times 2^{1/3} d\left(\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}\right)^{1/3}\left(1+e^{2 i(c+d x)}\right)^3\left(A+2 C+A \cos [2(c+d x)]\right) \operatorname{Sec}[c+d x]^{5/3}(b \operatorname{Sec}[c+d x])^{1/3}\right)$$

■ **Problem 12: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c+d x]\left(A+C \operatorname{Sec}[c+d x]^2\right)}{(b \operatorname{Sec}[c+d x])^{1/3}} d x$$

Optimal (type 5, 92 leaves, 4 steps):

$$-\frac{3(5 A+2 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{5 d(b \operatorname{Sec}[c+d x])^{1/3} \sqrt{\sin [c+d x]^2}} + \frac{3 C(b \operatorname{Sec}[c+d x])^{2/3} \tan [c+d x]}{5 b d}$$

Result (type 5, 168 leaves):

$$\left(3(b \operatorname{Sec}[c+d x])^{2/3}\left(A+C \operatorname{Sec}[c+d x]^2\right)\right. \\ \left.\left(-i 2^{2/3}(5 A+2 C)\left(\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}\right)^{2/3}\left(1+e^{2 i(c+d x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3},-e^{2 i(c+d x)}\right]+2 C \operatorname{Sec}[c+d x]^{5/3} \sin [c+d x]\right)\right) \Bigg/ \\ (5 b d(A+2 C+A \cos [2(c+d x)]) \operatorname{Sec}[c+d x]^{8/3})$$

■ **Problem 13: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A+C \operatorname{Sec}[c+d x]^2}{(b \operatorname{Sec}[c+d x])^{1/3}} d x$$

Optimal (type 5, 90 leaves, 3 steps):

$$-\frac{3 b(2 A-C) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos [c+d x]^2\right] \sin [c+d x]}{8 d(b \operatorname{Sec}[c+d x])^{4/3} \sqrt{\sin [c+d x]^2}} + \frac{3 C \tan [c+d x]}{2 d(b \operatorname{Sec}[c+d x])^{1/3}}$$

Result (type 5, 98 leaves):

$$-\frac{3 i\left(A-C+A e^{2 i(c+d x)}+(-2 A+C)\left(1+e^{2 i(c+d x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6},-e^{2 i(c+d x)}\right]\right)}{d\left(1+e^{2 i(c+d x)}\right)(b \operatorname{Sec}[c+d x])^{1/3}}$$

■ **Problem 14: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c+d x] (A+C \sec [c+d x])^2}{(b \sec [c+d x])^{1/3}} dx$$

Optimal (type 5, 88 leaves, 4 steps) :

$$-\frac{3(A+4C) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{4 d (b \sec [c+d x])^{1/3} \sqrt{\sin [c+d x]^2}} + \frac{3 A b \tan [c+d x]}{4 d (b \sec [c+d x])^{4/3}}$$

Result (type 5, 121 leaves) :

$$-\left(3 i e^{-i(c+d x)}\left(A\left(-1+e^{4 i(c+d x)}\right)+2(A+4 C) e^{2 i(c+d x)}\left(1+e^{2 i(c+d x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2 i(c+d x)}\right]\right)\right) / \left(8 d\left(1+e^{2 i(c+d x)}\right)(b \sec [c+d x])^{1/3}\right)$$

■ **Problem 16: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x]^2 (A+C \sec [c+d x])^2}{(b \sec [c+d x])^{4/3}} dx$$

Optimal (type 5, 95 leaves, 4 steps) :

$$-\frac{3(5 A+2 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos [c+d x]^2\right] \sin [c+d x]}{5 b d (b \sec [c+d x])^{1/3} \sqrt{\sin [c+d x]^2}} + \frac{3 C (b \sec [c+d x])^{2/3} \tan [c+d x]}{5 b^2 d}$$

Result (type 5, 165 leaves) :

$$\left(3(A+C \sec [c+d x])^2\right. \\ \left. - i 2^{2/3}(5 A+2 C)\left(\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}\right)^{2/3}\left(1+e^{2 i(c+d x)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2 i(c+d x)}\right]+2 C \sec [c+d x]^{5/3} \sin [c+d x]\right) / \left(5 d(A+2 C+A \cos [2(c+d x)]) \sec [c+d x]^{2/3}(b \sec [c+d x])^{4/3}\right)$$

■ **Problem 17: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec [c+d x] (A+C \sec [c+d x])^2}{(b \sec [c+d x])^{4/3}} dx$$

Optimal (type 5, 92 leaves, 4 steps) :

$$-\frac{3(2 A-C) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos [c+d x]^2\right] \sin [c+d x]}{8 d (b \sec [c+d x])^{4/3} \sqrt{\sin [c+d x]^2}} + \frac{3 C \tan [c+d x]}{2 b d (b \sec [c+d x])^{1/3}}$$

Result (type 5, 101 leaves) :

$$- \frac{3 i (A - C + A e^{2 i (c+dx)} + (-2 A + C) (1 + e^{2 i (c+dx)})^{2/3} \text{Hypergeometric2F1}[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2 i (c+dx)}])}{b d (1 + e^{2 i (c+dx)}) (b \text{Sec}[c + dx])^{1/3}}$$

- **Problem 18: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \text{Sec}[c + dx]^2}{(b \text{Sec}[c + dx])^{4/3}} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$- \frac{3 (A + 4 C) \text{Hypergeometric2F1}[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \text{Cos}[c + dx]^2] \text{Sin}[c + dx]}{4 b d (b \text{Sec}[c + dx])^{1/3} \sqrt{\text{Sin}[c + dx]^2}} + \frac{3 A \text{Tan}[c + dx]}{4 d (b \text{Sec}[c + dx])^{4/3}}$$

Result (type 5, 124 leaves):

$$- \left(\frac{3 i e^{-i (c+dx)} \left(A (-1 + e^{4 i (c+dx)}) + 2 (A + 4 C) e^{2 i (c+dx)} (1 + e^{2 i (c+dx)})^{2/3} \text{Hypergeometric2F1}[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2 i (c+dx)}] \right)}{(8 b d (1 + e^{2 i (c+dx)}) (b \text{Sec}[c + dx])^{1/3}} \right) /$$

- **Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + dx]^m (b \text{Sec}[c + dx])^{4/3} (A + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 5, 146 leaves, 4 steps):

$$\frac{3 b C \text{Sec}[c + dx]^{2+m} (b \text{Sec}[c + dx])^{1/3} \text{Sin}[c + dx]}{d (7 + 3 m)} + \left(\frac{3 b (C (4 + 3 m) + A (7 + 3 m)) \text{Hypergeometric2F1}[\frac{1}{2}, \frac{1}{6} (-1 - 3 m), \frac{1}{6} (5 - 3 m), \text{Cos}[c + dx]^2] \text{Sec}[c + dx]^m (b \text{Sec}[c + dx])^{1/3} \text{Sin}[c + dx]}{d (1 + 3 m) (7 + 3 m) \sqrt{\text{Sin}[c + dx]^2}} \right) /$$

Result (type 5, 333 leaves):

$$\begin{aligned}
& - \left(3 i 2^{\frac{7}{3}+m} e^{-\frac{1}{3} i d (4+3 m) x} \left(\frac{e^{i (c+d x)}}{1+e^{2 i (c+d x)}} \right)^{\frac{4}{3}+m} \left(1+e^{2 i (c+d x)} \right)^{\frac{4}{3}+m} \left(\frac{2 (A+2 C) e^{\frac{1}{3} i (6 c+d (10+3 m) x)} \operatorname{Hypergeometric2F1} \left[\frac{5}{3}+\frac{m}{2}, \frac{10}{3}+m, \frac{8}{3}+\frac{m}{2}, -e^{2 i (c+d x)} \right]}{10+3 m} \right. \right. \\
& \quad \left. \left. + \frac{A e^{4 i c+\frac{1}{3} i d (16+3 m) x} \operatorname{Hypergeometric2F1} \left[\frac{8}{3}+\frac{m}{2}, \frac{10}{3}+m, \frac{1}{6} (22+3 m), -e^{2 i (c+d x)} \right]}{16+3 m} \right. \right. \\
& \quad \left. \left. + \frac{A e^{\frac{1}{3} i d (4+3 m) x} \operatorname{Hypergeometric2F1} \left[\frac{10}{3}+m, \frac{1}{6} (4+3 m), \frac{5}{3}+\frac{m}{2}, -e^{2 i (c+d x)} \right]}{4+3 m} \right) \right) \\
& \left. (b \operatorname{Sec}[c+d x])^{4/3} (A+C \operatorname{Sec}[c+d x]^2) \right) / \left(d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+d x]^{10/3} \right)
\end{aligned}$$

- **Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^m (b \operatorname{Sec}[c+d x])^{2/3} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 5, 146 leaves, 4 steps):

$$\frac{3 C \operatorname{Sec}[c+d x]^{1+m} (b \operatorname{Sec}[c+d x])^{2/3} \operatorname{Sin}[c+d x]}{d (5+3 m)}$$

$$\begin{aligned}
& \left(3 (C (2+3 m) + A (5+3 m)) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (1-3 m), \frac{1}{6} (7-3 m), \operatorname{Cos}[c+d x]^2 \right] \operatorname{Sec}[c+d x]^{-1+m} (b \operatorname{Sec}[c+d x])^{2/3} \operatorname{Sin}[c+d x] \right) / \\
& \left(d (1-3 m) (5+3 m) \sqrt{\operatorname{Sin}[c+d x]^2} \right)
\end{aligned}$$

Result (type 5, 336 leaves):

$$\begin{aligned}
& - \left(3 i 2^{\frac{5}{3}+m} e^{-\frac{1}{3} i d (2+3 m) x} \left(\frac{e^{i (c+d x)}}{1+e^{2 i (c+d x)}} \right)^{\frac{2}{3}+m} \left(1+e^{2 i (c+d x)} \right)^{\frac{2}{3}+m} \left(\frac{A e^{4 i c+\frac{1}{3} i d (14+3 m) x} \operatorname{Hypergeometric2F1} \left[\frac{7}{3}+\frac{m}{2}, \frac{8}{3}+m, \frac{1}{6} (20+3 m), -e^{2 i (c+d x)} \right]}{14+3 m} \right. \right. \\
& \quad \left. \left. + \frac{1}{((2+3 m) (8+3 m))} e^{\frac{1}{3} i d (2+3 m) x} \left(A (8+3 m) \operatorname{Hypergeometric2F1} \left[\frac{8}{3}+m, \frac{1}{6} (2+3 m), \frac{1}{6} (8+3 m), -e^{2 i (c+d x)} \right] + \right. \right. \\
& \quad \left. \left. 2 (A+2 C) e^{2 i (c+d x)} (2+3 m) \operatorname{Hypergeometric2F1} \left[\frac{8}{3}+m, \frac{1}{6} (8+3 m), \frac{7}{3}+\frac{m}{2}, -e^{2 i (c+d x)} \right] \right) \right) \\
& \left. (b \operatorname{Sec}[c+d x])^{2/3} (A+C \operatorname{Sec}[c+d x]^2) \right) / \left(d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+d x]^{8/3} \right)
\end{aligned}$$

- **Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^m (b \sec[c+dx])^{1/3} (A+C \sec[c+dx]^2) dx}{(b \sec[c+dx])^{1/3}}$$

Optimal (type 5, 144 leaves, 4 steps):

$$\frac{3 C \sec[c+dx]^{1+m} (b \sec[c+dx])^{1/3} \sin[c+dx]}{d(4+3m)}$$

$$\left(\frac{3(C+3Cm+A(4+3m)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2-3m), \frac{1}{6}(8-3m), \cos[c+dx]^2\right] \sec[c+dx]^{-1+m} (b \sec[c+dx])^{1/3} \sin[c+dx]}{d(2-3m)(4+3m) \sqrt{\sin[c+dx]^2}} \right) /$$

Result (type 5, 336 leaves):

$$\begin{aligned} & - \frac{1}{(A+2C+A \cos[2c+2dx]) \sec[c+dx]^{7/3}} \\ & + \frac{3 i 2^{\frac{4}{3}+m} e^{-\frac{1}{3} i d(1+3m)x} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{\frac{1}{3}+m} (1+e^{2i(c+dx)})^{\frac{1}{3}+m} \left(\frac{A e^{\frac{1}{3} i(d+3d m)x} \operatorname{Hypergeometric2F1}\left[\frac{7}{3}+m, \frac{1}{6}(1+3m), \frac{1}{6}(7+3m), -e^{2i(c+dx)}\right]}{d+3dm} \right)}{d(7+3m)(13+3m)} \\ & + \frac{1}{d(7+3m)(13+3m)} e^{\frac{1}{3} i(6c+d(7+3m)x)} \left(2(A+2C)(13+3m) \operatorname{Hypergeometric2F1}\left[\frac{7}{3}+m, \frac{1}{6}(7+3m), \frac{1}{6}(13+3m), -e^{2i(c+dx)}\right] + \right. \\ & \left. A e^{2i(c+dx)} (7+3m) \operatorname{Hypergeometric2F1}\left[\frac{7}{3}+m, \frac{1}{6}(13+3m), \frac{1}{6}(19+3m), -e^{2i(c+dx)}\right] \right) \left((b \sec[c+dx])^{1/3} (A+C \sec[c+dx]^2) \right) \end{aligned}$$

- **Problem 24: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^m (A+C \sec[c+dx]^2) dx}{(b \sec[c+dx])^{1/3}}$$

Optimal (type 5, 147 leaves, 4 steps):

$$\frac{3 C \sec[c+dx]^{1+m} \sin[c+dx]}{d(2+3m) (b \sec[c+dx])^{1/3}}$$

$$\left(\frac{3(C(1-3m)-A(2+3m)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(4-3m), \frac{1}{6}(10-3m), \cos[c+dx]^2\right] \sec[c+dx]^{-1+m} \sin[c+dx]}{d(4-3m)(2+3m) (b \sec[c+dx])^{1/3} \sqrt{\sin[c+dx]^2}} \right) /$$

Result (type 5, 311 leaves):

$$\begin{aligned}
& - \left(\left(3 \operatorname{Im} 2^{\frac{2}{3}+m} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{-\frac{1}{3}+m} (1+e^{2i(c+dx)})^{-\frac{1}{3}+m} \left(A (55+48m+9m^2) \operatorname{Hypergeometric2F1} \left[\frac{5}{3}+m, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), -e^{2i(c+dx)} \right] + \right. \right. \\
& \quad e^{2i(c+dx)} (-1+3m) \left(2(A+2C)(11+3m) \operatorname{Hypergeometric2F1} \left[\frac{5}{3}+m, \frac{1}{6}(5+3m), \frac{1}{6}(11+3m), -e^{2i(c+dx)} \right] + \right. \\
& \quad \left. \left. A e^{2i(c+dx)} (5+3m) \operatorname{Hypergeometric2F1} \left[\frac{5}{3}+m, \frac{1}{6}(11+3m), \frac{1}{6}(17+3m), -e^{2i(c+dx)} \right] \right) \right) (A+C \operatorname{Sec}[c+dx]^2) \right) / \\
& \left. \left(d(-1+3m)(5+3m)(11+3m)(A+2C+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/3} (b \operatorname{Sec}[c+dx])^{1/3} \right) \right)
\end{aligned}$$

- **Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^m (A+C \operatorname{Sec}[c+dx]^2)}{(b \operatorname{Sec}[c+dx])^{2/3}} dx$$

Optimal (type 5, 145 leaves, 4 steps):

$$\begin{aligned}
& \frac{3 C \operatorname{Sec}[c+dx]^{1+m} \operatorname{Sin}[c+dx]}{d(1+3m)(b \operatorname{Sec}[c+dx])^{2/3}} - \\
& \left(3(A-C(2-3m)+3Am) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6}(5-3m), \frac{1}{6}(11-3m), \operatorname{Cos}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^{-1+m} \operatorname{Sin}[c+dx] \right) / \\
& \left(d(5-3m)(1+3m)(b \operatorname{Sec}[c+dx])^{2/3} \sqrt{\operatorname{Sin}[c+dx]^2} \right)
\end{aligned}$$

Result (type 5, 311 leaves):

$$\begin{aligned}
& - \left(\left(3 \operatorname{Im} 2^{\frac{1}{3}+m} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{-\frac{2}{3}+m} (1+e^{2i(c+dx)})^{-\frac{2}{3}+m} \left(A e^{4i(c+dx)} (-8+6m+9m^2) \operatorname{Hypergeometric2F1} \left[\frac{5}{3}+\frac{m}{2}, \frac{4}{3}+m, \frac{8}{3}+\frac{m}{2}, -e^{2i(c+dx)} \right] + \right. \right. \\
& \quad (10+3m) \left(A(4+3m) \operatorname{Hypergeometric2F1} \left[\frac{4}{3}+m, \frac{1}{6}(-2+3m), \frac{1}{6}(4+3m), -e^{2i(c+dx)} \right] + \right. \\
& \quad \left. \left. 2(A+2C) e^{2i(c+dx)} (-2+3m) \operatorname{Hypergeometric2F1} \left[\frac{4}{3}+m, \frac{1}{6}(4+3m), \frac{5}{3}+\frac{m}{2}, -e^{2i(c+dx)} \right] \right) \right) (A+C \operatorname{Sec}[c+dx]^2) \right) / \\
& \left. \left(d(-2+3m)(4+3m)(10+3m)(A+2C+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{4/3} (b \operatorname{Sec}[c+dx])^{2/3} \right) \right)
\end{aligned}$$

- **Problem 26: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^m (A+C \operatorname{Sec}[c+dx]^2)}{(b \operatorname{Sec}[c+dx])^{4/3}} dx$$

Optimal (type 5, 148 leaves, 4 steps) :

$$\frac{3 C \operatorname{Sec}[c+d x]^m \operatorname{Sin}[c+d x]}{b d (1-3 m) (b \operatorname{Sec}[c+d x])^{1/3}} - \left(\frac{3 (A+C (4-3 m) - 3 A m) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6} (7-3 m), \frac{1}{6} (13-3 m), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sec}[c+d x]^{-2+m} \operatorname{Sin}[c+d x]}{b d (1-3 m) (7-3 m) (b \operatorname{Sec}[c+d x])^{1/3} \sqrt{\operatorname{Sin}[c+d x]^2}} \right) /$$

Result (type 5, 340 leaves) :

$$\begin{aligned} & - \left(3 i 2^{-\frac{1}{3}+m} e^{-\frac{1}{3} i (6 c+d (2+3 m) x)} \left(\frac{e^{i (c+d x)}}{1+e^{2 i (c+d x)}} \right)^{\frac{2}{3}+m} (1+e^{2 i (c+d x)})^{\frac{2}{3}+m} \left(\frac{A e^{\frac{1}{3} i d (-4+3 m) x} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}+m, \frac{1}{6} (-4+3 m), \frac{1}{6} (2+3 m), -e^{2 i (c+d x)}\right]}{-4+3 m} \right. \right. \\ & \quad \left. \left. + \frac{1}{((2+3 m) (8+3 m))} e^{\frac{1}{3} i (6 c+d (2+3 m) x)} \left(2 (A+2 C) (8+3 m) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}+m, \frac{1}{6} (2+3 m), \frac{1}{6} (8+3 m), -e^{2 i (c+d x)}\right] + \right. \right. \\ & \quad \left. \left. A e^{2 i (c+d x)} (2+3 m) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}+m, \frac{1}{6} (8+3 m), \frac{7}{3}+\frac{m}{2}, -e^{2 i (c+d x)}\right] \right) \right) \right) \\ & \left. (A+C \operatorname{Sec}[c+d x]^2) \right) / \left(d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+d x]^{2/3} (b \operatorname{Sec}[c+d x])^{4/3} \right) \end{aligned}$$

■ **Problem 27: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^m (b \operatorname{Sec}[c+d x])^n (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 5, 145 leaves, 4 steps) :

$$\frac{C \operatorname{Sec}[c+d x]^{1+m} (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x]}{d (1+m+n)} - \left(\frac{(C (m+n) + A (1+m+n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1-m-n), \frac{1}{2} (3-m-n), \operatorname{Cos}[c+d x]^2\right] \operatorname{Sec}[c+d x]^{-1+m} (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x]}{d (1-m-n) (1+m+n) \sqrt{\operatorname{Sin}[c+d x]^2}} \right) /$$

Result (type 5, 303 leaves) :

$$\begin{aligned}
& - \frac{1}{d (A + 2 C + A \cos [2 c + 2 d x])} \\
& \left(i 2^{1+m+n} e^{-i d (m+n) x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{m+n} \left(1 + e^{2 i (c+d x)} \right)^{m+n} \left(\frac{A e^{i d (m+n) x} \text{Hypergeometric2F1} \left[\frac{m+n}{2}, 2+m+n, \frac{1}{2} (2+m+n), -e^{2 i (c+d x)} \right]}{m+n} \right) + \right. \\
& \left. e^{2 i c} \left(\frac{2 (A + 2 C) e^{i d (2+m+n) x} \text{Hypergeometric2F1} \left[\frac{1}{2} (2+m+n), 2+m+n, \frac{1}{2} (4+m+n), -e^{2 i (c+d x)} \right]}{2+m+n} \right) + \right. \\
& \left. \left. \frac{A e^{i (2 c+d (4+m+n) x)} \text{Hypergeometric2F1} \left[2+m+n, \frac{1}{2} (4+m+n), \frac{1}{2} (6+m+n), -e^{2 i (c+d x)} \right]}{4+m+n} \right) \right) \\
& \text{Sec}[c+d x]^{-2-n} (b \text{Sec}[c+d x])^n (A + C \text{Sec}[c+d x]^2)
\end{aligned}$$

- **Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c+d x]^2 (b \text{Sec}[c+d x])^n (A + C \text{Sec}[c+d x]^2) dx$$

Optimal (type 5, 120 leaves, 4 steps):

$$\begin{aligned}
& \left((C (2+n) + A (3+n)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (-1-n), \frac{1-n}{2}, \cos [c+d x]^2 \right] (b \text{Sec}[c+d x])^{1+n} \sin [c+d x] \right) / \\
& \left(b d (1+n) (3+n) \sqrt{\sin [c+d x]^2} \right) + \frac{C (b \text{Sec}[c+d x])^{2+n} \tan [c+d x]}{b^2 d (3+n)}
\end{aligned}$$

Result (type 5, 289 leaves):

$$\begin{aligned}
& - \left(i 2^{3+n} e^{2 i c - i d n x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^n \left(1 + e^{2 i (c+d x)} \right)^n \right. \\
& \left(\frac{A e^{i d (2+n) x} \text{Hypergeometric2F1} \left[\frac{2+n}{2}, 4+n, \frac{4+n}{2}, -e^{2 i (c+d x)} \right]}{2+n} + 1 / ((4+n) (6+n)) e^{i (2 c+d (4+n) x)} \left(2 (A + 2 C) (6+n) \right. \right. \\
& \left. \left. \text{Hypergeometric2F1} \left[\frac{4+n}{2}, 4+n, \frac{6+n}{2}, -e^{2 i (c+d x)} \right] + A e^{2 i (c+d x)} (4+n) \text{Hypergeometric2F1} \left[4+n, \frac{6+n}{2}, \frac{8+n}{2}, -e^{2 i (c+d x)} \right] \right) \right) \\
& \left. \text{Sec}[c+d x]^{-2-n} (b \text{Sec}[c+d x])^n (A + C \text{Sec}[c+d x]^2) \right) / (d (A + 2 C + A \cos [2 c + 2 d x]))
\end{aligned}$$

- **Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c+d x] (b \text{Sec}[c+d x])^n (A + C \text{Sec}[c+d x]^2) dx$$

Optimal (type 5, 109 leaves, 4 steps):

$$\frac{(C(1+n) + A(2+n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \cos[c+dx]^2\right] (b \operatorname{Sec}[c+dx])^n \sin[c+dx]}{dn(2+n)\sqrt{\sin[c+dx]^2}} + \frac{C(b \operatorname{Sec}[c+dx])^{1+n} \tan[c+dx]}{bd(2+n)}$$

Result (type 5, 284 leaves):

$$\begin{aligned} & - \left(i 2^{2+n} e^{i(c-dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^n (1+e^{2i(c+dx)})^n \right. \\ & \left(\frac{A e^{i d(1+n)x} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, -e^{2i(c+dx)}\right]}{1+n} + \frac{2(A+2C) e^{i(2c+d(3+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, 3+n, \frac{5+n}{2}, -e^{2i(c+dx)}\right]}{3+n} \right. \\ & \left. \left. + \frac{A e^{i(4c+d(5+n)x)} \operatorname{Hypergeometric2F1}\left[3+n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2i(c+dx)}\right]}{5+n} \right) \right) \\ & \left. \operatorname{Sec}[c+dx]^{-2-n} (b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2) \right) / (d(A+2C+A \cos[2c+2dx])) \end{aligned}$$

■ **Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 113 leaves, 3 steps):

$$\frac{b(A+An+Cn) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos[c+dx]^2\right] (b \operatorname{Sec}[c+dx])^{-1+n} \sin[c+dx]}{d(1-n)(1+n)\sqrt{\sin[c+dx]^2}} + \frac{C(b \operatorname{Sec}[c+dx])^n \tan[c+dx]}{d(1+n)}$$

Result (type 5, 262 leaves):

$$\begin{aligned} & - \frac{1}{dn(2+n)(4+n)(A+2C+A \cos[2c+2dx])} \\ & i 2^{1+n} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^n (1+e^{2i(c+dx)})^n \left(A(8+6n+n^2) \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, 2+n, \frac{2+n}{2}, -e^{2i(c+dx)}\right] + \right. \\ & \left. 2(A+2C) e^{2i(c+dx)} n(4+n) \operatorname{Hypergeometric2F1}\left[\frac{2+n}{2}, 2+n, \frac{4+n}{2}, -e^{2i(c+dx)}\right] + \right. \\ & \left. A e^{4i(c+dx)} n(2+n) \operatorname{Hypergeometric2F1}\left[2+n, \frac{4+n}{2}, \frac{6+n}{2}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}[c+dx]^{-2-n} (b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2) \end{aligned}$$

■ **Problem 31: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] (b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 117 leaves, 4 steps):

$$\frac{b^2 (C (1 - n) - A n) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \cos[c + dx]^2\right] (b \sec[c + dx])^{-2+n} \sin[c + dx]}{d (2 - n) n \sqrt{\sin[c + dx]^2}} + \frac{b C (b \sec[c + dx])^{-1+n} \tan[c + dx]}{d n}$$

Result (type 6, 6049 leaves) : Display of huge result suppressed!

- **Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (b \sec[c + dx])^n (A + C \sec[c + dx]^2) dx$$

Optimal (type 5, 132 leaves, 4 steps) :

$$-\left(b^3 (A (1 - n) + C (2 - n)) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3 - n}{2}, \frac{5 - n}{2}, \cos[c + dx]^2\right] (b \sec[c + dx])^{-3+n} \sin[c + dx] \right) /$$

$$\left(d (1 - n) (3 - n) \sqrt{\sin[c + dx]^2} \right) - \frac{b^2 C (b \sec[c + dx])^{-2+n} \tan[c + dx]}{d (1 - n)}$$

Result (type 6, 8395 leaves) : Display of huge result suppressed!

- **Problem 33: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (b \sec[c + dx])^n (A + C \sec[c + dx]^2) dx$$

Optimal (type 5, 132 leaves, 4 steps) :

$$-\left(b^4 (A (2 - n) + C (3 - n)) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4 - n}{2}, \frac{6 - n}{2}, \cos[c + dx]^2\right] (b \sec[c + dx])^{-4+n} \sin[c + dx] \right) /$$

$$\left(d (2 - n) (4 - n) \sqrt{\sin[c + dx]^2} \right) - \frac{b^3 C (b \sec[c + dx])^{-3+n} \tan[c + dx]}{d (2 - n)}$$

Result (type 6, 12608 leaves) :

$$-\left(\left(6 b \sec[c + dx]^{1-n} (b \sec[c + dx])^{-1+n} \right. \right.$$

$$\left. \left(\frac{1}{2} A \cos[2(c + dx)] \sec[c + dx]^{-1+n} + \cos[c + dx] \left(\frac{1}{2} A \sec[c + dx]^n + C \sec[c + dx]^n \right) \right) \tan\left[\frac{1}{2}(c + dx)\right] \left(\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \right)^n \right.$$

$$\left. \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^{-4+n} \left(\left(A \text{AppellF1}\left[\frac{1}{2}, n, 1 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^3 \right) / \right. \right.$$

$$\left. \left(3 \text{AppellF1}\left[\frac{1}{2}, n, 1 - n, \frac{3}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] + 2 \left((-1 + n) \text{AppellF1}\left[\frac{3}{2}, n, 2 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, \right. \right. \right. \right.$$

$$\left. \left. \left. -\tan\left[\frac{1}{2}(c + dx)\right]^2 \right) + n \text{AppellF1}\left[\frac{3}{2}, 1 + n, 1 - n, \frac{5}{2}, \tan\left[\frac{1}{2}(c + dx)\right]^2, -\tan\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \tan\left[\frac{1}{2}(c + dx)\right]^2 \right) +$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n, 4-n, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + 2 \left((-4+n) \operatorname{AppellF1}\left[\frac{3}{2}, n, 5-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] + n \operatorname{AppellF1}\left[\frac{3}{2}, 1+n, 4-n, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right)^2 \right)^2 \right)^2 \right)^2$$

■ **Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{5/2} (b \operatorname{Sec}[c+dx])^n (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 142 leaves, 4 steps):

$$\frac{2 C \operatorname{Sec}[c+dx]^{7/2} (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d (7+2n)} +$$

$$\left(2 (C (5+2n) + A (7+2n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-3-2n), \frac{1}{4}(1-2n), \operatorname{Cos}[c+dx]^2\right] \operatorname{Sec}[c+dx]^{3/2} (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx] \right) / \left(d (3+2n) (7+2n) \sqrt{\operatorname{Sin}[c+dx]^2} \right)$$

Result (type 5, 341 leaves):

$$-\frac{1}{d (A + 2 C + A \operatorname{Cos}[2c + 2dx])} + \frac{i 2^{\frac{9}{2}+n} e^{2ic - \frac{1}{2}id(1+2n)x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{1}{2}+n} (1 + e^{2i(c+dx)})^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2}id(5+2n)x} \operatorname{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), -e^{2i(c+dx)}\right]}{5+2n} + \frac{1}{(9+2n)(13+2n)} e^{\frac{1}{2}i(4c+d(9+2n)x} \left(2(A+2C)(13+2n) \operatorname{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{1}{4}(9+2n), \frac{1}{4}(13+2n), -e^{2i(c+dx)}\right] + A e^{2i(c+dx)} (9+2n) \operatorname{Hypergeometric2F1}\left[\frac{9}{2}+n, \frac{1}{4}(13+2n), \frac{1}{4}(17+2n), -e^{2i(c+dx)}\right] \right)} \right) \operatorname{Sec}[c+dx]^{-2-n} (b \operatorname{Sec}[c+dx])^n (A + C \operatorname{Sec}[c+dx]^2)$$

■ **Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (b \operatorname{Sec}[c+dx])^n (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 142 leaves, 4 steps):

$$\frac{2 C \operatorname{Sec}[c+d x]^{5/2} (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x]}{d(5+2 n)} +$$

$$\left(2 (C(3+2 n) + A(5+2 n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(-1-2 n), \frac{1}{4}(3-2 n), \operatorname{Cos}[c+d x]^2\right] \sqrt{\operatorname{Sec}[c+d x]} (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x] \right) /$$

$$\left(d(1+2 n)(5+2 n) \sqrt{\operatorname{Sin}[c+d x]^2} \right)$$

Result (type 5, 335 leaves):

$$\frac{1}{d(A+2 C+A \operatorname{Cos}[2 c+2 d x])}$$

$$i 2^{7/2+n} e^{-\frac{1}{2} i d(3+2 n) x} \left(\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}} \right)^{\frac{3}{2}+n} \left(1+e^{2 i(c+d x)} \right)^{\frac{3}{2}+n} \left(\frac{A e^{\frac{1}{2} i d(3+2 n) x} \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+n, \frac{1}{4}(3+2 n), \frac{1}{4}(7+2 n), -e^{2 i(c+d x)}\right]}{3+2 n} + \right.$$

$$\left. \frac{1}{(7+2 n)(11+2 n)} e^{\frac{1}{2} i(4 c+d(7+2 n) x)} \left(2(A+2 C)(11+2 n) \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+n, \frac{1}{4}(7+2 n), \frac{1}{4}(11+2 n), -e^{2 i(c+d x)}\right] + A e^{2 i(c+d x)} \right. \right.$$

$$\left. \left. (7+2 n) \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+n, \frac{1}{4}(11+2 n), \frac{1}{4}(15+2 n), -e^{2 i(c+d x)}\right] \right) \right) \operatorname{Sec}[c+d x]^{-2-n} (b \operatorname{Sec}[c+d x])^n (A+C \operatorname{Sec}[c+d x]^2)$$

■ **Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+d x]} (b \operatorname{Sec}[c+d x])^n (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 5, 140 leaves, 4 steps):

$$\frac{2 C \operatorname{Sec}[c+d x]^{3/2} (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x]}{d(3+2 n)} -$$

$$\left(2 (C+2 C n+A(3+2 n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1-2 n), \frac{1}{4}(5-2 n), \operatorname{Cos}[c+d x]^2\right] (b \operatorname{Sec}[c+d x])^n \operatorname{Sin}[c+d x] \right) /$$

$$\left(d(1-2 n)(3+2 n) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{\operatorname{Sin}[c+d x]^2} \right)$$

Result (type 5, 336 leaves):

$$\begin{aligned}
& - \frac{1}{A + 2C + A \cos[2c + 2dx]} \\
& i 2^{\frac{5}{2}+n} e^{-\frac{1}{2} i d (1+2n) x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{1}{2}+n} (1 + e^{2i(c+dx)})^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2} i (d+2dn) x} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), -e^{2i(c+dx)}\right]}{d + 2dn} \right) + \\
& \frac{1}{d(5+2n)(9+2n)} e^{\frac{1}{2} i (4c+d(5+2n)x)} \left(2(A+2C)(9+2n) \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), -e^{2i(c+dx)}\right] + A e^{2i(c+dx)} \right. \\
& \left. (5+2n) \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4}(9+2n), \frac{1}{4}(13+2n), -e^{2i(c+dx)}\right] \right) \Bigg) \sec[c+dx]^{-2-n} (b \sec[c+dx])^n (A + C \sec[c+dx])^2
\end{aligned}$$

- **Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \sec[c+dx])^n (A + C \sec[c+dx])^2}{\sqrt{\sec[c+dx]}} dx$$

Optimal (type 5, 141 leaves, 4 steps):

$$\begin{aligned}
& \frac{2C \sqrt{\sec[c+dx]} (b \sec[c+dx])^n \sin[c+dx]}{d(1+2n)} - \\
& \left(2(A-C(1-2n) + 2An) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3-2n), \frac{1}{4}(7-2n), \cos[c+dx]^2\right] (b \sec[c+dx])^n \sin[c+dx] \right) / \\
& \left(d(3-2n)(1+2n) \sec[c+dx]^{3/2} \sqrt{\sin[c+dx]^2} \right)
\end{aligned}$$

Result (type 5, 311 leaves):

$$\begin{aligned}
& - \frac{1}{d(-1+2n)(3+2n)(7+2n)(A+2C+A \cos[2c+2dx])} \\
& i 2^{\frac{3}{2}+n} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{-\frac{1}{2}+n} (1 + e^{2i(c+dx)})^{-\frac{1}{2}+n} \left(A(21+20n+4n^2) \text{Hypergeometric2F1}\left[\frac{3}{2} + n, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), -e^{2i(c+dx)}\right] + \right. \\
& e^{2i(c+dx)}(-1+2n) \left(2(A+2C)(7+2n) \text{Hypergeometric2F1}\left[\frac{3}{2} + n, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), -e^{2i(c+dx)}\right] + A e^{2i(c+dx)}(3+2n) \right. \\
& \left. \left. \text{Hypergeometric2F1}\left[\frac{3}{2} + n, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), -e^{2i(c+dx)}\right] \right) \right) \Bigg) \sec[c+dx]^{-2-n} (b \sec[c+dx])^n (A + C \sec[c+dx])^2
\end{aligned}$$

- **Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \sec[c+dx])^n (A + C \sec[c+dx])^2}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 5, 140 leaves, 4 steps):

$$-\frac{2C(b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d(1-2n)\sqrt{\operatorname{Sec}[c+dx]}} - \left(\frac{2(A+C(3-2n)-2An) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(5-2n), \frac{1}{4}(9-2n), \operatorname{Cos}[c+dx]^2\right] (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d(1-2n)(5-2n) \operatorname{Sec}[c+dx]^{5/2} \sqrt{\operatorname{Sin}[c+dx]^2}} \right)$$

Result (type 5, 343 leaves):

$$-\frac{1}{A+2C+A \operatorname{Cos}[2c+2dx]} + \frac{i 2^{\frac{1}{2}+n} e^{-\frac{1}{2}i(4c+d(1+2n)x}}{\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}} \left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2}i d(-3+2n)x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), -e^{2i(c+dx)}\right]}{d(-3+2n)}\right) + \frac{1}{d(1+2n)(5+2n)} e^{\frac{1}{2}i(4c+d(1+2n)x}} \left(2(A+2C)(5+2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), -e^{2i(c+dx)}\right] + A e^{2i(c+dx)} (1+2n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}+n, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), -e^{2i(c+dx)}\right]\right) \operatorname{Sec}[c+dx]^{-2-n} (b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2)$$

■ **Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \operatorname{Sec}[c+dx])^n (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{5/2}} dx$$

Optimal (type 5, 142 leaves, 4 steps):

$$-\frac{2C(b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d(3-2n) \operatorname{Sec}[c+dx]^{3/2}} - \left(\frac{2(A(3-2n)+C(5-2n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(7-2n), \frac{1}{4}(11-2n), \operatorname{Cos}[c+dx]^2\right] (b \operatorname{Sec}[c+dx])^n \operatorname{Sin}[c+dx]}{d(3-2n)(7-2n) \operatorname{Sec}[c+dx]^{7/2} \sqrt{\operatorname{Sin}[c+dx]^2}} \right)$$

Result (type 5, 338 leaves):

$$\begin{aligned}
& - \frac{1}{d (A + 2 C + A \cos [2 c + 2 d x])} i 2^{-\frac{1}{2}+n} e^{-\frac{1}{2} i (4 c+d (-1+2 n) x)} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{-\frac{1}{2}+n} \\
& (1 + e^{2 i (c+d x)})^{-\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2} i d (-5+2 n) x} \text{Hypergeometric2F1} \left[-\frac{1}{2} + n, \frac{1}{4} (-5 + 2 n), \frac{1}{4} (-1 + 2 n), -e^{2 i (c+d x)} \right]}{-5 + 2 n} + \frac{1}{-3 + 4 n + 4 n^2} \right. \\
& \left. e^{\frac{1}{2} i (4 c+d (-1+2 n) x)} \left(2 (A + 2 C) (3 + 2 n) \text{Hypergeometric2F1} \left[-\frac{1}{2} + n, \frac{1}{4} (-1 + 2 n), \frac{1}{4} (3 + 2 n), -e^{2 i (c+d x)} \right] + A e^{2 i (c+d x)} (-1 + 2 n) \right. \right. \\
& \left. \left. \text{Hypergeometric2F1} \left[-\frac{1}{2} + n, \frac{1}{4} (3 + 2 n), \frac{1}{4} (7 + 2 n), -e^{2 i (c+d x)} \right] \right) \right) \text{Sec}[c + d x]^{-2-n} (b \text{Sec}[c + d x])^n (A + C \text{Sec}[c + d x]^2)
\end{aligned}$$

■ **Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^2 (b \text{Sec}[c + d x])^{2/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 (11 A + 8 C) \text{Hypergeometric2F1} \left[-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos [c + d x]^2 \right] (b \text{Sec}[c + d x])^{5/3} \sin [c + d x]}{55 b d \sqrt{\sin [c + d x]^2}} + \\
& \frac{3 B \text{Hypergeometric2F1} \left[-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos [c + d x]^2 \right] (b \text{Sec}[c + d x])^{8/3} \sin [c + d x]}{8 b^2 d \sqrt{\sin [c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{8/3} \tan [c + d x]}{11 b^2 d}
\end{aligned}$$

Result (type 5, 461 leaves):

$$\begin{aligned}
& - \left(\left(3 i e^{-i (c+d x)} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{2/3} \left(275 B (1 + e^{2 i (c+d x)}) + 275 B (-1 + e^{2 i c}) (1 + e^{2 i (c+d x)})^{2/3} \text{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2 i (c+d x)} \right] + \right. \right. \right. \\
& \left. \left. 16 (11 A + 8 C) e^{i (c+d x)} (-1 + e^{2 i c}) (1 + e^{2 i (c+d x)})^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2 i (c+d x)} \right] \right) (b \text{Sec}[c + d x])^{2/3} \right. \\
& \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / \left(220 \times 2^{1/3} d (-1 + e^{2 i c}) (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \text{Sec}[c + d x]^{8/3} \right) + \\
& \left(\cos [c + d x]^2 (b \text{Sec}[c + d x])^{2/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right. \\
& \left. \left(\frac{15 B \cos [d x] \text{Csc}[c]}{8 d} + \frac{6 C \text{Sec}[c] \text{Sec}[c + d x]^3 \sin [d x]}{11 d} + \frac{3 \text{Sec}[c] \text{Sec}[c + d x]^2 (8 C \sin [c] + 11 B \sin [d x])}{44 d} + \right. \right. \\
& \left. \left. \frac{3 \text{Sec}[c] \text{Sec}[c + d x] (55 B \sin [c] + 88 A \sin [d x] + 64 C \sin [d x])}{220 d} + \frac{6 (11 A + 8 C) \tan [c]}{55 d} \right) \right) / (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])
\end{aligned}$$

- **Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (b \text{Sec}[c + d x])^{2/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 151 leaves, 7 steps):

$$\frac{3 (8 A + 5 C) \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{2/3} \text{Sin}[c + d x]}{16 d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 B \text{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{5/3} \text{Sin}[c + d x]}{5 b d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{5/3} \text{Tan}[c + d x]}{8 b d}$$

Result (type 5, 689 leaves):

$$\begin{aligned} & \frac{1}{b} \left(- \left(3 A e^{-i (2 c + d x)} \left(\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}} \right)^{2/3} \text{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) (1 + e^{2 i (c + d x)})^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2 i (c + d x)}\right] \right) \right. \right. \\ & \quad \left. \left. (b \text{Sec}[c + d x])^{5/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \right) / \left(2^{1/3} d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{11/3} \right) - \\ & \left(15 C e^{-i (2 c + d x)} \left(\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}} \right)^{2/3} \text{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) (1 + e^{2 i (c + d x)})^{2/3} \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2 i (c + d x)}\right] \right) \right. \\ & \quad \left. (b \text{Sec}[c + d x])^{5/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / \left(8 \times 2^{1/3} d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{11/3} \right) - \\ & \left(6 i 2^{2/3} B \left(\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}} \right)^{2/3} (1 + e^{2 i (c + d x)})^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2 i (c + d x)}\right] (b \text{Sec}[c + d x])^{5/3} \right. \\ & \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / \left(5 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{11/3} \right) + \\ & \left(\text{Cos}[c + d x]^3 (b \text{Sec}[c + d x])^{5/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \left(\frac{3 (8 A + 5 C) \text{Cos}[d x] \text{Csc}[c]}{8 d} + \frac{3 C \text{Sec}[c] \text{Sec}[c + d x]^2 \text{Sin}[d x]}{4 d} \right. \right. \\ & \quad \left. \left. + \frac{3 \text{Sec}[c] \text{Sec}[c + d x] (5 C \text{Sin}[c] + 8 B \text{Sin}[d x])}{20 d} + \frac{6 B \text{Tan}[c]}{5 d} \right) \right) / \left((A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \right) \end{aligned}$$

- **Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \text{Sec}[c + d x])^{2/3} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 5, 146 leaves, 6 steps):

$$\frac{3 b (5 A + 2 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos[c + d x]^2\right] \sin[c + d x]}{5 d (b \sec[c + d x])^{1/3} \sqrt{\sin[c + d x]^2}} + \frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos[c + d x]^2\right] (b \sec[c + d x])^{2/3} \sin[c + d x]}{2 d \sqrt{\sin[c + d x]^2}} + \frac{3 C (b \sec[c + d x])^{2/3} \tan[c + d x]}{5 d}$$

Result (type 5, 311 leaves):

$$\left((b \sec[c + d x])^{2/3} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right. \\ \left. - \left(3 i 2^{2/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{2/3} \left(5 B (1 + e^{2i(c+dx)}) + 5 B (-1 + e^{2i c}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)}\right] + \right. \right. \right. \\ \left. \left. (5 A + 2 C) e^{i(c+dx)} (-1 + e^{2i c}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)}\right] \right) \right) / (d (-1 + e^{2i c}) \sec[c + d x]^{8/3}) + \\ \left. \frac{3 \cos[c + d x] (5 B \cos[d x] \cos[c + d x] \csc[c] + 2 C \sin[c + d x])}{d} \right) / (5 (A + 2 C + 2 B \cos[c + d x] + A \cos[2(c + d x)]))$$

- **Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + d x]^2 (b \sec[c + d x])^{4/3} (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\frac{3 (13 A + 10 C) \operatorname{Hypergeometric2F1}\left[-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos[c + d x]^2\right] (b \sec[c + d x])^{7/3} \sin[c + d x]}{91 b d \sqrt{\sin[c + d x]^2}} + \frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \cos[c + d x]^2\right] (b \sec[c + d x])^{10/3} \sin[c + d x]}{10 b^2 d \sqrt{\sin[c + d x]^2}} + \frac{3 C (b \sec[c + d x])^{10/3} \tan[c + d x]}{13 b^2 d}$$

Result (type 5, 759 leaves):

$$\begin{aligned}
& - \left(12 \times 2^{1/3} A e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (7d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3}) - \\
& \left(120 \times 2^{1/3} C e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] \right) \right) \\
& \quad \left. (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (91d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3}) - \\
& \left(21 i B \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)} \right] (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (10 \times 2^{2/3} d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3}) + \\
& \left(\operatorname{Cos}[c+dx]^3 (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(\frac{24(13A+10C) \operatorname{Cos}[dx] \operatorname{Csc}[c]}{91d} + \frac{6C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 \operatorname{Sin}[dx]}{13d} + \right. \right. \\
& \quad \left. \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (10C \operatorname{Sin}[c] + 13B \operatorname{Sin}[dx])}{65d} + \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (520A \operatorname{Sin}[c] + 400C \operatorname{Sin}[c] + 637B \operatorname{Sin}[dx])}{1820d} + \right. \\
& \quad \left. \left. \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (91B \operatorname{Sin}[c] + 130A \operatorname{Sin}[dx] + 100C \operatorname{Sin}[dx])}{455d} + \frac{21B \operatorname{Tan}[c]}{20d} \right) \right) / (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx])
\end{aligned}$$

- **Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx] (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 151 leaves, 7 steps):

$$\begin{aligned}
& \frac{3(10A+7C) \operatorname{Hypergeometric2F1} \left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \operatorname{Cos}[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{4/3} \operatorname{Sin}[c+dx]}{40d \sqrt{\operatorname{Sin}[c+dx]^2}} + \\
& \frac{3B \operatorname{Hypergeometric2F1} \left[-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \operatorname{Cos}[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{7/3} \operatorname{Sin}[c+dx]}{7bd \sqrt{\operatorname{Sin}[c+dx]^2}} + \frac{3C (b \operatorname{Sec}[c+dx])^{7/3} \operatorname{Tan}[c+dx]}{10bd}
\end{aligned}$$

Result (type 5, 465 leaves):

$$\frac{1}{b} \left(- \left(\left(3 i e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \left(160 B (1+e^{2i(c+dx)}) + 160 B (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \text{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] + \right. \right. \right. \\ \left. \left. \left. 7 (10 A + 7 C) e^{i(c+dx)} (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \text{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)} \right] \right) (b \text{Sec}[c+dx])^{7/3} \right. \right. \\ \left. \left. (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) \right) \right) / \left(70 \times 2^{2/3} d (-1+e^{2ic}) (A + 2 C + 2 B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx]) \text{Sec}[c+dx]^{13/3} \right) + \\ \left(\text{Cos}[c+dx]^4 (b \text{Sec}[c+dx])^{7/3} (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) \left(\frac{24 B \text{Cos}[dx] \text{Csc}[c]}{7 d} + \frac{3 C \text{Sec}[c] \text{Sec}[c+dx]^3 \text{Sin}[dx]}{5 d} + \right. \right. \\ \left. \left. \frac{3 \text{Sec}[c] \text{Sec}[c+dx]^2 (7 C \text{Sin}[c] + 10 B \text{Sin}[dx])}{35 d} + \frac{3 \text{Sec}[c] \text{Sec}[c+dx] (40 B \text{Sin}[c] + 70 A \text{Sin}[dx] + 49 C \text{Sin}[dx])}{140 d} + \right. \right. \\ \left. \left. \frac{3 (10 A + 7 C) \text{Tan}[c]}{20 d} \right) \right) / (A + 2 C + 2 B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx]) \right)$$

- **Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \text{Sec}[c+dx])^{4/3} (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 5, 146 leaves, 6 steps):

$$\frac{3 b (7 A + 4 C) \text{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \text{Cos}[c+dx]^2 \right] (b \text{Sec}[c+dx])^{1/3} \text{Sin}[c+dx]}{7 d \sqrt{\text{Sin}[c+dx]^2}} + \\ \frac{3 B \text{Hypergeometric2F1} \left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \text{Cos}[c+dx]^2 \right] (b \text{Sec}[c+dx])^{4/3} \text{Sin}[c+dx]}{4 d \sqrt{\text{Sin}[c+dx]^2}} + \frac{3 C (b \text{Sec}[c+dx])^{4/3} \text{Tan}[c+dx]}{7 d}$$

Result (type 5, 683 leaves):

$$\begin{aligned}
& - \left(3 \times 2^{1/3} A e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3} \right) - \\
& \left(12 \times 2^{1/3} C e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(7d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3} \right) - \\
& \left(3iB \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)} \right] (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(2^{2/3} d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{10/3} \right) + \\
& \left(\operatorname{Cos}[c+dx]^3 (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(\frac{6(7A+4C) \operatorname{Cos}[dx] \operatorname{Csc}[c]}{7d} + \frac{6C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \operatorname{Sin}[dx]}{7d} + \right. \right. \\
& \quad \left. \left. \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (4C \operatorname{Sin}[c] + 7B \operatorname{Sin}[dx])}{14d} + \frac{3B \operatorname{Tan}[c]}{2d} \right) \right) / (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx])
\end{aligned}$$

■ **Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] (b \operatorname{Sec}[c+dx])^{4/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 146 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3b^2(4A+C) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \operatorname{Cos}[c+dx]^2 \right] \operatorname{Sin}[c+dx]}{8d (b \operatorname{Sec}[c+dx])^{2/3} \sqrt{\operatorname{Sin}[c+dx]^2}} + \\
& \frac{3bB \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \operatorname{Cos}[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Sin}[c+dx]^2}} + \frac{3bC (b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{4d}
\end{aligned}$$

Result (type 5, 303 leaves):

$$\begin{aligned}
& \left(3b (b \operatorname{Sec}[c+dx])^{1/3} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left(-1 / (-1+e^{2ic}) i 2^{1/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{1/3} \left(4B (1+e^{2i(c+dx)}) + 4B (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \right. \right. \\
& \quad \left. \left. \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)} \right] + (4A+C) e^{i(c+dx)} (-1+e^{2ic}) (1+e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)} \right] \right) + \\
& \quad \left. \left. \operatorname{Sec}[c+dx]^{1/3} (4B \operatorname{Cos}[dx] \operatorname{Csc}[c] + C \operatorname{Tan}[c+dx]) \right) \right) / \left(2d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2(c+dx)]) \operatorname{Sec}[c+dx]^{7/3} \right)
\end{aligned}$$

■ **Problem 53: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(b \text{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\frac{3 (7 A + 4 C) \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{1/3} \text{Sin}[c + d x]}{7 b d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 B \text{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{4/3} \text{Sin}[c + d x]}{4 b^2 d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{4/3} \text{Tan}[c + d x]}{7 b^2 d}$$

Result (type 5, 299 leaves):

$$\begin{aligned} & - \left(\left(3 b e^{-i (2 c + d x)} (-1 + e^{2 i c}) \text{Csc}[c] \left(-28 A - 16 C - 7 B e^{i (c + d x)} - 56 A e^{2 i (c + d x)} - 40 C e^{2 i (c + d x)} - \right. \right. \right. \\ & \quad 28 A e^{4 i (c + d x)} - 8 C e^{4 i (c + d x)} + 7 B e^{5 i (c + d x)} + 4 (7 A + 4 C) (1 + e^{2 i (c + d x)})^{7/3} \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2 i (c + d x)}\right] + \\ & \quad \left. \left. 7 B e^{i (c + d x)} (1 + e^{2 i (c + d x)})^{7/3} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2 i (c + d x)}\right] \right) (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / \\ & \left. (28 d (1 + e^{2 i (c + d x)})^2 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 (c + d x)]) (b \text{Sec}[c + d x])^{5/3} \right) \end{aligned}$$

■ **Problem 54: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(b \text{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 147 leaves, 7 steps):

$$\frac{3 (4 A + C) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \text{Cos}[c + d x]^2\right] \text{Sin}[c + d x]}{8 d (b \text{Sec}[c + d x])^{2/3} \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 B \text{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \text{Cos}[c + d x]^2\right] (b \text{Sec}[c + d x])^{1/3} \text{Sin}[c + d x]}{b d \sqrt{\text{Sin}[c + d x]^2}} + \frac{3 C (b \text{Sec}[c + d x])^{1/3} \text{Tan}[c + d x]}{4 b d}$$

Result (type 5, 305 leaves):

$$\left(3 (b \operatorname{Sec}[c + d x])^{1/3} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-1 / (-1 + e^{2 i c}) \right) i 2^{1/3} e^{-i (c+d x)} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{1/3} \left(4 B (1 + e^{2 i (c+d x)}) + 4 B (-1 + e^{2 i c}) (1 + e^{2 i (c+d x)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \right. \right. \right. \\ \left. \left. \left. \frac{1}{3}, \frac{2}{3}, -e^{2 i (c+d x)} \right] + (4 A + C) e^{i (c+d x)} (-1 + e^{2 i c}) (1 + e^{2 i (c+d x)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2 i (c+d x)} \right] \right) \right) + \\ \left. \operatorname{Sec}[c + d x]^{1/3} (4 B \operatorname{Cos}[d x] \operatorname{Csc}[c] + C \operatorname{Tan}[c + d x]) \right) \Big/ (2 b d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 (c + d x)]) \operatorname{Sec}[c + d x]^{7/3})$$

■ **Problem 55: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(b \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{3 b (A - 2 C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \operatorname{Cos}[c + d x]^2 \right] \operatorname{Sin}[c + d x]}{5 d (b \operatorname{Sec}[c + d x])^{5/3} \sqrt{\operatorname{Sin}[c + d x]^2}} - \\ \frac{3 B \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \operatorname{Cos}[c + d x]^2 \right] \operatorname{Sin}[c + d x]}{2 d (b \operatorname{Sec}[c + d x])^{2/3} \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{3 C \operatorname{Tan}[c + d x]}{d (b \operatorname{Sec}[c + d x])^{2/3}}$$

Result (type 5, 154 leaves):

$$-\frac{1}{4 b d} 3 i e^{-i (c+d x)} \left(A - 4 C + A e^{2 i (c+d x)} - 2 (A - 2 C) (1 + e^{2 i (c+d x)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2 i (c+d x)} \right] + \right. \\ \left. 4 B e^{i (c+d x)} (1 + e^{2 i (c+d x)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2 i (c+d x)} \right] \right) (b \operatorname{Sec}[c + d x])^{1/3}$$

■ **Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(b \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 147 leaves, 7 steps):

$$\frac{3 (4 A + C) \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \operatorname{Cos}[c + d x]^2 \right] \operatorname{Sin}[c + d x]}{8 d (b \operatorname{Sec}[c + d x])^{2/3} \sqrt{\operatorname{Sin}[c + d x]^2}} + \\ \frac{3 B \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \operatorname{Cos}[c + d x]^2 \right] (b \operatorname{Sec}[c + d x])^{1/3} \operatorname{Sin}[c + d x]}{b d \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 b d}$$

Result (type 5, 305 leaves):

$$\left(3 (b \operatorname{Sec}[c + d x])^{1/3} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-1 / (-1 + e^{2 i c}) i 2^{1/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1 + e^{2 i(c+dx)}} \right)^{1/3} \left(4 B (1 + e^{2 i(c+dx)}) + 4 B (-1 + e^{2 i c}) (1 + e^{2 i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \right. \right. \right. \right. \\ \left. \left. \left. \frac{1}{3}, \frac{2}{3}, -e^{2 i(c+dx)} \right] + (4 A + C) e^{i(c+dx)} (-1 + e^{2 i c}) (1 + e^{2 i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2 i(c+dx)} \right] \right) \right. \\ \left. \left. \operatorname{Sec}[c + d x]^{1/3} (4 B \operatorname{Cos}[d x] \operatorname{Csc}[c] + C \operatorname{Tan}[c + d x]) \right) \right) / (2 b d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2(c + d x)]) \operatorname{Sec}[c + d x]^{7/3})$$

■ **Problem 57: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(b \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\frac{3 (7 A + 4 C) \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \operatorname{Cos}[c + d x]^2 \right] (b \operatorname{Sec}[c + d x])^{1/3} \operatorname{Sin}[c + d x]}{7 b d \sqrt{\operatorname{Sin}[c + d x]^2}} + \\ \frac{3 B \operatorname{Hypergeometric2F1} \left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \operatorname{Cos}[c + d x]^2 \right] (b \operatorname{Sec}[c + d x])^{4/3} \operatorname{Sin}[c + d x]}{4 b^2 d \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{4/3} \operatorname{Tan}[c + d x]}{7 b^2 d}$$

Result (type 5, 299 leaves):

$$- \left(\left(3 b e^{-i(2c+dx)} (-1 + e^{2 i c}) \operatorname{Csc}[c] \left(-28 A - 16 C - 7 B e^{i(c+dx)} - 56 A e^{2 i(c+dx)} - 40 C e^{2 i(c+dx)} - \right. \right. \right. \\ \left. \left. \left. 28 A e^{4 i(c+dx)} - 8 C e^{4 i(c+dx)} + 7 B e^{5 i(c+dx)} + 4 (7 A + 4 C) (1 + e^{2 i(c+dx)})^{7/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2 i(c+dx)} \right] + \right. \right. \\ \left. \left. \left. 7 B e^{i(c+dx)} (1 + e^{2 i(c+dx)})^{7/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2 i(c+dx)} \right] \right) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \right. \\ \left. \left. (28 d (1 + e^{2 i(c+dx)})^2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2(c + d x)]) (b \operatorname{Sec}[c + d x])^{5/3} \right) \right)$$

■ **Problem 58: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(b \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\frac{3 (10 A + 7 C) \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos[c + dx]^2\right] (b \operatorname{Sec}[c + dx])^{4/3} \sin[c + dx]}{40 b^2 d \sqrt{\sin[c + dx]^2}} + \frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos[c + dx]^2\right] (b \operatorname{Sec}[c + dx])^{7/3} \sin[c + dx]}{7 b^3 d \sqrt{\sin[c + dx]^2}} + \frac{3 C (b \operatorname{Sec}[c + dx])^{7/3} \tan[c + dx]}{10 b^3 d}$$

Result (type 5, 333 leaves):

$$\left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(- \left(3 i 2^{1/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{1/3} \left(160 B (1 + e^{2i(c+dx)}) + 160 B (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, -e^{2i(c+dx)}\right] + 7 (10 A + 7 C) e^{i(c+dx)} (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{3}, \frac{7}{6}, -e^{2i(c+dx)}\right] \right) \right) / (d (-1 + e^{2ic}) \operatorname{Sec}[c + dx]^{4/3}) + 1 / d^3 (160 B \cos[dx] \cos[c + dx] \operatorname{Csc}[c] + 7 (10 A + 7 C) \sin[c + dx] + 4 (10 B + 7 C \operatorname{Sec}[c + dx]) \tan[c + dx]) \right) / (140 (A + 2 C + 2 B \cos[c + dx] + A \cos[2(c + dx)]) (b \operatorname{Sec}[c + dx])^{2/3})$$

■ **Problem 59: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(b \operatorname{Sec}[c + dx])^{4/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$- \frac{3 (5 A + 2 C) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos[c + dx]^2\right] \sin[c + dx]}{5 b d (b \operatorname{Sec}[c + dx])^{1/3} \sqrt{\sin[c + dx]^2}} + \frac{3 B \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos[c + dx]^2\right] (b \operatorname{Sec}[c + dx])^{2/3} \sin[c + dx]}{2 b^2 d \sqrt{\sin[c + dx]^2}} + \frac{3 C (b \operatorname{Sec}[c + dx])^{2/3} \tan[c + dx]}{5 b^2 d}$$

Result (type 5, 299 leaves):

$$\left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(- \left(3 i 2^{2/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{2/3} \left(5 B (1 + e^{2i(c+dx)}) + 5 B (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] + (5 A + 2 C) e^{i(c+dx)} (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) \right) / (d (-1 + e^{2ic}) \operatorname{Sec}[c + dx]^{2/3}) + \frac{3 (5 B \operatorname{Cos}[dx] \operatorname{Csc}[c] + 2 C \operatorname{Tan}[c + dx])}{d} \right) / (5 (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2(c + dx)]) (b \operatorname{Sec}[c + dx])^{4/3})$$

■ **Problem 60: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(b \operatorname{Sec}[c + dx])^{4/3}} dx$$

Optimal (type 5, 149 leaves, 7 steps):

$$\frac{3 (2 A - C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \operatorname{Cos}[c + dx]^2 \right] \operatorname{Sin}[c + dx]}{8 d (b \operatorname{Sec}[c + dx])^{4/3} \sqrt{\operatorname{Sin}[c + dx]^2}} - \frac{3 B \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \operatorname{Cos}[c + dx]^2 \right] \operatorname{Sin}[c + dx]}{b d (b \operatorname{Sec}[c + dx])^{1/3} \sqrt{\operatorname{Sin}[c + dx]^2}} + \frac{3 C \operatorname{Tan}[c + dx]}{2 b d (b \operatorname{Sec}[c + dx])^{1/3}}$$

Result (type 5, 174 leaves):

$$\frac{1}{2 b^2 d} 3 e^{-i(3c+2dx)} \left(A - C + A e^{2i(c+dx)} + (-2A + C) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] + B e^{i(c+dx)} (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) (b \operatorname{Sec}[c + dx])^{2/3} (-i \operatorname{Cos}[2c + dx] + \operatorname{Sin}[2c + dx])$$

■ **Problem 61: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{(b \operatorname{Sec}[c + dx])^{4/3}} dx$$

Optimal (type 5, 146 leaves, 6 steps):

$$\frac{3 B \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \operatorname{Cos}[c + dx]^2 \right] \operatorname{Sin}[c + dx]}{4 d (b \operatorname{Sec}[c + dx])^{4/3} \sqrt{\operatorname{Sin}[c + dx]^2}} - \frac{3 (A + 4 C) \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \operatorname{Cos}[c + dx]^2 \right] \operatorname{Sin}[c + dx]}{4 b d (b \operatorname{Sec}[c + dx])^{1/3} \sqrt{\operatorname{Sin}[c + dx]^2}} + \frac{3 A \operatorname{Tan}[c + dx]}{4 d (b \operatorname{Sec}[c + dx])^{4/3}}$$

Result (type 5, 143 leaves):

$$\left(3 \left(8 i B \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] - i (A + 4 C) e^{i(c+dx)} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] + (1 + e^{2i(c+dx)})^{1/3} (-4 i B + A \sin[c + dx]) \right) \right) / \left(4 b d (1 + e^{2i(c+dx)})^{1/3} (b \sec[c + dx])^{1/3} \right)$$

■ **Problem 62: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(b \sec[c + dx])^{4/3}} dx$$

Optimal (type 5, 149 leaves, 7 steps):

$$\frac{3(2A - C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos[c + dx]^2 \right] \sin[c + dx]}{8 d (b \sec[c + dx])^{4/3} \sqrt{\sin[c + dx]^2}} - \frac{3 B \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos[c + dx]^2 \right] \sin[c + dx]}{b d (b \sec[c + dx])^{1/3} \sqrt{\sin[c + dx]^2}} + \frac{3 C \tan[c + dx]}{2 b d (b \sec[c + dx])^{1/3}}$$

Result (type 5, 174 leaves):

$$\frac{1}{2 b^2 d} 3 e^{-i(3c+2dx)} \left(A - C + A e^{2i(c+dx)} + (-2A + C) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] + B e^{i(c+dx)} (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) (b \sec[c + dx])^{2/3} (-i \cos[2c + dx] + \sin[2c + dx])$$

■ **Problem 63: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(b \sec[c + dx])^{4/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps):

$$\frac{3(5A + 2C) \operatorname{Hypergeometric2F1} \left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos[c + dx]^2 \right] \sin[c + dx]}{5 b d (b \sec[c + dx])^{1/3} \sqrt{\sin[c + dx]^2}} + \frac{3 B \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos[c + dx]^2 \right] (b \sec[c + dx])^{2/3} \sin[c + dx]}{2 b^2 d \sqrt{\sin[c + dx]^2}} + \frac{3 C (b \sec[c + dx])^{2/3} \tan[c + dx]}{5 b^2 d}$$

Result (type 5, 299 leaves):

$$\left((A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(- \left(3 i 2^{2/3} e^{-i(c+dx)} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{2/3} \left(5 B (1 + e^{2i(c+dx)}) + 5 B (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] + \right. \right. \right. \\ \left. \left. \left. (5 A + 2 C) e^{i(c+dx)} (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] \right) \right) \right) / (d (-1 + e^{2ic}) \operatorname{Sec}[c + d x]^{2/3}) + \\ \left. \frac{3 (5 B \operatorname{Cos}[d x] \operatorname{Csc}[c] + 2 C \operatorname{Tan}[c + d x])}{d} \right) / (5 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2(c + d x)]) (b \operatorname{Sec}[c + d x])^{4/3})$$

- **Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(b \operatorname{Sec}[c + d x])^{4/3}} dx$$

Optimal (type 5, 154 leaves, 7 steps) :

$$\frac{3 (8 A + 5 C) \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \operatorname{Cos}[c + d x]^2 \right] (b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Sin}[c + d x]}{16 b^2 d \sqrt{\operatorname{Sin}[c + d x]^2}} + \\ \frac{3 B \operatorname{Hypergeometric2F1} \left[-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \operatorname{Cos}[c + d x]^2 \right] (b \operatorname{Sec}[c + d x])^{5/3} \operatorname{Sin}[c + d x]}{5 b^3 d \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{3 C (b \operatorname{Sec}[c + d x])^{5/3} \operatorname{Tan}[c + d x]}{8 b^3 d}$$

Result (type 5, 677 leaves) :

$$\begin{aligned}
& - \left(3 A e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(2^{1/3} d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{2/3} (b \operatorname{Sec}[c+dx])^{4/3} - \right. \\
& \quad \left. \left(15 C e^{-i(2c+dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[-\frac{1}{6}, \frac{2}{3}, \frac{5}{6}, -e^{2i(c+dx)} \right] \right) \right) \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(8 \times 2^{1/3} d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{2/3} (b \operatorname{Sec}[c+dx])^{4/3} - \right. \\
& \quad \left. \left(6 i 2^{2/3} B \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{2/3} (1 + e^{2i(c+dx)})^{2/3} \operatorname{Hypergeometric2F1} \left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -e^{2i(c+dx)} \right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \right) / \\
& \quad \left(5 d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{2/3} (b \operatorname{Sec}[c+dx])^{4/3} + \left((A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \right. \\
& \quad \left. \left. \left(\frac{3(8A+5C) \operatorname{Cos}[dx] \operatorname{Csc}[c]}{8d} + \frac{3C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \operatorname{Sin}[dx]}{4d} + \frac{3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5C \operatorname{Sin}[c] + 8B \operatorname{Sin}[dx])}{20d} + \frac{6B \operatorname{Tan}[c]}{5d} \right) \right) \right) / \\
& \quad \left((A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (b \operatorname{Sec}[c+dx])^{4/3} \right)
\end{aligned}$$

■ **Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^m (b \operatorname{Sec}[c+dx])^{4/3} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 b C \operatorname{Sec}[c+dx]^{2+m} (b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Sin}[c+dx]}{d (7+3m)} + \\
& \left(3 b (C (4+3m) + A (7+3m)) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (-1-3m), \frac{1}{6} (5-3m), \operatorname{Cos}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^m (b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Sin}[c+dx] \right) / \\
& \quad \left(d (1+3m) (7+3m) \sqrt{\operatorname{Sin}[c+dx]^2} \right) + \\
& \left(3 b B \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (-4-3m), \frac{1}{6} (2-3m), \operatorname{Cos}[c+dx]^2 \right] \operatorname{Sec}[c+dx]^{1+m} (b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Sin}[c+dx] \right) / \\
& \quad \left(d (4+3m) \sqrt{\operatorname{Sin}[c+dx]^2} \right)
\end{aligned}$$

Result (type 5, 484 leaves):

$$\begin{aligned}
& - \frac{1}{d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{10/3}} \\
& 3 i 2^{\frac{7}{3}+m} e^{-\frac{1}{3} i d (4+3m) x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{4}{3}+m} (1 + e^{2i(c+dx)})^{\frac{4}{3}+m} \left(\frac{2(A + 2C) e^{\frac{1}{3} i (6c+d(10+3m)x}}{\text{Hypergeometric2F1} \left[\frac{5}{3} + \frac{m}{2}, \frac{10}{3} + m, \frac{8}{3} + \frac{m}{2}, -e^{2i(c+dx)} \right]}{10 + 3m}} \right) + \\
& \frac{A e^{4 i c + \frac{1}{3} i d (16+3m) x} \text{Hypergeometric2F1} \left[\frac{8}{3} + \frac{m}{2}, \frac{10}{3} + m, \frac{1}{6} (22 + 3m), -e^{2i(c+dx)} \right]}{16 + 3m} + \\
& \frac{A e^{\frac{1}{3} i d (4+3m) x} \text{Hypergeometric2F1} \left[\frac{10}{3} + m, \frac{1}{6} (4 + 3m), \frac{5}{3} + \frac{m}{2}, -e^{2i(c+dx)} \right]}{4 + 3m} + \\
& \frac{2B e^{\frac{1}{3} i (3c+d(7+3m)x}} \text{Hypergeometric2F1} \left[\frac{10}{3} + m, \frac{1}{6} (7 + 3m), \frac{1}{6} (13 + 3m), -e^{2i(c+dx)} \right]}{7 + 3m} + \\
& \left. \frac{2B e^{\frac{1}{3} i (9c+d(13+3m)x}} \text{Hypergeometric2F1} \left[\frac{10}{3} + m, \frac{1}{6} (13 + 3m), \frac{1}{6} (19 + 3m), -e^{2i(c+dx)} \right]}{13 + 3m} \right) \\
& (b \sec [c + dx])^{4/3} (A + B \sec [c + dx] + C \sec [c + dx]^2)
\end{aligned}$$

- **Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [c + dx]^m (b \sec [c + dx])^{2/3} (A + B \sec [c + dx] + C \sec [c + dx]^2) dx$$

Optimal (type 5, 227 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 C \sec [c + dx]^{1+m} (b \sec [c + dx])^{2/3} \sin [c + dx]}{d (5 + 3m)} - \\
& \left(3 (C (2 + 3m) + A (5 + 3m)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (1 - 3m), \frac{1}{6} (7 - 3m), \cos [c + dx]^2 \right] \sec [c + dx]^{-1+m} (b \sec [c + dx])^{2/3} \sin [c + dx] \right) / \\
& \left(d (1 - 3m) (5 + 3m) \sqrt{\sin [c + dx]^2} \right) + \\
& \left(3 B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{6} (-2 - 3m), \frac{1}{6} (4 - 3m), \cos [c + dx]^2 \right] \sec [c + dx]^m (b \sec [c + dx])^{2/3} \sin [c + dx] \right) / \left(d (2 + 3m) \sqrt{\sin [c + dx]^2} \right)
\end{aligned}$$

Result (type 5, 547 leaves):

$$\begin{aligned}
& - \frac{1}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sec[c + dx]^{8/3}} \\
& 3i 2^{\frac{5}{3}+m} e^{-\frac{1}{3}id(2+3m)x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{2}{3}+m} (1 + e^{2i(c+dx)})^{\frac{2}{3}+m} \left(\frac{A e^{4i c + \frac{1}{3}id(14+3m)x} \operatorname{Hypergeometric2F1}\left[\frac{7}{3} + \frac{m}{2}, \frac{8}{3} + m, \frac{1}{6}(20 + 3m), -e^{2i(c+dx)}\right]}{14 + 3m} \right) + \\
& \frac{A e^{\frac{1}{3}id(2+3m)x} \operatorname{Hypergeometric2F1}\left[\frac{8}{3} + m, \frac{1}{6}(2 + 3m), \frac{1}{6}(8 + 3m), -e^{2i(c+dx)}\right]}{2 + 3m} + \\
& \frac{2B e^{\frac{1}{3}i(3c+d(5+3m)x)} \operatorname{Hypergeometric2F1}\left[\frac{8}{3} + m, \frac{1}{6}(5 + 3m), \frac{1}{6}(11 + 3m), -e^{2i(c+dx)}\right]}{5 + 3m} + \\
& \frac{2A e^{\frac{1}{3}i(6c+d(8+3m)x)} \operatorname{Hypergeometric2F1}\left[\frac{8}{3} + m, \frac{1}{6}(8 + 3m), \frac{7}{3} + \frac{m}{2}, -e^{2i(c+dx)}\right]}{8 + 3m} + \\
& \frac{4C e^{\frac{1}{3}i(6c+d(8+3m)x)} \operatorname{Hypergeometric2F1}\left[\frac{8}{3} + m, \frac{1}{6}(8 + 3m), \frac{7}{3} + \frac{m}{2}, -e^{2i(c+dx)}\right]}{8 + 3m} + \\
& \left. \frac{2B e^{\frac{1}{3}i(9c+d(11+3m)x)} \operatorname{Hypergeometric2F1}\left[\frac{8}{3} + m, \frac{1}{6}(11 + 3m), \frac{1}{6}(17 + 3m), -e^{2i(c+dx)}\right]}{11 + 3m} \right) \\
& (b \sec[c + dx])^{2/3} (A + B \sec[c + dx] + C \sec[c + dx]^2)
\end{aligned}$$

■ **Problem 67: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^m (b \sec[c + dx])^{1/3} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 5, 225 leaves, 7 steps):

$$\begin{aligned}
& \frac{3C \sec[c + dx]^{1+m} (b \sec[c + dx])^{1/3} \sin[c + dx]}{d(4 + 3m)} - \\
& \left(3(C + 3Cm + A(4 + 3m)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(2 - 3m), \frac{1}{6}(8 - 3m), \cos[c + dx]^2\right] \sec[c + dx]^{-1+m} (b \sec[c + dx])^{1/3} \sin[c + dx] \right) / \\
& \left(d(2 - 3m)(4 + 3m) \sqrt{\sin[c + dx]^2} \right) + \\
& \left(3B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(-1 - 3m), \frac{1}{6}(5 - 3m), \cos[c + dx]^2\right] \sec[c + dx]^m (b \sec[c + dx])^{1/3} \sin[c + dx] \right) / \left(d(1 + 3m) \sqrt{\sin[c + dx]^2} \right)
\end{aligned}$$

Result (type 5, 494 leaves):

$$\begin{aligned}
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sec[c + dx]^{7/3}} \\
& 3i 2^{\frac{4}{3}+m} e^{-\frac{1}{3}i d (1+3m)x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{1}{3}+m} \left(1 + e^{2i(c+dx)} \right)^{\frac{1}{3}+m} \left(\frac{2B e^{\frac{1}{3}i(9c+d(10+3m)x}} \operatorname{Hypergeometric2F1}\left[\frac{5}{3} + \frac{m}{2}, \frac{7}{3} + m, \frac{8}{3} + \frac{m}{2}, -e^{2i(c+dx)}\right]}{d(10+3m)} \right) + \\
& \frac{A e^{\frac{1}{3}i(d+3d m)x} \operatorname{Hypergeometric2F1}\left[\frac{7}{3} + m, \frac{1}{6}(1+3m), \frac{1}{6}(7+3m), -e^{2i(c+dx)}\right]}{d+3dm} + \frac{1}{d} \\
& e^{ic} \left(\frac{2B e^{\frac{1}{3}i d(4+3m)x} \operatorname{Hypergeometric2F1}\left[\frac{7}{3} + m, \frac{1}{6}(4+3m), \frac{5}{3} + \frac{m}{2}, -e^{2i(c+dx)}\right]}{4+3m} + 1 / ((7+3m)(13+3m)) e^{\frac{1}{3}i(3c+d(7+3m)x)} \right. \\
& \left. \left(2(A+2C)(13+3m) \operatorname{Hypergeometric2F1}\left[\frac{7}{3} + m, \frac{1}{6}(7+3m), \frac{1}{6}(13+3m), -e^{2i(c+dx)}\right] + A e^{2i(c+dx)}(7+3m) \right. \right. \\
& \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{7}{3} + m, \frac{1}{6}(13+3m), \frac{1}{6}(19+3m), -e^{2i(c+dx)}\right] \right) \right) \left. \right) \left. \right) (b \sec[c + dx])^{1/3} (A + B \sec[c + dx] + C \sec[c + dx]^2)
\end{aligned}$$

- **Problem 68: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^m (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(b \sec[c + dx])^{1/3}} dx$$

Optimal (type 5, 228 leaves, 7 steps):

$$\begin{aligned}
& \frac{3C \sec[c + dx]^{1+m} \sin[c + dx]}{d(2+3m)(b \sec[c + dx])^{1/3}} + \\
& \left(3(C(1-3m) - A(2+3m)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(4-3m), \frac{1}{6}(10-3m), \cos[c + dx]^2\right] \sec[c + dx]^{-1+m} \sin[c + dx] \right) / \\
& \left(d(4-3m)(2+3m)(b \sec[c + dx])^{1/3} \sqrt{\sin[c + dx]^2} \right) - \\
& \frac{3B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{6}(1-3m), \frac{1}{6}(7-3m), \cos[c + dx]^2\right] \sec[c + dx]^m \sin[c + dx]}{d(1-3m)(b \sec[c + dx])^{1/3} \sqrt{\sin[c + dx]^2}}
\end{aligned}$$

Result (type 5, 548 leaves):

$$\begin{aligned}
& - \left(\left(3 i 2^{-\frac{1}{3}+m} e^{-\frac{1}{3} i (6c+d(2+3m)x)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{\frac{2}{3}+m} \right. \right. \\
& \quad \left. \left(1+e^{2i(c+dx)} \right)^{\frac{2}{3}+m} \left(\frac{A e^{\frac{1}{3} i d (-4+3m)x} \text{Hypergeometric2F1} \left[\frac{2}{3}+m, \frac{1}{6}(-4+3m), \frac{1}{6}(2+3m), -e^{2i(c+dx)} \right]}{-4+3m} \right. \right. + \\
& \quad \left. \left. \frac{2 B e^{\frac{1}{3} i (3c+d(-1+3m)x)} \text{Hypergeometric2F1} \left[\frac{2}{3}+m, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), -e^{2i(c+dx)} \right]}{-1+3m} \right. \right. + \\
& \quad \left. \left. e^{2ic} \left(\frac{2(A+2C) e^{\frac{1}{3} i d (2+3m)x} \text{Hypergeometric2F1} \left[\frac{2}{3}+m, \frac{1}{6}(2+3m), \frac{1}{6}(8+3m), -e^{2i(c+dx)} \right]}{2+3m} \right. \right. + \\
& \quad \left. \left. \frac{2 B e^{\frac{1}{3} i (3c+d(5+3m)x)} \text{Hypergeometric2F1} \left[\frac{2}{3}+m, \frac{1}{6}(5+3m), \frac{1}{6}(11+3m), -e^{2i(c+dx)} \right]}{5+3m} \right. \right. + \\
& \quad \left. \left. \left. \frac{A e^{\frac{1}{3} i (6c+d(8+3m)x)} \text{Hypergeometric2F1} \left[\frac{2}{3}+m, \frac{1}{6}(8+3m), \frac{7}{3}+\frac{m}{2}, -e^{2i(c+dx)} \right]}{8+3m} \right) \right) \right) \left(A+B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2 \right) \Big/ \\
& \quad \left. \left(d(A+2C+2B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx]) \text{Sec}[c+dx]^{2/3} (b \text{Sec}[c+dx])^{4/3} \right) \right)
\end{aligned}$$

■ **Problem 71: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sec}[c+dx]^m (b \text{Sec}[c+dx])^n (A+B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 5, 226 leaves, 7 steps):

$$\frac{C \text{Sec}[c+dx]^{1+m} (b \text{Sec}[c+dx])^n \text{Sin}[c+dx]}{d(1+m+n)} -$$

$$\left((C(m+n) + A(1+m+n)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2}(1-m-n), \frac{1}{2}(3-m-n), \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^{-1+m} (b \text{Sec}[c+dx])^n \text{Sin}[c+dx] \right) \Big/ \\
\left(d(1-m-n)(1+m+n) \sqrt{\text{Sin}[c+dx]^2} \right) + \frac{1}{d(m+n) \sqrt{\text{Sin}[c+dx]^2}}$$

$$B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2}(-m-n), \frac{1}{2}(2-m-n), \text{Cos}[c+dx]^2 \right] \text{Sec}[c+dx]^m (b \text{Sec}[c+dx])^n \text{Sin}[c+dx]$$

Result (type 5, 436 leaves):

$$\begin{aligned}
& - \frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \\
& i 2^{1+m+n} e^{-i d (m+n) x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{m+n} \left(1 + e^{2 i (c+d x)} \right)^{m+n} \left(\frac{A e^{i d (m+n) x} \text{Hypergeometric2F1} \left[\frac{m+n}{2}, 2+m+n, \frac{1}{2} (2+m+n), -e^{2 i (c+d x)} \right]}{m+n} \right) + \\
& \frac{2 B e^{i (c+d (1+m+n) x)} \text{Hypergeometric2F1} \left[\frac{1}{2} (1+m+n), 2+m+n, \frac{1}{2} (3+m+n), -e^{2 i (c+d x)} \right]}{1+m+n} + \\
& e^{2 i c} \left(\frac{2 (A + 2 C) e^{i d (2+m+n) x} \text{Hypergeometric2F1} \left[\frac{1}{2} (2+m+n), 2+m+n, \frac{1}{2} (4+m+n), -e^{2 i (c+d x)} \right]}{2+m+n} + \right. \\
& \frac{2 B e^{i (c+d (3+m+n) x)} \text{Hypergeometric2F1} \left[2+m+n, \frac{1}{2} (3+m+n), \frac{1}{2} (5+m+n), -e^{2 i (c+d x)} \right]}{3+m+n} + \\
& \left. \frac{A e^{i (2 c+d (4+m+n) x)} \text{Hypergeometric2F1} \left[2+m+n, \frac{1}{2} (4+m+n), \frac{1}{2} (6+m+n), -e^{2 i (c+d x)} \right]}{4+m+n} \right) \Bigg) \\
& \sec [c + d x]^{-2-n} (b \sec [c + d x])^n (A + B \sec [c + d x] + C \sec [c + d x]^2)
\end{aligned}$$

- **Problem 72: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^2 (b \sec [c + d x])^n (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 5, 189 leaves, 7 steps):

$$\begin{aligned}
& \left((C (2+n) + A (3+n)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (-1-n), \frac{1-n}{2}, \cos [c + d x]^2 \right] (b \sec [c + d x])^{1+n} \sin [c + d x] \right) / \\
& \left(b d (1+n) (3+n) \sqrt{\sin [c + d x]^2} \right) + \\
& \frac{B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (-2-n), -\frac{n}{2}, \cos [c + d x]^2 \right] (b \sec [c + d x])^{2+n} \sin [c + d x]}{b^2 d (2+n) \sqrt{\sin [c + d x]^2}} + \frac{C (b \sec [c + d x])^{2+n} \tan [c + d x]}{b^2 d (3+n)}
\end{aligned}$$

Result (type 5, 462 leaves):

$$\begin{aligned}
& - \frac{1}{d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i^{2^{3+n}} e^{2i c - i d n x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^n (1 + e^{2i(c+dx)})^n \\
& \left(\frac{A e^{i d (2+n)x} \operatorname{Hypergeometric2F1}\left[\frac{2+n}{2}, 4+n, \frac{4+n}{2}, -e^{2i(c+dx)}\right]}{2+n} + \frac{2B e^{i(c+d(3+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, 4+n, \frac{5+n}{2}, -e^{2i(c+dx)}\right]}{3+n} + \right. \\
& \frac{2A e^{i(2c+d(4+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{4+n}{2}, 4+n, \frac{6+n}{2}, -e^{2i(c+dx)}\right]}{4+n} + \frac{4C e^{i(2c+d(4+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{4+n}{2}, 4+n, \frac{6+n}{2}, -e^{2i(c+dx)}\right]}{4+n} + \\
& \left. \frac{2B e^{i(3c+d(5+n)x)} \operatorname{Hypergeometric2F1}\left[4+n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2i(c+dx)}\right]}{5+n} + \frac{A e^{i(4c+d(6+n)x)} \operatorname{Hypergeometric2F1}\left[4+n, \frac{6+n}{2}, \frac{8+n}{2}, -e^{2i(c+dx)}\right]}{6+n} \right) \\
& \operatorname{Sec}[c + dx]^{-2-n} (b \operatorname{Sec}[c + dx])^n (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)
\end{aligned}$$

- **Problem 73: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx] (b \operatorname{Sec}[c + dx])^n (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 5, 182 leaves, 7 steps):

$$\begin{aligned}
& \frac{(C(1+n) + A(2+n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \cos[c + dx]^2\right] (b \operatorname{Sec}[c + dx])^n \sin[c + dx]}{dn(2+n) \sqrt{\sin[c + dx]^2}} + \\
& \frac{B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}(-1-n), \frac{1-n}{2}, \cos[c + dx]^2\right] (b \operatorname{Sec}[c + dx])^{1+n} \sin[c + dx]}{bd(1+n) \sqrt{\sin[c + dx]^2}} + \frac{C(b \operatorname{Sec}[c + dx])^{1+n} \tan[c + dx]}{bd(2+n)}
\end{aligned}$$

Result (type 5, 460 leaves):

$$\begin{aligned}
& - \frac{1}{d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i^{2^{2+n}} e^{i(c-dn)x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^n (1 + e^{2i(c+dx)})^n \\
& \left(\frac{A e^{i d (1+n)x} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 3+n, \frac{3+n}{2}, -e^{2i(c+dx)}\right]}{1+n} + \frac{2B e^{i(c+d(2+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{2+n}{2}, 3+n, \frac{4+n}{2}, -e^{2i(c+dx)}\right]}{2+n} + \right. \\
& \frac{2A e^{i(2c+d(3+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, 3+n, \frac{5+n}{2}, -e^{2i(c+dx)}\right]}{3+n} + \frac{4C e^{i(2c+d(3+n)x)} \operatorname{Hypergeometric2F1}\left[\frac{3+n}{2}, 3+n, \frac{5+n}{2}, -e^{2i(c+dx)}\right]}{3+n} + \\
& \left. \frac{2B e^{i(3c+d(4+n)x)} \operatorname{Hypergeometric2F1}\left[3+n, \frac{4+n}{2}, \frac{6+n}{2}, -e^{2i(c+dx)}\right]}{4+n} + \frac{A e^{i(4c+d(5+n)x)} \operatorname{Hypergeometric2F1}\left[3+n, \frac{5+n}{2}, \frac{7+n}{2}, -e^{2i(c+dx)}\right]}{5+n} \right) \\
& \operatorname{Sec}[c + dx]^{-2-n} (b \operatorname{Sec}[c + dx])^n (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)
\end{aligned}$$

- **Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (b \operatorname{Sec}[c + dx])^n (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 5, 175 leaves, 6 steps):

$$\frac{b (A + A n + C n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{-1+n} \operatorname{Sin}[c + d x]}{d (1-n) (1+n) \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \operatorname{Cos}[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d n \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{c (b \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]}{d (1+n)}$$

Result (type 5, 401 leaves):

$$\frac{1}{d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} i^{2^{1+n}} e^{-i d n x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^n (1 + e^{2 i (c+d x)})^n$$

$$\left(\frac{A e^{i d n x} \operatorname{Hypergeometric2F1}\left[\frac{n}{2}, 2+n, \frac{2+n}{2}, -e^{2 i (c+d x)}\right]}{n} + \frac{2 B e^{i (c+d (1+n) x)} \operatorname{Hypergeometric2F1}\left[\frac{1+n}{2}, 2+n, \frac{3+n}{2}, -e^{2 i (c+d x)}\right]}{1+n} + e^{2 i c} \right.$$

$$\left. \left(\frac{2 (A + 2 C) e^{i d (2+n) x} \operatorname{Hypergeometric2F1}\left[\frac{2+n}{2}, 2+n, \frac{4+n}{2}, -e^{2 i (c+d x)}\right]}{2+n} + \frac{2 B e^{i (c+d (3+n) x)} \operatorname{Hypergeometric2F1}\left[2+n, \frac{3+n}{2}, \frac{5+n}{2}, -e^{2 i (c+d x)}\right]}{3+n} \right. \right.$$

$$\left. \left. \frac{A e^{i (2 c+d (4+n) x)} \operatorname{Hypergeometric2F1}\left[2+n, \frac{4+n}{2}, \frac{6+n}{2}, -e^{2 i (c+d x)}\right]\right)}{4+n} \right) \operatorname{Sec}[c + d x]^{-2-n} (b \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)$$

- **Problem 75: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (b \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 191 leaves, 7 steps):

$$\frac{b^2 (C (1-n) - A n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \operatorname{Cos}[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{-2+n} \operatorname{Sin}[c + d x]}{d (2-n) n \sqrt{\operatorname{Sin}[c + d x]^2}} - \frac{b B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \operatorname{Cos}[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^{-1+n} \operatorname{Sin}[c + d x]}{d (1-n) \sqrt{\operatorname{Sin}[c + d x]^2}} + \frac{b C (b \operatorname{Sec}[c + d x])^{-1+n} \operatorname{Tan}[c + d x]}{d n}$$

Result (type 6, 6083 leaves): Display of huge result suppressed!

- **Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^2 (b \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 208 leaves, 7 steps):

$$\begin{aligned}
& - \left(b^3 (A (1-n) + C (2-n)) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \cos[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{-3+n} \sin[c+dx] \right) / \\
& \left(d (1-n) (3-n) \sqrt{\sin[c+dx]^2} \right) - \\
& \frac{b^2 B \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \cos[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{-2+n} \sin[c+dx]}{d (2-n) \sqrt{\sin[c+dx]^2}} - \frac{b^2 C (b \operatorname{Sec}[c+dx])^{-2+n} \tan[c+dx]}{d (1-n)}
\end{aligned}$$

Result (type 6, 12574 leaves) : Display of huge result suppressed!

■ **Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 (b \operatorname{Sec}[c+dx])^n (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 208 leaves, 7 steps) :

$$\begin{aligned}
& - \left(b^4 (A (2-n) + C (3-n)) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{4-n}{2}, \frac{6-n}{2}, \cos[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{-4+n} \sin[c+dx] \right) / \\
& \left(d (2-n) (4-n) \sqrt{\sin[c+dx]^2} \right) - \\
& \frac{b^3 B \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \cos[c+dx]^2 \right] (b \operatorname{Sec}[c+dx])^{-3+n} \sin[c+dx]}{d (3-n) \sqrt{\sin[c+dx]^2}} - \frac{b^3 C (b \operatorname{Sec}[c+dx])^{-3+n} \tan[c+dx]}{d (2-n)}
\end{aligned}$$

Result (type 6, 18886 leaves) : Display of huge result suppressed!

■ **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{5/2} (b \operatorname{Sec}[c+dx])^n (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 5, 223 leaves, 7 steps) :

$$\begin{aligned}
& \frac{2 C \operatorname{Sec}[c+dx]^{7/2} (b \operatorname{Sec}[c+dx])^n \sin[c+dx]}{d (7+2n)} + \\
& \left(2 (C (5+2n) + A (7+2n)) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4} (-3-2n), \frac{1}{4} (1-2n), \cos[c+dx]^2 \right] \operatorname{Sec}[c+dx]^{3/2} (b \operatorname{Sec}[c+dx])^n \sin[c+dx] \right) / \\
& \left(d (3+2n) (7+2n) \sqrt{\sin[c+dx]^2} \right) + \\
& \left(2 B \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4} (-5-2n), \frac{1}{4} (-1-2n), \cos[c+dx]^2 \right] \operatorname{Sec}[c+dx]^{5/2} (b \operatorname{Sec}[c+dx])^n \sin[c+dx] \right) / \\
& \left(d (5+2n) \sqrt{\sin[c+dx]^2} \right)
\end{aligned}$$

Result (type 5, 493 leaves) :

$$\begin{aligned}
& - \frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \\
& i 2^{\frac{9}{2}+n} e^{2 i c - \frac{1}{2} i d (1+2 n) x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{\frac{1}{2}+n} (1 + e^{2 i (c+d x)})^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2} i d (5+2 n) x} \text{Hypergeometric2F1} \left[\frac{9}{2} + n, \frac{1}{4} (5 + 2 n), \frac{1}{4} (9 + 2 n), -e^{2 i (c+d x)} \right]}{5 + 2 n} \right. \\
& \frac{2 B e^{\frac{1}{2} i (2 c+d (7+2 n) x)} \text{Hypergeometric2F1} \left[\frac{9}{2} + n, \frac{1}{4} (7 + 2 n), \frac{1}{4} (11 + 2 n), -e^{2 i (c+d x)} \right]}{7 + 2 n} + \\
& e^{2 i c} \left(\frac{2 (A + 2 C) e^{\frac{1}{2} i d (9+2 n) x} \text{Hypergeometric2F1} \left[\frac{9}{2} + n, \frac{1}{4} (9 + 2 n), \frac{1}{4} (13 + 2 n), -e^{2 i (c+d x)} \right]}{9 + 2 n} + \right. \\
& \frac{2 B e^{\frac{1}{2} i (2 c+d (11+2 n) x)} \text{Hypergeometric2F1} \left[\frac{9}{2} + n, \frac{1}{4} (11 + 2 n), \frac{1}{4} (15 + 2 n), -e^{2 i (c+d x)} \right]}{11 + 2 n} + \\
& \left. \left. \frac{A e^{\frac{1}{2} i (4 c+d (13+2 n) x)} \text{Hypergeometric2F1} \left[\frac{9}{2} + n, \frac{1}{4} (13 + 2 n), \frac{1}{4} (17 + 2 n), -e^{2 i (c+d x)} \right]}{13 + 2 n} \right) \right) \\
& \sec [c + d x]^{-2-n} (b \sec [c + d x])^n (A + B \sec [c + d x] + C \sec [c + d x]^2)
\end{aligned}$$

- **Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^{3/2} (b \sec [c + d x])^n (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 5, 223 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 C \sec [c + d x]^{5/2} (b \sec [c + d x])^n \sin [c + d x]}{d (5 + 2 n)} + \\
& \left(2 (C (3 + 2 n) + A (5 + 2 n)) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4} (-1 - 2 n), \frac{1}{4} (3 - 2 n), \cos [c + d x]^2 \right] \sqrt{\sec [c + d x]} (b \sec [c + d x])^n \sin [c + d x] \right) / \\
& \left(d (1 + 2 n) (5 + 2 n) \sqrt{\sin [c + d x]^2} \right) + \\
& \left(2 B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4} (-3 - 2 n), \frac{1}{4} (1 - 2 n), \cos [c + d x]^2 \right] \sec [c + d x]^{3/2} (b \sec [c + d x])^n \sin [c + d x] \right) / \left(d (3 + 2 n) \sqrt{\sin [c + d x]^2} \right)
\end{aligned}$$

Result (type 5, 487 leaves):

$$\begin{aligned}
& - \frac{1}{d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} \\
& i 2^{\frac{7}{2}+n} e^{-\frac{1}{2} i d (3+2 n) x} \left(\frac{e^{i (c+d x)}}{1 + e^{2 i (c+d x)}} \right)^{\frac{3}{2}+n} \left(1 + e^{2 i (c+d x)} \right)^{\frac{3}{2}+n} \left(\frac{A e^{\frac{1}{2} i d (3+2 n) x} \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + n, \frac{1}{4} (3 + 2 n), \frac{1}{4} (7 + 2 n), -e^{2 i (c+d x)}\right]}{3 + 2 n} \right) + \\
& \frac{2 B e^{\frac{1}{2} i (2 c+d (5+2 n) x)} \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + n, \frac{1}{4} (5 + 2 n), \frac{1}{4} (9 + 2 n), -e^{2 i (c+d x)}\right]}{5 + 2 n} + \\
& e^{2 i c} \left(\frac{2 (A + 2 C) e^{\frac{1}{2} i d (7+2 n) x} \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + n, \frac{1}{4} (7 + 2 n), \frac{1}{4} (11 + 2 n), -e^{2 i (c+d x)}\right]}{7 + 2 n} \right) + \\
& \frac{2 B e^{\frac{1}{2} i (2 c+d (9+2 n) x)} \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + n, \frac{1}{4} (9 + 2 n), \frac{1}{4} (13 + 2 n), -e^{2 i (c+d x)}\right]}{9 + 2 n} + \\
& \left. \frac{A e^{\frac{1}{2} i (4 c+d (11+2 n) x)} \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + n, \frac{1}{4} (11 + 2 n), \frac{1}{4} (15 + 2 n), -e^{2 i (c+d x)}\right]}{11 + 2 n} \right) \Bigg) \\
& \operatorname{Sec}[c + d x]^{-2-n} (b \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)
\end{aligned}$$

- **Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c + d x]} (b \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 5, 221 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 C \operatorname{Sec}[c + d x]^{3/2} (b \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d (3 + 2 n)} - \\
& \left(2 (C + 2 C n + A (3 + 2 n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} (1 - 2 n), \frac{1}{4} (5 - 2 n), \operatorname{Cos}[c + d x]^2\right] (b \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \right) / \\
& \left(d (1 - 2 n) (3 + 2 n) \sqrt{\operatorname{Sec}[c + d x]} \sqrt{\operatorname{Sin}[c + d x]^2} \right) + \\
& \left(2 B \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} (-1 - 2 n), \frac{1}{4} (3 - 2 n), \operatorname{Cos}[c + d x]^2\right] \sqrt{\operatorname{Sec}[c + d x]} (b \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \right) / \left(d (1 + 2 n) \sqrt{\operatorname{Sin}[c + d x]^2} \right)
\end{aligned}$$

Result (type 5, 492 leaves):

$$\begin{aligned}
& - \frac{1}{A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]} \\
& i 2^{\frac{5}{2}+n} e^{-\frac{1}{2} i d (1+2n) x} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{1}{2}+n} (1 + e^{2i(c+dx)})^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2} i (d+2dn) x} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4} (1 + 2n), \frac{1}{4} (5 + 2n), -e^{2i(c+dx)}\right]}{d + 2dn} \right) + \\
& \frac{1}{d} e^{i c} \left(\frac{2B e^{\frac{1}{2} i d (3+2n) x} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4} (3 + 2n), \frac{1}{4} (7 + 2n), -e^{2i(c+dx)}\right]}{3 + 2n} \right) + \\
& e^{i c} \left(\frac{2(A + 2C) e^{\frac{1}{2} i d (5+2n) x} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4} (5 + 2n), \frac{1}{4} (9 + 2n), -e^{2i(c+dx)}\right]}{5 + 2n} \right) + \\
& \frac{2B e^{\frac{1}{2} i (2c+d(7+2n) x)} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4} (7 + 2n), \frac{1}{4} (11 + 2n), -e^{2i(c+dx)}\right]}{7 + 2n} + \\
& \frac{A e^{\frac{1}{2} i (4c+d(9+2n) x)} \text{Hypergeometric2F1}\left[\frac{5}{2} + n, \frac{1}{4} (9 + 2n), \frac{1}{4} (13 + 2n), -e^{2i(c+dx)}\right]}{9 + 2n} \Bigg) \Bigg) \Bigg) \\
& \sec[c + dx]^{-2-n} (b \sec[c + dx])^n (A + B \sec[c + dx] + C \sec[c + dx]^2)
\end{aligned}$$

- **Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \sec[c + dx])^n (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{\sec[c + dx]}} dx$$

Optimal (type 5, 222 leaves, 7 steps):

$$\begin{aligned}
& \frac{2C \sqrt{\sec[c + dx]} (b \sec[c + dx])^n \sin[c + dx]}{d(1 + 2n)} - \\
& \left(\frac{2(A - C(1 - 2n) + 2An) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(3 - 2n), \frac{1}{4}(7 - 2n), \cos[c + dx]^2\right] (b \sec[c + dx])^n \sin[c + dx]}{d(3 - 2n)(1 + 2n) \sec[c + dx]^{3/2} \sqrt{\sin[c + dx]^2}} \right) - \\
& \frac{2B \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(1 - 2n), \frac{1}{4}(5 - 2n), \cos[c + dx]^2\right] (b \sec[c + dx])^n \sin[c + dx]}{d(1 - 2n) \sqrt{\sec[c + dx]} \sqrt{\sin[c + dx]^2}}
\end{aligned}$$

Result (type 5, 548 leaves):

$$\begin{aligned}
& - \left(\left(i 2^{\frac{3}{2}+n} e^{-\frac{i}{2} (2c+d(1+2n)x)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}} \right)^{\frac{1}{2}+n} (1+e^{2i(c+dx)})^{\frac{1}{2}+n} \right. \right. \\
& \quad \left(A e^{\frac{i}{2} d(-1+2n)x} (105+352n+344n^2+128n^3+16n^4) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), -e^{2i(c+dx)} \right] + \right. \\
& \quad e^{ic}(-1+2n) \left(2 B e^{\frac{i}{2} d(1+2n)x} (105+142n+60n^2+8n^3) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), -e^{2i(c+dx)} \right] + \right. \\
& \quad e^{\frac{i}{2} (2c+d(3+2n)x)} (1+2n) \left(2 (A+2C) (35+24n+4n^2) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), -e^{2i(c+dx)} \right] + \right. \\
& \quad e^{i(c+dx)} (3+2n) \left(2 B (7+2n) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), -e^{2i(c+dx)} \right] + \right. \\
& \quad \left. \left. \left. \left. \left. \left. A e^{i(c+dx)} (5+2n) \text{Hypergeometric2F1} \left[\frac{3}{2}+n, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), -e^{2i(c+dx)} \right] \right] \right] \right] \right] \right] \right] \right) \\
& \quad \left. \left. \left. \left. \left. \left. \text{Sec}[c+dx]^{-2-n} (b \text{Sec}[c+dx])^n (A+B \text{Sec}[c+dx]+C \text{Sec}[c+dx]^2) \right] \right] \right] \right] \right] \right] \left/ (d(-1+2n)(1+2n)(3+2n)(5+2n) \right. \\
& \quad \left. (7+2n)(A+2C+2B \text{Cos}[c+dx]+A \text{Cos}[2c+2dx]) \right) \right)
\end{aligned}$$

- **Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \text{Sec}[c+dx])^n (A+B \text{Sec}[c+dx]+C \text{Sec}[c+dx]^2)}{\text{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 5, 221 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2C(b \text{Sec}[c+dx])^n \text{Sin}[c+dx]}{d(1-2n)\sqrt{\text{Sec}[c+dx]}} - \\
& \left(2(A+C(3-2n)-2An) \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4}(5-2n), \frac{1}{4}(9-2n), \text{Cos}[c+dx]^2 \right] (b \text{Sec}[c+dx])^n \text{Sin}[c+dx] \right) \left/ \right. \\
& \left(d(1-2n)(5-2n) \text{Sec}[c+dx]^{5/2} \sqrt{\text{Sin}[c+dx]^2} \right) - \\
& \frac{2B \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{4}(3-2n), \frac{1}{4}(7-2n), \text{Cos}[c+dx]^2 \right] (b \text{Sec}[c+dx])^n \text{Sin}[c+dx]}{d(3-2n) \text{Sec}[c+dx]^{3/2} \sqrt{\text{Sin}[c+dx]^2}}
\end{aligned}$$

Result (type 5, 502 leaves):

$$\begin{aligned}
& - \frac{1}{A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]} \\
& i 2^{\frac{1}{2}+n} e^{-\frac{1}{2}i(4c+d(1+2n)x)} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{\frac{1}{2}+n} \left(1 + e^{2i(c+dx)} \right)^{\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2}id(-3+2n)x} \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), -e^{2i(c+dx)}\right]}{d(-3 + 2n)} \right) + \\
& \frac{2B e^{\frac{1}{2}i(2c+d(-1+2n)x)} \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), -e^{2i(c+dx)}\right]}{d(-1 + 2n)} + \\
& e^{2ic} \left(\frac{2(A + 2C) e^{\frac{1}{2}id(1+2n)x} \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), -e^{2i(c+dx)}\right]}{d + 2dn} \right) + \frac{1}{d(3 + 2n)(5 + 2n)} \\
& e^{\frac{1}{2}i(2c+d(3+2n)x)} \left(2B(5 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1}{4}(3 + 2n), \frac{1}{4}(7 + 2n), -e^{2i(c+dx)}\right] + A e^{i(c+dx)}(3 + 2n) \text{Hypergeometric2F1}\left[\frac{1}{2} + n, \frac{1}{4}(5 + 2n), \frac{1}{4}(9 + 2n), -e^{2i(c+dx)}\right] \right) \Bigg) \Bigg) \text{Sec}[c + dx]^{-2-n} (b \text{Sec}[c + dx])^n (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2)
\end{aligned}$$

- **Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(b \text{Sec}[c + dx])^n (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2)}{\text{Sec}[c + dx]^{5/2}} dx$$

Optimal (type 5, 223 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2C(b \text{Sec}[c + dx])^n \text{Sin}[c + dx]}{d(3 - 2n) \text{Sec}[c + dx]^{3/2}} - \\
& \left(2(A(3 - 2n) + C(5 - 2n)) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(7 - 2n), \frac{1}{4}(11 - 2n), \text{Cos}[c + dx]^2\right] (b \text{Sec}[c + dx])^n \text{Sin}[c + dx] \right) / \\
& \left(d(3 - 2n)(7 - 2n) \text{Sec}[c + dx]^{7/2} \sqrt{\text{Sin}[c + dx]^2} \right) - \\
& \frac{2B \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4}(5 - 2n), \frac{1}{4}(9 - 2n), \text{Cos}[c + dx]^2\right] (b \text{Sec}[c + dx])^n \text{Sin}[c + dx]}{d(5 - 2n) \text{Sec}[c + dx]^{5/2} \sqrt{\text{Sin}[c + dx]^2}}
\end{aligned}$$

Result (type 5, 502 leaves):

$$\begin{aligned}
& - \frac{1}{A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]} \left(\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}} \right)^{-\frac{1}{2}+n} \\
& \left(1 + e^{2i(c+dx)} \right)^{-\frac{1}{2}+n} \left(\frac{A e^{\frac{1}{2}id(-5+2n)x} \text{Hypergeometric2F1}\left[-\frac{1}{2} + n, \frac{1}{4}(-5 + 2n), \frac{1}{4}(-1 + 2n), -e^{2i(c+dx)}\right]}{d(-5 + 2n)} + \right. \\
& \frac{2B e^{\frac{1}{2}i(2c+d(-3+2n)x)} \text{Hypergeometric2F1}\left[-\frac{1}{2} + n, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), -e^{2i(c+dx)}\right]}{d(-3 + 2n)} + \\
& e^{2ic} \left(\frac{2(A + 2C) e^{\frac{1}{2}id(-1+2n)x} \text{Hypergeometric2F1}\left[-\frac{1}{2} + n, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), -e^{2i(c+dx)}\right]}{d(-1 + 2n)} + \right. \\
& \frac{2B e^{\frac{1}{2}i(2c+dx+2dn)x} \text{Hypergeometric2F1}\left[-\frac{1}{2} + n, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), -e^{2i(c+dx)}\right]}{d + 2dn} + \\
& \left. \left. \frac{A e^{\frac{1}{2}i(4c+d(3+2n)x)} \text{Hypergeometric2F1}\left[-\frac{1}{2} + n, \frac{1}{4}(3 + 2n), \frac{1}{4}(7 + 2n), -e^{2i(c+dx)}\right]}{d(3 + 2n)} \right) \right) \\
& \sec[c + dx]^{-2-n} (b \sec[c + dx])^n (A + B \sec[c + dx] + C \sec[c + dx]^2)
\end{aligned}$$

■ **Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^3 (a + a \sec[c + dx]) (A + C \sec[c + dx]^2) dx$$

Optimal (type 3, 140 leaves, 7 steps):

$$\begin{aligned}
& \frac{a(4A + 3C) \text{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a(5A + 4C) \tan[c + dx]}{5d} + \frac{a(4A + 3C) \sec[c + dx] \tan[c + dx]}{8d} + \\
& \frac{aC \sec[c + dx]^3 \tan[c + dx]}{4d} + \frac{aC \sec[c + dx]^4 \tan[c + dx]}{5d} + \frac{a(5A + 4C) \tan[c + dx]^3}{15d}
\end{aligned}$$

Result (type 3, 426 leaves):

$$\begin{aligned}
& - \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
& \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \\
& \frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{3 a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{3 a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{2 a A \operatorname{Tan}[c+dx]}{3 d} + \\
& \frac{8 a C \operatorname{Tan}[c+dx]}{15 d} + \frac{a A \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d} + \frac{4 a C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{15 d} + \frac{a C \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx]}{5 d}
\end{aligned}$$

■ **Problem 85: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^2 (a + a \operatorname{Sec}[c+dx]) (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\begin{aligned}
& \frac{a (4 A + 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{8 d} + \frac{a (3 A + 2 C) \operatorname{Tan}[c+dx]}{3 d} + \\
& \frac{a (4 A + 3 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{8 d} + \frac{a C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d} + \frac{a C \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{4 d}
\end{aligned}$$

Result (type 3, 377 leaves):

$$\begin{aligned}
& - \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \\
& \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{3 a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \frac{a A}{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \\
& \frac{3 a C}{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{a A \operatorname{Tan}[c+dx]}{d} + \frac{2 a C \operatorname{Tan}[c+dx]}{3 d} + \frac{a C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d}
\end{aligned}$$

■ **Problem 87: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+dx]) (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$a A x + \frac{a (2 A + C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a C \operatorname{Tan}[c + d x]}{d} + \frac{a C \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 218 leaves):

$$a A x - \frac{a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} -$$

$$\frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} +$$

$$\frac{a C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{a C}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{a C \operatorname{Tan}[c + d x]}{d}$$

■ **Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x]) (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 42 leaves, 5 steps):

$$a A x + \frac{a C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{a A \operatorname{Sin}[c + d x]}{d} + \frac{a C \operatorname{Tan}[c + d x]}{d}$$

Result (type 3, 112 leaves):

$$a A x - \frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{d} + \frac{a A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d} + \frac{a C \operatorname{Tan}[c + d x]}{d}$$

■ **Problem 94: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{a^2 (12 A + 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^2 (12 A + 7 C) \operatorname{Tan}[c + d x]}{6 d} +$$

$$\frac{a^2 (12 A + 7 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{24 d} - \frac{C (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} + \frac{C (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 a d}$$

Result (type 3, 291 leaves):

$$\begin{aligned}
& - \frac{1}{384 d (A + 2 C + A \cos[2(c + dx)])} a^2 (1 + \cos[c + dx])^2 (C + A \cos[c + dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \operatorname{Sec}[c + dx]^4 \\
& \left(24 (12 A + 7 C) \cos[c + dx]^4 \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \right. \\
& \quad \operatorname{Sec}[c] (-48 (3 A + 2 C) \sin[c] + 3 (4 A + 15 C) \sin[dx] + 12 A \sin[2c + dx] + 45 C \sin[2c + dx] + \\
& \quad 144 A \sin[c + 2dx] + 128 C \sin[c + 2dx] - 48 A \sin[3c + 2dx] + 12 A \sin[2c + 3dx] + 21 C \sin[2c + 3dx] + \\
& \quad \left. 12 A \sin[4c + 3dx] + 21 C \sin[4c + 3dx] + 48 A \sin[3c + 4dx] + 32 C \sin[3c + 4dx] \right)
\end{aligned}$$

■ **Problem 95: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx])^2 (A + C \operatorname{Sec}[c + dx])^2 dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$a^2 A x + \frac{a^2 (2 A + C) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{a^2 (A + C) \tan[c + dx]}{d} + \frac{C (a + a \operatorname{Sec}[c + dx])^2 \tan[c + dx]}{3 d} + \frac{C (a^2 + a^2 \operatorname{Sec}[c + dx]) \tan[c + dx]}{3 d}$$

Result (type 3, 1090 leaves):

$$\begin{aligned}
& \frac{A x \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{2(A+2 C+A \cos [2 c+2 d x])} + \\
& \left((-2 A-C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\right) / \\
& (2 d(A+2 C+A \cos [2 c+2 d x])) + \\
& \left((2 A+C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\right) / \\
& (2 d(A+2 C+A \cos [2 c+2 d x])) + \frac{C \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \sin \left[\frac{d x}{2}\right]}{12 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\left(7 C \cos \left[\frac{c}{2}\right]-5 C \sin \left[\frac{c}{2}\right]\right)}{24 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\left(3 A \sin \left[\frac{d x}{2}\right]+5 C \sin \left[\frac{d x}{2}\right]\right)}{6 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)} + \\
& \frac{C \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \sin \left[\frac{d x}{2}\right]}{12 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3} + \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\left(-7 C \cos \left[\frac{c}{2}\right]-5 C \sin \left[\frac{c}{2}\right]\right)}{24 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2} + \\
& \frac{\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)\left(3 A \sin \left[\frac{d x}{2}\right]+5 C \sin \left[\frac{d x}{2}\right]\right)}{6 d(A+2 C+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)}
\end{aligned}$$

■ **Problem 96: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x] (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) dx$$

Optimal (type 3, 112 leaves, 6 steps):

$$\begin{aligned}
& 2 a^2 A x + \frac{a^2 (2 A+3 C) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{A (a+a \sec [c+d x])^2 \sin [c+d x]}{d} - \\
& \frac{a^2 (2 A-3 C) \tan [c+d x]}{2 d} - \frac{(2 A-C)\left(a^2+a^2 \sec [c+d x]\right) \tan [c+d x]}{2 d}
\end{aligned}$$

Result (type 3, 330 leaves):

$$\frac{1}{8(A+2C+A\cos[2(c+dx)])} a^2 \cos[c+dx]^4 \sec\left[\frac{1}{2}(c+dx)\right]^4 (1+\sec[c+dx])^2 (A+C\sec[c+dx]^2)$$

$$\left(8Ax - \frac{2(2A+3C)\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{2(2A+3C)\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{d} + \frac{4A\cos[dx]\sin[c]}{d} + \frac{4A\cos[c]\sin[dx]}{d} + \frac{C}{d\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{8C\sin\left[\frac{dx}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} - \frac{C}{d\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{8C\sin\left[\frac{dx}{2}\right]}{d\left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} \right)$$

■ **Problem 97: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a+a\sec[c+dx])^2 (A+C\sec[c+dx]^2) dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$\frac{1}{2} a^2 (3A+2C)x + \frac{2a^2 C \operatorname{ArcTanh}[\sin[c+dx]]}{d} + \frac{a^2 (3A-2C) \sin[c+dx]}{2d} + \frac{A \cos[c+dx] (a+a\sec[c+dx])^2 \sin[c+dx]}{2d} - \frac{(A-2C) (a^2+a^2\sec[c+dx]) \sin[c+dx]}{2d}$$

Result (type 3, 292 leaves):

$$-\left(\left(a^2 \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(4\cos[dx] \left(3Adx+2Cdx-4C\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4C\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \right) + 4\cos[2c+dx] \left(3Adx+2Cdx-4C\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 4C\log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + A\sin[dx] + 16C\sin[dx] + A\sin[2c+dx] + 8A\sin[c+2dx] + 8A\sin[3c+2dx] + A\sin[2c+3dx] + A\sin[4c+3dx] \right) \right) / \left(16d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left(1 + \tan\left[\frac{1}{2}(c+dx)\right] \right) \right)$$

■ **Problem 103: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx] (a+a\sec[c+dx])^3 (A+C\sec[c+dx]^2) dx$$

Optimal (type 3, 157 leaves, 11 steps):

$$\frac{a^3 (20A+13C) \operatorname{ArcTanh}[\sin[c+dx]]}{8d} + \frac{a^3 (20A+13C) \tan[c+dx]}{5d} + \frac{3a^3 (20A+13C) \sec[c+dx] \tan[c+dx]}{40d} - \frac{C(a+a\sec[c+dx])^3 \tan[c+dx]}{20d} + \frac{C(a+a\sec[c+dx])^4 \tan[c+dx]}{5ad} + \frac{a^3 (20A+13C) \tan[c+dx]^3}{60d}$$

Result (type 3, 323 leaves) :

$$-\frac{1}{7680 d (A + 2 C + A \cos[2(c + dx)])} a^3 (1 + \cos[c + dx])^3 (C + A \cos[c + dx])^2 \sec\left[\frac{1}{2}(c + dx)\right]^6 \sec[c + dx]^5$$

$$\left(240 (20 A + 13 C) \cos[c + dx]^5 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \right.$$

$$\left. \sec[c] (80 (34 A + 29 C) \sin[dx] - 240 (7 A + 3 C) \sin[2c + dx] + 360 A \sin[c + 2dx] + 750 C \sin[c + 2dx] + 360 A \sin[3c + 2dx] + 750 C \sin[3c + 2dx] + 1840 A \sin[2c + 3dx] + 1520 C \sin[2c + 3dx] - 360 A \sin[4c + 3dx] + 180 A \sin[3c + 4dx] + 195 C \sin[3c + 4dx] + 180 A \sin[5c + 4dx] + 195 C \sin[5c + 4dx] + 440 A \sin[4c + 5dx] + 304 C \sin[4c + 5dx]) \right)$$

■ **Problem 104: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c + dx])^3 (A + C \sec[c + dx])^2 dx$$

Optimal (type 3, 147 leaves, 7 steps) :

$$a^3 Ax + \frac{a^3 (28 A + 15 C) \operatorname{ArcTanh}[\sin[c + dx]]}{8 d} + \frac{5 a^3 (4 A + 3 C) \tan[c + dx]}{8 d} +$$

$$\frac{C (a + a \sec[c + dx])^3 \tan[c + dx]}{4 d} + \frac{C (a^2 + a^2 \sec[c + dx])^2 \tan[c + dx]}{4 a d} + \frac{(4 A + 5 C) (a^3 + a^3 \sec[c + dx]) \tan[c + dx]}{8 d}$$

Result (type 3, 363 leaves) :

$$\frac{1}{256 d (A + 2 C + A \cos[2(c + dx)])} a^3 (1 + \cos[c + dx])^3 (C + A \cos[c + dx])^2 \sec\left[\frac{1}{2}(c + dx)\right]^6 \sec[c + dx]^4$$

$$\left(-8 (28 A + 15 C) \cos[c + dx]^4 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\left. \sec[c] (24 A dx \cos[c] + 16 A dx \cos[c + 2dx] + 16 A dx \cos[3c + 2dx] + 4 A dx \cos[3c + 4dx] + 4 A dx \cos[5c + 4dx] - 72 A \sin[c] - 72 C \sin[c] + 4 A \sin[dx] + 23 C \sin[dx] + 4 A \sin[2c + dx] + 23 C \sin[2c + dx] + 72 A \sin[c + 2dx] + 88 C \sin[c + 2dx] - 24 A \sin[3c + 2dx] - 8 C \sin[3c + 2dx] + 4 A \sin[2c + 3dx] + 15 C \sin[2c + 3dx] + 4 A \sin[4c + 3dx] + 15 C \sin[4c + 3dx] + 24 A \sin[3c + 4dx] + 24 C \sin[3c + 4dx]) \right)$$

■ **Problem 105: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + a \sec[c + dx])^3 (A + C \sec[c + dx])^2 dx$$

Optimal (type 3, 145 leaves, 7 steps) :

$$3 a^3 Ax + \frac{a^3 (6 A + 5 C) \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} + \frac{A (a + a \sec[c + dx])^3 \sin[c + dx]}{d} +$$

$$\frac{5 a^3 C \tan[c + dx]}{2 d} - \frac{(3 A - C) (a^2 + a^2 \sec[c + dx])^2 \tan[c + dx]}{3 a d} - \frac{(6 A - 5 C) (a^3 + a^3 \sec[c + dx]) \tan[c + dx]}{6 d}$$

Result (type 3, 1250 leaves) :

$$\begin{aligned}
 & \frac{3 A x \cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2)}{4(A+2 C+A \cos [2 c+2 d x])} + \\
 & \left((-6 A-5 C) \cos [c+d x]^5 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \right) / \\
 & (8 d(A+2 C+A \cos [2 c+2 d x])) + \\
 & \left((6 A+5 C) \cos [c+d x]^5 \log \left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \right) / \\
 & (8 d(A+2 C+A \cos [2 c+2 d x])) + \frac{A \cos [d x] \cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \sin [c]}{4 d(A+2 C+A \cos [2 c+2 d x])} + \\
 & \frac{A \cos [c] \cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \sin [d x]}{4 d(A+2 C+A \cos [2 c+2 d x])} + \\
 & \frac{C \cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \sin \left[\frac{d x}{2}\right]}{24 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3 + \\
 & \frac{\cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) (5 C \cos \left[\frac{c}{2}\right]-4 C \sin \left[\frac{c}{2}\right])}{24 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2 + \\
 & \frac{\cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) (3 A \sin \left[\frac{d x}{2}\right]+11 C \sin \left[\frac{d x}{2}\right])}{12 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right) + \\
 & \frac{C \cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \sin \left[\frac{d x}{2}\right]}{24 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3 + \\
 & \frac{\cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) (-5 C \cos \left[\frac{c}{2}\right]-4 C \sin \left[\frac{c}{2}\right])}{24 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2 + \\
 & \frac{\cos [c+d x]^5 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) (3 A \sin \left[\frac{d x}{2}\right]+11 C \sin \left[\frac{d x}{2}\right])}{12 d(A+2 C+A \cos [2 c+2 d x])} \left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right) \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)
 \end{aligned}$$

■ **Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) dx$$

Optimal (type 3, 162 leaves, 6 steps) :

$$\frac{1}{2} a^3 (7A + 2C) x + \frac{a^3 (2A + 7C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} + \frac{5a^3 (A - C) \operatorname{Sin}[c + dx]}{2d} +$$

$$\frac{A \operatorname{Cos}[c + dx] (a + a \operatorname{Sec}[c + dx])^3 \operatorname{Sin}[c + dx]}{2d} - \frac{(A - C) (a^2 + a^2 \operatorname{Sec}[c + dx])^2 \operatorname{Sin}[c + dx]}{2ad} - \frac{(A - 4C) (a^3 + a^3 \operatorname{Sec}[c + dx]) \operatorname{Sin}[c + dx]}{2d}$$

Result (type 3, 1074 leaves):

$$\frac{(7A + 2C) x \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2)}{8(A + 2C + A \operatorname{Cos}[2c + 2dx])} +$$

$$\left(\frac{(-2A - 7C) \operatorname{Cos}[c + dx]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2)}{(8d(A + 2C + A \operatorname{Cos}[2c + 2dx]))} \right) /$$

$$\left(\frac{(2A + 7C) \operatorname{Cos}[c + dx]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2)}{(8d(A + 2C + A \operatorname{Cos}[2c + 2dx]))} \right) /$$

$$(8d(A + 2C + A \operatorname{Cos}[2c + 2dx])) + \frac{3A \operatorname{Cos}[dx] \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c]}{4d(A + 2C + A \operatorname{Cos}[2c + 2dx])} +$$

$$\frac{A \operatorname{Cos}[2dx] \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[2c]}{16d(A + 2C + A \operatorname{Cos}[2c + 2dx])} +$$

$$\frac{3A \operatorname{Cos}[c] \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[dx]}{4d(A + 2C + A \operatorname{Cos}[2c + 2dx])} +$$

$$\frac{A \operatorname{Cos}[2c] \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[2dx]}{16d(A + 2C + A \operatorname{Cos}[2c + 2dx])} +$$

$$\frac{C \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2)}{16d(A + 2C + A \operatorname{Cos}[2c + 2dx])} \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2 +$$

$$\frac{3C \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}\left[\frac{dx}{2}\right]}{4d(A + 2C + A \operatorname{Cos}[2c + 2dx])} \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right) -$$

$$\frac{C \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2)}{16d(A + 2C + A \operatorname{Cos}[2c + 2dx])} \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2 +$$

$$\frac{3C \operatorname{Cos}[c + dx]^5 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}\left[\frac{dx}{2}\right]}{4d(A + 2C + A \operatorname{Cos}[2c + 2dx])} \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right)$$

■ **Problem 107: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^3 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$\frac{1}{2} a^3 (5 A + 6 C) x + \frac{3 a^3 C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^3 A \operatorname{Sin}[c + d x]}{2 d} + \frac{A \operatorname{Cos}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{3 d} +$$

$$\frac{A \operatorname{Cos}[c + d x] (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 a d} - \frac{(5 A - 6 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{6 d}$$

Result (type 3, 1014 leaves):

$$\begin{aligned}
& a^3 \left(\frac{(5A + 6C) x \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2)}{8(A + 2C + A \cos[2c + 2dx])} - \right. \\
& \frac{3C \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2)}{4d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{3C \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2)}{4d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{(15A + 4C) \cos[dx] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[c]}{16d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{3A \cos[2dx] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[2c]}{16d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[3dx] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[3c]}{48d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{(15A + 4C) \cos[c] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[dx]}{16d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{3A \cos[2c] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[2dx]}{16d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[3c] \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin[3dx]}{48d(A + 2C + A \cos[2c + 2dx])} + \\
& \left. \frac{C \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{4d(A + 2C + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right) \\
& \left. \frac{C \cos[c + dx]^2 (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{4d(A + 2C + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 111: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^4 (A + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 228 leaves, 15 steps):

$$\frac{a^4 (14 A + 11 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{4 d} + \frac{16 a^4 (14 A + 11 C) \operatorname{Tan}[c + d x]}{35 d} + \frac{27 a^4 (14 A + 11 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{140 d} +$$

$$\frac{a^4 (14 A + 11 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{70 d} + \frac{(21 A + 4 C) (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{105 d} +$$

$$\frac{C \operatorname{Sec}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{7 d} + \frac{2 C (a + a \operatorname{Sec}[c + d x])^5 \operatorname{Tan}[c + d x]}{21 a d} + \frac{8 a^4 (14 A + 11 C) \operatorname{Tan}[c + d x]^3}{105 d}$$

Result (type 3, 574 leaves):

$$\left((-14 A - 11 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) \right) /$$

$$(32 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left((14 A + 11 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) \right) /$$

$$(32 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\frac{1}{215040 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])} \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2)$$

$$(50960 A \operatorname{Sin}[d x] + 46480 C \operatorname{Sin}[d x] - 30380 A \operatorname{Sin}[2 c + d x] - 17080 C \operatorname{Sin}[2 c + d x] + 10710 A \operatorname{Sin}[c + 2 d x] + 16415 C \operatorname{Sin}[c + 2 d x] +$$

$$10710 A \operatorname{Sin}[3 c + 2 d x] + 16415 C \operatorname{Sin}[3 c + 2 d x] + 41244 A \operatorname{Sin}[2 c + 3 d x] + 37296 C \operatorname{Sin}[2 c + 3 d x] -$$

$$7560 A \operatorname{Sin}[4 c + 3 d x] - 840 C \operatorname{Sin}[4 c + 3 d x] + 7560 A \operatorname{Sin}[3 c + 4 d x] + 7700 C \operatorname{Sin}[3 c + 4 d x] + 7560 A \operatorname{Sin}[5 c + 4 d x] +$$

$$7700 C \operatorname{Sin}[5 c + 4 d x] + 15848 A \operatorname{Sin}[4 c + 5 d x] + 12712 C \operatorname{Sin}[4 c + 5 d x] - 420 A \operatorname{Sin}[6 c + 5 d x] + 1470 A \operatorname{Sin}[5 c + 6 d x] +$$

$$1155 C \operatorname{Sin}[5 c + 6 d x] + 1470 A \operatorname{Sin}[7 c + 6 d x] + 1155 C \operatorname{Sin}[7 c + 6 d x] + 2324 A \operatorname{Sin}[6 c + 7 d x] + 1816 C \operatorname{Sin}[6 c + 7 d x])$$

■ **Problem 112: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 188 leaves, 14 steps):

$$\frac{7 a^4 (10 A + 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 d} + \frac{4 a^4 (10 A + 7 C) \operatorname{Tan}[c + d x]}{5 d} + \frac{27 a^4 (10 A + 7 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{80 d} +$$

$$\frac{a^4 (10 A + 7 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{40 d} - \frac{C (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{30 d} + \frac{C (a + a \operatorname{Sec}[c + d x])^5 \operatorname{Tan}[c + d x]}{6 a d} + \frac{2 a^4 (10 A + 7 C) \operatorname{Tan}[c + d x]^3}{15 d}$$

Result (type 3, 530 leaves):

$$\begin{aligned}
& - \left(7 (10 A + 7 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right] - \operatorname{Sin} \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) \right) / \\
& (128 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])) + \\
& \left(7 (10 A + 7 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right] + \operatorname{Sin} \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) \right) / \\
& (128 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])) + \frac{1}{61440 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x])} \operatorname{Sec}[c] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) \\
& (-16000 A \operatorname{Sin}[c] - 11520 C \operatorname{Sin}[c] + 1860 A \operatorname{Sin}[d x] + 3750 C \operatorname{Sin}[d x] + 1860 A \operatorname{Sin}[2 c + d x] + 3750 C \operatorname{Sin}[2 c + d x] + 17280 A \operatorname{Sin}[c + 2 d x] + \\
& 15360 C \operatorname{Sin}[c + 2 d x] - 6720 A \operatorname{Sin}[3 c + 2 d x] - 1920 C \operatorname{Sin}[3 c + 2 d x] + 2670 A \operatorname{Sin}[2 c + 3 d x] + 3845 C \operatorname{Sin}[2 c + 3 d x] + \\
& 2670 A \operatorname{Sin}[4 c + 3 d x] + 3845 C \operatorname{Sin}[4 c + 3 d x] + 8640 A \operatorname{Sin}[3 c + 4 d x] + 6912 C \operatorname{Sin}[3 c + 4 d x] - 960 A \operatorname{Sin}[5 c + 4 d x] + \\
& 810 A \operatorname{Sin}[4 c + 5 d x] + 735 C \operatorname{Sin}[4 c + 5 d x] + 810 A \operatorname{Sin}[6 c + 5 d x] + 735 C \operatorname{Sin}[6 c + 5 d x] + 1600 A \operatorname{Sin}[5 c + 6 d x] + 1152 C \operatorname{Sin}[5 c + 6 d x])
\end{aligned}$$

■ **Problem 113: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 177 leaves, 8 steps):

$$\begin{aligned}
& a^4 A x + \frac{a^4 (12 A + 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a^4 (10 A + 7 C) \operatorname{Tan}[c + d x]}{2 d} + \frac{a C (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{5 d} + \\
& \frac{C (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{5 d} + \frac{(5 A + 7 C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{15 d} + \frac{(8 A + 7 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{6 d}
\end{aligned}$$

Result (type 3, 418 leaves):

$$\begin{aligned}
& \frac{1}{3840 d (A + 2 C + A \operatorname{Cos}[2 (c + d x)])} a^4 (1 + \operatorname{Cos}[c + d x])^4 (C + A \operatorname{Cos}[c + d x]^2) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^8 \operatorname{Sec}[c + d x]^5 \\
& \left(-240 (12 A + 7 C) \operatorname{Cos}[c + d x]^5 \left(\operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] - \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \right) + \right. \\
& \left. \operatorname{Sec}[c] (150 A d x \operatorname{Cos}[d x] + 150 A d x \operatorname{Cos}[2 c + d x] + 75 A d x \operatorname{Cos}[2 c + 3 d x] + 75 A d x \operatorname{Cos}[4 c + 3 d x] + 15 A d x \operatorname{Cos}[4 c + 5 d x] + 15 A d x \right. \\
& \left. \operatorname{Cos}[6 c + 5 d x] + 1220 A \operatorname{Sin}[d x] + 1180 C \operatorname{Sin}[d x] - 780 A \operatorname{Sin}[2 c + d x] - 480 C \operatorname{Sin}[2 c + d x] + 120 A \operatorname{Sin}[c + 2 d x] + 330 C \operatorname{Sin}[c + 2 d x] + \right. \\
& \left. 120 A \operatorname{Sin}[3 c + 2 d x] + 330 C \operatorname{Sin}[3 c + 2 d x] + 820 A \operatorname{Sin}[2 c + 3 d x] + 800 C \operatorname{Sin}[2 c + 3 d x] - 180 A \operatorname{Sin}[4 c + 3 d x] - 30 C \operatorname{Sin}[4 c + 3 d x] + \right. \\
& \left. 60 A \operatorname{Sin}[3 c + 4 d x] + 105 C \operatorname{Sin}[3 c + 4 d x] + 60 A \operatorname{Sin}[5 c + 4 d x] + 105 C \operatorname{Sin}[5 c + 4 d x] + 200 A \operatorname{Sin}[4 c + 5 d x] + 166 C \operatorname{Sin}[4 c + 5 d x]) \right)
\end{aligned}$$

■ **Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$4 a^4 A x + \frac{a^4 (52 A + 35 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{A (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{d} + \frac{5 a^4 (4 A + 7 C) \operatorname{Tan}[c + d x]}{8 d} -$$

$$\frac{a (4 A - C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d} - \frac{(12 A - 7 C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} - \frac{(12 A - 35 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{24 d}$$

Result (type 3, 379 leaves):

$$\frac{1}{1536 d (A + 2 C + A \operatorname{Cos}[2 (c + d x)])} a^4 (C + A \operatorname{Cos}[c + d x]^2) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^8 (1 + \operatorname{Sec}[c + d x])^4$$

$$\left(-24 (52 A + 35 C) \operatorname{Cos}[c + d x]^4 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + \right.$$

$$\left. \operatorname{Sec}[c] (288 A d x \operatorname{Cos}[c] + 192 A d x \operatorname{Cos}[c + 2 d x] + 192 A d x \operatorname{Cos}[3 c + 2 d x] + 48 A d x \operatorname{Cos}[3 c + 4 d x] + 48 A d x \operatorname{Cos}[5 c + 4 d x] - \right.$$

$$288 A \operatorname{Sin}[c] - 480 C \operatorname{Sin}[c] + 24 A \operatorname{Sin}[d x] + 105 C \operatorname{Sin}[d x] + 24 A \operatorname{Sin}[2 c + d x] + 105 C \operatorname{Sin}[2 c + d x] + 288 A \operatorname{Sin}[c + 2 d x] +$$

$$544 C \operatorname{Sin}[c + 2 d x] - 96 A \operatorname{Sin}[3 c + 2 d x] - 96 C \operatorname{Sin}[3 c + 2 d x] + 30 A \operatorname{Sin}[2 c + 3 d x] + 81 C \operatorname{Sin}[2 c + 3 d x] + 30 A \operatorname{Sin}[4 c + 3 d x] +$$

$$\left. 81 C \operatorname{Sin}[4 c + 3 d x] + 96 A \operatorname{Sin}[3 c + 4 d x] + 160 C \operatorname{Sin}[3 c + 4 d x] + 6 A \operatorname{Sin}[4 c + 5 d x] + 6 A \operatorname{Sin}[6 c + 5 d x]) \right)$$

■ **Problem 115: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$\frac{1}{2} a^4 (13 A + 2 C) x + \frac{2 a^4 (2 A + 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^4 (A - 2 C) \operatorname{Sin}[c + d x]}{2 d} - \frac{a (3 A - 2 C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{6 d} +$$

$$\frac{A \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{2 d} - \frac{(A - 2 C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 d} + \frac{(3 A + 22 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{6 d}$$

Result (type 3, 1420 leaves):

$$\begin{aligned}
& \frac{(13A + 2C) \times \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{16(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{\left((-2A - 3C) \cos[c + dx]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \right)}{(4d(A + 2C + A \cos[2c + 2dx]))} + \\
& \frac{\left((2A + 3C) \cos[c + dx]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \right)}{(4d(A + 2C + A \cos[2c + 2dx]))} + \frac{A \cos[dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[c]}{2d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[2dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[2c]}{32d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[dx]}{2d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[2c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[2dx]}{32d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{48d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \left(13C \cos\left[\frac{c}{2}\right] - 11C \sin\left[\frac{c}{2}\right]\right)}{96d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \left(3A \sin\left[\frac{dx}{2}\right] + 20C \sin\left[\frac{dx}{2}\right]\right)}{24d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{48d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \frac{\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \left(-13C \cos\left[\frac{c}{2}\right] - 11C \sin\left[\frac{c}{2}\right]\right)}{96d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \left(3A \sin\left[\frac{dx}{2}\right] + 20C \sin\left[\frac{dx}{2}\right]\right)}{24d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 116: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) dx$$

Optimal (type 3, 198 leaves, 7 steps):

$$2 a^4 (3 A + 2 C) x + \frac{a^4 (2 A + 13 C) \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} + \frac{5 a^4 (2 A - C) \sin[c + dx]}{2 d} + \frac{2 a A \cos[c + dx] (a + a \sec[c + dx])^3 \sin[c + dx]}{3 d} +$$

$$\frac{A \cos[c + dx]^2 (a + a \sec[c + dx])^4 \sin[c + dx]}{3 d} - \frac{(2 A - C) (a^2 + a^2 \sec[c + dx])^2 \sin[c + dx]}{2 d} - \frac{(4 A - 9 C) (a^4 + a^4 \sec[c + dx]) \sin[c + dx]}{3 d}$$

Result (type 3, 1250 leaves):

$$\begin{aligned}
& \frac{(3A + 2C) \times \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{4(A + 2C + A \cos[2c + 2dx])} + \\
& \left(\frac{(-2A - 13C) \cos[c + dx]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{(16d(A + 2C + A \cos[2c + 2dx]))} \right) / \\
& \left(\frac{(2A + 13C) \cos[c + dx]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{(16d(A + 2C + A \cos[2c + 2dx]))} \right) / \\
& \frac{(27A + 4C) \cos[dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[c]}{32d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[2dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[2c]}{8d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[3dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[3c]}{96d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{(27A + 4C) \cos[c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[dx]}{32d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[2c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[2dx]}{8d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{A \cos[3c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin[3dx]}{96d(A + 2C + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{32d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{2d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2)}{32d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{2d(A + 2C + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)}
\end{aligned}$$

■ **Problem 121: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + dx]^4 (A + C \text{Sec}[c + dx]^2)}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 3, 165 leaves, 7 steps):

$$\frac{3(4A + 5C) \text{ArcTanh}[\text{Sin}[c + dx]]}{8ad} - \frac{(3A + 4C) \text{Tan}[c + dx]}{ad} + \frac{3(4A + 5C) \text{Sec}[c + dx] \text{Tan}[c + dx]}{8ad} +$$

$$\frac{(4A + 5C) \text{Sec}[c + dx]^3 \text{Tan}[c + dx]}{4ad} - \frac{(A + C) \text{Sec}[c + dx]^4 \text{Tan}[c + dx]}{d(a + a \text{Sec}[c + dx])} - \frac{(3A + 4C) \text{Tan}[c + dx]^3}{3ad}$$

Result (type 3, 792 leaves):

$$- \frac{3(4A + 5C) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Cos}[c + dx] \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + C \text{Sec}[c + dx]^2)}{2d(A + 2C + A \text{Cos}[2c + 2dx]) (a + a \text{Sec}[c + dx])} +$$

$$\frac{3(4A + 5C) \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Cos}[c + dx] \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + C \text{Sec}[c + dx]^2)}{2d(A + 2C + A \text{Cos}[2c + 2dx]) (a + a \text{Sec}[c + dx])} +$$

$$\frac{1}{192d(A + 2C + A \text{Cos}[2c + 2dx]) (a + a \text{Sec}[c + dx])} \text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c] \text{Sec}[c + dx]^3 (A + C \text{Sec}[c + dx]^2)$$

$$\left(-60A \text{Sin}\left[\frac{dx}{2}\right] - 75C \text{Sin}\left[\frac{dx}{2}\right] - 60A \text{Sin}\left[\frac{3dx}{2}\right] - 91C \text{Sin}\left[\frac{3dx}{2}\right] + 204A \text{Sin}\left[c - \frac{dx}{2}\right] + 219C \text{Sin}\left[c - \frac{dx}{2}\right] - 60A \text{Sin}\left[c + \frac{dx}{2}\right] +$$

$$21C \text{Sin}\left[c + \frac{dx}{2}\right] + 84A \text{Sin}\left[2c + \frac{dx}{2}\right] + 165C \text{Sin}\left[2c + \frac{dx}{2}\right] + 36A \text{Sin}\left[c + \frac{3dx}{2}\right] + 5C \text{Sin}\left[c + \frac{3dx}{2}\right] + 36A \text{Sin}\left[2c + \frac{3dx}{2}\right] +$$

$$69C \text{Sin}\left[2c + \frac{3dx}{2}\right] + 132A \text{Sin}\left[3c + \frac{3dx}{2}\right] + 165C \text{Sin}\left[3c + \frac{3dx}{2}\right] - 156A \text{Sin}\left[c + \frac{5dx}{2}\right] - 211C \text{Sin}\left[c + \frac{5dx}{2}\right] -$$

$$60A \text{Sin}\left[2c + \frac{5dx}{2}\right] - 115C \text{Sin}\left[2c + \frac{5dx}{2}\right] - 60A \text{Sin}\left[3c + \frac{5dx}{2}\right] - 51C \text{Sin}\left[3c + \frac{5dx}{2}\right] + 36A \text{Sin}\left[4c + \frac{5dx}{2}\right] +$$

$$45C \text{Sin}\left[4c + \frac{5dx}{2}\right] - 12A \text{Sin}\left[2c + \frac{7dx}{2}\right] - 19C \text{Sin}\left[2c + \frac{7dx}{2}\right] + 12A \text{Sin}\left[3c + \frac{7dx}{2}\right] + 5C \text{Sin}\left[3c + \frac{7dx}{2}\right] +$$

$$12A \text{Sin}\left[4c + \frac{7dx}{2}\right] + 21C \text{Sin}\left[4c + \frac{7dx}{2}\right] + 36A \text{Sin}\left[5c + \frac{7dx}{2}\right] + 45C \text{Sin}\left[5c + \frac{7dx}{2}\right] - 48A \text{Sin}\left[3c + \frac{9dx}{2}\right] -$$

$$64C \text{Sin}\left[3c + \frac{9dx}{2}\right] - 24A \text{Sin}\left[4c + \frac{9dx}{2}\right] - 40C \text{Sin}\left[4c + \frac{9dx}{2}\right] - 24A \text{Sin}\left[5c + \frac{9dx}{2}\right] - 24C \text{Sin}\left[5c + \frac{9dx}{2}\right] \Big)$$

■ **Problem 122: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + dx]^3 (A + C \text{Sec}[c + dx]^2)}{a + a \text{Sec}[c + dx]} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$-\frac{(2A+3C)\operatorname{ArcTanh}[\sin[c+dx]]}{2ad} + \frac{(3A+4C)\tan[c+dx]}{ad} - \frac{(2A+3C)\sec[c+dx]\tan[c+dx]}{2ad} - \frac{(A+C)\sec[c+dx]^3\tan[c+dx]}{d(a+a\sec[c+dx])} + \frac{(3A+4C)\tan[c+dx]^3}{3ad}$$

Result (type 3, 1090 leaves):

$$\begin{aligned} & \frac{2(2A+3C)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+C\sec[c+dx]^2)}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])} - \\ & \frac{2(2A+3C)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\log\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+C\sec[c+dx]^2)}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])} + \\ & \frac{4\cos\left[\frac{c}{2}+\frac{dx}{2}\right]\cos[c+dx]\sec\left[\frac{c}{2}\right](A+C\sec[c+dx]^2)(A\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])} + \\ & \frac{2C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)\sin\left[\frac{dx}{2}\right]}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^3} - \\ & \frac{2\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)(C\cos\left[\frac{c}{2}\right]-2C\sin\left[\frac{c}{2}\right])}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^2} + \\ & \frac{4\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)(3A\sin\left[\frac{dx}{2}\right]+5C\sin\left[\frac{dx}{2}\right])}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]-\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)} + \\ & \frac{2C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)\sin\left[\frac{dx}{2}\right]}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^3} + \\ & \frac{2\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)(C\cos\left[\frac{c}{2}\right]+2C\sin\left[\frac{c}{2}\right])}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)^2} + \\ & \frac{4\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx](A+C\sec[c+dx]^2)(3A\sin\left[\frac{dx}{2}\right]+5C\sin\left[\frac{dx}{2}\right])}{d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])\left(\cos\left[\frac{c}{2}\right]+\sin\left[\frac{c}{2}\right]\right)\left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right)} \end{aligned}$$

■ **Problem 123: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^2(A+C\sec[c+dx]^2)}{a+a\sec[c+dx]} dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$\frac{(2A + 3C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2ad} - \frac{(A + 2C) \operatorname{Tan}[c + dx]}{ad} + \frac{(2A + 3C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2ad} - \frac{(A + C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{d(a + a \operatorname{Sec}[c + dx])}$$

Result (type 3, 316 leaves):

$$\frac{1}{ad(A + 2C + A \operatorname{Cos}[2(c + dx)])(1 + \operatorname{Sec}[c + dx])} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \operatorname{Cos}[c + dx] (A + C \operatorname{Sec}[c + dx])^2 \left(-4(A + C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right] + \right. \\ \left. \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \left(-2(2A + 3C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 4A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\ \left. 6C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \frac{C}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{C}{\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \right. \\ \left. (4C \operatorname{Sin}[dx]) \right) / \left(\left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \right)$$

■ **Problem 124: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx])^2}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{ad} + \frac{C \operatorname{Tan}[c + dx]}{ad} + \frac{(A + C) \operatorname{Tan}[c + dx]}{ad(1 + \operatorname{Sec}[c + dx])}$$

Result (type 3, 227 leaves):

$$\left(4 \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \operatorname{Cos}[c + dx] (A + C \operatorname{Sec}[c + dx])^2 \right. \\ \left. \left((A + C) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\ \left. \operatorname{Sin}[dx] \right) / \left(\left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) / \\ (ad(A + 2C + A \operatorname{Cos}[2(c + dx)])(1 + \operatorname{Sec}[c + dx]))$$

■ **Problem 125: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{Ax}{a} + \frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{ad} - \frac{(A + C) \operatorname{Tan}[c + dx]}{ad(1 + \operatorname{Sec}[c + dx])}$$

Result (type 3, 143 leaves) :

$$- \left(4 \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] (C + A \operatorname{Cos} [c + d x])^2 \right. \\ \left. - \operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] \left(A d x - C \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] + C \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \right) \right) + \\ \left. (A + C) \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sin} \left[\frac{d x}{2} \right] \right) / (a d (1 + \operatorname{Cos} [c + d x]) (A + 2 C + A \operatorname{Cos} [2 (c + d x)]))$$

■ **Problem 126: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos} [c + d x] (A + C \operatorname{Sec} [c + d x])^2}{a + a \operatorname{Sec} [c + d x]} dx$$

Optimal (type 3, 52 leaves, 4 steps) :

$$-\frac{A x}{a} + \frac{(2 A + C) \operatorname{Sin} [c + d x]}{a d} - \frac{(A + C) \operatorname{Sin} [c + d x]}{d (a + a \operatorname{Sec} [c + d x])}$$

Result (type 3, 108 leaves) :

$$\frac{1}{4 a d} \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right] \\ \left(-2 A d x \operatorname{Cos} \left[\frac{d x}{2} \right] - 2 A d x \operatorname{Cos} \left[c + \frac{d x}{2} \right] + 5 A \operatorname{Sin} \left[\frac{d x}{2} \right] + 4 C \operatorname{Sin} \left[\frac{d x}{2} \right] + A \operatorname{Sin} \left[c + \frac{d x}{2} \right] + A \operatorname{Sin} \left[c + \frac{3 d x}{2} \right] + A \operatorname{Sin} \left[2 c + \frac{3 d x}{2} \right] \right)$$

■ **Problem 130: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec} [c + d x]^4 (A + C \operatorname{Sec} [c + d x])^2}{(a + a \operatorname{Sec} [c + d x])^2} dx$$

Optimal (type 3, 172 leaves, 7 steps) :

$$-\frac{(2 A + 5 C) \operatorname{ArcTanh} [\operatorname{Sin} [c + d x]]}{a^2 d} + \frac{(5 A + 12 C) \operatorname{Tan} [c + d x]}{a^2 d} - \frac{(2 A + 5 C) \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x]}{a^2 d} - \\ \frac{2 (2 A + 5 C) \operatorname{Sec} [c + d x]^3 \operatorname{Tan} [c + d x]}{3 a^2 d (1 + \operatorname{Sec} [c + d x])} - \frac{(A + C) \operatorname{Sec} [c + d x]^4 \operatorname{Tan} [c + d x]}{3 d (a + a \operatorname{Sec} [c + d x])^2} + \frac{(5 A + 12 C) \operatorname{Tan} [c + d x]^3}{3 a^2 d}$$

Result (type 3, 623 leaves) :

$$\frac{1}{24 a^2 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^2} \cos\left[\frac{1}{2}(c + dx)\right] (A + C \sec[c + dx])^2$$

$$\left(192 (2A + 5C) \cos\left[\frac{1}{2}(c + dx)\right]^3 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3 \left(-3 (8A + C) \sin\left[\frac{dx}{2}\right] + (66A + 155C) \sin\left[\frac{3dx}{2}\right] - 60A \sin\left[c - \frac{dx}{2}\right] - 153C \sin\left[c - \frac{dx}{2}\right] + 24A \sin\left[c + \frac{dx}{2}\right] + \right.$$

$$21C \sin\left[c + \frac{dx}{2}\right] - 60A \sin\left[2c + \frac{dx}{2}\right] - 135C \sin\left[2c + \frac{dx}{2}\right] - 4A \sin\left[c + \frac{3dx}{2}\right] + 25C \sin\left[c + \frac{3dx}{2}\right] + 36A \sin\left[2c + \frac{3dx}{2}\right] +$$

$$45C \sin\left[2c + \frac{3dx}{2}\right] - 34A \sin\left[3c + \frac{3dx}{2}\right] - 85C \sin\left[3c + \frac{3dx}{2}\right] + 42A \sin\left[c + \frac{5dx}{2}\right] + 99C \sin\left[c + \frac{5dx}{2}\right] + 21C \sin\left[2c + \frac{5dx}{2}\right] +$$

$$24A \sin\left[3c + \frac{5dx}{2}\right] + 33C \sin\left[3c + \frac{5dx}{2}\right] - 18A \sin\left[4c + \frac{5dx}{2}\right] - 45C \sin\left[4c + \frac{5dx}{2}\right] + 24A \sin\left[2c + \frac{7dx}{2}\right] + 57C \sin\left[2c + \frac{7dx}{2}\right] +$$

$$3A \sin\left[3c + \frac{7dx}{2}\right] + 18C \sin\left[3c + \frac{7dx}{2}\right] + 15A \sin\left[4c + \frac{7dx}{2}\right] + 24C \sin\left[4c + \frac{7dx}{2}\right] - 6A \sin\left[5c + \frac{7dx}{2}\right] - 15C \sin\left[5c + \frac{7dx}{2}\right] +$$

$$\left. \left. 10A \sin\left[3c + \frac{9dx}{2}\right] + 24C \sin\left[3c + \frac{9dx}{2}\right] + 3A \sin\left[4c + \frac{9dx}{2}\right] + 11C \sin\left[4c + \frac{9dx}{2}\right] + 7A \sin\left[5c + \frac{9dx}{2}\right] + 13C \sin\left[5c + \frac{9dx}{2}\right] \right) \right)$$

■ **Problem 131: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + C \sec[c + dx])^2}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 150 leaves, 7 steps):

$$\frac{(2A + 7C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2 d} - \frac{4(A + 4C) \tan[c + dx]}{3a^2 d} +$$

$$\frac{(2A + 7C) \sec[c + dx] \tan[c + dx]}{2a^2 d} - \frac{2(A + 4C) \sec[c + dx]^2 \tan[c + dx]}{3a^2 d (1 + \sec[c + dx])} - \frac{(A + C) \sec[c + dx]^3 \tan[c + dx]}{3d (a + a \sec[c + dx])^2}$$

Result (type 3, 513 leaves):

$$\begin{aligned}
& - \frac{1}{24 a^2 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^2} \cos\left[\frac{1}{2}(c + dx)\right] (A + C \sec[c + dx])^2 \\
& \left(96 (2A + 7C) \cos\left[\frac{1}{2}(c + dx)\right]^3 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\
& \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^2 \left(-2 (10A + 7C) \sin\left[\frac{dx}{2}\right] + (22A + 97C) \sin\left[\frac{3dx}{2}\right] - 36A \sin\left[c - \frac{dx}{2}\right] - 126C \sin\left[c - \frac{dx}{2}\right] + \right. \\
& 36A \sin\left[c + \frac{dx}{2}\right] + 42C \sin\left[c + \frac{dx}{2}\right] - 20A \sin\left[2c + \frac{dx}{2}\right] - 98C \sin\left[2c + \frac{dx}{2}\right] - 18A \sin\left[c + \frac{3dx}{2}\right] - 3C \sin\left[c + \frac{3dx}{2}\right] + \\
& 22A \sin\left[2c + \frac{3dx}{2}\right] + 37C \sin\left[2c + \frac{3dx}{2}\right] - 18A \sin\left[3c + \frac{3dx}{2}\right] - 63C \sin\left[3c + \frac{3dx}{2}\right] + 18A \sin\left[c + \frac{5dx}{2}\right] + 75C \sin\left[c + \frac{5dx}{2}\right] - \\
& 6A \sin\left[2c + \frac{5dx}{2}\right] + 15C \sin\left[2c + \frac{5dx}{2}\right] + 18A \sin\left[3c + \frac{5dx}{2}\right] + 39C \sin\left[3c + \frac{5dx}{2}\right] - 6A \sin\left[4c + \frac{5dx}{2}\right] - 21C \sin\left[4c + \frac{5dx}{2}\right] + \\
& \left. \left. 8A \sin\left[2c + \frac{7dx}{2}\right] + 32C \sin\left[2c + \frac{7dx}{2}\right] + 12C \sin\left[3c + \frac{7dx}{2}\right] + 8A \sin\left[4c + \frac{7dx}{2}\right] + 20C \sin\left[4c + \frac{7dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 132: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^2 (A + C \sec[c + dx])^2}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 99 leaves, 6 steps):

$$- \frac{2C \operatorname{ArcTanh}[\sin[c + dx]]}{a^2 d} + \frac{(A + 4C) \tan[c + dx]}{3 a^2 d} + \frac{2C \tan[c + dx]}{a^2 d (1 + \sec[c + dx])} - \frac{(A + C) \sec[c + dx]^2 \tan[c + dx]}{3 d (a + a \sec[c + dx])^2}$$

Result (type 3, 280 leaves):

$$\begin{aligned}
& \frac{1}{3 a^2 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^2} \\
& 4 \cos\left[\frac{1}{2}(c + dx)\right] (A + C \sec[c + dx])^2 \left((A + C) \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 2 (A + 7C) \cos\left[\frac{1}{2}(c + dx)\right]^2 \sec\left[\frac{c}{2}\right] \sin\left[\frac{dx}{2}\right] + 6C \cos\left[\frac{1}{2}(c + dx)\right]^3 \right. \\
& \left(2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - 2 \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \sin[dx] \right) / \left(\left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \right. \\
& \left. \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) + (A + C) \cos\left[\frac{1}{2}(c + dx)\right] \tan\left[\frac{c}{2}\right]
\end{aligned}$$

■ **Problem 133: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] (A + C \sec[c + dx])^2}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^2 d} + \frac{(A - 5 C) \operatorname{Tan}[c + d x]}{3 a^2 d (1 + \operatorname{Sec}[c + d x])} + \frac{(A + C) \operatorname{Tan}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 3, 377 leaves):

$$\begin{aligned} & - \frac{1}{6 a^2 d (1 + \operatorname{Sec}[c + d x])^2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 \\ & \left(3 C \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 3 C \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\ & 9 C \operatorname{Cos}\left[\frac{d x}{2}\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\ & 9 C \operatorname{Cos}\left[c + \frac{d x}{2}\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\ & 3 C \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 3 C \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \\ & 6 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 18 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 6 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 6 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 4 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 8 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] \end{aligned}$$

■ **Problem 134: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{A x}{a^2} - \frac{2(2A - C) \operatorname{Tan}[c + d x]}{3 a^2 d (1 + \operatorname{Sec}[c + d x])} - \frac{(A + C) \operatorname{Tan}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 3, 141 leaves):

$$\begin{aligned} & \frac{1}{24 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 \left(9 A d x \operatorname{Cos}\left[\frac{d x}{2}\right] + 9 A d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 3 A d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] + \right. \\ & \left. 3 A d x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] - 18 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 6 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 12 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 10 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 2 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] \right) \end{aligned}$$

■ **Problem 135: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x] (A + C \operatorname{Sec}[c + d x]^2)}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 82 leaves, 5 steps):

$$-\frac{2 A x}{a^2} + \frac{(10 A + C) \operatorname{Sin}[c + d x]}{3 a^2 d} - \frac{2 A \operatorname{Sin}[c + d x]}{a^2 d (1 + \operatorname{Sec}[c + d x])} - \frac{(A + C) \operatorname{Sin}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 3, 195 leaves):

$$\frac{1}{48 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\left(-36 A dx \operatorname{Cos}\left[\frac{dx}{2}\right] - 36 A dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] - 12 A dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] - 12 A dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 66 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 12 C \operatorname{Sin}\left[\frac{dx}{2}\right] - 30 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 12 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 41 A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 8 C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 9 A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 3 A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 3 A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right]\right)$$

■ **Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 (A+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$\frac{(7A+2C)x}{2a^2} - \frac{4(4A+C)\operatorname{Sin}[c+dx]}{3a^2d} + \frac{(7A+2C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{2a^2d} - \frac{2(4A+C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{3a^2d(1+\operatorname{Sec}[c+dx])} - \frac{(A+C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{3d(a+a \operatorname{Sec}[c+dx])^2}$$

Result (type 3, 281 leaves):

$$\frac{1}{48 a^2 d (1 + \operatorname{Sec}[c+dx])^2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^2$$

$$\left(36(7A+2C)dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 36(7A+2C)dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 84A dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 24C dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 84A dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 24C dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] - 381A \operatorname{Sin}\left[\frac{dx}{2}\right] - 144C \operatorname{Sin}\left[\frac{dx}{2}\right] + 147A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 96C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 239A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 80C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 63A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 15A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 15A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 3A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] + 3A \operatorname{Sin}\left[4c + \frac{7dx}{2}\right]\right)$$

■ **Problem 137: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^3 (A+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$-\frac{(5A+2C)x}{a^2} + \frac{(12A+5C)\operatorname{Sin}[c+dx]}{a^2d} - \frac{(5A+2C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{a^2d} - \frac{2(5A+2C)\operatorname{Cos}[c+dx]^2\operatorname{Sin}[c+dx]}{3a^2d(1+\operatorname{Sec}[c+dx])} - \frac{(A+C)\operatorname{Cos}[c+dx]^2\operatorname{Sin}[c+dx]}{3d(a+a \operatorname{Sec}[c+dx])^2} - \frac{(12A+5C)\operatorname{Sin}[c+dx]^3}{3a^2d}$$

Result (type 3, 349 leaves):

$$\frac{1}{48 a^2 d (1 + \operatorname{Sec}[c + d x])^2} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2$$

$$\left(-72 (5 A + 2 C) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 72 (5 A + 2 C) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] - 120 A d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 48 C d x \operatorname{Cos}\left[c + \frac{3 d x}{2}\right] - 120 A d x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] - \right.$$

$$48 C d x \operatorname{Cos}\left[2 c + \frac{3 d x}{2}\right] + 516 A \operatorname{Sin}\left[\frac{d x}{2}\right] + 264 C \operatorname{Sin}\left[\frac{d x}{2}\right] - 156 A \operatorname{Sin}\left[c + \frac{d x}{2}\right] - 120 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 342 A \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] +$$

$$164 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 118 A \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 36 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] + 30 A \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 12 C \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] +$$

$$\left. 30 A \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 12 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] - 3 A \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] - 3 A \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] + A \operatorname{Sin}\left[4 c + \frac{9 d x}{2}\right] + A \operatorname{Sin}\left[5 c + \frac{9 d x}{2}\right] \right)$$

■ **Problem 138: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4 (A + C \operatorname{Sec}[c + d x]^2)}{(a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 3, 198 leaves, 8 steps):

$$\frac{(2 A + 13 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^3 d} - \frac{2 (11 A + 76 C) \operatorname{Tan}[c + d x]}{15 a^3 d} + \frac{(2 A + 13 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a^3 d} -$$

$$\frac{(A + C) \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^3} - \frac{(A + 11 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} - \frac{(11 A + 76 C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{15 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 3, 632 leaves):

$$\begin{aligned}
& - \frac{1}{240 a^3 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^3} \cos\left[\frac{1}{2}(c + dx)\right] \sec[c + dx] (A + C \sec[c + dx]^2) \\
& \left(1920 (2A + 13C) \cos\left[\frac{1}{2}(c + dx)\right]^5 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\
& \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^2 \left(-5 (98A + 247C) \sin\left[\frac{dx}{2}\right] + 5 (106A + 761C) \sin\left[\frac{3dx}{2}\right] - 654A \sin\left[c - \frac{dx}{2}\right] - 4329C \sin\left[c - \frac{dx}{2}\right] + \right. \\
& 654A \sin\left[c + \frac{dx}{2}\right] + 1989C \sin\left[c + \frac{dx}{2}\right] - 490A \sin\left[2c + \frac{dx}{2}\right] - 3575C \sin\left[2c + \frac{dx}{2}\right] - 350A \sin\left[c + \frac{3dx}{2}\right] - 475C \sin\left[c + \frac{3dx}{2}\right] + \\
& 530A \sin\left[2c + \frac{3dx}{2}\right] + 2005C \sin\left[2c + \frac{3dx}{2}\right] - 350A \sin\left[3c + \frac{3dx}{2}\right] - 2275C \sin\left[3c + \frac{3dx}{2}\right] + 378A \sin\left[c + \frac{5dx}{2}\right] + \\
& 2673C \sin\left[c + \frac{5dx}{2}\right] - 150A \sin\left[2c + \frac{5dx}{2}\right] + 105C \sin\left[2c + \frac{5dx}{2}\right] + 378A \sin\left[3c + \frac{5dx}{2}\right] + 1593C \sin\left[3c + \frac{5dx}{2}\right] - \\
& 150A \sin\left[4c + \frac{5dx}{2}\right] - 975C \sin\left[4c + \frac{5dx}{2}\right] + 190A \sin\left[2c + \frac{7dx}{2}\right] + 1325C \sin\left[2c + \frac{7dx}{2}\right] - 30A \sin\left[3c + \frac{7dx}{2}\right] + \\
& 255C \sin\left[3c + \frac{7dx}{2}\right] + 190A \sin\left[4c + \frac{7dx}{2}\right] + 875C \sin\left[4c + \frac{7dx}{2}\right] - 30A \sin\left[5c + \frac{7dx}{2}\right] - 195C \sin\left[5c + \frac{7dx}{2}\right] + \\
& \left. \left. 44A \sin\left[3c + \frac{9dx}{2}\right] + 304C \sin\left[3c + \frac{9dx}{2}\right] + 90C \sin\left[4c + \frac{9dx}{2}\right] + 44A \sin\left[5c + \frac{9dx}{2}\right] + 214C \sin\left[5c + \frac{9dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 145 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 C \operatorname{ArcTanh}[\sin[c + dx]]}{a^3 d} + \frac{(2 A + 27 C) \tan[c + dx]}{15 a^3 d} - \\
& \frac{(A + C) \sec[c + dx]^3 \tan[c + dx]}{5 d (a + a \sec[c + dx])^3} + \frac{(A - 9 C) \sec[c + dx]^2 \tan[c + dx]}{15 a d (a + a \sec[c + dx])^2} + \frac{3 C \tan[c + dx]}{d (a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 3, 457 leaves):

$$\frac{1}{60 a^3 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^3} \cos\left[\frac{1}{2}(c + dx)\right] \sec[c + dx] (A + C \sec[c + dx])^2$$

$$\left(2880 C \cos\left[\frac{1}{2}(c + dx)\right]^5 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx] \left(-5(4A + 51C) \sin\left[\frac{dx}{2}\right] + (22A + 567C) \sin\left[\frac{3dx}{2}\right] - 10A \sin\left[c - \frac{dx}{2}\right] - 600C \sin\left[c - \frac{dx}{2}\right] + 10A \sin\left[c + \frac{dx}{2}\right] + \right.$$

$$375C \sin\left[c + \frac{dx}{2}\right] - 20A \sin\left[2c + \frac{dx}{2}\right] - 480C \sin\left[2c + \frac{dx}{2}\right] - 60C \sin\left[c + \frac{3dx}{2}\right] + 22A \sin\left[2c + \frac{3dx}{2}\right] + 402C \sin\left[2c + \frac{3dx}{2}\right] -$$

$$225C \sin\left[3c + \frac{3dx}{2}\right] + 10A \sin\left[c + \frac{5dx}{2}\right] + 315C \sin\left[c + \frac{5dx}{2}\right] + 30C \sin\left[2c + \frac{5dx}{2}\right] + 10A \sin\left[3c + \frac{5dx}{2}\right] + 240C \sin\left[3c + \frac{5dx}{2}\right] -$$

$$\left. \left. 45C \sin\left[4c + \frac{5dx}{2}\right] + 2A \sin\left[2c + \frac{7dx}{2}\right] + 72C \sin\left[2c + \frac{7dx}{2}\right] + 15C \sin\left[3c + \frac{7dx}{2}\right] + 2A \sin\left[4c + \frac{7dx}{2}\right] + 57C \sin\left[4c + \frac{7dx}{2}\right] \right) \right)$$

■ **Problem 142: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 106 leaves, 4 steps):

$$\frac{Ax}{a^3} - \frac{(A + C) \tan[c + dx]}{5d(a + a \sec[c + dx])^3} - \frac{(7A - 3C) \tan[c + dx]}{15ad(a + a \sec[c + dx])^2} - \frac{(22A - 3C) \tan[c + dx]}{15d(a^3 + a^3 \sec[c + dx])}$$

Result (type 3, 227 leaves):

$$\frac{1}{480 a^3 d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^5 \left(150 A dx \cos\left[\frac{dx}{2}\right] + 150 A dx \cos\left[c + \frac{dx}{2}\right] + 75 A dx \cos\left[c + \frac{3dx}{2}\right] + 75 A dx \cos\left[2c + \frac{3dx}{2}\right] + 15 A dx \cos\left[2c + \frac{5dx}{2}\right] + \right.$$

$$15 A dx \cos\left[3c + \frac{5dx}{2}\right] - 370 A \sin\left[\frac{dx}{2}\right] + 30 C \sin\left[\frac{dx}{2}\right] + 270 A \sin\left[c + \frac{dx}{2}\right] - 30 C \sin\left[c + \frac{dx}{2}\right] -$$

$$\left. \left. 230 A \sin\left[c + \frac{3dx}{2}\right] + 30 C \sin\left[c + \frac{3dx}{2}\right] + 90 A \sin\left[2c + \frac{3dx}{2}\right] - 64 A \sin\left[2c + \frac{5dx}{2}\right] + 6 C \sin\left[2c + \frac{5dx}{2}\right] \right)$$

■ **Problem 143: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + C \sec[c + dx])^2}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 120 leaves, 6 steps):

$$-\frac{3Ax}{a^3} + \frac{2(36A + C) \sin[c + dx]}{15a^3 d} - \frac{(A + C) \sin[c + dx]}{5d(a + a \sec[c + dx])^3} - \frac{(9A - C) \sin[c + dx]}{15ad(a + a \sec[c + dx])^2} - \frac{3A \sin[c + dx]}{d(a^3 + a^3 \sec[c + dx])}$$

Result (type 3, 283 leaves):

$$\begin{aligned}
& - \frac{1}{960 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \\
& \left(900 A dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 900 A dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 450 A dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 450 A dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 90 A dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + \right. \\
& \quad 90 A dx \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] - 1755 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 160 C \operatorname{Sin}\left[\frac{dx}{2}\right] + 1125 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 120 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 1215 A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - \\
& \quad 80 C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 225 A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 60 C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 363 A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - \\
& \quad \left. 28 C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 75 A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 15 A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 15 A \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 144: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^2 (A+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\frac{(13A+2C)x}{2a^3} - \frac{2(76A+11C)\operatorname{Sin}[c+dx]}{15a^3d} + \frac{(13A+2C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{2a^3d} - \frac{(A+C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{5d(a+a \operatorname{Sec}[c+dx])^3} - \frac{(11A+C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{15ad(a+a \operatorname{Sec}[c+dx])^2} - \frac{(76A+11C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{15d(a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 3, 385 leaves):

$$\begin{aligned}
& \frac{1}{3840 a^3 d} \\
& \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \left(600 (13A+2C) dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 600 (13A+2C) dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 3900 A dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 600 C dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + \right. \\
& \quad 3900 A dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 600 C dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 780 A dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + 120 C dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + 780 A dx \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + \\
& \quad 120 C dx \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] - 12760 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 2960 C \operatorname{Sin}\left[\frac{dx}{2}\right] + 7560 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 2160 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 9230 A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - \\
& \quad 1840 C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 930 A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 720 C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 2782 A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 512 C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - \\
& \quad \left. 750 A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 105 A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 105 A \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] + 15 A \operatorname{Sin}\left[4c + \frac{9dx}{2}\right] + 15 A \operatorname{Sin}\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 145: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^3 (A+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 216 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(23A + 6C)x}{2a^3} + \frac{4(34A + 9C)\sin[c + dx]}{5a^3d} - \frac{(23A + 6C)\cos[c + dx]\sin[c + dx]}{2a^3d} - \frac{(A + C)\cos[c + dx]^2\sin[c + dx]}{5d(a + a\sec[c + dx])^3} - \\
& \frac{(13A + 3C)\cos[c + dx]^2\sin[c + dx]}{15ad(a + a\sec[c + dx])^2} - \frac{(23A + 6C)\cos[c + dx]^2\sin[c + dx]}{3d(a^3 + a^3\sec[c + dx])} - \frac{4(34A + 9C)\sin[c + dx]^3}{15a^3d}
\end{aligned}$$

Result (type 3, 455 leaves):

$$\begin{aligned}
& - \frac{1}{3840a^3d} \\
& \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^5 \left(600(23A + 6C)dx \cos\left[\frac{dx}{2}\right] + 600(23A + 6C)dx \cos\left[c + \frac{dx}{2}\right] + 6900Adx \cos\left[c + \frac{3dx}{2}\right] + 1800Cdx \cos\left[c + \frac{3dx}{2}\right] + \right. \\
& 6900Adx \cos\left[2c + \frac{3dx}{2}\right] + 1800Cdx \cos\left[2c + \frac{3dx}{2}\right] + 1380Adx \cos\left[2c + \frac{5dx}{2}\right] + 360Cdx \cos\left[2c + \frac{5dx}{2}\right] + \\
& 1380Adx \cos\left[3c + \frac{5dx}{2}\right] + 360Cdx \cos\left[3c + \frac{5dx}{2}\right] - 20410A \sin\left[\frac{dx}{2}\right] - 7020C \sin\left[\frac{dx}{2}\right] + 11110A \sin\left[c + \frac{dx}{2}\right] + 4500C \sin\left[c + \frac{dx}{2}\right] - \\
& 15380A \sin\left[c + \frac{3dx}{2}\right] - 4860C \sin\left[c + \frac{3dx}{2}\right] + 380A \sin\left[2c + \frac{3dx}{2}\right] + 900C \sin\left[2c + \frac{3dx}{2}\right] - 4777A \sin\left[2c + \frac{5dx}{2}\right] - \\
& 1452C \sin\left[2c + \frac{5dx}{2}\right] - 1625A \sin\left[3c + \frac{5dx}{2}\right] - 300C \sin\left[3c + \frac{5dx}{2}\right] - 230A \sin\left[3c + \frac{7dx}{2}\right] - 60C \sin\left[3c + \frac{7dx}{2}\right] - \\
& \left. 230A \sin\left[4c + \frac{7dx}{2}\right] - 60C \sin\left[4c + \frac{7dx}{2}\right] + 20A \sin\left[4c + \frac{9dx}{2}\right] + 20A \sin\left[5c + \frac{9dx}{2}\right] - 5A \sin\left[5c + \frac{11dx}{2}\right] - 5A \sin\left[6c + \frac{11dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 146: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^5 (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 232 leaves, 9 steps):

$$\begin{aligned}
& \frac{(2A + 21C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^4d} - \frac{32(5A + 54C) \tan[c + dx]}{105a^4d} + \frac{(2A + 21C) \sec[c + dx] \tan[c + dx]}{2a^4d} - \frac{(10A + 129C) \sec[c + dx]^3 \tan[c + dx]}{105a^4d(1 + \sec[c + dx])^2} - \\
& \frac{16(5A + 54C) \sec[c + dx]^2 \tan[c + dx]}{105a^4d(1 + \sec[c + dx])} - \frac{(A + C) \sec[c + dx]^5 \tan[c + dx]}{7d(a + a \sec[c + dx])^4} - \frac{2C \sec[c + dx]^4 \tan[c + dx]}{5ad(a + a \sec[c + dx])^3}
\end{aligned}$$

Result (type 3, 890 leaves):

$$\begin{aligned}
& - \frac{16 (2A + 21C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + C \sec[c + dx]^2)}{d (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4} + \\
& \frac{16 (2A + 21C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + C \sec[c + dx]^2)}{d (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4} + \\
& \frac{1}{3360 d (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^4 (A + C \sec[c + dx]^2) \\
& \left(14140 A \sin\left[\frac{dx}{2}\right] + 73206 C \sin\left[\frac{dx}{2}\right] - 15160 A \sin\left[\frac{3dx}{2}\right] - 166668 C \sin\left[\frac{3dx}{2}\right] + 17220 A \sin\left[c - \frac{dx}{2}\right] + \right. \\
& 183162 C \sin\left[c - \frac{dx}{2}\right] - 17220 A \sin\left[c + \frac{dx}{2}\right] - 100842 C \sin\left[c + \frac{dx}{2}\right] + 14140 A \sin\left[2c + \frac{dx}{2}\right] + 155526 C \sin\left[2c + \frac{dx}{2}\right] + \\
& 9800 A \sin\left[c + \frac{3dx}{2}\right] + 37380 C \sin\left[c + \frac{3dx}{2}\right] - 15160 A \sin\left[2c + \frac{3dx}{2}\right] - 101148 C \sin\left[2c + \frac{3dx}{2}\right] + 9800 A \sin\left[3c + \frac{3dx}{2}\right] + \\
& 102900 C \sin\left[3c + \frac{3dx}{2}\right] - 10920 A \sin\left[c + \frac{5dx}{2}\right] - 119364 C \sin\left[c + \frac{5dx}{2}\right] + 4760 A \sin\left[2c + \frac{5dx}{2}\right] + 8820 C \sin\left[2c + \frac{5dx}{2}\right] - \\
& 10920 A \sin\left[3c + \frac{5dx}{2}\right] - 78204 C \sin\left[3c + \frac{5dx}{2}\right] + 4760 A \sin\left[4c + \frac{5dx}{2}\right] + 49980 C \sin\left[4c + \frac{5dx}{2}\right] - 5890 A \sin\left[2c + \frac{7dx}{2}\right] - \\
& 64053 C \sin\left[2c + \frac{7dx}{2}\right] + 1470 A \sin\left[3c + \frac{7dx}{2}\right] - 3885 C \sin\left[3c + \frac{7dx}{2}\right] - 5890 A \sin\left[4c + \frac{7dx}{2}\right] - 44733 C \sin\left[4c + \frac{7dx}{2}\right] + \\
& 1470 A \sin\left[5c + \frac{7dx}{2}\right] + 15435 C \sin\left[5c + \frac{7dx}{2}\right] - 2030 A \sin\left[3c + \frac{9dx}{2}\right] - 21987 C \sin\left[3c + \frac{9dx}{2}\right] + 210 A \sin\left[4c + \frac{9dx}{2}\right] - \\
& 3675 C \sin\left[4c + \frac{9dx}{2}\right] - 2030 A \sin\left[5c + \frac{9dx}{2}\right] - 16107 C \sin\left[5c + \frac{9dx}{2}\right] + 210 A \sin\left[6c + \frac{9dx}{2}\right] + 2205 C \sin\left[6c + \frac{9dx}{2}\right] - \\
& \left. 320 A \sin\left[4c + \frac{11dx}{2}\right] - 3456 C \sin\left[4c + \frac{11dx}{2}\right] - 840 C \sin\left[5c + \frac{11dx}{2}\right] - 320 A \sin\left[6c + \frac{11dx}{2}\right] - 2616 C \sin\left[6c + \frac{11dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 147: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 183 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4C \operatorname{ArcTanh}[\sin[c + dx]]}{a^4 d} + \frac{2(3A + 122C) \tan[c + dx]}{105 a^4 d} + \frac{(3A - 88C) \sec[c + dx]^2 \tan[c + dx]}{105 a^4 d (1 + \sec[c + dx])^2} + \\
& \frac{4C \tan[c + dx]}{a^4 d (1 + \sec[c + dx])} - \frac{(A + C) \sec[c + dx]^4 \tan[c + dx]}{7d (a + a \sec[c + dx])^4} + \frac{2(A - 6C) \sec[c + dx]^3 \tan[c + dx]}{35 a d (a + a \sec[c + dx])^3}
\end{aligned}$$

Result (type 3, 544 leaves):

$$\frac{1}{840 a^4 d (A + 2 C + A \cos[2(c + dx)]) (1 + \sec[c + dx])^4} \cos\left[\frac{1}{2}(c + dx)\right] \sec[c + dx]^2 (A + C \sec[c + dx])^2$$

$$\left(107520 C \cos\left[\frac{1}{2}(c + dx)\right]^7 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx] \left(-70(3A + 154C) \sin\left[\frac{dx}{2}\right] + 28(9A + 671C) \sin\left[\frac{3dx}{2}\right] - 126A \sin\left[c - \frac{dx}{2}\right] - 20524C \sin\left[c - \frac{dx}{2}\right] + \right.$$

$$126A \sin\left[c + \frac{dx}{2}\right] + 14644C \sin\left[c + \frac{dx}{2}\right] - 210A \sin\left[2c + \frac{dx}{2}\right] - 16660C \sin\left[2c + \frac{dx}{2}\right] - 4690C \sin\left[c + \frac{3dx}{2}\right] +$$

$$252A \sin\left[2c + \frac{3dx}{2}\right] + 14378C \sin\left[2c + \frac{3dx}{2}\right] - 9100C \sin\left[3c + \frac{3dx}{2}\right] + 132A \sin\left[c + \frac{5dx}{2}\right] + 11668C \sin\left[c + \frac{5dx}{2}\right] -$$

$$630C \sin\left[2c + \frac{5dx}{2}\right] + 132A \sin\left[3c + \frac{5dx}{2}\right] + 9358C \sin\left[3c + \frac{5dx}{2}\right] - 2940C \sin\left[4c + \frac{5dx}{2}\right] + 42A \sin\left[2c + \frac{7dx}{2}\right] +$$

$$4228C \sin\left[2c + \frac{7dx}{2}\right] + 315C \sin\left[3c + \frac{7dx}{2}\right] + 42A \sin\left[4c + \frac{7dx}{2}\right] + 3493C \sin\left[4c + \frac{7dx}{2}\right] - 420C \sin\left[5c + \frac{7dx}{2}\right] +$$

$$\left. \left. 6A \sin\left[3c + \frac{9dx}{2}\right] + 664C \sin\left[3c + \frac{9dx}{2}\right] + 105C \sin\left[4c + \frac{9dx}{2}\right] + 6A \sin\left[5c + \frac{9dx}{2}\right] + 559C \sin\left[5c + \frac{9dx}{2}\right] \right) \right)$$

■ **Problem 151: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{Ax}{a^4} - \frac{(55A - 8C) \tan[c + dx]}{105 a^4 d (1 + \sec[c + dx])^2} - \frac{8(20A - C) \tan[c + dx]}{105 a^4 d (1 + \sec[c + dx])} - \frac{(A + C) \tan[c + dx]}{7 d (a + a \sec[c + dx])^4} - \frac{2(5A - 2C) \tan[c + dx]}{35 a d (a + a \sec[c + dx])^3}$$

Result (type 3, 315 leaves):

$$\frac{1}{13440 a^4 d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^7$$

$$\left(3675 A dx \cos\left[\frac{dx}{2}\right] + 3675 A dx \cos\left[c + \frac{dx}{2}\right] + 2205 A dx \cos\left[c + \frac{3dx}{2}\right] + 2205 A dx \cos\left[2c + \frac{3dx}{2}\right] + 735 A dx \cos\left[2c + \frac{5dx}{2}\right] + \right.$$

$$735 A dx \cos\left[3c + \frac{5dx}{2}\right] + 105 A dx \cos\left[3c + \frac{7dx}{2}\right] + 105 A dx \cos\left[4c + \frac{7dx}{2}\right] - 9940 A \sin\left[\frac{dx}{2}\right] + 560 C \sin\left[\frac{dx}{2}\right] +$$

$$8260 A \sin\left[c + \frac{dx}{2}\right] - 350 C \sin\left[c + \frac{dx}{2}\right] - 7140 A \sin\left[c + \frac{3dx}{2}\right] + 336 C \sin\left[c + \frac{3dx}{2}\right] + 3780 A \sin\left[2c + \frac{3dx}{2}\right] - 210 C \sin\left[2c + \frac{3dx}{2}\right] -$$

$$\left. \left. 2800 A \sin\left[2c + \frac{5dx}{2}\right] + 182 C \sin\left[2c + \frac{5dx}{2}\right] + 840 A \sin\left[3c + \frac{5dx}{2}\right] - 520 A \sin\left[3c + \frac{7dx}{2}\right] + 26 C \sin\left[3c + \frac{7dx}{2}\right] \right)$$

■ **Problem 152: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$-\frac{4Ax}{a^4} + \frac{2(332A + 3C)\sin[c + dx]}{105a^4d} - \frac{(88A - 3C)\sin[c + dx]}{105a^4d(1 + \sec[c + dx])^2} - \frac{4A\sin[c + dx]}{a^4d(1 + \sec[c + dx])} - \frac{(A + C)\sin[c + dx]}{7d(a + a\sec[c + dx])^4} - \frac{2(6A - C)\sin[c + dx]}{35ad(a + a\sec[c + dx])^3}$$

Result (type 3, 371 leaves):

$$\begin{aligned} & -\frac{1}{26880a^4d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^7 \\ & \left(29400Adx \cos\left[\frac{dx}{2}\right] + 29400Adx \cos\left[c + \frac{dx}{2}\right] + 17640Adx \cos\left[c + \frac{3dx}{2}\right] + 17640Adx \cos\left[2c + \frac{3dx}{2}\right] + 5880Adx \cos\left[2c + \frac{5dx}{2}\right] + \right. \\ & 5880Adx \cos\left[3c + \frac{5dx}{2}\right] + 840Adx \cos\left[3c + \frac{7dx}{2}\right] + 840Adx \cos\left[4c + \frac{7dx}{2}\right] - 60830A \sin\left[\frac{dx}{2}\right] - 2520C \sin\left[\frac{dx}{2}\right] + \\ & 46130A \sin\left[c + \frac{dx}{2}\right] + 2520C \sin\left[c + \frac{dx}{2}\right] - 46116A \sin\left[c + \frac{3dx}{2}\right] - 1764C \sin\left[c + \frac{3dx}{2}\right] + 18060A \sin\left[2c + \frac{3dx}{2}\right] + \\ & 1260C \sin\left[2c + \frac{3dx}{2}\right] - 19292A \sin\left[2c + \frac{5dx}{2}\right] - 588C \sin\left[2c + \frac{5dx}{2}\right] + 2100A \sin\left[3c + \frac{5dx}{2}\right] + 420C \sin\left[3c + \frac{5dx}{2}\right] - \\ & \left. 3791A \sin\left[3c + \frac{7dx}{2}\right] - 144C \sin\left[3c + \frac{7dx}{2}\right] - 735A \sin\left[4c + \frac{7dx}{2}\right] - 105A \sin\left[4c + \frac{9dx}{2}\right] - 105A \sin\left[5c + \frac{9dx}{2}\right] \right) \end{aligned}$$

■ **Problem 153: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 215 leaves, 8 steps):

$$\begin{aligned} & \frac{(21A + 2C)x}{2a^4} - \frac{32(54A + 5C)\sin[c + dx]}{105a^4d} + \frac{(21A + 2C)\cos[c + dx]\sin[c + dx]}{2a^4d} - \frac{(129A + 10C)\cos[c + dx]\sin[c + dx]}{105a^4d(1 + \sec[c + dx])^2} - \\ & \frac{16(54A + 5C)\cos[c + dx]\sin[c + dx]}{105a^4d(1 + \sec[c + dx])} - \frac{(A + C)\cos[c + dx]\sin[c + dx]}{7d(a + a\sec[c + dx])^4} - \frac{2A\cos[c + dx]\sin[c + dx]}{5ad(a + a\sec[c + dx])^3} \end{aligned}$$

Result (type 3, 505 leaves):

$$\frac{1}{107520 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7$$

$$\left(14700(21A+2C)dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 14700(21A+2C)dx \operatorname{Cos}\left[c+\frac{dx}{2}\right] + 185220Adx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + 17640Cdx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + \right.$$

$$185220Adx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 17640Cdx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 61740Adx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] + 5880Cdx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] +$$

$$61740Adx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] + 5880Cdx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] + 8820Adx \operatorname{Cos}\left[3c+\frac{7dx}{2}\right] + 840Cdx \operatorname{Cos}\left[3c+\frac{7dx}{2}\right] + 8820Adx \operatorname{Cos}\left[4c+\frac{7dx}{2}\right] +$$

$$840Cdx \operatorname{Cos}\left[4c+\frac{7dx}{2}\right] - 539490A \operatorname{Sin}\left[\frac{dx}{2}\right] - 79520C \operatorname{Sin}\left[\frac{dx}{2}\right] + 386190A \operatorname{Sin}\left[c+\frac{dx}{2}\right] + 66080C \operatorname{Sin}\left[c+\frac{dx}{2}\right] -$$

$$422478A \operatorname{Sin}\left[c+\frac{3dx}{2}\right] - 57120C \operatorname{Sin}\left[c+\frac{3dx}{2}\right] + 132930A \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] + 30240C \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] - 181461A \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] -$$

$$22400C \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] + 3675A \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] + 6720C \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] - 36003A \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] - 4160C \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] -$$

$$\left.9555A \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] - 945A \operatorname{Sin}\left[4c+\frac{9dx}{2}\right] - 945A \operatorname{Sin}\left[5c+\frac{9dx}{2}\right] + 105A \operatorname{Sin}\left[5c+\frac{11dx}{2}\right] + 105A \operatorname{Sin}\left[6c+\frac{11dx}{2}\right]\right)$$

■ **Problem 154: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^3 (A+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^4} dx$$

Optimal (type 3, 248 leaves, 9 steps):

$$-\frac{2(11A+2C)x}{a^4} + \frac{4(454A+83C) \operatorname{Sin}[c+dx]}{35a^4d} - \frac{2(11A+2C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{a^4d} - \frac{(178A+31C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{105a^4d(1+\operatorname{Sec}[c+dx])^2} -$$

$$\frac{4(11A+2C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3a^4d(1+\operatorname{Sec}[c+dx])} - \frac{(A+C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{7d(a+a \operatorname{Sec}[c+dx])^4} - \frac{2(8A+C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{35ad(a+a \operatorname{Sec}[c+dx])^3} - \frac{4(454A+83C) \operatorname{Sin}[c+dx]^3}{105a^4d}$$

Result (type 3, 575 leaves):

$$\begin{aligned}
& - \frac{1}{107520 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7 \\
& \left(58800(11A+2C)dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 58800(11A+2C)dx \operatorname{Cos}\left[c+\frac{dx}{2}\right] + 388080Adx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + 70560Cdx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + \right. \\
& 388080Adx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 70560Cdx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 129360Adx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] + 23520Cdx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] + \\
& 129360Adx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] + 23520Cdx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] + 18480Adx \operatorname{Cos}\left[3c+\frac{7dx}{2}\right] + 3360Cdx \operatorname{Cos}\left[3c+\frac{7dx}{2}\right] + \\
& 18480Adx \operatorname{Cos}\left[4c+\frac{7dx}{2}\right] + 3360Cdx \operatorname{Cos}\left[4c+\frac{7dx}{2}\right] - 1010660A \operatorname{Sin}\left[\frac{dx}{2}\right] - 243320C \operatorname{Sin}\left[\frac{dx}{2}\right] + \\
& 687260A \operatorname{Sin}\left[c+\frac{dx}{2}\right] + 184520C \operatorname{Sin}\left[c+\frac{dx}{2}\right] - 814107A \operatorname{Sin}\left[c+\frac{3dx}{2}\right] - 184464C \operatorname{Sin}\left[c+\frac{3dx}{2}\right] + \\
& 204645A \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] + 72240C \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] - 357609A \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] - 77168C \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] - \\
& 18025A \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] + 8400C \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] - 72522A \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] - 15164C \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] - \\
& 24010A \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] - 2940C \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] - 2310A \operatorname{Sin}\left[4c+\frac{9dx}{2}\right] - 420C \operatorname{Sin}\left[4c+\frac{9dx}{2}\right] - 2310A \operatorname{Sin}\left[5c+\frac{9dx}{2}\right] - \\
& \left. 420C \operatorname{Sin}\left[5c+\frac{9dx}{2}\right] + 175A \operatorname{Sin}\left[5c+\frac{11dx}{2}\right] + 175A \operatorname{Sin}\left[6c+\frac{11dx}{2}\right] - 35A \operatorname{Sin}\left[6c+\frac{13dx}{2}\right] - 35A \operatorname{Sin}\left[7c+\frac{13dx}{2}\right] \right)
\end{aligned}$$

- **Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] \sqrt{a+a \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 94 leaves, 5 steps):

$$\frac{\sqrt{a} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{A \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d} - \frac{a(A-2C) \operatorname{Tan}[c+dx]}{d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 398 leaves):

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(\frac{1}{2}(-A+4C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}A \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{d}$$

$$\frac{1}{d} 4(-3-2\sqrt{2}) A \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$\frac{\sqrt{a} (3A+8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a A \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{A \operatorname{Cos}[c+dx] \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 410 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{8} A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4} A \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8} A \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right]\right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}}\right) (3A + 8C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

- **Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{\sqrt{a} (5A + 8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a (5A + 8C) \operatorname{Sin}[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aA \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{A \operatorname{Cos}[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 4, 439 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(-\frac{1}{48}(11A+24C)\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{6}(2A+3C)\operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16}A\operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{24}A\operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]\right) + \\
& \frac{1}{d}\left(1 + \frac{3}{2\sqrt{2}}\right)(5A+8C)\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]\operatorname{Sec}[c+dx]\sqrt{a(1+\operatorname{Sec}[c+dx])}\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

■ **Problem 163: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 \sqrt{a+a\operatorname{Sec}[c+dx]} (A+C\operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 196 leaves, 6 steps):

$$\begin{aligned}
& \frac{\sqrt{a}(35A+48C)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{64d} + \frac{a(35A+48C)\operatorname{Sin}[c+dx]}{64d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \\
& \frac{a(35A+48C)\operatorname{Cos}[c+dx]\operatorname{Sin}[c+dx]}{96d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{aA\operatorname{Cos}[c+dx]^2\operatorname{Sin}[c+dx]}{24d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{A\operatorname{Cos}[c+dx]^3\sqrt{a+a\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{4d}
\end{aligned}$$

Result (type 4, 460 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{384}(41A+48C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \quad \left. \frac{1}{48}(11A+12C) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{128}(15A+16C) \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{48}A \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + \frac{1}{64}A \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right] \right) + \\
& \quad \frac{1}{(-64+48\sqrt{2})d} (35A+48C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \quad \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \quad \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

- **Problem 169: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^2 (a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2}(7A+8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a^2(5A-8C) \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} - \\
& \frac{a(A-4C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2d} + \frac{A \operatorname{Cos}[c+dx] (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2d}
\end{aligned}$$

Result (type 4, 424 leaves):

$$\frac{1}{2} \left(\frac{1}{d} \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \left(-\frac{1}{8}(5A-16C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{3}{4}A \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}A \sin\left[\frac{5}{2}(c+dx)\right] \right) + \right.$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (7A+8C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right] \right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\left. \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right)$$

- **Problem 171: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+a\sec[c+dx])^{3/2} (A+C\sec[c+dx]^2) dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{a^{3/2} (75A+112C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{64d} + \frac{a^2 (75A+112C) \sin[c+dx]}{64d \sqrt{a+a\sec[c+dx]}} + \frac{a^2 (13A+16C) \cos[c+dx] \sin[c+dx]}{32d \sqrt{a+a\sec[c+dx]}} +$$

$$\frac{aA \cos[c+dx]^2 \sqrt{a+a\sec[c+dx]} \sin[c+dx]}{8d} + \frac{A \cos[c+dx]^3 (a+a\sec[c+dx])^{3/2} \sin[c+dx]}{4d}$$

Result (type 4, 534 leaves):

$$\begin{aligned}
& \left(\cos[c+dx]^3 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+C\operatorname{Sec}[c+dx]^2) \left(-\frac{1}{128} (43A+80C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{3}{16} (3A+4C) \sin\left[\frac{3}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{128} (23A+16C) \sin\left[\frac{5}{2}(c+dx)\right] + \frac{1}{16} A \sin\left[\frac{7}{2}(c+dx)\right] + \frac{1}{64} A \sin\left[\frac{9}{2}(c+dx)\right] \right) \right) / (d(A+2C+A\cos[2c+2dx])) + \\
& \frac{1}{(-64+48\sqrt{2})d(A+2C+A\cos[2c+2dx])} (75A+112C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \\
& \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \cos[c+dx]^2 \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+C\operatorname{Sec}[c+dx]^2) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

- **Problem 172: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 (a+a\operatorname{Sec}[c+dx])^{3/2} (A+C\operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 245 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{3/2} (133A+176C) \operatorname{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{128d} + \frac{a^2 (133A+176C) \sin[c+dx]}{128d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{a^2 (133A+176C) \cos[c+dx] \sin[c+dx]}{192d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \\
& \frac{a^2 (67A+80C) \cos[c+dx]^2 \sin[c+dx]}{240d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{3aA\cos[c+dx]^3 \sqrt{a+a\operatorname{Sec}[c+dx]} \sin[c+dx]}{40d} + \frac{A\cos[c+dx]^4 (a+a\operatorname{Sec}[c+dx])^{3/2} \sin[c+dx]}{5d}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^3 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} (A+C \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. - \frac{(1019 A+1360 C) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{3840} + \frac{1}{480} (239 A+280 C) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + \frac{1}{256} (49 A+48 C) \operatorname{Sin}\left[\frac{5}{2}(c+d x)\right] + \right. \\
& \quad \left. \frac{1}{240} (17 A+10 C) \operatorname{Sin}\left[\frac{7}{2}(c+d x)\right] + \frac{3}{128} A \operatorname{Sin}\left[\frac{9}{2}(c+d x)\right] + \frac{1}{160} A \operatorname{Sin}\left[\frac{11}{2}(c+d x)\right] \right) / (d(A+2 C+A \cos [2 c+2 d x])) + \\
& \quad \frac{1}{64 d(A+2 C+A \cos [2 c+2 d x])} (4+3 \sqrt{2})(133 A+176 C) \cos \left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \\
& \quad \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]^2 \\
& \quad \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right) \\
& \quad \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\operatorname{Sec}[c+d x]))^{3/2} (A+C \operatorname{Sec}[c+d x]^2) \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}
\end{aligned}$$

- **Problem 178: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+a \operatorname{Sec}[c+d x])^{5/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (19 A+8 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{4 d} + \frac{a^3 (27 A-56 C) \operatorname{Sin}[c+d x]}{12 d \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{a^2 (A-8 C) \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 d} - \\
& \frac{a (3 A-4 C) (a+a \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{6 d} + \frac{A \cos [c+d x] (a+a \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{2 d}
\end{aligned}$$

Result (type 4, 455 leaves):

$$\frac{1}{2} \left(\frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \right. \\ \left. \left(-\frac{1}{48} (27A-128C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} C \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \frac{5}{8} A \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16} A \sin\left[\frac{5}{2}(c+dx)\right] \right) \right) + \\ \frac{1}{d} \left(1 + \frac{3}{2\sqrt{2}} \right) (19A+8C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right] \right) \\ \cos[c+dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\ \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \\ \left. \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right)$$

- **Problem 180: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+a\sec[c+dx])^{5/2} (A+C\sec[c+dx])^2 dx$$

Optimal (type 3, 200 leaves, 6 steps):

$$\frac{a^{5/2} (163A+304C) \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{64d} + \frac{a^3 (299A+432C) \sin[c+dx]}{192d\sqrt{a+a\sec[c+dx]}} + \frac{a^2 (17A+16C) \cos[c+dx] \sqrt{a+a\sec[c+dx]} \sin[c+dx]}{32d} + \\ \frac{5aA\cos[c+dx]^2 (a+a\sec[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{A\cos[c+dx]^3 (a+a\sec[c+dx])^{5/2} \sin[c+dx]}{4d}$$

Result (type 4, 535 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \left(-\frac{1}{768} (265 A+432 C) \sin\left[\frac{1}{2}(c+d x)\right] + \right. \right. \\
& \quad \left. \left. \frac{5}{96} (11 A+12 C) \sin\left[\frac{3}{2}(c+d x)\right] + \frac{1}{256} (47 A+16 C) \sin\left[\frac{5}{2}(c+d x)\right] + \frac{5}{96} A \sin\left[\frac{7}{2}(c+d x)\right] + \frac{1}{128} A \sin\left[\frac{9}{2}(c+d x)\right] \right) \right) / \\
& (d(A+2 C+A \cos [2 c+2 d x])) + \frac{1}{64 d(A+2 C+A \cos [2 c+2 d x])} (4+3 \sqrt{2})(163 A+304 C) \cos\left[\frac{1}{4}(c+d x)\right]^4 \\
& \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]}{1+\cos\left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]^3 \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \sqrt{3-2 \sqrt{2}-\tan\left[\frac{1}{4}(c+d x)\right]^2}
\end{aligned}$$

- **Problem 181: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^5 (a+a \operatorname{Sec}[c+d x])^{5/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 245 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2} (283 A+400 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{128 d} + \frac{a^3 (283 A+400 C) \sin [c+d x]}{128 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \\
& \frac{a^3 (787 A+1040 C) \cos [c+d x] \sin [c+d x]}{960 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 (79 A+80 C) \cos [c+d x]^2 \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{240 d} + \\
& \frac{a A \cos [c+d x]^3 (a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{8 d} + \frac{A \cos [c+d x]^4 (a+a \operatorname{Sec}[c+d x])^{5/2} \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 4, 556 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \right. \\
& \left. - \frac{(2309 A+3760 C) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{7680} + \frac{1}{960} (509 A+640 C) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + \frac{5}{512} (19 A+16 C) \operatorname{Sin}\left[\frac{5}{2}(c+d x)\right] + \right. \\
& \left. \frac{1}{240} (16 A+5 C) \operatorname{Sin}\left[\frac{7}{2}(c+d x)\right] + \frac{5}{256} A \operatorname{Sin}\left[\frac{9}{2}(c+d x)\right] + \frac{1}{320} A \operatorname{Sin}\left[\frac{11}{2}(c+d x)\right] \right) / (d(A+2 C+A \cos [2 c+2 d x])) + \\
& \frac{1}{64 d(A+2 C+A \cos [2 c+2 d x])} \left(2+\frac{3}{\sqrt{2}}\right) (283 A+400 C) \cos \left[\frac{1}{4}(c+d x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \\
& \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]^3 \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}
\end{aligned}$$

■ **Problem 182: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^6 (a+a \operatorname{Sec}[c+d x])^{5/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 290 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^{5/2} (1015 A+1304 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{512 d} + \frac{a^3 (1015 A+1304 C) \operatorname{Sin}[c+d x]}{512 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^3 (1015 A+1304 C) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{768 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \\
& \frac{a^3 (109 A+136 C) \operatorname{Cos}[c+d x]^2 \operatorname{Sin}[c+d x]}{192 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2 (23 A+24 C) \operatorname{Cos}[c+d x]^3 \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{96 d} + \\
& \frac{a A \operatorname{Cos}[c+d x]^4 (a+a \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{12 d} + \frac{A \operatorname{Cos}[c+d x]^5 (a+a \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{6 d}
\end{aligned}$$

Result (type 4, 577 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \right. \\
& \left. \left(-\frac{(1589 A+2120 C) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{6144} + \frac{11}{384} (17 A+20 C) \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + \frac{(1145 A+1128 C) \operatorname{Sin}\left[\frac{5}{2}(c+d x)\right]}{6144} + \right. \right. \\
& \left. \left. \frac{1}{384} (29 A+20 C) \operatorname{Sin}\left[\frac{7}{2}(c+d x)\right] + \frac{(83 A+24 C) \operatorname{Sin}\left[\frac{9}{2}(c+d x)\right]}{3072} + \frac{1}{128} A \operatorname{Sin}\left[\frac{11}{2}(c+d x)\right] + \frac{1}{768} A \operatorname{Sin}\left[\frac{13}{2}(c+d x)\right] \right) \right) / \\
& (d(A+2 C+A \cos [2 c+2 d x])) + \frac{1}{512 d(A+2 C+A \cos [2 c+2 d x])} (4+3 \sqrt{2}) (1015 A+1304 C) \cos \left[\frac{1}{4}(c+d x)\right]^4 \\
& \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]}{1+\cos \left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]^3 \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \sqrt{3-2 \sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+d x)\right]^2}
\end{aligned}$$

■ **Problem 183: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^4 (A+C \operatorname{Sec}[c+d x]^2)}{\sqrt{a+a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 236 leaves, 7 steps):

$$\begin{aligned}
& -\frac{\sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} + \frac{4(147 A+143 C) \operatorname{Tan}[c+d x]}{315 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{2(21 A+19 C) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{105 d \sqrt{a+a \operatorname{Sec}[c+d x]}} \\
& \frac{2 C \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{63 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{2 C \operatorname{Sec}[c+d x]^4 \operatorname{Tan}[c+d x]}{9 d \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{2(21 A+29 C) \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{315 a d}
\end{aligned}$$

Result (type 3, 474 leaves):

$$\frac{1}{(A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])}} \cos[c + dx]^2 \sqrt{(1 + \cos[c + dx]) \sec[c + dx]} \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx]^2)$$

$$\left(\frac{8(-84A - 126C + 273A \cos[c] + 257C \cos[c]) \sin\left[\frac{c}{2}\right]}{315d \left(\cos\left[\frac{c}{2}\right] + \cos\left[\frac{3c}{2}\right]\right)} + \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (357A \sin\left[\frac{dx}{2}\right] + 383C \sin\left[\frac{dx}{2}\right])}{315d} + \right.$$

$$\frac{4C \sec[c] \sec[c + dx]^4 \sin[dx]}{9d} + \frac{4 \sec[c] \sec[c + dx] (63A \sin[c] + 97C \sin[c] - 84A \sin[dx] - 126C \sin[dx])}{315d} -$$

$$\left. \frac{4 \sec[c] \sec[c + dx]^2 (40C \sin[c] - 63A \sin[dx] - 97C \sin[dx])}{315d} + \frac{4 \sec[c] \sec[c + dx]^3 (7C \sin[c] - 8C \sin[dx])}{63d} \right) +$$

$$\left(2\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \sec[c + dx]}}\right] \cos[c + dx]^4 \sqrt{-1 + \sec[c + dx]} (1 + \sec[c + dx])^2 (A + C \sec[c + dx]^2) \sin[c + dx] \right) /$$

$$\left(d(1 + \cos[c + dx]) \sqrt{1 - \cos[c + dx]^2} (A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])} \sqrt{\cos[c + dx]^2 (-1 + \sec[c + dx]) (1 + \sec[c + dx])} \right)$$

■ **Problem 184: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + C \sec[c + dx]^2)}{\sqrt{a + a \sec[c + dx]}} dx$$

Optimal (type 3, 193 leaves, 6 steps):

$$\frac{\sqrt{2} (A + C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{\sqrt{a} d} - \frac{4(35A + 37C) \tan[c + dx]}{105d \sqrt{a + a \sec[c + dx]}} -$$

$$\frac{2C \sec[c + dx]^2 \tan[c + dx]}{35d \sqrt{a + a \sec[c + dx]}} + \frac{2C \sec[c + dx]^3 \tan[c + dx]}{7d \sqrt{a + a \sec[c + dx]}} + \frac{2(35A + 31C) \sqrt{a + a \sec[c + dx]} \tan[c + dx]}{105ad}$$

Result (type 3, 432 leaves):

$$\left(\cos [c+d x]^2 \sqrt{(1+\cos [c+d x]) \sec [c+d x]} \sqrt{1+\sec [c+d x]} (A+C \sec [c+d x]^2) \right. \\ \left. - \frac{8(-35 A-49 C+35 A \cos [c]+43 C \cos [c]) \sin \left[\frac{c}{2}\right]}{105 d\left(\cos \left[\frac{c}{2}\right]+\cos \left[\frac{3 c}{2}\right]\right)} - \frac{8 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]\left(35 A \sin \left[\frac{d x}{2}\right]+46 C \sin \left[\frac{d x}{2}\right]\right)}{105 d} + \right. \\ \left. \frac{4 C \sec [c] \sec [c+d x]^3 \sin [d x]}{7 d} - \frac{4 \sec [c] \sec [c+d x]\left(18 C \sin [c]-35 A \sin [d x]-49 C \sin [d x]\right)}{105 d} + \right. \\ \left. \frac{4 \sec [c] \sec [c+d x]^2\left(5 C \sin [c]-6 C \sin [d x]\right)}{35 d} \right) / \left((A+2 C+A \cos [2 c+2 d x]) \sqrt{a(1+\sec [c+d x])} \right) - \\ \left(2 \sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [c+d x]}}\right] \cos [c+d x]^4 \sqrt{-1+\sec [c+d x]} (1+\sec [c+d x])^2 (A+C \sec [c+d x]^2) \sin [c+d x] \right) / \\ \left(d(1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} (A+2 C+A \cos [2 c+2 d x]) \sqrt{a(1+\sec [c+d x])} \sqrt{\cos [c+d x]^2(-1+\sec [c+d x]) (1+\sec [c+d x])} \right)$$

■ **Problem 185: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c+d x]^2 (A+C \sec [c+d x]^2)}{\sqrt{a+a \sec [c+d x]}} dx$$

Optimal (type 3, 152 leaves, 5 steps):

$$-\frac{\sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{2} \sqrt{a+a \sec [c+d x]}}\right]}{\sqrt{a} d} + \frac{2(15 A+14 C) \tan [c+d x]}{15 d \sqrt{a+a \sec [c+d x]}} + \frac{2 C \sec [c+d x]^2 \tan [c+d x]}{5 d \sqrt{a+a \sec [c+d x]}} - \frac{2 C \sqrt{a+a \sec [c+d x]} \tan [c+d x]}{15 a d}$$

Result (type 3, 392 leaves):

$$\left(\cos [c+d x]^2 \sqrt{(1+\cos [c+d x]) \sec [c+d x]} \sqrt{1+\sec [c+d x]} (A+C \sec [c+d x]^2) \right. \\ \left. \frac{8(-4 C+15 A \cos [c]+13 C \cos [c]) \sin \left[\frac{c}{2}\right]}{15 d\left(\cos \left[\frac{c}{2}\right]+\cos \left[\frac{3 c}{2}\right]\right)} + \frac{4 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]\left(15 A \sin \left[\frac{d x}{2}\right]+17 C \sin \left[\frac{d x}{2}\right]\right)}{15 d} + \frac{4 C \sec [c] \sec [c+d x]^2 \sin [d x]}{5 d} + \right. \\ \left. \frac{4 \sec [c] \sec [c+d x]\left(3 C \sin [c]-4 C \sin [d x]\right)}{15 d} \right) / \left((A+2 C+A \cos [2 c+2 d x]) \sqrt{a(1+\sec [c+d x])} \right) + \\ \left(2 \sqrt{2}(A+C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [c+d x]}}\right] \cos [c+d x]^4 \sqrt{-1+\sec [c+d x]} (1+\sec [c+d x])^2 (A+C \sec [c+d x]^2) \sin [c+d x] \right) / \\ \left(d(1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} (A+2 C+A \cos [2 c+2 d x]) \sqrt{a(1+\sec [c+d x])} \sqrt{\cos [c+d x]^2(-1+\sec [c+d x]) (1+\sec [c+d x])} \right)$$

■ **Problem 192: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^4 (A + C \text{Sec}[c + d x]^2)}{(a + a \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 259 leaves, 7 steps):

$$\frac{(11 A + 19 C) \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c + d x]}{\sqrt{2} \sqrt{a + a \text{Sec}[c + d x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A + C) \text{Sec}[c + d x]^4 \text{Tan}[c + d x]}{2 d (a + a \text{Sec}[c + d x])^{3/2}} - \frac{(455 A + 799 C) \text{Tan}[c + d x]}{105 a d \sqrt{a + a \text{Sec}[c + d x]}} -$$

$$\frac{(35 A + 67 C) \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{70 a d \sqrt{a + a \text{Sec}[c + d x]}} + \frac{(7 A + 11 C) \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{14 a d \sqrt{a + a \text{Sec}[c + d x]}} + \frac{(245 A + 397 C) \sqrt{a + a \text{Sec}[c + d x]} \text{Tan}[c + d x]}{210 a^2 d}$$

Result (type 3, 528 leaves):

$$\frac{1}{(A + 2 C + A \text{Cos}[2 c + 2 d x]) (a (1 + \text{Sec}[c + d x]))^{3/2}} \text{Cos}[c + d x]^2 \sqrt{(1 + \text{Cos}[c + d x]) \text{Sec}[c + d x]}$$

$$(1 + \text{Sec}[c + d x])^{3/2} (A + C \text{Sec}[c + d x]^2) \left(- \frac{2 (-140 A - 448 C + 665 A \text{Cos}[c] + 1201 C \text{Cos}[c]) \text{Sin}\left[\frac{c}{2}\right]}{105 d (\text{Cos}\left[\frac{c}{2}\right] + \text{Cos}\left[\frac{3c}{2}\right])} + \right.$$

$$\frac{\text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A \text{Sin}\left[\frac{c}{2}\right] + C \text{Sin}\left[\frac{c}{2}\right])}{2 d} + \frac{\text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-805 A \text{Sin}\left[\frac{dx}{2}\right] - 1649 C \text{Sin}\left[\frac{dx}{2}\right])}{105 d} +$$

$$\frac{\text{Sec}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \text{Sin}\left[\frac{dx}{2}\right] + C \text{Sin}\left[\frac{dx}{2}\right])}{2 d} + \frac{4 C \text{Sec}[c] \text{Sec}[c + d x]^3 \text{Sin}[d x]}{7 d} -$$

$$\left. \frac{4 \text{Sec}[c] \text{Sec}[c + d x] (39 C \text{Sin}[c] - 35 A \text{Sin}[d x] - 112 C \text{Sin}[d x])}{105 d} + \frac{4 \text{Sec}[c] \text{Sec}[c + d x]^2 (5 C \text{Sin}[c] - 13 C \text{Sin}[d x])}{35 d} \right) -$$

$$\left((11 A + 19 C) \text{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \text{Sec}[c + d x]}}\right] \text{Cos}[c + d x]^4 \sqrt{-1 + \text{Sec}[c + d x]} (1 + \text{Sec}[c + d x])^3 (A + C \text{Sec}[c + d x]^2) \text{Sin}[c + d x] \right) /$$

$$\left(\sqrt{2} d (1 + \text{Cos}[c + d x]) \sqrt{1 - \text{Cos}[c + d x]^2} (A + 2 C + A \text{Cos}[2 c + 2 d x]) \right.$$

$$\left. (a (1 + \text{Sec}[c + d x]))^{3/2} \sqrt{\text{Cos}[c + d x]^2 (-1 + \text{Sec}[c + d x]) (1 + \text{Sec}[c + d x])} \right)$$

■ **Problem 193: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3 (A + C \text{Sec}[c + d x]^2)}{(a + a \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 214 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(7A + 15C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A+C) \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{2d (a+a \operatorname{Sec}[c+dx])^{3/2}} + \\
& \frac{(15A + 31C) \operatorname{Tan}[c+dx]}{5ad \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{(5A + 9C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{10ad \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{(5A + 13C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{10a^2 d}
\end{aligned}$$

Result (type 3, 490 leaves):

$$\begin{aligned}
& \frac{1}{(A + 2C + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c + dx]))^{3/2}} \\
& \operatorname{Cos}[c + dx]^2 \sqrt{(1 + \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]} (1 + \operatorname{Sec}[c + dx])^{3/2} (A + C \operatorname{Sec}[c + dx]^2) \left(\frac{2(-12C + 25A \operatorname{Cos}[c] + 49C \operatorname{Cos}[c]) \operatorname{Sin}\left[\frac{c}{2}\right]}{5d (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Cos}\left[\frac{3c}{2}\right])} + \right. \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-A \operatorname{Sin}\left[\frac{c}{2}\right] - C \operatorname{Sin}\left[\frac{c}{2}\right])}{2d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \operatorname{Sin}\left[\frac{dx}{2}\right] - C \operatorname{Sin}\left[\frac{dx}{2}\right])}{2d} + \\
& \left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (25A \operatorname{Sin}\left[\frac{dx}{2}\right] + 61C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5d} + \frac{4C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[dx]}{5d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (C \operatorname{Sin}[c] - 3C \operatorname{Sin}[dx])}{5d} \right) + \\
& \left(\frac{(7A + 15C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \operatorname{Sec}[c + dx]}}\right] \operatorname{Cos}[c + dx]^4 \sqrt{-1 + \operatorname{Sec}[c + dx]} (1 + \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c + dx]}{\sqrt{2} d (1 + \operatorname{Cos}[c + dx]) \sqrt{1 - \operatorname{Cos}[c + dx]}^2 (A + 2C + A \operatorname{Cos}[2c + 2dx])} \right) / \\
& (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \sqrt{\operatorname{Cos}[c + dx]^2 (-1 + \operatorname{Sec}[c + dx]) (1 + \operatorname{Sec}[c + dx])}
\end{aligned}$$

■ **Problem 194: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2 (A + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\begin{aligned}
& \frac{(3A + 11C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A+C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{2d (a+a \operatorname{Sec}[c+dx])^{3/2}} - \frac{(3A + 13C) \operatorname{Tan}[c+dx]}{3ad \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{(3A + 7C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{6a^2 d}
\end{aligned}$$

Result (type 3, 458 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^2 \sqrt{(1+\cos [c+d x]) \sec [c+d x]} (1+\sec [c+d x])^{3/2} \right. \\
& (A+C \sec [c+d x])^2 \left(-\frac{2(-4 C+3 A \cos [c]+19 C \cos [c]) \sin \left[\frac{c}{2}\right]}{3 d\left(\cos \left[\frac{c}{2}\right]+\cos \left[\frac{3 c}{2}\right]\right)}+\frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^2\left(A \sin \left[\frac{c}{2}\right]+C \sin \left[\frac{c}{2}\right]\right)}{2 d} \right. \\
& \left. \left. +\frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]\left(-3 A \sin \left[\frac{d x}{2}\right]-23 C \sin \left[\frac{d x}{2}\right]\right)}{3 d}+\frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^3\left(A \sin \left[\frac{d x}{2}\right]+C \sin \left[\frac{d x}{2}\right]\right)}{2 d}+\frac{4 C \sec [c] \sec [c+d x] \sin [d x]}{3 d} \right) \right) / \\
& \left((A+2 C+A \cos [2 c+2 d x]) (a(1+\sec [c+d x]))^{3/2} \right)-\left((3 A+11 C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [c+d x]}}\right] \cos [c+d x]^4 \right. \\
& \left. \sqrt{-1+\sec [c+d x]} (1+\sec [c+d x])^3 (A+C \sec [c+d x])^2 \sin [c+d x] \right) / \\
& \left(\sqrt{2} d(1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} (A+2 C+A \cos [2 c+2 d x]) (a(1+\sec [c+d x]))^{3/2} \right. \\
& \left. \sqrt{\cos [c+d x]^2(-1+\sec [c+d x])(1+\sec [c+d x])} \right)
\end{aligned}$$

■ **Problem 204: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \sec [c+d x]^2}{(a+a \sec [c+d x])^{5/2}} dx$$

Optimal (type 3, 162 leaves, 7 steps):

$$\frac{2 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right]}{a^{5/2} d}-\frac{(43 A-5 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{2} \sqrt{a+a \sec [c+d x]}}\right]}{16 \sqrt{2} a^{5/2} d}-\frac{(A+C) \tan [c+d x]}{4 d(a+a \sec [c+d x])^{5/2}}-\frac{(11 A-5 C) \tan [c+d x]}{16 a d(a+a \sec [c+d x])^{3/2}}$$

Result (type 3, 725 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^2 (1+\operatorname{Sec}[c+d x])^{5 / 2} (A+C \operatorname{Sec}[c+d x]^2) \right. \\
& \left. - \left(\sqrt{2} (-11 A+5 C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\operatorname{Sec}[c+d x]}}\right] \cos [c+d x]^2 \sqrt{-1+\operatorname{Sec}[c+d x]} (1+\operatorname{Sec}[c+d x])^{3 / 2} \sin [c+d x] \right) \right) / \\
& \left(d (1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} \sqrt{\cos [c+d x]^2 (-1+\operatorname{Sec}[c+d x]) (1+\operatorname{Sec}[c+d x])} \right) + \\
& \left(32 A \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\operatorname{Sec}[c+d x]}}\right] + \operatorname{ArcTan}\left[\frac{-2+\sqrt{1+\operatorname{Sec}[c+d x]}}{\sqrt{-1+\operatorname{Sec}[c+d x]}}\right] - \operatorname{ArcTan}\left[\frac{2+\sqrt{1+\operatorname{Sec}[c+d x]}}{\sqrt{-1+\operatorname{Sec}[c+d x]}}\right] \right) \right. \\
& \left. \cos [c+d x]^2 \sqrt{-1+\operatorname{Sec}[c+d x]} (1+\operatorname{Sec}[c+d x])^{3 / 2} \sin [c+d x] \right) / \\
& \left. \left(d (1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} \sqrt{\cos [c+d x]^2 (-1+\operatorname{Sec}[c+d x]) (1+\operatorname{Sec}[c+d x])} \right) \right) / \\
& \left(16 (A+2 C+A \cos [2 c+2 d x]) (a (1+\operatorname{Sec}[c+d x]))^{5 / 2} \right) + \frac{1}{(A+2 C+A \cos [2 c+2 d x]) (a (1+\operatorname{Sec}[c+d x]))^{5 / 2}} \\
& \frac{\cos [c+d x]^2 \sqrt{(1+\cos [c+d x]) \operatorname{Sec}[c+d x]}}{(1+\operatorname{Sec}[c+d x])^{5 / 2} (A+C \operatorname{Sec}[c+d x]^2)} \\
& \left(\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (-A \sin \left[\frac{c}{2}\right]-C \sin \left[\frac{c}{2}\right])}{8 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 (19 A \sin \left[\frac{c}{2}\right]+3 C \sin \left[\frac{c}{2}\right])}{16 d} + \right. \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^5 (-A \sin \left[\frac{d x}{2}\right]-C \sin \left[\frac{d x}{2}\right])}{8 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right] (-15 A \sin \left[\frac{d x}{2}\right]+C \sin \left[\frac{d x}{2}\right])}{8 d} + \\
& \left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^3 (19 A \sin \left[\frac{d x}{2}\right]+3 C \sin \left[\frac{d x}{2}\right])}{16 d} - \frac{(15 A-C) \tan \left[\frac{c}{2}\right]}{8 d} \right)
\end{aligned}$$

■ **Problem 205: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] (A+C \operatorname{Sec}[c+d x]^2)}{(a+a \operatorname{Sec}[c+d x])^{5 / 2}} dx$$

Optimal (type 3, 199 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5 A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} + \frac{(115 A + 3 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \\
& \frac{(A+C) \operatorname{Sin}[c+dx]}{4 d (a+a \operatorname{Sec}[c+dx])^{5/2}} - \frac{(15 A-C) \operatorname{Sin}[c+dx]}{16 a d (a+a \operatorname{Sec}[c+dx])^{3/2}} + \frac{(35 A+3 C) \operatorname{Sin}[c+dx]}{16 a^2 d \sqrt{a+a \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 3, 739 leaves):

$$\begin{aligned}
& \left(\operatorname{Cos}[c+dx]^2 (1+\operatorname{Sec}[c+dx])^{5/2} (A+C \operatorname{Sec}[c+dx])^2 \right. \\
& \left. \left(\left(\sqrt{2} (-35 A-3 C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\operatorname{Sec}[c+dx]}}\right] \operatorname{Cos}[c+dx]^2 \sqrt{-1+\operatorname{Sec}[c+dx]} (1+\operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx] \right) / \right. \right. \\
& \left. \left(d (1+\operatorname{Cos}[c+dx]) \sqrt{1-\operatorname{Cos}[c+dx]^2} \sqrt{\operatorname{Cos}[c+dx]^2 (-1+\operatorname{Sec}[c+dx]) (1+\operatorname{Sec}[c+dx])} \right) - \right. \\
& \left. \left(80 A \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\operatorname{Sec}[c+dx]}}\right] + \operatorname{ArcTan}\left[\frac{-2+\sqrt{1+\operatorname{Sec}[c+dx]}}{\sqrt{-1+\operatorname{Sec}[c+dx]}}\right] - \operatorname{ArcTan}\left[\frac{2+\sqrt{1+\operatorname{Sec}[c+dx]}}{\sqrt{-1+\operatorname{Sec}[c+dx]}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Cos}[c+dx]^2 \sqrt{-1+\operatorname{Sec}[c+dx]} (1+\operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx] \right) / \right. \\
& \left. \left. \left(d (1+\operatorname{Cos}[c+dx]) \sqrt{1-\operatorname{Cos}[c+dx]^2} \sqrt{\operatorname{Cos}[c+dx]^2 (-1+\operatorname{Sec}[c+dx]) (1+\operatorname{Sec}[c+dx])} \right) \right) \right) / \\
& \left(16 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a (1+\operatorname{Sec}[c+dx]))^{5/2} \right) + \frac{1}{(A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a (1+\operatorname{Sec}[c+dx]))^{5/2}} \\
& \operatorname{Cos}[c+dx]^2 \sqrt{(1+\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]} \\
& (1+\operatorname{Sec}[c+dx])^{5/2} (A+C \operatorname{Sec}[c+dx])^2 \\
& \left(\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 (-27 A \operatorname{Sin}\left[\frac{c}{2}\right]-11 C \operatorname{Sin}\left[\frac{c}{2}\right])}{16 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 (A \operatorname{Sin}\left[\frac{c}{2}\right]+C \operatorname{Sin}\left[\frac{c}{2}\right])}{8 d} + \frac{2 A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{d} + \right. \\
& \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^3 (-27 A \operatorname{Sin}\left[\frac{dx}{2}\right]-11 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{16 d} + \frac{7 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right]+C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} + \\
& \left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right]+C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} + \frac{2 A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{d} + \frac{7 (A+C) \operatorname{Tan}\left[\frac{c}{2}\right]}{8 d} \right)
\end{aligned}$$

■ **Problem 207: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx]) (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 4, 205 leaves, 9 steps) :

$$\begin{aligned}
 & - \frac{2 a (5 A + 3 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \\
 & \frac{2 a (7 A + 5 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{2 a (5 A + 3 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \\
 & \frac{2 a (7 A + 5 C) \sec [c + d x]^{3/2} \sin [c + d x]}{21 d} + \frac{2 a C \sec [c + d x]^{5/2} \sin [c + d x]}{5 d} + \frac{2 a C \sec [c + d x]^{7/2} \sin [c + d x]}{7 d}
 \end{aligned}$$

Result (type 5, 624 leaves) :

$$\begin{aligned}
 & a \left(- \left(2 \sqrt{2} A e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \cos [c + d x]^2 \operatorname{Csc}[c] \right. \right. \\
 & \left. \left. \left(1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right]\right) (A + C \sec [c + d x]^2) \right) \right) / \\
 & (d (A + 2 C + A \cos [2 c + 2 d x])) - \left(6 \sqrt{2} C e^{-i(2 c + d x)} \sqrt{\frac{e^{i(c + d x)}}{1 + e^{2 i(c + d x)}}} \cos [c + d x]^2 \operatorname{Csc}[c] \right. \\
 & \left. \left(1 + e^{2 i(c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right]\right) (A + C \sec [c + d x]^2) \right) / \\
 & (5 d (A + 2 C + A \cos [2 c + 2 d x])) + \frac{4 A \cos [c + d x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} (A + C \sec [c + d x]^2)}{3 d (A + 2 C + A \cos [2 c + 2 d x])} + \\
 & \frac{20 C \cos [c + d x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} (A + C \sec [c + d x]^2)}{21 d (A + 2 C + A \cos [2 c + 2 d x])} + \\
 & \left((A + C \sec [c + d x]^2) \left(\frac{4 (5 A + 3 C) \cos [d x] \operatorname{Csc}[c]}{5 d} + \frac{4 C \sec [c] \sec [c + d x]^3 \sin [d x]}{7 d} + \right. \right. \\
 & \left. \frac{4 \sec [c] \sec [c + d x]^2 (5 C \sin [c] + 7 C \sin [d x])}{35 d} + \frac{4 \sec [c] \sec [c + d x] (21 C \sin [c] + 35 A \sin [d x] + 25 C \sin [d x])}{105 d} \right. \\
 & \left. \left. \frac{4 (7 A + 5 C) \tan [c]}{21 d} \right) \right) / \left((A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{3/2} \right)
 \end{aligned}$$

■ **Problem 208: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{\sec[c+dx]} (a + a \sec[c+dx]) (A + C \sec[c+dx]^2) dx}{\sqrt{\sec[c+dx]}}$$

Optimal (type 4, 172 leaves, 8 steps):

$$\frac{2 a (5 A + 3 C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5 d} + \frac{2 a (3 A + C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 d} + \frac{2 a (5 A + 3 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{5 d} + \frac{2 a C \sec[c+dx]^{3/2} \sin[c+dx]}{3 d} + \frac{2 a C \sec[c+dx]^{5/2} \sin[c+dx]}{5 d}$$

Result (type 5, 282 leaves):

$$\frac{1}{15 d (1 + e^{2i(c+dx)})^2 (A + 2 C + A \cos[2(c+dx)]) \sec[c+dx]^{3/2}} \left(2 a e^{-i(2c+dx)} (-1 + e^{2ic}) \operatorname{Csc}[c] \left(15 A + 9 C + 5 C e^{i(c+dx)} + 30 A e^{2i(c+dx)} + 24 C e^{2i(c+dx)} + 15 A e^{4i(c+dx)} + 3 C e^{4i(c+dx)} - 5 C e^{5i(c+dx)} - 5 i (3 A + C) e^{i(c+dx)} (1 + e^{2i(c+dx)})^2 \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] - 3 (5 A + 3 C) (1 + e^{2i(c+dx)})^{5/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) (A + C \sec[c+dx]^2) \right)$$

■ **Problem 209: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx]) (A + C \sec[c+dx]^2) dx}{\sqrt{\sec[c+dx]}}$$

Optimal (type 4, 135 leaves, 7 steps):

$$\frac{2 a (A - C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{d} + \frac{2 a (3 A + C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 d} + \frac{2 a C \sqrt{\sec[c+dx]} \sin[c+dx]}{d} + \frac{2 a C \sec[c+dx]^{3/2} \sin[c+dx]}{3 d}$$

Result (type 5, 197 leaves):

$$\frac{1}{3 d} a e^{-i(2c+dx)} \sec[c+dx]^{3/2} \left(-3 i A + 3 i C - 3 i A \cos[2(c+dx)] + 3 i C \cos[2(c+dx)] + 2 (3 A + C) \cos[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 3 i (A - C) e^{-2i(c+dx)} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 C \sin[c+dx] + 3 C \sin[2(c+dx)] \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 210: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 135 leaves, 7 steps):

$$\frac{2 a (A - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} +$$

$$\frac{2 a (A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a C \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d}$$

Result (type 5, 182 leaves):

$$\frac{1}{3 d} a e^{-i(2 c + d x)} \sqrt{\operatorname{Sec}[c + d x]}$$

$$\left(-6 i A \operatorname{Cos}[c + d x] + 6 i C \operatorname{Cos}[c + d x] + 2 (A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 6 i (A - C) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \right. \\ \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + 6 C \operatorname{Sin}[c + d x] + A \operatorname{Sin}[2(c + d x)] \right) (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])$$

■ **Problem 211: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 141 leaves, 7 steps):

$$\frac{2 a (3 A + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} +$$

$$\frac{2 a (A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 a A \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 183 leaves):

$$\frac{1}{30 d} a e^{-i(2 c + d x)} \sqrt{\operatorname{Sec}[c + d x]}$$

$$\left(20 (A + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 12 i (3 A + 5 C) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + \right. \\ \left. 2 \operatorname{Cos}[c + d x] (-6 i (3 A + 5 C) + 10 A \operatorname{Sin}[c + d x] + 3 A \operatorname{Sin}[2(c + d x)]) \right) (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])$$

■ **Problem 212: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 174 leaves, 8 steps):

$$\frac{2 a (3 A + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \frac{2 a (5 A + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 a A \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 a (5 A + 7 C) \operatorname{Sin}[c + d x]}{21 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 202 leaves):

$$\frac{1}{420 d} a e^{-i (2 c + d x)} \sqrt{\operatorname{Sec}[c + d x]} \left(40 (5 A + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] + 168 i (3 A + 5 C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 2 \operatorname{Cos}[c + d x] (-84 i (3 A + 5 C) + 5 (23 A + 28 C) \operatorname{Sin}[c + d x] + 42 A \operatorname{Sin}[2 (c + d x)] + 15 A \operatorname{Sin}[3 (c + d x)]) \right) (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])$$

■ **Problem 213: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 205 leaves, 9 steps):

$$\frac{2 a (7 A + 9 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} + \frac{2 a (5 A + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{2 a A \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 a (7 A + 9 C) \operatorname{Sin}[c + d x]}{45 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 a (5 A + 7 C) \operatorname{Sin}[c + d x]}{21 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 218 leaves):

$$\frac{1}{2520 d} a e^{-i (2 c + d x)} \sqrt{\operatorname{Sec}[c + d x]} \left(240 (5 A + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] + 336 i (7 A + 9 C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 2 \operatorname{Cos}[c + d x] (-1176 i A - 1512 i C + 30 (23 A + 28 C) \operatorname{Sin}[c + d x] + 14 (19 A + 18 C) \operatorname{Sin}[2 (c + d x)] + 90 A \operatorname{Sin}[3 (c + d x)] + 35 A \operatorname{Sin}[4 (c + d x)]) \right) (\operatorname{Cos}[2 c + d x] + i \operatorname{Sin}[2 c + d x])$$

- **Problem 214: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^{3/2} (a + a \sec[c + dx])^2 (A + C \sec[c + dx]^2) dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\begin{aligned} & - \frac{16 a^2 (3 A + 2 C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{15 d} + \\ & \frac{4 a^2 (7 A + 5 C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{21 d} + \frac{16 a^2 (3 A + 2 C) \sqrt{\sec[c + dx]} \sin[c + dx]}{15 d} + \\ & \frac{4 a^2 (7 A + 5 C) \sec[c + dx]^{3/2} \sin[c + dx]}{21 d} + \frac{2 a^2 (21 A + 19 C) \sec[c + dx]^{5/2} \sin[c + dx]}{105 d} + \\ & \frac{2 C \sec[c + dx]^{5/2} (a + a \sec[c + dx])^2 \sin[c + dx]}{9 d} + \frac{8 C \sec[c + dx]^{5/2} (a^2 + a^2 \sec[c + dx]) \sin[c + dx]}{63 d} \end{aligned}$$

Result (type 5, 801 leaves):

$$\begin{aligned}
& - \left(4 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+A \cos[2c+2dx])) - \\
& \left(8 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \right) / (15d(A+2C+A \cos[2c+2dx])) + \\
& \frac{2A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2)}{3d(A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} + \\
& \frac{10C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2)}{21d(A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \left(\frac{8(3A+2C) \cos[dx] \operatorname{Csc}[c]}{15d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 \sin[dx]}{9d} + \right. \right. \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (7C \sin[c] + 18C \sin[dx])}{63d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (90C \sin[c] + 63A \sin[dx] + 112C \sin[dx])}{315d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (63A \sin[c] + 112C \sin[c] + 210A \sin[dx] + 150C \sin[dx])}{315d} \right) \\
& \quad \left. \left. \frac{2(7A+5C) \tan[c]}{21d} \right) \right) / ((A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2})
\end{aligned}$$

- **Problem 215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 237 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^2 (5 A + 3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{8 a^2 (7 A + 3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \\
& \frac{4 a^2 (5 A + 3 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{2 a^2 (35 A + 33 C) \sec [c+d x]^{3 / 2} \sin [c+d x]}{105 d} + \\
& \frac{2 C \sec [c+d x]^{3 / 2} (a+a \sec [c+d x])^2 \sin [c+d x]}{7 d} + \frac{8 C \sec [c+d x]^{3 / 2} (a^2+a^2 \sec [c+d x]) \sin [c+d x]}{35 d}
\end{aligned}$$

Result (type 5, 757 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \cos [c+d x]^4 \operatorname{Csc}[c] \left(1+e^{2 i(c+d x)} + (-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \right) / (d(A+2 C+A \cos [2 c+2 d x])) - \\
& \left(3 \sqrt{2} C e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \cos [c+d x]^4 \operatorname{Csc}[c] \left(1+e^{2 i(c+d x)} + (-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \right) / (5 d(A+2 C+A \cos [2 c+2 d x])) + \\
& \frac{4 A \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{3 d(A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7 / 2}} + \\
& \frac{4 C \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{7 d(A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7 / 2}} + \\
& \left(\sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \right. \\
& \quad \left(\frac{2(5 A+3 C) \cos [d x] \operatorname{Csc}[c]}{5 d} + \frac{C \sec [c] \sec [c+d x]^3 \sin [d x]}{7 d} + \frac{\sec [c] \sec [c+d x]^2 (5 C \sin [c] + 14 C \sin [d x])}{35 d} + \right. \\
& \quad \left. \frac{\sec [c] \sec [c+d x] (42 C \sin [c] + 35 A \sin [d x] + 60 C \sin [d x])}{105 d} + \frac{(7 A+12 C) \tan [c]}{21 d} \right) \left. \right) / ((A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7 / 2})
\end{aligned}$$

■ **Problem 216: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{\sqrt{\sec [c+d x]}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{16 a^2 C \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5 d} + \\
 & \frac{4 a^2 (3 A + C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 d} + \frac{2 a^2 (15 A + 17 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{15 d} + \\
 & \frac{2 C \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^2 \sin[c+dx]}{5 d} + \frac{8 C \sqrt{\sec[c+dx]} (a^2 + a^2 \sec[c+dx]) \sin[c+dx]}{15 d}
 \end{aligned}$$

Result (type 5, 436 leaves):

$$\begin{aligned}
 & - \frac{1}{15 d (-1 + e^{2 i c}) (A + 2 C + A \cos[2 c + 2 d x])} 2 i \sqrt{2} e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2 i(c+dx)}}} \\
 & \cos[c+dx]^4 \left(12 C (1 + e^{2 i(c+dx)}) + 12 C (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+dx)}\right] + \right. \\
 & \left. 5 (3 A + C) e^{i(c+dx)} (-1 + e^{2 i c}) \sqrt{1 + e^{2 i(c+dx)}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2 i(c+dx)}\right] \right) \\
 & \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c+dx])^2 (A + C \sec[c+dx])^2 + \left(\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c+dx])^2 (A + C \sec[c+dx])^2 \right. \\
 & \left(- \frac{(-5 A - 16 C + 5 A \cos[2 c]) \cos[dx] \operatorname{Csc}[c]}{10 d} + \frac{A \cos[c] \sin[dx]}{d} + \frac{C \sec[c] \sec[c+dx]^2 \sin[dx]}{5 d} + \right. \\
 & \left. \left. \frac{\sec[c] \sec[c+dx] (3 C \sin[c] + 10 C \sin[dx])}{15 d} + \frac{2 C \tan[c]}{3 d} \right) \right) / ((A + 2 C + A \cos[2 c + 2 d x]) \sec[c+dx])^{7/2}
 \end{aligned}$$

■ **Problem 217: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx])^2 (A + C \sec[c+dx])^2}{\sec[c+dx]^{3/2}} dx$$

Optimal (type 4, 198 leaves, 8 steps):

$$\begin{aligned}
 & \frac{4 a^2 (A - C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{d} + \frac{8 a^2 (A + C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3 d} - \\
 & \frac{2 a^2 (A - 5 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{3 d} + \frac{2 A (a + a \sec[c+dx])^2 \sin[c+dx]}{3 d \sqrt{\sec[c+dx]}} - \frac{2 (A - C) \sqrt{\sec[c+dx]} (a^2 + a^2 \sec[c+dx]) \sin[c+dx]}{3 d}
 \end{aligned}$$

Result (type 5, 215 leaves):

$$\frac{1}{6d}$$

$$a^2 e^{-i(2c+dx)} \operatorname{Sec}[c+dx]^{3/2} \left(-12iA + 12iC - 12iA \cos[2(c+dx)] + 12iC \cos[2(c+dx)] + 16(A+C) \cos[c+dx]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 12i(A-C) e^{-2i(c+dx)} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + A \sin[c+dx] + 4C \sin[c+dx] + 12C \sin[2(c+dx)] + A \sin[3(c+dx)] \right) (\cos[2c+dx] + i \sin[2c+dx])$$

■ **Problem 218: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^2 (A + C \operatorname{Sec}[c+dx])^2}{\operatorname{Sec}[c+dx]^{5/2}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{16a^2 A \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{4a^2 (A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3d} - \frac{2a^2 (7A-15C) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{15d} + \frac{2A (a + a \operatorname{Sec}[c+dx])^2 \sin[c+dx]}{5d \operatorname{Sec}[c+dx]^{3/2}} + \frac{8A (a^2 + a^2 \operatorname{Sec}[c+dx]) \sin[c+dx]}{15d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 5, 209 leaves):

$$\frac{1}{30d \sqrt{\operatorname{Sec}[c+dx]}} a^2 \left(\cos\left[\frac{c}{2}\right] - i \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2}\right] + i \sin\left[\frac{c}{2}\right] \right) \left(-96iA + \frac{192iA \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1 + e^{2i(c+dx)}}} - \frac{80i(A+3C) e^{i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right]}{\sqrt{1 + e^{2i(c+dx)}}} + 40A \sin[c+dx] + 3A \operatorname{Sec}[c+dx] \sin[3(c+dx)] + 3A \tan[c+dx] + 60C \tan[c+dx] \right)$$

■ **Problem 219: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^2 (A + C \operatorname{Sec}[c+dx])^2}{\operatorname{Sec}[c+dx]^{7/2}} dx$$

Optimal (type 4, 204 leaves, 8 steps):

$$\frac{4a^2 (3A+5C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5d} + \frac{8a^2 (3A+7C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{21d} + \frac{2a^2 (33A+35C) \sin[c+dx]}{105d \sqrt{\operatorname{Sec}[c+dx]}} + \frac{2A (a + a \operatorname{Sec}[c+dx])^2 \sin[c+dx]}{7d \operatorname{Sec}[c+dx]^{5/2}} + \frac{8A (a^2 + a^2 \operatorname{Sec}[c+dx]) \sin[c+dx]}{35d \operatorname{Sec}[c+dx]^{3/2}}$$

Result (type 5, 203 leaves) :

$$\frac{1}{420 d} a^2 e^{-i (2 c+d x)} \sqrt{\sec [c+d x]} \left(160 (3 A+7 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right. \\ \left. 336 i (3 A+5 C) e^{-i (c+d x)} \sqrt{1+e^{2 i (c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c+d x)}\right] + \right. \\ \left. 2 \cos [c+d x] (-504 i A-840 i C+5 (51 A+28 C) \sin [c+d x]+84 A \sin [2(c+d x)]+15 A \sin [3(c+d x)]) \right) (\cos [2 c+d x]+i \sin [2 c+d x])$$

■ **Problem 220: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sec [c+d x])^2 (A+C \sec [c+d x])^2}{\sec [c+d x]^{9/2}} dx$$

Optimal (type 4, 237 leaves, 9 steps) :

$$\frac{16 a^2 (2 A+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} + \frac{4 a^2 (5 A+7 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \\ \frac{2 a^2 (19 A+21 C) \sin [c+d x]}{105 d \sec [c+d x]^{3/2}} + \frac{4 a^2 (5 A+7 C) \sin [c+d x]}{21 d \sqrt{\sec [c+d x]}} + \frac{2 A (a+a \sec [c+d x])^2 \sin [c+d x]}{9 d \sec [c+d x]^{7/2}} + \frac{8 A (a^2+a^2 \sec [c+d x]) \sin [c+d x]}{63 d \sec [c+d x]^{5/2}}$$

Result (type 5, 850 leaves) :

$$\begin{aligned}
& \left(8 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \right) / (15d(A+2C+A \operatorname{Cos}[2c+2dx])) + \\
& \left(4 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+A \operatorname{Cos}[2c+2dx])) + \\
& \frac{10 A \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2)}{21d(A+2C+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} + \\
& \frac{2 C \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2)}{3d(A+2C+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} + \\
& \frac{1}{(A+2C+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{(347A+558C+421A \operatorname{Cos}[2c]+594C \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}[c]}{720d} + \frac{(13A+14C) \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{42d} + \right. \\
& \quad \frac{(79A+36C) \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{720d} + \frac{A \operatorname{Cos}[4dx] \operatorname{Sin}[4c]}{28d} + \frac{A \operatorname{Cos}[5dx] \operatorname{Sin}[5c]}{144d} + \frac{(421A+594C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{360d} + \\
& \quad \left. \frac{(13A+14C) \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{42d} + \frac{(79A+36C) \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{720d} + \frac{A \operatorname{Cos}[4c] \operatorname{Sin}[4dx]}{28d} + \frac{A \operatorname{Cos}[5c] \operatorname{Sin}[5dx]}{144d} \right)
\end{aligned}$$

■ **Problem 221: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^2 (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{11/2}} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\frac{4 a^2 (7 A+9 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} +$$

$$\frac{8 a^2 (25 A+33 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{231 d} + \frac{2 a^2 (89 A+99 C) \sin [c+d x]}{693 d \sec [c+d x]^{5/2}} + \frac{4 a^2 (7 A+9 C) \sin [c+d x]}{45 d \sec [c+d x]^{3/2}} +$$

$$\frac{8 a^2 (25 A+33 C) \sin [c+d x]}{231 d \sqrt{\sec [c+d x]}} + \frac{2 A (a+a \sec [c+d x])^2 \sin [c+d x]}{11 d \sec [c+d x]^{9/2}} + \frac{8 A (a^2+a^2 \sec [c+d x]) \sin [c+d x]}{99 d \sec [c+d x]^{7/2}}$$

Result (type 5, 896 leaves):

$$\left(7 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c+d x]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \right) / (15 d (A+2 C+A \cos [2 c+2 d x])) +$$

$$\left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c+d x]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \right) / (5 d (A+2 C+A \cos [2 c+2 d x])) +$$

$$\frac{100 A \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{231 d (A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7/2}} +$$

$$\frac{4 C \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2)}{7 d (A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7/2}} +$$

$$\frac{1}{(A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{7/2}}$$

$$\sec \left[\frac{c}{2} + \frac{dx}{2} \right]^4 (a+a \sec [c+d x])^2 (A+C \sec [c+d x]^2) \left(-\frac{(149 A+198 C+187 A \cos [2 c]+234 C \cos [2 c]) \cos [d x] \operatorname{Csc}[c]}{360 d} + \right.$$

$$\frac{(2185 A+2376 C) \cos [2 d x] \sin [2 c]}{7392 d} + \frac{(43 A+36 C) \cos [3 d x] \sin [3 c]}{360 d} + \frac{(27 A+11 C) \cos [4 d x] \sin [4 c]}{616 d} +$$

$$\frac{A \cos [5 d x] \sin [5 c]}{72 d} + \frac{A \cos [6 d x] \sin [6 c]}{352 d} + \frac{(187 A+234 C) \cos [c] \sin [d x]}{180 d} + \frac{(2185 A+2376 C) \cos [2 c] \sin [2 d x]}{7392 d} +$$

$$\left. \frac{(43 A+36 C) \cos [3 c] \sin [3 d x]}{360 d} + \frac{(27 A+11 C) \cos [4 c] \sin [4 d x]}{616 d} + \frac{A \cos [5 c] \sin [5 d x]}{72 d} + \frac{A \cos [6 c] \sin [6 d x]}{352 d} \right)$$

- **Problem 222: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^{3/2} (a + a \text{Sec}[c + d x])^3 (\text{A} + \text{C} \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 319 leaves, 11 steps):

$$\begin{aligned} & - \frac{4 a^3 (7 A + 5 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (143 A + 105 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{231 d} + \\ & \frac{4 a^3 (7 A + 5 C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{5 d} + \frac{4 a^3 (143 A + 105 C) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{231 d} + \\ & \frac{8 a^3 (44 A + 35 C) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{385 d} + \frac{2 C \text{Sec}[c + d x]^{5/2} (a + a \text{Sec}[c + d x])^3 \text{Sin}[c + d x]}{11 d} + \\ & \frac{4 C \text{Sec}[c + d x]^{5/2} (a^2 + a^2 \text{Sec}[c + d x])^2 \text{Sin}[c + d x]}{33 a d} + \frac{2 (33 A + 35 C) \text{Sec}[c + d x]^{5/2} (a^3 + a^3 \text{Sec}[c + d x]) \text{Sin}[c + d x]}{231 d} \end{aligned}$$

Result (type 5, 841 leaves):

$$\begin{aligned}
& - \left(7 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A + 2C + A \cos[2c + 2dx]) \right) - \\
& \left(C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A + 2C + A \cos[2c + 2dx]) \right) + \\
& \frac{13 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2)}{21 d (A + 2C + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{5 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2)}{11 d (A + 2C + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{9/2}} \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2) \left(\frac{(7A + 5C) \cos[dx] \operatorname{Csc}[c]}{5d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^5 \sin[dx]}{22d} + \right. \\
& \quad \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 (3C \sin[c] + 11C \sin[dx])}{66d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (77C \sin[c] + 33A \sin[dx] + 126C \sin[dx])}{462d} + \\
& \quad \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (165A \sin[c] + 630C \sin[c] + 693A \sin[dx] + 770C \sin[dx])}{2310d} + \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (693A \sin[c] + 770C \sin[c] + 1430A \sin[dx] + 1050C \sin[dx])}{2310d} + \frac{(143A + 105C) \tan[c]}{231d} \right)
\end{aligned}$$

■ **Problem 223: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[c+dx]} (a + a \operatorname{Sec}[c+dx])^3 (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 286 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 a^3 (27 A + 17 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{15 d} + \\
& \frac{4 a^3 (21 A + 11 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{4 a^3 (27 A + 17 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{15 d} + \\
& \frac{8 a^3 (21 A + 16 C) \sec [c + d x]^{3/2} \sin [c + d x]}{105 d} + \frac{2 C \sec [c + d x]^{3/2} (a + a \sec [c + d x])^3 \sin [c + d x]}{9 d} + \\
& \frac{4 C \sec [c + d x]^{3/2} (a^2 + a^2 \sec [c + d x])^2 \sin [c + d x]}{21 a d} + \frac{2 (63 A + 73 C) \sec [c + d x]^{3/2} (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{315 d}
\end{aligned}$$

Result (type 5, 798 leaves):

$$\begin{aligned}
& - \left(9 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) \right) / \left(5 \sqrt{2} d (A + 2 C + A \cos [2 c + 2 d x]) \right) - \\
& \left(17 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) \right) / \left(15 \sqrt{2} d (A + 2 C + A \cos [2 c + 2 d x]) \right) + \\
& \frac{A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2)}{d (A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2}} + \\
& \frac{11 C \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2)}{21 d (A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2}} + \\
& \left(\sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) \left(\frac{(27 A + 17 C) \cos [d x] \operatorname{Csc}[c]}{15 d} + \frac{C \sec [c] \sec [c + d x]^4 \sin [d x]}{18 d} + \right. \right. \\
& \quad \left. \frac{\sec [c] \sec [c + d x]^3 (7 C \sin [c] + 27 C \sin [d x])}{126 d} + \frac{\sec [c] \sec [c + d x]^2 (135 C \sin [c] + 63 A \sin [d x] + 238 C \sin [d x])}{630 d} + \right. \\
& \quad \left. \frac{\sec [c] \sec [c + d x] (63 A \sin [c] + 238 C \sin [c] + 315 A \sin [d x] + 330 C \sin [d x])}{630 d} + \right. \\
& \quad \left. \left. \frac{(21 A + 22 C) \tan [c]}{42 d} \right) \right) / \left((A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2} \right)
\end{aligned}$$

- **Problem 224: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 253 leaves, 9 steps):

$$\begin{aligned} & - \frac{4 a^3 (5 A + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (35 A + 13 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \\ & \frac{8 a^3 (70 A + 53 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{105 d} + \frac{2 C \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{7 d} + \\ & \frac{12 C \sqrt{\operatorname{Sec}[c + d x]} (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{35 a d} + \frac{2 (5 A + 7 C) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 778 leaves):

$$\begin{aligned}
& - \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) - \\
& \left(7 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \frac{5 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{3 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{13 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{21 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \left(-\frac{(-25A-28C+5A \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{20 d} + \right. \right. \\
& \quad \frac{A \cos[c] \sin[dx]}{2 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \sin[dx]}{14 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (5C \sin[c] + 21C \sin[dx])}{70 d} + \\
& \quad \left. \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (63C \sin[c] + 35A \sin[dx] + 130C \sin[dx])}{210 d} + \frac{(7A+26C) \tan[c]}{42 d} \right) \right) / \left((A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right)
\end{aligned}$$

■ **Problem 225: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 4, 259 leaves, 9 steps):

$$\begin{aligned}
& \frac{4 a^3 (5 A - 9 C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5 d} + \frac{4 a^3 (5 A + 3 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3 d} + \\
& \frac{4 a^3 (5 A + 21 C) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{15 d} + \frac{2 A (a+a \operatorname{Sec}[c+dx])^3 \sin[c+dx]}{3 d \sqrt{\operatorname{Sec}[c+dx]}} - \\
& \frac{2 (5 A - 3 C) \sqrt{\operatorname{Sec}[c+dx]} (a^2+a^2 \operatorname{Sec}[c+dx])^2 \sin[c+dx]}{15 a d} - \frac{2 (5 A - 9 C) \sqrt{\operatorname{Sec}[c+dx]} (a^3+a^3 \operatorname{Sec}[c+dx]) \sin[c+dx]}{15 d}
\end{aligned}$$

Result (type 5, 764 leaves) :

$$\begin{aligned}
 & \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
 & \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) - \\
 & \left(9 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
 & \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
 & \frac{5 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{3 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
 & \frac{C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
 & \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right. \\
 & \quad \left(-\frac{(5A-36C+15A \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{20 d} + \frac{A \cos[2dx] \sin[2c]}{12 d} + \frac{3A \cos[c] \sin[dx]}{2 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{10 d} + \right. \\
 & \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (C \sin[c] + 5C \sin[dx])}{10 d} + \frac{A \cos[2c] \sin[2dx]}{12 d} + \frac{C \tan[c]}{2 d} \right) \left. \right) / \left((A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right)
 \end{aligned}$$

■ **Problem 226: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{5/2}} dx$$

Optimal (type 4, 253 leaves, 9 steps) :

$$\frac{4 a^3 (9 A - 5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{4 a^3 (3 A + 5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} - \frac{8 a^3 (3 A - 10 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{15 d} +$$

$$\frac{2 A (a + a \sec [c+d x])^3 \sin [c+d x]}{5 d \sec [c+d x]^{3/2}} + \frac{4 A (a^2 + a^2 \sec [c+d x])^2 \sin [c+d x]}{5 a d \sqrt{\sec [c+d x]}} - \frac{2 (9 A - 5 C) \sqrt{\sec [c+d x]} (a^3 + a^3 \sec [c+d x]) \sin [c+d x]}{15 d}$$

Result (type 5, 245 leaves) :

$$\frac{1}{60 d} a^3 e^{-i(2 c+d x)} \sec [c+d x]^{3/2}$$

$$\left(-216 i A + 120 i C - 216 i A \cos [2(c+d x)] + 120 i C \cos [2(c+d x)] + 80 (3 A + 5 C) \cos [c+d x]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right.$$

$$24 i (9 A - 5 C) e^{-2 i(c+d x)} (1 + e^{2 i(c+d x)})^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 30 A \sin [c+d x] + 40 C \sin [c+d x] +$$

$$\left. 6 A \sin [2(c+d x)] + 180 C \sin [2(c+d x)] + 30 A \sin [3(c+d x)] + 3 A \sin [4(c+d x)] \right) (\cos [2 c+d x] + i \sin [2 c+d x])$$

■ **Problem 227: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec [c+d x])^3 (A + C \sec [c+d x])^2}{\sec [c+d x]^{7/2}} dx$$

Optimal (type 4, 253 leaves, 9 steps) :

$$\frac{4 a^3 (7 A + 5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{4 a^3 (13 A + 35 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} - \frac{4 a^3 (41 A - 35 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{105 d} +$$

$$\frac{2 A (a + a \sec [c+d x])^3 \sin [c+d x]}{7 d \sec [c+d x]^{5/2}} + \frac{12 A (a^2 + a^2 \sec [c+d x])^2 \sin [c+d x]}{35 a d \sec [c+d x]^{3/2}} + \frac{2 (7 A + 5 C) (a^3 + a^3 \sec [c+d x]) \sin [c+d x]}{15 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 231 leaves) :

$$\frac{1}{420 d} a^3 e^{-i(2 c+d x)} \sqrt{\sec [c+d x]} \left(-2352 i A \cos [c+d x] - 1680 i C \cos [c+d x] + 80 (13 A + 35 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right.$$

$$336 i (7 A + 5 C) e^{-i(c+d x)} \sqrt{1 + e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] + 126 A \sin [c+d x] + 840 C \sin [c+d x] +$$

$$\left. 550 A \sin [2(c+d x)] + 140 C \sin [2(c+d x)] + 126 A \sin [3(c+d x)] + 15 A \sin [4(c+d x)] \right) (\cos [2 c+d x] + i \sin [2 c+d x])$$

- **Problem 228: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 253 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^3 (17 A + 27 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} + \\ & \frac{4 a^3 (11 A + 21 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{8 a^3 (16 A + 21 C) \operatorname{Sin}[c + d x]}{105 d \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{2 A (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{4 A (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{21 a d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 (73 A + 63 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{315 d \operatorname{Sec}[c + d x]^{3/2}} \end{aligned}$$

Result (type 5, 847 leaves):

$$\begin{aligned}
& \left(17 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(15 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \left(9 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \frac{11 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{21 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{1}{(A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{(743 A + 1278 C + 889 A \cos[2c] + 1314 C \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{1440 d} + \frac{(53 A + 42 C) \cos[2dx] \sin[2c]}{168 d} + \right. \\
& \quad \frac{(151 A + 36 C) \cos[3dx] \sin[3c]}{1440 d} + \frac{3 A \cos[4dx] \sin[4c]}{112 d} + \frac{A \cos[5dx] \sin[5c]}{288 d} + \frac{(889 A + 1314 C) \cos[c] \sin[dx]}{720 d} + \\
& \quad \left. \frac{(53 A + 42 C) \cos[2c] \sin[2dx]}{168 d} + \frac{(151 A + 36 C) \cos[3c] \sin[3dx]}{1440 d} + \frac{3 A \cos[4c] \sin[4dx]}{112 d} + \frac{A \cos[5c] \sin[5dx]}{288 d} \right)
\end{aligned}$$

■ **Problem 229: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{11/2}} dx$$

Optimal (type 4, 286 leaves, 10 steps):

$$\begin{aligned}
& \frac{4 a^3 (5 A + 7 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \\
& \frac{4 a^3 (105 A + 143 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{8 a^3 (35 A + 44 C) \sin [c + d x]}{385 d \sec [c + d x]^{3/2}} + \frac{4 a^3 (105 A + 143 C) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}} + \\
& \frac{2 A (a + a \sec [c + d x])^3 \sin [c + d x]}{11 d \sec [c + d x]^{9/2}} + \frac{4 A (a^2 + a^2 \sec [c + d x])^2 \sin [c + d x]}{33 a d \sec [c + d x]^{7/2}} + \frac{2 (35 A + 33 C) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{231 d \sec [c + d x]^{5/2}}
\end{aligned}$$

Result (type 5, 893 leaves):

$$\begin{aligned}
& \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \left(7 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(5\sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \frac{5 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{11 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{13 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{21 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{1}{(A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \left(-\frac{(215 A+318 C+265 A \cos[2c]+354 C \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{480 d} + \right. \\
& \frac{(4473 A+4840 C) \cos[2dx] \sin[2c]}{14784 d} + \frac{(55 A+36 C) \cos[3dx] \sin[3c]}{480 d} + \frac{(49 A+11 C) \cos[4dx] \sin[4c]}{1232 d} + \\
& \frac{A \cos[5dx] \sin[5c]}{96 d} + \frac{A \cos[6dx] \sin[6c]}{704 d} + \frac{(265 A+354 C) \cos[c] \sin[dx]}{240 d} + \frac{(4473 A+4840 C) \cos[2c] \sin[2dx]}{14784 d} + \\
& \left. \frac{(55 A+36 C) \cos[3c] \sin[3dx]}{480 d} + \frac{(49 A+11 C) \cos[4c] \sin[4dx]}{1232 d} + \frac{A \cos[5c] \sin[5dx]}{96 d} + \frac{A \cos[6c] \sin[6dx]}{704 d} \right)
\end{aligned}$$

- **Problem 230: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{13/2}} dx$$

Optimal (type 4, 319 leaves, 11 steps):

$$\begin{aligned}
& \frac{4 a^3 (175 A + 221 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{195 d} + \\
& \frac{4 a^3 (95 A + 121 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{40 a^3 (118 A + 143 C) \sin [c + d x]}{9009 d \sec [c + d x]^{5/2}} + \\
& \frac{4 a^3 (175 A + 221 C) \sin [c + d x]}{585 d \sec [c + d x]^{3/2}} + \frac{4 a^3 (95 A + 121 C) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}} + \frac{2 A (a + a \sec [c + d x])^3 \sin [c + d x]}{13 d \sec [c + d x]^{11/2}} + \\
& \frac{12 A (a^2 + a^2 \sec [c + d x])^2 \sin [c + d x]}{143 a d \sec [c + d x]^{9/2}} + \frac{2 (145 A + 143 C) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{1287 d \sec [c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 942 leaves):

$$\begin{aligned}
& \left(35 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(39 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \left(17 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(15 \sqrt{2} d (A+2C+A \cos[2c+2dx]) \right) + \\
& \frac{95 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{231 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{11 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2)}{21 d (A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \frac{1}{(A+2C+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{(59375 A + 77272 C + 75025 A \cos[2c] + 92456 C \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{149760 d} + \frac{(4267 A + 4664 C) \cos[2dx] \sin[2c]}{14784 d} + \right. \\
& \quad \frac{(9005 A + 7852 C) \cos[3dx] \sin[3c]}{74880 d} + \frac{(59 A + 33 C) \cos[4dx] \sin[4c]}{1232 d} + \frac{(245 A + 52 C) \cos[5dx] \sin[5c]}{14976 d} + \frac{3 A \cos[6dx] \sin[6c]}{704 d} + \\
& \quad \frac{A \cos[7dx] \sin[7c]}{1664 d} + \frac{(75025 A + 92456 C) \cos[c] \sin[dx]}{74880 d} + \frac{(4267 A + 4664 C) \cos[2c] \sin[2dx]}{14784 d} + \frac{(9005 A + 7852 C) \cos[3c] \sin[3dx]}{74880 d} \\
& \quad \left. \frac{(59 A + 33 C) \cos[4c] \sin[4dx]}{1232 d} + \frac{(245 A + 52 C) \cos[5c] \sin[5dx]}{14976 d} + \frac{3 A \cos[6c] \sin[6dx]}{704 d} + \frac{A \cos[7c] \sin[7dx]}{1664 d} \right)
\end{aligned}$$

- **Problem 231: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2} (A+C \operatorname{Sec}[c+dx]^2)}{a+a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3(5A+7C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{5ad} - \\
& \frac{(3A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{3ad} + \frac{3(5A+7C)\sqrt{\sec[c+dx]}\sin[c+dx]}{5ad} - \\
& \frac{(3A+5C)\sec[c+dx]^{3/2}\sin[c+dx]}{3ad} + \frac{(5A+7C)\sec[c+dx]^{5/2}\sin[c+dx]}{5ad} - \frac{(A+C)\sec[c+dx]^{7/2}\sin[c+dx]}{d(a+a\sec[c+dx])}
\end{aligned}$$

Result (type 5, 828 leaves):

$$\begin{aligned}
& - \left(3\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\sec\left[\frac{c}{2}\right](A+C\sec[c+dx]^2) \right) / \\
& (d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])) - \left(21\sqrt{2}Ce^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\sec\left[\frac{c}{2}\right](A+C\sec[c+dx]^2) \right) / \\
& (5d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])) - \\
& \frac{2A\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sec\left[\frac{c}{2}\right](A+C\sec[c+dx]^2)\sin[c]}{d(A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx])} - \\
& \frac{10C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sec\left[\frac{c}{2}\right](A+C\sec[c+dx]^2)\sin[c]}{3d(A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx])} + \\
& \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2(A+C\sec[c+dx]^2) \left(\frac{6(5A+7C)\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}\right]}{5d} - \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}+\frac{dx}{2}\right](A\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{d} \right. \right. \\
& \left. \left. \frac{8C\sec[c]\sec[c+dx]^2\sin[dx]}{5d} + \frac{8\sec[c]\sec[c+dx](3C\sin[c]-5C\sin[dx])}{15d} - \frac{4(2C+3A\cos[c]+5C\cos[c])\sec[c]\tan\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& ((A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx]))
\end{aligned}$$

- **Problem 232: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + C \text{Sec}[c + d x]^2)}{a + a \text{Sec}[c + d x]} dx$$

Optimal (type 4, 190 leaves, 8 steps):

$$\frac{(A + 3 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} + \frac{(3 A + 5 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{3 a d} - \frac{(A + 3 C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{a d} + \frac{(3 A + 5 C) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 a d} - \frac{(A + C) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{d (a + a \text{Sec}[c + d x])}$$

Result (type 5, 791 leaves):

$$\begin{aligned}
& \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) + \left(3\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) + \\
& \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx]^2) \sin[c]}{d(A + 2C + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx])} + \\
& \frac{10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx]^2) \sin[c]}{3d(A + 2C + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx])} + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + C \operatorname{Sec}[c+dx]^2) \right. \\
& \left. \left(-\frac{2(A + 3C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{3d} \right. \right. \\
& \left. \left. \frac{4(2C + 3A \cos[c] + 5C \cos[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \left((A + 2C + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right)
\end{aligned}$$

- **Problem 233: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]} (A + C \operatorname{Sec}[c+dx]^2)}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \\
& \frac{(A-C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \frac{(A+3C) \sqrt{\sec[c+dx]} \sin[c+dx]}{ad} - \frac{(A+C) \sec[c+dx]^{3/2} \sin[c+dx]}{d(a+a \sec[c+dx])}
\end{aligned}$$

Result (type 5, 755 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad (d(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])) - \left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad (d(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])) + \\
& \quad \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{d(A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx])} - \\
& \quad \frac{2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{d(A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx])} + \\
& \quad \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + C \sec[c+dx]^2) \left(\frac{2(A+3C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{4(A+C) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \quad ((A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx]))
\end{aligned}$$

■ **Problem 234: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c+dx]^2}{\sqrt{\sec[c+dx]} (a + a \sec[c+dx])} dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{(3A+C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} - \frac{(A-C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{ad} - \frac{(A+C)\sqrt{\sec[c+dx]}\sin[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 5, 772 leaves):

$$\begin{aligned} & \left(3\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\ & (d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])) + \left(\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\ & (d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])) - \\ & \frac{2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{d(A + 2C + A \cos[2c + 2dx]) \sqrt{\sec[c+dx]} (a + a \sec[c+dx])} + \\ & \frac{2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{d(A + 2C + A \cos[2c + 2dx]) \sqrt{\sec[c+dx]} (a + a \sec[c+dx])} + \\ & \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + C \sec[c+dx]^2) \left(-\frac{2(2A + C + A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right) \right. \\ & \left. \frac{8A \cos[c] \sin[dx]}{d} + \frac{4(A+C) \operatorname{Tan}\left[\frac{c}{2}\right]}{d} \right) / \left((A + 2C + A \cos[2c + 2dx]) \sqrt{\sec[c+dx]} (a + a \sec[c+dx]) \right) \end{aligned}$$

■ **Problem 235: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c+dx]^2}{\sec[c+dx]^{3/2} (a + a \sec[c+dx])} dx$$

Optimal (type 4, 162 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(3A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} + \\
& \frac{(5A+3C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3ad} + \frac{(5A+3C) \sin[c+dx]}{3ad \sqrt{\sec[c+dx]}} - \frac{(A+C) \sin[c+dx]}{d \sqrt{\sec[c+dx]} (a+a \sec[c+dx])}
\end{aligned}$$

Result (type 5, 809 leaves):

$$\begin{aligned}
& - \left(3 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad (d(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])) - \left(\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad (d(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])) + \\
& \quad \frac{10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{3d(A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx])} + \\
& \quad \frac{2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \sin[c]}{d(A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx])} + \\
& \quad \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + C \sec[c+dx]^2) \left(\frac{2(2A+C+A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4A \cos[2dx] \sin[2c]}{3d} - \right. \right. \\
& \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{8A \cos[c] \sin[dx]}{d} + \frac{4A \cos[2c] \sin[2dx]}{3d} - \frac{4(A+C) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \quad (d(A+2C+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+a \sec[c+dx]))
\end{aligned}$$

■ **Problem 236: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c+dx]^2}{\sec[c+dx]^{5/2} (a+a \sec[c+dx])} dx$$

Optimal (type 4, 199 leaves, 8 steps):

$$\frac{3(7A+5C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]} - (5A+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{5ad} + \frac{(7A+5C)\sin[c+dx]}{5ad\sec[c+dx]^{3/2}} - \frac{(5A+3C)\sin[c+dx]}{3ad\sqrt{\sec[c+dx]}} - \frac{(A+C)\sin[c+dx]}{d\sec[c+dx]^{3/2}(a+a\sec[c+dx])}$$

Result (type 5, 865 leaves):

$$\left(21\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx]^2) \right) / \\ (5d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])) + \left(3\sqrt{2}Ce^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx]^2) \right) / \\ (d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])) - \\ \frac{10A\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx]^2)\sin[c]}{3d(A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx])} - \\ \frac{2C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx]^2)\sin[c]}{d(A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx])} + \\ \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2(A+C\sec[c+dx]^2) \left(-\frac{(51A+40C+33A\cos[2c]+20C\cos[2c])\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}\right]}{10d} - \frac{4A\cos[2dx]\sin[2c]}{3d} + \right. \right. \\ \left. \frac{2A\cos[3dx]\sin[3c]}{5d} + \frac{4\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right](A\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{d} + \frac{2(33A+20C)\cos[c]\sin[dx]}{5d} - \frac{4A\cos[2c]\sin[2dx]}{3d} + \right. \\ \left. \left. \frac{2A\cos[3c]\sin[3dx]}{5d} + \frac{4(A+C)\tan\left[\frac{c}{2}\right]}{d} \right) \right) / \left((A+2C+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+a\sec[c+dx]) \right)$$

- **Problem 237: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{5/2} (A + C \text{Sec}[c + d x]^2)}{(a + a \text{Sec}[c + d x])^2} dx$$

Optimal (type 4, 229 leaves, 9 steps):

$$\begin{aligned} & \frac{(A + 7 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a^2 d} + \\ & \frac{2(A + 5 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{3 a^2 d} - \frac{(A + 7 C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{a^2 d} + \\ & \frac{2(A + 5 C) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 a^2 d} - \frac{(A + 7 C) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{3 a^2 d (1 + \text{Sec}[c + d x])} - \frac{(A + C) \text{Sec}[c + d x]^{7/2} \text{Sin}[c + d x]}{3 d (a + a \text{Sec}[c + d x])^2} \end{aligned}$$

Result (type 5, 860 leaves):

$$\begin{aligned}
& \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(14 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(8 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(40 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4(A + 7C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} \right) \right. \\
& \left. \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{16 C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{3d} + \right. \\
& \left. \frac{16(C + A \cos[c] + 5C \cos[c]) \operatorname{Sec}[c] \tan\left[\frac{c}{2}\right]}{3d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) / \left((A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)
\end{aligned}$$

- **Problem 238: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{3/2} (A + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 191 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 C \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^2 d} + \frac{(A-5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 a^2 d} + \\
& \frac{4 C \sqrt{\sec [c+d x]} \sin [c+d x]}{a^2 d} + \frac{(A-5 C) \sec [c+d x]^{3 / 2} \sin [c+d x]}{3 a^2 d(1+\sec [c+d x])} - \frac{(A+C) \sec [c+d x]^{5 / 2} \sin [c+d x]}{3 d(a+a \sec [c+d x])^2}
\end{aligned}$$

Result (type 5, 643 leaves):

$$\begin{aligned}
& - \left(8 \sqrt{2} C e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1+e^{2 i(c+d x)}+(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i(c+d x)}\right]\right) \sec \left[\frac{c}{2}\right] (A+C \sec [c+d x]^2) \right) / \\
& \quad (d(A+2 C+A \cos [2 c+2 d x]) (a+a \sec [c+d x])^2) + \\
& \quad \left(4 A \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \sqrt{\cos [c+d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2}\right] \sqrt{\sec [c+d x]} (A+C \sec [c+d x]^2) \sin [c] \right) / \\
& \quad (3 d(A+2 C+A \cos [2 c+2 d x]) (a+a \sec [c+d x])^2) - \\
& \quad \left(20 C \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \sqrt{\cos [c+d x]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sec \left[\frac{c}{2}\right] \sqrt{\sec [c+d x]} (A+C \sec [c+d x]^2) \sin [c] \right) / \\
& \quad (3 d(A+2 C+A \cos [2 c+2 d x]) (a+a \sec [c+d x])^2) + \left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]^4 \sqrt{\sec [c+d x]} (A+C \sec [c+d x]^2) \right. \\
& \quad \left. \left(\frac{16 C \cos [d x] \operatorname{Csc}\left[\frac{c}{2}\right] \sec \left[\frac{c}{2}\right]}{d} + \frac{8 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right] (A \sin \left[\frac{d x}{2}\right]-5 C \sin \left[\frac{d x}{2}\right])}{3 d} - \frac{4 \sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^3 (A \sin \left[\frac{d x}{2}\right]+C \sin \left[\frac{d x}{2}\right])}{3 d} \right. \right. \\
& \quad \left. \left. \frac{8(-A+5 C) \tan \left[\frac{c}{2}\right]}{3 d} - \frac{4(A+C) \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^2 \tan \left[\frac{c}{2}\right]}{3 d} \right) \right) / ((A+2 C+A \cos [2 c+2 d x]) (a+a \sec [c+d x])^2)
\end{aligned}$$

■ **Problem 239: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec [c+d x]} (A+C \sec [c+d x]^2)}{(a+a \sec [c+d x])^2} dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A-C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2 d} + \\
& \frac{2(A+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{3a^2 d} + \frac{(A-C) \sqrt{\sec[c+dx]} \sin[c+dx]}{a^2 d (1+\sec[c+dx])} - \frac{(A+C) \sec[c+dx]^{3/2} \sin[c+dx]}{3d (a+a \sec[c+dx])^2}
\end{aligned}$$

Result (type 5, 835 leaves):

$$\begin{aligned}
& - \left(2\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad \left(d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])^2 \right) + \left(2\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c+dx]^2) \right) / \\
& \quad \left(d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])^2 \right) + \\
& \quad \left(8A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c+dx]} (A + C \sec[c+dx]^2) \sin[c] \right) / \\
& \quad \left(3d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])^2 \right) + \\
& \quad \left(8C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c+dx]} (A + C \sec[c+dx]^2) \sin[c] \right) / \\
& \quad \left(3d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])^2 \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\sec[c+dx]} (A + C \sec[c+dx]^2) \right. \\
& \quad \left. \left(\frac{4(A-C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (2A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} \right. \right. \\
& \quad \left. \left. \frac{16(2A-C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} + \frac{4(A+C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \left((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c+dx])^2 \right)
\end{aligned}$$

- **Problem 240: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 170 leaves, 7 steps):

$$\frac{4 A \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a^2 d} - \frac{(5 A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3 a^2 d} - \frac{(5 A - C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{3 a^2 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A + C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{3 d (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 5, 659 leaves):

$$\left(8 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \right) / \left(d (A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right) - \\ \left(20 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\ \left(3 d (A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right) + \\ \left(4 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\ \left(3 d (A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right) + \\ \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4 A (3 + \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (7 A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3 d} + \frac{16 A \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{8 (7 A + C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} - \frac{4 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)$$

- **Problem 241: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 201 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(7A+C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{a^2d} + \frac{2(5A+C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{3a^2d} + \\
& \frac{2(5A+C)\sin[c+dx]}{3a^2d\sqrt{\sec[c+dx]}} - \frac{(7A+C)\sin[c+dx]}{3a^2d\sqrt{\sec[c+dx]}(1+\sec[c+dx])} - \frac{(A+C)\sin[c+dx]}{3d\sqrt{\sec[c+dx]}(a+a\sec[c+dx])^2}
\end{aligned}$$

Result (type 5, 888 leaves):

$$\begin{aligned}
& - \left(14\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^4\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx])^2 \right) / \\
& \quad (d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])^2) - \left(2\sqrt{2}Ce^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^4\operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right)\operatorname{Sec}\left[\frac{c}{2}\right](A+C\sec[c+dx])^2 \right) / \\
& \quad (d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])^2) + \\
& \quad \left(40A\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^4\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right]\sqrt{\sec[c+dx]}(A+C\sec[c+dx])^2\sin[c] \right) / \\
& \quad (3d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])^2) + \\
& \quad \left(8C\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^4\sqrt{\cos[c+dx]}\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\operatorname{Sec}\left[\frac{c}{2}\right]\sqrt{\sec[c+dx]}(A+C\sec[c+dx])^2\sin[c] \right) / \\
& \quad (3d(A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])^2) + \\
& \quad \left(\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^4\sqrt{\sec[c+dx]}(A+C\sec[c+dx])^2 \left(\frac{4(5A+C+2A\cos[2c])\cos[dx]\operatorname{Csc}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{8A\cos[2dx]\sin[2c]}{3d} + \right. \right. \\
& \quad \left. \frac{4\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^3(A\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{3d} - \frac{16\operatorname{Sec}\left[\frac{c}{2}\right]\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right](5A\sin\left[\frac{dx}{2}\right]+2C\sin\left[\frac{dx}{2}\right])}{3d} - \frac{32A\cos[c]\sin[dx]}{d} + \right. \\
& \quad \left. \left. \frac{8A\cos[2c]\sin[2dx]}{3d} - \frac{16(5A+2C)\tan\left[\frac{c}{2}\right]}{3d} + \frac{4(A+C)\operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2\tan\left[\frac{c}{2}\right]}{3d} \right) \right) / ((A+2C+A\cos[2c+2dx])(a+a\sec[c+dx])^2)
\end{aligned}$$

- **Problem 242: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 236 leaves, 9 steps):

$$\frac{4 (14 A + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - 5 (3 A + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 a^2 d} + \frac{4 (14 A + 5 C) \operatorname{Sin}[c + d x]}{15 a^2 d \operatorname{Sec}[c + d x]^{3/2}} - \frac{5 (3 A + C) \operatorname{Sin}[c + d x]}{3 a^2 d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{(3 A + C) \operatorname{Sin}[c + d x]}{a^2 d \operatorname{Sec}[c + d x]^{3/2} (1 + \operatorname{Sec}[c + d x])} - \frac{3 a^2 d}{(A + C) \operatorname{Sin}[c + d x]} - \frac{3 d \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^2}{(A + C) \operatorname{Sin}[c + d x]}$$

Result (type 5, 941 leaves):

$$\begin{aligned}
& \left(112 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(8 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) \\
& \left(-\frac{(151A + 60C + 73A \cos[2c] + 20C \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{16A \cos[2dx] \sin[2c]}{3d} + \frac{4A \cos[3dx] \sin[3c]}{5d} - \right. \\
& \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (13A \sin\left[\frac{dx}{2}\right] + 7C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4(73A + 20C) \cos[c] \sin[dx]}{5d} - \\
& \left. \frac{16A \cos[2c] \sin[2dx]}{3d} + \frac{4A \cos[3c] \sin[3dx]}{5d} + \frac{8(13A + 7C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 243: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{7/2} (A + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 282 leaves, 10 steps):

$$\begin{aligned}
& \frac{(9A + 119C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10a^3d} + \frac{(A + 11C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{2a^3d} - \\
& \frac{(9A + 119C) \sqrt{\sec[c + dx]} \sin[c + dx]}{10a^3d} + \frac{(A + 11C) \sec[c + dx]^{3/2} \sin[c + dx]}{2a^3d} - \\
& \frac{(A + C) \sec[c + dx]^{9/2} \sin[c + dx]}{5d(a + a \sec[c + dx])^3} - \frac{2C \sec[c + dx]^{7/2} \sin[c + dx]}{3ad(a + a \sec[c + dx])^2} - \frac{(9A + 119C) \sec[c + dx]^{5/2} \sin[c + dx]}{30d(a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 964 leaves):

$$\begin{aligned}
& \left(18 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(238 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(44C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \left(-\frac{4(9A + 119C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \right. \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{dx}{2}\right] + 13C \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \sin\left[\frac{dx}{2}\right] + 29C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{32C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{3d} + \\
& \quad \left. \frac{8(4C + 3A \cos[c] + 33C \cos[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} + \frac{8(3A + 13C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 244: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^{5/2} (A + C \operatorname{Sec}[c+dx]^2)}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 249 leaves, 9 steps):

$$\frac{(A - 49 C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} +$$

$$\frac{(A - 13 C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6 a^3 d} - \frac{(A - 49 C) \sqrt{\sec[c + dx]} \sin[c + dx]}{10 a^3 d} -$$

$$\frac{(A + C) \sec[c + dx]^{7/2} \sin[c + dx]}{5 d (a + a \sec[c + dx])^3} + \frac{2 (A - 4 C) \sec[c + dx]^{5/2} \sin[c + dx]}{15 a d (a + a \sec[c + dx])^2} + \frac{(A - 13 C) \sec[c + dx]^{3/2} \sin[c + dx]}{6 d (a^3 + a^3 \sec[c + dx])}$$

Result (type 5, 933 leaves):

$$\left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx]^2) \right) /$$

$$(5 d (A + 2 C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) - \left(98 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx]^2) \right) /$$

$$(5 d (A + 2 C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) +$$

$$\left(4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \sin[c] \right) /$$

$$(3 d (A + 2 C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) -$$

$$\left(52 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \sin[c] \right) /$$

$$(3 d (A + 2 C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \left(-\frac{4 (A - 49 C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{5 d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 13 C \sin\left[\frac{dx}{2}\right])}{3 d} \right. \right.$$

$$\left. \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - 4 C \sin\left[\frac{dx}{2}\right])}{15 d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5 d} - \frac{8 (-A + 13 C) \tan\left[\frac{c}{2}\right]}{3 d} \right. \right.$$

$$\left. \left. \frac{16 (A - 4 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15 d} - \frac{4 (A + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5 d} \right) \right) / \left((A + 2 C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)$$

- **Problem 245: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + C \text{Sec}[c + d x]^2)}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 4, 220 leaves, 8 steps):

$$\begin{aligned} & - \frac{(A - 9 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{10 a^3 d} + \frac{(A + 3 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{6 a^3 d} \\ & - \frac{(A + C) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{5 d (a + a \text{Sec}[c + d x])^3} + \frac{2 (2 A - 3 C) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{15 a d (a + a \text{Sec}[c + d x])^2} + \frac{(A - 9 C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{10 d (a^3 + a^3 \text{Sec}[c + d x])} \end{aligned}$$

Result (type 5, 932 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(18 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \left(\frac{4(A - 9C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (7A \sin\left[\frac{dx}{2}\right] - 3C \sin\left[\frac{dx}{2}\right])}{15d} \right. \right. \\
& \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8(A + 3C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right. \right. \\
& \quad \left. \left. \frac{8(7A - 3C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right) \right) / ((A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3)
\end{aligned}$$

- **Problem 246: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]} (A + C \operatorname{Sec}[c+dx]^2)}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 222 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(9A - C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10a^3d} + \frac{(3A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6a^3d} \\
& \frac{(A + C) \sec[c + dx]^{3/2} \sin[c + dx]}{5d(a + a \sec[c + dx])^3} + \frac{2(3A - 2C) \sqrt{\sec[c + dx]} \sin[c + dx]}{15ad(a + a \sec[c + dx])^2} + \frac{(3A + C) \sqrt{\sec[c + dx]} \sin[c + dx]}{6d(a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 934 leaves):

$$\begin{aligned}
& - \left(18\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \left(2\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \\
& \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c + dx]^{3/2} (A + C \sec[c + dx]^2) \left(\frac{4(9A - C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{5d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (9A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{3d} \right. \right. \\
& \left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (6A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{15d} - \frac{8(9A - C) \tan\left[\frac{c}{2}\right]}{3d} \right. \\
& \left. \left. \frac{16(6A + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) \right) / \left((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)
\end{aligned}$$

- **Problem 247: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 226 leaves, 8 steps):

$$\frac{(49 A - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} - (13 A - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{10 a^3 d} - \frac{6 a^3 d}{(A + C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]} - \frac{2 (4 A - C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a d (a + a \operatorname{Sec}[c + d x])^2} - \frac{(13 A - C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{6 d (a^3 + a^3 \operatorname{Sec}[c + d x])}$$

Result (type 5, 955 leaves):

$$\begin{aligned}
& \left(98 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(2\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(52A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{4(39A - C + 10A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \right. \\
& \quad \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (23A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (17A \sin\left[\frac{dx}{2}\right] + 7C \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \left. \frac{32A \cos[c] \sin[dx]}{d} + \frac{8(23A + C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{8(17A + 7C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 248: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 249 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(119 A + 9 C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} + \\
& \frac{(11 A + C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{2 a^3 d} + \frac{(11 A + C) \sin[c + dx]}{2 a^3 d \sqrt{\sec[c + dx]}} - \\
& \frac{(A + C) \sin[c + dx]}{5 d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^3} - \frac{2 A \sin[c + dx]}{3 a d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2} - \frac{(119 A + 9 C) \sin[c + dx]}{30 d \sqrt{\sec[c + dx]} (a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 988 leaves):

$$\begin{aligned}
& - \left(238 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(18 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(44 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \\
& \left(\frac{4(89A + 9C + 30A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \frac{16A \cos[2dx] \sin[2c]}{3d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \right. \\
& \quad \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (11A \sin\left[\frac{dx}{2}\right] + 6C \sin\left[\frac{dx}{2}\right])}{15d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (43A \sin\left[\frac{dx}{2}\right] + 9C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{96A \cos[c] \sin[dx]}{d} + \\
& \quad \left. \frac{16A \cos[2c] \sin[2dx]}{3d} - \frac{8(43A + 9C) \tan\left[\frac{c}{2}\right]}{3d} + \frac{16(11A + 6C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 249: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 290 leaves, 10 steps):

$$\begin{aligned}
& \frac{7 (33 A + 7 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{10 a^3 d} - \\
& \frac{(63 A + 13 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{6 a^3 d} + \frac{7 (33 A + 7 C) \sin [c + d x]}{30 a^3 d \sec [c + d x]^{3/2}} - \frac{(63 A + 13 C) \sin [c + d x]}{6 a^3 d \sqrt{\sec [c + d x]}} - \\
& \frac{(A + C) \sin [c + d x]}{5 d \sec [c + d x]^{3/2} (a + a \sec [c + d x])^3} - \frac{2 (6 A + C) \sin [c + d x]}{15 a d \sec [c + d x]^{3/2} (a + a \sec [c + d x])^2} - \frac{(63 A + 13 C) \sin [c + d x]}{10 d \sec [c + d x]^{3/2} (a^3 + a^3 \sec [c + d x])}
\end{aligned}$$

Result (type 5, 1032 leaves):

$$\begin{aligned}
& \left(462 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(98 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(84 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(52 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{2(329A + 78C + 133A \cos[2c] + 20C \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{16A \cos[2dx] \sin[2c]}{d} + \frac{8A \cos[3dx] \sin[3c]}{5d} + \right. \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{184 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \\
& \quad \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (27A \sin\left[\frac{dx}{2}\right] + 17C \sin\left[\frac{dx}{2}\right])}{15d} + \frac{8(133A + 20C) \cos[c] \sin[dx]}{5d} - \frac{16A \cos[2c] \sin[2dx]}{d} + \\
& \quad \left. \frac{8A \cos[3c] \sin[3dx]}{5d} + \frac{184(3A + C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{8(27A + 17C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 261: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^{3/2} (A + C \operatorname{Sec}[c + dx]^2)}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 171 leaves, 5 steps) :

$$\frac{a^{3/2} (8A + 7C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a^2 (8A - 5C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{3aC \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d} + \frac{C \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 3, 378 leaves) :

$$\left((8A + 7C) \operatorname{Cos}[c+dx]^3 \left(-\operatorname{Log}[1 + \operatorname{Sec}[c+dx]] + \operatorname{Log}\left[\sqrt{\operatorname{Sec}[c+dx]} + \operatorname{Sec}[c+dx]\right]^{3/2} + \sqrt{1 + \operatorname{Sec}[c+dx]} \sqrt{-1 + \operatorname{Sec}[c+dx]^2} \right) \right)$$

$$\left(a (1 + \operatorname{Sec}[c+dx]) \right)^{3/2} \sqrt{-1 + \operatorname{Sec}[c+dx]^2} (A + C \operatorname{Sec}[c+dx]^2) \operatorname{Sin}[c+dx] \Bigg/$$

$$(2d (1 - \operatorname{Cos}[c+dx]^2) (A + 2C + A \operatorname{Cos}[2c + 2dx]) (1 + \operatorname{Sec}[c+dx])^{3/2}) +$$

$$\left(\sqrt{(1 + \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]} (a (1 + \operatorname{Sec}[c+dx]) \right)^{3/2} (A + C \operatorname{Sec}[c+dx]^2)$$

$$\left(\frac{4A \operatorname{Cos}[dx] \operatorname{Sin}[c]}{d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-8A \operatorname{Sin}\left[\frac{dx}{2}\right] + 5C \operatorname{Sin}\left[\frac{dx}{2}\right]^2)}{2d} + \frac{4A \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} -$$

$$\frac{(-2C + 8A \operatorname{Cos}[c] - 7C \operatorname{Cos}[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{2d} \right) \Bigg/ \left((A + 2C + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c+dx]^{3/2} (1 + \operatorname{Sec}[c+dx])^{3/2} \right)$$

■ **Problem 262: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^{3/2} (A + C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 3, 169 leaves, 5 steps) :

$$\frac{3a^{3/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a^2 (8A - 3C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}} -$$

$$\frac{a (2A - 3C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d} + \frac{2A (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 382 leaves) :

$$\begin{aligned} & \left(6 C \cos [c+d x]^3 \left(-\log [1+\sec [c+d x]] + \log \left[\sqrt{\sec [c+d x]} + \sec [c+d x]^{3/2} + \sqrt{1+\sec [c+d x]} \sqrt{-1+\sec [c+d x]^2} \right] \right) (a(1+\sec [c+d x]))^{3/2} \right. \\ & \quad \left. \sqrt{-1+\sec [c+d x]^2} (A+C \sec [c+d x]^2) \sin [c+d x] \right) / \left(d(1-\cos [c+d x]^2) (A+2 C+A \cos [2 c+2 d x]) (1+\sec [c+d x])^{3/2} \right) + \\ & \left(\sqrt{(1+\cos [c+d x]) \sec [c+d x]} (a(1+\sec [c+d x]))^{3/2} (A+C \sec [c+d x]^2) \left(\frac{16 A \cos [d x] \sin [c]}{3 d} + \frac{2 A \cos [2 d x] \sin [2 c]}{3 d} - \right. \right. \\ & \quad \left. \left. \frac{2 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (8 A \sin \left[\frac{d x}{2} \right] - 3 C \sin \left[\frac{d x}{2} \right])}{3 d} + \frac{16 A \cos [c] \sin [d x]}{3 d} + \frac{2 A \cos [2 c] \sin [2 d x]}{3 d} - \frac{2(8 A - 3 C) \tan \left[\frac{c}{2} \right]}{3 d} \right) \right) / \\ & \left((A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} (1+\sec [c+d x])^{3/2} \right) \end{aligned}$$

■ **Problem 263: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \sec [c+d x])^{3/2} (A+C \sec [c+d x]^2)}{\sec [c+d x]^{5/2}} dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\begin{aligned} & \frac{2 a^{3/2} C \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}} \right]}{d} + \frac{2 a^2 (4 A+5 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d \sqrt{a+a \sec [c+d x]}} + \\ & \frac{2 a A \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{5 d \sqrt{\sec [c+d x]}} + \frac{2 A (a+a \sec [c+d x])^{3/2} \sin [c+d x]}{5 d \sec [c+d x]^{3/2}} \end{aligned}$$

Result (type 3, 428 leaves):

$$\begin{aligned} & \left(4 C \cos [c+d x]^3 \left(-\log [1+\sec [c+d x]] + \log \left[\sqrt{\sec [c+d x]} + \sec [c+d x]^{3/2} + \sqrt{1+\sec [c+d x]} \sqrt{-1+\sec [c+d x]^2} \right] \right) (a(1+\sec [c+d x]))^{3/2} \right. \\ & \quad \left. \sqrt{-1+\sec [c+d x]^2} (A+C \sec [c+d x]^2) \sin [c+d x] \right) / \left(d(1-\cos [c+d x]^2) (A+2 C+A \cos [2 c+2 d x]) (1+\sec [c+d x])^{3/2} \right) + \\ & \left(\sqrt{(1+\cos [c+d x]) \sec [c+d x]} (a(1+\sec [c+d x]))^{3/2} (A+C \sec [c+d x]^2) \right. \\ & \quad \left(\frac{(17 A+20 C) \cos [d x] \sin [c]}{5 d} + \frac{4 A \cos [2 d x] \sin [2 c]}{5 d} + \frac{A \cos [3 d x] \sin [3 c]}{5 d} - \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (4 A \sin \left[\frac{d x}{2} \right] + 5 C \sin \left[\frac{d x}{2} \right])}{5 d} + \right. \\ & \quad \left. \frac{(17 A+20 C) \cos [c] \sin [d x]}{5 d} + \frac{4 A \cos [2 c] \sin [2 d x]}{5 d} + \frac{A \cos [3 c] \sin [3 d x]}{5 d} - \frac{4(4 A+5 C) \tan \left[\frac{c}{2} \right]}{5 d} \right) \right) / \\ & \left((A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} (1+\sec [c+d x])^{3/2} \right) \end{aligned}$$

■ **Problem 272: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\frac{5 a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a^3 (64 A + 15 C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15 d \sqrt{a+a \operatorname{Sec}[c+dx]}} -$$

$$\frac{a^2 (16 A - 15 C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15 d} + \frac{2 a A (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 A (a + a \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}}$$

Result (type 3, 428 leaves):

$$\left(10 C \operatorname{Cos}[c + d x]^3 \left(-\operatorname{Log}[1 + \operatorname{Sec}[c + d x]] + \operatorname{Log}\left[\sqrt{\operatorname{Sec}[c + d x]} + \operatorname{Sec}[c + d x]^{3/2} + \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{-1 + \operatorname{Sec}[c + d x]^2}\right] \right) (a (1 + \operatorname{Sec}[c + d x]))^{5/2} \right.$$

$$\left. \sqrt{-1 + \operatorname{Sec}[c + d x]^2} (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}[c + d x] \right) / \left(d (1 - \operatorname{Cos}[c + d x]^2) (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (1 + \operatorname{Sec}[c + d x])^{5/2} \right) +$$

$$\left(\sqrt{(1 + \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]} (a (1 + \operatorname{Sec}[c + d x]))^{5/2} (A + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left(\frac{(131 A + 60 C) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{15 d} + \frac{22 A \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{15 d} + \frac{A \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{5 d} - \frac{2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (64 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 15 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{15 d} + \right.$$

$$\left. \frac{(131 A + 60 C) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{15 d} + \frac{22 A \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{15 d} + \frac{A \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{5 d} - \frac{2 (64 A + 15 C) \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} \right) \left. \right) /$$

$$\left((A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{3/2} (1 + \operatorname{Sec}[c + d x])^{5/2} \right)$$

■ **Problem 273: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 3, 210 leaves, 6 steps):

$$\frac{2 a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2 a^3 (32 A + 49 C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{21 d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{2 a^2 (8 A + 7 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{21 d \sqrt{\operatorname{Sec}[c+dx]}} + \frac{2 a A (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 A (a + a \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}}$$

Result (type 3, 474 leaves):

$$\begin{aligned}
& \left(4 C \cos [c+d x]^3 \left(-\operatorname{Log}[1+\operatorname{Sec}[c+d x]] + \operatorname{Log}\left[\sqrt{\operatorname{Sec}[c+d x]} + \operatorname{Sec}[c+d x]^{3/2} + \sqrt{1+\operatorname{Sec}[c+d x]} \sqrt{-1+\operatorname{Sec}[c+d x]^2}\right]\right) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right. \\
& \quad \left. \sqrt{-1+\operatorname{Sec}[c+d x]^2} (A+C \operatorname{Sec}[c+d x]^2) \sin [c+d x]\right) / \left(d \left(1-\cos [c+d x]^2\right) (A+2 C+A \cos [2 c+2 d x]) (1+\operatorname{Sec}[c+d x])^{5/2}\right) + \\
& \quad \frac{1}{(A+2 C+A \cos [2 c+2 d x]) \operatorname{Sec}[c+d x]^{3/2} (1+\operatorname{Sec}[c+d x])^{5/2}} \sqrt{(1+\cos [c+d x]) \operatorname{Sec}[c+d x]} (a(1+\operatorname{Sec}[c+d x]))^{5/2} \\
& \quad (A+C \operatorname{Sec}[c+d x]^2) \left(\frac{(137 A+196 C) \cos [d x] \sin [c]}{21 d} + \frac{(31 A+14 C) \cos [2 d x] \sin [2 c]}{21 d} + \frac{3 A \cos [3 d x] \sin [3 c]}{7 d} + \right. \\
& \quad \frac{A \cos [4 d x] \sin [4 c]}{14 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right] \left(32 A \sin \left[\frac{d x}{2}\right]+49 C \sin \left[\frac{d x}{2}\right]\right)}{21 d} + \frac{(137 A+196 C) \cos [c] \sin [d x]}{21 d} + \\
& \quad \left. \frac{(31 A+14 C) \cos [2 c] \sin [2 d x]}{21 d} + \frac{3 A \cos [3 c] \sin [3 d x]}{7 d} + \frac{A \cos [4 c] \sin [4 d x]}{14 d} - \frac{4(32 A+49 C) \tan \left[\frac{c}{2}\right]}{21 d} \right)
\end{aligned}$$

■ **Problem 277: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{5/2} (A+C \operatorname{Sec}[c+d x]^2)}{\sqrt{a+a \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 3, 226 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(8 A+9 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{8 \sqrt{a} d} + \frac{\sqrt{2} (A+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{\sqrt{a} d} + \\
& \frac{(8 A+7 C) \operatorname{Sec}[c+d x]^{3/2} \sin [c+d x]}{8 d \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{C \operatorname{Sec}[c+d x]^{5/2} \sin [c+d x]}{12 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{C \operatorname{Sec}[c+d x]^{7/2} \sin [c+d x]}{3 d \sqrt{a+a \operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 3, 793 leaves):

$$\begin{aligned}
& \left(\sqrt{(1 + \cos[c + dx]) \sec[c + dx]} \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx])^2 \right. \\
& \left. \left(-\frac{C(-2 + \cos[c]) \sin\left[\frac{c}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \cos\left[\frac{3c}{2}\right]\right)} - \frac{3C \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{2d} + \frac{C \sec[c] \sec[c + dx] \sin[dx]}{d} \right) \right) / \\
& \left((A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} \sqrt{a(1 + \sec[c + dx])} \right) + \\
& \frac{1}{4(A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])}} \cos[c + dx]^2 \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx])^2 \\
& \left(-\left(C \cos[c + dx]^2 \left(\log\left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}\right] - \log\left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}\right] \right) (1 + \sec[c + dx]) \sqrt{-1 + \sec[c + dx]^2} \right. \\
& \left. \sin[c + dx] \right) / \left(2d(1 + \cos[c + dx]) \sqrt{2 - 2\cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2} \right) - \frac{1}{4d(1 + \cos[c + dx]) (1 - \cos[c + dx]^2)} \\
& (-8A - 7C) \cos[c + dx]^2 \left(-8 \log[1 + \sec[c + dx]] + 8 \log\left[\sqrt{\sec[c + dx]} + \sec[c + dx]^{3/2} + \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}\right] + \sqrt{2} \right. \\
& \left. \left(-\log\left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}\right] + \log\left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}\right] \right) \right) (1 + \sec[c + dx]) \sqrt{-1 + \sec[c + dx]^2} \sin[c + dx] \Big)
\end{aligned}$$

■ **Problem 279: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]} (A + C \sec[c + dx])^2}{\sqrt{a + a \sec[c + dx]}} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (A + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c + dx]} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{\sqrt{a} d} + \frac{C \sec[c + dx]^{3/2} \sin[c + dx]}{d \sqrt{a + a \sec[c + dx]}}$$

Result (type 3, 717 leaves):

$$\begin{aligned}
& \left((2A + C) \cos[c + dx]^4 \left(\log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] - \right. \right. \\
& \quad \left. \left. \log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] \right) \right. \\
& \quad \left. (1 + \sec[c + dx])^{3/2} \sqrt{-1 + \sec[c + dx]^2} (A + C \sec[c + dx]^2) \sin[c + dx] \right) / \\
& \left(2d (1 + \cos[c + dx]) \sqrt{2 - 2 \cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2} (A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])} \right) - \\
& \left(C \cos[c + dx]^4 \left(-8 \log[1 + \sec[c + dx]] + 8 \log \left[\sqrt{\sec[c + dx]} + \sec[c + dx]^{3/2} + \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] + \right. \right. \\
& \quad \left. \left. \sqrt{2} \left(-\log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] + \right. \right. \\
& \quad \left. \left. \log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] \right) \right) \right) \\
& \quad \left. (1 + \sec[c + dx])^{3/2} \sqrt{-1 + \sec[c + dx]^2} (A + C \sec[c + dx]^2) \sin[c + dx] \right) / \\
& \left(4d (1 + \cos[c + dx]) (1 - \cos[c + dx]^2) (A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])} \right) + \\
& \left(\sqrt{(1 + \cos[c + dx]) \sec[c + dx]} \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx]^2) \left(\frac{2C \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{2C \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \left((A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} \sqrt{a(1 + \sec[c + dx])} \right)
\end{aligned}$$

■ **Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{\sqrt{\sec[c + dx]} \sqrt{a + a \sec[c + dx]}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\frac{2C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{\sqrt{a} d} - \frac{\sqrt{2} (A+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right]}{\sqrt{a} d} + \frac{2A \sqrt{\sec[c+dx]} \sin[c+dx]}{d \sqrt{a+a \sec[c+dx]}}$$

Result (type 3, 504 leaves):

$$\begin{aligned}
& - \frac{1}{4 d (-1 + \operatorname{Sec}[c + d x]) \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{a (1 + \operatorname{Sec}[c + d x])}} \\
& \operatorname{Tan}[c + d x] \left(\frac{8 A}{\sqrt{\frac{1}{1 + \operatorname{Cos}[c + d x]}}} - 8 A \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]} + 8 C \operatorname{Log}[1 + \operatorname{Sec}[c + d x]] \sqrt{\operatorname{Tan}[c + d x]^2} - \right. \\
& 8 C \operatorname{Log}\left[\sqrt{\operatorname{Sec}[c + d x]} + \operatorname{Sec}[c + d x]^{3/2} + \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]^2}\right] \sqrt{\operatorname{Tan}[c + d x]^2} + \\
& \sqrt{2} A \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + d x] - 3 \operatorname{Sec}[c + d x]^2 - 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]^2}\right] \sqrt{\operatorname{Tan}[c + d x]^2} + \\
& \sqrt{2} C \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + d x] - 3 \operatorname{Sec}[c + d x]^2 - 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]^2}\right] \sqrt{\operatorname{Tan}[c + d x]^2} - \\
& \sqrt{2} A \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + d x] - 3 \operatorname{Sec}[c + d x]^2 + 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]^2}\right] \sqrt{\operatorname{Tan}[c + d x]^2} - \\
& \left. \sqrt{2} C \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + d x] - 3 \operatorname{Sec}[c + d x]^2 + 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + d x]} \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{\operatorname{Tan}[c + d x]^2}\right] \sqrt{\operatorname{Tan}[c + d x]^2} \right)
\end{aligned}$$

■ **Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{3/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 136 leaves, 4 steps):

$$\frac{\sqrt{2} (A + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} + \frac{2 A \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]}} - \frac{2 A \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 462 leaves):

$$\begin{aligned}
& \left((A + C) \cos[c + dx]^4 \left(\log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2 \sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] - \right. \right. \\
& \quad \left. \left. \log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2 \sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] \right) \right. \\
& \quad \left. (1 + \sec[c + dx])^{3/2} \sqrt{-1 + \sec[c + dx]^2} (A + C \sec[c + dx]^2) \sin[c + dx] \right) / \\
& \left(d (1 + \cos[c + dx]) \sqrt{2 - 2 \cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2} (A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])} \right) + \\
& \left(\sqrt{(1 + \cos[c + dx]) \sec[c + dx]} \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx]^2) \left(-\frac{8A \cos[dx] \sin[c]}{3d} + \frac{2A \cos[2dx] \sin[2c]}{3d} + \right. \right. \\
& \quad \left. \left. \frac{8A \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{3d} - \frac{8A \cos[c] \sin[dx]}{3d} + \frac{2A \cos[2c] \sin[2dx]}{3d} + \frac{8A \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& \left((A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} \sqrt{a(1 + \sec[c + dx])} \right)
\end{aligned}$$

■ **Problem 282: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{\sec[c + dx]^{5/2} \sqrt{a + a \sec[c + dx]}} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} (A + C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec[c + dx]} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}} \right]}{\sqrt{a} d} + \frac{2A \sin[c + dx]}{5d \sec[c + dx]^{3/2} \sqrt{a + a \sec[c + dx]}} - \\
& \frac{2A \sin[c + dx]}{15d \sqrt{\sec[c + dx]} \sqrt{a + a \sec[c + dx]}} + \frac{2(13A + 15C) \sqrt{\sec[c + dx]} \sin[c + dx]}{15d \sqrt{a + a \sec[c + dx]}}
\end{aligned}$$

Result (type 3, 528 leaves):

$$\begin{aligned}
& - \left(\left((A+C) \cos[c+dx]^4 \left(\log \left[1 - 2 \sec[c+dx] - 3 \sec[c+dx]^2 - 2\sqrt{2} \sqrt{\sec[c+dx]} \sqrt{1+\sec[c+dx]} \sqrt{-1+\sec[c+dx]^2} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \log \left[1 - 2 \sec[c+dx] - 3 \sec[c+dx]^2 + 2\sqrt{2} \sqrt{\sec[c+dx]} \sqrt{1+\sec[c+dx]} \sqrt{-1+\sec[c+dx]^2} \right] \right) \right. \right. \\
& \quad \left. \left. (1+\sec[c+dx])^{3/2} \sqrt{-1+\sec[c+dx]^2} (A+C \sec[c+dx]^2) \sin[c+dx] \right) \right) / \\
& \left(d(1+\cos[c+dx]) \sqrt{2-2\cos[c+dx]^2} \sqrt{1-\cos[c+dx]^2} (A+2C+A\cos[2c+2dx]) \sqrt{a(1+\sec[c+dx])} \right) + \\
& \left(\sqrt{(1+\cos[c+dx]) \sec[c+dx]} \sqrt{1+\sec[c+dx]} (A+C \sec[c+dx]^2) \right. \\
& \left(\frac{(71A+60C) \cos[dx] \sin[c]}{15d} - \frac{8A \cos[2dx] \sin[2c]}{15d} + \frac{A \cos[3dx] \sin[3c]}{5d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (17A \sin\left[\frac{dx}{2}\right] + 15C \sin\left[\frac{dx}{2}\right])}{15d} + \right. \\
& \quad \left. \frac{(71A+60C) \cos[c] \sin[dx]}{15d} - \frac{8A \cos[2c] \sin[2dx]}{15d} + \frac{A \cos[3c] \sin[3dx]}{5d} - \frac{4(17A+15C) \tan\left[\frac{c}{2}\right]}{15d} \right) \right) / \\
& \left((A+2C+A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \right)
\end{aligned}$$

■ **Problem 283: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \sec[c+dx]^2}{\sec[c+dx]^{7/2} \sqrt{a+a \sec[c+dx]}} dx$$

Optimal (type 3, 224 leaves, 6 steps):

$$\begin{aligned}
& \frac{\sqrt{2} (A+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right]}{\sqrt{a} d} + \frac{2A \sin[c+dx]}{7d \sec[c+dx]^{5/2} \sqrt{a+a \sec[c+dx]}} - \\
& \frac{2A \sin[c+dx]}{35d \sec[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}} + \frac{2(31A+35C) \sin[c+dx]}{105d \sqrt{\sec[c+dx]} \sqrt{a+a \sec[c+dx]}} - \frac{2(43A+35C) \sqrt{\sec[c+dx]} \sin[c+dx]}{105d \sqrt{a+a \sec[c+dx]}}
\end{aligned}$$

Result (type 3, 573 leaves):

$$\begin{aligned}
& \left((A + C) \cos[c + dx]^4 \left(\log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] - \right. \right. \\
& \quad \left. \left. \log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] \right) \right. \\
& \quad \left. (1 + \sec[c + dx])^{3/2} \sqrt{-1 + \sec[c + dx]^2} (A + C \sec[c + dx]^2) \sin[c + dx] \right) / \\
& \left(d (1 + \cos[c + dx]) \sqrt{2 - 2 \cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2} (A + 2C + A \cos[2c + 2dx]) \sqrt{a(1 + \sec[c + dx])} \right) + \\
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} \sqrt{a(1 + \sec[c + dx])}} \\
& \sqrt{(1 + \cos[c + dx]) \sec[c + dx]} \sqrt{1 + \sec[c + dx]} (A + C \sec[c + dx]^2) \left(-\frac{2(193A + 140C) \cos[dx] \sin[c]}{105d} + \frac{(113A + 70C) \cos[2dx] \sin[2c]}{105d} - \right. \\
& \quad \frac{6A \cos[3dx] \sin[3c]}{35d} + \frac{A \cos[4dx] \sin[4c]}{14d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (46A \sin\left[\frac{dx}{2}\right] + 35C \sin\left[\frac{dx}{2}\right])}{105d} - \frac{2(193A + 140C) \cos[c] \sin[dx]}{105d} + \\
& \quad \left. \frac{(113A + 70C) \cos[2c] \sin[2dx]}{105d} - \frac{6A \cos[3c] \sin[3dx]}{35d} + \frac{A \cos[4c] \sin[4dx]}{14d} + \frac{8(46A + 35C) \tan\left[\frac{c}{2}\right]}{105d} \right)
\end{aligned}$$

■ **Problem 284: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{3/2} (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{3/2}} dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{3C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{a^{3/2} d} + \frac{(A+9C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A+C) \sec[c+dx]^{5/2} \sin[c+dx]}{2d(a+a \sec[c+dx])^{3/2}} + \frac{(A+3C) \sec[c+dx]^{3/2} \sin[c+dx]}{2ad\sqrt{a+a \sec[c+dx]}}$$

Result (type 3, 800 leaves):

$$\frac{1}{2 (A + 2 C + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{3/2}} \cos [c + d x]^2 (1 + \sec [c + d x])^{3/2} (A + C \sec [c + d x]^2) \left((A + 3 C) \cos [c + d x]^2 \left(\log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 - 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] - \log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 + 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] \right) (1 + \sec [c + d x]) \sqrt{-1 + \sec [c + d x]^2} \sin [c + d x] \right) / \left(2 d (1 + \cos [c + d x]) \sqrt{2 - 2 \cos [c + d x]^2} \sqrt{1 - \cos [c + d x]^2} \right) - \frac{1}{2 d (1 + \cos [c + d x]) (1 - \cos [c + d x]^2)} 3 C \cos [c + d x]^2 \left(-8 \log [1 + \sec [c + d x]] + 8 \log [\sqrt{\sec [c + d x]} + \sec [c + d x]^{3/2} + \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] + \sqrt{2} \left(-\log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 - 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] + \log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 + 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] \right) \right) (1 + \sec [c + d x]) \sqrt{-1 + \sec [c + d x]^2} \sin [c + d x] \right) + \left(\sqrt{(1 + \cos [c + d x]) \sec [c + d x]} (1 + \sec [c + d x])^{3/2} (A + C \sec [c + d x]^2) \left(\frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}]^2 (-A \sin [\frac{c}{2}] - C \sin [\frac{c}{2}])}{2 d} + \frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}]^3 (-A \sin [\frac{dx}{2}] - C \sin [\frac{dx}{2}])}{2 d} + \frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}] (A \sin [\frac{dx}{2}] + 3 C \sin [\frac{dx}{2}])}{d} + \frac{(A + 3 C) \tan [\frac{c}{2}]}{d} \right) \right) / \left((A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{3/2} (a (1 + \sec [c + d x]))^{3/2} \right)$$

■ **Problem 285: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec [c + d x]} (A + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^{3/2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$\frac{2 C \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \sec [c + d x]}} \right]}{a^{3/2} d} + \frac{(3 A - 5 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + d x]} \sin [c + d x]}{\sqrt{2} \sqrt{a + a \sec [c + d x]}} \right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A + C) \sec [c + d x]^{3/2} \sin [c + d x]}{2 d (a + a \sec [c + d x])^{3/2}}$$

Result (type 3, 795 leaves):

$$\frac{1}{2 (A + 2 C + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{3/2}} \cos [c + d x]^2 (1 + \sec [c + d x])^{3/2} (A + C \sec [c + d x]^2) \left((3 A - C) \cos [c + d x]^2 \left(\log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 - 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] - \log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 + 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] \right) (1 + \sec [c + d x]) \sqrt{-1 + \sec [c + d x]^2} \sin [c + d x] \right) / \left(2 d (1 + \cos [c + d x]) \sqrt{2 - 2 \cos [c + d x]^2} \sqrt{1 - \cos [c + d x]^2} \right) + \frac{1}{d (1 + \cos [c + d x]) (1 - \cos [c + d x]^2)} C \cos [c + d x]^2 \left(-8 \log [1 + \sec [c + d x]] + 8 \log [\sqrt{\sec [c + d x]} + \sec [c + d x]^{3/2} + \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] + \sqrt{2} \left(-\log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 - 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] + \log [1 - 2 \sec [c + d x] - 3 \sec [c + d x]^2 + 2 \sqrt{2} \sqrt{\sec [c + d x]} \sqrt{1 + \sec [c + d x]} \sqrt{-1 + \sec [c + d x]^2}] \right) \right) (1 + \sec [c + d x]) \sqrt{-1 + \sec [c + d x]^2} \sin [c + d x] \right) + \left(\sqrt{(1 + \cos [c + d x]) \sec [c + d x]} (1 + \sec [c + d x])^{3/2} (A + C \sec [c + d x]^2) \left(\frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}]^2 (A \sin [\frac{c}{2}] + C \sin [\frac{c}{2}])}{2 d} + \frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}] (-A \sin [\frac{dx}{2}] - C \sin [\frac{dx}{2}])}{d} + \frac{\sec [\frac{c}{2}] \sec [\frac{c}{2} + \frac{dx}{2}]^3 (A \sin [\frac{dx}{2}] + C \sin [\frac{dx}{2}])}{2 d} - \frac{(A + C) \tan [\frac{c}{2}]}{d} \right) \right) / \left((A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{3/2} (a (1 + \sec [c + d x]))^{3/2} \right)$$

■ **Problem 288: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec [c + d x]^2}{\sec [c + d x]^{5/2} (a + a \sec [c + d x])^{3/2}} dx$$

Optimal (type 3, 248 leaves, 6 steps):

$$-\frac{(15 A + 7 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + d x]} \sin [c + d x]}{\sqrt{2} \sqrt{a + a \sec [c + d x]}} \right]}{2 \sqrt{2} a^{3/2} d} - \frac{(A + C) \sin [c + d x]}{2 d \sec [c + d x]^{3/2} (a + a \sec [c + d x])^{3/2}} + \frac{(9 A + 5 C) \sin [c + d x]}{10 a d \sec [c + d x]^{3/2} \sqrt{a + a \sec [c + d x]}} - \frac{(13 A + 5 C) \sin [c + d x]}{10 a d \sqrt{\sec [c + d x]} \sqrt{a + a \sec [c + d x]}} + \frac{(49 A + 25 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{10 a d \sqrt{a + a \sec [c + d x]}}$$

Result (type 3, 630 leaves):

$$\begin{aligned}
& - \left(\left((15A + 7C) \cos[c + dx]^4 \left(\log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] - \right. \right. \right. \\
& \quad \left. \left. \left. \log \left[1 - 2 \sec[c + dx] - 3 \sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2} \right] \right) \right) \right. \\
& \quad \left. (1 + \sec[c + dx])^{5/2} \sqrt{-1 + \sec[c + dx]^2} (A + C \sec[c + dx]^2) \sin[c + dx] \right) / \\
& \quad \left(4d (1 + \cos[c + dx]) \sqrt{2 - 2 \cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2} (A + 2C + A \cos[2c + 2dx]) (a (1 + \sec[c + dx]))^{3/2} \right) + \\
& \quad \frac{1}{(A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} (a (1 + \sec[c + dx]))^{3/2}} \\
& \quad \sqrt{(1 + \cos[c + dx]) \sec[c + dx]} (1 + \sec[c + dx])^{3/2} (A + C \sec[c + dx]^2) \\
& \quad \left(\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-A \sin\left[\frac{c}{2}\right] - C \sin\left[\frac{c}{2}\right])}{2d} + \frac{(57A + 20C) \cos[dx] \sin[c]}{5d} - \frac{6A \cos[2dx] \sin[2c]}{5d} + \right. \\
& \quad \frac{A \cos[3dx] \sin[3c]}{5d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{2d} - \frac{3 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (17A \sin\left[\frac{dx}{2}\right] + 5C \sin\left[\frac{dx}{2}\right])}{5d} \\
& \quad \left. \frac{(57A + 20C) \cos[c] \sin[dx]}{5d} - \frac{6A \cos[2c] \sin[2dx]}{5d} + \frac{A \cos[3c] \sin[3dx]}{5d} - \frac{3(17A + 5C) \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 289: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{5/2} (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{5/2}} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{a^{5/2} d} + \frac{(3A + 115C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c + dx]} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \\
& \frac{(A + C) \sec[c + dx]^{7/2} \sin[c + dx]}{4d (a + a \sec[c + dx])^{5/2}} + \frac{(A - 15C) \sec[c + dx]^{5/2} \sin[c + dx]}{16ad (a + a \sec[c + dx])^{3/2}} + \frac{(3A + 35C) \sec[c + dx]^{3/2} \sin[c + dx]}{16a^2 d \sqrt{a + a \sec[c + dx]}}
\end{aligned}$$

Result (type 3, 903 leaves):

$$\begin{aligned}
& \frac{1}{16 (A + 2C + A \cos[2c + 2dx]) (a (1 + \sec[c + dx]))^{5/2}} \\
& \cos[c + dx]^2 (1 + \sec[c + dx])^{5/2} (A + C \sec[c + dx]^2) \left(\frac{1}{2d (1 + \cos[c + dx]) \sqrt{2 - 2\cos[c + dx]^2} \sqrt{1 - \cos[c + dx]^2}} \right. \\
& (3A + 35C) \cos[c + dx]^2 \left(\log[1 - 2\sec[c + dx] - 3\sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}] - \right. \\
& \left. \log[1 - 2\sec[c + dx] - 3\sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}] \right) \\
& (1 + \sec[c + dx]) \sqrt{-1 + \sec[c + dx]^2} \sin[c + dx] - \frac{1}{d (1 + \cos[c + dx]) (1 - \cos[c + dx]^2)} \\
& 20C \cos[c + dx]^2 \left(-8 \log[1 + \sec[c + dx]] + 8 \log[\sqrt{\sec[c + dx]} + \sec[c + dx]^{3/2} + \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}] + \right. \\
& \left. \sqrt{2} \left(-\log[1 - 2\sec[c + dx] - 3\sec[c + dx]^2 - 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}] + \right. \right. \\
& \left. \left. \log[1 - 2\sec[c + dx] - 3\sec[c + dx]^2 + 2\sqrt{2} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \sqrt{-1 + \sec[c + dx]^2}] \right) \right) \\
& (1 + \sec[c + dx]) \sqrt{-1 + \sec[c + dx]^2} \sin[c + dx] \left. \right) + \left(\sqrt{(1 + \cos[c + dx]) \sec[c + dx]} (1 + \sec[c + dx])^{5/2} \right. \\
& (A + C \sec[c + dx]^2) \left(\frac{\sec[\frac{c}{2}] \sec[\frac{c}{2} + \frac{dx}{2}]^2 (A \sin[\frac{c}{2}] - 15C \sin[\frac{c}{2}])}{16d} + \frac{\sec[\frac{c}{2}] \sec[\frac{c}{2} + \frac{dx}{2}]^4 (-A \sin[\frac{c}{2}] - C \sin[\frac{c}{2}])}{8d} + \right. \\
& \frac{\sec[\frac{c}{2}] \sec[\frac{c}{2} + \frac{dx}{2}]^3 (A \sin[\frac{dx}{2}] - 15C \sin[\frac{dx}{2}])}{16d} + \frac{\sec[\frac{c}{2}] \sec[\frac{c}{2} + \frac{dx}{2}]^5 (-A \sin[\frac{dx}{2}] - C \sin[\frac{dx}{2}])}{8d} + \\
& \left. \left. \frac{\sec[\frac{c}{2}] \sec[\frac{c}{2} + \frac{dx}{2}] (3A \sin[\frac{dx}{2}] + 35C \sin[\frac{dx}{2}])}{8d} + \frac{(3A + 35C) \tan[\frac{c}{2}]}{8d} \right) \right) / \\
& ((A + 2C + A \cos[2c + 2dx]) \sec[c + dx]^{3/2} (a (1 + \sec[c + dx]))^{5/2})
\end{aligned}$$

■ **Problem 290: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{3/2} (A + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{5/2}} dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$\frac{2 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} + \frac{(5 A - 43 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A+C) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{4 d (a+a \operatorname{Sec}[c+dx])^{5/2}} + \frac{(5 A - 11 C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{16 a d (a+a \operatorname{Sec}[c+dx])^{3/2}}$$

Result (type 3, 901 leaves):

$$\frac{1}{16 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a (1+\operatorname{Sec}[c+dx]))^{5/2}}$$

$$\operatorname{Cos}[c+dx]^2 (1+\operatorname{Sec}[c+dx])^{5/2} (A+C \operatorname{Sec}[c+dx]^2) \left(\frac{1}{2 d (1+\operatorname{Cos}[c+dx]) \sqrt{2-2 \operatorname{Cos}[c+dx]^2} \sqrt{1-\operatorname{Cos}[c+dx]^2}} \right.$$

$$\left. (5 A - 11 C) \operatorname{Cos}[c+dx]^2 \left(\operatorname{Log}\left[1-2 \operatorname{Sec}[c+dx]-3 \operatorname{Sec}[c+dx]^2-2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{-1+\operatorname{Sec}[c+dx]^2}\right] - \right.$$

$$\left. \operatorname{Log}\left[1-2 \operatorname{Sec}[c+dx]-3 \operatorname{Sec}[c+dx]^2+2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{-1+\operatorname{Sec}[c+dx]^2}\right] \right)$$

$$(1+\operatorname{Sec}[c+dx]) \sqrt{-1+\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] + \frac{1}{d (1+\operatorname{Cos}[c+dx]) (1-\operatorname{Cos}[c+dx]^2)}$$

$$8 C \operatorname{Cos}[c+dx]^2 \left(-8 \operatorname{Log}[1+\operatorname{Sec}[c+dx]] + 8 \operatorname{Log}\left[\sqrt{\operatorname{Sec}[c+dx]} + \operatorname{Sec}[c+dx]^{3/2} + \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{-1+\operatorname{Sec}[c+dx]^2}\right] + \right.$$

$$\left. \sqrt{2} \left(-\operatorname{Log}\left[1-2 \operatorname{Sec}[c+dx]-3 \operatorname{Sec}[c+dx]^2-2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{-1+\operatorname{Sec}[c+dx]^2}\right] + \right.$$

$$\left. \operatorname{Log}\left[1-2 \operatorname{Sec}[c+dx]-3 \operatorname{Sec}[c+dx]^2+2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{-1+\operatorname{Sec}[c+dx]^2}\right] \right)$$

$$(1+\operatorname{Sec}[c+dx]) \sqrt{-1+\operatorname{Sec}[c+dx]^2} \operatorname{Sin}[c+dx] \left. \right) + \left(\sqrt{(1+\operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]} (1+\operatorname{Sec}[c+dx])^{5/2} \right.$$

$$(A+C \operatorname{Sec}[c+dx]^2) \left(\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^4 (A \operatorname{Sin}\left[\frac{c}{2}\right]+C \operatorname{Sin}\left[\frac{c}{2}\right])}{8 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 (-9 A \operatorname{Sin}\left[\frac{c}{2}\right]+7 C \operatorname{Sin}\left[\frac{c}{2}\right])}{16 d} + \right.$$

$$\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right] (5 A \operatorname{Sin}\left[\frac{dx}{2}\right]-11 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right]+C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} +$$

$$\left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^3 (-9 A \operatorname{Sin}\left[\frac{dx}{2}\right]+7 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{16 d} + \frac{(5 A - 11 C) \operatorname{Tan}\left[\frac{c}{2}\right]}{8 d} \right) \Bigg/$$

$$((A+2 C+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+dx]^{3/2} (a (1+\operatorname{Sec}[c+dx]))^{5/2})$$

■ **Problem 293: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 246 leaves, 6 steps):

$$\frac{(163 A + 19 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A + C) \operatorname{Sin}[c + dx]}{4 d \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{5/2}} -$$

$$\frac{(17 A + C) \operatorname{Sin}[c + dx]}{16 a d \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{3/2}} + \frac{5 (19 A + 3 C) \operatorname{Sin}[c + dx]}{48 a^2 d \sqrt{\operatorname{Sec}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]}} - \frac{(299 A + 27 C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{48 a^2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 680 leaves):

$$\left((163 A + 19 C) \operatorname{Cos}[c + dx]^4 \left(\operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + dx] - 3 \operatorname{Sec}[c + dx]^2 - 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + dx]} \sqrt{1 + \operatorname{Sec}[c + dx]} \sqrt{-1 + \operatorname{Sec}[c + dx]^2}\right] - \right. \right.$$

$$\left. \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c + dx] - 3 \operatorname{Sec}[c + dx]^2 + 2 \sqrt{2} \sqrt{\operatorname{Sec}[c + dx]} \sqrt{1 + \operatorname{Sec}[c + dx]} \sqrt{-1 + \operatorname{Sec}[c + dx]^2}\right] \right)$$

$$\left. (1 + \operatorname{Sec}[c + dx])^{7/2} \sqrt{-1 + \operatorname{Sec}[c + dx]^2} (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c + dx] \right) /$$

$$\left(32 d (1 + \operatorname{Cos}[c + dx]) \sqrt{2 - 2 \operatorname{Cos}[c + dx]^2} \sqrt{1 - \operatorname{Cos}[c + dx]^2} (A + 2 C + A \operatorname{Cos}[2 c + 2 dx]) (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \right) +$$

$$\frac{1}{(A + 2 C + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{3/2} (a (1 + \operatorname{Sec}[c + dx]))^{5/2}}$$

$$\sqrt{(1 + \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]} (1 + \operatorname{Sec}[c + dx])^{5/2} (A + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (-A \operatorname{Sin}\left[\frac{c}{2}\right] - C \operatorname{Sin}\left[\frac{c}{2}\right])}{8 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (33 A \operatorname{Sin}\left[\frac{c}{2}\right] + 17 C \operatorname{Sin}\left[\frac{c}{2}\right])}{16 d} - \frac{32 A \operatorname{Cos}[dx] \operatorname{Sin}[c]}{3 d} + \right.$$

$$\frac{2 A \operatorname{Cos}[2 dx] \operatorname{Sin}[2 c]}{3 d} + \frac{13 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (13 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 3 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{24 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (-A \operatorname{Sin}\left[\frac{dx}{2}\right] - C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} +$$

$$\left. \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (33 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 17 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{16 d} - \frac{32 A \operatorname{Cos}[c] \operatorname{Sin}[dx]}{3 d} + \frac{2 A \operatorname{Cos}[2 c] \operatorname{Sin}[2 dx]}{3 d} + \frac{13 (13 A - 3 C) \operatorname{Tan}\left[\frac{c}{2}\right]}{24 d} \right)$$

■ **Problem 294: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 295 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(283 A + 75 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A+C) \operatorname{Sin}[c+dx]}{4 d \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{5/2}} - \frac{(21 A+5 C) \operatorname{Sin}[c+dx]}{16 a d \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2}} + \\
& \frac{(157 A+45 C) \operatorname{Sin}[c+dx]}{80 a^2 d \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{(787 A+195 C) \operatorname{Sin}[c+dx]}{240 a^2 d \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{(2671 A+735 C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{240 a^2 d \sqrt{a+a \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 3, 722 leaves):

$$\begin{aligned}
& - \left(\left((283 A + 75 C) \operatorname{Cos}[c+dx]^4 \left(\operatorname{Log}\left[1 - 2 \operatorname{Sec}[c+dx] - 3 \operatorname{Sec}[c+dx]^2 - 2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1 + \operatorname{Sec}[c+dx]} \sqrt{-1 + \operatorname{Sec}[c+dx]^2}\right] - \right. \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[1 - 2 \operatorname{Sec}[c+dx] - 3 \operatorname{Sec}[c+dx]^2 + 2 \sqrt{2} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1 + \operatorname{Sec}[c+dx]} \sqrt{-1 + \operatorname{Sec}[c+dx]^2}\right] \right) \right) \\
& \quad \left. (1 + \operatorname{Sec}[c+dx])^{7/2} \sqrt{-1 + \operatorname{Sec}[c+dx]^2} (A + C \operatorname{Sec}[c+dx]^2) \operatorname{Sin}[c+dx] \right) / \\
& \quad \left(32 d (1 + \operatorname{Cos}[c+dx]) \sqrt{2 - 2 \operatorname{Cos}[c+dx]^2} \sqrt{1 - \operatorname{Cos}[c+dx]^2} (A + 2 C + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2} \right) + \\
& \quad \frac{1}{(A + 2 C + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c+dx]^{3/2} (a (1 + \operatorname{Sec}[c+dx]))^{5/2}} \\
& \quad \sqrt{(1 + \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]} (1 + \operatorname{Sec}[c+dx])^{5/2} (A + C \operatorname{Sec}[c+dx]^2) \\
& \quad \left(\frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (-41 A \operatorname{Sin}\left[\frac{c}{2}\right] - 25 C \operatorname{Sin}\left[\frac{c}{2}\right])}{16 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A \operatorname{Sin}\left[\frac{c}{2}\right] + C \operatorname{Sin}\left[\frac{c}{2}\right])}{8 d} + \frac{(331 A + 60 C) \operatorname{Cos}[dx] \operatorname{Sin}[c]}{15 d} - \right. \\
& \quad \frac{28 A \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{15 d} + \frac{A \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{5 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-2069 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 165 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{120 d} + \\
& \quad \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (-41 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 25 C \operatorname{Sin}\left[\frac{dx}{2}\right])}{16 d} + \frac{\operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{8 d} + \\
& \quad \left. \frac{(331 A + 60 C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{15 d} - \frac{28 A \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{15 d} + \frac{A \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{5 d} - \frac{(2069 A + 165 C) \operatorname{Tan}\left[\frac{c}{2}\right]}{120 d} \right)
\end{aligned}$$

■ **Problem 295: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c+dx])^{2/3} (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 6, 434 leaves, 10 steps):

$$\frac{3 C (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{7 d \sqrt{1 - \operatorname{Sec}[c + d x]}} +$$

$$\frac{3 C (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d (1 + \operatorname{Sec}[c + d x])} - \left(3^{3/4} C \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (a + a \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(5 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 8, 29 leaves) :

$$\int (a + a \operatorname{Sec}[c + d x])^{2/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 296: Unable to integrate problem.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 6, 384 leaves, 9 steps) :

$$\frac{3 C \operatorname{Tan}[c + d x]}{2 d (a + a \operatorname{Sec}[c + d x])^{1/3}} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{d \sqrt{1 - \operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{1/3}} +$$

$$\left(3^{3/4} C \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(2 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 8, 29 leaves) :

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

■ **Problem 297: Unable to integrate problem.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{4/3}} dx$$

Optimal (type 6, 396 leaves, 9 steps):

$$-\frac{3(A+C)\operatorname{Tan}[c+dx]}{5d(a+a\operatorname{Sec}[c+dx])^{4/3}} + \frac{3\sqrt{2}A\operatorname{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2}(1+\operatorname{Sec}[c+dx]), 1+\operatorname{Sec}[c+dx]\right]\operatorname{Tan}[c+dx]}{ad\sqrt{1-\operatorname{Sec}[c+dx]}(a+a\operatorname{Sec}[c+dx])^{1/3}} +$$

$$\left(3^{3/4}(A-4C)\operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3}-(1-\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}{2^{1/3}-(1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right.$$

$$\left.(2^{1/3}-(1+\operatorname{Sec}[c+dx])^{1/3})\sqrt{\frac{2^{2/3}+2^{1/3}(1+\operatorname{Sec}[c+dx])^{1/3}+(1+\operatorname{Sec}[c+dx])^{2/3}}{(2^{1/3}-(1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3})^2}}\operatorname{Tan}[c+dx]\right)/$$

$$\left(5\times 2^{1/3}ad(1-\operatorname{Sec}[c+dx])(a+a\operatorname{Sec}[c+dx])^{1/3}\sqrt{\frac{(1+\operatorname{Sec}[c+dx])^{1/3}(2^{1/3}-(1+\operatorname{Sec}[c+dx])^{1/3})}{(2^{1/3}-(1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3})^2}}\right)$$

Result (type 8, 29 leaves):

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{4/3}} dx$$

■ **Problem 298: Unable to integrate problem.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{7/3}} dx$$

Optimal (type 6, 457 leaves, 10 steps):

$$\begin{aligned}
& - \frac{3 (A + C) \operatorname{Tan}[c + d x]}{11 d (a + a \operatorname{Sec}[c + d x])^{7/3}} - \frac{3 (4 A - 7 C) \operatorname{Tan}[c + d x]}{55 a^2 d (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3}} - \\
& \frac{3 \sqrt{2} A \operatorname{AppellF1}\left[-\frac{5}{6}, \frac{1}{2}, 1, \frac{1}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{5 a^2 d \sqrt{1 - \operatorname{Sec}[c + d x]} (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(3^{3/4} (4 A - 7 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(55 \times 2^{1/3} a^2 d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{7/3}} dx$$

■ **Problem 299: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 815 leaves, 12 steps):

$$\begin{aligned}
& \frac{3 a C (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{7 d} + \frac{1}{11 d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& 3 \sqrt{2} a \operatorname{AppellF1}\left[\frac{11}{6}, \frac{1}{2}, 1, \frac{17}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x] + \\
& \frac{3 C (a + a \operatorname{Sec}[c + d x])^{4/3} \operatorname{Tan}[c + d x]}{7 d} - \frac{15 (1 + \sqrt{3}) a C (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{7 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3})) (1 + \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(15 \times 2^{1/3} 3^{1/4} a C \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(7 d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(5 \times 3^{3/4} (1 - \sqrt{3}) a C \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(7 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^{4/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 300: Unable to integrate problem.**

$$\int (a + a \operatorname{Sec}[c + d x])^{1/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 774 leaves, 11 steps):

$$\begin{aligned}
& \frac{3 C (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{5 d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& \frac{3 (1 + \sqrt{3}) C (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3})) (1 + \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(3 \times 3^{1/4} C \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(2 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(3^{3/4} (1 - \sqrt{3}) C \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(4 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int (a + a \operatorname{Sec}[c + d x])^{1/3} (A + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 301: Unable to integrate problem.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 6, 791 leaves, 11 steps):

$$\begin{aligned}
& - \frac{3 (A + C) \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])^{2/3}} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{5 a d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& \frac{3 (1 + \sqrt{3}) (A + 2 C) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{a d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3})) (1 + \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(3 \times 2^{1/3} 3^{1/4} (A + 2 C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\
& \left. (a + a \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) / \\
& \left(a d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(3^{3/4} (1 - \sqrt{3}) (A + 2 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2} \operatorname{Tan}[c + d x]} \right) / \\
& \left(2^{2/3} a d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{2/3}} dx$$

■ **Problem 302: Unable to integrate problem.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{5/3}} dx$$

Optimal (type 6, 841 leaves, 12 steps):

$$\begin{aligned}
& - \frac{3(A+C)\tan[c+dx]}{7d(a+a\sec[c+dx])^{5/3}} - \frac{3(2A-5C)\tan[c+dx]}{7ad(a+a\sec[c+dx])^{2/3}} - \frac{3\sqrt{2} \operatorname{AppellF1}\left[-\frac{1}{6}, \frac{1}{2}, 1, \frac{5}{6}, \frac{1}{2}(1+\sec[c+dx]), 1+\sec[c+dx]\right] \tan[c+dx]}{ad\sqrt{1-\sec[c+dx]}(a+a\sec[c+dx])^{2/3}} \\
& + \frac{3(1+\sqrt{3})(2A-5C)(1+\sec[c+dx])^{1/3}\tan[c+dx]}{7ad(a+a\sec[c+dx])^{2/3}\left(2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}\right)} \\
& \left(3 \times 2^{1/3} 3^{1/4} (2A-5C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3}-(1-\sqrt{3})(1+\sec[c+dx])^{1/3}}{2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right. \\
& \left. (1+\sec[c+dx])^{1/3} \left(2^{1/3}-(1+\sec[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3}+2^{1/3}(1+\sec[c+dx])^{1/3}+(1+\sec[c+dx])^{2/3}}{\left(2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}\right)^2}} \tan[c+dx] \right) / \\
& \left(7ad(1-\sec[c+dx])(a+a\sec[c+dx])^{2/3} \sqrt{-\frac{(1+\sec[c+dx])^{1/3}\left(2^{1/3}-(1+\sec[c+dx])^{1/3}\right)}{\left(2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}\right)^2}} \right) + \\
& \left(3^{3/4}(1-\sqrt{3})(2A-5C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3}-(1-\sqrt{3})(1+\sec[c+dx])^{1/3}}{2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] (1+\sec[c+dx])^{1/3} \right. \\
& \left. \left(2^{1/3}-(1+\sec[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3}+2^{1/3}(1+\sec[c+dx])^{1/3}+(1+\sec[c+dx])^{2/3}}{\left(2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}\right)^2}} \tan[c+dx] \right) / \\
& \left(7 \times 2^{2/3} ad(1-\sec[c+dx])(a+a\sec[c+dx])^{2/3} \sqrt{-\frac{(1+\sec[c+dx])^{1/3}\left(2^{1/3}-(1+\sec[c+dx])^{1/3}\right)}{\left(2^{1/3}-(1+\sqrt{3})(1+\sec[c+dx])^{1/3}\right)^2}} \right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{A+C\sec[c+dx]^2}{(a+a\sec[c+dx])^{5/3}} dx$$

■ **Problem 303: Unable to integrate problem.**

$$\int \sec[c+dx]^m (a+a\sec[c+dx])^n (A+C\sec[c+dx]^2) dx$$

Optimal (type 6, 244 leaves, 8 steps):

$$\frac{C \operatorname{Sec}[c + d x]^{1+m} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d (1 + m + n)} + \frac{1}{d (1 + m + n)}$$

$$2^{\frac{3}{2}+n} C n \operatorname{AppellF1}\left[\frac{1}{2}, 1 - m, -\frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right] (1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x] +$$

$$\frac{1}{d (1 + m + n)} 2^{\frac{1}{2}+n} (C (m - n) + A (1 + m + n)) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - m, \frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right]$$

$$(1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]$$

Result (type 8, 35 leaves):

$$\int \operatorname{Sec}[c + d x]^m (a + a \operatorname{Sec}[c + d x])^n (A + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 304: Unable to integrate problem.**

$$\int \operatorname{Sec}[c + d x]^{-1-n} (a + a \operatorname{Sec}[c + d x])^n (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 253 leaves, 8 steps):

$$\frac{A \operatorname{Sec}[c + d x]^{-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d (1 + n)}$$

$$\left((C - A n + C n) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - n, -n, 1 - n, -\frac{2 \operatorname{Sec}[c + d x]}{1 - \operatorname{Sec}[c + d x]}\right] \operatorname{Sec}[c + d x]^{1-n} \left(\frac{1 + \operatorname{Sec}[c + d x]}{1 - \operatorname{Sec}[c + d x]}\right)^{\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \right) /$$

$$(d n (1 + n) (1 + \operatorname{Sec}[c + d x])) + \frac{1}{d}$$

$$2^{\frac{3}{2}+n} C \operatorname{AppellF1}\left[\frac{1}{2}, 1 + n, -\frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right] (1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]$$

Result (type 8, 39 leaves):

$$\int \operatorname{Sec}[c + d x]^{-1-n} (a + a \operatorname{Sec}[c + d x])^n (A + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 306: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (a + a \operatorname{Sec}[c + d x]) (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{a (4 B + 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a (B + C) \operatorname{Tan}[c + d x]}{d} +$$

$$\frac{a (4 B + 3 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a C \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{a (B + C) \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 337 leaves):

$$\begin{aligned}
& - \frac{1}{192d} a \operatorname{Sec}[c+dx]^4 \left(36B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + 27C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 12(4B+3C) \cos[2(c+dx)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \\
& 3(4B+3C) \cos[4(c+dx)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& 36B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 27C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] - 24B \sin[c+dx] - 66C \sin[c+dx] - \\
& 64B \sin[2(c+dx)] - 64C \sin[2(c+dx)] - 24B \sin[3(c+dx)] - 18C \sin[3(c+dx)] - 16B \sin[4(c+dx)] - 16C \sin[4(c+dx)] \Big)
\end{aligned}$$

■ **Problem 307: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx] (a + a \operatorname{Sec}[c+dx]) (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 86 leaves, 7 steps):

$$\frac{a(B+C) \operatorname{ArcTanh}[\sin[c+dx]]}{2d} + \frac{a(3B+2C) \tan[c+dx]}{3d} + \frac{a(B+C) \operatorname{Sec}[c+dx] \tan[c+dx]}{2d} + \frac{aC \operatorname{Sec}[c+dx]^2 \tan[c+dx]}{3d}$$

Result (type 3, 181 leaves):

$$\begin{aligned}
& - \frac{1}{24d} a \operatorname{Sec}[c+dx]^3 \left(9(B+C) \cos[c+dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \\
& 3(B+C) \cos[3(c+dx)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - \\
& \left. 4(3B+4C+3(B+C) \cos[c+dx] + (3B+2C) \cos[2(c+dx)]) \sin[c+dx] \right)
\end{aligned}$$

■ **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+dx]) (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{a(2B+C) \operatorname{ArcTanh}[\sin[c+dx]]}{2d} + \frac{a(B+C) \tan[c+dx]}{d} + \frac{aC \operatorname{Sec}[c+dx] \tan[c+dx]}{2d}$$

Result (type 3, 154 leaves):

$$\frac{1}{4d} a \left(-2(2B+C) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. 4B \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right] + 2C \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. \frac{c}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} - \frac{c}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + 4(B+C) \operatorname{Tan}[c+dx] \right)$$

■ **Problem 309: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] (a + a \sec[c+dx]) (B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 32 leaves, 5 steps):

$$a B x + \frac{a(B+C) \operatorname{ArcTanh}[\sin[c+dx]]}{d} + \frac{a C \operatorname{Tan}[c+dx]}{d}$$

Result (type 3, 159 leaves):

$$a B x - \frac{a B \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} - \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \\ \frac{a B \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{a C \operatorname{Tan}[c+dx]}{d}$$

■ **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a + a \sec[c+dx]) (B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 32 leaves, 4 steps):

$$a(B+C)x + \frac{a C \operatorname{ArcTanh}[\sin[c+dx]]}{d} + \frac{a B \sin[c+dx]}{d}$$

Result (type 3, 104 leaves):

$$a B x + a C x - \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{a B \cos[dx] \sin[c]}{d} + \frac{a B \cos[c] \sin[dx]}{d}$$

■ **Problem 314: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^2 (a + a \sec[c+dx])^2 (B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 169 leaves, 8 steps):

$$\frac{a^2 (7B + 6C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^2 (10B + 9C) \tan[c + dx]}{5d} + \frac{a^2 (7B + 6C) \sec[c + dx] \tan[c + dx]}{8d} +$$

$$\frac{a^2 (5B + 6C) \sec[c + dx]^3 \tan[c + dx]}{20d} + \frac{C \sec[c + dx]^3 (a^2 + a^2 \sec[c + dx]) \tan[c + dx]}{5d} + \frac{a^2 (10B + 9C) \tan[c + dx]^3}{15d}$$

Result (type 3, 391 leaves):

$$-\frac{1}{1920d}$$

$$a^2 \sec[c + dx]^5 \left(105B \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 90C \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) +$$

$$150(7B + 6C) \cos[c + dx] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) +$$

$$75(7B + 6C) \cos[3(c + dx)] \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) -$$

$$105B \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - 90C \cos[5(c + dx)] \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] -$$

$$640B \sin[c + dx] - 960C \sin[c + dx] - 660B \sin[2(c + dx)] - 840C \sin[2(c + dx)] - 800B \sin[3(c + dx)] -$$

$$720C \sin[3(c + dx)] - 210B \sin[4(c + dx)] - 180C \sin[4(c + dx)] - 160B \sin[5(c + dx)] - 144C \sin[5(c + dx)] \Big)$$

■ **Problem 315: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + a \sec[c + dx])^2 (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 138 leaves, 8 steps):

$$\frac{a^2 (8B + 7C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^2 (8B + 7C) \tan[c + dx]}{6d} +$$

$$\frac{a^2 (8B + 7C) \sec[c + dx] \tan[c + dx]}{24d} + \frac{(4B - C) (a + a \sec[c + dx])^2 \tan[c + dx]}{12d} + \frac{C (a + a \sec[c + dx])^3 \tan[c + dx]}{4ad}$$

Result (type 3, 339 leaves):

$$\begin{aligned}
& - \frac{1}{192 d} a^2 \operatorname{Sec}[c + d x]^4 \left(72 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 63 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 12 (8 B + 7 C) \operatorname{Cos}[2(c + d x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 3 (8 B + 7 C) \operatorname{Cos}[4(c + d x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\
& 72 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 63 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 48 B \operatorname{Sin}[c + d x] - 90 C \operatorname{Sin}[c + d x] - \\
& 112 B \operatorname{Sin}[2(c + d x)] - 128 C \operatorname{Sin}[2(c + d x)] - 48 B \operatorname{Sin}[3(c + d x)] - 42 C \operatorname{Sin}[3(c + d x)] - 40 B \operatorname{Sin}[4(c + d x)] - 32 C \operatorname{Sin}[4(c + d x)] \Big)
\end{aligned}$$

■ **Problem 317: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^2 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$a^2 B x + \frac{a^2 (4 B + 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{a^2 (2 B + 3 C) \operatorname{Tan}[c + d x]}{2 d} + \frac{C (a^2 + a^2 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 277 leaves):

$$\begin{aligned}
& \frac{1}{16} a^2 (1 + \operatorname{Cos}[c + d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^4 \\
& \left(4 B x - \frac{2 (4 B + 3 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \frac{2 (4 B + 3 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{d} + \right. \\
& \left. \frac{c}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{4 (B + 2 C) \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} - \right. \\
& \left. \frac{c}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{4 (B + 2 C) \operatorname{Sin}\left[\frac{d x}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} \right)
\end{aligned}$$

■ **Problem 323: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^3 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 163 leaves, 12 steps):

$$\begin{aligned}
& \frac{a^3 (15 B + 13 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^3 (15 B + 13 C) \operatorname{Tan}[c + d x]}{5 d} + \frac{3 a^3 (15 B + 13 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{40 d} + \\
& \frac{(5 B - C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{20 d} + \frac{C (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{5 a d} + \frac{a^3 (15 B + 13 C) \operatorname{Tan}[c + d x]^3}{60 d}
\end{aligned}$$

Result (type 3, 391 leaves) :

$$\begin{aligned}
 & -\frac{1}{1920 d} a^3 \operatorname{Sec}[c+d x]^5 \\
 & \left(225 B \operatorname{Cos}[5(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 195 C \operatorname{Cos}[5(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
 & 150(15 B+13 C) \operatorname{Cos}[c+d x] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
 & 75(15 B+13 C) \operatorname{Cos}[3(c+d x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) - \\
 & 225 B \operatorname{Cos}[5(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 195 C \operatorname{Cos}[5(c+d x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \\
 & 1200 B \operatorname{Sin}[c+d x] - 1600 C \operatorname{Sin}[c+d x] - 1140 B \operatorname{Sin}[2(c+d x)] - 1500 C \operatorname{Sin}[2(c+d x)] - 1560 B \operatorname{Sin}[3(c+d x)] - \\
 & 1520 C \operatorname{Sin}[3(c+d x)] - 450 B \operatorname{Sin}[4(c+d x)] - 390 C \operatorname{Sin}[4(c+d x)] - 360 B \operatorname{Sin}[5(c+d x)] - 304 C \operatorname{Sin}[5(c+d x)] \Big)
 \end{aligned}$$

■ **Problem 324: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c+d x])^3 (B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 125 leaves, 11 steps) :

$$\begin{aligned}
 & \frac{5 a^3 (4 B+3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{8 d} + \frac{a^3 (4 B+3 C) \operatorname{Tan}[c+d x]}{d} + \\
 & \frac{3 a^3 (4 B+3 C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{8 d} + \frac{C (a + a \operatorname{Sec}[c+d x])^3 \operatorname{Tan}[c+d x]}{4 d} + \frac{a^3 (4 B+3 C) \operatorname{Tan}[c+d x]^3}{12 d}
 \end{aligned}$$

Result (type 3, 339 leaves) :

$$\begin{aligned}
 & \frac{1}{192 d} a^3 \operatorname{Sec}[c+d x]^4 \left(-180 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 135 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) - \\
 & 60(4 B+3 C) \operatorname{Cos}[2(c+d x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) - \\
 & 15(4 B+3 C) \operatorname{Cos}[4(c+d x)] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
 & 180 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 135 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 72 B \operatorname{Sin}[c+d x] + 138 C \operatorname{Sin}[c+d x] + \\
 & 208 B \operatorname{Sin}[2(c+d x)] + 240 C \operatorname{Sin}[2(c+d x)] + 72 B \operatorname{Sin}[3(c+d x)] + 90 C \operatorname{Sin}[3(c+d x)] + 88 B \operatorname{Sin}[4(c+d x)] + 72 C \operatorname{Sin}[4(c+d x)] \Big)
 \end{aligned}$$

■ **Problem 325: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x] (a + a \operatorname{Sec}[c+d x])^3 (B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 111 leaves, 7 steps) :

$$a^3 B x + \frac{a^3 (7 B + 5 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{5 a^3 (B + C) \operatorname{Tan}[c + d x]}{2 d} +$$

$$\frac{a C (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{(3 B + 5 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{6 d}$$

Result (type 3, 772 leaves) :

$$a^3 \left(\frac{1}{8} B x (1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 + \frac{(-7 B - 5 C) (1 + \operatorname{Cos}[c + d x])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6}{16 d} +$$

$$\frac{(7 B + 5 C) (1 + \operatorname{Cos}[c + d x])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6}{16 d} + \frac{C (1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sin}\left[\frac{d x}{2}\right]}{48 d (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^3} +$$

$$\frac{(1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (3 B \operatorname{Cos}\left[\frac{c}{2}\right] + 10 C \operatorname{Cos}\left[\frac{c}{2}\right] - 3 B \operatorname{Sin}\left[\frac{c}{2}\right] - 8 C \operatorname{Sin}\left[\frac{c}{2}\right])}{96 d (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^2} +$$

$$\frac{(1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (9 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 11 C \operatorname{Sin}\left[\frac{d x}{2}\right])}{24 d (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])} + \frac{C (1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sin}\left[\frac{d x}{2}\right]}{48 d (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^3} +$$

$$\frac{(1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (-3 B \operatorname{Cos}\left[\frac{c}{2}\right] - 10 C \operatorname{Cos}\left[\frac{c}{2}\right] - 3 B \operatorname{Sin}\left[\frac{c}{2}\right] - 8 C \operatorname{Sin}\left[\frac{c}{2}\right])}{96 d (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])^2} +$$

$$\left. \frac{(1 + \operatorname{Cos}[c + d x])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (9 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 11 C \operatorname{Sin}\left[\frac{d x}{2}\right])}{24 d (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right])} \right)$$

■ **Problem 327: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^3 (a + a \operatorname{Sec}[c + d x])^3 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 117 leaves, 6 steps) :

$$\frac{1}{2} a^3 (7 B + 6 C) x + \frac{a^3 (B + 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^3 B \operatorname{Sin}[c + d x]}{2 d} +$$

$$\frac{a B \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 d} - \frac{(B - 2 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{2 d}$$

Result (type 3, 272 leaves) :

$$\frac{1}{32} a^3 (1 + \cos[c + dx])^3 \sec\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left(2(7B + 6C)x - \frac{4(B + 3C) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{4(B + 3C) \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{4(3B + C) \cos[dx] \sin[c]}{d} + \frac{B \cos[2dx] \sin[2c]}{d} + \frac{4(3B + C) \cos[c] \sin[dx]}{d} + \frac{B \cos[2c] \sin[2dx]}{d} + \frac{4C \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{4C \sin\left[\frac{dx}{2}\right]}{d \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} \right)$$

■ **Problem 332: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (B \sec[c + dx] + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$\frac{3(B - C) \operatorname{ArcTanh}[\sin[c + dx]]}{2ad} - \frac{(3B - 4C) \tan[c + dx]}{ad} + \frac{3(B - C) \sec[c + dx] \tan[c + dx]}{2ad} + \frac{(B - C) \sec[c + dx]^3 \tan[c + dx]}{d(a + a \sec[c + dx])} - \frac{(3B - 4C) \tan[c + dx]^3}{3ad}$$

Result (type 3, 550 leaves):

$$\begin{aligned}
& - \frac{1}{24 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^3 \\
& \left(9 B \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 9 C \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 9 B \operatorname{Cos}\left[\frac{7}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 9 C \operatorname{Cos}\left[\frac{7}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \\
& 27 (B - C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \\
& 27 (B - C) \operatorname{Cos}\left[\frac{3}{2}(c + d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - \\
& 9 B \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \\
& 9 C \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 9 B \operatorname{Cos}\left[\frac{7}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + \\
& 9 C \operatorname{Cos}\left[\frac{7}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 12 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 18 B \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] - \\
& 30 C \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] - 6 B \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right] + 2 C \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right] + 12 B \operatorname{Sin}\left[\frac{7}{2}(c + d x)\right] - 16 C \operatorname{Sin}\left[\frac{7}{2}(c + d x)\right] \Big)
\end{aligned}$$

■ **Problem 333: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 108 leaves, 7 steps):

$$- \frac{(2 B - 3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a d} + \frac{2 (B - C) \operatorname{Tan}[c + d x]}{a d} - \frac{(2 B - 3 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a d} + \frac{(B - C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 3, 383 leaves):

$$\frac{1}{4 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}[c + d x]^2$$

$$\left(2 B \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 3 C \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) +$$

$$2(2 B - 3 C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) +$$

$$(2 B - 3 C) \operatorname{Cos}\left[\frac{3}{2}(c + d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right) -$$

$$2 B \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 3 C \operatorname{Cos}\left[\frac{5}{2}(c + d x)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] +$$

$$4 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 2 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 C \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right] + 4 B \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right] - 4 C \operatorname{Sin}\left[\frac{5}{2}(c + d x)\right]$$

■ **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x] (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 62 leaves, 6 steps):

$$\frac{(B - C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d} + \frac{C \operatorname{Tan}[c + d x]}{a d} - \frac{(B - C) \operatorname{Tan}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 3, 234 leaves):

$$-\left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]\right)$$

$$\left((B - C) \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) + (B - C) \operatorname{Cos}\left[\frac{3}{2}(c + d x)\right]\right)$$

$$\left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 2(C - (B - 2C) \operatorname{Cos}[c + d x]) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) /$$

$$\left(a d (1 + \operatorname{Cos}[c + d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)\right)$$

■ **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a d} + \frac{(B - C) \operatorname{Tan}[c + d x]}{a d (1 + \operatorname{Sec}[c + d x])}$$

Result (type 3, 106 leaves):

$$\frac{1}{a d (1 + \cos[c + d x])} - 2 \cos\left[\frac{1}{2}(c + d x)\right] \left(C \cos\left[\frac{1}{2}(c + d x)\right] \left(-\log\left[\cos\left[\frac{1}{2}(c + d x)\right] - \sin\left[\frac{1}{2}(c + d x)\right]\right] + \log\left[\cos\left[\frac{1}{2}(c + d x)\right] + \sin\left[\frac{1}{2}(c + d x)\right]\right] \right) + (B - C) \sin\left[\frac{1}{2}(c + d x)\right] \right)$$

■ **Problem 336: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x] (B \sec[c + d x] + C \sec[c + d x]^2)}{a + a \sec[c + d x]} dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$\frac{B x}{a} - \frac{(B - C) \tan[c + d x]}{d (a + a \sec[c + d x])}$$

Result (type 3, 72 leaves):

$$\frac{\cos\left[\frac{1}{2}(c + d x)\right] \sec\left[\frac{c}{2}\right] (B d x \cos\left[\frac{d x}{2}\right] + B d x \cos\left[c + \frac{d x}{2}\right] + 2(-B + C) \sin\left[\frac{d x}{2}\right])}{a d (1 + \cos[c + d x])}$$

■ **Problem 338: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^3 (B \sec[c + d x] + C \sec[c + d x]^2)}{a + a \sec[c + d x]} dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$\frac{(3B - 2C)x}{2a} - \frac{2(B - C) \sin[c + d x]}{a d} + \frac{(3B - 2C) \cos[c + d x] \sin[c + d x]}{2 a d} - \frac{(B - C) \cos[c + d x] \sin[c + d x]}{d (a + a \sec[c + d x])}$$

Result (type 3, 197 leaves):

$$\frac{1}{8 a d (1 + \cos[c + d x])} \cos\left[\frac{1}{2}(c + d x)\right] \sec\left[\frac{c}{2}\right] \left(4(3B - 2C) d x \cos\left[\frac{d x}{2}\right] + 4(3B - 2C) d x \cos\left[c + \frac{d x}{2}\right] - 20B \sin\left[\frac{d x}{2}\right] + 20C \sin\left[\frac{d x}{2}\right] - 4B \sin\left[c + \frac{d x}{2}\right] + 4C \sin\left[c + \frac{d x}{2}\right] - 3B \sin\left[c + \frac{3 d x}{2}\right] + 4C \sin\left[c + \frac{3 d x}{2}\right] - 3B \sin\left[2c + \frac{3 d x}{2}\right] + 4C \sin\left[2c + \frac{3 d x}{2}\right] + B \sin\left[2c + \frac{5 d x}{2}\right] + B \sin\left[3c + \frac{5 d x}{2}\right] \right)$$

■ **Problem 339: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^4 (B \sec[c + d x] + C \sec[c + d x]^2)}{a + a \sec[c + d x]} dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$-\frac{3(B - C)x}{2a} + \frac{(4B - 3C) \sin[c + d x]}{a d} - \frac{3(B - C) \cos[c + d x] \sin[c + d x]}{2 a d} - \frac{(B - C) \cos[c + d x]^2 \sin[c + d x]}{d (a + a \sec[c + d x])} - \frac{(4B - 3C) \sin[c + d x]^3}{3 a d}$$

Result (type 3, 249 leaves) :

$$\frac{1}{24 a d (1 + \operatorname{Cos}[c + d x])} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right] \left(-36 (B - C) d x \operatorname{Cos}\left[\frac{d x}{2}\right] - 36 (B - C) d x \operatorname{Cos}\left[c + \frac{d x}{2}\right] + 69 B \operatorname{Sin}\left[\frac{d x}{2}\right] - 60 C \operatorname{Sin}\left[\frac{d x}{2}\right] + 21 B \operatorname{Sin}\left[c + \frac{d x}{2}\right] - \right. \\ \left. 12 C \operatorname{Sin}\left[c + \frac{d x}{2}\right] + 18 B \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] - 9 C \operatorname{Sin}\left[c + \frac{3 d x}{2}\right] + 18 B \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - 9 C \operatorname{Sin}\left[2 c + \frac{3 d x}{2}\right] - \right. \\ \left. 2 B \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] + 3 C \operatorname{Sin}\left[2 c + \frac{5 d x}{2}\right] - 2 B \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + 3 C \operatorname{Sin}\left[3 c + \frac{5 d x}{2}\right] + B \operatorname{Sin}\left[3 c + \frac{7 d x}{2}\right] + B \operatorname{Sin}\left[4 c + \frac{7 d x}{2}\right] \right)$$

■ **Problem 340: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 156 leaves, 8 steps) :

$$-\frac{(4 B - 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 a^2 d} + \frac{2 (5 B - 8 C) \operatorname{Tan}[c + d x]}{3 a^2 d} - \\ \frac{(4 B - 7 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 a^2 d} + \frac{(5 B - 8 C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 a^2 d (1 + \operatorname{Sec}[c + d x])} + \frac{(B - C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{3 d (a + a \operatorname{Sec}[c + d x])^2}$$

Result (type 3, 379 leaves) :

$$\frac{1}{6 a^2 d} \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right]^4 \operatorname{Sec}[c + d x]^2 \\ \left(3 (4 B - 7 C) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + 8 (B + 5 C) \operatorname{Csc}[c + d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^4 - \right. \\ \left. 64 (B - C) \operatorname{Csc}[c + d x]^5 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^8 - 128 C \operatorname{Csc}[c + d x]^7 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]^{12} + (26 B - 44 C) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \right. \\ \left. 6 (4 B - 7 C) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 - \right. \\ \left. 8 (5 B - 8 C) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + 3 (4 B - 7 C) \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \right. \\ \left. \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^4 + \left(14 B - 20 C + B \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 \right)$$

■ **Problem 341: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 108 leaves, 7 steps) :

$$\frac{(B-2C) \operatorname{ArcTanh}[\sin[c+dx]]}{a^2 d} - \frac{(B-4C) \tan[c+dx]}{3 a^2 d} - \frac{(B-2C) \tan[c+dx]}{a^2 d (1+\sec[c+dx])} + \frac{(B-C) \sec[c+dx]^2 \tan[c+dx]}{3 d (a+a \sec[c+dx])^2}$$

Result (type 3, 245 leaves):

$$\frac{1}{3 a^2 d} \cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \left(-3(B-2C) \left(\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) - \right. \\ \left. 4(B-C) \csc[c+dx]^3 \sin\left[\frac{1}{2}(c+dx)\right]^4 + 16(B-C) \csc[c+dx]^5 \sin\left[\frac{1}{2}(c+dx)\right]^8 + (-4B+13C) \tan\left[\frac{1}{2}(c+dx)\right] + 3(B-2C) \right. \\ \left. \left(\log\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \tan\left[\frac{1}{2}(c+dx)\right]^2 + (4B-7C) \tan\left[\frac{1}{2}(c+dx)\right]^3 \right)$$

■ **Problem 344: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] (B \sec[c+dx] + C \sec[c+dx]^2)}{(a+a \sec[c+dx])^2} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$\frac{Bx}{a^2} - \frac{(4B-C) \tan[c+dx]}{3 a^2 d (1+\sec[c+dx])} - \frac{(B-C) \tan[c+dx]}{3 d (a+a \sec[c+dx])^2}$$

Result (type 3, 153 leaves):

$$\frac{1}{24 a^2 d} \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c+dx)\right]^3 \left(9 B dx \cos\left[\frac{dx}{2}\right] + 9 B dx \cos\left[c + \frac{dx}{2}\right] + 3 B dx \cos\left[c + \frac{3 dx}{2}\right] + 3 B dx \cos\left[2c + \frac{3 dx}{2}\right] - \right. \\ \left. 18 B \sin\left[\frac{dx}{2}\right] + 6 C \sin\left[\frac{dx}{2}\right] + 12 B \sin\left[c + \frac{dx}{2}\right] - 6 C \sin\left[c + \frac{dx}{2}\right] - 10 B \sin\left[c + \frac{3 dx}{2}\right] + 4 C \sin\left[c + \frac{3 dx}{2}\right] \right)$$

■ **Problem 345: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (B \sec[c+dx] + C \sec[c+dx]^2)}{(a+a \sec[c+dx])^2} dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$-\frac{(2B-C)x}{a^2} + \frac{2(5B-2C) \sin[c+dx]}{3 a^2 d} - \frac{(2B-C) \sin[c+dx]}{a^2 d (1+\sec[c+dx])} - \frac{(B-C) \sin[c+dx]}{3 d (a+a \sec[c+dx])^2}$$

Result (type 3, 245 leaves):

$$\frac{1}{12 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-18 (2 B - C) d x \cos \left[\frac{d x}{2} \right] - 18 (2 B - C) d x \cos \left[c + \frac{d x}{2} \right] - 12 B d x \cos \left[c + \frac{3 d x}{2} \right] + 6 C d x \cos \left[c + \frac{3 d x}{2} \right] - 12 B d x \cos \left[2 c + \frac{3 d x}{2} \right] + 6 C d x \cos \left[2 c + \frac{3 d x}{2} \right] + 66 B \sin \left[\frac{d x}{2} \right] - 36 C \sin \left[\frac{d x}{2} \right] - 30 B \sin \left[c + \frac{d x}{2} \right] + 24 C \sin \left[c + \frac{d x}{2} \right] + 41 B \sin \left[c + \frac{3 d x}{2} \right] - 20 C \sin \left[c + \frac{3 d x}{2} \right] + 9 B \sin \left[2 c + \frac{3 d x}{2} \right] + 3 B \sin \left[2 c + \frac{5 d x}{2} \right] + 3 B \sin \left[3 c + \frac{5 d x}{2} \right] \right)$$

■ **Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 143 leaves, 7 steps):

$$\frac{(7 B - 4 C) x}{2 a^2} - \frac{2 (8 B - 5 C) \sin [c + d x]}{3 a^2 d} + \frac{(7 B - 4 C) \cos [c + d x] \sin [c + d x]}{2 a^2 d} - \frac{(8 B - 5 C) \cos [c + d x] \sin [c + d x]}{3 a^2 d (1 + \sec [c + d x])} - \frac{(B - C) \cos [c + d x] \sin [c + d x]}{3 d (a + a \sec [c + d x])^2}$$

Result (type 3, 315 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(36 (7 B - 4 C) d x \cos \left[\frac{d x}{2} \right] + 36 (7 B - 4 C) d x \cos \left[c + \frac{d x}{2} \right] + 84 B d x \cos \left[c + \frac{3 d x}{2} \right] - 48 C d x \cos \left[c + \frac{3 d x}{2} \right] + 84 B d x \cos \left[2 c + \frac{3 d x}{2} \right] - 48 C d x \cos \left[2 c + \frac{3 d x}{2} \right] - 381 B \sin \left[\frac{d x}{2} \right] + 264 C \sin \left[\frac{d x}{2} \right] + 147 B \sin \left[c + \frac{d x}{2} \right] - 120 C \sin \left[c + \frac{d x}{2} \right] - 239 B \sin \left[c + \frac{3 d x}{2} \right] + 164 C \sin \left[c + \frac{3 d x}{2} \right] - 63 B \sin \left[2 c + \frac{3 d x}{2} \right] + 36 C \sin \left[2 c + \frac{3 d x}{2} \right] - 15 B \sin \left[2 c + \frac{5 d x}{2} \right] + 12 C \sin \left[2 c + \frac{5 d x}{2} \right] - 15 B \sin \left[3 c + \frac{5 d x}{2} \right] + 12 C \sin \left[3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[3 c + \frac{7 d x}{2} \right] + 3 B \sin \left[4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 347: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^4 (B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 170 leaves, 8 steps):

$$-\frac{(10 B - 7 C) x}{2 a^2} + \frac{4 (3 B - 2 C) \sin [c + d x]}{a^2 d} - \frac{(10 B - 7 C) \cos [c + d x] \sin [c + d x]}{2 a^2 d} - \frac{(10 B - 7 C) \cos [c + d x]^2 \sin [c + d x]}{3 a^2 d (1 + \sec [c + d x])} - \frac{(B - C) \cos [c + d x]^2 \sin [c + d x]}{3 d (a + a \sec [c + d x])^2} - \frac{4 (3 B - 2 C) \sin [c + d x]^3}{3 a^2 d}$$

Result (type 3, 369 leaves):

$$\frac{1}{48 a^2 d (1 + \cos [c + d x])^2} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-36 (10 B - 7 C) d x \cos \left[\frac{d x}{2} \right] - 36 (10 B - 7 C) d x \cos \left[c + \frac{d x}{2} \right] - 120 B d x \cos \left[c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[c + \frac{3 d x}{2} \right] - 120 B d x \cos \left[2 c + \frac{3 d x}{2} \right] + 84 C d x \cos \left[2 c + \frac{3 d x}{2} \right] + 516 B \sin \left[\frac{d x}{2} \right] - 381 C \sin \left[\frac{d x}{2} \right] - 156 B \sin \left[c + \frac{d x}{2} \right] + 147 C \sin \left[c + \frac{d x}{2} \right] + 342 B \sin \left[c + \frac{3 d x}{2} \right] - 239 C \sin \left[c + \frac{3 d x}{2} \right] + 118 B \sin \left[2 c + \frac{3 d x}{2} \right] - 63 C \sin \left[2 c + \frac{3 d x}{2} \right] + 30 B \sin \left[2 c + \frac{5 d x}{2} \right] - 15 C \sin \left[2 c + \frac{5 d x}{2} \right] + 30 B \sin \left[3 c + \frac{5 d x}{2} \right] - 15 C \sin \left[3 c + \frac{5 d x}{2} \right] - 3 B \sin \left[3 c + \frac{7 d x}{2} \right] + 3 C \sin \left[3 c + \frac{7 d x}{2} \right] - 3 B \sin \left[4 c + \frac{7 d x}{2} \right] + 3 C \sin \left[4 c + \frac{7 d x}{2} \right] + B \sin \left[4 c + \frac{9 d x}{2} \right] + B \sin \left[5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 348: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^4 (B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^3} dx$$

Optimal (type 3, 202 leaves, 9 steps):

$$-\frac{(6 B - 13 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 a^3 d} + \frac{8 (9 B - 19 C) \tan [c + d x]}{15 a^3 d} - \frac{(6 B - 13 C) \sec [c + d x] \tan [c + d x]}{2 a^3 d} + \frac{(B - C) \sec [c + d x]^4 \tan [c + d x]}{5 d (a + a \sec [c + d x])^3} + \frac{(6 B - 11 C) \sec [c + d x]^3 \tan [c + d x]}{15 a d (a + a \sec [c + d x])^2} + \frac{4 (9 B - 19 C) \sec [c + d x]^2 \tan [c + d x]}{15 d (a^3 + a^3 \sec [c + d x])}$$

Result (type 3, 428 leaves):

$$\frac{1}{60 a^3 d} \cos \left[\frac{1}{2} (c + d x) \right]^4 \sec [c + d x]^2 \left(30 (6 B - 13 C) \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) + 16 (12 B + 13 C) \operatorname{Csc} [c + d x]^3 \sin \left[\frac{1}{2} (c + d x) \right]^4 + 4 (87 B - 197 C) \tan \left[\frac{1}{2} (c + d x) \right] + (-21 B + 31 C + (24 B - 34 C) \cos [c + d x]) \sec \left[\frac{1}{2} (c + d x) \right]^4 \tan \left[\frac{1}{2} (c + d x) \right] - 60 (6 B - 13 C) \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^2 - 64 (9 B - 19 C) \tan \left[\frac{1}{2} (c + d x) \right]^3 - (-6 B + 11 C + (12 B - 17 C) \cos [c + d x]) \sec \left[\frac{1}{2} (c + d x) \right]^4 \tan \left[\frac{1}{2} (c + d x) \right]^3 + 30 (6 B - 13 C) \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) \tan \left[\frac{1}{2} (c + d x) \right]^4 + \left(228 B - 428 C + 3 (B - C) \sec \left[\frac{1}{2} (c + d x) \right]^4 \right) \tan \left[\frac{1}{2} (c + d x) \right]^5 \right)$$

■ **Problem 353: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] \left(B \sec [c+d x]+C \sec [c+d x]^2\right)}{\left(a+a \sec [c+d x]\right)^3} d x$$

Optimal (type 3, 108 leaves, 5 steps):

$$\frac{B x}{a^3} - \frac{(B-C) \tan [c+d x]}{5 d\left(a+a \sec [c+d x]\right)^3} - \frac{(7 B-2 C) \tan [c+d x]}{15 a d\left(a+a \sec [c+d x]\right)^2} - \frac{2(11 B-C) \tan [c+d x]}{15 d\left(a^3+a^3 \sec [c+d x]\right)}$$

Result (type 3, 241 leaves):

$$\frac{1}{480 a^3 d} \sec \left[\frac{c}{2}\right] \sec \left[\frac{1}{2}(c+d x)\right]^5 \left(150 B d x \cos \left[\frac{d x}{2}\right]+150 B d x \cos \left[c+\frac{d x}{2}\right]+75 B d x \cos \left[c+\frac{3 d x}{2}\right]+75 B d x \cos \left[2 c+\frac{3 d x}{2}\right]+15 B d x \cos \left[2 c+\frac{5 d x}{2}\right]+15 B d x \cos \left[3 c+\frac{5 d x}{2}\right]-370 B \sin \left[\frac{d x}{2}\right]+80 C \sin \left[\frac{d x}{2}\right]+270 B \sin \left[c+\frac{d x}{2}\right]-60 C \sin \left[c+\frac{d x}{2}\right]-230 B \sin \left[c+\frac{3 d x}{2}\right]+40 C \sin \left[c+\frac{3 d x}{2}\right]+90 B \sin \left[2 c+\frac{3 d x}{2}\right]-30 C \sin \left[2 c+\frac{3 d x}{2}\right]-64 B \sin \left[2 c+\frac{5 d x}{2}\right]+14 C \sin \left[2 c+\frac{5 d x}{2}\right]\right)$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^2 \left(B \sec [c+d x]+C \sec [c+d x]^2\right)}{\left(a+a \sec [c+d x]\right)^3} d x$$

Optimal (type 3, 136 leaves, 7 steps):

$$-\frac{(3 B-C) x}{a^3} + \frac{2(36 B-11 C) \sin [c+d x]}{15 a^3 d} - \frac{(B-C) \sin [c+d x]}{5 d\left(a+a \sec [c+d x]\right)^3} - \frac{(9 B-4 C) \sin [c+d x]}{15 a d\left(a+a \sec [c+d x]\right)^2} - \frac{(3 B-C) \sin [c+d x]}{d\left(a^3+a^3 \sec [c+d x]\right)}$$

Result (type 3, 365 leaves):

$$\frac{1}{120 a^3 d\left(1+\cos [c+d x]\right)^3} \cos \left[\frac{1}{2}(c+d x)\right] \sec \left[\frac{c}{2}\right] \left(-300(3 B-C) d x \cos \left[\frac{d x}{2}\right]-300(3 B-C) d x \cos \left[c+\frac{d x}{2}\right]-450 B d x \cos \left[c+\frac{3 d x}{2}\right]+150 C d x \cos \left[c+\frac{3 d x}{2}\right]-450 B d x \cos \left[2 c+\frac{3 d x}{2}\right]+150 C d x \cos \left[2 c+\frac{3 d x}{2}\right]-90 B d x \cos \left[2 c+\frac{5 d x}{2}\right]+30 C d x \cos \left[2 c+\frac{5 d x}{2}\right]-90 B d x \cos \left[3 c+\frac{5 d x}{2}\right]+30 C d x \cos \left[3 c+\frac{5 d x}{2}\right]+1755 B \sin \left[\frac{d x}{2}\right]-740 C \sin \left[\frac{d x}{2}\right]-1125 B \sin \left[c+\frac{d x}{2}\right]+540 C \sin \left[c+\frac{d x}{2}\right]+1215 B \sin \left[c+\frac{3 d x}{2}\right]-460 C \sin \left[c+\frac{3 d x}{2}\right]-225 B \sin \left[2 c+\frac{3 d x}{2}\right]+180 C \sin \left[2 c+\frac{3 d x}{2}\right]+363 B \sin \left[2 c+\frac{5 d x}{2}\right]-128 C \sin \left[2 c+\frac{5 d x}{2}\right]+75 B \sin \left[3 c+\frac{5 d x}{2}\right]+15 B \sin \left[3 c+\frac{7 d x}{2}\right]+15 B \sin \left[4 c+\frac{7 d x}{2}\right]\right)$$

■ **Problem 355: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^3 (B \sec [c+d x]+C \sec [c+d x]^2)}{(a+a \sec [c+d x])^3} d x$$

Optimal (type 3, 187 leaves, 8 steps):

$$\frac{(13 B-6 C) x}{2 a^3}-\frac{8(19 B-9 C) \sin [c+d x]}{15 a^3 d}+\frac{(13 B-6 C) \cos [c+d x] \sin [c+d x]}{2 a^3 d}-\frac{(B-C) \cos [c+d x] \sin [c+d x]}{5 d(a+a \sec [c+d x])^3}-\frac{(11 B-6 C) \cos [c+d x] \sin [c+d x]}{15 a d(a+a \sec [c+d x])^2}-\frac{4(19 B-9 C) \cos [c+d x] \sin [c+d x]}{15 d\left(a^3+a^3 \sec [c+d x]\right)}$$

Result (type 3, 435 leaves):

$$\frac{1}{480 a^3 d(1+\cos [c+d x])^3} \cos \left[\frac{1}{2}(c+d x)\right] \sec \left[\frac{c}{2}\right]$$

$$\left(600(13 B-6 C) d x \cos \left[\frac{d x}{2}\right]+600(13 B-6 C) d x \cos \left[c+\frac{d x}{2}\right]+3900 B d x \cos \left[c+\frac{3 d x}{2}\right]-1800 C d x \cos \left[c+\frac{3 d x}{2}\right]+3900 B d x \cos \left[2 c+\frac{3 d x}{2}\right]-1800 C d x \cos \left[2 c+\frac{3 d x}{2}\right]+780 B d x \cos \left[2 c+\frac{5 d x}{2}\right]-360 C d x \cos \left[2 c+\frac{5 d x}{2}\right]+780 B d x \cos \left[3 c+\frac{5 d x}{2}\right]-360 C d x \cos \left[3 c+\frac{5 d x}{2}\right]-12760 B \sin \left[\frac{d x}{2}\right]+7020 C \sin \left[\frac{d x}{2}\right]+7560 B \sin \left[c+\frac{d x}{2}\right]-4500 C \sin \left[c+\frac{d x}{2}\right]-9230 B \sin \left[c+\frac{3 d x}{2}\right]+4860 C \sin \left[c+\frac{3 d x}{2}\right]+930 B \sin \left[2 c+\frac{3 d x}{2}\right]-900 C \sin \left[2 c+\frac{3 d x}{2}\right]-2782 B \sin \left[2 c+\frac{5 d x}{2}\right]+1452 C \sin \left[2 c+\frac{5 d x}{2}\right]-750 B \sin \left[3 c+\frac{5 d x}{2}\right]+300 C \sin \left[3 c+\frac{5 d x}{2}\right]-105 B \sin \left[3 c+\frac{7 d x}{2}\right]+60 C \sin \left[3 c+\frac{7 d x}{2}\right]-105 B \sin \left[4 c+\frac{7 d x}{2}\right]+60 C \sin \left[4 c+\frac{7 d x}{2}\right]+15 B \sin \left[4 c+\frac{9 d x}{2}\right]+15 B \sin \left[5 c+\frac{9 d x}{2}\right]\right)$$

■ **Problem 361: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x] \sqrt{a+a \sec [c+d x]} (B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 66 leaves, 5 steps):

$$\frac{2 \sqrt{a} B \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right]}{d}+\frac{2 a C \tan [c+d x]}{d \sqrt{a+a \sec [c+d x]}}$$

Result (type 4, 363 leaves):

$$\begin{aligned}
& -\frac{1}{d} 8 (-3 - 2\sqrt{2}) B \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec\left[\frac{1}{2}(c + dx)\right] \sec[c + dx]} \\
& \sqrt{a(1 + \sec[c + dx])} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} + \frac{2C\sqrt{a(1 + \sec[c + dx])} \tan\left[\frac{1}{2}(c + dx)\right]}{d}
\end{aligned}$$

- **Problem 362: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 \sqrt{a + a \sec[c + dx]} (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 68 leaves, 4 steps):

$$\frac{\sqrt{a} (B + 2C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{d} + \frac{a B \sin[c + dx]}{d \sqrt{a + a \sec[c + dx]}}$$

Result (type 4, 396 leaves):

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{2}B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}B \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{d} -$$

$$\frac{1}{d} 4(-3-2\sqrt{2})(B+2C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}$$

■ **Problem 363: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$\frac{\sqrt{a} (3B+4C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a(3B+4C) \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 418 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{8}(B+4C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}(B+2C) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}B \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (3B+4C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

- **Problem 364: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 \sqrt{a+a\operatorname{Sec}[c+dx]} (B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 160 leaves, 6 steps):

$$\frac{\sqrt{a} (5B+6C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a(5B+6C) \operatorname{Sin}[c+dx]}{8d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{a(5B+6C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{12d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{aB \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3d\sqrt{a+a\operatorname{Sec}[c+dx]}}$$

Result (type 4, 443 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(-\frac{1}{48}(11B+6C)\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{12}(4B+3C)\operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16}(B+2C)\operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{24}B\operatorname{Sin}\left[\frac{7}{2}(c+dx)\right]\right) + \\
& \frac{1}{d}\left(1 + \frac{3}{2\sqrt{2}}\right)(5B+6C)\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]\operatorname{Sec}[c+dx]\sqrt{a(1+\operatorname{Sec}[c+dx])}\sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

■ **Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] (a+a\operatorname{Sec}[c+dx])^{3/2} (B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 105 leaves, 6 steps):

$$\frac{2a^{3/2}B\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2a^2(3B+4C)\operatorname{Tan}[c+dx]}{3d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{2aC\sqrt{a+a\operatorname{Sec}[c+dx]}\operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 408 leaves):

$$\frac{1}{d} \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \left(\frac{1}{3}(3B+5C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3}C \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] \right) -$$

$$\frac{1}{d} 4(-3-2\sqrt{2})B \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2 \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 370: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a+a\sec[c+dx])^{3/2} (B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$\frac{a^{3/2}(3B+2C) \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{d} + \frac{a^2(B-2C)\sin[c+dx]}{d\sqrt{a+a\sec[c+dx]}} + \frac{2aC\sqrt{a+a\sec[c+dx]}\sin[c+dx]}{d}$$

Result (type 4, 408 leaves):

$$\frac{1}{d} \cos[c+dx] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} \left(\frac{1}{4}(-B+4C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}B \sin\left[\frac{3}{2}(c+dx)\right]\right) -$$

$$\frac{1}{d} 2(-3-2\sqrt{2})(3B+2C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}}$$

■ **Problem 372: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+a \operatorname{Sec}[c+dx])^{3/2} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 164 leaves, 6 steps):

$$\frac{a^{3/2} (11B+14C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a^2 (11B+14C) \sin[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (7B+6C) \cos[c+dx] \sin[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \cos[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} \sin[c+dx]}{3d}$$

Result (type 4, 3073 leaves):

$$a \left(-\frac{1}{d} 2(-3-2\sqrt{2})C \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \right.$$

$$\left. \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) (1+\cos[c+dx]) \right.$$

$$\left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right)$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} + \\
& \frac{1}{4\sqrt{1 + \operatorname{Sec}[c + dx]}} B(1 + \cos[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \operatorname{Sec}[c + dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(-\frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \frac{1}{d} 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \right. \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]} \\
& \operatorname{Sec}[c + dx] \sqrt{1 + \operatorname{Sec}[c + dx]} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \left. + \frac{1}{2\sqrt{1 + \operatorname{Sec}[c + dx]}} \right) \\
& c(1 + \cos[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \operatorname{Sec}[c + dx])} \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(-\frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \right. \\
& \frac{1}{d} 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \\
& \left. \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right) \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& \frac{1}{2\sqrt{1+\operatorname{Sec}[c+dx]}} B(1+\cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(-\frac{1}{8}\sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}\sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}\sin\left[\frac{5}{2}(c+dx)\right]\right)}{d} + \right. \\
& \frac{1}{d} 3 \left(2 + \frac{3}{\sqrt{2}}\right) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2} + \\
& \frac{1}{4\sqrt{1+\operatorname{Sec}[c+dx]}} C(1+\cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(-\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right]\right)}{2d} + \frac{1}{2d\sqrt{\operatorname{Sec}[c+dx]}} \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{1}{2} \sqrt{\operatorname{Sec}[c+dx]} \left(\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{5}{2}(c+dx)\right] \right) + 4(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) + \\
& \frac{1}{4\sqrt{1 + \sec[c + dx]}} B(1 + \cos[c + dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \sec[c + dx])} \\
& \left(\frac{1}{2} \left(\frac{\sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(-\frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \frac{1}{d} 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \right. \right. \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left. \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right. \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec\left[\frac{1}{2}(c + dx)\right]} \\
& \left. \left. \sec[c + dx] \sqrt{1 + \sec[c + dx]} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) + \right. \\
& \left. \frac{1}{2} \left(\frac{\sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(2 \sin\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{3}{2}(c + dx)\right] + \sin\left[\frac{7}{2}(c + dx)\right]\right)}{6d} + \frac{1}{2d\sqrt{\sec[c + dx]}} \right)
\end{aligned}$$

$$\begin{aligned} & \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{1}{2} \sqrt{\operatorname{Sec}[c+dx]} \left(\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{5}{2}(c+dx)\right] \right) + 4(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \right. \\ & \left. \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \right. \\ & \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\ & \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}} \right) \right) \end{aligned}$$

- **Problem 373: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 (a+a \operatorname{Sec}[c+dx])^{3/2} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\begin{aligned} & \frac{a^{3/2} (75B + 88C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64d} + \frac{a^2 (75B + 88C) \sin[c+dx]}{64d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (75B + 88C) \cos[c+dx] \sin[c+dx]}{96d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \\ & \frac{a^2 (9B + 8C) \cos[c+dx]^2 \sin[c+dx]}{24d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{aB \cos[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} \sin[c+dx]}{4d} \end{aligned}$$

Result (type 4, 4534 leaves):

$$\begin{aligned} & a \left(-\frac{1}{d} 2(-3-2\sqrt{2}) C \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \right. \\ & \left. \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) (1+\cos[c+dx]) \right. \\ & \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} + \\
& \frac{1}{4\sqrt{1 + \operatorname{Sec}[c + dx]}} B(1 + \cos[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \operatorname{Sec}[c + dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(-\frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \frac{1}{d} 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \right. \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]} \\
& \left. \operatorname{Sec}[c + dx] \sqrt{1 + \operatorname{Sec}[c + dx]} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2} \right) + \\
& \frac{1}{8\sqrt{1 + \operatorname{Sec}[c + dx]}} 3C(1 + \cos[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \operatorname{Sec}[c + dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(-\frac{1}{2} \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \sin\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \frac{1}{d} 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \right. \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \text{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \text{Sec}\left[\frac{1}{2}(c+dx)\right]} \\
& \left. \text{Sec}[c+dx] \sqrt{1+\text{Sec}[c+dx]} \sqrt{3-2\sqrt{2} - \text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \frac{1}{8\sqrt{1+\text{Sec}[c+dx]}} 3B(1+\text{Cos}[c+dx]) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1+\text{Sec}[c+dx])} \\
& \left(\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\text{Sec}[c+dx]} \left(-\frac{1}{8}\text{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}\text{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}\text{Sin}\left[\frac{5}{2}(c+dx)\right]\right)}{d} + \right. \\
& \left. \frac{1}{d} 3 \left(2 + \frac{3}{\sqrt{2}}\right) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2} + (10-7\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\text{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \right) \\
& \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1+\sqrt{2} - (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \text{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \text{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \\
& \left. \text{Sec}\left[\frac{1}{2}(c+dx)\right] \text{Sec}[c+dx] \sqrt{1+\text{Sec}[c+dx]} \sqrt{3-2\sqrt{2} - \text{Tan}\left[\frac{1}{4}(c+dx)\right]^2} \right) + \\
& \frac{1}{4\sqrt{1+\text{Sec}[c+dx]}} C(1+\text{Cos}[c+dx]) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1+\text{Sec}[c+dx])} \\
& \left(\frac{\text{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\text{Sec}[c+dx]} \left(-\text{Sin}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{2d} + \frac{1}{2d\sqrt{\text{Sec}[c+dx]}} \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{1}{2} \sqrt{\operatorname{Sec}[c+dx]} \left(\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{5}{2}(c+dx)\right] \right) + 4(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \right. \\
& \left. \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \right. \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\
& \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}} \right) \right) + \\
& \frac{1}{8\sqrt{1+\operatorname{Sec}[c+dx]}} c(1+\cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2}+\frac{dx}{2}\right]^2 \sqrt{a(1+\operatorname{Sec}[c+dx])} \\
& \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} (2\sin\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{3}{2}(c+dx)\right] + \sin\left[\frac{7}{2}(c+dx)\right])}{6d} + \frac{1}{2d\sqrt{\operatorname{Sec}[c+dx]}} \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{1}{2} \sqrt{\operatorname{Sec}[c+dx]} \left(\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{5}{2}(c+dx)\right] \right) + 4(-3-2\sqrt{2}) \cos\left[\frac{1}{4}(c+dx)\right]^4 \right. \\
& \left. \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \right. \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\
& \left. \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4 \sqrt{1 + \operatorname{Sec}[c + dx]}} B(1 + \operatorname{Cos}[c + dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \operatorname{Sec}[c + dx])} \\
& \left(\frac{1}{2} \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(-\frac{1}{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right]\right)}{d} - \right. \right. \\
& \frac{1}{d} 4(-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \\
& \left. \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right) \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + \right. \right. \\
& \left. \left. 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c + dx)\right]^2} \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \sqrt{1 + \operatorname{Sec}[c + dx]} \sqrt{3 - 2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]^2} \right) + \\
& \frac{1}{2} \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{7}{2}(c + dx)\right]\right)}{6d} + \frac{1}{2d \sqrt{\operatorname{Sec}[c + dx]}} \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \operatorname{Sec}[c + dx]} \left(\frac{1}{2} \sqrt{\operatorname{Sec}[c + dx]} \left(\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{5}{2}(c + dx)\right]\right) + 4(-3 - 2\sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right]^4 \right. \right. \\
& \left. \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]\right) \right. \\
& \left. \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3 + 2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) \right) \right) + \\
& \frac{1}{8\sqrt{1 + \sec[c + dx]}} B(1 + \cos[c + dx]) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a(1 + \sec[c + dx])} \\
& \left(\frac{1}{2} \left(\frac{\sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(-\sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{3}{2}(c + dx)\right]\right)}{2d} + \frac{1}{2d\sqrt{\sec[c + dx]}} \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(\frac{1}{2} \sqrt{\sec[c + dx]} \left(\sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{5}{2}(c + dx)\right] \right) + 4(-3 - 2\sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right]^4 \right. \right. \right. \\
& \left. \left. \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \right. \right. \\
& \left. \left. \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right. \right. \\
& \left. \left. \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2 \sec[c + dx]^{3/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}} \right) \right) \right) + \\
& \frac{1}{2} \left(\frac{\sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(2 \sin\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{3}{2}(c + dx)\right] + \sin\left[\frac{7}{2}(c + dx)\right]\right)}{6d} + \frac{1}{2d\sqrt{\sec[c + dx]}} \right. \\
& \left. \sec\left[\frac{1}{2}(c + dx)\right] \sqrt{1 + \sec[c + dx]} \left(-\frac{1}{8} \sqrt{\sec[c + dx]} \left(3 \sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{5}{2}(c + dx)\right] - 2 \sin\left[\frac{9}{2}(c + dx)\right]\right) + \right. \right. \\
& \left. \left. 3 \left(2 + \frac{3}{\sqrt{2}}\right) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \right) \right)
\end{aligned}$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\text{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\text{Cos}\left[\frac{1}{2}(c+dx)\right]\right)\text{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \text{Sec}[c+dx]^{3/2} \sqrt{3-2\sqrt{2}-\text{Tan}\left[\frac{1}{4}(c+dx)\right]^2}} \right)$$

- **Problem 378: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c+dx] (a+a \text{Sec}[c+dx])^{5/2} (B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 3, 142 leaves, 7 steps):

$$\frac{2 a^{5/2} B \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c+dx]}{\sqrt{a+a \text{Sec}[c+dx]}}\right]}{d} + \frac{2 a^3 (35 B + 32 C) \text{Tan}[c+dx]}{15 d \sqrt{a+a \text{Sec}[c+dx]}} + \frac{2 a^2 (5 B + 8 C) \sqrt{a+a \text{Sec}[c+dx]} \text{Tan}[c+dx]}{15 d} + \frac{2 a C (a+a \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{5 d}$$

Result (type 4, 455 leaves):

$$\frac{1}{d} \text{Cos}[c + dx]^2 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} \left(\frac{1}{30} (40B + 43C) \text{Sin}\left[\frac{1}{2}(c + dx)\right] + \frac{1}{10} C \text{Sec}[c + dx]^2 \text{Sin}\left[\frac{1}{2}(c + dx)\right] + \frac{1}{30} \text{Sec}[c + dx] \left(5B \text{Sin}\left[\frac{1}{2}(c + dx)\right] + 14C \text{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) -$$

$$\frac{1}{d} 2(-3 - 2\sqrt{2}) B \text{Cos}\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \text{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \text{Cos}\left[\frac{1}{2}(c + dx)\right]}{1 + \text{Cos}\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \text{Cos}\left[\frac{1}{2}(c + dx)\right] \right) \text{Cos}[c + dx]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\text{Tan}\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \text{Cos}\left[\frac{1}{2}(c + dx)\right]\right) \text{Sec}\left[\frac{1}{4}(c + dx)\right]^2 \text{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \text{Sec}[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \text{Tan}\left[\frac{1}{4}(c + dx)\right]^2}}$$

■ **Problem 379: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^2 (a + a \text{Sec}[c + dx])^{5/2} (B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 3, 143 leaves, 6 steps):

$$\frac{a^{5/2} (5B + 2C) \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[c + dx]}{\sqrt{a + a \text{Sec}[c + dx]}}\right]}{d} - \frac{a^3 (3B + 14C) \text{Sin}[c + dx]}{3d \sqrt{a + a \text{Sec}[c + dx]}} +$$

$$\frac{2a^2 (B + 2C) \sqrt{a + a \text{Sec}[c + dx]} \text{Sin}[c + dx]}{d} + \frac{2aC (a + a \text{Sec}[c + dx])^{3/2} \text{Sin}[c + dx]}{3d}$$

Result (type 4, 434 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \left(\frac{1}{24} (9B + 32C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{6} C \sec[c + dx] \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{8} B \sin\left[\frac{3}{2}(c + dx)\right] \right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (5B + 2C) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right] \right) \cos[c + dx]$$

$$\left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 380: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^{5/2} (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{a^{5/2} (19B + 20C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{4d} + \frac{a^3 (9B - 4C) \sin[c + dx]}{4d \sqrt{a + a \sec[c + dx]}} -$$

$$\frac{a^2 (B - 4C) \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{2d} + \frac{aB \cos[c + dx] (a + a \sec[c + dx])^{3/2} \sin[c + dx]}{2d}$$

Result (type 4, 437 leaves):

$$\frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \left(\frac{3}{32}(-3B+4C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{16}(5B+2C) \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{32}B \sin\left[\frac{5}{2}(c+dx)\right] \right) +$$

$$\frac{1}{8d} (4+3\sqrt{2})(19B+20C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\cos[c+dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^5 (a+a\sec[c+dx])^{5/2} (B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\frac{a^{5/2}(163B+200C) \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{64d} + \frac{a^3(163B+200C) \sin[c+dx]}{64d\sqrt{a+a\sec[c+dx]}} + \frac{a^3(95B+104C) \cos[c+dx] \sin[c+dx]}{96d\sqrt{a+a\sec[c+dx]}} +$$

$$\frac{a^2(11B+8C) \cos[c+dx]^2 \sqrt{a+a\sec[c+dx]} \sin[c+dx]}{24d} + \frac{aB \cos[c+dx]^3 (a+a\sec[c+dx])^{3/2} \sin[c+dx]}{4d}$$

Result (type 4, 479 leaves):

$$\begin{aligned} & \frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \left(-\frac{(265B+376C)\sin\left[\frac{1}{2}(c+dx)\right]}{1536} + \right. \\ & \left. \frac{1}{192}(55B+64C)\sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{512}(47B+40C)\sin\left[\frac{5}{2}(c+dx)\right] + \frac{1}{192}(5B+2C)\sin\left[\frac{7}{2}(c+dx)\right] + \frac{1}{256}B\sin\left[\frac{9}{2}(c+dx)\right] \right) + \\ & \frac{1}{64d} \left(2 + \frac{3}{\sqrt{2}} \right) (163B+200C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right] \right) \\ & \cos[c+dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\ & \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \\ & \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \end{aligned}$$

■ **Problem 383: Result unnecessarily involves higher level functions.**

$$\int \cos[c+dx]^6 (a+a\sec[c+dx])^{5/2} (B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 254 leaves, 8 steps):

$$\begin{aligned} & \frac{a^{5/2}(283B+326C)\text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{128d} + \frac{a^3(283B+326C)\sin[c+dx]}{128d\sqrt{a+a\sec[c+dx]}} + \\ & \frac{a^3(283B+326C)\cos[c+dx]\sin[c+dx]}{192d\sqrt{a+a\sec[c+dx]}} + \frac{a^3(157B+170C)\cos[c+dx]^2\sin[c+dx]}{240d\sqrt{a+a\sec[c+dx]}} + \\ & \frac{a^2(13B+10C)\cos[c+dx]^3\sqrt{a+a\sec[c+dx]}\sin[c+dx]}{40d} + \frac{aB\cos[c+dx]^4(a+a\sec[c+dx])^{3/2}\sin[c+dx]}{5d} \end{aligned}$$

Result (type 4, 500 leaves):

$$\frac{1}{d} \cos[c + dx]^2 \sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2}$$

$$\left(-\frac{(2309B + 2650C) \sin\left[\frac{1}{2}(c + dx)\right]}{15360} + \frac{(509B + 550C) \sin\left[\frac{3}{2}(c + dx)\right]}{1920} + \frac{(95B + 94C) \sin\left[\frac{5}{2}(c + dx)\right]}{1024} + \right.$$

$$\left. \frac{1}{960} (32B + 25C) \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{512} (5B + 2C) \sin\left[\frac{9}{2}(c + dx)\right] + \frac{1}{640} B \sin\left[\frac{11}{2}(c + dx)\right] \right) +$$

$$\frac{1}{256d} (4 + 3\sqrt{2}) (283B + 326C) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right)$$

$$\cos[c + dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2}$$

$$\sec\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \sec[c + dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 404: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] (B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{(5B + 19C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(B - C) \tan[c + dx]}{4d (a + a \sec[c + dx])^{5/2}} + \frac{(5B - 13C) \tan[c + dx]}{16ad (a + a \sec[c + dx])^{3/2}}$$

Result (type 3, 256 leaves):

$$\frac{(5B + 19C) \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \cos\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{Sec}[c + dx]^{5/2} \sqrt{1 + \operatorname{Sec}[c + dx]} + 4d \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}}}{d(a(1 + \operatorname{Sec}[c + dx]))^{5/2}} \cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^3 \left(-\frac{1}{2}(-B + 9C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(-B \sin\left[\frac{1}{2}(c + dx)\right] + C \sin\left[\frac{1}{2}(c + dx)\right]\right) + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(3B \sin\left[\frac{1}{2}(c + dx)\right] + 5C \sin\left[\frac{1}{2}(c + dx)\right]\right)\right)$$

■ **Problem 405: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{(a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{(3B + 5C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{2} \sqrt{a + \operatorname{Sec}[c + dx]}}\right]}{16\sqrt{2} a^{5/2} d} + \frac{(B - C) \tan[c + dx]}{4d(a + a \operatorname{Sec}[c + dx])^{5/2}} + \frac{(3B + 5C) \tan[c + dx]}{16ad(a + a \operatorname{Sec}[c + dx])^{3/2}}$$

Result (type 3, 298 leaves):

$$\left((3B + 5C) \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right] \cos\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{Sec}[c + dx]^{3/2} \sqrt{1 + \operatorname{Sec}[c + dx]} (B + C \operatorname{Sec}[c + dx]) \right) / \left(4d(C + B \cos[c + dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}} \right) + \left(\cos\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx]^2 (B + C \operatorname{Sec}[c + dx]) \left(\frac{1}{2}(7B + C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(B \sin\left[\frac{1}{2}(c + dx)\right] - C \sin\left[\frac{1}{2}(c + dx)\right] \right) + \frac{1}{4} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(-11B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) / (d(C + B \cos[c + dx]) (a(1 + \operatorname{Sec}[c + dx]))^{5/2})$$

■ **Problem 408: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^3 (a + a \operatorname{Sec}[c + dx]) (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{a(4A + 3(B + C)) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a(5A + 5B + 4C) \tan[c + dx]}{5d} + \frac{a(4A + 3(B + C)) \operatorname{Sec}[c + dx] \tan[c + dx]}{8d} + \frac{a(B + C) \operatorname{Sec}[c + dx]^3 \tan[c + dx]}{4d} + \frac{aC \operatorname{Sec}[c + dx]^4 \tan[c + dx]}{5d} + \frac{a(5A + 5B + 4C) \tan[c + dx]^3}{15d}$$

Result (type 3, 660 leaves) :

$$\begin{aligned}
 & - \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{3 a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} - \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
 & \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{3 a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \\
 & \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{3 a B} + \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{3 a C} + \frac{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a A} + \\
 & \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a C} + \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a A} - \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{3 a B} - \\
 & \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4}{3 a C} - \frac{4 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{a A} - \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{3 a B} - \\
 & \frac{16 d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2}{3 d} + \frac{2 a A \operatorname{Tan}[c+dx]}{3 d} + \frac{2 a B \operatorname{Tan}[c+dx]}{3 d} + \frac{8 a C \operatorname{Tan}[c+dx]}{15 d} + \\
 & \frac{a A \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d} + \frac{a B \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d} + \frac{4 a C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{15 d} + \frac{a C \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx]}{5 d}
 \end{aligned}$$

■ **Problem 409: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^2 (a + a \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 127 leaves, 7 steps) :

$$\begin{aligned}
 & \frac{a (4 A + 4 B + 3 C) \operatorname{ArcTanh}\left[\sin\left[\frac{1}{2}(c+dx)\right]\right]}{8 d} + \frac{a (3 A + 2 (B + C)) \operatorname{Tan}[c+dx]}{3 d} + \\
 & \frac{a (4 A + 4 B + 3 C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{8 d} + \frac{a (B + C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{3 d} + \frac{a C \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{4 d}
 \end{aligned}$$

Result (type 3, 545 leaves) :

$$\begin{aligned}
& - \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} - \\
& \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a A \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{2d} + \\
& \frac{3 a C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right]}{8d} + \frac{a C}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} + \frac{a A}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{a B}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{3 a C}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a C}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^4} - \\
& \frac{a A}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{a B}{4d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} - \frac{3 a C}{16d \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2} + \\
& \frac{a A \tan[c+dx]}{d} + \frac{2 a B \tan[c+dx]}{3d} + \frac{2 a C \tan[c+dx]}{3d} + \frac{a B \sec[c+dx]^2 \tan[c+dx]}{3d} + \frac{a C \sec[c+dx]^2 \tan[c+dx]}{3d}
\end{aligned}$$

■ **Problem 410: Result more than twice size of optimal antiderivative.**

$$\int \sec[c+dx] (a + a \sec[c+dx]) (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 92 leaves, 6 steps):

$$\frac{a (2 A + B + C) \operatorname{ArcTanh}[\sin[c+dx]]}{2d} + \frac{a (3 A + 3 B + 2 C) \tan[c+dx]}{3d} + \frac{a (B + C) \sec[c+dx] \tan[c+dx]}{2d} + \frac{a C \sec[c+dx]^2 \tan[c+dx]}{3d}$$

Result (type 3, 995 leaves):

$$\begin{aligned}
& a \left(\frac{(-2A - B - C) \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \\
& \frac{(2A + B + C) \cos[c + dx]^2 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{\cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (3B \cos\left[\frac{c}{2}\right] + 4C \cos\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right] - 2C \sin\left[\frac{c}{2}\right])}{6d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \frac{2 \cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (3A \sin\left[\frac{dx}{2}\right] + 3B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right])}{3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} + \\
& \frac{C \cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^3} + \\
& \frac{\cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (-3B \cos\left[\frac{c}{2}\right] - 4C \cos\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right] - 2C \sin\left[\frac{c}{2}\right])}{6d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \left. \frac{2 \cos[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (3A \sin\left[\frac{dx}{2}\right] + 3B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right])}{3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])} \right)
\end{aligned}$$

■ **Problem 411: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + dx]) (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$a A x + \frac{a (2A + 2B + C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{a (B + C) \tan[c + dx]}{d} + \frac{a C \operatorname{Sec}[c + dx] \tan[c + dx]}{2d}$$

Result (type 3, 305 leaves):

$$\left(a \cos [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\ \left. \left(4 A x - \frac{2 (2 A + 2 B + C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \frac{2 (2 A + 2 B + C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right]}{d} + \right. \right. \\ \left. \frac{c}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (B + C) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)} - \right. \\ \left. \frac{c}{d \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \frac{4 (B + C) \sin \left[\frac{d x}{2} \right]}{d \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)} \right) \Bigg) / \\ (2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)]))$$

■ **Problem 412: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + a \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$a (A + B) x + \frac{a (B + C) \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a A \sin [c + d x]}{d} + \frac{a C \tan [c + d x]}{d}$$

Result (type 3, 187 leaves):

$$a A x + a B x - \frac{a B \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right]}{d} - \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right]}{d} + \frac{a B \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right]}{d} + \\ \frac{a C \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right]}{d} + \frac{a A \cos [d x] \sin [c]}{d} + \frac{a A \cos [c] \sin [d x]}{d} + \frac{a C \tan [c + d x]}{d}$$

■ **Problem 417: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^3 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 3, 222 leaves, 8 steps):

$$\frac{a^2 (14 A + 12 B + 11 C) \operatorname{ArcTanh}[\sin [c + d x]]}{16 d} + \frac{a^2 (10 A + 9 B + 8 C) \tan [c + d x]}{5 d} + \frac{a^2 (14 A + 12 B + 11 C) \sec [c + d x] \tan [c + d x]}{16 d} + \\ \frac{a^2 (10 A + 12 B + 9 C) \sec [c + d x]^3 \tan [c + d x]}{40 d} + \frac{C \sec [c + d x]^3 (a + a \sec [c + d x])^2 \tan [c + d x]}{6 d} + \\ \frac{(3 B + C) \sec [c + d x]^3 (a^2 + a^2 \sec [c + d x]) \tan [c + d x]}{15 d} + \frac{a^2 (10 A + 9 B + 8 C) \tan [c + d x]^3}{15 d}$$

Result (type 3, 959 leaves):

$$\begin{aligned}
& \left((-14A - 12B - 11C) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left((14A + 12B + 11C) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \frac{C \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c + dx]^2 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{12d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec[c + dx] (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) (5C \sin[c] + 6B \sin[dx] + 12C \sin[dx]) \right) / \\
& (60d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (24B \sin[c] + 48C \sin[c] + 30A \sin[dx] + 60B \sin[dx] + 55C \sin[dx]) \right) / (240d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left(\cos[c + dx] \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (30A \sin[c] + 60B \sin[c] + 55C \sin[c] + 80A \sin[dx] + 72B \sin[dx] + 64C \sin[dx]) \right) / \\
& (240d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^2 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (160A \sin[c] + 144B \sin[c] + 128C \sin[c] + 210A \sin[dx] + 180B \sin[dx] + 165C \sin[dx]) \right) / \\
& (480d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^3 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (210A \sin[c] + 180B \sin[c] + 165C \sin[c] + 320A \sin[dx] + 288B \sin[dx] + 256C \sin[dx]) \right) / \\
& (480d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))
\end{aligned}$$

■ **Problem 418: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^2 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 190 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^2 (8A + 7B + 6C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{a^2 (8A + 7B + 6C) \tan[c + dx]}{6d} + \frac{a^2 (8A + 7B + 6C) \sec[c + dx] \tan[c + dx]}{24d} + \\
& \frac{(20A - 5B + 6C) (a + a \sec[c + dx])^2 \tan[c + dx]}{60d} + \frac{C \sec[c + dx]^2 (a + a \sec[c + dx])^2 \tan[c + dx]}{5d} + \frac{(5B + 2C) (a + a \sec[c + dx])^3 \tan[c + dx]}{20ad}
\end{aligned}$$

Result (type 3, 417 leaves):

$$\begin{aligned}
& - \frac{1}{3840 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)])} a^2 (1 + \cos [c + d x])^2 (C + B \cos [c + d x] + A \cos [c + d x]^2) \sec \left[\frac{1}{2} (c + d x) \right]^4 \sec [c + d x]^5 \\
& \left(240 (8 A + 7 B + 6 C) \cos [c + d x]^5 \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) - \right. \\
& \quad \left. \sec [c] (80 (16 A + 14 B + 15 C) \sin [d x] - 240 (3 A + 2 B + C) \sin [2 c + d x] + 240 A \sin [c + 2 d x] + 330 B \sin [c + 2 d x] + 420 C \sin [c + 2 d x] + \right. \\
& \quad 240 A \sin [3 c + 2 d x] + 330 B \sin [3 c + 2 d x] + 420 C \sin [3 c + 2 d x] + 880 A \sin [2 c + 3 d x] + 800 B \sin [2 c + 3 d x] + \\
& \quad 720 C \sin [2 c + 3 d x] - 120 A \sin [4 c + 3 d x] + 120 A \sin [3 c + 4 d x] + 105 B \sin [3 c + 4 d x] + 90 C \sin [3 c + 4 d x] + 120 A \sin [5 c + 4 d x] + \\
& \quad \left. 105 B \sin [5 c + 4 d x] + 90 C \sin [5 c + 4 d x] + 200 A \sin [4 c + 5 d x] + 160 B \sin [4 c + 5 d x] + 144 C \sin [4 c + 5 d x]) \right)
\end{aligned}$$

■ **Problem 419: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x] (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 3, 147 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^2 (12 A + 8 B + 7 C) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a^2 (12 A + 8 B + 7 C) \tan [c + d x]}{6 d} + \\
& \frac{a^2 (12 A + 8 B + 7 C) \sec [c + d x] \tan [c + d x]}{24 d} + \frac{(4 B - C) (a + a \sec [c + d x])^2 \tan [c + d x]}{12 d} + \frac{C (a + a \sec [c + d x])^3 \tan [c + d x]}{4 a d}
\end{aligned}$$

Result (type 3, 386 leaves):

$$\begin{aligned}
& - \frac{1}{384 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)])} a^2 (1 + \cos [c + d x])^2 (C + B \cos [c + d x] + A \cos [c + d x]^2) \sec \left[\frac{1}{2} (c + d x) \right]^4 \sec [c + d x]^4 \\
& \left(24 (12 A + 8 B + 7 C) \cos [c + d x]^4 \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) - \right. \\
& \quad \left. \sec [c] (-24 (6 A + 5 B + 4 C) \sin [c] + 3 (4 A + 8 B + 15 C) \sin [d x] + 12 A \sin [2 c + d x] + 24 B \sin [2 c + d x] + \right. \\
& \quad 45 C \sin [2 c + d x] + 144 A \sin [c + 2 d x] + 136 B \sin [c + 2 d x] + 128 C \sin [c + 2 d x] - 48 A \sin [3 c + 2 d x] - \\
& \quad 24 B \sin [3 c + 2 d x] + 12 A \sin [2 c + 3 d x] + 24 B \sin [2 c + 3 d x] + 21 C \sin [2 c + 3 d x] + 12 A \sin [4 c + 3 d x] + \\
& \quad \left. 24 B \sin [4 c + 3 d x] + 21 C \sin [4 c + 3 d x] + 48 A \sin [3 c + 4 d x] + 40 B \sin [3 c + 4 d x] + 32 C \sin [3 c + 4 d x]) \right)
\end{aligned}$$

■ **Problem 420: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 3, 120 leaves, 6 steps):

$$\begin{aligned}
& a^2 A x + \frac{a^2 (4 A + 3 B + 2 C) \operatorname{ArcTanh}[\sin [c + d x]]}{2 d} + \frac{a^2 (2 A + 3 B + 2 C) \tan [c + d x]}{2 d} + \\
& \frac{C (a + a \sec [c + d x])^2 \tan [c + d x]}{3 d} + \frac{(3 B + 2 C) (a^2 + a^2 \sec [c + d x]) \tan [c + d x]}{6 d}
\end{aligned}$$

Result (type 3, 1307 leaves):

$$\begin{aligned}
& \frac{A x \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{2(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])}+ \\
& \left(\frac{(-4 A-3 B-2 C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{(4 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))}+\right. \\
& \left.\frac{(4 A+3 B+2 C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{(4 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))}+\right. \\
& \left.\frac{C \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right) \sin \left[\frac{d x}{2}\right]}{12 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3}+\right. \\
& \left.\frac{\left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)\left(3 B \cos \left[\frac{c}{2}\right]+7 C \cos \left[\frac{c}{2}\right]-3 B \sin \left[\frac{c}{2}\right]-5 C \sin \left[\frac{c}{2}\right]\right)\right)}{\left(24 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2\right)}+ \\
& \left.\frac{\left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)\left(3 A \sin \left[\frac{d x}{2}\right]+6 B \sin \left[\frac{d x}{2}\right]+5 C \sin \left[\frac{d x}{2}\right]\right)\right)}{\left(6 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]-\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]-\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)\right)}+\right. \\
& \left.\frac{C \cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right) \sin \left[\frac{d x}{2}\right]}{12 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^3}+\right. \\
& \left.\frac{\left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)\left(-3 B \cos \left[\frac{c}{2}\right]-7 C \cos \left[\frac{c}{2}\right]-3 B \sin \left[\frac{c}{2}\right]-5 C \sin \left[\frac{c}{2}\right]\right)\right)}{\left(24 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)^2\right)}+ \\
& \left.\frac{\left(\cos [c+d x]^4 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^4\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)\left(3 A \sin \left[\frac{d x}{2}\right]+6 B \sin \left[\frac{d x}{2}\right]+5 C \sin \left[\frac{d x}{2}\right]\right)\right)}{\left(6 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{c}{2}\right]+\sin \left[\frac{c}{2}\right]\right)\left(\cos \left[\frac{c}{2}+\frac{d x}{2}\right]+\sin \left[\frac{c}{2}+\frac{d x}{2}\right]\right)\right)}\right)
\end{aligned}$$

■ **Problem 421: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]\left(a+a \sec [c+d x]\right)^2\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right) d x$$

Optimal (type 3, 121 leaves, 6 steps):

$$a^2 (2A + B)x + \frac{a^2 (2A + 4B + 3C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} +$$

$$\frac{A(a + a \sec[c + dx])^2 \sin[c + dx]}{d} - \frac{a^2 (2A - 2B - 3C) \tan[c + dx]}{2d} - \frac{(2A - C)(a^2 + a^2 \sec[c + dx]) \tan[c + dx]}{2d}$$

Result (type 3, 1091 leaves):

$$\frac{(2A + B)x \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} +$$

$$\left(\frac{(-2A - 4B - 3C) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} + \right.$$

$$\left. \frac{(2A + 4B + 3C) \cos[c + dx]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} + \right.$$

$$\frac{A \cos[dx] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c]}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} +$$

$$\frac{A \cos[c] \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} +$$

$$\frac{C \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} +$$

$$\left(\frac{\cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right]\right)}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} - \right.$$

$$\left. \frac{C \cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \right.$$

$$\left(\frac{\cos[c + dx]^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right]\right)}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} \right)$$

■ **Problem 422: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 128 leaves, 5 steps) :

$$\frac{1}{2} a^2 (3A + 4B + 2C) x + \frac{a^2 (B + 2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{d} + \frac{a^2 (3A + 2B - 2C) \operatorname{Sin}[c + dx]}{2d} + \frac{A \operatorname{Cos}[c + dx] (a + a \operatorname{Sec}[c + dx])^2 \operatorname{Sin}[c + dx]}{2d} - \frac{(A - 2C) (a^2 + a^2 \operatorname{Sec}[c + dx]) \operatorname{Sin}[c + dx]}{2d}$$

Result (type 3, 1016 leaves) :

$$a^2 \left(\frac{(3A + 4B + 2C) x \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{4 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx])} + \frac{\left((-B - 2C) \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right)}{(2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{\left((B + 2C) \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right)}{(2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{\left((2A + B) \operatorname{Cos}[dx] \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right)}{(2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{\left(A \operatorname{Cos}[2dx] \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[2c] \right)}{(8d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{\left((2A + B) \operatorname{Cos}[c] \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[dx] \right)}{(2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{\left(A \operatorname{Cos}[2c] \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[2dx] \right)}{(8d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]))} + \frac{C \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}\left[\frac{dx}{2}\right]}{2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} + \frac{C \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^2 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}\left[\frac{dx}{2}\right]}{2d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right])} \right)$$

■ **Problem 427: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^3 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 274 leaves, 9 steps) :

$$\begin{aligned}
& \frac{a^3 (26 A + 23 B + 21 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 d} + \frac{a^3 (133 A + 119 B + 108 C) \operatorname{Tan}[c + d x]}{35 d} + \\
& \frac{a^3 (26 A + 23 B + 21 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{16 d} + \frac{a^3 (154 A + 147 B + 129 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{280 d} + \\
& \frac{C \operatorname{Sec}[c + d x]^3 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{7 d} + \frac{(7 B + 3 C) \operatorname{Sec}[c + d x]^3 (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{42 a d} + \\
& \frac{(3 A + 4 B + 3 C) \operatorname{Sec}[c + d x]^3 (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{15 d} + \frac{a^3 (133 A + 119 B + 108 C) \operatorname{Tan}[c + d x]^3}{105 d}
\end{aligned}$$

Result (type 3, 1087 leaves):

$$\begin{aligned}
& \left((-26A - 23B - 21C) \cos[c + dx]^5 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (64d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left((26A + 23B + 21C) \cos[c + dx]^5 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (64d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \frac{C \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c + dx]^2 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{28d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c + dx] (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) (6C \sin[c] + 7B \sin[dx] + 21C \sin[dx]) \right) / \\
& (168d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (35B \sin[c] + 105C \sin[c] + 42A \sin[dx] + 126B \sin[dx] + 162C \sin[dx]) \right) / (840d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left(\cos[c + dx] \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (168A \sin[c] + 504B \sin[c] + 648C \sin[c] + 630A \sin[dx] + 805B \sin[dx] + 735C \sin[dx]) \right) / \\
& (3360d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^2 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (630A \sin[c] + 805B \sin[c] + 735C \sin[c] + 1064A \sin[dx] + 952B \sin[dx] + 864C \sin[dx]) \right) / \\
& (3360d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^3 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (2128A \sin[c] + 1904B \sin[c] + 1728C \sin[c] + 2730A \sin[dx] + 2415B \sin[dx] + 2205C \sin[dx]) \right) / \\
& (6720d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^4 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (2730A \sin[c] + 2415B \sin[c] + 2205C \sin[c] + 4256A \sin[dx] + 3808B \sin[dx] + 3456C \sin[dx]) \right) / \\
& (6720d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))
\end{aligned}$$

■ **Problem 428: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx]^2 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 216 leaves, 12 steps):

$$\frac{a^3 (30 A + 26 B + 23 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 d} + \frac{a^3 (30 A + 26 B + 23 C) \operatorname{Tan}[c + d x]}{10 d} +$$

$$\frac{3 a^3 (30 A + 26 B + 23 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{80 d} + \frac{(30 A - 6 B + 7 C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{120 d} +$$

$$\frac{C \operatorname{Sec}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{6 d} + \frac{(2 B + C) (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{10 a d} + \frac{a^3 (30 A + 26 B + 23 C) \operatorname{Tan}[c + d x]^3}{120 d}$$

Result (type 3, 959 leaves):

$$\left((-30 A - 26 B - 23 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)\right) /$$

$$(64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left((30 A + 26 B + 23 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)\right) /$$

$$(64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\frac{C \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}[c + d x] (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}[d x]}{24 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} +$$

$$\left(\operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (5 C \operatorname{Sin}[c] + 6 B \operatorname{Sin}[d x] + 18 C \operatorname{Sin}[d x]) \right) /$$

$$(120 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left(\operatorname{Cos}[c + d x] \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. (24 B \operatorname{Sin}[c] + 72 C \operatorname{Sin}[c] + 30 A \operatorname{Sin}[d x] + 90 B \operatorname{Sin}[d x] + 115 C \operatorname{Sin}[d x]) \right) / (480 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left(\operatorname{Cos}[c + d x]^2 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. (30 A \operatorname{Sin}[c] + 90 B \operatorname{Sin}[c] + 115 C \operatorname{Sin}[c] + 120 A \operatorname{Sin}[d x] + 152 B \operatorname{Sin}[d x] + 136 C \operatorname{Sin}[d x]) \right) /$$

$$(480 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \left(\operatorname{Cos}[c + d x]^3 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (240 A \operatorname{Sin}[c] + 304 B \operatorname{Sin}[c] + 272 C \operatorname{Sin}[c] + 450 A \operatorname{Sin}[d x] + 390 B \operatorname{Sin}[d x] + 345 C \operatorname{Sin}[d x]) \right) /$$

$$(960 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \left(\operatorname{Cos}[c + d x]^4 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (450 A \operatorname{Sin}[c] + 390 B \operatorname{Sin}[c] + 345 C \operatorname{Sin}[c] + 720 A \operatorname{Sin}[d x] + 608 B \operatorname{Sin}[d x] + 544 C \operatorname{Sin}[d x]) \right) /$$

$$(960 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]))$$

■ **Problem 429: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x] (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 175 leaves, 11 steps):

$$\frac{a^3 (20 A + 15 B + 13 C) \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{a^3 (20 A + 15 B + 13 C) \text{Tan}[c + d x]}{5 d} + \frac{3 a^3 (20 A + 15 B + 13 C) \text{Sec}[c + d x] \text{Tan}[c + d x]}{40 d} + \frac{(5 B - C) (a + a \text{Sec}[c + d x])^3 \text{Tan}[c + d x]}{20 d} + \frac{C (a + a \text{Sec}[c + d x])^4 \text{Tan}[c + d x]}{5 a d} + \frac{a^3 (20 A + 15 B + 13 C) \text{Tan}[c + d x]^3}{60 d}$$

Result (type 3, 629 leaves):

$$\left((-20 A - 15 B - 13 C) \text{Cos}[c + d x]^5 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / (32 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) + \left((20 A + 15 B + 13 C) \text{Cos}[c + d x]^5 \text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / (32 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) + \frac{1}{7680 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} \text{Sec}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) (2720 A \text{Sin}[d x] + 2400 B \text{Sin}[d x] + 2320 C \text{Sin}[d x] - 1680 A \text{Sin}[2 c + d x] - 1200 B \text{Sin}[2 c + d x] - 720 C \text{Sin}[2 c + d x] + 360 A \text{Sin}[c + 2 d x] + 570 B \text{Sin}[c + 2 d x] + 750 C \text{Sin}[c + 2 d x] + 360 A \text{Sin}[3 c + 2 d x] + 570 B \text{Sin}[3 c + 2 d x] + 750 C \text{Sin}[3 c + 2 d x] + 1840 A \text{Sin}[2 c + 3 d x] + 1680 B \text{Sin}[2 c + 3 d x] + 1520 C \text{Sin}[2 c + 3 d x] - 360 A \text{Sin}[4 c + 3 d x] - 120 B \text{Sin}[4 c + 3 d x] + 180 A \text{Sin}[3 c + 4 d x] + 225 B \text{Sin}[3 c + 4 d x] + 195 C \text{Sin}[3 c + 4 d x] + 180 A \text{Sin}[5 c + 4 d x] + 225 B \text{Sin}[5 c + 4 d x] + 195 C \text{Sin}[5 c + 4 d x] + 440 A \text{Sin}[4 c + 5 d x] + 360 B \text{Sin}[4 c + 5 d x] + 304 C \text{Sin}[4 c + 5 d x])$$

■ **Problem 430: Result more than twice size of optimal antiderivative.**

$$\int (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 162 leaves, 7 steps):

$$a^3 A x + \frac{a^3 (28 A + 20 B + 15 C) \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{5 a^3 (4 A + 4 B + 3 C) \text{Tan}[c + d x]}{8 d} + \frac{C (a + a \text{Sec}[c + d x])^3 \text{Tan}[c + d x]}{4 d} + \frac{(4 B + 3 C) (a^2 + a^2 \text{Sec}[c + d x])^2 \text{Tan}[c + d x]}{12 a d} + \frac{(12 A + 20 B + 15 C) (a^3 + a^3 \text{Sec}[c + d x]) \text{Tan}[c + d x]}{24 d}$$

Result (type 3, 464 leaves):

$$\frac{1}{768 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2(c + dx)])} a^3 (1 + \cos[c + dx])^3 (C + B \cos[c + dx] + A \cos[c + dx]^2) \sec\left[\frac{1}{2}(c + dx)\right]^6 \sec[c + dx]^4$$

$$\left(-24 (28 A + 20 B + 15 C) \cos[c + dx]^4 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right.$$

$$\left. \sec[c] (72 A dx \cos[c] + 48 A dx \cos[c + 2 dx] + 48 A dx \cos[3 c + 2 dx] + 12 A dx \cos[3 c + 4 dx] + 12 A dx \cos[5 c + 4 dx] - \right.$$

$$216 A \sin[c] - 264 B \sin[c] - 216 C \sin[c] + 12 A \sin[dx] + 36 B \sin[dx] + 69 C \sin[dx] + 12 A \sin[2 c + dx] +$$

$$36 B \sin[2 c + dx] + 69 C \sin[2 c + dx] + 216 A \sin[c + 2 dx] + 280 B \sin[c + 2 dx] + 264 C \sin[c + 2 dx] - 72 A \sin[3 c + 2 dx] -$$

$$72 B \sin[3 c + 2 dx] - 24 C \sin[3 c + 2 dx] + 12 A \sin[2 c + 3 dx] + 36 B \sin[2 c + 3 dx] + 45 C \sin[2 c + 3 dx] + 12 A \sin[4 c + 3 dx] +$$

$$\left. 36 B \sin[4 c + 3 dx] + 45 C \sin[4 c + 3 dx] + 72 A \sin[3 c + 4 dx] + 88 B \sin[3 c + 4 dx] + 72 C \sin[3 c + 4 dx] \right)$$

■ **Problem 431: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 156 leaves, 7 steps):

$$a^3 (3 A + B) x + \frac{a^3 (6 A + 7 B + 5 C) \operatorname{ArcTanh}[\sin[c + dx]]}{2 d} + \frac{A (a + a \sec[c + dx])^3 \sin[c + dx]}{d} +$$

$$\frac{5 a^3 (B + C) \tan[c + dx]}{2 d} - \frac{(3 A - C) (a^2 + a^2 \sec[c + dx])^2 \tan[c + dx]}{3 a d} - \frac{(6 A - 3 B - 5 C) (a^3 + a^3 \sec[c + dx]) \tan[c + dx]}{6 d}$$

Result (type 3, 1503 leaves):

$$\begin{aligned}
& \frac{(3A + B) \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{4(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\frac{(-6A - 7B - 5C) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} + \right. \\
& \left. \frac{(6A + 7B + 5C) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} \right) / \\
& \frac{A \cos[dx] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c]}{4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{A \cos[c] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{24d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \left(\frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(3B \cos\left[\frac{c}{2}\right] + 10C \cos\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{48d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \right. \\
& \left. \frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(3A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right] + 11C \sin\left[\frac{dx}{2}\right]\right)}{12d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} \right) + \\
& \frac{C \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{24d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^3} + \\
& \left(\frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-3B \cos\left[\frac{c}{2}\right] - 10C \cos\left[\frac{c}{2}\right] - 3B \sin\left[\frac{c}{2}\right] - 8C \sin\left[\frac{c}{2}\right]\right)}{48d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)^2} + \right. \\
& \left. \frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(3A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right] + 11C \sin\left[\frac{dx}{2}\right]\right)}{12d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right]\right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right)} \right) /
\end{aligned}$$

■ **Problem 432: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^2 (a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 171 leaves, 6 steps):

$$\frac{1}{2} a^3 (7 A + 6 B + 2 C) x + \frac{a^3 (2 A + 6 B + 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{5 a^3 (A - C) \operatorname{Sin}[c + d x]}{2 d} +$$

$$\frac{A \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{2 d} - \frac{(A - C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{2 a d} - \frac{(A - 2 B - 4 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{2 d}$$

Result (type 3, 1302 leaves):

$$\begin{aligned}
& \frac{(7A + 6B + 2C) \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{8(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\frac{(-2A - 6B - 7C) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} \right) / \\
& \left(\frac{(2A + 6B + 7C) \cos[c + dx]^5 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} \right) / \\
& \left(\frac{(3A + B) \cos[dx] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c]}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} \right) / \\
& \frac{A \cos[2dx] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2c]}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\frac{(3A + B) \cos[c] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))} \right) / \\
& \frac{A \cos[2c] \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2dx]}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2 + \\
& \left(\frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right] \right)}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right))} \right) - \\
& \frac{C \cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2 + \\
& \left(\frac{\cos[c + dx]^5 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right] \right)}{(4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right))} \right) /
\end{aligned}$$

■ **Problem 433: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 169 leaves, 6 steps) :

$$\frac{1}{2} a^3 (5A + 7B + 6C) x + \frac{a^3 (B + 3C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{d} + \frac{5a^3 (A + B) \operatorname{Sin}[c + dx]}{2d} + \frac{A \operatorname{Cos}[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^3 \operatorname{Sin}[c + dx]}{3d} + \frac{(A + B) \operatorname{Cos}[c + dx] (a^2 + a^2 \operatorname{Sec}[c + dx])^2 \operatorname{Sin}[c + dx]}{2ad} - \frac{(5A + 3B - 6C) (a^3 + a^3 \operatorname{Sec}[c + dx]) \operatorname{Sin}[c + dx]}{6d}$$

Result (type 3, 379 leaves) :

$$\frac{1}{48 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2(c + dx)])} a^3 \operatorname{Cos}[c + dx]^2 (1 + \operatorname{Cos}[c + dx])^3 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^6$$

$$\left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(6(5A + 7B + 6C)x - \frac{12(B + 3C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{12(B + 3C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{d} + \frac{3(15A + 4(3B + C)) \operatorname{Cos}[dx] \operatorname{Sin}[c]}{d} + \frac{3(3A + B) \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{d} + \frac{A \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{d} + \frac{3(15A + 4(3B + C)) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{3(3A + B) \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{d} + \frac{A \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{d} + \frac{12C \operatorname{Sin}\left[\frac{dx}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{12C \operatorname{Sin}\left[\frac{dx}{2}\right]}{d \left(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)} \right)$$

■ **Problem 438: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 252 leaves, 15 steps) :

$$\frac{a^4 (56A + 49B + 44C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{16d} + \frac{4a^4 (56A + 49B + 44C) \operatorname{Tan}[c + dx]}{35d} + \frac{27a^4 (56A + 49B + 44C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{560d} + \frac{a^4 (56A + 49B + 44C) \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{280d} + \frac{(42A - 7B + 8C) (a + a \operatorname{Sec}[c + dx])^4 \operatorname{Tan}[c + dx]}{210d} + \frac{C \operatorname{Sec}[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^4 \operatorname{Tan}[c + dx]}{7d} + \frac{(7B + 4C) (a + a \operatorname{Sec}[c + dx])^5 \operatorname{Tan}[c + dx]}{42ad} + \frac{2a^4 (56A + 49B + 44C) \operatorname{Tan}[c + dx]^3}{105d}$$

Result (type 3, 1087 leaves) :

$$\begin{aligned}
& \left((-56A - 49B - 44C) \cos[c + dx]^6 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (128d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left((56A + 49B + 44C) \cos[c + dx]^6 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (128d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \frac{C \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 \sec[c + dx] (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{56d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left(\sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) (6C \sin[c] + 7B \sin[dx] + 28C \sin[dx]) \right) / \\
& (336d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left(\cos[c + dx] \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (35B \sin[c] + 140C \sin[c] + 42A \sin[dx] + 168B \sin[dx] + 288C \sin[dx]) \right) / (1680d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left(\cos[c + dx]^2 \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. (168A \sin[c] + 672B \sin[c] + 1152C \sin[c] + 840A \sin[dx] + 1435B \sin[dx] + 1540C \sin[dx]) \right) / \\
& (6720d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^3 \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (840A \sin[c] + 1435B \sin[c] + 1540C \sin[c] + 1904A \sin[dx] + 2016B \sin[dx] + 1816C \sin[dx]) \right) / \\
& (6720d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^4 \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (3808A \sin[c] + 4032B \sin[c] + 3632C \sin[c] + 5880A \sin[dx] + 5145B \sin[dx] + 4620C \sin[dx]) \right) / \\
& (13440d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \left(\cos[c + dx]^5 \sec[c] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^8 (a + a \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) (5880A \sin[c] + 5145B \sin[c] + 4620C \sin[c] + 9296A \sin[dx] + 8064B \sin[dx] + 7264C \sin[dx]) \right) / \\
& (13440d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]))
\end{aligned}$$

■ **Problem 439: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 209 leaves, 14 steps):

$$\frac{7 a^4 (10 A + 8 B + 7 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{16 d} + \frac{4 a^4 (10 A + 8 B + 7 C) \operatorname{Tan}[c + d x]}{5 d} +$$

$$\frac{27 a^4 (10 A + 8 B + 7 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{80 d} + \frac{a^4 (10 A + 8 B + 7 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{40 d} +$$

$$\frac{(6 B - C) (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{30 d} + \frac{C (a + a \operatorname{Sec}[c + d x])^5 \operatorname{Tan}[c + d x]}{6 a d} + \frac{2 a^4 (10 A + 8 B + 7 C) \operatorname{Tan}[c + d x]^3}{15 d}$$

Result (type 3, 961 leaves):

$$-\left(7 (10 A + 8 B + 7 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)\right) /$$

$$(128 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left(7 (10 A + 8 B + 7 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)\right) /$$

$$(128 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \frac{C \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}[d x]}{48 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} +$$

$$\left(\operatorname{Cos}[c + d x] \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (5 C \operatorname{Sin}[c] + 6 B \operatorname{Sin}[d x] + 24 C \operatorname{Sin}[d x])\right) /$$

$$(240 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left(\operatorname{Cos}[c + d x]^2 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (24 B \operatorname{Sin}[c] + 96 C \operatorname{Sin}[c] + 30 A \operatorname{Sin}[d x] + 120 B \operatorname{Sin}[d x] + 205 C \operatorname{Sin}[d x])\right) / (960 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) +$$

$$\left(\operatorname{Cos}[c + d x]^3 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (30 A \operatorname{Sin}[c] + 120 B \operatorname{Sin}[c] + 205 C \operatorname{Sin}[c] + 160 A \operatorname{Sin}[d x] + 272 B \operatorname{Sin}[d x] + 288 C \operatorname{Sin}[d x])\right) /$$

$$(960 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \left(\operatorname{Cos}[c + d x]^4 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (320 A \operatorname{Sin}[c] + 544 B \operatorname{Sin}[c] + 576 C \operatorname{Sin}[c] + 810 A \operatorname{Sin}[d x] + 840 B \operatorname{Sin}[d x] + 735 C \operatorname{Sin}[d x])\right) /$$

$$(1920 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \left(\operatorname{Cos}[c + d x]^5 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (810 A \operatorname{Sin}[c] + 840 B \operatorname{Sin}[c] + 735 C \operatorname{Sin}[c] + 1600 A \operatorname{Sin}[d x] + 1328 B \operatorname{Sin}[d x] + 1152 C \operatorname{Sin}[d x])\right) /$$

$$(1920 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]))$$

■ **Problem 440: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 195 leaves, 8 steps):

$$a^4 A x + \frac{a^4 (48 A + 35 B + 28 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a^4 (40 A + 35 B + 28 C) \operatorname{Tan}[c + d x]}{8 d} + \frac{a (5 B + 4 C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{20 d} + \frac{C (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{5 d} + \frac{(20 A + 35 B + 28 C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{60 d} + \frac{(32 A + 35 B + 28 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{24 d}$$

Result (type 3, 725 leaves):

$$\left((-48 A - 35 B - 28 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \\ (64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \\ \left((48 A + 35 B + 28 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \\ (64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \\ \frac{1}{15360 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} \operatorname{Cos}[c + d x] \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\ (600 A d x \operatorname{Cos}[d x] + 600 A d x \operatorname{Cos}[2 c + d x] + 300 A d x \operatorname{Cos}[2 c + 3 d x] + 300 A d x \operatorname{Cos}[4 c + 3 d x] + 60 A d x \operatorname{Cos}[4 c + 5 d x] + \\ 60 A d x \operatorname{Cos}[6 c + 5 d x] + 4880 A \operatorname{Sin}[d x] + 5120 B \operatorname{Sin}[d x] + 4720 C \operatorname{Sin}[d x] - 3120 A \operatorname{Sin}[2 c + d x] - 2880 B \operatorname{Sin}[2 c + d x] - \\ 1920 C \operatorname{Sin}[2 c + d x] + 480 A \operatorname{Sin}[c + 2 d x] + 930 B \operatorname{Sin}[c + 2 d x] + 1320 C \operatorname{Sin}[c + 2 d x] + 480 A \operatorname{Sin}[3 c + 2 d x] + 930 B \operatorname{Sin}[3 c + 2 d x] + \\ 1320 C \operatorname{Sin}[3 c + 2 d x] + 3280 A \operatorname{Sin}[2 c + 3 d x] + 3520 B \operatorname{Sin}[2 c + 3 d x] + 3200 C \operatorname{Sin}[2 c + 3 d x] - 720 A \operatorname{Sin}[4 c + 3 d x] - \\ 480 B \operatorname{Sin}[4 c + 3 d x] - 120 C \operatorname{Sin}[4 c + 3 d x] + 240 A \operatorname{Sin}[3 c + 4 d x] + 405 B \operatorname{Sin}[3 c + 4 d x] + 420 C \operatorname{Sin}[3 c + 4 d x] + \\ 240 A \operatorname{Sin}[5 c + 4 d x] + 405 B \operatorname{Sin}[5 c + 4 d x] + 420 C \operatorname{Sin}[5 c + 4 d x] + 800 A \operatorname{Sin}[4 c + 5 d x] + 800 B \operatorname{Sin}[4 c + 5 d x] + 664 C \operatorname{Sin}[4 c + 5 d x])$$

■ **Problem 441: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + a \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 196 leaves, 8 steps):

$$a^4 (4 A + B) x + \frac{a^4 (52 A + 48 B + 35 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{A (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{d} + \frac{5 a^4 (4 A + 8 B + 7 C) \operatorname{Tan}[c + d x]}{8 d} - \frac{a (4 A - C) (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d} - \frac{(12 A - 4 B - 7 C) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} - \frac{(12 A - 32 B - 35 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{24 d}$$

Result (type 3, 738 leaves):

$$\left((-52A - 48B - 35C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) /$$

$$(64d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left((52A + 48B + 35C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) /$$

$$(64d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\frac{1}{1536d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^2 \sec[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$(288Adx \cos[c] + 72Bdx \cos[c] + 192Adx \cos[c + 2dx] + 48Bdx \cos[c + 2dx] + 192Adx \cos[3c + 2dx] + 48Bdx \cos[3c + 2dx] +$$

$$48Adx \cos[3c + 4dx] + 12Bdx \cos[3c + 4dx] + 48Adx \cos[5c + 4dx] + 12Bdx \cos[5c + 4dx] - 288A \sin[c] -$$

$$480B \sin[c] - 480C \sin[c] + 24A \sin[dx] + 48B \sin[dx] + 105C \sin[dx] + 24A \sin[2c + dx] + 48B \sin[2c + dx] +$$

$$105C \sin[2c + dx] + 288A \sin[c + 2dx] + 496B \sin[c + 2dx] + 544C \sin[c + 2dx] - 96A \sin[3c + 2dx] - 144B \sin[3c + 2dx] -$$

$$96C \sin[3c + 2dx] + 30A \sin[2c + 3dx] + 48B \sin[2c + 3dx] + 81C \sin[2c + 3dx] + 30A \sin[4c + 3dx] + 48B \sin[4c + 3dx] +$$

$$81C \sin[4c + 3dx] + 96A \sin[3c + 4dx] + 160B \sin[3c + 4dx] + 160C \sin[3c + 4dx] + 6A \sin[4c + 5dx] + 6A \sin[6c + 5dx])$$

■ **Problem 442: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\frac{1}{2} a^4 (13A + 8B + 2C) x + \frac{a^4 (8A + 13B + 12C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{5a^4 (A - B - 2C) \sin[c + dx]}{2d} -$$

$$\frac{a (3A - 2C) (a + a \sec[c + dx])^3 \sin[c + dx]}{6d} + \frac{A \cos[c + dx] (a + a \sec[c + dx])^4 \sin[c + dx]}{2d} -$$

$$\frac{(A - B - 2C) (a^2 + a^2 \sec[c + dx])^2 \sin[c + dx]}{2d} + \frac{(3A + 18B + 22C) (a^4 + a^4 \sec[c + dx]) \sin[c + dx]}{6d}$$

Result (type 3, 739 leaves):

$$\left((-8A - 13B - 12C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left((8A + 13B + 12C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\frac{1}{1536d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^3 \operatorname{Sec}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) +$$

$$(468Adx \cos[dx] + 288Bdx \cos[dx] + 72Cdx \cos[dx] + 468Adx \cos[2c + dx] + 288Bdx \cos[2c + dx] + 72Cdx \cos[2c + dx] +$$

$$156Adx \cos[2c + 3dx] + 96Bdx \cos[2c + 3dx] + 24Cdx \cos[2c + 3dx] + 156Adx \cos[4c + 3dx] + 96Bdx \cos[4c + 3dx] +$$

$$24Cdx \cos[4c + 3dx] + 102A \sin[dx] + 384B \sin[dx] + 672C \sin[dx] - 42A \sin[2c + dx] - 192B \sin[2c + dx] -$$

$$288C \sin[2c + dx] + 96A \sin[c + 2dx] + 48B \sin[c + 2dx] + 96C \sin[c + 2dx] + 96A \sin[3c + 2dx] + 48B \sin[3c + 2dx] +$$

$$96C \sin[3c + 2dx] + 57A \sin[2c + 3dx] + 192B \sin[2c + 3dx] + 320C \sin[2c + 3dx] + 9A \sin[4c + 3dx] +$$

$$48A \sin[3c + 4dx] + 12B \sin[3c + 4dx] + 48A \sin[5c + 4dx] + 12B \sin[5c + 4dx] + 3A \sin[4c + 5dx] + 3A \sin[6c + 5dx])$$

■ **Problem 443: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^3 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\frac{1}{2} a^4 (12A + 13B + 8C) x + \frac{a^4 (2A + 8B + 13C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{5a^4 (2A + B - C) \sin[c + dx]}{2d} +$$

$$\frac{a(4A + 3B) \cos[c + dx] (a + a \operatorname{Sec}[c + dx])^3 \sin[c + dx]}{6d} + \frac{A \cos[c + dx]^2 (a + a \operatorname{Sec}[c + dx])^4 \sin[c + dx]}{3d} -$$

$$\frac{(2A + B - C) (a^2 + a^2 \operatorname{Sec}[c + dx])^2 \sin[c + dx]}{2d} - \frac{(8A - 3B - 18C) (a^4 + a^4 \operatorname{Sec}[c + dx]) \sin[c + dx]}{6d}$$

Result (type 3, 1518 leaves):

$$\frac{(12A + 13B + 8C) x \cos[c + dx]^6 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{16(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} +$$

$$\left((-2A - 8B - 13C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left((2A + 8B + 13C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left((27A + 16B + 4C) \cos[dx] \cos[c + dx]^6 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) /$$

$$(32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\begin{aligned}
& \left((4A + B) \cos[2dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2c] \right) / \\
& (32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \frac{A \cos[3dx] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[3c]}{96d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \left((27A + 16B + 4C) \cos[c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx] \right) / \\
& (32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \left((4A + B) \cos[2c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2dx] \right) / \\
& (32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \\
& \frac{A \cos[3c] \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[3dx]}{96d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \left(\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \left(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right) - \\
& \frac{C \cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{32d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right])^2} + \\
& \left(\cos[c + dx]^6 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \left(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 444: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 217 leaves, 7 steps):

$$\frac{1}{8} a^4 (35 A + 48 B + 52 C) x + \frac{a^4 (B + 4 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{5 a^4 (7 A + 8 B + 4 C) \operatorname{Sin}[c + d x]}{8 d} +$$

$$\frac{a (A + B) \operatorname{Cos}[c + d x]^2 (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{3 d} + \frac{A \operatorname{Cos}[c + d x]^3 (a + a \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{4 d} +$$

$$\frac{(7 A + 8 B + 4 C) \operatorname{Cos}[c + d x] (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{8 d} - \frac{(35 A + 32 B - 12 C) (a^4 + a^4 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{24 d}$$

Result (type 3, 1436 leaves):

$$\begin{aligned}
& a^4 \left(\frac{(35A + 48B + 52C) x \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{64 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \\
& \left. \left(\frac{(-B - 4C) \cos[c + dx]^2 (1 + \cos[c + dx])^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{8d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(B + 4C) \cos[c + dx]^2 (1 + \cos[c + dx])^4 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{8d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(28A + 27B + 16C) \cos[dx] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c]}{32d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(7A + 4B + C) \cos[2dx] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2c]}{32d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(4A + B) \cos[3dx] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[3c]}{96d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{A \cos[4dx] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[4c]}{256d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(28A + 27B + 16C) \cos[c] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{32d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(7A + 4B + C) \cos[2c] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2dx]}{32d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{(4A + B) \cos[3c] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[3dx]}{96d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \left(\frac{A \cos[4c] \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[4dx]}{256d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} + \right. \right. \\
& \left. \frac{C \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{8d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) + \right. \\
& \left. \left. \frac{C \cos[c + dx]^2 (1 + \cos[c + dx])^4 \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{dx}{2}\right]}{8d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 449: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\frac{3(4A - 4B + 5C) \operatorname{ArcTanh}[\sin[c + dx]]}{8ad} - \frac{(3A - 4B + 4C) \tan[c + dx]}{ad} + \frac{3(4A - 4B + 5C) \sec[c + dx] \tan[c + dx]}{8ad} +$$

$$\frac{(4A - 4B + 5C) \sec[c + dx]^3 \tan[c + dx]}{4ad} - \frac{(A - B + C) \sec[c + dx]^4 \tan[c + dx]}{d(a + a \sec[c + dx])} - \frac{(3A - 4B + 4C) \tan[c + dx]^3}{3ad}$$

Result (type 3, 1099 leaves):

$$\begin{aligned}
& - \left(3 (4A - 4B + 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \\
& \left(3 (4A - 4B + 5C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \\
& \frac{1}{192d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3 \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-60A \sin\left[\frac{dx}{2}\right] + 108B \sin\left[\frac{dx}{2}\right] - 75C \sin\left[\frac{dx}{2}\right] - 60A \sin\left[\frac{3dx}{2}\right] + 124B \sin\left[\frac{3dx}{2}\right] - \right. \\
& 91C \sin\left[\frac{3dx}{2}\right] + 204A \sin\left[c - \frac{dx}{2}\right] - 252B \sin\left[c - \frac{dx}{2}\right] + 219C \sin\left[c - \frac{dx}{2}\right] - 60A \sin\left[c + \frac{dx}{2}\right] + 12B \sin\left[c + \frac{dx}{2}\right] + \\
& 21C \sin\left[c + \frac{dx}{2}\right] + 84A \sin\left[2c + \frac{dx}{2}\right] - 132B \sin\left[2c + \frac{dx}{2}\right] + 165C \sin\left[2c + \frac{dx}{2}\right] + 36A \sin\left[c + \frac{3dx}{2}\right] + 28B \sin\left[c + \frac{3dx}{2}\right] + \\
& 5C \sin\left[c + \frac{3dx}{2}\right] + 36A \sin\left[2c + \frac{3dx}{2}\right] - 36B \sin\left[2c + \frac{3dx}{2}\right] + 69C \sin\left[2c + \frac{3dx}{2}\right] + 132A \sin\left[3c + \frac{3dx}{2}\right] - 132B \sin\left[3c + \frac{3dx}{2}\right] + \\
& 165C \sin\left[3c + \frac{3dx}{2}\right] - 156A \sin\left[c + \frac{5dx}{2}\right] + 220B \sin\left[c + \frac{5dx}{2}\right] - 211C \sin\left[c + \frac{5dx}{2}\right] - 60A \sin\left[2c + \frac{5dx}{2}\right] + \\
& 124B \sin\left[2c + \frac{5dx}{2}\right] - 115C \sin\left[2c + \frac{5dx}{2}\right] - 60A \sin\left[3c + \frac{5dx}{2}\right] + 60B \sin\left[3c + \frac{5dx}{2}\right] - 51C \sin\left[3c + \frac{5dx}{2}\right] + \\
& 36A \sin\left[4c + \frac{5dx}{2}\right] - 36B \sin\left[4c + \frac{5dx}{2}\right] + 45C \sin\left[4c + \frac{5dx}{2}\right] - 12A \sin\left[2c + \frac{7dx}{2}\right] + 28B \sin\left[2c + \frac{7dx}{2}\right] - 19C \sin\left[2c + \frac{7dx}{2}\right] + \\
& 12A \sin\left[3c + \frac{7dx}{2}\right] + 4B \sin\left[3c + \frac{7dx}{2}\right] + 5C \sin\left[3c + \frac{7dx}{2}\right] + 12A \sin\left[4c + \frac{7dx}{2}\right] - 12B \sin\left[4c + \frac{7dx}{2}\right] + 21C \sin\left[4c + \frac{7dx}{2}\right] + \\
& 36A \sin\left[5c + \frac{7dx}{2}\right] - 36B \sin\left[5c + \frac{7dx}{2}\right] + 45C \sin\left[5c + \frac{7dx}{2}\right] - 48A \sin\left[3c + \frac{9dx}{2}\right] + 64B \sin\left[3c + \frac{9dx}{2}\right] - 64C \sin\left[3c + \frac{9dx}{2}\right] - \\
& \left. 24A \sin\left[4c + \frac{9dx}{2}\right] + 40B \sin\left[4c + \frac{9dx}{2}\right] - 40C \sin\left[4c + \frac{9dx}{2}\right] - 24A \sin\left[5c + \frac{9dx}{2}\right] + 24B \sin\left[5c + \frac{9dx}{2}\right] - 24C \sin\left[5c + \frac{9dx}{2}\right] \right)
\end{aligned}$$

■ **Problem 450: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(2A - 3B + 3C) \operatorname{ArcTanh}[\sin[c + dx]]}{2ad} + \frac{(3A - 3B + 4C) \tan[c + dx]}{ad} - \\
& \frac{(2A - 3B + 3C) \sec[c + dx] \tan[c + dx]}{2ad} - \frac{(A - B + C) \sec[c + dx]^3 \tan[c + dx]}{d(a + a \sec[c + dx])} + \frac{(3A - 3B + 4C) \tan[c + dx]^3}{3ad}
\end{aligned}$$

Result (type 3, 898 leaves) :

$$\frac{\left(2(2A-3B+3C)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]-\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+B\sec[c+dx]+C\sec[c+dx]^2)\right)/}{(d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx]))-}$$

$$\frac{\left(2(2A-3B+3C)\cos\left[\frac{c}{2}+\frac{dx}{2}\right]^2\cos[c+dx]\operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{dx}{2}\right]+\sin\left[\frac{c}{2}+\frac{dx}{2}\right]\right](A+B\sec[c+dx]+C\sec[c+dx]^2)\right)/}{(d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx]))+}$$

$$\frac{1}{24d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx])}\cos\left[\frac{c}{2}+\frac{dx}{2}\right]\sec\left[\frac{c}{2}\right]\sec[c]\sec[c+dx]^2$$

$$(A+B\sec[c+dx]+C\sec[c+dx]^2)\left(-6A\sin\left[\frac{dx}{2}\right]+6B\sin\left[\frac{dx}{2}\right]+6C\sin\left[\frac{dx}{2}\right]+30A\sin\left[\frac{3dx}{2}\right]-27B\sin\left[\frac{3dx}{2}\right]+39C\sin\left[\frac{3dx}{2}\right]-\right.$$

$$12A\sin\left[c-\frac{dx}{2}\right]+12B\sin\left[c-\frac{dx}{2}\right]-24C\sin\left[c-\frac{dx}{2}\right]-6A\sin\left[c+\frac{dx}{2}\right]+6B\sin\left[c+\frac{dx}{2}\right]-6C\sin\left[c+\frac{dx}{2}\right]-24A\sin\left[2c+\frac{dx}{2}\right]+$$

$$24B\sin\left[2c+\frac{dx}{2}\right]-24C\sin\left[2c+\frac{dx}{2}\right]+12A\sin\left[c+\frac{3dx}{2}\right]-9B\sin\left[c+\frac{3dx}{2}\right]+21C\sin\left[c+\frac{3dx}{2}\right]+12A\sin\left[2c+\frac{3dx}{2}\right]-$$

$$9B\sin\left[2c+\frac{3dx}{2}\right]+9C\sin\left[2c+\frac{3dx}{2}\right]-6A\sin\left[3c+\frac{3dx}{2}\right]+9B\sin\left[3c+\frac{3dx}{2}\right]-9C\sin\left[3c+\frac{3dx}{2}\right]+6A\sin\left[c+\frac{5dx}{2}\right]-$$

$$3B\sin\left[c+\frac{5dx}{2}\right]+7C\sin\left[c+\frac{5dx}{2}\right]+3B\sin\left[2c+\frac{5dx}{2}\right]+C\sin\left[2c+\frac{5dx}{2}\right]+3B\sin\left[3c+\frac{5dx}{2}\right]-3C\sin\left[3c+\frac{5dx}{2}\right]-$$

$$6A\sin\left[4c+\frac{5dx}{2}\right]+9B\sin\left[4c+\frac{5dx}{2}\right]-9C\sin\left[4c+\frac{5dx}{2}\right]+12A\sin\left[2c+\frac{7dx}{2}\right]-12B\sin\left[2c+\frac{7dx}{2}\right]+16C\sin\left[2c+\frac{7dx}{2}\right]+$$

$$\left.6A\sin\left[3c+\frac{7dx}{2}\right]-6B\sin\left[3c+\frac{7dx}{2}\right]+10C\sin\left[3c+\frac{7dx}{2}\right]+6A\sin\left[4c+\frac{7dx}{2}\right]-6B\sin\left[4c+\frac{7dx}{2}\right]+6C\sin\left[4c+\frac{7dx}{2}\right]\right)$$

■ **Problem 451: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^2(A+B\sec[c+dx]+C\sec[c+dx]^2)}{a+a\sec[c+dx]} dx$$

Optimal (type 3, 119 leaves, 6 steps) :

$$\frac{(2A-2B+3C)\operatorname{ArcTanh}[\sin[c+dx]]}{2ad} - \frac{(A-2B+2C)\tan[c+dx]}{ad} + \frac{(2A-2B+3C)\sec[c+dx]\tan[c+dx]}{2ad} - \frac{(A-B+C)\sec[c+dx]^2\tan[c+dx]}{d(a+a\sec[c+dx])}$$

Result (type 3, 900 leaves) :

$$\begin{aligned}
& - \left(2 (2A - 2B + 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) + \\
& \left(2 (2A - 2B + 3C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) - \\
& \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \cos[c + dx] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) + \\
& \quad \frac{C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} \\
& \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right) - \\
& \quad \frac{C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right)^2} \\
& \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx]) \left(\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right] \right) \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right] \right) \right)
\end{aligned}$$

■ **Problem 452: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{(B - C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a d} + \frac{C \operatorname{Tan}[c + dx]}{a d} + \frac{(A - B + C) \operatorname{Tan}[c + dx]}{a d (1 + \operatorname{Sec}[c + dx])}$$

Result (type 3, 255 leaves):

$$\frac{1}{a d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)]) (1 + \sec [c + d x])} 4 \cos \left[\frac{1}{2} (c + d x) \right] \cos [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2) \\ \left((A - B + C) \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] + \cos \left[\frac{1}{2} (c + d x) \right] \left(- (B - C) \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) \right) + \\ (C \sin [d x]) \left/ \left(\left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) \right)$$

- **Problem 453: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x] + C \sec [c + d x]^2}{a + a \sec [c + d x]} dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$\frac{A x}{a} + \frac{C \operatorname{ArcTanh}[\sin [c + d x]]}{a d} - \frac{(A - B + C) \tan [c + d x]}{a d (1 + \sec [c + d x])}$$

Result (type 3, 163 leaves):

$$\left(4 \cos \left[\frac{1}{2} (c + d x) \right] (C + B \cos [c + d x] + A \cos [c + d x]^2) \right. \\ \left. \left(\cos \left[\frac{1}{2} (c + d x) \right] \left(A d x - C \log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + C \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) - \right. \\ \left. (A - B + C) \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] \right) \left/ (a d (1 + \cos [c + d x]) (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)])) \right)$$

- **Problem 456: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^3 (A + B \sec [c + d x] + C \sec [c + d x]^2)}{a + a \sec [c + d x]} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$- \frac{(3 A - 3 B + 2 C) x}{2 a} + \frac{(4 A - 3 B + 3 C) \sin [c + d x]}{a d} - \\ \frac{(3 A - 3 B + 2 C) \cos [c + d x] \sin [c + d x]}{2 a d} - \frac{(A - B + C) \cos [c + d x]^2 \sin [c + d x]}{d (a + a \sec [c + d x])} - \frac{(4 A - 3 B + 3 C) \sin [c + d x]^3}{3 a d}$$

Result (type 3, 307 leaves):

$$\frac{1}{24 a d (1 + \cos [c + d x])} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(-12 (3 A - 3 B + 2 C) d x \cos \left[\frac{d x}{2} \right] - 12 (3 A - 3 B + 2 C) d x \cos \left[c + \frac{d x}{2} \right] + 69 A \sin \left[\frac{d x}{2} \right] - 60 B \sin \left[\frac{d x}{2} \right] + 60 C \sin \left[\frac{d x}{2} \right] + 21 A \sin \left[c + \frac{d x}{2} \right] - 12 B \sin \left[c + \frac{d x}{2} \right] + 12 C \sin \left[c + \frac{d x}{2} \right] + 18 A \sin \left[c + \frac{3 d x}{2} \right] - 9 B \sin \left[c + \frac{3 d x}{2} \right] + 12 C \sin \left[c + \frac{3 d x}{2} \right] + 18 A \sin \left[2 c + \frac{3 d x}{2} \right] - 9 B \sin \left[2 c + \frac{3 d x}{2} \right] + 12 C \sin \left[2 c + \frac{3 d x}{2} \right] - 2 A \sin \left[2 c + \frac{5 d x}{2} \right] + 3 B \sin \left[2 c + \frac{5 d x}{2} \right] - 2 A \sin \left[3 c + \frac{5 d x}{2} \right] + 3 B \sin \left[3 c + \frac{5 d x}{2} \right] + A \sin \left[3 c + \frac{7 d x}{2} \right] + A \sin \left[4 c + \frac{7 d x}{2} \right] \right)$$

■ **Problem 457: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^4 (A + B \sec [c + d x] + C \sec [c + d x]^2)}{a + a \sec [c + d x]} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{3 (5 A - 4 B + 4 C) x}{8 a} - \frac{(4 A - 4 B + 3 C) \sin [c + d x]}{a d} + \frac{3 (5 A - 4 B + 4 C) \cos [c + d x] \sin [c + d x]}{8 a d} + \frac{(5 A - 4 B + 4 C) \cos [c + d x]^3 \sin [c + d x]}{4 a d} - \frac{(A - B + C) \cos [c + d x]^3 \sin [c + d x]}{d (a + a \sec [c + d x])} + \frac{(4 A - 4 B + 3 C) \sin [c + d x]^3}{3 a d}$$

Result (type 3, 393 leaves):

$$\frac{1}{192 a d (1 + \cos [c + d x])} \cos \left[\frac{1}{2} (c + d x) \right] \sec \left[\frac{c}{2} \right] \left(72 (5 A - 4 B + 4 C) d x \cos \left[\frac{d x}{2} \right] + 72 (5 A - 4 B + 4 C) d x \cos \left[c + \frac{d x}{2} \right] - 552 A \sin \left[\frac{d x}{2} \right] + 552 B \sin \left[\frac{d x}{2} \right] - 480 C \sin \left[\frac{d x}{2} \right] - 168 A \sin \left[c + \frac{d x}{2} \right] + 168 B \sin \left[c + \frac{d x}{2} \right] - 96 C \sin \left[c + \frac{d x}{2} \right] - 120 A \sin \left[c + \frac{3 d x}{2} \right] + 144 B \sin \left[c + \frac{3 d x}{2} \right] - 72 C \sin \left[c + \frac{3 d x}{2} \right] - 120 A \sin \left[2 c + \frac{3 d x}{2} \right] + 144 B \sin \left[2 c + \frac{3 d x}{2} \right] - 72 C \sin \left[2 c + \frac{3 d x}{2} \right] + 40 A \sin \left[2 c + \frac{5 d x}{2} \right] - 16 B \sin \left[2 c + \frac{5 d x}{2} \right] + 24 C \sin \left[2 c + \frac{5 d x}{2} \right] + 40 A \sin \left[3 c + \frac{5 d x}{2} \right] - 16 B \sin \left[3 c + \frac{5 d x}{2} \right] + 24 C \sin \left[3 c + \frac{5 d x}{2} \right] - 5 A \sin \left[3 c + \frac{7 d x}{2} \right] + 8 B \sin \left[3 c + \frac{7 d x}{2} \right] - 5 A \sin \left[4 c + \frac{7 d x}{2} \right] + 8 B \sin \left[4 c + \frac{7 d x}{2} \right] + 3 A \sin \left[4 c + \frac{9 d x}{2} \right] + 3 A \sin \left[5 c + \frac{9 d x}{2} \right] \right)$$

■ **Problem 458: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^4 (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$- \frac{(4A - 7B + 10C) \operatorname{ArcTanh}[\sin[c + dx]]}{2a^2 d} + \frac{(5A - 8B + 12C) \tan[c + dx]}{a^2 d} - \frac{(4A - 7B + 10C) \sec[c + dx] \tan[c + dx]}{2a^2 d} - \frac{(4A - 7B + 10C) \sec[c + dx]^3 \tan[c + dx]}{3a^2 d (1 + \sec[c + dx])} - \frac{(A - B + C) \sec[c + dx]^4 \tan[c + dx]}{3d (a + a \sec[c + dx])^2} + \frac{(5A - 8B + 12C) \tan[c + dx]^3}{3a^2 d}$$

Result (type 3, 1069 leaves):

$$\frac{4(4A - 7B + 10C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} - \frac{4(4A - 7B + 10C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} + \frac{1}{48d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-48A \sin\left[\frac{dx}{2}\right] + 45B \sin\left[\frac{dx}{2}\right] - 6C \sin\left[\frac{dx}{2}\right] + 132A \sin\left[\frac{3dx}{2}\right] - 201B \sin\left[\frac{3dx}{2}\right] + 310C \sin\left[\frac{3dx}{2}\right] - 120A \sin\left[c - \frac{dx}{2}\right] + 195B \sin\left[c - \frac{dx}{2}\right] - 306C \sin\left[c - \frac{dx}{2}\right] + 48A \sin\left[c + \frac{dx}{2}\right] - 51B \sin\left[c + \frac{dx}{2}\right] + 42C \sin\left[c + \frac{dx}{2}\right] - 120A \sin\left[2c + \frac{dx}{2}\right] + 189B \sin\left[2c + \frac{dx}{2}\right] - 270C \sin\left[2c + \frac{dx}{2}\right] - 8A \sin\left[c + \frac{3dx}{2}\right] - B \sin\left[c + \frac{3dx}{2}\right] + 50C \sin\left[c + \frac{3dx}{2}\right] + 72A \sin\left[2c + \frac{3dx}{2}\right] - 81B \sin\left[2c + \frac{3dx}{2}\right] + 90C \sin\left[2c + \frac{3dx}{2}\right] - 68A \sin\left[3c + \frac{3dx}{2}\right] + 119B \sin\left[3c + \frac{3dx}{2}\right] - 170C \sin\left[3c + \frac{3dx}{2}\right] + 84A \sin\left[c + \frac{5dx}{2}\right] - 129B \sin\left[c + \frac{5dx}{2}\right] + 198C \sin\left[c + \frac{5dx}{2}\right] - 9B \sin\left[2c + \frac{5dx}{2}\right] + 42C \sin\left[2c + \frac{5dx}{2}\right] + 48A \sin\left[3c + \frac{5dx}{2}\right] - 57B \sin\left[3c + \frac{5dx}{2}\right] + 66C \sin\left[3c + \frac{5dx}{2}\right] - 36A \sin\left[4c + \frac{5dx}{2}\right] + 63B \sin\left[4c + \frac{5dx}{2}\right] - 90C \sin\left[4c + \frac{5dx}{2}\right] + 48A \sin\left[2c + \frac{7dx}{2}\right] - 75B \sin\left[2c + \frac{7dx}{2}\right] + 114C \sin\left[2c + \frac{7dx}{2}\right] + 6A \sin\left[3c + \frac{7dx}{2}\right] - 15B \sin\left[3c + \frac{7dx}{2}\right] + 36C \sin\left[3c + \frac{7dx}{2}\right] + 30A \sin\left[4c + \frac{7dx}{2}\right] - 39B \sin\left[4c + \frac{7dx}{2}\right] + 48C \sin\left[4c + \frac{7dx}{2}\right] - 12A \sin\left[5c + \frac{7dx}{2}\right] + 21B \sin\left[5c + \frac{7dx}{2}\right] - 30C \sin\left[5c + \frac{7dx}{2}\right] + 20A \sin\left[3c + \frac{9dx}{2}\right] - 32B \sin\left[3c + \frac{9dx}{2}\right] + 48C \sin\left[3c + \frac{9dx}{2}\right] + 6A \sin\left[4c + \frac{9dx}{2}\right] - 12B \sin\left[4c + \frac{9dx}{2}\right] + 22C \sin\left[4c + \frac{9dx}{2}\right] + 14A \sin\left[5c + \frac{9dx}{2}\right] - 20B \sin\left[5c + \frac{9dx}{2}\right] + 26C \sin\left[5c + \frac{9dx}{2}\right] \right)$$

■ **Problem 459: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\frac{(2A - 4B + 7C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2a^2 d} - \frac{2(2A - 5B + 8C) \operatorname{Tan}[c + dx]}{3a^2 d} +$$

$$\frac{(2A - 4B + 7C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2a^2 d} - \frac{(2A - 5B + 8C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3a^2 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B + C) \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{3d (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 3, 901 leaves):

$$- \frac{4(2A - 4B + 7C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} +$$

$$\frac{4(2A - 4B + 7C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} +$$

$$\frac{1}{24d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2}$$

$$\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(20A \operatorname{Sin}\left[\frac{dx}{2}\right] - 14B \operatorname{Sin}\left[\frac{dx}{2}\right] + 14C \operatorname{Sin}\left[\frac{dx}{2}\right] - 22A \operatorname{Sin}\left[\frac{3dx}{2}\right] + 64B \operatorname{Sin}\left[\frac{3dx}{2}\right] - 97C \operatorname{Sin}\left[\frac{3dx}{2}\right] + 36A \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 84B \operatorname{Sin}\left[c - \frac{dx}{2}\right] + \right.$$

$$126C \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 36A \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 42B \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 42C \operatorname{Sin}\left[c + \frac{dx}{2}\right] + 20A \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 56B \operatorname{Sin}\left[2c + \frac{dx}{2}\right] +$$

$$98C \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 18A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 6B \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 3C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 22A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 34B \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] -$$

$$37C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 18A \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] - 36B \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] + 63C \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] - 18A \operatorname{Sin}\left[c + \frac{5dx}{2}\right] + 48B \operatorname{Sin}\left[c + \frac{5dx}{2}\right] -$$

$$75C \operatorname{Sin}\left[c + \frac{5dx}{2}\right] + 6A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 6B \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 15C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] - 18A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 30B \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] -$$

$$39C \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 6A \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] - 12B \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] + 21C \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] - 8A \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] + 20B \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] -$$

$$\left. 32C \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] + 6B \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 12C \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] - 8A \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] + 14B \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] - 20C \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 460: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 3, 112 leaves, 6 steps):

$$\frac{(B - 2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{a^2 d} + \frac{(A - B + 4C) \operatorname{Tan}[c + dx]}{3a^2 d} - \frac{(B - 2C) \operatorname{Tan}[c + dx]}{a^2 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B + C) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d (a + a \operatorname{Sec}[c + dx])^2}$$

Result (type 3, 312 leaves):

1

$$\frac{3 a^2 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)]) (1 + \sec [c + d x])^2}{}$$

$$4 \cos \left[\frac{1}{2} (c + d x) \right] (A + B \sec [c + d x] + C \sec [c + d x]^2) \left((A - B + C) \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] + 2 (A - 4 B + 7 C) \cos \left[\frac{1}{2} (c + d x) \right]^2 \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] + \right. \\ \left. \cos \left[\frac{1}{2} (c + d x) \right]^3 \left(-6 (B - 2 C) \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) + \right. \\ \left. (6 C \sin [d x]) \right) / \left(\left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) \right) + \\ (A - B + C) \cos \left[\frac{1}{2} (c + d x) \right] \tan \left[\frac{c}{2} \right]$$

■ **Problem 461: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$\frac{C \operatorname{ArcTanh}[\sin [c + d x]]}{a^2 d} + \frac{(A + 2 B - 5 C) \tan [c + d x]}{3 a^2 d (1 + \sec [c + d x])} + \frac{(A - B + C) \tan [c + d x]}{3 d (a + a \sec [c + d x])^2}$$

Result (type 3, 219 leaves):

$$- \left(\left(4 \cos \left[\frac{1}{2} (c + d x) \right] (C + B \cos [c + d x] + A \cos [c + d x]^2) \right. \right. \\ \left. \left(6 C \cos \left[\frac{1}{2} (c + d x) \right]^3 \left(\log \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - \log \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) + \right. \\ \left. (A - B + C) \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] - 2 (2 A + B - 4 C) \cos \left[\frac{1}{2} (c + d x) \right]^2 \sec \left[\frac{c}{2} \right] \sin \left[\frac{d x}{2} \right] + (A - B + C) \cos \left[\frac{1}{2} (c + d x) \right] \tan \left[\frac{c}{2} \right] \right) \right) / \\ (3 a^2 d (1 + \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 (c + d x)]))$$

■ **Problem 462: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x] + C \sec [c + d x]^2}{(a + a \sec [c + d x])^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{A x}{a^2} - \frac{(4 A - B - 2 C) \tan [c + d x]}{3 a^2 d (1 + \sec [c + d x])} - \frac{(A - B + C) \tan [c + d x]}{3 d (a + a \sec [c + d x])^2}$$

Result (type 3, 175 leaves):

$$\frac{1}{24 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^3$$

$$\left(9 A d x \operatorname{Cos}\left[\frac{d x}{2}\right]+9 A d x \operatorname{Cos}\left[c+\frac{d x}{2}\right]+3 A d x \operatorname{Cos}\left[c+\frac{3 d x}{2}\right]+3 A d x \operatorname{Cos}\left[2 c+\frac{3 d x}{2}\right]-18 A \operatorname{Sin}\left[\frac{d x}{2}\right]+6 B \operatorname{Sin}\left[\frac{d x}{2}\right]+6 C \operatorname{Sin}\left[\frac{d x}{2}\right]+12 A \operatorname{Sin}\left[c+\frac{d x}{2}\right]-6 B \operatorname{Sin}\left[c+\frac{d x}{2}\right]-10 A \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]+4 B \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]+2 C \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]\right)$$

■ **Problem 463: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x](A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{(a+a \operatorname{Sec}[c+d x])^2} d x$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{(2 A-B) x}{a^2}+\frac{(10 A-4 B+C) \operatorname{Sin}[c+d x]}{3 a^2 d}-\frac{(2 A-B) \operatorname{Sin}[c+d x]}{a^2 d(1+\operatorname{Sec}[c+d x])}-\frac{(A-B+C) \operatorname{Sin}[c+d x]}{3 d(a+a \operatorname{Sec}[c+d x])^2}$$

Result (type 3, 279 leaves):

$$\frac{1}{12 a^2 d(1+\operatorname{Cos}[c+d x])^2}$$

$$\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] \operatorname{Sec}\left[\frac{c}{2}\right]\left(-18(2 A-B) d x \operatorname{Cos}\left[\frac{d x}{2}\right]-18(2 A-B) d x \operatorname{Cos}\left[c+\frac{d x}{2}\right]-12 A d x \operatorname{Cos}\left[c+\frac{3 d x}{2}\right]+6 B d x \operatorname{Cos}\left[c+\frac{3 d x}{2}\right]-12 A d x \operatorname{Cos}\left[2 c+\frac{3 d x}{2}\right]+6 B d x \operatorname{Cos}\left[2 c+\frac{3 d x}{2}\right]+66 A \operatorname{Sin}\left[\frac{d x}{2}\right]-36 B \operatorname{Sin}\left[\frac{d x}{2}\right]+12 C \operatorname{Sin}\left[\frac{d x}{2}\right]-30 A \operatorname{Sin}\left[c+\frac{d x}{2}\right]+24 B \operatorname{Sin}\left[c+\frac{d x}{2}\right]-12 C \operatorname{Sin}\left[c+\frac{d x}{2}\right]+41 A \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]-20 B \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]+8 C \operatorname{Sin}\left[c+\frac{3 d x}{2}\right]+9 A \operatorname{Sin}\left[2 c+\frac{3 d x}{2}\right]+3 A \operatorname{Sin}\left[2 c+\frac{5 d x}{2}\right]+3 A \operatorname{Sin}\left[3 c+\frac{5 d x}{2}\right]\right)$$

■ **Problem 464: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]^2(A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{(a+a \operatorname{Sec}[c+d x])^2} d x$$

Optimal (type 3, 156 leaves, 6 steps):

$$\frac{(7 A-4 B+2 C) x}{2 a^2}-\frac{2(8 A-5 B+2 C) \operatorname{Sin}[c+d x]}{3 a^2 d}+\frac{(7 A-4 B+2 C) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{2 a^2 d}-\frac{(8 A-5 B+2 C) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{3 a^2 d(1+\operatorname{Sec}[c+d x])}-\frac{(A-B+C) \operatorname{Cos}[c+d x] \operatorname{Sin}[c+d x]}{3 d(a+a \operatorname{Sec}[c+d x])^2}$$

Result (type 3, 377 leaves):

$$\frac{1}{192 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \left(36(7A-4B+2C) dx \cos\left[\frac{dx}{2}\right] + 36(7A-4B+2C) dx \cos\left[c+\frac{dx}{2}\right] + 84A dx \cos\left[c+\frac{3dx}{2}\right] - 48B dx \cos\left[c+\frac{3dx}{2}\right] + 24C dx \cos\left[c+\frac{3dx}{2}\right] + 84A dx \cos\left[2c+\frac{3dx}{2}\right] - 48B dx \cos\left[2c+\frac{3dx}{2}\right] + 24C dx \cos\left[2c+\frac{3dx}{2}\right] - 381A \sin\left[\frac{dx}{2}\right] + 264B \sin\left[\frac{dx}{2}\right] - 144C \sin\left[\frac{dx}{2}\right] + 147A \sin\left[c+\frac{dx}{2}\right] - 120B \sin\left[c+\frac{dx}{2}\right] + 96C \sin\left[c+\frac{dx}{2}\right] - 239A \sin\left[c+\frac{3dx}{2}\right] + 164B \sin\left[c+\frac{3dx}{2}\right] - 80C \sin\left[c+\frac{3dx}{2}\right] - 63A \sin\left[2c+\frac{3dx}{2}\right] + 36B \sin\left[2c+\frac{3dx}{2}\right] - 15A \sin\left[2c+\frac{5dx}{2}\right] + 12B \sin\left[2c+\frac{5dx}{2}\right] - 15A \sin\left[3c+\frac{5dx}{2}\right] + 12B \sin\left[3c+\frac{5dx}{2}\right] + 3A \sin\left[3c+\frac{7dx}{2}\right] + 3A \sin\left[4c+\frac{7dx}{2}\right] \right)$$

■ **Problem 465: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 3, 185 leaves, 7 steps):

$$-\frac{(10A-7B+4C)x}{2a^2} + \frac{(12A-8B+5C)\sin[c+dx]}{a^2 d} - \frac{(10A-7B+4C)\cos[c+dx]\sin[c+dx]}{2a^2 d} - \frac{(10A-7B+4C)\cos[c+dx]^2\sin[c+dx]}{3a^2 d(1+\operatorname{Sec}[c+dx])} - \frac{(A-B+C)\cos[c+dx]^2\sin[c+dx]}{3d(a+a \operatorname{Sec}[c+dx])^2} - \frac{(12A-8B+5C)\sin[c+dx]^3}{3a^2 d}$$

Result (type 3, 473 leaves):

$$\frac{1}{192 a^2 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \left(-36(10A-7B+4C) dx \cos\left[\frac{dx}{2}\right] - 36(10A-7B+4C) dx \cos\left[c+\frac{dx}{2}\right] - 120A dx \cos\left[c+\frac{3dx}{2}\right] + 84B dx \cos\left[c+\frac{3dx}{2}\right] - 48C dx \cos\left[c+\frac{3dx}{2}\right] - 120A dx \cos\left[2c+\frac{3dx}{2}\right] + 84B dx \cos\left[2c+\frac{3dx}{2}\right] - 48C dx \cos\left[2c+\frac{3dx}{2}\right] + 516A \sin\left[\frac{dx}{2}\right] - 381B \sin\left[\frac{dx}{2}\right] + 264C \sin\left[\frac{dx}{2}\right] - 156A \sin\left[c+\frac{dx}{2}\right] + 147B \sin\left[c+\frac{dx}{2}\right] - 120C \sin\left[c+\frac{dx}{2}\right] + 342A \sin\left[c+\frac{3dx}{2}\right] - 239B \sin\left[c+\frac{3dx}{2}\right] + 164C \sin\left[c+\frac{3dx}{2}\right] + 118A \sin\left[2c+\frac{3dx}{2}\right] - 63B \sin\left[2c+\frac{3dx}{2}\right] + 36C \sin\left[2c+\frac{3dx}{2}\right] + 30A \sin\left[2c+\frac{5dx}{2}\right] - 15B \sin\left[2c+\frac{5dx}{2}\right] + 12C \sin\left[2c+\frac{5dx}{2}\right] + 30A \sin\left[3c+\frac{5dx}{2}\right] - 15B \sin\left[3c+\frac{5dx}{2}\right] + 12C \sin\left[3c+\frac{5dx}{2}\right] - 3A \sin\left[3c+\frac{7dx}{2}\right] + 3B \sin\left[3c+\frac{7dx}{2}\right] - 3A \sin\left[4c+\frac{7dx}{2}\right] + 3B \sin\left[4c+\frac{7dx}{2}\right] + A \sin\left[4c+\frac{9dx}{2}\right] + A \sin\left[5c+\frac{9dx}{2}\right] \right)$$

■ **Problem 466: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + a \text{Sec}[c + d x])^3} dx$$

Optimal (type 3, 216 leaves, 8 steps):

$$\frac{(2A - 6B + 13C) \text{ArcTanh}[\text{Sin}[c + dx]]}{2a^3 d} - \frac{2(11A - 36B + 76C) \text{Tan}[c + dx]}{15a^3 d} + \frac{(2A - 6B + 13C) \text{Sec}[c + dx] \text{Tan}[c + dx]}{2a^3 d} - \frac{(A - B + C) \text{Sec}[c + dx]^4 \text{Tan}[c + dx]}{5d(a + a \text{Sec}[c + dx])^3} - \frac{(A - 6B + 11C) \text{Sec}[c + dx]^3 \text{Tan}[c + dx]}{15ad(a + a \text{Sec}[c + dx])^2} - \frac{(11A - 36B + 76C) \text{Sec}[c + dx]^2 \text{Tan}[c + dx]}{15d(a^3 + a^3 \text{Sec}[c + dx])}$$

Result (type 3, 1081 leaves):

$$\begin{aligned}
& - \left(8 (2A - 6B + 13C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \\
& \left(8 (2A - 6B + 13C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c] \sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \quad \left(490A \sin\left[\frac{dx}{2}\right] - 870B \sin\left[\frac{dx}{2}\right] + 1235C \sin\left[\frac{dx}{2}\right] - 530A \sin\left[\frac{3dx}{2}\right] + 1830B \sin\left[\frac{3dx}{2}\right] - 3805C \sin\left[\frac{3dx}{2}\right] + 654A \sin\left[c - \frac{dx}{2}\right] - \right. \\
& \quad 2094B \sin\left[c - \frac{dx}{2}\right] + 4329C \sin\left[c - \frac{dx}{2}\right] - 654A \sin\left[c + \frac{dx}{2}\right] + 1314B \sin\left[c + \frac{dx}{2}\right] - 1989C \sin\left[c + \frac{dx}{2}\right] + 490A \sin\left[2c + \frac{dx}{2}\right] - \\
& \quad 1650B \sin\left[2c + \frac{dx}{2}\right] + 3575C \sin\left[2c + \frac{dx}{2}\right] + 350A \sin\left[c + \frac{3dx}{2}\right] - 450B \sin\left[c + \frac{3dx}{2}\right] + 475C \sin\left[c + \frac{3dx}{2}\right] - 530A \sin\left[2c + \frac{3dx}{2}\right] + \\
& \quad 1230B \sin\left[2c + \frac{3dx}{2}\right] - 2005C \sin\left[2c + \frac{3dx}{2}\right] + 350A \sin\left[3c + \frac{3dx}{2}\right] - 1050B \sin\left[3c + \frac{3dx}{2}\right] + 2275C \sin\left[3c + \frac{3dx}{2}\right] - \\
& \quad 378A \sin\left[c + \frac{5dx}{2}\right] + 1278B \sin\left[c + \frac{5dx}{2}\right] - 2673C \sin\left[c + \frac{5dx}{2}\right] + 150A \sin\left[2c + \frac{5dx}{2}\right] - 90B \sin\left[2c + \frac{5dx}{2}\right] - 105C \sin\left[2c + \frac{5dx}{2}\right] - \\
& \quad 378A \sin\left[3c + \frac{5dx}{2}\right] + 918B \sin\left[3c + \frac{5dx}{2}\right] - 1593C \sin\left[3c + \frac{5dx}{2}\right] + 150A \sin\left[4c + \frac{5dx}{2}\right] - 450B \sin\left[4c + \frac{5dx}{2}\right] + \\
& \quad 975C \sin\left[4c + \frac{5dx}{2}\right] - 190A \sin\left[2c + \frac{7dx}{2}\right] + 630B \sin\left[2c + \frac{7dx}{2}\right] - 1325C \sin\left[2c + \frac{7dx}{2}\right] + 30A \sin\left[3c + \frac{7dx}{2}\right] + \\
& \quad 60B \sin\left[3c + \frac{7dx}{2}\right] - 255C \sin\left[3c + \frac{7dx}{2}\right] - 190A \sin\left[4c + \frac{7dx}{2}\right] + 480B \sin\left[4c + \frac{7dx}{2}\right] - 875C \sin\left[4c + \frac{7dx}{2}\right] + \\
& \quad 30A \sin\left[5c + \frac{7dx}{2}\right] - 90B \sin\left[5c + \frac{7dx}{2}\right] + 195C \sin\left[5c + \frac{7dx}{2}\right] - 44A \sin\left[3c + \frac{9dx}{2}\right] + 144B \sin\left[3c + \frac{9dx}{2}\right] - 304C \sin\left[3c + \frac{9dx}{2}\right] + \\
& \quad \left. \left. 30B \sin\left[4c + \frac{9dx}{2}\right] - 90C \sin\left[4c + \frac{9dx}{2}\right] - 44A \sin\left[5c + \frac{9dx}{2}\right] + 114B \sin\left[5c + \frac{9dx}{2}\right] - 214C \sin\left[5c + \frac{9dx}{2}\right] \right) \right) / \\
& \quad (240d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3)
\end{aligned}$$

■ **Problem 467: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 161 leaves, 7 steps):

$$\frac{(B-3C) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{a^3 d} + \frac{(2A-7B+27C) \operatorname{Tan}[c+dx]}{15 a^3 d} - \frac{(A-B+C) \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{5 d (a+a \operatorname{Sec}[c+dx])^3} + \frac{(A+4B-9C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{15 a d (a+a \operatorname{Sec}[c+dx])^2} - \frac{(B-3C) \operatorname{Tan}[c+dx]}{d (a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 3, 839 leaves):

$$\left(16 (-B+3C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\ (d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3) - \\ \left(16 (-B+3C) \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\ (d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3) + \\ \frac{1}{60 d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \\ (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-20 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 160 B \operatorname{Sin}\left[\frac{dx}{2}\right] - 255 C \operatorname{Sin}\left[\frac{dx}{2}\right] + 22 A \operatorname{Sin}\left[\frac{3dx}{2}\right] - 167 B \operatorname{Sin}\left[\frac{3dx}{2}\right] + \right. \\ 567 C \operatorname{Sin}\left[\frac{3dx}{2}\right] - 10 A \operatorname{Sin}\left[c - \frac{dx}{2}\right] + 170 B \operatorname{Sin}\left[c - \frac{dx}{2}\right] - 600 C \operatorname{Sin}\left[c - \frac{dx}{2}\right] + 10 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 170 B \operatorname{Sin}\left[c + \frac{dx}{2}\right] + \\ 375 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 20 A \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 160 B \operatorname{Sin}\left[2c + \frac{dx}{2}\right] - 480 C \operatorname{Sin}\left[2c + \frac{dx}{2}\right] + 75 B \operatorname{Sin}\left[c + \frac{3dx}{2}\right] - 60 C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + \\ 22 A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 167 B \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 402 C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] + 75 B \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] - 225 C \operatorname{Sin}\left[3c + \frac{3dx}{2}\right] + \\ 10 A \operatorname{Sin}\left[c + \frac{5dx}{2}\right] - 95 B \operatorname{Sin}\left[c + \frac{5dx}{2}\right] + 315 C \operatorname{Sin}\left[c + \frac{5dx}{2}\right] + 15 B \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 30 C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 10 A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - \\ 95 B \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 240 C \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] + 15 B \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] - 45 C \operatorname{Sin}\left[4c + \frac{5dx}{2}\right] + 2 A \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] - \\ \left. 22 B \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] + 72 C \operatorname{Sin}\left[2c + \frac{7dx}{2}\right] + 15 C \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] + 2 A \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] - 22 B \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] + 57 C \operatorname{Sin}\left[4c + \frac{7dx}{2}\right] \right)$$

■ **Problem 468: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 132 leaves, 5 steps):

$$\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{a^3 d} - \frac{(A-B+C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{5 d (a+a \operatorname{Sec}[c+dx])^3} - \frac{(3A+2B-7C) \operatorname{Tan}[c+dx]}{15 a d (a+a \operatorname{Sec}[c+dx])^2} + \frac{(6A+4B-29C) \operatorname{Tan}[c+dx]}{15 d (a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 3, 277 leaves):

$$- \left(\left((C + B \cos[c + dx] + A \cos[c + dx])^2 \right. \right. \\ \left. \left. \left(240 C \cos\left[\frac{1}{2}(c + dx)\right] \right)^6 \left(\log\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \log\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - \right. \\ \left. \cos\left[\frac{1}{2}(c + dx)\right] \sec\left[\frac{c}{2}\right] \left(5(3A + 4B - 29C) \sin\left[\frac{dx}{2}\right] - 15(A - 5C) \sin\left[c + \frac{dx}{2}\right] + 15A \sin\left[c + \frac{3dx}{2}\right] + 10B \sin\left[c + \frac{3dx}{2}\right] - \right. \right. \\ \left. \left. 95C \sin\left[c + \frac{3dx}{2}\right] + 15C \sin\left[2c + \frac{3dx}{2}\right] + 3A \sin\left[2c + \frac{5dx}{2}\right] + 2B \sin\left[2c + \frac{5dx}{2}\right] - 22C \sin\left[2c + \frac{5dx}{2}\right] \right) \right) \Big/ \\ \left. \left(15a^3 d (1 + \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2(c + dx)]) \right) \right)$$

■ **Problem 470: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$\frac{Ax}{a^3} - \frac{(A - B + C) \tan[c + dx]}{5d(a + a \sec[c + dx])^3} - \frac{(7A - 2B - 3C) \tan[c + dx]}{15ad(a + a \sec[c + dx])^2} - \frac{(22A - 2B - 3C) \tan[c + dx]}{15d(a^3 + a^3 \sec[c + dx])}$$

Result (type 3, 289 leaves):

$$\frac{1}{480a^3d} \\ \sec\left[\frac{c}{2}\right] \sec\left[\frac{1}{2}(c + dx)\right]^5 \left(150Adx \cos\left[\frac{dx}{2}\right] + 150Adx \cos\left[c + \frac{dx}{2}\right] + 75Adx \cos\left[c + \frac{3dx}{2}\right] + 75Adx \cos\left[2c + \frac{3dx}{2}\right] + 15Adx \cos\left[2c + \frac{5dx}{2}\right] + \right. \\ \left. 15Adx \cos\left[3c + \frac{5dx}{2}\right] - 370A \sin\left[\frac{dx}{2}\right] + 80B \sin\left[\frac{dx}{2}\right] + 30C \sin\left[\frac{dx}{2}\right] + 270A \sin\left[c + \frac{dx}{2}\right] - \right. \\ \left. 60B \sin\left[c + \frac{dx}{2}\right] - 30C \sin\left[c + \frac{dx}{2}\right] - 230A \sin\left[c + \frac{3dx}{2}\right] + 40B \sin\left[c + \frac{3dx}{2}\right] + 30C \sin\left[c + \frac{3dx}{2}\right] + \right. \\ \left. 90A \sin\left[2c + \frac{3dx}{2}\right] - 30B \sin\left[2c + \frac{3dx}{2}\right] - 64A \sin\left[2c + \frac{5dx}{2}\right] + 14B \sin\left[2c + \frac{5dx}{2}\right] + 6C \sin\left[2c + \frac{5dx}{2}\right] \right)$$

■ **Problem 471: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$-\frac{(3A - B)x}{a^3} + \frac{2(36A - 11B + C) \sin[c + dx]}{15a^3d} - \frac{(A - B + C) \sin[c + dx]}{5d(a + a \sec[c + dx])^3} - \frac{(9A - 4B - C) \sin[c + dx]}{15ad(a + a \sec[c + dx])^2} - \frac{(3A - B) \sin[c + dx]}{d(a^3 + a^3 \sec[c + dx])}$$

Result (type 3, 419 leaves):

$$\frac{1}{960 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\left(-300(3A-B)dx \cos\left[\frac{dx}{2}\right] - 300(3A-B)dx \cos\left[c + \frac{dx}{2}\right] - 450Adx \cos\left[c + \frac{3dx}{2}\right] + 150Bdx \cos\left[c + \frac{3dx}{2}\right] - 450Adx \cos\left[2c + \frac{3dx}{2}\right] + 150Bdx \cos\left[2c + \frac{3dx}{2}\right] - 90Adx \cos\left[2c + \frac{5dx}{2}\right] + 30Bdx \cos\left[2c + \frac{5dx}{2}\right] - 90Adx \cos\left[3c + \frac{5dx}{2}\right] + 30Bdx \cos\left[3c + \frac{5dx}{2}\right] + 1755A \sin\left[\frac{dx}{2}\right] - 740B \sin\left[\frac{dx}{2}\right] + 160C \sin\left[\frac{dx}{2}\right] - 1125A \sin\left[c + \frac{dx}{2}\right] + 540B \sin\left[c + \frac{dx}{2}\right] - 120C \sin\left[c + \frac{dx}{2}\right] + 1215A \sin\left[c + \frac{3dx}{2}\right] - 460B \sin\left[c + \frac{3dx}{2}\right] + 80C \sin\left[c + \frac{3dx}{2}\right] - 225A \sin\left[2c + \frac{3dx}{2}\right] + 180B \sin\left[2c + \frac{3dx}{2}\right] - 60C \sin\left[2c + \frac{3dx}{2}\right] + 363A \sin\left[2c + \frac{5dx}{2}\right] - 128B \sin\left[2c + \frac{5dx}{2}\right] + 28C \sin\left[2c + \frac{5dx}{2}\right] + 75A \sin\left[3c + \frac{5dx}{2}\right] + 15A \sin\left[3c + \frac{7dx}{2}\right] + 15A \sin\left[4c + \frac{7dx}{2}\right]\right)$$

■ **Problem 472: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 201 leaves, 7 steps):

$$\frac{(13A-6B+2C)x}{2a^3} - \frac{2(76A-36B+11C) \sin[c+dx]}{15a^3d} + \frac{(13A-6B+2C) \cos[c+dx] \sin[c+dx]}{2a^3d} - \frac{(A-B+C) \cos[c+dx] \sin[c+dx]}{5d(a+a \operatorname{Sec}[c+dx])^3} - \frac{(11A-6B+C) \cos[c+dx] \sin[c+dx]}{15ad(a+a \operatorname{Sec}[c+dx])^2} - \frac{(76A-36B+11C) \cos[c+dx] \sin[c+dx]}{15d(a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 3, 557 leaves):

$$\frac{1}{3840 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\left(600(13A-6B+2C)dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 600(13A-6B+2C)dx \operatorname{Cos}\left[c+\frac{dx}{2}\right] + 3900A dx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] - 1800B dx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + \right.$$

$$600C dx \operatorname{Cos}\left[c+\frac{3dx}{2}\right] + 3900A dx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] - 1800B dx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 600C dx \operatorname{Cos}\left[2c+\frac{3dx}{2}\right] + 780A dx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] -$$

$$360B dx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] + 120C dx \operatorname{Cos}\left[2c+\frac{5dx}{2}\right] + 780A dx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] - 360B dx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] + 120C dx \operatorname{Cos}\left[3c+\frac{5dx}{2}\right] -$$

$$12760A \operatorname{Sin}\left[\frac{dx}{2}\right] + 7020B \operatorname{Sin}\left[\frac{dx}{2}\right] - 2960C \operatorname{Sin}\left[\frac{dx}{2}\right] + 7560A \operatorname{Sin}\left[c+\frac{dx}{2}\right] - 4500B \operatorname{Sin}\left[c+\frac{dx}{2}\right] + 2160C \operatorname{Sin}\left[c+\frac{dx}{2}\right] -$$

$$9230A \operatorname{Sin}\left[c+\frac{3dx}{2}\right] + 4860B \operatorname{Sin}\left[c+\frac{3dx}{2}\right] - 1840C \operatorname{Sin}\left[c+\frac{3dx}{2}\right] + 930A \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] - 900B \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] + 720C \operatorname{Sin}\left[2c+\frac{3dx}{2}\right] -$$

$$2782A \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] + 1452B \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] - 512C \operatorname{Sin}\left[2c+\frac{5dx}{2}\right] - 750A \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] + 300B \operatorname{Sin}\left[3c+\frac{5dx}{2}\right] -$$

$$105A \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] + 60B \operatorname{Sin}\left[3c+\frac{7dx}{2}\right] - 105A \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] + 60B \operatorname{Sin}\left[4c+\frac{7dx}{2}\right] + 15A \operatorname{Sin}\left[4c+\frac{9dx}{2}\right] + 15A \operatorname{Sin}\left[5c+\frac{9dx}{2}\right] \left. \right)$$

■ **Problem 473: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+dx]^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 237 leaves, 8 steps):

$$-\frac{(23A-13B+6C)x}{2a^3} + \frac{4(34A-19B+9C) \operatorname{Sin}[c+dx]}{5a^3 d} - \frac{(23A-13B+6C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2a^3 d} - \frac{(A-B+C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{5d(a+a \operatorname{Sec}[c+dx])^3} -$$

$$\frac{(13A-8B+3C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{15ad(a+a \operatorname{Sec}[c+dx])^2} - \frac{(23A-13B+6C) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{3d(a^3+a^3 \operatorname{Sec}[c+dx])} - \frac{4(34A-19B+9C) \operatorname{Sin}[c+dx]^3}{15a^3 d}$$

Result (type 3, 655 leaves):

$$\frac{1}{3840 a^3 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\left(-600(23A-13B+6C)dx \cos\left[\frac{dx}{2}\right] - 600(23A-13B+6C)dx \cos\left[c+\frac{dx}{2}\right] - 6900A dx \cos\left[c+\frac{3dx}{2}\right] + 3900B dx \cos\left[c+\frac{3dx}{2}\right] -\right.$$

$$1800C dx \cos\left[c+\frac{3dx}{2}\right] - 6900A dx \cos\left[2c+\frac{3dx}{2}\right] + 3900B dx \cos\left[2c+\frac{3dx}{2}\right] - 1800C dx \cos\left[2c+\frac{3dx}{2}\right] -$$

$$1380A dx \cos\left[2c+\frac{5dx}{2}\right] + 780B dx \cos\left[2c+\frac{5dx}{2}\right] - 360C dx \cos\left[2c+\frac{5dx}{2}\right] - 1380A dx \cos\left[3c+\frac{5dx}{2}\right] + 780B dx \cos\left[3c+\frac{5dx}{2}\right] -$$

$$360C dx \cos\left[3c+\frac{5dx}{2}\right] + 20410A \sin\left[\frac{dx}{2}\right] - 12760B \sin\left[\frac{dx}{2}\right] + 7020C \sin\left[\frac{dx}{2}\right] - 11110A \sin\left[c+\frac{dx}{2}\right] + 7560B \sin\left[c+\frac{dx}{2}\right] -$$

$$4500C \sin\left[c+\frac{dx}{2}\right] + 15380A \sin\left[c+\frac{3dx}{2}\right] - 9230B \sin\left[c+\frac{3dx}{2}\right] + 4860C \sin\left[c+\frac{3dx}{2}\right] - 380A \sin\left[2c+\frac{3dx}{2}\right] +$$

$$930B \sin\left[2c+\frac{3dx}{2}\right] - 900C \sin\left[2c+\frac{3dx}{2}\right] + 4777A \sin\left[2c+\frac{5dx}{2}\right] - 2782B \sin\left[2c+\frac{5dx}{2}\right] + 1452C \sin\left[2c+\frac{5dx}{2}\right] +$$

$$1625A \sin\left[3c+\frac{5dx}{2}\right] - 750B \sin\left[3c+\frac{5dx}{2}\right] + 300C \sin\left[3c+\frac{5dx}{2}\right] + 230A \sin\left[3c+\frac{7dx}{2}\right] - 105B \sin\left[3c+\frac{7dx}{2}\right] +$$

$$60C \sin\left[3c+\frac{7dx}{2}\right] + 230A \sin\left[4c+\frac{7dx}{2}\right] - 105B \sin\left[4c+\frac{7dx}{2}\right] + 60C \sin\left[4c+\frac{7dx}{2}\right] - 20A \sin\left[4c+\frac{9dx}{2}\right] +$$

$$15B \sin\left[4c+\frac{9dx}{2}\right] - 20A \sin\left[5c+\frac{9dx}{2}\right] + 15B \sin\left[5c+\frac{9dx}{2}\right] + 5A \sin\left[5c+\frac{11dx}{2}\right] + 5A \sin\left[6c+\frac{11dx}{2}\right]\Big)$$

■ **Problem 474: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^5 (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^4} dx$$

Optimal (type 3, 254 leaves, 9 steps):

$$\frac{(2A-8B+21C) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2a^4 d} - \frac{8(20A-83B+216C) \operatorname{Tan}[c+dx]}{105a^4 d} +$$

$$\frac{(2A-8B+21C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{2a^4 d} - \frac{(10A-52B+129C) \operatorname{Sec}[c+dx]^3 \operatorname{Tan}[c+dx]}{105a^4 d (1+\operatorname{Sec}[c+dx])^2} -$$

$$\frac{4(20A-83B+216C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{105a^4 d (1+\operatorname{Sec}[c+dx])} - \frac{(A-B+C) \operatorname{Sec}[c+dx]^5 \operatorname{Tan}[c+dx]}{7d(a+a \operatorname{Sec}[c+dx])^4} + \frac{(B-2C) \operatorname{Sec}[c+dx]^4 \operatorname{Tan}[c+dx]}{5ad(a+a \operatorname{Sec}[c+dx])^3}$$

Result (type 3, 1322 leaves):

$$\begin{aligned}
& - \left(16 (2A - 8B + 21C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] - \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) + \\
& \left(16 (2A - 8B + 21C) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \log\left[\cos\left[\frac{c}{2} + \frac{dx}{2}\right] + \sin\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(A \sin\left[\frac{c}{2}\right] - B \sin\left[\frac{c}{2}\right] + C \sin\left[\frac{c}{2}\right] \right) \right) / \\
& \quad (7d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(10A \sin\left[\frac{c}{2}\right] - 17B \sin\left[\frac{c}{2}\right] + 24C \sin\left[\frac{c}{2}\right] \right) \right) / \\
& \quad (35d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(55A \sin\left[\frac{c}{2}\right] - 139B \sin\left[\frac{c}{2}\right] + 258C \sin\left[\frac{c}{2}\right] \right) \right) / \\
& \quad (105d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad (7d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(10A \sin\left[\frac{dx}{2}\right] - 17B \sin\left[\frac{dx}{2}\right] + 24C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad (35d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(55A \sin\left[\frac{dx}{2}\right] - 139B \sin\left[\frac{dx}{2}\right] + 258C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad (105d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) - \\
& \left(32 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sec\left[\frac{c}{2}\right] \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(160A \sin\left[\frac{dx}{2}\right] - 559B \sin\left[\frac{dx}{2}\right] + 1308C \sin\left[\frac{dx}{2}\right] \right) \right) / \\
& \quad (105d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4) + \\
& \frac{16C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec[c] \sec[c + dx]^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[dx]}{d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4} + \\
& \left(16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec[c] \sec[c + dx]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) (C \sin[c] + 2B \sin[dx] - 8C \sin[dx]) \right) / \\
& \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4)
\end{aligned}$$

■ **Problem 475: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^4} dx$$

Optimal (type 3, 204 leaves, 8 steps) :

$$\frac{(B - 4 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{a^4 d} + \frac{(6 A - 55 B + 244 C) \operatorname{Tan}[c + d x]}{105 a^4 d} + \frac{(3 A + 25 B - 88 C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{105 a^4 d (1 + \operatorname{Sec}[c + d x])^2} -$$

$$\frac{(B - 4 C) \operatorname{Tan}[c + d x]}{a^4 d (1 + \operatorname{Sec}[c + d x])} - \frac{(A - B + C) \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{7 d (a + a \operatorname{Sec}[c + d x])^4} + \frac{(2 A + 5 B - 12 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{35 a d (a + a \operatorname{Sec}[c + d x])^3}$$

Result (type 3, 1208 leaves) :

$$\left(32 (-B + 4 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) /$$

$$(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) -$$

$$\left(32 (-B + 4 C) \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) /$$

$$(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(4 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (A \operatorname{Sin}\left[\frac{c}{2}\right] - B \operatorname{Sin}\left[\frac{c}{2}\right] + C \operatorname{Sin}\left[\frac{c}{2}\right]) \right) /$$

$$(7 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(8 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (3 A \operatorname{Sin}\left[\frac{c}{2}\right] - 10 B \operatorname{Sin}\left[\frac{c}{2}\right] + 17 C \operatorname{Sin}\left[\frac{c}{2}\right]) \right) /$$

$$(35 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(16 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (6 A \operatorname{Sin}\left[\frac{c}{2}\right] - 55 B \operatorname{Sin}\left[\frac{c}{2}\right] + 139 C \operatorname{Sin}\left[\frac{c}{2}\right]) \right) /$$

$$(105 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(4 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right]) \right) /$$

$$(7 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(8 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (3 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 10 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 17 C \operatorname{Sin}\left[\frac{d x}{2}\right]) \right) /$$

$$(35 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(16 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (6 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 55 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 139 C \operatorname{Sin}\left[\frac{d x}{2}\right]) \right) /$$

$$(105 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\left(32 \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^7 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (6 A \operatorname{Sin}\left[\frac{d x}{2}\right] - 160 B \operatorname{Sin}\left[\frac{d x}{2}\right] + 559 C \operatorname{Sin}\left[\frac{d x}{2}\right]) \right) /$$

$$(105 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4) +$$

$$\frac{32 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}[d x]}{d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^4}$$

■ **Problem 479: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{(a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 3, 148 leaves, 5 steps):

$$\frac{Ax}{a^4} - \frac{(55A - 6B - 8C) \operatorname{Tan}[c + dx]}{105 a^4 d (1 + \operatorname{Sec}[c + dx])^2} - \frac{2(80A - 3B - 4C) \operatorname{Tan}[c + dx]}{105 a^4 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B + C) \operatorname{Tan}[c + dx]}{7d (a + a \operatorname{Sec}[c + dx])^4} - \frac{(10A - 3B - 4C) \operatorname{Tan}[c + dx]}{35 a d (a + a \operatorname{Sec}[c + dx])^3}$$

Result (type 3, 405 leaves):

$$\begin{aligned} & \frac{1}{13440 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^7 \\ & \left(3675 A dx \operatorname{Cos}\left[\frac{dx}{2}\right] + 3675 A dx \operatorname{Cos}\left[c + \frac{dx}{2}\right] + 2205 A dx \operatorname{Cos}\left[c + \frac{3dx}{2}\right] + 2205 A dx \operatorname{Cos}\left[2c + \frac{3dx}{2}\right] + 735 A dx \operatorname{Cos}\left[2c + \frac{5dx}{2}\right] + \right. \\ & 735 A dx \operatorname{Cos}\left[3c + \frac{5dx}{2}\right] + 105 A dx \operatorname{Cos}\left[3c + \frac{7dx}{2}\right] + 105 A dx \operatorname{Cos}\left[4c + \frac{7dx}{2}\right] - 9940 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 1260 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 560 C \operatorname{Sin}\left[\frac{dx}{2}\right] + \\ & 8260 A \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 1260 B \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 350 C \operatorname{Sin}\left[c + \frac{dx}{2}\right] - 7140 A \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 882 B \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + 336 C \operatorname{Sin}\left[c + \frac{3dx}{2}\right] + \\ & 3780 A \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 630 B \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 210 C \operatorname{Sin}\left[2c + \frac{3dx}{2}\right] - 2800 A \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 294 B \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + \\ & \left. 182 C \operatorname{Sin}\left[2c + \frac{5dx}{2}\right] + 840 A \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 210 B \operatorname{Sin}\left[3c + \frac{5dx}{2}\right] - 520 A \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] + 72 B \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] + 26 C \operatorname{Sin}\left[3c + \frac{7dx}{2}\right] \right) \end{aligned}$$

■ **Problem 480: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 3, 176 leaves, 7 steps):

$$\begin{aligned} & -\frac{(4A - B)x}{a^4} + \frac{2(332A - 80B + 3C) \operatorname{Sin}[c + dx]}{105 a^4 d} - \frac{(88A - 25B - 3C) \operatorname{Sin}[c + dx]}{105 a^4 d (1 + \operatorname{Sec}[c + dx])^2} - \\ & \frac{(4A - B) \operatorname{Sin}[c + dx]}{a^4 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B + C) \operatorname{Sin}[c + dx]}{7d (a + a \operatorname{Sec}[c + dx])^4} - \frac{(12A - 5B - 2C) \operatorname{Sin}[c + dx]}{35 a d (a + a \operatorname{Sec}[c + dx])^3} \end{aligned}$$

Result (type 3, 567 leaves):

$$\frac{1}{26880 a^4 d} \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^7 \left(-7350(4A-B)dx \cos\left[\frac{dx}{2}\right] - 7350(4A-B)dx \cos\left[c+\frac{dx}{2}\right] - 17640Adx \cos\left[c+\frac{3dx}{2}\right] + 4410Bdx \cos\left[c+\frac{3dx}{2}\right] - 17640Adx \cos\left[2c+\frac{3dx}{2}\right] + 4410Bdx \cos\left[2c+\frac{3dx}{2}\right] - 5880Adx \cos\left[2c+\frac{5dx}{2}\right] + 1470Bdx \cos\left[2c+\frac{5dx}{2}\right] - 5880Adx \cos\left[3c+\frac{5dx}{2}\right] + 1470Bdx \cos\left[3c+\frac{5dx}{2}\right] - 840Adx \cos\left[3c+\frac{7dx}{2}\right] + 210Bdx \cos\left[3c+\frac{7dx}{2}\right] - 840Adx \cos\left[4c+\frac{7dx}{2}\right] + 210Bdx \cos\left[4c+\frac{7dx}{2}\right] + 60830A \sin\left[\frac{dx}{2}\right] - 19880B \sin\left[\frac{dx}{2}\right] + 2520C \sin\left[\frac{dx}{2}\right] - 46130A \sin\left[c+\frac{dx}{2}\right] + 16520B \sin\left[c+\frac{dx}{2}\right] - 2520C \sin\left[c+\frac{dx}{2}\right] + 46116A \sin\left[c+\frac{3dx}{2}\right] - 14280B \sin\left[c+\frac{3dx}{2}\right] + 1764C \sin\left[c+\frac{3dx}{2}\right] - 18060A \sin\left[2c+\frac{3dx}{2}\right] + 7560B \sin\left[2c+\frac{3dx}{2}\right] - 1260C \sin\left[2c+\frac{3dx}{2}\right] + 19292A \sin\left[2c+\frac{5dx}{2}\right] - 5600B \sin\left[2c+\frac{5dx}{2}\right] + 588C \sin\left[2c+\frac{5dx}{2}\right] - 2100A \sin\left[3c+\frac{5dx}{2}\right] + 1680B \sin\left[3c+\frac{5dx}{2}\right] - 420C \sin\left[3c+\frac{5dx}{2}\right] + 3791A \sin\left[3c+\frac{7dx}{2}\right] - 1040B \sin\left[3c+\frac{7dx}{2}\right] + 144C \sin\left[3c+\frac{7dx}{2}\right] + 735A \sin\left[4c+\frac{7dx}{2}\right] + 105A \sin\left[4c+\frac{9dx}{2}\right] + 105A \sin\left[5c+\frac{9dx}{2}\right] \right)$$

■ **Problem 481: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^4} dx$$

Optimal (type 3, 239 leaves, 8 steps):

$$\frac{(21A-8B+2C)x}{2a^4} - \frac{8(21A-83B+20C)\sin[c+dx]}{105a^4d} + \frac{(21A-8B+2C)\cos[c+dx]\sin[c+dx]}{2a^4d} - \frac{(129A-52B+10C)\cos[c+dx]\sin[c+dx]}{105a^4d(1+\operatorname{Sec}[c+dx])^2} - \frac{4(21A-83B+20C)\cos[c+dx]\sin[c+dx]}{105a^4d(1+\operatorname{Sec}[c+dx])} - \frac{(A-B+C)\cos[c+dx]\sin[c+dx]}{7d(a+a \operatorname{Sec}[c+dx])^4} - \frac{(2A-B)\cos[c+dx]\sin[c+dx]}{5ad(a+a \operatorname{Sec}[c+dx])^3}$$

Result (type 3, 1290 leaves):

$$\begin{aligned}
& \frac{16 (21 A - 8 B + 2 C) x \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4} + \\
& \frac{\left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(A \sin\left[\frac{c}{2}\right] - B \sin\left[\frac{c}{2}\right] + C \sin\left[\frac{c}{2}\right]\right)\right)}{(7 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} - \\
& \frac{\left(8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(39 A \sin\left[\frac{c}{2}\right] - 32 B \sin\left[\frac{c}{2}\right] + 25 C \sin\left[\frac{c}{2}\right]\right)\right)}{(35 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} + \\
& \frac{\left(16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(447 A \sin\left[\frac{c}{2}\right] - 286 B \sin\left[\frac{c}{2}\right] + 160 C \sin\left[\frac{c}{2}\right]\right)\right)}{(105 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} + \\
& \frac{\left(4 \cos\left[\frac{c}{2} + \frac{dx}{2}\right] \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right]\right)\right)}{(7 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} - \\
& \frac{\left(8 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(39 A \sin\left[\frac{dx}{2}\right] - 32 B \sin\left[\frac{dx}{2}\right] + 25 C \sin\left[\frac{dx}{2}\right]\right)\right)}{(35 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} + \\
& \frac{\left(16 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(447 A \sin\left[\frac{dx}{2}\right] - 286 B \sin\left[\frac{dx}{2}\right] + 160 C \sin\left[\frac{dx}{2}\right]\right)\right)}{(105 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} - \\
& \frac{\left(32 \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^7 \sec\left[\frac{c}{2}\right] \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(1653 A \sin\left[\frac{dx}{2}\right] - 764 B \sin\left[\frac{dx}{2}\right] + 260 C \sin\left[\frac{dx}{2}\right]\right)\right)}{(105 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4)} + \\
& \frac{(4 A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{16 i \cos[c+dx]}{d} - \frac{16 \sin[c+dx]}{d}\right)}{(A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4} + \\
& \frac{(4 A - B) \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{16 i \cos[c+dx]}{d} - \frac{16 \sin[c+dx]}{d}\right)}{(A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4} + \\
& \frac{8 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \sec^2[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[2 c + 2 dx]}{d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \sec[c + dx])^4}
\end{aligned}$$

- **Problem 486: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\frac{2\sqrt{a} A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2a(3B+C) \operatorname{Tan}[c+dx]}{3d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2C\sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 506 leaves):

$$\left(\cos[c+dx]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\ \left. \left(\frac{4}{3}(3B+2C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{4}{3}C \operatorname{Sec}[c+dx] \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / (d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) - \\ \frac{1}{d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])} 16(-3-2\sqrt{2}) A \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2} + (10-7\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \\ \sqrt{\frac{-1+\sqrt{2} - (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2} + (-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \cos[c+dx] \\ \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\ \sqrt{\left(-1-\sqrt{2} + (2+\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]} \\ \sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sqrt{3-2\sqrt{2} - \operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 487: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\frac{\sqrt{a} (A+2B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{A\sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d} - \frac{a(A-2C) \operatorname{Tan}[c+dx]}{d\sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 4, 402 leaves):

$$\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(\frac{1}{2}(-A+4C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2}A \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)}{d}$$

$$\frac{1}{d} 4(-3-2\sqrt{2})(A+2B) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}}$$

- **Problem 488: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 117 leaves, 4 steps):

$$\frac{\sqrt{a} (3A+4B+8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a(A+4B) \operatorname{Sin}[c+dx]}{4d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{A \operatorname{Cos}[c+dx] \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 421 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{8}(A+4B) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4}(A+2B) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8}A \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] \right) +$$

$$\frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (3A+4B+8C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

- **Problem 489: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{\sqrt{a} (5A+6B+8C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a(5A+6B+8C) \operatorname{Sin}[c+dx]}{8d \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a(A+6B) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{A \operatorname{Cos}[c+dx]^2 \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 4, 452 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{48}(11A+6B+24C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{12}(4A+3B+6C) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16}(A+2B) \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{24}A \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] \right) + \frac{1}{d} \left(1 + \frac{3}{2\sqrt{2}} \right) (5A+6B+8C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \right) \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 490: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4 \sqrt{a+a \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 209 leaves, 6 steps):

$$\frac{\sqrt{a} (35A+40B+48C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64d} + \frac{a(35A+40B+48C) \operatorname{Sin}[c+dx]}{64d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a(35A+40B+48C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{96d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a(A+8B) \operatorname{Cos}[c+dx]^2 \operatorname{Sin}[c+dx]}{24d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{A \operatorname{Cos}[c+dx]^3 \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d}$$

Result (type 4, 476 leaves):

$$\frac{1}{d} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-\frac{1}{384}(41A+88B+48C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{48}(11A+16B+12C) \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right] + \frac{1}{128}(15A+8B+16C) \operatorname{Sin}\left[\frac{5}{2}(c+dx)\right] + \frac{1}{48}(A+2B) \operatorname{Sin}\left[\frac{7}{2}(c+dx)\right] + \frac{1}{64}A \operatorname{Sin}\left[\frac{9}{2}(c+dx)\right] \right) +$$

$$\frac{1}{(-64+48\sqrt{2})d} (35A+40B+48C) \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right)$$

$$\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2\sqrt{2}, -\operatorname{ArcSin}\left[\frac{\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right)$$

$$\sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+dx)\right]^2}$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \operatorname{Sec}[c+dx] \sqrt{a(1+\operatorname{Sec}[c+dx])} \sqrt{3-2\sqrt{2}-\operatorname{Tan}\left[\frac{1}{4}(c+dx)\right]^2}$$

■ **Problem 494: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+a \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 142 leaves, 6 steps):

$$\frac{2a^{3/2}A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{2a^2(15A+20B+12C) \operatorname{Tan}[c+dx]}{15d\sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{2a(5B+3C)\sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{15d} + \frac{2C(a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Tan}[c+dx]}{5d}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& \left(\cos[c+dx]^3 \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right. \\
& \quad \left. \left(\frac{2}{15}(15A+25B+18C)\sin\left[\frac{1}{2}(c+dx)\right] + \frac{2}{5}C\sec[c+dx]^2\sin\left[\frac{1}{2}(c+dx)\right] + \frac{2}{15}\sec[c+dx] \left(5B\sin\left[\frac{1}{2}(c+dx)\right] + 9C\sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) / \\
& \quad (d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])) - \frac{1}{d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])} 8(-3-2\sqrt{2})A\cos\left[\frac{1}{4}(c+dx)\right]^4 \\
& \quad \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \\
& \quad \cos[c+dx]^2 \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \quad \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right)\sec\left[\frac{1}{4}(c+dx)\right]^2\sec\left[\frac{1}{2}(c+dx)\right]^3} \\
& \quad (a(1+\sec[c+dx]))^{3/2} (A+B\sec[c+dx]+C\sec[c+dx]^2) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2}
\end{aligned}$$

- **Problem 496: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a+a\sec[c+dx])^{3/2} (A+B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (7A+12B+8C) \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{4d} + \frac{a^2 (5A+4B-8C) \sin[c+dx]}{4d\sqrt{a+a\sec[c+dx]}} - \\
& \frac{a(A-4C)\sqrt{a+a\sec[c+dx]}\sin[c+dx]}{2d} + \frac{A\cos[c+dx](a+a\sec[c+dx])^{3/2}\sin[c+dx]}{2d}
\end{aligned}$$

Result (type 4, 436 leaves):

$$\frac{1}{2} \left(\frac{1}{d} \cos[c+dx] \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \right. \\ \left. \left(-\frac{1}{8} (5A+4B-16C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{4} (3A+2B) \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{8} A \sin\left[\frac{5}{2}(c+dx)\right] \right) + \right. \\ \left. \frac{1}{d} \left(2 + \frac{3}{\sqrt{2}} \right) (7A+12B+8C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]}{1+\cos\left[\frac{1}{2}(c+dx)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right] \right) \right. \\ \left. \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + 2\text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \right. \\ \left. \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2})\cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \right. \\ \left. \sec\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\sec[c+dx]))^{3/2} \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(c+dx)\right]^2} \right)$$

- **Problem 498: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+a\sec[c+dx])^{3/2} (A+B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 215 leaves, 6 steps):

$$\frac{a^{3/2} (75A+88B+112C) \text{ArcTan}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{64d} + \frac{a^2 (75A+88B+112C) \sin[c+dx]}{64d\sqrt{a+a\sec[c+dx]}} + \frac{a^2 (39A+56B+48C) \cos[c+dx] \sin[c+dx]}{96d\sqrt{a+a\sec[c+dx]}} + \\ \frac{a(3A+8B) \cos[c+dx]^2 \sqrt{a+a\sec[c+dx]} \sin[c+dx]}{24d} + \frac{A \cos[c+dx]^3 (a+a\sec[c+dx])^{3/2} \sin[c+dx]}{4d}$$

Result (type 4, 586 leaves):

$$\left(\cos[c + dx]^3 \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} \right. \\ \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{1}{384} (129A + 136B + 240C) \sin\left[\frac{1}{2}(c + dx)\right] + \frac{1}{48} (27A + 28B + 36C) \sin\left[\frac{3}{2}(c + dx)\right] + \right. \right. \\ \left. \left. \frac{1}{128} (23A + 24B + 16C) \sin\left[\frac{5}{2}(c + dx)\right] + \frac{1}{48} (3A + 2B) \sin\left[\frac{7}{2}(c + dx)\right] + \frac{1}{64} A \sin\left[\frac{9}{2}(c + dx)\right] \right) \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) + \frac{1}{(-64 + 48\sqrt{2})d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \\ (75A + 88B + 112C) \cos\left[\frac{1}{4}(c + dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]}{1 + \cos\left[\frac{1}{2}(c + dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \cos[c + dx]^2 \\ \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c + dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \\ \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c + dx)\right]\right) \sec\left[\frac{1}{4}(c + dx)\right]^2} \\ \sec\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \sec[c + dx]))^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c + dx)\right]^2}$$

■ **Problem 499: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^5 (a + a \sec[c + dx])^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 263 leaves, 7 steps):

$$\frac{a^{3/2} (133A + 150B + 176C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{128d} + \frac{a^2 (133A + 150B + 176C) \sin[c + dx]}{128d \sqrt{a + a \sec[c + dx]}} + \\ \frac{a^2 (133A + 150B + 176C) \cos[c + dx] \sin[c + dx]}{192d \sqrt{a + a \sec[c + dx]}} + \frac{a^2 (67A + 90B + 80C) \cos[c + dx]^2 \sin[c + dx]}{240d \sqrt{a + a \sec[c + dx]}} + \\ \frac{a (3A + 10B) \cos[c + dx]^3 \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{40d} + \frac{A \cos[c + dx]^4 (a + a \sec[c + dx])^{3/2} \sin[c + dx]}{5d}$$

Result (type 4, 611 leaves):

$$\frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^3 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \operatorname{Sec} [c + d x]))^{3/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\ \left(-\frac{(1019 A + 1290 B + 1360 C) \sin \left[\frac{1}{2} (c + d x) \right]}{3840} + \frac{1}{480} (239 A + 270 B + 280 C) \sin \left[\frac{3}{2} (c + d x) \right] + \frac{1}{256} (49 A + 46 B + 48 C) \sin \left[\frac{5}{2} (c + d x) \right] + \right. \\ \left. \frac{1}{240} (17 A + 15 B + 10 C) \sin \left[\frac{7}{2} (c + d x) \right] + \frac{1}{128} (3 A + 2 B) \sin \left[\frac{9}{2} (c + d x) \right] + \frac{1}{160} A \sin \left[\frac{11}{2} (c + d x) \right] \right) + \\ \frac{1}{64 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} (4 + 3 \sqrt{2}) (133 A + 150 B + 176 C) \cos \left[\frac{1}{4} (c + d x) \right]^4 \\ \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]^2 \\ \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\ \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2} \\ \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \operatorname{Sec} [c + d x]))^{3/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 503: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec} [c + d x])^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) dx$$

Optimal (type 3, 182 leaves, 7 steps):

$$\frac{2 a^{5/2} A \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \operatorname{Sec} [c + d x]}} \right]}{d} + \frac{2 a^3 (245 A + 224 B + 160 C) \tan [c + d x]}{105 d \sqrt{a + a \operatorname{Sec} [c + d x]}} + \frac{2 a^2 (35 A + 56 B + 40 C) \sqrt{a + a \operatorname{Sec} [c + d x]} \tan [c + d x]}{105 d} + \\ \frac{2 a (7 B + 5 C) (a + a \operatorname{Sec} [c + d x])^{3/2} \tan [c + d x]}{35 d} + \frac{2 C (a + a \operatorname{Sec} [c + d x])^{5/2} \tan [c + d x]}{7 d}$$

Result (type 4, 606 leaves):

$$\frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^4 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2}$$

$$\left((A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \left(\frac{1}{105} (280 A + 301 B + 230 C) \sin \left[\frac{1}{2} (c + d x) \right] + \frac{1}{7} C \operatorname{Sec} [c + d x]^3 \sin \left[\frac{1}{2} (c + d x) \right] + \frac{1}{35} \operatorname{Sec} [c + d x]^2 \right. \right.$$

$$\left. \left. \left(7 B \sin \left[\frac{1}{2} (c + d x) \right] + 20 C \sin \left[\frac{1}{2} (c + d x) \right] \right) + \frac{1}{105} \operatorname{Sec} [c + d x] \left(35 A \sin \left[\frac{1}{2} (c + d x) \right] + 98 B \sin \left[\frac{1}{2} (c + d x) \right] + 115 C \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) \right) -$$

$$\frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 4 (-3 - 2 \sqrt{2}) A \cos \left[\frac{1}{4} (c + d x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}}$$

$$\sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]^3$$

$$\left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5}$$

$$(a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 505: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^2 (a + a \operatorname{Sec} [c + d x])^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) dx$$

Optimal (type 3, 197 leaves, 6 steps):

$$\frac{a^{5/2} (19 A + 20 B + 8 C) \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \operatorname{Sec} [c + d x]}} \right]}{4 d} + \frac{a^3 (27 A - 12 B - 56 C) \sin [c + d x]}{12 d \sqrt{a + a \operatorname{Sec} [c + d x]}} - \frac{a^2 (A - 4 B - 8 C) \sqrt{a + a \operatorname{Sec} [c + d x]} \sin [c + d x]}{2 d}$$

$$\frac{a (3 A - 4 C) (a + a \operatorname{Sec} [c + d x])^{3/2} \sin [c + d x]}{6 d} + \frac{A \cos [c + d x] (a + a \operatorname{Sec} [c + d x])^{5/2} \sin [c + d x]}{2 d}$$

Result (type 4, 467 leaves):

$$\frac{1}{2} \left(\frac{1}{d} \cos[c+dx]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \right. \\ \left. \left(-\frac{1}{48} (27A - 36B - 128C) \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{3} C \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{8} (5A + 2B) \sin\left[\frac{3}{2}(c+dx)\right] + \frac{1}{16} A \sin\left[\frac{5}{2}(c+dx)\right] \right) + \right. \\ \left. \frac{1}{d} \left(1 + \frac{3}{2\sqrt{2}} \right) (19A + 20B + 8C) \cos\left[\frac{1}{4}(c+dx)\right]^4 \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]}{1 + \cos\left[\frac{1}{2}(c+dx)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right] \right) \right. \\ \left. \cos[c+dx] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+dx)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] \right) \right. \\ \left. \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(c+dx)\right]\right) \sec\left[\frac{1}{4}(c+dx)\right]^2} \right. \\ \left. \sec\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\sec[c+dx]))^{5/2} \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(c+dx)\right]^2} \right)$$

■ **Problem 507: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a + a \sec[c+dx])^{5/2} (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 215 leaves, 6 steps):

$$\frac{a^{5/2} (163A + 200B + 304C) \text{ArcTan}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right]}{64d} + \\ \frac{a^3 (299A + 392B + 432C) \sin[c+dx]}{192d \sqrt{a+a \sec[c+dx]}} + \frac{a^2 (17A + 24B + 16C) \cos[c+dx] \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{32d} + \\ \frac{a (5A + 8B) \cos[c+dx]^2 (a+a \sec[c+dx])^{3/2} \sin[c+dx]}{24d} + \frac{A \cos[c+dx]^3 (a+a \sec[c+dx])^{5/2} \sin[c+dx]}{4d}$$

Result (type 4, 587 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^4 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. \left(-\frac{1}{768}(265 A+376 B+432 C) \sin\left[\frac{1}{2}(c+d x)\right] + \frac{1}{96}(55 A+64 B+60 C) \sin\left[\frac{3}{2}(c+d x)\right] + \frac{1}{256}(47 A+40 B+16 C) \sin\left[\frac{5}{2}(c+d x)\right] + \right. \right. \\
& \quad \left. \left. \frac{1}{96}(5 A+2 B) \sin\left[\frac{7}{2}(c+d x)\right] + \frac{1}{128} A \sin\left[\frac{9}{2}(c+d x)\right] \right) \right) / (d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) + \\
& \frac{1}{64 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} (4+3 \sqrt{2})(163 A+200 B+304 C) \cos\left[\frac{1}{4}(c+d x)\right]^4 \\
& \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]}{1+\cos\left[\frac{1}{2}(c+d x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \cos [c+d x]^3 \\
& \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] + 2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2}, -\operatorname{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(c+d x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right] \right) \\
& \sqrt{\left(-1+\sqrt{2}-(-2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos\left[\frac{1}{2}(c+d x)\right]\right) \operatorname{Sec}\left[\frac{1}{4}(c+d x)\right]^2} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \sqrt{3-2 \sqrt{2}-\tan\left[\frac{1}{4}(c+d x)\right]^2}
\end{aligned}$$

■ **Problem 508: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^5 (a+a \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 261 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2}(283 A+326 B+400 C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{128 d} + \frac{a^3(283 A+326 B+400 C) \sin [c+d x]}{128 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \\
& \frac{a^3(787 A+950 B+1040 C) \cos [c+d x] \sin [c+d x]}{960 d \sqrt{a+a \operatorname{Sec}[c+d x]}} + \frac{a^2(79 A+110 B+80 C) \cos [c+d x]^2 \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{240 d} + \\
& \frac{a(A+2 B) \cos [c+d x]^3 (a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{8 d} + \frac{A \cos [c+d x]^4 (a+a \operatorname{Sec}[c+d x])^{5/2} \sin [c+d x]}{5 d}
\end{aligned}$$

Result (type 4, 611 leaves):

$$\frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^4 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2)$$

$$\left(-\frac{(2309 A + 2650 B + 3760 C) \sin \left[\frac{1}{2} (c + d x) \right]}{7680} + \frac{1}{960} (509 A + 550 B + 640 C) \sin \left[\frac{3}{2} (c + d x) \right] + \frac{1}{512} (95 A + 94 B + 80 C) \sin \left[\frac{5}{2} (c + d x) \right] + \right.$$

$$\left. \frac{1}{480} (32 A + 25 B + 10 C) \sin \left[\frac{7}{2} (c + d x) \right] + \frac{1}{256} (5 A + 2 B) \sin \left[\frac{9}{2} (c + d x) \right] + \frac{1}{320} A \sin \left[\frac{11}{2} (c + d x) \right] \right) +$$

$$\frac{1}{64 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \left(2 + \frac{3}{\sqrt{2}} \right) (283 A + 326 B + 400 C) \cos \left[\frac{1}{4} (c + d x) \right]^4$$

$$\sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]^3$$

$$\left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right)$$

$$\sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2}$$

$$\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 509: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^6 (a + a \operatorname{Sec} [c + d x])^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) dx$$

Optimal (type 3, 311 leaves, 8 steps):

$$\frac{a^{5/2} (1015 A + 1132 B + 1304 C) \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [c + d x]}{\sqrt{a + a \operatorname{Sec} [c + d x]}} \right]}{512 d} + \frac{a^3 (1015 A + 1132 B + 1304 C) \sin [c + d x]}{512 d \sqrt{a + a \operatorname{Sec} [c + d x]}} +$$

$$\frac{a^3 (1015 A + 1132 B + 1304 C) \cos [c + d x] \sin [c + d x]}{768 d \sqrt{a + a \operatorname{Sec} [c + d x]}} + \frac{a^3 (545 A + 628 B + 680 C) \cos [c + d x]^2 \sin [c + d x]}{960 d \sqrt{a + a \operatorname{Sec} [c + d x]}} +$$

$$\frac{a^2 (115 A + 156 B + 120 C) \cos [c + d x]^3 \sqrt{a + a \operatorname{Sec} [c + d x]} \sin [c + d x]}{480 d} +$$

$$\frac{a (5 A + 12 B) \cos [c + d x]^4 (a + a \operatorname{Sec} [c + d x])^{3/2} \sin [c + d x]}{60 d} + \frac{A \cos [c + d x]^5 (a + a \operatorname{Sec} [c + d x])^{5/2} \sin [c + d x]}{6 d}$$

Result (type 4, 635 leaves) :

$$\frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^4 \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\ \left(- \frac{(7945 A + 9236 B + 10600 C) \operatorname{Sin} \left[\frac{1}{2} (c + d x) \right]}{30720} + \frac{(935 A + 1018 B + 1100 C) \operatorname{Sin} \left[\frac{3}{2} (c + d x) \right]}{1920} + \frac{(1145 A + 1140 B + 1128 C) \operatorname{Sin} \left[\frac{5}{2} (c + d x) \right]}{6144} + \right. \\ \left. \frac{(145 A + 128 B + 100 C) \operatorname{Sin} \left[\frac{7}{2} (c + d x) \right]}{1920} + \frac{(83 A + 60 B + 24 C) \operatorname{Sin} \left[\frac{9}{2} (c + d x) \right]}{3072} + \frac{1}{640} (5 A + 2 B) \operatorname{Sin} \left[\frac{11}{2} (c + d x) \right] + \frac{1}{768} A \operatorname{Sin} \left[\frac{13}{2} (c + d x) \right] \right) + \\ \frac{1}{512 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} (4 + 3 \sqrt{2}) (1015 A + 1132 B + 1304 C) \cos \left[\frac{1}{4} (c + d x) \right]^4 \\ \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right]}{1 + \cos \left[\frac{1}{2} (c + d x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \cos [c + d x]^3 \\ \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + 2 \operatorname{EllipticPi} \left[-3 + 2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\ \sqrt{\left(-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2} \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (c + d x) \right] \right) \operatorname{Sec} \left[\frac{1}{4} (c + d x) \right]^2} \\ \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^5 (a (1 + \operatorname{Sec} [c + d x]))^{5/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \sqrt{3 - 2 \sqrt{2} - \operatorname{Tan} \left[\frac{1}{4} (c + d x) \right]^2}$$

■ **Problem 522: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec} [c + d x] (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2)}{(a + a \operatorname{Sec} [c + d x])^{3/2}} dx$$

Optimal (type 3, 120 leaves, 4 steps) :

$$\frac{(A + 3 B - 7 C) \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Tan} [c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec} [c + d x]}} \right]}{2 \sqrt{2} a^{3/2} d} + \frac{(A - B + C) \operatorname{Tan} [c + d x]}{2 d (a + a \operatorname{Sec} [c + d x])^{3/2}} + \frac{2 C \operatorname{Tan} [c + d x]}{a d \sqrt{a + a \operatorname{Sec} [c + d x]}}$$

Result (type 3, 307 leaves) :

$$\left(2 (A + 3B - 7C) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \sqrt{\frac{\operatorname{Cos}[c + dx]}{1 + \operatorname{Cos}[c + dx]}} \sqrt{1 + \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) +$$

$$\left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right.$$

$$\left. \left(4 (A - B + 5C) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - 2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + C \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) /$$

$$(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c + dx]))^{3/2})$$

■ **Problem 526: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 284 leaves, 9 steps):

$$-\frac{(47A - 38B + 24C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{8a^{3/2}d} + \frac{(17A - 13B + 9C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{2\sqrt{2}a^{3/2}d} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^2 \operatorname{Sin}[c + dx]}{2d(a + a \operatorname{Sec}[c + dx])^{3/2}} +$$

$$\frac{(21A - 14B + 12C) \operatorname{Sin}[c + dx]}{8ad\sqrt{a + a \operatorname{Sec}[c + dx]}} - \frac{(13A - 12B + 6C) \operatorname{Cos}[c + dx] \operatorname{Sin}[c + dx]}{12ad\sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{(5A - 3B + 3C) \operatorname{Cos}[c + dx]^2 \operatorname{Sin}[c + dx]}{6ad\sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 831 leaves):

$$\begin{aligned}
& \left(\cos [c+d x]^2 (1+\sec [c+d x])^{3 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2) \right. \\
& \left. \left(\left(\sqrt{2}(-21 A+14 B-12 C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [c+d x]}}\right] \cos [c+d x]^2 \sqrt{-1+\sec [c+d x]} (1+\sec [c+d x])^{3 / 2} \sin [c+d x] \right) / \right. \right. \\
& \left. \left(d(1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} \sqrt{\cos [c+d x]^2(-1+\sec [c+d x]) (1+\sec [c+d x])} \right) - \right. \\
& \left. \left((47 A-38 B+24 C) \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [c+d x]}}\right] + \operatorname{ArcTan}\left[\frac{-2+\sqrt{1+\sec [c+d x]}}{\sqrt{-1+\sec [c+d x]}}\right] - \operatorname{ArcTan}\left[\frac{2+\sqrt{1+\sec [c+d x]}}{\sqrt{-1+\sec [c+d x]}}\right] \right) \right. \right. \\
& \left. \left. \cos [c+d x]^2 \sqrt{-1+\sec [c+d x]} (1+\sec [c+d x])^{3 / 2} \sin [c+d x] \right) / \right. \\
& \left. \left. \left(d(1+\cos [c+d x]) \sqrt{1-\cos [c+d x]^2} \sqrt{\cos [c+d x]^2(-1+\sec [c+d x]) (1+\sec [c+d x])} \right) \right) \right) / \\
& (8(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) (a(1+\sec [c+d x]))^{3 / 2}) + \\
& \frac{1}{(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) (a(1+\sec [c+d x]))^{3 / 2}} \\
& \frac{\cos [c+d x]^2 \sqrt{(1+\cos [c+d x]) \sec [c+d x]}}{(1+\sec [c+d x])^{3 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2)} \\
& \left(\frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^2 (-A \sin \left[\frac{c}{2}\right]+B \sin \left[\frac{c}{2}\right]-C \sin \left[\frac{c}{2}\right])}{2 d} + \frac{(25 A-14 B+8 C) \cos [d x] \sin [c]}{4 d} - \right. \\
& \frac{(11 A-6 B) \cos [2 d x] \sin [2 c]}{12 d} + \frac{A \cos [3 d x] \sin [3 c]}{6 d} + \frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right] (-61 A \sin \left[\frac{d x}{2}\right]+30 B \sin \left[\frac{d x}{2}\right]-12 C \sin \left[\frac{d x}{2}\right])}{12 d} + \\
& \frac{\sec \left[\frac{c}{2}\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^3 (-A \sin \left[\frac{d x}{2}\right]+B \sin \left[\frac{d x}{2}\right]-C \sin \left[\frac{d x}{2}\right])}{2 d} + \frac{(25 A-14 B+8 C) \cos [c] \sin [d x]}{4 d} - \\
& \left. \frac{(11 A-6 B) \cos [2 c] \sin [2 d x]}{12 d} + \frac{A \cos [3 c] \sin [3 d x]}{6 d} - \frac{(61 A-30 B+12 C) \tan \left[\frac{c}{2}\right]}{12 d} \right)
\end{aligned}$$

■ **Problem 529: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c+d x]^2 (A+B \sec [c+d x]+C \sec [c+d x]^2)}{(a+a \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 3, 179 leaves, 5 steps):

$$\frac{(5A + 19B - 75C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx]}{4d(a+a \operatorname{Sec}[c+dx])^{5/2}} - \frac{(3A + 5B - 13C) \operatorname{Tan}[c+dx]}{16ad(a+a \operatorname{Sec}[c+dx])^{3/2}} + \frac{(A - B + 9C) \operatorname{Tan}[c+dx]}{4a^2 d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 372 leaves):

$$\left((5A + 19B - 75C) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Sec}[c+dx]} \\ \left((A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Big/ \left(2d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a(1+\operatorname{Sec}[c+dx]))^{5/2}} \right) + \\ \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\ \left. \left((A - 9B + 49C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \left(3A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 5B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - 13C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) + \right. \\ \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \left(-A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] - C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) \Big/ \\ \left(d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right)$$

■ **Problem 530: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+a \operatorname{Sec}[c+dx])^{5/2}} dx$$

Optimal (type 3, 137 leaves, 4 steps):

$$\frac{(3A + 5B + 19C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx]}{4d(a+a \operatorname{Sec}[c+dx])^{5/2}} + \frac{(7A + B - 9C) \operatorname{Tan}[c+dx]}{16ad(a+a \operatorname{Sec}[c+dx])^{3/2}}$$

Result (type 3, 371 leaves):

$$\left((3A + 5B + 19C) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right] \cos\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \sqrt{\sec[c + dx]} \sqrt{1 + \sec[c + dx]} \right. \\ \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \left(2d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec\left[\frac{1}{2}(c + dx)\right]^2} (a(1 + \sec[c + dx]))^{5/2} \right) + \\ \left(\cos\left[\frac{1}{2}(c + dx)\right]^5 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\ \left. \left((7A + B - 9C) \sin\left[\frac{1}{2}(c + dx)\right] + \sec\left[\frac{1}{2}(c + dx)\right]^4 \left(A \sin\left[\frac{1}{2}(c + dx)\right] - B \sin\left[\frac{1}{2}(c + dx)\right] + C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) + \right. \\ \left. \frac{1}{2} \sec\left[\frac{1}{2}(c + dx)\right]^2 \left(-11A \sin\left[\frac{1}{2}(c + dx)\right] + 3B \sin\left[\frac{1}{2}(c + dx)\right] + 5C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) / \\ (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a(1 + \sec[c + dx]))^{5/2})$$

■ **Problem 531: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{(a + a \sec[c + dx])^{5/2}} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{2A \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{a^{5/2} d} - \frac{(43A - 3B - 5C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Tan}[c + dx]}{4d (a + a \sec[c + dx])^{5/2}} - \frac{(11A - 3B - 5C) \operatorname{Tan}[c + dx]}{16ad (a + a \sec[c + dx])^{3/2}}$$

Result (type 3, 418 leaves):

$$\left(\left((-43A + 3B + 5C) \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right] + 32\sqrt{2}A \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{\frac{\operatorname{Cos}[c+dx]}{1+\operatorname{Cos}[c+dx]}}}}\right] \right) \right. \\ \left. \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^4 \sqrt{\frac{\operatorname{Cos}[c + dx]}{1 + \operatorname{Cos}[c + dx]}} \sqrt{\operatorname{Sec}[c + dx]} \sqrt{1 + \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ \left(2d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{5/2}} \right) + \\ \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^5 \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left(- (15A - 7B - C) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 \left(-A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - C \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) + \right. \\ \left. \left. \frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \left(19A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] - 11B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 3C \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \right) / \\ \left(d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right)$$

■ **Problem 532: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$- \frac{(5A - 2B) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} + \frac{(115A - 43B + 3C) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16\sqrt{2} a^{5/2} d} - \\ \frac{(A - B + C) \operatorname{Sin}[c + dx]}{4d(a + a \operatorname{Sec}[c + dx])^{5/2}} - \frac{(15A - 7B - C) \operatorname{Sin}[c + dx]}{16ad(a + a \operatorname{Sec}[c + dx])^{3/2}} + \frac{(35A - 11B + 3C) \operatorname{Sin}[c + dx]}{16a^2 d \sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 839 leaves):

$$\begin{aligned}
& \left(\cos[c+dx]^2 (1+\sec[c+dx])^{5/2} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right. \\
& \left. \left(\left(\sqrt{2} (-35A+11B-3C) \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec[c+dx]}}\right] \cos[c+dx]^2 \sqrt{-1+\sec[c+dx]} (1+\sec[c+dx])^{3/2} \sin[c+dx] \right) / \right. \right. \\
& \left. \left(d(1+\cos[c+dx]) \sqrt{1-\cos[c+dx]^2} \sqrt{\cos[c+dx]^2(-1+\sec[c+dx])(1+\sec[c+dx])} \right) - \right. \\
& \left. \left((80A-32B) \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec[c+dx]}}\right] + \operatorname{ArcTan}\left[\frac{-2+\sqrt{1+\sec[c+dx]}}{\sqrt{-1+\sec[c+dx]}}\right] - \operatorname{ArcTan}\left[\frac{2+\sqrt{1+\sec[c+dx]}}{\sqrt{-1+\sec[c+dx]}}\right] \right) \right) \right. \\
& \left. \left. \cos[c+dx]^2 \sqrt{-1+\sec[c+dx]} (1+\sec[c+dx])^{3/2} \sin[c+dx] \right) / \right. \\
& \left. \left. \left(d(1+\cos[c+dx]) \sqrt{1-\cos[c+dx]^2} \sqrt{\cos[c+dx]^2(-1+\sec[c+dx])(1+\sec[c+dx])} \right) \right) \right) / \\
& \frac{(16(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}) + 1}{(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}} \\
& \frac{\cos[c+dx]^2 \sqrt{(1+\cos[c+dx])\sec[c+dx]}}{(1+\sec[c+dx])^{5/2} (A+B\sec[c+dx]+C\sec[c+dx]^2)} \\
& \left(\frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^2 (-27A\sin\left[\frac{c}{2}\right]+19B\sin\left[\frac{c}{2}\right]-11C\sin\left[\frac{c}{2}\right])}{16d} + \right. \\
& \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^4 (A\sin\left[\frac{c}{2}\right]-B\sin\left[\frac{c}{2}\right]+C\sin\left[\frac{c}{2}\right])}{8d} + \frac{2A\cos[dx]\sin[c]}{d} + \\
& \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^3 (-27A\sin\left[\frac{dx}{2}\right]+19B\sin\left[\frac{dx}{2}\right]-11C\sin\left[\frac{dx}{2}\right])}{16d} + \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right]^5 (A\sin\left[\frac{dx}{2}\right]-B\sin\left[\frac{dx}{2}\right]+C\sin\left[\frac{dx}{2}\right])}{8d} + \\
& \left. \frac{\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}+\frac{dx}{2}\right] (7A\sin\left[\frac{dx}{2}\right]-15B\sin\left[\frac{dx}{2}\right]+7C\sin\left[\frac{dx}{2}\right])}{8d} + \frac{2A\cos[c]\sin[dx]}{d} + \frac{(7A-15B+7C)\tan\left[\frac{c}{2}\right]}{8d} \right)
\end{aligned}$$

■ **Problem 534: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^{3/2} (a+a\sec[c+dx]) (A+B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 4, 217 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 a (5 A + 3 (B + C)) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \\
& \frac{2 a (7 A + 7 B + 5 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \frac{2 a (5 A + 3 (B + C)) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \\
& \frac{2 a (7 A + 7 B + 5 C) \sec [c + d x]^{3/2} \sin [c + d x]}{21 d} + \frac{2 a (B + C) \sec [c + d x]^{5/2} \sin [c + d x]}{5 d} + \frac{2 a C \sec [c + d x]^{7/2} \sin [c + d x]}{7 d}
\end{aligned}$$

Result (type 5, 997 leaves):

$$\begin{aligned}
& a \left(- \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^2 \operatorname{Csc}[c] \right. \right. \\
& \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \right. \\
& \left. (d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) - \left(6 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^2 \operatorname{Csc}[c] \right. \right. \\
& \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \right. \\
& \left. (5 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) - \left(6 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^2 \operatorname{Csc}[c] \right. \right. \\
& \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \right. \right. \\
& \left. \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / (5 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) + \right. \\
& \left. \frac{4 A \cos [c + d x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2)}{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} + \right. \\
& \left. \frac{4 B \cos [c + d x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2)}{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} + \right.
\end{aligned}$$

$$\frac{20 C \cos [c+d x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{21 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} +$$

$$\left((A+B \sec [c+d x]+C \sec [c+d x]^2) \left(\frac{4(5 A+3 B+3 C) \cos [d x] \operatorname{Csc}[c]}{5 d} + \frac{4 C \sec [c] \sec [c+d x]^3 \sin [d x]}{7 d} + \frac{4 \sec [c] \sec [c+d x]^2 (5 C \sin [c]+7 B \sin [d x]+7 C \sin [d x])}{35 d} + \frac{4 \sec [c] \sec [c+d x] (21 B \sin [c]+21 C \sin [c]+35 A \sin [d x]+35 B \sin [d x]+25 C \sin [d x])}{105 d} + \frac{4(7 A+7 B+5 C) \tan [c]}{21 d} \right) \right) /$$

$$\left((A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} \right)$$

- **Problem 535: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec [c+d x]} (a+a \sec [c+d x]) (A+B \sec [c+d x]+C \sec [c+d x]^2) dx$$

Optimal (type 4, 181 leaves, 8 steps):

$$- \frac{2 a (5 A+5 B+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{2 a (3 A+B+C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} +$$

$$\frac{2 a (5 A+5 B+3 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{5 d} + \frac{2 a (B+C) \sec [c+d x]^{3/2} \sin [c+d x]}{3 d} + \frac{2 a C \sec [c+d x]^{5/2} \sin [c+d x]}{5 d}$$

Result (type 5, 937 leaves):

$$\begin{aligned}
& a \left(- \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \right. \\
& \quad \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \right. \\
& \quad (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) - \left(2 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \\
& \quad \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \right. \\
& \quad (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) - \left(6 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \\
& \quad \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \right. \right. \\
& \quad \left. \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) + \right. \\
& \quad \frac{4A \cos[c+dx]^{5/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \quad \frac{4B \cos[c+dx]^{5/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \quad \frac{4C \cos[c+dx]^{5/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \quad \left((A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(\frac{4(5A + 5B + 3C) \cos[dx] \operatorname{Csc}[c]}{5d} + \frac{4C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{5d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (3C \sin[c] + 5B \sin[dx] + 5C \sin[dx])}{15d} + \right. \right. \\
& \quad \left. \left. \frac{4(B+C) \operatorname{Tan}[c]}{3d} \right) / \left((A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{3/2} \right) \right)
\end{aligned}$$

- **Problem 536: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 143 leaves, 7 steps):

$$\frac{2 a (A - B - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \frac{2 a (3 A + 3 B + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a (B + C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d} + \frac{2 a C \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 d}$$

Result (type 5, 908 leaves):

$$\begin{aligned}
& a \left(\left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \right. \\
& \quad \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) - \left(2 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) - \left(2 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^2 \operatorname{Csc}[c] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])) + \\
& \frac{4 A \cos[c+dx]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \frac{4 B \cos[c+dx]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \frac{4 C \cos[c+dx]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx])} + \\
& \left((A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-\frac{2(A - 2B - 2C + A \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{d} + \frac{4 A \cos[c] \sin[dx]}{d} + \right. \right. \\
& \quad \left. \left. \frac{4 C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{3d} + \frac{4 C \operatorname{Tan}[c]}{3d} \right) \right) / ((A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{3/2}) \Big)
\end{aligned}$$

■ **Problem 537: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 138 leaves, 7 steps):

$$\frac{2 a (A + B - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} +$$

$$\frac{2 a (A + 3 (B + C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 a C \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d}$$

Result (type 5, 162 leaves):

$$\frac{1}{3 d} a \sqrt{\operatorname{Sec}[c + d x]} \left(-6 i A \operatorname{Cos}[c + d x] - 6 i B \operatorname{Cos}[c + d x] + 6 i C \operatorname{Cos}[c + d x] + 2 (A + 3 (B + C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] + \right.$$

$$\left. 6 i (A + B - C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 6 C \operatorname{Sin}[c + d x] + A \operatorname{Sin}[2 (c + d x)] \right)$$

■ **Problem 538: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$\frac{2 a (3 A + 5 (B + C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} +$$

$$\frac{2 a (A + B + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 a (A + B) \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 156 leaves):

$$\frac{1}{30 d} a \sqrt{\operatorname{Sec}[c + d x]} \left(20 (A + B + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] + \right.$$

$$\left. 12 i (3 A + 5 (B + C)) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + \right.$$

$$\left. 2 \operatorname{Cos}[c + d x] (-6 i (3 A + 5 (B + C)) + 10 (A + B) \operatorname{Sin}[c + d x] + 3 A \operatorname{Sin}[2 (c + d x)]) \right)$$

■ **Problem 539: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 182 leaves, 8 steps):

$$\frac{2 a (3 (A+B) + 5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} +$$

$$\frac{2 a (5 A+7 (B+C)) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} +$$

$$\frac{2 a A \sin [c+d x]}{7 d \sec [c+d x]^{5/2}} + \frac{2 a (A+B) \sin [c+d x]}{5 d \sec [c+d x]^{3/2}} + \frac{2 a (5 A+7 (B+C)) \sin [c+d x]}{21 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 180 leaves):

$$\frac{1}{420 d} a \sqrt{\sec [c+d x]} \left(40 (5 A+7 (B+C)) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right.$$

$$168 i (3 A+3 B+5 C) e^{-i(c+d x)} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] +$$

$$\left. 2 \cos [c+d x] (-84 i (3 A+3 B+5 C) + 5 (23 A+28 (B+C)) \sin [c+d x] + 42 (A+B) \sin [2(c+d x)] + 15 A \sin [3(c+d x)]) \right)$$

■ **Problem 540: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x]) (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sec [c+d x]^{9/2}} dx$$

Optimal (type 4, 215 leaves, 9 steps):

$$\frac{2 a (7 A+9 (B+C)) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{15 d} +$$

$$\frac{2 a (5 (A+B)+7 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{21 d} + \frac{2 a A \sin [c+d x]}{9 d \sec [c+d x]^{7/2}} +$$

$$\frac{2 a (A+B) \sin [c+d x]}{7 d \sec [c+d x]^{5/2}} + \frac{2 a (7 A+9 (B+C)) \sin [c+d x]}{45 d \sec [c+d x]^{3/2}} + \frac{2 a (5 (A+B)+7 C) \sin [c+d x]}{21 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 210 leaves):

$$\frac{1}{2520 d} a \sqrt{\sec [c+d x]} \left(240 (5 A+5 B+7 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] + \right.$$

$$336 i (7 A+9 (B+C)) e^{-i(c+d x)} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c+d x)}\right] +$$

$$\left. 2 \cos [c+d x] (-1176 i A - 1512 i B - 1512 i C + 30 (23 A+23 B+28 C) \sin [c+d x] + 14 (19 A+18 (B+C)) \sin [2(c+d x)] + \right.$$

$$\left. 90 A \sin [3(c+d x)] + 90 B \sin [3(c+d x)] + 35 A \sin [4(c+d x)] \right)$$

■ **Problem 541: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^{3/2} (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 291 leaves, 10 steps):

$$\begin{aligned} & - \frac{4 a^2 (12 A + 9 B + 8 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{15 d} + \\ & \frac{4 a^2 (7 A + 6 B + 5 C) \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{21 d} + \frac{4 a^2 (12 A + 9 B + 8 C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{15 d} + \\ & \frac{4 a^2 (7 A + 6 B + 5 C) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{21 d} + \frac{2 a^2 (21 A + 27 B + 19 C) \text{Sec}[c + d x]^{5/2} \text{Sin}[c + d x]}{105 d} + \\ & \frac{2 C \text{Sec}[c + d x]^{5/2} (a + a \text{Sec}[c + d x])^2 \text{Sin}[c + d x]}{9 d} + \frac{2 (9 B + 4 C) \text{Sec}[c + d x]^{5/2} (a^2 + a^2 \text{Sec}[c + d x]) \text{Sin}[c + d x]}{63 d} \end{aligned}$$

Result (type 5, 1240 leaves):

$$\begin{aligned} & - \left(4 \sqrt{2} A e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \text{Cos}[c + d x]^4 \text{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \right. \\ & \quad \left. \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / (5 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) - \\ & \left(3 \sqrt{2} B e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \text{Cos}[c + d x]^4 \text{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \right. \\ & \quad \left. \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / (5 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) - \\ & \left(8 \sqrt{2} C e^{-i (2 c + d x)} \sqrt{\frac{e^{i (c + d x)}}{1 + e^{2 i (c + d x)}}} \text{Cos}[c + d x]^4 \text{Csc}[c] \left(1 + e^{2 i (c + d x)} + (-1 + e^{2 i c}) \sqrt{1 + e^{2 i (c + d x)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] \right) \right. \\ & \quad \left. \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / (15 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) + \\ & \left(2 A \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) / \end{aligned}$$

$$\begin{aligned}
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \right) + \\
& \left(4 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) \Bigg/ \\
& \left(7 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \right) + \\
& \left(10 C \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) \Bigg/ \\
& \left(21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \right) + \\
& \frac{1}{(A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\frac{2 (12 A + 9 B + 8 C) \cos [d x] \operatorname{Csc}[c]}{15 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \sin [d x]}{9 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (7 C \sin [c] + 9 B \sin [d x] + 18 C \sin [d x])}{63 d} \right) + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (45 B \sin [c] + 90 C \sin [c] + 63 A \sin [d x] + 126 B \sin [d x] + 112 C \sin [d x])}{315 d} + \frac{1}{315 d} \\
& \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (63 A \sin [c] + 126 B \sin [c] + 112 C \sin [c] + 210 A \sin [d x] + 180 B \sin [d x] + 150 C \sin [d x]) + \frac{2 (7 A + 6 B + 5 C) \tan [c]}{21 d} \Bigg)
\end{aligned}$$

- **Problem 542: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec [c + d x]} (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 255 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^2 (5 A + 4 B + 3 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \\
& \frac{4 a^2 (14 A + 7 B + 6 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{21 d} + \\
& \frac{4 a^2 (5 A + 4 B + 3 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{5 d} + \frac{2 a^2 (35 A + 49 B + 33 C) \operatorname{Sec}[c + d x]^{3/2} \sin [c + d x]}{105 d} + \\
& \frac{2 C \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^2 \sin [c + d x]}{7 d} + \frac{2 (7 B + 4 C) \operatorname{Sec}[c + d x]^{3/2} (a^2 + a^2 \operatorname{Sec}[c + d x]) \sin [c + d x]}{35 d}
\end{aligned}$$

Result (type 5, 1184 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) - \\
& \left(4 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) - \\
& \left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) + \\
& \left(4 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(2 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(4 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (7d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left(\frac{2(5A+4B+3C) \cos[dx] \operatorname{Csc}[c]}{5d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \sin[dx]}{7d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (5C \sin[c] + 7B \sin[dx] + 14C \sin[dx])}{35d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (21B \sin[c] + 42C \sin[c] + 35A \sin[dx] + 70B \sin[dx] + 60C \sin[dx])}{105d} + \frac{(7A+14B+12C) \tan[c]}{21d} \right) \left. \right) / \\
& ((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2})
\end{aligned}$$

- **Problem 543: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$\begin{aligned} & - \frac{4 a^2 (5 B + 4 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^2 (3 A + 2 B + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \frac{2 a^2 (15 A + 25 B + 17 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \\ & \frac{2 C \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{5 d} + \frac{2 (5 B + 4 C) \sqrt{\operatorname{Sec}[c + d x]} (a^2 + a^2 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 942 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) - \\
& \left(4 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) + \\
& \left(2A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(4B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(2C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-\frac{(-5A-20B-16C+5A \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{10d} + \frac{A \cos[c] \sin[dx]}{d} + \right. \right. \\
& \quad \left. \left. \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{5d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (3C \sin[c] + 5B \sin[dx] + 10C \sin[dx])}{15d} + \frac{(B+2C) \tan[c]}{3d} \right) \right) / \\
& \quad ((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2})
\end{aligned}$$

■ **Problem 544: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 4, 208 leaves, 8 steps):

$$\frac{4 a^2 (A-C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{d} + \frac{4 a^2 (2 A+3 B+2 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} - \frac{2 a^2 (A-3 B-5 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 d} + \frac{2 A\left(a+a \sec [c+d x]\right)^2 \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}} - \frac{2(A-C) \sqrt{\sec [c+d x]}\left(a^2+a^2 \sec [c+d x]\right) \sin [c+d x]}{3 d}$$

Result (type 5, 233 leaves):

$$\frac{1}{6 d} a^2 e^{-i(2 c+d x)} \sec [c+d x]^{3 / 2} \left(-12 i A+12 i C-12 i A \cos [2(c+d x)]+12 i C \cos [2(c+d x)]+8(2 A+3 B+2 C) \cos [c+d x]^{3 / 2} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]+12 i(A-C) e^{-2 i(c+d x)}\left(1+e^{2 i(c+d x)}\right)^{3 / 2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i(c+d x)}\right]+A \sin [c+d x]+4 C \sin [c+d x]+6 B \sin [2(c+d x)]+12 C \sin [2(c+d x)]+A \sin [3(c+d x)]\right) (\cos [2 c+d x]+i \sin [2 c+d x])$$

■ **Problem 545: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^2(A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sec [c+d x]^{5 / 2}} d x$$

Optimal (type 4, 214 leaves, 8 steps):

$$\frac{4 a^2(4 A+5 B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 d} + \frac{4 a^2(A+2 B+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 d} - \frac{2 a^2(7 A+5 B-15 C) \sqrt{\sec [c+d x]} \sin [c+d x]}{15 d} + \frac{2 A\left(a+a \sec [c+d x]\right)^2 \sin [c+d x]}{5 d \sec [c+d x]^{3 / 2}} + \frac{2(4 A+5 B)\left(a^2+a^2 \sec [c+d x]\right) \sin [c+d x]}{15 d \sqrt{\sec [c+d x]}}$$

Result (type 5, 187 leaves):

$$\frac{1}{30 d} a^2 \sqrt{\sec [c+d x]} \left(-96 i A \cos [c+d x]-120 i B \cos [c+d x]+40(A+2 B+3 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]+24 i(4 A+5 B) e^{-i(c+d x)} \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i(c+d x)}\right]+3 A \sin [c+d x]+60 C \sin [c+d x]+20 A \sin [2(c+d x)]+10 B \sin [2(c+d x)]+3 A \sin [3(c+d x)]\right)$$

■ **Problem 546: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 219 leaves, 8 steps):

$$\begin{aligned} & \frac{4 a^2 (3 A + 4 B + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^2 (6 A + 7 B + 14 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{2 a^2 (33 A + 49 B + 35 C) \operatorname{Sin}[c + d x]}{105 d \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{2 A (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 (4 A + 7 B) (a^2 + a^2 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{35 d \operatorname{Sec}[c + d x]^{3/2}} \end{aligned}$$

Result (type 5, 189 leaves):

$$\begin{aligned} & \frac{1}{420 d} a^2 \sqrt{\operatorname{Sec}[c + d x]} \left(80 (6 A + 7 (B + 2 C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right. \\ & \quad \left. 336 i (3 A + 4 B + 5 C) e^{-i(c + d x)} \sqrt{1 + e^{2 i(c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i(c + d x)}\right] + \right. \\ & \quad \left. 2 \operatorname{Cos}[c + d x] (-504 i A - 672 i B - 840 i C + 5 (51 A + 28 (2 B + C)) \operatorname{Sin}[c + d x] + 42 (2 A + B) \operatorname{Sin}[2(c + d x)] + 15 A \operatorname{Sin}[3(c + d x)]) \right) \end{aligned}$$

■ **Problem 547: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 255 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^2 (8 A + 9 B + 12 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} + \\ & \frac{4 a^2 (5 A + 6 B + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{2 a^2 (19 A + 27 B + 21 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Sec}[c + d x]^{3/2}} + \\ & \frac{4 a^2 (5 A + 6 B + 7 C) \operatorname{Sin}[c + d x]}{21 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 A (a + a \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{2 (4 A + 9 B) (a^2 + a^2 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{63 d \operatorname{Sec}[c + d x]^{5/2}} \end{aligned}$$

Result (type 5, 248 leaves):

$$\frac{1}{2520 d} a^2 e^{-i(2c+dx)} \sqrt{\text{Sec}[c+dx]} \left(480 (5A+6B+7C) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 672 i (8A+9B+12C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \\ \left. 2 \text{Cos}[c+dx] (-2688 i A - 3024 i B - 4032 i C + 30 (46A+51B+56C) \text{Sin}[c+dx] + 14 (37A+36B+18C) \text{Sin}[2(c+dx)] + \right. \\ \left. 180A \text{Sin}[3(c+dx)] + 90B \text{Sin}[3(c+dx)] + 35A \text{Sin}[4(c+dx)]) \right) (\text{Cos}[2c+dx] + i \text{Sin}[2c+dx])$$

■ **Problem 548: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c+dx])^2 (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2)}{\text{Sec}[c+dx]^{11/2}} dx$$

Optimal (type 4, 291 leaves, 10 steps):

$$\frac{4 a^2 (7 A + 8 B + 9 C) \sqrt{\text{Cos}[c+dx]} \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{15 d} + \\ \frac{4 a^2 (50 A + 55 B + 66 C) \sqrt{\text{Cos}[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\text{Sec}[c+dx]}}{231 d} + \\ \frac{2 a^2 (89 A + 121 B + 99 C) \text{Sin}[c+dx]}{693 d \text{Sec}[c+dx]^{5/2}} + \frac{4 a^2 (7 A + 8 B + 9 C) \text{Sin}[c+dx]}{45 d \text{Sec}[c+dx]^{3/2}} + \frac{4 a^2 (50 A + 55 B + 66 C) \text{Sin}[c+dx]}{231 d \sqrt{\text{Sec}[c+dx]}} + \\ \frac{2 A (a + a \text{Sec}[c+dx])^2 \text{Sin}[c+dx]}{11 d \text{Sec}[c+dx]^{9/2}} + \frac{2 (4 A + 11 B) (a^2 + a^2 \text{Sec}[c+dx]) \text{Sin}[c+dx]}{99 d \text{Sec}[c+dx]^{7/2}}$$

Result (type 5, 1337 leaves):

$$\left(7 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \text{Cos}[c+dx]^4 \text{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ \left. \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c+dx])^2 (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) \right) / (15 d (A + 2 C + 2 B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx])) + \\ \left(8 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \text{Cos}[c+dx]^4 \text{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ \left. \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c+dx])^2 (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) \right) / (15 d (A + 2 C + 2 B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx])) +$$

$$\begin{aligned}
& \left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^4 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx])) + \\
& \left(100 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (231d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(10 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (21d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \left(4 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (7d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}) + \\
& \frac{1}{(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a+a \operatorname{Sec}[c+dx])^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{(298A+347B+396C+374A \cos[2c] + 421B \cos[2c] + 468C \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{720d} + \frac{(2185A+2288B+2376C) \cos[2dx] \sin[2c]}{7392d} + \right. \\
& \frac{(86A+79B+72C) \cos[3dx] \sin[3c]}{720d} + \frac{(27A+22B+11C) \cos[4dx] \sin[4c]}{616d} + \frac{(2A+B) \cos[5dx] \sin[5c]}{144d} + \\
& \frac{A \cos[6dx] \sin[6c]}{352d} + \frac{(374A+421B+468C) \cos[c] \sin[dx]}{360d} + \frac{(2185A+2288B+2376C) \cos[2c] \sin[2dx]}{7392d} + \\
& \left. \frac{(86A+79B+72C) \cos[3c] \sin[3dx]}{720d} + \frac{(27A+22B+11C) \cos[4c] \sin[4dx]}{616d} + \frac{(2A+B) \cos[5c] \sin[5dx]}{144d} + \frac{A \cos[6c] \sin[6dx]}{352d} \right)
\end{aligned}$$

■ **Problem 549: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\begin{aligned}
& \frac{4 a^3 (21 A + 17 B + 15 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{15 d} + \\
& \frac{4 a^3 (143 A + 121 B + 105 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \\
& \frac{4 a^3 (21 A + 17 B + 15 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{15 d} + \frac{4 a^3 (143 A + 121 B + 105 C) \sec [c + d x]^{3/2} \sin [c + d x]}{231 d} + \\
& \frac{4 a^3 (264 A + 253 B + 210 C) \sec [c + d x]^{5/2} \sin [c + d x]}{1155 d} + \frac{2 C \sec [c + d x]^{5/2} (a + a \sec [c + d x])^3 \sin [c + d x]}{11 d} + \\
& \frac{2 (11 B + 6 C) \sec [c + d x]^{5/2} (a^2 + a^2 \sec [c + d x])^2 \sin [c + d x]}{99 a d} + \frac{2 (99 A + 143 B + 105 C) \sec [c + d x]^{5/2} (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{693 d}
\end{aligned}$$

Result (type 5, 1292 leaves):

$$\begin{aligned}
& - \left(7 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(5 \sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) - \\
& \left(17 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(15 \sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) - \\
& \left(C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(\sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) + \\
& \left(13 A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \quad (21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2}) +
\end{aligned}$$

$$\begin{aligned}
& \left(11 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (21 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(5 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (11 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \frac{1}{(A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c+dx])^3 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \\
& \left(\frac{(21 A + 17 B + 15 C) \cos[dx] \operatorname{Csc}[c]}{15 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^5 \sin[dx]}{22 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 (9 C \sin[c] + 11 B \sin[dx] + 33 C \sin[dx])}{198 d} \right) + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (77 B \sin[c] + 231 C \sin[c] + 99 A \sin[dx] + 297 B \sin[dx] + 378 C \sin[dx])}{1386 d} + \frac{1}{6930 d} \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \\
& (495 A \sin[c] + 1485 B \sin[c] + 1890 C \sin[c] + 2079 A \sin[dx] + 2618 B \sin[dx] + 2310 C \sin[dx]) + \frac{1}{6930 d} \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \\
& (2079 A \sin[c] + 2618 B \sin[c] + 2310 C \sin[c] + 4290 A \sin[dx] + 3630 B \sin[dx] + 3150 C \sin[dx]) + \frac{(143 A + 121 B + 105 C) \tan[c]}{231 d} \Big)
\end{aligned}$$

- **Problem 550: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 307 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 a^3 (27 A + 21 B + 17 C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15 d} + \\
& \frac{4 a^3 (21 A + 13 B + 11 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21 d} + \frac{4 a^3 (27 A + 21 B + 17 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{15 d} + \\
& \frac{4 a^3 (42 A + 41 B + 32 C) \operatorname{Sec}[c+dx]^{3/2} \sin[c+dx]}{105 d} + \frac{2 C \operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])^3 \sin[c+dx]}{9 d} + \\
& \frac{2 (3 B + 2 C) \operatorname{Sec}[c+dx]^{3/2} (a^2 + a^2 \operatorname{Sec}[c+dx])^2 \sin[c+dx]}{21 a d} + \frac{2 (63 A + 99 B + 73 C) \operatorname{Sec}[c+dx]^{3/2} (a^3 + a^3 \operatorname{Sec}[c+dx]) \sin[c+dx]}{315 d}
\end{aligned}$$

Result (type 5, 1237 leaves):

$$- \left(9 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right)$$

$$\begin{aligned}
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(5 \sqrt{2} d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \right) - \\
& \left(7 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos[c + dx]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(5 \sqrt{2} d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \right) - \\
& \left(17 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos[c + dx]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(15 \sqrt{2} d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \right) + \\
& \left(A \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2}) + \\
& \left(13 B \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (21 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2}) + \\
& \left(11 C \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (21 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2}) + \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2}} \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(\frac{(27A + 21B + 17C) \cos[dx] \operatorname{Csc}[c]}{15d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^4 \sin[dx]}{18d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 (7C \sin[c] + 9B \sin[dx] + 27C \sin[dx])}{126d} \right) + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (45B \sin[c] + 135C \sin[c] + 63A \sin[dx] + 189B \sin[dx] + 238C \sin[dx])}{630d} + \frac{1}{630d} \\
& \left. \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (63A \sin[c] + 189B \sin[c] + 238C \sin[c] + 315A \sin[dx] + 390B \sin[dx] + 330C \sin[dx]) + \frac{(21A + 26B + 22C) \tan[c]}{42d} \right)
\end{aligned}$$

■ **Problem 551: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 271 leaves, 9 steps):

$$\begin{aligned} & - \frac{4 a^3 (5 A + 9 B + 7 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (35 A + 21 B + 13 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \\ & \frac{4 a^3 (140 A + 147 B + 106 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{105 d} + \frac{2 C \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{7 d} + \\ & \frac{2 (7 B + 6 C) \sqrt{\operatorname{Sec}[c + d x]} (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{35 a d} + \frac{2 (5 A + 9 B + 7 C) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 1202 leaves):

$$\begin{aligned}
& - \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) - \\
& \left(9 B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) - \\
& \left(7 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) + \\
& \left(5 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3 d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(13 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (21 d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-\frac{(-25A-36B-28C+5A \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{20 d} + \right. \right. \\
& \quad \frac{A \cos[c] \sin[dx]}{2 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \sin[dx]}{14 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (5C \sin[c] + 7B \sin[dx] + 21C \sin[dx])}{70 d} + \\
& \quad \left. \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (21B \sin[c] + 63C \sin[c] + 35A \sin[dx] + 105B \sin[dx] + 130C \sin[dx])}{210 d} + \frac{(7A+21B+26C) \tan[c]}{42 d} \right) \right) / \\
& \quad ((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2})
\end{aligned}$$

- **Problem 552: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 271 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^3 (5 A - 5 B - 9 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (5 A + 5 B + 3 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \\ & \frac{4 a^3 (5 A + 20 B + 21 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \frac{2 A (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} - \\ & \frac{2 (5 A - 3 C) \sqrt{\operatorname{Sec}[c + d x]} (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{15 a d} - \frac{2 (5 A - 5 B - 9 C) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 1190 leaves):

$$\begin{aligned}
& \left(A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) - \\
& \left(B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) - \\
& \left(9 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5 \sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) + \\
& \left(5 A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(5 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \\
& \left(C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) + \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \left(-\frac{(5A-25B-36C+15A \cos[2c]+5B \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{20d} + \frac{A \cos[2dx] \sin[2c]}{12d} + \right. \\
& \quad \left. \frac{(3A+B) \cos[c] \sin[dx]}{2d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{10d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (3C \sin[c] + 5B \sin[dx] + 15C \sin[dx])}{30d} + \right. \\
& \quad \left. \frac{A \cos[2c] \sin[2dx]}{12d} + \frac{(B+3C) \tan[c]}{6d} \right) / \left((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right)
\end{aligned}$$

- **Problem 553: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 270 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^3 (9 A + 5 B - 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (3 A + 5 (B + C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} - \\ & \frac{4 a^3 (6 A - 5 B - 20 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \frac{2 A (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} + \\ & \frac{2 (6 A + 5 B) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{15 a d \sqrt{\operatorname{Sec}[c + d x]}} - \frac{2 (9 A + 5 B - 5 C) \sqrt{\operatorname{Sec}[c + d x]} (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d} \end{aligned}$$

Result (type 5, 1203 leaves):

$$\begin{aligned}
& \left(9 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) + \\
& \left(B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) - \\
& \left(C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^5 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \right) + \\
& \left(A \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \\
& \left(5 B \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(3 d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \\
& \left(5 C \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(3 d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \left(\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a \operatorname{Sec}[c+dx])^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left(-\frac{(71A+20B-100C+73A \cos[2c]+60B \cos[2c]+20C \cos[2c]) \cos[dx] \operatorname{Csc}[c]}{80d} + \frac{(3A+B) \cos[2dx] \sin[2c]}{12d} + \frac{A \cos[3dx] \sin[3c]}{40d} + \right. \\
& \quad \left. \frac{(73A+60B+20C) \cos[c] \sin[dx]}{40d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \sin[dx]}{6d} + \frac{(3A+B) \cos[2c] \sin[2dx]}{12d} + \frac{A \cos[3c] \sin[3dx]}{40d} + \frac{C \tan[c]}{6d} \right) \left. \right) / \left((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right)
\end{aligned}$$

■ **Problem 554: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 271 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^3 (7 A + 9 B + 5 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 d} + \\ & \frac{4 a^3 (13 A + 21 B + 35 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} - \frac{4 a^3 (41 A + 42 B - 35 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{105 d} + \\ & \frac{2 A (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 (6 A + 7 B) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{35 a d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 (7 A + 9 B + 5 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Sec}[c + d x]}} \end{aligned}$$

Result (type 5, 279 leaves):

$$\begin{aligned} & \frac{1}{420 d} a^3 e^{-i(2c+dx)} \sqrt{\operatorname{Sec}[c + d x]} \\ & \left(-2352 i A \operatorname{Cos}[c + d x] - 3024 i B \operatorname{Cos}[c + d x] - 1680 i C \operatorname{Cos}[c + d x] + 80 (13 A + 21 B + 35 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + \right. \\ & 336 i (7 A + 9 B + 5 C) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 126 A \operatorname{Sin}[c + d x] + \\ & 42 B \operatorname{Sin}[c + d x] + 840 C \operatorname{Sin}[c + d x] + 550 A \operatorname{Sin}[2(c + d x)] + 420 B \operatorname{Sin}[2(c + d x)] + 140 C \operatorname{Sin}[2(c + d x)] + \\ & \left. 126 A \operatorname{Sin}[3(c + d x)] + 42 B \operatorname{Sin}[3(c + d x)] + 15 A \operatorname{Sin}[4(c + d x)] \right) (\operatorname{Cos}[2c + dx] + i \operatorname{Sin}[2c + dx]) \end{aligned}$$

■ **Problem 555: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 271 leaves, 9 steps):

$$\begin{aligned} & \frac{4 a^3 (17 A + 21 B + 27 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} + \\ & \frac{4 a^3 (11 A + 13 B + 21 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{21 d} + \frac{4 a^3 (32 A + 41 B + 42 C) \operatorname{Sin}[c + d x]}{105 d \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{2 A (a + a \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{2 (2 A + 3 B) (a^2 + a^2 \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{21 a d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 (73 A + 99 B + 63 C) (a^3 + a^3 \operatorname{Sec}[c + d x]) \operatorname{Sin}[c + d x]}{315 d \operatorname{Sec}[c + d x]^{3/2}} \end{aligned}$$

Result (type 5, 214 leaves):

$$\frac{1}{2520 d} a^3 \sqrt{\sec[c+dx]} \left(480 (11 A + 13 B + 21 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right. \\ \left. 672 i (17 A + 21 B + 27 C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + \right. \\ \left. 2 \cos[c+dx] (-5712 i A - 7056 i B - 9072 i C + 30 (97 A + 107 B + 84 C) \sin[c+dx] + 14 (73 A + 54 B + 18 C) \sin[2(c+dx)] + \right. \\ \left. 270 A \sin[3(c+dx)] + 90 B \sin[3(c+dx)] + 35 A \sin[4(c+dx)] \right) \Bigg)$$

■ **Problem 556: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sec[c+dx]^{11/2}} dx$$

Optimal (type 4, 307 leaves, 10 steps):

$$\frac{4 a^3 (15 A + 17 B + 21 C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15 d} + \\ \frac{4 a^3 (105 A + 121 B + 143 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{231 d} + \\ \frac{4 a^3 (210 A + 253 B + 264 C) \sin[c+dx]}{1155 d \sec[c+dx]^{3/2}} + \frac{4 a^3 (105 A + 121 B + 143 C) \sin[c+dx]}{231 d \sqrt{\sec[c+dx]}} + \frac{2 A (a + a \sec[c+dx])^3 \sin[c+dx]}{11 d \sec[c+dx]^{9/2}} + \\ \frac{2 (6 A + 11 B) (a^2 + a^2 \sec[c+dx])^2 \sin[c+dx]}{99 a d \sec[c+dx]^{7/2}} + \frac{2 (105 A + 143 B + 99 C) (a^3 + a^3 \sec[c+dx]) \sin[c+dx]}{693 d \sec[c+dx]^{5/2}}$$

Result (type 5, 247 leaves):

$$\frac{1}{55440 d} a^3 \sqrt{\sec[c+dx]} \left(960 (105 A + 121 B + 143 C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + 14784 i (15 A + 17 B + 21 C) e^{-i(c+dx)} \sqrt{1+e^{2i(c+dx)}} \right. \\ \left. \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] + 2 \cos[c+dx] (-110880 i A - 125664 i B - 155232 i C + \right. \\ \left. 30 (1953 A + 2134 B + 2354 C) \sin[c+dx] + 308 (75 A + 73 B + 54 C) \sin[2(c+dx)] + 8505 A \sin[3(c+dx)] + \right. \\ \left. 5940 B \sin[3(c+dx)] + 1980 C \sin[3(c+dx)] + 2310 A \sin[4(c+dx)] + 770 B \sin[4(c+dx)] + 315 A \sin[5(c+dx)] \right) \Bigg)$$

■ **Problem 557: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sec[c+dx]^{13/2}} dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\begin{aligned}
& \frac{4 a^3 (175 A + 195 B + 221 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{195 d} + \\
& \frac{4 a^3 (95 A + 105 B + 121 C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{231 d} + \frac{20 a^3 (236 A + 273 B + 286 C) \sin [c + d x]}{9009 d \sec [c + d x]^{5/2}} + \\
& \frac{4 a^3 (175 A + 195 B + 221 C) \sin [c + d x]}{585 d \sec [c + d x]^{3/2}} + \frac{4 a^3 (95 A + 105 B + 121 C) \sin [c + d x]}{231 d \sqrt{\sec [c + d x]}} + \frac{2 A (a + a \sec [c + d x])^3 \sin [c + d x]}{13 d \sec [c + d x]^{11/2}} + \\
& \frac{2 (6 A + 13 B) (a^2 + a^2 \sec [c + d x])^2 \sin [c + d x]}{143 a d \sec [c + d x]^{9/2}} + \frac{2 (145 A + 195 B + 143 C) (a^3 + a^3 \sec [c + d x]) \sin [c + d x]}{1287 d \sec [c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 1386 leaves):

$$\begin{aligned}
& \left(35 A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(39 \sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) + \\
& \left(B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(\sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) + \\
& \left(17 C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos [c + d x]^5 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \left. \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \left(15 \sqrt{2} d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) + \\
& \left(95 A \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \left(231 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2} \right) + \\
& \left(5 B \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sec \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \left(11 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{9/2} \right) +
\end{aligned}$$

$$\left(11 C \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)\right) /$$

$$\left(21 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \operatorname{Sec}[c+d x]^{9 / 2}\right)+$$

$$\frac{1}{(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \operatorname{Sec}[c+d x]^{9 / 2}} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)$$

$$\left(-\frac{1}{149760 d}(59375 A+67080 B+77272 C+75025 A \cos [2 c]+82680 B \cos [2 c]+92456 C \cos [2 c]) \cos [d x] \operatorname{Csc}[c]+$$

$$\frac{(4267 A+4473 B+4664 C) \cos [2 d x] \sin [2 c]}{14784 d}+\frac{(9005 A+8580 B+7852 C) \cos [3 d x] \sin [3 c]}{74880 d}+\right.$$

$$\frac{(59 A+49 B+33 C) \cos [4 d x] \sin [4 c]}{1232 d}+\frac{(245 A+156 B+52 C) \cos [5 d x] \sin [5 c]}{14976 d}+\frac{(3 A+B) \cos [6 d x] \sin [6 c]}{704 d}+$$

$$\frac{A \cos [7 d x] \sin [7 c]}{1664 d}+\frac{(75025 A+82680 B+92456 C) \cos [c] \sin [d x]}{74880 d}+\frac{(4267 A+4473 B+4664 C) \cos [2 c] \sin [2 d x]}{14784 d}+$$

$$\frac{(9005 A+8580 B+7852 C) \cos [3 c] \sin [3 d x]}{74880 d}+\frac{(59 A+49 B+33 C) \cos [4 c] \sin [4 d x]}{1232 d}+$$

$$\left.\frac{(245 A+156 B+52 C) \cos [5 c] \sin [5 d x]}{14976 d}+\frac{(3 A+B) \cos [6 c] \sin [6 d x]}{704 d}+\frac{A \cos [7 c] \sin [7 d x]}{1664 d}\right)$$

■ **Problem 558: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{5 / 2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{a+a \operatorname{Sec}[c+d x]} d x$$

Optimal (type 4, 250 leaves, 9 steps):

$$\frac{3(5 A-5 B+7 C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{5 a d}$$

$$-\frac{(3 A-5 B+5 C) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 a d}+\frac{3(5 A-5 B+7 C) \sqrt{\operatorname{Sec}[c+d x]} \sin [c+d x]}{5 a d}$$

$$-\frac{(3 A-5 B+5 C) \operatorname{Sec}[c+d x]^{3 / 2} \sin [c+d x]}{3 a d}+\frac{(5 A-5 B+7 C) \operatorname{Sec}[c+d x]^{5 / 2} \sin [c+d x]}{5 a d}-\frac{(A-B+C) \operatorname{Sec}[c+d x]^{7 / 2} \sin [c+d x]}{d(a+a \operatorname{Sec}[c+d x])}$$

Result (type 5, 1278 leaves):

$$-\left(3 \sqrt{2} A e^{-i(2 c+d x)} \sqrt{\frac{e^{i(c+d x)}}{1+e^{2 i(c+d x)}}} \cos \left[\frac{c}{2}+\frac{d x}{2}\right]^2 \cos [c+d x] \operatorname{Csc}\left[\frac{c}{2}\right]\right.$$

$$\left.\left(1+e^{2 i(c+d x)}+(-1+e^{2 i c}) \sqrt{1+e^{2 i(c+d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i(c+d x)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)\right) /$$

$$\begin{aligned}
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \sec[c + dx])) + \left(3\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \sec[c + dx])) - \\
& \left(21\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& (5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \sec[c + dx])) - \\
& \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right) + \\
& \left(10B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right) - \\
& \left(10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. \left(\frac{6(5A - 5B + 7C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \sin[dx]}{5d} \right. \right. \\
& \left. \left. + \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (3C \sin[c] + 5B \sin[dx] - 5C \sin[dx])}{15d} - \frac{4(-2B + 2C + 3A \cos[c] - 5B \cos[c] + 5C \cos[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& \left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right)
\end{aligned}$$

■ **Problem 559: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 4, 205 leaves, 8 steps) :

$$\frac{(A - 3B + 3C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{ad} + \frac{(3A - 3B + 5C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3ad} - \frac{(A - 3B + 3C) \sqrt{\sec[c + dx]} \sin[c + dx]}{ad} + \frac{(3A - 3B + 5C) \sec[c + dx]^{3/2} \sin[c + dx]}{3ad} - \frac{(A - B + C) \sec[c + dx]^{5/2} \sin[c + dx]}{d(a + a \sec[c + dx])}$$

Result (type 5, 1229 leaves) :

$$\left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) - \left(3\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \left(3\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \\ \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx])) - \\ \left(2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\ (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx])) + \\ \left(10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) /$$

$$\left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{\sec [c + d x]} (a + a \sec [c + d x]) \right) + \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\ \left. - \frac{2 (A - 3 B + 3 C) \cos [d x] \operatorname{Csc} \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right]}{d} + \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[\frac{d x}{2} \right] - B \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{d} + \right. \\ \left. \frac{8 C \sec [c] \sec [c + d x] \sin [d x]}{3 d} + \frac{4 (2 C + 3 A \cos [c] - 3 B \cos [c] + 5 C \cos [c]) \sec [c] \tan \left[\frac{c}{2} \right]}{3 d} \right) \Bigg) / \\ \left((A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{\sec [c + d x]} (a + a \sec [c + d x]) \right)$$

- **Problem 560: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2)}{a + a \sec [c + d x]} dx$$

Optimal (type 4, 162 leaves, 7 steps):

$$- \frac{(A - B + 3 C) \sqrt{\cos [c + d x]} \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{a d} + \frac{(A + B - C) \sqrt{\cos [c + d x]} \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right] \sqrt{\sec [c + d x]}}{a d} + \\ \frac{(A - B + 3 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{a d} - \frac{(A - B + C) \sec [c + d x]^{3/2} \sin [c + d x]}{d (a + a \sec [c + d x])}$$

Result (type 5, 1190 leaves):

$$\begin{aligned}
& - \left(\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])) + \left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])) - \\
& \left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])) + \\
& \left(2 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx])) + \\
& \left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx])) - \\
& \left(2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx])) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(\frac{2(A - B + 3C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{4(A - B + C) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& ((A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]))
\end{aligned}$$

- **Problem 561: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\frac{(3A - B + C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} - \frac{(A - B - C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a d} - \frac{(A - B + C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d (a + a \operatorname{Sec}[c + d x])}$$

Result (type 5, 1208 leaves):

$$\begin{aligned} & \left(3 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) - \left(\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) + \left(\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) - \\ & \left(2A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\ & (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])) + \\ & \left(2B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \end{aligned}$$

$$\begin{aligned}
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right) + \\
& \left(2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{2(2A - B + C + A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{d} + \right. \right. \\
& \quad \left. \left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8A \cos[c] \sin[dx]}{d} + \frac{4(A - B + C) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right)
\end{aligned}$$

■ **Problem 562: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\sec[c + dx]^{3/2} (a + a \sec[c + dx])} dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$\begin{aligned}
& -\frac{(3A - 3B + C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{ad} + \\
& \frac{(5A - 3B + 3C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3ad} + \frac{(5A - 3B + 3C) \sin[c + dx]}{3ad \sqrt{\sec[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])}
\end{aligned}$$

Result (type 5, 1256 leaves):

$$\begin{aligned}
& -\left(3\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx]) \right) + \left(3\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx]) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) + \\
& \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(3 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) - \\
& \left(2 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) + \\
& \left(2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(\frac{2(2A - 2B + C + A \cos[2c] - B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{4A \cos[2dx] \sin[2c]}{3d} - \right. \right. \\
& \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \frac{8(A - B) \cos[c] \sin[dx]}{d} + \frac{4A \cos[2c] \sin[2dx]}{3d} - \frac{4(A - B + C) \tan\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \left((A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right)
\end{aligned}$$

■ **Problem 563: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{\operatorname{Sec}[c+dx]^{5/2} (a + a \operatorname{Sec}[c+dx])} dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$\begin{aligned}
& \frac{3(7A - 5B + 5C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5ad} - \\
& \frac{(5A - 5B + 3C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3ad} + \\
& \frac{(7A - 5B + 5C) \sin[c+dx]}{5ad \operatorname{Sec}[c+dx]^{3/2}} - \frac{(5A - 5B + 3C) \sin[c+dx]}{3ad \sqrt{\operatorname{Sec}[c+dx]}} - \frac{(A - B + C) \sin[c+dx]}{d \operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])}
\end{aligned}$$

Result (type 5, 1321 leaves) :

$$\begin{aligned}
& \left(21 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) - \left(3 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) + \left(3 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])) - \\
& \left(10 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) + \\
& \left(10 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) - \\
& \left(2 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \left(d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c+dx]} (a + a \operatorname{Sec}[c+dx]) \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-\frac{(51A - 40B + 40C + 33A \cos[2c] - 20B \cos[2c] + 20C \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{10d} \right. \right. \\
& \quad \left. \left. + \frac{4(A - B) \cos[2dx] \sin[2c]}{3d} + \frac{2A \cos[3dx] \sin[3c]}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right) \right)
\end{aligned}$$

$$\left. \left(\frac{2(33A - 20B + 20C) \cos[c] \sin[dx]}{5d} - \frac{4(A - B) \cos[2c] \sin[2dx]}{3d} + \frac{2A \cos[3c] \sin[3dx]}{5d} + \frac{4(A - B + C) \tan\left[\frac{c}{2}\right]}{d} \right) \right/$$

$$\left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + a \sec[c + dx]) \right)$$

■ **Problem 564: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\sec[c + dx]^{7/2} (a + a \sec[c + dx])} dx$$

Optimal (type 4, 250 leaves, 9 steps):

$$\begin{aligned} & - \frac{3(7A - 7B + 5C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{5ad} + \\ & \frac{5(9A - 7B + 7C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{21ad} + \frac{(9A - 7B + 7C) \sin[c + dx]}{7ad \sec[c + dx]^{5/2}} - \\ & \frac{(7A - 7B + 5C) \sin[c + dx]}{5ad \sec[c + dx]^{3/2}} + \frac{5(9A - 7B + 7C) \sin[c + dx]}{21ad \sqrt{\sec[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \sec[c + dx]^{5/2} (a + a \sec[c + dx])} \end{aligned}$$

Result (type 5, 1377 leaves):

$$\begin{aligned} & - \left(21\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\ & (5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \left(21\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\ & \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\ & (5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) - \\ & \left(3\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right) \end{aligned}$$

$$\begin{aligned}
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \Bigg) / \\
& (d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])) + \\
& \left(30 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\
& \left(7 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx]) \right) - \\
& \left(10 B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\
& \left(3 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx]) \right) + \\
& \left(10 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) / \\
& \left(3 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx]) \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\frac{(51A - 51B + 40C + 33A \operatorname{Cos}[2c] - 33B \operatorname{Cos}[2c] + 20C \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{10d} + \right. \right. \\
& \quad \frac{2(27A - 14B + 14C) \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{21d} - \frac{2(A - B) \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{5d} + \frac{A \operatorname{Cos}[4dx] \operatorname{Sin}[4c]}{7d} - \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{d} - \frac{2(33A - 33B + 20C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{5d} + \\
& \quad \left. \left. \frac{2(27A - 14B + 14C) \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{21d} - \frac{2(A - B) \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{5d} + \frac{A \operatorname{Cos}[4c] \operatorname{Sin}[4dx]}{7d} - \frac{4(A - B + C) \operatorname{Tan}\left[\frac{c}{2}\right]}{d} \right) \right) / \\
& \left((A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx]) \right)
\end{aligned}$$

- **Problem 565: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$\begin{aligned}
& \frac{(A - 4B + 7C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{a^2 d} + \\
& \frac{(2A - 5B + 10C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{3a^2 d} - \frac{(A - 4B + 7C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{a^2 d} + \\
& \frac{(2A - 5B + 10C) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{3a^2 d} - \frac{(A - 4B + 7C) \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{3a^2 d (1 + \operatorname{Sec}[c + dx])} - \frac{(A - B + C) \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{3d (a + a \operatorname{Sec}[c + dx])^2}
\end{aligned}$$

Result (type 5, 1311 leaves):

$$\begin{aligned}
& \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \left(8 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(14 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(8 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(40 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \left(-\frac{4(A - 4B + 7C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \right. \\
& \quad \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (2A \sin\left[\frac{dx}{2}\right] - 5B \sin\left[\frac{dx}{2}\right] + 8C \sin\left[\frac{dx}{2}\right])}{3d} \right) +
\end{aligned}$$

$$\left. \left(\frac{16 C \operatorname{Sec}[c] \operatorname{Sec}[c+d x] \operatorname{Sin}[d x]}{3 d} + \frac{8 (2 C + 2 A \operatorname{Cos}[c] - 5 B \operatorname{Cos}[c] + 10 C \operatorname{Cos}[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} + \frac{4 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) /$$

$$\left((A + 2 C + 2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c+d x])^2 \right)$$

- **Problem 566: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^{3/2} (A + B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2)}{(a + a \operatorname{Sec}[c+d x])^2} dx$$

Optimal (type 4, 207 leaves, 8 steps):

$$\frac{(B - 4 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]} + (A + 2 B - 5 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\operatorname{Sec}[c+d x]}}{a^2 d} - \frac{(B - 4 C) \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{a^2 d} + \frac{(A + 2 B - 5 C) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 a^2 d (1 + \operatorname{Sec}[c+d x])} - \frac{(A - B + C) \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{3 d (a + a \operatorname{Sec}[c+d x])^2}$$

Result (type 5, 1072 leaves):

$$\begin{aligned}
& \left(2\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \left(8\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(8B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \left(\frac{4(-B + 4C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \right. \\
& \quad \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right] - 5C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \\
& \quad \left. \frac{8(A + 2B - 5C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) / \left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)
\end{aligned}$$

■ **Problem 567: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 173 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A - C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \\
& \frac{(2A + B + 2C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} + \frac{(A - C) \sqrt{\sec[c + dx]} \sin[c + dx]}{a^2 d (1 + \sec[c + dx])} - \frac{(A - B + C) \sec[c + dx]^{3/2} \sin[c + dx]}{3d (a + a \sec[c + dx])^2}
\end{aligned}$$

Result (type 5, 1073 leaves):

$$\begin{aligned}
& - \left(2\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(2\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(8A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(4B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(8C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \quad \left(\frac{4(A - C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (4A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] - 2C \sin\left[\frac{dx}{2}\right])}{3d} + \right. \\
& \quad \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{8(4A - B - 2C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} + \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& ((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2)
\end{aligned}$$

■ **Problem 568: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 184 leaves, 7 steps):

$$\frac{(4A - B) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]} - (5A - 2B - C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} - \frac{(5A - 2B - C) \sqrt{\sec[c + dx]} \sin[c + dx] - (A - B + C) \sqrt{\sec[c + dx]} \sin[c + dx]}{3 a^2 d (1 + \sec[c + dx])} - \frac{(A - B + C) \sqrt{\sec[c + dx]} \sin[c + dx]}{3 d (a + a \sec[c + dx])^2}$$

Result (type 5, 1090 leaves):

$$\begin{aligned}
& \left(8 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2) - \left(2 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2) - \\
& \left(20 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(8 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(4 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(-\frac{4(3A - B + A \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \right. \right. \\
& \quad \left. \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (7A \sin\left[\frac{dx}{2}\right] - 4B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \right. \\
& \quad \left. \left. \frac{16 A \cos[c] \sin[dx]}{d} + \frac{8(7A - 4B + C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& ((A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])^2)
\end{aligned}$$

■ **Problem 569:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{\operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 220 leaves, 8 steps) :

$$\begin{aligned}
 & - \frac{(7A - 4B + C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{a^2 d} + \\
 & \frac{(10A - 5B + 2C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{3a^2 d} + \frac{(10A - 5B + 2C) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]}} - \\
 & \frac{(7A - 4B + C) \sin[c + dx]}{3a^2 d \sqrt{\sec[c + dx]} (1 + \sec[c + dx])} - \frac{(A - B + C) \sin[c + dx]}{3d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2}
 \end{aligned}$$

Result (type 5, 1346 leaves) :

$$\begin{aligned}
 & - \left(14 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
 & \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
 & \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2) + \left(8 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
 & \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
 & \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2) - \left(2 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
 & \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
 & \quad (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2) + \\
 & \quad \left(40 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
 & \quad (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2) - \\
 & \quad \left(20 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
 & \quad (3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2) +
 \end{aligned}$$

$$\left(8 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) /$$

$$(3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^2) +$$

$$\frac{1}{(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)$$

$$\left(\frac{4(5A-3B+C+2A \cos[2c] - B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{d} + \frac{8A \cos[2dx] \sin[2c]}{3d} + \right.$$

$$\left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (10A \sin\left[\frac{dx}{2}\right] - 7B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right])}{3d} \right.$$

$$\left. \frac{16(2A-B) \cos[c] \sin[dx]}{d} + \frac{8A \cos[2c] \sin[2dx]}{3d} - \frac{8(10A-7B+4C) \tan\left[\frac{c}{2}\right]}{3d} + \frac{4(A-B+C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)$$

■ **Problem 570: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{\operatorname{Sec}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 4, 254 leaves, 9 steps):

$$\frac{(56A-35B+20C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5a^2d} -$$

$$\frac{5(3A-2B+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{3a^2d} + \frac{(56A-35B+20C) \sin[c+dx]}{15a^2d \operatorname{Sec}[c+dx]^{3/2}} -$$

$$\frac{5(3A-2B+C) \sin[c+dx]}{3a^2d \sqrt{\operatorname{Sec}[c+dx]}} - \frac{(3A-2B+C) \sin[c+dx]}{a^2d \operatorname{Sec}[c+dx]^{3/2} (1+\operatorname{Sec}[c+dx])} - \frac{(A-B+C) \sin[c+dx]}{3d \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^2}$$

Result (type 5, 1408 leaves):

$$\left(112 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\left. \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$(5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^2) - \left(14 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right.$$

$$\begin{aligned}
& \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \Bigg/ \\
& (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \left(8\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \\
& (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) \Bigg/ \\
& (d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \left(40B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) \Bigg/ \\
& (3d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) - \\
& \left(20C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}[c] \right) \Bigg/ \\
& (3d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2) + \\
& \frac{1}{(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(-\frac{(151A - 100B + 60C + 73A \operatorname{Cos}[2c] - 40B \operatorname{Cos}[2c] + 20C \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \right. \\
& \frac{8(2A - B) \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{3d} + \frac{4A \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3d} + \\
& \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (13A \operatorname{Sin}\left[\frac{dx}{2}\right] - 10B \operatorname{Sin}\left[\frac{dx}{2}\right] + 7C \operatorname{Sin}\left[\frac{dx}{2}\right])}{3d} + \frac{4(73A - 40B + 20C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{5d} - \\
& \left. \frac{8(2A - B) \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{3d} + \frac{4A \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{5d} + \frac{8(13A - 10B + 7C) \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} - \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

■ **Problem 571: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{7/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 308 leaves, 10 steps):

$$\frac{(9A - 49B + 119C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10a^3 d} +$$

$$\frac{(3A - 13B + 33C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6a^3 d} - \frac{(9A - 49B + 119C) \sqrt{\sec[c + dx]} \sin[c + dx]}{10a^3 d} +$$

$$\frac{(3A - 13B + 33C) \sec[c + dx]^{3/2} \sin[c + dx]}{6a^3 d} - \frac{(A - B + C) \sec[c + dx]^{9/2} \sin[c + dx]}{5d(a + a \sec[c + dx])^3} +$$

$$\frac{(B - 2C) \sec[c + dx]^{7/2} \sin[c + dx]}{3ad(a + a \sec[c + dx])^2} - \frac{(9A - 49B + 119C) \sec[c + dx]^{5/2} \sin[c + dx]}{30d(a^3 + a^3 \sec[c + dx])^2}$$

Result (type 5, 1432 leaves):

$$\left(18\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right.$$

$$\left. \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3) - \left(98\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \right.$$

$$\left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right.$$

$$\left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$(5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3) + \left(238\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right.$$

$$\left. \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right)$$

$$\begin{aligned}
& \left. \frac{(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(5d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \operatorname{Sec}[c + dx])^3)} \right) + \\
& \left(\frac{4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c]}{(d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \operatorname{Sec}[c + dx])^3)} \right) - \\
& \left(\frac{52B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c]}{(3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \operatorname{Sec}[c + dx])^3)} \right) + \\
& \left(\frac{44C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin[c]}{(d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \operatorname{Sec}[c + dx])^3)} \right) \Big/ \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + a \operatorname{Sec}[c + dx])^3} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4(9A - 49B + 119C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} + \right. \\
& \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3A \sin\left[\frac{dx}{2}\right] - 8B \sin\left[\frac{dx}{2}\right] + 13C \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \sin\left[\frac{dx}{2}\right] - 13B \sin\left[\frac{dx}{2}\right] + 29C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{32C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{3d} + \\
& \left. \frac{8(4C + 3A \cos[c] - 13B \cos[c] + 33C \cos[c]) \operatorname{Sec}[c] \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} + \frac{8(3A - 8B + 13C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 572: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 269 leaves, 9 steps):

$$\begin{aligned}
& \frac{(A + 9B - 49C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{10a^3d} + \\
& \frac{(A + 3B - 13C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\operatorname{Sec}[c + dx]}}{6a^3d} - \frac{(A + 9B - 49C) \sqrt{\operatorname{Sec}[c + dx]} \sin[c + dx]}{10a^3d} - \\
& \frac{(A - B + C) \operatorname{Sec}[c + dx]^{7/2} \sin[c + dx]}{5d(a + a \operatorname{Sec}[c + dx])^3} + \frac{(2A + 3B - 8C) \operatorname{Sec}[c + dx]^{5/2} \sin[c + dx]}{15ad(a + a \operatorname{Sec}[c + dx])^2} + \frac{(A + 3B - 13C) \operatorname{Sec}[c + dx]^{3/2} \sin[c + dx]}{6d(a^3 + a^3 \operatorname{Sec}[c + dx])}
\end{aligned}$$

Result (type 5, 1400 leaves) :

$$\begin{aligned}
& \left(2 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(18 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \right. \\
& \quad \left. \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(98 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right. \\
& \quad \left. \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(4B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(52C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& (3d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)
\end{aligned}$$

$$\left(-\frac{4(A+9B-49C)\cos\left[\frac{c}{2}\right]\csc\left[\frac{c}{2}\right]\sec\left[\frac{c}{2}\right]}{5d} + \frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right](A\sin\left[\frac{dx}{2}\right] + 3B\sin\left[\frac{dx}{2}\right] - 13C\sin\left[\frac{dx}{2}\right])}{3d} + \frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3(2A\sin\left[\frac{dx}{2}\right] + 3B\sin\left[\frac{dx}{2}\right] - 8C\sin\left[\frac{dx}{2}\right])}{15d} - \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5(A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right])}{5d} - \frac{8(-A-3B+13C)\tan\left[\frac{c}{2}\right]}{3d} + \frac{8(2A+3B-8C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2\tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A-B+C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4\tan\left[\frac{c}{2}\right]}{5d} \right)$$

■ **Problem 573: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2}(A+B\sec[c+dx]+C\sec[c+dx]^2)}{(a+a\sec[c+dx])^3} dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$-\frac{(A-B-9C)\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{10a^3d} + \frac{(A+B+3C)\sqrt{\cos[c+dx]}\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]\sqrt{\sec[c+dx]}}{6a^3d} - \frac{(A-B+C)\sec[c+dx]^{5/2}\sin[c+dx]}{5d(a+a\sec[c+dx])^3} + \frac{(4A+B-6C)\sec[c+dx]^{3/2}\sin[c+dx]}{15ad(a+a\sec[c+dx])^2} + \frac{(A-B-9C)\sqrt{\sec[c+dx]}\sin[c+dx]}{10d(a^3+a^3\sec[c+dx])}$$

Result (type 5, 1395 leaves):

$$-\left(2\sqrt{2}Ae^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6\csc\left[\frac{c}{2}\right] \right. \\ \left. \left(1+e^{2i(c+dx)} + (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right]\sec[c+dx] \right. \\ \left. (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \left(5d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx])^3 \right) + \left(2\sqrt{2}Be^{-i(2c+dx)} \right. \\ \left. \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6\csc\left[\frac{c}{2}\right] \left(1+e^{2i(c+dx)} + (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ \left. \sec\left[\frac{c}{2}\right]\sec[c+dx](A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \left(5d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx])^3 \right) +$$

$$\begin{aligned}
& \left(18 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right]\right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3 \right) + \\
& \quad \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \quad \left(3d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3 \right) + \\
& \quad \left(4B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \quad \left(3d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3 \right) + \\
& \quad \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) / \\
& \quad \left(d(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3 \right) + \\
& \quad \frac{1}{(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \\
& \quad \left(\frac{4(A - B - 9C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (7A \sin\left[\frac{dx}{2}\right] - 2B \sin\left[\frac{dx}{2}\right] - 3C \sin\left[\frac{dx}{2}\right])}{15d} \right) + \\
& \quad \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right] + 3C \sin\left[\frac{dx}{2}\right])}{3d} + \\
& \quad \left(\frac{8(A + B + 3C) \tan\left[\frac{c}{2}\right]}{3d} - \frac{8(7A - 2B - 3C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

- **Problem 574: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a + a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 231 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(9A+B-C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{10a^3d} + \frac{(3A+B+C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{6a^3d} \\
& - \frac{(A-B+C) \sec[c+dx]^{3/2} \sin[c+dx]}{5d(a+a\sec[c+dx])^3} + \frac{(6A-B-4C) \sqrt{\sec[c+dx]} \sin[c+dx]}{15ad(a+a\sec[c+dx])^2} + \frac{(3A+B+C) \sqrt{\sec[c+dx]} \sin[c+dx]}{6d(a^3+a^3\sec[c+dx])}
\end{aligned}$$

Result (type 5, 1401 leaves):

$$\begin{aligned}
& - \left(18\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c+dx] \right. \\
& \quad \left. (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \left(5d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \sec[c+dx])^3 - \left(2\sqrt{2} B e^{-i(2c+dx)} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \right. \\
& \quad \left. \left. \sec\left[\frac{c}{2}\right] \sec[c+dx] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) \right) / \\
& \quad \left(5d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \sec[c+dx])^3 + \right. \\
& \quad \left. \left(2\sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \right. \\
& \quad \left. \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c+dx] \right. \right. \\
& \quad \left. \left. (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) \right) / \left(5d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \sec[c+dx])^3 + \right. \\
& \quad \left. \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{3/2} (A + B \sec[c+dx] + C \sec[c+dx]^2) \sin[c] \right) \right) / \\
& \quad \left(d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \sec[c+dx])^3 + \right. \\
& \quad \left. \left(4B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c+dx]^{3/2} (A + B \sec[c+dx] + C \sec[c+dx]^2) \sin[c] \right) \right) / \\
& \quad \left(3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + a \sec[c+dx])^3 + \right.
\end{aligned}$$

$$\left(4 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sin[c] \right) /$$

$$(3d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3) +$$

$$\frac{1}{(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)$$

$$\left(\frac{4(9A+B-C) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (9A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{3d} - \right.$$

$$\frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (12A \sin\left[\frac{dx}{2}\right] - 7B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right])}{15d} -$$

$$\left. \frac{8(9A-B-C) \tan\left[\frac{c}{2}\right]}{3d} + \frac{8(12A-7B+2C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A-B+C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)$$

■ **Problem 575: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{\sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 4, 241 leaves, 8 steps):

$$\frac{(49A-9B-C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{10a^3d} - \frac{(13A-3B-C) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{6a^3d}$$

$$\frac{(A-B+C) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{5d(a+a \operatorname{Sec}[c+dx])^3} - \frac{(8A-3B-2C) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{15ad(a+a \operatorname{Sec}[c+dx])^2} - \frac{(13A-3B-C) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{6d(a^3+a^3 \operatorname{Sec}[c+dx])}$$

Result (type 5, 1419 leaves):

$$\left(98\sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right.$$

$$\operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right)$$

$$\left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$(5d(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \operatorname{Sec}[c+dx])^3) - \left(18\sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \right.$$

Optimal (type 4, 274 leaves, 9 steps) :

$$\begin{aligned}
 & - \frac{(119 A - 49 B + 9 C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} + \\
 & \frac{(33 A - 13 B + 3 C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6 a^3 d} + \frac{(33 A - 13 B + 3 C) \sin[c + dx]}{6 a^3 d \sqrt{\sec[c + dx]}} - \\
 & \frac{(A - B + C) \sin[c + dx]}{5 d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^3} - \frac{(2 A - B) \sin[c + dx]}{3 a d \sqrt{\sec[c + dx]} (a + a \sec[c + dx])^2} - \frac{(119 A - 49 B + 9 C) \sin[c + dx]}{30 d \sqrt{\sec[c + dx]} (a^3 + a^3 \sec[c + dx])}
 \end{aligned}$$

Result (type 5, 1467 leaves) :

$$\begin{aligned}
 & - \left(238 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
 & \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] \right. \\
 & \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \left(5 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) (a + a \sec[c + dx])^3 \right) + \left(98 \sqrt{2} B e^{-i(2c+dx)} \right. \\
 & \quad \left. \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
 & \quad \left. \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
 & \quad \left(5 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) (a + a \sec[c + dx])^3 \right) - \\
 & \left(18 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
 & \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \sec\left[\frac{c}{2}\right] \sec[c + dx] \right. \\
 & \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \left(5 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) (a + a \sec[c + dx])^3 \right) + \\
 & \left(44 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right) - \\
& \left(52 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(3 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right) + \\
& \left(4 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\cos[c + dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sec\left[\frac{c}{2}\right] \sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin[c] \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right) + \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(\frac{4 (89A - 39B + 9C + 30A \cos[2c] - 10B \cos[2c]) \cos[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]}{5d} + \frac{16A \cos[2dx] \sin[2c]}{3d} - \right. \\
& \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (43A \sin\left[\frac{dx}{2}\right] - 23B \sin\left[\frac{dx}{2}\right] + 9C \sin\left[\frac{dx}{2}\right])}{3d} + \\
& \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (22A \sin\left[\frac{dx}{2}\right] - 17B \sin\left[\frac{dx}{2}\right] + 12C \sin\left[\frac{dx}{2}\right])}{15d} - \frac{32 (3A - B) \cos[c] \sin[dx]}{d} + \frac{16A \cos[2c] \sin[2dx]}{3d} - \\
& \left. \frac{8 (43A - 23B + 9C) \tan\left[\frac{c}{2}\right]}{3d} + \frac{8 (22A - 17B + 12C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4 (A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)
\end{aligned}$$

■ **Problem 577: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\sec[c + dx]^{5/2} (a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 313 leaves, 10 steps):

$$\begin{aligned}
& \frac{7 (33A - 17B + 7C) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{10 a^3 d} - \\
& \frac{(63A - 33B + 13C) \sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right] \sqrt{\sec[c + dx]}}{6 a^3 d} + \frac{7 (33A - 17B + 7C) \sin[c + dx]}{30 a^3 d \sec[c + dx]^{3/2}} - \frac{(63A - 33B + 13C) \sin[c + dx]}{6 a^3 d \sqrt{\sec[c + dx]}} - \\
& \frac{(A - B + C) \sin[c + dx]}{5 d \sec[c + dx]^{3/2} (a + a \sec[c + dx])^3} - \frac{(12A - 7B + 2C) \sin[c + dx]}{15 a d \sec[c + dx]^{3/2} (a + a \sec[c + dx])^2} - \frac{(63A - 33B + 13C) \sin[c + dx]}{10 d \sec[c + dx]^{3/2} (a^3 + a^3 \sec[c + dx])}
\end{aligned}$$

Result (type 5, 1525 leaves):

$$\left(462 \sqrt{2} A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right)$$

$$\begin{aligned}
& \left. \left(\operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \right) / \\
& (5d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \left(238 \sqrt{2} B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \right. \\
& \left. \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \left(98 \sqrt{2} C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1 + e^{2i(c+dx)}}} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right. \\
& \left. \operatorname{Csc}\left[\frac{c}{2}\right] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] \right. \\
& \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(84 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sin}[c] \right) / \\
& (d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \left(44 B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sin}[c] \right) / \\
& (d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) - \\
& \left(52 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sin}[c] \right) / \\
& (3d(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3) + \\
& \frac{1}{(A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a + a \operatorname{Sec}[c+dx])^3} \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[c+dx]^{3/2} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \\
& \left(-\frac{2(329A - 178B + 78C + 133A \operatorname{Cos}[2c] - 60B \operatorname{Cos}[2c] + 20C \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5d} \right) -
\end{aligned}$$

$$\frac{16(3A-B)\cos[2dx]\sin[2c]}{3d} + \frac{8A\cos[3dx]\sin[3c]}{5d} + \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right])}{5d} -$$

$$\frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (27A\sin\left[\frac{dx}{2}\right] - 22B\sin\left[\frac{dx}{2}\right] + 17C\sin\left[\frac{dx}{2}\right])}{15d} + \frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right] (69A\sin\left[\frac{dx}{2}\right] - 43B\sin\left[\frac{dx}{2}\right] + 23C\sin\left[\frac{dx}{2}\right])}{3d} +$$

$$\frac{8(133A - 60B + 20C)\cos[c]\sin[dx]}{5d} - \frac{16(3A-B)\cos[2c]\sin[2dx]}{3d} + \frac{8A\cos[3c]\sin[3dx]}{5d} +$$

$$\left. \frac{8(69A - 43B + 23C)\tan\left[\frac{c}{2}\right]}{3d} - \frac{8(27A - 22B + 17C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A-B+C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right)$$

- **Problem 578: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c+dx]^{5/2} \sqrt{a+a\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) dx$$

Optimal (type 3, 227 leaves, 6 steps):

$$\frac{\sqrt{a} (48A + 40B + 35C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{64d} + \frac{a(48A + 40B + 35C) \sec[c+dx]^{3/2} \sin[c+dx]}{64d \sqrt{a+a\sec[c+dx]}}$$

$$\frac{a(48A + 40B + 35C) \sec[c+dx]^{5/2} \sin[c+dx]}{96d \sqrt{a+a\sec[c+dx]}} + \frac{a(8B+C) \sec[c+dx]^{7/2} \sin[c+dx]}{24d \sqrt{a+a\sec[c+dx]}} + \frac{C \sec[c+dx]^{7/2} \sqrt{a+a\sec[c+dx]} \sin[c+dx]}{4d}$$

Result (type 3, 2038 leaves):

$$-\left(\frac{1}{256} + \frac{i}{256}\right) \left((-1+i) + \sqrt{2}\right) \left((144+48i)A + 48\sqrt{2}A + (120+40i)B + 40\sqrt{2}B + (105+35i)C + 35\sqrt{2}C\right)$$

$$\operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\left.(A+B\sec[c+dx]+C\sec[c+dx]^2)\right) / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sec[c+dx]^{5/2}\right) -$$

$$\left(\frac{1}{256} - \frac{i}{256}\right) \left((1+i) + \sqrt{2}\right) \left((-144+48i)A + 48\sqrt{2}A - (120-40i)B + 40\sqrt{2}B - (105-35i)C + 35\sqrt{2}C\right)$$

$$\operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]}\right] \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\begin{aligned}
& \left. \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \right/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left((96A + 48i\sqrt{2}A + 80B + 40i\sqrt{2}B + 70C + 35i\sqrt{2}C) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \left/ \left(128 (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \right) - \right. \\
& \left(\left(\frac{1}{512} - \frac{i}{512} \right) \left((-1 + i) + \sqrt{2} \right) \left((144 + 48i)A + 48\sqrt{2}A + (120 + 40i)B + 40\sqrt{2}B + (105 + 35i)C + 35\sqrt{2}C \right) \right. \\
& \left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \left/ \right. \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left(\left(\frac{1}{512} + \frac{i}{512} \right) \left((1 + i) + \sqrt{2} \right) \left((-144 + 48i)A + 48\sqrt{2}A - (120 - 40i)B + 40\sqrt{2}B - (105 - 35i)C + 35\sqrt{2}C \right) \right. \\
& \left. \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \left/ \right. \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \frac{(8B + 7C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{48d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{(48A + 40B + 35C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{64d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} + \\
& \frac{(-8B - 7C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{48d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \frac{(-48A - 40B - 35C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{64d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(16A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 8B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 11C \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right) \right) \left/ \right. \\
& \left(32d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^2 \right) +
\end{aligned}$$

$$\begin{aligned} & \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \right. \\ & \quad \left. \left(16A\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 8B\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + 11C\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\ & \left(32d(A+2C+2B\operatorname{Cos}[c+dx]+A\operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) + \\ & \frac{c\sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8d(A+2C+2B\operatorname{Cos}[c+dx]+A\operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4} + \\ & \frac{c\sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{8d(A+2C+2B\operatorname{Cos}[c+dx]+A\operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4} \end{aligned}$$

■ **Problem 579: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+a\operatorname{Sec}[c+dx]} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 179 leaves, 5 steps):

$$\begin{aligned} & \frac{\sqrt{a} (8A+6B+5C) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{8d} + \frac{a(8A+6B+5C)\operatorname{Sec}[c+dx]^{3/2}\operatorname{Sin}[c+dx]}{8d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \\ & \frac{a(6B+C)\operatorname{Sec}[c+dx]^{5/2}\operatorname{Sin}[c+dx]}{12d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{C\operatorname{Sec}[c+dx]^{5/2}\sqrt{a+a\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{3d} \end{aligned}$$

Result (type 3, 1772 leaves):

$$\begin{aligned} & - \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((-1+i) + \sqrt{2} \right) \left((24+8i)A + 8\sqrt{2}A + (18+6i)B + 6\sqrt{2}B + (15+5i)C + 5\sqrt{2}C \right) \right. \\ & \quad \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right. \\ & \quad \left. (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\operatorname{Cos}[c+dx]+A\operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \right) - \\ & \left(\left(\frac{1}{32} - \frac{i}{32} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A - (18-6i)B + 6\sqrt{2}B - (15-5i)C + 5\sqrt{2}C \right) \right) \end{aligned}$$

$$\begin{aligned}
& \text{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} \\
& \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \right) + \\
& \left((16A + 8i\sqrt{2}A + 12B + 6i\sqrt{2}B + 10C + 5i\sqrt{2}C) \log \left[\sqrt{2} + 2 \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} \right. \\
& \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \left(16 (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \right) - \\
& \left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A + (18 + 6i)B + 6\sqrt{2}B + (15 + 5i)C + 5\sqrt{2}C \right) \right. \\
& \left. \log \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \right) + \\
& \left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i)A + 8\sqrt{2}A - (18 - 6i)B + 6\sqrt{2}B - (15 - 5i)C + 5\sqrt{2}C \right) \right. \\
& \left. \log \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \right) + \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{6d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(8A + 6B + 5C) \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{8d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)} - \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{6d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(-8A - 6B - 5C) \sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{8d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \left(\sec \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \sec [c + dx])} (A + B \sec [c + dx] + C \sec [c + dx]^2) \left(2B \sin \left[\frac{1}{2} (c + dx) \right] + C \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) \Bigg/ \\
& \left(4d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^2 \right) +
\end{aligned}$$

$$\left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \left(2B\sin\left[\frac{1}{2}(c+dx)\right]+C\sin\left[\frac{1}{2}(c+dx)\right]\right) \right) / \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{5/2} \left(\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right)^2 \right)$$

■ **Problem 580: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a\operatorname{Sec}[c+dx]} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{\sqrt{a} (8A+4B+3C) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a(4B+C)\operatorname{Sec}[c+dx]^{3/2}\sin[c+dx]}{4d\sqrt{a+a\operatorname{Sec}[c+dx]}} + \frac{C\operatorname{Sec}[c+dx]^{3/2}\sqrt{a+a\operatorname{Sec}[c+dx]}\sin[c+dx]}{2d}$$

Result (type 3, 1490 leaves):

$$\begin{aligned} & - \left(\left(\frac{1}{16} + \frac{i}{16} \right) \left((-1+i) + \sqrt{2} \right) \left((24+8i)A + 8\sqrt{2}A + (12+4i)B + 4\sqrt{2}B + (9+3i)C + 3\sqrt{2}C \right) \right. \\ & \quad \left. \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right. \\ & \quad \left. (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \right] / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{5/2} \right) - \\ & \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A - (12-4i)B + 4\sqrt{2}B - (9-3i)C + 3\sqrt{2}C \right) \right. \\ & \quad \left. \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right. \\ & \quad \left. (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \right] / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{5/2} \right) + \\ & \left((16A+8i\sqrt{2}A+8B+4i\sqrt{2}B+6C+3i\sqrt{2}C) \operatorname{Log}\left[\sqrt{2}+2\sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \right. \\ & \quad \left. (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \right] / \left(8(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{5/2} \right) - \\ & \left(\left(\frac{1}{32} - \frac{i}{32} \right) \left((-1+i) + \sqrt{2} \right) \left((24+8i)A + 8\sqrt{2}A + (12+4i)B + 4\sqrt{2}B + (9+3i)C + 3\sqrt{2}C \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left. \text{Log} \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \text{Sec} \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2) \right) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \right) + \\
& \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i) A + 8\sqrt{2} A - (12 - 4i) B + 4\sqrt{2} B - (9 - 3i) C + 3\sqrt{2} C \right) \right. \\
& \left. \text{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \text{Sec} \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2) \right) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \right) + \\
& \frac{(4B + 3C) \text{Sec} \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2)}{4d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{(-4B - 3C) \text{Sec} \left[\frac{1}{2} (c + dx) \right] \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2)}{4d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{C \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2) \text{Tan} \left[\frac{1}{2} (c + dx) \right]}{2d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^2} + \\
& \frac{C \sqrt{a (1 + \text{Sec} [c + dx])} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2) \text{Tan} \left[\frac{1}{2} (c + dx) \right]}{2d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \text{Sec} [c + dx]^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^2}
\end{aligned}$$

- **Problem 581: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \text{Sec} [c + dx]} (A + B \text{Sec} [c + dx] + C \text{Sec} [c + dx]^2)}{\sqrt{\text{Sec} [c + dx]}} dx$$

Optimal (type 3, 119 leaves, 4 steps):

$$\frac{\sqrt{a} (2B + C) \text{ArcSinh} \left[\frac{\sqrt{a} \text{Tan} [c + dx]}{\sqrt{a + a \text{Sec} [c + dx]}} \right]}{d} + \frac{a (2A - C) \sqrt{\text{Sec} [c + dx]} \sin [c + dx]}{d \sqrt{a + a \text{Sec} [c + dx]}} + \frac{C \sqrt{\text{Sec} [c + dx]} \sqrt{a + a \text{Sec} [c + dx]} \sin [c + dx]}{d}$$

Result (type 3, 627 leaves):

$$\frac{1}{d (A + 2C + 2B \cos[c + dx] + A \cos[2(c + dx)]) \sec[c + dx]^{5/2}} \left(\frac{1}{16} + \frac{i}{16} \right) \sqrt{a(1 + \sec[c + dx])} (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\frac{1}{i + \sqrt{2}} 2i\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2B + C) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (-1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + dx)\right]}{(1 + \sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \sec\left[\frac{1}{2}(c + dx)\right] - \right.$$

$$\left. \frac{1}{i + \sqrt{2}} 2\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2B + C) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2}) \sin\left[\frac{1}{4}(c + dx)\right]}{(-1 + \sqrt{2}) \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \sec\left[\frac{1}{2}(c + dx)\right] + \right.$$

$$\left. \frac{(4 + 4i) \left(-2i + \sqrt{2} \right) (2B + C) \operatorname{Log} \left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{1}{2}(c + dx)\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}} \right.$$

$$\left. i\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2B + C) \operatorname{Log} \left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{1}{2}(c + dx)\right] + \right.$$

$$\left. \frac{1}{i + \sqrt{2}} \sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2B + C) \operatorname{Log} \left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c + dx)\right] \right] \sec\left[\frac{1}{2}(c + dx)\right] - \right.$$

$$\left. \frac{(8 - 8i) C \sec\left[\frac{1}{2}(c + dx)\right]^2}{-1 + \tan\left[\frac{1}{2}(c + dx)\right]} + (32 - 32i) A \tan\left[\frac{1}{2}(c + dx)\right] - \frac{(8 - 8i) C \sec\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]} \right)$$

■ **Problem 582: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + a \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sec[c + dx]^{3/2}} dx$$

Optimal (type 3, 120 leaves, 4 steps):

$$\frac{2\sqrt{a} C \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}} \right]}{d} + \frac{2a(A + 3B) \sqrt{\sec[c + dx]} \sin[c + dx]}{3d \sqrt{a + a \sec[c + dx]}} + \frac{2A \sqrt{a + a \sec[c + dx]} \sin[c + dx]}{3d \sqrt{\sec[c + dx]}}$$

Result (type 3, 347 leaves):

$$\frac{1}{12 d \sqrt{\sec[c+d x]}} \sec\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1+\sec[c+d x])}$$

$$\left(-6 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right] - (-1+\sqrt{2}) \sin\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+d x)\right] - \sin\left[\frac{1}{4}(c+d x)\right]}\right] - 6 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right] - (1+\sqrt{2}) \sin\left[\frac{1}{4}(c+d x)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+d x)\right] - \sin\left[\frac{1}{4}(c+d x)\right]}\right] + \right.$$

$$6 \sqrt{2} C \operatorname{Log}\left[\sqrt{2} + 2 \sin\left[\frac{1}{2}(c+d x)\right]\right] - 3 \sqrt{2} C \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+d x)\right]\right] -$$

$$\left. 3 \sqrt{2} C \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+d x)\right]\right] + 12 A \sin\left[\frac{1}{2}(c+d x)\right] + 24 B \sin\left[\frac{1}{2}(c+d x)\right] + 4 A \sin\left[\frac{3}{2}(c+d x)\right] \right)$$

- **Problem 586: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[c+d x]^{5/2} (a+a \sec[c+d x])^{3/2} (A+B \sec[c+d x]+C \sec[c+d x]^2) dx$$

Optimal (type 3, 283 leaves, 7 steps):

$$\frac{a^{3/2} (176 A + 150 B + 133 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+d x]}{\sqrt{a+a \sec[c+d x]}}\right]}{128 d} + \frac{a^2 (176 A + 150 B + 133 C) \sec[c+d x]^{3/2} \sin[c+d x]}{128 d \sqrt{a+a \sec[c+d x]}}$$

$$\frac{a^2 (176 A + 150 B + 133 C) \sec[c+d x]^{5/2} \sin[c+d x]}{192 d \sqrt{a+a \sec[c+d x]}} + \frac{a^2 (80 A + 90 B + 67 C) \sec[c+d x]^{7/2} \sin[c+d x]}{240 d \sqrt{a+a \sec[c+d x]}}$$

$$\frac{a (10 B + 3 C) \sec[c+d x]^{7/2} \sqrt{a+a \sec[c+d x]} \sin[c+d x]}{40 d} + \frac{C \sec[c+d x]^{7/2} (a+a \sec[c+d x])^{3/2} \sin[c+d x]}{5 d}$$

Result (type 3, 2352 leaves):

$$-\left(\left(\frac{1}{1024} + \frac{i}{1024} \right) \left((-1+i) + \sqrt{2} \right) \left((528+176 i) A + 176 \sqrt{2} A + (450+150 i) B + 150 \sqrt{2} B + (399+133 i) C + 133 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right] - \sin\left[\frac{1}{4}(c+d x)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c+d x)\right]}{-\cos\left[\frac{1}{4}(c+d x)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c+d x)\right] - \sin\left[\frac{1}{4}(c+d x)\right]}\right] \sec\left[\frac{1}{2}(c+d x)\right]^3 (a(1+\sec[c+d x]))^{3/2} \right.$$

$$\left. (A+B \sec[c+d x]+C \sec[c+d x]^2) \right) / \left(\sqrt{2} (i+\sqrt{2}) d (A+2 C+2 B \cos[c+d x]+A \cos[2 c+2 d x]) \sec[c+d x]^{7/2} \right) -$$

$$\left(\left(\frac{1}{1024} - \frac{i}{1024} \right) \left((1+i) + \sqrt{2} \right) \left((-528+176 i) A + 176 \sqrt{2} A - (450-150 i) B + 150 \sqrt{2} B - (399-133 i) C + 133 \sqrt{2} C \right) \right)$$

$$\begin{aligned}
& \text{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} \\
& \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \right) + \\
& \left((352A + 176i\sqrt{2}A + 300B + 150i\sqrt{2}B + 266C + 133i\sqrt{2}C) \log \left[\sqrt{2} + 2 \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} \right. \\
& \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \left(512 (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \right) - \\
& \left(\left(\frac{1}{2048} - \frac{i}{2048} \right) \left((-1 + i) + \sqrt{2} \right) \left((528 + 176i)A + 176\sqrt{2}A + (450 + 150i)B + 150\sqrt{2}B + (399 + 133i)C + 133\sqrt{2}C \right) \right. \\
& \left. \log \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \right) + \\
& \left(\left(\frac{1}{2048} + \frac{i}{2048} \right) \left((1 + i) + \sqrt{2} \right) \left((-528 + 176i)A + 176\sqrt{2}A - (450 - 150i)B + 150\sqrt{2}B - (399 - 133i)C + 133\sqrt{2}C \right) \right. \\
& \left. \log \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \Bigg/ \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \right) + \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{40 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^5} + \\
& \frac{(16A + 30B + 29C) \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{192 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(176A + 150B + 133C) \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{256 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)} - \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{40 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^5} + \\
& \frac{(-16A - 30B - 29C) \sec \left[\frac{1}{2} (c + dx) \right]^3 (a (1 + \sec [c + dx]))^{3/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{192 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{7/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(-176 A - 150 B - 133 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{256 d (A+2 C+2 B \cos [c+dx]+A \cos [2 c+2 dx]) \operatorname{Sec}[c+dx]^{7/2} \left(\cos \left[\frac{1}{2}(c+dx)\right]+\sin \left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(2 B \sin \left[\frac{1}{2}(c+dx)\right]+3 C \sin \left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(32 d (A+2 C+2 B \cos [c+dx]+A \cos [2 c+2 dx]) \operatorname{Sec}[c+dx]^{7/2} \left(\cos \left[\frac{1}{2}(c+dx)\right]-\sin \left[\frac{1}{2}(c+dx)\right]\right)^4\right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(2 B \sin \left[\frac{1}{2}(c+dx)\right]+3 C \sin \left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(32 d (A+2 C+2 B \cos [c+dx]+A \cos [2 c+2 dx]) \operatorname{Sec}[c+dx]^{7/2} \left(\cos \left[\frac{1}{2}(c+dx)\right]+\sin \left[\frac{1}{2}(c+dx)\right]\right)^4\right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(48 A \sin \left[\frac{1}{2}(c+dx)\right]+38 B \sin \left[\frac{1}{2}(c+dx)\right]+37 C \sin \left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(128 d (A+2 C+2 B \cos [c+dx]+A \cos [2 c+2 dx]) \operatorname{Sec}[c+dx]^{7/2} \left(\cos \left[\frac{1}{2}(c+dx)\right]-\sin \left[\frac{1}{2}(c+dx)\right]\right)^2\right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(48 A \sin \left[\frac{1}{2}(c+dx)\right]+38 B \sin \left[\frac{1}{2}(c+dx)\right]+37 C \sin \left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(128 d (A+2 C+2 B \cos [c+dx]+A \cos [2 c+2 dx]) \operatorname{Sec}[c+dx]^{7/2} \left(\cos \left[\frac{1}{2}(c+dx)\right]+\sin \left[\frac{1}{2}(c+dx)\right]\right)^2\right)
\end{aligned}$$

■ **Problem 587: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{3/2} (112 A + 88 B + 75 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64 d} + \\
& \frac{a^2 (112 A + 88 B + 75 C) \operatorname{Sec}[c+dx]^{3/2} \sin [c+dx]}{64 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (48 A + 56 B + 39 C) \operatorname{Sec}[c+dx]^{5/2} \sin [c+dx]}{96 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \\
& \frac{a (8 B + 3 C) \operatorname{Sec}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \sin [c+dx]}{24 d} + \frac{C \operatorname{Sec}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \sin [c+dx]}{4 d}
\end{aligned}$$

Result (type 3, 2084 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{512} + \frac{i}{512} \right) \left((-1 + i) + \sqrt{2} \right) \left((336 + 112 i) A + 112 \sqrt{2} A + (264 + 88 i) B + 88 \sqrt{2} B + (225 + 75 i) C + 75 \sqrt{2} C \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4} (c + d x) \right]}{-\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right]} \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
& \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right] / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) - \\
& \left(\left(\frac{1}{512} - \frac{i}{512} \right) \left((1 + i) + \sqrt{2} \right) \left((-336 + 112 i) A + 112 \sqrt{2} A - (264 - 88 i) B + 88 \sqrt{2} B - (225 - 75 i) C + 75 \sqrt{2} C \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \text{Sin} \left[\frac{1}{4} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4} (c + d x) \right]}{\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right]} \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
& \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right] / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
& \left((224 A + 112 i \sqrt{2} A + 176 B + 88 i \sqrt{2} B + 150 C + 75 i \sqrt{2} C) \text{Log} \left[\sqrt{2} + 2 \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
& \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right] / \left(256 (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) - \\
& \left(\left(\frac{1}{1024} - \frac{i}{1024} \right) \left((-1 + i) + \sqrt{2} \right) \left((336 + 112 i) A + 112 \sqrt{2} A + (264 + 88 i) B + 88 \sqrt{2} B + (225 + 75 i) C + 75 \sqrt{2} C \right) \right. \\
& \quad \left. \text{Log} \left[2 - \sqrt{2} \text{Cos} \left[\frac{1}{2} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right] / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
& \left(\left(\frac{1}{1024} + \frac{i}{1024} \right) \left((1 + i) + \sqrt{2} \right) \left((-336 + 112 i) A + 112 \sqrt{2} A - (264 - 88 i) B + 88 \sqrt{2} B - (225 - 75 i) C + 75 \sqrt{2} C \right) \right. \\
& \quad \left. \text{Log} \left[2 + \sqrt{2} \text{Cos} \left[\frac{1}{2} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right] / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
& \quad \frac{(8 B + 15 C) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{96 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \left(\text{Cos} \left[\frac{1}{2} (c + d x) \right] - \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(112 A + 88 B + 75 C) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{128 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} + \\
& \frac{(-8 B - 15 C) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{96 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} + \\
& \frac{(-112 A - 88 B - 75 C) \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{128 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \quad \left. \left(16 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 24 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 19 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) / \\
& \left(64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 \right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^3 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \quad \left. \left(16 A \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 24 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 19 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) / \\
& \left(64 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 \right) + \\
& \frac{C \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{16 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \\
& \frac{C \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^2 (a(1 + \operatorname{Sec}[c + d x]))^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]}{16 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4}
\end{aligned}$$

■ **Problem 588: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (24 A + 14 B + 11 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{8 d} + \frac{a^2 (24 A + 30 B + 19 C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{24 d \sqrt{a + a \operatorname{Sec}[c + d x]}} + \\
& \frac{a (2 B + C) \operatorname{Sec}[c + d x]^{3/2} \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{4 d} + \frac{C \operatorname{Sec}[c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{3 d}
\end{aligned}$$

Result (type 3, 1796 leaves) :

$$\begin{aligned}
 & - \left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((-1 + i) + \sqrt{2} \right) \left((72 + 24 i) A + 24 \sqrt{2} A + (42 + 14 i) B + 14 \sqrt{2} B + (33 + 11 i) C + 11 \sqrt{2} C \right) \right. \\
 & \quad \left. \text{ArcTan} \left[\frac{\text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4} (c + d x) \right]}{-\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right]} \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
 & \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) - \\
 & \left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((1 + i) + \sqrt{2} \right) \left((-72 + 24 i) A + 24 \sqrt{2} A - (42 - 14 i) B + 14 \sqrt{2} B - (33 - 11 i) C + 11 \sqrt{2} C \right) \right. \\
 & \quad \left. \text{ArcTan} \left[\frac{\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \text{Sin} \left[\frac{1}{4} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{4} (c + d x) \right]}{\text{Cos} \left[\frac{1}{4} (c + d x) \right] + \sqrt{2} \text{Cos} \left[\frac{1}{4} (c + d x) \right] - \text{Sin} \left[\frac{1}{4} (c + d x) \right]} \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
 & \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
 & \left((48 A + 24 i \sqrt{2} A + 28 B + 14 i \sqrt{2} B + 22 C + 11 i \sqrt{2} C) \text{Log} \left[\sqrt{2} + 2 \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} \right. \\
 & \quad \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \Bigg/ \left(32 (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) - \\
 & \left(\left(\frac{1}{128} - \frac{i}{128} \right) \left((-1 + i) + \sqrt{2} \right) \left((72 + 24 i) A + 24 \sqrt{2} A + (42 + 14 i) B + 14 \sqrt{2} B + (33 + 11 i) C + 11 \sqrt{2} C \right) \right. \\
 & \quad \left. \text{Log} \left[2 - \sqrt{2} \text{Cos} \left[\frac{1}{2} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \Bigg/ \\
 & \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
 & \left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((1 + i) + \sqrt{2} \right) \left((-72 + 24 i) A + 24 \sqrt{2} A - (42 - 14 i) B + 14 \sqrt{2} B - (33 - 11 i) C + 11 \sqrt{2} C \right) \right. \\
 & \quad \left. \text{Log} \left[2 + \sqrt{2} \text{Cos} \left[\frac{1}{2} (c + d x) \right] - \sqrt{2} \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \Bigg/ \\
 & \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \right) + \\
 & \quad \frac{C \text{Sec} \left[\frac{1}{2} (c + d x) \right]^3 (a (1 + \text{Sec}[c + d x]))^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{12 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2} \left(\text{Cos} \left[\frac{1}{2} (c + d x) \right] - \text{Sin} \left[\frac{1}{2} (c + d x) \right] \right)^3} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{(8A + 14B + 11C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} - \\
& \frac{C \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{12d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \\
& \frac{(-8A - 14B - 11C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{16d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(2B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) / \\
& \left(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(2B \sin\left[\frac{1}{2}(c + dx)\right] + 3C \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) / \\
& \left(8d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \right)
\end{aligned}$$

■ **Problem 589: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{\sqrt{\operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 183 leaves, 5 steps):

$$\begin{aligned}
& \frac{a^{3/2} (8A + 12B + 7C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{4d} + \frac{a^2 (8A - 4B - 5C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{4d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \\
& \frac{a (4B + 3C) \sqrt{\operatorname{Sec}[c + dx]} \sqrt{a + a \operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{4d} + \frac{C \sqrt{\operatorname{Sec}[c + dx]} (a + a \operatorname{Sec}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{2d}
\end{aligned}$$

Result (type 3, 1627 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A + (36 + 12i)B + 12\sqrt{2}B + (21 + 7i)C + 7\sqrt{2}C \right) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(\left(\frac{1}{32} - \frac{i}{32} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i) A + 8\sqrt{2} A - (36 - 12i) B + 12\sqrt{2} B - (21 - 7i) C + 7\sqrt{2} C \right) \right. \\
& \left. \operatorname{ArcTan} \left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \left(\left(16A + 8i\sqrt{2}A + 24B + 12i\sqrt{2}B + 14C + 7i\sqrt{2}C \right) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(16(i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(\left(\frac{1}{64} - \frac{i}{64} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i) A + 8\sqrt{2} A + (36 + 12i) B + 12\sqrt{2} B + (21 + 7i) C + 7\sqrt{2} C \right) \right. \\
& \left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \left(\left(\frac{1}{64} + \frac{i}{64} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i) A + 8\sqrt{2} A - (36 - 12i) B + 12\sqrt{2} B - (21 - 7i) C + 7\sqrt{2} C \right) \right. \\
& \left. \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \frac{(4B + 7C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{8d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} + \\
& \frac{(-4B - 7C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^3 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{8d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} + \\
& \frac{2A \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 (a (1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2}} +
\end{aligned}$$

$$\frac{C \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4d(A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2} (\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right])^2} +$$

$$\frac{C \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (a(1+\operatorname{Sec}[c+dx]))^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{4d(A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{7/2} (\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right])^2}$$

- **Problem 590: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 3, 177 leaves, 5 steps):

$$\frac{a^{3/2} (2B+3C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{d} + \frac{a^2 (8A+6B-3C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d \sqrt{a+a \operatorname{Sec}[c+dx]}} -$$

$$\frac{a (2A-3C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3d} + \frac{2A (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 1406 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{8} + \frac{i}{8} \right) \left((-1 + i) + \sqrt{2} \right) \left((6 + 2i) B + 2\sqrt{2} B + (9 + 3i) C + 3\sqrt{2} C \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right] \Bigg/ \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(\left(\frac{1}{8} - \frac{i}{8} \right) \left((1 + i) + \sqrt{2} \right) \left((-6 + 2i) B + 2\sqrt{2} B - (9 - 3i) C + 3\sqrt{2} C \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \right. \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right] \Bigg/ \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \left(\left(4B + 2i\sqrt{2} B + 6C + 3i\sqrt{2} C \right) \operatorname{Log} \left[\sqrt{2} + 2 \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \\
& \quad \left(4(i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((-1 + i) + \sqrt{2} \right) \left((6 + 2i) B + 2\sqrt{2} B + (9 + 3i) C + 3\sqrt{2} C \right) \operatorname{Log} \left[2 - \sqrt{2} \cos \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 \right. \\
& \quad \left. (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \left(\left(\frac{1}{16} + \frac{i}{16} \right) \left((1 + i) + \sqrt{2} \right) \left((-6 + 2i) B + 2\sqrt{2} B - (9 - 3i) C + 3\sqrt{2} C \right) \operatorname{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 \right. \\
& \quad \left. (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right) + \\
& \frac{C \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} - \\
& \frac{C \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{2d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} + \\
& \frac{A \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^3 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin\left[\frac{3}{2}(c + dx)\right]}{3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2}} + \\
& \left((3A + 2B) \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^2 (a(1 + \operatorname{Sec}[c + dx]))^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Tan} \left[\frac{1}{2}(c + dx) \right] \right) \Bigg/ \\
& \quad \left(d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2} \right)
\end{aligned}$$

- **Problem 591: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\frac{2 a^{3/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{d} + \frac{2 a^2 (12 A + 20 B + 15 C) \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{15 d \sqrt{a+a \operatorname{Sec}[c+d x]}} +$$

$$\frac{2 a (3 A + 5 B) \sqrt{a+a \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{15 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 A (a+a \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}}$$

Result (type 3, 387 leaves):

$$\frac{1}{60 d \sqrt{\operatorname{Sec}[c+d x]}} a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{a(1+\operatorname{Sec}[c+d x])}$$

$$\left(-30 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right] - (-1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(1+\sqrt{2}) \cos\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] - 30 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\cos\left[\frac{1}{4}(c+d x)\right] - (1+\sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{(-1+\sqrt{2}) \cos\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}\right] + \right.$$

$$30 \sqrt{2} C \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 15 \sqrt{2} C \operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] -$$

$$15 \sqrt{2} C \operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 120 A \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 180 B \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] +$$

$$\left. 120 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 30 A \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + 20 B \operatorname{Sin}\left[\frac{3}{2}(c+d x)\right] + 6 A \operatorname{Sin}\left[\frac{5}{2}(c+d x)\right] \right)$$

- **Problem 595: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 333 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^{5/2} (1304 A + 1132 B + 1015 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{512 d} + \frac{a^3 (1304 A + 1132 B + 1015 C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{512 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \\
& \frac{a^3 (1304 A + 1132 B + 1015 C) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{768 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^3 (680 A + 628 B + 545 C) \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{960 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \\
& \frac{a^2 (120 A + 156 B + 115 C) \operatorname{Sec}[c+dx]^{7/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{480 d} + \\
& \frac{a (12 B + 5 C) \operatorname{Sec}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{60 d} + \frac{C \operatorname{Sec}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{6 d}
\end{aligned}$$

Result (type 3, 1361 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{8192} + \frac{i}{8192} \right) \left((-1+i) + \sqrt{2} \right) \left((3912 + 1304 i) A + 1304 \sqrt{2} A + (3396 + 1132 i) B + 1132 \sqrt{2} B + (3045 + 1015 i) C + 1015 \sqrt{2} C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right] / \left(\sqrt{2} (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{8192} - \frac{i}{8192} \right) \left((1+i) + \sqrt{2} \right) \left((-3912 + 1304 i) A + 1304 \sqrt{2} A - (3396 - 1132 i) B + 1132 \sqrt{2} B - (3045 - 1015 i) C + 1015 \sqrt{2} C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right] / \left(\sqrt{2} (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \\
& \left((2608 A + 1304 i \sqrt{2} A + 2264 B + 1132 i \sqrt{2} B + 2030 C + 1015 i \sqrt{2} C) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right] / \left(4096 (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{16384} - \frac{i}{16384} \right) \left((-1+i) + \sqrt{2} \right) \left((3912 + 1304 i) A + 1304 \sqrt{2} A + (3396 + 1132 i) B + 1132 \sqrt{2} B + (3045 + 1015 i) C + 1015 \sqrt{2} C \right) \right. \\
& \quad \left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right] /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \left(\left(\frac{1}{16384} + \frac{i}{16384} \right) \left((1+i) + \sqrt{2} \right) \left((-3912 + 1304i) A + 1304\sqrt{2} A - (3396 - 1132i) B + 1132\sqrt{2} B - (3045 - 1015i) C + 1015\sqrt{2} C \right) \right. \\
& \quad \left. \operatorname{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right] \Bigg) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \frac{1}{491520 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 \operatorname{Sec}[c + dx]^{3/2} (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-96720 A \sin \left[\frac{1}{2} (c + dx) \right] - 78120 B \sin \left[\frac{1}{2} (c + dx) \right] - 47250 C \sin \left[\frac{1}{2} (c + dx) \right] + \right. \\
& \quad 164240 A \sin \left[\frac{3}{2} (c + dx) \right] + 167944 B \sin \left[\frac{3}{2} (c + dx) \right] + 184490 C \sin \left[\frac{3}{2} (c + dx) \right] - 7560 A \sin \left[\frac{5}{2} (c + dx) \right] + 13980 B \sin \left[\frac{5}{2} (c + dx) \right] + \\
& \quad 28275 C \sin \left[\frac{5}{2} (c + dx) \right] + 101160 A \sin \left[\frac{7}{2} (c + dx) \right] + 98484 B \sin \left[\frac{7}{2} (c + dx) \right] + 88305 C \sin \left[\frac{7}{2} (c + dx) \right] + 6520 A \sin \left[\frac{9}{2} (c + dx) \right] + \\
& \quad \left. 5660 B \sin \left[\frac{9}{2} (c + dx) \right] + 5075 C \sin \left[\frac{9}{2} (c + dx) \right] + 19560 A \sin \left[\frac{11}{2} (c + dx) \right] + 16980 B \sin \left[\frac{11}{2} (c + dx) \right] + 15225 C \sin \left[\frac{11}{2} (c + dx) \right] \right)
\end{aligned}$$

■ **Problem 596: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 281 leaves, 7 steps):

$$\begin{aligned}
& \frac{a^{5/2} (400A + 326B + 283C) \operatorname{ArcSinh} \left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}} \right]}{128d} + \frac{a^3 (400A + 326B + 283C) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{128d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \\
& \frac{a^3 (1040A + 950B + 787C) \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{960d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{a^2 (80A + 110B + 79C) \operatorname{Sec}[c + dx]^{5/2} \sqrt{a + a \operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{240d} + \\
& \frac{a (2B + C) \operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{8d} + \frac{C \operatorname{Sec}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^{5/2} \operatorname{Sin}[c + dx]}{5d}
\end{aligned}$$

Result (type 3, 2352 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{2048} + \frac{i}{2048} \right) \left((-1+i) + \sqrt{2} \right) \left((1200 + 400i) A + 400\sqrt{2} A + (978 + 326i) B + 326\sqrt{2} B + (849 + 283i) C + 283\sqrt{2} C \right) \right. \\
& \quad \left. \operatorname{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{-\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \operatorname{Sec}[c + dx]))^{5/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{2048} - \frac{i}{2048} \right) \left((1 + i) + \sqrt{2} \right) \left((-1200 + 400i)A + 400\sqrt{2}A - (978 - 326i)B + 326\sqrt{2}B - (849 - 283i)C + 283\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{ArcTan} \left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \left(\left(800A + 400i\sqrt{2}A + 652B + 326i\sqrt{2}B + 566C + 283i\sqrt{2}C \right) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(1024 (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{4096} - \frac{i}{4096} \right) \left((-1 + i) + \sqrt{2} \right) \left((1200 + 400i)A + 400\sqrt{2}A + (978 + 326i)B + 326\sqrt{2}B + (849 + 283i)C + 283\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \left(\left(\frac{1}{4096} + \frac{i}{4096} \right) \left((1 + i) + \sqrt{2} \right) \left((-1200 + 400i)A + 400\sqrt{2}A - (978 - 326i)B + 326\sqrt{2}B - (849 - 283i)C + 283\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \quad \frac{C \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{80 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^5} + \\
& \quad \frac{(16A + 46B + 59C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{384 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^3} + \\
& \quad \frac{(400A + 326B + 283C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{512 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{C \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{80 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^5} + \\
& \frac{(-16 A-46 B-59 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{384 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{(-400 A-326 B-283 C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{512 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(2 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+5 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left(64 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4 \right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(2 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+5 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left(64 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^4 \right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(80 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+86 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+75 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left(256 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(80 A \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+86 B \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]+75 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \left(256 d (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)^2 \right)
\end{aligned}$$

■ **Problem 597: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\frac{a^{5/2} (304 A + 200 B + 163 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{64 d} +$$

$$\frac{a^3 (432 A + 392 B + 299 C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{192 d \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (16 A + 24 B + 17 C) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{32 d} +$$

$$\frac{a (8 B + 5 C) \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{24 d} + \frac{C \operatorname{Sec}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{4 d}$$

Result (type 3, 2084 leaves):

$$- \left(\left(\frac{1}{1024} + \frac{i}{1024} \right) \left((-1+i) + \sqrt{2} \right) \left((912+304i) A + 304 \sqrt{2} A + (600+200i) B + 200 \sqrt{2} B + (489+163i) C + 163 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) -$$

$$\left(\left(\frac{1}{1024} - \frac{i}{1024} \right) \left((1+i) + \sqrt{2} \right) \left((-912+304i) A + 304 \sqrt{2} A - (600-200i) B + 200 \sqrt{2} B - (489-163i) C + 163 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(\sqrt{2} (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) +$$

$$\left((608A+304i\sqrt{2}A+400B+200i\sqrt{2}B+326C+163i\sqrt{2}C) \operatorname{Log}\left[\sqrt{2}+2\operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} \right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(512 (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) -$$

$$\left(\left(\frac{1}{2048} - \frac{i}{2048} \right) \left((-1+i) + \sqrt{2} \right) \left((912+304i) A + 304 \sqrt{2} A + (600+200i) B + 200 \sqrt{2} B + (489+163i) C + 163 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{Log}\left[2-\sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(\sqrt{2} (i+\sqrt{2}) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) +$$

$$\left(\left(\frac{1}{2048} + \frac{i}{2048} \right) \left((1+i) + \sqrt{2} \right) \left((-912+304i) A + 304 \sqrt{2} A - (600-200i) B + 200 \sqrt{2} B - (489-163i) C + 163 \sqrt{2} C \right) \right.$$

$$\begin{aligned}
& \left. \left(\log \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) \right) / \\
& \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \right) + \\
& \frac{(8B + 23C) \sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{192 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(176A + 200B + 163C) \sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{256 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \frac{(-8B - 23C) \sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{192 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^3} + \\
& \frac{(-176A - 200B - 163C) \sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2)}{256 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)} + \\
& \left(\sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right. \\
& \quad \left. \left(16A \sin \left[\frac{1}{2} (c + dx) \right] + 40B \sin \left[\frac{1}{2} (c + dx) \right] + 43C \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) / \\
& \left(128 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^2 \right) + \\
& \left(\sec \left[\frac{1}{2} (c + dx) \right]^5 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \right. \\
& \quad \left. \left(16A \sin \left[\frac{1}{2} (c + dx) \right] + 40B \sin \left[\frac{1}{2} (c + dx) \right] + 43C \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) / \\
& \left(128 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^2 \right) + \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right]^4 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \tan \left[\frac{1}{2} (c + dx) \right]}{32 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right)^4} + \\
& \frac{C \sec \left[\frac{1}{2} (c + dx) \right]^4 (a (1 + \sec [c + dx]))^{5/2} (A + B \sec [c + dx] + C \sec [c + dx]^2) \tan \left[\frac{1}{2} (c + dx) \right]}{32 d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sec [c + dx]^{9/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right)^4}
\end{aligned}$$

■ **Problem 598:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\frac{a^{5/2} (40 A + 38 B + 25 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{8 d} + \frac{a^3 (24 A - 54 B - 49 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{24 d \sqrt{a + a \operatorname{Sec}[c + d x]}} +$$

$$\frac{a^2 (24 A + 42 B + 31 C) \sqrt{\operatorname{Sec}[c + d x]} \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{24 d} +$$

$$\frac{a (6 B + 5 C) \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{12 d} + \frac{C \sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{3 d}$$

Result (type 3, 1894 leaves):

$$- \left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((-1 + i) + \sqrt{2} \right) \left((120 + 40 i) A + 40 \sqrt{2} A + (114 + 38 i) B + 38 \sqrt{2} B + (75 + 25 i) C + 25 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right] / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \right) -$$

$$\left(\left(\frac{1}{128} - \frac{i}{128} \right) \left((1 + i) + \sqrt{2} \right) \left((-120 + 40 i) A + 40 \sqrt{2} A - (114 - 38 i) B + 38 \sqrt{2} B - (75 - 25 i) C + 25 \sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + d x)\right]} \right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right] / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \right) +$$

$$\left((80 A + 40 i \sqrt{2} A + 76 B + 38 i \sqrt{2} B + 50 C + 25 i \sqrt{2} C) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]^5 (a (1 + \operatorname{Sec}[c + d x]))^{5/2} \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right] / \left(64 (i + \sqrt{2}) d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \right) -$$

$$\left(\left(\frac{1}{256} - \frac{i}{256} \right) \left((-1 + i) + \sqrt{2} \right) \left((120 + 40 i) A + 40 \sqrt{2} A + (114 + 38 i) B + 38 \sqrt{2} B + (75 + 25 i) C + 25 \sqrt{2} C \right) \right.$$

$$\begin{aligned}
& \frac{\operatorname{Log}\left[2 - \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{\left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2}\right) +} \\
& \left(\frac{1}{256} + \frac{i}{256}\right) \left((1+i)+\sqrt{2}\right) \left((-120+40i)A+40\sqrt{2}A-(114-38i)B+38\sqrt{2}B-(75-25i)C+25\sqrt{2}C\right)}{\operatorname{Log}\left[2 + \sqrt{2} \cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \sin\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{\left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2}\right) +} \\
& C\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{24d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{(8A+22B+25C)\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{32d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)} - \\
& \frac{C\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{24d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^3} + \\
& \frac{(-8A-22B-25C)\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{32d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)} + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \left(2B\sin\left[\frac{1}{2}(c+dx)\right] + 5C\sin\left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(16d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right) + \\
& \left(\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \left(2B\sin\left[\frac{1}{2}(c+dx)\right] + 5C\sin\left[\frac{1}{2}(c+dx)\right]\right)\right) / \\
& \left(16d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right)^2\right) + \\
& \frac{A\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\operatorname{Sec}[c+dx]^{9/2}}
\end{aligned}$$

■ **Problem 599: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\operatorname{Sec}[c+dx])^{5/2} (A+B\operatorname{Sec}[c+dx]+C\operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 3, 233 leaves, 6 steps):

$$\frac{a^{5/2} (8A + 20B + 19C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4d} + \frac{a^3 (56A + 12B - 27C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12d \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

$$\frac{a^2 (8A - 12B - 21C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12d} -$$

$$\frac{a (4A - 3C) \sqrt{\operatorname{Sec}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{6d} + \frac{2A (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 3, 1736 leaves):

$$-\left(\left(\frac{1}{64} + \frac{i}{64}\right) \left((-1+i) + \sqrt{2}\right) \left((24+8i)A + 8\sqrt{2}A + (60+20i)B + 20\sqrt{2}B + (57+19i)C + 19\sqrt{2}C\right)\right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}\right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)\right) / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}\right) -$$

$$\left(\left(\frac{1}{64} - \frac{i}{64}\right) \left((1+i) + \sqrt{2}\right) \left((-24+8i)A + 8\sqrt{2}A - (60-20i)B + 20\sqrt{2}B - (57-19i)C + 19\sqrt{2}C\right)\right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}\right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)\right) / \left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}\right) +$$

$$\left(\left(16A + 8i\sqrt{2}A + 40B + 20i\sqrt{2}B + 38C + 19i\sqrt{2}C\right) \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2}\right.$$

$$\left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)\right) / \left(32(i+\sqrt{2})d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}\right) -$$

$$\left(\left(\frac{1}{128} - \frac{i}{128}\right) \left((-1+i) + \sqrt{2}\right) \left((24+8i)A + 8\sqrt{2}A + (60+20i)B + 20\sqrt{2}B + (57+19i)C + 19\sqrt{2}C\right)\right.$$

$$\left. \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)\right) /$$

$$\left(\sqrt{2}(i+\sqrt{2})d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}\right) +$$

$$\begin{aligned}
& \left(\left(\frac{1}{128} + \frac{i}{128} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A - (60-20i)B + 20\sqrt{2}B - (57-19i)C + 19\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c+dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c+dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2) \right] \Bigg) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \\
& \quad \frac{(4B+11C) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2)}{16d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right)} + \\
& \quad \frac{(-4B-11C) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2)}{16d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right)} + \\
& \quad \frac{A \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^5 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2) \sin \left[\frac{3}{2} (c+dx) \right]}{6d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}} + \\
& \quad \left((5A+2B) \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right] \right) / \\
& \quad \left(2d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \right) + \\
& \quad \frac{C \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{8d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right)^2} + \\
& \quad \frac{C \operatorname{Sec} \left[\frac{1}{2} (c+dx) \right]^4 (a(1+\operatorname{Sec}[c+dx]))^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{8d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \left(\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right)^2}
\end{aligned}$$

- **Problem 600: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a+a\operatorname{Sec}[c+dx])^{5/2} (A+B\operatorname{Sec}[c+dx] + C\operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{5/2}} dx$$

Optimal (type 3, 223 leaves, 6 steps):

$$\begin{aligned}
& \frac{a^{5/2} (2B+5C) \operatorname{ArcSinh} \left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a\operatorname{Sec}[c+dx]}} \right]}{d} + \frac{a^3 (64A+70B+15C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15d\sqrt{a+a\operatorname{Sec}[c+dx]}} - \\
& \frac{a^2 (16A+10B-15C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+a\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{15d} + \\
& \frac{2a(A+B)(a+a\operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3d\sqrt{\operatorname{Sec}[c+dx]}} + \frac{2A(a+a\operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{5d\operatorname{Sec}[c+dx]^{3/2}}
\end{aligned}$$

Result (type 3, 1519 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{16} + \frac{i}{16} \right) \left((-1 + i) + \sqrt{2} \right) \left((6 + 2i)B + 2\sqrt{2}B + (15 + 5i)C + 5\sqrt{2}C \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c + dx)\right]}{-\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \right) \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((1 + i) + \sqrt{2} \right) \left((-6 + 2i)B + 2\sqrt{2}B - (15 - 5i)C + 5\sqrt{2}C \right) \operatorname{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c + dx)\right]}{\cos\left[\frac{1}{4}(c + dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right]} \right] \right) \\
& \quad \left. \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \left((4B + 2i\sqrt{2}B + 10C + 5i\sqrt{2}C) \operatorname{Log} \left[\sqrt{2} + 2 \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \\
& \quad \left(8 (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) - \\
& \left(\left(\frac{1}{32} - \frac{i}{32} \right) \left((-1 + i) + \sqrt{2} \right) \left((6 + 2i)B + 2\sqrt{2}B + (15 + 5i)C + 5\sqrt{2}C \right) \operatorname{Log} \left[2 - \sqrt{2} \cos \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 \right. \\
& \quad \left. (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((1 + i) + \sqrt{2} \right) \left((-6 + 2i)B + 2\sqrt{2}B - (15 - 5i)C + 5\sqrt{2}C \right) \operatorname{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2}(c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2}(c + dx) \right] \right] \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 \right. \\
& \quad \left. (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \frac{C \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} (\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right])} - \\
& \frac{C \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{4d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} (\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right])} + \\
& \left((5A + 2B) \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin \left[\frac{3}{2}(c + dx) \right] \right) \Bigg/ \\
& \quad \left(12d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2} \right) + \\
& \frac{A \operatorname{Sec} \left[\frac{1}{2}(c + dx) \right]^5 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sin \left[\frac{5}{2}(c + dx) \right]}{20d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2}} +
\end{aligned}$$

$$\left((5A + 5B + 2C) \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^4 (a(1 + \operatorname{Sec}[c + dx]))^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) / \\ (2d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{9/2})$$

■ **Problem 601: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{\operatorname{Sec}[c + dx]^{7/2}} dx$$

Optimal (type 3, 222 leaves, 6 steps):

$$\frac{2a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \operatorname{Sec}[c + dx]}}\right]}{d} + \frac{2a^3 (160A + 224B + 245C) \sqrt{\operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{105d \sqrt{a + a \operatorname{Sec}[c + dx]}} + \\ \frac{2a^2 (40A + 56B + 35C) \sqrt{a + a \operatorname{Sec}[c + dx]} \operatorname{Sin}[c + dx]}{105d \sqrt{\operatorname{Sec}[c + dx]}} + \frac{2a(5A + 7B) (a + a \operatorname{Sec}[c + dx])^{3/2} \operatorname{Sin}[c + dx]}{35d \operatorname{Sec}[c + dx]^{3/2}} + \frac{2A (a + a \operatorname{Sec}[c + dx])^{5/2} \operatorname{Sin}[c + dx]}{7d \operatorname{Sec}[c + dx]^{5/2}}$$

Result (type 3, 428 leaves):

$$\frac{1}{420d \sqrt{\operatorname{Sec}[c + dx]}} a^2 \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \sqrt{a(1 + \operatorname{Sec}[c + dx])} \\ \left(-210i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}\right] - 210i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]}\right] + \right. \\ 210 \sqrt{2} C \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - 105 \sqrt{2} C \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \\ 105 \sqrt{2} C \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 1575A \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 2100B \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + 2100C \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \\ \left. 385A \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right] + 350B \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right] + 140C \operatorname{Sin}\left[\frac{3}{2}(c + dx)\right] + 105A \operatorname{Sin}\left[\frac{5}{2}(c + dx)\right] + 42B \operatorname{Sin}\left[\frac{5}{2}(c + dx)\right] + 15A \operatorname{Sin}\left[\frac{7}{2}(c + dx)\right] \right)$$

■ **Problem 605: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{\sqrt{a + a \operatorname{Sec}[c + dx]}} dx$$

Optimal (type 3, 241 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(8A - 14B + 9C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{8\sqrt{a}d} + \frac{\sqrt{2}(A - B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a}d} + \\
& \frac{(8A - 2B + 7C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{8d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{(6B - C) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{12d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{C \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{3d\sqrt{a+a \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 3, 2012 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((1+i) - i\sqrt{2} \right) \left((24+8i)A + 8\sqrt{2}A - (42+14i)B - 14\sqrt{2}B + (27+9i)C + 9\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) + \\
& \left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A + (42-14i)B - 14\sqrt{2}B - (27-9i)C + 9\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) - \\
& \left(4(A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) + \\
& \left(4(A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) + \\
& \left((-16A - 8i\sqrt{2}A + 28B + 14i\sqrt{2}B - 18C - 9i\sqrt{2}C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(8(i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) + \\
& \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((1+i) - i\sqrt{2} \right) \left((24+8i)A + 8\sqrt{2}A - (42+14i)B - 14\sqrt{2}B + (27+9i)C + 9\sqrt{2}C \right) \right. \\
& \quad \left. \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\frac{1}{32} + \frac{i}{32} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A + (42-14i)B - 14\sqrt{2}B - (27-9i)C + 9\sqrt{2}C \right) \right. \\
& \quad \left. \cos\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2\right) \right) / \\
& \quad \left(\sqrt{2} \left(i + \sqrt{2} \right) d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \right) + \\
& \quad \left(C \cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \right) / \\
& \quad \left(3d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \right) + \\
& \quad \left((8A - 2B + 7C) \cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \right) / \\
& \quad \left(4d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) - \\
& \quad \left(C \cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \right) / \\
& \quad \left(3d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^3 \right) + \\
& \quad \left((-8A + 2B - 7C) \cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \right) / \\
& \quad \left(4d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& \quad \left(\cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \left(2B\sin\left[\frac{1}{2}(c+dx)\right] - C\sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \quad \left(2d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) + \\
& \quad \left(\cos\left[\frac{1}{2}(c+dx)\right] \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right) \left(2B\sin\left[\frac{1}{2}(c+dx)\right] - C\sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / \\
& \quad \left(2d \left(A + 2C + 2B\cos[c+dx] + A\cos[2c+2dx] \right) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right)
\end{aligned}$$

■ **Problem 606: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2} \left(A + B\sec[c+dx] + C\sec[c+dx]^2 \right)}{\sqrt{a+a\sec[c+dx]}} dx$$

Optimal (type 3, 195 leaves, 7 steps):

$$\frac{(8A - 4B + 7C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4\sqrt{a}d} - \frac{\sqrt{2}(A - B + C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{\sqrt{a}d} +$$

$$\frac{(4B - C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{4d\sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{C \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{2d\sqrt{a+a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 1732 leaves):

$$- \left(\left(\frac{1}{8} + \frac{i}{8} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A - (12 + 4i)B - 4\sqrt{2}B + (21 + 7i)C + 7\sqrt{2}C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) -$$

$$\left(\left(\frac{1}{8} - \frac{i}{8} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i)A + 8\sqrt{2}A + (12 - 4i)B - 4\sqrt{2}B - (21 - 7i)C + 7\sqrt{2}C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) +$$

$$\left(4(A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) -$$

$$\left(4(A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) +$$

$$\left((16A + 8i\sqrt{2}A - 8B - 4i\sqrt{2}B + 14C + 7i\sqrt{2}C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(4(i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) -$$

$$\left(\left(\frac{1}{16} - \frac{i}{16} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A - (12 + 4i)B - 4\sqrt{2}B + (21 + 7i)C + 7\sqrt{2}C \right) \right.$$

$$\left. \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) /$$

$$\left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a(1 + \operatorname{Sec}[c+dx])} \right) +$$

$$\begin{aligned}
& \left(\left(\frac{1}{16} + \frac{i}{16} \right) \left((1+i) + \sqrt{2} \right) \left((-24+8i)A + 8\sqrt{2}A + (12-4i)B - 4\sqrt{2}B - (21-7i)C + 7\sqrt{2}C \right) \right. \\
& \quad \left. \cos\left[\frac{1}{2}(c+dx)\right] \log\left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right]\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \right) + \\
& \quad \left((4B-C) \cos\left[\frac{1}{2}(c+dx)\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \right) / \\
& \quad \left(2d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) + \\
& \quad \left(C \cos\left[\frac{1}{2}(c+dx)\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right] \right) / \\
& \quad \left(d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) + \\
& \quad \left(C \cos\left[\frac{1}{2}(c+dx)\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right] \right) / \\
& \quad \left(d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right)^2 \right) + \\
& \quad \left((-4B+C) \cos\left[\frac{1}{2}(c+dx)\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \right) / \\
& \quad \left(2d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a(1+\sec[c+dx])} \left(\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right] \right) \right)
\end{aligned}$$

- **Problem 607: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx]^2)}{\sqrt{a+a\sec[c+dx]}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(2B-C) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right]}{\sqrt{a}d} + \frac{\sqrt{2}(A-B+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{\sec[c+dx]}\sin[c+dx]}{\sqrt{2}\sqrt{a+a\sec[c+dx]}}\right]}{\sqrt{a}d} + \frac{C\sec[c+dx]^{3/2}\sin[c+dx]}{d\sqrt{a+a\sec[c+dx]}}$$

Result (type 3, 644 leaves):

$$\left(\left(\frac{1}{8} + \frac{i}{8} \right) \cos \left[\frac{1}{2} (c + dx) \right] (A + B \sec [c + dx] + C \sec [c + dx]^2) \right.$$

$$\left. \frac{2i\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2B - C) \operatorname{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] - (-1 + \sqrt{2}) \sin \left[\frac{1}{4} (c + dx) \right]}{(1 + \sqrt{2}) \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} - \right.$$

$$\left. \frac{2\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2B - C) \operatorname{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] - (1 + \sqrt{2}) \sin \left[\frac{1}{4} (c + dx) \right]}{(-1 + \sqrt{2}) \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right]}{i + \sqrt{2}} - \right.$$

$$(16 - 16i) (A - B + C) \log \left[\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right] + (16 - 16i) (A - B + C) \log \left[\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right] +$$

$$\frac{(4 + 4i) \left(-2i + \sqrt{2} \right) (2B - C) \log \left[\sqrt{2} + 2 \sin \left[\frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$i\sqrt{2} \left((-1 + i) + \sqrt{2} \right) \left((3 + i) + \sqrt{2} \right) (2B - C) \log \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] +$$

$$\frac{\sqrt{2} \left((-3 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) (2B - C) \log \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right]}{i + \sqrt{2}} +$$

$$\left. \left. \frac{(8 - 8i) C}{\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right]} - \frac{(8 - 8i) C}{\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right]} \right) \right) /$$

$$\left(d (A + 2C + 2B \cos [c + dx] + A \cos [2(c + dx)]) \sec [c + dx]^{3/2} \sqrt{a(1 + \sec [c + dx])} \right)$$

■ **Problem 608: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + dx] + C \sec [c + dx]^2}{\sqrt{\sec [c + dx]} \sqrt{a + a \sec [c + dx]}} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{2C \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c + dx]}{\sqrt{a + a \sec [c + dx]}} \right]}{\sqrt{a} d} - \frac{\sqrt{2} (A - B + C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + dx]} \sin [c + dx]}{\sqrt{2} \sqrt{a + a \sec [c + dx]}} \right]}{\sqrt{a} d} + \frac{2A \sqrt{\sec [c + dx]} \sin [c + dx]}{d \sqrt{a + a \sec [c + dx]}}$$

Result (type 3, 477 leaves) :

$$\frac{1}{2 d \sqrt{a} (1 + \operatorname{Sec}[c + d x])} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] \sqrt{\operatorname{Sec}[c + d x]}$$

$$\left(-2 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (-1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] - 2 i \sqrt{2} C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - (1 + \sqrt{2}) \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}{(-1 + \sqrt{2}) \operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]}\right] + \right.$$

$$4 A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] - 4 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] +$$

$$4 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] - 4 A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] +$$

$$4 B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] - 4 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{4} (c + d x)\right]\right] + 2 \sqrt{2} C \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] -$$

$$\left. \sqrt{2} C \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \sqrt{2} C \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 8 A \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 612: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Sec}[c + d x]} (a A + (A b + a B) \operatorname{Sec}[c + d x] + b B \operatorname{Sec}[c + d x]^2)}{\sqrt{a + a \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 3, 152 leaves, 6 steps) :

$$\frac{(2 A b + 2 a B - b B) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} + \frac{\sqrt{2} (a - b) (A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{\sqrt{a} d} + \frac{b B \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{d \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 638 leaves) :

$$\frac{1}{d \sqrt{a} (1 + \operatorname{Sec}[c + dx])}$$

$$\left(\frac{1}{16} + \frac{i}{16} \right) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \sqrt{\operatorname{Sec}[c + dx]} \left(\frac{2i\sqrt{2}((-3+i) + \sqrt{2})((1+i) + \sqrt{2})(2Ab + 2aB - bB) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (-1+\sqrt{2})\operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(1+\sqrt{2})\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right]}{i + \sqrt{2}} \right)}{i + \sqrt{2}} \right.$$

$$\left. \frac{2\sqrt{2}((-1+i) + \sqrt{2})((3+i) + \sqrt{2})(2Ab + 2aB - bB) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - (1+\sqrt{2})\operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{(-1+\sqrt{2})\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}\right]}{i + \sqrt{2}} \right) -$$

$$(16 - 16i)(a - b)(A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right] + (16 - 16i)(a - b)(A - B) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right] +$$

$$\frac{(4 + 4i)(-2i + \sqrt{2})(2Ab + 2aB - bB) \operatorname{Log}\left[\sqrt{2} + 2\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{i + \sqrt{2}} + \frac{1}{i + \sqrt{2}}$$

$$i\sqrt{2}((-1+i) + \sqrt{2})((3+i) + \sqrt{2})(2Ab + 2aB - bB) \operatorname{Log}\left[2 - \sqrt{2}\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] +$$

$$\frac{1}{i + \sqrt{2}}\sqrt{2}((-3+i) + \sqrt{2})((1+i) + \sqrt{2})(2Ab + 2aB - bB) \operatorname{Log}\left[2 + \sqrt{2}\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \sqrt{2}\operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] +$$

$$\left. \frac{(8 - 8i)bB}{\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]} - \frac{(8 - 8i)bB}{\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]} \right)$$

■ **Problem 613: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^{3/2}} dx$$

Optimal (type 3, 260 leaves, 8 steps):

$$\frac{(8A - 12B + 19C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{4a^{3/2}d} - \frac{(5A - 9B + 13C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{2\sqrt{2}a^{3/2}d}$$

$$+ \frac{(A - B + C) \operatorname{Sec}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{2d(a + a \operatorname{Sec}[c + dx])^{3/2}} - \frac{(2A - 6B + 7C) \operatorname{Sec}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{4ad\sqrt{a + a \operatorname{Sec}[c + dx]}} + \frac{(A - B + 2C) \operatorname{Sec}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{2ad\sqrt{a + a \operatorname{Sec}[c + dx]}}$$

Result (type 3, 1996 leaves):

$$\begin{aligned}
& - \left(\left(\frac{1}{4} + \frac{i}{4} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A - (36 + 12i)B - 12\sqrt{2}B + (57 + 19i)C + 19\sqrt{2}C \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{-\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{1}{2}(c+dx)\right]^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) - \\
& \left(\left(\frac{1}{4} - \frac{i}{4} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i)A + 8\sqrt{2}A + (36 - 12i)B - 12\sqrt{2}B - (57 - 19i)C + 19\sqrt{2}C \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{4}(c+dx)\right]}{\cos\left[\frac{1}{4}(c+dx)\right] + \sqrt{2}\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]} \right] \cos\left[\frac{1}{2}(c+dx)\right]^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) + \\
& \left(2(5A - 9B + 13C) \cos\left[\frac{1}{2}(c+dx)\right]^3 \text{Log} \left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) - \\
& \left(2(5A - 9B + 13C) \cos\left[\frac{1}{2}(c+dx)\right]^3 \text{Log} \left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) + \\
& \left((16A + 8i\sqrt{2}A - 24B - 12i\sqrt{2}B + 38C + 19i\sqrt{2}C) \cos\left[\frac{1}{2}(c+dx)\right]^3 \text{Log} \left[\sqrt{2} + 2\sin\left[\frac{1}{2}(c+dx)\right] \right] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(2(i + \sqrt{2}) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) - \\
& \left(\left(\frac{1}{8} - \frac{i}{8} \right) \left((-1 + i) + \sqrt{2} \right) \left((24 + 8i)A + 8\sqrt{2}A - (36 + 12i)B - 12\sqrt{2}B + (57 + 19i)C + 19\sqrt{2}C \right) \right. \\
& \quad \left. \cos\left[\frac{1}{2}(c+dx)\right]^3 \text{Log} \left[2 - \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) + \\
& \left(\left(\frac{1}{8} + \frac{i}{8} \right) \left((1 + i) + \sqrt{2} \right) \left((-24 + 8i)A + 8\sqrt{2}A + (36 - 12i)B - 12\sqrt{2}B - (57 - 19i)C + 19\sqrt{2}C \right) \right. \\
& \quad \left. \cos\left[\frac{1}{2}(c+dx)\right]^3 \text{Log} \left[2 + \sqrt{2}\cos\left[\frac{1}{2}(c+dx)\right] - \sqrt{2}\sin\left[\frac{1}{2}(c+dx)\right] \right] (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a(1 + \sec[c+dx]))^{3/2} \right) + \\
& \left((-A + B - C) \cos\left[\frac{1}{2}(c+dx)\right]^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] - \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \right) + \\
& \left((A - B + C) \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \right) + \\
& \left((4B - 5C) \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) + \\
& \left(2C \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{1}{2}(c + dx)\right] \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \right) + \\
& \left(2C \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sin\left[\frac{1}{2}(c + dx)\right] \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right)^2 \right) + \\
& \left((-4B + 5C) \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)
\end{aligned}$$

■ **Problem 614: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{3/2}} dx$$

Optimal (type 3, 202 leaves, 7 steps):

$$\frac{(2B - 3C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{a^{3/2} d} + \frac{(A - 5B + 9C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c + dx]} \sin[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \\
\frac{(A - B + C) \sec[c + dx]^{5/2} \sin[c + dx]}{2d (a + a \sec[c + dx])^{3/2}} + \frac{(A - B + 3C) \sec[c + dx]^{3/2} \sin[c + dx]}{2ad \sqrt{a + a \sec[c + dx]}}$$

Result (type 3, 1669 leaves):

$$\begin{aligned}
& - \left((1 - i) \left((1 + i) - i \sqrt{2} \right) \left((-6 - 2i) B - 2 \sqrt{2} B + (9 + 3i) C + 3 \sqrt{2} C \right) \right. \\
& \quad \left. \text{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{-\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \cos \left[\frac{1}{2} (c + dx) \right]^3 (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) + \\
& \left((1 - i) \left((1 + i) + \sqrt{2} \right) \left((6 - 2i) B - 2 \sqrt{2} B - (9 - 3i) C + 3 \sqrt{2} C \right) \text{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \right. \\
& \quad \left. \cos \left[\frac{1}{2} (c + dx) \right]^3 (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) - \\
& \left(2 (A - 5B + 9C) \cos \left[\frac{1}{2} (c + dx) \right]^3 \text{Log} \left[\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right] (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) + \\
& \left(2 (A - 5B + 9C) \cos \left[\frac{1}{2} (c + dx) \right]^3 \text{Log} \left[\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right] (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) + \\
& \left(2 (4B + 2i \sqrt{2} B - 6C - 3i \sqrt{2} C) \cos \left[\frac{1}{2} (c + dx) \right]^3 \text{Log} \left[\sqrt{2} + 2 \sin \left[\frac{1}{2} (c + dx) \right] \right] (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left((i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) + \\
& \left(\left(\frac{1}{2} + \frac{i}{2} \right) \left((1 + i) - i \sqrt{2} \right) \left((-6 - 2i) B - 2 \sqrt{2} B + (9 + 3i) C + 3 \sqrt{2} C \right) \cos \left[\frac{1}{2} (c + dx) \right]^3 \text{Log} \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \right. \\
& \quad \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) - \\
& \left(\left(\frac{1}{2} + \frac{i}{2} \right) \left((1 + i) + \sqrt{2} \right) \left((6 - 2i) B - 2 \sqrt{2} B - (9 - 3i) C + 3 \sqrt{2} C \right) \cos \left[\frac{1}{2} (c + dx) \right]^3 \text{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] \right. \\
& \quad \left. (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \right) + \\
& \left((A - B + C) \cos \left[\frac{1}{2} (c + dx) \right]^3 (A + B \sec [c + dx] + C \sec [c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) \sqrt{\sec [c + dx]} (a (1 + \sec [c + dx]))^{3/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right)^2 \right) +
\end{aligned}$$

$$\left((-A + B - C) \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{4}(c + dx)\right] + \sin\left[\frac{1}{4}(c + dx)\right] \right)^2 \right) +$$

$$\left(4C \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right] \right) \right) -$$

$$\left(4C \cos\left[\frac{1}{2}(c + dx)\right]^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a(1 + \sec[c + dx]))^{3/2} \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right] \right) \right)$$

■ **Problem 615: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^{3/2}} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$\frac{2C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + dx]}{\sqrt{a + a \sec[c + dx]}}\right]}{a^{3/2} d} + \frac{(3A + B - 5C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c + dx]} \operatorname{Sin}[c + dx]}{\sqrt{2} \sqrt{a + a \sec[c + dx]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sec[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{2d (a + a \sec[c + dx])^{3/2}}$$

Result (type 3, 1259 leaves):

$$\begin{aligned}
& - \left(2 \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) C \operatorname{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{-\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \cos \left[\frac{1}{2} (c + dx) \right]^3 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) - \\
& \left(2 \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) C \operatorname{ArcTan} \left[\frac{\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{4} (c + dx) \right]}{\cos \left[\frac{1}{4} (c + dx) \right] + \sqrt{2} \cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right]} \right] \cos \left[\frac{1}{2} (c + dx) \right]^3 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) - \\
& \left(2 (3A + B - 5C) \cos \left[\frac{1}{2} (c + dx) \right]^3 \operatorname{Log} \left[\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \left(2 (3A + B - 5C) \cos \left[\frac{1}{2} (c + dx) \right]^3 \operatorname{Log} \left[\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \frac{4 \sqrt{2} C \cos \left[\frac{1}{2} (c + dx) \right]^3 \operatorname{Log} \left[\sqrt{2} + 2 \sin \left[\frac{1}{2} (c + dx) \right] \right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2}} + \\
& \left(i \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) C \cos \left[\frac{1}{2} (c + dx) \right]^3 \operatorname{Log} \left[2 - \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \left(i \left((-1 + i) + \sqrt{2} \right) \left((1 + i) + \sqrt{2} \right) C \cos \left[\frac{1}{2} (c + dx) \right]^3 \operatorname{Log} \left[2 + \sqrt{2} \cos \left[\frac{1}{2} (c + dx) \right] - \sqrt{2} \sin \left[\frac{1}{2} (c + dx) \right] \right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\
& \left((-A + B - C) \cos \left[\frac{1}{2} (c + dx) \right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right)^2 \right) + \\
& \left((A - B + C) \cos \left[\frac{1}{2} (c + dx) \right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\
& \quad \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right)^2 \right)
\end{aligned}$$

■ **Problem 616: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 3, 161 leaves, 4 steps):

$$-\frac{(7A - 3B - C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{\sqrt{2} \sqrt{a + a \operatorname{Sec}[c + d x]}}\right]}{2\sqrt{2} a^{3/2} d} - \frac{(A - B + C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{2d (a + a \operatorname{Sec}[c + d x])^{3/2}} + \frac{(5A - B + C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{2ad \sqrt{a + a \operatorname{Sec}[c + d x]}}$$

Result (type 3, 578 leaves):

$$\begin{aligned} & \left(2 (7A - 3B - C) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & \left(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) - \\ & \left(2 (7A - 3B - C) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right]\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & \left(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \right) + \\ & \left((A - B + C) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & \left(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] \right)^2 \right) + \\ & \left((-A + B - C) \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \\ & \left(d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2} \left(\operatorname{Cos}\left[\frac{1}{4}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c + dx)\right] \right)^2 \right) + \\ & \frac{16A \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]}{d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{\operatorname{Sec}[c + dx]} (a (1 + \operatorname{Sec}[c + dx]))^{3/2}} \end{aligned}$$

■ **Problem 619: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^{5/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 3, 254 leaves, 8 steps):

$$\frac{(2B - 5C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{a^{5/2} d} + \frac{(3A - 43B + 115C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A - B + C) \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{4d (a + a \operatorname{Sec}[c+dx])^{5/2}} + \frac{(A + 7B - 15C) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{16ad (a + a \operatorname{Sec}[c+dx])^{3/2}} + \frac{(3A - 11B + 35C) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{16a^2 d \sqrt{a + a \operatorname{Sec}[c+dx]}}$$

Result (type 3, 1923 leaves):

$$- \left((1-i) \sqrt{2} \left((1+i) - i \sqrt{2} \right) \left((-6-2i) B - 2\sqrt{2} B + (15+5i) C + 5\sqrt{2} C \right) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/$$

$$\left((i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2} \right) +$$

$$\left((1-i) \sqrt{2} \left((1+i) + \sqrt{2} \right) \left((6-2i) B - 2\sqrt{2} B - (15-5i) C + 5\sqrt{2} C \right) \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]} \right] \right.$$

$$\left. \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/$$

$$\left((i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2} \right) +$$

$$\left((-3A + 43B - 115C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/$$

$$(2d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2}) +$$

$$\left((3A - 43B + 115C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/$$

$$(2d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2}) +$$

$$\left(4 (4B + 2i \sqrt{2} B - 10C - 5i \sqrt{2} C) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/$$

$$\left((i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2} \right) +$$

$$\left((1+i) \left((1+i) - i \sqrt{2} \right) \left((-6-2i) B - 2\sqrt{2} B + (15+5i) C + 5\sqrt{2} C \right) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right.$$

$$\left. \sqrt{\operatorname{Sec}[c+dx]} (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) \Bigg/ \left(\sqrt{2} (i + \sqrt{2}) d (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a (1 + \operatorname{Sec}[c+dx]))^{5/2} \right) -$$

$$\left((1+i) \left((1+i) + \sqrt{2} \right) \left((6-2i) B - 2\sqrt{2} B - (15-5i) C + 5\sqrt{2} C \right) \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] \right)$$

$$\begin{aligned}
& \left. \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \right) / \left(\sqrt{2} \left(i + \sqrt{2} \right) d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \right) + \\
& \left(\left(A - B + C \right) \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(4d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right) \right)^4 + \\
& \left(\left(3A - 11B + 19C \right) \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(4d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] - \sin \left[\frac{1}{4} (c + dx) \right] \right) \right)^2 + \\
& \left(\left(-A + B - C \right) \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(4d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right) \right)^4 + \\
& \left(\left(-3A + 11B - 19C \right) \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(4d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{4} (c + dx) \right] + \sin \left[\frac{1}{4} (c + dx) \right] \right) \right)^2 + \\
& \left(8C \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) - \\
& \left(8C \cos \left[\frac{1}{2} (c + dx) \right] \right)^5 \sqrt{\sec[c+dx]} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right) \Big/ \\
& \left(d \left(A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx] \right) \left(a \left(1 + \sec[c+dx] \right) \right)^{5/2} \left(\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right) \right)
\end{aligned}$$

- **Problem 620: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]^{3/2} \left(A + B \sec[c+dx] + C \sec[c+dx]^2 \right)}{\left(a + a \sec[c+dx] \right)^{5/2}} dx$$

Optimal (type 3, 201 leaves, 7 steps):

$$\frac{2 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{a^{5/2} d} + \frac{(5 A+3 B-43 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right]}{16 \sqrt{2} a^{5/2} d} -$$

$$\frac{(A-B+C) \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{4 d(a+a \operatorname{Sec}[c+d x])^{5/2}} + \frac{(5 A+3 B-11 C) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{16 a d(a+a \operatorname{Sec}[c+d x])^{3/2}}$$

Result (type 3, 1521 leaves):

$$- \left(4 \left((-1+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{-\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{\operatorname{Sec}[c+d x]} \right.$$

$$\left. (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) / \left(d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) -$$

$$\left(4 \left((-1+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) C \operatorname{ArcTan}\left[\frac{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]}{\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] + \sqrt{2} \operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]} \right] \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \right.$$

$$\left. \sqrt{\operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) / \left(d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left((-5 A-3 B+43 C) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right] \sqrt{\operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) /$$

$$\left(2 d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left((5 A+3 B-43 C) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{4}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{4}(c+d x)\right]\right] \sqrt{\operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) /$$

$$\left(2 d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left(8 \sqrt{2} C \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[\sqrt{2} + 2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \sqrt{\operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) /$$

$$\left(d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left(2 i \left((-1+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) C \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[2 - \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \sqrt{\operatorname{Sec}[c+d x]} \right.$$

$$\left. (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) / \left(d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left(2 i \left((-1+i) + \sqrt{2} \right) \left((1+i) + \sqrt{2} \right) C \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Log}\left[2 + \sqrt{2} \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \sqrt{2} \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \sqrt{\operatorname{Sec}[c+d x]} \right.$$

$$\left. (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) / \left(d(A+2 C+2 B \operatorname{Cos}[c+d x] + A \operatorname{Cos}[2 c+2 d x]) (a(1+\operatorname{Sec}[c+d x]))^{5/2} \right) +$$

$$\left((-A+B-C) \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{\operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \right) /$$

$$\begin{aligned}
& \left(4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \left(\cos \left[\frac{1}{4} (c + d x) \right] - \sin \left[\frac{1}{4} (c + d x) \right] \right)^4 \right) + \\
& \left((5 A + 3 B - 11 C) \cos \left[\frac{1}{2} (c + d x) \right]^5 \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \left(4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \left(\cos \left[\frac{1}{4} (c + d x) \right] - \sin \left[\frac{1}{4} (c + d x) \right] \right)^2 \right) + \\
& \left((A - B + C) \cos \left[\frac{1}{2} (c + d x) \right]^5 \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \left(4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \left(\cos \left[\frac{1}{4} (c + d x) \right] + \sin \left[\frac{1}{4} (c + d x) \right] \right)^4 \right) + \\
& \left((-5 A - 3 B + 11 C) \cos \left[\frac{1}{2} (c + d x) \right]^5 \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2) \right) / \\
& \left(4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \left(\cos \left[\frac{1}{4} (c + d x) \right] + \sin \left[\frac{1}{4} (c + d x) \right] \right)^2 \right)
\end{aligned}$$

■ **Problem 622: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + d x] + C \sec [c + d x]^2}{\sqrt{\sec [c + d x]} (a + a \sec [c + d x])^{5/2}} dx$$

Optimal (type 3, 211 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(75 A - 19 B - 5 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + d x]} \sin [c + d x]}{\sqrt{2} \sqrt{a + a \sec [c + d x]}} \right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{4 d (a + a \sec [c + d x])^{5/2}} - \\
& \frac{(13 A - 5 B - 3 C) \sqrt{\sec [c + d x]} \sin [c + d x]}{16 a d (a + a \sec [c + d x])^{3/2}} + \frac{(49 A - 9 B + C) \sqrt{\sec [c + d x]} \sin [c + d x]}{16 a^2 d \sqrt{a + a \sec [c + d x]}}
\end{aligned}$$

Result (type 3, 836 leaves):

$$\begin{aligned}
& \left((75 A - 19 B - 5 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad (2 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2}) + \\
& \left((-75 A + 19 B + 5 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Log}\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad (2 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2}) + \\
& \left((-A + B - C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((21 A - 13 B + 5 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) + \\
& \left((A - B + C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((-21 A + 13 B - 5 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4 d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) + \\
& \frac{32 A \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right]}{d (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) (a (1 + \sec[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 623: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c+dx] + C \sec[c+dx]^2}{\sec[c+dx]^{3/2} (a + a \sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 261 leaves, 6 steps):

$$\begin{aligned}
& \frac{(163 A - 75 B + 19 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\sec[c+dx]} \sin[c+dx]}{\sqrt{2} \sqrt{a+a \sec[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \sin[c+dx]}{4 d \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^{5/2}} - \\
& \frac{(17 A - 9 B + C) \sin[c+dx]}{16 a d \sqrt{\sec[c+dx]} (a + a \sec[c+dx])^{3/2}} + \frac{(95 A - 39 B + 15 C) \sin[c+dx]}{48 a^2 d \sqrt{\sec[c+dx]} \sqrt{a + a \sec[c+dx]}} - \frac{(299 A - 147 B + 27 C) \sqrt{\sec[c+dx]} \sin[c+dx]}{48 a^2 d \sqrt{a + a \sec[c+dx]}}
\end{aligned}$$

Result (type 3, 943 leaves):

$$\begin{aligned}
& \left((-163A + 75B - 19C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \log\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad (2d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2}) + \\
& \left((163A - 75B + 19C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \log\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad (2d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2}) + \\
& \left((A - B + C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((-29A + 21B - 13C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) + \\
& \left((-A + B - C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((29A - 21B + 13C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \right) / \\
& \quad \left(4d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) - \\
& \frac{16(5A - 2B) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right]}{d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2}} + \\
& \frac{16A \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) \sin\left[\frac{3}{2}(c+dx)\right]}{3d(A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a(1 + \sec[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 624: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c+dx] + C \sec[c+dx]^2}{\sec[c+dx]^{5/2} (a + a \sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 313 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(283 A - 163 B + 75 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+dx]}}\right]}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \operatorname{Sin}[c+dx]}{4 d \operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])^{5/2}} - \frac{(21 A - 13 B + 5 C) \operatorname{Sin}[c+dx]}{16 a d \operatorname{Sec}[c+dx]^{3/2} (a + a \operatorname{Sec}[c+dx])^{3/2}} + \\
& \frac{(157 A - 85 B + 45 C) \operatorname{Sin}[c+dx]}{80 a^2 d \operatorname{Sec}[c+dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c+dx]}} - \frac{(787 A - 475 B + 195 C) \operatorname{Sin}[c+dx]}{240 a^2 d \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a + a \operatorname{Sec}[c+dx]}} + \frac{(2671 A - 1495 B + 735 C) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{240 a^2 d \sqrt{a + a \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 3, 1053 leaves):

$$\begin{aligned}
& \left((283 A - 163 B + 75 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \log\left[\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad (2d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}) + \\
& \left((-283 A + 163 B - 75 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \log\left[\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right]\right] \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad (2d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}) + \\
& \left((-A+B-C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((37 A - 29 B + 21 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] - \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) + \\
& \left((A-B+C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^4 \right) + \\
& \left((-37 A + 29 B - 21 C) \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \right) / \\
& \quad \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2} \left(\cos\left[\frac{1}{4}(c+dx)\right] + \sin\left[\frac{1}{4}(c+dx)\right] \right)^2 \right) + \\
& \frac{16(10A-5B+2C)\cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right]}{d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}} - \\
& \frac{8(5A-2B)\cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \sin\left[\frac{3}{2}(c+dx)\right]}{3d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}} + \\
& \frac{8A\cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \sin\left[\frac{5}{2}(c+dx)\right]}{5d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a(1+\sec[c+dx]))^{5/2}}
\end{aligned}$$

■ **Problem 625: Result more than twice size of optimal antiderivative.**

$$\int (a + a \sec[c+dx])^{2/3} (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 6, 446 leaves, 10 steps):

$$\frac{3 C (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{5 d} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 1, \frac{13}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{7 d \sqrt{1 - \operatorname{Sec}[c + d x]}} +$$

$$\frac{3 (5 B + 2 C) (a + a \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{10 d (1 + \operatorname{Sec}[c + d x])} -$$

$$\left(3^{3/4} (5 B + 2 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{2/3} \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(10 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x]) \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 6, 7349 leaves) : Display of huge result suppressed!

■ **Problem 626: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 6, 390 leaves, 9 steps) :

$$\frac{3 C \operatorname{Tan}[c + d x]}{2 d (a + a \operatorname{Sec}[c + d x])^{1/3}} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{d \sqrt{1 - \operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{1/3}} -$$

$$\left(3^{3/4} (2 B - C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right.$$

$$\left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) /$$

$$\left(2 \times 2^{1/3} d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)$$

Result (type 6, 7485 leaves) : Display of huge result suppressed!

■ **Problem 627: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{4/3}} dx$$

Optimal (type 6, 402 leaves, 9 steps):

$$\begin{aligned} & -\frac{3(A - B + C) \operatorname{Tan}[c + d x]}{5 d (a + a \operatorname{Sec}[c + d x])^{4/3}} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{a d \sqrt{1 - \operatorname{Sec}[c + d x]} (a + a \operatorname{Sec}[c + d x])^{1/3}} + \\ & \left(3^{3/4} (A - B - 4 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\ & \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\ & \left(5 \times 2^{1/3} a d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) \end{aligned}$$

Result (type 6, 7586 leaves): Display of huge result suppressed!

■ **Problem 628: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{7/3}} dx$$

Optimal (type 6, 466 leaves, 10 steps):

$$\begin{aligned}
& - \frac{3 (A - B + C) \operatorname{Tan}[c + d x]}{11 d (a + a \operatorname{Sec}[c + d x])^{7/3}} - \frac{3 (4 A - 4 B - 7 C) \operatorname{Tan}[c + d x]}{55 a^2 d (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3}} - \\
& \frac{3 \sqrt{2} A \operatorname{AppellF1}\left[-\frac{5}{6}, \frac{1}{2}, 1, \frac{1}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] \operatorname{Tan}[c + d x]}{5 a^2 d \sqrt{1 - \operatorname{Sec}[c + d x]} (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(3^{3/4} (4 A - 4 B - 7 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(55 \times 2^{1/3} a^2 d (1 - \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 7682 leaves) : Display of huge result suppressed!

■ **Problem 629: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{4/3} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 839 leaves, 12 steps) :

$$\begin{aligned}
& \frac{3 a (7 B + 4 C) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{28 d} + \frac{1}{11 d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& 3 \sqrt{2} a \operatorname{AppellF1}\left[\frac{11}{6}, \frac{1}{2}, 1, \frac{17}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (1 + \operatorname{Sec}[c + d x]) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x] + \\
& \frac{3 C (a + a \operatorname{Sec}[c + d x])^{4/3} \operatorname{Tan}[c + d x]}{7 d} - \frac{15 (1 + \sqrt{3}) a (7 B + 4 C) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{28 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})} + \\
& \left(15 \times 3^{1/4} a (7 B + 4 C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] \right. \\
& \quad \left. (a + a \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(14 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(5 \times 3^{3/4} (1 - \sqrt{3}) a (7 B + 4 C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \quad \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(28 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 5459 leaves): Display of huge result suppressed!

■ **Problem 630: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^{1/3} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 786 leaves, 11 steps):

$$\begin{aligned}
& \frac{3 C (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d} + \frac{3 \sqrt{2} \operatorname{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, \frac{1}{2} (1 + \operatorname{Sec}[c + d x]), 1 + \operatorname{Sec}[c + d x]\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{5 d \sqrt{1 - \operatorname{Sec}[c + d x]}} \\
& \frac{3 (1 + \sqrt{3}) (4 B + C) (a + a \operatorname{Sec}[c + d x])^{1/3} \operatorname{Tan}[c + d x]}{4 d (1 + \operatorname{Sec}[c + d x])^{2/3} (2^{1/3} - (1 + \sqrt{3})) (1 + \operatorname{Sec}[c + d x])^{1/3}} + \\
& \left(3 \times 3^{1/4} (4 B + C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(2 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right) + \\
& \left(3^{3/4} (1 - \sqrt{3}) (4 B + C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1 - \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}{2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3}}\right], \frac{1}{4} (2 + \sqrt{3})\right] (a + a \operatorname{Sec}[c + d x])^{1/3} \right. \\
& \left. (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3}) \sqrt{\frac{2^{2/3} + 2^{1/3} (1 + \operatorname{Sec}[c + d x])^{1/3} + (1 + \operatorname{Sec}[c + d x])^{2/3}}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \operatorname{Tan}[c + d x] \right) / \\
& \left(4 \times 2^{2/3} d (1 - \operatorname{Sec}[c + d x]) (1 + \operatorname{Sec}[c + d x])^{2/3} \sqrt{-\frac{(1 + \operatorname{Sec}[c + d x])^{1/3} (2^{1/3} - (1 + \operatorname{Sec}[c + d x])^{1/3})}{(2^{1/3} - (1 + \sqrt{3}) (1 + \operatorname{Sec}[c + d x])^{1/3})^2}} \right)
\end{aligned}$$

Result (type 6, 5345 leaves): Display of huge result suppressed!

■ **Problem 631: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + a \operatorname{Sec}[c + d x])^{2/3}} dx$$

Optimal (type 6, 803 leaves, 11 steps):

$$\begin{aligned}
& - \frac{3(A-B+C) \operatorname{Tan}[c+dx]}{d(a+a \operatorname{Sec}[c+dx])^{2/3}} + \frac{3\sqrt{2} A \operatorname{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, \frac{1}{2}(1+\operatorname{Sec}[c+dx]), 1+\operatorname{Sec}[c+dx]\right] (a+a \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{5ad\sqrt{1-\operatorname{Sec}[c+dx]}} \\
& \frac{3(1+\sqrt{3})(A-B+2C)(a+a \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{ad(1+\operatorname{Sec}[c+dx])^{2/3} \left(2^{1/3} - (1+\sqrt{3}) (1+\operatorname{Sec}[c+dx])^{1/3}\right)} + \\
& \left(3 \times 2^{1/3} 3^{1/4} (A-B+2C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1-\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}{2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right. \\
& \quad \left. (a+a \operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3} + 2^{1/3}(1+\operatorname{Sec}[c+dx])^{1/3} + (1+\operatorname{Sec}[c+dx])^{2/3}}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \operatorname{Tan}[c+dx] \right) / \\
& \left(ad(1-\operatorname{Sec}[c+dx])(1+\operatorname{Sec}[c+dx])^{2/3} \sqrt{-\frac{(1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right)}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \right) + \\
& \left(3^{3/4} (1-\sqrt{3})(A-B+2C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1-\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}{2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] (a+a \operatorname{Sec}[c+dx])^{1/3} \right. \\
& \quad \left. \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3} + 2^{1/3}(1+\operatorname{Sec}[c+dx])^{1/3} + (1+\operatorname{Sec}[c+dx])^{2/3}}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \operatorname{Tan}[c+dx] \right) / \\
& \left(2^{2/3} ad(1-\operatorname{Sec}[c+dx])(1+\operatorname{Sec}[c+dx])^{2/3} \sqrt{-\frac{(1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right)}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \right)
\end{aligned}$$

Result (type 6, 5389 leaves): Display of huge result suppressed!

■ **Problem 632: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2}{(a+a \operatorname{Sec}[c+dx])^{5/3}} dx$$

Optimal (type 6, 856 leaves, 12 steps):

$$\begin{aligned}
& - \frac{3(A-B+C) \operatorname{Tan}[c+dx]}{7d(a+a \operatorname{Sec}[c+dx])^{5/3}} - \frac{3(2A-2B-5C) \operatorname{Tan}[c+dx]}{7ad(a+a \operatorname{Sec}[c+dx])^{2/3}} - \frac{3\sqrt{2} \operatorname{AppellF1}\left[-\frac{1}{6}, \frac{1}{2}, 1, \frac{5}{6}, \frac{1}{2}(1+\operatorname{Sec}[c+dx]), 1+\operatorname{Sec}[c+dx]\right] \operatorname{Tan}[c+dx]}{ad\sqrt{1-\operatorname{Sec}[c+dx]}(a+a \operatorname{Sec}[c+dx])^{2/3}} \\
& \frac{3(1+\sqrt{3})(2A-2B-5C)(1+\operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{7ad(a+a \operatorname{Sec}[c+dx])^{2/3} \left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)} + \\
& \left(3 \times 2^{1/3} 3^{1/4} (2A-2B-5C) \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1-\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}{2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right. \\
& \quad \left. (1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3} + 2^{1/3}(1+\operatorname{Sec}[c+dx])^{1/3} + (1+\operatorname{Sec}[c+dx])^{2/3}}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \operatorname{Tan}[c+dx] \right) / \\
& \left(7ad(1-\operatorname{Sec}[c+dx])(a+a \operatorname{Sec}[c+dx])^{2/3} \sqrt{-\frac{(1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right)}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \right) + \\
& \left(3^{3/4} (1-\sqrt{3})(2A-2B-5C) \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{2^{1/3} - (1-\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}{2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right. \\
& \quad \left. (1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right) \sqrt{\frac{2^{2/3} + 2^{1/3}(1+\operatorname{Sec}[c+dx])^{1/3} + (1+\operatorname{Sec}[c+dx])^{2/3}}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \operatorname{Tan}[c+dx] \right) / \\
& \left(7 \times 2^{2/3} ad(1-\operatorname{Sec}[c+dx])(a+a \operatorname{Sec}[c+dx])^{2/3} \sqrt{-\frac{(1+\operatorname{Sec}[c+dx])^{1/3} \left(2^{1/3} - (1+\operatorname{Sec}[c+dx])^{1/3}\right)}{\left(2^{1/3} - (1+\sqrt{3})(1+\operatorname{Sec}[c+dx])^{1/3}\right)^2}} \right)
\end{aligned}$$

Result (type 6, 5507 leaves): Display of huge result suppressed!

■ **Problem 633: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+dx]^m (a+a \operatorname{Sec}[c+dx])^n (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 6, 259 leaves, 8 steps):

$$\frac{C \operatorname{Sec}[c + d x]^{1+m} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d (1 + m + n)} + \frac{1}{d (1 + m + n)}$$

$$2^{\frac{3}{2}+n} (C n + B (1 + m + n)) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - m, -\frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right]$$

$$(1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x] + \frac{1}{d (1 + m + n)} 2^{\frac{1}{2}+n} (C (m - n) + A (1 + m + n) - B (1 + m + n))$$

$$\operatorname{AppellF1}\left[\frac{1}{2}, 1 - m, \frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right] (1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]$$

Result (type 8, 43 leaves):

$$\int \operatorname{Sec}[c + d x]^m (a + a \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 634: Unable to integrate problem.**

$$\int \operatorname{Sec}[c + d x]^{-1-n} (a + a \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 258 leaves, 8 steps):

$$\frac{A \operatorname{Sec}[c + d x]^{-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x]}{d (1 + n)} +$$

$$\left((A n + B (1 + n) - C (1 + n)) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - n, -n, 1 - n, -\frac{2 \operatorname{Sec}[c + d x]}{1 - \operatorname{Sec}[c + d x]}\right] \operatorname{Sec}[c + d x]^{1-n} \right.$$

$$\left. \left(\frac{1 + \operatorname{Sec}[c + d x]}{1 - \operatorname{Sec}[c + d x]} \right)^{\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Sin}[c + d x] \right) / (d n (1 + n) (1 + \operatorname{Sec}[c + d x])) + \frac{1}{d}$$

$$2^{\frac{3}{2}+n} C \operatorname{AppellF1}\left[\frac{1}{2}, 1 + n, -\frac{1}{2} - n, \frac{3}{2}, 1 - \operatorname{Sec}[c + d x], \frac{1}{2} (1 - \operatorname{Sec}[c + d x])\right] (1 + \operatorname{Sec}[c + d x])^{-\frac{1}{2}-n} (a + a \operatorname{Sec}[c + d x])^n \operatorname{Tan}[c + d x]$$

Result (type 8, 47 leaves):

$$\int \operatorname{Sec}[c + d x]^{-1-n} (a + a \operatorname{Sec}[c + d x])^n (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

■ **Problem 636: Result more than twice size of optimal antiderivative.**

$$\int (a + a \operatorname{Sec}[c + d x])^m (B - C + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 6, 171 leaves, 8 steps):

$$\left(\sqrt{2} (B-C) \operatorname{AppellF1} \left[\frac{3}{2} + m, \frac{1}{2}, 1, \frac{5}{2} + m, \frac{1}{2} (1 + \operatorname{Sec}[c + dx]), 1 + \operatorname{Sec}[c + dx] \right] (1 + \operatorname{Sec}[c + dx]) (a + a \operatorname{Sec}[c + dx])^m \operatorname{Tan}[c + dx] \right) /$$

$$\left(d (3 + 2m) \sqrt{1 - \operatorname{Sec}[c + dx]} \right) + \frac{1}{d}$$

$$2^{\frac{3}{2} + m} C \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{1}{2} - m, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + dx]) \right] (1 + \operatorname{Sec}[c + dx])^{-\frac{1}{2} - m} (a + a \operatorname{Sec}[c + dx])^m \operatorname{Tan}[c + dx]$$

Result (type 6, 2582 leaves):

$$\left(2^{1+m} \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right] \operatorname{Cos}[c + dx] \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec}[c + dx] \right)^m (1 + \operatorname{Sec}[c + dx])^{-1-m} (a (1 + \operatorname{Sec}[c + dx]))^{1+m} (B - C + C \operatorname{Sec}[c + dx]) \right.$$

$$\left. (2 C \operatorname{Sec}[c + dx]^2 (1 + \operatorname{Sec}[c + dx])^m + \operatorname{Sec}[c + dx] (2 B (1 + \operatorname{Sec}[c + dx])^m - 2 C (1 + \operatorname{Sec}[c + dx])^m) \right) \operatorname{Sin} \left[\frac{1}{2} (c + dx) \right]$$

$$\left(\left((B - C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + 2 C \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \right.$$

$$\left. \left(\operatorname{Cos}[c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \right)^m + \left(3 (B - C) \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^2 \right) \right) /$$

$$\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] - \right.$$

$$\left. m \operatorname{AppellF1} \left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right) \right) /$$

$$\left(a d (C + B \operatorname{Cos}[c + dx] - C \operatorname{Cos}[c + dx]) \left(2^m \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec}[c + dx] \right)^m \right. \right.$$

$$\left. \left(\left((B - C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + 2 C \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \right. \right.$$

$$\left. \left(\operatorname{Cos}[c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \right)^m + \left(3 (B - C) \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^2 \right) \right) /$$

$$\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] - 2 \left(\operatorname{AppellF1} \left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] - \right.$$

$$\left. m \operatorname{AppellF1} \left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2, -\operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right) \right) +$$

$$2^{1+m} \left(\operatorname{Cos} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sec}[c + dx] \right)^m \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \left(m \left((B - C) \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 1 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] + \right. \right.$$

$$\left. 2 C \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, 2 + m, \frac{3}{2}, \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 \right] \right) \left(\operatorname{Cos}[c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \right)^{-1+m}$$

$$\left(-\operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Sin}[c + dx] + \operatorname{Cos}[c + dx] \operatorname{Sec} \left[\frac{1}{2} (c + dx) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right] \right) -$$

$$\begin{aligned} & \frac{1}{2} (B - C) \operatorname{Csc}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \left(1 - \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right)^{-1-m} \right) \right) + \\ & 2^{1+m} m \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx] \right)^{-1+m} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \left(\left((B - C) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] + \right. \right. \\ & \quad \left. \left. 2 C \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 2 + m, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \left(\operatorname{Cos}[c + dx] \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^2 \right)^m + \right. \\ & \quad \left. \left(3 (B - C) \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) / \\ & \quad \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, m, 1, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] - 2 \left(\operatorname{AppellF1}\left[\frac{3}{2}, m, 2, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right. \right. \\ & \quad \left. \left. m \operatorname{AppellF1}\left[\frac{3}{2}, 1 + m, 1, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 \right) \right) \\ & \quad \left. \left(-\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] \operatorname{Sec}[c + dx] \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right]^2 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx] \right) \right) \right) \end{aligned}$$

■ **Problem 637: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^3 (a + b \operatorname{Sec}[c + dx]) (A + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 140 leaves, 7 steps):

$$\frac{a(4A + 3C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} + \frac{b(5A + 4C) \operatorname{Tan}[c + dx]}{5d} + \frac{a(4A + 3C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{8d} +$$

$$\frac{aC \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{4d} + \frac{bC \operatorname{Sec}[c + dx]^4 \operatorname{Tan}[c + dx]}{5d} + \frac{b(5A + 4C) \operatorname{Tan}[c + dx]^3}{15d}$$

Result (type 3, 426 leaves):

$$\begin{aligned} & -\frac{aA \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{2d} - \frac{3aC \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \\ & \frac{aA \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{3aC \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right]}{8d} + \frac{aC}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} + \\ & \frac{aA}{4d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{3aC}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{aC}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^4} - \\ & \frac{aA}{4d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{3aC}{16d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{2Ab \operatorname{Tan}[c + dx]}{3d} + \\ & \frac{8bC \operatorname{Tan}[c + dx]}{15d} + \frac{Ab \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3d} + \frac{4bC \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{15d} + \frac{bC \operatorname{Sec}[c + dx]^4 \operatorname{Tan}[c + dx]}{5d} \end{aligned}$$

■ **Problem 638: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[c + d x]^2 (a + b \text{Sec}[c + d x]) (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{b(4A + 3C) \text{ArcTanh}[\text{Sin}[c + d x]]}{8d} + \frac{a(3A + 2C) \text{Tan}[c + d x]}{3d} +$$

$$\frac{b(4A + 3C) \text{Sec}[c + d x] \text{Tan}[c + d x]}{8d} + \frac{aC \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3d} + \frac{bC \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{4d}$$

Result (type 3, 377 leaves):

$$-\frac{Ab \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)]]}{2d} - \frac{3bC \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)]]}{8d} + \frac{Ab \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)]]}{2d} +$$

$$\frac{3bC \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)]]}{8d} + \frac{bC}{16d (\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)])^4} + \frac{Ab}{4d (\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)])^2} +$$

$$\frac{3bC}{16d (\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)])^2} - \frac{bC}{16d (\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)])^4} - \frac{Ab}{4d (\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)])^2} -$$

$$\frac{3bC}{16d (\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)])^2} + \frac{aA \text{Tan}[c + dx]}{d} + \frac{2aC \text{Tan}[c + dx]}{3d} + \frac{aC \text{Sec}[c + dx]^2 \text{Tan}[c + dx]}{3d}$$

■ **Problem 640: Result more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[c + d x]) (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$aAx + \frac{b(2A + C) \text{ArcTanh}[\text{Sin}[c + d x]]}{2d} + \frac{aC \text{Tan}[c + d x]}{d} + \frac{bC \text{Sec}[c + d x] \text{Tan}[c + d x]}{2d}$$

Result (type 3, 218 leaves):

$$aAx - \frac{Ab \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] - \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d} + \frac{Ab \text{Log}[\text{Cos}[\frac{c}{2} + \frac{dx}{2}] + \text{Sin}[\frac{c}{2} + \frac{dx}{2}]]}{d} -$$

$$\frac{bC \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)]]}{2d} + \frac{bC \text{Log}[\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)]]}{2d} +$$

$$\frac{bC}{4d (\text{Cos}[\frac{1}{2}(c + dx)] - \text{Sin}[\frac{1}{2}(c + dx)])^2} - \frac{bC}{4d (\text{Cos}[\frac{1}{2}(c + dx)] + \text{Sin}[\frac{1}{2}(c + dx)])^2} + \frac{aC \text{Tan}[c + dx]}{d}$$

■ **Problem 641: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x] (a + b \sec [c + d x]) (A + C \sec [c + d x]^2) dx$$

Optimal (type 3, 42 leaves, 5 steps):

$$A b x + \frac{a C \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{a A \sin [c + d x]}{d} + \frac{b C \tan [c + d x]}{d}$$

Result (type 3, 112 leaves):

$$A b x - \frac{a C \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a C \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] + \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a A \cos [d x] \sin [c]}{d} + \frac{a A \cos [c] \sin [d x]}{d} + \frac{b C \tan [c + d x]}{d}$$

■ **Problem 642: Result more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^2 (a + b \sec [c + d x]) (A + C \sec [c + d x]^2) dx$$

Optimal (type 3, 58 leaves, 5 steps):

$$\frac{1}{2} a (A + 2 C) x + \frac{b C \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{A b \sin [c + d x]}{d} + \frac{a A \cos [c + d x] \sin [c + d x]}{2 d}$$

Result (type 3, 131 leaves):

$$a C x + \frac{a A (c + d x)}{2 d} - \frac{b C \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] - \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{b C \operatorname{Log}\left[\cos \left[\frac{c}{2} + \frac{d x}{2}\right] + \sin \left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{A b \cos [d x] \sin [c]}{d} + \frac{A b \cos [c] \sin [d x]}{d} + \frac{a A \sin [2 (c + d x)]}{4 d}$$

■ **Problem 647: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x] (a + b \sec [c + d x])^2 (A + C \sec [c + d x]^2) dx$$

Optimal (type 3, 170 leaves, 7 steps):

$$\frac{(4 a^2 (2 A + C) + b^2 (4 A + 3 C)) \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{a (12 A b^2 - a^2 C + 8 b^2 C) \tan [c + d x]}{6 b d} - \frac{(2 a^2 C - 3 b^2 (4 A + 3 C)) \sec [c + d x] \tan [c + d x]}{24 d} - \frac{a C (a + b \sec [c + d x])^2 \tan [c + d x]}{12 b d} + \frac{C (a + b \sec [c + d x])^3 \tan [c + d x]}{4 b d}$$

Result (type 3, 1123 leaves):

$$\begin{aligned}
& \left((-8 a^2 A - 4 A b^2 - 4 a^2 C - 3 b^2 C) \cos [c + d x]^4 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \right) / \\
& \quad (4 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x])) + \\
& \left((8 a^2 A + 4 A b^2 + 4 a^2 C + 3 b^2 C) \cos [c + d x]^4 \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \right) / \\
& \quad (4 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x])) + \\
& \quad \frac{b^2 C \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2)}{8 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^4} + \\
& \quad \frac{(12 A b^2 + 12 a^2 C + 8 a b C + 9 b^2 C) \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2)}{24 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \\
& \quad \frac{2 a b C \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \sin \left[\frac{1}{2} (c + d x) \right]}{3 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^3} - \\
& \quad \frac{b^2 C \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2)}{8 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^4} + \\
& \quad \frac{2 a b C \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \sin \left[\frac{1}{2} (c + d x) \right]}{3 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^3} + \\
& \quad \frac{(-12 A b^2 - 12 a^2 C - 8 a b C - 9 b^2 C) \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2)}{24 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} + \\
& \left(4 \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \left(3 a A b \sin \left[\frac{1}{2} (c + d x) \right] + 2 a b C \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) / \\
& \quad \left(3 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) + \\
& \left(4 \cos [c + d x]^4 (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) \left(3 a A b \sin \left[\frac{1}{2} (c + d x) \right] + 2 a b C \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) / \\
& \quad \left(3 d (b + a \cos [c + d x])^2 (A + 2 C + A \cos [2 c + 2 d x]) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right) \right)
\end{aligned}$$

■ **Problem 648: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec} [c + d x])^2 (A + C \operatorname{Sec} [c + d x]^2) dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$a^2 A x + \frac{a b (2 A + C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{(3 A b^2 + 2 (a^2 + b^2) C) \operatorname{Tan}[c + d x]}{3 d} + \frac{a b C \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{3 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 242 leaves):

$$\frac{1}{12 d} \operatorname{Sec}[c + d x]^3 \left(9 a \operatorname{Cos}[c + d x] \left(a A (c + d x) - b (2 A + C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + b (2 A + C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + \right. \\ \left. 3 a \operatorname{Cos}[3 (c + d x)] \left(a A (c + d x) - b (2 A + C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + b (2 A + C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + \right. \\ \left. 2 (3 A b^2 + 3 a^2 C + 4 b^2 C + 6 a b C \operatorname{Cos}[c + d x] + (3 A b^2 + 3 a^2 C + 2 b^2 C) \operatorname{Cos}[2 (c + d x)]) \operatorname{Sin}[c + d x] \right)$$

■ **Problem 649: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x])^2 dx$$

Optimal (type 3, 109 leaves, 6 steps):

$$2 a A b x + \frac{(2 A b^2 + (2 a^2 + b^2) C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{A (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Sin}[c + d x]}{d} - \frac{2 a b (A - C) \operatorname{Tan}[c + d x]}{d} - \frac{b^2 (2 A - C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 352 leaves):

$$\frac{1}{4 d} \operatorname{Sec}[c + d x]^2 \left(4 a A b c + 4 a A b d x - 2 A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - 2 a^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \right. \\ \left. b^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + 2 A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \right. \\ \left. 2 a^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + b^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \right. \\ \left. \operatorname{Cos}[2 (c + d x)] \left(4 a A b (c + d x) - (2 A b^2 + (2 a^2 + b^2) C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] + \right. \right. \\ \left. \left. (2 A b^2 + 2 a^2 C + b^2 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) + (a^2 A + 2 b^2 C) \operatorname{Sin}[c + d x] + 4 a b C \operatorname{Sin}[2 (c + d x)] + a^2 A \operatorname{Sin}[3 (c + d x)] \right)$$

■ **Problem 656: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x])^2 dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\begin{aligned}
 & a^3 A x + \frac{b (12 a^2 (2 A + C) + b^2 (4 A + 3 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a (6 A b^2 + (a^2 + 4 b^2) C) \operatorname{Tan}[c + d x]}{2 d} + \\
 & \frac{b (2 a^2 C + b^2 (4 A + 3 C)) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{a C (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{4 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}
 \end{aligned}$$

Result (type 3, 1241 leaves):

$$\begin{aligned}
& \frac{2 a^3 A (c+d x) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} + \\
& \left(\frac{(-24 a^2 A b-4 A b^3-12 a^2 b C-3 b^3 C) \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{4 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} \right) + \\
& \left(\frac{(24 a^2 A b+4 A b^3+12 a^2 b C+3 b^3 C) \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{4 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} \right) + \\
& \frac{b^3 C \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^4} + \\
& \frac{(4 A b^3+12 a^2 b C+4 a b^2 C+3 b^3 C) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^2} + \\
& \frac{a b^2 C \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^3} - \\
& \frac{b^3 C \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^4} + \\
& \frac{a b^2 C \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^3} + \\
& \frac{(-4 A b^3-12 a^2 b C-4 a b^2 C-3 b^3 C) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right])^2} + \\
& \left(\frac{2 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \left(3 a A b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+a^3 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+2 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right) + \\
& \left(\frac{2 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \left(3 a A b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+a^3 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+2 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{d (b+a \operatorname{Cos}[c+d x])^3 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right)
\end{aligned}$$

■ **Problem 665: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 227 leaves, 8 steps) :

$$a^4 A x + \frac{a b (4 a^2 (2 A + C) + b^2 (4 A + 3 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} +$$

$$\frac{(6 a^4 C + 2 b^4 (5 A + 4 C) + a^2 b^2 (85 A + 56 C)) \operatorname{Tan}[c + d x]}{15 d} + \frac{a b (40 A b^2 + 6 a^2 C + 29 b^2 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{30 d} +$$

$$\frac{(3 a^2 C + b^2 (5 A + 4 C)) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{15 d} + \frac{a C (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{5 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{5 d}$$

Result (type 3, 503 leaves) :

$$\frac{1}{120 d (A + 2 C + A \operatorname{Cos}[2 (c + d x)])} (C + A \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^5$$

$$\left(150 a^4 A (c + d x) \operatorname{Cos}[c + d x] + 75 a^4 A (c + d x) \operatorname{Cos}[3 (c + d x)] + 15 a^4 A c \operatorname{Cos}[5 (c + d x)] + 15 a^4 A d x \operatorname{Cos}[5 (c + d x)] - \right.$$

$$120 a b (4 a^2 (2 A + C) + b^2 (4 A + 3 C)) \operatorname{Cos}[c + d x]^5 \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right]\right] \right) +$$

$$180 a^2 A b^2 \operatorname{Sin}[c + d x] + 40 A b^4 \operatorname{Sin}[c + d x] + 30 a^4 C \operatorname{Sin}[c + d x] + 240 a^2 b^2 C \operatorname{Sin}[c + d x] + 80 b^4 C \operatorname{Sin}[c + d x] + 120 a A b^3 \operatorname{Sin}[2 (c + d x)] +$$

$$120 a^3 b C \operatorname{Sin}[2 (c + d x)] + 210 a b^3 C \operatorname{Sin}[2 (c + d x)] + 270 a^2 A b^2 \operatorname{Sin}[3 (c + d x)] + 50 A b^4 \operatorname{Sin}[3 (c + d x)] + 45 a^4 C \operatorname{Sin}[3 (c + d x)] +$$

$$300 a^2 b^2 C \operatorname{Sin}[3 (c + d x)] + 40 b^4 C \operatorname{Sin}[3 (c + d x)] + 60 a A b^3 \operatorname{Sin}[4 (c + d x)] + 60 a^3 b C \operatorname{Sin}[4 (c + d x)] + 45 a b^3 C \operatorname{Sin}[4 (c + d x)] +$$

$$\left. 90 a^2 A b^2 \operatorname{Sin}[5 (c + d x)] + 10 A b^4 \operatorname{Sin}[5 (c + d x)] + 15 a^4 C \operatorname{Sin}[5 (c + d x)] + 60 a^2 b^2 C \operatorname{Sin}[5 (c + d x)] + 8 b^4 C \operatorname{Sin}[5 (c + d x)] \right)$$

■ **Problem 666: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^4 (A + C \operatorname{Sec}[c + d x])^2 dx$$

Optimal (type 3, 229 leaves, 8 steps) :

$$4 a^3 A b x + \frac{(8 a^4 C + 24 a^2 b^2 (2 A + C) + b^4 (4 A + 3 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{A (a + b \operatorname{Sec}[c + d x])^4 \operatorname{Sin}[c + d x]}{d} -$$

$$\frac{a b (a^2 (12 A - 19 C) - 8 b^2 (3 A + 2 C)) \operatorname{Tan}[c + d x]}{6 d} - \frac{b^2 (a^2 (24 A - 26 C) - 3 b^2 (4 A + 3 C)) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{24 d} -$$

$$\frac{a b (12 A - 7 C) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} - \frac{b (4 A - C) (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 1357 leaves) :

$$\begin{aligned}
& \frac{8 a^3 A b (c+d x) \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} + \\
& \left(\frac{(-48 a^2 A b^2 - 4 A b^4 - 8 a^4 C - 24 a^2 b^2 C - 3 b^4 C) \operatorname{Cos}[c+d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{4 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} \right) + \\
& \left(\frac{(48 a^2 A b^2 + 4 A b^4 + 8 a^4 C + 24 a^2 b^2 C + 3 b^4 C) \operatorname{Cos}[c+d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{4 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x])} \right) + \\
& \frac{b^4 C \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\
& \frac{(12 A b^4 + 72 a^2 b^2 C + 16 a b^3 C + 9 b^4 C) \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{24 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \frac{4 a b^3 C \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{3 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} - \\
& \frac{b^4 C \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\
& \frac{4 a b^3 C \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{3 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \\
& \frac{(-12 A b^4 - 72 a^2 b^2 C - 16 a b^3 C - 9 b^4 C) \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2)}{24 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} + \\
& \left(\frac{8 \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) \left(3 a A b^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 3 a^3 b C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 2 a b^3 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{3 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right) + \\
& \left(\frac{8 \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) \left(3 a A b^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 3 a^3 b C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] + 2 a b^3 C \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{3 d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right) + \\
& \frac{2 a^4 A \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}[c+d x]}{d (b+a \operatorname{Cos}[c+d x])^4 (A+2 C+A \operatorname{Cos}[2 c+2 d x])}
\end{aligned}$$

■ **Problem 667: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+b \operatorname{Sec}[c+d x])^4 (A+C \operatorname{Sec}[c+d x])^2 dx$$

Optimal (type 3, 219 leaves, 8 steps):

$$\frac{1}{2} a^2 (12 A b^2 + a^2 (A+2 C)) x + \frac{2 a b (2 A b^2 + (2 a^2 + b^2) C) \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} + \frac{2 A b (a+b \operatorname{Sec}[c+d x])^3 \operatorname{Sin}[c+d x]}{d} +$$

$$\frac{A \cos [c+d x] (a+b \operatorname{Sec}[c+d x])^4 \operatorname{Sin}[c+d x]}{2 d} - \frac{b^2 (a^2 (39 A-34 C)-2 b^2 (3 A+2 C)) \operatorname{Tan}[c+d x]}{6 d} -$$

$$\frac{a b^3 (9 A-4 C) \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{3 d} - \frac{b^2 (15 A-2 C) (a+b \operatorname{Sec}[c+d x])^2 \operatorname{Tan}[c+d x]}{6 d}$$

Result (type 3, 864 leaves):

$$a^2 (a^2 A+12 A b^2+2 a^2 C) (c+d x) \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4$$

$$\frac{2 d (b+a \cos [c+d x])^4}{2 d (b+a \cos [c+d x])^4} -$$

$$\frac{2 (2 a A b^3+2 a^3 b C+a b^3 C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^4}{d (b+a \cos [c+d x])^4} +$$

$$\frac{2 (2 a A b^3+2 a^3 b C+a b^3 C) \cos [c+d x]^4 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^4}{d (b+a \cos [c+d x])^4} +$$

$$\frac{(12 a b^3 C+b^4 C) \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4}{12 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + \frac{b^4 C \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \sin \left[\frac{1}{2}(c+d x)\right]}{6 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^3} +$$

$$\frac{b^4 C \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \sin \left[\frac{1}{2}(c+d x)\right]}{6 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^3} + \frac{(-12 a b^3 C-b^4 C) \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4}{12 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} +$$

$$\left(\cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \left(3 A b^4 \sin \left[\frac{1}{2}(c+d x)\right]+18 a^2 b^2 C \sin \left[\frac{1}{2}(c+d x)\right]+2 b^4 C \sin \left[\frac{1}{2}(c+d x)\right]\right)\right) /$$

$$\left(3 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)\right) +$$

$$\left(\cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \left(3 A b^4 \sin \left[\frac{1}{2}(c+d x)\right]+18 a^2 b^2 C \sin \left[\frac{1}{2}(c+d x)\right]+2 b^4 C \sin \left[\frac{1}{2}(c+d x)\right]\right)\right) /$$

$$\left(3 d (b+a \cos [c+d x])^4 \left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)\right) +$$

$$\frac{4 a^3 A b \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \operatorname{Sin}[c+d x]}{d (b+a \cos [c+d x])^4} + \frac{a^4 A \cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^4 \operatorname{Sin}[2(c+d x)]}{4 d (b+a \cos [c+d x])^4}$$

■ **Problem 673: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 (a^2 - b^2 \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 158 leaves, 8 steps):

$$a^5 x + \frac{b (24 a^4 - 8 a^2 b^2 - 3 b^4) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{a b^2 (5 a^2 - 4 b^2) \operatorname{Tan}[c + d x]}{2 d} +$$

$$\frac{b^3 (2 a^2 - 3 b^2) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} - \frac{a b^2 (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{4 d} - \frac{b^2 (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 1299 leaves):

$$\begin{aligned}
& \frac{2 a^5 (c+d x) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x])} + \\
& \left(\frac{(-24 a^4 b+8 a^2 b^3+3 b^5) \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{(4 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]))} \right) / \\
& \left(\frac{(24 a^4 b-8 a^2 b^3-3 b^5) \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{(4 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]))} \right) - \\
& \frac{b^5 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} + \\
& \frac{(-8 a^2 b^3-4 a b^4-3 b^5) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\
& \frac{a b^4 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \\
& \frac{b^5 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4} - \\
& \frac{a b^4 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]}{d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^3} + \\
& \frac{(8 a^2 b^3+4 a b^4+3 b^5) \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2)}{8 d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2} - \\
& \left(\frac{4 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2) \left(-a^3 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+a b^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right) / \\
& \left(\frac{4 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Sec}[c+d x])^3 (a^2-b^2 \operatorname{Sec}[c+d x]^2) \left(-a^3 b^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]+a b^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{d (b+a \operatorname{Cos}[c+d x])^3 (a^2-2 b^2+a^2 \operatorname{Cos}[2 c+2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)} \right) /
\end{aligned}$$

■ **Problem 675: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+d x]) (a^2-b^2 \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 75 leaves, 6 steps):

$$a^3 x + \frac{b (2 a^2 - b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} - \frac{a b^2 \operatorname{Tan}[c + d x]}{2 d} - \frac{b^2 (a + b \operatorname{Sec}[c + d x]) \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 230 leaves):

$$a^3 x - \frac{a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} + \frac{a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} +$$

$$\frac{b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} -$$

$$\frac{a b^2 \operatorname{Tan}[c + d x]}{d} - \frac{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{b^3} + \frac{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2}{b^3}$$

- **Problem 676: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3 (A + C \operatorname{Sec}[c + d x]^2)}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 186 leaves, 8 steps):

$$- \frac{a (2 A b^2 + (2 a^2 + b^2) C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 b^4 d} + \frac{2 a^2 (A b^2 + a^2 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^4 \sqrt{a+b} d} +$$

$$\frac{(3 a^2 C + b^2 (3 A + 2 C)) \operatorname{Tan}[c + d x]}{3 b^3 d} - \frac{a C \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 b^2 d} + \frac{C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 b d}$$

Result (type 3, 657 leaves):

$$\frac{1}{6 b^4 d (A + 2 C + A \cos [2 (c + d x)]) (a + b \sec [c + d x])} \cos [c + d x] (b + a \cos [c + d x]) (A + C \sec [c + d x])^2$$

$$\left(6 a (2 A b^2 + (2 a^2 + b^2) C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] - 6 a (2 A b^2 + (2 a^2 + b^2) C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] - \right.$$

$$\frac{24 i a^2 (A b^2 + a^2 C) \operatorname{ArcTan} \left[\frac{(i \cos [c] + \sin [c]) (a \sin [c] + (-b + a \cos [c]) \tan \left[\frac{d x}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos [c] - i \sin [c])^2}} \right] (\cos [c] - i \sin [c])}{\sqrt{a^2 - b^2} \sqrt{(\cos [c] - i \sin [c])^2}} +$$

$$\frac{2 b^3 C \sin \left[\frac{d x}{2} \right]}{(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right])^3} + \frac{b^2 C ((-3 a + b) \cos \left[\frac{c}{2} \right] + (3 a + b) \sin \left[\frac{c}{2} \right])}{(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right])^2} +$$

$$\frac{4 b (3 A b^2 + 3 a^2 C + 2 b^2 C) \sin \left[\frac{d x}{2} \right]}{(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right])} + \frac{2 b^3 C \sin \left[\frac{d x}{2} \right]}{(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right])^3} +$$

$$\left. \frac{b^2 C ((3 a - b) \cos \left[\frac{c}{2} \right] + (3 a + b) \sin \left[\frac{c}{2} \right])}{(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right])^2} + \frac{4 b (3 A b^2 + 3 a^2 C + 2 b^2 C) \sin \left[\frac{d x}{2} \right]}{(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right])} \right)$$

- **Problem 677: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^2 (A + C \sec [c + d x])^2}{a + b \sec [c + d x]} dx$$

Optimal (type 3, 137 leaves, 7 steps):

$$\frac{(2 a^2 C + b^2 (2 A + C)) \operatorname{ArcTanh}[\sin [c + d x]]}{2 b^3 d} - \frac{2 a (A b^2 + a^2 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{\sqrt{a-b} b^3 \sqrt{a+b} d} - \frac{a C \tan [c + d x]}{b^2 d} + \frac{C \sec [c + d x] \tan [c + d x]}{2 b d}$$

Result (type 3, 428 leaves):

$$\frac{1}{2 b^3 d (A + 2 C + A \cos [2 (c + d x)]) (a + b \sec [c + d x]) (A + C \sec [c + d x])^2} \cos [c + d x] (b + a \cos [c + d x]) (A + C \sec [c + d x])^2$$

$$\left(-2 (2 A b^2 + (2 a^2 + b^2) C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right] + 2 (2 A b^2 + (2 a^2 + b^2) C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right] \right) +$$

$$\frac{8 a (A b^2 + a^2 C) \operatorname{ArcTan} \left[\frac{(i \cos [c] + \sin [c]) (a \sin [c] + (-b + a \cos [c]) \tan \left[\frac{d x}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos [c] - i \sin [c])^2}} \right] (i \cos [c] + \sin [c])}{\sqrt{a^2 - b^2} \sqrt{(\cos [c] - i \sin [c])^2}} +$$

$$\frac{b^2 c}{\left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} - \frac{4 a b C \sin \left[\frac{d x}{2} \right]}{\left(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] - \sin \left[\frac{1}{2} (c + d x) \right] \right)} -$$

$$\left. \frac{b^2 c}{\left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)^2} - \frac{4 a b C \sin \left[\frac{d x}{2} \right]}{\left(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right] \right) \left(\cos \left[\frac{1}{2} (c + d x) \right] + \sin \left[\frac{1}{2} (c + d x) \right] \right)} \right)$$

■ **Problem 678: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x] (A + C \sec [c + d x])^2}{a + b \sec [c + d x]} dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$-\frac{a C \operatorname{ArcTanh}[\sin [c + d x]]}{b^2 d} + \frac{2 (A b^2 + a^2 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right]}{\sqrt{a-b} b^2 \sqrt{a+b} d} + \frac{C \tan [c + d x]}{b d}$$

Result (type 3, 331 leaves):

$$\begin{aligned}
& \frac{1}{b^2 d (A + 2C + A \cos[2(c + dx)]) (a + b \sec[c + dx])} \\
& 2 \cos[c + dx] (b + a \cos[c + dx]) (A + C \sec[c + dx])^2 \left(a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right] - \right. \\
& \left. a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right] - \frac{2 i (A b^2 + a^2 C) \operatorname{ArcTan} \left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan \left[\frac{dx}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} \right] (\cos[c] - i \sin[c])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} + \right. \\
& \left. \frac{b C \sin \left[\frac{dx}{2} \right]}{(\cos \left[\frac{c}{2} \right] - \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right])} + \frac{b C \sin \left[\frac{dx}{2} \right]}{(\cos \left[\frac{c}{2} \right] + \sin \left[\frac{c}{2} \right]) (\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right])} \right)
\end{aligned}$$

- **Problem 679: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{a + b \sec[c + dx]} dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$\frac{Ax}{a} + \frac{C \operatorname{ArcTanh}[\sin[c + dx]]}{bd} - \frac{2(Ab^2 + a^2C) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2} (c + dx) \right]}{\sqrt{a+b}} \right]}{a \sqrt{a-b} b \sqrt{a+b} d}$$

Result (type 3, 239 leaves):

$$\begin{aligned}
& \left(2 (C + A \cos[c + dx])^2 \right. \\
& \left. \left(\sqrt{a^2 - b^2} \left(A b dx - a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right] + a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right] \right) \sqrt{(\cos[c] - i \sin[c])^2} + \right. \\
& \left. \left. 2 (A b^2 + a^2 C) \operatorname{ArcTan} \left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan \left[\frac{dx}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} \right] (i \cos[c] + \sin[c]) \right] \right) \Bigg) / \\
& \left(a b \sqrt{a^2 - b^2} d (A + 2C + A \cos[2(c + dx)]) \sqrt{(\cos[c] - i \sin[c])^2} \right)
\end{aligned}$$

■ **Problem 685: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^2 (A + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 3, 153 leaves, 7 steps):

$$-\frac{2 a C \operatorname{ArcTanh}[\sin[c + dx]]}{b^3 d} - \frac{2 (A b^4 - 2 a^4 C + 3 a^2 b^2 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{3/2} b^3 (a+b)^{3/2} d} + \frac{C \tan[c + dx]}{b^2 d} + \frac{a (A b^2 + a^2 C) \tan[c + dx]}{b^2 (a^2 - b^2) d (a + b \sec[c + dx])}$$

Result (type 3, 336 leaves):

$$\frac{1}{b^3 d (A + 2 C + A \cos[2(c + dx)]) (a + b \sec[c + dx])^2} \left(\frac{2 (A b^4 - 2 a^4 C + 3 a^2 b^2 C) \operatorname{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right] (b + a \cos[c + dx])}{(a^2 - b^2)^{3/2}} + \right. \\ \left. 2 a C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - 2 a C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right. \\ \left. \frac{b C (b + a \cos[c + dx]) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} + \frac{b C (b + a \cos[c + dx]) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} + \frac{a b (A b^2 + a^2 C) \sin[c + dx]}{(a - b)(a + b)} \right)$$

■ **Problem 686: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] (A + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\frac{C \operatorname{ArcTanh}[\sin[c + dx]]}{b^2 d} + \frac{2 a (A b^2 - a^2 C + 2 b^2 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{3/2} b^2 (a+b)^{3/2} d} - \frac{(A b^2 + a^2 C) \tan[c + dx]}{b (a^2 - b^2) d (a + b \sec[c + dx])}$$

Result (type 3, 331 leaves):

$$\frac{1}{b^2 d (A + 2C + A \cos[2(c + dx)]) (a + b \sec[c + dx])^2} 2 (b + a \cos[c + dx]) (A + C \sec[c + dx])^2$$

$$\left(-C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right.$$

$$\left. \left(2a (-Ab^2 + (a^2 - 2b^2)C) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}}\right] (b + a \cos[c + dx]) (i \cos[c] + \sin[c]) \right) \right) /$$

$$\left((a^2 - b^2)^{3/2} \sqrt{(\cos[c] - i \sin[c])^2} \right) + \frac{b (Ab^2 + a^2 C) (b \sin[c] - a \sin[dx])}{a (a - b) (a + b) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right])} \right)$$

- **Problem 687: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 3, 125 leaves, 5 steps):

$$\frac{Ax}{a^2} - \frac{2b(2a^2A - Ab^2 + a^2C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(Ab^2 + a^2C) \tan[c + dx]}{a(a^2 - b^2)d(a + b \sec[c + dx])}$$

Result (type 3, 270 leaves):

$$\left(2 (b + a \cos[c + dx]) (A + C \sec[c + dx])^2 \right.$$

$$\left. \left(Ax (b + a \cos[c + dx]) + \left(2b (-Ab^2 + a^2 (2A + C)) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}}\right] \right) \right) \right) / \left((a^2 - b^2)^{3/2} d \sqrt{(\cos[c] - i \sin[c])^2} \right) +$$

$$\left. \frac{(Ab^2 + a^2 C) (-b \sin[c] + a \sin[dx])}{(a - b) (a + b) d (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right])} \right) / \left(a^2 (A + 2C + A \cos[2(c + dx)]) (a + b \sec[c + dx])^2 \right)$$

- **Problem 691: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + C \sec[c + dx])^2}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 3, 381 leaves, 9 steps) :

$$\frac{(2 A b^2 + (12 a^2 + b^2) C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 b^5 d} - \frac{a (6 A b^6 + a^4 b^2 (2 A - 29 C) - 5 a^2 b^4 (A - 4 C) + 12 a^6 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{(a-b)^{5/2} b^5 (a+b)^{5/2} d} -$$

$$\frac{a (a^2 b^2 (2 A - 21 C) - b^4 (5 A - 6 C) + 12 a^4 C) \operatorname{Tan}[c + d x]}{2 b^4 (a^2 - b^2)^2 d} + \frac{(a^2 b^2 (A - 10 C) - b^4 (4 A - C) + 6 a^4 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 b^3 (a^2 - b^2)^2 d} -$$

$$\frac{(A b^2 + a^2 C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{2 b (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^2} + \frac{(3 A b^4 - 4 a^4 C + 7 a^2 b^2 C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{2 b^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 1028 leaves) :

$$\begin{aligned}
& \left(2a \left(2a^4 A b^2 - 5a^2 A b^4 + 6A b^6 + 12a^6 C - 29a^4 b^2 C + 20a^2 b^4 C \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2 - b^2}} \right] (b+a \operatorname{Cos}[c+dx])^3 \right. \\
& \quad \left. \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) \right) / \left(b^5 \sqrt{a^2 - b^2} (-a^2 + b^2)^2 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \right) + \\
& \left((-2Ab^2 - 12a^2 C - b^2 C) (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(b^5 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \right) + \\
& \left((2Ab^2 + 12a^2 C + b^2 C) (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad \left(b^5 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \right) + \\
& \quad \frac{C (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2)}{2b^3 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)^2} - \\
& \quad \frac{6aC (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]}{b^4 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)} - \\
& \quad \frac{C (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2)}{2b^3 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)^2} - \\
& \quad \frac{6aC (b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right]}{b^4 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \left(\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right)} + \\
& \quad \frac{(b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) (a^2 A b^2 \operatorname{Sin}[c+dx] + a^4 C \operatorname{Sin}[c+dx])}{b^3 (-a+b) (a+b) d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3} + \\
& \quad \left((b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c+dx] (A+C \operatorname{Sec}[c+dx]^2) (-2a^4 A b^2 \operatorname{Sin}[c+dx] + 5a^2 A b^4 \operatorname{Sin}[c+dx] - 6a^6 C \operatorname{Sin}[c+dx] + 9a^4 b^2 C \operatorname{Sin}[c+dx]) \right) / \\
& \quad \left(b^4 (-a+b)^2 (a+b)^2 d (A+2C+A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^3 \right)
\end{aligned}$$

- **Problem 693: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2 (A+C \operatorname{Sec}[c+dx]^2)}{(a+b \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 212 leaves, 7 steps):

$$\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{b^3 d} - \frac{a (3 A b^4 + (2 a^4 - 5 a^2 b^2 + 6 b^4) C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{5/2} b^3 (a+b)^{5/2} d} +$$

$$\frac{a (A b^2 + a^2 C) \operatorname{Tan}[c + d x]}{2 b^2 (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^2} + \frac{(2 A b^4 - 3 a^4 C + a^2 b^2 (A + 6 C)) \operatorname{Tan}[c + d x]}{2 b^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 895 leaves) :

$$- \frac{2 C (b + a \operatorname{Cos}[c + d x])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2}{b^3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^3} +$$

$$\frac{2 C (b + a \operatorname{Cos}[c + d x])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2}{b^3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^3} +$$

$$\left((3 A b^4 + 2 a^4 C - 5 a^2 b^2 C + 6 b^4 C) (b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2 \right)$$

$$\left(\left(2 i a \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right] \right) \right) \right)$$

$$\operatorname{Cos}[c] \left/ \left(b^3 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]} \right) \right. +$$

$$\left(2 a \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right] \right) \right)$$

$$\operatorname{Sin}[c] \left/ \left(b^3 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]} \right) \right) \left/ \right)$$

$$\left((-a^2 + b^2)^2 (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^3 \right) + \frac{1}{2 a b^2 (-a^2 + b^2)^2 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^3}$$

$$(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2$$

$$(-2 a^4 A b^2 \operatorname{Sin}[c] - 5 a^2 A b^4 \operatorname{Sin}[c] - 2 A b^6 \operatorname{Sin}[c] + 2 a^6 C \operatorname{Sin}[c] - a^4 b^2 C \operatorname{Sin}[c] - 10 a^2 b^4 C \operatorname{Sin}[c] +$$

$$5 a^3 A b^3 \operatorname{Sin}[dx] + 4 a A b^5 \operatorname{Sin}[dx] - 7 a^5 b C \operatorname{Sin}[dx] + 16 a^3 b^3 C \operatorname{Sin}[dx] - 3 a^3 A b^3 \operatorname{Sin}[2 c + d x] + a^5 b C \operatorname{Sin}[2 c + d x] -$$

$$4 a^3 b^3 C \operatorname{Sin}[2 c + d x] + 2 a^4 A b^2 \operatorname{Sin}[c + 2 d x] + a^2 A b^4 \operatorname{Sin}[c + 2 d x] - 2 a^6 C \operatorname{Sin}[c + 2 d x] + 5 a^4 b^2 C \operatorname{Sin}[c + 2 d x])$$

■ **Problem 694: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2}{(a + b \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 3, 177 leaves, 6 steps) :

$$\frac{(a^2 (2A + C) + b^2 (A + 2C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{5/2} (a+b)^{5/2} d} - \frac{(Ab^2 + a^2 C) \operatorname{Tan}[c+dx]}{2b(a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^2} - \frac{a(3Ab^2 - a^2 C + 4b^2 C) \operatorname{Tan}[c+dx]}{2b(a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+dx])}$$

Result (type 3, 342 leaves) :

$$\left((b + a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx] (A + C \operatorname{Sec}[c+dx])^2 \right. \\ \left. - \left(4i (a^2 (2A + C) + b^2 (A + 2C)) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (a \operatorname{Sin}[c] + (-b + a \operatorname{Cos}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] \right) \right. \\ \left. (b + a \operatorname{Cos}[c+dx])^2 (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) \right) / \left((a^2 - b^2)^{5/2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2} + 1 / (a^3 - ab^2)^2 (a \operatorname{Sec}[c] \right. \\ \left. ((2Ab^4 + a^4 C - a^2 b^2 (11A + 10C)) \operatorname{Sin}[dx] + (-2Ab^4 + a^4 C + a^2 b^2 (5A + 2C)) \operatorname{Sin}[2c+dx] + ab(Ab^2 - a^2 (4A + 3C)) \operatorname{Sin}[c+2dx]) + \right. \\ \left. b(a^2 + 2b^2) (-Ab^2 + a^2 (4A + 3C)) \operatorname{Tan}[c]) \right) / (2d(A + 2C + A \operatorname{Cos}[2(c+dx)]) (a+b \operatorname{Sec}[c+dx])^3)$$

■ **Problem 695: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^3} dx$$

Optimal (type 3, 202 leaves, 6 steps) :

$$\frac{Ax}{a^3} + \frac{b(5a^2 Ab^2 - 2Ab^4 - 3a^4(2A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^3 (a-b)^{5/2} (a+b)^{5/2} d} + \\ \frac{(Ab^2 + a^2 C) \operatorname{Tan}[c+dx]}{2a(a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^2} - \frac{(2Ab^4 - a^4 C - a^2 b^2 (5A + 2C)) \operatorname{Tan}[c+dx]}{2a^2 (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+dx])}$$

Result (type 3, 926 leaves) :

$$\left((6 a^4 A - 5 a^2 A b^2 + 2 A b^4 + 3 a^4 C) (b + a \cos[c + dx])^3 \sec[c + dx] (A + C \sec[c + dx])^2 \right. \\ \left. \left(\left(2 i b \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right) \right. \right. \\ \left. \left. \cos[c] \right) / \left(a^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) + \right. \\ \left. \left(2 b \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right) \right. \\ \left. \sin[c] \right) / \left(a^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) \left. \right) / \\ \left((-a^2 + b^2)^2 (A + 2C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3 \right) + \frac{1}{2 a^3 (a^2 - b^2)^2 d (A + 2C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3}$$

$(b + a \cos[c + dx]) \sec[c]$

$\sec[c + dx] (A + C \sec[c + dx])^2$

$$(2 a^6 A d x \cos[c] - 6 a^2 A b^4 d x \cos[c] + 4 A b^6 d x \cos[c] + 4 a^5 A b d x \cos[dx] - 8 a^3 A b^3 d x \cos[dx] + 4 a A b^5 d x \cos[dx] + \\ 4 a^5 A b d x \cos[2c + dx] - 8 a^3 A b^3 d x \cos[2c + dx] + 4 a A b^5 d x \cos[2c + dx] + a^6 A d x \cos[c + 2dx] - \\ 2 a^4 A b^2 d x \cos[c + 2dx] + a^2 A b^4 d x \cos[c + 2dx] + a^6 A d x \cos[3c + 2dx] - 2 a^4 A b^2 d x \cos[3c + 2dx] + \\ a^2 A b^4 d x \cos[3c + 2dx] - 6 a^4 A b^2 \sin[c] - 9 a^2 A b^4 \sin[c] + 6 A b^6 \sin[c] - 2 a^6 C \sin[c] - 5 a^4 b^2 C \sin[c] - 2 a^2 b^4 C \sin[c] + \\ 17 a^3 A b^3 \sin[dx] - 8 a A b^5 \sin[dx] + 5 a^5 b C \sin[dx] + 4 a^3 b^3 C \sin[dx] - 7 a^3 A b^3 \sin[2c + dx] + 4 a A b^5 \sin[2c + dx] - \\ 3 a^5 b C \sin[2c + dx] + 6 a^4 A b^2 \sin[c + 2dx] - 3 a^2 A b^4 \sin[c + 2dx] + 2 a^6 C \sin[c + 2dx] + a^4 b^2 C \sin[c + 2dx])$$

■ **Problem 696: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + C \sec[c + dx])^2}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 3, 266 leaves, 7 steps):

$$\frac{3 A b x}{a^4} - \frac{(15 a^2 A b^4 - 6 A b^6 - 2 a^6 C - a^4 b^2 (12 A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4 (a-b)^{5/2} (a+b)^{5/2} d} - \\ \frac{(11 a^2 A b^2 - 6 A b^4 - a^4 (2 A - 3 C)) \sin[c + dx]}{2 a^3 (a^2 - b^2)^2 d} + \frac{(A b^2 + a^2 C) \sin[c + dx]}{2 a (a^2 - b^2) d (a + b \sec[c + dx])^2} - \frac{(3 A b^4 - 2 a^4 C - a^2 b^2 (6 A + C)) \sin[c + dx]}{2 a^2 (a^2 - b^2)^2 d (a + b \sec[c + dx])}$$

Result (type 3, 1186 leaves):

$$\frac{1}{(-a^2 + b^2)^2 (A + 2C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3} \left(12a^4 A b^2 - 15a^2 A b^4 + 6A b^6 + 2a^6 C + a^4 b^2 C \right) (b + a \cos[c + dx])^3 \sec[c + dx] (A + C \sec[c + dx])^2 \left(-2i \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right] \cos[c] \right) / \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) - \left(2 \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right] \sin[c] \right) / \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) \Bigg) +$$

$$\frac{1}{4a^4 (a^2 - b^2)^2 d (A + 2C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3} (b + a \cos[c + dx]) \sec[c] \sec[c + dx] (A + C \sec[c + dx])^2 \left(-12a^6 A b d x \cos[c] + 36a^2 A b^5 d x \cos[c] - 24A b^7 d x \cos[c] - 24a^5 A b^2 d x \cos[dx] + 48a^3 A b^4 d x \cos[dx] - 24a A b^6 d x \cos[dx] - 24a^5 A b^2 d x \cos[2c + dx] + 48a^3 A b^4 d x \cos[2c + dx] - 24a A b^6 d x \cos[2c + dx] - 6a^6 A b d x \cos[c + 2dx] + 12a^4 A b^3 d x \cos[c + 2dx] - 6a^2 A b^5 d x \cos[c + 2dx] - 6a^6 A b d x \cos[3c + 2dx] + 12a^4 A b^3 d x \cos[3c + 2dx] - 6a^2 A b^5 d x \cos[3c + 2dx] + 16a^4 A b^3 \sin[c] + 22a^2 A b^5 \sin[c] - 20A b^7 \sin[c] + 8a^6 b C \sin[c] + 14a^4 b^3 C \sin[c] - 4a^2 b^5 C \sin[c] + a^7 A \sin[dx] + 2a^5 A b^2 \sin[dx] - 53a^3 A b^4 \sin[dx] + 32a A b^6 \sin[dx] - 22a^5 b^2 C \sin[dx] + 4a^3 b^4 C \sin[dx] + a^7 A \sin[2c + dx] + 2a^5 A b^2 \sin[2c + dx] + 11a^3 A b^4 \sin[2c + dx] - 8a A b^6 \sin[2c + dx] + 10a^5 b^2 C \sin[2c + dx] - 4a^3 b^4 C \sin[2c + dx] + 4a^6 A b \sin[c + 2dx] - 24a^4 A b^3 \sin[c + 2dx] + 14a^2 A b^5 \sin[c + 2dx] - 8a^6 b C \sin[c + 2dx] + 2a^4 b^3 C \sin[c + 2dx] + 4a^6 A b \sin[3c + 2dx] - 8a^4 A b^3 \sin[3c + 2dx] + 4a^2 A b^5 \sin[3c + 2dx] + a^7 A \sin[2c + 3dx] - 2a^5 A b^2 \sin[2c + 3dx] + a^3 A b^4 \sin[2c + 3dx] + a^7 A \sin[4c + 3dx] - 2a^5 A b^2 \sin[4c + 3dx] + a^3 A b^4 \sin[4c + 3dx] \right)$$

■ **Problem 698: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + C \sec[c + dx])^2}{(a + b \sec[c + dx])^4} dx$$

Optimal (type 3, 378 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a C \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{b^5 d} - \frac{(2 A b^8 - 8 a^8 C + 28 a^6 b^2 C - 35 a^4 b^4 C + a^2 b^6 (3 A + 20 C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} b^5 (a+b)^{7/2} d} \\
& \frac{(5 A b^4 - (12 a^4 - 23 a^2 b^2 + 6 b^4) C) \operatorname{Tan}[c+d x]}{6 b^4 (a^2 - b^2)^2 d} - \frac{(A b^2 + a^2 C) \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^3} + \\
& \frac{(3 A b^4 - 4 a^4 C + a^2 b^2 (2 A + 9 C)) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{6 b^2 (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+d x])^2} + \frac{a (2 A b^6 + 4 a^6 C - 11 a^4 b^2 C + 3 a^2 b^4 (A + 4 C)) \operatorname{Tan}[c+d x]}{2 b^4 (a^2 - b^2)^3 d (a+b \operatorname{Sec}[c+d x])}
\end{aligned}$$

Result (type 3, 874 leaves):

$$\begin{aligned}
& - \left(2 (3 a^2 A b^6 + 2 A b^8 - 8 a^8 C + 28 a^6 b^2 C - 35 a^4 b^4 C + 20 a^2 b^6 C) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2 - b^2}}\right] (b+a \operatorname{Cos}[c+d x])^4 \right. \\
& \left. \operatorname{Sec}[c+d x]^2 (A+C \operatorname{Sec}[c+d x])^2 \right) / \left(b^5 \sqrt{a^2 - b^2} (-a^2 + b^2)^3 d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a+b \operatorname{Sec}[c+d x])^4 \right) + \\
& \frac{8 a C (b+a \operatorname{Cos}[c+d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^2 (A+C \operatorname{Sec}[c+d x])^2}{b^5 d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a+b \operatorname{Sec}[c+d x])^4} - \\
& \frac{8 a C (b+a \operatorname{Cos}[c+d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \operatorname{Sec}[c+d x]^2 (A+C \operatorname{Sec}[c+d x])^2}{b^5 d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a+b \operatorname{Sec}[c+d x])^4} + \\
& \frac{1}{24 b^4 (-a^2 + b^2)^3 d (A+2 C+A \operatorname{Cos}[2 c+2 d x]) (a+b \operatorname{Sec}[c+d x])^4} \\
& (b+a \operatorname{Cos}[c+d x]) \operatorname{Sec}[c+d x]^3 (A+C \operatorname{Sec}[c+d x])^2 (-6 a^4 A b^5 \operatorname{Sin}[c+d x] - 54 a^2 A b^7 \operatorname{Sin}[c+d x] - 120 a^8 b C \operatorname{Sin}[c+d x] + \\
& 294 a^6 b^3 C \operatorname{Sin}[c+d x] - 174 a^4 b^5 C \operatorname{Sin}[c+d x] - 108 a^2 b^7 C \operatorname{Sin}[c+d x] + 48 b^9 C \operatorname{Sin}[c+d x] - 16 a^5 A b^4 \operatorname{Sin}[2(c+d x)] - \\
& 2 a^3 A b^6 \operatorname{Sin}[2(c+d x)] - 72 a A b^8 \operatorname{Sin}[2(c+d x)] - 48 a^9 C \operatorname{Sin}[2(c+d x)] - 40 a^7 b^2 C \operatorname{Sin}[2(c+d x)] + 370 a^5 b^4 C \operatorname{Sin}[2(c+d x)] - \\
& 444 a^3 b^6 C \operatorname{Sin}[2(c+d x)] + 72 a b^8 C \operatorname{Sin}[2(c+d x)] - 6 a^4 A b^5 \operatorname{Sin}[3(c+d x)] - 54 a^2 A b^7 \operatorname{Sin}[3(c+d x)] - 120 a^8 b C \operatorname{Sin}[3(c+d x)] + \\
& 342 a^6 b^3 C \operatorname{Sin}[3(c+d x)] - 318 a^4 b^5 C \operatorname{Sin}[3(c+d x)] + 36 a^2 b^7 C \operatorname{Sin}[3(c+d x)] - 4 a^5 A b^4 \operatorname{Sin}[4(c+d x)] - \\
& 11 a^3 A b^6 \operatorname{Sin}[4(c+d x)] - 24 a^9 C \operatorname{Sin}[4(c+d x)] + 68 a^7 b^2 C \operatorname{Sin}[4(c+d x)] - 65 a^5 b^4 C \operatorname{Sin}[4(c+d x)] + 6 a^3 b^6 C \operatorname{Sin}[4(c+d x)])
\end{aligned}$$

■ **Problem 699: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^3 (A+C \operatorname{Sec}[c+d x])^2}{(a+b \operatorname{Sec}[c+d x])^4} dx$$

Optimal (type 3, 313 leaves, 8 steps):

$$\frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{b^4 d} + \frac{a (a^2 b^4 (A - 8 C) - 2 a^6 C + 7 a^4 b^2 C + 4 b^6 (A + 2 C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} b^4 (a+b)^{7/2} d} - \frac{(A b^2 + a^2 C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 b (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^3} -$$

$$\frac{a (2 A b^4 - 3 a^4 C + a^2 b^2 (3 A + 8 C)) \operatorname{Tan}[c + d x]}{6 b^3 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^2} - \frac{(4 A b^6 + 9 a^6 C + 2 a^2 b^4 (7 A + 17 C) - a^4 b^2 (3 A + 28 C)) \operatorname{Tan}[c + d x]}{6 b^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 1092 leaves):

$$\frac{2 C (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2}{b^4 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4} +$$

$$\frac{2 C (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2}{b^4 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4} +$$

$$\frac{1}{(-a^2 + b^2)^3 (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4}$$

$$\frac{(a^2 A b^4 + 4 A b^6 - 2 a^6 C + 7 a^4 b^2 C - 8 a^2 b^4 C + 8 b^6 C) (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2}{\left(\left(2 i a \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}}\right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right]\right)\right) \operatorname{Cos}[c]\right) /$$

$$\left(b^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}\right) +$$

$$\left(2 a \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}}\right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right]\right)\right) \operatorname{Sin}[c]\right) /$$

$$\left(b^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}\right) -$$

$$\frac{(2 (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2 (A b^3 \operatorname{Sin}[c] + a^2 b C \operatorname{Sin}[c] - a A b^2 \operatorname{Sin}[d x] - a^3 C \operatorname{Sin}[d x])) /}{(3 a b (-a^2 + b^2) d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4) +$$

$$\frac{(b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2}{(-5 a A b^3 \operatorname{Sin}[c] + a^3 b C \operatorname{Sin}[c] - 6 a b^3 C \operatorname{Sin}[c] + 3 a^2 A b^2 \operatorname{Sin}[d x] + 2 A b^4 \operatorname{Sin}[d x] - 3 a^4 C \operatorname{Sin}[d x] + 8 a^2 b^2 C \operatorname{Sin}[d x]) /}$$

$$\frac{(3 b^2 (-a^2 + b^2)^2 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4) +}{(b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x])^2 (-3 a^3 A b^3 \operatorname{Sin}[c] - 12 a A b^5 \operatorname{Sin}[c] - 3 a^5 b C \operatorname{Sin}[c] + 6 a^3 b^3 C \operatorname{Sin}[c] -$$

$$\frac{18 a b^5 C \operatorname{Sin}[c] + 13 a^2 A b^4 \operatorname{Sin}[d x] + 2 A b^6 \operatorname{Sin}[d x] + 6 a^6 C \operatorname{Sin}[d x] - 17 a^4 b^2 C \operatorname{Sin}[d x] + 26 a^2 b^4 C \operatorname{Sin}[d x]) /}{(3 b^3 (-a^2 + b^2)^3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4)}$$

- **Problem 701: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x] (A + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^4} dx$$

Optimal (type 3, 252 leaves, 7 steps):

$$\frac{a (a^2 (2 A + C) + b^2 (3 A + 4 C)) \text{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} (a+b)^{7/2} d} - \frac{(A b^2 + a^2 C) \tan[c + d x]}{3 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^3} -$$

$$\frac{a (5 A b^2 - a^2 C + 6 b^2 C) \tan[c + d x]}{6 b (a^2 - b^2)^2 d (a + b \text{Sec}[c + d x])^2} + \frac{(a^4 C - 2 b^4 (2 A + 3 C) - a^2 b^2 (11 A + 10 C)) \tan[c + d x]}{6 b (a^2 - b^2)^3 d (a + b \text{Sec}[c + d x])}$$

Result (type 3, 868 leaves):

$$\left((2 a^2 A + 3 A b^2 + a^2 C + 4 b^2 C) (b + a \text{Cos}[c + d x])^4 \text{Sec}[c + d x]^2 (A + C \text{Sec}[c + d x]^2) \right.$$

$$\left. \left(\left(2 i a \text{ArcTan}\left[\text{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\text{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]}} - \frac{i \text{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]}} \right) \left(-i b \text{Sin}\left[\frac{d x}{2}\right] + i a \text{Sin}\left[c + \frac{d x}{2}\right] \right) \right) \right.$$

$$\left. \left. \text{Cos}[c] \right) / \left(\sqrt{a^2 - b^2} d \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]} \right) + \right.$$

$$\left. \left(2 a \text{ArcTan}\left[\text{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\text{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]}} - \frac{i \text{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]}} \right) \left(-i b \text{Sin}\left[\frac{d x}{2}\right] + i a \text{Sin}\left[c + \frac{d x}{2}\right] \right) \right) \right.$$

$$\left. \left. \text{Sin}[c] \right) / \left(\sqrt{a^2 - b^2} d \sqrt{\text{Cos}[2 c] - i \text{Sin}[2 c]} \right) \right) / \left((-a^2 + b^2)^3 (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + b \text{Sec}[c + d x])^4 \right) +$$

$$\left(2 (b + a \text{Cos}[c + d x]) \text{Sec}[c] \text{Sec}[c + d x]^2 (A + C \text{Sec}[c + d x]^2) (A b^4 \text{Sin}[c] + a^2 b^2 C \text{Sin}[c] - a A b^3 \text{Sin}[d x] - a^3 b C \text{Sin}[d x]) \right) /$$

$$\left(3 a^3 (a^2 - b^2) d (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + b \text{Sec}[c + d x])^4 \right) +$$

$$\left((b + a \text{Cos}[c + d x])^2 \text{Sec}[c] \text{Sec}[c + d x]^2 (A + C \text{Sec}[c + d x]^2) \right.$$

$$\left. (-11 a^2 A b^3 \text{Sin}[c] + 6 A b^5 \text{Sin}[c] - 5 a^4 b C \text{Sin}[c] + 9 a^3 A b^2 \text{Sin}[d x] - 4 a A b^4 \text{Sin}[d x] + 3 a^5 C \text{Sin}[d x] + 2 a^3 b^2 C \text{Sin}[d x]) \right) /$$

$$\left(3 a^3 (a^2 - b^2)^2 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + b \text{Sec}[c + d x])^4 \right) +$$

$$\left((b + a \text{Cos}[c + d x])^3 \text{Sec}[c] \text{Sec}[c + d x]^2 (A + C \text{Sec}[c + d x]^2) (27 a^4 A b^2 \text{Sin}[c] - 18 a^2 A b^4 \text{Sin}[c] + 6 A b^6 \text{Sin}[c] + 3 a^6 C \text{Sin}[c] + \right.$$

$$\left. 12 a^4 b^2 C \text{Sin}[c] - 18 a^5 A b \text{Sin}[d x] + 5 a^3 A b^3 \text{Sin}[d x] - 2 a A b^5 \text{Sin}[d x] - 13 a^5 b C \text{Sin}[d x] - 2 a^3 b^3 C \text{Sin}[d x]) \right) /$$

$$\left(3 a^3 (a^2 - b^2)^3 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + b \text{Sec}[c + d x])^4 \right)$$

■ **Problem 702: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 292 leaves, 7 steps):

$$\frac{A x}{a^4} - \frac{b (7 a^2 A b^4 - 2 A b^6 - a^4 b^2 (8 A - C) + 4 a^6 (2 A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4 (a-b)^{7/2} (a+b)^{7/2} d} + \frac{(A b^2 + a^2 C) \operatorname{Tan}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^3} - \frac{(3 A b^4 - 2 a^4 C - a^2 b^2 (8 A + 3 C)) \operatorname{Tan}[c + d x]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^2} - \frac{(17 a^2 A b^4 - 6 A b^6 - 2 a^6 C - 13 a^4 b^2 (2 A + C)) \operatorname{Tan}[c + d x]}{6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 995 leaves):

$$\frac{2 A x (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2)}{a^4 (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4} + \frac{1}{(-a^2 + b^2)^3 (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4}$$

$$(-8 a^6 A + 8 a^4 A b^2 - 7 a^2 A b^4 + 2 A b^6 - 4 a^6 C - a^4 b^2 C) (b + a \operatorname{Cos}[c + d x])^4 \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2)$$

$$\left(\left(2 i b \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{d x}{2}\right] + i a \operatorname{Sin}\left[c + \frac{d x}{2}\right] \right) \right) \operatorname{Cos}[c] \right) /$$

$$\left(a^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]} \right) +$$

$$\left(2 b \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{d x}{2}\right] + i a \operatorname{Sin}\left[c + \frac{d x}{2}\right] \right) \right) \operatorname{Sin}[c] \right) /$$

$$\left(a^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]} \right) \left. \right) -$$

$$\left(2 (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2) (A b^5 \operatorname{Sin}[c] + a^2 b^3 C \operatorname{Sin}[c] - a A b^4 \operatorname{Sin}[d x] - a^3 b^2 C \operatorname{Sin}[d x]) \right) /$$

$$(3 a^4 (a^2 - b^2) d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4) +$$

$$\left((b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2) (14 a^2 A b^4 \operatorname{Sin}[c] - 9 A b^6 \operatorname{Sin}[c] + \right.$$

$$8 a^4 b^2 C \operatorname{Sin}[c] - 3 a^2 b^4 C \operatorname{Sin}[c] - 12 a^3 A b^3 \operatorname{Sin}[d x] + 7 a A b^5 \operatorname{Sin}[d x] - 6 a^5 b C \operatorname{Sin}[d x] + a^3 b^3 C \operatorname{Sin}[d x]) \left. \right) /$$

$$(3 a^4 (a^2 - b^2)^2 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4) +$$

$$\left((b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + C \operatorname{Sec}[c + d x]^2) (-48 a^4 A b^3 \operatorname{Sin}[c] + 51 a^2 A b^5 \operatorname{Sin}[c] - 18 A b^7 \operatorname{Sin}[c] - 12 a^6 b C \operatorname{Sin}[c] - \right.$$

$$3 a^4 b^3 C \operatorname{Sin}[c] + 36 a^5 A b^2 \operatorname{Sin}[d x] - 32 a^3 A b^4 \operatorname{Sin}[d x] + 11 a A b^6 \operatorname{Sin}[d x] + 6 a^7 C \operatorname{Sin}[d x] + 10 a^5 b^2 C \operatorname{Sin}[d x] - a^3 b^4 C \operatorname{Sin}[d x]) \left. \right) /$$

$$(3 a^4 (a^2 - b^2)^3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^4)$$

- **Problem 703: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x] (A+C \operatorname{Sec}[c+d x]^2)}{(a+b \operatorname{Sec}[c+d x])^4} dx$$

Optimal (type 3, 367 leaves, 8 steps):

$$\begin{aligned} & -\frac{4 A b x}{a^5} - \frac{(35 a^4 A b^4 - 28 a^2 A b^6 + 8 A b^8 - 2 a^8 C - a^6 b^2 (20 A + 3 C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{a^5 (a-b)^{7/2} (a+b)^{7/2} d} + \\ & \frac{(68 a^2 A b^4 - 24 A b^6 + a^6 (6 A - 11 C) - a^4 b^2 (65 A + 4 C)) \operatorname{Sin}[c+d x]}{6 a^4 (a^2 - b^2)^3 d} + \frac{(A b^2 + a^2 C) \operatorname{Sin}[c+d x]}{3 a (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^3} - \\ & \frac{(4 A b^4 - 3 a^4 C - a^2 b^2 (9 A + 2 C)) \operatorname{Sin}[c+d x]}{6 a^2 (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+d x])^2} - \frac{(11 a^2 A b^4 - 4 A b^6 - 2 a^6 C - 3 a^4 b^2 (4 A + C)) \operatorname{Sin}[c+d x]}{2 a^3 (a^2 - b^2)^3 d (a+b \operatorname{Sec}[c+d x])} \end{aligned}$$

Result (type 3, 1089 leaves):

$$\begin{aligned}
& - \frac{8 A b x (b + a \cos [c + d x])^4 \sec [c + d x]^2 (A + C \sec [c + d x]^2)}{a^5 (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4} + \frac{1}{(-a^2 + b^2)^3 (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4} \\
& (-20 a^6 A b^2 + 35 a^4 A b^4 - 28 a^2 A b^6 + 8 A b^8 - 2 a^8 C - 3 a^6 b^2 C) (b + a \cos [c + d x])^4 \sec [c + d x]^2 (A + C \sec [c + d x]^2) \\
& \left(- \left(2 i \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \cos [c] \right) / \right. \\
& \left. \left(a^5 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) - \right. \\
& \left. \left(2 \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \right) \sin [c] \right) / \right. \\
& \left. \left(a^5 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) \right) + \\
& \left(2 (b + a \cos [c + d x]) \sec [c] \sec [c + d x]^2 (A + C \sec [c + d x]^2) (A b^6 \sin [c] + a^2 b^4 C \sin [c] - a A b^5 \sin [d x] - a^3 b^3 C \sin [d x]) \right) / \\
& \left(3 a^5 (a^2 - b^2) d (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4 \right) + \\
& \left((b + a \cos [c + d x])^2 \sec [c] \sec [c + d x]^2 (A + C \sec [c + d x]^2) (-17 a^2 A b^5 \sin [c] + 12 A b^7 \sin [c] - \right. \\
& \left. 11 a^4 b^3 C \sin [c] + 6 a^2 b^5 C \sin [c] + 15 a^3 A b^4 \sin [d x] - 10 a A b^6 \sin [d x] + 9 a^5 b^2 C \sin [d x] - 4 a^3 b^4 C \sin [d x]) \right) / \\
& \left(3 a^5 (a^2 - b^2)^2 d (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4 \right) + \frac{1}{3 a^5 (a^2 - b^2)^3 d (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4} \\
& (b + a \cos [c + d x])^3 \sec [c] \sec [c + d x]^2 (A + C \sec [c + d x]^2) \\
& (75 a^4 A b^4 \sin [c] - 96 a^2 A b^6 \sin [c] + 36 A b^8 \sin [c] + 27 a^6 b^2 C \sin [c] - 18 a^4 b^4 C \sin [c] + 6 a^2 b^6 C \sin [c] - \\
& 60 a^5 A b^3 \sin [d x] + 71 a^3 A b^5 \sin [d x] - 26 a A b^7 \sin [d x] - 18 a^7 b C \sin [d x] + 5 a^5 b^3 C \sin [d x] - 2 a^3 b^5 C \sin [d x]) + \\
& \frac{2 A (b + a \cos [c + d x])^4 \sec [c + d x] (A + C \sec [c + d x]^2) \tan [c + d x]}{a^4 d (A + 2 C + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4}
\end{aligned}$$

■ **Problem 704: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^2 (A + C \sec [c + d x]^2)}{(a + b \sec [c + d x])^4} dx$$

Optimal (type 3, 513 leaves, 9 steps):

$$\frac{(20 A b^2 + a^2 (A + 2 C)) x}{2 a^6} + \frac{(20 A b^9 - a^2 b^7 (69 A - 2 C) - 8 a^6 b^3 (5 A - C) + 7 a^4 b^5 (12 A - C) - 8 a^8 b C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^6 \sqrt{a-b} \sqrt{a+b} (a^2 - b^2)^3 d} +$$

$$\frac{b (60 A b^6 - a^6 (24 A - 26 C) + a^4 b^2 (146 A - 17 C) - a^2 b^4 (167 A - 6 C)) \operatorname{Sin}[c+dx]}{6 a^5 (a^2 - b^2)^3 d} -$$

$$\frac{(10 A b^6 - a^6 (A - 6 C) + a^4 b^2 (23 A - 2 C) - a^2 b^4 (27 A - C)) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{2 a^4 (a^2 - b^2)^3 d} + \frac{(A b^2 + a^2 C) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c+dx])^3} -$$

$$\frac{(5 A b^4 - 4 a^4 C - a^2 b^2 (10 A + C)) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c+dx])^2} + \frac{(20 A b^6 - a^2 b^4 (53 A - 2 C) + 12 a^6 C + a^4 b^2 (48 A + C)) \operatorname{Cos}[c+dx] \operatorname{Sin}[c+dx]}{6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c+dx])}$$

Result(type3, 1452 leaves):

$$\frac{1}{a^6 \sqrt{a^2 - b^2} (-a^2 + b^2)^3 d} b (-40 a^6 A b^2 + 84 a^4 A b^4 - 69 a^2 A b^6 + 20 A b^8 - 8 a^8 C + 8 a^6 b^2 C - 7 a^4 b^4 C + 2 a^2 b^6 C) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2 - b^2}}\right] -$$

$$\frac{1}{96 a^6 (a^2 - b^2)^3 d (b + a \operatorname{Cos}[c+dx])^3}$$

$$(-72 a^{10} A b (c+dx) - 1272 a^8 A b^3 (c+dx) + 3288 a^6 A b^5 (c+dx) - 1512 a^4 A b^7 (c+dx) - 1392 a^2 A b^9 (c+dx) +$$

$$960 A b^{11} (c+dx) - 144 a^{10} b C (c+dx) + 336 a^8 b^3 C (c+dx) - 144 a^6 b^5 C (c+dx) - 144 a^4 b^7 C (c+dx) + 96 a^2 b^9 C (c+dx) -$$

$$36 a^{11} A (c+dx) \operatorname{Cos}[c+dx] - 756 a^9 A b^2 (c+dx) \operatorname{Cos}[c+dx] - 396 a^7 A b^4 (c+dx) \operatorname{Cos}[c+dx] + 6084 a^5 A b^6 (c+dx) \operatorname{Cos}[c+dx] -$$

$$7776 a^3 A b^8 (c+dx) \operatorname{Cos}[c+dx] + 2880 a A b^{10} (c+dx) \operatorname{Cos}[c+dx] - 72 a^{11} C (c+dx) \operatorname{Cos}[c+dx] - 72 a^9 b^2 C (c+dx) \operatorname{Cos}[c+dx] +$$

$$648 a^7 b^4 C (c+dx) \operatorname{Cos}[c+dx] - 792 a^5 b^6 C (c+dx) \operatorname{Cos}[c+dx] + 288 a^3 b^8 C (c+dx) \operatorname{Cos}[c+dx] - 72 a^{10} A b (c+dx) \operatorname{Cos}[2(c+dx)] -$$

$$1224 a^8 A b^3 (c+dx) \operatorname{Cos}[2(c+dx)] + 4104 a^6 A b^5 (c+dx) \operatorname{Cos}[2(c+dx)] - 4248 a^4 A b^7 (c+dx) \operatorname{Cos}[2(c+dx)] +$$

$$1440 a^2 A b^9 (c+dx) \operatorname{Cos}[2(c+dx)] - 144 a^{10} b C (c+dx) \operatorname{Cos}[2(c+dx)] + 432 a^8 b^3 C (c+dx) \operatorname{Cos}[2(c+dx)] -$$

$$432 a^6 b^5 C (c+dx) \operatorname{Cos}[2(c+dx)] + 144 a^4 b^7 C (c+dx) \operatorname{Cos}[2(c+dx)] - 12 a^{11} A (c+dx) \operatorname{Cos}[3(c+dx)] -$$

$$204 a^9 A b^2 (c+dx) \operatorname{Cos}[3(c+dx)] + 684 a^7 A b^4 (c+dx) \operatorname{Cos}[3(c+dx)] - 708 a^5 A b^6 (c+dx) \operatorname{Cos}[3(c+dx)] +$$

$$240 a^3 A b^8 (c+dx) \operatorname{Cos}[3(c+dx)] - 24 a^{11} C (c+dx) \operatorname{Cos}[3(c+dx)] + 72 a^9 b^2 C (c+dx) \operatorname{Cos}[3(c+dx)] -$$

$$72 a^7 b^4 C (c+dx) \operatorname{Cos}[3(c+dx)] + 24 a^5 b^6 C (c+dx) \operatorname{Cos}[3(c+dx)] - 6 a^{11} A \operatorname{Sin}[c+dx] + 270 a^9 A b^2 \operatorname{Sin}[c+dx] -$$

$$750 a^7 A b^4 \operatorname{Sin}[c+dx] - 1086 a^5 A b^6 \operatorname{Sin}[c+dx] + 2232 a^3 A b^8 \operatorname{Sin}[c+dx] - 960 a A b^{10} \operatorname{Sin}[c+dx] - 144 a^9 b^2 C \operatorname{Sin}[c+dx] -$$

$$288 a^7 b^4 C \operatorname{Sin}[c+dx] + 228 a^5 b^6 C \operatorname{Sin}[c+dx] - 96 a^3 b^8 C \operatorname{Sin}[c+dx] + 60 a^{10} A b \operatorname{Sin}[2(c+dx)] + 372 a^8 A b^3 \operatorname{Sin}[2(c+dx)] -$$

$$2772 a^6 A b^5 \operatorname{Sin}[2(c+dx)] + 3300 a^4 A b^7 \operatorname{Sin}[2(c+dx)] - 1200 a^2 A b^9 \operatorname{Sin}[2(c+dx)] - 480 a^8 b^3 C \operatorname{Sin}[2(c+dx)] +$$

$$360 a^6 b^5 C \operatorname{Sin}[2(c+dx)] - 120 a^4 b^7 C \operatorname{Sin}[2(c+dx)] - 9 a^{11} A \operatorname{Sin}[3(c+dx)] + 279 a^9 A b^2 \operatorname{Sin}[3(c+dx)] -$$

$$1143 a^7 A b^4 \operatorname{Sin}[3(c+dx)] + 1253 a^5 A b^6 \operatorname{Sin}[3(c+dx)] - 440 a^3 A b^8 \operatorname{Sin}[3(c+dx)] - 144 a^9 b^2 C \operatorname{Sin}[3(c+dx)] +$$

$$128 a^7 b^4 C \operatorname{Sin}[3(c+dx)] - 44 a^5 b^6 C \operatorname{Sin}[3(c+dx)] + 30 a^{10} A b \operatorname{Sin}[4(c+dx)] - 90 a^8 A b^3 \operatorname{Sin}[4(c+dx)] + 90 a^6 A b^5 \operatorname{Sin}[4(c+dx)] -$$

$$30 a^4 A b^7 \operatorname{Sin}[4(c+dx)] - 3 a^{11} A \operatorname{Sin}[5(c+dx)] + 9 a^9 A b^2 \operatorname{Sin}[5(c+dx)] - 9 a^7 A b^4 \operatorname{Sin}[5(c+dx)] + 3 a^5 A b^6 \operatorname{Sin}[5(c+dx)]])$$

■ **Problem 705: Result more than twice size of optimal antiderivative.**

$$\int \frac{a^2 - b^2 \operatorname{Sec}[c+dx]^2}{a + b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$a x - \frac{b \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d}$$

Result (type 3, 73 leaves):

$$a x + \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d} - \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right]}{d}$$

■ **Problem 709: Attempted integration timed out after 120 seconds.**

$$\int \sec[c + d x]^3 \sqrt{a + b \sec[c + d x]} (A + C \sec[c + d x]^2) dx$$

Optimal (type 4, 467 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{315 b^5 d} 2 (a - b) \sqrt{a + b} (16 a^4 C + 6 a^2 b^2 (7 A + 4 C) - 21 b^4 (9 A + 7 C)) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b (1 - \sec[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \sec[c + d x])}{a - b}} + \frac{1}{315 b^4 d} 2 (a - b) \sqrt{a + b} (16 a^3 C + 12 a^2 b C + 6 a b^2 (7 A + 6 C) + 21 b^3 (9 A + 7 C)) \\ & \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \sec[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \sec[c + d x])}{a - b}} + \\ & \frac{2 a (21 A b^2 + 8 a^2 C + 13 b^2 C) \sqrt{a + b \sec[c + d x]} \operatorname{Tan}[c + d x]}{315 b^3 d} - \frac{2 (6 a^2 C - 7 b^2 (9 A + 7 C)) \sec[c + d x] \sqrt{a + b \sec[c + d x]} \operatorname{Tan}[c + d x]}{315 b^2 d} + \\ & \frac{2 a C \sec[c + d x]^2 \sqrt{a + b \sec[c + d x]} \operatorname{Tan}[c + d x]}{63 b d} + \frac{2 C \sec[c + d x]^3 \sqrt{a + b \sec[c + d x]} \operatorname{Tan}[c + d x]}{9 d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 710: Unable to integrate problem.**

$$\int \sec[c + d x]^2 \sqrt{a + b \sec[c + d x]} (A + C \sec[c + d x]^2) dx$$

Optimal (type 4, 375 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 b^4 d} 2 a (a-b) \sqrt{a+b} (35 A b^2 + 8 a^2 C + 19 b^2 C) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{105 b^3 d} \\
& 2(a-b) \sqrt{a+b} (35 A b^2 + (8 a^2 + 6 a b + 25 b^2) C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2(8 a^2 C + 5 b^2(7 A + 5 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{105 b^2 d} - \\
& \frac{8 a C (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{35 b^2 d} + \frac{2 C \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{7 b d}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \operatorname{Sec}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} (A+C \operatorname{Sec}[c+d x]^2) dx$$

■ **Problem 711: Unable to integrate problem.**

$$\int \operatorname{Sec}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 308 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{15 b^3 d} 2(a-b) \sqrt{a+b} (2 a^2 C - 3 b^2(5 A + 3 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{15 b^2 d} \\
& 2(a-b) \sqrt{a+b} (15 A b + 2 a C + 9 b C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \\
& \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{4 a C \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{15 b d} + \frac{2 C (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{5 b d}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \operatorname{Sec}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} (A+C \operatorname{Sec}[c+d x]^2) dx$$

■ **Problem 712: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{a+b \operatorname{Sec}[c+d x]} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 355 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{1}{3b^2d} 2a(a-b)\sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3bd} \\
 & 2\sqrt{a+b} (3Ab - (a-b)C) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
 & \frac{2A\sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d} + \frac{2C\sqrt{a+b \operatorname{Sec}[c+dx]} \tan[c+dx]}{3d}
 \end{aligned}$$

Result (type 4, 570 leaves):

$$\begin{aligned}
 & \frac{1}{3b\sqrt{\frac{-a+b}{a+b}} d (b+a \cos[c+dx]) (A+2C+A \cos[2c+2dx])} 4 \cos\left[\frac{1}{2}(c+dx)\right]^2 \cos[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx])^2 \\
 & \left(2ia(a-b)C \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] + \right. \\
 & 2i(a-b)b(3A+C) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] - \\
 & 12iaAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] - \\
 & \left. a \sqrt{\frac{-a+b}{a+b}} C \cos[c+dx] (b+a \cos[c+dx]) \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) + \\
 & \frac{\cos[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx])^2 \left(\frac{4aC \sin[c+dx]}{3b} + \frac{4}{3} C \tan[c+dx]\right)}{d(A+2C+A \cos[2c+2dx])}
 \end{aligned}$$

■ **Problem 713: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 4, 352 leaves, 6 steps):

$$\frac{1}{bd} (a-b) \sqrt{a+b} (A-2C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{bd}$$

$$\sqrt{a+b} (Ab+2(a-b)C) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{Ab \sqrt{a+b} \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{ad} + \frac{A \sqrt{a+b \operatorname{Sec}[c+dx]} \sin[c+dx]}{d}$$

Result (type 4, 727 leaves):

$$\frac{2 C \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d} +$$

$$\left(\sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(a A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a A \right. \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 4 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + a A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 2 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right.$$

$$2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} -$$

$$2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + (a+b)(A-2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2(A b - (a+b) C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right]$$

$$\left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \Bigg/$$

$$\left(d \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right)$$

- **Problem 714: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 411 leaves, 7 steps):

$$\frac{A(a-b)\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4ad}}{4ad} + \frac{1}{4ad}$$

$$\sqrt{a+b}(Ab+2a(A+4C))\operatorname{Cot}[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{4a^2d}\sqrt{a+b}(Ab^2-4a^2(A+2C))\operatorname{Cot}[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{\frac{-b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{Ab\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{4ad} + \frac{A\operatorname{Cos}[c+dx]\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1417 leaves):

$$\frac{A\sqrt{a+b}\operatorname{Sec}[c+dx]\operatorname{Sin}[2(c+dx)]}{4d} +$$

$$\left(\sqrt{a+b}\operatorname{Sec}[c+dx]\left(-aAb\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - Ab^2\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 2aAb\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 -\right.\right.$$

$$\left.aAb\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + Ab^2\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 8ia^2A\operatorname{EllipticPi}\left[-\frac{a+b}{a-b},\right.\right.$$

$$\left.i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$2iAb^2\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 16ia^2C\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\begin{aligned}
& 8 i a^2 A \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 i A b^2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 16 i a^2 C \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i A (a-b) b \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i (a-b) (Ab+2a(A+2C)) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) \Bigg) / \\
& \left(4 a \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^2\right) \\
& \left(1+\tan\left[\frac{1}{2}(c+dx)\right]\right)^{3/2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}}
\end{aligned}$$

■ **Problem 715: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 \sqrt{a+b \sec[c+dx]} (A+C \sec[c+dx]^2) dx$$

Optimal (type 4, 502 leaves, 8 steps) :

$$\begin{aligned}
 & -\frac{1}{24 a^2 b d} (a-b) \sqrt{a+b} (3 A b^2 - 8 a^2 (2 A + 3 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{24 a^2 d} \sqrt{a+b} (2 a A b - 3 A b^2 + 8 a^2 (2 A + 3 C)) \\
 & \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
 & \frac{1}{8 a^3 d} b \sqrt{a+b} (A b^2 + 4 a^2 (A + 2 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
 & \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{(3 A b^2 - 8 a^2 (2 A + 3 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 a^2 d} + \\
 & \frac{A b \operatorname{Cos}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 a d} + \frac{A \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d}
 \end{aligned}$$

Result (type 4, 1347 leaves) :

$$\begin{aligned}
 & \frac{\sqrt{a+b \operatorname{Sec}[c+d x]} \left(\frac{1}{12} A \operatorname{Sin}[c+d x] + \frac{A b \operatorname{Sin}[2(c+d x)]}{24 a} + \frac{1}{12} A \operatorname{Sin}[3(c+d x)] \right)}{d} + \\
 & \left(\sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
 & \left. \left(-16 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 16 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 3 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 24 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right. \right. \\
 & \left. \left. 24 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 32 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 6 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 48 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 16 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \right. \right. \\
 & \left. \left. 16 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 24 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 24 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 \right) \right. \\
 & \left. + 24 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) +
 \end{aligned}$$

$$\begin{aligned}
& 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a^2 b C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 48 a^2 b C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& (a+b) (-3 A b^2 + 8 a^2 (2 A + 3 C)) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2 a b (14 a A - A b + 24 a C) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(24 a^2 d \sqrt{b+a \text{Cos}[c+dx]} \sqrt{\text{Sec}[c+dx]} \sqrt{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 716: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 \sqrt{a+b \text{Sec}[c+dx]} (A+C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 587 leaves, 9 steps):

$$\frac{1}{192 a^3 d} (a-b) \sqrt{a+b} (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{192 a^3 d}$$

$$\sqrt{a+b} (10 a A b^2 - 15 A b^3 - 24 a^3 (3 A + 4 C) - 4 a^2 b (7 A + 12 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{64 a^4 d} \sqrt{a+b} (5 A b^4 + 8 a^2 b^2 (A + 2 C) - 16 a^4 (3 A + 4 C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{b(15 A b^2 + 4 a^2 (7 A + 12 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{192 a^3 d} - \frac{(5 A b^2 - 12 a^2 (3 A + 4 C)) \operatorname{Cos}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{96 a^2 d} +$$

$$\frac{A b \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 a d} + \frac{A \operatorname{Cos}[c+d x]^3 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d}$$

Result (type 4, 4121 leaves):

$$\frac{\sqrt{a+b \operatorname{Sec}[c+d x]} \left(\frac{A b \operatorname{Sin}[c+d x]}{96 a} + \frac{(48 a^2 A - 5 A b^2 + 48 a^2 C) \operatorname{Sin}[2(c+d x)]}{192 a^2} + \frac{A b \operatorname{Sin}[3(c+d x)]}{96 a} + \frac{1}{32} A \operatorname{Sin}[4(c+d x)] \right)}{d} +$$

$$\left(\frac{3 a A}{8 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{A b^2}{96 a \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a C}{2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{25 A b \sqrt{\operatorname{Sec}[c+d x]}}{96 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \right.$$

$$\frac{5 A b^3 \sqrt{\operatorname{Sec}[c+d x]}}{384 a^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{3 b C \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{7 A b \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{96 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{5 A b^3 \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{128 a^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} +$$

$$\left. \frac{b C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} \right) \sqrt{a+b \operatorname{Sec}[c+d x]} \frac{\left(b(15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right)}{192 a^3 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} +$$

$$\begin{aligned}
& \left(b (a+b) (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2 a (2 a A b^2 + 5 A b^3 + 24 a^3 (3 A + 4 C) - 12 a^2 b (3 A + 4 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6 (-5 A b^4 - 8 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^4} \Bigg/ \\
& \left(192 a^3 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 \right)^2 \right) \right) \Bigg/ \\
& \left(d \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(\frac{b (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}}}{384 a^3 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}}} - \right. \right. \\
& \left. \left(b (a+b) (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] - \right. \right. \\
& 2 a (2 a A b^2 + 5 A b^3 + 24 a^3 (3 A + 4 C) - 12 a^2 b (3 A + 4 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6 (-5 A b^4 - 8 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^4}
\end{aligned}$$

$$\begin{aligned}
& \left(-2 b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 2 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right) / \\
& \left(192 a^3 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right)^2 \right) - \\
& \left(b(a+b) (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2 a (2 a A b^2 + 5 A b^3 + 24 a^3 (3 A + 4 C) - 12 a^2 b (3 A + 4 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6 (-5 A b^4 - 8 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} / \\
& \left(192 a^3 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) + \\
& \left(b(a+b) (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2 a (2 a A b^2 + 5 A b^3 + 24 a^3 (3 A + 4 C) - 12 a^2 b (3 A + 4 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6 (-5 A b^4 - 8 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \Bigg/ \left(384 a^3 (a+b) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \\
& \left. \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) - \frac{1}{384 a^3 \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right)} \\
& \left(b (a+b) (15 A b^2 + 4 a^2 (7 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2 a (2 a A b^2 + 5 A b^3 + 24 a^3 (3 A + 4 C) - 12 a^2 b (3 A + 4 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6 (-5 A b^4 - 8 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} - \\
& \left(b (15 A b^2 + 4 a^2 (7 A + 12 C)) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \right) \Bigg/ \left(384 a^3 \left(\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \\
& \left(b (15 A b^2 + 4 a^2 (7 A + 12 C)) \tan\left[\frac{1}{2}(c+dx)\right] \left(\frac{-a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \right) \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \left(384 a^3 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left(b(a+b) (15Ab^2 + 4a^2(7A+12C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2a(2aAb^2 + 5Ab^3 + 24a^3(3A+4C) - 12a^2b(3A+4C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - 6(-5Ab^4 - 8a^2b^2(A+2C) + \\
& 16a^4(3A+4C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left(\frac{-a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] (a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2)}{(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2)^2} \right) \Bigg) / \\
& \left(384 a^3 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) + \\
& \left(\sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left(- \frac{a(2aAb^2 + 5Ab^3 + 24a^3(3A+4C) - 12a^2b(3A+4C)) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{3(-5Ab^4 - 8a^2b^2(A+2C) + 16a^4(3A+4C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \left. \frac{b(a+b)(15Ab^2 + 4a^2(7A+12C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \Bigg/ \\
& \left. \left(192a^3 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right) \Bigg)
\end{aligned}$$

■ **Problem 717: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 550 leaves, 8 steps):

$$\frac{1}{1155 b^5 d} 4 a (a-b) \sqrt{a+b} (8 a^4 C + 3 a^2 b^2 (11 A + 6 C) - b^4 (451 A + 348 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{1155 b^4 d}$$

$$2(a-b) \sqrt{a+b} (16 a^4 C + 12 a^3 b C + 6 a^2 b^2 (11 A + 8 C) - 25 b^4 (11 A + 9 C) + 3 a b^3 (209 A + 157 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2(8 a^4 C + 25 b^4 (11 A + 9 C) + a^2 b^2 (33 A + 19 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{1155 b^3 d} +$$

$$\frac{4 a (132 A b^2 - 3 a^2 C + 101 b^2 C) \operatorname{Sec}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{1155 b^2 d} +$$

$$\frac{2(a^2 C + 3 b^2 (11 A + 9 C)) \operatorname{Sec}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{231 b d} +$$

$$\frac{2 a C \operatorname{Sec}[c+d x]^3 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{33 d} + \frac{2 C \operatorname{Sec}[c+d x]^3 (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{11 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 718: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+d x]^2 (a+b \operatorname{Sec}[c+d x])^{3/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 454 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} \left(8 a^4 C + 21 b^4 (9 A + 7 C) + 3 a^2 b^2 (21 A + 11 C) \right) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} \left(8 a^3 C + 6 a^2 b C - 21 b^3 (9 A + 7 C) + 3 a b^2 (21 A + 13 C) \right) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{2 a (63 A b^2 + 8 a^2 C + 39 b^2 C) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{315 b^2 d} + \frac{2 (8 a^2 C + 7 b^2 (9 A + 7 C)) (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{315 b^2 d} - \\
& \frac{8 a C (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{63 b^2 d} + \frac{2 C \text{Sec}[c+d x] (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{9 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 719: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+d x] (a+b \text{Sec}[c+d x])^{3/2} (A+C \text{Sec}[c+d x])^2 dx$$

Optimal (type 4, 374 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{105 b^3 d} 4 a (a-b) \sqrt{a+b} \left(70 A b^2 - 3 a^2 C + 41 b^2 C \right) \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{1}{105 b^2 d} 2 (a-b) \sqrt{a+b} \left(105 a A b - 35 A b^2 + 6 a^2 C + 57 a b C - 25 b^2 C \right) \\
& \quad \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{2 (6 a^2 C - 5 b^2 (7 A + 5 C)) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{105 b d} - \frac{4 a C (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{35 b d} + \frac{2 C (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{7 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 720: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^{3/2} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 415 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{5 b^2 d} 2 (a - b) \sqrt{a + b} (a^2 C + b^2 (5 A + 3 C)) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{5 b d} 2 \sqrt{a + b} (a^2 C - 2 a b (5 A + 2 C) + b^2 (5 A + 3 C)) \\ & \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{d} \\ & 2 a A \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} + \\ & \frac{2 a C \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{5 d} + \frac{2 C (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]}{5 d} \end{aligned}$$

Result (type 4, 1023 leaves):

$$\begin{aligned} & -\left(\left(4 (a + b \operatorname{Sec}[c + d x])^{3/2} (A + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \right. \\ & \left. \left(5 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 5 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\ & \left. \left. 3 b^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 10 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 2 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 6 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 5 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - \right. \right. \\ & \left. \left. 5 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 3 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 3 b^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + \right. \right. \\ & \left. \left. 10 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} + \right. \right. \\ & \left. \left. 10 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \right) \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) \left(5Ab^2+(a^2+3b^2)C\right) \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& b \left(a^2(-5A+C)+2ab(5A+2C)+b^2(5A+3C)\right) \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left(5bd(b+a \cos[c+dx])^{3/2}(A+2C+A \cos[2c+2dx]) \sec[c+dx]^{7/2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) + \\
& \left(\cos[c+dx]^3(a+b \sec[c+dx])^{3/2}(A+C \sec[c+dx]^2) \left(\frac{4(5Ab^2+a^2C+3b^2C) \sin[c+dx]}{5b} + \frac{8}{5}aC \tan[c+dx] + \right. \right. \\
& \left. \left. \frac{4}{5}bC \sec[c+dx] \tan[c+dx] \right) \right) / (d(b+a \cos[c+dx])(A+2C+A \cos[2c+2dx]))
\end{aligned}$$

- **Problem 722: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a+b \sec[c+dx])^{3/2} (A+C \sec[c+dx]^2) dx$$

Optimal (type 4, 414 leaves, 7 steps):

$$\frac{1}{4d} (a-b) \sqrt{a+b} (5A-8C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4d}$$

$$\sqrt{a+b} (2aA+5Ab+16aC-8bC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{4ad} \sqrt{a+b} (3Ab^2+4a^2(A+2C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{3Ab\sqrt{a+b \operatorname{Sec}[c+dx]} \sin[c+dx]}{4d} + \frac{A \cos[c+dx] (a+b \operatorname{Sec}[c+dx])^{3/2} \sin[c+dx]}{2d}$$

Result (type 4, 1618 leaves):

$$\frac{1}{2} \left(\frac{\cos[c+dx] (a+b \operatorname{Sec}[c+dx])^{3/2} (4bC \sin[c+dx] + \frac{1}{2} aA \sin[2(c+dx)])}{d(b+a \cos[c+dx])} - \right.$$

$$\left. \left((a+b \operatorname{Sec}[c+dx])^{3/2} \left(5aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] + 5Ab^2 \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] - 8ab \sqrt{\frac{-a+b}{a+b}} C \tan\left[\frac{1}{2}(c+dx)\right] - \right. \right. \right.$$

$$8b^2 \sqrt{\frac{-a+b}{a+b}} C \tan\left[\frac{1}{2}(c+dx)\right] - 10aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 + 16ab \sqrt{\frac{-a+b}{a+b}} C \tan\left[\frac{1}{2}(c+dx)\right]^3 +$$

$$5aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 - 5Ab^2 \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 - 8ab \sqrt{\frac{-a+b}{a+b}} C \tan\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$8b^2 \sqrt{\frac{-a+b}{a+b}} C \tan\left[\frac{1}{2}(c+dx)\right]^5 - 8ia^2A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6iAb^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 16ia^2C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right.$$

$$\begin{aligned}
& i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \right. \\
& \left. \left. 16 i a^2 C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\right. \right. \right. \\
& \left. \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i(a-b)b(5A-8C) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \right. \right. \right. \\
& \left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \right. \\
& \left. \left. 2 i(a-b)(b(A-4C)+2a(A+2C)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \frac{a+b}{a-b}\right] \right. \right. \right. \\
& \left. \left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \right) \right) \Bigg/ \\
& \left(2 \sqrt{\frac{-a+b}{a+b}} d(b+a \operatorname{Cos}[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right. \\
& \left. \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) \right) \Bigg)
\end{aligned}$$

Problem 723: Result more than twice size of optimal antiderivative.

$$\int \cos [c+d x]^3 (a+b \sec [c+d x])^{3 / 2} (A+C \sec [c+d x]^2) d x$$

Optimal (type 4, 504 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{24 a b d} (a-b) \sqrt{a+b} (3 A b^2+8 a^2(2 A+3 C)) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{1}{24 a d} \sqrt{a+b} (16 a^2 A+14 a A b+3 A b^2+24 a^2 C+48 a b C) \\ & \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{1}{8 a^2 d} b \sqrt{a+b} (A b^2-12 a^2(A+2 C)) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{(3 A b^2+8 a^2(2 A+3 C)) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{24 a d} + \\ & \frac{A b \cos [c+d x] \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 d} + \frac{A \cos [c+d x]^2 (a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{3 d} \end{aligned}$$

Result (type 4, 1393 leaves):

$$\begin{aligned} & \left(\cos [c+d x]^3 (a+b \sec [c+d x])^{3 / 2} (A+C \sec [c+d x]^2) \left(\frac{1}{6} a A \sin [c+d x] + \frac{7}{12} A b \sin [2(c+d x)] + \frac{1}{6} a A \sin [3(c+d x)] \right) \right) / \\ & (d(b+a \cos [c+d x])(A+2 C+A \cos [2 c+2 d x])) + \left((a+b \sec [c+d x])^{3 / 2} (A+C \sec [c+d x]^2) \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right. \\ & \left. \left(16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right] + 16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right] + 3 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right] + 3 A b^3 \tan \left[\frac{1}{2}(c+d x)\right] + 24 a^3 C \tan \left[\frac{1}{2}(c+d x)\right] + \right. \right. \\ & \left. \left. 24 a^2 b C \tan \left[\frac{1}{2}(c+d x)\right] - 32 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^3 - 6 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^3 - 48 a^3 C \tan \left[\frac{1}{2}(c+d x)\right]^3 + 16 a^3 A \tan \left[\frac{1}{2}(c+d x)\right]^5 - \right. \right. \\ & \left. \left. 16 a^2 A b \tan \left[\frac{1}{2}(c+d x)\right]^5 + 3 a A b^2 \tan \left[\frac{1}{2}(c+d x)\right]^5 - 3 A b^3 \tan \left[\frac{1}{2}(c+d x)\right]^5 + 24 a^3 C \tan \left[\frac{1}{2}(c+d x)\right]^5 - 24 a^2 b C \tan \left[\frac{1}{2}(c+d x)\right]^5 \right) \right. \end{aligned}$$

$$\begin{aligned}
& 72 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 144 a^2 b C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 72 a^2 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 144 a^2 b C \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (3 A b^2 + 8 a^2 (2 A + 3 C)) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 a b (26 a A - 7 A b + 48 a C - 24 b C) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(12 a d (b + a \text{Cos}[c+dx])^{3/2} (A + 2 C + A \text{Cos}[2c+2dx]) \text{Sec}[c+dx]^{7/2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right)
\end{aligned}$$

$$\sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

■ **Problem 724: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 (a + b \sec[c + dx])^{3/2} (A + C \sec[c + dx]^2) dx$$

Optimal (type 4, 583 leaves, 9 steps):

$$\begin{aligned} & -\frac{1}{64 a^2 d} (a - b) \sqrt{a + b} (3 A b^2 - 4 a^2 (13 A + 20 C)) \cot[c + dx] \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{64 a^2 d} \\ & \sqrt{a + b} (2 a A b^2 - 3 A b^3 + 8 a^3 (3 A + 4 C) + a^2 (52 A b + 80 b C)) \cot[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \frac{1}{64 a^3 d} \sqrt{a + b} (3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \\ & \cot[c + dx] \text{EllipticPi}\left[\frac{a + b}{a}, \text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \\ & \frac{b(3 A b^2 - 4 a^2 (13 A + 20 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{64 a^2 d} + \frac{(A b^2 + 4 a^2 (3 A + 4 C)) \cos[c + dx] \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{32 a d} \\ & \frac{A b \cos[c + dx]^2 \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{8 d} + \frac{A \cos[c + dx]^3 (a + b \sec[c + dx])^{3/2} \sin[c + dx]}{4 d} \end{aligned}$$

Result (type 4, 4278 leaves):

$$\begin{aligned} & \left(\cos[c + dx]^3 (a + b \sec[c + dx])^{3/2} (A + C \sec[c + dx]^2) \right. \\ & \left. \left(\frac{3}{16} A b \sin[c + dx] + \frac{(16 a^2 A + A b^2 + 16 a^2 C) \sin[2(c + dx)]}{32 a} + \frac{3}{16} A b \sin[3(c + dx)] + \frac{1}{16} a A \sin[4(c + dx)] \right) \right) / \\ & (d (b + a \cos[c + dx]) (A + 2 C + A \cos[2c + 2dx])) + \end{aligned}$$

$$\left(\left(\frac{3 a^2 A}{4 \sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{19 A b^2}{16 \sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{a^2 C}{\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \right. \right.$$

$$\frac{2 b^2 C}{\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{19 a A b \sqrt{\sec [c+d x]}}{16 \sqrt{b+a \cos [c+d x]}} - \frac{A b^3 \sqrt{\sec [c+d x]}}{64 a \sqrt{b+a \cos [c+d x]}} + \frac{7 a b C \sqrt{\sec [c+d x]}}{4 \sqrt{b+a \cos [c+d x]}} +$$

$$\left. \frac{13 a A b \cos [2 (c+d x)] \sqrt{\sec [c+d x]}}{16 \sqrt{b+a \cos [c+d x]}} - \frac{3 A b^3 \cos [2 (c+d x)] \sqrt{\sec [c+d x]}}{64 a \sqrt{b+a \cos [c+d x]}} + \frac{5 a b C \cos [2 (c+d x)] \sqrt{\sec [c+d x]}}{4 \sqrt{b+a \cos [c+d x]}} \right)$$

$$(a+b \sec [c+d x])^{3/2} (A+C \sec [c+d x])^2 \left(\frac{b \left(-3 A b^2 + a^2 (52 A + 80 C) \right) \tan \left[\frac{1}{2} (c+d x) \right] \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2}}}{32 a^2 \sqrt{\frac{1+\tan \left[\frac{1}{2} (c+d x) \right]^2}{1-\tan \left[\frac{1}{2} (c+d x) \right]^2}}} - \right.$$

$$\left. \left(\left(b (a+b) \left(3 A b^2 - 4 a^2 (13 A + 20 C) \right) \text{EllipticE} \left[\text{ArcSin} \left[\tan \left[\frac{1}{2} (c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] + \right. \right.$$

$$2 \left(a \left(-A b^3 + 8 a^3 (3 A + 4 C) - 4 a^2 b (3 A + 4 C) + 2 a b^2 (19 A + 32 C) \right) \text{EllipticF} \left[\text{ArcSin} \left[\tan \left[\frac{1}{2} (c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] +$$

$$\left. \left. \left(3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C) \right) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\tan \left[\frac{1}{2} (c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] \right) \right)$$

$$\sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2}} \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^4} \right) /$$

$$\left. \left. \left(32 a^2 \sqrt{\frac{1}{1-\tan \left[\frac{1}{2} (c+d x) \right]^2}} \left(b-b \tan \left[\frac{1}{2} (c+d x) \right] \right)^4 + a \left(-1+\tan \left[\frac{1}{2} (c+d x) \right] \right)^2 \right)^2 \right) \right) \right) /$$

$$\left(d (b + a \cos [c + d x])^{3/2} (A + 2 C + A \cos [2 c + 2 d x]) \sec [c + d x]^{7/2} \right.$$

$$\left. \frac{b (-3 A b^2 + a^2 (52 A + 80 C)) \sec \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1+\tan \left[\frac{1}{2} (c+d x) \right]^2}}}{64 a^2 \sqrt{\frac{1+\tan \left[\frac{1}{2} (c+d x) \right]^2}{1-\tan \left[\frac{1}{2} (c+d x) \right]^2}}} + \right.$$

$$\left(b (a + b) (3 A b^2 - 4 a^2 (13 A + 20 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right.$$

$$2 \left(a (-A b^3 + 8 a^3 (3 A + 4 C) - 4 a^2 b (3 A + 4 C) + 2 a b^2 (19 A + 32 C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right.$$

$$\left. \left. (3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \right)$$

$$\sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^4}$$

$$\left(-2 b \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]^3 + 2 a \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \left(-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) /$$

$$\left(32 a^2 \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}} \left(b - b \tan \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right)^2 \right) +$$

$$\left(b (a + b) (3 A b^2 - 4 a^2 (13 A + 20 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right.$$

$$\begin{aligned}
& 2 \left(a \left(-A b^3 + 8 a^3 (3 A + 4 C) - 4 a^2 b (3 A + 4 C) + 2 a b^2 (19 A + 32 C) \right) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad \left. (3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \\
& \text{Tan} \left[\frac{1}{2} (c + d x) \right]^3 \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \Big/ \\
& \left(32 a^2 \sqrt{\frac{1}{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \left(b - b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) - \\
& \left(b (a + b) (3 A b^2 - 4 a^2 (13 A + 20 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad 2 \left(a \left(-A b^3 + 8 a^3 (3 A + 4 C) - 4 a^2 b (3 A + 4 C) + 2 a b^2 (19 A + 32 C) \right) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad \left. (3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \Big/ \\
& \left(-a \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[\frac{1}{2} (c + d x) \right] + b \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[\frac{1}{2} (c + d x) \right] \right) \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \\
& \sqrt{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \Big/ \left(64 a^2 (a + b) \sqrt{\frac{1}{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right. \\
& \quad \left. \left(b - b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) + \frac{1}{64 a^2 \left(b - b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right)} \\
& \left(b (a + b) (3 A b^2 - 4 a^2 (13 A + 20 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad 2 \left(a \left(-A b^3 + 8 a^3 (3 A + 4 C) - 4 a^2 b (3 A + 4 C) + 2 a b^2 (19 A + 32 C) \right) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad \left. (3 A b^4 + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} - \\
& \left(b(-3Ab^2+a^2(52A+80C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \left(64a^2 \left(\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \\
& \left(b(-3Ab^2+a^2(52A+80C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \left(64a^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) - \\
& \left(b(a+b)(3Ab^2-4a^2(13A+20C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \left. 2 \left(a(-Ab^3+8a^3(3A+4C)-4a^2b(3A+4C)+2ab^2(19A+32C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. (3Ab^4+24a^2b^2(A+2C)+16a^4(3A+4C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \\
& \left(64 a^2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right) - \\
& \left(\sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left(\frac{b(a+b) \left(3Ab^2 - 4a^2(13A+20C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} + \right. \right. \\
& \left. \left. 2 \frac{a \left(-Ab^3 + 8a^3(3A+4C) - 4a^2b(3A+4C) + 2ab^2(19A+32C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \right. \right. \\
& \left. \left. \frac{\left(3Ab^4 + 24a^2b^2(A+2C) + 16a^4(3A+4C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1 - \frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} \right) \right) \Bigg/
\end{aligned}$$

$$\left(\left(32 a^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left(b - b \tan\left[\frac{1}{2}(c + dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^2 \right) \right) \right)$$

■ **Problem 725: Attempted integration timed out after 120 seconds.**

$$\int \sec[c + dx]^3 (a + b \sec[c + dx])^{5/2} (A + C \sec[c + dx]^2) dx$$

Optimal (type 4, 650 leaves, 9 steps):

$$\frac{1}{45045 b^5 d} 2 (a - b) \sqrt{a + b} (240 a^6 C - 1617 b^6 (13 A + 11 C) + 10 a^4 b^2 (143 A + 76 C) - 3 a^2 b^4 (13299 A + 10223 C))$$

$$\cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{45045 b^4 d}$$

$$2 (a - b) \sqrt{a + b} (240 a^5 C + 180 a^4 b C + 1617 b^5 (13 A + 11 C) + 10 a^3 b^2 (143 A + 94 C) + 15 a^2 b^3 (1573 A + 1175 C) - 6 a b^4 (2717 A + 2174 C))$$

$$\cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} +$$

$$\frac{2 a (120 a^4 C + 5 a^2 b^2 (143 A + 79 C) + b^4 (23309 A + 18973 C)) \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{45045 b^3 d} -$$

$$\frac{2 (90 a^4 C - 539 b^4 (13 A + 11 C) - 15 a^2 b^2 (715 A + 543 C)) \sec[c + dx] \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{45045 b^2 d} +$$

$$\frac{2 a (2717 A b^2 + 15 a^2 C + 2209 b^2 C) \sec[c + dx]^2 \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{9009 b d} +$$

$$\frac{2 (15 a^2 C + 11 b^2 (13 A + 11 C)) \sec[c + dx]^3 \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{1287 d} +$$

$$\frac{10 a C \sec[c + dx]^3 (a + b \sec[c + dx])^{3/2} \tan[c + dx]}{143 d} + \frac{2 C \sec[c + dx]^3 (a + b \sec[c + dx])^{5/2} \tan[c + dx]}{13 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 726: Attempted integration timed out after 120 seconds.**

$$\int \sec[c + dx]^2 (a + b \sec[c + dx])^{5/2} (A + C \sec[c + dx]^2) dx$$

Optimal (type 4, 534 leaves, 8 steps) :

$$\begin{aligned}
 & - \frac{1}{693 b^4 d} 2 a (a-b) \sqrt{a+b} \left(8 a^4 C + 3 a^2 b^2 (33 A + 17 C) + 3 b^4 (319 A + 247 C) \right) \\
 & \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
 & \frac{1}{693 b^3 d} 2 (a-b) \sqrt{a+b} \left(8 a^4 C + 6 a^3 b C + 15 b^4 (11 A + 9 C) + 3 a^2 b^2 (33 A + 19 C) - 6 a b^3 (132 A + 101 C) \right) \text{Cot}[c+dx] \\
 & \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
 & \frac{2 \left(8 a^4 C + 15 b^4 (11 A + 9 C) + 3 a^2 b^2 (33 A + 19 C) \right) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{693 b^2 d} + \\
 & \frac{2 a \left(99 A b^2 + 8 a^2 C + 67 b^2 C \right) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{693 b^2 d} + \frac{2 \left(8 a^2 C + 9 b^2 (11 A + 9 C) \right) (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{693 b^2 d} - \\
 & \frac{8 a C (a+b \text{Sec}[c+dx])^{7/2} \text{Tan}[c+dx]}{99 b^2 d} + \frac{2 C \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{7/2} \text{Tan}[c+dx]}{11 b d}
 \end{aligned}$$

Result (type 1, 1 leaves) :

???

■ **Problem 727: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{5/2} (A+C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 454 leaves, 7 steps) :

$$\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (10 a^4 C - 21 b^4 (9 A + 7 C) - 3 a^2 b^2 (161 A + 93 C))$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{315 b^2 d} 2 (a-b) \sqrt{a+b} (10 a^3 C + 21 b^3 (9 A + 7 C) + 15 a^2 b (21 A + 11 C) - 6 a b^2 (28 A + 19 C)) \text{Cot}[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{4 a (84 A b^2 - 5 a^2 C + 57 b^2 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{315 b d} - \frac{2 (10 a^2 C - 7 b^2 (9 A + 7 C)) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{315 b d}$$

$$\frac{4 a C (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{63 b d} + \frac{2 C (a+b \text{Sec}[c+dx])^{7/2} \text{Tan}[c+dx]}{9 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 729: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c+dx] (a+b \text{Sec}[c+dx])^{5/2} (A+C \text{Sec}[c+dx])^2 dx$$

Optimal (type 4, 478 leaves, 8 steps):

$$\frac{1}{15 b d} (a-b) \sqrt{a+b} (a^2 (15 A - 46 C) - 6 b^2 (5 A + 3 C)) \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{1}{15 b d} \sqrt{a+b} (a^2 b (15 A - 46 C) + 30 a^3 C - 6 b^3 (5 A + 3 C) + 2 a b^2 (45 A + 17 C))$$

$$\text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{d}$$

$$5 a A b \sqrt{a+b} \text{Cot}[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} +$$

$$\frac{A (a+b \text{Sec}[c+dx])^{5/2} \text{Sin}[c+dx]}{d} - \frac{a b (15 A - 16 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{15 d} - \frac{b (5 A - 2 C) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{5 d}$$

Result (type 4, 1129 leaves):

$$\left(2 (a + b \operatorname{Sec}[c + d x])^{5/2} (A + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right.$$

$$\left(15 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + 15 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 30 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 30 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 46 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \right.$$

$$46 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 18 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 18 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - 30 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + 60 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 +$$

$$92 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + 36 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + 15 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - 15 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - 30 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 +$$

$$30 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - 46 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 + 46 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - 18 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 + 18 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 -$$

$$150 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} -$$

$$150 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}$$

$$\sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} + (a + b) (a^2 (15 A - 46 C) - 6 b^2 (5 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right]$$

$$\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} +$$

$$2 (15 a^3 C + 3 b^3 (5 A + 3 C) + a b^2 (45 A + 17 C) + a^2 (-45 A b + 23 b C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right]$$

$$\left. \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} \right) \right) /$$

$$\left(15 d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right)^{3/2} \right.$$

$$\left(\frac{\sqrt{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} + \left(\cos[c+dx]^4 (a+b \sec[c+dx])^{5/2} (A+C \sec[c+dx]^2) \left(\frac{4}{15} (15Ab^2 + 23a^2C + 9b^2C) \sin[c+dx] + \frac{44}{15} abc \tan[c+dx] + \frac{4}{5} b^2C \sec[c+dx] \tan[c+dx] \right) \right) \right) / (d(b+a \cos[c+dx])^2 (A+2C+A \cos[2c+2dx]))$$

■ **Problem 731: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 (a+b \sec[c+dx])^{5/2} (A+C \sec[c+dx]^2) dx$$

Optimal (type 4, 507 leaves, 8 steps):

$$\frac{1}{24bd} (a-b) \sqrt{a+b} (3b^2(11A-16C) + 8a^2(2A+3C)) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{24d} \sqrt{a+b} (16a^2A + 26aAb + 33Ab^2 + 24a^2C + 144abc - 48b^2C) \\ \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ \frac{1}{8ad} 5b\sqrt{a+b} (Ab^2 + 4a^2(A+2C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{(15Ab^2 + 8a^2(2A+3C)) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{24d} + \\ \frac{5Ab \cos[c+dx] (a+b \sec[c+dx])^{3/2} \sin[c+dx]}{12d} + \frac{A \cos[c+dx]^2 (a+b \sec[c+dx])^{5/2} \sin[c+dx]}{3d}$$

Result (type 4, 1513 leaves):

$$\left(\cos[c+dx]^4 (a+b \sec[c+dx])^{5/2} (A+C \sec[c+dx]^2) \left(\frac{1}{6} (a^2A + 24b^2C) \sin[c+dx] + \frac{13}{12} abc \sin[2(c+dx)] + \frac{1}{6} a^2A \sin[3(c+dx)] \right) \right) / (d(b+a \cos[c+dx])^2 (A+2C+A \cos[2c+2dx])) + \\ \left((a+b \sec[c+dx])^{5/2} (A+C \sec[c+dx]^2) \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(16a^3A \tan\left[\frac{1}{2}(c+dx)\right] + 16a^2Ab \tan\left[\frac{1}{2}(c+dx)\right] + \right. \right.$$

$$\begin{aligned}
& 33 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 33 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 24 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 24 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 48 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& 48 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 32 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 66 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 48 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 96 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \\
& 16 a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 16 a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 33 a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 33 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 24 a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 24 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 48 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 48 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 120 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 30 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 240 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 240 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -
\end{aligned}$$

$$2 b (24 b^2 (A - C) - a b (13 A + 72 C) + a^2 (38 A + 72 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right]$$

$$\sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} \right) \Bigg| \Bigg|$$

$$\left(12 d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2\right)^{3/2}\right.$$

$$\left.\sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}}\right)$$

■ **Problem 734: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \operatorname{Sec}[c + d x]} (a^2 - b^2 \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 353 leaves, 7 steps):

$$\frac{2 a (a - b) \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}}{3 d} + \frac{1}{3 d}$$

$$2 \sqrt{a + b} (3 a^2 + a b - b^2) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{1}{d} 2 a^2 \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 b^2 \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 4, 799 leaves):

$$\begin{aligned}
& - \left(\left(4 \sqrt{a+b \operatorname{Sec}[c+dx]} (a^2 - b^2 \operatorname{Sec}[c+dx]^2) \right. \right. \\
& \quad \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(i a (a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \quad \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& \quad i (3 a^3 - 3 a^2 b - a b^2 + b^3) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \quad \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6 i a^3 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& \quad \left. \left. a b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right) \Bigg) / \\
& \quad \left(3 \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \operatorname{Cos}[c+dx]} (a^2 - 2 b^2 + a^2 \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \quad \left. \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \Bigg) + \\
& \quad \frac{\operatorname{Cos}[c+dx]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} (a^2 - b^2 \operatorname{Sec}[c+dx]^2) \left(-\frac{4}{3} a b \operatorname{Sin}[c+dx] - \frac{4}{3} b^2 \operatorname{Tan}[c+dx] \right)}{d (a^2 - 2 b^2 + a^2 \operatorname{Cos}[2c+2dx])}
\end{aligned}$$

■ **Problem 735: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^3 (A + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 393 leaves, 6 steps):

$$\frac{1}{105 b^5 d} 4 a (a - b) \sqrt{a + b} (35 A b^2 + 24 a^2 C + 22 b^2 C) \cot[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{105 b^4 d}$$

$$2 \sqrt{a + b} (48 a^3 C - 12 a^2 b C + 5 b^3 (7 A + 5 C) + 2 a b^2 (35 A + 22 C)) \cot[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{2(24 a^2 C + 5 b^2 (7 A + 5 C)) \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{105 b^3 d}$$

$$\frac{12 a C \sec[c + dx] \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{35 b^2 d} + \frac{2 C \sec[c + dx]^2 \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{7 b d}$$

Result (type 8, 37 leaves):

$$\int \frac{\sec[c + dx]^3 (A + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

■ **Problem 736: Unable to integrate problem.**

$$\int \frac{\sec[c + dx]^2 (A + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 320 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{15b^4d} 2(a-b)\sqrt{a+b} (8a^2C + 3b^2(5A+3C)) \operatorname{Cot}[c+dx] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{15b^3d} \\
& 2\sqrt{a+b} (8a^2C - 2abC + 3b^2(5A+3C)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \\
& \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{8aC\sqrt{a+b\operatorname{Sec}[c+dx]}\operatorname{Tan}[c+dx]}{15b^2d} + \frac{2C\operatorname{Sec}[c+dx]\sqrt{a+b\operatorname{Sec}[c+dx]}\operatorname{Tan}[c+dx]}{5bd}
\end{aligned}$$

Result (type 8, 37 leaves):

$$\int \frac{\operatorname{Sec}[c+dx]^2 (A+C\operatorname{Sec}[c+dx]^2)}{\sqrt{a+b\operatorname{Sec}[c+dx]}} dx$$

■ **Problem 737: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[c+dx] (A+C\operatorname{Sec}[c+dx]^2)}{\sqrt{a+b\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 253 leaves, 4 steps):

$$\begin{aligned}
& \frac{1}{3b^3d} 4a(a-b)\sqrt{a+b} C \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3b^2d} \\
& 2\sqrt{a+b} (3Ab + (2a+b)C) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \\
& \frac{2C\sqrt{a+b\operatorname{Sec}[c+dx]}\operatorname{Tan}[c+dx]}{3bd}
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \frac{\operatorname{Sec}[c+dx] (A+C\operatorname{Sec}[c+dx]^2)}{\sqrt{a+b\operatorname{Sec}[c+dx]}} dx$$

■ **Problem 738: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+C\operatorname{Sec}[c+dx]^2}{\sqrt{a+b\operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 313 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{b^2 d} 2 (a-b) \sqrt{a+b} C \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} \\
& \frac{2 \sqrt{a+b} C \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b d} \\
& \frac{2 A \sqrt{a+b} \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a d}
\end{aligned}$$

Result (type 4, 914 leaves):

$$\begin{aligned}
& \frac{4 C \cos [c+d x] (b+a \cos [c+d x]) (A+C \sec [c+d x]^2) \sin [c+d x]}{b d (A+2 C+A \cos [2 c+2 d x]) \sqrt{a+b \sec [c+d x]}} + \\
& \left(4 \sqrt{b+a \cos [c+d x]} (A+C \sec [c+d x]^2) \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \\
& \left. -a \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2}(c+d x)\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} -b \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2}(c+d x)\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} + \right. \\
& \left. a \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} -b \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} - \right. \\
& \left. 2 i A b \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right. \\
& \left. 2 i A b \operatorname{EllipticPi} \left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \right. \\
& \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} +i(a-b) C \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right. \\
& \left. \left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} +i b(A+C) \right. \\
& \left. \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) / \\
& \left(b \sqrt{\frac{-a+b}{a+b}} d (A+2 C+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)
\end{aligned}$$

■ **Problem 740: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2 (A + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 411 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{4a^2d} 3A(a-b)\sqrt{a+b} \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{4a^2d} \\ & A(2a-3b)\sqrt{a+b} \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{1}{4a^3d} \sqrt{a+b} (3Ab^2 + 4a^2(A+2C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \\ & \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{3Ab\sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4a^2d} + \frac{A \cos[c+dx] \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{2ad} \end{aligned}$$

Result (type 4, 1475 leaves):

$$\begin{aligned} & \frac{A(b+a \cos[c+dx]) \sec[c+dx] \sin[2(c+dx)]}{4ad\sqrt{a+b \sec[c+dx]}} - \left(\sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\ & \left(3aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] + 3Ab^2 \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right] - 6aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^3 + 3aAb \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 - \right. \\ & \left. 3Ab^2 \sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]^5 + 8ia^2A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ & \left. \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 6iAb^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \\ & \left. \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 16ia^2C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \right. \end{aligned}$$

Problem 741: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[c+dx]^3 (A+C \sec[c+dx]^2)}{\sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 506 leaves, 8 steps):

$$\frac{1}{24 a^3 b d} (a-b) \sqrt{a+b} (15 A b^2 + 8 a^2 (2 A + 3 C)) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{1}{24 a^3 d} \sqrt{a+b} (10 a A b - 15 A b^2 - 8 a^2 (2 A + 3 C))$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{8 a^4 d} b \sqrt{a+b} (5 A b^2 + 4 a^2 (A + 2 C)) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{(15 A b^2 + 8 a^2 (2 A + 3 C)) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{24 a^3 d}$$

$$\frac{5 A b \cos[c+dx] \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{12 a^2 d} + \frac{A \cos[c+dx]^2 \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{3 a d}$$

Result (type 4, 1363 leaves):

$$\frac{(b+a \cos[c+dx]) \sec[c+dx] \left(\frac{A \sin[c+dx]}{12 a} - \frac{5 A b \sin[2(c+dx)]}{24 a^2} + \frac{A \sin[3(c+dx)]}{12 a} \right)}{d \sqrt{a+b \sec[c+dx]}}$$

$$\left(\sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(16 a^3 A \tan\left[\frac{1}{2}(c+dx)\right] + 16 a^2 A b \tan\left[\frac{1}{2}(c+dx)\right] + 15 a A b^2 \tan\left[\frac{1}{2}(c+dx)\right] + 15 A b^3 \tan\left[\frac{1}{2}(c+dx)\right] + 24 a^3 C \tan\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$24 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right] - 32 a^3 A \tan\left[\frac{1}{2}(c+dx)\right]^3 - 30 a A b^2 \tan\left[\frac{1}{2}(c+dx)\right]^3 - 48 a^3 C \tan\left[\frac{1}{2}(c+dx)\right]^3 + 16 a^3 A \tan\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$16 a^2 A b \tan\left[\frac{1}{2}(c+dx)\right]^5 + 15 a A b^2 \tan\left[\frac{1}{2}(c+dx)\right]^5 - 15 A b^3 \tan\left[\frac{1}{2}(c+dx)\right]^5 + 24 a^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 - 24 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$\begin{aligned}
& 24 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 48 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 48 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (15 A b^2 + 8 a^2 (2 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 a A b (2 a+5 b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg/ \\
& \left(24 a^3 d \sqrt{a+b \operatorname{Sec}[c+dx]} \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)
\end{aligned}$$

■ Problem 742: Attempted integration timed out after 120 seconds.

$$\int \frac{\sec[c+dx]^3 (A+C \sec[c+dx]^2)}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 460 leaves, 6 steps):

$$-\frac{1}{5b^5 \sqrt{a+b} d} 2 (2a^2 b^2 (5A-4C) + 16a^4 C - b^4 (5A+3C)) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{1}{5b^4 \sqrt{a+b} d}$$

$$2 (16a^3 C + 12a^2 b C + 2ab^2 (5A+2C) + b^3 (5A+3C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{2(Ab^2+a^2C) \sec[c+dx]^2 \tan[c+dx]}{b(a^2-b^2) d \sqrt{a+b \sec[c+dx]}}$$

$$\frac{2a(5Ab^2+8a^2C-3b^2C) \sqrt{a+b \sec[c+dx]} \tan[c+dx]}{5b^3(a^2-b^2)d} + \frac{2(5Ab^2+6a^2C-b^2C) \sec[c+dx] \sqrt{a+b \sec[c+dx]} \tan[c+dx]}{5b^2(a^2-b^2)d}$$

Result (type 1, 1 leaves):

???

■ **Problem 743: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c+dx]^2 (A+C \sec[c+dx]^2)}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 327 leaves, 5 steps):

$$\frac{1}{3b^4 \sqrt{a+b} d} 2a (3Ab^2 + 8a^2C - 5b^2C) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{3b^3 \sqrt{a+b} d}$$

$$2 (3Ab^2 + (8a^2 + 6ab + b^2)C) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{2a(Ab^2+a^2C) \tan[c+dx]}{b^2(a^2-b^2)d \sqrt{a+b \sec[c+dx]}} + \frac{2C \sqrt{a+b \sec[c+dx]} \tan[c+dx]}{3b^2 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 744: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c + dx] (A + C \sec[c + dx])^2}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 279 leaves, 4 steps):

$$\begin{aligned} & -\frac{1}{b^3 \sqrt{a+b} d} 2 (A b^2 + 2 a^2 C - b^2 C) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{b^2 \sqrt{a+b} d} 2 (A b - (2 a + b) C) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\ & \frac{2 (A b^2 + a^2 C) \tan[c + dx]}{b (a^2 - b^2) d \sqrt{a+b \sec[c+dx]}} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 745: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 381 leaves, 6 steps):

$$\begin{aligned} & \frac{2 (A b^2 + a^2 C) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a b^2 \sqrt{a+b} d} - \\ & \frac{2 (A b - a C) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a b \sqrt{a+b} d} - \\ & \frac{2 A \sqrt{a+b} \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a^2 d} + \frac{2 (A b^2 + a^2 C) \tan[c + dx]}{a (a^2 - b^2) d \sqrt{a+b \sec[c+dx]}} \end{aligned}$$

Result (type 4, 1127 leaves):

$$\frac{(b + a \cos[c + dx])^2 (A + C \sec[c + dx])^2 \left(\frac{4 (A b^2 + a^2 C) \sin[c + dx]}{a b (-a^2 + b^2)} + \frac{4 (A b^2 \sin[c + dx] + a^2 C \sin[c + dx])}{a (a^2 - b^2) (b + a \cos[c + dx])} \right)}{d (A + 2 C + A \cos[2 c + 2 d x]) (a + b \sec[c + dx])^{3/2}}$$

$$\left(4 (b + a \cos [c + d x])^{3/2} (A + C \sec [c + d x])^2 \sqrt{\frac{1}{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}} \right.$$

$$\left(a A b^2 \tan \left[\frac{1}{2} (c + d x) \right] + A b^3 \tan \left[\frac{1}{2} (c + d x) \right] + a^3 C \tan \left[\frac{1}{2} (c + d x) \right] + a^2 b C \tan \left[\frac{1}{2} (c + d x) \right] - 2 a A b^2 \tan \left[\frac{1}{2} (c + d x) \right]^3 - \right.$$

$$2 a^3 C \tan \left[\frac{1}{2} (c + d x) \right]^3 + a A b^2 \tan \left[\frac{1}{2} (c + d x) \right]^5 - A b^3 \tan \left[\frac{1}{2} (c + d x) \right]^5 + a^3 C \tan \left[\frac{1}{2} (c + d x) \right]^5 - a^2 b C \tan \left[\frac{1}{2} (c + d x) \right]^5 -$$

$$2 a^2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} +$$

$$2 A b^3 \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} -$$

$$2 a^2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \tan^2 \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}$$

$$\sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 2 A b^3 \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\tan^2 \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} +$$

$$(a + b) (A b^2 + a^2 C) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \tan^2 \left[\frac{1}{2} (c + d x) \right]^2 \right)$$

$$\sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - a b (a + b) (A + C) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\left. \left. \sqrt{1 - \tan^2 \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \tan^2 \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \tan^2 \left[\frac{1}{2} (c + d x) \right] + b \tan^2 \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right) \right) \right) /$$

$$\left(a (-a^2 b + b^3) d (A + 2C + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + b \sec[c + dx])^{3/2} \left(1 + \tan\left[\frac{1}{2}(c + dx)\right] \right)^2 \right)^{3/2}$$

$$\sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

■ **Problem 746: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^{3/2}} dx$$

Optimal (type 4, 431 leaves, 7 steps):

$$-\frac{1}{a^2 b \sqrt{a+b} d} (3Ab^2 - a^2(A-2C)) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{1}{a^2 b \sqrt{a+b} d} (aAb + 3Ab^2 + 2a^2C) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{1}{a^3 d} 3Ab \sqrt{a+b} \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c + dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a-b}} +$$

$$\frac{A \sin[c + dx]}{a d \sqrt{a+b \sec[c + dx]}} - \frac{b(3Ab^2 - a^2(A-2C)) \tan[c + dx]}{a^2(a^2 - b^2) d \sqrt{a+b \sec[c + dx]}}$$

Result (type 4, 1259 leaves):

$$\frac{(b + a \cos[c + dx])^2 (A + C \sec[c + dx]^2) \left(\frac{4(Ab^2 + a^2C) \sin[c + dx]}{a^2(a^2 - b^2)} - \frac{4(Ab^3 \sin[c + dx] + a^2 b C \sin[c + dx])}{a^2(a^2 - b^2)(b + a \cos[c + dx])} \right)}{d (A + 2C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^{3/2}}$$

$$\left(2 (b + a \cos[c + dx])^{3/2} (A + C \sec[c + dx]^2) \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right.$$

$$\left. \left(a^3 A \tan\left[\frac{1}{2}(c + dx)\right] + a^2 A b \tan\left[\frac{1}{2}(c + dx)\right] - 3 a A b^2 \tan\left[\frac{1}{2}(c + dx)\right] - 3 A b^3 \tan\left[\frac{1}{2}(c + dx)\right] - 2 a^3 C \tan\left[\frac{1}{2}(c + dx)\right] \right) -$$

$$\int \frac{\cos [c+d x]^2 (A+C \sec [c+d x]^2)}{(a+b \sec [c+d x])^{3/2}} dx$$

Optimal (type 4, 501 leaves, 8 steps):

$$\frac{1}{4 a^3 \sqrt{a+b} d} (15 A b^2 - a^2 (7 A - 8 C)) \cot [c+d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{4 a^3 \sqrt{a+b} d}$$

$$(5 a A b + 15 A b^2 - 2 a^2 (A - 4 C)) \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}}$$

$$\frac{1}{4 a^4 d} \sqrt{a+b} (15 A b^2 + 4 a^2 (A + 2 C)) \cot [c+d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}}$$

$$\sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{5 A b \sin [c+d x]}{4 a^2 d \sqrt{a+b \sec [c+d x]}} + \frac{A \cos [c+d x] \sin [c+d x]}{2 a d \sqrt{a+b \sec [c+d x]}} + \frac{b^2 (15 A b^2 - a^2 (7 A - 8 C)) \tan [c+d x]}{4 a^3 (a^2 - b^2) d \sqrt{a+b \sec [c+d x]}}$$

Result (type 4, 41966 leaves): Display of huge result suppressed!

■ **Problem 748: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec [c+d x]^3 (A+C \sec [c+d x]^2)}{(a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 488 leaves, 6 steps):

$$\frac{1}{3 b^5 \sqrt{a+b} (a^2-b^2) d} 4 a (a^2 b^2 (A-14 C)-b^4 (3 A-4 C)+8 a^4 C) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 b^4 \sqrt{a+b} (a^2-b^2) d}$$

$$2 (2 a^2 b^2 (A-8 C)+3 a b^3 (A-3 C)+16 a^4 C+12 a^3 b C-b^4 (3 A+C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{2 (A b^2+a^2 C) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{3 b (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}}$$

$$\frac{4 a (2 A b^4-3 a^4 C+5 a^2 b^2 C) \operatorname{Tan}[c+d x]}{3 b^3 (a^2-b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 (A b^2+2 a^2 C-b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{3 b^3 (a^2-b^2) d}$$

Result (type 1, 1 leaves):

???

■ **Problem 749: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c+d x]^2 (A+C \operatorname{Sec}[c+d x]^2)}{(a+b \operatorname{Sec}[c+d x])^{5/2}} dx$$

Optimal (type 4, 408 leaves, 5 steps):

$$\frac{1}{3 b^4 \sqrt{a+b} (a^2-b^2) d} 2 (3 b^4 (A-C)-8 a^4 C+a^2 b^2 (A+15 C)) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{3 b^3 \sqrt{a+b} (a^2-b^2) d}$$

$$2 (3 b^3 (A-C)+8 a^3 C+6 a^2 b C-a b^2 (A+9 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 a (A b^2+a^2 C) \operatorname{Tan}[c+d x]}{3 b^2 (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 (3 A b^4-5 a^4 C+a^2 b^2 (A+9 C)) \operatorname{Tan}[c+d x]}{3 b^2 (a^2-b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 750: Unable to integrate problem.**

$$\int \frac{\sec[c + dx] (A + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^{5/2}} dx$$

Optimal (type 4, 378 leaves, 5 steps) :

$$\begin{aligned} & -\frac{1}{3(a-b)b^3(a+b)^{3/2}d} \\ & 4a(2Ab^2 - a^2C + 3b^2C) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\ & \frac{1}{3b^2\sqrt{a+b}(a^2-b^2)d} 2(2a^2C + 3ab(A+C) - b^2(A+3C)) \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{2(Ab^2 + a^2C) \tan[c + dx]}{3b(a^2-b^2)d(a+b\sec[c+dx])^{3/2}} - \frac{4a(2Ab^2 - a^2C + 3b^2C) \tan[c + dx]}{3b(a^2-b^2)^2d\sqrt{a+b\sec[c+dx]}} \end{aligned}$$

Result (type 8, 35 leaves) :

$$\int \frac{\sec[c + dx] (A + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^{5/2}} dx$$

■ **Problem 751: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + b \sec[c + dx])^{5/2}} dx$$

Optimal (type 4, 517 leaves, 7 steps) :

$$\begin{aligned}
& - \frac{1}{3 a^2 b^2 \sqrt{a+b} (a^2 - b^2) d} \\
& 2 (3 A b^4 - a^4 C - a^2 b^2 (7 A + 3 C)) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{3 a^2 (a-b) b (a+b)^{3/2} d} 2 (6 a^2 A b - a A b^2 - 3 A b^3 - a^3 C + 3 a^2 b C) \operatorname{Cot}[c + d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{2 A \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^3 d} + \\
& \frac{2 (A b^2 + a^2 C) \operatorname{Tan}[c + d x]}{3 a (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} - \frac{2 (3 A b^4 - a^4 C - a^2 b^2 (7 A + 3 C)) \operatorname{Tan}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 4, 1727 leaves):

$$\begin{aligned}
& \left((b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x] (A + C \operatorname{Sec}[c + d x])^2 \left(\frac{4 (-7 a^2 A b^2 + 3 A b^4 - a^4 C - 3 a^2 b^2 C) \operatorname{Sin}[c + d x]}{3 a^2 b (-a^2 + b^2)^2} - \right. \right. \\
& \left. \frac{4 (A b^3 \operatorname{Sin}[c + d x] + a^2 b C \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])^2} + \frac{8 (4 a^2 A b^2 \operatorname{Sin}[c + d x] - 2 A b^4 \operatorname{Sin}[c + d x] + a^4 C \operatorname{Sin}[c + d x] + a^2 b^2 C \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2)^2 (b + a \operatorname{Cos}[c + d x])} \right) \Bigg) / \\
& (d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^{5/2}) - \left(4 (b + a \operatorname{Cos}[c + d x])^{5/2} \sqrt{\operatorname{Sec}[c + d x]} (A + C \operatorname{Sec}[c + d x])^2 \right. \\
& \left. \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \\
& \left(7 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 7 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 3 a A b^4 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 3 A b^5 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + a^5 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right. \\
& \left. a^4 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 14 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 6 a A b^4 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 6 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 7 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 7 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \\
& 3 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 3 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 6 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 12 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 12 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + (a+b) (-3 A b^4 + a^4 C + a^2 b^2 (7 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& a b (a+b) (-2 A b^2 + 3 a b (A+C) + a^2 (3 A+C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(3 a^2 b (a^2 - b^2)^2 d (A + 2 C + A \cos[2c + 2dx]) (a + b \sec[c + dx])^{5/2} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 752: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] (A + C \sec[c+dx])^2}{(a + b \sec[c+dx])^{5/2}} dx$$

Optimal (type 4, 559 leaves, 8 steps):

$$-\frac{1}{3 a^3 (a-b) b (a+b)^{3/2} d} (26 a^2 A b^2 - 15 A b^4 - a^4 (3 A - 8 C)) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{3 a^3 (a-b) b (a+b)^{3/2} d} (21 a^2 A b^2 - 5 a A b^3 - 15 A b^4 + a^3 b (3 A - 2 C) + 6 a^4 C) \cot[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{a^4 d}$$

$$5 A b \sqrt{a+b} \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{A \sin[c+dx]}{a d (a+b \sec[c+dx])^{3/2}} - \frac{b(5 A b^2 - a^2 (3 A - 2 C)) \tan[c+dx]}{3 a^2 (a^2 - b^2) d (a+b \sec[c+dx])^{3/2}} - \frac{b(26 a^2 A b^2 - 15 A b^4 - a^4 (3 A - 8 C)) \tan[c+dx]}{3 a^3 (a^2 - b^2)^2 d \sqrt{a+b \sec[c+dx]}}$$

Result (type 4, 1714 leaves):

$$\left((b + a \cos[c+dx])^3 \sec[c+dx] (A + C \sec[c+dx])^2 \right) \left(-\frac{8(-5 a^2 A b^2 + 3 A b^4 - 2 a^4 C) \sin[c+dx]}{3 a^3 (-a^2 + b^2)^2} + \right.$$

$$\left. \frac{4 (A b^4 \sin[c + d x] + a^2 b^2 C \sin[c + d x])}{3 a^3 (a^2 - b^2) (b + a \cos[c + d x])^2} + \frac{4 (-11 a^2 A b^3 \sin[c + d x] + 7 A b^5 \sin[c + d x] - 5 a^4 b C \sin[c + d x] + a^2 b^3 C \sin[c + d x])}{3 a^3 (a^2 - b^2)^2 (b + a \cos[c + d x])} \right) /$$

$$(d (A + 2 C + A \cos[2 c + 2 d x]) (a + b \sec[c + d x])^{5/2}) - \left(2 (b + a \cos[c + d x])^{5/2} \sqrt{\sec[c + d x]} (A + C \sec[c + d x])^2 \right)$$

$$\sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + d x)\right]^2 + b \tan\left[\frac{1}{2}(c + d x)\right]^2}{1 + \tan\left[\frac{1}{2}(c + d x)\right]^2}}$$

$$\left(3 a^5 A \tan\left[\frac{1}{2}(c + d x)\right] + 3 a^4 A b \tan\left[\frac{1}{2}(c + d x)\right] - 26 a^3 A b^2 \tan\left[\frac{1}{2}(c + d x)\right] - 26 a^2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right] + 15 a A b^4 \tan\left[\frac{1}{2}(c + d x)\right] + \right.$$

$$15 A b^5 \tan\left[\frac{1}{2}(c + d x)\right] - 8 a^5 C \tan\left[\frac{1}{2}(c + d x)\right] - 8 a^4 b C \tan\left[\frac{1}{2}(c + d x)\right] - 6 a^5 A \tan\left[\frac{1}{2}(c + d x)\right]^3 + 52 a^3 A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^3 -$$

$$30 a A b^4 \tan\left[\frac{1}{2}(c + d x)\right]^3 + 16 a^5 C \tan\left[\frac{1}{2}(c + d x)\right]^3 + 3 a^5 A \tan\left[\frac{1}{2}(c + d x)\right]^5 - 3 a^4 A b \tan\left[\frac{1}{2}(c + d x)\right]^5 - 26 a^3 A b^2 \tan\left[\frac{1}{2}(c + d x)\right]^5 +$$

$$26 a^2 A b^3 \tan\left[\frac{1}{2}(c + d x)\right]^5 + 15 a A b^4 \tan\left[\frac{1}{2}(c + d x)\right]^5 - 15 A b^5 \tan\left[\frac{1}{2}(c + d x)\right]^5 - 8 a^5 C \tan\left[\frac{1}{2}(c + d x)\right]^5 + 8 a^4 b C \tan\left[\frac{1}{2}(c + d x)\right]^5 +$$

$$30 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + d x)\right]^2 + b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} -$$

$$60 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + d x)\right]^2 + b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} +$$

$$30 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + d x)\right]^2 + b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} +$$

$$30 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \tan\left[\frac{1}{2}(c + d x)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c + d x)\right]^2}$$

$$\sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + d x)\right]^2 + b \tan\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} - 60 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] \tan\left[\frac{1}{2}(c + d x)\right]^2$$

$$\begin{aligned}
& - \frac{1}{12 a^4 \sqrt{a+b} (a^2 - b^2) d} (105 A b^4 + a^4 (33 A - 56 C) - 2 a^2 b^2 (85 A - 12 C)) \\
& \quad \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a-b}} + \\
& \frac{1}{12 a^4 \sqrt{a+b} (a^2 - b^2) d} (35 a A b^3 + 105 A b^4 + 6 a^4 (A - 8 C) - 3 a^2 b^2 (45 A - 8 C) - a^3 (27 A b - 8 b C)) \text{Cot}[c + d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a-b}} - \\
& \frac{1}{4 a^5 d} \sqrt{a+b} (35 A b^2 + 4 a^2 (A + 2 C)) \text{Cot}[c + d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c + d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a-b}} - \frac{7 A b \text{Sin}[c + d x]}{4 a^2 d (a+b \text{Sec}[c + d x])^{3/2}} + \frac{A \text{Cos}[c + d x] \text{Sin}[c + d x]}{2 a d (a+b \text{Sec}[c + d x])^{3/2}} + \\
& \frac{b^2 (35 A b^2 - a^2 (27 A - 8 C)) \text{Tan}[c + d x]}{12 a^3 (a^2 - b^2) d (a+b \text{Sec}[c + d x])^{3/2}} - \frac{b^2 (105 A b^4 + a^4 (33 A - 56 C) - 2 a^2 b^2 (85 A - 12 C)) \text{Tan}[c + d x]}{12 a^4 (a^2 - b^2)^2 d \sqrt{a+b \text{Sec}[c + d x]}}
\end{aligned}$$

Result (type 4, 4981 leaves):

$$\begin{aligned}
& \frac{1}{2} \left(\frac{1}{d (a+b \text{Sec}[c + d x])^{5/2}} \right. \\
& \quad (b + a \text{Cos}[c + d x])^3 \text{Sec}[c + d x]^3 \left(\frac{4 b (-13 a^2 A b^2 + 9 A b^4 - 7 a^4 C + 3 a^2 b^2 C) \text{Sin}[c + d x]}{3 a^4 (a^2 - b^2)^2} - \frac{4 (A b^5 \text{Sin}[c + d x] + a^2 b^3 C \text{Sin}[c + d x])}{3 a^4 (a^2 - b^2) (b + a \text{Cos}[c + d x])^2} - \right. \\
& \quad \left. \frac{8 (-7 a^2 A b^4 \text{Sin}[c + d x] + 5 A b^6 \text{Sin}[c + d x] - 4 a^4 b^2 C \text{Sin}[c + d x] + 2 a^2 b^4 C \text{Sin}[c + d x])}{3 a^4 (a^2 - b^2)^2 (b + a \text{Cos}[c + d x])} + \frac{A \text{Sin}[2 (c + d x)]}{2 a^3} \right) + \\
& \quad \left((b + a \text{Cos}[c + d x])^{5/2} \left(\frac{a^2 A}{(a^2 - b^2)^2 \sqrt{b + a \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \frac{4 A b^2}{(a^2 - b^2)^2 \sqrt{b + a \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} - \right. \right. \\
& \quad \left. \left. \frac{7 A b^4}{3 a^2 (a^2 - b^2)^2 \sqrt{b + a \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \frac{2 a^2 C}{(a^2 - b^2)^2 \sqrt{b + a \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 c}{3 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]} \sqrt{\sec [c + d x]}} - \frac{9 a A b \sqrt{\sec [c + d x]}}{4 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} + \frac{31 A b^3 \sqrt{\sec [c + d x]}}{6 a (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - \\
& \frac{35 A b^5 \sqrt{\sec [c + d x]}}{12 a^3 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} + \frac{2 a b c \sqrt{\sec [c + d x]}}{3 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - \frac{2 b^3 c \sqrt{\sec [c + d x]}}{3 a (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - \\
& \frac{11 a A b \cos [2 (c + d x)] \sqrt{\sec [c + d x]}}{4 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} + \frac{85 A b^3 \cos [2 (c + d x)] \sqrt{\sec [c + d x]}}{6 a (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - \frac{35 A b^5 \cos [2 (c + d x)] \sqrt{\sec [c + d x]}}{4 a^3 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} + \\
& \left. \frac{14 a b c \cos [2 (c + d x)] \sqrt{\sec [c + d x]}}{3 (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} - \frac{2 b^3 c \cos [2 (c + d x)] \sqrt{\sec [c + d x]}}{a (a^2 - b^2)^2 \sqrt{b + a \cos [c + d x]}} \right) \sec [c + d x]^{5/2} \\
& - \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \tan \left[\frac{1}{2} (c + d x) \right] \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right) / \\
& \left(6 a^4 (a^2 - b^2)^2 \sqrt{\frac{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right) - \\
& \left((a + b) \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& 2 a (21 a A b^3 - 35 A b^4 + a^2 b^2 (33 A - 8 C) + 6 a^4 (A + 2 C) + a^3 (-9 A b + 12 b C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right] \right], \frac{a - b}{a + b} \right) + \\
& \left. 6 (a - b)^2 (a + b) (35 A b^2 + 4 a^2 (A + 2 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right] \right], \frac{a - b}{a + b} \right) \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^4} \right) / \\
& \left(6 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}} \left(b - b \tan \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(d (a + b \operatorname{Sec}[c + d x])^{5/2} - \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right. \right. \\
& \quad \left. \left. \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right) / \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right) + \right. \\
& \quad \left((a + b) \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \quad \left. \left. 2 a (21 a A b^3 - 35 A b^4 + a^2 b^2 (33 A - 8 C) + 6 a^4 (A + 2 C) + a^3 (-9 A b + 12 b C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \quad \left. \left. 6 (a - b)^2 (a + b) (35 A b^2 + 4 a^2 (A + 2 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \right) \right. \\
& \quad \left. \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^4} \right. \\
& \quad \left. \left(-2 b \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 2 a \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \right) \right) / \\
& \quad \left(6 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2 \right) \right) + \\
& \quad \left((a + b) \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \quad \left. \left. 2 a (21 a A b^3 - 35 A b^4 + a^2 b^2 (33 A - 8 C) + 6 a^4 (A + 2 C) + a^3 (-9 A b + 12 b C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \quad \left. \left. 6 (a - b)^2 (a + b) (35 A b^2 + 4 a^2 (A + 2 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \right) \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \tan\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \\
& \left(6a^4(a^2-b^2)^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4} \left(b-b\tan\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) - \\
& \left(b(105Ab^4+a^4(33A-56C)+2a^2b^2(-85A+12C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \quad 2a(21aAb^3-35Ab^4+a^2b^2(33A-8C)+6a^4(A+2C)+a^3(-9Ab+12bC)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \\
& \quad \left. 6(a-b)^2(a+b)(35Ab^2+4a^2(A+2C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4} \right) / \left(12a^4(a^2-b^2)^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \left(b-b\tan\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2 \right) \right) + \\
& \left((a+b) \left(b(105Ab^4+a^4(33A-56C)+2a^2b^2(-85A+12C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad 2a(21aAb^3-35Ab^4+a^2b^2(33A-8C)+6a^4(A+2C)+a^3(-9Ab+12bC)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \\
& \quad \left. \left. 6(a-b)^2(a+b)(35Ab^2+4a^2(A+2C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^4} \left/ \left(12 a^4 (a^2 - b^2)^2 \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right. + \\
& \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1 - \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \left/ \right. \\
& \left(12 a^4 (a^2 - b^2)^2 \left(\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) - \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \right. \\
& \tan\left[\frac{1}{2}(c+dx)\right] \left(\frac{-a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a + b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)}{\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \left/ \right. \\
& \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) - \\
& \left((a+b) \left(b (105 A b^4 + a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 2 a (21 a A b^3 - \right. \right. \\
& \left. \left. 35 A b^4 + a^2 b^2 (33 A - 8 C) + 6 a^4 (A + 2 C) + a^3 (-9 A b + 12 b C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 6 (a-b)^2 (a+b) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left((35 A b^2 + 4 a^2 (A + 2 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right. \\
& \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \left(\frac{-a \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] + b \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} - \right. \\
& \left. \left. \frac{\operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right] \left(a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)}{\left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2} \right) \right) / \left(12 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \right. \\
& \left. \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \left(b - b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) - \\
& \left((a + b) \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \right. \\
& \left. \left(a \left(21 a A b^3 - 35 A b^4 + a^2 b^2 (33 A - 8 C) + 6 a^4 (A + 2 C) + a^3 (-9 A b + 12 b C) \right) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \right) / \left(\sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right. \right. \\
& \left. \left. \sqrt{1 - \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right) - \frac{3 (a - b)^2 (a + b) (35 A b^2 + 4 a^2 (A + 2 C)) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{\sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{1 - \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}}} + \left(b (105 A b^4 + \right. \right. \\
& \left. \left. a^4 (33 A - 56 C) + 2 a^2 b^2 (-85 A + 12 C) \right) \operatorname{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \frac{(a - b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right) / \left(2 \sqrt{1 - \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \right) \right) \right) /
\end{aligned}$$

$$\left(\left(6 a^4 (a^2 - b^2)^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left(b - b \tan\left[\frac{1}{2}(c + dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c + dx)\right]^2 \right)^2 \right) \right) \right) \right)$$

■ **Problem 754: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{(a + b \sec[c + dx])^{7/2}} dx$$

Optimal (type 4, 626 leaves, 8 steps):

$$- \left(2 (41 a^2 A b^4 - 15 A b^6 - 3 a^6 C - 29 a^4 b^2 (2 A + C)) \cot[c + dx] \right)$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} \Big/ (15 a^3 b^2 \sqrt{a + b} (a^2 - b^2)^2 d) +$$

$$\left(2 (36 a^2 A b^3 - 5 a A b^4 - 15 A b^5 + 3 a^5 C + a^3 b^2 (13 A + 5 C) - 3 a^4 b (15 A + 8 C)) \cot[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \right)$$

$$\sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} \Big/ (15 a^3 b \sqrt{a + b} (a^2 - b^2)^2 d) -$$

$$\frac{2 A \sqrt{a + b} \cot[c + dx] \text{EllipticPi}\left[\frac{a + b}{a}, \text{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}}}{a^4 d} +$$

$$\frac{2 (A b^2 + a^2 C) \tan[c + dx]}{5 a (a^2 - b^2) d (a + b \sec[c + dx])^{5/2}} - \frac{2 (5 A b^4 - 3 a^4 C - a^2 b^2 (13 A + 5 C)) \tan[c + dx]}{15 a^2 (a^2 - b^2)^2 d (a + b \sec[c + dx])^{3/2}} -$$

$$\frac{2 (41 a^2 A b^4 - 15 A b^6 - 3 a^6 C - 29 a^4 b^2 (2 A + C)) \tan[c + dx]}{15 a^3 (a^2 - b^2)^3 d \sqrt{a + b \sec[c + dx]}}$$

Result (type 4, 2204 leaves):

$$\left((b + a \cos[c + dx])^4 \sec[c + dx]^2 (A + C \sec[c + dx])^2 \right) \left(\frac{4 (58 a^4 A b^2 - 41 a^2 A b^4 + 15 A b^6 + 3 a^6 C + 29 a^4 b^2 C) \sin[c + dx]}{15 a^3 b (-a^2 + b^2)^3} + \right)$$

$$\frac{4 (A b^4 \sin[c + d x] + a^2 b^2 C \sin[c + d x])}{5 a^3 (a^2 - b^2) (b + a \cos[c + d x])^3} + \frac{4 (-19 a^2 A b^3 \sin[c + d x] + 11 A b^5 \sin[c + d x] - 9 a^4 b C \sin[c + d x] + a^2 b^3 C \sin[c + d x])}{15 a^3 (a^2 - b^2)^2 (b + a \cos[c + d x])^2} +$$

$$\left(4 (74 a^4 A b^2 \sin[c + d x] - 65 a^2 A b^4 \sin[c + d x] + 23 A b^6 \sin[c + d x] + 9 a^6 C \sin[c + d x] + 25 a^4 b^2 C \sin[c + d x] - 2 a^2 b^4 C \sin[c + d x]) \right) /$$

$$\left(15 a^3 (a^2 - b^2)^3 (b + a \cos[c + d x]) \right) \Bigg) / \left(d (A + 2 C + A \cos[2 c + 2 d x]) (a + b \sec[c + d x])^{7/2} \right) -$$

$$\left(4 (b + a \cos[c + d x])^{7/2} \sec[c + d x]^{3/2} (A + C \sec[c + d x])^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2}} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2} (c + d x)\right]^2 + b \tan\left[\frac{1}{2} (c + d x)\right]^2}{1 + \tan\left[\frac{1}{2} (c + d x)\right]^2}} \right.$$

$$\left(58 a^5 A b^2 \tan\left[\frac{1}{2} (c + d x)\right] + 58 a^4 A b^3 \tan\left[\frac{1}{2} (c + d x)\right] - 41 a^3 A b^4 \tan\left[\frac{1}{2} (c + d x)\right] - 41 a^2 A b^5 \tan\left[\frac{1}{2} (c + d x)\right] + \right.$$

$$15 a A b^6 \tan\left[\frac{1}{2} (c + d x)\right] + 15 A b^7 \tan\left[\frac{1}{2} (c + d x)\right] + 3 a^7 C \tan\left[\frac{1}{2} (c + d x)\right] + 3 a^6 b C \tan\left[\frac{1}{2} (c + d x)\right] + 29 a^5 b^2 C \tan\left[\frac{1}{2} (c + d x)\right] +$$

$$29 a^4 b^3 C \tan\left[\frac{1}{2} (c + d x)\right] - 116 a^5 A b^2 \tan\left[\frac{1}{2} (c + d x)\right]^3 + 82 a^3 A b^4 \tan\left[\frac{1}{2} (c + d x)\right]^3 - 30 a A b^6 \tan\left[\frac{1}{2} (c + d x)\right]^3 -$$

$$6 a^7 C \tan\left[\frac{1}{2} (c + d x)\right]^3 - 58 a^5 b^2 C \tan\left[\frac{1}{2} (c + d x)\right]^3 + 58 a^5 A b^2 \tan\left[\frac{1}{2} (c + d x)\right]^5 - 58 a^4 A b^3 \tan\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$41 a^3 A b^4 \tan\left[\frac{1}{2} (c + d x)\right]^5 + 41 a^2 A b^5 \tan\left[\frac{1}{2} (c + d x)\right]^5 + 15 a A b^6 \tan\left[\frac{1}{2} (c + d x)\right]^5 - 15 A b^7 \tan\left[\frac{1}{2} (c + d x)\right]^5 +$$

$$3 a^7 C \tan\left[\frac{1}{2} (c + d x)\right]^5 - 3 a^6 b C \tan\left[\frac{1}{2} (c + d x)\right]^5 + 29 a^5 b^2 C \tan\left[\frac{1}{2} (c + d x)\right]^5 - 29 a^4 b^3 C \tan\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$30 a^6 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2} (c + d x)\right]^2 + b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} +$$

$$90 a^4 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2} (c + d x)\right]^2 + b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} -$$

$$90 a^2 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2} (c + d x)\right]^2 + b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} +$$

$$30 A b^7 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2} (c + d x)\right]\right], \frac{a - b}{a + b}\right] \sqrt{1 - \tan\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2} (c + d x)\right]^2 + b \tan\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} -$$

$$\begin{aligned}
& 30 a^6 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 90 a^4 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 90 a^2 A b^5 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 A b^7 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-41 a^2 A b^4 + 15 A b^6 + 3 a^6 C + 29 a^4 b^2 (2A+C)) \text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& a b (a+b) (-6 a A b^3 + 10 A b^4 + 3 a^4 (5A+C) + 6 a^3 b (5A+4C) + a^2 b^2 (-17A+5C)) \text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) \Bigg/ \\
& \left(15 a^3 b (a^2 - b^2)^3 d (A + 2C + A \text{Cos}[2c + 2dx]) (a+b \text{Sec}[c+dx])^{7/2} \sqrt{1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \left. \left(a \left(-1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 755: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a^2 - b^2 \operatorname{Sec}[c + d x]^2}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 303 leaves, 7 steps) :

$$\frac{2(a-b)\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d} +$$

$$\frac{2b\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d} -$$

$$\frac{2a\sqrt{a+b}\operatorname{Cot}[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{d}$$

Result (type 4, 939 leaves) :

$$\frac{4 b \cos [c+d x] (b+a \cos [c+d x])\left(a^2-b^2 \sec [c+d x]^2\right) \sin [c+d x]}{d\left(a^2-2 b^2+a^2 \cos [2 c+2 d x]\right) \sqrt{a+b \sec [c+d x]}}$$

$$\left(4 \sqrt{b+a \cos [c+d x]}\left(a^2-b^2 \sec [c+d x]^2\right) \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}\right. \\ \left(-a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}-b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}+\right. \\ \left.a b \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}-b^2 \sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}+\right. \\ \left.2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+\right. \\ \left.2 i a^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2\right. \\ \left.\sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+i(a-b) b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right]\right. \\ \left.\left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-i\left(a^2-b^2\right)\right. \\ \left.\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right]\left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^2 \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right)\right) / \\ \left(\sqrt{\frac{-a+b}{a+b}} d\left(a^2-2 b^2+a^2 \cos [2 c+2 d x]\right) \sec [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]}\left(1+\tan \left[\frac{1}{2}(c+d x)\right]\right)^{3 / 2}\right. \\ \left.\sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}}\right)$$

- **Problem 757: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a^2 - b^2 \operatorname{Sec}[c + dx]^2}{(a + b \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 4, 338 leaves, 7 steps):

$$\frac{4 \operatorname{Cot}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{\sqrt{a+b} d} -$$

$$\frac{4 \operatorname{Cot}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{\sqrt{a+b} d} -$$

$$\frac{2 \sqrt{a+b} \operatorname{Cot}[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a d} + \frac{4 b^2 \operatorname{Tan}[c + dx]}{(a^2 - b^2) d \sqrt{a + b \operatorname{Sec}[c + dx]}}$$

Result (type 4, 790 leaves):

$$\begin{aligned}
& \frac{(b + a \cos[c + dx])^2 \sec[c + dx] (a - b \sec[c + dx]) \left(\frac{4b \sin[c + dx]}{-a^2 + b^2} - \frac{4b^2 \sin[c + dx]}{(-a^2 + b^2)(b + a \cos[c + dx])} \right)}{d (-b + a \cos[c + dx]) (a + b \sec[c + dx])^{3/2}} + \\
& \left(2 (b + a \cos[c + dx])^{3/2} \sqrt{\sec[c + dx]} (a - b \sec[c + dx]) \right. \\
& \left(2 i (a - b) b \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + dx) \right] \right], \frac{a + b}{a - b} \sqrt{1 - \tan \left[\frac{1}{2} (c + dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c + dx) \right]^2 \right) \right. \right. \\
& \left. \left. \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + dx) \right]^2 + b \tan \left[\frac{1}{2} (c + dx) \right]^2}{a + b}} - i (a^2 + 2ab - 3b^2) \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + dx) \right] \right], \frac{a + b}{a - b} \right] \right. \right. \\
& \left. \left. \sqrt{1 - \tan \left[\frac{1}{2} (c + dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c + dx) \right]^2 \right) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + dx) \right]^2 + b \tan \left[\frac{1}{2} (c + dx) \right]^2}{a + b}} + \right. \right. \\
& \left. \left. 2 i (a^2 - b^2) \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + dx) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + dx) \right]^2} \right. \right. \\
& \left. \left. \left(1 + \tan \left[\frac{1}{2} (c + dx) \right]^2 \right) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + dx) \right]^2 + b \tan \left[\frac{1}{2} (c + dx) \right]^2}{a + b}} - \right. \right. \\
& \left. \left. 2 b \sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + dx) \right] \left(b - b \tan \left[\frac{1}{2} (c + dx) \right]^4 + a \left(-1 + \tan \left[\frac{1}{2} (c + dx) \right]^2 \right)^2 \right) \right) \right) \Bigg/ \\
& \left(\sqrt{\frac{-a + b}{a + b}} (a^2 - b^2) d (-b + a \cos[c + dx]) (a + b \sec[c + dx])^{3/2} \sqrt{\frac{1 + \tan \left[\frac{1}{2} (c + dx) \right]^2}{1 - \tan \left[\frac{1}{2} (c + dx) \right]^2}} \right. \\
& \left. \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + dx) \right]^2 + b \tan \left[\frac{1}{2} (c + dx) \right]^2}{1 + \tan \left[\frac{1}{2} (c + dx) \right]^2}} \left(-1 + \tan \left[\frac{1}{2} (c + dx) \right]^4 \right) \right)
\end{aligned}$$

- **Problem 758: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a^2 - b^2 \operatorname{Sec}[c + dx]^2}{(a + b \operatorname{Sec}[c + dx])^{7/2}} dx$$

Optimal (type 4, 445 leaves, 8 steps):

$$\frac{2(11a^2 - 3b^2) \operatorname{Cot}[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{3a(a-b)(a+b)^{3/2}d} - \frac{1}{3a(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2(9a^2 - 2ab - 3b^2) \operatorname{Cot}[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{3a(a-b)(a+b)^{3/2}d} -$$

$$+ \frac{2\sqrt{a+b} \operatorname{Cot}[c + dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{a^2 d} +$$

$$\frac{4b^2 \operatorname{Tan}[c + dx]}{3(a^2 - b^2)d(a+b \operatorname{Sec}[c + dx])^{3/2}} + \frac{2b^2(11a^2 - 3b^2) \operatorname{Tan}[c + dx]}{3a(a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c + dx]}}$$

Result (type 4, 1849 leaves):

$$\left((b + a \operatorname{Cos}[c + dx])^3 \operatorname{Sec}[c + dx]^2 (a - b \operatorname{Sec}[c + dx]) \right.$$

$$\left. \left(\frac{2b(-11a^2 + 3b^2) \operatorname{Sin}[c + dx]}{3a(-a^2 + b^2)^2} - \frac{4b^3 \operatorname{Sin}[c + dx]}{3a(a^2 - b^2)(b + a \operatorname{Cos}[c + dx])^2} - \frac{2(-13a^2 b^2 \operatorname{Sin}[c + dx] + 5b^4 \operatorname{Sin}[c + dx])}{3a(a^2 - b^2)^2 (b + a \operatorname{Cos}[c + dx])} \right) \right) /$$

$$(d(-b + a \operatorname{Cos}[c + dx])(a + b \operatorname{Sec}[c + dx])^{5/2}) +$$

$$\left(2(b + a \operatorname{Cos}[c + dx])^{5/2} \operatorname{Sec}[c + dx]^{3/2} (a - b \operatorname{Sec}[c + dx]) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^2}} \right.$$

$$\left. \left(11a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] + 11a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - 3ab^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - 3b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] - \right.$$

$$\left. 22a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^3 + 6ab^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^3 + 11a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]^5 - \right.$$

$$\begin{aligned}
& 11 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 6 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 12 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i a^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 12 i a^2 b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i b^4 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& i b (11 a^3 - 11 a^2 b - 3 a b^2 + 3 b^3) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$i \left(3a^4 + 9a^3b - 17a^2b^2 - ab^3 + 6b^4\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) \Bigg/$$

$$\left(3a \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^2 d (-b + a \cos[c+dx]) (a+b \sec[c+dx])^{5/2} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}}\right.$$

$$\left.\left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)$$

■ **Problem 759: Unable to integrate problem.**

$$\int \frac{A + C \sec[c+dx]^2}{\sqrt{\sec[c+dx]} (a+b \sec[c+dx])} dx$$

Optimal (type 4, 145 leaves, 8 steps):

$$\frac{2A \sqrt{\cos[c+dx]} \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]} - 2Ab \sqrt{\cos[c+dx]} \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{ad} +$$

$$\frac{2(Ab^2 + a^2C) \sqrt{\cos[c+dx]} \text{EllipticPi}\left[\frac{2a}{a+b}, \frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{a^2(a+b)d}$$

Result (type 8, 37 leaves):

$$\int \frac{A + C \sec[c+dx]^2}{\sqrt{\sec[c+dx]} (a+b \sec[c+dx])} dx$$

■ **Problem 760: Unable to integrate problem.**

$$\int \frac{A + C \sec[c+dx]^2}{\sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 213 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 A b \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{a d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{2 c \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{a d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 8, 39 leaves):

$$\int \frac{A + C \operatorname{Sec}[c+d x]^2}{\sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

■ **Problem 761: Attempted integration timed out after 120 seconds.**

$$\int (a+b \operatorname{Sec}[c+d x])^{2/3} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 9, 242 leaves, 8 steps):

$$\begin{aligned}
& \left(\sqrt{2} (a+b) c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2}(1-\operatorname{Sec}[c+d x]), \frac{b(1-\operatorname{Sec}[c+d x])}{a+b}\right] (a+b \operatorname{Sec}[c+d x])^{2/3} \operatorname{Tan}[c+d x] \right) / \\
& \left(b d \sqrt{1+\operatorname{Sec}[c+d x]} \left(\frac{a+b \operatorname{Sec}[c+d x]}{a+b} \right)^{2/3} \right) - \\
& \frac{\sqrt{2} a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1-\operatorname{Sec}[c+d x]), \frac{b(1-\operatorname{Sec}[c+d x])}{a+b}\right] (a+b \operatorname{Sec}[c+d x])^{2/3} \operatorname{Tan}[c+d x]}{b d \sqrt{1+\operatorname{Sec}[c+d x]} \left(\frac{a+b \operatorname{Sec}[c+d x]}{a+b} \right)^{2/3}} + \\
& A \operatorname{Unintegrable}\left[(a+b \operatorname{Sec}[c+d x])^{2/3}, x\right]
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 763: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + C \operatorname{Sec}[c+d x]^2}{(a+b \operatorname{Sec}[c+d x])^{1/3}} dx$$

Optimal (type 9, 239 leaves, 8 steps):

$$\frac{\sqrt{2} C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{2/3}} -$$

$$\frac{\sqrt{2} a C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{1/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}} + \text{A Unintegrable}\left[\frac{1}{(a + b \operatorname{Sec}[c + d x])^{1/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 766: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (a + b \operatorname{Sec}[c + d x]) (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$\frac{(4 a B + 3 b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]}{d} +$$

$$\frac{(4 a B + 3 b C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{b C \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]^3}{3 d}$$

Result (type 3, 403 leaves):

$$-\frac{a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{3 b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{a B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} +$$

$$\frac{3 b C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{b C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{a B}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{3 b C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{a B}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} -$$

$$\frac{3 b C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{2 b B \operatorname{Tan}[c + d x]}{3 d} + \frac{2 a C \operatorname{Tan}[c + d x]}{3 d} + \frac{b B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d}$$

■ **Problem 768: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x]) (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{(2 a B + b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{(b B + a C) \operatorname{Tan}[c + d x]}{d} + \frac{b C \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d}$$

Result (type 3, 164 leaves):

$$\frac{1}{4d} \left(-2(2aB + bC) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right] + \right. \\ \left. 4aB \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right] + 2bC \operatorname{Log} \left[\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right] + \right. \\ \left. \frac{bC}{\left(\cos \left[\frac{1}{2}(c + dx) \right] - \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} - \frac{bC}{\left(\cos \left[\frac{1}{2}(c + dx) \right] + \sin \left[\frac{1}{2}(c + dx) \right] \right)^2} + 4(bB + aC) \operatorname{Tan}[c + dx] \right)$$

■ **Problem 769: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + b \sec[c + dx]) (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 35 leaves, 5 steps):

$$aBx + \frac{(bB + aC) \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{bC \operatorname{Tan}[c + dx]}{d}$$

Result (type 3, 159 leaves):

$$aBx - \frac{bB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} - \frac{aC \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \\ \frac{bB \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aC \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{bC \operatorname{Tan}[c + dx]}{d}$$

■ **Problem 770: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^2 (a + b \sec[c + dx]) (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$(bB + aC)x + \frac{bC \operatorname{ArcTanh}[\sin[c + dx]]}{d} + \frac{aB \sin[c + dx]}{d}$$

Result (type 3, 104 leaves):

$$bBx + aCx - \frac{bC \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{bC \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right]}{d} + \frac{aB \cos[dx] \sin[c]}{d} + \frac{aB \cos[c] \sin[dx]}{d}$$

■ **Problem 776: Result more than twice size of optimal antiderivative.**

$$\int \sec[c + dx] (a + b \sec[c + dx])^2 (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$\frac{(8abB + 4a^2C + 3b^2C) \operatorname{ArcTanh}[\sin[c + dx]]}{8d} + \frac{(4a^2bB + 4b^3B - a^3C + 8ab^2C) \tan[c + dx]}{6bd} +$$

$$\frac{(8abB - 2a^2C + 9b^2C) \sec[c + dx] \tan[c + dx]}{24d} + \frac{(4bB - aC)(a + b \sec[c + dx])^2 \tan[c + dx]}{12bd} + \frac{C(a + b \sec[c + dx])^3 \tan[c + dx]}{4bd}$$

Result (type 3, 457 leaves):

$$\frac{1}{48d} \left(-6(8abB + 4a^2C + 3b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 6(8abB + 4a^2C + 3b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right.$$

$$\frac{3b^2C}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} + \frac{12a^2C + 8ab(3B + C) + b^2(4B + 9C)}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{8b(bB + 2aC) \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} +$$

$$\frac{16(3a^2B + 2b^2B + 4abC) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} - \frac{3b^2C}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^4} +$$

$$\left. \frac{8b(bB + 2aC) \sin\left[\frac{1}{2}(c + dx)\right]}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} - \frac{12a^2C + 8ab(3B + C) + b^2(4B + 9C)}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \frac{16(3a^2B + 2b^2B + 4abC) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} \right)$$

■ **Problem 778: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx] (a + b \sec[c + dx])^2 (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$a^2 B x + \frac{(4abB + 2a^2C + b^2C) \operatorname{ArcTanh}[\sin[c + dx]]}{2d} + \frac{b(2bB + 3aC) \tan[c + dx]}{2d} + \frac{bC(a + b \sec[c + dx]) \tan[c + dx]}{2d}$$

Result (type 3, 225 leaves):

$$\frac{1}{4d} \left(4a^2 B c + 4a^2 B dx - 2(4abB + 2a^2C + b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + 8abB \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right.$$

$$4a^2 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 2b^2 C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] +$$

$$\left. \frac{b^2 C}{\left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} - \frac{b^2 C}{\left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + 4b(bB + 2aC) \tan[c + dx] \right)$$

■ **Problem 786: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[c + dx])^3 (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$\frac{(8 a^3 B + 12 a b^2 B + 12 a^2 b C + 3 b^3 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(16 a^2 b B + 4 b^3 B + 3 a^3 C + 12 a b^2 C) \operatorname{Tan}[c + d x]}{6 d} +$$

$$\frac{b (20 a b B + 6 a^2 C + 9 b^2 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{24 d} + \frac{(4 b B + 3 a C) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 639 leaves):

$$\frac{(-8 a^3 B - 12 a b^2 B - 12 a^2 b C - 3 b^3 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} +$$

$$\frac{(8 a^3 B + 12 a b^2 B + 12 a^2 b C + 3 b^3 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{b^3 C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} +$$

$$\frac{36 a b^2 B + 4 b^3 B + 36 a^2 b C + 12 a b^2 C + 9 b^3 C}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b^3 C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{-36 a b^2 B - 4 b^3 B - 36 a^2 b C - 12 a b^2 C - 9 b^3 C}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{b^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} + \frac{b^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{6 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3} +$$

$$\frac{9 a^2 b B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 b^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a^3 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)} +$$

$$\frac{9 a^2 b B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 b^3 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 3 a^3 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 6 a b^2 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)}$$

■ **Problem 787: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^3 (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 137 leaves, 7 steps):

$$a^3 B x + \frac{(6 a^2 b B + b^3 B + 2 a^3 C + 3 a b^2 C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} +$$

$$\frac{b (9 a b B + 8 a^2 C + 2 b^2 C) \operatorname{Tan}[c + d x]}{3 d} + \frac{b^2 (3 b B + 5 a C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} + \frac{b C (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 392 leaves):

$$\frac{1}{12d} \left(12a^3B(c+dx) - 6(6a^2bB + b^3B + 2a^3C + 3ab^2C) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. 6(6a^2bB + b^3B + 2a^3C + 3ab^2C) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right] + \frac{b^2(9aC + b(3B+C))}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + \right. \\ \left. \frac{2b^3C \sin \left[\frac{1}{2}(c+dx) \right]}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^3} + \frac{4b(9abB + 9a^2C + 2b^2C) \sin \left[\frac{1}{2}(c+dx) \right]}{\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right]} + \right. \\ \left. \frac{2b^3C \sin \left[\frac{1}{2}(c+dx) \right]}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^3} - \frac{b^2(9aC + b(3B+C))}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + \frac{4b(9abB + 9a^2C + 2b^2C) \sin \left[\frac{1}{2}(c+dx) \right]}{\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right]} \right)$$

■ **Problem 788: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^3 (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$a^2(3bB+aC)x + \frac{b(6abB+6a^2C+b^2C) \operatorname{ArcTanh}[\sin[c+dx]]}{2d} + \\ \frac{aB(a+b \operatorname{Sec}[c+dx])^2 \sin[c+dx]}{d} - \frac{b(2a^2B-b^2B-3abC) \tan[c+dx]}{d} - \frac{b^2(2aB-bC) \operatorname{Sec}[c+dx] \tan[c+dx]}{2d}$$

Result (type 3, 277 leaves):

$$\frac{1}{4d} \left(4a^2(3bB+aC)(c+dx) - 2b(6abB+6a^2C+b^2C) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right] + \right. \\ \left. 2b(6abB+6a^2C+b^2C) \operatorname{Log} \left[\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right] + \frac{b^3C}{\left(\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + \right. \\ \left. \frac{4b^2(bB+3aC) \sin \left[\frac{1}{2}(c+dx) \right]}{\cos \left[\frac{1}{2}(c+dx) \right] - \sin \left[\frac{1}{2}(c+dx) \right]} - \frac{b^3C}{\left(\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right] \right)^2} + \frac{4b^2(bB+3aC) \sin \left[\frac{1}{2}(c+dx) \right]}{\cos \left[\frac{1}{2}(c+dx) \right] + \sin \left[\frac{1}{2}(c+dx) \right]} + 4a^3B \sin[c+dx] \right)$$

■ **Problem 793: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^3 (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{a+b \operatorname{Sec}[c+dx]} dx$$

Optimal (type 3, 187 leaves, 9 steps):

$$\frac{(2a^2 + b^2)(bB - aC) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^4 d} - \frac{2a^3(bB - aC) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^4 \sqrt{a+b} d} -$$

$$\frac{(3abB - 3a^2C - 2b^2C) \operatorname{Tan}[c + dx]}{3b^3 d} + \frac{(bB - aC) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2b^2 d} + \frac{C \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{3bd}$$

Result (type 3, 422 leaves):

$$\frac{1}{12b^4 d} \left(\frac{24a^3(bB - aC) \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{\sqrt{a^2-b^2}} + 6(2a^2 + b^2)(-bB + aC) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] - \right.$$

$$6(2a^2 + b^2)(-bB + aC) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \frac{b^2(-3aC + b(3B + C))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} +$$

$$\frac{2b^3 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} + \frac{4b(-3abB + 3a^2C + 2b^2C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} +$$

$$\left. \frac{2b^3 C \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^3} - \frac{b^2(-3aC + b(3B + C))}{\left(\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)^2} + \frac{4b(-3abB + 3a^2C + 2b^2C) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]} \right)$$

■ **Problem 794: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^2 (B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 143 leaves, 8 steps):

$$- \frac{(2abB - 2a^2C - b^2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^3 d} + \frac{2a^2(bB - aC) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^3 \sqrt{a+b} d} + \frac{(bB - aC) \operatorname{Tan}[c + dx]}{b^2 d} + \frac{C \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2bd}$$

Result (type 3, 300 leaves):

$$\frac{1}{4 b^3 d} \left(\frac{8 a^2 (-b B + a C) \operatorname{ArcTanh} \left[\frac{(-a+b) \tan \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right]}{\sqrt{a^2-b^2}} - 2 (-2 a b B + 2 a^2 C + b^2 C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right] \right) +$$

$$2 (-2 a b B + 2 a^2 C + b^2 C) \operatorname{Log} \left[\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right] + \frac{b^2 C}{\left(\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right)^2} +$$

$$\left. \frac{4 b (b B - a C) \sin \left[\frac{1}{2} (c+dx) \right]}{\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right]} - \frac{b^2 C}{\left(\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right)^2} + \frac{4 b (b B - a C) \sin \left[\frac{1}{2} (c+dx) \right]}{\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right]} \right)$$

■ **Problem 814: Attempted integration timed out after 120 seconds.**

$$\int \sec [c+dx]^3 \sqrt{a+b \sec [c+dx]} (B \sec [c+dx] + C \sec [c+dx]^2) dx$$

Optimal (type 4, 485 leaves, 8 steps):

$$-\frac{1}{315 b^5 d} 2 (a-b) \sqrt{a+b} (24 a^3 b B + 57 a b^3 B - 16 a^4 C - 24 a^2 b^2 C + 147 b^4 C)$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a+b \sec [c+dx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\sec [c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec [c+dx])}{a-b}} -$$

$$\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 B - 49 C) + 18 a b^2 (B - 2 C) + 12 a^2 b (2 B - C) - 16 a^3 C) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a+b \sec [c+dx]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\sec [c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec [c+dx])}{a-b}} -$$

$$\frac{2 (12 a^2 b B - 75 b^3 B - 8 a^3 C - 13 a b^2 C) \sqrt{a+b \sec [c+dx]} \operatorname{Tan}[c+dx]}{315 b^3 d} + \frac{2 (9 a b B - 6 a^2 C + 49 b^2 C) \sec [c+dx] \sqrt{a+b \sec [c+dx]} \operatorname{Tan}[c+dx]}{315 b^2 d} +$$

$$\frac{2 (9 b B + a C) \sec [c+dx]^2 \sqrt{a+b \sec [c+dx]} \operatorname{Tan}[c+dx]}{63 b d} + \frac{2 C \sec [c+dx]^3 \sqrt{a+b \sec [c+dx]} \operatorname{Tan}[c+dx]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 815: Attempted integration timed out after 120 seconds.**

$$\int \sec [c+dx]^2 \sqrt{a+b \sec [c+dx]} (B \sec [c+dx] + C \sec [c+dx]^2) dx$$

Optimal (type 4, 397 leaves, 7 steps):

$$\frac{1}{105 b^4 d} 2 (a-b) \sqrt{a+b} (14 a^2 b B - 63 b^3 B - 8 a^3 C - 19 a b^2 C) \cot [c+d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{1}{105 b^3 d}$$

$$2 (a-b) \sqrt{a+b} (b^2 (63 B - 25 C) + 2 a b (7 B - 3 C) - 8 a^2 C) \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{2(7 a b B - 4 a^2 C + 25 b^2 C) \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{105 b^2 d} +$$

$$\frac{2(7 b B + a C) \sec [c+d x] \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{35 b d} + \frac{2 C \sec [c+d x]^2 \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{7 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 816: Unable to integrate problem.**

$$\int \sec [c+d x] \sqrt{a+b \sec [c+d x]} (B \sec [c+d x] + C \sec [c+d x]^2) dx$$

Optimal (type 4, 314 leaves, 6 steps):

$$-\frac{1}{15 b^3 d} 2 (a-b) \sqrt{a+b} (5 a b B - 2 a^2 C + 9 b^2 C) \cot [c+d x]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{15 b^2 d}$$

$$2 (a-b) \sqrt{a+b} (5 b B - 2 a C - 9 b C) \cot [c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}}$$

$$\sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{2(5 b B - 2 a C) \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{15 b d} + \frac{2 C (a+b \sec [c+d x])^{3/2} \tan [c+d x]}{5 b d}$$

Result (type 8, 42 leaves):

$$\int \sec [c+d x] \sqrt{a+b \sec [c+d x]} (B \sec [c+d x] + C \sec [c+d x]^2) dx$$

■ **Problem 817: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a+b \sec [c+d x]} (B \sec [c+d x] + C \sec [c+d x]^2) dx$$

Optimal (type 4, 256 leaves, 5 steps) :

$$\begin{aligned}
 & -\frac{1}{3b^2d} 2(a-b)\sqrt{a+b} (3bB+aC) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\
 & \frac{1}{3bd} 2(a-b)\sqrt{a+b} (3B-C) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\
 & \frac{2C\sqrt{a+b\sec[c+dx]} \tan[c+dx]}{3d}
 \end{aligned}$$

Result (type 1, 1 leaves) :

???

- **Problem 818: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx] \sqrt{a+b\sec[c+dx]} (B\sec[c+dx] + C\sec[c+dx]^2) dx$$

Optimal (type 4, 320 leaves, 6 steps) :

$$\begin{aligned}
 & -\frac{1}{bd} 2(a-b)\sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{bd} \\
 & 2\sqrt{a+b} (b(B-C) + aC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\
 & \frac{2\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{d}
 \end{aligned}$$

Result (type 4, 863 leaves) :

$$\begin{aligned}
& \frac{2 C \sqrt{a+b} \operatorname{Sec}[c+d x] \operatorname{Sin}[c+d x]}{d} + \\
& \left(2 \sqrt{a+b} \operatorname{Sec}[c+d x] \left(a \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \right. \right. \\
& \quad a \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 2 i a B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \quad \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \\
& \quad 2 i a B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \quad \left. \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - i (a-b) C \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right. \\
& \quad \left. \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right. \\
& \quad \left. i (a-b) (B-C) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\
& \quad \left. \left. \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \right) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a} \operatorname{Cos}[c+d x] \sqrt{\operatorname{Sec}[c+d x]} \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \right. \\
& \quad \left. \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^{3/2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right)
\end{aligned}$$

- **Problem 819: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \left(B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2 \right) d x$$

Optimal (type 4, 344 leaves, 7 steps):

$$\frac{(a-b) \sqrt{a+b} B \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{b d} +$$

$$\frac{\sqrt{a+b}(B+2 C) \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{d} - \frac{1}{a d}$$

$$\frac{\sqrt{a+b}(b B+2 a C) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{d} +$$

$$\frac{B \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{d}$$

Result (type 4, 1107 leaves):

$$\left(\sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right.$$

$$\left(a \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + a \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right.$$

$$b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 2 i b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}$$

$$\left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 i a C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right.$$

$$\left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right.$$

$$\begin{aligned}
& 2 i b B \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} - 4 i a C \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \\
& \tan \left[\frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} - \\
& i (a-b) B \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} + 2 i (a-b) C \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} \right) \Bigg) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos [c+dx]} \sqrt{\sec [c+dx]} \sqrt{\frac{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2}{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2}} \left(b - b \tan \left[\frac{1}{2} (c+dx) \right]^4 + a \left(-1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right)^2 \right) \right)
\end{aligned}$$

■ **Problem 820: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+dx]^3 \sqrt{a+b \sec [c+dx]} (B \sec [c+dx] + C \sec [c+dx]^2) dx$$

Optimal (type 4, 429 leaves, 8 steps) :

$$\frac{1}{4abd} (a-b) \sqrt{a+b} (bB+4aC) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{4ad} \sqrt{a+b} (bB+2a(B+2C)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{4a^2d} \sqrt{a+b} (4a^2B-b^2B+4abC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{(bB+4aC) \sqrt{a+b \operatorname{Sec}[c+dx]} \sin[c+dx]}{4ad} + \frac{B \cos[c+dx] \sqrt{a+b \operatorname{Sec}[c+dx]} \sin[c+dx]}{2d}$$

Result (type 4, 1161 leaves):

$$\frac{B \sqrt{a+b \operatorname{Sec}[c+dx]} \sin[2(c+dx)]}{4d} + \left(\sqrt{a+b \operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4a^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4abC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 2abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - \right.$$

$$8a^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - b^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 4a^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4abC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$8a^2B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$2b^2B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$8abC \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$8a^2B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2b^2 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 8ab \text{C EllipticPi}\left[-1, -\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b)(bB+4aC) \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2a(2aB-bB+4bC) \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) / \\
& \left(4ad \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

■ **Problem 821: Attempted integration timed out after 120 seconds.**

$$\int \sec[c+dx]^3 (a+b \sec[c+dx])^{3/2} (B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 573 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{3465 b^5 d} 2 (a-b) \sqrt{a+b} (88 a^4 b B + 363 a^2 b^3 B + 1617 b^5 B - 48 a^5 C - 108 a^3 b^2 C + 2088 a b^4 C) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{3465 b^4 d} 2 (a-b) \sqrt{a+b} (3 a b^3 (143 B - 471 C) - 3 b^4 (539 B - 225 C) + 6 a^2 b^2 (11 B - 24 C) + 4 a^3 b (22 B - 9 C) - 48 a^4 C) \\
& \quad \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{2 (88 a^3 b B + 429 a b^3 B - 48 a^4 C - 144 a^2 b^2 C + 675 b^4 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{3465 b^3 d} + \\
& \frac{2 (88 a^2 b B + 539 b^3 B - 48 a^3 C - 204 a b^2 C) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{3465 b^3 d} - \frac{2 (44 a b B - 24 a^2 C - 81 b^2 C) (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{693 b^3 d} + \\
& \frac{2 (11 b B - 6 a C) \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{99 b^2 d} + \frac{2 C \text{Sec}[c+dx]^2 (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{11 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 822: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+dx]^2 (a+b \text{Sec}[c+dx])^{3/2} (B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 475 leaves, 8 steps):

$$\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (18 a^3 b B - 246 a b^3 B - 8 a^4 C - 33 a^2 b^2 C - 147 b^4 C)$$

$$\text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 B - 49 C) - 3 a b^2 (57 B - 13 C) - 6 a^2 b (3 B - C) + 8 a^3 C) \text{Cot}[c+dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} -$$

$$\frac{2 (18 a^2 b B - 75 b^3 B - 8 a^3 C - 39 a b^2 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{315 b^2 d} - \frac{2 (18 a b B - 8 a^2 C - 49 b^2 C) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{315 b^2 d} +$$

$$\frac{2 (9 b B - 4 a C) (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{63 b^2 d} + \frac{2 C \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{9 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 823: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{3/2} (B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 387 leaves, 7 steps):

$$-\frac{1}{105 b^3 d} 2 (a-b) \sqrt{a+b} (21 a^2 b B + 63 b^3 B - 6 a^3 C + 82 a b^2 C) \text{Cot}[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{105 b^2 d}$$

$$2 (a-b) \sqrt{a+b} (a b (21 B - 57 C) - b^2 (63 B - 25 C) - 6 a^2 C) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{2 (21 a b B - 6 a^2 C + 25 b^2 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{105 b d} +$$

$$\frac{2 (7 b B - 2 a C) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{35 b d} + \frac{2 C (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{7 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 824: Attempted integration timed out after 120 seconds.**

$$\int (a + b \operatorname{Sec}[c + d x])^{3/2} (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$-\frac{1}{15 b^2 d} 2 (a - b) \sqrt{a + b} (20 a b B + 3 a^2 C + 9 b^2 C) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} + \frac{1}{15 b d}$$

$$2 (a - b) \sqrt{a + b} (15 a B - 5 b B - 3 a C + 9 b C) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}}$$

$$\sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} + \frac{2 (5 b B + 3 a C) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{15 d} + \frac{2 C (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]}{5 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 826: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Sec}[c + d x])^{3/2} (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 361 leaves, 7 steps):

$$\frac{1}{b d} (a - b) \sqrt{a + b} (a B - 2 b C) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} + \frac{1}{d}$$

$$\sqrt{a + b} (2 b (B - C) + a (B + 4 C)) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}$$

$$\frac{1}{d} \sqrt{a + b} (3 b B + 2 a C) \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} +$$

$$\frac{a B \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d}$$

Result (type 4, 979 leaves):

$$\frac{2 b C \cos [c+d x] (a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{d (b+a \cos [c+d x])} +$$

$$\left((a+b \sec [c+d x])^{3 / 2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \left(a^2 B \tan \left[\frac{1}{2}(c+d x)\right] + a b B \tan \left[\frac{1}{2}(c+d x)\right] - 2 a b C \tan \left[\frac{1}{2}(c+d x)\right] - 2 b^2 C \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ \left. \left. 2 a^2 B \tan \left[\frac{1}{2}(c+d x)\right]^3 + 4 a b C \tan \left[\frac{1}{2}(c+d x)\right]^3 + a^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 - a b B \tan \left[\frac{1}{2}(c+d x)\right]^5 - 2 a b C \tan \left[\frac{1}{2}(c+d x)\right]^5 + \right. \right. \\ \left. \left. 2 b^2 C \tan \left[\frac{1}{2}(c+d x)\right]^5 - 6 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 a b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \right. \right. \\ \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \right. \\ \left. \left. (a+b)(a B-2 b C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \right. \right. \\ \left. \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2(2 a b(B-C)+a^2 C-b^2(B+C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \right. \\ \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) / \\ \left(d (b+a \cos [c+d x])^{3 / 2} \sec [c+d x]^{3 / 2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)^{3 / 2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)$$

- **Problem 827: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^3 (a+b \sec [c+d x])^{3 / 2} (B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 428 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{4 b d} (a-b) \sqrt{a+b} (5 b B+4 a C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{1}{4 d} \sqrt{a+b} (2 a B+5 b B+4 a C+8 b C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{4 a d} \sqrt{a+b} (4 a^2 B+3 b^2 B+12 a b C) \operatorname{Cot}[c+d x] \\ & \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{(5 b B+4 a C) \sqrt{a+b \sec [c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{a B \cos [c+d x] \sqrt{a+b \sec [c+d x]} \operatorname{Sin}[c+d x]}{2 d} \end{aligned}$$

Result (type 4, 1580 leaves):

$$\begin{aligned} & \frac{a B \sqrt{a+b \sec [c+d x]} \operatorname{Sin}[2(c+d x)]}{4 d} - \\ & \left(\sqrt{a+b \sec [c+d x]} \left(5 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 5 b^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 4 a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \right. \right. \\ & 4 a b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 10 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 8 a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + \\ & 5 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 5 b^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 4 a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\ & \left. 4 a b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 8 i a^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\ & \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 i b^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 24 i a b C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \\
& i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\left. \right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 8 i a^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i b^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 24 i a b C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i (a-b) (5 b B + 4 a C) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i (a-b) (2 a B + b (B + 4 C)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left. \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) \right) \Bigg/ \\
& \left(4 \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)
\end{aligned}$$

$$\left(1 + \tan\left[\frac{1}{2}(c + dx)\right]\right)^{3/2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}}$$

■ **Problem 828: Result more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^4 (a + b \sec[c + dx])^{3/2} (B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 520 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{24abd} (a - b) \sqrt{a + b} (16a^2B + 3b^2B + 30abc) \cot[c + dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{24ad} \sqrt{a + b} (16a^2B + 14abB + 3b^2B + 12a^2C + 30abc) \\ & \cot[c + dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{8a^2d} \sqrt{a + b} (12a^2bB - b^3B + 8a^3C + 6ab^2C) \cot[c + dx] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{(16a^2B + 3b^2B + 30abc) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{24ad} + \\ & \frac{(7bB + 6aC) \cos[c + dx] \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{12d} + \frac{aB \cos[c + dx]^2 \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{3d} \end{aligned}$$

Result (type 4, 1532 leaves):

$$\begin{aligned} & \frac{\sqrt{a + b \sec[c + dx]} \left(\frac{1}{12} a B \sin[c + dx] + \frac{1}{24} (7bB + 6aC) \sin[2(c + dx)] + \frac{1}{12} a B \sin[3(c + dx)]\right)}{d} + \\ & \left(\sqrt{a + b \sec[c + dx]} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \left(16a^3B \tan\left[\frac{1}{2}(c + dx)\right] + 16a^2bB \tan\left[\frac{1}{2}(c + dx)\right] + 3ab^2B \tan\left[\frac{1}{2}(c + dx)\right] + \right. \right. \\ & \quad \left. \left. 3b^3B \tan\left[\frac{1}{2}(c + dx)\right] + 30a^2bC \tan\left[\frac{1}{2}(c + dx)\right] + 30ab^2C \tan\left[\frac{1}{2}(c + dx)\right] - 32a^3B \tan\left[\frac{1}{2}(c + dx)\right]^3 - \right. \right. \\ & \quad \left. \left. 6ab^2B \tan\left[\frac{1}{2}(c + dx)\right]^3 - 60a^2bC \tan\left[\frac{1}{2}(c + dx)\right]^3 + 16a^3B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 16a^2bB \tan\left[\frac{1}{2}(c + dx)\right]^5 + \right. \right. \end{aligned}$$

$$\begin{aligned}
& 3 a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 3 b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 30 a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 30 a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 72 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 48 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 36 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 72 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 6 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 48 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 36 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + (a+b) (16 a^2 B + 3 b^2 B + 30 a b C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} -
\end{aligned}$$

$$2 a \left(a b (26 B - 6 C) + 12 a^2 C + b^2 (-7 B + 24 C) \right) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\sqrt{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2} \left(1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \right) \Bigg| \Bigg|$$

$$\left(24 a d \sqrt{b + a \text{Cos} [c + d x]} \sqrt{\text{Sec} [c + d x]} \left(1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^{3/2} \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \right)$$

■ **Problem 829: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec} [c + d x]^2 (a + b \text{Sec} [c + d x])^{5/2} (B \text{Sec} [c + d x] + C \text{Sec} [c + d x]^2) dx$$

Optimal (type 4, 565 leaves, 9 steps):

$$\frac{1}{3465 b^4 d} 2 (a - b) \sqrt{a + b} (110 a^4 b B - 3069 a^2 b^3 B - 1617 b^5 B - 40 a^5 C - 255 a^3 b^2 C - 3705 a b^4 C)$$

$$\text{Cot} [c + d x] \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{a + b \text{Sec} [c + d x]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b (1 - \text{Sec} [c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec} [c + d x])}{a - b}} -$$

$$\frac{1}{3465 b^3 d} 2 (a - b) \sqrt{a + b} (6 a b^3 (209 B - 505 C) - 3 b^4 (539 B - 225 C) - a^3 b (110 B - 30 C) - 15 a^2 b^2 (121 B - 19 C) + 40 a^4 C)$$

$$\text{Cot} [c + d x] \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{a + b \text{Sec} [c + d x]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b (1 - \text{Sec} [c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec} [c + d x])}{a - b}} -$$

$$\frac{2 (110 a^3 b B - 1254 a b^3 B - 40 a^4 C - 285 a^2 b^2 C - 675 b^4 C) \sqrt{a + b \text{Sec} [c + d x]} \text{Tan} [c + d x]}{3465 b^2 d} -$$

$$\frac{2 (110 a^2 b B - 539 b^3 B - 40 a^3 C - 335 a b^2 C) (a + b \text{Sec} [c + d x])^{3/2} \text{Tan} [c + d x]}{3465 b^2 d} - \frac{2 (22 a b B - 8 a^2 C - 81 b^2 C) (a + b \text{Sec} [c + d x])^{5/2} \text{Tan} [c + d x]}{693 b^2 d} +$$

$$\frac{2 (11 b B - 4 a C) (a + b \text{Sec} [c + d x])^{7/2} \text{Tan} [c + d x]}{99 b^2 d} + \frac{2 C \text{Sec} [c + d x] (a + b \text{Sec} [c + d x])^{7/2} \text{Tan} [c + d x]}{11 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 830: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec} [c + d x] (a + b \text{Sec} [c + d x])^{5/2} (B \text{Sec} [c + d x] + C \text{Sec} [c + d x]^2) dx$$

Optimal (type 4, 469 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (45 a^3 b B + 435 a b^3 B - 10 a^4 C + 279 a^2 b^2 C + 147 b^4 C) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{315 b^2 d} 2 (a-b) \sqrt{a+b} (3 b^3 (25 B - 49 C) - 6 a b^2 (60 B - 19 C) + 15 a^2 b (3 B - 11 C) - 10 a^3 C) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \\
& \frac{2 (45 a^2 b B + 75 b^3 B - 10 a^3 C + 114 a b^2 C) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{315 b d} + \frac{2 (45 a b B - 10 a^2 C + 49 b^2 C) (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{315 b d} + \\
& \frac{2 (9 b B - 2 a C) (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{63 b d} + \frac{2 C (a+b \text{Sec}[c+d x])^{7/2} \text{Tan}[c+d x]}{9 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 831: Attempted integration timed out after 120 seconds.**

$$\int (a+b \text{Sec}[c+d x])^{5/2} (B \text{Sec}[c+d x] + C \text{Sec}[c+d x]^2) dx$$

Optimal (type 4, 384 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{105 b^2 d} 2 (a-b) \sqrt{a+b} (161 a^2 b B + 63 b^3 B + 15 a^3 C + 145 a b^2 C) \text{Cot}[c+d x] \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{1}{105 b d} \\
& 2 (a-b) \sqrt{a+b} (b^2 (63 B - 25 C) - 8 a b (7 B - 15 C) + 15 a^2 (7 B - C)) \text{Cot}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} + \frac{2 (56 a b B + 15 a^2 C + 25 b^2 C) \sqrt{a+b \text{Sec}[c+d x]} \text{Tan}[c+d x]}{105 d} + \\
& \frac{2 (7 b B + 5 a C) (a+b \text{Sec}[c+d x])^{3/2} \text{Tan}[c+d x]}{35 d} + \frac{2 C (a+b \text{Sec}[c+d x])^{5/2} \text{Tan}[c+d x]}{7 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 833: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 (a+b \sec [c+d x])^{5 / 2} (B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 433 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{3 b d} (a-b) \sqrt{a+b} (3 a^2 B-6 b^2 B-14 a b C) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{1}{3 d} \sqrt{a+b} (2 a b(9 B-7 C)-2 b^2(3 B-C)+3 a^2(B+6 C)) \\ & \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{d} \\ & a \sqrt{a+b} (5 b B+2 a C) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{a B(a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{d} - \frac{b(3 a B-2 b C) \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{3 d} \end{aligned}$$

Result (type 4, 1146 leaves):

$$\begin{aligned} & \left((a+b \sec [c+d x])^{5 / 2} \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right. \\ & \left. \left(3 a^3 B \tan \left[\frac{1}{2}(c+d x)\right] + 3 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right] - 6 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right] - 6 b^3 B \tan \left[\frac{1}{2}(c+d x)\right] - 14 a^2 b C \tan \left[\frac{1}{2}(c+d x)\right] - \right. \right. \\ & 14 a b^2 C \tan \left[\frac{1}{2}(c+d x)\right] - 6 a^3 B \tan \left[\frac{1}{2}(c+d x)\right]^3 + 12 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^3 + 28 a^2 b C \tan \left[\frac{1}{2}(c+d x)\right]^3 + 3 a^3 B \tan \left[\frac{1}{2}(c+d x)\right]^5 - \\ & 3 a^2 b B \tan \left[\frac{1}{2}(c+d x)\right]^5 - 6 a b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 + 6 b^3 B \tan \left[\frac{1}{2}(c+d x)\right]^5 - 14 a^2 b C \tan \left[\frac{1}{2}(c+d x)\right]^5 + 14 a b^2 C \tan \left[\frac{1}{2}(c+d x)\right]^5 - \\ & 30 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\ & \left. 12 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \end{aligned}$$

$$\begin{aligned}
& 30 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (3 a^2 B - 6 b^2 B - 14 a b C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (9 a^2 b (B-C) + 3 a^3 C - b^3 (3 B+C) - a b^2 (9 B+7 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) \right) / \\
& \left(3 d (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right) + \\
& \frac{\operatorname{Cos}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^{5/2} \left(\frac{2}{3} b (3 b B+7 a C) \operatorname{Sin}[c+dx] + \frac{2}{3} b^2 C \operatorname{Tan}[c+dx]\right)}{d (b+a \operatorname{Cos}[c+dx])^2}
\end{aligned}$$

■ **Problem 834: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^{5/2} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 450 leaves, 8 steps):

$$\frac{1}{4bd} (a-b) \sqrt{a+b} (9abB + 4a^2C - 8b^2C) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{4d} \sqrt{a+b} (8b^2(B-C) + 2a^2(B+2C) + 3ab(3B+8C))$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{4d}$$

$$\sqrt{a+b} (4a^2B + 15b^2B + 20abC) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{a(7bB + 4aC) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4d} + \frac{aB \operatorname{Cos}[c+dx] (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2d}$$

Result (type 4, 1338 leaves):

$$\frac{\operatorname{Cos}[c+dx]^2 (a+b \operatorname{Sec}[c+dx])^{5/2} (2b^2C \operatorname{Sin}[c+dx] + \frac{1}{4}a^2B \operatorname{Sin}[2(c+dx)])}{d(b+a \operatorname{Cos}[c+dx])^2} +$$

$$\left((a+b \operatorname{Sec}[c+dx])^{5/2} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(9a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 9ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 4a^3C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$4a^2bC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 8ab^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 8b^3C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 18a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 -$$

$$8a^3C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 16ab^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 9a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 9ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$4a^3C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4a^2bC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 8ab^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 8b^3C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$8a^3B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$30ab^2B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$40a^2bC \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$\begin{aligned}
& 8 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 40 a^2 b C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (9 a b B + 4 a^2 C - 8 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 \left(2 a^3 B - a^2 b (B - 12 C) + 12 a b^2 (B - C) - 4 b^3 (B + C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Big/ \\
& \left(4 d (b+a \operatorname{Cos}[c+dx])^{5/2} \operatorname{Sec}[c+dx]^{5/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)
\end{aligned}$$

■ **Problem 835: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^4 (a+b \operatorname{Sec}[c+dx])^{5/2} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 518 leaves, 9 steps):

$$\frac{1}{24bd} (a-b) \sqrt{a+b} (16a^2B + 33b^2B + 54abC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{24d} \sqrt{a+b} (4a^2(4B+3C) + 3b^2(11B+16C) + ab(26B+54C))$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} -$$

$$\frac{1}{8ad} \sqrt{a+b} (20a^2bB + 5b^3B + 8a^3C + 30ab^2C) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\operatorname{Sec}[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{(16a^2B + 33b^2B + 54abC) \sqrt{a+b}\operatorname{Sec}[c+dx] \operatorname{Sin}[c+dx]}{24d} +$$

$$\frac{a(3bB + 2aC) \operatorname{Cos}[c+dx] \sqrt{a+b}\operatorname{Sec}[c+dx] \operatorname{Sin}[c+dx]}{4d} + \frac{aB \operatorname{Cos}[c+dx]^2 (a+b)\operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 4, 1546 leaves):

$$\frac{\sqrt{a+b}\operatorname{Sec}[c+dx] \left(\frac{1}{12} a^2 B \operatorname{Sin}[c+dx] + \frac{1}{24} a (13bB + 6aC) \operatorname{Sin}[2(c+dx)] + \frac{1}{12} a^2 B \operatorname{Sin}[3(c+dx)] \right)}{d} +$$

$$\left(\sqrt{a+b}\operatorname{Sec}[c+dx] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(16a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 16a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 33ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 33b^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 54a^2bC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$54ab^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 32a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 66ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 108a^2bC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 16a^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$16a^2bB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 33ab^2B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 33b^3B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 54a^2bC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 54ab^2C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$\left. 120a^2bB \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right)$$

$$\begin{aligned}
& 30 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 48 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 180 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 120 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 30 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 48 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 180 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (16 a^2 B + 33 b^2 B + 54 a b C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (a^2 b (38 B - 6 C) + 24 b^3 (B - C) + 12 a^3 C + a b^2 (-13 B + 72 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) /
\end{aligned}$$

$$\left(24 d \sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]} \left(1 + \tan \left[\frac{1}{2} (c+d x) \right] \right)^{3/2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2 + b \tan \left[\frac{1}{2} (c+d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c+d x) \right]^2}} \right)$$

■ **Problem 837: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec [c+d x]^3 (B \sec [c+d x] + C \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 411 leaves, 7 steps):

$$-\frac{1}{105 b^5 d} 2 (a-b) \sqrt{a+b} (56 a^2 b B + 63 b^3 B - 48 a^3 C - 44 a b^2 C) \cot [c+d x]$$

$$\text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{105 b^4 d}$$

$$2 \sqrt{a+b} (b^3 (63 B - 25 C) - 48 a^3 C + 4 a^2 b (14 B + 3 C) - 2 a b^2 (7 B + 22 C)) \cot [c+d x] \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}} \right], \frac{a+b}{a-b} \right]$$

$$\sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{2(28 a b B - 24 a^2 C - 25 b^2 C) \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{105 b^3 d} +$$

$$\frac{2(7 b B - 6 a C) \sec [c+d x] \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{35 b^2 d} + \frac{2 C \sec [c+d x]^2 \sqrt{a+b \sec [c+d x]} \tan [c+d x]}{7 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 838: Unable to integrate problem.**

$$\int \frac{\sec [c+d x]^2 (B \sec [c+d x] + C \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 329 leaves, 6 steps):

$$\frac{1}{15 b^4 d} 2 (a-b) \sqrt{a+b} (10 a b B - 8 a^2 C - 9 b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{15 b^3 d} 2 \sqrt{a+b} (b^2 (5 B - 9 C) - 8 a^2 C + 2 a b (5 B + C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2(5 b B - 4 a C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{15 b^2 d} + \frac{2 C \operatorname{Sec}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{5 b d}$$

Result (type 8, 44 leaves):

$$\int \frac{\operatorname{Sec}[c+d x]^2 (B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2)}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

■ **Problem 839: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c+d x] (B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2)}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 261 leaves, 5 steps):

$$-\frac{1}{3 b^3 d}$$

$$2(a-b) \sqrt{a+b} (3 b B - 2 a C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{3 b^2 d} 2 \sqrt{a+b} (3 b B - 2 a C - b C) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 C \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{3 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 840: Unable to integrate problem.**

$$\int \frac{B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2}{\sqrt{a+b \operatorname{Sec}[c+d x]}} dx$$

Optimal (type 4, 210 leaves, 4 steps):

$$-\frac{1}{b^2 d} 2(a-b)\sqrt{a+b} C \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{2\sqrt{a+b}(B-C)\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{bd}$$

Result (type 8, 36 leaves):

$$\int \frac{B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{\sqrt{a+b \operatorname{Sec}[c+dx]}} dx$$

■ **Problem 842: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{\sqrt{a+b \operatorname{Sec}[c+dx]}} dx$$

Optimal (type 4, 348 leaves, 7 steps):

$$\frac{(a-b)\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{abd} +$$

$$\frac{\sqrt{a+b} B \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{ad} + \frac{1}{a^2 d}$$

$$\frac{\sqrt{a+b}(bB-2aC)\cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}}}{ad} +$$

$$\frac{B\sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{ad}$$

Result (type 4, 1027 leaves):

$$\begin{aligned}
& \left(\sqrt{b+a \operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \left. \left(a \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. a \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + b \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \right. \right. \\
& \left. \left. 2 i b \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \right. \right. \\
& \left. \left. 4 i a \operatorname{C EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \right. \right. \\
& \left. \left. 2 i b \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right. \right. \\
& \left. \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 4 i a \operatorname{C EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i (a-b) \operatorname{B EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \right. \right. \\
& \left. \left. \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 i (b \operatorname{B}-a \operatorname{C}) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg/ \\
& \left(a \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b \operatorname{Sec}[c+dx]} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

■ **Problem 843: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^3 (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 471 leaves, 7 steps):

$$\begin{aligned} & \frac{1}{15 b^5 \sqrt{a + b} d} 2 (40 a^3 b B - 25 a b^3 B - 48 a^4 C + 24 a^2 b^2 C + 9 b^4 C) \text{Cot}[c + d x] \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a - b}} + \frac{1}{15 b^4 \sqrt{a + b} d} \\ & 2 (b^3 (5 B - 9 C) + 4 a^2 b (10 B - 9 C) + 6 a b^2 (5 B - 2 C) - 48 a^3 C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 a (b B - a C) \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{b (a^2 - b^2) d \sqrt{a + b \text{Sec}[c + d x]}} + \\ & \frac{2 (20 a^2 b B - 5 b^3 B - 24 a^3 C + 9 a b^2 C) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{15 b^3 (a^2 - b^2) d} - \frac{2 (5 a b B - 6 a^2 C + b^2 C) \text{Sec}[c + d x] \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{5 b^2 (a^2 - b^2) d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 844: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^2 (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 329 leaves, 6 steps):

$$\begin{aligned} & - \frac{1}{3 b^4 \sqrt{a + b} d} 2 (6 a^2 b B - 3 b^3 B - 8 a^3 C + 5 a b^2 C) \text{Cot}[c + d x] \\ & \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a - b}} - \frac{1}{3 b^3 \sqrt{a + b} d} \\ & 2 (2 a + b) (3 b B - 4 a C - b C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b(1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b(1 + \text{Sec}[c + d x])}{a - b}} - \\ & \frac{2 a^2 (b B - a C) \text{Tan}[c + d x]}{b^2 (a^2 - b^2) d \sqrt{a + b \text{Sec}[c + d x]}} + \frac{2 C \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{3 b^2 d} \end{aligned}$$

Result (type 1, 1 leaves) :

???

■ **Problem 845: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x] (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 275 leaves, 5 steps) :

$$\frac{1}{b^3 \sqrt{a+b} d} 2 (a b B - 2 a^2 C + b^2 C) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{b^2 \sqrt{a+b} d} 2 (b(B-C) - 2 a C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (b B - a C) \text{Tan}[c + d x]}{b (a^2 - b^2) d \sqrt{a+b \text{Sec}[c+d x]}}$$

Result (type 1, 1 leaves) :

???

■ **Problem 846: Attempted integration timed out after 120 seconds.**

$$\int \frac{B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 254 leaves, 5 steps) :

$$\frac{2 (b B - a C) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}}}{b^2 \sqrt{a+b} d} +$$

$$\frac{2 (B + C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}}}{b \sqrt{a+b} d} - \frac{2 (b B - a C) \text{Tan}[c + d x]}{(a^2 - b^2) d \sqrt{a+b \text{Sec}[c+d x]}}$$

Result (type 1, 1 leaves) :

???

■ **Problem 847: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cos}[c + d x] (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 376 leaves, 7 steps) :

$$\frac{2 (b B - a C) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} -$$

$$\frac{2 (b B - a C) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a b \sqrt{a+b} d} -$$

$$\frac{2 \sqrt{a+b} B \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c+d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 d} + \frac{2 b (b B - a C) \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}$$

Result (type 4, 1445 leaves) :

$$\frac{(b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x] \left(\frac{2(-bB+aC) \operatorname{Sin}[c+d x]}{a(a^2-b^2)} - \frac{2(-b^2 B \operatorname{Sin}[c+d x] + a b C \operatorname{Sin}[c+d x])}{a(a^2-b^2)(b+a \operatorname{Cos}[c+d x])} \right)}{d \sqrt{a+b} \operatorname{Sec}[c + d x]} +$$

$$\left(2 \sqrt{b+a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right.$$

$$\left(a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] + b^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - a b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \right.$$

$$2 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + 2 a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^3 + a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 -$$

$$b^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 - a^2 \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 + a b \sqrt{\frac{-a+b}{a+b}} C \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^5 -$$

$$2 i a^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}$$

$$\left. \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a+b}} + 2 i b^2 B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a+b}{a-b}\right] \right)$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2i a^2 \text{B EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 2i b^2 \text{B EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i(a-b)(-bB+aC) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i(a-b)(2bB+a(B-C)) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) \Bigg/ \\
& \left(a \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2) d \sqrt{a+b \sec[c+dx]} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right. \right. \\
& \left. \left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right) \right)
\end{aligned}$$

■ **Problem 848: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^2 (B \sec[c+dx] + C \sec[c+dx]^2)}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 427 leaves, 8 steps):

$$\frac{1}{a^2 b \sqrt{a+b} d} (a^2 B - 3 b^2 B + 2 a b C) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{a^2 \sqrt{a+b} d} (3 b B + a(B-2C)) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{a^3 d} \sqrt{a+b} (3 b B - 2 a C) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{B \sin[c+dx]}{a d \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{b(a^2 B - 3 b^2 B + 2 a b C) \tan[c+dx]}{a^2 (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+dx]}}$$

Result (type 4, 1613 leaves):

$$\frac{(b+a \cos[c+dx])^2 \operatorname{Sec}[c+dx]^2 \left(-\frac{2b(bB-aC) \sin[c+dx]}{a^2(-a^2+b^2)} + \frac{2(-b^3 B \sin[c+dx] + a b^2 C \sin[c+dx])}{a^2(a^2-b^2)(b+a \cos[c+dx])} \right)}{d(a+b \operatorname{Sec}[c+dx])^{3/2}} -$$

$$\left((b+a \cos[c+dx])^{3/2} \operatorname{Sec}[c+dx]^{3/2} \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(a^3 B \tan\left[\frac{1}{2}(c+dx)\right] + a^2 b B \tan\left[\frac{1}{2}(c+dx)\right] - 3 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right] - 3 b^3 B \tan\left[\frac{1}{2}(c+dx)\right] + 2 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right] + \right.$$

$$2 a b^2 C \tan\left[\frac{1}{2}(c+dx)\right] - 2 a^3 B \tan\left[\frac{1}{2}(c+dx)\right]^3 + 6 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right]^3 - 4 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^3 + a^3 B \tan\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$a^2 b B \tan\left[\frac{1}{2}(c+dx)\right]^5 - 3 a b^2 B \tan\left[\frac{1}{2}(c+dx)\right]^5 + 3 b^3 B \tan\left[\frac{1}{2}(c+dx)\right]^5 + 2 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^5 - 2 a b^2 C \tan\left[\frac{1}{2}(c+dx)\right]^5 +$$

$$6 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$6 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$4 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +$$

$$\begin{aligned}
& 4 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 6 a^2 b B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 b^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 4 a^3 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 4 a b^2 C \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) \left(a^2 B - 3 b^2 B + 2 a b C\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 a (a+b) (-b B + a C) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(a^2 (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^{3/2} \sqrt{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(a \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ Problem 849: Attempted integration timed out after 120 seconds.

$$\int \frac{\sec[c + dx]^3 (B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^{5/2}} dx$$

Optimal (type 4, 509 leaves, 7 steps):

$$\begin{aligned} & - \frac{1}{3(a-b)b^5(a+b)^{3/2}d} \left(8a^4bB - 15a^2b^3B + 3b^5B - 16a^5C + 28a^3b^2C - 8ab^4C \right) \\ & \quad \cot[c + dx] \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \\ & \frac{1}{3b^4\sqrt{a+b}(a^2-b^2)d} \left(a^3b(8B-12C) - 9ab^3(B-C) - b^4(3B-C) - 16a^4C + 2a^2b^2(3B+8C) \right) \cot[c + dx] \\ & \quad \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{2a(bB - aC) \sec[c + dx]^2 \tan[c + dx]}{3b(a^2 - b^2)d(a + b \sec[c + dx])^{3/2}} - \\ & \frac{2a^2(3a^2bB - 7b^3B - 6a^3C + 10ab^2C) \tan[c + dx]}{3b^3(a^2 - b^2)^2d\sqrt{a + b \sec[c + dx]}} - \frac{2(abB - 2a^2C + b^2C) \sqrt{a + b \sec[c + dx]} \tan[c + dx]}{3b^3(a^2 - b^2)d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 850: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c + dx]^2 (B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^{5/2}} dx$$

Optimal (type 4, 417 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{3(a-b)b^4(a+b)^{3/2}d} \left(2a^3bB - 6ab^3B - 8a^4C + 15a^2b^2C - 3b^4C \right) \cot[c + dx] \\ & \quad \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} + \frac{1}{3b^3\sqrt{a+b}(a^2-b^2)d} \\ & \quad 2 \left(2a^2b(B-3C) - 3b^3(B-C) - 8a^3C + 3ab^2(B+3C) \right) \cot[c + dx] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \sec[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \\ & \quad \sqrt{\frac{b(1 - \sec[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \sec[c + dx])}{a - b}} - \frac{2a^2(bB - aC) \tan[c + dx]}{3b^2(a^2 - b^2)d(a + b \sec[c + dx])^{3/2}} + \frac{2a(2a^2bB - 6b^3B - 5a^3C + 9ab^2C) \tan[c + dx]}{3b^2(a^2 - b^2)^2d\sqrt{a + b \sec[c + dx]}} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 851: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x] (B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 387 leaves, 6 steps):

$$\frac{1}{3 (a - b) b^3 (a + b)^{3/2} d} + 2 (a^2 b B + 3 b^3 B + 2 a^3 C - 6 a b^2 C) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{1}{3 b^2 \sqrt{a + b} (a^2 - b^2) d} 2 (2 a^2 C - 3 b^2 (B + C) + a b (B + 3 C)) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 a (b B - a C) \text{Tan}[c + d x]}{3 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} + \frac{2 (a^2 b B + 3 b^3 B + 2 a^3 C - 6 a b^2 C) \text{Tan}[c + d x]}{3 b (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 852: Unable to integrate problem.**

$$\int \frac{B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 353 leaves, 6 steps):

$$-\frac{1}{3 (a - b) b^2 (a + b)^{3/2} d} 2 (4 a b B - a^2 C - 3 b^2 C) \text{Cot}[c + d x] + \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{1}{3 (a - b) b (a + b)^{3/2} d} + 2 (3 a B - b B + a C - 3 b C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} - \frac{2 (b B - a C) \text{Tan}[c + d x]}{3 (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} - \frac{2 (4 a b B - a^2 C - 3 b^2 C) \text{Tan}[c + d x]}{3 (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}}$$

Result (type 8, 36 leaves):

$$\int \frac{B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

- **Problem 853: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + d x] (B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 495 leaves, 8 steps):

$$\frac{1}{3 a^2 (a - b) b (a + b)^{3/2} d}$$

$$2 (7 a^2 b B - 3 b^3 B - 4 a^3 C) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{1}{3 a^2 (a - b) b (a + b)^{3/2} d} 2 (6 a^2 b B - a b^2 B - 3 b^3 B - 3 a^3 C + a^2 b C) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 \sqrt{a + b} B \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}}}{a^3 d} +$$

$$\frac{2 b (b B - a C) \operatorname{Tan}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 b (7 a^2 b B - 3 b^3 B - 4 a^3 C) \operatorname{Tan}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \operatorname{Sec}[c + d x]}}$$

Result (type 4, 2039 leaves):

$$\frac{1}{d (a + b \operatorname{Sec}[c + d x])^{5/2}} (b + a \operatorname{Cos}[c + d x])^3 \operatorname{Sec}[c + d x]^3 \left(\frac{2 (-7 a^2 b B + 3 b^3 B + 4 a^3 C) \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2} - \right.$$

$$\left. \frac{2 (b^3 B \operatorname{Sin}[c + d x] - a b^2 C \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])^2} - \frac{2 (-8 a^2 b^2 B \operatorname{Sin}[c + d x] + 4 b^4 B \operatorname{Sin}[c + d x] + 5 a^3 b C \operatorname{Sin}[c + d x] - a b^3 C \operatorname{Sin}[c + d x])}{3 a^2 (a^2 - b^2)^2 (b + a \operatorname{Cos}[c + d x])} \right) +$$

$$\left(2 (b + a \operatorname{Cos}[c + d x])^{5/2} \operatorname{Sec}[c + d x]^{5/2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right)$$

$$\begin{aligned}
& \left(7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] + 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] - \right. \\
& 3 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right] - 4 a^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right] - 4 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& 14 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 6 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 8 a^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \\
& 7 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 3 b^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{B Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4 a^4 \sqrt{\frac{-a+b}{a+b}} \operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 4 a^3 b \sqrt{\frac{-a+b}{a+b}} \operatorname{C Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 6 i a^4 \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 12 i a^2 b^2 \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 i b^4 \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 i a^4 \operatorname{B EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} +
\end{aligned}$$

$$\begin{aligned}
& 12 i a^2 b^2 \text{B EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \\
& \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} - 6 i b^4 \text{B EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \\
& \tan \left[\frac{1}{2} (c+dx) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} + \\
& i (a-b) (-7 a^2 b B + 3 b^3 B + 4 a^3 C) \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} + \\
& i (a-b) (-4 a b^2 B - 6 b^3 B + 3 a^3 (B-C) + a^2 b (9 B+C)) \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+dx) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2} \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{a+b - a \tan \left[\frac{1}{2} (c+dx) \right]^2 + b \tan \left[\frac{1}{2} (c+dx) \right]^2}{a+b}} \right) \Bigg) / \\
& \left(3 a^2 \sqrt{\frac{-a+b}{a+b}} (a^2 - b^2)^2 d (a+b \text{Sec}[c+dx])^{5/2} \left(-1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \sqrt{\frac{1 + \tan \left[\frac{1}{2} (c+dx) \right]^2}{1 - \tan \left[\frac{1}{2} (c+dx) \right]^2}} \right. \right. \\
& \left. \left. \left(a \left(-1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) - b \left(1 + \tan \left[\frac{1}{2} (c+dx) \right]^2 \right) \right) \right) \right)
\end{aligned}$$

■ **Problem 854: Attempted integration timed out after 120 seconds.**

$$\int \frac{B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2}{(a+b \text{Sec}[c+dx])^{7/2}} dx$$

Optimal (type 4, 446 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{15(a-b)^2 b^2 (a+b)^{5/2} d} (23 a^2 b B + 9 b^3 B - 3 a^3 C - 29 a b^2 C) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{15 b \sqrt{a+b} (a^2-b^2)^2 d} (3 a^2 (5 B+C) - 8 a b (B+3 C) + b^2 (9 B+5 C)) \\
& \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{2(bB-aC) \operatorname{Tan}[c+dx]}{5(a^2-b^2) d (a+b \operatorname{Sec}[c+dx])^{5/2}} - \frac{2(8 a b B - 3 a^2 C - 5 b^2 C) \operatorname{Tan}[c+dx]}{15(a^2-b^2)^2 d (a+b \operatorname{Sec}[c+dx])^{3/2}} - \frac{2(23 a^2 b B + 9 b^3 B - 3 a^3 C - 29 a b^2 C) \operatorname{Tan}[c+dx]}{15(a^2-b^2)^3 d \sqrt{a+b \operatorname{Sec}[c+dx]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 857: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+dx])^{2/3} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 6, 229 leaves, 8 steps):

$$\begin{aligned}
& \left(\sqrt{2} (a+b) C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2} (1-\operatorname{Sec}[c+dx]), \frac{b(1-\operatorname{Sec}[c+dx])}{a+b}\right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx] \right) / \\
& \left(b d \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \right) + \\
& \left(\sqrt{2} (bB-aC) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1-\operatorname{Sec}[c+dx]), \frac{b(1-\operatorname{Sec}[c+dx])}{a+b}\right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx] \right) / \\
& \left(b d \sqrt{1+\operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \right)
\end{aligned}$$

Result (type 6, 33208 leaves): Display of huge result suppressed!

■ **Problem 858: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+dx])^{1/3} (B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 6, 229 leaves, 8 steps):

$$\left(\sqrt{2} (a+b) C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx] \right) /$$

$$\left(b d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \right) +$$

$$\left(\sqrt{2} (bB - aC) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx] \right) /$$

$$\left(b d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \right)$$

Result (type 6, 33 199 leaves) : Display of huge result suppressed!

■ **Problem 859: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^{1/3}} dx$$

Optimal (type 6, 226 leaves, 8 steps) :

$$\frac{\sqrt{2} C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{2/3} \operatorname{Tan}[c+dx]}{b d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3}} +$$

$$\frac{\sqrt{2} (bB - aC) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3} \operatorname{Tan}[c+dx]}{b d \sqrt{1 + \operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{1/3}}$$

Result (type 6, 18676 leaves) : Display of huge result suppressed!

■ **Problem 860: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2}{(a+b \operatorname{Sec}[c+dx])^{2/3}} dx$$

Optimal (type 6, 226 leaves, 8 steps) :

$$\frac{\sqrt{2} C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] (a+b \operatorname{Sec}[c+dx])^{1/3} \operatorname{Tan}[c+dx]}{b d \sqrt{1 + \operatorname{Sec}[c+dx]} \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{1/3}} +$$

$$\frac{\sqrt{2} (bB - aC) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c+dx]), \frac{b(1 - \operatorname{Sec}[c+dx])}{a+b} \right] \left(\frac{a+b \operatorname{Sec}[c+dx]}{a+b} \right)^{2/3} \operatorname{Tan}[c+dx]}{b d \sqrt{1 + \operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx])^{2/3}}$$

Result (type 6, 18666 leaves) : Display of huge result suppressed!

■ **Problem 861: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^3 (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 165 leaves, 7 steps) :

$$\frac{(4 a A + 3 b B + 3 a C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(5 A b + 5 a B + 4 b C) \operatorname{Tan}[c + d x]}{5 d} + \frac{(4 a A + 3 b B + 3 a C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} +$$

$$\frac{(b B + a C) \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d} + \frac{b C \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d} + \frac{(5 A b + 5 a B + 4 b C) \operatorname{Tan}[c + d x]^3}{15 d}$$

Result (type 3, 660 leaves) :

$$\frac{a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} - \frac{3 b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} - \frac{3 a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} +$$

$$\frac{a A \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{2 d} + \frac{3 b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} + \frac{3 a C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right]}{8 d} +$$

$$\frac{b B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{a C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} + \frac{a A}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{3 b B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} + \frac{3 a C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{b B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} -$$

$$\frac{a C}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4} - \frac{a A}{4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} - \frac{3 b B}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2} +$$

$$\frac{A b \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{a B \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{4 b C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{15 d} + \frac{b C \operatorname{Sec}[c + d x]^4 \operatorname{Tan}[c + d x]}{5 d}$$

■ **Problem 862: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^2 (a + b \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 137 leaves, 7 steps) :

$$\frac{(4 A b + 4 a B + 3 b C) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(3 a A + 2 b B + 2 a C) \operatorname{Tan}[c + d x]}{3 d} +$$

$$\frac{(4 A b + 4 a B + 3 b C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{(b B + a C) \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 d} + \frac{b C \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 545 leaves) :

$$\begin{aligned}
& \frac{A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}-\frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}- \\
& \frac{3 b C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}+\frac{A b \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+\frac{a B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{2 d}+ \\
& \frac{3 b C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right]}{8 d}+\frac{b C}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}+\frac{A b}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+ \\
& \frac{a B}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+\frac{3 b C}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]-\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{b C}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^4}- \\
& \frac{A b}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{a B}{4 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}-\frac{3 b C}{16 d\left(\cos\left[\frac{1}{2}(c+d x)\right]+\sin\left[\frac{1}{2}(c+d x)\right]\right)^2}+ \\
& \frac{a A \tan [c+d x]}{d}+\frac{2 b B \tan [c+d x]}{3 d}+\frac{2 a C \tan [c+d x]}{3 d}+\frac{b B \sec [c+d x]^2 \tan [c+d x]}{3 d}+\frac{a C \sec [c+d x]^2 \tan [c+d x]}{3 d}
\end{aligned}$$

■ **Problem 865: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x](a+b \sec [c+d x])(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 52 leaves, 5 steps):

$$(A b+a B) x+\frac{(b B+a C) \operatorname{ArcTanh}[\sin [c+d x]]}{d}+\frac{a A \sin [c+d x]}{d}+\frac{b C \tan [c+d x]}{d}$$

Result (type 3, 187 leaves):

$$\begin{aligned}
A b x+a B x-\frac{b B \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d}-\frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]-\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d}+\frac{b B \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d}+ \\
\frac{a C \operatorname{Log}\left[\cos\left[\frac{c}{2}+\frac{d x}{2}\right]+\sin\left[\frac{c}{2}+\frac{d x}{2}\right]\right]}{d}+\frac{a A \cos [d x] \sin [c]}{d}+\frac{a A \cos [c] \sin [d x]}{d}+\frac{b C \tan [c+d x]}{d}
\end{aligned}$$

■ **Problem 872: Result more than twice size of optimal antiderivative.**

$$\int (a+b \sec [c+d x])^2(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 134 leaves, 6 steps):

$$\begin{aligned}
a^2 A x+\frac{\left(2 a^2 B+b^2 B+2 a b(2 A+C)\right) \operatorname{ArcTanh}[\sin [c+d x]]}{2 d}+ \\
\frac{\left(3 A b^2+6 a b B+2 a^2 C+2 b^2 C\right) \tan [c+d x]}{3 d}+\frac{b(3 b B+2 a C) \sec [c+d x] \tan [c+d x]}{6 d}+\frac{C(a+b \sec [c+d x])^2 \tan [c+d x]}{3 d}
\end{aligned}$$

Result (type 3, 322 leaves):

$$\frac{1}{24 d} \operatorname{Sec}[c+d x]^3 \left(9 \operatorname{Cos}[c+d x] \left(2 a^2 A (c+d x) - (2 a^2 B + b^2 B + 2 a b (2 A + C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right) + (2 a^2 B + b^2 B + 2 a b (2 A + C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right) + 3 \operatorname{Cos}[3(c+d x)] \left(2 a^2 A (c+d x) - (2 a^2 B + b^2 B + 2 a b (2 A + C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right) + (2 a^2 B + b^2 B + 2 a b (2 A + C)) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right]\right) + 4 \left(3 A b^2 + 6 a b B + 3 a^2 C + 4 b^2 C + 3 b (b B + 2 a C) \operatorname{Cos}[c+d x] + (3 A b^2 + 6 a b B + 3 a^2 C + 2 b^2 C) \operatorname{Cos}[2(c+d x)] \right) \operatorname{Sin}[c+d x] \right)$$

■ **Problem 873: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+d x] (a+b \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$a(2Ab+aB)x + \frac{(2Ab^2+4abB+2a^2C+b^2C)\operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{2d} + \frac{A(a+b \operatorname{Sec}[c+dx])^2 \operatorname{Sin}[c+dx]}{d} - \frac{b(2aA-bB-2aC)\operatorname{Tan}[c+dx]}{d} - \frac{b^2(2A-C)\operatorname{Sec}[c+dx]\operatorname{Tan}[c+dx]}{2d}$$

Result (type 3, 453 leaves):

$$\frac{1}{4 d} \operatorname{Sec}[c+d x]^2 \left(4 a A b c + 2 a^2 B c + 4 a A b d x + 2 a^2 B d x - 2 A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 4 a b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - 2 a^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - b^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 2 A b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 4 a b B \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 2 a^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + b^2 C \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + \operatorname{Cos}[2(c+d x)] \left(2 a (2 A b + a B) (c+d x) - (2 A b^2 + 4 a b B + 2 a^2 C + b^2 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + (2 A b^2 + 4 a b B + 2 a^2 C + b^2 C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + (a^2 A + 2 b^2 C) \operatorname{Sin}[c+d x] + 2 b^2 B \operatorname{Sin}[2(c+d x)] + 4 a b C \operatorname{Sin}[2(c+d x)] + a^2 A \operatorname{Sin}[3(c+d x)] \right)$$

■ **Problem 880: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$a^3 A x + \frac{(8 a^3 B + 12 a b^2 B + 12 a^2 b (2 A + C) + b^3 (4 A + 3 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{(16 a^2 b B + 4 b^3 B + 3 a^3 C + 6 a b^2 (3 A + 2 C)) \operatorname{Tan}[c + d x]}{6 d} + \frac{b (12 A b^2 + 20 a b B + 6 a^2 C + 9 b^2 C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{24 d} + \frac{(4 b B + 3 a C) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{12 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 687 leaves):

$$\left((-24 a^2 A b - 4 A b^3 - 8 a^3 B - 12 a b^2 B - 12 a^2 b C - 3 b^3 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \left(4 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \right) + \left((24 a^2 A b + 4 A b^3 + 8 a^3 B + 12 a b^2 B + 12 a^2 b C + 3 b^3 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \left(4 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \right) + \frac{1}{48 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (36 a^3 A (c + d x) + 48 a^3 A (c + d x) \operatorname{Cos}[2(c + d x)] + 12 a^3 A (c + d x) \operatorname{Cos}[4(c + d x)] + 12 A b^3 \operatorname{Sin}[c + d x] + 36 a b^2 B \operatorname{Sin}[c + d x] + 36 a^2 b C \operatorname{Sin}[c + d x] + 33 b^3 C \operatorname{Sin}[c + d x] + 72 a A b^2 \operatorname{Sin}[2(c + d x)] + 72 a^2 b B \operatorname{Sin}[2(c + d x)] + 32 b^3 B \operatorname{Sin}[2(c + d x)] + 24 a^3 C \operatorname{Sin}[2(c + d x)] + 96 a b^2 C \operatorname{Sin}[2(c + d x)] + 12 A b^3 \operatorname{Sin}[3(c + d x)] + 36 a b^2 B \operatorname{Sin}[3(c + d x)] + 36 a^2 b C \operatorname{Sin}[3(c + d x)] + 9 b^3 C \operatorname{Sin}[3(c + d x)] + 36 a A b^2 \operatorname{Sin}[4(c + d x)] + 36 a^2 b B \operatorname{Sin}[4(c + d x)] + 8 b^3 B \operatorname{Sin}[4(c + d x)] + 12 a^3 C \operatorname{Sin}[4(c + d x)] + 24 a b^2 C \operatorname{Sin}[4(c + d x)])$$

■ **Problem 881: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 192 leaves, 7 steps):

$$a^2 (3 A b + a B) x + \frac{(6 a^2 b B + b^3 B + 2 a^3 C + 3 a b^2 (2 A + C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} + \frac{A (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Sin}[c + d x]}{d} + \frac{b (9 a b B - a^2 (6 A - 8 C) + b^2 (3 A + 2 C)) \operatorname{Tan}[c + d x]}{3 d} - \frac{b^2 (6 a A - 3 b B - 5 a C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{6 d} - \frac{b (3 A - C) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{3 d}$$

Result (type 3, 1335 leaves):

$$\begin{aligned}
& \frac{2 a^2 (3 A b + a B) (c + d x) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} + \\
& \left((-6 a A b^2 - 6 a^2 b B - b^3 B - 2 a^3 C - 3 a b^2 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / (d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \\
& \left((6 a A b^2 + 6 a^2 b B + b^3 B + 2 a^3 C + 3 a b^2 C) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Sec}[c + d x])^3 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / (d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])) + \\
& \left((3 b^3 B + 9 a b^2 C + b^3 C) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \\
& \left(6 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 \right) + \\
& \left(b^3 C \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^3 \right) + \\
& \left(b^3 C \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^3 \right) + \\
& \left((-3 b^3 B - 9 a b^2 C - b^3 C) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \\
& \left(6 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right)^2 \right) + \\
& \left(2 \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \quad \left. \left(3 A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 b C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 b^3 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) + \\
& \left(2 \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Sec}[c + d x])^3 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \quad \left. \left(3 A b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a b^2 B \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 9 a^2 b C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 2 b^3 C \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \right) /
\end{aligned}$$

$$\frac{\left(3 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)\right)+2 a^3 A \cos [c+d x]^5 (a+b \sec [c+d x])^3 (A+B \sec [c+d x]+C \sec [c+d x]^2) \sin [c+d x]}{d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])}$$

■ **Problem 887: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^2 (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) dx$$

Optimal (type 3, 491 leaves, 10 steps):

$$\begin{aligned} & \frac{(8 a^4 B+36 a^2 b^2 B+5 b^4 B+8 a^3 b(4 A+3 C)+4 a b^3(6 A+5 C)) \operatorname{ArcTanh}[\sin [c+d x]]}{16 d}-\frac{1}{420 b^2 d} \\ & (28 a^5 b B-847 a^3 b^3 B-896 a b^5 B-8 a^6 C-32 b^6(7 A+6 C)-4 a^4 b^2(42 A+23 C)-32 a^2 b^4(49 A+39 C)) \tan [c+d x]- \\ & \frac{1}{1680 b d}(56 a^4 b B-1246 a^2 b^3 B-525 b^5 B-16 a^5 C-48 a^3 b^2(7 A+4 C)-4 a b^4(406 A+333 C)) \sec [c+d x] \tan [c+d x]- \\ & \frac{(28 a^3 b B-371 a b^3 B-8 a^4 C-32 b^4(7 A+6 C)-12 a^2 b^2(14 A+9 C))(a+b \sec [c+d x])^2 \tan [c+d x]}{840 b^2 d}- \\ & \frac{(28 a^2 b B-175 b^3 B-8 a^3 C-4 a b^2(42 A+31 C))(a+b \sec [c+d x])^3 \tan [c+d x]}{840 b^2 d}+ \\ & \frac{(42 A b^2-7 a b B+2 a^2 C+36 b^2 C)(a+b \sec [c+d x])^4 \tan [c+d x]}{210 b^2 d}+ \\ & \frac{(7 b B-2 a C)(a+b \sec [c+d x])^5 \tan [c+d x]}{42 b^2 d}+\frac{C \sec [c+d x](a+b \sec [c+d x])^5 \tan [c+d x]}{7 b d} \end{aligned}$$

Result (type 3, 1348 leaves):

$$\begin{aligned}
& \left((-32 a^3 A b - 24 a A b^3 - 8 a^4 B - 36 a^2 b^2 B - 5 b^4 B - 24 a^3 b C - 20 a b^3 C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] \right. \\
& \quad \left. (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(8 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \right) + \\
& \left((32 a^3 A b + 24 a A b^3 + 8 a^4 B + 36 a^2 b^2 B + 5 b^4 B + 24 a^3 b C + 20 a b^3 C) \cos[c + dx]^6 \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right. \\
& \quad \left. (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) / \left(8 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \right) + \\
& \frac{(a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (b^4 B \sin[c + dx] + 4 a b^3 C \sin[c + dx])}{3 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])} + \\
& \frac{(\cos[c + dx]^2 (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (24 a A b^3 \sin[c + dx] + 36 a^2 b^2 B \sin[c + dx] + 5 b^4 B \sin[c + dx] + 24 a^3 b C \sin[c + dx] + 20 a b^3 C \sin[c + dx]))}{(12 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))} + \\
& \frac{(\cos[c + dx]^4 (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (32 a^3 A b \sin[c + dx] + 24 a A b^3 \sin[c + dx] + 8 a^4 B \sin[c + dx] + 36 a^2 b^2 B \sin[c + dx] + 5 b^4 B \sin[c + dx] + 24 a^3 b C \sin[c + dx] + 20 a b^3 C \sin[c + dx]))}{(8 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) + (2 \cos[c + dx] (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (7 a b^4 \sin[c + dx] + 28 a b^3 B \sin[c + dx] + 42 a^2 b^2 C \sin[c + dx] + 6 b^4 C \sin[c + dx]))} / \\
& \frac{(35 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))}{(2 \cos[c + dx]^3 (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (210 a^2 A b^2 \sin[c + dx] + 28 A b^4 \sin[c + dx] + 140 a^3 b B \sin[c + dx] + 112 a b^3 B \sin[c + dx] + 35 a^4 C \sin[c + dx] + 168 a^2 b^2 C \sin[c + dx] + 24 b^4 C \sin[c + dx]))} / \\
& \frac{(105 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))}{(2 \cos[c + dx]^5 (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) (105 a^4 A \sin[c + dx] + 420 a^2 A b^2 \sin[c + dx] + 56 A b^4 \sin[c + dx] + 280 a^3 b B \sin[c + dx] + 224 a b^3 B \sin[c + dx] + 70 a^4 C \sin[c + dx] + 336 a^2 b^2 C \sin[c + dx] + 48 b^4 C \sin[c + dx]))} / \\
& \frac{(105 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))}{2 b^4 C (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Tan}[c + dx]} \\
& \frac{7 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])}{7 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])}
\end{aligned}$$

■ **Problem 889: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 290 leaves, 8 steps):

$$\begin{aligned}
& a^4 A x + \frac{(8 a^4 B + 24 a^2 b^2 B + 3 b^4 B + 16 a^3 b (2 A + C) + 4 a b^3 (4 A + 3 C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \\
& \frac{(95 a^3 b B + 80 a b^3 B + 12 a^4 C + 4 b^4 (5 A + 4 C) + 2 a^2 b^2 (85 A + 56 C)) \operatorname{Tan}[c + d x]}{30 d} + \\
& \frac{b (130 a^2 b B + 45 b^3 B + 24 a^3 C + 4 a b^2 (40 A + 29 C)) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{120 d} + \\
& \frac{(20 A b^2 + 35 a b B + 12 a^2 C + 16 b^2 C) (a + b \operatorname{Sec}[c + d x])^2 \operatorname{Tan}[c + d x]}{60 d} + \\
& \frac{(5 b B + 4 a C) (a + b \operatorname{Sec}[c + d x])^3 \operatorname{Tan}[c + d x]}{20 d} + \frac{C (a + b \operatorname{Sec}[c + d x])^4 \operatorname{Tan}[c + d x]}{5 d}
\end{aligned}$$

Result (type 3, 915 leaves):

$$\begin{aligned}
& \left((-32 a^3 A b - 16 a A b^3 - 8 a^4 B - 24 a^2 b^2 B - 3 b^4 B - 16 a^3 b C - 12 a b^3 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right. \\
& \quad \left. (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \left(4 d (b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \right) + \\
& \left((32 a^3 A b + 16 a A b^3 + 8 a^4 B + 24 a^2 b^2 B + 3 b^4 B + 16 a^3 b C + 12 a b^3 C) \operatorname{Cos}[c + d x]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right. \\
& \quad \left. (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) / \left(4 d (b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \right) + \\
& (\operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \quad (600 a^4 A (c + d x) \operatorname{Cos}[c + d x] + 300 a^4 A (c + d x) \operatorname{Cos}[3 (c + d x)] + 60 a^4 A (c + d x) \operatorname{Cos}[5 (c + d x)] + 720 a^2 A b^2 \operatorname{Sin}[c + d x] + \\
& \quad 160 A b^4 \operatorname{Sin}[c + d x] + 480 a^3 b B \operatorname{Sin}[c + d x] + 640 a b^3 B \operatorname{Sin}[c + d x] + 120 a^4 C \operatorname{Sin}[c + d x] + 960 a^2 b^2 C \operatorname{Sin}[c + d x] + \\
& \quad 320 b^4 C \operatorname{Sin}[c + d x] + 480 a A b^3 \operatorname{Sin}[2 (c + d x)] + 720 a^2 b^2 B \operatorname{Sin}[2 (c + d x)] + 210 b^4 B \operatorname{Sin}[2 (c + d x)] + 480 a^3 b C \operatorname{Sin}[2 (c + d x)] + \\
& \quad 840 a b^3 C \operatorname{Sin}[2 (c + d x)] + 1080 a^2 A b^2 \operatorname{Sin}[3 (c + d x)] + 200 A b^4 \operatorname{Sin}[3 (c + d x)] + 720 a^3 b B \operatorname{Sin}[3 (c + d x)] + 800 a b^3 B \operatorname{Sin}[3 (c + d x)] + \\
& \quad 180 a^4 C \operatorname{Sin}[3 (c + d x)] + 1200 a^2 b^2 C \operatorname{Sin}[3 (c + d x)] + 160 b^4 C \operatorname{Sin}[3 (c + d x)] + 240 a A b^3 \operatorname{Sin}[4 (c + d x)] + 360 a^2 b^2 B \operatorname{Sin}[4 (c + d x)] + \\
& \quad 45 b^4 B \operatorname{Sin}[4 (c + d x)] + 240 a^3 b C \operatorname{Sin}[4 (c + d x)] + 180 a b^3 C \operatorname{Sin}[4 (c + d x)] + 360 a^2 A b^2 \operatorname{Sin}[5 (c + d x)] + 40 A b^4 \operatorname{Sin}[5 (c + d x)] + \\
& \quad 240 a^3 b B \operatorname{Sin}[5 (c + d x)] + 160 a b^3 B \operatorname{Sin}[5 (c + d x)] + 60 a^4 C \operatorname{Sin}[5 (c + d x)] + 240 a^2 b^2 C \operatorname{Sin}[5 (c + d x)] + 32 b^4 C \operatorname{Sin}[5 (c + d x)]) / \\
& (480 d (b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]))
\end{aligned}$$

■ **Problem 890: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x] (a + b \operatorname{Sec}[c + d x])^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 3, 273 leaves, 8 steps):

$$\begin{aligned}
& a^3 (4Ab + aB) x + \frac{(32a^3bB + 16ab^3B + 8a^4C + 24a^2b^2(2A + C) + b^4(4A + 3C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{8d} + \frac{A(a + b \operatorname{Sec}[c + dx])^4 \operatorname{Sin}[c + dx]}{d} + \\
& \frac{b(34a^2bB + 4b^3B - a^3(12A - 19C) + 8ab^2(3A + 2C)) \operatorname{Tan}[c + dx]}{6d} + \frac{b^2(32abB - a^2(24A - 26C) + 3b^2(4A + 3C)) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{24d} - \\
& \frac{b(12aA - 4bB - 7aC)(a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]}{12d} - \frac{b(4A - C)(a + b \operatorname{Sec}[c + dx])^3 \operatorname{Tan}[c + dx]}{4d}
\end{aligned}$$

Result (type 3, 813 leaves):

$$\begin{aligned}
& \left((-48a^2Ab^2 - 4Ab^4 - 32a^3bB - 16ab^3B - 8a^4C - 24a^2b^2C - 3b^4C) \operatorname{Cos}[c + dx]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \\
& \quad (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \Bigg/ (4d(b + a \operatorname{Cos}[c + dx])^4 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx])) + \\
& \left((48a^2Ab^2 + 4Ab^4 + 32a^3bB + 16ab^3B + 8a^4C + 24a^2b^2C + 3b^4C) \operatorname{Cos}[c + dx]^6 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) \\
& \quad (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \Bigg/ (4d(b + a \operatorname{Cos}[c + dx])^4 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx])) + \\
& \frac{1}{48d(b + a \operatorname{Cos}[c + dx])^4 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx])} \operatorname{Cos}[c + dx]^2 (a + b \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \quad (144a^3Ab(c + dx) + 36a^4B(c + dx) + 192a^3Ab(c + dx) \operatorname{Cos}[2(c + dx)] + 48a^4B(c + dx) \operatorname{Cos}[2(c + dx)] + \\
& \quad 48a^3Ab(c + dx) \operatorname{Cos}[4(c + dx)] + 12a^4B(c + dx) \operatorname{Cos}[4(c + dx)] + 12a^4A \operatorname{Sin}[c + dx] + 12Ab^4 \operatorname{Sin}[c + dx] + \\
& \quad 48ab^3B \operatorname{Sin}[c + dx] + 72a^2b^2C \operatorname{Sin}[c + dx] + 33b^4C \operatorname{Sin}[c + dx] + 96aAb^3 \operatorname{Sin}[2(c + dx)] + 144a^2b^2B \operatorname{Sin}[2(c + dx)] + \\
& \quad 32b^4B \operatorname{Sin}[2(c + dx)] + 96a^3bC \operatorname{Sin}[2(c + dx)] + 128ab^3C \operatorname{Sin}[2(c + dx)] + 18a^4A \operatorname{Sin}[3(c + dx)] + 12Ab^4 \operatorname{Sin}[3(c + dx)] + \\
& \quad 48ab^3B \operatorname{Sin}[3(c + dx)] + 72a^2b^2C \operatorname{Sin}[3(c + dx)] + 9b^4C \operatorname{Sin}[3(c + dx)] + 48aAb^3 \operatorname{Sin}[4(c + dx)] + \\
& \quad 72a^2b^2B \operatorname{Sin}[4(c + dx)] + 8b^4B \operatorname{Sin}[4(c + dx)] + 48a^3bC \operatorname{Sin}[4(c + dx)] + 32ab^3C \operatorname{Sin}[4(c + dx)] + 6a^4A \operatorname{Sin}[5(c + dx)])
\end{aligned}$$

■ **Problem 898: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx])^2 (abB - a^2C + b^2B \operatorname{Sec}[c + dx] + b^2C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 149 leaves, 7 steps):

$$\begin{aligned}
& a^3 (bB - aC) x + \frac{b(6a^2bB + b^3B - 4a^3C + 2ab^2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} + \\
& \frac{b^2(9abB - a^2C + 2b^2C) \operatorname{Tan}[c + dx]}{3d} + \frac{b^3(3bB + 2aC) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{6d} + \frac{b^2C(a + b \operatorname{Sec}[c + dx])^2 \operatorname{Tan}[c + dx]}{3d}
\end{aligned}$$

Result (type 3, 462 leaves):

$$\begin{aligned}
& - \frac{a^3 (-bB + aC) (c + dx)}{d} + \frac{(-6a^2b^2B - b^4B + 4a^3bC - 2ab^3C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \\
& \frac{(6a^2b^2B + b^4B - 4a^3bC + 2ab^3C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right]}{2d} + \frac{3b^4B + 6ab^3C + b^4C}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{b^4C \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{b^4C \sin\left[\frac{1}{2}(c + dx)\right]}{6d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^3} + \frac{-3b^4B - 6ab^3C - b^4C}{12d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)^2} + \\
& \frac{9ab^3B \sin\left[\frac{1}{2}(c + dx)\right] + 2b^4C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right)} + \frac{9ab^3B \sin\left[\frac{1}{2}(c + dx)\right] + 2b^4C \sin\left[\frac{1}{2}(c + dx)\right]}{3d \left(\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right)}
\end{aligned}$$

■ **Problem 899: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + dx]) (abB - a^2C + b^2B \operatorname{Sec}[c + dx] + b^2C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$a^2 (bB - aC) x + \frac{b(4abB - 2a^2C + b^2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} + \frac{b^2(2bB + aC) \operatorname{Tan}[c + dx]}{2d} + \frac{b^2C(a + b \operatorname{Sec}[c + dx]) \operatorname{Tan}[c + dx]}{2d}$$

Result (type 3, 379 leaves):

$$\begin{aligned}
& - \frac{1}{4d} \operatorname{Sec}[c + dx]^2 \left(-2a^2bBc + 2a^3cC - 2a^2bBdx + 2a^3Cdx + 4ab^2B \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& \quad \left. 2a^2bC \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + b^3C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - 4ab^2B \right. \\
& \quad \left. \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + 2a^2bC \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] - b^3C \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) + \\
& \quad \cos[2(c + dx)] \left(2a^2(-bB + aC)(c + dx) + b(4abB - 2a^2C + b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\
& \quad \left. b(4abB - 2a^2C + b^2C) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] \right) - 2b^3C \sin[c + dx] - 2b^3B \sin[2(c + dx)] - 2ab^2C \sin[2(c + dx)] \Big)
\end{aligned}$$

■ **Problem 900: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 215 leaves, 8 steps):

$$\frac{(2 a^2 b B + b^3 B - 2 a^3 C - a b^2 (2 A + C)) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 b^4 d} + \frac{2 a^2 (A b^2 - a (b B - a C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^4 \sqrt{a+b} d} +$$

$$\frac{(3 A b^2 - 3 a b B + 3 a^2 C + 2 b^2 C) \operatorname{Tan}[c + d x]}{3 b^3 d} + \frac{(b B - a C) \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 b^2 d} + \frac{C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x]}{3 b d}$$

Result (type 3, 965 leaves):

$$\left((2 a A b^2 - 2 a^2 b B - b^3 B + 2 a^3 C + a b^2 C) \operatorname{Cos}[c + d x] (b + a \operatorname{Cos}[c + d x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right) /$$

$$(b^4 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])) +$$

$$\left((-2 a A b^2 + 2 a^2 b B + b^3 B - 2 a^3 C - a b^2 C) \operatorname{Cos}[c + d x] (b + a \operatorname{Cos}[c + d x]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{d x}{2}\right]\right] \right)$$

$$(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) / (b^4 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])) +$$

$$\left((A b^2 - a b B + a^2 C) \operatorname{Cos}[c + d x] (b + a \operatorname{Cos}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right)$$

$$\left(- \left(4 i a^2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{d x}{2}\right] + i a \operatorname{Sin}\left[c + \frac{d x}{2}\right] \right) \right) \right)$$

$$\operatorname{Cos}[c] / (b^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}) -$$

$$\left(4 a^2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{d x}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2 - b^2} \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{d x}{2}\right] + i a \operatorname{Sin}\left[c + \frac{d x}{2}\right] \right) \right)$$

$$\operatorname{Sin}[c] / (b^4 \sqrt{a^2 - b^2} d \sqrt{\operatorname{Cos}[2 c] - i \operatorname{Sin}[2 c]}) \right) / ((A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])) +$$

$$\left((b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) (12 A b^2 \operatorname{Sin}[d x] - 12 a b B \operatorname{Sin}[d x] + 12 a^2 C \operatorname{Sin}[d x] + \right.$$

$$12 b^2 C \operatorname{Sin}[d x] - 6 A b^2 \operatorname{Sin}[2 c + d x] + 6 a b B \operatorname{Sin}[2 c + d x] - 6 a^2 C \operatorname{Sin}[2 c + d x] + 3 b^2 B \operatorname{Sin}[c + 2 d x] - 3 a b C \operatorname{Sin}[c + 2 d x] + 3 b^2 B$$

$$\operatorname{Sin}[3 c + 2 d x] - 3 a b C \operatorname{Sin}[3 c + 2 d x] + 6 A b^2 \operatorname{Sin}[2 c + 3 d x] - 6 a b B \operatorname{Sin}[2 c + 3 d x] + 6 a^2 C \operatorname{Sin}[2 c + 3 d x] + 4 b^2 C \operatorname{Sin}[2 c + 3 d x]) \right) /$$

$$(12 b^3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x]))$$

■ **Problem 901: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 153 leaves, 7 steps) :

$$\frac{(b^2 (2A + C) - 2a (bB - aC)) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^3 d} -$$

$$\frac{2a (Ab^2 - a (bB - aC)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^3 \sqrt{a+b} d} + \frac{(bB - aC) \operatorname{Tan}[c + dx]}{b^2 d} + \frac{C \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2bd}$$

Result (type 3, 472 leaves) :

$$\left(\operatorname{Cos}[c + dx] (b + a \operatorname{Cos}[c + dx]) (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \right)$$

$$\left(-2 (2Ab^2 - 2abB + 2a^2C + b^2C) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + 2 (2Ab^2 - 2abB + 2a^2C + b^2C) \right.$$

$$\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \frac{8a (Ab^2 + a (-bB + aC)) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (a \operatorname{Sin}[c] + (-b + a \operatorname{Cos}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right]}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}} (i \operatorname{Cos}[c] + \operatorname{Sin}[c]) \right. +$$

$$\left. \frac{b^2 C}{(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])^2} + \frac{4b (bB - aC) \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} - \right.$$

$$\left. \frac{b^2 C}{(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])^2} + \frac{4b (bB - aC) \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} \right) \Bigg/$$

$$(2b^3 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2(c + dx)]) (a + b \operatorname{Sec}[c + dx]))$$

■ **Problem 902: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 106 leaves, 6 steps) :

$$\frac{(bB - aC) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{b^2 d} + \frac{2 (Ab^2 - a(bB - aC)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{\sqrt{a-b} b^2 \sqrt{a+b} d} + \frac{C \operatorname{Tan}[c + dx]}{bd}$$

Result (type 3, 365 leaves):

$$\frac{1}{b^2 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2(c + dx)]) (a + b \operatorname{Sec}[c + dx])} 2 \operatorname{Cos}[c + dx] (b + a \operatorname{Cos}[c + dx]) (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(- (bB - aC) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + (bB - aC) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \right.$$

$$\frac{2i (Ab^2 + a(-bB + aC)) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (a \operatorname{Sin}[c] + (-b + a \operatorname{Cos}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right] (\operatorname{Cos}[c] - i \operatorname{Sin}[c])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}} + \right.$$

$$\left. \frac{bC \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} + \frac{bC \operatorname{Sin}\left[\frac{dx}{2}\right]}{(\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right])} \right)$$

■ **Problem 903: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{a + b \operatorname{Sec}[c + dx]} dx$$

Optimal (type 3, 94 leaves, 6 steps):

$$\frac{Ax}{a} + \frac{C \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{bd} - \frac{2 (Ab^2 - a(bB - aC)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a \sqrt{a-b} b \sqrt{a+b} d}$$

Result (type 3, 261 leaves):

$$\left(2 (C + B \cos[c + dx] + A \cos[c + dx])^2 \right. \\ \left. \left(\sqrt{a^2 - b^2} \left(A b dx - a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] - \sin \left[\frac{1}{2} (c + dx) \right] \right] + a C \log \left[\cos \left[\frac{1}{2} (c + dx) \right] + \sin \left[\frac{1}{2} (c + dx) \right] \right] \right) \sqrt{(\cos[c] - i \sin[c])^2} + \right. \\ \left. 2 (A b^2 + a (-bB + aC)) \operatorname{ArcTan} \left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan \left[\frac{dx}{2} \right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}} \right] (i \cos[c] + \sin[c]) \right) \right) / \\ \left(a b \sqrt{a^2 - b^2} d (A + 2C + 2B \cos[c + dx] + A \cos[2(c + dx)]) \sqrt{(\cos[c] - i \sin[c])^2} \right)$$

■ **Problem 908: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 3, 407 leaves, 9 steps):

$$\frac{(6 a^2 b B + b^3 B - 8 a^3 C - 2 a b^2 (2 A + C)) \operatorname{ArcTanh}[\sin[c + dx]]}{2 b^5 d} + \\ \frac{2 a^2 (2 a^2 A b^2 - 3 A b^4 - 3 a^3 b B + 4 a b^3 B + 4 a^4 C - 5 a^2 b^2 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2} (c + dx) \right]}{\sqrt{a+b}} \right]}{(a-b)^{3/2} b^5 (a+b)^{3/2} d} - \\ \frac{(9 a^3 b B - 6 a b^3 B - a^2 b^2 (6 A - 7 C) - 12 a^4 C + b^4 (3 A + 2 C)) \tan[c + dx]}{3 b^4 (a^2 - b^2) d} + \frac{(3 a^2 b B - b^3 B - 2 a b^2 (A - C) - 4 a^3 C) \sec[c + dx] \tan[c + dx]}{2 b^3 (a^2 - b^2) d} + \\ \frac{(3 A b^2 - 3 a b B + 4 a^2 C - b^2 C) \sec[c + dx]^2 \tan[c + dx]}{3 b^2 (a^2 - b^2) d} - \frac{(A b^2 - a (b B - a C)) \sec[c + dx]^3 \tan[c + dx]}{b (a^2 - b^2) d (a + b \sec[c + dx])}$$

Result (type 3, 973 leaves):

$$\begin{aligned}
& - \left(4 a^2 (-2 a^2 A b^2 + 3 A b^4 + 3 a^3 b B - 4 a b^3 B - 4 a^4 C + 5 a^2 b^2 C) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right] (b+a \operatorname{Cos}[c+dx])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(b^5 \sqrt{a^2-b^2} (-a^2+b^2) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^2 \right) + \\
& \left((4 a A b^2 - 6 a^2 b B - b^3 B + 8 a^3 C + 2 a b^2 C) (b+a \operatorname{Cos}[c+dx])^2 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] - \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (b^5 d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^2) + \\
& \left((-4 a A b^2 + 6 a^2 b B + b^3 B - 8 a^3 C - 2 a b^2 C) (b+a \operatorname{Cos}[c+dx])^2 \operatorname{Log} \left[\operatorname{Cos} \left[\frac{1}{2} (c+dx) \right] + \operatorname{Sin} \left[\frac{1}{2} (c+dx) \right] \right] (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& \quad (b^5 d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^2) + \\
& \quad \left((b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^3 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) (-6 a^2 A b^3 \operatorname{Sin}[c+dx] + 6 A b^5 \operatorname{Sin}[c+dx] + 9 a^3 b^2 B \operatorname{Sin}[c+dx] - \right. \\
& \quad 9 a b^4 B \operatorname{Sin}[c+dx] - 12 a^4 b C \operatorname{Sin}[c+dx] + 12 b^5 C \operatorname{Sin}[c+dx] - 12 a^3 A b^2 \operatorname{Sin}[2(c+dx)] + 6 a A b^4 \operatorname{Sin}[2(c+dx)] + \\
& \quad 18 a^4 b B \operatorname{Sin}[2(c+dx)] - 18 a^2 b^3 B \operatorname{Sin}[2(c+dx)] + 6 b^5 B \operatorname{Sin}[2(c+dx)] - 24 a^5 C \operatorname{Sin}[2(c+dx)] + 22 a^3 b^2 C \operatorname{Sin}[2(c+dx)] - \\
& \quad 4 a b^4 C \operatorname{Sin}[2(c+dx)] - 6 a^2 A b^3 \operatorname{Sin}[3(c+dx)] + 6 A b^5 \operatorname{Sin}[3(c+dx)] + 9 a^3 b^2 B \operatorname{Sin}[3(c+dx)] - 9 a b^4 B \operatorname{Sin}[3(c+dx)] - \\
& \quad 12 a^4 b C \operatorname{Sin}[3(c+dx)] + 8 a^2 b^3 C \operatorname{Sin}[3(c+dx)] + 4 b^5 C \operatorname{Sin}[3(c+dx)] - 6 a^3 A b^2 \operatorname{Sin}[4(c+dx)] + 3 a A b^4 \operatorname{Sin}[4(c+dx)] + \\
& \quad \left. 9 a^4 b B \operatorname{Sin}[4(c+dx)] - 6 a^2 b^3 B \operatorname{Sin}[4(c+dx)] - 12 a^5 C \operatorname{Sin}[4(c+dx)] + 7 a^3 b^2 C \operatorname{Sin}[4(c+dx)] + 2 a b^4 C \operatorname{Sin}[4(c+dx)]) \right) / \\
& \quad (12 b^4 (-a^2+b^2) d (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) (a+b \operatorname{Sec}[c+dx])^2)
\end{aligned}$$

■ **Problem 910: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{(a+b \operatorname{Sec}[c+dx])^2} dx$$

Optimal (type 3, 177 leaves, 7 steps):

$$\frac{(bB - 2aC) \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{b^3 d} -$$

$$\frac{2 (A b^4 + a^3 b B - 2 a b^3 B - 2 a^4 C + 3 a^2 b^2 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{(a-b)^{3/2} b^3 (a+b)^{3/2} d} + \frac{C \operatorname{Tan}[c+dx]}{b^2 d} + \frac{a (A b^2 - a (bB - aC)) \operatorname{Tan}[c+dx]}{b^2 (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])}$$

Result (type 3, 382 leaves):

$$\frac{1}{b^3 d (A + 2C + 2B \cos[c + dx] + A \cos[2(c + dx)]) (a + b \sec[c + dx])^2} 2 (b + a \cos[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx])^2$$

$$\left(\frac{2 (A b^4 + a (a^2 b B - 2 b^3 B - 2 a^3 C + 3 a b^2 C)) \operatorname{ArcTanh}\left[\frac{(-a+b) \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^2-b^2}}\right]}{(a^2 - b^2)^{3/2}} - (bB - 2aC) (b + a \cos[c + dx]) \right.$$

$$\left. \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + (bB - 2aC) (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right.$$

$$\left. \frac{bC (b + a \cos[c + dx]) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]} + \frac{bC (b + a \cos[c + dx]) \sin\left[\frac{1}{2}(c + dx)\right]}{\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]} + \frac{ab (Ab^2 + a(-bB + aC)) \sin[c + dx]}{(a - b)(a + b)} \right)$$

■ **Problem 911: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$\frac{C \operatorname{ArcTanh}[\sin[c + dx]]}{b^2 d} + \frac{2 (a A b^2 - b^3 B - a^3 C + 2 a b^2 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{3/2} b^2 (a+b)^{3/2} d} - \frac{(A b^2 - a (b B - a C)) \tan[c + dx]}{b (a^2 - b^2) d (a + b \sec[c + dx])}$$

Result (type 3, 356 leaves):

$$\frac{1}{b^2 d (A + 2C + 2B \cos[c + dx] + A \cos[2(c + dx)]) (a + b \sec[c + dx])^2} 2 (b + a \cos[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx])^2$$

$$\left(-C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] - \sin\left[\frac{1}{2}(c + dx)\right]\right] + C (b + a \cos[c + dx]) \operatorname{Log}\left[\cos\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{1}{2}(c + dx)\right]\right] + \right.$$

$$\left(2 (b^3 B + a^3 C - a b^2 (A + 2C)) \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (a \sin[c] + (-b + a \cos[c]) \tan\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\cos[c] - i \sin[c])^2}}\right] (b + a \cos[c + dx]) (i \cos[c] + \sin[c]) \right) /$$

$$\left((a^2 - b^2)^{3/2} \sqrt{(\cos[c] - i \sin[c])^2} + \frac{b (A b^2 + a (-b B + a C)) (b \sin[c] - a \sin[dx])}{a (a - b) (a + b) (\cos\left[\frac{c}{2}\right] - \sin\left[\frac{c}{2}\right]) (\cos\left[\frac{c}{2}\right] + \sin\left[\frac{c}{2}\right])} \right)$$

- **Problem 912: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{A x}{a^2} - \frac{2 (2 a^2 A b - A b^3 - a^3 B + a^2 b C) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^2 (a-b)^{3/2} (a+b)^{3/2} d} + \frac{(A b^2 - a (b B - a C)) \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 299 leaves):

$$\left(2 (b + a \operatorname{Cos}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(A x (b + a \operatorname{Cos}[c + d x]) - \left(2 i (A b^3 + a^3 B - a^2 b (2 A + C)) \operatorname{ArcTan}\left[\frac{(i \operatorname{Cos}[c] + \operatorname{Sin}[c]) (a \operatorname{Sin}[c] + (-b + a \operatorname{Cos}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right])}{\sqrt{a^2 - b^2} \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2}}\right]} \right) \right. \right. \\ \left. \left. (b + a \operatorname{Cos}[c + d x]) (\operatorname{Cos}[c] - i \operatorname{Sin}[c]) \right) \right) / \left((a^2 - b^2)^{3/2} d \sqrt{(\operatorname{Cos}[c] - i \operatorname{Sin}[c])^2} \right) + \\ \left. \frac{(A b^2 + a (-b B + a C)) (-b \operatorname{Sin}[c] + a \operatorname{Sin}[d x])}{(a - b) (a + b) d (\operatorname{Cos}\left[\frac{c}{2}\right] - \operatorname{Sin}\left[\frac{c}{2}\right]) (\operatorname{Cos}\left[\frac{c}{2}\right] + \operatorname{Sin}\left[\frac{c}{2}\right])} \right) \right) / (a^2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 (c + d x)]) (a + b \operatorname{Sec}[c + d x])^2)$$

- **Problem 916: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 3, 465 leaves, 9 steps):

$$\frac{(2Ab^2 - 6abB + 12a^2C + b^2C) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2b^5d} - \frac{1}{(a-b)^{5/2}b^5(a+b)^{5/2}d}$$

$$a \left(6Ab^6 - 6a^5bB + 15a^3b^3B - 12ab^5B + a^4b^2(2A - 29C) - 5a^2b^4(A - 4C) + 12a^6C \right) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a+b}}\right] +$$

$$\frac{(6a^4bB - 11a^2b^3B + 2b^5B - a^3b^2(2A - 21C) + ab^4(5A - 6C) - 12a^5C) \operatorname{Tan}[c + dx]}{2b^4(a^2 - b^2)^2d} -$$

$$\frac{(3a^3bB - 6ab^3B - a^2b^2(A - 10C) + b^4(4A - C) - 6a^4C) \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2b^3(a^2 - b^2)^2d} -$$

$$\frac{(Ab^2 - a(bB - aC)) \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{2b(a^2 - b^2)d(a + b \operatorname{Sec}[c + dx])^2} + \frac{(3Ab^4 + a(2a^2bB - 5b^3B - 4a^3C + 7ab^2C)) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{2b^2(a^2 - b^2)^2d(a + b \operatorname{Sec}[c + dx])}$$

Result (type 3, 1124 leaves):

$$\left(2a(2a^4Ab^2 - 5a^2Ab^4 + 6Ab^6 - 6a^5bB + 15a^3b^3B - 12ab^5B + 12a^6C - 29a^4b^2C + 20a^2b^4C) \right.$$

$$\left. \operatorname{ArcTanh}\left[\frac{(-a+b) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 - b^2}}\right] (b + a \operatorname{Cos}[c + dx])^3 \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$\left(b^5 \sqrt{a^2 - b^2} (-a^2 + b^2)^2 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + b \operatorname{Sec}[c + dx])^3 \right) +$$

$$\left((-2Ab^2 + 6abB - 12a^2C - b^2C) (b + a \operatorname{Cos}[c + dx])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$\left(b^5 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + b \operatorname{Sec}[c + dx])^3 \right) +$$

$$\left((2Ab^2 - 6abB + 12a^2C + b^2C) (b + a \operatorname{Cos}[c + dx])^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) /$$

$$\left(b^5 d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + b \operatorname{Sec}[c + dx])^3 + (b + a \operatorname{Cos}[c + dx]) \operatorname{Sec}[c + dx]^3 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right.$$

$$\left. (-6a^4Ab^3 \operatorname{Sin}[c + dx] + 12a^2Ab^5 \operatorname{Sin}[c + dx] + 18a^5b^2B \operatorname{Sin}[c + dx] - 32a^3b^4B \operatorname{Sin}[c + dx] + 8ab^6B \operatorname{Sin}[c + dx] - \right.$$

$$\left. 36a^6bC \operatorname{Sin}[c + dx] + 72a^4b^3C \operatorname{Sin}[c + dx] - 38a^2b^5C \operatorname{Sin}[c + dx] + 8b^7C \operatorname{Sin}[c + dx] - 4a^5Ab^2 \operatorname{Sin}[2(c + dx)] + \right.$$

$$\left. 10a^3Ab^4 \operatorname{Sin}[2(c + dx)] + 12a^6bB \operatorname{Sin}[2(c + dx)] - 14a^4b^3B \operatorname{Sin}[2(c + dx)] - 12a^2b^5B \operatorname{Sin}[2(c + dx)] + 8b^7B \operatorname{Sin}[2(c + dx)] - \right.$$

$$\left. 24a^7C \operatorname{Sin}[2(c + dx)] + 26a^5b^2C \operatorname{Sin}[2(c + dx)] + 20a^3b^4C \operatorname{Sin}[2(c + dx)] - 16ab^6C \operatorname{Sin}[2(c + dx)] - 6a^4Ab^3 \operatorname{Sin}[3(c + dx)] + \right.$$

$$\left. 12a^2Ab^5 \operatorname{Sin}[3(c + dx)] + 18a^5b^2B \operatorname{Sin}[3(c + dx)] - 32a^3b^4B \operatorname{Sin}[3(c + dx)] + 8ab^6B \operatorname{Sin}[3(c + dx)] - 36a^6bC \operatorname{Sin}[3(c + dx)] + \right.$$

$$\left. 64a^4b^3C \operatorname{Sin}[3(c + dx)] - 22a^2b^5C \operatorname{Sin}[3(c + dx)] - 2a^5Ab^2 \operatorname{Sin}[4(c + dx)] + 5a^3Ab^4 \operatorname{Sin}[4(c + dx)] + 6a^6bB \operatorname{Sin}[4(c + dx)] - \right.$$

$$\left. 11a^4b^3B \operatorname{Sin}[4(c + dx)] + 2a^2b^5B \operatorname{Sin}[4(c + dx)] - 12a^7C \operatorname{Sin}[4(c + dx)] + 21a^5b^2C \operatorname{Sin}[4(c + dx)] - 6a^3b^4C \operatorname{Sin}[4(c + dx)] \right) /$$

$$(8b^4(-a^2 + b^2)^2d(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx])(a + b \operatorname{Sec}[c + dx])^3)$$

- **Problem 918: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^3} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{C \text{ArcTanh}[\text{Sin}[c + d x]]}{b^3 d} + \frac{(a^2 b^3 B + 2 b^5 B - 2 a^5 C + 5 a^3 b^2 C - 3 a b^4 (A + 2 C)) \text{ArcTanh}\left[\frac{\sqrt{a-b} \text{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{5/2} b^3 (a+b)^{5/2} d} +$$

$$\frac{a (A b^2 - a (b B - a C)) \text{Tan}[c + d x]}{2 b^2 (a^2 - b^2) d (a + b \text{Sec}[c + d x])^2} + \frac{(2 A b^4 + a^3 b B - 4 a b^3 B - 3 a^4 C + a^2 b^2 (A + 6 C)) \text{Tan}[c + d x]}{2 b^2 (a^2 - b^2)^2 d (a + b \text{Sec}[c + d x])}$$

Result (type 3, 1071 leaves):

$$\begin{aligned}
& - \left(2 C (b + a \cos[c + dx])^3 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] - \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right) / \\
& \quad (b^3 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3) + \\
& \quad \frac{2 C (b + a \cos[c + dx])^3 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{dx}{2} \right] + \sin \left[\frac{c}{2} + \frac{dx}{2} \right] \right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{b^3 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3} + \\
& \quad \left((-3 a A b^4 + a^2 b^3 B + 2 b^5 B - 2 a^5 C + 5 a^3 b^2 C - 6 a b^4 C) (b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \quad \left. - \left(2 i \operatorname{ArcTan} \left[\sec \left[\frac{dx}{2} \right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin \left[\frac{dx}{2} \right] + i a \sin \left[c + \frac{dx}{2} \right] \right) \right] \right. \right. \\
& \quad \left. \left. \cos[c] \right) / (b^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]}) - \right. \\
& \quad \left. \left(2 \operatorname{ArcTan} \left[\sec \left[\frac{dx}{2} \right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin \left[\frac{dx}{2} \right] + i a \sin \left[c + \frac{dx}{2} \right] \right) \right] \right) \sin[c] \right) / \\
& \quad \left(b^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) \left. \right) / ((-a^2 + b^2)^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3) + \\
& \quad ((b + a \cos[c + dx]) \sec[c] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \quad (-2 a^4 A b^2 \sin[c] - 5 a^2 A b^4 \sin[c] - 2 A b^6 \sin[c] + 3 a^3 b^3 B \sin[c] + 6 a b^5 B \sin[c] + 2 a^6 C \sin[c] - a^4 b^2 C \sin[c] - 10 a^2 b^4 C \sin[c] + \\
& \quad 5 a^3 A b^3 \sin[dx] + 4 a A b^5 \sin[dx] + a^4 b^2 B \sin[dx] - 10 a^2 b^4 B \sin[dx] - 7 a^5 b C \sin[dx] + 16 a^3 b^3 C \sin[dx] - \\
& \quad 3 a^3 A b^3 \sin[2c + dx] + a^4 b^2 B \sin[2c + dx] + 2 a^2 b^4 B \sin[2c + dx] + a^5 b C \sin[2c + dx] - 4 a^3 b^3 C \sin[2c + dx] + \\
& \quad 2 a^4 A b^2 \sin[c + 2dx] + a^2 A b^4 \sin[c + 2dx] - 3 a^3 b^3 B \sin[c + 2dx] - 2 a^6 C \sin[c + 2dx] + 5 a^4 b^2 C \sin[c + 2dx]) / \\
& \quad (2 a b^2 (-a^2 + b^2)^2 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3)
\end{aligned}$$

■ **Problem 919: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 3, 202 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(3 a b B - a^2 (2 A + C) - b^2 (A + 2 C)) \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]}{\sqrt{a+b}} \right]}{(a - b)^{5/2} (a + b)^{5/2} d} - \\
& \frac{(A b^2 - a (b B - a C)) \operatorname{Tan}[c + dx]}{2 b (a^2 - b^2) d (a + b \sec[c + dx])^2} + \frac{(a^2 b B + 2 b^3 B + a^3 C - a b^2 (3 A + 4 C)) \operatorname{Tan}[c + dx]}{2 b (a^2 - b^2)^2 d (a + b \sec[c + dx])}
\end{aligned}$$

Result (type 3, 800 leaves) :

$$\left((2 a^2 A + A b^2 - 3 a b B + a^2 C + 2 b^2 C) (b + a \cos[c + d x])^3 \sec[c + d x] (A + B \sec[c + d x] + C \sec[c + d x])^2 \right. \\ \left. - \left(2 i \operatorname{ArcTan}\left[\sec\left[\frac{d x}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2 c] - i \sin[2 c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2 c] - i \sin[2 c]}} \right) \left(-i b \sin\left[\frac{d x}{2}\right] + i a \sin\left[c + \frac{d x}{2}\right] \right) \right) \right. \\ \left. \cos[c] \right) / \left(\sqrt{a^2 - b^2} d \sqrt{\cos[2 c] - i \sin[2 c]} \right) - \\ \left(2 \operatorname{ArcTan}\left[\sec\left[\frac{d x}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2 c] - i \sin[2 c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2 c] - i \sin[2 c]}} \right) \left(-i b \sin\left[\frac{d x}{2}\right] + i a \sin\left[c + \frac{d x}{2}\right] \right) \right) \sin[c] \right) / \\ \left(\sqrt{a^2 - b^2} d \sqrt{\cos[2 c] - i \sin[2 c]} \right) \Bigg) / \\ \left((-a^2 + b^2)^2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) (a + b \sec[c + d x])^3 + \right. \\ \left((b + a \cos[c + d x]) \sec[c] \sec[c + d x] (A + B \sec[c + d x] + C \sec[c + d x])^2 + \right. \\ \left(4 a^4 A b \sin[c] + 7 a^2 A b^3 \sin[c] - 2 A b^5 \sin[c] - 2 a^5 B \sin[c] - 5 a^3 b^2 B \sin[c] - 2 a b^4 B \sin[c] + 3 a^4 b C \sin[c] + \right. \\ \left. 6 a^2 b^3 C \sin[c] - 11 a^3 A b^2 \sin[d x] + 2 a A b^4 \sin[d x] + 5 a^4 b B \sin[d x] + 4 a^2 b^3 B \sin[d x] + a^5 C \sin[d x] - 10 a^3 b^2 C \sin[d x] + \right. \\ \left. 5 a^3 A b^2 \sin[2 c + d x] - 2 a A b^4 \sin[2 c + d x] - 3 a^4 b B \sin[2 c + d x] + a^5 C \sin[2 c + d x] + 2 a^3 b^2 C \sin[2 c + d x] - \right. \\ \left. 4 a^4 A b \sin[c + 2 d x] + a^2 A b^3 \sin[c + 2 d x] + 2 a^5 B \sin[c + 2 d x] + a^3 b^2 B \sin[c + 2 d x] - 3 a^4 b C \sin[c + 2 d x] \right) \Bigg) / \\ \left(2 a^2 (a^2 - b^2)^2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) (a + b \sec[c + d x])^3 \right)$$

■ **Problem 920: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + d x] + C \sec[c + d x]^2}{(a + b \sec[c + d x])^3} dx$$

Optimal (type 3, 229 leaves, 6 steps) :

$$\frac{A x}{a^3} + \frac{(5 a^2 A b^3 - 2 A b^5 + 2 a^5 B + a^3 b^2 B - 3 a^4 b (2 A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^3 (a-b)^{5/2} (a+b)^{5/2} d} + \\ \frac{(A b^2 - a (b B - a C)) \tan[c + d x]}{2 a (a^2 - b^2) d (a + b \sec[c + d x])^2} - \frac{(2 A b^4 + 3 a^3 b B - a^4 C - a^2 b^2 (5 A + 2 C)) \tan[c + d x]}{2 a^2 (a^2 - b^2)^2 d (a + b \sec[c + d x])}$$

Result (type 3, 1093 leaves) :

$$\left((6 a^4 A b - 5 a^2 A b^3 + 2 A b^5 - 2 a^5 B - a^3 b^2 B + 3 a^4 b C) (b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\ \left. \left(\left(2 i \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right) \cos[c] \right) / \right. \\ \left. \left(a^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) + \right. \\ \left. \left(2 \operatorname{ArcTan}\left[\sec\left[\frac{dx}{2}\right]\right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} - \frac{i \sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i \sin[2c]}} \right) \left(-i b \sin\left[\frac{dx}{2}\right] + i a \sin\left[c + \frac{dx}{2}\right] \right) \right) \sin[c] \right) / \right. \\ \left. \left(a^3 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i \sin[2c]} \right) \right) \Bigg) / \\ \left((-a^2 + b^2)^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3 \right) + \\ \left((b + a \cos[c + dx]) \sec[c] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\ \left(2 a^6 A dx \cos[c] - 6 a^2 A b^4 dx \cos[c] + 4 A b^6 dx \cos[c] + 4 a^5 A b dx \cos[dx] - 8 a^3 A b^3 dx \cos[dx] + 4 a A b^5 dx \cos[dx] + \right. \\ \left. 4 a^5 A b dx \cos[2c + dx] - 8 a^3 A b^3 dx \cos[2c + dx] + 4 a A b^5 dx \cos[2c + dx] + a^6 A dx \cos[c + 2dx] - 2 a^4 A b^2 dx \cos[c + 2dx] + \right. \\ \left. a^2 A b^4 dx \cos[c + 2dx] + a^6 A dx \cos[3c + 2dx] - 2 a^4 A b^2 dx \cos[3c + 2dx] + a^2 A b^4 dx \cos[3c + 2dx] - 6 a^4 A b^2 \sin[c] - \right. \\ \left. 9 a^2 A b^4 \sin[c] + 6 A b^6 \sin[c] + 4 a^5 b B \sin[c] + 7 a^3 b^3 B \sin[c] - 2 a b^5 B \sin[c] - 2 a^6 C \sin[c] - 5 a^4 b^2 C \sin[c] - 2 a^2 b^4 C \sin[c] + \right. \\ \left. 17 a^3 A b^3 \sin[dx] - 8 a A b^5 \sin[dx] - 11 a^4 b^2 B \sin[dx] + 2 a^2 b^4 B \sin[dx] + 5 a^5 b C \sin[dx] + 4 a^3 b^3 C \sin[dx] - 7 a^3 A b^3 \sin[2c + dx] + \right. \\ \left. 4 a A b^5 \sin[2c + dx] + 5 a^4 b^2 B \sin[2c + dx] - 2 a^2 b^4 B \sin[2c + dx] - 3 a^5 b C \sin[2c + dx] + 6 a^4 A b^2 \sin[c + 2dx] - \right. \\ \left. 3 a^2 A b^4 \sin[c + 2dx] - 4 a^5 b B \sin[c + 2dx] + a^3 b^3 B \sin[c + 2dx] + 2 a^6 C \sin[c + 2dx] + a^4 b^2 C \sin[c + 2dx] \right) \Bigg) / \\ \left(2 a^3 (a^2 - b^2)^2 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3 \right)$$

■ **Problem 921: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 3, 330 leaves, 7 steps):

$$\frac{(3 A b - a B) x}{a^4} - \frac{(15 a^2 A b^4 - 6 A b^6 + 6 a^5 b B - 5 a^3 b^3 B + 2 a b^5 B - 2 a^6 C - a^4 b^2 (12 A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \tan\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4 (a-b)^{5/2} (a+b)^{5/2} d} - \\ \frac{(11 a^2 A b^2 - 6 A b^4 - 5 a^3 b B + 2 a b^3 B - a^4 (2 A - 3 C)) \sin[c + dx]}{2 a^3 (a^2 - b^2)^2 d} + \\ \frac{(A b^2 - a (b B - a C)) \sin[c + dx]}{2 a (a^2 - b^2) d (a + b \sec[c + dx])^2} - \frac{(3 A b^4 + 4 a^3 b B - a b^3 B - 2 a^4 C - a^2 b^2 (6 A + C)) \sin[c + dx]}{2 a^2 (a^2 - b^2)^2 d (a + b \sec[c + dx])}$$

Result (type 3, 1015 leaves) :

$$\begin{aligned}
 & - \frac{2 (3 A b - a B) x (b + a \cos [c + d x])^3 \sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2)}{a^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^3} + \\
 & \left((12 a^4 A b^2 - 15 a^2 A b^4 + 6 A b^6 - 6 a^5 b B + 5 a^3 b^3 B - 2 a b^5 B + 2 a^6 C + a^4 b^2 C) (b + a \cos [c + d x])^3 \sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
 & \left. - \left(2 i \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \right] \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right) \right. \\
 & \left. \cos [c] \right) / \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) - \\
 & \left(2 \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \right] \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right) \sin [c] \right) / \\
 & \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) \Big) / \\
 & \left((-a^2 + b^2)^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^3 \right) + \\
 & \left((b + a \cos [c + d x]) \sec [c] \sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
 & \left. (-A b^5 \sin [c] + a b^4 B \sin [c] - a^2 b^3 C \sin [c] + a A b^4 \sin [d x] - a^2 b^3 B \sin [d x] + a^3 b^2 C \sin [d x]) \right) / \\
 & \left(a^4 (a^2 - b^2) d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^3 \right) + \\
 & \left((b + a \cos [c + d x])^2 \sec [c] \sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
 & \left. (9 a^2 A b^4 \sin [c] - 6 A b^6 \sin [c] - 7 a^3 b^3 B \sin [c] + 4 a b^5 B \sin [c] + 5 a^4 b^2 C \sin [c] - 2 a^2 b^4 C \sin [c] - \right. \\
 & \left. 8 a^3 A b^3 \sin [d x] + 5 a A b^5 \sin [d x] + 6 a^4 b^2 B \sin [d x] - 3 a^2 b^4 B \sin [d x] - 4 a^5 b C \sin [d x] + a^3 b^3 C \sin [d x]) \right) / \\
 & \left(a^4 (a^2 - b^2)^2 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^3 \right) + \\
 & \frac{2 A (b + a \cos [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \tan [c + d x]}{a^3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^3}
 \end{aligned}$$

■ **Problem 922: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + b \sec [c + d x])^3} dx$$

Optimal (type 3, 453 leaves, 8 steps) :

$$\frac{(12 A b^2 - 6 a b B + a^2 (A + 2 C)) x}{2 a^5} - \frac{1}{a^5 (a - b)^{5/2} (a + b)^{5/2} d}$$

$$b (12 A b^6 - 12 a^5 b B + 15 a^3 b^3 B - 6 a b^5 B - a^2 b^4 (29 A - 2 C) + 5 a^4 b^2 (4 A - C) + 6 a^6 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a - b} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a + b}}\right] -$$

$$\frac{(12 A b^5 - 2 a^5 B + 11 a^3 b^2 B - 6 a b^4 B + a^4 b (6 A - 5 C) - a^2 b^3 (21 A - 2 C)) \operatorname{Sin}[c + d x]}{2 a^4 (a^2 - b^2)^2 d} +$$

$$\frac{(6 A b^4 + 6 a^3 b B - 3 a b^3 B + a^4 (A - 4 C) - a^2 b^2 (10 A - C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 a^3 (a^2 - b^2)^2 d} +$$

$$\frac{(A b^2 - a (b B - a C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^2} + \frac{(7 a^2 A b^2 - 4 A b^4 - 5 a^3 b B + 2 a b^3 B + 3 a^4 C) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{2 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])}$$

Result (type 3, 1150 leaves):

$$\frac{1}{a^5 \sqrt{a^2 - b^2} (-a^2 + b^2)^2 d}$$

$$b (20 a^4 A b^2 - 29 a^2 A b^4 + 12 A b^6 - 12 a^5 b B + 15 a^3 b^3 B - 6 a b^5 B + 6 a^6 C - 5 a^4 b^2 C + 2 a^2 b^4 C) \operatorname{ArcTanh}\left[\frac{(-a + b) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 - b^2}}\right] +$$

$$\frac{1}{16 a^5 (a^2 - b^2)^2 d (b + a \operatorname{Cos}[c + d x])^2}$$

$$(4 a^8 A (c + d x) + 48 a^6 A b^2 (c + d x) - 12 a^4 A b^4 (c + d x) - 136 a^2 A b^6 (c + d x) + 96 A b^8 (c + d x) - 24 a^7 b B (c + d x) + 72 a^3 b^5 B (c + d x) -$$

$$48 a b^7 B (c + d x) + 8 a^8 C (c + d x) - 24 a^4 b^4 C (c + d x) + 16 a^2 b^6 C (c + d x) + 16 a^7 A b (c + d x) \operatorname{Cos}[c + d x] + 160 a^5 A b^3 (c + d x) \operatorname{Cos}[c + d x] -$$

$$368 a^3 A b^5 (c + d x) \operatorname{Cos}[c + d x] + 192 a A b^7 (c + d x) \operatorname{Cos}[c + d x] - 96 a^6 b^2 B (c + d x) \operatorname{Cos}[c + d x] + 192 a^4 b^4 B (c + d x) \operatorname{Cos}[c + d x] -$$

$$96 a^2 b^6 B (c + d x) \operatorname{Cos}[c + d x] + 32 a^7 b C (c + d x) \operatorname{Cos}[c + d x] - 64 a^5 b^3 C (c + d x) \operatorname{Cos}[c + d x] + 32 a^3 b^5 C (c + d x) \operatorname{Cos}[c + d x] +$$

$$4 a^8 A (c + d x) \operatorname{Cos}[2 (c + d x)] + 40 a^6 A b^2 (c + d x) \operatorname{Cos}[2 (c + d x)] - 92 a^4 A b^4 (c + d x) \operatorname{Cos}[2 (c + d x)] + 48 a^2 A b^6 (c + d x) \operatorname{Cos}[2 (c + d x)] -$$

$$24 a^7 b B (c + d x) \operatorname{Cos}[2 (c + d x)] + 48 a^5 b^3 B (c + d x) \operatorname{Cos}[2 (c + d x)] - 24 a^3 b^5 B (c + d x) \operatorname{Cos}[2 (c + d x)] + 8 a^8 C (c + d x) \operatorname{Cos}[2 (c + d x)] -$$

$$16 a^6 b^2 C (c + d x) \operatorname{Cos}[2 (c + d x)] + 8 a^4 b^4 C (c + d x) \operatorname{Cos}[2 (c + d x)] - 8 a^7 A b \operatorname{Sin}[c + d x] - 32 a^5 A b^3 \operatorname{Sin}[c + d x] +$$

$$160 a^3 A b^5 \operatorname{Sin}[c + d x] - 96 a A b^7 \operatorname{Sin}[c + d x] + 4 a^8 B \operatorname{Sin}[c + d x] + 8 a^6 b^2 B \operatorname{Sin}[c + d x] - 84 a^4 b^4 B \operatorname{Sin}[c + d x] + 48 a^2 b^6 B \operatorname{Sin}[c + d x] +$$

$$40 a^5 b^3 C \operatorname{Sin}[c + d x] - 16 a^3 b^5 C \operatorname{Sin}[c + d x] + 2 a^8 A \operatorname{Sin}[2 (c + d x)] - 48 a^6 A b^2 \operatorname{Sin}[2 (c + d x)] + 130 a^4 A b^4 \operatorname{Sin}[2 (c + d x)] -$$

$$72 a^2 A b^6 \operatorname{Sin}[2 (c + d x)] + 16 a^7 b B \operatorname{Sin}[2 (c + d x)] - 64 a^5 b^3 B \operatorname{Sin}[2 (c + d x)] + 36 a^3 b^5 B \operatorname{Sin}[2 (c + d x)] + 24 a^6 b^2 C \operatorname{Sin}[2 (c + d x)] -$$

$$12 a^4 b^4 C \operatorname{Sin}[2 (c + d x)] - 8 a^7 A b \operatorname{Sin}[3 (c + d x)] + 16 a^5 A b^3 \operatorname{Sin}[3 (c + d x)] - 8 a^3 A b^5 \operatorname{Sin}[3 (c + d x)] + 4 a^8 B \operatorname{Sin}[3 (c + d x)] -$$

$$8 a^6 b^2 B \operatorname{Sin}[3 (c + d x)] + 4 a^4 b^4 B \operatorname{Sin}[3 (c + d x)] + a^8 A \operatorname{Sin}[4 (c + d x)] - 2 a^6 A b^2 \operatorname{Sin}[4 (c + d x)] + a^4 A b^4 \operatorname{Sin}[4 (c + d x)])$$

■ **Problem 923: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^4 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^4} dx$$

Optimal (type 3, 470 leaves, 9 steps) :

$$\frac{(bB - 4aC) \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{b^5 d} - \frac{1}{(a-b)^{7/2} b^5 (a+b)^{7/2} d}$$

$$\left(2Ab^8 + 2a^7 bB - 7a^5 b^3 B + 8a^3 b^5 B - 8ab^7 B - 8a^8 C + 28a^6 b^2 C - 35a^4 b^4 C + a^2 b^6 (3A + 20C) \right) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a+b}} \right] -$$

$$\frac{(5Ab^4 + 3a^3 bB - 8ab^3 B - 12a^4 C + 23a^2 b^2 C - 6b^4 C) \operatorname{Tan}[c + dx]}{6b^4 (a^2 - b^2)^2 d} - \frac{(Ab^2 - a(bB - aC)) \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{3b(a^2 - b^2) d (a + b \operatorname{Sec}[c + dx])^3} +$$

$$\frac{(3Ab^4 + a^3 bB - 6ab^3 B - 4a^4 C + a^2 b^2 (2A + 9C)) \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]}{6b^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + dx])^2} +$$

$$\frac{a(2Ab^6 - a^5 bB + 2a^3 b^3 B - 6ab^5 B + 4a^6 C - 11a^4 b^2 C + 3a^2 b^4 (A + 4C)) \operatorname{Tan}[c + dx]}{2b^4 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + dx])}$$

Result (type 3, 1197 leaves) :

$$\begin{aligned}
& - \left(2 \left(3 a^2 A b^6 + 2 A b^8 + 2 a^7 b B - 7 a^5 b^3 B + 8 a^3 b^5 B - 8 a b^7 B - 8 a^8 C + 28 a^6 b^2 C - 35 a^4 b^4 C + 20 a^2 b^6 C \right) \right. \\
& \quad \left. \text{ArcTanh} \left[\frac{(-a+b) \tan \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right] (b+a \cos [c+dx])^4 \sec [c+dx]^2 (A+B \sec [c+dx] + C \sec [c+dx]^2) \right) / \\
& \quad \left(b^5 \sqrt{a^2-b^2} (-a^2+b^2)^3 d (A+2C+2B \cos [c+dx] + A \cos [2c+2dx]) (a+b \sec [c+dx])^4 \right) - \\
& \quad \left(2 (bB-4aC) (b+a \cos [c+dx])^4 \log \left[\cos \left[\frac{1}{2} (c+dx) \right] - \sin \left[\frac{1}{2} (c+dx) \right] \right] \sec [c+dx]^2 (A+B \sec [c+dx] + C \sec [c+dx]^2) \right) / \\
& \quad \left(b^5 d (A+2C+2B \cos [c+dx] + A \cos [2c+2dx]) (a+b \sec [c+dx])^4 \right) + \\
& \quad \left(2 (bB-4aC) (b+a \cos [c+dx])^4 \log \left[\cos \left[\frac{1}{2} (c+dx) \right] + \sin \left[\frac{1}{2} (c+dx) \right] \right] \sec [c+dx]^2 (A+B \sec [c+dx] + C \sec [c+dx]^2) \right) / \\
& \quad \left(b^5 d (A+2C+2B \cos [c+dx] + A \cos [2c+2dx]) (a+b \sec [c+dx])^4 \right) + \\
& \quad \left((b+a \cos [c+dx]) \sec [c+dx]^3 (A+B \sec [c+dx] + C \sec [c+dx]^2) \right. \\
& \quad \left(-6 a^4 A b^5 \sin [c+dx] - 54 a^2 A b^7 \sin [c+dx] + 30 a^7 b^2 B \sin [c+dx] - 90 a^5 b^4 B \sin [c+dx] + 120 a^3 b^6 B \sin [c+dx] - \right. \\
& \quad \left. 120 a^8 b C \sin [c+dx] + 294 a^6 b^3 C \sin [c+dx] - 174 a^4 b^5 C \sin [c+dx] - 108 a^2 b^7 C \sin [c+dx] + 48 b^9 C \sin [c+dx] - \right. \\
& \quad \left. 16 a^5 A b^4 \sin [2(c+dx)] - 2 a^3 A b^6 \sin [2(c+dx)] - 72 a A b^8 \sin [2(c+dx)] + 12 a^8 b B \sin [2(c+dx)] + 10 a^6 b^3 B \sin [2(c+dx)] - \right. \\
& \quad \left. 76 a^4 b^5 B \sin [2(c+dx)] + 144 a^2 b^7 B \sin [2(c+dx)] - 48 a^9 C \sin [2(c+dx)] - 40 a^7 b^2 C \sin [2(c+dx)] + 370 a^5 b^4 C \sin [2(c+dx)] - \right. \\
& \quad \left. 444 a^3 b^6 C \sin [2(c+dx)] + 72 a b^8 C \sin [2(c+dx)] - 6 a^4 A b^5 \sin [3(c+dx)] - 54 a^2 A b^7 \sin [3(c+dx)] + 30 a^7 b^2 B \sin [3(c+dx)] - 90 \right. \\
& \quad \left. a^5 b^4 B \sin [3(c+dx)] + 120 a^3 b^6 B \sin [3(c+dx)] - 120 a^8 b C \sin [3(c+dx)] + 342 a^6 b^3 C \sin [3(c+dx)] - 318 a^4 b^5 C \sin [3(c+dx)] + \right. \\
& \quad \left. 36 a^2 b^7 C \sin [3(c+dx)] - 4 a^5 A b^4 \sin [4(c+dx)] - 11 a^3 A b^6 \sin [4(c+dx)] + 6 a^8 b B \sin [4(c+dx)] - 17 a^6 b^3 B \sin [4(c+dx)] + \right. \\
& \quad \left. 26 a^4 b^5 B \sin [4(c+dx)] - 24 a^9 C \sin [4(c+dx)] + 68 a^7 b^2 C \sin [4(c+dx)] - 65 a^5 b^4 C \sin [4(c+dx)] + 6 a^3 b^6 C \sin [4(c+dx)] \right) \left. \right) / \\
& \quad \left(24 b^4 (-a^2+b^2)^3 d (A+2C+2B \cos [c+dx] + A \cos [2c+2dx]) (a+b \sec [c+dx])^4 \right)
\end{aligned}$$

■ **Problem 924: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c+dx]^3 (A+B \sec [c+dx] + C \sec [c+dx]^2)}{(a+b \sec [c+dx])^4} dx$$

Optimal (type 3, 358 leaves, 8 steps):

$$\begin{aligned}
& \frac{C \text{ArcTanh}[\sin [c+dx]]}{b^4 d} - \frac{\left(3 a^2 b^5 B + 2 b^7 B - a^3 b^4 (A-8C) + 2 a^7 C - 7 a^5 b^2 C - 4 a b^6 (A+2C) \right) \text{ArcTanh} \left[\frac{\sqrt{a-b} \tan \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a+b}} \right]}{(a-b)^{7/2} b^4 (a+b)^{7/2} d} \\
& \frac{(A b^2 - a (b B - a C)) \sec [c+dx]^2 \tan [c+dx]}{3 b (a^2 - b^2) d (a+b \sec [c+dx])^3} - \frac{a (2 A b^4 - 5 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 8 C)) \tan [c+dx]}{6 b^3 (a^2 - b^2)^2 d (a+b \sec [c+dx])^2} \\
& \frac{(4 A b^6 + a^3 b^3 B - 16 a b^5 B + 9 a^6 C + 2 a^2 b^4 (7 A + 17 C) - a^4 b^2 (3 A + 28 C)) \tan [c+dx]}{6 b^3 (a^2 - b^2)^3 d (a+b \sec [c+dx])}
\end{aligned}$$

Result (type 3, 1302 leaves) :

$$\begin{aligned}
 & - \left(2 C (b + a \cos [c + d x])^4 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] - \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \right) / \\
 & \quad (b^4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4) + \\
 & \left(2 C (b + a \cos [c + d x])^4 \operatorname{Log} \left[\cos \left[\frac{c}{2} + \frac{d x}{2} \right] + \sin \left[\frac{c}{2} + \frac{d x}{2} \right] \right] \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \right) / \\
 & \quad (b^4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4) + \\
 & \left(-a^3 A b^4 - 4 a A b^6 + 3 a^2 b^5 B + 2 b^7 B + 2 a^7 C - 7 a^5 b^2 C + 8 a^3 b^4 C - 8 a b^6 C \right) (b + a \cos [c + d x])^4 \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\
 & \quad \left(- \left(2 i \operatorname{ArcTan} \left[\operatorname{Sec} \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \right) \right) \\
 & \quad \cos [c] \Big) / \left(b^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) - \\
 & \left(2 \operatorname{ArcTan} \left[\operatorname{Sec} \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \sin [c] \right) / \\
 & \quad \left(b^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) \Big) / \left((-a^2 + b^2)^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4 \right) - \\
 & \left(2 (b + a \cos [c + d x]) \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \right. \\
 & \quad \left. (A b^3 \sin [c] - a b^2 B \sin [c] + a^2 b C \sin [c] - a A b^2 \sin [d x] + a^2 b B \sin [d x] - a^3 C \sin [d x]) \right) / \\
 & \left(3 a b (-a^2 + b^2) d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4 \right) + \\
 & \left((b + a \cos [c + d x])^2 \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) (-5 a A b^3 \sin [c] + 2 a^2 b^2 B \sin [c] + 3 b^4 B \sin [c] + \right. \\
 & \quad \left. a^3 b C \sin [c] - 6 a b^3 C \sin [c] + 3 a^2 A b^2 \sin [d x] + 2 A b^4 \sin [d x] - 5 a b^3 B \sin [d x] - 3 a^4 C \sin [d x] + 8 a^2 b^2 C \sin [d x]) \right) / \\
 & \left(3 b^2 (-a^2 + b^2)^2 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4 \right) + \\
 & \left((b + a \cos [c + d x])^3 \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^2 (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \right. \\
 & \quad \left. (-3 a^3 A b^3 \sin [c] - 12 a A b^5 \sin [c] + 9 a^2 b^4 B \sin [c] + 6 b^6 B \sin [c] - 3 a^5 b C \sin [c] + 6 a^3 b^3 C \sin [c] - 18 a b^5 C \sin [c] + \right. \\
 & \quad \left. 13 a^2 A b^4 \sin [d x] + 2 A b^6 \sin [d x] - 4 a^3 b^3 B \sin [d x] - 11 a b^5 B \sin [d x] + 6 a^6 C \sin [d x] - 17 a^4 b^2 C \sin [d x] + 26 a^2 b^4 C \sin [d x]) \right) / \\
 & \left(3 b^3 (-a^2 + b^2)^3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \operatorname{Sec} [c + d x])^4 \right)
 \end{aligned}$$

■ **Problem 926: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec} [c + d x] (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2)}{(a + b \operatorname{Sec} [c + d x])^4} dx$$

Optimal (type 3, 299 leaves, 7 steps) :

$$\begin{aligned}
 & - \frac{(4 a^2 b B + b^3 B - a^3 (2 A + C) - a b^2 (3 A + 4 C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{(a-b)^{7/2} (a+b)^{7/2} d} - \frac{(A b^2 - a (b B - a C)) \operatorname{Tan}[c+dx]}{3 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+dx])^3} + \\
 & \frac{(2 a^2 b B + 3 b^3 B + a^3 C - a b^2 (5 A + 6 C)) \operatorname{Tan}[c+dx]}{6 b (a^2 - b^2)^2 d (a+b \operatorname{Sec}[c+dx])^2} + \frac{(2 a^3 b B + 13 a b^3 B + a^4 C - 2 b^4 (2 A + 3 C) - a^2 b^2 (11 A + 10 C)) \operatorname{Tan}[c+dx]}{6 b (a^2 - b^2)^3 d (a+b \operatorname{Sec}[c+dx])}
 \end{aligned}$$

Result (type 3, 1069 leaves) :

$$\begin{aligned}
 & \left((-2 a^3 A - 3 a A b^2 + 4 a^2 b B + b^3 B - a^3 C - 4 a b^2 C) (b+a \operatorname{Cos}[c+dx])^4 \operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx])^2 \right. \\
 & \left. \left(- \left(2 i \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2-b^2} \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2-b^2} \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right] \right) \right) \right. \right. \\
 & \left. \left. \operatorname{Cos}[c] \right) / \left(\sqrt{a^2-b^2} d \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]} \right) - \right. \\
 & \left. \left(2 \operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{dx}{2}\right]\right] \left(\frac{\operatorname{Cos}[c]}{\sqrt{a^2-b^2} \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]}} - \frac{i \operatorname{Sin}[c]}{\sqrt{a^2-b^2} \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]}} \right) \left(-i b \operatorname{Sin}\left[\frac{dx}{2}\right] + i a \operatorname{Sin}\left[c + \frac{dx}{2}\right] \right) \right) \operatorname{Sin}[c] \right) / \\
 & \left. \left(\sqrt{a^2-b^2} d \sqrt{\operatorname{Cos}[2c] - i \operatorname{Sin}[2c]} \right) \right) / \\
 & \left((-a^2 + b^2)^3 (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a+b \operatorname{Sec}[c+dx])^4 \right) + \\
 & \left(2 (b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx])^2 \right. \\
 & \left. (A b^4 \operatorname{Sin}[c] - a b^3 B \operatorname{Sin}[c] + a^2 b^2 C \operatorname{Sin}[c] - a A b^3 \operatorname{Sin}[dx] + a^2 b^2 B \operatorname{Sin}[dx] - a^3 b C \operatorname{Sin}[dx]) \right) / \\
 & \left(3 a^3 (a^2 - b^2) d (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a+b \operatorname{Sec}[c+dx])^4 \right) + \\
 & \left((b+a \operatorname{Cos}[c+dx])^2 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx])^2 \right. \\
 & \left. (-11 a^2 A b^3 \operatorname{Sin}[c] + 6 A b^5 \operatorname{Sin}[c] + 8 a^3 b^2 B \operatorname{Sin}[c] - 3 a b^4 B \operatorname{Sin}[c] - 5 a^4 b C \operatorname{Sin}[c] + 9 a^3 A b^2 \operatorname{Sin}[dx] - \right. \\
 & \left. 4 a A b^4 \operatorname{Sin}[dx] - 6 a^4 b B \operatorname{Sin}[dx] + a^2 b^3 B \operatorname{Sin}[dx] + 3 a^5 C \operatorname{Sin}[dx] + 2 a^3 b^2 C \operatorname{Sin}[dx]) \right) / \\
 & \left(3 a^3 (a^2 - b^2)^2 d (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a+b \operatorname{Sec}[c+dx])^4 \right) + \\
 & \left((b+a \operatorname{Cos}[c+dx])^3 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx])^2 \right. \\
 & \left. (27 a^4 A b^2 \operatorname{Sin}[c] - 18 a^2 A b^4 \operatorname{Sin}[c] + 6 A b^6 \operatorname{Sin}[c] - 12 a^5 b B \operatorname{Sin}[c] - 3 a^3 b^3 B \operatorname{Sin}[c] + 3 a^6 C \operatorname{Sin}[c] + 12 a^4 b^2 C \operatorname{Sin}[c] - 18 a^5 A b \operatorname{Sin}[dx] + \right. \\
 & \left. 5 a^3 A b^3 \operatorname{Sin}[dx] - 2 a A b^5 \operatorname{Sin}[dx] + 6 a^6 B \operatorname{Sin}[dx] + 10 a^4 b^2 B \operatorname{Sin}[dx] - a^2 b^4 B \operatorname{Sin}[dx] - 13 a^5 b C \operatorname{Sin}[dx] - 2 a^3 b^3 C \operatorname{Sin}[dx]) \right) / \\
 & \left(3 a^3 (a^2 - b^2)^3 d (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c + 2dx]) (a+b \operatorname{Sec}[c+dx])^4 \right)
 \end{aligned}$$

- **Problem 927: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{(a + b \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 3, 336 leaves, 7 steps):

$$\frac{Ax}{a^4} - \frac{(7a^2Ab^5 - 2Ab^7 - 2a^7B - 3a^5b^2B - a^4b^3(8A - C) + 4a^6b(2A + C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a+b}}\right]}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{(Ab^2 - a(bB - aC)) \operatorname{Tan}[c + dx]}{3a(a^2 - b^2)d(a + b \operatorname{Sec}[c + dx])^3} - \frac{(3Ab^4 + 5a^3bB - 2a^4C - a^2b^2(8A + 3C)) \operatorname{Tan}[c + dx]}{6a^2(a^2 - b^2)^2d(a + b \operatorname{Sec}[c + dx])^2} - \frac{(17a^2Ab^4 - 6Ab^6 + 11a^5bB + 4a^3b^3B - 2a^6C - 13a^4b^2(2A + C)) \operatorname{Tan}[c + dx]}{6a^3(a^2 - b^2)^3d(a + b \operatorname{Sec}[c + dx])}$$

Result (type 3, 1230 leaves):

$$\begin{aligned}
& \frac{2 A x (b + a \cos [c + d x])^4 \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2)}{a^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4} + \\
& \left((-8 a^6 A b + 8 a^4 A b^3 - 7 a^2 A b^5 + 2 A b^7 + 2 a^7 B + 3 a^5 b^2 B - 4 a^6 b C - a^4 b^3 C) (b + a \cos [c + d x])^4 \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\left(2 i \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \cos [c] \right) / \right. \\
& \left. \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) + \right. \\
& \left. \left(2 \operatorname{ArcTan} \left[\sec \left[\frac{d x}{2} \right] \left(\frac{\cos [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} - \frac{i \sin [c]}{\sqrt{a^2 - b^2} \sqrt{\cos [2 c] - i \sin [2 c]}} \right) \left(-i b \sin \left[\frac{d x}{2} \right] + i a \sin \left[c + \frac{d x}{2} \right] \right) \right] \sin [c] \right) / \right. \\
& \left. \left(a^4 \sqrt{a^2 - b^2} d \sqrt{\cos [2 c] - i \sin [2 c]} \right) \right) / \left((-a^2 + b^2)^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4 \right) - \\
& (2 (b + a \cos [c + d x]) \sec [c] \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& (A b^5 \sin [c] - a b^4 B \sin [c] + a^2 b^3 C \sin [c] - a A b^4 \sin [d x] + a^2 b^3 B \sin [d x] - a^3 b^2 C \sin [d x])) / \\
& (3 a^4 (a^2 - b^2) d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4) + \\
& ((b + a \cos [c + d x])^2 \sec [c] \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& (14 a^2 A b^4 \sin [c] - 9 A b^5 \sin [c] - 11 a^3 b^3 B \sin [c] + 6 a b^5 B \sin [c] + 8 a^4 b^2 C \sin [c] - 3 a^2 b^4 C \sin [c] - \\
& 12 a^3 A b^3 \sin [d x] + 7 a A b^5 \sin [d x] + 9 a^4 b^2 B \sin [d x] - 4 a^2 b^4 B \sin [d x] - 6 a^5 b C \sin [d x] + a^3 b^3 C \sin [d x])) / \\
& (3 a^4 (a^2 - b^2)^2 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4) + \\
& ((b + a \cos [c + d x])^3 \sec [c] \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) (-48 a^4 A b^3 \sin [c] + 51 a^2 A b^5 \sin [c] - 18 A b^7 \sin [c] + \\
& 27 a^5 b^2 B \sin [c] - 18 a^3 b^4 B \sin [c] + 6 a b^6 B \sin [c] - 12 a^6 b C \sin [c] - 3 a^4 b^3 C \sin [c] + 36 a^5 A b^2 \sin [d x] - 32 a^3 A b^4 \sin [d x] + \\
& 11 a A b^6 \sin [d x] - 18 a^6 b B \sin [d x] + 5 a^4 b^3 B \sin [d x] - 2 a^2 b^5 B \sin [d x] + 6 a^7 C \sin [d x] + 10 a^5 b^2 C \sin [d x] - a^3 b^4 C \sin [d x])) / \\
& (3 a^4 (a^2 - b^2)^3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + b \sec [c + d x])^4)
\end{aligned}$$

- **Problem 928: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + b \sec [c + d x])^4} dx$$

Optimal (type 3, 471 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(4 A b - a B) x}{a^5} - \frac{1}{a^5 (a - b)^{7/2} (a + b)^{7/2} d} \\
& (35 a^4 A b^4 - 28 a^2 A b^6 + 8 A b^8 + 8 a^7 b B - 8 a^5 b^3 B + 7 a^3 b^5 B - 2 a b^7 B - 2 a^8 C - a^6 b^2 (20 A + 3 C)) \operatorname{ArcTanh}\left[\frac{\sqrt{a - b} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a + b}}\right] + \\
& \frac{(68 a^2 A b^4 - 24 A b^6 + 26 a^5 b B - 17 a^3 b^3 B + 6 a b^5 B + a^6 (6 A - 11 C) - a^4 b^2 (65 A + 4 C)) \operatorname{Sin}[c + d x]}{6 a^4 (a^2 - b^2)^3 d} + \\
& \frac{(A b^2 - a (b B - a C)) \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^3} - \frac{(4 A b^4 + 6 a^3 b B - a b^3 B - 3 a^4 C - a^2 b^2 (9 A + 2 C)) \operatorname{Sin}[c + d x]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^2} - \\
& \frac{(11 a^2 A b^4 - 4 A b^6 + 6 a^5 b B - 2 a^3 b^3 B + a b^5 B - 2 a^6 C - 3 a^4 b^2 (4 A + C)) \operatorname{Sin}[c + d x]}{2 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + d x])}
\end{aligned}$$

Result (type 3, 1367 leaves):

$$\begin{aligned}
& - \frac{2(4Ab - aB)x(b + a\cos[c + dx])^4 \sec[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2)}{a^5(A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx])(a + b\sec[c + dx])^4} + \\
& \left(\frac{-20a^6Ab^2 + 35a^4Ab^4 - 28a^2Ab^6 + 8Ab^8 + 8a^7bB - 8a^5b^3B + 7a^3b^5B - 2ab^7B - 2a^8C - 3a^6b^2C}{(b + a\cos[c + dx])^4 \sec[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2)} \right. \\
& \left. \left(- \left(2i \operatorname{ArcTan} \left[\sec \left[\frac{dx}{2} \right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i\sin[2c]}} - \frac{i\sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i\sin[2c]}} \right) \left(-ib\sin\left[\frac{dx}{2}\right] + ia\sin\left[c + \frac{dx}{2}\right] \right) \right] \right) \right. \\
& \left. \left. \cos[c] \right) / \left(a^5 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i\sin[2c]} \right) - \right. \\
& \left. \left(2 \operatorname{ArcTan} \left[\sec \left[\frac{dx}{2} \right] \left(\frac{\cos[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i\sin[2c]}} - \frac{i\sin[c]}{\sqrt{a^2 - b^2} \sqrt{\cos[2c] - i\sin[2c]}} \right) \right] \left(-ib\sin\left[\frac{dx}{2}\right] + ia\sin\left[c + \frac{dx}{2}\right] \right) \right) \sin[c] \right) / \\
& \left. \left(a^5 \sqrt{a^2 - b^2} d \sqrt{\cos[2c] - i\sin[2c]} \right) \right) / \\
& \left((-a^2 + b^2)^3 (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) (a + b\sec[c + dx])^4 \right) + \\
& \left(2(b + a\cos[c + dx]) \sec[c] \sec[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2) \right. \\
& \left. (Ab^6 \sin[c] - ab^5 B \sin[c] + a^2 b^4 C \sin[c] - aAb^5 \sin[dx] + a^2 b^4 B \sin[dx] - a^3 b^3 C \sin[dx]) \right) / \\
& \left(3a^5 (a^2 - b^2) d (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) (a + b\sec[c + dx])^4 \right) + \\
& \left((b + a\cos[c + dx])^2 \sec[c] \sec[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2) \right. \\
& \left. (-17a^2 Ab^5 \sin[c] + 12Ab^7 \sin[c] + 14a^3 b^4 B \sin[c] - 9ab^6 B \sin[c] - 11a^4 b^3 C \sin[c] + 6a^2 b^5 C \sin[c] + \right. \\
& \left. 15a^3 Ab^4 \sin[dx] - 10aAb^6 \sin[dx] - 12a^4 b^3 B \sin[dx] + 7a^2 b^5 B \sin[dx] + 9a^5 b^2 C \sin[dx] - 4a^3 b^4 C \sin[dx]) \right) / \\
& \left(3a^5 (a^2 - b^2)^2 d (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) (a + b\sec[c + dx])^4 \right) + \\
& \left((b + a\cos[c + dx])^3 \sec[c] \sec[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2) \right. \\
& \left. (75a^4 Ab^4 \sin[c] - 96a^2 Ab^6 \sin[c] + 36Ab^8 \sin[c] - 48a^5 b^3 B \sin[c] + 51a^3 b^5 B \sin[c] - 18ab^7 B \sin[c] + \right. \\
& \left. 27a^6 b^2 C \sin[c] - 18a^4 b^4 C \sin[c] + 6a^2 b^6 C \sin[c] - 60a^5 Ab^3 \sin[dx] + 71a^3 Ab^5 \sin[dx] - 26aAb^7 \sin[dx] + \right. \\
& \left. 36a^6 b^2 B \sin[dx] - 32a^4 b^4 B \sin[dx] + 11a^2 b^6 B \sin[dx] - 18a^7 bC \sin[dx] + 5a^5 b^3 C \sin[dx] - 2a^3 b^5 C \sin[dx]) \right) / \\
& \left(3a^5 (a^2 - b^2)^3 d (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) (a + b\sec[c + dx])^4 \right) + \\
& \frac{2A(b + a\cos[c + dx])^4 \sec[c + dx] (A + B\sec[c + dx] + C\sec[c + dx]^2) \tan[c + dx]}{a^4 d (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) (a + b\sec[c + dx])^4}
\end{aligned}$$

■ **Problem 929: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^2 (A + B\sec[c + dx] + C\sec[c + dx]^2)}{(a + b\sec[c + dx])^4} dx$$

Optimal (type 3, 648 leaves, 9 steps) :

$$\frac{(20 A b^2 - 8 a b B + a^2 (A + 2 C)) x}{2 a^6} +$$

$$\left(b (20 A b^3 + 20 a^7 b B - 35 a^5 b^3 B + 28 a^3 b^5 B - 8 a b^7 B - a^2 b^6 (69 A - 2 C) - 8 a^6 b^2 (5 A - C) + 7 a^4 b^4 (12 A - C) - 8 a^8 C) \right.$$

$$\left. \operatorname{ArcTanh} \left[\frac{\sqrt{a-b} \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a+b}} \right] \right) / \left(a^6 \sqrt{a-b} \sqrt{a+b} (a^2 - b^2)^3 d \right) + \frac{1}{6 a^5 (a^2 - b^2)^3 d}$$

$$(60 A b^7 + 6 a^7 B - 65 a^5 b^2 B + 68 a^3 b^4 B - 24 a b^6 B + a^4 b^3 (146 A - 17 C) - a^2 b^5 (167 A - 6 C) - a^6 (24 A b - 26 b C)) \operatorname{Sin}[c + d x] -$$

$$\frac{1}{2 a^4 (a^2 - b^2)^3 d} (10 A b^6 - 12 a^5 b B + 11 a^3 b^3 B - 4 a b^5 B - a^6 (A - 6 C) + a^4 b^2 (23 A - 2 C) - a^2 b^4 (27 A - C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x] +$$

$$\frac{(A b^2 - a (b B - a C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^3} - \frac{(5 A b^4 + 7 a^3 b B - 2 a b^3 B - 4 a^4 C - a^2 b^2 (10 A + C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x]}{6 a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + d x])^2} +$$

$$\left((20 A b^6 - 27 a^5 b B + 20 a^3 b^3 B - 8 a b^5 B - a^2 b^4 (53 A - 2 C) + 12 a^6 C + a^4 b^2 (48 A + C)) \operatorname{Cos}[c + d x] \operatorname{Sin}[c + d x] \right) /$$

$$(6 a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + d x]))$$

Result (type 3, 658 leaves) :

$$\frac{(a^2 A + 20 A b^2 - 8 a b B + 2 a^2 C) (c + d x)}{2 a^6 d} + \frac{1}{a^6 \sqrt{a^2 - b^2} (-a^2 + b^2)^3 d}$$

$$b (-40 a^6 A b^2 + 84 a^4 A b^4 - 69 a^2 A b^6 + 20 A b^8 + 20 a^7 b B - 35 a^5 b^3 B + 28 a^3 b^5 B - 8 a b^7 B - 8 a^8 C + 8 a^6 b^2 C - 7 a^4 b^4 C + 2 a^2 b^6 C)$$

$$\operatorname{ArcTanh} \left[\frac{(-a + b) \operatorname{Tan} \left[\frac{1}{2} (c + d x) \right]}{\sqrt{a^2 - b^2}} \right] + \frac{(4 A b - a B) \left(-\frac{i \operatorname{Cos}[c + d x]}{2 a^5} - \frac{\operatorname{Sin}[c + d x]}{2 a^5} \right)}{d} +$$

$$\frac{(4 A b - a B) \left(\frac{i \operatorname{Cos}[c + d x]}{2 a^5} - \frac{\operatorname{Sin}[c + d x]}{2 a^5} \right)}{d} + \frac{A b^6 \operatorname{Sin}[c + d x] - a b^5 B \operatorname{Sin}[c + d x] + a^2 b^4 C \operatorname{Sin}[c + d x]}{3 a^5 (a^2 - b^2) d (b + a \operatorname{Cos}[c + d x])^3} +$$

$$\left(-18 a^2 A b^5 \operatorname{Sin}[c + d x] + 13 A b^7 \operatorname{Sin}[c + d x] + 15 a^3 b^4 B \operatorname{Sin}[c + d x] - 10 a b^6 B \operatorname{Sin}[c + d x] - 12 a^4 b^3 C \operatorname{Sin}[c + d x] + 7 a^2 b^5 C \operatorname{Sin}[c + d x] \right) /$$

$$(6 a^5 (a^2 - b^2)^2 d (b + a \operatorname{Cos}[c + d x])^2) +$$

$$(90 a^4 A b^4 \operatorname{Sin}[c + d x] - 122 a^2 A b^6 \operatorname{Sin}[c + d x] + 47 A b^8 \operatorname{Sin}[c + d x] - 60 a^5 b^3 B \operatorname{Sin}[c + d x] + 71 a^3 b^5 B \operatorname{Sin}[c + d x] - 26 a b^7 B \operatorname{Sin}[c + d x] +$$

$$36 a^6 b^2 C \operatorname{Sin}[c + d x] - 32 a^4 b^4 C \operatorname{Sin}[c + d x] + 11 a^2 b^6 C \operatorname{Sin}[c + d x]) / (6 a^5 (a^2 - b^2)^3 d (b + a \operatorname{Cos}[c + d x])) + \frac{A \operatorname{Sin}[2 (c + d x)]}{4 a^4 d}$$

■ **Problem 930: Result more than twice size of optimal antiderivative.**

$$\int \frac{a b B - a^2 C + b^2 B \operatorname{Sec}[c + d x] + b^2 C \operatorname{Sec}[c + d x]^2}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 3, 24 leaves, 3 steps) :

$$(bB - aC)x + \frac{bC \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{d}$$

Result (type 3, 81 leaves) :

$$bBx - aCx - \frac{bC \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{bC \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d}$$

■ **Problem 934: Result more than twice size of optimal antiderivative.**

$$\int \frac{a b B - a^2 C + b^2 B \operatorname{Sec}[c + dx] + b^2 C \operatorname{Sec}[c + dx]^2}{(a + b \operatorname{Sec}[c + dx])^5} dx$$

Optimal (type 3, 336 leaves, 8 steps) :

$$\frac{(bB - aC)x}{a^4} - \frac{1}{a^4 (a - b)^{7/2} (a + b)^{7/2} d}$$

$$b \left(8 a^6 b B - 8 a^4 b^3 B + 7 a^2 b^5 B - 2 b^7 B - 10 a^7 C + 5 a^5 b^2 C - 7 a^3 b^4 C + 2 a b^6 C \right) \operatorname{ArcTanh}\left[\frac{\sqrt{a - b} \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a + b}} \right] +$$

$$\frac{b^2 (bB - 2aC) \operatorname{Tan}[c + dx]}{3a (a^2 - b^2) d (a + b \operatorname{Sec}[c + dx])^3} + \frac{b^2 (8a^2 b B - 3b^3 B - 13a^3 C + 3a b^2 C) \operatorname{Tan}[c + dx]}{6a^2 (a^2 - b^2)^2 d (a + b \operatorname{Sec}[c + dx])^2} +$$

$$\frac{b^2 (26a^4 b B - 17a^2 b^3 B + 6b^5 B - 37a^5 C + 13a^3 b^2 C - 6a b^4 C) \operatorname{Tan}[c + dx]}{6a^3 (a^2 - b^2)^3 d (a + b \operatorname{Sec}[c + dx])}$$

Result (type 3, 1287 leaves) :

$$\left(b \left(-8 a^6 b B + 8 a^4 b^3 B - 7 a^2 b^5 B + 2 b^7 B + 10 a^7 C - 5 a^5 b^2 C + 7 a^3 b^4 C - 2 a b^6 C \right) \operatorname{ArcTanh} \left[\frac{(-a+b) \operatorname{Tan} \left[\frac{1}{2} (c+dx) \right]}{\sqrt{a^2-b^2}} \right] (b+a \operatorname{Cos}[c+dx])^4 \right. \\ \left. \operatorname{Sec}[c+dx]^3 (bB-aC+bC \operatorname{Sec}[c+dx]) \right) / \left(a^4 \sqrt{a^2-b^2} (-a^2+b^2)^3 d (bC+bB \operatorname{Cos}[c+dx] - aC \operatorname{Cos}[c+dx]) (a+b \operatorname{Sec}[c+dx])^4 \right) + \\ \left((b+a \operatorname{Cos}[c+dx]) \operatorname{Sec}[c+dx]^3 (bB-aC+bC \operatorname{Sec}[c+dx]) \left(36 a^8 b^2 B (c+dx) - 84 a^6 b^4 B (c+dx) + 36 a^4 b^6 B (c+dx) + \right. \right. \\ \left. 36 a^2 b^8 B (c+dx) - 24 b^{10} B (c+dx) - 36 a^9 b C (c+dx) + 84 a^7 b^3 C (c+dx) - 36 a^5 b^5 C (c+dx) - 36 a^3 b^7 C (c+dx) + \right. \\ \left. 24 a b^9 C (c+dx) + 18 a^9 b B (c+dx) \operatorname{Cos}[c+dx] + 18 a^7 b^3 B (c+dx) \operatorname{Cos}[c+dx] - 162 a^5 b^5 B (c+dx) \operatorname{Cos}[c+dx] + \right. \\ \left. 198 a^3 b^7 B (c+dx) \operatorname{Cos}[c+dx] - 72 a b^9 B (c+dx) \operatorname{Cos}[c+dx] - 18 a^{10} C (c+dx) \operatorname{Cos}[c+dx] - 18 a^8 b^2 C (c+dx) \operatorname{Cos}[c+dx] + \right. \\ \left. 162 a^6 b^4 C (c+dx) \operatorname{Cos}[c+dx] - 198 a^4 b^6 C (c+dx) \operatorname{Cos}[c+dx] + 72 a^2 b^8 C (c+dx) \operatorname{Cos}[c+dx] + 36 a^8 b^2 B (c+dx) \operatorname{Cos}[2(c+dx)] - \right. \\ \left. 108 a^6 b^4 B (c+dx) \operatorname{Cos}[2(c+dx)] + 108 a^4 b^6 B (c+dx) \operatorname{Cos}[2(c+dx)] - 36 a^2 b^8 B (c+dx) \operatorname{Cos}[2(c+dx)] - \right. \\ \left. 36 a^9 b C (c+dx) \operatorname{Cos}[2(c+dx)] + 108 a^7 b^3 C (c+dx) \operatorname{Cos}[2(c+dx)] - 108 a^5 b^5 C (c+dx) \operatorname{Cos}[2(c+dx)] + \right. \\ \left. 36 a^3 b^7 C (c+dx) \operatorname{Cos}[2(c+dx)] + 6 a^9 b B (c+dx) \operatorname{Cos}[3(c+dx)] - 18 a^7 b^3 B (c+dx) \operatorname{Cos}[3(c+dx)] + \right. \\ \left. 18 a^5 b^5 B (c+dx) \operatorname{Cos}[3(c+dx)] - 6 a^3 b^7 B (c+dx) \operatorname{Cos}[3(c+dx)] - 6 a^{10} C (c+dx) \operatorname{Cos}[3(c+dx)] + \right. \\ \left. 18 a^8 b^2 C (c+dx) \operatorname{Cos}[3(c+dx)] - 18 a^6 b^4 C (c+dx) \operatorname{Cos}[3(c+dx)] + 6 a^4 b^6 C (c+dx) \operatorname{Cos}[3(c+dx)] + 36 a^7 b^3 B \operatorname{Sin}[c+dx] + \right. \\ \left. 72 a^5 b^5 B \operatorname{Sin}[c+dx] - 57 a^3 b^7 B \operatorname{Sin}[c+dx] + 24 a b^9 B \operatorname{Sin}[c+dx] - 54 a^8 b^2 C \operatorname{Sin}[c+dx] - 111 a^6 b^4 C \operatorname{Sin}[c+dx] + \right. \\ \left. 39 a^4 b^6 C \operatorname{Sin}[c+dx] - 24 a^2 b^8 C \operatorname{Sin}[c+dx] + 120 a^6 b^4 B \operatorname{Sin}[2(c+dx)] - 90 a^4 b^6 B \operatorname{Sin}[2(c+dx)] + 30 a^2 b^8 B \operatorname{Sin}[2(c+dx)] - \right. \\ \left. 174 a^7 b^3 C \operatorname{Sin}[2(c+dx)] + 84 a^5 b^5 C \operatorname{Sin}[2(c+dx)] - 30 a^3 b^7 C \operatorname{Sin}[2(c+dx)] + 36 a^7 b^3 B \operatorname{Sin}[3(c+dx)] - 32 a^5 b^5 B \operatorname{Sin}[3(c+dx)] + \right. \\ \left. 11 a^3 b^7 B \operatorname{Sin}[3(c+dx)] - 54 a^8 b^2 C \operatorname{Sin}[3(c+dx)] + 37 a^6 b^4 C \operatorname{Sin}[3(c+dx)] - 13 a^4 b^6 C \operatorname{Sin}[3(c+dx)] \right) \left. \right) / \\ (24 a^4 (a^2-b^2)^3 d (bC+bB \operatorname{Cos}[c+dx] - aC \operatorname{Cos}[c+dx]) (a+b \operatorname{Sec}[c+dx])^4)$$

■ **Problem 935: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+dx]^3 \sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 517 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^5 d} 2 (a-b) \sqrt{a+b} \left(24 a^3 b B + 57 a b^3 B - 16 a^4 C - 6 a^2 b^2 (7 A + 4 C) + 21 b^4 (9 A + 7 C) \right) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} \left(12 a^2 b (2 B - C) - 16 a^3 C - 6 a b^2 (7 A - 3 B + 6 C) - 3 b^3 (63 A - 25 B + 49 C) \right) \text{Cot}[c+dx] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
& \frac{2 (12 a^2 b B - 75 b^3 B - 8 a^3 C - a b^2 (21 A + 13 C)) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{315 b^3 d} + \\
& \frac{2 (63 A b^2 + 9 a b B - 6 a^2 C + 49 b^2 C) \text{Sec}[c+dx] \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{315 b^2 d} + \\
& \frac{2 (9 b B + a C) \text{Sec}[c+dx]^2 \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{63 b d} + \frac{2 C \text{Sec}[c+dx]^3 \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{9 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 936: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+dx]^2 \sqrt{a+b \text{Sec}[c+dx]} (A+B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 413 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{105 b^4 d} 2 (a-b) \sqrt{a+b} \left(14 a^2 b B - 63 b^3 B - 8 a^3 C - a b^2 (35 A + 19 C) \right) \text{Cot}[c+dx] \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{105 b^3 d} \\
& 2 (a-b) \sqrt{a+b} \left(35 A b^2 - b^2 (63 B - 25 C) + 8 a^2 C - a (14 b B - 6 b C) \right) \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \frac{2 (35 A b^2 - 14 a b B + 8 a^2 C + 25 b^2 C) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{105 b^2 d} + \\
& \frac{2 (7 b B - 4 a C) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{35 b^2 d} + \frac{2 C \text{Sec}[c+dx] (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{7 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 937: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c + dx] \sqrt{a + b \text{Sec}[c + dx]} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 324 leaves, 5 steps):

$$-\frac{1}{15 b^3 d} 2 (a - b) \sqrt{a + b} (3 b^2 (5 A + 3 C) + a (5 b B - 2 a C)) \text{Cot}[c + dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + dx])}{a - b}} + \frac{1}{15 b^2 d}$$

$$2 (a - b) \sqrt{a + b} (15 A b - 5 b B + 2 a C + 9 b C) \text{Cot}[c + dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + dx])}{a + b}}$$

$$\sqrt{-\frac{b (1 + \text{Sec}[c + dx])}{a - b}} + \frac{2 (5 b B - 2 a C) \sqrt{a + b \text{Sec}[c + dx]} \text{Tan}[c + dx]}{15 b d} + \frac{2 C (a + b \text{Sec}[c + dx])^{3/2} \text{Tan}[c + dx]}{5 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 938: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \text{Sec}[c + dx]} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 366 leaves, 6 steps):

$$-\frac{1}{3 b^2 d} 2 (a - b) \sqrt{a + b} (3 b B + a C) \text{Cot}[c + dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + dx])}{a - b}} +$$

$$\frac{1}{3 b d} 2 \sqrt{a + b} (3 A b + (a - b) (5 B - C)) \text{Cot}[c + dx]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + dx])}{a - b}} -$$

$$\frac{2 A \sqrt{a + b} \text{Cot}[c + dx] \text{EllipticPi}\left[\frac{a + b}{a}, \text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + dx]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + dx])}{a - b}}}{d} + \frac{2 C \sqrt{a + b \text{Sec}[c + dx]} \text{Tan}[c + dx]}{3 d}$$

Result (type 4, 5313 leaves):

$$\begin{aligned}
& \left(\cos[c+dx]^2 \sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx]+C \sec[c+dx]^2) \left(\frac{4(3bB+aC) \sin[c+dx]}{3b} + \frac{4}{3} C \tan[c+dx] \right) \right) / \\
& (d(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) - \\
& \left(4 \left(\frac{2aA}{\sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{2bB}{\sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]}} - \frac{2aC}{3\sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]}} + \frac{2Ab \sqrt{\sec[c+dx]}}{\sqrt{b+a \cos[c+dx]}} - \right. \right. \\
& \left. \left. \frac{2a^2 C \sqrt{\sec[c+dx]}}{3b \sqrt{b+a \cos[c+dx]}} + \frac{2bC \sqrt{\sec[c+dx]}}{3 \sqrt{b+a \cos[c+dx]}} - \frac{2aB \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{\sqrt{b+a \cos[c+dx]}} - \frac{2a^2 C \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{3b \sqrt{b+a \cos[c+dx]}} \right) \right) \\
& \sqrt{\cos\left[\frac{1}{2}(c+dx)\right]^2 \sec[c+dx] \sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx]+C \sec[c+dx]^2)} \\
& \left(12aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& 2(a+b)(3bB+aC) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 - \\
& 2b(a(-3A+3B+C)+b(3A+3B+C)) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sec\left[\frac{1}{2}(c+dx)\right]^2 + 3abB \tan\left[\frac{1}{2}(c+dx)\right] + 3b^2B \tan\left[\frac{1}{2}(c+dx)\right] + a^2C \tan\left[\frac{1}{2}(c+dx)\right] + abC \tan\left[\frac{1}{2}(c+dx)\right] + \\
& 12aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 - \\
& 6abB \tan\left[\frac{1}{2}(c+dx)\right]^3 - 2a^2C \tan\left[\frac{1}{2}(c+dx)\right]^3 + 3abB \tan\left[\frac{1}{2}(c+dx)\right]^5 - \\
& \left. \left. 3b^2B \tan\left[\frac{1}{2}(c+dx)\right]^5 + a^2C \tan\left[\frac{1}{2}(c+dx)\right]^5 - abC \tan\left[\frac{1}{2}(c+dx)\right]^5 \right) \right) / \\
& \left(3bd(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \left(\sec\left[\frac{1}{2}(c+dx)\right]^2 \right)^{3/2} \sec[c+dx]^{5/2} \right)
\end{aligned}$$

$$\left(-\frac{1}{3 b (b + a \cos [c + d x])^{3/2} \left(\sec \left[\frac{1}{2} (c + d x) \right] \right)^{3/2}} 2 a \sqrt{\cos \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x] \sin [c + d x]} \right.$$

$$\left(12 a A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right.$$

$$2 (a + b) (3 b B + a C) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 -$$

$$2 b (a (-3 A + 3 B + C) + b (3 A + 3 B + C)) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\sec \left[\frac{1}{2} (c + d x) \right]^2 + 3 a b B \tan \left[\frac{1}{2} (c + d x) \right] + 3 b^2 B \tan \left[\frac{1}{2} (c + d x) \right] + a^2 C \tan \left[\frac{1}{2} (c + d x) \right] + a b C \tan \left[\frac{1}{2} (c + d x) \right] +$$

$$12 a A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \tan \left[\frac{1}{2} (c + d x) \right]^2 -$$

$$6 a b B \tan \left[\frac{1}{2} (c + d x) \right]^3 - 2 a^2 C \tan \left[\frac{1}{2} (c + d x) \right]^3 + 3 a b B \tan \left[\frac{1}{2} (c + d x) \right]^5 - 3 b^2 B \tan \left[\frac{1}{2} (c + d x) \right]^5 + a^2 C \tan \left[\frac{1}{2} (c + d x) \right]^5 -$$

$$a b C \tan \left[\frac{1}{2} (c + d x) \right]^5 \left. \right) + \frac{1}{b \sqrt{b + a \cos [c + d x]} \left(\sec \left[\frac{1}{2} (c + d x) \right] \right)^{3/2}} 2 \sqrt{\cos \left[\frac{1}{2} (c + d x) \right]^2 \sec [c + d x] \tan \left[\frac{1}{2} (c + d x) \right]}$$

$$\left(12 a A b \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right.$$

$$2 (a + b) (3 b B + a C) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \sec \left[\frac{1}{2} (c + d x) \right]^2 -$$

$$2 b (a (-3 A + 3 B + C) + b (3 A + 3 B + C)) \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\sec \left[\frac{1}{2} (c + d x) \right]^2 + 3 a b B \tan \left[\frac{1}{2} (c + d x) \right] + 3 b^2 B \tan \left[\frac{1}{2} (c + d x) \right] + a^2 C \tan \left[\frac{1}{2} (c + d x) \right] + a b C \tan \left[\frac{1}{2} (c + d x) \right] +$$

$$\begin{aligned}
& 12 a A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 - \\
& 6 a b B \tan \left[\frac{1}{2}(c+d x)\right]^3 - 2 a^2 C \tan \left[\frac{1}{2}(c+d x)\right]^3 + 3 a b B \tan \left[\frac{1}{2}(c+d x)\right]^5 - 3 b^2 B \tan \left[\frac{1}{2}(c+d x)\right]^5 + \\
& a^2 C \tan \left[\frac{1}{2}(c+d x)\right]^5 - a b C \tan \left[\frac{1}{2}(c+d x)\right]^5 \left. \right) - \frac{1}{3 b \sqrt{b+a \cos [c+d x]} \left(\sec \left[\frac{1}{2}(c+d x)\right]\right)^{3 / 2}} \\
& 4 \sqrt{\cos \left[\frac{1}{2}(c+d x)\right]^2 \sec [c+d x]} \left(\frac{3}{2} a b B \sec \left[\frac{1}{2}(c+d x)\right]^2 + \frac{3}{2} b^2 B \sec \left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{2} a^2 C \sec \left[\frac{1}{2}(c+d x)\right]^2 + \right. \\
& \left. \frac{1}{2} a b C \sec \left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} 6 a A b \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \left. \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]} \right) + \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} (a+b)(3 b B+a C) \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \right. \\
& \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sec \left[\frac{1}{2}(c+d x)\right]^2 \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]} \right) - \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \right. \\
& \left. b(a(-3 A+3 B+C)+b(3 A+3 B+C)) \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \left. \sec \left[\frac{1}{2}(c+d x)\right]^2 \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]} \right) + \frac{1}{\sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 6 a A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \right. \\
& \left. \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} (a+b)(3 b B+a C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right) - \frac{1}{\sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} \\
& b(a(-3 A+3 B+C)+b(3 A+3 B+C)) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right) + 2(a+b)(3 b B+a C) \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 b(a(-3 A+3 B+C)+b(3 A+3 B+C)) \\
& \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \\
& 12 a A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 9 a b B \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 - 3 a^2 C \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} \\
& 6 a A b \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]}\right) \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + \frac{1}{\sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} 6 a A b \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + \frac{15}{2} a b B \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4 - \\
& \frac{15}{2} b^2 B \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4 + \frac{5}{2} a^2 C \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4 - \frac{5}{2} a b C \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4 -
\end{aligned}$$

$$\begin{aligned}
& \frac{b(a(-3A+3B+C) + b(3A+3B+C)) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{1-\frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \\
& \frac{6aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \\
& \frac{6aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \left((a+b)(3bB+aC) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \right. \\
& \left. \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{1-\frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) / \left(\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right) - \\
& \left(2 \left(12aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& 2(a+b)(3bB+aC) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 - \\
& 2b(a(-3A+3B+C) + b(3A+3B+C)) \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + 3aAb \tan\left[\frac{1}{2}(c+dx)\right] + 3b^2B \tan\left[\frac{1}{2}(c+dx)\right] + a^2C \tan\left[\frac{1}{2}(c+dx)\right] + abC \tan\left[\frac{1}{2}(c+dx)\right] + \right. \\
& 12aAb \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \sqrt{\frac{b+a\cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 - \\
& \left. 6aAb \tan\left[\frac{1}{2}(c+dx)\right]^3 - 2a^2C \tan\left[\frac{1}{2}(c+dx)\right]^3 + 3aAb \tan\left[\frac{1}{2}(c+dx)\right]^5 - 3b^2B \tan\left[\frac{1}{2}(c+dx)\right]^5 + a^2C \tan\left[\frac{1}{2}(c+dx)\right]^5 - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 C \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d} + \\
& \left(\sqrt{a+b \operatorname{Sec}[c+d x]} \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(a A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 2 a A \right. \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 4 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + a A \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - A b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 2 a C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 2 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 \right. \\
& \quad \left. - 2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right. \\
& \quad \left. 4 a B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \right. \\
& \quad \left. 2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right. \\
& \quad \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \right. \\
& \quad \left. (a+b)(A-2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \right. \\
& \quad \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2(A b+a(B-C)-b(B+C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \quad \left. \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \right) / \\
& \left(d \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^{3/2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right)
\end{aligned}$$

■ **Problem 940: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^2 \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 435 leaves, 7 steps):

$$\frac{1}{4 a b d}$$

$$\begin{aligned} & (a-b) \sqrt{a+b} (A b+4 a B) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{1}{4 a d} \\ & \sqrt{a+b} (A b+2 a(A+2 B+4 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \\ & \frac{1}{4 a^2 d} \sqrt{a+b} (A b^2-4 a b B-4 a^2(A+2 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \\ & \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} + \frac{(A b+4 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{4 a d} + \frac{A \cos [c+d x] \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{2 d} \end{aligned}$$

Result (type 4, 1842 leaves):

$$\frac{A \sqrt{a+b \sec [c+d x]} \sin [2(c+d x)]}{4 d} +$$

$$\left(\sqrt{a+b \sec [c+d x]} \left(-a A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - A b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 4 a^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - \right.$$

$$\left. 4 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 2 a A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 8 a^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \right.$$

$$\left. a A b \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + A b^2 \sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 4 a^2 \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + \right.$$

$$\left. 4 a b \sqrt{\frac{-a+b}{a+b}} B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \right)$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \\
& \left. i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 16 i a^2 C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 16 i a^2 C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + i(a-b)(A b+4 a B) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2i(a-b)(Ab+2a(A+2C)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4a \sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos[c+dx]} \sqrt{\sec[c+dx]} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right. \\
& \left.\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}}\right) \right)
\end{aligned}$$

■ **Problem 941: Result more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^3 \sqrt{a+b \sec[c+dx]} (A+B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 538 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{24 a^2 b d} (a-b) \sqrt{a+b} \left(3 A b^2 - 6 a b B - 8 a^2 (2 A + 3 C)\right) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{24 a^2 d} \sqrt{a+b} \left(3 A b^2 - 2 a b (A+3 B) - 4 a^2 (4 A+3 B+6 C)\right) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{8 a^3 d} \\
& \sqrt{a+b} \left(A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A+2 C)\right) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{(3 A b^2 - 6 a b B - 8 a^2 (2 A + 3 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 a^2 d} + \\
& \frac{(A b + 6 a B) \operatorname{Cos}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 a d} + \frac{A \operatorname{Cos}[c+d x]^2 \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 4, 4114 leaves):

$$\begin{aligned}
& \frac{\sqrt{a+b \operatorname{Sec}[c+d x]} \left(\frac{1}{12} A \operatorname{Sin}[c+d x] + \frac{(A b + 6 a B) \operatorname{Sin}[2(c+d x)]}{24 a} + \frac{1}{12} A \operatorname{Sin}[3(c+d x)]\right)}{d} + \\
& \left(\frac{7 A b}{12 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a B}{2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{b C}{\sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a A \sqrt{\operatorname{Sec}[c+d x]}}{3 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \right. \\
& \frac{A b^2 \sqrt{\operatorname{Sec}[c+d x]}}{48 a \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{3 b B \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{a C \sqrt{\operatorname{Sec}[c+d x]}}{2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{a A \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{3 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \\
& \left. \frac{A b^2 \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{16 a \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{b B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{a C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{2 \sqrt{b+a \operatorname{Cos}[c+d x]}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a+b} \operatorname{Sec}[c+dx] \left(\frac{(-3Ab^2 + 6abB + 8a^2(2A+3C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}}{24a^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}} + \right. \\
& \left. \left((a+b) (-3Ab^2 + 6abB + 8a^2(2A+3C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \right. \\
& \quad 2 \left(a (-Ab^2 + 12a^2B + 2ab(7A-3B+12C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \quad \left. \left. 3 (Ab^3 + 8a^3B - 2ab^2B + 4a^2b(A+2C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Big/ \\
& \left. \left(24a^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right) \Big/ \\
& \left(d \sqrt{b+a} \operatorname{Cos}[c+dx] \sqrt{\operatorname{Sec}[c+dx]} \left(\frac{(-3Ab^2 + 6abB + 8a^2(2A+3C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}}{48a^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}} - \right. \right. \\
& \left. \left((a+b) (-3Ab^2 + 6abB + 8a^2(2A+3C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \right. \\
& \quad \left. \left. 2 \left(a (-Ab^2 + 12a^2B + 2ab(7A-3B+12C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \left(A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C) \right) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \\
& \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \\
& \left(-2 b \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[\frac{1}{2} (c + d x) \right]^3 + 2 a \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \text{Tan} \left[\frac{1}{2} (c + d x) \right] \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \Bigg) / \\
& \left(24 a^2 \sqrt{\frac{1}{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \left(b - b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) - \\
& \left((a + b) (-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \\
& 2 \left(a (-A b^2 + 12 a^2 B + 2 a b (7 A - 3 B + 12 C)) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \left. 3 \left(A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C) \right) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 \\
& \left. \text{Tan} \left[\frac{1}{2} (c + d x) \right]^3 \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 + b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}{1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \right) / \\
& \left(24 a^2 \sqrt{\frac{1}{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4} \left(b - b \text{Tan} \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \text{Tan} \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) + \\
& \left((a + b) (-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \\
& 2 \left(a (-A b^2 + 12 a^2 B + 2 a b (7 A - 3 B + 12 C)) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \left. 3 \left(A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C) \right) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left/ \left(48 a^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \right. \\
& \left. \left. \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) - \frac{1}{48 a^2 \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right)} \right. \\
& \left((a+b) (-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2 \left(a (-A b^2 + 12 a^2 B + 2 a b (7 A - 3 B + 12 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \left. \left. 3 (A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} - \\
& \left((-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \left/ \left(48 a^2 \left(\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) \right. \\
& \left. \left((-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \\
& \left(48 a^2 \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left(\left((a+b) (-3 A b^2 + 6 a b B + 8 a^2 (2 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \right. \\
& \quad \left. \left. 2 \left(a (-A b^2 + 12 a^2 B + 2 a b (7 A - 3 B + 12 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 3 (A b^3 + 8 a^3 B - 2 a b^2 B + 4 a^2 b (A + 2 C)) \right. \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \quad \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}^4 \left(\frac{-a \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \quad \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \Bigg/ \\
& \left(48 a^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) + \\
& \left(\sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3465 b^5 d} 2 (a-b) \sqrt{a+b} (88 a^4 b B + 363 a^2 b^3 B + 1617 b^5 B - 48 a^5 C - 18 a^3 b^2 (11 A + 6 C) + 6 a b^4 (451 A + 348 C)) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \frac{1}{3465 b^4 d} \\
& 2 (a-b) \sqrt{a+b} (4 a^3 b (22 B - 9 C) - 48 a^4 C - 6 a^2 b^2 (33 A - 11 B + 24 C) + 3 b^4 (275 A - 539 B + 225 C) - 3 a b^3 (627 A - 143 B + 471 C)) \\
& \quad \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{3465 b^3 d} 2 (44 a^3 b B - 968 a b^3 B - 24 a^4 C - 75 b^4 (11 A + 9 C) - 3 a^2 b^2 (33 A + 19 C)) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx] + \\
& \frac{2 (33 a^2 b B + 539 b^3 B - 18 a^3 C + 6 a b^2 (132 A + 101 C)) \text{Sec}[c+dx] \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{3465 b^2 d} + \\
& \frac{2 (99 A b^2 + 110 a b B + 3 a^2 C + 81 b^2 C) \text{Sec}[c+dx]^2 \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{693 b d} + \\
& \frac{2 (11 b B + 3 a C) \text{Sec}[c+dx]^3 \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{99 d} + \frac{2 C \text{Sec}[c+dx]^3 (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{11 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 943: Attempted integration timed out after 120 seconds.**

$$\int \text{Sec}[c+dx]^2 (a+b \text{Sec}[c+dx])^{3/2} (A+B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 505 leaves, 7 steps):

$$\frac{1}{315 b^4 d} 2 (a-b) \sqrt{a+b} (18 a^3 b B - 246 a b^3 B - 8 a^4 C - 21 b^4 (9 A + 7 C) - 3 a^2 b^2 (21 A + 11 C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} (6 a^2 b (3 B - C) - 8 a^3 C - 3 a b^2 (21 A - 57 B + 13 C) + 3 b^3 (63 A - 25 B + 49 C)) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{2 (18 a^2 b B - 75 b^3 B - 8 a^3 C - 3 a b^2 (21 A + 13 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{315 b^2 d} +$$

$$\frac{2 (63 A b^2 - 18 a b B + 8 a^2 C + 49 b^2 C) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{315 b^2 d} +$$

$$\frac{2 (9 b B - 4 a C) (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{63 b^2 d} + \frac{2 C \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{9 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 944: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Sec}[c+d x] (a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 406 leaves, 6 steps):

$$-\frac{1}{105 b^3 d} 2 (a-b) \sqrt{a+b} (21 a^2 b B + 63 b^3 B - 6 a^3 C + 2 a b^2 (70 A + 41 C)) \operatorname{Cot}[c+d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 b^2 d}$$

$$2 (a-b) \sqrt{a+b} (6 a^2 C + 3 a b (35 A - 7 B + 19 C) - b^2 (35 A - 63 B + 25 C)) \operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{2 (35 A b^2 + 21 a b B - 6 a^2 C + 25 b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{105 b d} +$$

$$\frac{2 (7 b B - 2 a C) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{35 b d} + \frac{2 C (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{7 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 945: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 443 leaves, 7 steps):

$$-\frac{1}{15 b^2 d} 2 (a - b) \sqrt{a + b} (15 A b^2 + 20 a b B + 3 a^2 C + 9 b^2 C) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} +$$

$$\frac{1}{15 b d} 2 \sqrt{a + b} (3 a^2 (5 B - C) + 2 a b (15 A - 10 B + 6 C) - b^2 (15 A - 5 B + 9 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} - \frac{1}{d}$$

$$2 a A \sqrt{a + b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a + b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[c + d x])}{a - b}} +$$

$$\frac{2 (5 b B + 3 a C) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Tan}[c + d x]}{15 d} + \frac{2 C (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Tan}[c + d x]}{5 d}$$

Result (type 4, 1192 leaves):

$$-\left(\left(4 (a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \right.$$

$$\left. \left(15 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 15 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 20 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 20 a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 3 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right.$$

$$3 a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 9 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 9 b^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 30 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 40 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 -$$

$$6 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 18 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 15 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 15 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 20 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$\left. \left. 20 a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 3 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 3 a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 9 a b^2 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 9 b^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 \right) + \right.$$

$$\begin{aligned}
& 30 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + (a+b) (15 A b^2 + 20 a b B + 3 a^2 C + 9 b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \right. \\
& \left. \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& b (3 a^2 (-5 A + 5 B + C) + 2 a b (15 A + 10 B + 6 C) + b^2 (15 A + 5 B + 9 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right) \right] / \\
& \left(15 b d (b+a \operatorname{Cos}[c+dx])^{3/2} (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+dx]^{7/2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left(\operatorname{Cos}[c+dx]^3 (a+b \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \\
& \left. \left(\frac{4 (15 A b^2 + 20 a b B + 3 a^2 C + 9 b^2 C) \operatorname{Sin}[c+dx]}{15 b} + \frac{4}{15} \operatorname{Sec}[c+dx] (5 b B \operatorname{Sin}[c+dx] + 6 a C \operatorname{Sin}[c+dx]) + \frac{4}{5} b C \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) \right) / \\
& (d (b+a \operatorname{Cos}[c+dx]) (A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 d x]))
\end{aligned}$$

■ **Problem 946: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] (a+b \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 426 leaves, 7 steps):

$$\frac{1}{3bd} (a-b) \sqrt{a+b} (3aA - 6bB - 8aC) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} + \frac{1}{3bd} \sqrt{a+b} (ab(3A+12B-8C) + 6a^2C + 2b^2(3A-3B+C))$$

$$\operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{d}$$

$$\sqrt{a+b} (3Ab + 2aB) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{A(a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{d} - \frac{b(3A-2C) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{3d}$$

Result (type 4, 1208 leaves):

$$\left(2(a+b \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(3a^2 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 3aAb \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 6abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 6b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 8a^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right.$$

$$8abC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 6a^2 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 12abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 16a^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 3a^2 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$3aAb \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 6abB \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 6b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 8a^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 8abC \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 -$$

$$18aAb \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$12a^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} -$$

$$18aAb \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-12 a^2 \operatorname{B EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+ \\
& (a+b)(3 a A-6 b B-8 a C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}- \\
& 2(2 a b(3 A-3 B-2 C)+3 a^2(B-C)-b^2(3 A+3 B+C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \left.\left.\left.\left.\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}\right)\right]\right)\right) / \\
& \left(3 d(b+a \operatorname{Cos}[c+dx])^{3 / 2}(A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]) \operatorname{Sec}[c+dx]^{7 / 2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^{3 / 2}\right. \\
& \left.\left.\sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}}\right)+\right. \\
& \left.\left(\operatorname{Cos}[c+dx]^3(a+b \operatorname{Sec}[c+dx])^{3 / 2}(A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)\left(\frac{4}{3}(3 b B+4 a C) \operatorname{Sin}[c+dx]+\frac{4}{3} b C \operatorname{Tan}[c+dx]\right)\right)\right) / \\
& (d(b+a \operatorname{Cos}[c+dx])(A+2 C+2 B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2 c+2 dx]))
\end{aligned}$$

■ **Problem 949: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx]^4(a+b \operatorname{Sec}[c+dx])^{3 / 2}(A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 650 leaves, 9 steps):

$$\begin{aligned}
& -\frac{1}{192 a^2 b d} (a-b) \sqrt{a+b} \left(9 A b^3 - 128 a^3 B - 24 a b^2 B - 12 a^2 b (13 A + 20 C)\right) \\
& \quad \text{Cot}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \\
& \frac{1}{192 a^2 d} \sqrt{a+b} \left(9 A b^3 - 6 a b^2 (A+4 B) - 8 a^3 (9 A+16 B+12 C) - 4 a^2 b (39 A+28 B+60 C)\right) \text{Cot}[c+d x] \\
& \quad \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \frac{1}{64 a^3 d} \\
& \sqrt{a+b} \left(3 A b^4 + 96 a^3 b B - 8 a b^3 B + 24 a^2 b^2 (A+2 C) + 16 a^4 (3 A+4 C)\right) \text{Cot}[c+d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \quad \sqrt{\frac{b(1-\text{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+d x])}{a-b}} - \frac{(9 A b^3 - 128 a^3 B - 24 a b^2 B - 12 a^2 b (13 A + 20 C)) \sqrt{a+b \text{Sec}[c+d x]} \text{Sin}[c+d x]}{192 a^2 d} + \\
& \quad \frac{(3 A b^2 + 56 a b B + 12 a^2 (3 A + 4 C)) \text{Cos}[c+d x] \sqrt{a+b \text{Sec}[c+d x]} \text{Sin}[c+d x]}{96 a d} + \\
& \quad \frac{(3 A b + 8 a B) \text{Cos}[c+d x]^2 \sqrt{a+b \text{Sec}[c+d x]} \text{Sin}[c+d x]}{24 d} + \frac{A \text{Cos}[c+d x]^3 (a+b \text{Sec}[c+d x])^{3/2} \text{Sin}[c+d x]}{4 d}
\end{aligned}$$

Result (type 4, 4781 leaves):

$$\begin{aligned}
& \left(\text{Cos}[c+d x]^3 (a+b \text{Sec}[c+d x])^{3/2} (A+B \text{Sec}[c+d x] + C \text{Sec}[c+d x]^2) \left(\frac{1}{48} (9 A b + 8 a B) \text{Sin}[c+d x] + \right. \right. \\
& \quad \left. \left. \frac{(48 a^2 A + 3 A b^2 + 56 a b B + 48 a^2 C) \text{Sin}[2(c+d x)]}{96 a} + \frac{1}{48} (9 A b + 8 a B) \text{Sin}[3(c+d x)] + \frac{1}{16} a A \text{Sin}[4(c+d x)] \right) \right) / \\
& (d (b+a \text{Cos}[c+d x]) (A+2 C+2 B \text{Cos}[c+d x] + A \text{Cos}[2 c+2 d x])) + \\
& \left(\left(\frac{3 a^2 A}{4 \sqrt{b+a \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{19 A b^2}{16 \sqrt{b+a \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{13 a b B}{6 \sqrt{b+a \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \right. \right. \\
& \quad \left. \frac{a^2 C}{\sqrt{b+a \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{2 b^2 C}{\sqrt{b+a \text{Cos}[c+d x]} \sqrt{\text{Sec}[c+d x]}} + \frac{19 a A b \sqrt{\text{Sec}[c+d x]}}{16 \sqrt{b+a \text{Cos}[c+d x]}} - \frac{A b^3 \sqrt{\text{Sec}[c+d x]}}{64 a \sqrt{b+a \text{Cos}[c+d x]}} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a^2 B \sqrt{\operatorname{Sec}[c+d x]}}{3 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{17 b^2 B \sqrt{\operatorname{Sec}[c+d x]}}{24 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{7 a b C \sqrt{\operatorname{Sec}[c+d x]}}{4 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{13 a A b \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{16 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \\
& \frac{3 A b^3 \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{64 a \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2 a^2 B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{3 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{b^2 B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{8 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \\
& \left. \frac{5 a b C \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{4 \sqrt{b+a \operatorname{Cos}[c+d x]}} \right) (a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x])^2 \left(\frac{1}{96 a^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}} \right. \\
& \left. (-9 A b^3+128 a^3 B+24 a b^2 B+12 a^2 b(13 A+20 C)) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} + \right. \\
& \left. \left((a+b)(-9 A b^3+128 a^3 B+24 a b^2 B+12 a^2 b(13 A+20 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] - \right. \right. \\
& \left. 2 a(-3 A b^3+24 a^3(3 A+4 C)-4 a^2 b(9 A-52 B+12 C)+2 a b^2(57 A-28 B+96 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& \left. 6(3 A b^4+96 a^3 b B-8 a b^3 B+24 a^2 b^2(A+2 C)+16 a^4(3 A+4 C)) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4} \right) / \\
& \left. \left(96 a^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)^2 \right) \right) \right) /
\end{aligned}$$

$$\left(d (b + a \cos [c + d x])^{3/2} (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{7/2} \left(\frac{1}{192 a^2 \sqrt{\frac{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}}} \right. \right. \right.$$

$$\left. \left. \left. (-9 A b^3 + 128 a^3 B + 24 a b^2 B + 12 a^2 b (13 A + 20 C)) \sec \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} - \right. \right. \right.$$

$$\left((a + b) (-9 A b^3 + 128 a^3 B + 24 a b^2 B + 12 a^2 b (13 A + 20 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right.$$

$$2 a (-3 A b^3 + 24 a^3 (3 A + 4 C) - 4 a^2 b (9 A - 52 B + 12 C) + 2 a b^2 (57 A - 28 B + 96 C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] -$$

$$6 (3 A b^4 + 96 a^3 b B - 8 a b^3 B + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\left. \left. \left. \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^4} \right. \right. \right.$$

$$\left. \left. \left. \left(-2 b \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right]^3 + 2 a \sec \left[\frac{1}{2} (c + d x) \right]^2 \tan \left[\frac{1}{2} (c + d x) \right] \left(-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right) \right) \right) \right) /$$

$$\left(96 a^2 \sqrt{\frac{1}{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2}} \left(b - b \tan \left[\frac{1}{2} (c + d x) \right]^4 + a \left(-1 + \tan \left[\frac{1}{2} (c + d x) \right]^2 \right)^2 \right) \right) -$$

$$\left((a + b) (-9 A b^3 + 128 a^3 B + 24 a b^2 B + 12 a^2 b (13 A + 20 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right.$$

$$2 a (-3 A b^3 + 24 a^3 (3 A + 4 C) - 4 a^2 b (9 A - 52 B + 12 C) + 2 a b^2 (57 A - 28 B + 96 C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] -$$

$$6 (3 A b^4 + 96 a^3 b B - 8 a b^3 B + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right]$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \Big/ \\
& \left(96 a^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)\right) + \\
& \left((a+b) (-9 A b^3+128 a^3 B+24 a b^2 B+12 a^2 b (13 A+20 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& \quad 2 a (-3 A b^3+24 a^3 (3 A+4 C)-4 a^2 b (9 A-52 B+12 C)+2 a b^2 (57 A-28 B+96 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \\
& \quad \left. 6 (3 A b^4+96 a^3 b B-8 a b^3 B+24 a^2 b^2 (A+2 C)+16 a^4 (3 A+4 C)) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \Big/ \left(192 a^2 (a+b) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \\
& \quad \left. \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)\right) - \frac{1}{192 a^2 \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right)} \\
& \left((a+b) (-9 A b^3+128 a^3 B+24 a b^2 B+12 a^2 b (13 A+20 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& \quad 2 a (-3 A b^3+24 a^3 (3 A+4 C)-4 a^2 b (9 A-52 B+12 C)+2 a b^2 (57 A-28 B+96 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \\
& \quad \left. 6 (3 A b^4+96 a^3 b B-8 a b^3 B+24 a^2 b^2 (A+2 C)+16 a^4 (3 A+4 C)) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} - \\
& \left((-9Ab^3+128a^3B+24ab^2B+12a^2b(13A+20C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \left. \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \left(192a^2 \left(\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) + \left((-9Ab^3+128a^3B+24ab^2B+12a^2b(13A+20C)) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right. \\
& \left. \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right. \\
& \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) / \\
& \left(192a^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left((a+b) (-9Ab^3+128a^3B+24ab^2B+12a^2b(13A+20C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& \left. 2a(-3Ab^3+24a^3(3A+4C)-4a^2b(9A-52B+12C)+2ab^2(57A-28B+96C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& \left. 6(3Ab^4+96a^3bB-8ab^3B+24a^2b^2(A+2C)+16a^4(3A+4C)) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4}} \right. \\
& \left. \frac{\left(-a \sec\left[\frac{1}{2}(c+dx)\right]\right)^2 \tan\left[\frac{1}{2}(c+dx)\right] + b \sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right]}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \left. \frac{\sec\left[\frac{1}{2}(c+dx)\right]^2 \tan\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) / \\
& \left(192 a^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b-b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right) + \\
& \left(\sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4} \right. \\
& \left. - \left(a \left(-3 A b^3 + 24 a^3 (3 A + 4 C) - 4 a^2 b (9 A - 52 B + 12 C) + 2 a b^2 (57 A - 28 B + 96 C) \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) / \left(\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)^2 \right. \\
& \left. \sqrt{1-\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) + \frac{3 \left(3 A b^4 + 96 a^3 b B - 8 a b^3 B + 24 a^2 b^2 (A + 2 C) + 16 a^4 (3 A + 4 C) \right) \sec\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1-\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} + \\
& \left((a+b) \left(-9 A b^3 + 128 a^3 B + 24 a b^2 B + 12 a^2 b (13 A + 20 C) \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) /
\end{aligned}$$

$$\left(2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \right) \left/ \left(96 a^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right.$$

■ **Problem 950: Attempted integration timed out after 120 seconds.**

$$\int \sec[c+dx]^2 (a+b \sec[c+dx])^{5/2} (A+B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 610 leaves, 8 steps):

$$\frac{1}{3465 b^4 d} 2 (a-b) \sqrt{a+b} (110 a^4 b B - 3069 a^2 b^3 B - 1617 b^5 B - 40 a^5 C - 15 a^3 b^2 (33 A + 17 C) - 15 a b^4 (319 A + 247 C))$$

$$\cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{3465 b^3 d}$$

$$2 (a-b) \sqrt{a+b} (10 a^3 b (11 B - 3 C) - 40 a^4 C - 15 a^2 b^2 (33 A - 121 B + 19 C) - 3 b^4 (275 A - 539 B + 225 C) + 6 a b^3 (660 A - 209 B + 505 C))$$

$$\cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{1}{3465 b^2 d} 2 (110 a^3 b B - 1254 a b^3 B - 40 a^4 C - 75 b^4 (11 A + 9 C) - 15 a^2 b^2 (33 A + 19 C)) \sqrt{a+b \sec[c+dx]} \tan[c+dx] -$$

$$\frac{2 (110 a^2 b B - 539 b^3 B - 40 a^3 C - 5 a b^2 (99 A + 67 C)) (a+b \sec[c+dx])^{3/2} \tan[c+dx]}{3465 b^2 d} +$$

$$\frac{2 (99 A b^2 - 22 a b B + 8 a^2 C + 81 b^2 C) (a+b \sec[c+dx])^{5/2} \tan[c+dx]}{693 b^2 d} +$$

$$\frac{2 (11 b B - 4 a C) (a+b \sec[c+dx])^{7/2} \tan[c+dx]}{99 b^2 d} + \frac{2 C \sec[c+dx] (a+b \sec[c+dx])^{7/2} \tan[c+dx]}{11 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 951: Attempted integration timed out after 120 seconds.**

$$\int \sec[c+dx] (a+b \sec[c+dx])^{5/2} (A+B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 502 leaves, 7 steps):

$$\begin{aligned}
& -\frac{1}{315 b^3 d} 2 (a-b) \sqrt{a+b} \left(45 a^3 b B + 435 a b^3 B - 10 a^4 C + 21 b^4 (9 A + 7 C) + 3 a^2 b^2 (161 A + 93 C) \right) \\
& \quad \text{Cot}[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{1}{315 b^2 d} 2 (a-b) \sqrt{a+b} \left(10 a^3 C + 15 a^2 b (21 A - 3 B + 11 C) - 6 a b^2 (28 A - 60 B + 19 C) + 3 b^3 (63 A - 25 B + 49 C) \right) \\
& \quad \text{Cot}[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\text{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\text{Sec}[c+dx])}{a-b}} + \\
& \frac{2 \left(45 a^2 b B + 75 b^3 B - 10 a^3 C + 6 a b^2 (28 A + 19 C) \right) \sqrt{a+b \text{Sec}[c+dx]} \text{Tan}[c+dx]}{315 b d} + \\
& \frac{2 \left(63 A b^2 + 45 a b B - 10 a^2 C + 49 b^2 C \right) (a+b \text{Sec}[c+dx])^{3/2} \text{Tan}[c+dx]}{315 b d} + \\
& \frac{2 \left(9 b B - 2 a C \right) (a+b \text{Sec}[c+dx])^{5/2} \text{Tan}[c+dx]}{63 b d} + \frac{2 C (a+b \text{Sec}[c+dx])^{7/2} \text{Tan}[c+dx]}{9 b d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 952: Result more than twice size of optimal antiderivative.**

$$\int (a+b \text{Sec}[c+dx])^{5/2} (A+B \text{Sec}[c+dx]+C \text{Sec}[c+dx]^2) dx$$

Optimal (type 4, 521 leaves, 8 steps):

$$\begin{aligned}
& - \frac{1}{105 b^2 d} 2 (a-b) \sqrt{a+b} (161 a^2 b B + 63 b^3 B + 15 a^3 C + 5 a b^2 (49 A + 29 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{105 b d} \\
& 2 \sqrt{a+b} (15 a^3 (7 B - C) + b^3 (35 A - 63 B + 25 C) + a^2 b (315 A - 161 B + 135 C) - a b^2 (245 A - 119 B + 145 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{d} \\
& 2 a^2 A \sqrt{a+b} \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{2 (35 A b^2 + 56 a b B + 15 a^2 C + 25 b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{105 d} + \\
& \frac{2 (7 b B + 5 a C) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Tan}[c+d x]}{35 d} + \frac{2 C (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Tan}[c+d x]}{7 d}
\end{aligned}$$

Result (type 4, 1405 leaves):

$$\begin{aligned}
& - \left(\left(4 (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x] + C \operatorname{Sec}[c+d x]^2) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \right. \\
& \left. \left(245 a^2 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 245 a A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 161 a^3 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 161 a^2 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \right. \right. \\
& 63 a b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 63 b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 15 a^4 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 15 a^3 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + \\
& 145 a^2 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] + 145 a b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right] - 490 a^2 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 322 a^3 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - \\
& 126 a b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 30 a^4 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 - 290 a^2 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^3 + 245 a^2 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& 245 a A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 161 a^3 b B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 161 a^2 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 63 a b^3 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \\
& \left. \left. 63 b^4 B \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 15 a^4 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - 15 a^3 b C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 + 145 a^2 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^5 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 145 a b^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + 210 a^3 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 210 a^3 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (161 a^2 b B + 63 b^3 B + 15 a^3 C + 5 a b^2 (49 A + 29 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& b (-15 a^3 (7 A - 7 B - C) + b^3 (35 A + 63 B + 25 C) + a^2 b (315 A + 161 B + 135 C) + a b^2 (245 A + 119 B + 145 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(105 b d (b + a \cos[c+dx])^{5/2} (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sec[c+dx]^{9/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) \Bigg) + \\
& \frac{1}{d (b + a \cos[c+dx])^2 (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \cos[c+dx]^4} \\
& (a + b \sec[c+dx])^{5/2} (A + B \sec[c+dx] + C \sec[c+dx]^2) \\
& \left(\frac{4 (245 a A b^2 + 161 a^2 b B + 63 b^3 B + 15 a^3 C + 145 a b^2 C) \sin[c+dx]}{105 b} + \right. \\
& \left. \frac{4}{35} \sec[c+dx]^2 (7 b^2 B \sin[c+dx] + 15 a b C \sin[c+dx]) + \right.
\end{aligned}$$

$$\frac{4}{105} \operatorname{Sec}[c+dx] \left(35 A b^2 \operatorname{Sin}[c+dx] + 77 a b B \operatorname{Sin}[c+dx] + 45 a^2 C \operatorname{Sin}[c+dx] + 25 b^2 C \operatorname{Sin}[c+dx] \right) + \frac{4}{7} b^2 C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \Bigg)$$

■ **Problem 953: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c+dx] (a+b \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 505 leaves, 8 steps):

$$-\frac{1}{15bd} (a-b) \sqrt{a+b} (70abB - a^2(15A - 46C) + 6b^2(5A + 3C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{1}{15bd} \sqrt{a+b} (a^2b(15A + 90B - 46C) + 30a^3C - 2b^3(15A - 5B + 9C) + 2ab^2(45A - 35B + 17C)) \operatorname{Cot}[c+dx]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{1}{d}$$

$$a \sqrt{a+b} (5Ab + 2aB) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} +$$

$$\frac{A(a+b \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{d} - \frac{b(15aA - 10bB - 16aC) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Tan}[c+dx]}{15d} - \frac{b(5A - 2C)(a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Tan}[c+dx]}{5d}$$

Result (type 4, 1498 leaves):

$$\left(2(a+b \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left(15a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 15a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 30a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 30A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 70a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right.$$

$$70a b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 46a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 46a^2 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 18a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 18b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right.$$

$$30a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 60a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 140a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 92a^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 36a b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \right.$$

$$15a^3 A \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 15a^2 A b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 30a A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 30A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 70a^2 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \right.$$

$$\begin{aligned}
& 70 a^2 b^2 B \tan\left[\frac{1}{2}(c+dx)\right]^5 - 46 a^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + 46 a^2 b C \tan\left[\frac{1}{2}(c+dx)\right]^5 - 18 a b^2 C \tan\left[\frac{1}{2}(c+dx)\right]^5 + 18 b^3 C \tan\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 150 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 60 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 150 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 60 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-70 a b B + a^2 (15 A - 46 C) - 6 b^2 (5 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 2 (a^2 b (45 A - 45 B - 23 C) + 15 a^3 (B - C) - b^3 (15 A + 5 B + 9 C) - a b^2 (45 A + 35 B + 17 C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(15 d (b + a \cos[c+dx])^{5/2} (A + 2 C + 2 B \cos[c+dx] + A \cos[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right) +
\end{aligned}$$

$$\left(\cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \left(\frac{4}{15} (15 A b^2+35 a b B+23 a^2 C+9 b^2 C) \sin [c+d x]+ \right. \right. \\ \left. \left. \frac{4}{15} \operatorname{Sec}[c+d x] (5 b^2 B \sin [c+d x]+11 a b C \sin [c+d x])+\frac{4}{5} b^2 C \operatorname{Sec}[c+d x] \tan [c+d x] \right) \right) / \\ (d(b+a \cos [c+d x])^2(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

■ **Problem 955: Result more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^3 (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 549 leaves, 8 steps):

$$\frac{1}{24 b d} (a-b) \sqrt{a+b} (54 a b B+3 b^2 (11 A-16 C)+8 a^2 (2 A+3 C)) \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{24 d} \sqrt{a+b} (3 b^2 (11 A+16(B-C))+4 a^2 (4 A+3 B+6 C)+2 a b (13 A+27 B+72 C)) \\ \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{8 a d} \\ \sqrt{a+b} (5 A b^3+8 a^3 B+30 a b^2 B+20 a^2 b(A+2 C)) \cot [c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{(15 A b^2+42 a b B+8 a^2 (2 A+3 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \sin [c+d x]}{24 d} + \\ \frac{(5 A b+6 a B) \cos [c+d x] (a+b \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{12 d} + \frac{A \cos [c+d x]^2 (a+b \operatorname{Sec}[c+d x])^{5/2} \sin [c+d x]}{3 d}$$

Result (type 4, 2922 leaves):

$$\left(\cos [c+d x]^4 (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \left(\frac{1}{6} (a^2 A+24 b^2 C) \sin [c+d x]+ \frac{1}{12} a (13 A b+6 a B) \sin [2(c+d x)]+ \frac{1}{6} a^2 A \sin [3(c+d x)] \right) \right) / \\ (d(b+a \cos [c+d x])^2(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) + \\ \left(\cos [c+d x]^5 \left(\frac{19 a^2 A b}{6 \sqrt{b+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 A b^3}{\sqrt{b+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a^3 B}{\sqrt{b+a \cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \right. \right.$$

$$\begin{aligned} & \frac{6 a b^2 B}{\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{6 a^2 b C}{\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} - \frac{2 b^3 C}{\sqrt{b+a \cos [c+d x]} \sqrt{\sec [c+d x]}} + \frac{2 a^3 A \sqrt{\sec [c+d x]}}{3 \sqrt{b+a \cos [c+d x]}} + \\ & \frac{59 a A b^2 \sqrt{\sec [c+d x]}}{24 \sqrt{b+a \cos [c+d x]}} + \frac{11 a^2 b B \sqrt{\sec [c+d x]}}{4 \sqrt{b+a \cos [c+d x]}} + \frac{2 b^3 B \sqrt{\sec [c+d x]}}{\sqrt{b+a \cos [c+d x]}} + \frac{a^3 C \sqrt{\sec [c+d x]}}{\sqrt{b+a \cos [c+d x]}} + \frac{4 a b^2 C \sqrt{\sec [c+d x]}}{\sqrt{b+a \cos [c+d x]}} + \\ & \frac{2 a^3 A \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{3 \sqrt{b+a \cos [c+d x]}} + \frac{11 a A b^2 \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{8 \sqrt{b+a \cos [c+d x]}} + \frac{9 a^2 b B \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{4 \sqrt{b+a \cos [c+d x]}} + \\ & \left. \frac{a^3 C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{\sqrt{b+a \cos [c+d x]}} - \frac{2 a b^2 C \cos [2(c+d x)] \sqrt{\sec [c+d x]}}{\sqrt{b+a \cos [c+d x]}} \right) (a+b \sec [c+d x])^{5/2} (A+B \sec [c+d x]+C \sec [c+d x])^2 \end{aligned}$$

$$\left(\frac{1}{\sqrt{\cos [c+d x] \sec \left[\frac{1}{2}(c+d x) \right]^2}} \left((a+b) (54 a b B+3 b^2 (11 A-16 C)+8 a^2 (2 A+3 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x) \right] \right] \right], \frac{a-b}{a+b} \right) - \right.$$

$$2 (12 a^3 B+a b^2 (-13 A+72 (B-C))+24 b^3 (A-B-C)+2 a^2 b (19 A-3 B+36 C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] -$$

$$6 (5 A b^3+8 a^3 B+30 a b^2 B+20 a^2 b (A+2 C)) \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] \Bigg)$$

$$\left. \left. \frac{\sqrt{\frac{(b+a \cos [c+d x]) \sec \left[\frac{1}{2}(c+d x) \right]^2}{a+b}} + (54 a b B+3 b^2 (11 A-16 C)+8 a^2 (2 A+3 C)) (b+a \cos [c+d x]) \tan \left[\frac{1}{2}(c+d x) \right] \right]}{\right)} \right)$$

$$\left(12 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \left(\frac{1}{24 (b+a \cos [c+d x])^{3/2} \sqrt{\sec [c+d x]}} a \sin [c+d x] \right) \right.$$

$$\left(\frac{1}{\sqrt{\cos [c+d x] \sec \left[\frac{1}{2}(c+d x) \right]^2}} \left((a+b) (54 a b B+3 b^2 (11 A-16 C)+8 a^2 (2 A+3 C)) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x) \right] \right] \right], \frac{a-b}{a+b} \right) - \right.$$

$$2 (12 a^3 B+a b^2 (-13 A+72 (B-C))+24 b^3 (A-B-C)+2 a^2 b (19 A-3 B+36 C)) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x) \right] \right] \right], \frac{a-b}{a+b} \right] -$$

$$\begin{aligned}
& 6 \left(5 A b^3 + 8 a^3 B + 30 a b^2 B + 20 a^2 b (A + 2 C) \right) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \\
& \left. \sqrt{\frac{(b + a \text{Cos}[c + d x]) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) (b + a \text{Cos}[c + d x]) \text{Tan} \left[\frac{1}{2} (c + d x) \right]} \right) - \\
& \frac{1}{24 \sqrt{b + a \text{Cos}[c + d x]}} \sqrt{\text{Sec}[c + d x] \text{Sin}[c + d x]} \left(\frac{1}{\sqrt{\text{Cos}[c + d x] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2}} \right. \\
& \left. \left((a + b) (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \right. \\
& \left. \left. 2 (12 a^3 B + a b^2 (-13 A + 72 (B - C)) + 24 b^3 (A - B - C) + 2 a^2 b (19 A - 3 B + 36 C)) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \right. \\
& \left. \left. 6 (5 A b^3 + 8 a^3 B + 30 a b^2 B + 20 a^2 b (A + 2 C)) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \right. \\
& \left. \sqrt{\frac{(b + a \text{Cos}[c + d x]) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) (b + a \text{Cos}[c + d x]) \text{Tan} \left[\frac{1}{2} (c + d x) \right]} \right) + \\
& \frac{1}{12 \sqrt{b + a \text{Cos}[c + d x]} \sqrt{\text{Sec}[c + d x]}} \left(\frac{1}{2} (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) (b + a \text{Cos}[c + d x]) \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2 - \right. \\
& a (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) \text{Sin}[c + d x] \text{Tan} \left[\frac{1}{2} (c + d x) \right] - \frac{1}{2 (\text{Cos}[c + d x] \text{Sec} \left[\frac{1}{2} (c + d x) \right]^2)^{3/2}} \\
& \left. \left((a + b) (54 a b B + 3 b^2 (11 A - 16 C) + 8 a^2 (2 A + 3 C)) \text{EllipticE} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \right. \\
& \left. \left. 2 (12 a^3 B + a b^2 (-13 A + 72 (B - C)) + 24 b^3 (A - B - C) + 2 a^2 b (19 A - 3 B + 36 C)) \text{EllipticF} \left[\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] - \right. \right. \\
& \left. \left. 6 (5 A b^3 + 8 a^3 B + 30 a b^2 B + 20 a^2 b (A + 2 C)) \text{EllipticPi} \left[-1, -\text{ArcSin} \left[\text{Tan} \left[\frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \left(-\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sin [c+d x] + \cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right]\right) + \\
& \frac{1}{2 \sqrt{\cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}} \\
& \left((a+b) \left(54 a b B+3 b^2(11 A-16 C)+8 a^2(2 A+3 C)\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] - \right. \\
& 2\left(12 a^3 B+a b^2(-13 A+72(B-C))+24 b^3(A-B-C)+2 a^2 b(19 A-3 B+36 C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] - \\
& \left. 6\left(5 A b^3+8 a^3 B+30 a b^2 B+20 a^2 b(A+2 C)\right) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right]\right) \\
& \left(-\frac{a \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sin [c+d x]}{a+b} + \frac{(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \tan \left[\frac{1}{2}(c+d x)\right]}{a+b}\right) + \\
& \left(\sqrt{\frac{(b+a \cos [c+d x]) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \left(-\left((12 a^3 B+a b^2(-13 A+72(B-C))+24 b^3(A-B-C)+2 a^2 b(19 A-3 B+36 C)\right) \right.\right. \\
& \left.\left.\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2\right) / \left(\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{1-\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right)\right) + \\
& \frac{3\left(5 A b^3+8 a^3 B+30 a b^2 B+20 a^2 b(A+2 C)\right) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}{\sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}\left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{1-\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}} + \left((a+b) \left(54 a b B+3 b^2(11 A-16 C)+8 a^2(2 A+3 C)\right) \right. \\
& \left.\left.\operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\frac{(a-b) \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}}\right) / \left(2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}\right)\right) / \left(\sqrt{\cos [c+d x] \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^2}\right)\right)
\end{aligned}$$

■ **Problem 957: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x]^5 (a+b \sec [c+d x])^{5 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 774 leaves, 10 steps):

$$\begin{aligned} & -\frac{1}{1920 a^2 b d} (a-b) \sqrt{a+b} \left(45 A b^4 - 2840 a^3 b B - 150 a b^3 B - 256 a^4 (4 A+5 C) - 12 a^2 b^2 (141 A+220 C)\right) \\ & \quad \cot [c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \frac{1}{1920 a^2 d} \\ & \quad \sqrt{a+b} \left(45 A b^4 - 30 a b^3 (A+5 B) - 16 a^4 (64 A+45 B+80 C) - 8 a^3 b (193 A+355 B+260 C) - 4 a^2 b^2 (423 A+295 B+660 C)\right) \\ & \quad \cot [c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \\ & \quad \frac{1}{128 a^3 d} \sqrt{a+b} \left(3 A b^5 + 96 a^5 B + 240 a^3 b^2 B - 10 a b^4 B + 40 a^2 b^3 (A+2 C) + 80 a^4 b (3 A+4 C)\right) \cot [c+d x] \\ & \quad \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec [c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec [c+d x])}{a+b}} \sqrt{-\frac{b(1+\sec [c+d x])}{a-b}} - \\ & \quad \frac{1}{1920 a^2 d} \left(45 A b^4 - 2840 a^3 b B - 150 a b^3 B - 256 a^4 (4 A+5 C) - 12 a^2 b^2 (141 A+220 C)\right) \sqrt{a+b \sec [c+d x]} \sin [c+d x] + \\ & \quad \frac{\left(15 A b^3 + 360 a^3 B + 590 a b^2 B + 4 a^2 b (193 A+260 C)\right) \cos [c+d x] \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{960 a d} + \\ & \quad \frac{\left(15 A b^2 + 110 a b B + 16 a^2 (4 A+5 C)\right) \cos [c+d x]^2 \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{240 d} + \\ & \quad \frac{(A b+2 a B) \cos [c+d x]^3 (a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{8 d} + \frac{A \cos [c+d x]^4 (a+b \sec [c+d x])^{5 / 2} \sin [c+d x]}{5 d} \end{aligned}$$

Result (type 4, 942 leaves):

$$\begin{aligned} & \frac{1}{d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} \cos [c+d x]^4 (a+b \sec [c+d x])^{5 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2) \\ & \left(\frac{1}{480} (88 a^2 A+93 A b^2+170 a b B+80 a^2 C) \sin [c+d x] + \frac{(1024 a^2 A b+15 A b^3+480 a^3 B+590 a b^2 B+1040 a^2 b C) \sin [2(c+d x)]}{960 a} + \right. \\ & \left. \frac{1}{480} (100 a^2 A+93 A b^2+170 a b B+80 a^2 C) \sin [3(c+d x)] + \frac{1}{160} a (21 A b+10 a B) \sin [4(c+d x)] + \frac{1}{40} a^2 A \sin [5(c+d x)] \right) - \end{aligned}$$

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right.$$

$$\left. - \left(-45 A b^4 + 2840 a^3 b B + 150 a b^3 B + 256 a^4 (4 A + 5 C) + 12 a^2 b^2 (141 A + 220 C) \right) \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] - \right.$$

$$\left. \frac{1}{\sqrt{\frac{-a+b}{a+b} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right)^2 \right)}} i \right.$$

$$\left((a - b) \left(-45 A b^4 + 2840 a^3 b B + 150 a b^3 B + 256 a^4 (4 A + 5 C) + 12 a^2 b^2 (141 A + 220 C) \right) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right], \right. \right.$$

$$\left. \frac{a+b}{a-b} \right] - 2 (a - b) \left(-45 A b^4 - 30 a b^3 (A - 5 B) + 720 a^4 B + 4 a^2 b^2 (129 A + 185 B + 180 C) + 8 a^3 b (161 A + 45 B + 220 C) \right)$$

$$\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right], \frac{a+b}{a-b} \right] + 30 \left(3 A b^5 + 96 a^5 B + 240 a^3 b^2 B - 10 a b^4 B + \right.$$

$$\left. 40 a^2 b^3 (A + 2 C) + 80 a^4 b (3 A + 4 C) \right) \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right] \right], \frac{a+b}{a-b} \right]$$

$$\left. \left. \left(-1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 \right) \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{a + b}} \right) \right) \right) /$$

$$\left(960 a^2 d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2} \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]^2}} \right)$$

■ Problem 958: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Sec}[c + d x]^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\sqrt{a + b \text{Sec}[c + d x]}} dx$$

Optimal (type 4, 429 leaves, 6 steps):

$$-\frac{1}{105 b^5 d} 2 (a - b) \sqrt{a + b} (56 a^2 b B + 63 b^3 B - 48 a^3 C - 2 a b^2 (35 A + 22 C))$$

$$\text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{1}{105 b^4 d} 2 \sqrt{a + b} (48 a^3 C - 4 a^2 b (14 B + 3 C) + 2 a b^2 (35 A + 7 B + 22 C) + b^3 (35 A - 63 B + 25 C)) \text{Cot}[c + d x]$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{2 (35 A b^2 - 28 a b B + 24 a^2 C + 25 b^2 C) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{105 b^3 d} +$$

$$\frac{2 (7 b B - 6 a C) \text{Sec}[c + d x] \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{35 b^2 d} + \frac{2 C \text{Sec}[c + d x]^2 \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{7 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 959: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\sqrt{a + b \text{Sec}[c + d x]}} dx$$

Optimal (type 4, 342 leaves, 5 steps):

$$-\frac{1}{15 b^4 d} 2 (a - b) \sqrt{a + b} (15 A b^2 - 10 a b B + 8 a^2 C + 9 b^2 C) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right]$$

$$\sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} - \frac{1}{15 b^3 d} 2 \sqrt{a + b} (15 A b^2 - b^2 (5 B - 9 C) + 8 a^2 C - 2 a b (5 B + C))$$

$$\text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} +$$

$$\frac{2 (5 b B - 4 a C) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{15 b^2 d} + \frac{2 C \text{Sec}[c + d x] \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{5 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 960: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\sqrt{a + b \text{Sec}[c + d x]}} dx$$

Optimal (type 4, 267 leaves, 4 steps):

$$\begin{aligned} & -\frac{1}{3 b^3 d} \\ & 2 (a - b) \sqrt{a + b} (3 b B - 2 a C) \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \\ & \frac{1}{3 b^2 d} 2 \sqrt{a + b} (3 A b - b (3 B - C) + 2 a C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \\ & \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \frac{2 C \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{3 b d} \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 961: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2}{\sqrt{a + b \text{Sec}[c + d x]}} dx$$

Optimal (type 4, 317 leaves, 5 steps):

$$\begin{aligned} & -\frac{1}{b^2 d} 2 (a - b) \sqrt{a + b} C \text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} + \\ & \frac{2 \sqrt{a + b} (B - C) \text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}}}{b d} \\ & \frac{2 A \sqrt{a + b} \text{Cot}[c + d x] \text{EllipticPi}\left[\frac{a + b}{a}, \text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}}}{a d} \end{aligned}$$

Result (type 4, 762 leaves):

$$\begin{aligned}
& \frac{4 C \cos [c+d x] (b+a \cos [c+d x]) (A+B \sec [c+d x]+C \sec [c+d x]^2) \sin [c+d x]}{b d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{a+b \sec [c+d x]}} - \\
& \left(4 \sqrt{b+a \cos [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) \sqrt{\frac{1}{1-\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right. \\
& \left(a C \tan \left[\frac{1}{2}(c+d x)\right] + b C \tan \left[\frac{1}{2}(c+d x)\right] - 2 a C \tan \left[\frac{1}{2}(c+d x)\right]^3 + a C \tan \left[\frac{1}{2}(c+d x)\right]^5 - b C \tan \left[\frac{1}{2}(c+d x)\right]^5 + \right. \\
& 2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 2 A b \operatorname{EllipticPi} \left[-1, -\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \tan \left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + (a+b) C \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \\
& \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + b (A-B-C) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\tan \left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \left. \left. \sqrt{1-\tan \left[\frac{1}{2}(c+d x)\right]^2} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \right) \right) / \\
& \left(b d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)^{3/2} \right. \\
& \left. \sqrt{\frac{a+b-a \tan \left[\frac{1}{2}(c+d x)\right]^2+b \tan \left[\frac{1}{2}(c+d x)\right]^2}{1+\tan \left[\frac{1}{2}(c+d x)\right]^2}} \right)
\end{aligned}$$

■ **Problem 962: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 358 leaves, 6 steps):

$$\frac{A(a-b)\sqrt{a+b}\cot[c+dx]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{abd}}{\sqrt{a+b}(Ab+2aC)\cot[c+dx]\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{a^2d}} + \frac{\sqrt{a+b}(Ab-2aB)\cot[c+dx]\operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sec[c+dx]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]\sqrt{\frac{b(1-\sec[c+dx])}{a+b}}\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{A\sqrt{a+b}\sec[c+dx]\sin[c+dx]}{ad}}$$

Result (type 4, 861 leaves):

$$\left(2\sqrt{b+a\cos[c+dx]}(B+A\cos[c+dx]+C\sec[c+dx])\sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}}\right. \\ \left. \left(aA\tan\left[\frac{1}{2}(c+dx)\right]+Ab\tan\left[\frac{1}{2}(c+dx)\right]-2aA\tan\left[\frac{1}{2}(c+dx)\right]^3+aA\tan\left[\frac{1}{2}(c+dx)\right]^5-Ab\tan\left[\frac{1}{2}(c+dx)\right]^5\right)+\right. \\ \left.2Ab\operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right],\frac{a-b}{a+b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}-\right. \\ \left.4aB\operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right],\frac{a-b}{a+b}\right]\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}+\right. \\ \left.2Ab\operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right],\frac{a-b}{a+b}\right]\tan\left[\frac{1}{2}(c+dx)\right]^2\sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}\right)$$

$$\begin{aligned}
& -\frac{1}{4a^2bd} \\
& (a-b)\sqrt{a+b} (3Ab-4aB) \operatorname{Cot}[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{4a^2d} \sqrt{a+b} (3Ab-2a(A+2B)) \operatorname{Cot}[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \\
& \frac{1}{4a^3d} \sqrt{a+b} (3Ab^2-4abB+4a^2(A+2C)) \operatorname{Cot}[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b\operatorname{Sec}[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+dx])}{a+b}} \\
& \sqrt{-\frac{b(1+\operatorname{Sec}[c+dx])}{a-b}} - \frac{(3Ab-4aB)\sqrt{a+b\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{4a^2d} + \frac{A\operatorname{Cos}[c+dx]\sqrt{a+b\operatorname{Sec}[c+dx]}\operatorname{Sin}[c+dx]}{2ad}
\end{aligned}$$

Result (type 4, 1905 leaves):

$$\begin{aligned}
& \frac{A(b+a\operatorname{Cos}[c+dx])\operatorname{Sec}[c+dx]\operatorname{Sin}[2(c+dx)]}{4ad\sqrt{a+b\operatorname{Sec}[c+dx]}} + \\
& \left(\sqrt{b+a\operatorname{Cos}[c+dx]}\sqrt{\operatorname{Sec}[c+dx]}\sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(-3aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right) - \right. \\
& 3Ab^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right] + 4a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right] + 4ab\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right] + 6aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^3 - \\
& 8a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^3 - 3aAb\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 + 3Ab^2\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 4a^2\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^5 - 4ab\sqrt{\frac{-a+b}{a+b}}B\tan\left[\frac{1}{2}(c+dx)\right]^5 - 8ia^2A\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, \right. \\
& \left. i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a\tan\left[\frac{1}{2}(c+dx)\right]^2+b\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& \left. 6iAb^2\operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}}\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 16 i a^2 C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-8 i a^2 A \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 6 i A b^2 \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+8 i a b B \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}- \\
& 16 i a^2 C \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}-i(a-b)(-3 A b+4 a B) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}+
\end{aligned}$$

$$\begin{aligned}
& 2 i (3 A b^2 - a b (A + 4 B) + 2 a^2 (A + 2 C)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(4 a^2 \sqrt{\frac{-a+b}{a+b}} d \sqrt{a+b \operatorname{Sec}[c+d x]} \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}} \right. \right. \\
& \left. \left. \left(a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) - b \left(1 + \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right)\right)\right) \right)
\end{aligned}$$

■ **Problem 964: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c+d x]^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{(a+b \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 4, 510 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{15 b^5 \sqrt{a+b} d} 2 (40 a^3 b B - 25 a b^3 B - 6 a^2 b^2 (5 A - 4 C) - 48 a^4 C + 3 b^4 (5 A + 3 C)) \\
& \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+d x])}{a-b}} + \\
& \frac{1}{15 b^4 \sqrt{a+b} d} 2 (a^2 b (40 B - 36 C) - 48 a^3 C - 6 a b^2 (5 A - 5 B + 2 C) - b^3 (15 A - 5 B + 9 C)) \operatorname{Cot}[c+d x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c+d x])}{a-b}} - \\
& \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sec}[c+d x]^2 \operatorname{Tan}[c+d x]}{b (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 (20 a^2 b B - 5 b^3 B - 3 a b^2 (5 A - 3 C) - 24 a^3 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{15 b^3 (a^2 - b^2) d} + \\
& \frac{2 (5 A b^2 - 5 a b B + 6 a^2 C - b^2 C) \operatorname{Sec}[c+d x] \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Tan}[c+d x]}{5 b^2 (a^2 - b^2) d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 965: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c+dx]^2 (A+B\sec[c+dx]+C\sec[c+dx]^2)}{(a+b\sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 352 leaves, 5 steps):

$$-\frac{1}{3b^4\sqrt{a+b}d} 2(6a^2bB-3b^3B-ab^2(3A-5C)-8a^3C)\cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{3b^3\sqrt{a+b}d}$$

$$2(3Ab^2-(2a+b)(b(3B-C)-4aC))\cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}}$$

$$\sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{2a(Ab^2-a(bB-aC))\tan[c+dx]}{b^2(a^2-b^2)d\sqrt{a+b\sec[c+dx]}} + \frac{2C\sqrt{a+b\sec[c+dx]}\tan[c+dx]}{3b^2d}$$

Result (type 1, 1 leaves):

???

■ **Problem 966: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sec[c+dx] (A+B\sec[c+dx]+C\sec[c+dx]^2)}{(a+b\sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 293 leaves, 4 steps):

$$-\frac{1}{b^3\sqrt{a+b}d}$$

$$2(Ab^2-abB+2a^2C-b^2C)\cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} +$$

$$\frac{1}{b^2\sqrt{a+b}d} 2(Ab+b(B-C)-2aC)\cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{2(Ab^2-a(bB-aC))\tan[c+dx]}{b(a^2-b^2)d\sqrt{a+b\sec[c+dx]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 967: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 395 leaves, 6 steps):

$$\frac{1}{a b^2 \sqrt{a+b} d} 2 (A b^2 - a (b B - a C)) \operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{1}{a b \sqrt{a+b} d} 2 (A b - a (B + C)) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} -$$

$$\frac{2 A \sqrt{a+b} \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}}}{a^2 d} + \frac{2 (A b^2 - a (b B - a C)) \operatorname{Tan}[c + d x]}{a (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 4, 1275 leaves):

$$\left((b + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \left(\frac{4 (A b^2 - a b B + a^2 C) \operatorname{Sin}[c + d x]}{a b (-a^2 + b^2)} + \frac{4 (A b^2 \operatorname{Sin}[c + d x] - a b B \operatorname{Sin}[c + d x] + a^2 C \operatorname{Sin}[c + d x])}{a (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])} \right) \right) /$$

$$(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^{3/2}) -$$

$$\left(4 (b + a \operatorname{Cos}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right.$$

$$\left(a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right.$$

$$a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 2 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 2 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - 2 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 -$$

$$\left. 2 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{a+b}} \right) +$$

$$\begin{aligned}
& - \frac{1}{a^2 b \sqrt{a+b} d} (3 A b^2 - 2 a b B - a^2 (A - 2 C)) \operatorname{Cot}[c + d x] \\
& \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c + d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}} + \frac{1}{a^2 b \sqrt{a+b} d} \\
& (3 A b^2 + a b (A - 2 B) + 2 a^2 C) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c + d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}} + \\
& \frac{1}{a^3 d} \sqrt{a+b} (3 A b - 2 a B) \operatorname{Cot}[c + d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[c + d x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c + d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c + d x])}{a-b}} + \\
& \frac{A \operatorname{Sin}[c + d x]}{a d \sqrt{a+b} \operatorname{Sec}[c + d x]} - \frac{b(3 A b^2 - 2 a b B - a^2 (A - 2 C)) \operatorname{Tan}[c + d x]}{a^2 (a^2 - b^2) d \sqrt{a+b} \operatorname{Sec}[c + d x]}
\end{aligned}$$

Result (type 4, 1814 leaves):

$$\begin{aligned}
& \left((b + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \left. \left(\frac{4 (A b^2 - a b B + a^2 C) \operatorname{Sin}[c + d x]}{a^2 (a^2 - b^2)} - \frac{4 (A b^3 \operatorname{Sin}[c + d x] - a b^2 B \operatorname{Sin}[c + d x] + a^2 b C \operatorname{Sin}[c + d x])}{a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])} \right) \right) / \\
& (d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + d x])^{3/2}) - \\
& \left(2 (b + a \operatorname{Cos}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \sqrt{\frac{1}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right. \\
& \left. \left(a^3 A \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + a^2 A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 3 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + 2 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] + \right. \right. \\
& 2 a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 2 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 2 a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - 2 a^3 A \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 6 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 - \\
& 4 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + 4 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^3 + a^3 A \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - a^2 A b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 3 a A b^2 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + \\
& \left. \left. 3 A b^3 \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 2 a^2 b B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 2 a b^2 B \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 - 2 a^3 C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 + 2 a^2 b C \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^5 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 6 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 6 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - \\
& 4 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 4 a b^2 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& 6 a^2 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 6 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 4 a^3 B \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + 4 a b^2 B \operatorname{EllipticPi}\left[-1, \right. \\
& \left. -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} + \\
& (a+b) (-3 A b^2+2 a b B+a^2(A-2 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2} \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2\right) \\
& \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}} - 2 a(a+b)(-A b+a(B-C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right) /$$

$$\left(a^2 (a^2 - b^2) d (A + 2C + 2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{\sec[c+dx]} (a+b \sec[c+dx])^{3/2} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right.$$

$$\left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)$$

■ **Problem 969: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c+dx]^2 (A + B \sec[c+dx] + C \sec[c+dx]^2)}{(a+b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 552 leaves, 8 steps):

$$\frac{1}{4 a^3 b \sqrt{a+b} d} (15 A b^3 + 4 a^3 B - 12 a b^2 B - a^2 (7 A b - 8 b C)) \cot[c+dx]$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{1}{4 a^3 \sqrt{a+b} d}$$

$$(15 A b^2 + a b (5 A - 12 B) - 2 a^2 (A + 2 B - 4 C)) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \frac{1}{4 a^4 d} \sqrt{a+b} (15 A b^2 - 12 a b B + 4 a^2 (A + 2 C)) \cot[c+dx]$$

$$\text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} -$$

$$\frac{(5 A b - 4 a B) \sin[c+dx]}{4 a^2 d \sqrt{a+b \sec[c+dx]}} + \frac{A \cos[c+dx] \sin[c+dx]}{2 a d \sqrt{a+b \sec[c+dx]}} + \frac{b (15 A b^3 + 4 a^3 B - 12 a b^2 B - a^2 (7 A b - 8 b C)) \tan[c+dx]}{4 a^3 (a^2 - b^2) d \sqrt{a+b \sec[c+dx]}}$$

Result (type 4, 746 leaves):

$$\begin{aligned}
& \frac{1}{2} \left(\frac{1}{d (a + b \operatorname{Sec}[c + d x])^{3/2}} (b + a \operatorname{Cos}[c + d x])^2 \operatorname{Sec}[c + d x]^2 \right. \\
& \left. \left(\frac{4 b (A b^2 - a b B + a^2 C) \operatorname{Sin}[c + d x]}{a^3 (-a^2 + b^2)} + \frac{4 (A b^4 \operatorname{Sin}[c + d x] - a b^3 B \operatorname{Sin}[c + d x] + a^2 b^2 C \operatorname{Sin}[c + d x])}{a^3 (a^2 - b^2) (b + a \operatorname{Cos}[c + d x])} + \frac{A \operatorname{Sin}[2 (c + d x)]}{2 a^2} \right) + \right. \\
& \left. \frac{1}{2 a^3 (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2}} \sqrt{\frac{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}} \right) \\
& \left((15 A b^3 + 4 a^3 B - 12 a b^2 B + a^2 (-7 A b + 8 b C)) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - \frac{1}{\sqrt{\frac{-a+b}{a+b} \left(b - b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right)^2}} \right) \right. \\
& \left. i (a - b) \left((-15 A b^3 - 4 a^3 B + 12 a b^2 B + a^2 b (7 A - 8 C)) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right], \frac{a+b}{a-b} \right] + \right. \right. \\
& \left. \left. 2 (15 A b^3 + 2 a b^2 (5 A - 6 B) + 2 a^3 (A + 2 C) + a^2 b (A - 8 B + 8 C)) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right], \frac{a+b}{a-b} \right] - \right. \right. \\
& \left. \left. 2 (a + b) (15 A b^2 - 12 a b B + 4 a^2 (A + 2 C)) \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] \right], \frac{a+b}{a-b} \right] \right) \right) \\
& \left. \left(-1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 \right) \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2}{a + b}} \right) \right)
\end{aligned}$$

■ **Problem 970: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 549 leaves, 6 steps) :

$$-\frac{1}{3 b^5 \sqrt{a+b} (a^2 - b^2) d} 2 (8 a^4 b B - 15 a^2 b^3 B + 3 b^5 B - 2 a^3 b^2 (A - 14 C) + 2 a b^4 (3 A - 4 C) - 16 a^5 C)$$

$$\text{Cot}[c + d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} -$$

$$\frac{1}{3 b^4 \sqrt{a+b} (a^2 - b^2) d} 2 (a^3 b (8 B - 12 C) - 2 a^2 b^2 (A - 3 B - 8 C) - 3 a b^3 (A + 3 B - 3 C) - 16 a^4 C + b^4 (3 A - 3 B + C))$$

$$\text{Cot}[c + d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a + b \text{Sec}[c + d x]}}{\sqrt{a + b}}\right], \frac{a + b}{a - b}\right] \sqrt{\frac{b (1 - \text{Sec}[c + d x])}{a + b}} \sqrt{-\frac{b (1 + \text{Sec}[c + d x])}{a - b}} -$$

$$\frac{2 (A b^2 - a (b B - a C)) \text{Sec}[c + d x]^2 \text{Tan}[c + d x]}{3 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} - \frac{2 a (4 A b^4 + a (3 a^2 b B - 7 b^3 B - 6 a^3 C + 10 a b^2 C)) \text{Tan}[c + d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}} +$$

$$\frac{2 (A b^2 - a b B + 2 a^2 C - b^2 C) \sqrt{a + b \text{Sec}[c + d x]} \text{Tan}[c + d x]}{3 b^3 (a^2 - b^2) d}$$

Result (type 1, 1 leaves) :

???

■ **Problem 971: Attempted integration timed out after 120 seconds.**

$$\int \frac{\text{Sec}[c + d x]^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 449 leaves, 5 steps) :

$$\frac{1}{3 b^4 \sqrt{a+b} (a^2 - b^2) d} 2 (2 a^3 b B - 6 a b^3 B + 3 b^4 (A - C) - 8 a^4 C + a^2 b^2 (A + 15 C))$$

$$\operatorname{Cot}[c + d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{1}{3 b^3 \sqrt{a+b} (a^2 - b^2) d} 2 (2 a^2 b (B - 3 C) - 3 b^3 (A + B - C) - 8 a^3 C + a b^2 (A + 3 B + 9 C)) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 a (A b^2 - a (b B - a C)) \operatorname{Tan}[c + d x]}{3 b^2 (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 (3 A b^4 + 2 a^3 b B - 6 a b^3 B - 5 a^4 C + a^2 b^2 (A + 9 C)) \operatorname{Tan}[c + d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 972: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 416 leaves, 5 steps):

$$-\frac{1}{3 (a-b) b^3 (a+b)^{3/2} d} 2 (4 a A b^2 - a^2 b B - 3 b^3 B - 2 a^3 C + 6 a b^2 C) \operatorname{Cot}[c + d x]$$

$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 b^2 \sqrt{a+b} (a^2 - b^2) d}$$

$$2 (2 a^2 C + a b (3 A + B + 3 C) - b^2 (A + 3 (B + C))) \operatorname{Cot}[c + d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}}$$

$$\sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{2 (A b^2 - a (b B - a C)) \operatorname{Tan}[c + d x]}{3 b (a^2 - b^2) d (a + b \operatorname{Sec}[c + d x])^{3/2}} + \frac{2 (a^2 b B + 3 b^3 B + 2 a^3 C - 2 a b^2 (2 A + 3 C)) \operatorname{Tan}[c + d x]}{3 b (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 973: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{(a + b \operatorname{Sec}[c + dx])^{5/2}} dx$$

Optimal (type 4, 541 leaves, 7 steps):

$$\frac{1}{3 a^2 (a-b) b^2 (a+b)^{3/2} d} {}_2 F_1 \left(7 a^2 A b^2 - 3 A b^4 - 4 a^3 b B + a^4 C + 3 a^2 b^2 C, \operatorname{Cot}[c + dx] \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \operatorname{Sec}[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \right)$$

$$\sqrt{\frac{b(1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c + dx])}{a - b}} + \frac{1}{3 a^2 b \sqrt{a + b} (a^2 - b^2) d} {}_2 F_1 \left(a A b^2 + 3 A b^3 + a^3 (3 B + C) - a^2 b (6 A + B + 3 C) \right)$$

$$\operatorname{Cot}[c + dx] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{a + b \operatorname{Sec}[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b(1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c + dx])}{a - b}} -$$

$$\frac{2 A \sqrt{a + b} \operatorname{Cot}[c + dx] \operatorname{EllipticPi} \left[\frac{a + b}{a}, \operatorname{ArcSin} \left[\frac{\sqrt{a + b \operatorname{Sec}[c + dx]}}{\sqrt{a + b}} \right], \frac{a + b}{a - b} \right] \sqrt{\frac{b(1 - \operatorname{Sec}[c + dx])}{a + b}} \sqrt{-\frac{b(1 + \operatorname{Sec}[c + dx])}{a - b}}}{a^3 d} +$$

$$\frac{2 (A b^2 - a (b B - a C)) \operatorname{Tan}[c + dx]}{3 a (a^2 - b^2) d (a + b \operatorname{Sec}[c + dx])^{3/2}} - \frac{2 (3 A b^4 + 4 a^3 b B - a^4 C - a^2 b^2 (7 A + 3 C)) \operatorname{Tan}[c + dx]}{3 a^2 (a^2 - b^2)^2 d \sqrt{a + b \operatorname{Sec}[c + dx]}}$$

Result (type 4, 1919 leaves):

$$\left((b + a \operatorname{Cos}[c + dx])^3 \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right.$$

$$\left(\frac{4 (-7 a^2 A b^2 + 3 A b^4 + 4 a^3 b B - a^4 C - 3 a^2 b^2 C) \operatorname{Sin}[c + dx]}{3 a^2 b (-a^2 + b^2)^2} - \frac{4 (A b^3 \operatorname{Sin}[c + dx] - a b^2 B \operatorname{Sin}[c + dx] + a^2 b C \operatorname{Sin}[c + dx])}{3 a^2 (a^2 - b^2) (b + a \operatorname{Cos}[c + dx])^2} + \right.$$

$$\left. \left. (4 (8 a^2 A b^2 \operatorname{Sin}[c + dx] - 4 A b^4 \operatorname{Sin}[c + dx] - 5 a^3 b B \operatorname{Sin}[c + dx] + a b^3 B \operatorname{Sin}[c + dx] + 2 a^4 C \operatorname{Sin}[c + dx] + 2 a^2 b^2 C \operatorname{Sin}[c + dx])) \right) \right) / \left(d (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 d x]) (a + b \operatorname{Sec}[c + dx])^{5/2} \right) -$$

$$\left(4 (b + a \operatorname{Cos}[c + dx])^{5/2} \sqrt{\operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \sqrt{\frac{1}{1 - \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2}} \right.$$

$$\left. \sqrt{\frac{a + b - a \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2 + b \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2}{1 + \operatorname{Tan} \left[\frac{1}{2} (c + dx) \right]^2}} \right)$$

$$\begin{aligned}
& \left(7 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 7 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 3 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - 4 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \right. \\
& 4 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 3 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + 3 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] - \\
& 14 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 6 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 8 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 2 a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 - 6 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + \\
& 7 a^3 A b^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 7 a^2 A b^3 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a A b^4 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 A b^5 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 4 a^4 b B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + \\
& 4 a^3 b^2 B \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + a^5 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - a^4 b C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 + 3 a^3 b^2 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - 3 a^2 b^3 C \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^5 - \\
& 6 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 12 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 6 a^4 A b \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 12 a^2 A b^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 6 A b^5 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& (a+b) (-3 A b^4 - 4 a^3 b B + a^4 C + a^2 b^2 (7 A + 3 C)) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& ab(a+b) \left(-2Ab^2 + a^2(3A-3B+C) + ab(3A-B+3C)\right) \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \Bigg) \Bigg) / \\
& \left(3a^2b(a^2-b^2)^2d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+b\sec[c+dx])^{5/2} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \\
& \left. \left(a\left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b\left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right)\right)\right)
\end{aligned}$$

■ **Problem 974: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx] (A+B\sec[c+dx]+C\sec[c+dx]^2)}{(a+b\sec[c+dx])^{5/2}} dx$$

Optimal (type 4, 618 leaves, 8 steps):

$$\begin{aligned}
& -\frac{1}{3a^3(a-b)b(a+b)^{3/2}d} (26a^2Ab^2 - 15Ab^4 - 14a^3bB + 6ab^3B - a^4(3A-8C)) \\
& \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} - \\
& \frac{1}{3a^3b\sqrt{a+b}(a^2-b^2)d} (15Ab^4 + ab^3(5A-6B) - a^2b^2(21A+2B) - 6a^4C - a^3b(3A-2(6B+C))) \cot[c+dx] \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \frac{1}{a^4d} \\
& \sqrt{a+b}(5Ab-2aB) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}} + \\
& \frac{A\sin[c+dx]}{ad(a+b\sec[c+dx])^{3/2}} - \frac{b(5Ab^2-2abB-a^2(3A-2C))\tan[c+dx]}{3a^2(a^2-b^2)d(a+b\sec[c+dx])^{3/2}} - \frac{b(26a^2Ab^2-15Ab^4-14a^3bB+6ab^3B-a^4(3A-8C))\tan[c+dx]}{3a^3(a^2-b^2)^2d\sqrt{a+b\sec[c+dx]}}
\end{aligned}$$

Result (type 4, 2631 leaves):

$$\begin{aligned}
 & \left((b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
 & \left. - \frac{4(-10a^2Ab^2 + 6Ab^4 + 7a^3bB - 3ab^3B - 4a^4C) \sin[c + dx]}{3a^3(-a^2 + b^2)^2} + \frac{4(Ab^4 \sin[c + dx] - ab^3B \sin[c + dx] + a^2b^2C \sin[c + dx])}{3a^3(a^2 - b^2)(b + a \cos[c + dx])^2} + \right. \\
 & \left. (4(-11a^2Ab^3 \sin[c + dx] + 7Ab^5 \sin[c + dx] + 8a^3b^2B \sin[c + dx] - 4ab^4B \sin[c + dx] - 5a^4bC \sin[c + dx] + a^2b^3C \sin[c + dx])) \right) / \\
 & \left. (3a^3(a^2 - b^2)^2(b + a \cos[c + dx])) \right) / (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])(a + b \sec[c + dx])^{5/2}) - \\
 & \left(2(b + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx])^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
 & \left. \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c + dx)\right]^2}} \right. \\
 & \left(3a^5A \tan\left[\frac{1}{2}(c + dx)\right] + 3a^4Ab \tan\left[\frac{1}{2}(c + dx)\right] - 26a^3Ab^2 \tan\left[\frac{1}{2}(c + dx)\right] - 26a^2Ab^3 \tan\left[\frac{1}{2}(c + dx)\right] + 15aAb^4 \tan\left[\frac{1}{2}(c + dx)\right] + \right. \\
 & 15Ab^5 \tan\left[\frac{1}{2}(c + dx)\right] + 14a^4bB \tan\left[\frac{1}{2}(c + dx)\right] + 14a^3b^2B \tan\left[\frac{1}{2}(c + dx)\right] - 6a^2b^3B \tan\left[\frac{1}{2}(c + dx)\right] - 6ab^4B \tan\left[\frac{1}{2}(c + dx)\right] - \\
 & 8a^5C \tan\left[\frac{1}{2}(c + dx)\right] - 8a^4bC \tan\left[\frac{1}{2}(c + dx)\right] - 6a^5A \tan\left[\frac{1}{2}(c + dx)\right]^3 + 52a^3Ab^2 \tan\left[\frac{1}{2}(c + dx)\right]^3 - 30aAb^4 \tan\left[\frac{1}{2}(c + dx)\right]^3 - \\
 & 28a^4bB \tan\left[\frac{1}{2}(c + dx)\right]^3 + 12a^2b^3B \tan\left[\frac{1}{2}(c + dx)\right]^3 + 16a^5C \tan\left[\frac{1}{2}(c + dx)\right]^3 + 3a^5A \tan\left[\frac{1}{2}(c + dx)\right]^5 - 3a^4Ab \tan\left[\frac{1}{2}(c + dx)\right]^5 - \\
 & 26a^3Ab^2 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 26a^2Ab^3 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 15aAb^4 \tan\left[\frac{1}{2}(c + dx)\right]^5 - 15Ab^5 \tan\left[\frac{1}{2}(c + dx)\right]^5 + 14a^4bB \tan\left[\frac{1}{2}(c + dx)\right]^5 - \\
 & 14a^3b^2B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 6a^2b^3B \tan\left[\frac{1}{2}(c + dx)\right]^5 + 6ab^4B \tan\left[\frac{1}{2}(c + dx)\right]^5 - 8a^5C \tan\left[\frac{1}{2}(c + dx)\right]^5 + 8a^4bC \tan\left[\frac{1}{2}(c + dx)\right]^5 + \\
 & 30a^4Ab \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} - \\
 & \left. 60a^2Ab^3 \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c + dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1 - \tan\left[\frac{1}{2}(c + dx)\right]^2} \sqrt{\frac{a + b - a \tan\left[\frac{1}{2}(c + dx)\right]^2 + b \tan\left[\frac{1}{2}(c + dx)\right]^2}{a + b}} \right) +
 \end{aligned}$$

$$\begin{aligned}
& 30 a b^5 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 12 a^5 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 24 a^3 b^2 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 12 a b^4 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 30 a^4 A b \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 60 a^2 A b^3 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 30 a b^5 \text{EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\
& \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\
& 12 a^5 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + 24 a^3 b^2 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \\
& \sqrt{1-\text{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \text{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 12 a b^4 \text{B EllipticPi}\left[-1, -\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right]
\end{aligned}$$

$$\begin{aligned} & \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\ & (a+b) \left(-26 a^2 A b^2 + 15 A b^4 + 14 a^3 b B - 6 a b^3 B + a^4 (3A - 8C)\right) \text{EllipticE}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\ & \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - \\ & 2 a (a+b) \left(5 A b^3 - a b^2 (3A + 2B) + 3 a^3 (B - C) - a^2 b (6A - 3B + C)\right) \text{EllipticF}\left[\text{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \\ & \left. \sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \right] / \\ & \left(3 a (a^3 - a b^2)^2 d (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) (a + b \sec[c+dx])^{5/2} \sqrt{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2} \right. \\ & \left. \left(a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) - b \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \right) \end{aligned}$$

■ **Problem 977: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a b B - a^2 C + b^2 B \sec[c+dx] + b^2 C \sec[c+dx]^2}{\sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 316 leaves, 6 steps):

$$\begin{aligned} & -\frac{1}{d} (a-b) \sqrt{a+b} C \cot[c+dx] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c+dx])}{a-b}} + \frac{1}{d} \\ & 2 b \sqrt{a+b} (B-C) \cot[c+dx] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c+dx])}{a-b}} - \frac{1}{d} \\ & 2 \sqrt{a+b} (bB - aC) \cot[c+dx] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1 - \sec[c+dx])}{a+b}} \sqrt{-\frac{b(1 + \sec[c+dx])}{a-b}} \end{aligned}$$

Result (type 4, 1232 leaves):

$$\begin{aligned}
& \frac{2 b C \cos [c+d x] \sqrt{a+b \sec [c+d x]} (b B-a C+b C \sec [c+d x]) \sin [c+d x]}{d (b C+b B \cos [c+d x]-a C \cos [c+d x])} + \\
& \left(2 \sqrt{a+b \sec [c+d x]} (b B-a C+b C \sec [c+d x]) \left(a b \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2} (c+d x) \right] + b^2 \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2} (c+d x) \right] - \right. \right. \\
& 2 a b \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2} (c+d x) \right]^3 + a b \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2} (c+d x) \right]^5 - b^2 \sqrt{\frac{-a+b}{a+b}} C \tan \left[\frac{1}{2} (c+d x) \right]^5 + \\
& 2 i a b B \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2+b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - 2 i a^2 C \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2+b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} + \\
& 2 i a b B \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \tan \left[\frac{1}{2} (c+d x) \right]^2 \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^2} \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2+b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - 2 i a^2 C \text{EllipticPi} \left[-\frac{a+b}{a-b}, i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \\
& \tan \left[\frac{1}{2} (c+d x) \right]^2 \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^2} \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2+b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} - \\
& i (a-b) b C \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right] \sqrt{1-\tan \left[\frac{1}{2} (c+d x) \right]^2} \left(1+\tan \left[\frac{1}{2} (c+d x) \right]^2 \right) \\
& \sqrt{\frac{a+b-a \tan \left[\frac{1}{2} (c+d x) \right]^2+b \tan \left[\frac{1}{2} (c+d x) \right]^2}{a+b}} + i (a-b) (a C+b (-B+C)) \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{-a+b}{a+b}} \tan \left[\frac{1}{2} (c+d x) \right] \right], \frac{a+b}{a-b} \right]
\end{aligned}$$

$$\left(\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg/$$

$$\left(\sqrt{\frac{-a+b}{a+b}} d \sqrt{b+a \cos[c+dx]} (bC + bB \cos[c+dx] - aC \cos[c+dx]) \sec[c+dx]^{3/2} \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right.$$

$$\left. \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^2 \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]\right)^{3/2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1 + \tan\left[\frac{1}{2}(c+dx)\right]^2}} \right)$$

■ **Problem 979: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a b B - a^2 C + b^2 B \sec[c+dx] + b^2 C \sec[c+dx]^2}{(a+b \sec[c+dx])^{5/2}} dx$$

Optimal (type 4, 379 leaves, 7 steps):

$$\frac{2(bB - 2aC) \cot[c+dx] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a \sqrt{a+b} d}$$

$$\frac{2(bB - 2aC) \cot[c+dx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a \sqrt{a+b} d} - \frac{1}{a^2 d}$$

$$\frac{2\sqrt{a+b} (bB - aC) \cot[c+dx] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[c+dx]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[c+dx])}{a+b}} \sqrt{-\frac{b(1+\sec[c+dx])}{a-b}}}{a (a^2 - b^2) d \sqrt{a+b \sec[c+dx]}} +$$

$$\frac{2b^2 (bB - 2aC) \tan[c+dx]}{a (a^2 - b^2) d \sqrt{a+b \sec[c+dx]}}$$

Result (type 4, 2090 leaves):

$$\left((b+a \cos[c+dx])^2 \sec[c+dx] (bB - aC + bC \sec[c+dx]) \left(\frac{2b(bB - 2aC) \sin[c+dx]}{a(-a^2 + b^2)} - \frac{2(-b^3 B \sin[c+dx] + 2ab^2 C \sin[c+dx])}{a(a^2 - b^2)(b+a \cos[c+dx])} \right) \right) \Bigg/$$

$$(d(bC + bB \cos[c+dx] - aC \cos[c+dx]) (a+b \sec[c+dx])^{3/2}) -$$

$$\begin{aligned}
& \left(2 (b + a \cos [c + d x])^{3/2} \sqrt{\sec [c + d x]} (b B - a C + b C \sec [c + d x]) \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{1 + \tan \left[\frac{1}{2} (c + d x) \right]^2}} \right. \\
& \left. - a b^2 \sqrt{\frac{-a + b}{a + b}} B \tan \left[\frac{1}{2} (c + d x) \right] - b^3 \sqrt{\frac{-a + b}{a + b}} B \tan \left[\frac{1}{2} (c + d x) \right] + 2 a^2 b \sqrt{\frac{-a + b}{a + b}} C \tan \left[\frac{1}{2} (c + d x) \right] + 2 a b^2 \sqrt{\frac{-a + b}{a + b}} C \tan \left[\frac{1}{2} (c + d x) \right] + \right. \\
& 2 a b^2 \sqrt{\frac{-a + b}{a + b}} B \tan \left[\frac{1}{2} (c + d x) \right]^3 - 4 a^2 b \sqrt{\frac{-a + b}{a + b}} C \tan \left[\frac{1}{2} (c + d x) \right]^3 - a b^2 \sqrt{\frac{-a + b}{a + b}} B \tan \left[\frac{1}{2} (c + d x) \right]^5 + \\
& b^3 \sqrt{\frac{-a + b}{a + b}} B \tan \left[\frac{1}{2} (c + d x) \right]^5 + 2 a^2 b \sqrt{\frac{-a + b}{a + b}} C \tan \left[\frac{1}{2} (c + d x) \right]^5 - 2 a b^2 \sqrt{\frac{-a + b}{a + b}} C \tan \left[\frac{1}{2} (c + d x) \right]^5 + \\
& 2 i a^2 b B \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - 2 i b^3 B \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} - 2 i a^3 C \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, \right. \\
& \left. i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + \\
& 2 i a b^2 C \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \\
& \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} + 2 i a^2 b B \operatorname{EllipticPi} \left[-\frac{a + b}{a - b}, i \operatorname{ArcSinh} \left[\sqrt{\frac{-a + b}{a + b}} \tan \left[\frac{1}{2} (c + d x) \right] \right], \frac{a + b}{a - b} \right] \\
& \tan \left[\frac{1}{2} (c + d x) \right]^2 \sqrt{1 - \tan \left[\frac{1}{2} (c + d x) \right]^2} \sqrt{\frac{a + b - a \tan \left[\frac{1}{2} (c + d x) \right]^2 + b \tan \left[\frac{1}{2} (c + d x) \right]^2}{a + b}} -
\end{aligned}$$

$$\begin{aligned}
& 2 i b^3 \text{B EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - 2 i a^3 \text{C EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& 2 i a b^2 \text{C EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} - i(a-b)b(-bB+2aC) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} + \\
& i(a-b)(-2b^2B-ab(B-3C)+a^2C) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^2} \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2+b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right) \Bigg) / \\
& \left(a \sqrt{\frac{-a+b}{a+b}} (a^2-b^2) d (bC+bB \cos[c+dx]-aC \cos[c+dx]) (a+b \sec[c+dx])^{3/2} \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right. \\
& \left. \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(a \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) - b \left(1+\tan\left[\frac{1}{2}(c+dx)\right]^2\right) \right) \right)
\end{aligned}$$

■ **Problem 980: Result more than twice size of optimal antiderivative.**

$$\int \frac{a b B - a^2 C + b^2 B \sec[c+dx] + b^2 C \sec[c+dx]^2}{(a+b \sec[c+dx])^{7/2}} dx$$

Optimal (type 4, 519 leaves, 8 steps) :

$$\frac{1}{3 a^2 (a-b) (a+b)^{3/2} d} 2 (7 a^2 b B - 3 b^3 B - 11 a^3 C + 3 a b^2 C) \operatorname{Cot}[c+d x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} + \frac{1}{3 a^2 \sqrt{a+b} (a^2-b^2) d} 2 (3 b^3 B + a b^2 (B-3 C) + 9 a^3 C - 2 a^2 b (3 B+C))$$

$$\operatorname{Cot}[c+d x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} - \frac{1}{a^3 d}$$

$$2 \sqrt{a+b} (b B - a C) \operatorname{Cot}[c+d x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \operatorname{Sec}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\operatorname{Sec}[c+d x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[c+d x])}{a-b}} +$$

$$\frac{2 b^2 (b B - 2 a C) \operatorname{Tan}[c+d x]}{3 a (a^2-b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 b^2 (7 a^2 b B - 3 b^3 B - 11 a^3 C + 3 a b^2 C) \operatorname{Tan}[c+d x]}{3 a^2 (a^2-b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 4, 4657 leaves) :

$$\left((b+a \operatorname{Cos}[c+d x])^3 \operatorname{Sec}[c+d x]^2 (b B - a C + b C \operatorname{Sec}[c+d x]) \left(\frac{2 b (-7 a^2 b B + 3 b^3 B + 11 a^3 C - 3 a b^2 C) \operatorname{Sin}[c+d x]}{3 a^2 (-a^2+b^2)^2} - \right. \right.$$

$$\left. \frac{2 (b^4 B \operatorname{Sin}[c+d x] - 2 a b^3 C \operatorname{Sin}[c+d x])}{3 a^2 (a^2-b^2) (b+a \operatorname{Cos}[c+d x])^2} - \frac{2 (-8 a^2 b^3 B \operatorname{Sin}[c+d x] + 4 b^5 B \operatorname{Sin}[c+d x] + 13 a^3 b^2 C \operatorname{Sin}[c+d x] - 5 a b^4 C \operatorname{Sin}[c+d x])}{3 a^2 (a^2-b^2)^2 (b+a \operatorname{Cos}[c+d x])} \right) \Bigg) /$$

$$(d (b C + b B \operatorname{Cos}[c+d x] - a C \operatorname{Cos}[c+d x]) (a+b \operatorname{Sec}[c+d x])^{5/2}) + (b+a \operatorname{Cos}[c+d x])^{5/2}$$

$$\left(\frac{a^2 b B}{(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{b^3 B}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} - \frac{a^3 C}{(a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} - \right.$$

$$\frac{5 a b^2 C}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{a b^2 B \sqrt{\operatorname{Sec}[c+d x]}}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} - \frac{b^4 B \sqrt{\operatorname{Sec}[c+d x]}}{3 a (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} -$$

$$\frac{2 a^2 b C \sqrt{\operatorname{Sec}[c+d x]}}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{2 b^3 C \sqrt{\operatorname{Sec}[c+d x]}}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} + \frac{7 a b^2 B \operatorname{Cos}[2(c+d x)] \sqrt{\operatorname{Sec}[c+d x]}}{3 (a^2-b^2)^2 \sqrt{b+a \operatorname{Cos}[c+d x]}} -$$

$$\begin{aligned}
& \left. \frac{b^4 B \cos[2(c+dx)] \sqrt{\sec[c+dx]} - 11 a^2 b C \cos[2(c+dx)] \sqrt{\sec[c+dx]} + b^3 C \cos[2(c+dx)] \sqrt{\sec[c+dx]}}{a(a^2-b^2)^2 \sqrt{b+a \cos[c+dx]} - 3(a^2-b^2)^2 \sqrt{b+a \cos[c+dx]} + (a^2-b^2)^2 \sqrt{b+a \cos[c+dx]}} \right) \sec[c+dx]^{3/2} \\
& (bB - aC + bC \sec[c+dx]) \left(- \frac{2b(-7a^2bB + 3b^3B + 11a^3C - 3ab^2C) \tan\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}}}{3a^2(a^2-b^2)^2 \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}}} \right) + \left(2(a+b) \right. \\
& \left. \left(-b(-7a^2bB + 3b^3B + 11a^3C - 3ab^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + a(2b^3B - 3a^2b(B-2C) + 3a^3C - ab^2(3B+C)) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 6(a-b)^2(a+b)(-bB+aC) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right. \\
& \left. \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\tan\left[\frac{1}{2}(c+dx)\right]^4} \right) / \\
& \left. \left(3a^2(a^2-b^2)^2 \sqrt{\frac{1}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}} \left(b-b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1+\tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) \right) / \\
& \left(d(bC + bB \cos[c+dx] - aC \cos[c+dx]) (a+b \sec[c+dx])^{5/2} \right. \\
& \left. - \frac{b(-7a^2bB + 3b^3B + 11a^3C - 3ab^2C) \sec\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(c+dx)\right]^2 + b \tan\left[\frac{1}{2}(c+dx)\right]^2}{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}}}{3a^2(a^2-b^2)^2 \sqrt{\frac{1+\tan\left[\frac{1}{2}(c+dx)\right]^2}{1-\tan\left[\frac{1}{2}(c+dx)\right]^2}}} \right) - \left(2(a+b) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-b (-7 a^2 b B + 3 b^3 B + 11 a^3 C - 3 a b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + a (2 b^3 B - 3 a^2 b (B - 2 C) + 3 a^3 C - a b^2 (3 B + C)) \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 6 (a-b)^2 (a+b) (-b B + a C) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \\
& \quad \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \\
& \quad \left(-2 b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 + 2 a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \Bigg) / \\
& \quad \left(3 a^2 (a^2 - b^2)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right)^2 \right) - \\
& \quad \left(2 (a+b) \left(-b (-7 a^2 b B + 3 b^3 B + 11 a^3 C - 3 a b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad \left. \left. a (2 b^3 B - 3 a^2 b (B - 2 C) + 3 a^3 C - a b^2 (3 B + C)) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad \left. \left. 6 (a-b)^2 (a+b) (-b B + a C) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right. \\
& \quad \left. \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) / \\
& \quad \left(3 a^2 (a^2 - b^2)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left(b - b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) + \\
& \quad \left(-b (-7 a^2 b B + 3 b^3 B + 11 a^3 C - 3 a b^2 C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + a (2 b^3 B - 3 a^2 b (B - 2 C) + 3 a^3 C - a b^2 (3 B + C)) \right. \\
& \quad \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 6 (a-b)^2 (a+b) (-b B + a C) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} \left/ \left(3a^2(a^2-b^2)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \right. \\
& \left. \left. \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right) - \frac{1}{3a^2(a^2-b^2)^2 \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right)} (a+b) \right. \\
& \left. \left(-b(-7a^2bB+3b^3B+11a^3C-3ab^2C) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + a(2b^3B-3a^2b(B-2C)+3a^3C-ab^2(3B+C)) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + 6(a-b)^2(a+b)(-bB+aC) \operatorname{EllipticPi}\left[-1, -\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \right. \\
& \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4} + \right. \\
& \left. \left(b(-7a^2bB+3b^3B+11a^3C-3ab^2C) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \right. \\
& \left. \left. \left(\frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} + \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) \right) \right/ \\
& \left(3a^2(a^2-b^2)^2 \left(\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} \right)^{3/2} \right) - \left(b(-7a^2bB+3b^3B+11a^3C-3ab^2C) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right. \\
& \left. \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] + b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) / \\
& \left(3 a^2 \left(a^2-b^2\right)^2 \sqrt{\frac{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right) + \\
& \left((a+b) \left(-b\left(-7 a^2 b B+3 b^3 B+11 a^3 C-3 a b^2 C\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \quad \left. a\left(2 b^3 B-3 a^2 b(B-2 C)+3 a^3 C-a b^2(3 B+C)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \quad \left. 6(a-b)^2(a+b)(-b B+a C) \operatorname{EllipticPi}\left[-1,-\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \\
& \sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}^4 \left(\frac{-a \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]+b \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2} - \right. \\
& \quad \left. \frac{\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left(a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)}{\left(1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2} \right) / \left(3 a^2 \left(a^2-b^2\right)^2 \sqrt{\frac{1}{1-\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right. \\
& \quad \left. \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \left(b-b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^4+a\left(-1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2\right)^2\right) \right) + \\
& \left(2(a+b) \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{a+b}} \sqrt{\frac{a+b-a \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2+b \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}{1+\operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2}} \right)
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]}^4 \left(\frac{a(2b^3B - 3a^2b(B-2C) + 3a^3C - ab^2(3B+C)) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{2\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]}^2 \sqrt{1 - \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \frac{3(a-b)^2(a+b)(-bB+aC) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]}^2 \left(1 + \tan\left[\frac{1}{2}(c+dx)\right]^2\right) \sqrt{1 - \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}} - \frac{b(-7a^2bB + 3b^3B + 11a^3C - 3ab^2C) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \sqrt{1 - \frac{(a-b)\tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{2\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]}^2} \right)}{\left(3a^2(a^2-b^2)^2 \sqrt{\frac{1}{1 - \tan\left[\frac{1}{2}(c+dx)\right]}^2} \left(b - b \tan\left[\frac{1}{2}(c+dx)\right]^4 + a \left(-1 + \tan\left[\frac{1}{2}(c+dx)\right]^2 \right)^2 \right) \right)} \right)$$

- **Problem 981: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{5/2} (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 266 leaves, 10 steps):

$$\begin{aligned} & - \frac{2(9Ab+9aB+7bC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{15d} + \\ & \frac{2(7aA+5bB+5aC) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{21d} + \\ & \frac{2(9Ab+9aB+7bC) \sqrt{\operatorname{Sec}[c+dx]} \sin[c+dx]}{15d} + \frac{2(7aA+5bB+5aC) \operatorname{Sec}[c+dx]^{3/2} \sin[c+dx]}{21d} + \\ & \frac{2(9Ab+9aB+7bC) \operatorname{Sec}[c+dx]^{5/2} \sin[c+dx]}{45d} + \frac{2(bB+aC) \operatorname{Sec}[c+dx]^{7/2} \sin[c+dx]}{7d} + \frac{2bC \operatorname{Sec}[c+dx]^{9/2} \sin[c+dx]}{9d} \end{aligned}$$

Result (type 5, 1232 leaves):

$$\begin{aligned}
& - \left(6 \sqrt{2} A b e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \operatorname{Csc}[c] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) (a + b \operatorname{Sec}[c+dx]) \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) - \\
& \left(6 \sqrt{2} a B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (5d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) - \left(14 \sqrt{2} b C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \right. \\
& \quad \left. \operatorname{Csc}[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) (a + b \operatorname{Sec}[c+dx]) \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (15d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) + \\
& \left(4 a A \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (3d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) + \\
& \left(20 b B \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (21d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) + \\
& \left(20 a C \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2}(c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \\
& (21d(b+a \cos[c+dx])(A+2C+2B \cos[c+dx]+A \cos[2c+2dx])) + \\
& \left((a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(\frac{4(9Ab+9aB+7bC) \cos[dx] \operatorname{Csc}[c]}{15d} + \right. \right. \\
& \quad \left. \frac{4bC \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^4 \sin[dx]}{9d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 (7bC \sin[c] + 9bB \sin[dx] + 9aC \sin[dx])}{63d} + \frac{1}{315d} \right)
\end{aligned}$$

$$\left(4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (63Ab \operatorname{Sin}[c] + 63aB \operatorname{Sin}[c] + 49bC \operatorname{Sin}[c] + 105aA \operatorname{Sin}[dx] + 75bB \operatorname{Sin}[dx] + 75aC \operatorname{Sin}[dx]) + \frac{1}{315d} \right. \\ \left. 4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (45bB \operatorname{Sin}[c] + 45aC \operatorname{Sin}[c] + 63Ab \operatorname{Sin}[dx] + 63aB \operatorname{Sin}[dx] + 49bC \operatorname{Sin}[dx]) + \frac{4(7aA + 5bB + 5aC) \operatorname{Tan}[c]}{21d} \right) / \\ ((b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2})$$

■ **Problem 982: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+dx]^{3/2} (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 230 leaves, 9 steps):

$$\begin{aligned} & - \frac{2(5aA+3bB+3aC) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{5d} + \\ & \frac{2(7Ab+7aB+5bC) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]}}{21d} + \frac{2(5aA+3bB+3aC) \sqrt{\operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{5d} + \\ & \frac{2(7Ab+7aB+5bC) \operatorname{Sec}[c+dx]^{3/2} \operatorname{Sin}[c+dx]}{21d} + \frac{2(bB+aC) \operatorname{Sec}[c+dx]^{5/2} \operatorname{Sin}[c+dx]}{5d} + \frac{2bC \operatorname{Sec}[c+dx]^{7/2} \operatorname{Sin}[c+dx]}{7d} \end{aligned}$$

Result (type 5, 1170 leaves):

$$\begin{aligned} & - \left(2\sqrt{2} aA e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \right. \\ & \left. \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) (a+b \operatorname{Sec}[c+dx]) \right. \\ & \left. (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx])) - \\ & \left(6\sqrt{2} bB e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\ & \left. (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / (5d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx])) - \\ & \left(6\sqrt{2} aC e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& - \left(2 \sqrt{2} A b e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \csc[c] \right. \\
& \quad \left. \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) (a + b \operatorname{Sec}[c+dx]) \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) - \\
& \left(2 \sqrt{2} a B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \csc[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) - \\
& \left(6 \sqrt{2} b C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \cos[c+dx]^3 \csc[c] \left(1 + e^{2i(c+dx)} + (-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)} \right] \right) \right. \\
& \quad \left. (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(5d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) + \\
& \left(4 a A \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2} (c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) + \\
& \left(4 b B \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2} (c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(3d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) + \\
& \left(4 a C \cos[c+dx]^{7/2} \operatorname{EllipticF} \left[\frac{1}{2} (c+dx), 2 \right] \sqrt{\operatorname{Sec}[c+dx]} (a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(3d (b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \right) + \\
& \left((a + b \operatorname{Sec}[c+dx]) (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \left(\frac{4(5Ab + 5aB + 3bC) \cos[dx] \csc[c]}{5d} + \frac{4bC \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 \sin[dx]}{5d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (3bC \sin[c] + 5bB \sin[dx] + 5aC \sin[dx])}{15d} + \frac{4(bB + aC) \tan[c]}{3d} \right) \right) / \left((b + a \cos[c+dx]) (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c+dx]^{5/2} \right)
\end{aligned}$$

- **Problem 984: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 152 leaves, 7 steps):

$$\begin{aligned} & - \frac{2 (b B - a (A - C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{d} + \\ & \frac{2 (3 A b + 3 a B + b C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 d} + \\ & \frac{2 (b B + a C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{d} + \frac{2 b C \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 d} \end{aligned}$$

Result (type 5, 1074 leaves):

$$\begin{aligned}
& \left(2\sqrt{2} a A e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) - \\
& \left(2\sqrt{2} b B e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) - \\
& \left(2\sqrt{2} a C e^{-i(2c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \operatorname{Cos}[c+dx]^3 \operatorname{Csc}[c] \left(1+e^{2i(c+dx)} + (-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] \right) \right. \\
& \quad \left. (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) + \\
& \left(4 a b \operatorname{Cos}[c+dx]^{7/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) + \\
& \left(4 a B \operatorname{Cos}[c+dx]^{7/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) + \\
& \left(4 b C \operatorname{Cos}[c+dx]^{7/2} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\operatorname{Sec}[c+dx]} (a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right) / \left(d(b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \right) + \\
& \quad \left((a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \right. \\
& \quad \left. \left(-\frac{2(aA-2bB-2aC+aA \operatorname{Cos}[2c]) \operatorname{Cos}[dx] \operatorname{Csc}[c]}{d} + \frac{4aA \operatorname{Cos}[c] \operatorname{Sin}[dx]}{d} + \frac{4bC \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{3d} + \frac{4bC \operatorname{Tan}[c]}{3d} \right) \right) / \left((b+a \operatorname{Cos}[c+dx]) (A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \right)
\end{aligned}$$

■ **Problem 985: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b \operatorname{Sec}[c+dx]) (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{\operatorname{Sec}[c+dx]^{3/2}} dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$\frac{2 (A b + a B - b C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{d} + \frac{2 (3 b B + a (A + 3 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a A \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}} + \frac{2 b C \sqrt{\sec [c + d x]} \sin [c + d x]}{d}$$

Result (type 5, 176 leaves):

$$\frac{1}{3 d} \sqrt{\sec [c + d x]} \left(-6 i A b \cos [c + d x] - 6 i a B \cos [c + d x] + 6 i b C \cos [c + d x] + 2 (3 b B + a (A + 3 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 6 i (A b + a B - b C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 6 b C \sin [c + d x] + a A \sin [2 (c + d x)] \right)$$

■ **Problem 986: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2)}{\sec [c + d x]^{5/2}} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$\frac{2 (3 a A + 5 b B + 5 a C) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{5 d} + \frac{2 (A b + a B + 3 b C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]}}{3 d} + \frac{2 a A \sin [c + d x]}{5 d \sec [c + d x]^{3/2}} + \frac{2 (A b + a B) \sin [c + d x]}{3 d \sqrt{\sec [c + d x]}}$$

Result (type 5, 173 leaves):

$$\frac{1}{30 d} \sqrt{\sec [c + d x]} \left(20 (A b + a B + 3 b C) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] + 12 i (3 a A + 5 b B + 5 a C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] + 2 \cos [c + d x] (-6 i (3 a A + 5 b B + 5 a C) + 10 (A b + a B) \sin [c + d x] + 3 a A \sin [2 (c + d x)]) \right)$$

■ **Problem 987: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2)}{\sec [c + d x]^{7/2}} dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$\frac{2(3Ab + 3aB + 5bC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{5d} +$$

$$\frac{2(5aA + 7bB + 7aC) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} +$$

$$\frac{2aA \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{2(Ab + aB) \sin[c+dx]}{5d \sec[c+dx]^{3/2}} + \frac{2(5aA + 7bB + 7aC) \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 198 leaves):

$$\frac{1}{420d} \sqrt{\sec[c+dx]} \left(40(5aA + 7bB + 7aC) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] + \right.$$

$$168i(3Ab + 3aB + 5bC) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] +$$

$$\left. 2 \cos[c+dx] (-84i(3Ab + 3aB + 5bC) + 5(23aA + 28bB + 28aC) \sin[c+dx] + 42(Ab + aB) \sin[2(c+dx)] + 15aA \sin[3(c+dx)]) \right)$$

■ **Problem 988: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b \sec[c+dx]) (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sec[c+dx]^{9/2}} dx$$

Optimal (type 4, 230 leaves, 9 steps):

$$\frac{2(7aA + 9bB + 9aC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{15d} +$$

$$\frac{2(5Ab + 5aB + 7bC) \sqrt{\cos[c+dx]} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right] \sqrt{\sec[c+dx]}}{21d} + \frac{2aA \sin[c+dx]}{9d \sec[c+dx]^{7/2}} +$$

$$\frac{2(Ab + aB) \sin[c+dx]}{7d \sec[c+dx]^{5/2}} + \frac{2(7aA + 9bB + 9aC) \sin[c+dx]}{45d \sec[c+dx]^{3/2}} + \frac{2(5Ab + 5aB + 7bC) \sin[c+dx]}{21d \sqrt{\sec[c+dx]}}$$

Result (type 5, 229 leaves):

$$\frac{1}{2520d} \sqrt{\sec[c+dx]} \left(240(5Ab + 5aB + 7bC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] + \right.$$

$$336i(7aA + 9bB + 9aC) e^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right] +$$

$$2 \cos[c+dx] (-1176iaA - 1512ibB - 1512iaC + 30(23Ab + 23aB + 28bC) \sin[c+dx] + 14(19aA + 18bB + 18aC) \sin[2(c+dx)] +$$

$$\left. 90Ab \sin[3(c+dx)] + 90aB \sin[3(c+dx)] + 35aA \sin[4(c+dx)]) \right)$$

■ **Problem 989: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{11/2}} dx$$

Optimal (type 4, 266 leaves, 10 steps):

$$\frac{2 (7 A b + 7 a B + 9 b C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{15 d} +$$

$$\frac{10 (9 a A + 11 b B + 11 a C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{231 d} + \frac{2 a A \operatorname{Sin}[c + d x]}{11 d \operatorname{Sec}[c + d x]^{9/2}} + \frac{2 (A b + a B) \operatorname{Sin}[c + d x]}{9 d \operatorname{Sec}[c + d x]^{7/2}} +$$

$$\frac{2 (9 a A + 11 b B + 11 a C) \operatorname{Sin}[c + d x]}{77 d \operatorname{Sec}[c + d x]^{5/2}} + \frac{2 (7 A b + 7 a B + 9 b C) \operatorname{Sin}[c + d x]}{45 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{10 (9 a A + 11 b B + 11 a C) \operatorname{Sin}[c + d x]}{231 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 5, 265 leaves):

$$\frac{1}{55440 d} \sqrt{\operatorname{Sec}[c + d x]} \left(2400 (9 a A + 11 b B + 11 a C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] + \right.$$

$$7392 i (7 A b + 7 a B + 9 b C) e^{-i (c + d x)} \sqrt{1 + e^{2 i (c + d x)}} \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i (c + d x)}\right] +$$

$$2 \operatorname{Cos}[c + d x] (-25872 i A b - 25872 i a B - 33264 i b C + 30 (435 a A + 506 b B + 506 a C) \operatorname{Sin}[c + d x] +$$

$$308 (19 A b + 19 a B + 18 b C) \operatorname{Sin}[2 (c + d x)] + 2565 a A \operatorname{Sin}[3 (c + d x)] + 1980 b B \operatorname{Sin}[3 (c + d x)] +$$

$$\left. 1980 a C \operatorname{Sin}[3 (c + d x)] + 770 A b \operatorname{Sin}[4 (c + d x)] + 770 a B \operatorname{Sin}[4 (c + d x)] + 315 a A \operatorname{Sin}[5 (c + d x)] \right)$$

■ **Problem 1012: Attempted integration timed out after 120 seconds.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{a + b \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 296 leaves, 11 steps):

$$\frac{2 (5 A b^2 - 5 a b B + 5 a^2 C + 3 b^2 C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{5 b^3 d} +$$

$$\frac{2 (b B - a C) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{3 b^2 d} -$$

$$\frac{2 a (A b^2 - a (b B - a C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{b^3 (a + b) d} +$$

$$\frac{2 (5 A b^2 - 5 a b B + 5 a^2 C + 3 b^2 C) \sqrt{\operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{5 b^3 d} + \frac{2 (b B - a C) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{3 b^2 d} + \frac{2 C \operatorname{Sec}[c + d x]^{5/2} \operatorname{Sin}[c + d x]}{5 b d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1013: Unable to integrate problem.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{a + b \text{Sec}[c + d x]} dx$$

Optimal (type 4, 218 leaves, 10 steps):

$$\begin{aligned} & - \frac{2 (b B - a C) \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{b^2 d} + \frac{2 C \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{3 b d} + \\ & \frac{2 (A b^2 - a (b B - a C)) \sqrt{\text{Cos}[c + d x]} \text{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{b^2 (a + b) d} + \\ & \frac{2 (b B - a C) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{b^2 d} + \frac{2 C \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 b d} \end{aligned}$$

Result (type 8, 45 leaves):

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{a + b \text{Sec}[c + d x]} dx$$

■ **Problem 1014: Unable to integrate problem.**

$$\int \frac{\sqrt{\text{Sec}[c + d x]} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{a + b \text{Sec}[c + d x]} dx$$

Optimal (type 4, 178 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 C \sqrt{\text{Cos}[c + d x]} \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{b d} + \frac{2 A \sqrt{\text{Cos}[c + d x]} \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a d} - \\ & \frac{2 (A b^2 - a (b B - a C)) \sqrt{\text{Cos}[c + d x]} \text{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\text{Sec}[c + d x]}}{a b (a + b) d} + \frac{2 C \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{b d} \end{aligned}$$

Result (type 8, 45 leaves):

$$\int \frac{\sqrt{\text{Sec}[c + d x]} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{a + b \text{Sec}[c + d x]} dx$$

■ **Problem 1015: Unable to integrate problem.**

$$\int \frac{A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2}{\sqrt{\text{Sec}[c + d x]} (a + b \text{Sec}[c + d x])} dx$$

Optimal (type 4, 157 leaves, 8 steps):

$$\frac{2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a d}-\frac{2(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^2 d}+\frac{2\left(A b^2-a(b B-a C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^2(a+b) d}$$

Result (type 8, 45 leaves):

$$\int \frac{A+B \sec [c+d x]+C \sec [c+d x]^2}{\sqrt{\sec [c+d x]}(a+b \sec [c+d x])} d x$$

■ **Problem 1016: Unable to integrate problem.**

$$\int \frac{A+B \sec [c+d x]+C \sec [c+d x]^2}{\sec [c+d x]^{3 / 2}(a+b \sec [c+d x])} d x$$

Optimal (type 4, 207 leaves, 9 steps):

$$-\frac{2(A b-a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^2 d}+\frac{2\left(3 A b^2-3 a b B+a^2(A+3 C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 a^3 d}-\frac{2 b\left(A b^2-a(b B-a C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^3(a+b) d}+\frac{2 A \sin [c+d x]}{3 a d \sqrt{\sec [c+d x]}}$$

Result (type 8, 45 leaves):

$$\int \frac{A+B \sec [c+d x]+C \sec [c+d x]^2}{\sec [c+d x]^{3 / 2}(a+b \sec [c+d x])} d x$$

■ **Problem 1017: Unable to integrate problem.**

$$\int \frac{A+B \sec [c+d x]+C \sec [c+d x]^2}{\sec [c+d x]^{5 / 2}(a+b \sec [c+d x])} d x$$

Optimal (type 4, 269 leaves, 10 steps):

$$-\frac{2\left(5 A b^2-5 a b B+a^2(3 A+5 C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{5 a^3 d}+\frac{2\left(3 A b^3-a^3 B-3 a b^2 B+a^2 b(A+3 C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{3 a^4 d}+\frac{2 b^2\left(A b^2-a(b B-a C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a+b}, \frac{1}{2}(c+d x), 2\right] \sqrt{\sec [c+d x]}}{a^4(a+b) d}+\frac{2 A \sin [c+d x]}{5 a d \sec [c+d x]^{3 / 2}}-\frac{2(A b-a B) \sin [c+d x]}{3 a^2 d \sqrt{\sec [c+d x]}}$$

Result (type 8, 45 leaves) :

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{5/2} (a + b \operatorname{Sec}[c + d x])} dx$$

■ **Problem 1018: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{7/2} (a + b \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 342 leaves, 11 steps) :

$$\begin{aligned} & - \frac{2 (5 A b^3 - 3 a^3 B - 5 a b^2 B + a^2 b (3 A + 5 C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} + \frac{1}{21 a^5 d}}{5 a^4 d} \\ & + \frac{2 (21 A b^4 - 7 a^3 b B - 21 a b^3 B + 7 a^2 b^2 (A + 3 C) + a^4 (5 A + 7 C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]} -}{a^5 (a + b) d} \\ & - \frac{2 b^3 (A b^2 - a (b B - a C)) \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2} (c + d x), 2\right] \sqrt{\operatorname{Sec}[c + d x]}}{a^5 (a + b) d} + \\ & \frac{2 A \operatorname{Sin}[c + d x]}{7 a d \operatorname{Sec}[c + d x]^{5/2}} - \frac{2 (A b - a B) \operatorname{Sin}[c + d x]}{5 a^2 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 (7 A b^2 - 7 a b B + a^2 (5 A + 7 C)) \operatorname{Sin}[c + d x]}{21 a^3 d \sqrt{\operatorname{Sec}[c + d x]}} \end{aligned}$$

Result (type 1, 1 leaves) :

???

■ **Problem 1019: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 447 leaves, 11 steps) :

$$\begin{aligned}
& - \frac{(3 a^2 b B - 2 b^3 B - a b^2 (A - 4 C) - 5 a^3 C) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{b^3 (a^2 - b^2) d} + \\
& \frac{(3 A b^2 - 3 a b B + 5 a^2 C - 2 b^2 C) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{3 b^2 (a^2 - b^2) d} - \frac{1}{(a - b) b^3 (a + b)^2 d} \\
& (3 A b^4 + 3 a^3 b B - 5 a b^3 B - a^2 b^2 (A - 7 C) - 5 a^4 C) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} + \\
& \frac{(3 a^2 b B - 2 b^3 B - a b^2 (A - 4 C) - 5 a^3 C) \sqrt{\sec[c + d x]} \sin[c + d x]}{b^3 (a^2 - b^2) d} + \\
& \frac{(3 A b^2 - 3 a b B + 5 a^2 C - 2 b^2 C) \sec[c + d x]^{3/2} \sin[c + d x]}{3 b^2 (a^2 - b^2) d} - \frac{(A b^2 - a (b B - a C)) \sec[c + d x]^{5/2} \sin[c + d x]}{b (a^2 - b^2) d (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 931 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left(- \left(2 (12 a A b^3 - 24 a^2 b^2 B + 12 b^4 B + 40 a^3 b C - 28 a b^3 C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) \right) + \\
& \left(2 (9 a^2 A b^2 - 12 A b^4 - 27 a^3 b B + 30 a b^3 B + 45 a^4 C - 44 a^2 b^2 C - 4 b^4 C) \cos[c + dx]^2 \right. \\
& \quad \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) \\
& \quad \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) - \\
& \left(2 (3 a^2 A b^2 - 9 a^3 b B + 6 a b^3 B + 15 a^4 C - 12 a^2 b^2 C) \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2 a b - 2 a b \sec[c + dx]^2 + \right. \right. \\
& \quad 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \\
& \quad \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - \right. \\
& \quad \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \sin[c + dx] \left. \right) / \\
& \quad \left(a^2 b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \left. \right) / \\
& (6 (a - b) b^3 (a + b) d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2) + \\
& \left((b + a \cos[c + dx])^2 \right. \\
& \quad \sqrt{\sec[c + dx]} \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \quad \left(\frac{2 (a A b^2 - 3 a^2 b B + 2 b^3 B + 5 a^3 C - 4 a b^2 C) \sin[c + dx]}{b^3 (-a^2 + b^2)} - \frac{2 (a A b^2 \sin[c + dx] - a^2 b B \sin[c + dx] + a^3 C \sin[c + dx])}{b^2 (-a^2 + b^2) (b + a \cos[c + dx])} + \frac{4 C \tan[c + dx]}{3 b^2} \right) \left. \right) / \\
& (d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2)
\end{aligned}$$

■ **Problem 1020: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 363 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(A b^2 - a b B + 3 a^2 C - 2 b^2 C) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{b^2 (a^2 - b^2) d} \\
& \frac{(A b^2 - a (b B - a C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a b (a^2 - b^2) d} + \frac{1}{a (a - b) b^2 (a + b)^2 d} \\
& (A b^4 + a^3 b B - 3 a b^3 B - 3 a^4 C + a^2 b^2 (A + 5 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} + \\
& \frac{(A b^2 - a b B + 3 a^2 C - 2 b^2 C) \sqrt{\sec[c + d x]} \sin[c + d x]}{b^2 (a^2 - b^2) d} - \frac{(A b^2 - a (b B - a C)) \sec[c + d x]^{3/2} \sin[c + d x]}{b (a^2 - b^2) d (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 865 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left(- \left(2 (4Ab^3 - 4ab^2B + 8a^2bC - 4b^3C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) \right) + \\
& \left(2 (-aAb^2 - 3a^2bB + 4b^3B + 9a^3C - 10ab^2C) \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \\
& (b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) - \left(2 (aAb^2 - a^2bB + 3a^3C - 2ab^2C) \cos[2(c + dx)] (a + b \sec[c + dx]) \right. \\
& \left(2ab - 2ab \sec[c + dx]^2 + 2ab \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a(a - 2b) \right. \\
& \quad \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \\
& \quad \left. \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \right) \\
& \left. \left. \sin[c + dx] \right) / \left(a^2 b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \right) / \\
& (2b^2 (-a + b) (a + b) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2) + \\
& \left((b + a \cos[c + dx])^2 \right. \\
& \quad \left. \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{b^2 (-a^2 + b^2)} + \frac{2 (Ab^2 \sin[c + dx] - abB \sin[c + dx] + a^2 C \sin[c + dx])}{b (-a^2 + b^2) (b + a \cos[c + dx])} \right) / \\
& (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2)
\end{aligned}$$

■ **Problem 1021: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 299 leaves, 9 steps):

$$\frac{(A b^2 - a (b B - a C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a b (a^2 - b^2) d} -$$

$$\frac{(A b^2 + a b B - a^2 (2 A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a^2 (a^2 - b^2) d} + \frac{1}{a^2 (a - b) b (a + b)^2 d}$$

$$\frac{(A b^4 + a^3 b B + a b^3 B + a^4 C - 3 a^2 b^2 (A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{b (a^2 - b^2) d (a + b \sec[c + d x])} -$$

$$\frac{(A b^2 - a (b B - a C)) \sqrt{\sec[c + d x]} \sin[c + d x]}{b (a^2 - b^2) d (a + b \sec[c + d x])}$$

Result (type 4, 829 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^2 (A + B \sec[c + dx] + C \sec^2[c + dx]) \right. \\
& \left. - \left(2 (4aAb - 4b^2B + 4abc) \cos[c + dx]^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] \right) / (a(b + a \cos[c + dx]) (1 - \cos^2[c + dx])) + \right. \\
& \left. \left(2 (-Ab^2 + abB + 3a^2C - 4b^2C) \cos[c + dx]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \right) \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] \right) / (b(b + a \cos[c + dx]) (1 - \cos^2[c + dx])) - \right. \\
& \left. \left(2 (Ab^2 - abB + a^2C) \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2ab - 2ab \sec[c + dx]^2 + 2ab \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \right. \right. \right. \\
& \quad \left. \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + a(a - 2b) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + \right. \right. \\
& \quad \left. \left. a^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} - \right. \right. \\
& \quad \left. \left. 2b^2 \operatorname{EllipticPi}\left[-\frac{b}{a}, -\operatorname{ArcSin}\left[\sqrt{\sec[c + dx]}\right], -1\right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} \right) \sin[c + dx] \right) / \\
& \quad \left. \left(a^2 b (b + a \cos[c + dx]) (1 - \cos^2[c + dx]) \sqrt{\sec[c + dx]} (2 - \sec^2[c + dx]) \right) \right) / \\
& \left(2(a - b)b(a + b)d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2 + \right. \\
& \left. \left((b + a \cos[c + dx])^2 \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec^2[c + dx])}{\frac{2(Ab^2 - abB + a^2C) \sin[c + dx]}{ab(-a^2 + b^2)} + \frac{2(Ab^2 \sin[c + dx] - abB \sin[c + dx] + a^2C \sin[c + dx])}{a(a^2 - b^2)(b + a \cos[c + dx])}} \right) \right) / \\
& \left. (d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2 \right)
\end{aligned}$$

■ **Problem 1022: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec^2[c + dx]}{\sqrt{\sec[c + dx]} (a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 317 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(3 A b^2 - a b B - a^2 (2 A - C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a^2 (a^2 - b^2) d} + \\
& \frac{(3 A b^3 + 2 a^3 B - a b^2 B - a^2 b (4 A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a^3 (a^2 - b^2) d} - \frac{1}{a^3 (a - b) (a + b)^2 d} \\
& \frac{(3 A b^4 + 3 a^3 b B - a b^3 B - a^4 C - a^2 b^2 (5 A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} +}{a (a^2 - b^2) d (a + b \sec[c + d x])} \\
& \frac{(A b^2 - a (b B - a C)) \sqrt{\sec[c + d x]} \sin[c + d x]}{a (a^2 - b^2) d (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 835 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^2 (A + B \sec[c + dx] + C \sec^2[c + dx]) \right. \\
& \left(- \left(2 (4 a A b - 4 a^2 B + 4 a b C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos^2[c + dx])) + \right. \\
& \left(2 (-2 a^2 A + A b^2 + a b B - a^2 C) \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) \right. \\
& \quad \left. (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] \right) / (b (b + a \cos[c + dx]) (1 - \cos^2[c + dx])) - \\
& \left(2 (-2 a^2 A + 3 A b^2 - a b B + a^2 C) \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2 a b - 2 a b \sec^2[c + dx] + \right. \right. \\
& \quad 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \\
& \quad \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} - \right. \\
& \quad \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} \right) \sin[c + dx] \left. \right) / \\
& \quad \left. \left(a^2 b (b + a \cos[c + dx]) (1 - \cos^2[c + dx]) \sqrt{\sec[c + dx]} (2 - \sec^2[c + dx]) \right) \right) / \\
& (2 a (-a + b) (a + b) d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2) + \\
& \left((b + a \cos[c + dx])^2 \right. \\
& \quad \left. \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec^2[c + dx])}{a^2 (a^2 - b^2)} - \frac{2 (A b^3 \sin[c + dx] - a b^2 B \sin[c + dx] + a^2 b C \sin[c + dx])}{a^2 (a^2 - b^2) (b + a \cos[c + dx])} \right) / \\
& \left. (d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2) \right)
\end{aligned}$$

■ **Problem 1023: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec^2[c + dx]^2}{\sec^3[c + dx]^{3/2} (a + b \sec[c + dx])^2} dx$$

Optimal (type 4, 406 leaves, 10 steps):

$$\begin{aligned}
& \frac{(5 A b^3 + 2 a^3 B - 3 a b^2 B - a^2 b (4 A - C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{a^3 (a^2 - b^2) d} - \frac{1}{3 a^4 (a^2 - b^2) d} \\
& \frac{(15 A b^4 + 12 a^3 b B - 9 a b^3 B - a^2 b^2 (16 A - 3 C) - 2 a^4 (A + 3 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} +}{a^4 (a - b) (a + b)^2 d} b (5 A b^4 + 5 a^3 b B - 3 a b^3 B - a^2 b^2 (7 A - C) - 3 a^4 C) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} - \\
& \frac{(5 A b^2 - 3 a b B - a^2 (2 A - 3 C)) \sin[c + d x]}{3 a^2 (a^2 - b^2) d \sqrt{\sec[c + d x]}} + \frac{(A b^2 - a (b B - a C)) \sin[c + d x]}{a (a^2 - b^2) d \sqrt{\sec[c + d x]} (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 887 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^2 (A + B \sec[c + dx] + C \sec^2[c + dx]) \right. \\
& \left(- \left(2 (4a^3 A + 8aAb^2 - 12a^2 bB + 12a^3 C) \cos^2[c + dx] \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos^2[c + dx])) + \right. \\
& \left(2 (-8a^2 Ab + 5Ab^3 + 6a^3 B - 3ab^2 B - 3a^2 bC) \cos^2[c + dx] \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) (a + b \sec[c + dx]) \sqrt{1 - \sec^2[c + dx]} \sin[c + dx] / (b (b + a \cos[c + dx]) (1 - \cos^2[c + dx])) - \right. \\
& \left(2 (-12a^2 Ab + 15Ab^3 + 6a^3 B - 9ab^2 B + 3a^2 bC) \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2ab - 2ab \sec[c + dx]^2 + \right. \right. \\
& \quad 2ab \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + a(a - 2b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \\
& \quad \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} - \right. \\
& \quad \left. 2b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec^2[c + dx]} \right) \sin[c + dx] / \\
& \quad \left. \left(a^2 b (b + a \cos[c + dx]) (1 - \cos^2[c + dx]) \sqrt{\sec[c + dx]} (2 - \sec^2[c + dx]) \right) \right) / \\
& (6a^2 (a - b) (a + b) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2) + \\
& \left((b + a \cos[c + dx])^2 \right. \\
& \quad \left. \sqrt{\sec[c + dx]} \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec^2[c + dx]) \right. \\
& \quad \left. \left(\frac{2b (Ab^2 - abB + a^2 C) \sin[c + dx]}{a^3 (-a^2 + b^2)} + \frac{2 (Ab^4 \sin[c + dx] - ab^3 B \sin[c + dx] + a^2 b^2 C \sin[c + dx])}{a^3 (a^2 - b^2) (b + a \cos[c + dx])} + \frac{2A \sin[2(c + dx)]}{3a^2} \right) \right) / \\
& (d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^2)
\end{aligned}$$

■ **Problem 1027: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec^2[c + dx])}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 4, 469 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{4 a b^2 (a^2 - b^2)^2 d} (A b^4 - a^3 b B - 5 a b^3 B - 3 a^4 C + a^2 b^2 (5 A + 9 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} + \\
& \frac{(A b^4 + 3 a^3 b B + 3 a b^3 B + a^4 C - 7 a^2 b^2 (A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]}}{4 a^2 b (a^2 - b^2)^2 d} - \frac{1}{4 a^2 (a - b)^2 b^2 (a + b)^3 d} \\
& (A b^6 - a^5 b B + 10 a^3 b^3 B + 3 a b^5 B - 3 a^4 b^2 (A - 2 C) - 3 a^6 C - 5 a^2 b^4 (2 A + 3 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} - \\
& \frac{(A b^2 - a (b B - a C)) \sec[c + d x]^{3/2} \sin[c + d x]}{2 b (a^2 - b^2) d (a + b \sec[c + d x])^2} + \frac{(A b^4 - a^3 b B - 5 a b^3 B - 3 a^4 C + a^2 b^2 (5 A + 9 C)) \sqrt{\sec[c + d x]} \sin[c + d x]}{4 b^2 (a^2 - b^2)^2 d (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 1051 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
& \left(- \left(2 (-24 a A b^3 + 8 a^2 b^2 B + 16 b^4 B + 8 a^3 b C - 32 a b^3 C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) \right) / \left(a (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) + \\
& \left(2 (a^2 A b^2 + 5 A b^4 + 3 a^3 b B - 9 a b^3 B + 9 a^4 C - 19 a^2 b^2 C + 16 b^4 C) \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \\
& \left(b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) - \left(2 (-5 a^2 A b^2 - A b^4 + a^3 b B + 5 a b^3 B + 3 a^4 C - 9 a^2 b^2 C) \cos[2(c + dx)] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \left(2 a b - 2 a b \sec[c + dx]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + \right. \right. \right. \\
& \quad \left. \left. a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], \right. \right. \right. \\
& \quad \left. \left. -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \\
& \quad \left. \left. \left. \sin[c + dx] \right) \right) / \left(a^2 b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \right) / \\
& \left(8 (a - b)^2 b^2 (a + b)^2 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3 \right) + \\
& \left((b + a \cos[c + dx])^3 \right. \\
& \quad \sec[c + dx]^{3/2} \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \quad \left(\frac{(5 a^2 A b^2 + A b^4 - a^3 b B - 5 a b^3 B - 3 a^4 C + 9 a^2 b^2 C) \sin[c + dx]}{2 a b^2 (-a^2 + b^2)^2} + \frac{A b^2 \sin[c + dx] - a b B \sin[c + dx] + a^2 C \sin[c + dx]}{a (a^2 - b^2) (b + a \cos[c + dx])^2} + \right. \\
& \quad \left. \frac{(-7 a^2 A b^2 \sin[c + dx] + A b^4 \sin[c + dx] + 3 a^3 b B \sin[c + dx] + 3 a b^3 B \sin[c + dx] + a^4 C \sin[c + dx] - 7 a^2 b^2 C \sin[c + dx])}{(2 a b (-a^2 + b^2)^2 (b + a \cos[c + dx]))} \right) / \\
& \quad \left. (d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 1028: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx])^2}{(a + b \sec[c + dx])^3} dx$$

Optimal (type 4, 478 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{1}{4 a^2 b (a^2 - b^2)^2 d} (3 A b^4 + 5 a^3 b B + a b^3 B - a^4 C - a^2 b^2 (9 A + 5 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} + \frac{1}{4 a^3 (a^2 - b^2)^2 d} \\
 & (3 A b^4 - 7 a^3 b B + a b^3 B - a^2 b^2 (5 A - 3 C) + a^4 (8 A + 3 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} - \frac{1}{4 a^3 (a - b)^2 b (a + b)^3 d} \\
 & (3 A b^6 - 3 a^5 b B - 10 a^3 b^3 B + a b^5 B - 3 a^2 b^4 (2 A - C) - a^6 C + 5 a^4 b^2 (3 A + 2 C)) \sqrt{\cos [c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec [c + d x]} - \\
 & \frac{(A b^2 - a (b B - a C)) \sqrt{\sec [c + d x]} \sin [c + d x]}{2 b (a^2 - b^2) d (a + b \sec [c + d x])^2} + \frac{(A b^4 + 3 a^3 b B + 3 a b^3 B + a^4 C - 7 a^2 b^2 (A + C)) \sqrt{\sec [c + d x]} \sin [c + d x]}{4 a b (a^2 - b^2)^2 d (a + b \sec [c + d x])}
 \end{aligned}$$

Result (type 4, 1051 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left(- \left(2 (16 a^3 A b + 8 a A b^3 - 24 a^2 b^2 B + 8 a^3 b C + 16 a b^3 C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) \right) + \\
& \left(2 (-5 a^2 A b^2 - A b^4 + a^3 b B + 5 a b^3 B + 3 a^4 C - 9 a^2 b^2 C) \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \operatorname{EllipticPi} \left[-\frac{b}{a}, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) - \\
& \left(2 (9 a^2 A b^2 - 3 A b^4 - 5 a^3 b B - a b^3 B + a^4 C + 5 a^2 b^2 C) \cos[2(c + dx)] (a + b \sec[c + dx]) \left(2 a b - 2 a b \sec[c + dx]^2 + \right. \right. \\
& \quad 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \\
& \quad \left. \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - \right. \\
& \quad \left. 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \sin[c + dx] \right) / \\
& \quad \left. \left(a^2 b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \right) / \\
& (8 a (a - b)^2 b (a + b)^2 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \\
& \quad (a + b \sec[c + dx])^3) + \\
& \left((b + a \cos[c + dx])^3 \sec[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left(\frac{(-9 a^2 A b^2 + 3 A b^4 + 5 a^3 b B + a b^3 B - a^4 C - 5 a^2 b^2 C) \sin[c + dx]}{2 a^2 b (-a^2 + b^2)^2} - \frac{A b^3 \sin[c + dx] - a b^2 B \sin[c + dx] + a^2 b C \sin[c + dx]}{a^2 (a^2 - b^2) (b + a \cos[c + dx])^2} + \right. \\
& \quad \left. (11 a^2 A b^2 \sin[c + dx] - 5 A b^4 \sin[c + dx] - 7 a^3 b B \sin[c + dx] + a b^3 B \sin[c + dx] + 3 a^4 C \sin[c + dx] + 3 a^2 b^2 C \sin[c + dx]) / \right. \\
& \quad \left. (2 a^2 (a^2 - b^2)^2 (b + a \cos[c + dx])) \right) / \\
& \quad \left. (d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3) \right)
\end{aligned}$$

■ **Problem 1029: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\sqrt{\sec[c + dx]} (a + b \sec[c + dx])^3} dx$$

Optimal (type 4, 486 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{4 a^3 (a^2 - b^2)^2 d} (15 A b^4 + 9 a^3 b B - 3 a b^3 B + a^4 (8 A - 5 C) - a^2 b^2 (29 A + C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} - \\
& \frac{1}{4 a^4 (a^2 - b^2)^2 d} (15 A b^5 - 8 a^5 B + 5 a^3 b^2 B - 3 a b^4 B - a^2 b^3 (33 A + C) + a^4 b (24 A + 7 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} + \\
& \frac{1}{4 a^4 (a - b)^2 (a + b)^3 d} (15 A b^6 - 15 a^5 b B + 6 a^3 b^3 B - 3 a b^5 B + 3 a^6 C - a^2 b^4 (38 A + C) + 5 a^4 b^2 (7 A + 2 C)) \\
& \sqrt{\cos[c + d x]} \operatorname{EllipticPi}\left[\frac{2 a}{a + b}, \frac{1}{2}(c + d x), 2\right] \sqrt{\sec[c + d x]} + \frac{(A b^2 - a (b B - a C)) \sqrt{\sec[c + d x]} \sin[c + d x]}{2 a (a^2 - b^2) d (a + b \sec[c + d x])^2} - \\
& \frac{(5 A b^4 + 7 a^3 b B - a b^3 B - 3 a^4 C - a^2 b^2 (11 A + 3 C)) \sqrt{\sec[c + d x]} \sin[c + d x]}{4 a^2 (a^2 - b^2)^2 d (a + b \sec[c + d x])}
\end{aligned}$$

Result (type 4, 1064 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx])^3 \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
& \left(- \left(2 (-32 a^3 A b + 8 a A b^3 + 16 a^4 B + 8 a^2 b^2 B - 24 a^3 b C) \cos[c + dx]^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right. \right. \\
& \quad \left. \left. (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / (a (b + a \cos[c + dx]) (1 - \cos[c + dx]^2)) \right) + \\
& \left(2 (8 a^4 A - 7 a^2 A b^2 + 5 A b^4 - 5 a^3 b B - a b^3 B + a^4 C + 5 a^2 b^2 C) \cos[c + dx]^2 \left(\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] + \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \right) (a + b \sec[c + dx]) \sqrt{1 - \sec[c + dx]^2} \sin[c + dx] \right) / \\
& \left(b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \right) - \left(2 (8 a^4 A - 29 a^2 A b^2 + 15 A b^4 + 9 a^3 b B - 3 a b^3 B - 5 a^4 C - a^2 b^2 C) \cos[2(c + dx)] \right. \\
& \quad \left. (a + b \sec[c + dx]) \left(2 a b - 2 a b \sec[c + dx]^2 + 2 a b \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + \right. \right. \\
& \quad \left. \left. a (a - 2 b) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} + a^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], \right. \right. \\
& \quad \left. \left. -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} - 2 b^2 \operatorname{EllipticPi} \left[-\frac{b}{a}, -\operatorname{ArcSin} \left[\sqrt{\sec[c + dx]} \right], -1 \right] \sqrt{\sec[c + dx]} \sqrt{1 - \sec[c + dx]^2} \right) \\
& \quad \left. \left. \sin[c + dx] \right) / \left(a^2 b (b + a \cos[c + dx]) (1 - \cos[c + dx]^2) \sqrt{\sec[c + dx]} (2 - \sec[c + dx]^2) \right) \right) \Bigg) / \\
& (8 a^2 (a - b)^2 (a + b)^2 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3) + \\
& \left((b + a \cos[c + dx])^3 \right. \\
& \quad \sec[c + dx]^{3/2} \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \quad \left(- \frac{(-13 a^2 A b^2 + 7 A b^4 + 9 a^3 b B - 3 a b^3 B - 5 a^4 C - a^2 b^2 C) \sin[c + dx]}{2 a^3 (-a^2 + b^2)^2} - \frac{-A b^4 \sin[c + dx] + a b^3 B \sin[c + dx] - a^2 b^2 C \sin[c + dx]}{a^3 (a^2 - b^2) (b + a \cos[c + dx])^2} + \right. \\
& \quad \left. (-15 a^2 A b^3 \sin[c + dx] + 9 A b^5 \sin[c + dx] + 11 a^3 b^2 B \sin[c + dx] - 5 a b^4 B \sin[c + dx] - 7 a^4 b C \sin[c + dx] + a^2 b^3 C \sin[c + dx]) / \right. \\
& \quad \left. (2 a^3 (a^2 - b^2)^2 (b + a \cos[c + dx])) \right) \Bigg) / \\
& (d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^3)
\end{aligned}$$

■ **Problem 1031: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sec[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx])^2 dx$$

Optimal (type 4, 447 leaves, 14 steps):

$$\begin{aligned}
& \frac{(24 A b^2 + 18 a b B - a^2 C + 16 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{24 b d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \\
& \frac{(2 a^2 b B - 8 b^3 B - a^3 C - 4 a b^2 (2 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]}}{8 b^2 d \sqrt{a+b \operatorname{Sec}[c+dx]}} - \\
& \frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]}}{24 b^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\operatorname{Sec}[c+dx]}} + \\
& \frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24 b^2 d} + \\
& \frac{(6 b B + a C) \operatorname{Sec}[c+dx]^{3/2} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12 b d} + \frac{C \operatorname{Sec}[c+dx]^{5/2} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d}
\end{aligned}$$

Result (type 4, 782 leaves):

$$\begin{aligned}
& \left(\sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(\frac{2(24ab^2B+4a^2bC) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \right. \\
& \frac{2(24aAb^2-18a^2bB+48b^3B+9a^3C+8ab^2C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \\
& \left. \left(2i(-24aAb^2-6a^2bB+3a^3C-16ab^2C) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right. \right. \\
& \left. \left. \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \right. \right. \right. \right. \\
& \left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+dx] \right) / \\
& \left. \left. \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2+2b^2-4b(b+a \operatorname{Cos}[c+dx])+2(b+a \operatorname{Cos}[c+dx])^2) \right) \right) \right) / \right. \\
& \left. (48b^2d \sqrt{b+a \operatorname{Cos}[c+dx]} (A+2C+2B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2}) + \right. \\
& \left. \left(\sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \right. \\
& \left. \left. \left(\frac{\operatorname{Sec}[c+dx]^2 (6bB \operatorname{Sin}[c+dx]+aC \operatorname{Sin}[c+dx])}{6b} + \right. \right. \right. \\
& \left. \left. \left. \frac{\operatorname{Sec}[c+dx] (24Ab^2 \operatorname{Sin}[c+dx]+6abB \operatorname{Sin}[c+dx]-3a^2C \operatorname{Sin}[c+dx]+16b^2C \operatorname{Sin}[c+dx])}{12b^2} + \frac{2}{3} C \operatorname{Sec}[c+dx]^2 \operatorname{Tan}[c+dx] \right) \right) \right) / \\
& \left. (d(A+2C+2B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2}) \right)
\end{aligned}$$

■ **Problem 1032:** Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 346 leaves, 13 steps):

$$\begin{aligned}
 & \frac{(8 a A + 4 b B + 3 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
 & \frac{(8 A b^2 + 4 a b B - a^2 C + 4 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
 & \frac{(4 b B + a C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
 & \frac{(4 b B + a C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b d} + \frac{C \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 d}
 \end{aligned}$$

Result (type 4, 683 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \left(\frac{2(-16ab-4abC) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \right. \right. \\
& \frac{2(-16Ab^2-4abB+3a^2C-8b^2C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \\
& \left. \left(2i(4abB+a^2C) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right. \right. \\
& \left. \left. \left(-2b(a+b) \operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c+dx] \right) \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2+2b^2-4b(b+a \operatorname{Cos}[c+dx])+2(b+a \operatorname{Cos}[c+dx])^2) \right) \right) \right) / \\
& \left. \left(8bd \sqrt{b+a \operatorname{Cos}[c+dx]} (A+2C+2B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \right) + \right. \\
& \left. \left(\sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \right. \right. \\
& \left. \left. \left(\frac{\operatorname{Sec}[c+dx] (4bB \operatorname{Sin}[c+dx]+aC \operatorname{Sin}[c+dx])}{2b} + C \operatorname{Sec}[c+dx] \operatorname{Tan}[c+dx] \right) \right) \right) / \\
& \left. \left(d(A+2C+2B \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{5/2} \right) \right)
\end{aligned}$$

- **Problem 1033: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 258 leaves, 12 steps):

$$\frac{(2 a B + b C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{(2 b B + a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{(2 A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \frac{c \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{d}$$

Result (type 4, 624 leaves):

$$\begin{aligned}
& \frac{2 C \sqrt{a+b \operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sin}[c+d x]}{d(A+2 C+2 B \operatorname{Cos}[c+d x]+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+d x]^{3/2}} + \\
& \left(\sqrt{a+b \operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \left(\frac{2(4 A b+4 a B) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \right. \right. \\
& \frac{2(2 a A+4 b B+a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+d x]}} + \left. \left(2 i(2 a A-a C) \sqrt{\frac{a-a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+d x]}{a-b}} \right. \right. \\
& \left. \left. \operatorname{Cos}[2(c+d x)] \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+d x]}\right], \frac{-a+b}{a+b}\right]\right) \right) \operatorname{Sin}[c+d x] \right) / \\
& \left. \left. \left. \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\operatorname{Cos}[c+d x]}^2 \sqrt{\frac{a^2-a^2 \operatorname{Cos}[c+d x]^2}{a^2}} (-a^2+2 b^2-4 b(b+a \operatorname{Cos}[c+d x])+2(b+a \operatorname{Cos}[c+d x])^2) \right) \right) \right) \right) / \right. \\
& \left. \left(2 d \sqrt{b+a \operatorname{Cos}[c+d x]} (A+2 C+2 B \operatorname{Cos}[c+d x]+A \operatorname{Cos}[2 c+2 d x]) \operatorname{Sec}[c+d x]^{5/2} \right) \right)
\end{aligned}$$

■ **Problem 1034: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+d x]} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\operatorname{Sec}[c+d x]^{3/2}} dx$$

Optimal (type 4, 277 leaves, 12 steps):

$$\begin{aligned}
& \frac{2 (A b^2 - a^2 (A + 3 C)) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 a d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 b C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} + \\
& \frac{2 (A b + 3 a B) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{2 A \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d \sqrt{\sec [c+d x]}}
\end{aligned}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sec [c+d x]^{3/2}} dx$$

- **Problem 1035: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sec [c+d x]^{5/2}} dx$$

Optimal (type 4, 273 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 (a^2 - b^2) (2 A b - 5 a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{15 a^2 d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{2 (2 A b^2 - 5 a b B - 3 a^2 (3 A + 5 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{15 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \\
& \frac{2 A \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d \sec [c+d x]^{3/2}} + \frac{2 (A b + 5 a B) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a d \sqrt{\sec [c+d x]}}
\end{aligned}$$

Result (type 6, 3426 leaves):

$$\begin{aligned}
& \left(\sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) \right. \\
& \left. - \frac{4 (9 a^2 A - 2 A b^2 + 5 a b B + 15 a^2 C) \cot [c]}{15 a^2 d} + \frac{4 (A b + 5 a B) \cos [d x] \sin [c]}{15 a d} + \frac{2 A \cos [2 d x] \sin [2 c]}{5 d} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{4(Ab + 5aB)\cos[c]\sin[dx]}{15ad} + \frac{2A\cos[2c]\sin[2dx]}{5d} \right) \right) / \left((A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) \sec[c + dx]^{5/2} \right) - \\
& \left(28Ab \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] \left(b - a\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a\sqrt{1 + \cot[c]^2} \left(1 + \frac{b\csc[c]}{a\sqrt{1 + \cot[c]^2}} \right)} \right], \right. \\
& \left. \frac{\csc[c] \left(b - a\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a\sqrt{1 + \cot[c]^2} \left(-1 + \frac{b\csc[c]}{a\sqrt{1 + \cot[c]^2}} \right)} \right] \csc[c] \sqrt{a + b\sec[c + dx]} \\
& (A + B\sec[c + dx] + C\sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a\sqrt{1 + \cot[c]^2} - a\sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a\sqrt{1 + \cot[c]^2} - b\csc[c]}} \\
& \left. \sqrt{\frac{a\sqrt{1 + \cot[c]^2} + a\sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a\sqrt{1 + \cot[c]^2} + b\csc[c]}} \sqrt{b - a\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(15ad\sqrt{b + a\cos[c + dx]} (A + 2C + 2B\cos[c + dx] + A\cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sec[c + dx]^{5/2} \right) - \\
& \left(4B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] \left(b - a\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a\sqrt{1 + \cot[c]^2} \left(1 + \frac{b\csc[c]}{a\sqrt{1 + \cot[c]^2}} \right)} \right], \right. \\
& \left. \frac{\csc[c] \left(b - a\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a\sqrt{1 + \cot[c]^2} \left(-1 + \frac{b\csc[c]}{a\sqrt{1 + \cot[c]^2}} \right)} \right] \csc[c] \sqrt{a + b\sec[c + dx]} \\
& (A + B\sec[c + dx] + C\sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a\sqrt{1 + \cot[c]^2} - a\sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a\sqrt{1 + \cot[c]^2} - b\csc[c]}}
\end{aligned}$$

$$\left(\frac{\sqrt{\frac{a\sqrt{1+\cot[c]^2} + a\sqrt{1+\cot[c]^2}\sin[dx - \text{ArcTan}[\cot[c]]]}{a\sqrt{1+\cot[c]^2} + b\csc[c]}}}{\sqrt{b - a\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]]}} \right) /$$

$$\left(3d\sqrt{b+a\cos[c+dx]}(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sqrt{1+\cot[c]^2}\sec[c+dx]^{5/2} \right) -$$

$$\left(4bC\text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c](b - a\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]])}{a\sqrt{1+\cot[c]^2}\left(1 + \frac{b\csc[c]}{a\sqrt{1+\cot[c]^2}}\right)}\right], \right.$$

$$\left. \frac{\csc[c](b - a\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]])}{a\sqrt{1+\cot[c]^2}\left(-1 + \frac{b\csc[c]}{a\sqrt{1+\cot[c]^2}}\right)} \right] \csc[c]\sqrt{a+b\sec[c+dx]}$$

$$(A+B\sec[c+dx]+C\sec[c+dx]^2)\sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{\frac{a\sqrt{1+\cot[c]^2} - a\sqrt{1+\cot[c]^2}\sin[dx - \text{ArcTan}[\cot[c]]]}{a\sqrt{1+\cot[c]^2} - b\csc[c]}}$$

$$\left(\frac{\sqrt{\frac{a\sqrt{1+\cot[c]^2} + a\sqrt{1+\cot[c]^2}\sin[dx - \text{ArcTan}[\cot[c]]]}{a\sqrt{1+\cot[c]^2} + b\csc[c]}}}{\sqrt{b - a\sqrt{1+\cot[c]^2}\sin[c]\sin[dx - \text{ArcTan}[\cot[c]]]}} \right) /$$

$$\left(ad\sqrt{b+a\cos[c+dx]}(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sqrt{1+\cot[c]^2}\sec[c+dx]^{5/2} \right) -$$

$$\left(6aA\csc[c]\sqrt{a+b\sec[c+dx]}(A+B\sec[c+dx]+C\sec[c+dx]^2) \right.$$

$$\left. \left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c](b+a\cos[c]\cos[dx + \text{ArcTan}[\tan[c]])\sqrt{1+\tan[c]^2}}{a\sqrt{1+\tan[c]^2}\left(1 - \frac{b\sec[c]}{a\sqrt{1+\tan[c]^2}}\right)}\right], \right. \right.$$

$$\begin{aligned}
& \left. - \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]} \right/ \\
& \left(\sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} - a \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} + a \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{-b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \right) - \\
& \left. \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 a \operatorname{Cos}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2}}{\sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} \right) \right/ \\
& \left(5 d \sqrt{b + a \operatorname{Cos}[c + dx]} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{5/2} \right) + \\
& \left(4 A b^2 \operatorname{Csc}[c] \sqrt{a + b \operatorname{Sec}[c + dx]} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right) \\
& \left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \right] \right. \right. \\
& \left. \left. - \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]} \right) \right/
\end{aligned}$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(15 a d \sqrt{b + a \cos[c + d x]} (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{5/2}) -$$

$$\left(2 b B \csc[c] \sqrt{a + b \sec[c + d x]} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}}\right)}\right], \right.$$

$$\left. - \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}}\right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) / \\
& (3 d \sqrt{b + a \cos[c + d x]} (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{5/2}) - \\
& \left(2 a C \csc[c] \sqrt{a + b \sec[c + d x]} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right. \\
& \left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) -
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

$$\left(d \sqrt{b + a \cos[c + d x]} (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{5/2} \right)$$

■ **Problem 1036: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \sec[c + d x]} (A + B \sec[c + d x] + C \sec[c + d x]^2)}{\sec[c + d x]^{7/2}} dx$$

Optimal (type 4, 360 leaves, 10 steps):

$$\frac{2 (a^2 - b^2) (25 a^2 A + 8 A b^2 - 14 a b B + 35 a^2 C) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\sec[c + d x]}}{105 a^3 d \sqrt{a + b \sec[c + d x]}} +$$

$$\frac{2 (8 A b^3 + 63 a^3 B - 14 a b^2 B + a^2 b (19 A + 35 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{105 a^3 d \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \sqrt{\sec[c + d x]}} + \frac{2 A \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{7 d \sec[c + d x]^{5/2}} +$$

$$\frac{2 (A b + 7 a B) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{35 a d \sec[c + d x]^{3/2}} - \frac{2 (4 A b^2 - 7 a b B - 5 a^2 (5 A + 7 C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{105 a^2 d \sqrt{\sec[c + d x]}}$$

Result (type 6, 4441 leaves):

$$\left(\sqrt{a + b \sec[c + d x]} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right.$$

$$\left(- \frac{4 (19 a^2 A b + 8 A b^3 + 63 a^3 B - 14 a b^2 B + 35 a^2 b C) \cot[c]}{105 a^3 d} + \frac{(115 a^2 A - 16 A b^2 + 28 a b B + 140 a^2 C) \cos[d x] \sin[c]}{105 a^2 d} + \right.$$

$$\frac{2 (A b + 7 a B) \cos[2 d x] \sin[2 c]}{35 a d} + \frac{A \cos[3 d x] \sin[3 c]}{7 d} + \frac{(115 a^2 A - 16 A b^2 + 28 a b B + 140 a^2 C) \cos[c] \sin[d x]}{105 a^2 d} +$$

$$\left. \left. \frac{2 (A b + 7 a B) \cos[2 c] \sin[2 d x]}{35 a d} + \frac{A \cos[3 c] \sin[3 d x]}{7 d} \right) \right) / \left((A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{5/2} \right) -$$

$$\left(20 A \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.$$

$$\left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] \operatorname{Csc}[c] \sqrt{a + b \operatorname{Sec}[c + dx]}$$

$$(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}}$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$(21 d \sqrt{b + a \operatorname{Cos}[c + dx]} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{5/2}) -$$

$$\left(8 A b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.$$

$$\left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] \operatorname{Csc}[c] \sqrt{a + b \operatorname{Sec}[c + dx]}$$

$$(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}}$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\begin{aligned}
& \left(105 a^2 d \sqrt{b + a \cos[c + dx]} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sec[c + dx]^{5/2} \right) - \\
& \left(28 b B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right], \right. \\
& \left. \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(-1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right] \csc[c] \sqrt{a + b \sec[c + dx]} \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a \sqrt{1 + \cot[c]^2} - a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} - b \csc[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \cot[c]^2} + a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} + b \csc[c]}} \sqrt{b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(15 a d \sqrt{b + a \cos[c + dx]} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sec[c + dx]^{5/2} \right) - \\
& \left(4 C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right], \right. \\
& \left. \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(-1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right] \csc[c] \sqrt{a + b \sec[c + dx]} \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a \sqrt{1 + \cot[c]^2} - a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} - b \csc[c]}}
\end{aligned}$$

$$\left. \sqrt{\frac{a \sqrt{1 + \cot[c]^2} + a \sqrt{1 + \cot[c]^2} \sin[dx - \text{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} + b \csc[c]} \sqrt{b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]}} \right/$$

$$\left(3 d \sqrt{b + a \cos[c + dx]} (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sec[c + dx]^{5/2} \right) -$$

$$\left(38 A b \csc[c] \sqrt{a + b \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] \left(b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2}}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right. \right.$$

$$\left. \left. - \frac{\sec[c] \left(b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2}}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) \right/$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\left(\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left(\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2} \right) \sqrt{b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) \right/$$

$$\left(105 d \sqrt{b + a \cos[c + dx]} (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sec[c + dx]^{5/2} \right) -$$

$$\left(16 A b^3 \operatorname{Csc}[c] \sqrt{a + b \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right. \right. \right.$$

$$\left. \left. - \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} - a \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} + a \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{-b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \right) -$$

$$\left. \frac{\frac{\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 a \operatorname{Cos}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2}}{\sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} \right) /$$

$$(105 a^2 d \sqrt{b + a \operatorname{Cos}[c + d x]} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{5/2}) -$$

$$\left(6 a B \operatorname{Csc}[c] \sqrt{a + b \operatorname{Sec}[c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right], \right.$$

$$\left. - \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg) /$$

$$\left(\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left(\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2} \right) \Bigg) /$$

$$\sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}$$

$$\left(5 d \sqrt{b + a \text{Cos}[c + dx]} (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \text{Sec}[c + dx]^{5/2} \right) +$$

$$\left(4 b^2 B \text{Csc}[c] \sqrt{a + b \text{Sec}[c + dx]} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right], \right.$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\frac{\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}}}{\sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) -$$

$$\frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2}}{\sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \Bigg/$$

$$(15 a d \sqrt{b + a \text{Cos}[c + dx]} (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \text{Sec}[c + dx]^{5/2}) -$$

$$\left(2 b C \text{Csc}[c] \sqrt{a + b \text{Sec}[c + dx]} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \right) \right)$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\frac{\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}}}{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) -$$

$$\left(\frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \Bigg/$$

$$(3 d \sqrt{b + a \cos[c + dx]} (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) \sec[c + dx]^{5/2})$$

■ **Problem 1037: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sec[c + dx]^{9/2}} dx$$

Optimal (type 4, 457 leaves, 11 steps):

$$- \left(2 (a^2 - b^2) (16 A b^3 - 75 a^3 B - 24 a b^2 B + 6 a^2 b (6 A + 7 C)) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{\sec[c + dx]} \right) /$$

$$(315 a^4 d \sqrt{a + b \sec[c + dx]}) -$$

$$\left(2 (16 A b^4 - 57 a^3 b B - 24 a b^3 B + 6 a^2 b^2 (4 A + 7 C) - 21 a^4 (7 A + 9 C)) \text{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{a + b \sec[c + dx]} \right) /$$

$$\left(315 a^4 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \sqrt{\sec[c + dx]} \right) + \frac{2 A \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{9 d \sec[c + dx]^{7/2}} +$$

$$\frac{2 (A b + 9 a B) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{63 a d \sec[c + dx]^{5/2}} - \frac{2 (6 A b^2 - 9 a b B - 7 a^2 (7 A + 9 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{315 a^2 d \sec[c + dx]^{3/2}} +$$

$$\frac{2 (8 A b^3 + 75 a^3 B - 12 a b^2 B + a^2 b (13 A + 21 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{315 a^3 d \sqrt{\sec[c + dx]}}$$

Result (type 6, 5993 leaves): Display of huge result suppressed!

■ **Problem 1038: Result unnecessarily involves imaginary or complex numbers.**

$$\int \text{Sec}[c + dx]^{3/2} (a + b \text{Sec}[c + dx])^{3/2} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 551 leaves, 15 steps):

$$\left((136 a^2 b B + 128 b^3 B - 3 a^3 C + 12 a b^2 (28 A + 19 C)) \sqrt{\frac{b + a \text{Cos}[c + dx]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{\text{Sec}[c + dx]} \right) /$$

$$(192 b d \sqrt{a + b \text{Sec}[c + dx]}) -$$

$$\left((8 a^3 b B - 96 a b^3 B - 3 a^4 C - 24 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \sqrt{\frac{b + a \text{Cos}[c + dx]}{a + b}} \text{EllipticPi}\left[2, \frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{\text{Sec}[c + dx]} \right) /$$

$$(64 b^2 d \sqrt{a + b \text{Sec}[c + dx]}) - \frac{(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \text{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{a + b \text{Sec}[c + dx]}}{192 b^2 d \sqrt{\frac{b + a \text{Cos}[c + dx]}{a + b}} \sqrt{\text{Sec}[c + dx]}}$$

$$+ \frac{(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{\text{Sec}[c + dx]} \sqrt{a + b \text{Sec}[c + dx]} \text{Sin}[c + dx]}{192 b^2 d}$$

$$+ \frac{(48 A b^2 + 56 a b B + 3 a^2 C + 36 b^2 C) \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \text{Sin}[c + dx]}{96 b d}$$

$$+ \frac{(8 b B + 3 a C) \text{Sec}[c + dx]^{5/2} \sqrt{a + b \text{Sec}[c + dx]} \text{Sin}[c + dx]}{24 d} + \frac{C \text{Sec}[c + dx]^{5/2} (a + b \text{Sec}[c + dx])^{3/2} \text{Sin}[c + dx]}{4 d}$$

Result (type 4, 916 leaves):

$$\left((a + b \text{Sec}[c + dx])^{3/2} (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right.$$

$$\left. \frac{2 (192 a A b^3 + 224 a^2 b^2 B + 12 a^3 b C + 144 a b^3 C) \sqrt{\frac{b + a \text{Cos}[c + dx]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right]}{\sqrt{b + a \text{Cos}[c + dx]}} + \frac{1}{\sqrt{b + a \text{Cos}[c + dx]}} \right.$$

$$\left. + 2 (48 a^2 A b^2 + 384 A b^4 - 72 a^3 b B + 448 a b^3 B + 27 a^4 C - 12 a^2 b^2 C + 288 b^4 C) \sqrt{\frac{b + a \text{Cos}[c + dx]}{a + b}} \text{EllipticPi}\left[2, \frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] + \right.$$

$$\begin{aligned}
& \left(2 i (-240 a^2 A b^2 - 24 a^3 b B - 128 a b^3 B + 9 a^4 C - 156 a^2 b^2 C) \sqrt{\frac{a - a \cos [c + d x]}{a + b}} \sqrt{\frac{a + a \cos [c + d x]}{a - b}} \cos [2 (c + d x)] \right. \\
& \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \right. \right. \\
& \left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos [c + d x]}\right], \frac{-a + b}{a + b}\right]\right) \sin [c + d x] \Bigg) / \\
& \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \cos [c + d x]}^2 \sqrt{\frac{a^2 - a^2 \cos [c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \cos [c + d x]) + 2 (b + a \cos [c + d x])^2) \right) \Bigg) / \\
& (384 b^2 d (b + a \cos [c + d x])^{3/2} (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{7/2}) + \\
& \left((a + b \sec [c + d x])^{3/2} (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left(\frac{1}{12} \sec [c + d x]^3 (8 b B \sin [c + d x] + 9 a C \sin [c + d x]) + \right. \\
& \left. \frac{\sec [c + d x]^2 (48 A b^2 \sin [c + d x] + 56 a b B \sin [c + d x] + 3 a^2 C \sin [c + d x] + 36 b^2 C \sin [c + d x])}{48 b} + \frac{1}{96 b^2} \right. \\
& \left. \sec [c + d x] (240 a A b^2 \sin [c + d x] + 24 a^2 b B \sin [c + d x] + 128 b^3 B \sin [c + d x] - 9 a^3 C \sin [c + d x] + 156 a b^2 C \sin [c + d x]) + \right. \\
& \left. \left. \frac{1}{2} b C \sec [c + d x]^3 \tan [c + d x] \right) \right) / \\
& (d (b + a \cos [c + d x]) (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sec [c + d x]^{7/2})
\end{aligned}$$

■ **Problem 1039: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\sec [c + d x]} (a + b \sec [c + d x])^{3/2} (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 4, 446 leaves, 14 steps):

$$\begin{aligned}
& \frac{(42 a b B + 8 b^2 (3 A + 2 C) + a^2 (48 A + 17 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{24 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(6 a^2 b B + 8 b^3 B - a^3 C + 12 a b^2 (2 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{8 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{(24 A b^2 + 30 a b B + 3 a^2 C + 16 b^2 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{24 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{(24 A b^2 + 30 a b B + 3 a^2 C + 16 b^2 C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 b d} + \\
& \frac{(2 b B + a C) \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{C \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 4, 800 leaves):

$$\begin{aligned}
& - \left(\left((a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\frac{2(-96 a^2 A b - 24 a b^2 B - 28 a^2 b C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \right. \right. \\
& \quad \frac{1}{\sqrt{b+a \operatorname{Cos}[c+dx]}} 2(-120 a A b^2 - 6 a^2 b B - 48 b^3 B + 9 a^3 C - 56 a b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + \\
& \quad \left(2 i (24 a A b^2 + 30 a^2 b B + 3 a^3 C + 16 a b^2 C) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right. \\
& \quad \left. \left(-2 b (a+b) \operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c+dx] \right) / \\
& \quad \left. \left. \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c+dx]) + 2 (b + a \operatorname{Cos}[c+dx])^2) \right) \right) \right) / \right. \\
& \quad \left. (48 b d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{7/2}) + \right. \\
& \quad \left. \left((a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \right. \\
& \quad \left. \left(\frac{1}{6} \operatorname{Sec}[c + dx]^2 (6 b B \operatorname{Sin}[c + dx] + 7 a C \operatorname{Sin}[c + dx]) + \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Sec}[c + dx] (24 A b^2 \operatorname{Sin}[c + dx] + 30 a b B \operatorname{Sin}[c + dx] + 3 a^2 C \operatorname{Sin}[c + dx] + 16 b^2 C \operatorname{Sin}[c + dx])}{12 b} + \frac{2}{3} b C \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx] \right) \right) / \\
& \quad \left. (d (b + a \operatorname{Cos}[c + dx]) (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{7/2}) \right)
\end{aligned}$$

- **Problem 1040: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 353 leaves, 13 steps):

$$\begin{aligned} & \frac{(8 a^2 B + 4 b^2 B + a b (8 A + 7 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\ & \frac{(8 A b^2 + 12 a b B + 3 a^2 C + 4 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\ & \frac{(8 a A - 4 b B - 5 a C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\ & \frac{(4 b B + 3 a C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d} + \frac{C \sqrt{\operatorname{Sec}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{2 d} \end{aligned}$$

Result (type 4, 709 leaves):

$$\begin{aligned}
& \left((a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\frac{2 (32 a A b + 16 a^2 B + 4 a b C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \right. \right. \\
& \frac{2 (8 a^2 A + 16 A b^2 + 20 a b B + a^2 C + 8 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \\
& \left. \left(2 i (8 a^2 A - 4 a b B - 5 a^2 C) \sqrt{\frac{a - a \operatorname{Cos}[c + dx]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + dx]}{a - b}} \operatorname{Cos}[2 (c + dx)] \right. \right. \\
& \left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right], \right. \right. \\
& \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \left. \right) \operatorname{Sin}[c + dx] \left. \right) / \\
& \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c + dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + dx]) + 2 (b + a \operatorname{Cos}[c + dx])^2) \right) \right) / \\
& (8 d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{7/2}) + \\
& \left((a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\frac{1}{2} \operatorname{Sec}[c + dx] (4 b B \operatorname{Sin}[c + dx] + 5 a C \operatorname{Sin}[c + dx]) + b C \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx] \right) \right) / \\
& (d (b + a \operatorname{Cos}[c + dx]) (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{7/2})
\end{aligned}$$

- **Problem 1041: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{\operatorname{Sec}[c + dx]^{3/2}} dx$$

Optimal (type 4, 340 leaves, 13 steps):

$$\begin{aligned}
& \frac{(6 a b B - b^2 (2 A - 3 C) + 2 a^2 (A + 3 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{b(2 b B + 3 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(8 A b + 6 a B - 3 b C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} - \\
& \frac{b(2 A - 3 C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 d} + \frac{2 A (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Sec}[c+d x]}}
\end{aligned}$$

Result (type 4, 685 leaves):

$$\left((a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \left(\frac{2 (4 a^2 A + 12 A b^2 + 24 a b B + 12 a^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c + d x]}} + \right. \right.$$

$$\frac{2 (8 a A b + 6 a^2 B + 12 b^2 B + 15 a b C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c + d x]}} +$$

$$\left. \left(2 i (8 a A b + 6 a^2 B - 3 a b C) \sqrt{\frac{a - a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + d x]}{a - b}} \operatorname{Cos}[2 (c + d x)] \right. \right.$$

$$\left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right], \right. \right.$$

$$\left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] \right) \left. \right) \operatorname{Sin}[c + d x] \left. \right) /$$

$$\left(\left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \operatorname{Cos}[c + d x]^2} \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \right) /$$

$$\frac{(6 d (b + a \operatorname{Cos}[c + d x])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2}) + (a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \left(\frac{4}{3} a A \operatorname{Sin}[c + d x] + 2 b C \operatorname{Tan}[c + d x]\right)}{d (b + a \operatorname{Cos}[c + d x]) (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{7/2}}$$

■ **Problem 1042: Unable to integrate problem.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 356 leaves, 13 steps):

$$\begin{aligned}
& \frac{2 \left(3 A b^3 - 5 a^3 B + 5 a b^2 B - 3 a^2 b (A + 5 C) \right) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{15 a d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{2 b^2 C \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{2 \left(3 A b^2 + 20 a b B + 3 a^2 (3 A + 5 C) \right) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{15 a d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{2 \left(3 A b + 5 a B \right) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{15 d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 A (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{5 d \operatorname{Sec}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 8, 47 leaves):

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\operatorname{Sec}[c+d x]^{5/2}} dx$$

- **Problem 1043: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\operatorname{Sec}[c+d x]^{7/2}} dx$$

Optimal (type 4, 359 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 \left(a^2 - b^2 \right) \left(25 a^2 A - 6 A b^2 + 21 a b B + 35 a^2 C \right) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{105 a^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{2 \left(6 A b^3 - 63 a^3 B - 21 a b^2 B - 2 a^2 b (41 A + 70 C) \right) \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{105 a^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{2 \left(3 A b + 7 a B \right) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{35 d \operatorname{Sec}[c+d x]^{3/2}} + \\
& \frac{2 \left(3 A b^2 + 42 a b B + 5 a^2 (5 A + 7 C) \right) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{105 a d \sqrt{\operatorname{Sec}[c+d x]}} + \frac{2 A (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{7 d \operatorname{Sec}[c+d x]^{5/2}}
\end{aligned}$$

Result (type 6, 4862 leaves):

$$\begin{aligned}
& \left((a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4 (82 a^2 A b - 6 A b^3 + 63 a^3 B + 21 a b^2 B + 140 a^2 b C) \operatorname{Cot}[c]}{105 a^2 d} + \right. \right. \\
& \quad \frac{(115 a^2 A + 12 A b^2 + 168 a b B + 140 a^2 C) \operatorname{Cos}[dx] \operatorname{Sin}[c]}{105 a d} + \frac{2 (8 A b + 7 a B) \operatorname{Cos}[2 dx] \operatorname{Sin}[2 c]}{35 d} + \frac{a A \operatorname{Cos}[3 dx] \operatorname{Sin}[3 c]}{7 d} + \\
& \quad \left. \frac{(115 a^2 A + 12 A b^2 + 168 a b B + 140 a^2 C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{105 a d} + \frac{2 (8 A b + 7 a B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 dx]}{35 d} + \frac{a A \operatorname{Cos}[3 c] \operatorname{Sin}[3 dx]}{7 d} \right) \Bigg) / \\
& \quad \left((b + a \operatorname{Cos}[c + dx]) (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \quad \left(20 a A \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right], \right. \\
& \quad \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right] \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^{3/2} \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right. \\
& \quad \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(21 d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \quad \left(68 A b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right], \right. \\
& \quad \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right] \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^{3/2} \right. \\
& \quad \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(35 a d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(16 b B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^{3/2} \\
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(5 d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{7/2} \right) - \\
& \left(4 a C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \text{Csc}[c] (a + b \text{Sec}[c + dx])^{3/2} \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \text{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{7/2} \right) - \\
& \left(4 b^2 C \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \text{Csc}[c] (a + b \text{Sec}[c + dx])^{3/2} \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(a d (b + a \text{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{7/2} \right) -
\end{aligned}$$

$$\left(164 a A b Csc[c] (a + b Sec[c + d x])^{3/2} (A + B Sec[c + d x] + C Sec[c + d x]^2) \left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{Sec[c] \left(b + a Cos[c] Cos[d x + ArcTan[Tan[c]] \right] \sqrt{1 + Tan[c]^2} \right)}{a \sqrt{1 + Tan[c]^2} \left(1 - \frac{b Sec[c]}{a \sqrt{1 + Tan[c]^2} \right)} \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{Sec[c] \left(b + a Cos[c] Cos[d x + ArcTan[Tan[c]] \right] \sqrt{1 + Tan[c]^2} \right)}{a \sqrt{1 + Tan[c]^2} \left(-1 - \frac{b Sec[c]}{a \sqrt{1 + Tan[c]^2} \right)} \right) \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{Sin[d x + ArcTan[Tan[c]]] Tan[c]}{\sqrt{1 + Tan[c]^2}} \sqrt{\frac{a \sqrt{1 + Tan[c]^2} - a Cos[d x + ArcTan[Tan[c]]] \sqrt{1 + Tan[c]^2}}{b Sec[c] + a \sqrt{1 + Tan[c]^2}}} \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{\sqrt{a \sqrt{1 + Tan[c]^2} + a Cos[d x + ArcTan[Tan[c]]] \sqrt{1 + Tan[c]^2}}}{-b Sec[c] + a \sqrt{1 + Tan[c]^2}} \sqrt{b + a Cos[c] Cos[d x + ArcTan[Tan[c]]] \sqrt{1 + Tan[c]^2}} \right) \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{\frac{Sin[d x + ArcTan[Tan[c]]] Tan[c]}{\sqrt{1 + Tan[c]^2}} + \frac{2 a Cos[c] \left(b + a Cos[c] Cos[d x + ArcTan[Tan[c]]] \sqrt{1 + Tan[c]^2} \right)}{a^2 Cos[c]^2 + a^2 Sin[c]^2}}{\sqrt{b + a Cos[c] Cos[d x + ArcTan[Tan[c]]] \sqrt{1 + Tan[c]^2}}} \right) \right) \right) \right) \sqrt{$$

$$(105 d (b + a Cos[c + d x])^{3/2} (A + 2 C + 2 B Cos[c + d x] + A Cos[2 c + 2 d x]) Sec[c + d x]^{7/2} +$$

$$\left(4 A b^3 Csc[c] (a + b Sec[c + d x])^{3/2} (A + B Sec[c + d x] + C Sec[c + d x]^2) \right.$$

$$\left(\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{Sec[c] \left(b + a Cos[c] Cos[d x + ArcTan[Tan[c]] \right] \sqrt{1 + Tan[c]^2} \right)}{a \sqrt{1 + Tan[c]^2} \left(1 - \frac{b Sec[c]}{a \sqrt{1 + Tan[c]^2} \right)} \right) \right. \right. \right.$$

$$\left. \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\left(\sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} - a \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \right.} \right.$$

$$\left. \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} + a \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{-b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \right) - \right.$$

$$\left. \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 a \operatorname{Cos}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2}}{\sqrt{b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} \right) \right) /$$

$$(35 a d (b + a \operatorname{Cos}[c + dx])^{3/2} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \operatorname{Sec}[c + dx]^{7/2}) -$$

$$\left(6 a^2 B \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right)$$

$$\left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \right] \right) \right)$$

$$\left. \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2}} \right)} \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(5 d (b + a \cos[c + d x])^{3/2} (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{7/2}) -$$

$$\left(2 b^2 B \csc[c] (a + b \sec[c + d x])^{3/2} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \right) \right)$$

$$- \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\left(\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left(\frac{\frac{\sin[d x + \arctan[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(5 d (b + a \cos[c + d x])^{3/2} (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{7/2}) -$$

$$\left(8 a b C \csc[c] (a + b \sec[c + d x])^{3/2} (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \right) \right)$$

$$- \frac{\sec[c] (b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \sin[d x + \arctan[\tan[c]]] \tan[c] /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\left(\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \arctan[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2}}{\sqrt{b + a \text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) /$$

$$(3 d (b + a \text{Cos}[c + d x])^{3/2} (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Sec}[c + d x]^{7/2})$$

- **Problem 1044: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \text{Sec}[c + d x])^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\text{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 455 leaves, 11 steps):

$$\left(2 (a^2 - b^2) (8 A b^3 + 75 a^3 B - 18 a b^2 B + a^2 (39 A b + 63 b C)) \sqrt{\frac{b + a \text{Cos}[c + d x]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\text{Sec}[c + d x]} \right) /$$

$$(315 a^3 d \sqrt{a + b \text{Sec}[c + d x]}) +$$

$$\left(2 (8 A b^4 + 246 a^3 b B - 18 a b^3 B + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \text{Sec}[c + d x]} \right) /$$

$$\left(315 a^3 d \sqrt{\frac{b + a \text{Cos}[c + d x]}{a + b}} \sqrt{\text{Sec}[c + d x]} \right) + \frac{2 (A b + 3 a B) \sqrt{a + b \text{Sec}[c + d x]} \text{Sin}[c + d x]}{21 d \text{Sec}[c + d x]^{5/2}} +$$

$$\frac{2 (3 A b^2 + 72 a b B + 7 a^2 (7 A + 9 C)) \sqrt{a + b \text{Sec}[c + d x]} \text{Sin}[c + d x]}{315 a d \text{Sec}[c + d x]^{3/2}} -$$

$$\frac{2 (4 A b^3 - 75 a^3 B - 9 a b^2 B - 2 a^2 b (44 A + 63 C)) \sqrt{a + b \text{Sec}[c + d x]} \text{Sin}[c + d x]}{315 a^2 d \sqrt{\text{Sec}[c + d x]}} + \frac{2 A (a + b \text{Sec}[c + d x])^{3/2} \text{Sin}[c + d x]}{9 d \text{Sec}[c + d x]^{7/2}}$$

Result (type 6, 5997 leaves): Display of huge result suppressed!

- **Problem 1045: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{\text{Sec}[c + d x]} (a + b \text{Sec}[c + d x])^{5/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 550 leaves, 15 steps):

$$\begin{aligned}
& \frac{1}{192 d \sqrt{a+b} \operatorname{Sec}[c+d x]} (472 a^2 b B + 128 b^3 B + 4 a b^2 (132 A + 89 C) + a^3 (384 A + 133 C)) \\
& \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} + \frac{1}{64 b d \sqrt{a+b} \operatorname{Sec}[c+d x]} \\
& (40 a^3 b B + 160 a b^3 B - 5 a^4 C + 120 a^2 b^2 (2 A + C) + 16 b^4 (4 A + 3 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]} - \\
& \frac{(264 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (108 A + 71 C)) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b} \operatorname{Sec}[c+d x]}{192 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{(264 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (108 A + 71 C)) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b} \operatorname{Sec}[c+d x] \operatorname{Sin}[c+d x]}{192 b d} + \\
& \frac{(16 A b^2 + 24 a b B + 5 a^2 C + 12 b^2 C) \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b} \operatorname{Sec}[c+d x] \operatorname{Sin}[c+d x]}{32 d} + \\
& \frac{(8 b B + 5 a C) \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{24 d} + \frac{C \operatorname{Sec}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{4 d}
\end{aligned}$$

Result (type 4, 925 leaves):

$$\begin{aligned}
& - \left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \left(\frac{1}{\sqrt{b + a \operatorname{Cos}[c + d x]}} \right. \right. \\
& \quad 2 (-768 a^3 A b - 192 a A b^3 - 416 a^2 b^2 B - 236 a^3 b C - 144 a b^3 C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 a}{a + b}\right] + \frac{1}{\sqrt{b + a \operatorname{Cos}[c + d x]}} \\
& \quad 2 (-1008 a^2 A b^2 - 384 A b^4 + 24 a^3 b B - 832 a b^3 B + 45 a^4 C - 436 a^2 b^2 C - 288 b^4 C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 a}{a + b}\right] + \\
& \quad \left(2 i (432 a^2 A b^2 + 264 a^3 b B + 128 a b^3 B + 15 a^4 C + 284 a^2 b^2 C) \sqrt{\frac{a - a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + d x]}{a - b}} \operatorname{Cos}[2(c + d x)] \right. \\
& \quad \left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] \right) \operatorname{Sin}[c + d x] \right) / \\
& \quad \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \operatorname{Cos}[c + d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \left. \right) / \\
& \quad \left. (384 b d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2}) \right) + \\
& \quad \left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \quad \left(\frac{1}{12} \operatorname{Sec}[c + d x]^3 (8 b^2 B \operatorname{Sin}[c + d x] + 17 a b C \operatorname{Sin}[c + d x]) + \right. \\
& \quad \frac{1}{48} \operatorname{Sec}[c + d x]^2 (48 A b^2 \operatorname{Sin}[c + d x] + 104 a b B \operatorname{Sin}[c + d x] + 59 a^2 C \operatorname{Sin}[c + d x] + 36 b^2 C \operatorname{Sin}[c + d x]) + \frac{1}{96 b} \\
& \quad \operatorname{Sec}[c + d x] (432 a A b^2 \operatorname{Sin}[c + d x] + 264 a^2 b B \operatorname{Sin}[c + d x] + 128 b^3 B \operatorname{Sin}[c + d x] + 15 a^3 C \operatorname{Sin}[c + d x] + 284 a b^2 C \operatorname{Sin}[c + d x]) + \\
& \quad \left. \left. \frac{1}{2} b^2 C \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x] \right) \right) / \\
& \quad (d (b + a \operatorname{Cos}[c + d x])^2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2})
\end{aligned}$$

■ **Problem 1046: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 453 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{24 d \sqrt{a + b \operatorname{Sec}[c + d x]}} (48 a^3 B + 66 a b^2 B + 8 b^3 (3 A + 2 C) + a^2 b (96 A + 59 C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} + \\ & \frac{1}{8 d \sqrt{a + b \operatorname{Sec}[c + d x]}} (30 a^2 b B + 8 b^3 B + 5 a^3 C + 20 a b^2 (2 A + C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} - \\ & \frac{(54 a b B - a^2 (48 A - 33 C) + 8 b^2 (3 A + 2 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]}}{24 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{(24 A b^2 + 42 a b B + 15 a^2 C + 16 b^2 C) \sqrt{\operatorname{Sec}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{24 d} + \\ & \frac{(6 b B + 5 a C) \sqrt{\operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{12 d} + \frac{C \sqrt{\operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{3 d} \end{aligned}$$

Result (type 4, 817 leaves):

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \frac{2 (288 a^2 A b + 96 a^3 B + 24 a b^2 B + 52 a^2 b C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right]}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \frac{1}{\sqrt{b + a \operatorname{Cos}[c + d x]}} \right.$$

$$2 (48 a^3 A + 216 a A b^2 + 126 a^2 b B + 48 b^3 B - 3 a^3 C + 104 a b^2 C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] +$$

$$\left(2 i (48 a^3 A - 24 a A b^2 - 54 a^2 b B - 33 a^3 C - 16 a b^2 C) \sqrt{\frac{a - a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + d x]}{a - b}} \operatorname{Cos}[2 (c + d x)] \right.$$

$$\left. \left(-2 b (a + b) \operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] \right. \right.$$

$$\left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] \right) \operatorname{Sin}[c + d x] \right) /$$

$$\left(\left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \operatorname{Cos}[c + d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \right) /$$

$$(48 d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2}) +$$

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left(\frac{1}{6} \operatorname{Sec}[c + d x]^2 (6 b^2 B \operatorname{Sin}[c + d x] + 13 a b C \operatorname{Sin}[c + d x]) + \right.$$

$$\left. \frac{1}{12} \operatorname{Sec}[c + d x] (24 A b^2 \operatorname{Sin}[c + d x] + 54 a b B \operatorname{Sin}[c + d x] + 33 a^2 C \operatorname{Sin}[c + d x] + 16 b^2 C \operatorname{Sin}[c + d x]) + \frac{2}{3} b^2 C \operatorname{Sec}[c + d x]^2 \operatorname{Tan}[c + d x] \right) \right) /$$

$$(d (b + a \operatorname{Cos}[c + d x])^2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2})$$

■ **Problem 1047: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{3/2}} dx$$

Optimal (type 4, 427 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{12 d \sqrt{a + b \operatorname{Sec}[c + d x]}} (48 a^2 b B + 12 b^3 B + 8 a^3 (A + 3 C) + a b^2 (16 A + 33 C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} + \\ & \frac{b (8 A b^2 + 20 a b B + 15 a^2 C + 4 b^2 C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]}}{4 d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\ & \frac{(24 a^2 B - 12 b^2 B + a b (56 A - 27 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]}}{12 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]}} - \\ & \frac{b (8 a A - 12 b B - 21 a C) \sqrt{\operatorname{Sec}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{12 d} - \\ & \frac{b (4 A - 3 C) \sqrt{\operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{6 d} + \frac{2 A (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} \end{aligned}$$

Result (type 4, 766 leaves):

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \frac{2 (16 a^3 A + 144 a A b^2 + 144 a^2 b B + 48 a^3 C + 12 a b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c+dx]}} + \frac{1}{\sqrt{b+a \operatorname{Cos}[c+dx]}} \right.$$

$$2 (56 a^2 A b + 48 A b^3 + 24 a^3 B + 108 a b^2 B + 63 a^2 b C + 24 b^3 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] +$$

$$\left(2 i (56 a^2 A b + 24 a^3 B - 12 a b^2 B - 27 a^2 b C) \sqrt{\frac{a-a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c+dx]}{a-b}} \operatorname{Cos}[2(c+dx)] \right.$$

$$\left. \left(-2 b (a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right], \right. \right.$$

$$\left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c+dx]}\right], \frac{-a+b}{a+b}\right]\right) \operatorname{Sin}[c+dx] \right) /$$

$$\left(\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c+dx]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b+a \operatorname{Cos}[c+dx]) + 2 (b+a \operatorname{Cos}[c+dx])^2) \right) \right) /$$

$$(24 d (b+a \operatorname{Cos}[c+dx])^{5/2} (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2}) +$$

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \left(\frac{4}{3} a^2 A \operatorname{Sin}[c + d x] + \frac{1}{2} \operatorname{Sec}[c + d x] (4 b^2 B \operatorname{Sin}[c + d x] + 9 a b C \operatorname{Sin}[c + d x]) + b^2 C \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x] \right) \right) /$$

$$(d (b+a \operatorname{Cos}[c+dx])^2 (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \operatorname{Sec}[c+dx]^{9/2})$$

■ **Problem 1048: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{5/2}} dx$$

Optimal (type 4, 419 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{15 d \sqrt{a + b \operatorname{Sec}[c + d x]}} (10 a^3 B + 20 a b^2 B - b^3 (16 A - 15 C) + 4 a^2 b (4 A + 15 C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} + \\ & \frac{b^2 (2 b B + 5 a C) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]}}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\ & \frac{(70 a b B + b^2 (46 A - 15 C) + 6 a^2 (3 A + 5 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]}}{15 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]}} - \\ & \frac{b (16 A b + 10 a B - 15 b C) \sqrt{\operatorname{Sec}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 d} + \\ & \frac{2 (A b + a B) (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{3 d \sqrt{\operatorname{Sec}[c + d x]}} + \frac{2 A (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{5 d \operatorname{Sec}[c + d x]^{3/2}} \end{aligned}$$

Result (type 4, 755 leaves):

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \frac{2 (68 a^2 A b + 60 A b^3 + 20 a^3 B + 180 a b^2 B + 180 a^2 b C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \operatorname{Cos}[c + d x]}} + \frac{1}{\sqrt{b+a \operatorname{Cos}[c + d x]}} \right.$$

$$2 (18 a^3 A + 46 a A b^2 + 70 a^2 b B + 60 b^3 B + 30 a^3 C + 135 a b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c + d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a+b}\right] +$$

$$\left(2 i (18 a^3 A + 46 a A b^2 + 70 a^2 b B + 30 a^3 C - 15 a b^2 C) \sqrt{\frac{a-a \operatorname{Cos}[c + d x]}{a+b}} \sqrt{\frac{a+a \operatorname{Cos}[c + d x]}{a-b}} \operatorname{Cos}[2 (c + d x)] \right.$$

$$\left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c + d x]}\right], \frac{-a+b}{a+b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c + d x]}\right], \frac{-a+b}{a+b}\right], \right. \right.$$

$$\left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \operatorname{Cos}[c + d x]}\right], \frac{-a+b}{a+b}\right] \right) \operatorname{Sin}[c + d x] \right) /$$

$$\left(\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \operatorname{Cos}[c + d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \right) /$$

$$(30 d (b + a \operatorname{Cos}[c + d x])^{5/2} (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2}) +$$

$$\left((a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \left(\frac{4}{15} a (11 A b + 5 a B) \operatorname{Sin}[c + d x] + \frac{2}{5} a^2 A \operatorname{Sin}[2 (c + d x)] + 2 b^2 C \operatorname{Tan}[c + d x] \right) \right) /$$

$$(d (b + a \operatorname{Cos}[c + d x])^2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{9/2})$$

■ **Problem 1049: Unable to integrate problem.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

Optimal (type 4, 441 leaves, 14 steps):

$$\begin{aligned} & - \left(2 (15 A b^4 - 56 a^3 b B + 56 a b^3 B + 10 a^2 b^2 (A - 7 C) - 5 a^4 (5 A + 7 C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} \right) / \\ & \left(105 a d \sqrt{a + b \operatorname{Sec}[c + d x]} \right) + \frac{2 b^3 C \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]}}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\ & \frac{2 (15 A b^3 + 63 a^3 B + 161 a b^2 B + 5 a^2 b (29 A + 49 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]}}{105 a d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{2 (15 A b^2 + 56 a b B + 5 a^2 (5 A + 7 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{105 d \sqrt{\operatorname{Sec}[c + d x]}} + \\ & \frac{2 (5 A b + 7 a B) (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{35 d \operatorname{Sec}[c + d x]^{3/2}} + \frac{2 A (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{7 d \operatorname{Sec}[c + d x]^{5/2}} \end{aligned}$$

Result (type 8, 47 leaves):

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{7/2}} dx$$

■ **Problem 1050: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Sec}[c + d x]^{9/2}} dx$$

Optimal (type 4, 452 leaves, 11 steps):

$$\begin{aligned}
& - \left(2 (a^2 - b^2) (10 A b^3 - 75 a^3 B - 45 a b^2 B - 6 a^2 b (19 A + 28 C)) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{\sec[c + dx]} \right) / \\
& \quad \left(315 a^2 d \sqrt{a + b \sec[c + dx]} \right) - \\
& \left(2 (10 A b^4 - 435 a^3 b B - 45 a b^3 B - 21 a^4 (7 A + 9 C) - 3 a^2 b^2 (93 A + 161 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2 a}{a + b}\right] \sqrt{a + b \sec[c + dx]} \right) / \\
& \left(315 a^2 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \sqrt{\sec[c + dx]} \right) + \frac{2 (15 A b^2 + 90 a b B + 7 a^2 (7 A + 9 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{315 d \sec[c + dx]^{3/2}} + \\
& \frac{2 (5 A b^3 + 75 a^3 B + 135 a b^2 B + a^2 b (163 A + 231 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{315 a d \sqrt{\sec[c + dx]}} + \\
& \frac{2 (5 A b + 9 a B) (a + b \sec[c + dx])^{3/2} \sin[c + dx]}{63 d \sec[c + dx]^{5/2}} + \frac{2 A (a + b \sec[c + dx])^{5/2} \sin[c + dx]}{9 d \sec[c + dx]^{7/2}}
\end{aligned}$$

Result (type 6, 6410 leaves) : Display of huge result suppressed!

- **Problem 1051: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sec[c + dx])^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sec[c + dx]^{11/2}} dx$$

Optimal (type 4, 565 leaves, 12 steps) :

$$\begin{aligned}
& \left(2 (a^2 - b^2) (40 A b^4 + 1254 a^3 b B - 110 a b^3 B + 75 a^4 (9 A + 11 C) + 15 a^2 b^2 (19 A + 33 C)) \right. \\
& \left. \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} \right) / (3465 a^3 d \sqrt{a + b \operatorname{Sec}[c + d x]}) + \\
& \left(2 (40 A b^5 + 1617 a^5 B + 3069 a^3 b^2 B - 110 a b^4 B + 15 a^2 b^3 (17 A + 33 C) + 15 a^4 b (247 A + 319 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \right. \\
& \left. \sqrt{a + b \operatorname{Sec}[c + d x]} \right) / \left(3465 a^3 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]} \right) + \frac{2 (5 A b^2 + 44 a b B + 3 a^2 (9 A + 11 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{231 d \operatorname{Sec}[c + d x]^{5/2}} + \\
& \frac{2 (15 A b^3 + 539 a^3 B + 825 a b^2 B + 5 a^2 b (229 A + 297 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3465 a d \operatorname{Sec}[c + d x]^{3/2}} - \frac{1}{3465 a^2 d \sqrt{\operatorname{Sec}[c + d x]}} \\
& \frac{2 (20 A b^4 - 1793 a^3 b B - 55 a b^3 B - 75 a^4 (9 A + 11 C) - 5 a^2 b^2 (205 A + 297 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{99 d \operatorname{Sec}[c + d x]^{7/2}} + \frac{2 A (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{11 d \operatorname{Sec}[c + d x]^{9/2}}
\end{aligned}$$

Result (type 6, 7479 leaves) : Display of huge result suppressed !

■ **Problem 1052: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 350 leaves, 13 steps) :

$$\begin{aligned}
& \frac{(4 b B - a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(8 A b^2 - 4 a b B + 3 a^2 C + 4 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{4 b^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{(4 b B - 3 a C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 b^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} + \\
& \frac{(4 b B - 3 a C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 b^2 d} + \frac{C \operatorname{Sec}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{2 b d}
\end{aligned}$$

Result (type 4, 690 leaves):

$$\left(\sqrt{b+a \cos [c+d x]} \left(A+B \sec [c+d x]+C \sec [c+d x]^2 \right) \right. \\
\left. \frac{\left(8 a b C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \right. \right. \\
\left. \left. + \frac{2\left(16 A b^2-12 a b B+9 a^2 C+8 b^2 C\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{\sqrt{b+a \cos [c+d x]}} \right)}{\sqrt{b+a \cos [c+d x]}} \right. \\
\left. \left(2 i\left(-4 a b B+3 a^2 C\right) \sqrt{\frac{a-a \cos [c+d x]}{a+b}} \sqrt{\frac{a+a \cos [c+d x]}{a-b}} \cos [2(c+d x)] \right. \right. \\
\left. \left. \left(-2 b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right. \right. \right. \\
\left. \left. \left. + a\left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right], \right. \right. \right. \\
\left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1-\frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos [c+d x]}\right], \frac{-a+b}{a+b}\right] \right) \right) \sin [c+d x] \right) / \\
\left. \left. \left. \left(\sqrt{\frac{1}{a-b}} b \sqrt{1-\cos [c+d x]} \sqrt{\frac{a^2-a^2 \cos [c+d x]^2}{a^2}} \left(-a^2+2 b^2-4 b(b+a \cos [c+d x])+2(b+a \cos [c+d x])^2\right) \right) \right) \right) \right) / \\
\left(8 b^2 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sec [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \right) + \\
\left((b+a \cos [c+d x])\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right) \right. \\
\left. \left(\frac{\sec [c+d x]\left(4 b B \sin [c+d x]-3 a C \sin [c+d x]\right)+\frac{C \sec [c+d x] \tan [c+d x]}{b}}{2 b^2} \right) \right) / \\
\left(d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sec [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \right)$$

■ **Problem 1053: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec [c+d x]} \left(A+B \sec [c+d x]+C \sec [c+d x]^2 \right)}{\sqrt{a+b \sec [c+d x]}} d x$$

Optimal (type 4, 260 leaves, 12 steps):

$$\frac{(2A + C) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]} + (2bB - aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\sec[c+dx]}}{d \sqrt{a+b \sec[c+dx]} + b d \sqrt{a+b \sec[c+dx]}}$$

$$\frac{C \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + C \sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{b d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \sqrt{\sec[c+dx]} + b d}$$

Result (type 4, 623 leaves):

$$\frac{2C(b+a \cos[c+dx])(A+B \sec[c+dx]+C \sec[c+dx]^2) \sin[c+dx]}{b d (A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]}} +$$

$$\left(\sqrt{b+a \cos[c+dx]} (A+B \sec[c+dx]+C \sec[c+dx]^2) \frac{8Ab \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos[c+dx]}} + \right.$$

$$\frac{2(4bB-3aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a \cos[c+dx]}} - \left(2iaC \sqrt{\frac{a-a \cos[c+dx]}{a+b}} \sqrt{\frac{a+a \cos[c+dx]}{a-b}} \cos[2(c+dx)] \right.$$

$$\left. \left. \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \right. \right. \right.$$

$$\left. \left. \left. \frac{-a+b}{a+b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \cos[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \right) \sin[c+dx] \right) /$$

$$\left(\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos[c+dx]} \sqrt{\frac{a^2 - a^2 \cos[c+dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b+a \cos[c+dx]) + 2(b+a \cos[c+dx])^2) \right) \right) /$$

$$\left(2bd(A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \sec[c+dx]^{3/2} \sqrt{a+b \sec[c+dx]} \right)$$

■ **Problem 1054: Unable to integrate problem.**

$$\int \frac{A+B \sec[c+dx]+C \sec[c+dx]^2}{\sqrt{\sec[c+dx]} \sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 219 leaves, 11 steps):

$$\frac{2 (A b - a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{a d \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{d \sqrt{a+b \sec [c+d x]}} + \frac{2 A \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{A + B \sec [c+d x] + C \sec [c+d x]^2}{\sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 1055: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c+d x] + C \sec [c+d x]^2}{\sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]}} dx$$

Optimal (type 4, 216 leaves, 8 steps):

$$\frac{2 (2 A b^2 - 3 a b B + a^2 (A + 3 C)) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 a^2 d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2 (2 A b - 3 a B) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} + \frac{2 A \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a d \sqrt{\sec [c+d x]}}$$

Result (type 6, 1959 leaves):

$$\left((b+a \cos [c+d x]) (A+B \sec [c+d x]+C \sec [c+d x]^2) \left(-\frac{4(-2 A b+3 a B) \cot [c]}{3 a^2 d} + \frac{4 A \cos [d x] \sin [c]}{3 a d} + \frac{4 A \cos [c] \sin [d x]}{3 a d} \right) \right) /$$

$$\left((A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sec [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \right) -$$

$$\left(4 A \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc [c] \left(b-a \sqrt{1+\cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]\right)}{a \sqrt{1+\cot [c]^2} \left(1 + \frac{b \csc [c]}{a \sqrt{1+\cot [c]^2}} \right)} \right] \right)$$

$$\begin{aligned}
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \sqrt{b + a \text{Cos}[c + dx]} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(3 a d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \right) - \\
& \left(4 C \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \sqrt{b + a \text{Cos}[c + dx]} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(a d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \right) +
\end{aligned}$$

$$\left(4 A b \sqrt{b + a \cos [c + d x]} \operatorname{Csc}[c] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Sec}[c] \left(b + a \cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right) \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right], \right. \right.$$

$$\left. \left. - \frac{\operatorname{Sec}[c] \left(b + a \cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right) \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} - a \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Tan}[c]^2} + a \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{-b \operatorname{Sec}[c] + a \sqrt{1 + \operatorname{Tan}[c]^2}}} \sqrt{b + a \cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \right) -$$

$$\left. \frac{\operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 a \cos [c] \left(b + a \cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a^2 \cos [c]^2 + a^2 \operatorname{Sin}[c]^2} \right) /$$

$$\left. \sqrt{b + a \cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \right) /$$

$$(3 a d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \operatorname{Sec}[c + d x]^{3/2} \sqrt{a + b \operatorname{Sec}[c + d x]}) -$$

$$\left(2 B \sqrt{b + a \cos [c + d x]} \operatorname{Csc}[c] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right], \right. \right.$$

$$\left. \left. - \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) \sqrt{$$

$$\left(\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2}}{\sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \sqrt{$$

$$(d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]})$$

■ **Problem 1056: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2}{\text{Sec}[c + dx]^{5/2} \sqrt{a + b \text{Sec}[c + dx]}} dx$$

Optimal (type 4, 291 leaves, 9 steps):

$$\begin{aligned}
& - \left(2 (8 A b^3 - 5 a^3 B - 10 a b^2 B + a^2 b (7 A + 15 C)) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), \frac{2a}{a+b}\right] \sqrt{\sec[c + dx]} \right) / (15 a^3 d \sqrt{a + b \sec[c + dx]}) + \\
& \frac{2 (8 A b^2 - 10 a b B + 3 a^2 (3 A + 5 C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), \frac{2a}{a+b}\right] \sqrt{a + b \sec[c + dx]}}{15 a^3 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \sqrt{\sec[c + dx]}} + \\
& \frac{2 A \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{5 a d \sec[c + dx]^{3/2}} - \frac{2 (4 A b - 5 a B) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{15 a^2 d \sqrt{\sec[c + dx]}}
\end{aligned}$$

Result (type 6, 3039 leaves):

$$\begin{aligned}
& \left((b + a \cos[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{4 (9 a^2 A + 8 A b^2 - 10 a b B + 15 a^2 C) \cot[c]}{15 a^3 d} + \right. \right. \\
& \quad \left. \frac{4 (-4 A b + 5 a B) \cos[dx] \sin[c]}{15 a^2 d} + \frac{2 A \cos[2 dx] \sin[2 c]}{5 a d} + \frac{4 (-4 A b + 5 a B) \cos[c] \sin[dx]}{15 a^2 d} + \frac{2 A \cos[2 c] \sin[2 dx]}{5 a d} \right) / \\
& \quad \left((A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) \sec[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} \right) - \\
& \quad \left(8 A b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] (b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]])]}{a \sqrt{1 + \cot[c]^2} \left(1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}\right)}\right], \right. \\
& \quad \left. \frac{\csc[c] (b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]])]}{a \sqrt{1 + \cot[c]^2} \left(-1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}\right)}\right] \sqrt{b + a \cos[c + dx]} \csc[c] \right) \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a \sqrt{1 + \cot[c]^2} - a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} - b \csc[c]}} \\
& \quad \left. \sqrt{\frac{a \sqrt{1 + \cot[c]^2} + a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} + b \csc[c]}} \sqrt{b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \quad \left(15 a^2 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) \sqrt{1 + \cot[c]^2} \sec[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} \right) -
\end{aligned}$$

$$\left(4 B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]] \right]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \right)} \right] \sqrt{b + a \operatorname{Cos}[c + d x]} \operatorname{Csc}[c] \right.$$

$$\left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left(3 a d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + d x]^{3/2} \sqrt{a + b \operatorname{Sec}[c + d x]} \right) -$$

$$\left(6 A \sqrt{b + a \operatorname{Cos}[c + d x]} \operatorname{Csc}[c] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right) \right.$$

$$\left. - \frac{\operatorname{Sec}[c] \left(b + a \operatorname{Cos}[c] \operatorname{Cos}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]] \right] \sqrt{1 + \operatorname{Tan}[c]^2} \right)}{a \sqrt{1 + \operatorname{Tan}[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \operatorname{Tan}[c]^2} \right)} \right] \operatorname{Sin}[d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]}) -$$

$$\left(16 A b^2 \sqrt{b + a \cos[c + d x]} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right.$$

$$\left. -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) / \\
& \left(15 a^2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]} \right) + \\
& \left(4 b B \sqrt{b + a \cos[c + d x]} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right) \\
& \left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right) \\
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\right) -
\end{aligned}$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right) /$$

$$\left(3 a d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \text{Sec}[c + d x]^{3/2} \sqrt{a + b \text{Sec}[c + d x]} \right) -$$

$$\left(2 C \sqrt{b + a \cos[c + d x]} \text{Csc}[c] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right] \right) \right)$$

$$- \frac{\text{Sec}[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \left] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

$$\left(\sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right) /$$

$$\left(d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \text{Sec}[c + d x]^{3/2} \sqrt{a + b \text{Sec}[c + d x]} \right)$$

Problem 1057: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{7/2} \sqrt{a + b \operatorname{Sec}[c + d x]}} dx$$

Optimal (type 4, 380 leaves, 10 steps):

$$\left(2 \left(48 A b^4 - 49 a^3 b B - 56 a b^3 B + 5 a^4 (5 A + 7 C) + 2 a^2 b^2 (16 A + 35 C) \right) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} \right) /$$

$$\left(105 a^4 d \sqrt{a + b \operatorname{Sec}[c + d x]} \right) - \frac{2 \left(48 A b^3 - 63 a^3 B - 56 a b^2 B + a^2 (44 A b + 70 b C) \right) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]}}{105 a^4 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]}} +$$

$$\frac{2 A \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{7 a d \operatorname{Sec}[c + d x]^{5/2}} - \frac{2 (6 A b - 7 a B) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{35 a^2 d \operatorname{Sec}[c + d x]^{3/2}} +$$

$$\frac{2 \left(24 A b^2 - 28 a b B + 5 a^2 (5 A + 7 C) \right) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{105 a^3 d \sqrt{\operatorname{Sec}[c + d x]}}$$

Result (type 6, 4470 leaves):

$$\left((b + a \operatorname{Cos}[c + d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \left(-\frac{4 (-44 a^2 A b - 48 A b^3 + 63 a^3 B + 56 a b^2 B - 70 a^2 b C) \operatorname{Cot}[c]}{105 a^4 d} + \right. \right.$$

$$\frac{(115 a^2 A + 96 A b^2 - 112 a b B + 140 a^2 C) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{105 a^3 d} + \frac{2 (-6 A b + 7 a B) \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{35 a^2 d} + \frac{A \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{7 a d} +$$

$$\left. \frac{(115 a^2 A + 96 A b^2 - 112 a b B + 140 a^2 C) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{105 a^3 d} + \frac{2 (-6 A b + 7 a B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 d x]}{35 a^2 d} + \frac{A \operatorname{Cos}[3 c] \operatorname{Sin}[3 d x]}{7 a d} \right) /$$

$$\left((A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \operatorname{Sec}[c + d x]^{3/2} \sqrt{a + b \operatorname{Sec}[c + d x]} \right) -$$

$$\left(20 A \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]] \right]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right),$$

$$\frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]] \right]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] \sqrt{b + a \operatorname{Cos}[c + d x]} \operatorname{Csc}[c]$$

$$\begin{aligned}
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(21 a d (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + b \operatorname{Sec}[c + dx]} \right) + \\
& \left(16 A b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] \sqrt{b + a \operatorname{Cos}[c + dx]} \operatorname{Csc}[c] \\
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \right. \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(35 a^3 d (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + b \operatorname{Sec}[c + dx]} \right) - \\
& \left(8 b B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \sqrt{b + a \text{Cos}[c + dx]} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(15 a^2 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \right) - \\
& \left(4 C \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right] \sqrt{b + a \text{Cos}[c + dx]} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(3 a d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \right) +
\end{aligned}$$

$$\left(88 A b \sqrt{b + a \cos[c + dx]} \operatorname{Csc}[c] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \right. \right. \right.$$

$$\left. \left. \left. -\frac{\operatorname{Sec}[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2} \right]}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right), -\frac{\operatorname{Sec}[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2} \right]}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right)$$

$$\left. \left. \left. \frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \operatorname{Sec}[c] + a \sqrt{1 + \tan[c]^2}}} \right. \right.$$

$$\left. \left. \left. \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \operatorname{Sec}[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) - \right.$$

$$\left. \left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2}}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right) \right)$$

$$(105 a d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \operatorname{Sec}[c + dx]^{3/2} \sqrt{a + b \operatorname{Sec}[c + dx]}) +$$

$$\left(32 A b^3 \sqrt{b + a \cos[c + dx]} \operatorname{Csc}[c] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right.$$

$$\left. \left(\operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \right. \right. \right.$$

$$\left. \left. \left. -\frac{\operatorname{Sec}[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]] \right) \sqrt{1 + \tan[c]^2} \right)}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \operatorname{Sec}[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right), \right.$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\frac{\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}}}{\sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) -$$

$$\frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2}}{\sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \Bigg/$$

$$\left(35 a^3 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \text{Sec}[c + dx]^{3/2} \sqrt{a + b \text{Sec}[c + dx]} \right) -$$

$$\left(6 B \sqrt{b + a \text{Cos}[c + dx]} \text{Csc}[c] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \right) \right)$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]}) -$$

$$\left(16 b^2 B \sqrt{b + a \cos[c + d x]} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right.$$

$$\left. -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) / \\
& \left(15 a^2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]} \right) + \\
& \left(4 b C \sqrt{b + a \cos[c + d x]} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right) \\
& \left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right) \\
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\right) -
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \Bigg/$$

$$(3 a d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]})$$

- **Problem 1058: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\sec[c + d x]} (a A + (A b + a B) \sec[c + d x] + b B \sec[c + d x]^2)}{\sqrt{a + b \sec[c + d x]}} dx$$

Optimal (type 4, 253 leaves, 13 steps):

$$\frac{(2 a A + b B) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b} \sqrt{\sec[c + d x]}\right]}{d \sqrt{a + b \sec[c + d x]}} +$$

$$\frac{(2 A b + a B) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \text{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b} \sqrt{\sec[c + d x]}\right] \sqrt{\sec[c + d x]}}{d \sqrt{a + b \sec[c + d x]}} -$$

$$\frac{B \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b} \sqrt{a + b \sec[c + d x]}\right]}{d \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \sqrt{\sec[c + d x]}} + \frac{B \sqrt{\sec[c + d x]} \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{d}$$

Result (type 4, 521 leaves):

$$\begin{aligned}
& \frac{B (b + a \operatorname{Cos}[c + d x]) \operatorname{Sec}[c + d x]^{3/2} \operatorname{Sin}[c + d x]}{d \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\
& \frac{1}{4 d \sqrt{a + b \operatorname{Sec}[c + d x]}} \sqrt{b + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]} \left(\frac{8 a A \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right]}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \right. \\
& \frac{2 (4 A b + a B) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2 a}{a + b}\right]}{\sqrt{b + a \operatorname{Cos}[c + d x]}} - \left(2 i a B \sqrt{\frac{a - a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\frac{a + a \operatorname{Cos}[c + d x]}{a - b}} \operatorname{Cos}[2 (c + d x)] \right. \\
& \left. \left(-2 b (a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right] + a \left(2 b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \right. \right. \right. \\
& \left. \left. \left. \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \operatorname{Cos}[c + d x]}\right], \frac{-a + b}{a + b}\right]\right) \right) \operatorname{Sin}[c + d x] \Bigg) / \\
& \left. \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \operatorname{Cos}[c + d x]}^2 \sqrt{\frac{a^2 - a^2 \operatorname{Cos}[c + d x]^2}{a^2}} (-a^2 + 2 b^2 - 4 b (b + a \operatorname{Cos}[c + d x]) + 2 (b + a \operatorname{Cos}[c + d x])^2) \right) \right)
\end{aligned}$$

■ **Problem 1059: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sec}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{(a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 393 leaves, 13 steps):

$$\begin{aligned}
& \frac{C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]} + (2 b B-3 a C) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b d \sqrt{a+b \sec [c+d x]}} + \frac{(2 b B-3 a C) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{b^2 d \sqrt{a+b \sec [c+d x]}} - \\
& \frac{(2 A b^2-2 a b B+3 a^2 C-b^2 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{b^2\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} - \\
& \frac{2\left(A b^2-a(b B-a C)\right) \sec [c+d x]^{3 / 2} \sin [c+d x]}{b\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}} + \frac{\left(2 A b^2-2 a b B+3 a^2 C-b^2 C\right) \sqrt{\sec [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{b^2\left(a^2-b^2\right) d}
\end{aligned}$$

Result (type 4, 774 leaves):

$$\left((b + a \cos[c + dx])^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{2(4Ab^3 - 4ab^2B + 4a^2bC) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a\cos[c+dx]}} + \right. \right.$$

$$\frac{2(2aAb^2 - 6a^2bB + 4b^3B + 9a^3C - 7ab^2C) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{\sqrt{b+a\cos[c+dx]}} +$$

$$\left(2i(2aAb^2 - 2a^2bB + 3a^3C - ab^2C) \sqrt{\frac{a-a\cos[c+dx]}{a+b}} \sqrt{\frac{a+a\cos[c+dx]}{a-b}} \cos[2(c+dx)] \right.$$

$$\left. \left(-2b(a+b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a\cos[c+dx]}\right], \frac{-a+b}{a+b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a\cos[c+dx]}\right], \frac{-a+b}{a+b}\right], \right. \right.$$

$$\left. \left. \frac{-a+b}{a+b} \right) + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a\cos[c+dx]}\right], \frac{-a+b}{a+b}\right] \right) \left. \right) \sin[c+dx] \left. \right) /$$

$$\left(\left(\sqrt{\frac{1}{a-b}} b \sqrt{1 - \cos[c+dx]}^2 \sqrt{\frac{a^2 - a^2 \cos[c+dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b+a\cos[c+dx]) + 2(b+a\cos[c+dx])^2) \right) \right) /$$

$$\left(2b^2(-a+b)(a+b)d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+b\sec[c+dx])^{3/2} + \right.$$

$$\left((b+a\cos[c+dx])^2 (A+B\sec[c+dx]+C\sec[c+dx]^2) \right.$$

$$\left. \left(-\frac{4(aAb^2\sin[c+dx]-a^2bB\sin[c+dx]+a^3C\sin[c+dx])}{b^2(-a^2+b^2)(b+a\cos[c+dx])} + \frac{2C\tan[c+dx]}{b^2} \right) \right) /$$

$$\left(d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])\sqrt{\sec[c+dx]}(a+b\sec[c+dx])^{3/2} \right)$$

■ **Problem 1060: Unable to integrate problem.**

$$\int \frac{\sqrt{\sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{(a + b \sec[c+dx])^{3/2}} dx$$

Optimal (type 4, 311 leaves, 12 steps):

$$\left(4 A b \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.$$

$$\left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] (b + a \cos[c + d x])^{3/2} \operatorname{Csc}[c]$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}}$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left(a (a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right) -$$

$$\left(4 B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.$$

$$\left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] (b + a \cos[c + d x])^{3/2} \operatorname{Csc}[c]$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}}$$

$$\left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\begin{aligned}
& \left((a^2 - b^2) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sqrt{\sec[c + dx]} (a + b \sec[c + dx])^{3/2} \right) + \\
& \left(4 b C \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right], \right. \\
& \left. \frac{\csc[c] \left(b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right)}{a \sqrt{1 + \cot[c]^2} \left(-1 + \frac{b \csc[c]}{a \sqrt{1 + \cot[c]^2}} \right)} \right] (b + a \cos[c + dx])^{3/2} \csc[c] \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{\frac{a \sqrt{1 + \cot[c]^2} - a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} - b \csc[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \cot[c]^2} + a \sqrt{1 + \cot[c]^2} \sin[dx - \operatorname{ArcTan}[\cot[c]]]}{a \sqrt{1 + \cot[c]^2} + b \csc[c]}} \sqrt{b - a \sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(a (a^2 - b^2) d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \sqrt{\sec[c + dx]} (a + b \sec[c + dx])^{3/2} \right) - \\
& \left(2 a A (b + a \cos[c + dx])^{3/2} \csc[c] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. \left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{\sec[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2} \right)}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right. \right. \\
& \left. \left. - \frac{\sec[c] \left(b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2} \right)}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /
\end{aligned}$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$\left((a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right) +$$

$$\left(4 A b^2 (b + a \cos[c + d x])^{3/2} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \right) \right)$$

$$- \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\begin{aligned}
& \left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) / \\
& \left(a (a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right) - \\
& \left(2 b B (b + a \cos[c + d x])^{3/2} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right) \\
& \left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \right. \right. \\
& \left. \left. - \frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right) \\
& \left(\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) -
\end{aligned}$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right) /$$

$$\left((a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right) +$$

$$\left(2 a C (b + a \cos[c + d x])^{3/2} \text{Csc}[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right] \right) \right)$$

$$- \frac{\sec[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \left] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \sec[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \right)$$

$$\left(\sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \sec[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \cos[c] \left(b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right) /$$

$$\left((a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right)$$

Problem 1062: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Sec}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{2(8Ab^2 - 6abB + a^2(A + 3C)) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{\operatorname{Sec}[c+dx]} + \frac{2(8Ab^3 + 3a^3B - 6ab^2B - a^2(5Ab - 3bC)) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]} + 3a^3(a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \sqrt{\operatorname{Sec}[c+dx]} - \frac{2(Ab^2 - a(bB - aC)) \operatorname{Sin}[c+dx]}{a(a^2 - b^2) d \sqrt{\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} - \frac{2(4Ab^2 - 3abB - a^2(A - 3C)) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3a^2(a^2 - b^2) d \sqrt{\operatorname{Sec}[c+dx]}}$$

Result (type 6, 4557 leaves):

$$\left((b + a \operatorname{Cos}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\ \left. \left(-1 / (3 a^3 (a^2 - b^2) d) 2 (-5 a^2 A b + 11 A b^3 + 3 a^3 B - 9 a b^2 B + 6 a^2 b C - 5 a^2 A b \operatorname{Cos}[2 c] + 5 A b^3 \operatorname{Cos}[2 c] + 3 a^3 B \operatorname{Cos}[2 c] - 3 a b^2 B \operatorname{Cos}[2 c]) \right. \right. \\ \left. \left. \operatorname{Csc}[c] \operatorname{Sec}[c] + \frac{4 A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 a^2 d} + \frac{4 A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 a^2 d} + \right. \right. \\ \left. \left. (4 \operatorname{Sec}[c] (A b^4 \operatorname{Sin}[c] - a b^3 B \operatorname{Sin}[c] + a^2 b^2 C \operatorname{Sin}[c] - a A b^3 \operatorname{Sin}[d x] + a^2 b^2 B \operatorname{Sin}[d x] - a^3 b C \operatorname{Sin}[d x])) / \right. \right. \\ \left. \left. (a^3 (a^2 - b^2) d (b + a \operatorname{Cos}[c + d x])) \right) \right) / \\ \left((A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{\operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2} - \right. \\ \left. \left(4 A \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]\right]} \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right), \right. \\ \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]\right]}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right) \right) (b + a \operatorname{Cos}[c + d x])^{3/2} \operatorname{Csc}[c] \right)$$

$$\begin{aligned}
& \left(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2 \right) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 (a^2 - b^2) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \sqrt{\operatorname{Sec}[c + dx]} (a + b \operatorname{Sec}[c + dx])^{3/2} \right) - \\
& \left(8 A b^2 \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right. \\
& \left. \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(-1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right] (b + a \operatorname{Cos}[c + dx])^{3/2} \operatorname{Csc}[c] \\
& \left(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2 \right) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} - b \operatorname{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \operatorname{Cot}[c]^2} + a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{a \sqrt{1 + \operatorname{Cot}[c]^2} + b \operatorname{Csc}[c]}} \sqrt{b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 a^2 (a^2 - b^2) d (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \sqrt{\operatorname{Sec}[c + dx]} (a + b \operatorname{Sec}[c + dx])^{3/2} \right) + \\
& \left(4 b B \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[c] \left(b - a \sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right)}{a \sqrt{1 + \operatorname{Cot}[c]^2} \left(1 + \frac{b \operatorname{Csc}[c]}{a \sqrt{1 + \operatorname{Cot}[c]^2}} \right)} \right], \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \Big] (b + a \text{Cos}[c + dx])^{3/2} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left((a^2 - b^2) d (A + 2C + 2B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} \sqrt{\text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^{3/2} \right) - \\
& \left(4 \text{C AppellF1} \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \right], \right. \\
& \frac{\text{Csc}[c] \left(b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right)}{a \sqrt{1 + \text{Cot}[c]^2} \left(-1 + \frac{b \text{Csc}[c]}{a \sqrt{1 + \text{Cot}[c]^2}} \right)} \Big] (b + a \text{Cos}[c + dx])^{3/2} \text{Csc}[c] \\
& (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} - b \text{Csc}[c]}} \\
& \left. \sqrt{\frac{a \sqrt{1 + \text{Cot}[c]^2} + a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}{a \sqrt{1 + \text{Cot}[c]^2} + b \text{Csc}[c]}} \sqrt{b - a \sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left((a^2 - b^2) d (A + 2C + 2B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} \sqrt{\text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^{3/2} \right) +
\end{aligned}$$

$$\left(10 A b (b + a \cos[c + dx])^{3/2} \csc[c] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left(\left(\operatorname{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right. \right.$$

$$\left. \left. - \frac{\sec[c] (b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]) \sqrt{1 + \tan[c]^2}}{a^2 \cos[c]^2 + a^2 \sin[c]^2} \right) /$$

$$\left(3 (a^2 - b^2) d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{\sec[c + dx]} (a + b \sec[c + dx])^{3/2} \right) -$$

$$\left(16 A b^3 (b + a \cos[c + dx])^{3/2} \csc[c] (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right], \right.$$

$$\left. - \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\sqrt{1 + \text{Tan}[c]^2} \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left(\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2} \right) \Bigg/ \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \Bigg/$$

$$\left(3 a^2 (a^2 - b^2) d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{\text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^{3/2} \right) -$$

$$\left(2 a B (b + a \text{Cos}[c + dx])^{3/2} \text{Csc}[c] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]] \right) \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2} \right)} \right], \right.$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\frac{\sqrt{1 + \text{Tan}[c]^2}}{\sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} - a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}}} \right.$$

$$\left. \sqrt{\frac{a \sqrt{1 + \text{Tan}[c]^2} + a \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{-b \text{Sec}[c] + a \sqrt{1 + \text{Tan}[c]^2}}} \sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 a \text{Cos}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a^2 \text{Cos}[c]^2 + a^2 \text{Sin}[c]^2}}{\sqrt{b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \Bigg/$$

$$\left((a^2 - b^2) d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{\text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^{3/2} \right) +$$

$$\left(4 b^2 B (b + a \text{Cos}[c + dx])^{3/2} \text{Csc}[c] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \right) \right)$$

$$\left. \frac{\text{Sec}[c] \left(b + a \text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \right)}{a \sqrt{1 + \text{Tan}[c]^2} \left(-1 - \frac{b \text{Sec}[c]}{a \sqrt{1 + \text{Tan}[c]^2}} \right)} \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \Bigg/$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} - \sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) /$$

$$(a (a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2}) -$$

$$\left(2 b C (b + a \cos[c + d x])^{3/2} \csc[c] (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{AppellF1} \left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right], \right.$$

$$\left. -\frac{\sec[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a \sqrt{1 + \tan[c]^2} \left(-1 - \frac{b \sec[c]}{a \sqrt{1 + \tan[c]^2}} \right)} \right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 + \tan[c]^2} \sqrt{\frac{a \sqrt{1 + \tan[c]^2} - a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{b \sec[c] + a \sqrt{1 + \tan[c]^2}}} \right)$$

$$\left(\frac{\sqrt{\frac{a \sqrt{1 + \tan[c]^2} + a \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{-b \sec[c] + a \sqrt{1 + \tan[c]^2}} \sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}}{\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 a \cos[c] (b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2})}{a^2 \cos[c]^2 + a^2 \sin[c]^2}}{\sqrt{b + a \cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}}} \right) /$$

$$\left((a^2 - b^2) d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2} \right)$$

- **Problem 1063: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + d x] + C \sec[c + d x]^2}{\sec[c + d x]^{5/2} (a + b \sec[c + d x])^{3/2}} dx$$

Optimal (type 4, 461 leaves, 10 steps):

$$\begin{aligned} & - \left(2 (48 A b^3 - 5 a^3 B - 40 a b^2 B + 6 a^2 b (2 A + 5 C)) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b} \sqrt{\sec[c + d x]} \right] / \right. \\ & \quad \left. (15 a^4 d \sqrt{a + b \sec[c + d x]}) - \right. \\ & \quad \left(2 (48 A b^4 + 25 a^3 b B - 40 a b^3 B - 6 a^2 b^2 (4 A - 5 C) - 3 a^4 (3 A + 5 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b} \sqrt{a + b \sec[c + d x]} \right] / \right. \\ & \quad \left. \left(15 a^4 (a^2 - b^2) d \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \sqrt{\sec[c + d x]} \right) + \right. \\ & \quad \frac{2 (A b^2 - a (b B - a C)) \sin[c + d x]}{a (a^2 - b^2) d \sec[c + d x]^{3/2} \sqrt{a + b \sec[c + d x]}} - \frac{2 (6 A b^2 - 5 a b B - a^2 (A - 5 C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{5 a^2 (a^2 - b^2) d \sec[c + d x]^{3/2}} + \\ & \quad \left. \frac{2 (24 A b^3 + 5 a^3 B - 20 a b^2 B - a^2 (9 A b - 15 b C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{15 a^3 (a^2 - b^2) d \sqrt{\sec[c + d x]}} \right) \end{aligned}$$

Result (type 6, 6134 leaves): Display of huge result suppressed!

- **Problem 1064: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sec[c + d x]^{5/2} (A + B \sec[c + d x] + C \sec[c + d x]^2)}{(a + b \sec[c + d x])^{5/2}} dx$$

Optimal (type 4, 563 leaves, 14 steps):

$$\begin{aligned}
 & \frac{(2 A b^2 - 2 a b B + 5 a^2 C - 3 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{3 b^2 (a^2 - b^2) d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
 & \frac{(2 b B - 5 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\operatorname{Sec}[c+d x]}}{b^3 d \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
 & \frac{(8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{3 b^3 (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \sqrt{\operatorname{Sec}[c+d x]}} - \\
 & \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sec}[c+d x]^{5/2} \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 (3 A b^4 + 2 a^3 b B - 6 a b^3 B - 5 a^4 C + a^2 b^2 (A + 9 C)) \operatorname{Sec}[c+d x]^{3/2} \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
 & \frac{(8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \sqrt{\operatorname{Sec}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^3 (a^2 - b^2)^2 d}
 \end{aligned}$$

Result (type 4, 938 leaves):

$$\begin{aligned}
& - \left(\left((b + a \cos[c + dx])^{5/2} \sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{1}{\sqrt{b + a \cos[c + dx]}} \right. \right. \right. \\
& \quad 2(-4a^2Ab^3 - 12Ab^5 - 8a^3b^2B + 24ab^4B + 20a^4bC - 36a^2b^3C) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2a}{a + b}\right] + \frac{1}{\sqrt{b + a \cos[c + dx]}} \\
& \quad 2(-8aAb^4 - 18a^4bB + 38a^2b^3B - 12b^5B + 45a^5C - 86a^3b^2C + 33ab^4C) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2a}{a + b}\right] + \\
& \quad \left(2i(-8aAb^4 - 6a^4bB + 14a^2b^3B + 15a^5C - 26a^3b^2C + 3ab^4C) \sqrt{\frac{a - a \cos[c + dx]}{a + b}} \sqrt{\frac{a + a \cos[c + dx]}{a - b}} \cos[2(c + dx)] \right. \\
& \quad \left. \left(-2b(a + b) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos[c + dx]}\right], \frac{-a + b}{a + b}\right] + a \left(2b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \sqrt{b + a \cos[c + dx]}\right], \frac{-a + b}{a + b}\right] + a \operatorname{EllipticPi}\left[1 - \frac{a}{b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{1}{a - b}} \sqrt{b + a \cos[c + dx]}\right], \frac{-a + b}{a + b}\right] \right) \sin[c + dx] \right) / \\
& \quad \left(\sqrt{\frac{1}{a - b}} b \sqrt{1 - \cos[c + dx]}^2 \sqrt{\frac{a^2 - a^2 \cos[c + dx]^2}{a^2}} (-a^2 + 2b^2 - 4b(b + a \cos[c + dx]) + 2(b + a \cos[c + dx])^2) \right) \left. \right) / \\
& \quad \left(6(a - b)^2 b^3 (a + b)^2 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^{5/2} \right) + \\
& \quad \left((b + a \cos[c + dx])^3 \sqrt{\sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \quad \left(-\frac{4(aAb^2 \sin[c + dx] - a^2bB \sin[c + dx] + a^3C \sin[c + dx])}{3b^2(-a^2 + b^2)(b + a \cos[c + dx])^2} - \right. \\
& \quad \left(4(4aAb^4 \sin[c + dx] + 3a^4bB \sin[c + dx] - 7a^2b^3B \sin[c + dx] - 6a^5C \sin[c + dx] + 10a^3b^2C \sin[c + dx]) \right) / \\
& \quad \left. \left(3b^3(-a^2 + b^2)^2(b + a \cos[c + dx]) + \frac{2C \tan[c + dx]}{b^3} \right) \right) / \\
& \quad \left(d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + b \sec[c + dx])^{5/2} \right)
\end{aligned}$$

■ **Problem 1065: Unable to integrate problem.**

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 447 leaves, 13 steps):

$$\frac{2 (A b^2 - a (b B - a C)) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \text{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{\text{Sec}[c + d x]} +}{3 a b (a^2 - b^2) d \sqrt{a + b \text{Sec}[c + d x]}}$$

$$\frac{2 C \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \text{EllipticPi}\left[2, \frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{\text{Sec}[c + d x]}}{b^2 d \sqrt{a + b \text{Sec}[c + d x]}} -$$

$$\frac{2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2a}{a+b}\right] \sqrt{a + b \text{Sec}[c + d x]}}{3 a b^2 (a^2 - b^2)^2 d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \sqrt{\text{Sec}[c + d x]}} -$$

$$\frac{2 (A b^2 - a (b B - a C)) \text{Sec}[c + d x]^{3/2} \text{Sin}[c + d x]}{3 b (a^2 - b^2) d (a + b \text{Sec}[c + d x])^{3/2}} + \frac{2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \sqrt{\text{Sec}[c + d x]} \text{Sin}[c + d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{a + b \text{Sec}[c + d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{\text{Sec}[c + d x]^{3/2} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

■ **Problem 1066: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\text{Sec}[c + d x]} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{(a + b \text{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 378 leaves, 9 steps):

$$\frac{2 \left(2 A b^2 + a b B - a^2 (3 A + C) \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 a^2 \left(a^2 - b^2\right) d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2 \left(2 A b^3 + 3 a^3 B + a b^2 B - 2 a^2 b (3 A + 2 C) \right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^2 \left(a^2 - b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]}} -$$

$$\frac{2 \left(A b^2 - a (b B - a C) \right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 b \left(a^2 - b^2\right) d (a+b \sec [c+d x])^{3/2}} + \frac{2 \left(A b^4 + 2 a^3 b B + 2 a b^3 B + a^4 C - 5 a^2 b^2 (A + C) \right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 a b \left(a^2 - b^2\right)^2 d \sqrt{a+b \sec [c+d x]}}$$

Result (type 6, 5040 leaves) : Display of huge result suppressed!

- **Problem 1067: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c+d x] + C \sec [c+d x]^2}{\sqrt{\sec [c+d x]} (a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 401 leaves, 9 steps) :

$$\frac{2 \left(8 A b^3 + 3 a^3 B - 2 a b^2 B - a^2 b (9 A + C) \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{\sec [c+d x]}}{3 a^3 \left(a^2 - b^2\right) d \sqrt{a+b \sec [c+d x]}} +$$

$$\left(2 \left(8 A b^4 + 6 a^3 b B - 2 a b^3 B + 3 a^4 (A - C) - a^2 b^2 (15 A + C) \right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) /$$

$$\left(3 a^3 \left(a^2 - b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \sqrt{\sec [c+d x]} \right) + \frac{2 \left(A b^2 - a (b B - a C) \right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 a \left(a^2 - b^2\right) d (a+b \sec [c+d x])^{3/2}} -$$

$$\frac{2 \left(4 A b^4 + 5 a^3 b B - a b^3 B - 2 a^4 C - 2 a^2 b^2 (4 A + C) \right) \sqrt{\sec [c+d x]} \sin [c+d x]}{3 a^2 \left(a^2 - b^2\right)^2 d \sqrt{a+b \sec [c+d x]}}$$

Result (type 6, 6142 leaves) : Display of huge result suppressed!

- **Problem 1068: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c+d x] + C \sec [c+d x]^2}{\sec [c+d x]^{3/2} (a+b \sec [c+d x])^{5/2}} dx$$

Optimal (type 4, 521 leaves, 10 steps) :

$$\begin{aligned}
& - \left(2 (16 A b^4 + 9 a^3 b B - 8 a b^3 B - 2 a^2 b^2 (8 A - C) - a^4 (A + 3 C)) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\sec[c + d x]} \right) / \\
& \quad \left(3 a^4 (a^2 - b^2) d \sqrt{a + b \sec[c + d x]} \right) - \\
& \left(2 (16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B - 2 a^2 b^3 (14 A - C) + a^4 (8 A b - 6 b C)) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \sec[c + d x]} \right) / \\
& \quad \left(3 a^4 (a^2 - b^2)^2 d \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \sqrt{\sec[c + d x]} \right) + \\
& \frac{2 (A b^2 - a (b B - a C)) \sin[c + d x]}{3 a (a^2 - b^2) d \sqrt{\sec[c + d x]} (a + b \sec[c + d x])^{3/2}} + \frac{2 (10 a^2 A b^2 - 6 A b^4 - 7 a^3 b B + 3 a b^3 B + 4 a^4 C) \sin[c + d x]}{3 a^2 (a^2 - b^2)^2 d \sqrt{\sec[c + d x]} \sqrt{a + b \sec[c + d x]}} + \\
& \frac{2 (8 A b^4 + 8 a^3 b B - 4 a b^3 B + a^4 (A - 5 C) - a^2 b^2 (13 A - C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{3 a^3 (a^2 - b^2)^2 d \sqrt{\sec[c + d x]}}
\end{aligned}$$

Result (type 6, 7608 leaves): Display of huge result suppressed!

- **Problem 1069: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + d x] + C \sec[c + d x]^2}{\sec[c + d x]^{5/2} (a + b \sec[c + d x])^{5/2}} dx$$

Optimal (type 4, 663 leaves, 11 steps):

$$\begin{aligned}
& \left(2 (128 A b^5 + 5 a^5 B + 80 a^3 b^2 B - 80 a b^4 B - 4 a^2 b^3 (29 A - 10 C) - a^4 b (17 A + 45 C)) \right. \\
& \left. \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{\operatorname{Sec}[c + d x]} \right) / \left(15 a^5 (a^2 - b^2) d \sqrt{a + b \operatorname{Sec}[c + d x]} \right) + \\
& \left(2 (128 A b^6 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B + 5 a^4 b^2 (11 A - 15 C) - 4 a^2 b^4 (53 A - 10 C) + 3 a^6 (3 A + 5 C)) \right. \\
& \left. \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]} \right) / \left(15 a^5 (a^2 - b^2)^2 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \sqrt{\operatorname{Sec}[c + d x]} \right) + \\
& \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sin}[c + d x]}{3 a (a^2 - b^2) d \operatorname{Sec}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} - \frac{2 (8 A b^4 + 9 a^3 b B - 5 a b^3 B - 2 a^2 b^2 (6 A - C) - 6 a^4 C) \operatorname{Sin}[c + d x]}{3 a^2 (a^2 - b^2)^2 d \operatorname{Sec}[c + d x]^{3/2} \sqrt{a + b \operatorname{Sec}[c + d x]}} + \\
& \frac{2 (48 A b^4 + 50 a^3 b B - 30 a b^3 B + a^4 (3 A - 35 C) - a^2 b^2 (71 A - 15 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{15 a^3 (a^2 - b^2)^2 d \operatorname{Sec}[c + d x]^{3/2}} - \\
& \left(2 (64 A b^5 - 5 a^5 B + 65 a^3 b^2 B - 40 a b^4 B + 2 a^4 b (7 A - 20 C) - 2 a^2 b^3 (49 A - 10 C)) \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x] \right) / \\
& \left(15 a^4 (a^2 - b^2)^2 d \sqrt{\operatorname{Sec}[c + d x]} \right)
\end{aligned}$$

Result (type 6, 9192 leaves): Display of huge result suppressed!

■ **Problem 1070: Attempted integration timed out after 120 seconds.**

$$\int (a + b \operatorname{Sec}[c + d x])^{2/3} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 9, 247 leaves, 8 steps):

$$\begin{aligned}
& \left(\sqrt{2} (a + b) C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x] \right) / \\
& \left(b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \right) + \\
& \left(\sqrt{2} (b B - a C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b (1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x] \right) / \\
& \left(b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b} \right)^{2/3} \right) + A \operatorname{Unintegrable}[(a + b \operatorname{Sec}[c + d x])^{2/3}, x]
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1072: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{(a + b \operatorname{Sec}[c + d x])^{1/3}} dx$$

Optimal (type 9, 244 leaves, 8 steps):

$$\frac{\sqrt{2} C \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] (a + b \operatorname{Sec}[c + d x])^{2/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{2/3}} +$$

$$\frac{\sqrt{2} (b B - a C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2} (1 - \operatorname{Sec}[c + d x]), \frac{b(1 - \operatorname{Sec}[c + d x])}{a + b}\right] \left(\frac{a + b \operatorname{Sec}[c + d x]}{a + b}\right)^{1/3} \operatorname{Tan}[c + d x]}{b d \sqrt{1 + \operatorname{Sec}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{1/3}} +$$

$$A \operatorname{Unintegrable}\left[\frac{1}{(a + b \operatorname{Sec}[c + d x])^{1/3}}, x\right]$$

Result (type 1, 1 leaves):

???

■ **Problem 1078: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + d x]^{3/2} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 48 leaves, 3 steps):

$$\frac{2 (A + 3 C) \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 A \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sin}[c + d x]}{3 d}$$

Result (type 5, 124 leaves):

$$- \left(4 \sqrt{\operatorname{Cos}[c + d x]} (C + A \operatorname{Cos}[c + d x]^2) \operatorname{Sin}[c] \right.$$

$$\left. \left((A + 3 C) \sqrt{\operatorname{Cos}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2} \sqrt{\operatorname{Csc}[c]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \right.$$

$$\left. \left. \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] - A \operatorname{Csc}[c] \operatorname{Sin}[c + d x] \right) \right) / (3 d (A + 2 C + A \operatorname{Cos}[2 (c + d x)]))$$

■ **Problem 1079: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\operatorname{Cos}[c + d x]} (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 44 leaves, 3 steps):

$$\frac{2 (A - C) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 C \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Cos}[c + d x]}}$$

Result (type 5, 289 leaves) :

$$\left(\cos [c+d x]^2 (A+C \sec [c+d x])^2 \right. \\ \left. - \frac{4((A-2 C) \cos [d x]+A \cos [2 c+d x]) \operatorname{Csc}[c]}{d \sqrt{\cos [c+d x]}} + \left(2(A-C) \operatorname{Csc}\left[\frac{c}{2}\right] \left(3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] + \right. \right. \right. \\ \left. \left. e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \right) \operatorname{Sec}\left[\frac{c}{2}\right] \sqrt{e^{-i d x}(2(1+e^{2 i d x}) \cos [c]+2 i(-1+e^{2 i d x}) \sin [c])} \right. \right. \\ \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]} \right) \right] / \left(3 d((1+e^{2 i d x}) \cos [c]+i(-1+e^{2 i d x}) \sin [c]) \right) \left. \right) / (2(A+2 C+A \cos [2(c+d x)]))$$

■ **Problem 1083: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{9 / 2}(a+a \sec [c+d x])(A+C \sec [c+d x])^2 d x$$

Optimal (type 4, 165 leaves, 8 steps) :

$$\frac{2 a(7 A+9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \frac{2 a(5 A+7 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d} + \frac{2 a(5 A+7 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{21 d} + \\ \frac{2 a(7 A+9 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d} + \frac{2 a A \cos [c+d x]^{5 / 2} \sin [c+d x]}{7 d} + \frac{2 a A \cos [c+d x]^{7 / 2} \sin [c+d x]}{9 d}$$

Result (type 5, 918 leaves) :

$$a \left(\sqrt{\cos [c+d x]}(1+\cos [c+d x]) \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \right. \\ \left(-\frac{(7 A+9 C) \operatorname{Cot}[c]}{15 d} + \frac{(23 A+28 C) \cos [d x] \sin [c]}{84 d} + \frac{(19 A+18 C) \cos [2 d x] \sin [2 c]}{180 d} + \frac{A \cos [3 d x] \sin [3 c]}{28 d} + \frac{A \cos [4 d x] \sin [4 c]}{72 d} + \right. \\ \left. \frac{(23 A+28 C) \cos [c] \sin [d x]}{84 d} + \frac{(19 A+18 C) \cos [2 c] \sin [2 d x]}{180 d} + \frac{A \cos [3 c] \sin [3 d x]}{28 d} + \frac{A \cos [4 c] \sin [4 d x]}{72 d} \right) - \frac{1}{21 d \sqrt{1+\operatorname{Cot}[c]^2}} \\ 5 A(1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \\ \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\ \frac{1}{3 d \sqrt{1+\operatorname{Cot}[c]^2}} C(1+\cos [c+d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^2 \right)$$

$$\begin{aligned}
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{30d} 7A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \left(\frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{10d} 3C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \left(\frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 1084: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{7/2} (a + a \text{Sec}[c + dx]) (A + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$\frac{2 a (3 A + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{2 a (5 A + 7 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{2 a (5 A + 7 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} + \frac{2 a A \cos [c + d x]^{3/2} \sin [c + d x]}{5 d} + \frac{2 a A \cos [c + d x]^{5/2} \sin [c + d x]}{7 d}$$

Result (type 5, 872 leaves):

$$a \left(\sqrt{\cos [c + d x]} (1 + \cos [c + d x]) \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(-\frac{(3 A + 5 C) \cot [c]}{5 d} + \frac{(23 A + 28 C) \cos [d x] \sin [c]}{84 d} + \frac{A \cos [2 d x] \sin [2 c]}{10 d} + \right. \right.$$

$$\left. \frac{A \cos [3 d x] \sin [3 c]}{28 d} + \frac{(23 A + 28 C) \cos [c] \sin [d x]}{84 d} + \frac{A \cos [2 c] \sin [2 d x]}{10 d} + \frac{A \cos [3 c] \sin [3 d x]}{28 d} \right) - \frac{1}{21 d \sqrt{1 + \cot [c]^2}}$$

$$5 A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \cot [c]^2}} C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \frac{1}{10 d} 3 A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) - \frac{1}{2 d} C (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

- **Problem 1085: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{5/2} (a + a \text{Sec}[c + dx]) (A + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$\frac{2 a (3 A + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{5 d} + \frac{2 a (A + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{3 d} + \\ \frac{2 a A \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{3 d} + \frac{2 a A \text{Cos}[c + dx]^{3/2} \text{Sin}[c + dx]}{5 d}$$

Result (type 5, 824 leaves):

$$a \left(\sqrt{\text{Cos}[c + dx]} (1 + \text{Cos}[c + dx]) \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\ \left. \left(-\frac{(3 A + 5 C) \text{Cot}[c]}{5 d} + \frac{A \text{Cos}[dx] \text{Sin}[c]}{3 d} + \frac{A \text{Cos}[2 dx] \text{Sin}[2 c]}{10 d} + \frac{A \text{Cos}[c] \text{Sin}[dx]}{3 d} + \frac{A \text{Cos}[2 c] \text{Sin}[2 dx]}{10 d} \right) - \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} \right. \\ \left. A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \right. \\ \left. \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \right. \\ \left. \frac{1}{d \sqrt{1 + \text{Cot}[c]^2}} C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right) \right)$$

$$\begin{aligned}
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{10d} 3A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{2d} C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \\
& \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \\
& \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right)
\end{aligned}$$

■ **Problem 1086: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{3/2} (a + a \text{Sec}[c + dx]) (A + C \text{Sec}[c + dx])^2 dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$\frac{2a(A - C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{2a(A + 3C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2aC \text{Sin}[c + dx]}{d \sqrt{\text{Cos}[c + dx]}} + \frac{2aA \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{3d}$$

Result (type 5, 813 leaves) :

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \right. \\
& \left. \left(-\frac{(A-2C+A\cos[2c])\operatorname{Csc}[c]\operatorname{Sec}[c]}{2d} + \frac{A\cos[dx]\sin[c]}{3d} + \frac{A\cos[c]\sin[dx]}{3d} + \frac{C\operatorname{Sec}[c]\operatorname{Sec}[c+dx]\sin[dx]}{d} \right) - \frac{1}{3d\sqrt{1+\cot[c]^2}} \right. \\
& A(1+\cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{d\sqrt{1 + \cot[c]^2}} C(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{2d} A(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) + \frac{1}{2d} C(1 + \cos[c+dx])\operatorname{Csc}[c]\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) -$$

■ **Problem 1087: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \sec[c + d x]) (A + C \sec[c + d x]^2) dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$\frac{2 a (A - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (3 A + C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a C \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a C \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 817 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right. \\ \left. - \frac{(A - 2 C + A \cos[2 c]) \csc[c] \sec[c]}{2 d} + \frac{C \sec[c] \sec[c + d x]^2 \sin[d x]}{3 d} + \frac{\sec[c] \sec[c + d x] (C \sin[c] + 3 C \sin[d x])}{3 d} \right) - \frac{1}{d \sqrt{1 + \cot[c]^2}} \\ A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \\ \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \\ \frac{1}{3 d \sqrt{1 + \cot[c]^2}} C (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \\ \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \\ \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{2 d} A (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2d} \text{C} (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \\
\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

- **Problem 1088: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + dx]) (A + C \text{Sec}[c + dx]^2)}{\sqrt{\text{Cos}[c + dx]}} dx$$

Optimal (type 4, 132 leaves, 7 steps):

$$-\frac{2a(5A+3C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{2a(3A+C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \\
\frac{2aC \text{Sin}[c+dx]}{5d \text{Cos}[c+dx]^{5/2}} + \frac{2aC \text{Sin}[c+dx]}{3d \text{Cos}[c+dx]^{3/2}} + \frac{2a(5A+3C) \text{Sin}[c+dx]}{5d \sqrt{\text{Cos}[c+dx]}}$$

Result (type 5, 851 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos[c+dx]} (1 + \cos[c+dx]) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \left(\frac{(5A+3C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \operatorname{Sin}[dx]}{5d} + \right. \right. \\
& \quad \left. \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (3C \operatorname{Sin}[c] + 5C \operatorname{Sin}[dx])}{15d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5C \operatorname{Sin}[c] + 15A \operatorname{Sin}[dx] + 9C \operatorname{Sin}[dx])}{15d} \right) - \frac{1}{d \sqrt{1 + \operatorname{Cot}[c]^2}} \right. \\
& A (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \quad \frac{1}{3d \sqrt{1 + \operatorname{Cot}[c]^2}} C (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{2d} A (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left(\frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right) + \frac{1}{10d} 3C (1 + \cos[c+dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} \right) -$$

- **Problem 1089: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x]) (A + C \sec[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 165 leaves, 8 steps):

$$-\frac{2 a (5 A + 3 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (7 A + 5 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{2 a C \sin[c + d x]}{7 d \cos[c + d x]^{7/2}} + \frac{2 a C \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (7 A + 5 C) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{2 a (5 A + 3 C) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 895 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(\frac{(5 A + 3 C) \csc[c] \sec[c]}{5 d} + \frac{C \sec[c] \sec[c + d x]^4 \sin[d x]}{7 d} + \right. \right.$$

$$\frac{\sec[c] \sec[c + d x]^3 (5 C \sin[c] + 7 C \sin[d x])}{35 d} + \frac{\sec[c] \sec[c + d x]^2 (21 C \sin[c] + 35 A \sin[d x] + 25 C \sin[d x])}{105 d} +$$

$$\left. \frac{\sec[c] \sec[c + d x] (35 A \sin[c] + 25 C \sin[c] + 105 A \sin[d x] + 63 C \sin[d x])}{105 d} \right) - \frac{1}{3 d \sqrt{1 + \cot[c]^2}}$$

$$A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{21 d \sqrt{1 + \cot[c]^2}} 5 C (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2d} A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} \right) + \frac{1}{10d} 3 C (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} \right) \right)$$

- **Problem 1090: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{11/2} (a + a \sec[c + d x])^2 (A + C \sec[c + d x])^2 dx$$

Optimal (type 4, 230 leaves, 10 steps):

$$\frac{4 a^2 (7 A+9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \frac{8 a^2 (25 A+33 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{231 d} +$$

$$\frac{8 a^2 (25 A+33 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{231 d} + \frac{4 a^2 (7 A+9 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d} + \frac{2 a^2 (89 A+99 C) \cos [c+d x]^{5 / 2} \sin [c+d x]}{693 d} +$$

$$\frac{2 A \cos [c+d x]^{5 / 2} (a+a \cos [c+d x])^2 \sin [c+d x]}{11 d} + \frac{8 A \cos [c+d x]^{5 / 2} (a^2+a^2 \cos [c+d x]) \sin [c+d x]}{99 d}$$

Result(type 5, 976 leaves):

$$a^2 \left(\sqrt{\cos [c+d x]} (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \left(-\frac{(7 A+9 C) \cot [c]}{15 d} + \frac{(941 A+1122 C) \cos [d x] \sin [c]}{3696 d} + \frac{(19 A+18 C) \cos [2 d x] \sin [2 c]}{180 d} \right. \right.$$

$$\left. \left. \frac{(101 A+44 C) \cos [3 d x] \sin [3 c]}{2464 d} + \frac{A \cos [4 d x] \sin [4 c]}{72 d} + \frac{A \cos [5 d x] \sin [5 c]}{352 d} + \frac{(941 A+1122 C) \cos [c] \sin [d x]}{3696 d} + \right. \right.$$

$$\left. \left. \frac{(19 A+18 C) \cos [2 c] \sin [2 d x]}{180 d} + \frac{(101 A+44 C) \cos [3 c] \sin [3 d x]}{2464 d} + \frac{A \cos [4 c] \sin [4 d x]}{72 d} + \frac{A \cos [5 c] \sin [5 d x]}{352 d} \right) -$$

$$\frac{1}{231 d \sqrt{1+\cot [c]^2}} 50 A (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{7 d \sqrt{1+\cot [c]^2}}$$

$$2 C (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4$$

$$\operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{30 d} 7 A (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) - \frac{1}{10 d} 3 C (1 + \cos[c + d x])^2 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right\} \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) \right)$$

■ **Problem 1091: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{9/2} (a + a \sec[c + d x])^2 (A + C \sec[c + d x])^2 dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$\frac{16 a^2 (2 A + 3 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{15 d} + \frac{4 a^2 (5 A + 7 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^2 (5 A + 7 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a^2 (19 A + 21 C) \cos[c + d x]^{3/2} \sin[c + d x]}{105 d} +$$

$$\frac{2 A \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2 \sin[c + d x]}{9 d} + \frac{8 A \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{63 d}$$

Result (type 5, 1118 leaves):

$$\frac{1}{A + 2 C + A \cos[2 c + 2 d x]} \cos[c + d x]^{9/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + C \sec[c + d x])^2$$

$$\left(-\frac{8 (2 A + 3 C) \cot[c]}{15 d} + \frac{(23 A + 28 C) \cos[d x] \sin[c]}{42 d} + \frac{(37 A + 18 C) \cos[2 d x] \sin[2 c]}{180 d} + \frac{A \cos[3 d x] \sin[3 c]}{14 d} + \frac{A \cos[4 d x] \sin[4 c]}{72 d} + \right.$$

$$\begin{aligned}
& \left. \frac{(23 A + 28 C) \cos [c] \sin [d x]}{42 d} + \frac{(37 A + 18 C) \cos [2 c] \sin [2 d x]}{180 d} + \frac{A \cos [3 c] \sin [3 d x]}{14 d} + \frac{A \cos [4 c] \sin [4 d x]}{72 d} \right) - \\
& \frac{1}{21 d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} 10 A \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -} \\
& \frac{1}{3 d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} 2 C \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -} \\
& \left(8 A \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (15 d (A + 2 C + A \cos [2 c + 2 d x])) - \right.
\end{aligned}$$

$$\left(4 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x])^2 \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (5 d (A+2 C+A \cos [2 c+2 d x]))$$

■ **Problem 1092: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{7 / 2} (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x])^2 d x$$

Optimal (type 4, 164 leaves, 8 steps):

$$\frac{4 a^2 (3 A+5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+\frac{8 a^2 (3 A+7 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d}+\frac{2 a^2 (33 A+35 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{105 d}+$$

$$\frac{2 A \sqrt{\cos [c+d x]}(a+a \cos [c+d x])^2 \sin [c+d x]}{7 d}+\frac{8 A \sqrt{\cos [c+d x]}(a^2+a^2 \cos [c+d x]) \sin [c+d x]}{35 d}$$

Result (type 5, 1070 leaves):

$$\frac{1}{A+2 C+A \cos [2 c+2 d x]}$$

$$\cos [c+d x]^{9 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x])^2 \left(-\frac{2(3 A+5 C) \cot [c]}{5 d}+\frac{(51 A+28 C) \cos [d x] \sin [c]}{84 d}+\frac{A \cos [2 d x] \sin [2 c]}{5 d}+\frac{A \cos [3 d x] \sin [3 c]}{28 d}+\frac{(51 A+28 C) \cos [c] \sin [d x]}{84 d}+\frac{A \cos [2 c] \sin [2 d x]}{5 d}+\frac{A \cos [3 c] \sin [3 d x]}{28 d} \right) -$$

$$\frac{1}{7 d (A+2 C+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} 4 A \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2$$

$$\begin{aligned}
& \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{3 d (A + 2 C + A \text{Cos}[2 c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2}} 4 C \text{Cos}[c + dx]^4 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \left(3 A \text{Cos}[c + dx]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + C \text{Sec}[c + dx]^2) \right. \\
& \left. \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \right. \\
& \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (5 d (A + 2 C + A \text{Cos}[2 c + 2 dx])) - \right. \\
& \left. C \text{Cos}[c + dx]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + C \text{Sec}[c + dx]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left(\frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (d (A + 2 C + A \text{Cos}[2 c + 2 d x]))$$

- **Problem 1093: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x]^{5/2} (a + a \text{Sec}[c + d x])^2 (A + C \text{Sec}[c + d x])^2 dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$\frac{16 a^2 A \text{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{4 a^2 (A + 3 C) \text{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{3 d} + \frac{2 a^2 (7 A - 15 C) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{15 d} + \frac{2 C (a + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{d \sqrt{\text{Cos}[c + d x]}} + \frac{2 (A - 5 C) \sqrt{\text{Cos}[c + d x]} (a^2 + a^2 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{5 d}$$

Result (type 5, 799 leaves):

$$\frac{1}{A + 2 C + A \text{Cos}[2 c + 2 d x]} \text{Cos}[c + d x]^{9/2} \text{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \text{Sec}[c + d x])^2 (A + C \text{Sec}[c + d x])^2 \left(- \frac{(8 A - 5 C + 8 A \text{Cos}[2 c] + 5 C \text{Cos}[2 c]) \text{Csc}[c] \text{Sec}[c]}{10 d} + \frac{2 A \text{Cos}[d x] \text{Sin}[c]}{3 d} + \frac{A \text{Cos}[2 d x] \text{Sin}[2 c]}{10 d} + \frac{2 A \text{Cos}[c] \text{Sin}[d x]}{3 d} + \frac{C \text{Sec}[c] \text{Sec}[c + d x] \text{Sin}[d x]}{d} + \frac{A \text{Cos}[2 c] \text{Sin}[2 d x]}{10 d} \right) - \frac{1}{3 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 2 A \text{Cos}[c + d x]^4 \text{Csc}[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \text{Sec}[c + d x])^2 (A + C \text{Sec}[c + d x])^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} - \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} 2 C \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} -$$

$$\left(4 A \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right\] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}} \sqrt{1 + \tan [c]^2} \right) -$$

$$\left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}} \right) / (5 d (A + 2 C + A \cos [2 c + 2 d x]))$$

- **Problem 1094: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{3/2} (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 154 leaves, 8 steps):

$$\frac{4 a^2 (A - C) \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{8 a^2 (A + C) \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} +$$

$$\frac{2 a^2 (A - 5 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 C (a + a \cos [c + d x])^2 \sin [c + d x]}{3 d \cos [c + d x]^{3/2}} + \frac{8 C (a^2 + a^2 \cos [c + d x]) \sin [c + d x]}{3 d \sqrt{\cos [c + d x]}}$$

Result (type 5, 1040 leaves):

$$\frac{1}{A + 2 C + A \cos [2 c + 2 d x]}$$

$$\begin{aligned}
& \cos [c+d x]^{9 / 2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x]^2) \left(-\frac{(A-2 C+A \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{d}+\frac{A \cos [d x] \sin [c]}{3 d}+\right. \\
& \quad \left.\frac{A \cos [c] \sin [d x]}{3 d}+\frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 \sin [d x]}{3 d}+\frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x](C \sin [c]+6 C \sin [d x])}{3 d}\right)- \\
& \frac{1}{3 d(A+2 C+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} 4 A \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-} \\
& \frac{1}{3 d(A+2 C+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} 4 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]}-} \\
& \left(A \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+C \operatorname{Sec}[c+d x]^2)\right. \\
& \left.\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right. \\
& \left.\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2}}}\right)-\right. \\
& \left.\left.\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}}\right) / (d(A+2 C+A \cos [2 c+2 d x]))+\right.
\end{aligned}$$

$$\left(C \cos[c + dx]^4 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + C \sec[c + dx])^2 \right.$$

$$\left. \left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]\right]^2 \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]\right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / (d (A + 2C + A \cos[2c + 2dx]))$$

- **Problem 1095: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + dx]} (a + a \sec[c + dx])^2 (A + C \sec[c + dx])^2 dx$$

Optimal (type 4, 156 leaves, 8 steps):

$$-\frac{16 a^2 C \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} + \frac{4 a^2 (3 A + C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3 d} +$$

$$\frac{2 a^2 (15 A + 17 C) \sin[c + dx]}{15 d \sqrt{\cos[c + dx]}} + \frac{2 C (a + a \cos[c + dx])^2 \sin[c + dx]}{5 d \cos[c + dx]^{5/2}} + \frac{8 C (a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{15 d \cos[c + dx]^{3/2}}$$

Result (type 5, 800 leaves):

$$\frac{1}{A + 2C + A \cos[2c + 2dx]} \cos[c + dx]^{9/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2$$

$$(A + C \sec[c + dx])^2 \left(-\frac{(-5A - 16C + 5A \cos[2c]) \csc[c] \sec[c]}{10 d} + \frac{C \sec[c] \sec[c + dx]^3 \sin[dx]}{5 d} + \right.$$

$$\left. \frac{\sec[c] \sec[c + dx]^2 (3C \sin[c] + 10C \sin[dx])}{15 d} + \frac{\sec[c] \sec[c + dx] (10C \sin[c] + 15A \sin[dx] + 24C \sin[dx])}{15 d} \right) -$$

$$\frac{1}{d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2}} 2A \cos[c + dx]^4 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]\right]^2$$

$$\begin{aligned}
& \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c + dx])^2 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}} -} \\
& \frac{1}{3 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2}} 2 C \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c + dx])^2 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}} +} \\
& \left(4 C \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c + dx])^2 (A + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right\} \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2}} \right) - \right. \\
& \left. \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} \right) / (5 d (A + 2 C + A \cos[2c + 2dx])) \right)
\end{aligned}$$

- **Problem 1096: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c + dx])^2 (A + C \operatorname{Sec}[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^2 (5 A + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{8 a^2 (7 A + 3 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (35 A + 33 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \\
& \frac{4 a^2 (5 A + 3 C) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 C (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{7 d \operatorname{Cos}[c + d x]^{7/2}} + \frac{8 C (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{35 d \operatorname{Cos}[c + d x]^{5/2}}
\end{aligned}$$

Result (type 5, 1092 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + A \operatorname{Cos}[2 c + 2 d x]} \\
& \operatorname{Cos}[c + d x]^{9/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \left(\frac{2 (5 A + 3 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \operatorname{Sin}[d x]}{7 d} + \right. \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 C \operatorname{Sin}[c] + 14 C \operatorname{Sin}[d x])}{35 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (42 C \operatorname{Sin}[c] + 35 A \operatorname{Sin}[d x] + 60 C \operatorname{Sin}[d x])}{105 d} + \right. \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (35 A \operatorname{Sin}[c] + 60 C \operatorname{Sin}[c] + 210 A \operatorname{Sin}[d x] + 126 C \operatorname{Sin}[d x])}{105 d} \right) - \\
& \frac{1}{3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 4 A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -} \\
& \frac{1}{7 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 4 C \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} +} \\
& \left(A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (d (A + 2 C + A \cos[2 c + 2 d x])) +$$

$$\left(3 C \cos[c + d x]^4 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (5 d (A + 2 C + A \cos[2 c + 2 d x]))$$

■ **Problem 1097: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x])^2 (A + C \sec[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 230 leaves, 10 steps):

$$\begin{aligned}
& - \frac{16 a^2 (3 A + 2 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^2 (7 A + 5 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (21 A + 19 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{5/2}} + \\
& \frac{4 a^2 (7 A + 5 C) \operatorname{Sin}[c + d x]}{21 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{16 a^2 (3 A + 2 C) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 C (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{9 d \operatorname{Cos}[c + d x]^{9/2}} + \frac{8 C (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{63 d \operatorname{Cos}[c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 1137 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + A \operatorname{Cos}[2 c + 2 d x]} \\
& \operatorname{Cos}[c + d x]^{9/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x])^2 \left(\frac{8 (3 A + 2 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{15 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \operatorname{Sin}[d x]}{9 d} + \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (7 C \operatorname{Sin}[c] + 18 C \operatorname{Sin}[d x])}{63 d} + \frac{2 \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (35 A \operatorname{Sin}[c] + 25 C \operatorname{Sin}[c] + 84 A \operatorname{Sin}[d x] + 56 C \operatorname{Sin}[d x])}{105 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (90 C \operatorname{Sin}[c] + 63 A \operatorname{Sin}[d x] + 112 C \operatorname{Sin}[d x])}{315 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (63 A \operatorname{Sin}[c] + 112 C \operatorname{Sin}[c] + 210 A \operatorname{Sin}[d x] + 150 C \operatorname{Sin}[d x])}{315 d} \right) - \\
& \frac{1}{3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 2 A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{21 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 10 C \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \\
& \left(4 A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + C \operatorname{Sec}[c + d x])^2 \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \left(\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / (5 d (A + 2 C + A \cos [2 c + 2 d x])) + \left(8 C \cos [c + d x]^4 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + C \sec [c + d x]^2) \right) \left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \left(\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / (15 d (A + 2 C + A \cos [2 c + 2 d x]))$$

■ **Problem 1098: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{13/2} (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) dx$$

Optimal (type 4, 279 leaves, 11 steps):

$$\begin{aligned}
& \frac{4 a^3 (175 A + 221 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{195 d} + \frac{4 a^3 (95 A + 121 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{231 d} + \\
& \frac{4 a^3 (95 A + 121 C) \sqrt{\cos[c + dx]} \sin[c + dx]}{231 d} + \frac{4 a^3 (175 A + 221 C) \cos[c + dx]^{3/2} \sin[c + dx]}{585 d} + \\
& \frac{40 a^3 (118 A + 143 C) \cos[c + dx]^{5/2} \sin[c + dx]}{9009 d} + \frac{2 A \cos[c + dx]^{5/2} (a + a \cos[c + dx])^3 \sin[c + dx]}{13 d} + \\
& \frac{12 A \cos[c + dx]^{5/2} (a^2 + a^2 \cos[c + dx])^2 \sin[c + dx]}{143 a d} + \frac{2 (145 A + 143 C) \cos[c + dx]^{5/2} (a^3 + a^3 \cos[c + dx]) \sin[c + dx]}{1287 d}
\end{aligned}$$

Result (type 5, 1022 leaves):

$$\begin{aligned}
& a^3 \left(\sqrt{\cos[c + dx]} (1 + \cos[c + dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right. \\
& \left(- \frac{(175 A + 221 C) \cot[c]}{390 d} + \frac{(1811 A + 2134 C) \cos[dx] \sin[c]}{7392 d} + \frac{(7825 A + 7592 C) \cos[2 dx] \sin[2 c]}{74880 d} + \right. \\
& \frac{(215 A + 132 C) \cos[3 dx] \sin[3 c]}{4928 d} + \frac{(59 A + 13 C) \cos[4 dx] \sin[4 c]}{3744 d} + \frac{3 A \cos[5 dx] \sin[5 c]}{704 d} + \frac{A \cos[6 dx] \sin[6 c]}{1664 d} + \\
& \frac{(1811 A + 2134 C) \cos[c] \sin[dx]}{7392 d} + \frac{(7825 A + 7592 C) \cos[2 c] \sin[2 dx]}{74880 d} + \frac{(215 A + 132 C) \cos[3 c] \sin[3 dx]}{4928 d} + \\
& \left. \left. \frac{(59 A + 13 C) \cos[4 c] \sin[4 dx]}{3744 d} + \frac{3 A \cos[5 c] \sin[5 dx]}{704 d} + \frac{A \cos[6 c] \sin[6 dx]}{1664 d} \right) - \frac{1}{462 d \sqrt{1 + \cot[c]^2}} \right. \\
& 95 A (1 + \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{42 d \sqrt{1 + \cot[c]^2}} 11 C (1 + \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \left. \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{156 d} 35 A (1 + \cos[c + dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{60 d} 17 C (1 + \text{Cos}[c + dx])^3 \text{Csc}[c] \text{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \\
\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

■ **Problem 1099: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{11/2} (a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 246 leaves, 10 steps):

$$\frac{4 a^3 (5 A + 7 C) \text{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{5 d} + \frac{4 a^3 (105 A + 143 C) \text{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{231 d} + \frac{4 a^3 (105 A + 143 C) \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{231 d} + \\
\frac{8 a^3 (35 A + 44 C) \text{Cos}[c + dx]^{3/2} \text{Sin}[c + dx]}{385 d} + \frac{2 A \text{Cos}[c + dx]^{3/2} (a + a \text{Cos}[c + dx])^3 \text{Sin}[c + dx]}{11 d} + \\
\frac{4 A \text{Cos}[c + dx]^{3/2} (a^2 + a^2 \text{Cos}[c + dx])^2 \text{Sin}[c + dx]}{33 a d} + \frac{2 (35 A + 33 C) \text{Cos}[c + dx]^{3/2} (a^3 + a^3 \text{Cos}[c + dx]) \text{Sin}[c + dx]}{231 d}$$

Result (type 5, 976 leaves) :

$$\begin{aligned}
& a^3 \left(\sqrt{\cos[c+dx]} (1+\cos[c+dx])^3 \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \right. \\
& \left(-\frac{(5A+7C)\cot[c]}{10d} + \frac{(1953A+2354C)\cos[dx]\sin[c]}{7392d} + \frac{(25A+18C)\cos[2dx]\sin[2c]}{240d} + \frac{(189A+44C)\cos[3dx]\sin[3c]}{4928d} + \right. \\
& \frac{A\cos[4dx]\sin[4c]}{96d} + \frac{A\cos[5dx]\sin[5c]}{704d} + \frac{(1953A+2354C)\cos[c]\sin[dx]}{7392d} + \frac{(25A+18C)\cos[2c]\sin[2dx]}{240d} + \\
& \left. \frac{(189A+44C)\cos[3c]\sin[3dx]}{4928d} + \frac{A\cos[4c]\sin[4dx]}{96d} + \frac{A\cos[5c]\sin[5dx]}{704d} \right) - \frac{1}{22d\sqrt{1+\cot[c]^2}} \\
& 5A(1+\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{42d\sqrt{1+\cot[c]^2}} 13C(1+\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{4d} A(1+\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \\
& \left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) - \frac{1}{20d} 7C(1+\cos[c+dx])^3 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

- **Problem 1100: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x]^{9/2} (a + a \text{Sec}[c + d x])^3 (A + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 213 leaves, 9 steps):

$$\frac{4 a^3 (17 A + 27 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (11 A + 21 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\ \frac{8 a^3 (16 A + 21 C) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{105 d} + \frac{2 A \sqrt{\text{Cos}[c + d x]} (a + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{9 d} + \\ \frac{4 A \sqrt{\text{Cos}[c + d x]} (a^2 + a^2 \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{21 a d} + \frac{2 (73 A + 63 C) \sqrt{\text{Cos}[c + d x]} (a^3 + a^3 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{315 d}$$

Result (type 5, 1116 leaves):

$$\frac{1}{A + 2 C + A \text{Cos}[2 c + 2 d x]} \text{Cos}[c + d x]^{11/2} \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + C \text{Sec}[c + d x]^2) \\ \left(-\frac{(17 A + 27 C) \text{Cot}[c]}{15 d} + \frac{(97 A + 84 C) \text{Cos}[d x] \text{Sin}[c]}{168 d} + \frac{(73 A + 18 C) \text{Cos}[2 d x] \text{Sin}[2 c]}{360 d} + \frac{3 A \text{Cos}[3 d x] \text{Sin}[3 c]}{56 d} + \frac{A \text{Cos}[4 d x] \text{Sin}[4 c]}{144 d} + \right. \\ \left. \frac{(97 A + 84 C) \text{Cos}[c] \text{Sin}[d x]}{168 d} + \frac{(73 A + 18 C) \text{Cos}[2 c] \text{Sin}[2 d x]}{360 d} + \frac{3 A \text{Cos}[3 c] \text{Sin}[3 d x]}{56 d} + \frac{A \text{Cos}[4 c] \text{Sin}[4 d x]}{144 d} \right) - \\ \frac{1}{21 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 11 A \text{Cos}[c + d x]^5 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\ \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\begin{aligned}
& \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{d(A+2C+A\cos[2c+2dx])\sqrt{1+\cot[c]^2}} C \cos[c+dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a\sec[c+dx])^3 (A+C\sec[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \\
& \left(17A \cos[c+dx]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a\sec[c+dx])^3 (A+C\sec[c+dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1-\cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}} \right) / (30d(A+2C+A\cos[2c+2dx])) - \\
& \left(9C \cos[c+dx]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a+a\sec[c+dx])^3 (A+C\sec[c+dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right.
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (10 d (A + 2 C + A \cos[2 c + 2 d x]))$$

■ **Problem 1101: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{7/2} (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) dx$$

Optimal (type 4, 215 leaves, 9 steps):

$$\frac{4 a^3 (7 A + 5 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (13 A + 35 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^3 (41 A - 35 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 C (a + a \cos[c + d x])^3 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2 (A - 7 C) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{7 a d} + \frac{2 (11 A - 35 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{35 d}$$

Result (type 5, 1108 leaves):

$$\frac{1}{A + 2 C + A \cos[2 c + 2 d x]} \cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) - \left(-\frac{(14 A + 5 C + 14 A \cos[2 c] + 15 C \cos[2 c]) \csc[c] \sec[c]}{20 d} + \frac{(107 A + 28 C) \cos[d x] \sin[c]}{168 d} + \frac{3 A \cos[2 d x] \sin[2 c]}{20 d} + \frac{A \cos[3 d x] \sin[3 c]}{56 d} + \frac{(107 A + 28 C) \cos[c] \sin[d x]}{168 d} + \frac{C \sec[c] \sec[c + d x] \sin[d x]}{2 d} + \frac{3 A \cos[2 c] \sin[2 d x]}{20 d} + \frac{A \cos[3 c] \sin[3 d x]}{56 d} \right) - \frac{1}{21 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 13 A \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} - \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} - \frac{1}{3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 5 C \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \left(7 A \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} \right) / (10 d (A + 2 C + A \operatorname{Cos}[2 c + 2 dx])) - \right. \\
& \left. C \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \operatorname{Sec}[c + dx])^3 (A + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \right.
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{(2 d (A + 2 C + A \cos[2 c + 2 d x]))}$$

- **Problem 1102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{5/2} (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) dx$$

Optimal (type 4, 211 leaves, 9 steps):

$$\frac{4 a^3 (9 A - 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{4 a^3 (3 A + 5 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{8 a^3 (3 A - 10 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} +$$

$$\frac{2 C (a + a \cos[c + d x])^3 \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{4 C (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{a d \sqrt{\cos[c + d x]}} + \frac{2 (3 A - 35 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d}$$

Result (type 5, 1089 leaves):

$$\frac{1}{A + 2 C + A \cos[2 c + 2 d x]} \cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2)$$

$$\left(-\frac{(18 A - 25 C + 18 A \cos[2 c] + 5 C \cos[2 c]) \csc[c] \sec[c]}{20 d} + \frac{A \cos[d x] \sin[c]}{2 d} + \frac{A \cos[2 d x] \sin[2 c]}{20 d} + \right.$$

$$\left. \frac{A \cos[c] \sin[d x]}{2 d} + \frac{C \sec[c] \sec[c + d x]^2 \sin[d x]}{6 d} + \frac{\sec[c] \sec[c + d x] (C \sin[c] + 9 C \sin[d x])}{6 d} + \frac{A \cos[2 c] \sin[2 d x]}{20 d} \right) -$$

$$\frac{1}{d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} A \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 5 C \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\left(9 A \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+A \cos [2 c+2 d x])) + \right.$$

$$\left. C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (2 d (A+2 C+A \cos [2 c+2 d x])) \right)$$

■ **Problem 1103:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos [c+d x]^{3 / 2} (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) d x$$

Optimal (type 4, 213 leaves, 9 steps):

$$\frac{4 a^3 (5 A-9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d} + \frac{4 a^3 (5 A+3 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} - \frac{4 a^3 (5 A+21 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{15 d} + \frac{2 C (a+a \cos [c+d x])^3 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}} + \frac{4 C (a^2+a^2 \cos [c+d x])^2 \sin [c+d x]}{5 a d \cos [c+d x]^{3 / 2}} + \frac{2 (5 A+11 C) (a^3+a^3 \cos [c+d x]) \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}$$

Result (type 5, 1085 leaves):

$$\frac{1}{A+2 C+A \cos [2 c+2 d x]} \cos [c+d x]^{11 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \left(-\frac{(5 A-36 C+15 A \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{20 d} + \frac{A \cos [d x] \sin [c]}{6 d} + \frac{A \cos [c] \sin [d x]}{6 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{10 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 (C \sin [c]+5 C \sin [d x])}{10 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x] (5 C \sin [c]+5 A \sin [d x]+18 C \sin [d x])}{10 d} \right) - \frac{1}{3 d (A+2 C+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} 5 A \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{d (A+2 C+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \left(A \cos [c+d x]^5 \operatorname{Csc}[c] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+C \sec [c+d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \left(\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / (2 d (A + 2 C + A \cos [2 c + 2 d x])) + \left(9 C \cos [c + d x]^5 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) \right) \left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \right) \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \left(\frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / (10 d (A + 2 C + A \cos [2 c + 2 d x]))$$

■ **Problem 1104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos [c + d x]} (a + a \sec [c + d x])^3 (A + C \sec [c + d x]^2) dx$$

Optimal (type 4, 213 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a^3 (5 A + 7 C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} + \frac{4 a^3 (35 A + 13 C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d} + \frac{8 a^3 (70 A + 53 C) \text{Sin}[c + dx]}{105 d \sqrt{\text{Cos}[c + dx]}} + \\
& \frac{2 C (a + a \text{Cos}[c + dx])^3 \text{Sin}[c + dx]}{7 d \text{Cos}[c + dx]^{7/2}} + \frac{12 C (a^2 + a^2 \text{Cos}[c + dx])^2 \text{Sin}[c + dx]}{35 a d \text{Cos}[c + dx]^{5/2}} + \frac{2 (5 A + 7 C) (a^3 + a^3 \text{Cos}[c + dx]) \text{Sin}[c + dx]}{15 d \text{Cos}[c + dx]^{3/2}}
\end{aligned}$$

Result(type5, 1102 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + A \text{Cos}[2 c + 2 d x]} \text{Cos}[c + dx]^{11/2} \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 \\
& (A + C \text{Sec}[c + dx]^2) \left(- \frac{(-25 A - 28 C + 5 A \text{Cos}[2 c]) \text{Csc}[c] \text{Sec}[c]}{20 d} + \frac{C \text{Sec}[c] \text{Sec}[c + dx]^4 \text{Sin}[d x]}{14 d} + \right. \\
& \frac{\text{Sec}[c] \text{Sec}[c + dx]^3 (5 C \text{Sin}[c] + 21 C \text{Sin}[d x])}{70 d} + \frac{\text{Sec}[c] \text{Sec}[c + dx]^2 (63 C \text{Sin}[c] + 35 A \text{Sin}[d x] + 130 C \text{Sin}[d x])}{210 d} + \\
& \left. \frac{\text{Sec}[c] \text{Sec}[c + dx] (35 A \text{Sin}[c] + 130 C \text{Sin}[c] + 315 A \text{Sin}[d x] + 294 C \text{Sin}[d x])}{210 d} \right) - \\
& \frac{1}{3 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 5 A \text{Cos}[c + dx]^5 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]}} - \\
& \frac{1}{21 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 13 C \text{Cos}[c + dx]^5 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]}} + \\
& \left(A \text{Cos}[c + dx]^5 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (2 d (A + 2 C + A \text{Cos}[2 c + 2 dx])) + \\
\left(7 C \text{Cos}[c + dx]^5 \text{Csc}[c] \text{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2) \right) \\
\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right\} \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (10 d (A + 2 C + A \text{Cos}[2 c + 2 dx]))$$

■ **Problem 1105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + dx])^3 (A + C \text{Sec}[c + dx]^2)}{\sqrt{\text{Cos}[c + dx]}} dx$$

Optimal (type 4, 246 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 a^3 (27 A + 17 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (21 A + 11 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\
& \frac{8 a^3 (21 A + 16 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^3 (27 A + 17 C) \operatorname{Sin}[c + d x]}{15 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 C (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sin}[c + d x]}{9 d \operatorname{Cos}[c + d x]^{9/2}} + \\
& \frac{4 C (a^2 + a^2 \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{21 a d \operatorname{Cos}[c + d x]^{7/2}} + \frac{2 (63 A + 73 C) (a^3 + a^3 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{315 d \operatorname{Cos}[c + d x]^{5/2}}
\end{aligned}$$

Result (type 5, 1135 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + A \operatorname{Cos}[2 c + 2 d x]} \\
& \operatorname{Cos}[c + d x]^{11/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \left(\frac{(27 A + 17 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{15 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 \operatorname{Sin}[d x]}{18 d} + \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (7 C \operatorname{Sin}[c] + 27 C \operatorname{Sin}[d x])}{126 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (135 C \operatorname{Sin}[c] + 63 A \operatorname{Sin}[d x] + 238 C \operatorname{Sin}[d x])}{630 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (105 A \operatorname{Sin}[c] + 110 C \operatorname{Sin}[c] + 378 A \operatorname{Sin}[d x] + 238 C \operatorname{Sin}[d x])}{210 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (63 A \operatorname{Sin}[c] + 238 C \operatorname{Sin}[c] + 315 A \operatorname{Sin}[d x] + 330 C \operatorname{Sin}[d x])}{630 d} \right) - \\
& \frac{1}{d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} A \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{21 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 11 C \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \\
& \left(9 A \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left(\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (10 d (A + 2 C + A \cos[2 c + 2 d x])) + \left(17 C \cos[c + d x]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + C \sec[c + d x])^2 \right) \left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left(\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (30 d (A + 2 C + A \cos[2 c + 2 d x]))$$

■ **Problem 1106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x])^3 (A + C \sec[c + d x])^2}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 279 leaves, 11 steps):

$$\begin{aligned}
& - \frac{4 a^3 (7 A + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (143 A + 105 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} + \\
& \frac{8 a^3 (44 A + 35 C) \operatorname{Sin}[c + d x]}{385 d \operatorname{Cos}[c + d x]^{5/2}} + \frac{4 a^3 (143 A + 105 C) \operatorname{Sin}[c + d x]}{231 d \operatorname{Cos}[c + d x]^{3/2}} + \frac{4 a^3 (7 A + 5 C) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \\
& \frac{2 C (a + a \operatorname{Cos}[c + d x])^3 \operatorname{Sin}[c + d x]}{11 d \operatorname{Cos}[c + d x]^{11/2}} + \frac{4 C (a^2 + a^2 \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{33 a d \operatorname{Cos}[c + d x]^{9/2}} + \frac{2 (33 A + 35 C) (a^3 + a^3 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{231 d \operatorname{Cos}[c + d x]^{7/2}}
\end{aligned}$$

Result(type5, 1179 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + A \operatorname{Cos}[2 c + 2 d x]} \\
& \operatorname{Cos}[c + d x]^{11/2} \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \left(\frac{(7 A + 5 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^6 \operatorname{Sin}[d x]}{22 d} + \right. \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^5 (3 C \operatorname{Sin}[c] + 11 C \operatorname{Sin}[d x])}{66 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 (77 C \operatorname{Sin}[c] + 33 A \operatorname{Sin}[d x] + 126 C \operatorname{Sin}[d x])}{462 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (165 A \operatorname{Sin}[c] + 630 C \operatorname{Sin}[c] + 693 A \operatorname{Sin}[d x] + 770 C \operatorname{Sin}[d x])}{2310 d} + \\
& \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (693 A \operatorname{Sin}[c] + 770 C \operatorname{Sin}[c] + 1430 A \operatorname{Sin}[d x] + 1050 C \operatorname{Sin}[d x])}{2310 d} + \\
& \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x] (715 A \operatorname{Sin}[c] + 525 C \operatorname{Sin}[c] + 1617 A \operatorname{Sin}[d x] + 1155 C \operatorname{Sin}[d x])}{1155 d} \right) - \\
& \frac{1}{21 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 13 A \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{11 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 5 C \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \operatorname{Sec}[c + d x])^3 (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} +
\end{aligned}$$

$$\left(7 A \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+A \cos [2 c+2 d x])) + \right.$$

$$\left. C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (2 d (A+2 C+A \cos [2 c+2 d x])) \right.$$

■ **Problem 1107:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos[c + dx]^{7/2} (A + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 4, 192 leaves, 8 steps):

$$-\frac{3(7A+5C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5ad} + \frac{5(9A+7C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{21ad} + \frac{5(9A+7C) \sqrt{\cos[c+dx]} \sin[c+dx]}{21ad} - \frac{(7A+5C) \cos[c+dx]^{3/2} \sin[c+dx]}{5ad} + \frac{(9A+7C) \cos[c+dx]^{5/2} \sin[c+dx]}{7ad} - \frac{(A+C) \cos[c+dx]^{7/2} \sin[c+dx]}{d(a+a \cos[c+dx])}$$

Result (type 5, 1393 leaves):

$$-\frac{1}{10(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])} 21iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+C \sec[c+dx]^2) \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) - \frac{1}{2(A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])} 3iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+C \sec[c+dx]^2) \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \sqrt{1+e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1+e^{2idx}) \cos[c] + d(-1+e^{2idx}) \sin[c]) \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx]^{3/2} (A+C \sec[c+dx]^2) \left(\frac{4(5A+5C+16A \cos[c] + 10C \cos[c]) \operatorname{Csc}[c]}{5d} + \frac{2(51A+28C) \cos[dx] \sin[c]}{21d} - \frac{4A \cos[2dx] \sin[2c]}{5d} + \frac{2A \cos[3dx] \sin[3c]}{7d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{2(51A+28C) \cos[c] \sin[dx]}{21d} - \frac{4A \cos[2c] \sin[2dx]}{5d} + \frac{2A \cos[3c] \sin[3dx]}{7d} \right) / ((A+2C+A \cos[2c+2dx]) (a+a \sec[c+dx])) -$$

$$\begin{aligned}
& \left(30 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + C \operatorname{Sec}[c+dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(7 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c+dx]) \right) - \\
& \left(10 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c+dx]) \right)
\end{aligned}$$

■ **Problem 1108: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2} (A + C \operatorname{Sec}[c+dx]^2)}{a + a \operatorname{Sec}[c+dx]} dx$$

Optimal (type 4, 159 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 (7 A + 5 C) \operatorname{EllipticE}\left[\frac{1}{2} (c+dx), 2\right]}{5 a d} - \frac{(5 A + 3 C) \operatorname{EllipticF}\left[\frac{1}{2} (c+dx), 2\right]}{3 a d} - \\
& \frac{(5 A + 3 C) \sqrt{\cos[c+dx]} \sin[c+dx]}{3 a d} + \frac{(7 A + 5 C) \cos[c+dx]^{3/2} \sin[c+dx]}{5 a d} - \frac{(A + C) \cos[c+dx]^{5/2} \sin[c+dx]}{d (a + a \cos[c+dx])}
\end{aligned}$$

Result (type 5, 1345 leaves):

$$\begin{aligned}
& \frac{1}{10 (A + 2 C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c+dx])} 21 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c+dx])^2 \\
& \quad \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) + \\
& \frac{1}{2 (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])} 3 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c])} - \right. \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x]^{3/2} (A + C \sec[c + d x]^2) \left(-\frac{4 (5 A + 5 C + 16 A \cos[c] + 10 C \cos[c]) \operatorname{Csc}[c]}{5 d} - \frac{8 A \cos[d x] \sin[c]}{3 d} + \right. \right. \\
& \left. \left. \frac{4 A \cos[2 d x] \sin[2 c]}{5 d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{d} - \frac{8 A \cos[c] \sin[d x]}{3 d} + \frac{4 A \cos[2 c] \sin[2 d x]}{5 d} \right) \right) \Bigg) / \\
& ((A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])) + \left(10 A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right. \\
& \left. \frac{\sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \Bigg) / \\
& \left(3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x]) \right) + \\
& \left(2 C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \Bigg) / \\
& \left(d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x]) \right)
\end{aligned}$$

Problem 1109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos[c + dx]^{3/2} (A + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{(3A + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(5A + 3C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} + \frac{(5A + 3C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3ad} - \frac{(A + C) \cos[c + dx]^{3/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1300 leaves):

$$\begin{aligned} & -\frac{1}{2(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])} 3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx]^2) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\ & \frac{1}{2(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])} iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx]^2) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ & \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx]^{3/2} (A + C \sec[c + dx]^2) \left(\frac{4(A + C + 2A \cos[c]) \operatorname{Csc}[c]}{d} + \frac{8A \cos[dx] \sin[c]}{3d} + \right. \right. \\ & \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \frac{8A \cos[c] \sin[dx]}{3d} \right) \right) / ((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])) - \end{aligned}$$

$$\begin{aligned}
& \left(10 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right) - \\
& \quad \left(2 C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (A + 2 C + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx]) \right)
\end{aligned}$$

■ **Problem 1110: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2)}{a + a \operatorname{Sec}[c + dx]} dx$$

Optimal (type 4, 84 leaves, 5 steps):

$$\frac{(3A + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A + C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{d(a + a \operatorname{Cos}[c + dx])}$$

Result (type 5, 1270 leaves):

$$\begin{aligned}
& \frac{1}{2(A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])} 3i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx])^2 \\
& \quad \left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \operatorname{Cos}[c] + 2i(-1 + e^{2i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2i dx} \operatorname{Cos}[2c] + i e^{2i dx} \operatorname{Sin}[2c]} \right) / (3i d (1 + e^{2i dx}) \operatorname{Cos}[c] - 3d (-1 + e^{2i dx}) \operatorname{Sin}[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \operatorname{Cos}[c] + 2i(-1 + e^{2i dx}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2i dx} \operatorname{Cos}[2c] + i e^{2i dx} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2i dx}) \operatorname{Cos}[c] + d (-1 + e^{2i dx}) \operatorname{Sin}[c]) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2 (A + 2 C + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])} i C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] (A + C \sec [c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x]^{3/2} (A + C \sec [c + d x])^2 \left(-\frac{4 (A + C + 2 A \cos [c]) \csc [c]}{d} - \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{d} \right) \right) / \\
& \left((A + 2 C + A \cos [2 c + 2 d x]) (a + a \sec [c + d x]) \right) + \\
& \left(2 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \\
& \quad \sec \left[\frac{c}{2} \right] (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]}} \right) / \\
& \left(d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right) - \\
& \left(2 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \\
& \quad \sec \left[\frac{c}{2} \right] (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]}} \right) / \\
& \left(d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right)
\end{aligned}$$

■ **Problem 1111: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec [c + d x]^2}{\sqrt{\cos [c + d x]} (a + a \sec [c + d x])} dx$$

Optimal (type 4, 112 leaves, 6 steps):

$$-\frac{(A+3C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(A-C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(A+3C) \operatorname{Sin}[c+dx]}{ad \sqrt{\operatorname{Cos}[c+dx]}} - \frac{(A+C) \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Cos}[c+dx])}$$

Result (type 5, 1304 leaves):

$$\begin{aligned} & -\frac{1}{2(A+2C+A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+C \operatorname{Sec}[c+dx])^2 \\ & \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) - \\ & \frac{1}{2(A+2C+A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])} 3i C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+C \operatorname{Sec}[c+dx])^2 \\ & \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \\ & \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx]^{3/2} (A+C \operatorname{Sec}[c+dx])^2 \left(\frac{2(2C+A \operatorname{Cos}[c] + C \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \right. \right. \\ & \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{d} + \frac{8 C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} \right) \right) / ((A+2C+A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])) - \\ & \left(2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ & \quad \left. (A+C \operatorname{Sec}[c+dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \end{aligned}$$

$$\left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx]) \right) +$$

$$\left(2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right.$$

$$\left. \sec\left[\frac{c}{2}\right] (A + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx]) \right)$$

■ **Problem 1112: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \sec[c + dx])} dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\frac{(A + 3C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(3A + 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} +$$

$$\frac{(3A + 5C) \sin[c + dx]}{3ad \cos[c + dx]^{3/2}} - \frac{(A + 3C) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A + C) \sin[c + dx]}{d \cos[c + dx]^{3/2} (a + a \cos[c + dx])}$$

Result (type 5, 1337 leaves):

$$\frac{1}{2(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) -$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \Big) +$$

$$\frac{1}{2(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])} 3iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c + dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Bigg) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x]^{3/2} (A + C \sec [c + d x])^2 \left(-\frac{2 (2 C + A \cos [c] + C \cos [c]) \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \sec [c]}{d} - \right. \right. \\
& \quad \left. \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{d} + \frac{8 C \sec [c] \sec [c + d x]^2 \sin [d x]}{3 d} + \frac{8 \sec [c] \sec [c + d x] (C \sin [c] - 3 C \sin [d x])}{3 d} \right) \Bigg) / \\
& ((A + 2 C + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])) - \left(2 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \csc \left[\frac{c}{2} \right] \right. \\
& \quad \left. \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \sec \left[\frac{c}{2} \right] (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \right. \\
& \quad \left. \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) \Bigg) / \\
& \left(d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right) - \\
& \left(10 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right. \\
& \quad \left. \sec \left[\frac{c}{2} \right] (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) \Bigg) / \\
& \left(3 d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec [c + d x]) \right)
\end{aligned}$$

■ **Problem 1113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec [c + d x]^2}{\cos [c + d x]^{5/2} (a + a \sec [c + d x])} dx$$

Optimal (type 4, 192 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3(5A+7C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right] - (3A+5C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{5ad} + \\
& \frac{(5A+7C) \operatorname{Sin}[c+dx]}{5ad \operatorname{Cos}[c+dx]^{5/2}} - \frac{(3A+5C) \operatorname{Sin}[c+dx]}{3ad \operatorname{Cos}[c+dx]^{3/2}} + \frac{3(5A+7C) \operatorname{Sin}[c+dx]}{5ad \sqrt{\operatorname{Cos}[c+dx]}} - \frac{(A+C) \operatorname{Sin}[c+dx]}{d \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Cos}[c+dx])}
\end{aligned}$$

Result (type 5, 1382 leaves):

$$\begin{aligned}
& - \frac{1}{2(A+2C+A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx])} 3iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A+C \operatorname{Sec}[c+dx]^2) \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) - \\
& \frac{1}{10(A+2C+A \operatorname{Cos}[2c+2dx]) (a+a \operatorname{Sec}[c+dx]) (A+C \operatorname{Sec}[c+dx]^2)} 21iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1+e^{2idx}) \operatorname{Cos}[c] - 3d(-1+e^{2idx}) \operatorname{Sin}[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \operatorname{Cos}[c] + 2i(-1+e^{2idx}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1+e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1+e^{2idx}) \operatorname{Cos}[c] + d(-1+e^{2idx}) \operatorname{Sin}[c]) \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Cos}[c+dx]^{3/2} (A+C \operatorname{Sec}[c+dx]^2) \left(\frac{2(10A+16C+5A \operatorname{Cos}[c] + 5C \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5d} + \right. \right. \\
& \quad \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{d} + \frac{8C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \operatorname{Sin}[dx]}{5d} - \right. \\
& \quad \left. \left. \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5C \operatorname{Sin}[c] - 15A \operatorname{Sin}[dx] - 24C \operatorname{Sin}[dx])}{15d} + \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (3C \operatorname{Sin}[c] - 5C \operatorname{Sin}[dx])}{15d} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx]) \right) + \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \right. \\
& \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left. (A + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right] / \\
& \left(d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx]) \right) + \\
& \left(10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right] / \\
& \left(3d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx]) \right)
\end{aligned}$$

■ **Problem 1114: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + C \sec[c + dx])^2}{(a + a \sec[c + dx])^2} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\begin{aligned}
& \frac{4(14A + 5C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5a^2d} - \frac{5(3A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2d} - \frac{5(3A + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3a^2d} + \\
& \frac{4(14A + 5C) \cos[c + dx]^{3/2} \sin[c + dx]}{15a^2d} - \frac{(3A + C) \cos[c + dx]^{5/2} \sin[c + dx]}{a^2d(1 + \cos[c + dx])} - \frac{(A + C) \cos[c + dx]^{7/2} \sin[c + dx]}{3d(a + a \cos[c + dx])^2}
\end{aligned}$$

Result (type 5, 1398 leaves):

$$\begin{aligned}
& \frac{1}{5(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 56iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) -
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{(A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} 4 i C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \left(20 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left(A + C \operatorname{Sec}[c + d x] \right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \left(20 C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \right. \\
& \quad \operatorname{Sec} \left[\frac{c}{2} \right] (A + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \left(\operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\operatorname{Cos}[c + d x]} (A + C \operatorname{Sec}[c + d x])^2 \left(-\frac{16 (10 A + 5 C + 18 A \operatorname{Cos}[c] + 5 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} - \frac{32 A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \right. \right.
\end{aligned}$$

$$\frac{8 A \cos[2 d x] \sin[2 c]}{5 d} + \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3 \left(A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right]\right)}{3 d} - \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right] \left(2 A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right]\right)}{d} - \left. \left. \frac{32 A \cos[c] \sin[d x]}{3 d} + \frac{8 A \cos[2 c] \sin[2 d x]}{5 d} + \frac{4 (A + C) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^2 \right)$$

- **Problem 1115: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{3/2} (A + C \sec[c + d x])^2}{(a + a \sec[c + d x])^2} dx$$

Optimal (type 4, 161 leaves, 7 steps):

$$-\frac{(7 A + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} + \frac{2(5 A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} + \frac{2(5 A + C) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 a^2 d} - \frac{(7 A + C) \cos[c + d x]^{3/2} \sin[c + d x]}{3 a^2 d (1 + \cos[c + d x])} - \frac{(A + C) \cos[c + d x]^{5/2} \sin[c + d x]}{3 d (a + a \cos[c + d x])^2}$$

Result (type 5, 1355 leaves):

$$-\frac{1}{(A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^2} 7 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c + d x])^2$$

$$\left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\frac{1}{(A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^2} i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + C \sec[c + d x])^2$$

$$\left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\begin{aligned}
& \left(40 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) - \\
& \left(8 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} (A + C \operatorname{Sec}[c + dx])^2 \left(\frac{8 (3 A + C + 4 A \cos[c]) \operatorname{Csc}[c]}{d} + \frac{16 A \cos[dx] \sin[c]}{3 d} - \right. \right. \\
& \quad \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3 d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (3 A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} + \right. \\
& \quad \left. \frac{16 A \cos[c] \sin[dx]}{3 d} - \frac{4 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((A + 2 C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)
\end{aligned}$$

- **Problem 1116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + C \operatorname{Sec}[c + dx])^2}{(a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$\frac{4 A \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{a^2 d} - \frac{(5 A - C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{3 a^2 d} - \frac{(5 A - C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3 a^2 d (1 + \cos[c + dx])} - \frac{(A + C) \cos[c + dx]^{3/2} \sin[c + dx]}{3 d (a + a \cos[c + dx])^2}$$

Result (type 5, 934 leaves):

$$\begin{aligned}
& \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 4iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \left(20A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) - \\
& \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} (A + C \sec[c + dx])^2 \right. \\
& \quad \left. \left(-\frac{16A \operatorname{Cot}\left[\frac{c}{2}\right]}{d} - \frac{16A \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) \right) / \\
& \left((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2 \right)
\end{aligned}$$

- **Problem 1117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$-\frac{(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{2(A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \frac{(A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{a^2 d (1 + \operatorname{Cos}[c + dx])} - \frac{(A + C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{3d (a + a \operatorname{Cos}[c + dx])^2}$$

Result (type 5, 1322 leaves):

$$\begin{aligned} & -\frac{1}{(A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \\ & \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) + \\ & \frac{1}{(A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} i C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \\ & \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) - \\ & \left(8 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + dx]^2) \\ & \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ & \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \end{aligned}$$

$$\begin{aligned}
& \left(3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + d x])^2 \right) - \\
& \left(8 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 \right. \\
& \quad \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + d x])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + d x])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + d x]} (A + C \sec[c + d x])^2 \left(\frac{8 (A - C) \operatorname{Csc}[c]}{d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{d} - \right. \right. \\
& \quad \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3 d} - \frac{4 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3 d} \right) \right) / \left((A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^2 \right)
\end{aligned}$$

- **Problem 1118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + d x]^2}{\cos[c + d x]^{3/2} (a + a \sec[c + d x])^2} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$\begin{aligned}
& - \frac{4 C \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} + \frac{(A - 5 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} + \\
& \frac{4 C \sin[c + d x]}{a^2 d \sqrt{\cos[c + d x]}} + \frac{(A - 5 C) \sin[c + d x]}{3 a^2 d \sqrt{\cos[c + d x]} (1 + \cos[c + d x])} - \frac{(A + C) \sin[c + d x]}{3 d \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^2}
\end{aligned}$$

Result (type 5, 954 leaves):

$$\begin{aligned}
& - \frac{1}{(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 4i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \left(4A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \right. \\
& \quad \left. \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(3d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \left(20C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(3d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} (A + C \sec[c + dx])^2 \right. \\
& \quad \left(\frac{16C \operatorname{Cot}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \frac{16C \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \sin\left[\frac{dx}{2}\right]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \right. \\
& \quad \left. \frac{16C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) / \left((A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2 \right)
\end{aligned}$$

- **Problem 1119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + d x]^2}{\operatorname{Cos}[c + d x]^{5/2} (a + a \operatorname{Sec}[c + d x])^2} dx$$

Optimal (type 4, 189 leaves, 8 steps):

$$\frac{(A + 7 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a^2 d} + \frac{2(A + 5 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 a^2 d} + \frac{2(A + 5 C) \operatorname{Sin}[c + d x]}{3 a^2 d \operatorname{Cos}[c + d x]^{3/2}} - \frac{(A + 7 C) \operatorname{Sin}[c + d x]}{a^2 d \sqrt{\operatorname{Cos}[c + d x]}} - \frac{(A + 7 C) \operatorname{Sin}[c + d x]}{3 a^2 d \operatorname{Cos}[c + d x]^{3/2} (1 + \operatorname{Cos}[c + d x])} - \frac{(A + C) \operatorname{Sin}[c + d x]}{3 d \operatorname{Cos}[c + d x]^{3/2} (a + a \operatorname{Cos}[c + d x])^2}$$

Result (type 5, 1391 leaves):

$$\frac{1}{(A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} i A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + d x]^2) \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) + \frac{1}{(A + 2 C + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} 7 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + d x]^2) \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}\right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \left(8 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right)$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A+2C+A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx])^2 \right) - \\
& \left(40C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right. \\
& \quad \left. \sec\left[\frac{c}{2}\right] (A+C\sec[c+dx])^2 \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A+2C+A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} (A+C\sec[c+dx])^2 \left(-\frac{4(4C+A\cos[c]+3C\cos[c]) \text{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c]}{d} \right. \right. \\
& \quad \left. \frac{4\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right])}{3d} - \frac{8\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A\sin\left[\frac{dx}{2}\right] + 3C\sin\left[\frac{dx}{2}\right])}{d} + \frac{16C\sec[c] \sec[c+dx]^2 \sin[dx]}{3d} \right. \\
& \quad \left. \left. \frac{16\sec[c] \sec[c+dx] (C\sin[c] - 6C\sin[dx])}{3d} - \frac{4(A+C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) / \left((A+2C+A\cos[2c+2dx]) (a+a\sec[c+dx])^2 \right)
\end{aligned}$$

- **Problem 1120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2} (A+C\sec[c+dx])^2}{(a+a\sec[c+dx])^3} dx$$

Optimal (type 4, 250 leaves, 9 steps):

$$\begin{aligned}
& \frac{7(33A+7C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{10a^3d} - \frac{(63A+13C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{6a^3d} \\
& \frac{(63A+13C) \sqrt{\cos[c+dx]} \sin[c+dx]}{6a^3d} + \frac{7(33A+7C) \cos[c+dx]^{3/2} \sin[c+dx]}{30a^3d} \\
& \frac{(A+C) \cos[c+dx]^{9/2} \sin[c+dx]}{5d(a+a\cos[c+dx])^3} - \frac{2(6A+C) \cos[c+dx]^{7/2} \sin[c+dx]}{15ad(a+a\cos[c+dx])^2} - \frac{(63A+13C) \cos[c+dx]^{5/2} \sin[c+dx]}{10d(a^3+a^3\cos[c+dx])}
\end{aligned}$$

Result (type 5, 1507 leaves):

$$\begin{aligned}
& \frac{1}{5 (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^3} 231 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] (A + C \sec[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) + \\
& \frac{1}{5 (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^3} 49 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] (A + C \sec[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) + \\
& \left(84 A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] \right. \\
& \quad \left. (A + C \sec[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \quad \left(d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x])^3 \right) + \\
& \left(52 C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[c + d x] (A + C \sec[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) /
\end{aligned}$$

$$\left(3 d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + d x])^3 \right) +$$

$$\frac{1}{\sqrt{\cos[c + d x]} (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^3}$$

$$\cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (A + C \sec[c + d x])^2 \left(-\frac{8 (99 A + 29 C + 132 A \cos[c] + 20 C \cos[c]) \operatorname{Csc}[c]}{5 d} - \right.$$

$$\frac{32 A \cos[d x] \sin[c]}{d} + \frac{16 A \cos[2 d x] \sin[2 c]}{5 d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{5 d} +$$

$$\frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (12 A \sin\left[\frac{d x}{2}\right] + 7 C \sin\left[\frac{d x}{2}\right])}{15 d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right] (99 A \sin\left[\frac{d x}{2}\right] + 29 C \sin\left[\frac{d x}{2}\right])}{5 d} \left. - \right.$$

$$\left. \frac{32 A \cos[c] \sin[d x]}{d} + \frac{16 A \cos[2 c] \sin[2 d x]}{5 d} + \frac{16 (12 A + 7 C) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15 d} - \frac{4 (A + C) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5 d} \right)$$

- **Problem 1121: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{3/2} (A + C \sec[c + d x])^2}{(a + a \sec[c + d x])^3} dx$$

Optimal (type 4, 209 leaves, 8 steps):

$$-\frac{(119 A + 9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{10 a^3 d} + \frac{(11 A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{2 a^3 d} + \frac{(11 A + C) \sqrt{\cos[c + d x]} \sin[c + d x]}{2 a^3 d}$$

$$-\frac{(A + C) \cos[c + d x]^{7/2} \sin[c + d x]}{5 d (a + a \cos[c + d x])^3} - \frac{2 A \cos[c + d x]^{5/2} \sin[c + d x]}{3 a d (a + a \cos[c + d x])^2} - \frac{(119 A + 9 C) \cos[c + d x]^{3/2} \sin[c + d x]}{30 d (a^3 + a^3 \cos[c + d x])}$$

Result (type 5, 1470 leaves):

$$-\frac{1}{5 (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^3} 119 i A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + d x] (A + C \sec[c + d x])^2$$

$$\left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\begin{aligned}
& \frac{1}{5 (A + 2 C + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])^3} 9 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] (A + C \sec[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) - \\
& \left(44 A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] \right. \\
& \quad \left. (A + C \sec[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x])^3 \right) - \\
& \left(4 C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \operatorname{Sec}[c + d x] (A + C \sec[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (A + 2 C + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x])^3 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (A + C \sec[c + d x])^2 \right. \\
& \quad \left(\frac{8 (59 A + 9 C + 60 A \cos[c]) \operatorname{Csc}[c]}{5 d} + \frac{32 A \cos[d x] \sin[c]}{3 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^5 (A \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{5 d} - \right. \\
& \quad \left. \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^3 (19 A \sin\left[\frac{d x}{2}\right] + 9 C \sin\left[\frac{d x}{2}\right])}{15 d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (59 A \sin\left[\frac{d x}{2}\right] + 9 C \sin\left[\frac{d x}{2}\right])}{5 d} + \frac{32 A \cos[c] \sin[d x]}{3 d} - \right.
\end{aligned}$$

$$\left. \frac{8 (19 A + 9 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{4 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right) / \left(\sqrt{\operatorname{Cos}[c + dx]} (A + 2 C + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)$$

- **Problem 1122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\operatorname{Cos}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 186 leaves, 7 steps):

$$\frac{(49 A - C) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^3 d} - \frac{(13 A - C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{6 a^3 d} - \frac{(A + C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} - \frac{2 (4 A - C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} - \frac{(13 A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{6 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 1446 leaves):

$$\frac{1}{5 (A + 2 C + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3} 49 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \frac{1}{5 (A + 2 C + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3} i C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) + \left(52 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right)$$

$$\begin{aligned}
& \left((A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3d (A + 2C + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) - \\
& \left(4C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3d (A + 2C + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \left(-\frac{8(29A - C + 20A \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (29A \operatorname{Sin}\left[\frac{dx}{2}\right] - C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5d} \right. \right. \\
& \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right])}{5d} + \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (7A \operatorname{Sin}\left[\frac{dx}{2}\right] + 2C \operatorname{Sin}\left[\frac{dx}{2}\right])}{15d} \right. \right. \\
& \left. \left. \frac{16(7A + 2C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} - \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right) \right) / \left(\sqrt{\operatorname{Cos}[c + dx]} (A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 1123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 184 leaves, 7 steps):

$$\begin{aligned}
& -\frac{(9A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(3A + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \\
& \frac{(A + C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{5d(a + a \operatorname{Cos}[c + dx])^3} - \frac{2(3A - 2C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15ad(a + a \operatorname{Cos}[c + dx])^2} + \frac{(9A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10d(a^3 + a^3 \operatorname{Cos}[c + dx])}
\end{aligned}$$

Result (type 5, 1439 leaves):

$$-\frac{1}{5(A + 2C + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} 9iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) + \\
& \frac{1}{5 (A + 2 C + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^3} i C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \sec [c + d x] (A + C \sec [c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) - \\
& \left(4 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2 \right] \sec \left[\frac{c}{2} \right] \sec [c + d x] \right. \\
& \quad (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) - \\
& \left(4 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2 \right] \sec \left[\frac{c}{2} \right] \right. \\
& \quad \sec [c + d x] (A + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) / \\
& \left(3 d (A + 2 C + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) +
\end{aligned}$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \sec[c + dx])^2 \right. \\ \left. \left(\frac{8(9A - C) \operatorname{Csc}[c]}{5d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (9A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{5d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (9A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{15d} \right. \right. \\ \left. \left. + \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} - \frac{8(9A - C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) \right) / \\ \left(\sqrt{\cos[c + dx]} (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)$$

■ **Problem 1124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 180 leaves, 7 steps):

$$-\frac{(A - 9C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(A + 3C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \\ \frac{(A + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} + \frac{2(2A - 3C) \sqrt{\cos[c + dx]} \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} + \frac{(A - 9C) \sqrt{\cos[c + dx]} \sin[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1436 leaves):

$$-\frac{1}{5(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx])^2 \\ \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ \frac{1}{5(A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} 9i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + C \sec[c + dx])^2 \\ \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c])} \right) - \right. \\
& \left. \left(4A \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec}[c + dx] \right. \right. \\
& \left. \left. (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]}}}{\right) \right. \\
& \left. \left(3d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) - \right. \\
& \left. \left(4C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \right. \\
& \left. \left. \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]}}}{\right) \right. \\
& \left. \left(d(A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \right. \\
& \left. \left(\cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (A + C \operatorname{Sec}[c + dx])^2 \right. \right. \\
& \left. \left. \left(\frac{8(A - 9C) \operatorname{Csc}[c]}{5d} + \frac{8 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right] (A \sin \left[\frac{dx}{2} \right] - 9C \sin \left[\frac{dx}{2} \right])}{5d} + \frac{16 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^3 (2A \sin \left[\frac{dx}{2} \right] - 3C \sin \left[\frac{dx}{2} \right])}{15d} \right. \right. \\
& \left. \left. \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^5 (A \sin \left[\frac{dx}{2} \right] + C \sin \left[\frac{dx}{2} \right])}{5d} + \frac{16(2A - 3C) \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{15d} - \frac{4(A + C) \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Tan} \left[\frac{c}{2} \right]}{5d} \right) \right) \right. \\
& \left. \left(\sqrt{\cos[c + dx]} (A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3 \right) \right) \Bigg/
\end{aligned}$$

■ **Problem 1125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 209 leaves, 8 steps):

$$\frac{(A - 49 C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(A - 13 C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} - \frac{(A - 49 C) \text{Sin}[c + dx]}{10 a^3 d \sqrt{\text{Cos}[c + dx]}}$$

$$\frac{(A + C) \text{Sin}[c + dx]}{5 d \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^3} + \frac{2 (A - 4 C) \text{Sin}[c + dx]}{15 a d \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^2} + \frac{(A - 13 C) \text{Sin}[c + dx]}{6 d \sqrt{\text{Cos}[c + dx]} (a^3 + a^3 \text{Cos}[c + dx])}$$

Result (type 5, 1473 leaves):

$$\frac{1}{5 (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + a \text{Sec}[c + d x])^3} i A \text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x] (A + C \text{Sec}[c + d x])^2$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \text{Cos}[c] + d (-1 + e^{2 i d x}) \text{Sin}[c]) \right) -$$

$$\frac{1}{5 (A + 2 C + A \text{Cos}[2 c + 2 d x]) (a + a \text{Sec}[c + d x])^3} 49 i C \text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x] (A + C \text{Sec}[c + d x])^2$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \text{Cos}[c] + d (-1 + e^{2 i d x}) \text{Sin}[c]) \right) -$$

$$\left(4 A \text{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + d x] \right.$$

$$\left. (A + C \text{Sec}[c + d x])^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /$$

$$(3 d (A + 2 C + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^3) +$$

$$\left(52 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ \left. \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3 d (A + 2 C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \right. \\ \left(-\frac{4(-20C + A \cos[c] - 29C \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - 29C \sin\left[\frac{dx}{2}\right])}{5d} + \right. \\ \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + 11C \sin\left[\frac{dx}{2}\right])}{15d} + \frac{32C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \right. \\ \left. \left. \frac{8(A + 11C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15d} + \frac{4(A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5d} \right) \right) / \left(\sqrt{\cos[c + dx]} (A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)$$

- **Problem 1126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + C \operatorname{Sec}[c + dx]^2}{\cos[c + dx]^{7/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 242 leaves, 9 steps):

$$\frac{(9A + 119C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3 d} + \frac{(A + 11C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{2a^3 d} + \frac{(A + 11C) \sin[c + dx]}{2a^3 d \cos[c + dx]^{3/2}} - \frac{(9A + 119C) \sin[c + dx]}{10a^3 d \sqrt{\cos[c + dx]}} - \\ \frac{(A + C) \sin[c + dx]}{5d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^3} - \frac{2C \sin[c + dx]}{3ad \cos[c + dx]^{3/2} (a + a \cos[c + dx])^2} - \frac{(9A + 119C) \sin[c + dx]}{30d \cos[c + dx]^{3/2} (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 1505 leaves):

$$\frac{1}{5(A + 2C + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} {}_9F_1 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \\ \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} \right) + \\
& \frac{1}{5 (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} 119 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(3 i d (1 + e^{2idx}) \cos[c] - 3 d (-1 + e^{2idx}) \sin[c])} - \right. \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]}}{(-i d (1 + e^{2idx}) \cos[c] + d (-1 + e^{2idx}) \sin[c])} \right) \right) - \\
& \left(4 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] \right. \\
& \left. (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \right) / \\
& \left(d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^3 \right) - \\
& \left(44 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[c + dx] (A + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \right) / \\
& \left(d (A + 2C + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \frac{1}{\sqrt{\cos[c + dx]} (A + 2C + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4 (60 C + 9 A \cos[c] + 59 C \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5 d} - \right. \\
& \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5 d} - \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (3 A \sin\left[\frac{dx}{2}\right] + 8 C \sin\left[\frac{dx}{2}\right])}{15 d} \right)
\end{aligned}$$

$$\frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(9 A \operatorname{Sin}\left[\frac{dx}{2}\right] + 59 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{32 C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[dx]}{3 d} +$$

$$\frac{32 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \left(C \operatorname{Sin}[c] - 9 C \operatorname{Sin}[dx]\right)}{3 d} - \frac{16 (3 A + 8 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{4 (A + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d}$$

■ **Problem 1130: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c + dx]^{3/2} \sqrt{a + a \operatorname{Sec}[c + dx]} (A + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 3, 136 leaves, 5 steps):

$$\frac{2 \sqrt{a} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} +$$

$$\frac{2 a A \operatorname{Sin}[c+dx]}{3 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2 A \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d}$$

Result (type 5, 174 leaves):

$$\left(2 \sqrt{\operatorname{Cos}[c+dx]} (C + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-6 i C e^{\frac{1}{2} i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] -\right.\right.$$

$$\left.\left.2 i C e^{\frac{3}{2} i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] + A \left(3 \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] + \operatorname{Sin}\left[\frac{3}{2}(c+dx)\right]\right)\right)\right) / (3 d (A + 2 C + A \operatorname{Cos}[2(c+dx)]))$$

■ **Problem 1131: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a + a \operatorname{Sec}[c+dx]} (A + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 135 leaves, 5 steps):

$$\frac{\sqrt{a} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{a (2 A - C) \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 184 leaves):

$$\left(2 (C + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-3 i C e^{\frac{1}{2} i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] - i C e^{\frac{3}{2} i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] +\right.\right.$$

$$\left.\left.3 (C + 2 A \operatorname{Cos}[c+dx]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)\right) / (3 d \sqrt{\operatorname{Cos}[c+dx]} (A + 2 C + A \operatorname{Cos}[2(c+dx)]))$$

■ **Problem 1132: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]} (A + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{\sqrt{a} (8 A + 3 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{4 d} + \frac{a C \operatorname{Sin}[c + d x]}{4 d \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{C \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{2 d \operatorname{Cos}[c + d x]^{3/2}}$$

Result (type 5, 161 leaves):

$$\frac{1}{12 d} \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \sqrt{a (1 + \operatorname{Sec}[c + d x])} \left(-3 i (8 A + 3 C) e^{\frac{1}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - \right. \\ \left. i (8 A + 3 C) e^{\frac{3}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + 3 C \operatorname{Sec}[c + d x] (3 + 2 \operatorname{Sec}[c + d x]) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 1133: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]} (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 3, 189 leaves, 6 steps):

$$\frac{\sqrt{a} (8 A + 5 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c + d x]}{\sqrt{a + a \operatorname{Sec}[c + d x]}}\right] \sqrt{\operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}}{8 d} + \\ \frac{a C \operatorname{Sin}[c + d x]}{12 d \operatorname{Cos}[c + d x]^{5/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{a (8 A + 5 C) \operatorname{Sin}[c + d x]}{8 d \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + a \operatorname{Sec}[c + d x]}} + \frac{C \sqrt{a + a \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{3 d \operatorname{Cos}[c + d x]^{5/2}}$$

Result (type 5, 176 leaves):

$$\frac{1}{24 d} \sqrt{\operatorname{Cos}[c + d x]} \operatorname{Sec}\left[\frac{1}{2} (c + d x)\right] \sqrt{a (1 + \operatorname{Sec}[c + d x])} \\ \left(-3 i (8 A + 5 C) e^{\frac{1}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c + d x)}\right] - i (8 A + 5 C) e^{\frac{3}{2} i (c + d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c + d x)}\right] + \right. \\ \left. \operatorname{Sec}[c + d x] (24 A + 15 C + 10 C \operatorname{Sec}[c + d x] + 8 C \operatorname{Sec}[c + d x]^2) \operatorname{Sin}\left[\frac{1}{2} (c + d x)\right] \right)$$

■ **Problem 1134: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + a \operatorname{Sec}[c + d x]} (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{5/2}} dx$$

Optimal (type 3, 234 leaves, 7 steps):

$$\frac{\sqrt{a} (48 A + 35 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{64 d} + \frac{a C \operatorname{Sin}[c+dx]}{24 d \operatorname{Cos}[c+dx]^{7/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a (48 A + 35 C) \operatorname{Sin}[c+dx]}{96 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a (48 A + 35 C) \operatorname{Sin}[c+dx]}{64 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d \operatorname{Cos}[c+dx]^{7/2}}$$

Result (type 5, 195 leaves):

$$\frac{1}{192 d} \sqrt{\operatorname{Cos}[c+dx]} \operatorname{Sec}\left[\frac{1}{2} (c+dx)\right] \sqrt{a (1+\operatorname{Sec}[c+dx])}$$

$$\left(-3 i (48 A + 35 C) e^{\frac{1}{2} i (c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - i (48 A + 35 C) e^{\frac{3}{2} i (c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] + \right.$$

$$\left. \operatorname{Sec}[c+dx] (3 (48 A + 35 C) + (96 A + 70 C) \operatorname{Sec}[c+dx] + 56 C \operatorname{Sec}[c+dx]^2 + 48 C \operatorname{Sec}[c+dx]^3) \operatorname{Sin}\left[\frac{1}{2} (c+dx)\right] \right)$$

■ **Problem 1138: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 183 leaves, 6 steps):

$$\frac{2 a^{3/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{2 a^2 (4 A + 5 C) \operatorname{Sin}[c+dx]}{5 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{2 a A \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{5 d} + \frac{2 A \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{5 d}$$

Result (type 5, 198 leaves):

$$\frac{1}{30 d (A + 2 C + A \operatorname{Cos}[2 (c+dx)])} a \sqrt{\operatorname{Cos}[c+dx]} (1 + \operatorname{Cos}[c+dx]) (C + A \operatorname{Cos}[c+dx])^2$$

$$\operatorname{Sec}\left[\frac{1}{2} (c+dx)\right]^3 \sqrt{a (1+\operatorname{Sec}[c+dx])} \left(-60 i C e^{\frac{1}{2} i (c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - \right.$$

$$\left. 20 i C e^{\frac{3}{2} i (c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] + 6 (13 A + 10 C + 6 A \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2 (c+dx)]) \operatorname{Sin}\left[\frac{1}{2} (c+dx)\right] \right)$$

■ **Problem 1139: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 189 leaves, 6 steps):

$$\frac{3 a^{3/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} + \frac{a^2 (8 A - 3 C) \operatorname{Sin}[c+dx]}{3 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}}$$

$$\frac{a (2 A - 3 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{2 A \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3 d}$$

Result (type 5, 207 leaves):

$$\frac{1}{3 d \sqrt{\operatorname{Cos}[c+dx]} (A+2 C+A \operatorname{Cos}[2(c+dx)])} a (1+\operatorname{Cos}[c+dx]) (C+A \operatorname{Cos}[c+dx])^2$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-9 i C e^{\frac{1}{2} i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] -\right.$$

$$\left.3 i C e^{\frac{3}{2} i(c+dx)} \operatorname{Cos}[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] + (A+3 C+10 A \operatorname{Cos}[c+dx]+A \operatorname{Cos}[2(c+dx)]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 1140: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 191 leaves, 6 steps):

$$\frac{a^{3/2} (8 A + 7 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4 d} +$$

$$\frac{a^2 (8 A - 5 C) \operatorname{Sin}[c+dx]}{4 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{3 a C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{2 d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 224 leaves):

$$\frac{1}{12 d \operatorname{Cos}[c+dx]^{3/2} (A+2 C+A \operatorname{Cos}[2(c+dx)])} a (1+\operatorname{Cos}[c+dx]) (C+A \operatorname{Cos}[c+dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-3 i (8 A + 7 C) e^{\frac{1}{2} i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] -\right.$$

$$\left. i (8 A + 7 C) e^{\frac{3}{2} i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] + 3 (2 C + 7 C \operatorname{Cos}[c+dx] + 8 A \operatorname{Cos}[c+dx]^2) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 1141: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx])^2}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 3, 191 leaves, 6 steps):

$$\frac{a^{3/2} (24 A + 11 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{8 d} +$$

$$\frac{a^2 (24 A + 19 C) \operatorname{Sin}[c+dx]}{24 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{4 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{3 d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 5, 229 leaves):

$$\frac{1}{24 d \operatorname{Cos}[c+dx]^{5/2} (A+2 C+A \operatorname{Cos}[2(c+dx)])} a (1+\operatorname{Cos}[c+dx]) (C+A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a(1+\operatorname{Sec}[c+dx])}$$

$$\left(-3 i (24 A+11 C) e^{\frac{1}{2} i(c+dx)} \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] - i (24 A+11 C) e^{\frac{3}{2} i(c+dx)} \operatorname{Cos}[c+dx]^3\right.$$

$$\left.\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] + (8 C+22 C \operatorname{Cos}[c+dx]+3(8 A+11 C) \operatorname{Cos}[c+dx]^2) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 1142: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

Optimal (type 3, 238 leaves, 7 steps):

$$\frac{a^{3/2} (112 A + 75 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{64 d} + \frac{a^2 (16 A + 13 C) \operatorname{Sin}[c+dx]}{32 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (112 A + 75 C) \operatorname{Sin}[c+dx]}{64 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{8 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{4 d \operatorname{Cos}[c+dx]^{5/2}}$$

Result (type 5, 261 leaves):

$$\frac{1}{768 d \operatorname{Cos}[c+dx]^{7/2} (A+2 C+A \operatorname{Cos}[2(c+dx)])} a (1+\operatorname{Cos}[c+dx]) (C+A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\sqrt{a(1+\operatorname{Sec}[c+dx])} \left(-12 i (112 A+75 C) e^{\frac{1}{2} i(c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+dx)}\right] -\right.$$

$$\left.4 i (112 A+75 C) e^{\frac{3}{2} i(c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+dx)}\right] +\right.$$

$$\left.3(64 A+164 C+7(48 A+55 C) \operatorname{Cos}[c+dx]+4(16 A+25 C) \operatorname{Cos}[2(c+dx)]+112 A \operatorname{Cos}[3(c+dx)]+75 C \operatorname{Cos}[3(c+dx)]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]\right)$$

■ **Problem 1143: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \operatorname{Sec}[c+dx])^{3/2} (A+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{5/2}} dx$$

Optimal (type 3, 285 leaves, 8 steps) :

$$\frac{a^{3/2} (176 A + 133 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{128 d} +$$

$$\frac{a^2 (80 A + 67 C) \operatorname{Sin}[c+dx]}{240 d \operatorname{Cos}[c+dx]^{7/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (176 A + 133 C) \operatorname{Sin}[c+dx]}{192 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (176 A + 133 C) \operatorname{Sin}[c+dx]}{128 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{3 a C \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{40 d \operatorname{Cos}[c+dx]^{7/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{7/2}}$$

Result (type 5, 263 leaves) :

$$\frac{1}{1920 d \operatorname{Cos}[c+dx]^{9/2} (A + 2 C + A \operatorname{Cos}[2(c+dx)])} a (1 + \operatorname{Cos}[c+dx]) (C + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3$$

$$\sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-15 i (176 A + 133 C) e^{\frac{1}{2} i (c+dx)} \operatorname{Cos}[c+dx]^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - \right.$$

$$\left. 5 i (176 A + 133 C) e^{\frac{3}{2} i (c+dx)} \operatorname{Cos}[c+dx]^5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] + \right.$$

$$\left. (384 C + 912 C \operatorname{Cos}[c+dx] + 8 (80 A + 133 C) \operatorname{Cos}[c+dx]^2 + 10 (176 A + 133 C) \operatorname{Cos}[c+dx]^3 + 15 (176 A + 133 C) \operatorname{Cos}[c+dx]^4) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1147: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^{5/2} (A+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 230 leaves, 7 steps) :

$$\frac{2 a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} +$$

$$\frac{2 a^3 (32 A + 49 C) \operatorname{Sin}[c+dx]}{21 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2 a^2 (8 A + 7 C) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{21 d} +$$

$$\frac{2 a A \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{7 d} + \frac{2 A \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{7 d}$$

Result (type 5, 297 leaves) :

$$\left(C \cos [c+d x]^{9/2} \left(-2 i e^{\frac{1}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \frac{2}{3} i e^{\frac{3}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \right) / (2 d (A+2 C+A \cos [2 c+2 d x])) + \\ \left(\cos [c+d x]^{9/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+C \operatorname{Sec}[c+d x]^2) \left(\frac{5}{8} (3 A+4 C) \sin \left[\frac{1}{2}(c+d x)\right] + \right. \right. \\ \left. \left. \frac{1}{24} (11 A+4 C) \sin \left[\frac{3}{2}(c+d x)\right] + \frac{1}{8} A \sin \left[\frac{5}{2}(c+d x)\right] + \frac{1}{56} A \sin \left[\frac{7}{2}(c+d x)\right] \right) \right) / (d (A+2 C+A \cos [2 c+2 d x])) \right)$$

■ **Problem 1148: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{5/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 230 leaves, 7 steps):

$$\frac{5 a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d} + \\ \frac{a^3 (64 A+15 C) \sin [c+d x]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{a^2 (16 A-15 C) \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{15 d \sqrt{\cos [c+d x]}} + \\ \frac{2 a A \sqrt{\cos [c+d x]} (a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{3 d} + \frac{2 A \cos [c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{5/2} \sin [c+d x]}{5 d}$$

Result (type 5, 230 leaves):

$$\frac{1}{60 d \sqrt{\cos [c+d x]} (A+2 C+A \cos [2(c+d x)])} a^2 (1+\cos [c+d x])^2 (C+A \cos [c+d x]^2) \\ \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\operatorname{Sec}[c+d x])} \left(-150 i C e^{\frac{1}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \right. \\ \left. 50 i C e^{\frac{3}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + \right. \\ \left. (28 A+30 C+(181 A+60 C) \cos [c+d x]+28 A \cos [2(c+d x)]+3 A \cos [3(c+d x)]) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 1149: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{5/2} (A+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 244 leaves, 7 steps):

$$\frac{a^{5/2} (8A + 19C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4d} +$$

$$\frac{a^3 (56A - 27C) \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{a^2 (8A - 21C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]}} -$$

$$\frac{a (4A - 3C) (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{6d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{2A \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 5, 246 leaves):

$$\frac{1}{24d \operatorname{Cos}[c+dx]^{3/2} (A + 2C + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + A \operatorname{Cos}[c+dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-3i (8A + 19C) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - \right.$$

$$\left. i (8A + 19C) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + \right.$$

$$\left. (32A + 6C + (6A + 33C) \operatorname{Cos}[c+dx] + 32A \operatorname{Cos}[2(c+dx)] + 2A \operatorname{Cos}[3(c+dx)]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1150: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} (A+C \operatorname{Sec}[c+dx])^2 dx$$

Optimal (type 3, 238 leaves, 7 steps):

$$\frac{5a^{5/2} (8A + 5C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{8d} + \frac{a^3 (24A - 49C) \operatorname{Sin}[c+dx]}{24d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (24A + 31C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{5aC (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 244 leaves):

$$\frac{1}{48d \operatorname{Cos}[c+dx]^{5/2} (A + 2C + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + A \operatorname{Cos}[c+dx])^2 \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{a(1 + \operatorname{Sec}[c+dx])}$$

$$\left(-15i (8A + 5C) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - 5i (8A + 5C) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx]^3 \right.$$

$$\left. \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + (8C + 34C \operatorname{Cos}[c+dx] + 3(8A + 25C) \operatorname{Cos}[c+dx]^2 + 48A \operatorname{Cos}[c+dx]^3) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1151: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 238 leaves, 7 steps):

$$\frac{a^{5/2} (304 A + 163 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{64 d} + \frac{a^3 (432 A + 299 C) \operatorname{Sin}[c+dx]}{192 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (16 A + 17 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{32 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{5 a C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{24 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{4 d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 5, 248 leaves):

$$\frac{1}{384 d \operatorname{Cos}[c+dx]^{7/2} (A + 2 C + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-3 i (304 A + 163 C) e^{\frac{1}{2} i (c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - \right.$$

$$i (304 A + 163 C) e^{\frac{3}{2} i (c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] +$$

$$\left. (48 C + 184 C \operatorname{Cos}[c+dx] + (96 A + 326 C) \operatorname{Cos}[c+dx]^2 + (528 A + 489 C) \operatorname{Cos}[c+dx]^3) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1152: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 3, 285 leaves, 8 steps):

$$\frac{a^{5/2} (400 A + 283 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{128 d} +$$

$$\frac{a^3 (1040 A + 787 C) \operatorname{Sin}[c+dx]}{960 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^3 (400 A + 283 C) \operatorname{Sin}[c+dx]}{128 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (80 A + 79 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{240 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{a C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{8 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{5/2}}$$

Result (type 5, 267 leaves):

$$\frac{1}{3840 d \cos [c+d x]^{9/2} (A+2 C+A \cos [2(c+d x)])} a^2 (1+\cos [c+d x])^2 (C+A \cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\sqrt{a(1+\sec [c+d x])} \left(-15 i(400 A+283 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right] -\right.$$

$$\left.5 i(400 A+283 C) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right] +\right.$$

$$\left.(384 C+1392 C \cos [c+d x]+8(80 A+283 C) \cos [c+d x]^2+10(272 A+283 C) \cos [c+d x]^3+15(400 A+283 C) \cos [c+d x]^4\right) \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 1153: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{5/2} (A+C \sec [c+d x]^2)}{\cos [c+d x]^{5/2}} dx$$

Optimal (type 3, 332 leaves, 9 steps):

$$\frac{a^{5/2} (1304 A+1015 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{512 d} +$$

$$\frac{a^3 (136 A+109 C) \sin [c+d x]}{192 d \cos [c+d x]^{7/2} \sqrt{a+a \sec [c+d x]}} + \frac{a^3 (1304 A+1015 C) \sin [c+d x]}{768 d \cos [c+d x]^{5/2} \sqrt{a+a \sec [c+d x]}} + \frac{a^3 (1304 A+1015 C) \sin [c+d x]}{512 d \cos [c+d x]^{3/2} \sqrt{a+a \sec [c+d x]}} +$$

$$\frac{a^2 (24 A+23 C) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{96 d \cos [c+d x]^{7/2}} + \frac{a C (a+a \sec [c+d x])^{3/2} \sin [c+d x]}{12 d \cos [c+d x]^{7/2}} + \frac{C (a+a \sec [c+d x])^{5/2} \sin [c+d x]}{6 d \cos [c+d x]^{7/2}}$$

Result (type 5, 282 leaves):

$$\frac{1}{3072 d \cos [c+d x]^{11/2} (A+2 C+A \cos [2(c+d x)])} a^2 (1+\cos [c+d x])^2 (C+A \cos [c+d x])^2 \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\sec [c+d x])}$$

$$\left(-3 i(1304 A+1015 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right] -i(1304 A+1015 C) e^{\frac{3}{2} i(c+d x)}\right.$$

$$\left.\cos [c+d x]^6 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right] + (256 C+896 C \cos [c+d x]+48(8 A+29 C) \cos [c+d x]^2 +\right.$$

$$\left.8(184 A+203 C) \cos [c+d x]^3+(2608 A+2030 C) \cos [c+d x]^4+(3912 A+3045 C) \cos [c+d x]^5\right) \sin \left[\frac{1}{2}(c+d x)\right]$$

■ **Problem 1164: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \sec [c+d x]^2}{\sqrt{\cos [c+d x]} (a+a \sec [c+d x])^{3/2}} dx$$

Optimal (type 3, 185 leaves, 7 steps):

$$\frac{2 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{a^{3/2} d} + \frac{(3 A - 5 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{2 \sqrt{2} a^{3/2} d} - \frac{(A+C) \operatorname{Sin}[c+d x]}{2 d \operatorname{Cos}[c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{3/2}}$$

Result (type 3, 386 leaves):

$$\begin{aligned} & - \left(\left(2 \operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] (C+A \operatorname{Cos}[c+d x])^2 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] \right. \right. \\ & \quad \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]^2 \sqrt{1+\operatorname{Cos}[c+d x]} \left(\sqrt{2} (3 A-C) \operatorname{Log}[1+\operatorname{Cos}[c+d x]] + 8 C \operatorname{Log}\left[\sqrt{\operatorname{Cos}[c+d x]} (1+\operatorname{Cos}[c+d x])\right] \right) - \right. \\ & \quad \left. 2 \sqrt{2} C \operatorname{Log}\left[(1+\operatorname{Cos}[c+d x])^2\right] - 3 \sqrt{2} A \operatorname{Log}\left[2 \sqrt{1+\operatorname{Cos}[c+d x]} + \sqrt{2-2 \operatorname{Cos}[c+d x]^2}\right] + \right. \\ & \quad \left. \sqrt{2} C \operatorname{Log}\left[2 \sqrt{1+\operatorname{Cos}[c+d x]} + \sqrt{2-2 \operatorname{Cos}[c+d x]^2}\right] - 8 C \operatorname{Log}\left[1+\operatorname{Cos}[c+d x] + \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sin}[c+d x]^2}\right] + \right. \\ & \quad \left. \left. 2 \sqrt{2} C \operatorname{Log}\left[3+2 \operatorname{Cos}[c+d x] - \operatorname{Cos}[c+d x]^2 + 2 \sqrt{2} \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sin}[c+d x]^2}\right] \right) + (A+C) \sqrt{\operatorname{Sin}[c+d x]^2} \right) / \\ & \quad \left(a d \sqrt{\operatorname{Cos}[c+d x]} (1+\operatorname{Cos}[c+d x]) (A+2 C+A \operatorname{Cos}[2(c+d x)]) \sqrt{a(1+\operatorname{Sec}[c+d x])} \sqrt{\operatorname{Sin}[c+d x]^2} \right) \end{aligned}$$

■ **Problem 1165: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+C \operatorname{Sec}[c+d x]^2}{\operatorname{Cos}[c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 228 leaves, 8 steps):

$$\begin{aligned} & - \frac{3 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{a^{3/2} d} + \frac{(A+9 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{2 \sqrt{2} a^{3/2} d} \\ & \quad \frac{(A+C) \operatorname{Sin}[c+d x]}{2 d \operatorname{Cos}[c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{3/2}} + \frac{(A+3 C) \operatorname{Sin}[c+d x]}{2 a d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+a \operatorname{Sec}[c+d x]}} \end{aligned}$$

Result (type 3, 540 leaves):

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} (c + dx) \right] \right)^3 \sqrt{\cos [c + dx]} (A + C \sec [c + dx]^2) \\
& \left(8 C \sec [c + dx] \sin \left[\frac{1}{2} (c + dx) \right] + 2 \sec \left[\frac{1}{2} (c + dx) \right]^2 \left(A \sin \left[\frac{1}{2} (c + dx) \right] + C \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) \Bigg) \Bigg) \Bigg) / \\
& (d (A + 2 C + A \cos [2 c + 2 dx]) (a (1 + \sec [c + dx]))^{3/2}) - \left(\sqrt{2} (A + 3 C) \cos \left[\frac{1}{2} (c + dx) \right]^2 \sqrt{\cos [c + dx]} \sqrt{1 + \cos [c + dx]} \right. \\
& \left. \left(\log [1 + \cos [c + dx]] - \log \left[2 \sqrt{1 + \cos [c + dx]} + \sqrt{2 - 2 \cos [c + dx]^2} \right] \right) (A + C \sec [c + dx]^2) \sin [c + dx] \right) \Bigg) / \\
& \left(d \sqrt{1 - \cos [c + dx]^2} (A + 2 C + A \cos [2 c + 2 dx]) (a (1 + \sec [c + dx]))^{3/2} \right) + \left(3 C \cos \left[\frac{1}{2} (c + dx) \right]^2 \sqrt{\cos [c + dx]} \sqrt{1 + \cos [c + dx]} \right. \\
& \left. \left(-\sqrt{2} \log [(1 + \cos [c + dx])^2] + 4 \log [\sqrt{\cos [c + dx]} + \cos [c + dx]^{3/2}] - 4 \log [1 + \cos [c + dx] + \sqrt{1 + \cos [c + dx]} \sqrt{1 - \cos [c + dx]^2}] \right. \right. \\
& \left. \left. + \sqrt{2} \log [3 + 2 \cos [c + dx] - \cos [c + dx]^2 + 2 \sqrt{2} \sqrt{1 + \cos [c + dx]} \sqrt{1 - \cos [c + dx]^2}] \right) (A + C \sec [c + dx]^2) \sin [c + dx] \right) \Bigg) / \\
& \left(d \sqrt{1 - \cos [c + dx]^2} (A + 2 C + A \cos [2 c + 2 dx]) (a (1 + \sec [c + dx]))^{3/2} \right)
\end{aligned}$$

■ **Problem 1171: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + C \sec [c + dx]^2}{\cos [c + dx]^{3/2} (a + a \sec [c + dx])^{5/2}} dx$$

Optimal (type 3, 232 leaves, 8 steps):

$$\frac{2 C \operatorname{ArcSinh} \left[\frac{\sqrt{a} \tan [c + dx]}{\sqrt{a + a \sec [c + dx]}} \right] \sqrt{\cos [c + dx]} \sqrt{\sec [c + dx]} + (5 A - 43 C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + dx]} \sin [c + dx]}{\sqrt{2} \sqrt{a + a \sec [c + dx]}} \right] \sqrt{\cos [c + dx]} \sqrt{\sec [c + dx]}}{a^{5/2} d} + \frac{(A + C) \sin [c + dx]}{4 d \cos [c + dx]^{5/2} (a + a \sec [c + dx])^{5/2}} + \frac{(5 A - 11 C) \sin [c + dx]}{16 a d \cos [c + dx]^{3/2} (a + a \sec [c + dx])^{3/2}}$$

Result (type 3, 563 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^5 (A+C \sec[c+dx]^2) \left(16 C \sec[c+dx] \sin\left[\frac{1}{2}(c+dx)\right] + \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^4 \left(A \sin\left[\frac{1}{2}(c+dx)\right] + C \sin\left[\frac{1}{2}(c+dx)\right] \right) + \frac{1}{2} \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(3 A \sin\left[\frac{1}{2}(c+dx)\right] + 19 C \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) \Big/ \\
& \left(d \sqrt{\cos[c+dx]} (A+2C+A \cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2} \right) + \frac{1}{4d(A+2C+A \cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2}} \\
& \cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+C \sec[c+dx]^2) \\
& \left(-\frac{1}{\sqrt{1-\cos[c+dx]^2}} \sqrt{2} (3A+35C) \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \left(\log[1+\cos[c+dx]] - \log\left[2\sqrt{1+\cos[c+dx]} + \sqrt{2-2\cos[c+dx]^2}\right] \right) \right. \\
& \quad \left. \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] + \frac{1}{\sqrt{1-\cos[c+dx]^2}} 40C \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \right. \\
& \quad \left(-\sqrt{2} \log[(1+\cos[c+dx])^2] + 4 \log\left[\sqrt{\cos[c+dx]} + \cos[c+dx]^{3/2}\right] - 4 \log\left[1+\cos[c+dx] + \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] + \right. \\
& \quad \left. \left. \sqrt{2} \log\left[3+2\cos[c+dx] - \cos[c+dx]^2 + 2\sqrt{2} \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] \right)
\end{aligned}$$

- **Problem 1182: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c+dx]^{3/2} (A+B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 4, 65 leaves, 5 steps):

$$\frac{2B \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2(A+3C) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \frac{2A \sqrt{\cos[c+dx]} \sin[c+dx]}{3d}$$

Result (type 5, 682 leaves):

$$\begin{aligned}
& \frac{\cos [c+d x]^{5 / 2} \left(A+B \sec [c+d x]+C \sec [c+d x]^2 \right) \left(-\frac{4 B \cot [c]}{d}+\frac{4 A \cos [d x] \sin [c]}{3 d}+\frac{4 A \cos [c] \sin [d x]}{3 d} \right)}{A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]} - \\
& \left(4 A \cos [c+d x]^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2 \right) (A+B \sec [c+d x]+C \sec [c+d x]^2) \\
& \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \Bigg) / \left(3 d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 C \cos [c+d x]^2 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]\right]^2 \right) \\
& (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \Bigg) / \\
& \left(d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \left(2 B \cos [c+d x]^2 \csc [c] (A+B \sec [c+d x]+C \sec [c+d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]\right]^2 \right) \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \\
& \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \\
& \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (d(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))
\end{aligned}$$

- **Problem 1183: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (A+B \sec[c+dx]+C \sec[c+dx]^2) dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{2(A-C) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2B \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{2C \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 5, 759 leaves):

$$\frac{\cos[c+dx]^{5/2} (A+B \sec[c+dx]+C \sec[c+dx]^2) \left(-\frac{2(A-2C+A \cos[2c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{d} + \frac{4C \operatorname{Sec}[c] \operatorname{Sec}[c+dx] \operatorname{Sin}[dx]}{d} \right)}{A+2C+2B \cos[c+dx]+A \cos[2c+2dx]} -$$

$$\left(4B \cos[c+dx]^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right)$$

$$\left((A+B \sec[c+dx]+C \sec[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /$$

$$\left(d(A+2C+2B \cos[c+dx]+A \cos[2c+2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \left(2A \cos[c+dx]^2 \operatorname{Csc}[c] (A+B \sec[c+dx]+C \sec[c+dx]^2) \right.$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\left(\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2} \right) /$$

$$\left(\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \right) /$$

$$\begin{aligned}
& (d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) + \left(2 C \cos [c + d x]^2 \csc [c] (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) \right) / (d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]))
\end{aligned}$$

- **Problem 1187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{9/2} (a + a \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 4, 175 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 a (7 A + 9 (B + C)) \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{15 d} + \frac{2 a (5 (A + B) + 7 C) \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{21 d} + \frac{2 a (5 (A + B) + 7 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{21 d} \\
& + \frac{2 a (7 A + 9 (B + C)) \cos [c + d x]^{3/2} \sin [c + d x]}{45 d} + \frac{2 a (A + B) \cos [c + d x]^{5/2} \sin [c + d x]}{7 d} + \frac{2 a A \cos [c + d x]^{7/2} \sin [c + d x]}{9 d}
\end{aligned}$$

Result (type 5, 1292 leaves):

$$\begin{aligned}
& a \left(\sqrt{\cos [c + d x]} (1 + \cos [c + d x]) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \left(-\frac{(7 A + 9 B + 9 C) \cot [c]}{15 d} + \frac{(23 A + 23 B + 28 C) \cos [d x] \sin [c]}{84 d} + \right. \right. \\
& \left. \frac{(19 A + 18 B + 18 C) \cos [2 d x] \sin [2 c]}{180 d} + \frac{(A + B) \cos [3 d x] \sin [3 c]}{28 d} + \frac{A \cos [4 d x] \sin [4 c]}{72 d} + \frac{(23 A + 23 B + 28 C) \cos [c] \sin [d x]}{84 d} \right. \\
& \left. \left. \frac{(19 A + 18 B + 18 C) \cos [2 c] \sin [2 d x]}{180 d} + \frac{(A + B) \cos [3 c] \sin [3 d x]}{28 d} + \frac{A \cos [4 c] \sin [4 d x]}{72 d} \right) - \frac{1}{21 d \sqrt{1 + \cot [c]^2}} \right)
\end{aligned}$$

$$5 A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\ \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{21 d \sqrt{1 + \operatorname{Cot}[c]^2}}$$

$$5 B (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\ \frac{1}{3 d \sqrt{1 + \operatorname{Cot}[c]^2}} \operatorname{C}(1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\ \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{30 d} 7 A (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} - \frac{1}{10 d} 3 B (1 + \cos [c + d x]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{10 d} 3 C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)$$

- **Problem 1188: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{7/2} (a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 142 leaves, 7 steps):

$$\frac{2 a (3 (A + B) + 5 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (5 A + 7 (B + C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{2 a (5 A + 7 (B + C)) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a (A + B) \cos[c + d x]^{3/2} \sin[c + d x]}{5 d} + \frac{2 a A \cos[c + d x]^{5/2} \sin[c + d x]}{7 d}$$

Result (type 5, 1240 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \left(-\frac{(3 A + 3 B + 5 C) \cot[c]}{5 d} + \frac{(23 A + 28 B + 28 C) \cos[d x] \sin[c]}{84 d} + \frac{(A + B) \cos[2 d x] \sin[2 c]}{10 d} \right. \right.$$

$$\left. \left. \frac{A \cos[3 d x] \sin[3 c]}{28 d} + \frac{(23 A + 28 B + 28 C) \cos[c] \sin[d x]}{84 d} + \frac{(A + B) \cos[2 c] \sin[2 d x]}{10 d} + \frac{A \cos[3 c] \sin[3 d x]}{28 d} \right) - \frac{1}{21 d \sqrt{1 + \cot[c]^2}} \right.$$

$$\left. 5 A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \right)$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}}$$

$$B (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} C (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{10 d} 3 A (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} - \frac{1}{10 d} 3 B (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) - \frac{1}{2 d} C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)$$

- **Problem 1189: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{5/2} (a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 106 leaves, 6 steps):

$$\frac{2 a (3 A + 5 (B + C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (A + B + 3 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} +$$

$$\frac{2 a (A + B) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d} + \frac{2 a A \cos[c + d x]^{3/2} \sin[c + d x]}{5 d}$$

Result (type 5, 1186 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left(-\frac{(3 A + 5 B + 5 C) \cot[c]}{5 d} + \frac{(A + B) \cos[d x] \sin[c]}{3 d} + \frac{A \cos[2 d x] \sin[2 c]}{10 d} + \frac{(A + B) \cos[c] \sin[d x]}{3 d} + \frac{A \cos[2 c] \sin[2 d x]}{10 d} \right) - \right.$$

$$\left. \frac{1}{3 d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \right)$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{3d\sqrt{1 + \operatorname{Cot}[c]^2}}}$$

$$B(1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{d\sqrt{1 + \operatorname{Cot}[c]^2}} C(1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{10d} 3A(1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} - \frac{1}{2d} B(1 + \cos[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} - \frac{1}{2d} C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)$$

- **Problem 1190: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 98 leaves, 6 steps):

$$\frac{2 a (A + B - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (A + 3 (B + C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a C \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2 a A \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d}$$

Result (type 5, 1173 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left(- \frac{(A + B - 2 C + A \cos[2 c] + B \cos[2 c]) \csc[c] \sec[c]}{2 d} + \frac{A \cos[d x] \sin[c]}{3 d} + \frac{A \cos[c] \sin[d x]}{3 d} + \frac{C \sec[c] \sec[c + d x] \sin[d x]}{d} \right) - \right.$$

$$\frac{1}{3 d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + d x]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\left. \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \right)$$

$$\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} - \frac{1}{d \sqrt{1+\cot[c]^2}}$$

$$B (1 + \cos[c + dx]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec[dx - \text{ArcTan}[\cot[c]]]$$

$$\sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{d \sqrt{1+\cot[c]^2}} C (1 + \cos[c + dx]) \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} - \frac{1}{2d} A (1 + \cos[c + dx]) \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{2d} B (1 + \cos[c + dx]) \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} + \frac{1}{2d} C (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) \right)$$

- **Problem 1191: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 103 leaves, 6 steps):

$$\frac{2 a (A - B - C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 a (3 A + 3 B + C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a C \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a (B + C) \sin[c + d x]}{d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1180 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left(-\frac{(A - 2 B - 2 C + A \cos[2 c]) \text{Csc}[c] \text{Sec}[c]}{2 d} + \frac{C \text{Sec}[c] \text{Sec}[c + d x]^2 \sin[d x]}{3 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x] (C \sin[c] + 3 B \sin[d x] + 3 C \sin[d x])}{3 d} \right) - \right.$$

$$\left. \frac{1}{d \sqrt{1 + \cot[c]^2}} A (1 + \cos[c + d x]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \right)$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{d \sqrt{1 + \operatorname{Cot}[c]^2}}$$

$$B (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\frac{1}{3d \sqrt{1 + \operatorname{Cot}[c]^2}} C (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{2d} A (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2}}{\sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{1}{2d} B (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) + \frac{1}{2 d} C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right) \right)$$

- **Problem 1192: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2)}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 141 leaves, 7 steps):

$$-\frac{2 a (5 A + 5 B + 3 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (3 A + B + C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} +$$

$$\frac{2 a C \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (B + C) \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} + \frac{2 a (5 A + 5 B + 3 C) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1228 leaves):

$$a \left(\sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \right.$$

$$\left. \left(\frac{(5 A + 5 B + 3 C) \csc[c] \sec[c]}{5 d} + \frac{C \sec[c] \sec[c + d x]^3 \sin[d x]}{5 d} + \frac{\sec[c] \sec[c + d x]^2 (3 C \sin[c] + 5 B \sin[d x] + 5 C \sin[d x])}{15 d} \right) + \right.$$

$$\begin{aligned}
& \left. \frac{\text{Sec}[c] \text{Sec}[c + dx] (5 B \text{Sin}[c] + 5 C \text{Sin}[c] + 15 A \text{Sin}[dx] + 15 B \text{Sin}[dx] + 9 C \text{Sin}[dx])}{15 d} \right) - \frac{1}{d \sqrt{1 + \text{Cot}[c]^2}} \\
& A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} \\
& B (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} - \\
& \frac{1}{3 d \sqrt{1 + \text{Cot}[c]^2}} C (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} + \frac{1}{2 d} A (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
& \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) + \frac{1}{2 d} B (1 + \text{Cos}[c + dx]) \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) + \frac{1}{10 d} 3 C (1 + \cos[c + d x]) \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\tan[c]]]^2\right] \sin[d x + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) \right)$$

■ **Problem 1193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2)}{\cos[c + d x]^{3/2}} dx$$

Optimal (type 4, 177 leaves, 8 steps):

$$-\frac{2 a (5 A + 3 (B + C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \frac{2 a (7 A + 7 B + 5 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{2 a C \sin[c + d x]}{7 d \cos[c + d x]^{7/2}} + \frac{2 a (B + C) \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 a (7 A + 7 B + 5 C) \sin[c + d x]}{21 d \cos[c + d x]^{3/2}} + \frac{2 a (5 A + 3 (B + C)) \sin[c + d x]}{5 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1284 leaves):

$$a \sqrt{\cos[c + d x]} (1 + \cos[c + d x]) \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^2$$

$$\begin{aligned}
& \left(\frac{(5A + 3B + 3C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^4 \operatorname{Sin}[dx]}{7d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^3 (5C \operatorname{Sin}[c] + 7B \operatorname{Sin}[dx] + 7C \operatorname{Sin}[dx])}{35d} \right. \\
& \quad \left. + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 (21B \operatorname{Sin}[c] + 21C \operatorname{Sin}[c] + 35A \operatorname{Sin}[dx] + 35B \operatorname{Sin}[dx] + 25C \operatorname{Sin}[dx])}{105d} + \frac{1}{105d} \right. \\
& \quad \left. \operatorname{Sec}[c] \operatorname{Sec}[c + dx] (35A \operatorname{Sin}[c] + 35B \operatorname{Sin}[c] + 25C \operatorname{Sin}[c] + 105A \operatorname{Sin}[dx] + 63B \operatorname{Sin}[dx] + 63C \operatorname{Sin}[dx]) \right) - \frac{1}{3d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& A (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{3d \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& B (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \quad \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \quad \frac{1}{21d \sqrt{1 + \operatorname{Cot}[c]^2}} 5C (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \\
& \quad \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + \frac{1}{2d} A (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \\
& \left(\operatorname{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2 \right] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \\
& \left(\frac{\operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \operatorname{Cos}[c]^2 \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\operatorname{Cos}[c]^2 + \operatorname{Sin}[c]^2} \right) + \frac{1}{10d} 3B (1 + \operatorname{Cos}[c + dx]) \operatorname{Csc}[c] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \\
& \quad \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Tan}[c]^2}
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \left. \right) + \frac{1}{10 d} 3 C (1 + \cos[c + d x]) \text{Csc}[c] \text{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^2$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \left. \right)$$

■ **Problem 1194: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{11/2} (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 251 leaves, 10 steps):

$$\begin{aligned}
& \frac{4 a^2 (7 A+8 B+9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \\
& \frac{4 a^2 (50 A+55 B+66 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{231 d} + \frac{4 a^2 (50 A+55 B+66 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{231 d} + \\
& \frac{4 a^2 (7 A+8 B+9 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{45 d} + \frac{2 a^2 (89 A+121 B+99 C) \cos [c+d x]^{5 / 2} \sin [c+d x]}{693 d} + \\
& \frac{2 A \cos [c+d x]^{5 / 2} (a+a \cos [c+d x])^2 \sin [c+d x]}{11 d} + \frac{2 (4 A+11 B) \cos [c+d x]^{5 / 2} (a^2+a^2 \cos [c+d x]) \sin [c+d x]}{99 d}
\end{aligned}$$

Result (type 5, 1364 leaves):

$$\begin{aligned}
& a^2 \left(\sqrt{\cos [c+d x]} (1+\cos [c+d x])^2 \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \right. \\
& \left(-\frac{(7 A+8 B+9 C) \cot [c]}{15 d} + \frac{(941 A+1012 B+1122 C) \cos [d x] \sin [c]}{3696 d} + \frac{(38 A+37 B+36 C) \cos [2 d x] \sin [2 c]}{360 d} + \right. \\
& \frac{(101 A+88 B+44 C) \cos [3 d x] \sin [3 c]}{2464 d} + \frac{(2 A+B) \cos [4 d x] \sin [4 c]}{144 d} + \frac{A \cos [5 d x] \sin [5 c]}{352 d} + \frac{(941 A+1012 B+1122 C) \cos [c] \sin [d x]}{3696 d} + \\
& \left. \frac{(38 A+37 B+36 C) \cos [2 c] \sin [2 d x]}{360 d} + \frac{(101 A+88 B+44 C) \cos [3 c] \sin [3 d x]}{2464 d} + \frac{(2 A+B) \cos [4 c] \sin [4 d x]}{144 d} + \frac{A \cos [5 c] \sin [5 d x]}{352 d} \right) - \\
& \frac{1}{231 d \sqrt{1+\cot [c]^2}} 50 A (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{21 d \sqrt{1+\cot [c]^2}} \\
& 5 B (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \\
& \frac{1}{7 d \sqrt{1+\cot [c]^2}} 2 C (1+\cos [c+d x])^2 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 \\
& \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]}
\end{aligned}$$

$$\sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} - \frac{1}{30 d} 7 A (1 + \cos[c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{15 d} 4 B (1 + \cos[c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) - \frac{1}{10 d} 3 C (1 + \cos[c + d x])^2 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]\right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left(\frac{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}} \right)$$

- **Problem 1195: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{9/2} (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 215 leaves, 9 steps):

$$\frac{4 a^2 (8 A + 9 B + 12 C) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{15 d} + \frac{4 a^2 (5 A + 6 B + 7 C) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^2 (5 A + 6 B + 7 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{21 d} + \frac{2 a^2 (19 A + 27 B + 21 C) \cos[c + d x]^{3/2} \sin[c + d x]}{105 d} +$$

$$\frac{2 A \cos[c + d x]^{3/2} (a + a \cos[c + d x])^2 \sin[c + d x]}{9 d} + \frac{2 (4 A + 9 B) \cos[c + d x]^{3/2} (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{63 d}$$

Result (type 5, 1699 leaves):

$$\frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]}$$

$$\cos[c + d x]^{9/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2) \left(-\frac{2 (8 A + 9 B + 12 C) \cot[c]}{15 d} + \right.$$

$$\frac{(46 A + 51 B + 56 C) \cos[d x] \sin[c]}{84 d} + \frac{(37 A + 36 B + 18 C) \cos[2 d x] \sin[2 c]}{180 d} + \frac{(2 A + B) \cos[3 d x] \sin[3 c]}{28 d} + \frac{A \cos[4 d x] \sin[4 c]}{72 d} +$$

$$\left. \frac{(46 A + 51 B + 56 C) \cos[c] \sin[d x]}{84 d} + \frac{(37 A + 36 B + 18 C) \cos[2 c] \sin[2 d x]}{180 d} + \frac{(2 A + B) \cos[3 c] \sin[3 d x]}{28 d} + \frac{A \cos[4 c] \sin[4 d x]}{72 d} \right) -$$

$$\frac{1}{21 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 10 A \cos[c + d x]^4 \csc[c]$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{7 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}}$$

$$\begin{aligned}
& 4 B \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{3 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2}} \\
& 2 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \\
& \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \\
& \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \left(8 A \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (15 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) - \right.
\end{aligned}$$

$$\left(3 B \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (5 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

$$\left(4 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2 \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (5 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

■ **Problem 1196: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{7/2} (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 179 leaves, 8 steps):

$$\frac{4 a^2 (3 A + 4 B + 5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5 d} +$$

$$\frac{4 a^2 (6 A + 7 B + 14 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21 d} + \frac{2 a^2 (33 A + 49 B + 35 C) \sqrt{\cos[c + dx]} \sin[c + dx]}{105 d} +$$

$$\frac{2 A \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2 \sin[c + dx]}{7 d} + \frac{2 (4 A + 7 B) \sqrt{\cos[c + dx]} (a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{35 d}$$

Result (type 5, 2001 leaves):

$$\frac{1}{10 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])} 3 i A \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\frac{1}{5 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])} 2 i B \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2$$

$$(A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\frac{1}{2 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx])} i C \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2$$

$$(A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) + \\
& \left(\cos [c + d x]^{9/2} \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \right. \\
& \quad \left(-\frac{2 (3 A + 4 B + 5 C) \cot [c]}{5 d} + \frac{(51 A + 56 B + 28 C) \cos [d x] \sin [c]}{84 d} + \frac{(2 A + B) \cos [2 d x] \sin [2 c]}{10 d} + \right. \\
& \quad \left. \frac{A \cos [3 d x] \sin [3 c]}{28 d} + \frac{(51 A + 56 B + 28 C) \cos [c] \sin [d x]}{84 d} + \frac{(2 A + B) \cos [2 c] \sin [2 d x]}{10 d} + \frac{A \cos [3 c] \sin [3 d x]}{28 d} \right) \Big) / \\
& \frac{1}{(A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) - \frac{7 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}}{1}} \\
& 4 A \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \\
& \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} - \frac{1}{1}} \\
& \frac{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}}{1} \\
& 2 B \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \\
& \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} - \frac{1}{1}} \\
& \frac{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}}{1} \\
& 4 C \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2
\end{aligned}$$

$$\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]}$$

■ **Problem 1197: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{5/2} (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 170 leaves, 8 steps):

$$\frac{4a^2(4A + 5B) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \frac{4a^2(A + 2B + 3C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2a^2(7A + 5B - 15C) \sqrt{\cos[c + dx]} \sin[c + dx]}{15d} +$$

$$\frac{2C(a + a \cos[c + dx])^2 \sin[c + dx]}{d \sqrt{\cos[c + dx]}} + \frac{2(A - 5C) \sqrt{\cos[c + dx]} (a^2 + a^2 \cos[c + dx]) \sin[c + dx]}{5d}$$

Result (type 5, 1356 leaves):

$$\left(\cos[c + dx]^{9/2} \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left(-\frac{(8A + 10B - 5C + 8A \cos[2c] + 10B \cos[2c] + 5C \cos[2c]) \csc[c] \sec[c]}{10d} + \frac{(2A + B) \cos[dx] \sin[c]}{3d} + \right.$$

$$\left. \frac{A \cos[2dx] \sin[2c]}{10d} + \frac{(2A + B) \cos[c] \sin[dx]}{3d} + \frac{C \sec[c] \sec[c + dx] \sin[dx]}{d} + \frac{A \cos[2c] \sin[2dx]}{10d} \right) \Bigg) /$$

$$(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) - \frac{1}{3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2}}$$

$$2A \cos[c + dx]^4 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2}}$$

$$4B \cos[c + dx]^4 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\begin{aligned}
& \frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} -}{1} \\
& d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} \\
& 2 C \cos[c + d x]^4 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} -}{1} \\
& \left(4 A \cos[c + d x]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right. \\
& \left. \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right\} \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \right. \\
& \left. \left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right) - \\
& \left. \left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]} \sqrt{1 + \text{Tan}[c]^2}}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) - \right. \\
& \left. B \cos[c + d x]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \text{Sec}[c + d x])^2 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (d (A + 2C + 2B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]))$$

- **Problem 1198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{3/2} (a + a \text{Sec}[c + dx])^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 170 leaves, 8 steps):

$$\frac{4a^2(A-C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{d} + \frac{4a^2(2A+3B+2C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} + \\ \frac{2a^2(A-3B-5C) \sqrt{\text{Cos}[c+dx]} \text{Sin}[c+dx]}{3d} + \frac{2C(a+a \text{Cos}[c+dx])^2 \text{Sin}[c+dx]}{3d \text{Cos}[c+dx]^{3/2}} + \frac{2(3B+4C)(a^2+a^2 \text{Cos}[c+dx]) \text{Sin}[c+dx]}{3d \sqrt{\text{Cos}[c+dx]}}$$

Result (type 5, 1583 leaves):

$$\frac{1}{2(A+2C+2B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx])} i A \text{Cos}[c+dx]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c+dx])^2 (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2) \\ \left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \text{Cos}[c] + 2i(-1+e^{2idx}) \text{Sin}[c])} \right. \right. \\ \left. \left. \sqrt{1+e^{2idx} \text{Cos}[2c] + i e^{2idx} \text{Sin}[2c]} \right) / (3id(1+e^{2idx}) \text{Cos}[c] - 3d(-1+e^{2idx}) \text{Sin}[c]) - \right. \\ \left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx}) \text{Cos}[c] + 2i(-1+e^{2idx}) \text{Sin}[c])} \right. \right. \\ \left. \left. \sqrt{1+e^{2idx} \text{Cos}[2c] + i e^{2idx} \text{Sin}[2c]} \right) / (-id(1+e^{2idx}) \text{Cos}[c] + d(-1+e^{2idx}) \text{Sin}[c]) \right) - \\ \frac{1}{2(A+2C+2B \text{Cos}[c+dx] + A \text{Cos}[2c+2dx])} i C \text{Cos}[c+dx]^4 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c+dx])^2 \\ (A + B \text{Sec}[c+dx] + C \text{Sec}[c+dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) + \\
& \quad \left(\cos [c + d x]^{9/2} \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \left(-\frac{(2 A - B - 4 C + 2 A \cos [2 c] + B \cos [2 c]) \csc [c] \sec [c]}{2 d} + \right. \right. \\
& \quad \left. \left. \frac{A \cos [d x] \sin [c]}{3 d} + \frac{A \cos [c] \sin [d x]}{3 d} + \frac{C \sec [c] \sec [c + d x]^2 \sin [d x]}{3 d} + \frac{\sec [c] \sec [c + d x] (C \sin [c] + 3 B \sin [d x] + 6 C \sin [d x])}{3 d} \right) \right) / \\
& \quad (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) - \frac{1}{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& \quad 4 A \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \\
& \quad \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \quad \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} -} \\
& \quad \frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& \quad 2 B \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \\
& \quad \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \\
& \quad \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} -} \\
& \quad \frac{1}{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& \quad 4 C \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \\
& \quad \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]]
\end{aligned}$$

$$\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}$$

■ **Problem 1199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 174 leaves, 8 steps):

$$-\frac{4 a^2 (5 B + 4 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (3 A + 2 B + C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} +$$

$$\frac{2 a^2 (15 A + 25 B + 17 C) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \frac{2 C (a + a \cos[c + d x])^2 \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 (5 B + 4 C) (a^2 + a^2 \cos[c + d x]) \sin[c + d x]}{15 d \cos[c + d x]^{3/2}}$$

Result (type 5, 1599 leaves):

$$-\frac{1}{2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} i B \cos[c + d x]^4 \text{Csc}[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\frac{1}{5 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} 2 i C \cos[c + d x]^4 \text{Csc}[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) +$$

$$\left(\cos[c + d x]^{9/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \sec[c + d x])^2 (A + B \sec[c + d x] + C \sec[c + d x]^2) \left(-\frac{(-5 A - 20 B - 16 C + 5 A \cos[2 c]) \text{Csc}[c] \sec[c]}{10 d} + \right.$$

$$\frac{\frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^3 \operatorname{Sin}[dx]}{5d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx]^2 (3C \operatorname{Sin}[c] + 5B \operatorname{Sin}[dx] + 10C \operatorname{Sin}[dx])}{15d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+dx] (5B \operatorname{Sin}[c] + 10C \operatorname{Sin}[c] + 15A \operatorname{Sin}[dx] + 30B \operatorname{Sin}[dx] + 24C \operatorname{Sin}[dx])}{15d}}{1} - \frac{1}{d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \sqrt{1+\operatorname{Cot}[c]^2}}$$

$$2A \operatorname{Cos}[c+dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c+dx])^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)$$

$$\frac{\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - 1}{3d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \sqrt{1+\operatorname{Cot}[c]^2}}$$

$$4B \operatorname{Cos}[c+dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c+dx])^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)$$

$$\frac{\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - 1}{3d(A+2C+2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2c+2dx]) \sqrt{1+\operatorname{Cot}[c]^2}}$$

$$2C \operatorname{Cos}[c+dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \operatorname{Sec}[c+dx])^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\frac{\sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - 1}{1}$$

■ **Problem 1200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \operatorname{Sec}[c+dx])^2 (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2)}{\sqrt{\operatorname{Cos}[c+dx]}} dx$$

Optimal (type 4, 215 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{4 a^2 (5 A + 4 B + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^2 (14 A + 7 B + 6 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{2 a^2 (35 A + 49 B + 33 C) \operatorname{Sin}[c + d x]}{105 d \operatorname{Cos}[c + d x]^{3/2}} + \\
 & \frac{4 a^2 (5 A + 4 B + 3 C) \operatorname{Sin}[c + d x]}{5 d \sqrt{\operatorname{Cos}[c + d x]}} + \frac{2 C (a + a \operatorname{Cos}[c + d x])^2 \operatorname{Sin}[c + d x]}{7 d \operatorname{Cos}[c + d x]^{7/2}} + \frac{2 (7 B + 4 C) (a^2 + a^2 \operatorname{Cos}[c + d x]) \operatorname{Sin}[c + d x]}{35 d \operatorname{Cos}[c + d x]^{5/2}}
 \end{aligned}$$

Result (type 5, 2041 leaves):

$$\begin{aligned}
 & - \frac{1}{2 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} i A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
 & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])\right]^2 \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
 & \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\
 & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])\right]^2 \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
 & \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \\
 & \frac{1}{5 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)} 2 i B \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 \\
 & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])\right]^2 \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
 & \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\
 & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])\right]^2 \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
 & \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \\
 & \frac{1}{10 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)} 3 i C \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 \\
 & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])\right]^2 \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
 & \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right.
 \end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]} \operatorname{Cos}[c + d x]^{9/2} \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \operatorname{Sec}[c + d x])^2 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\frac{2 (5 A + 4 B + 3 C) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^4 \operatorname{Sin}[d x]}{7 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 (5 C \operatorname{Sin}[c] + 7 B \operatorname{Sin}[d x] + 14 C \operatorname{Sin}[d x])}{35 d} + \right. \\
& \quad \left. \frac{\operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (21 B \operatorname{Sin}[c] + 42 C \operatorname{Sin}[c] + 35 A \operatorname{Sin}[d x] + 70 B \operatorname{Sin}[d x] + 60 C \operatorname{Sin}[d x])}{105 d} + \frac{1}{105 d} \right. \\
& \quad \left. \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (35 A \operatorname{Sin}[c] + 70 B \operatorname{Sin}[c] + 60 C \operatorname{Sin}[c] + 210 A \operatorname{Sin}[d x] + 168 B \operatorname{Sin}[d x] + 126 C \operatorname{Sin}[d x]) \right) - \\
& \frac{1}{3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} 4 A \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \\
& \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \\
& \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 2 B \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \\
& \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \\
& \frac{1}{7 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2}} \\
& 4 C \operatorname{Cos}[c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2
\end{aligned}$$

$$\text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}$$

■ **Problem 1201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + dx])^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2)}{\text{Cos}[c + dx]^{3/2}} dx$$

Optimal (type 4, 251 leaves, 10 steps):

$$-\frac{4a^2(12A + 9B + 8C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15d} + \frac{4a^2(7A + 6B + 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21d} +$$

$$\frac{2a^2(21A + 27B + 19C) \text{Sin}[c + dx]}{105d \text{Cos}[c + dx]^{5/2}} + \frac{4a^2(7A + 6B + 5C) \text{Sin}[c + dx]}{21d \text{Cos}[c + dx]^{3/2}} + \frac{4a^2(12A + 9B + 8C) \text{Sin}[c + dx]}{15d \sqrt{\text{Cos}[c + dx]}} +$$

$$\frac{2C(a + a \text{Cos}[c + dx])^2 \text{Sin}[c + dx]}{9d \text{Cos}[c + dx]^{9/2}} + \frac{2(9B + 4C)(a^2 + a^2 \text{Cos}[c + dx]) \text{Sin}[c + dx]}{63d \text{Cos}[c + dx]^{7/2}}$$

Result (type 5, 1741 leaves):

$$\frac{1}{A + 2C + 2B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]} \text{Cos}[c + dx]^{9/2} \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2)$$

$$\left(\frac{2(12A + 9B + 8C) \text{Csc}[c] \text{Sec}[c]}{15d} + \frac{C \text{Sec}[c] \text{Sec}[c + dx]^5 \text{Sin}[dx]}{9d} + \frac{\text{Sec}[c] \text{Sec}[c + dx]^4 (7C \text{Sin}[c] + 9B \text{Sin}[dx] + 18C \text{Sin}[dx])}{63d} + \right.$$

$$\frac{1}{105d} 2 \text{Sec}[c] \text{Sec}[c + dx] (35A \text{Sin}[c] + 30B \text{Sin}[c] + 25C \text{Sin}[c] + 84A \text{Sin}[dx] + 63B \text{Sin}[dx] + 56C \text{Sin}[dx]) +$$

$$\frac{\text{Sec}[c] \text{Sec}[c + dx]^3 (45B \text{Sin}[c] + 90C \text{Sin}[c] + 63A \text{Sin}[dx] + 126B \text{Sin}[dx] + 112C \text{Sin}[dx])}{315d} + \frac{1}{315d}$$

$$\left. \text{Sec}[c] \text{Sec}[c + dx]^2 (63A \text{Sin}[c] + 126B \text{Sin}[c] + 112C \text{Sin}[c] + 210A \text{Sin}[dx] + 180B \text{Sin}[dx] + 150C \text{Sin}[dx]) \right) -$$

$$\frac{1}{3d(A + 2C + 2B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2}} 2A \text{Cos}[c + dx]^4 \text{Csc}[c]$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 (a + a \text{Sec}[c + dx])^2$$

$$(A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\begin{aligned}
& \frac{1}{7 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 4 B \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2 \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\
& \frac{1}{21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 10 C \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]\right]^2 \\
& \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \\
& \left(4 A \cos [c + d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 (a + a \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]\right]^2 \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (5 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) + \right.
\end{aligned}$$

$$\left(3 B \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (5 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) +$$

$$\left(8 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^4 (a+a \operatorname{Sec}[c+d x])^2 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (15 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

■ **Problem 1202: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{11 / 2} (a+a \sec [c+d x])^3 (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 267 leaves, 10 steps):

$$\frac{4 a^3 (15 A+17 B+21 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{15 d} + \frac{4 a^3 (105 A+121 B+143 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{231 d} +$$

$$\frac{4 a^3 (105 A+121 B+143 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{231 d} + \frac{4 a^3 (210 A+253 B+264 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{1155 d} +$$

$$\frac{2 A \cos [c+d x]^{3 / 2} (a+a \cos [c+d x])^3 \sin [c+d x]}{11 d} + \frac{2 (6 A+11 B) \cos [c+d x]^{3 / 2} \left(a^2+a^2 \cos [c+d x]\right)^2 \sin [c+d x]}{99 a d} +$$

$$\frac{2 (105 A+143 B+99 C) \cos [c+d x]^{3 / 2} \left(a^3+a^3 \cos [c+d x]\right) \sin [c+d x]}{693 d}$$

Result (type 5, 1364 leaves):

$$a^3 \left(\sqrt{\cos [c+d x]} (1+\cos [c+d x])^3 \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 \left(-\frac{(15 A+17 B+21 C) \cot [c]}{30 d} + \frac{(1953 A+2134 B+2354 C) \cos [d x] \sin [c]}{7392 d} + \right. \right.$$

$$\frac{(75 A+73 B+54 C) \cos [2 d x] \sin [2 c]}{720 d} + \frac{(189 A+132 B+44 C) \cos [3 d x] \sin [3 c]}{4928 d} + \frac{(3 A+B) \cos [4 d x] \sin [4 c]}{288 d} +$$

$$\frac{A \cos [5 d x] \sin [5 c]}{704 d} + \frac{(1953 A+2134 B+2354 C) \cos [c] \sin [d x]}{7392 d} + \frac{(75 A+73 B+54 C) \cos [2 c] \sin [2 d x]}{720 d} +$$

$$\left. \frac{(189 A+132 B+44 C) \cos [3 c] \sin [3 d x]}{4928 d} + \frac{(3 A+B) \cos [4 c] \sin [4 d x]}{288 d} + \frac{A \cos [5 c] \sin [5 d x]}{704 d} \right) - \frac{1}{22 d \sqrt{1+\cot [c]^2}}$$

$$5 A (1+\cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 \sec [d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{42 d \sqrt{1+\cot [c]^2}} 11 B (1+\cos [c+d x])^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \frac{1}{42 d \sqrt{1+\cot [c]^2}}$$

$$13 C (1 + \cos[c + dx])^3 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} - \frac{1}{4d} A (1 + \cos[c + dx])^3 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{60d} 17 B (1 + \cos[c + dx])^3 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{1}{20d} 7 C (1 + \cos[c + dx])^3 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) -$$

- **Problem 1203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{9/2} (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$\frac{4 a^3 (17 A + 21 B + 27 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (11 A + 13 B + 21 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^3 (32 A + 41 B + 42 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 A \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \sin[c + d x]}{9 d} +$$

$$\frac{2 (2 A + 3 B) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{21 a d} + \frac{2 (73 A + 99 B + 63 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{315 d}$$

Result (type 5, 1697 leaves):

$$\frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \left(-\frac{(17 A + 21 B + 27 C) \cot[c]}{15 d} + \right.$$

$$\frac{(97 A + 107 B + 84 C) \cos[d x] \sin[c]}{168 d} + \frac{(73 A + 54 B + 18 C) \cos[2 d x] \sin[2 c]}{360 d} + \frac{(3 A + B) \cos[3 d x] \sin[3 c]}{56 d} + \frac{A \cos[4 d x] \sin[4 c]}{144 d} +$$

$$\left. \frac{(97 A + 107 B + 84 C) \cos[c] \sin[d x]}{168 d} + \frac{(73 A + 54 B + 18 C) \cos[2 c] \sin[2 d x]}{360 d} + \frac{(3 A + B) \cos[3 c] \sin[3 d x]}{56 d} + \frac{A \cos[4 c] \sin[4 d x]}{144 d} \right) -$$

$$\frac{1}{21 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 11 A \cos[c + d x]^5 \csc[c]$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\begin{aligned}
& \frac{1}{21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 13 B \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\
& \frac{1}{d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& C \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \\
& \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\
& \left(17 A \cos [c + d x]^5 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (30 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) - \right.
\end{aligned}$$

$$\left(7 B \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

$$\left(9 C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

■ **Problem 1204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{7 / 2} (a+a \sec [c+d x])^3 (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 227 leaves, 9 steps):

$$\frac{4 a^3 (7 A+9 B+5 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d} + \frac{4 a^3 (13 A+21 B+35 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{21 d} +$$

$$\frac{4 a^3 (41 A+42 B-35 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{105 d} + \frac{2 C (a+a \cos [c+d x])^3 \sin [c+d x]}{d \sqrt{\cos [c+d x]}} +$$

$$\frac{2 (A-7 C) \sqrt{\cos [c+d x]} (a^2+a^2 \cos [c+d x])^2 \sin [c+d x]}{7 a d} + \frac{2 (11 A+7 B-35 C) \sqrt{\cos [c+d x]} (a^3+a^3 \cos [c+d x]) \sin [c+d x]}{35 d}$$

Result (type 5, 1688 leaves):

$$\frac{1}{A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]} \cos [c+d x]^{11 / 2} \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3$$

$$(A+B \sec [c+d x]+C \sec [c+d x]^2) \left(-\frac{(14 A+18 B+5 C+14 A \cos [2 c]+18 B \cos [2 c]+15 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{20 d} + \right.$$

$$\frac{(107 A+84 B+28 C) \cos [d x] \sin [c]}{168 d} + \frac{(3 A+B) \cos [2 d x] \sin [2 c]}{20 d} + \frac{A \cos [3 d x] \sin [3 c]}{56 d} +$$

$$\left. \frac{(107 A+84 B+28 C) \cos [c] \sin [d x]}{168 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+d x] \sin [d x]}{2 d} + \frac{(3 A+B) \cos [2 c] \sin [2 d x]}{20 d} + \frac{A \cos [3 c] \sin [3 d x]}{56 d} \right) -$$

$$\frac{1}{21 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} 13 A \cos [c+d x]^5 \operatorname{Csc}[c]$$

$$\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3$$

$$(A+B \sec [c+d x]+C \sec [c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]}$$

$$\sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -$$

$$\frac{1}{d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}} B \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \sec [c+d x])^3 (A+B \sec [c+d x]+C \sec [c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -$$

$$\begin{aligned}
& \frac{1}{3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 5 C \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\
& \left(7 A \cos [c + d x]^5 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (10 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) - \\
& \left(9 B \cos [c + d x]^5 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(C \cos[c + d x]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right.$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right.$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

- **Problem 1205: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{5/2} (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 226 leaves, 9 steps):

$$\frac{4 a^3 (9 A + 5 B - 5 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (3 A + 5 (B + C)) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} +$$

$$\frac{4 a^3 (6 A - 5 B - 20 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} + \frac{2 C (a + a \cos[c + d x])^3 \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} +$$

$$\frac{2 (B + 2 C) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{a d \sqrt{\cos[c + d x]}} + \frac{2 (3 A - 15 B - 35 C) \sqrt{\cos[c + d x]} (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d}$$

Result (type 5, 1672 leaves):

$$\left(\cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 \right.$$

$$\left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \left(-\frac{(18 A + 5 B - 25 C + 18 A \cos[2 c] + 15 B \cos[2 c] + 5 C \cos[2 c]) \csc[c] \sec[c]}{20 d} + \right. \right.$$

$$\left. \frac{(3 A + B) \cos[d x] \sin[c]}{6 d} + \frac{A \cos[2 d x] \sin[2 c]}{20 d} + \frac{(3 A + B) \cos[c] \sin[d x]}{6 d} + \frac{C \sec[c] \sec[c + d x]^2 \sin[d x]}{6 d} + \right.$$

$$\left. \frac{\sec[c] \sec[c + d x] (C \sin[c] + 3 B \sin[d x] + 9 C \sin[d x])}{6 d} + \frac{A \cos[2 c] \sin[2 d x]}{20 d} \right) \Bigg/$$

$$\frac{1}{d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot^2[c]}}$$

$$A \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot^2[c]} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot^2[c]}}$$

$$5 B \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right]$$

$$\sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]]$$

$$\frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot^2[c]} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot^2[c]}}$$

$$5 C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]$$

$$\operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} -$$

$$\left(9 A \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right.$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

$$\left(B \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right.$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\left(2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \right) +$$

$$\left(C \cos[c + d x]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\left(2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \right)}$$

■ **Problem 1206: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$\frac{4 a^3 (5 A - 5 B - 9 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (5 A + 5 B + 3 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{3 d} - \frac{4 a^3 (5 A + 20 B + 21 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{15 d} +$$

$$\frac{2 C (a + a \cos[c + d x])^3 \sin[c + d x]}{5 d \cos[c + d x]^{5/2}} + \frac{2 (5 B + 6 C) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{15 a d \cos[c + d x]^{3/2}} + \frac{2 (15 A + 35 B + 33 C) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}}$$

Result (type 5, 1673 leaves):

$$\frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \left(-\frac{(5 A - 25 B - 36 C + 15 A \cos[2 c] + 5 B \cos[2 c]) \csc[c] \sec[c]}{20 d} + \frac{A \cos[d x] \sin[c]}{6 d} + \right)$$

$$\begin{aligned}
& \frac{A \cos[c] \sin[dx]}{6d} + \frac{C \sec[c] \sec[c+dx]^3 \sin[dx]}{10d} + \frac{\sec[c] \sec[c+dx]^2 (3C \sin[c] + 5B \sin[dx] + 15C \sin[dx])}{30d} + \\
& \left. \frac{\sec[c] \sec[c+dx] (5B \sin[c] + 15C \sin[c] + 15A \sin[dx] + 45B \sin[dx] + 54C \sin[dx])}{30d} \right) - \\
& \frac{1}{3d (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2}} 5A \cos[c+dx]^5 \csc[c] \\
& \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 \\
& (A + B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{3d (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2}} \\
& 5B \cos[c+dx]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{d (A + 2C + 2B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2}} \\
& C \cos[c+dx]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \\
& (a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} -} \\
& \left(A \cos[c+dx]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (a + a \sec[c+dx])^3 (A + B \sec[c+dx] + C \sec[c+dx]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \left) / (2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(B \cos[c + d x]^5 \csc[c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \left) / (2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(9 C \cos[c + d x]^5 \csc[c] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / (10 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]))$$

- **Problem 1207: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\text{Cos}[c + d x]} (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 231 leaves, 9 steps):

$$-\frac{4 a^3 (5 A + 9 B + 7 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{4 a^3 (35 A + 21 B + 13 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^3 (140 A + 147 B + 106 C) \text{Sin}[c + d x]}{105 d \sqrt{\text{Cos}[c + d x]}} + \frac{2 C (a + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{7 d \text{Cos}[c + d x]^{7/2}} + \frac{2 (7 B + 6 C) (a^2 + a^2 \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{35 a d \text{Cos}[c + d x]^{5/2}} + \frac{2 (5 A + 9 B + 7 C) (a^3 + a^3 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{15 d \text{Cos}[c + d x]^{3/2}}$$

Result (type 5, 1692 leaves):

$$\frac{1}{A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]} \text{Cos}[c + d x]^{11/2} \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \left(-\frac{(-25 A - 36 B - 28 C + 5 A \text{Cos}[2 c]) \text{Csc}[c] \text{Sec}[c]}{20 d} + \frac{C \text{Sec}[c] \text{Sec}[c + d x]^4 \text{Sin}[d x]}{14 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^3 (5 C \text{Sin}[c] + 7 B \text{Sin}[d x] + 21 C \text{Sin}[d x])}{70 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^2 (21 B \text{Sin}[c] + 63 C \text{Sin}[c] + 35 A \text{Sin}[d x] + 105 B \text{Sin}[d x] + 130 C \text{Sin}[d x])}{210 d} + \frac{1}{210 d} \text{Sec}[c] \text{Sec}[c + d x] (35 A \text{Sin}[c] + 105 B \text{Sin}[c] + 130 C \text{Sin}[c] + 315 A \text{Sin}[d x] + 378 B \text{Sin}[d x] + 294 C \text{Sin}[d x]) \right) - \frac{1}{3 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 5 A \text{Cos}[c + d x]^5 \text{Csc}[c]$$

$$\begin{aligned}
& \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 \\
& \frac{(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} - \\
& \frac{1}{d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} \\
& B \text{Cos}[c + d x]^5 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \\
& (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} - \\
& \frac{1}{21 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} \\
& 13 C \text{Cos}[c + d x]^5 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} + \\
& \left(A \text{Cos}[c + d x]^5 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right. \\
& \left. \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \right. \\
& \left. \left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \right.
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (2 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(9 B \cos[c + d x]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(7 C \cos[c + d x]^5 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\left(10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])\right)}$$

- **Problem 1208: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2)}{\sqrt{\cos[c + d x]}} dx$$

Optimal (type 4, 267 leaves, 10 steps):

$$\begin{aligned} & - \frac{4 a^3 (27 A + 21 B + 17 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{4 a^3 (21 A + 13 B + 11 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\ & \frac{4 a^3 (42 A + 41 B + 32 C) \sin[c + d x]}{105 d \cos[c + d x]^{3/2}} + \frac{4 a^3 (27 A + 21 B + 17 C) \sin[c + d x]}{15 d \sqrt{\cos[c + d x]}} + \frac{2 C (a + a \cos[c + d x])^3 \sin[c + d x]}{9 d \cos[c + d x]^{9/2}} + \\ & \frac{2 (3 B + 2 C) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{21 a d \cos[c + d x]^{7/2}} + \frac{2 (63 A + 99 B + 73 C) (a^3 + a^3 \cos[c + d x]) \sin[c + d x]}{315 d \cos[c + d x]^{5/2}} \end{aligned}$$

Result (type 5, 1739 leaves):

$$\begin{aligned} & \frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{11/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \\ & \left(\frac{(27 A + 21 B + 17 C) \csc[c] \sec[c]}{15 d} + \frac{C \sec[c] \sec[c + d x]^5 \sin[d x]}{18 d} + \frac{\sec[c] \sec[c + d x]^4 (7 C \sin[c] + 9 B \sin[d x] + 27 C \sin[d x])}{126 d} \right. \\ & \left. \frac{\sec[c] \sec[c + d x]^3 (45 B \sin[c] + 135 C \sin[c] + 63 A \sin[d x] + 189 B \sin[d x] + 238 C \sin[d x])}{630 d} + \frac{1}{210 d} \right. \\ & \left. \sec[c] \sec[c + d x] (105 A \sin[c] + 130 B \sin[c] + 110 C \sin[c] + 378 A \sin[d x] + 294 B \sin[d x] + 238 C \sin[d x]) + \frac{1}{630 d} \right. \\ & \left. \sec[c] \sec[c + d x]^2 (63 A \sin[c] + 189 B \sin[c] + 238 C \sin[c] + 315 A \sin[d x] + 390 B \sin[d x] + 330 C \sin[d x]) \right) - \\ & \frac{1}{d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} A \cos[c + d x]^5 \csc[c] \\ & \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec[c + d x])^3 \\ & (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \\ & \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -} \end{aligned}$$

$$\begin{aligned}
& \frac{1}{21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 13 B \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} - \\
& \frac{1}{21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2}} \\
& 11 C \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \\
& \left(9 A \cos [c + d x]^5 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^6 (a + a \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / \right) (10 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) +
\end{aligned}$$

$$\left(7 B \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) +$$

$$\left(17 C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^6 (a+a \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (30 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

■ **Problem 1209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{13 / 2} (a+a \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 310 leaves, 11 steps):

$$\begin{aligned} & \frac{8 a^4 (185 A+208 B+247 C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{195 d} + \\ & \frac{8 a^4 (100 A+113 B+132 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{231 d} + \frac{8 a^4 (100 A+113 B+132 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{231 d} + \\ & \frac{4 a^4 (5255 A+6019 B+6721 C) \cos [c+d x]^{3 / 2} \sin [c+d x]}{15015 d} + \frac{2 a (8 A+13 B) \cos [c+d x]^{3 / 2} (a+a \cos [c+d x])^3 \sin [c+d x]}{143 d} + \\ & \frac{2 A \cos [c+d x]^{3 / 2} (a+a \cos [c+d x])^4 \sin [c+d x]}{13 d} + \frac{2 (13 A+17 B+11 C) \cos [c+d x]^{3 / 2} \left(a^2+a^2 \cos [c+d x]\right)^2 \sin [c+d x]}{99 d} + \\ & \frac{4 (1355 A+1612 B+1573 C) \cos [c+d x]^{3 / 2} \left(a^4+a^4 \cos [c+d x]\right) \sin [c+d x]}{9009 d} \end{aligned}$$

Result (type 5, 1416 leaves):

$$\begin{aligned} & a^4 \left(\sqrt{\cos [c+d x]} (1+\cos [c+d x])^4 \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \right. \\ & \left(- \frac{(185 A+208 B+247 C) \cot [c]}{390 d} + \frac{(3764 A+4087 B+4488 C) \cos [d x] \sin [c]}{14784 d} + \frac{(15625 A+15392 B+13208 C) \cos [2 d x] \sin [2 c]}{149760 d} \right. \\ & \frac{(404 A+321 B+176 C) \cos [3 d x] \sin [3 c]}{9856 d} + \frac{(98 A+52 B+13 C) \cos [4 d x] \sin [4 c]}{7488 d} + \frac{(4 A+B) \cos [5 d x] \sin [5 c]}{1408 d} \\ & \frac{A \cos [6 d x] \sin [6 c]}{3328 d} + \frac{(3764 A+4087 B+4488 C) \cos [c] \sin [d x]}{14784 d} + \frac{(15625 A+15392 B+13208 C) \cos [2 c] \sin [2 d x]}{149760 d} \\ & \frac{(404 A+321 B+176 C) \cos [3 c] \sin [3 d x]}{9856 d} + \frac{(98 A+52 B+13 C) \cos [4 c] \sin [4 d x]}{7488 d} \\ & \left. \left. \frac{(4 A+B) \cos [5 c] \sin [5 d x]}{1408 d} + \frac{A \cos [6 c] \sin [6 d x]}{3328 d} \right) - \frac{1}{231 d \sqrt{1+\cot [c]^2}} \right. \\ & 50 A (1+\cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^8 \sec [d x-\operatorname{ArcTan}[\cot [c]]] \\ & \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - \\ & \frac{1}{462 d \sqrt{1+\cot [c]^2}} 113 B (1+\cos [c+d x])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] \end{aligned}$$

$$\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{7d\sqrt{1 + \operatorname{Cot}[c]^2}}}$$

$$2C(1 + \cos[c + dx])^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8$$

$$\operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}$$

$$\sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - \frac{1}{156d} 37A(1 + \cos[c + dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} - \frac{1}{15d} 4B(1 + \cos[c + dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8$$

$$\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /$$

$$\left(\sqrt{1 - \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \sqrt{1 + \operatorname{Tan}[c]^2} \right) -$$

$$\frac{\frac{\sin[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1 + \operatorname{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}}} - \frac{1}{60d} 19C(1 + \cos[c + dx])^4 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) \right)$$

- **Problem 1210: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + dx]^{11/2} (a + a \text{Sec}[c + dx])^4 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) dx$$

Optimal (type 4, 274 leaves, 10 steps):

$$\frac{8 a^4 (16 A + 19 B + 24 C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{15 d} + \frac{8 a^4 (113 A + 132 B + 187 C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{231 d} + \\ \frac{4 a^4 (667 A + 803 B + 913 C) \sqrt{\text{Cos}[c + dx]} \text{Sin}[c + dx]}{1155 d} + \frac{2 a (8 A + 11 B) \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^3 \text{Sin}[c + dx]}{99 d} + \\ \frac{2 A \sqrt{\text{Cos}[c + dx]} (a + a \text{Cos}[c + dx])^4 \text{Sin}[c + dx]}{11 d} + \frac{2 (43 A + 55 B + 33 C) \sqrt{\text{Cos}[c + dx]} (a^2 + a^2 \text{Cos}[c + dx])^2 \text{Sin}[c + dx]}{231 d} + \\ \frac{4 (769 A + 946 B + 891 C) \sqrt{\text{Cos}[c + dx]} (a^4 + a^4 \text{Cos}[c + dx]) \text{Sin}[c + dx]}{3465 d}$$

Result (type 5, 1751 leaves):

$$\frac{1}{A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]} \text{Cos}[c + dx]^{13/2} \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \text{Sec}[c + dx])^4 \\ (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \left(-\frac{(16 A + 19 B + 24 C) \text{Cot}[c]}{15 d} + \frac{(4087 A + 4488 B + 4202 C) \text{Cos}[dx] \text{Sin}[c]}{7392 d} + \right. \\ \left. \frac{(148 A + 127 B + 72 C) \text{Cos}[2dx] \text{Sin}[2c]}{720 d} + \frac{(321 A + 176 B + 44 C) \text{Cos}[3dx] \text{Sin}[3c]}{4928 d} + \frac{(4 A + B) \text{Cos}[4dx] \text{Sin}[4c]}{288 d} + \right. \\ \left. \frac{A \text{Cos}[5dx] \text{Sin}[5c]}{704 d} + \frac{(4087 A + 4488 B + 4202 C) \text{Cos}[c] \text{Sin}[dx]}{7392 d} + \frac{(148 A + 127 B + 72 C) \text{Cos}[2c] \text{Sin}[2dx]}{720 d} + \right)$$

$$\begin{aligned}
& \left. \frac{(321 A + 176 B + 44 C) \cos[3 c] \sin[3 d x]}{4928 d} + \frac{(4 A + B) \cos[4 c] \sin[4 d x]}{288 d} + \frac{A \cos[5 c] \sin[5 d x]}{704 d} \right) - \\
& \frac{1}{231 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 113 A \cos[c + d x]^6 \csc[c] \\
& \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 \\
& (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{7 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} \\
& 4 B \cos[c + d x]^6 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \frac{1}{21 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} \\
& 17 C \cos[c + d x]^6 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} -} \\
& \left(8 A \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[d x + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) \right) /
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (15 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(19 B \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (30 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(4 C \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} - \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]))$$

- **Problem 1211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{9/2} (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\frac{8 a^4 (19 A + 24 B + 21 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{8 a^4 (12 A + 17 B + 28 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \frac{4 a^4 (73 A + 83 B + 7 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 a (A - 9 C) \sqrt{\cos[c + d x]} (a + a \cos[c + d x])^3 \sin[c + d x]}{9 d} + \frac{2 C (a + a \cos[c + d x])^4 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2 (5 A + 3 B - 21 C) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{21 d} + \frac{4 (86 A + 81 B - 126 C) \sqrt{\cos[c + d x]} (a^4 + a^4 \cos[c + d x]) \sin[c + d x]}{315 d}$$

Result (type 5, 1742 leaves):

$$\frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{13/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) - \left(\frac{(76 A + 96 B + 69 C + 76 A \cos[2 c] + 96 B \cos[2 c] + 99 C \cos[2 c]) \text{Csc}[c] \sec[c]}{120 d} + \frac{(204 A + 191 B + 112 C) \cos[d x] \sin[c]}{336 d} + \frac{(127 A + 72 B + 18 C) \cos[2 d x] \sin[2 c]}{720 d} + \frac{(4 A + B) \cos[3 d x] \sin[3 c]}{112 d} + \frac{A \cos[4 d x] \sin[4 c]}{288 d} + \frac{(204 A + 191 B + 112 C) \cos[c] \sin[d x]}{336 d} + \frac{C \sec[c] \sec[c + d x] \sin[d x]}{4 d} + \frac{(127 A + 72 B + 18 C) \cos[2 c] \sin[2 d x]}{720 d} + \frac{(4 A + B) \cos[3 c] \sin[3 d x]}{112 d} + \frac{A \cos[4 c] \sin[4 d x]}{288 d} \right) - \frac{1}{7 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 4 A \cos[c + d x]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\begin{aligned}
& \frac{\sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} - 1}{21 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) \sqrt{1+\cot^2[c]}} \\
& 17 B \cos[c + dx]^6 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \\
& \frac{\sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} - 1}{1} \\
& \frac{3 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2 dx]) \sqrt{1+\cot^2[c]}}{4 C \cos[c + dx]^6 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right]} \\
& \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \\
& \frac{\sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} - 1}{19 A \cos[c + dx]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2)} \\
& \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) \right) -
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (30 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(4 B \cos[c + d x]^6 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(7 C \cos[c + d x]^6 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\left(10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])\right)}$$

- **Problem 1212: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{7/2} (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 269 leaves, 10 steps):

$$\frac{8 a^4 (8 A + 7 B) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{8 a^4 (17 A + 28 B + 35 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} +$$

$$\frac{4 a^4 (83 A + 7 B - 175 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 a (3 B + 8 C) (a + a \cos[c + d x])^3 \sin[c + d x]}{3 d \sqrt{\cos[c + d x]}} + \frac{2 C (a + a \cos[c + d x])^4 \sin[c + d x]}{3 d \cos[c + d x]^{3/2}} +$$

$$\frac{2 (A - 7 B - 21 C) \sqrt{\cos[c + d x]} (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{7 d} + \frac{4 (27 A - 42 B - 175 C) \sqrt{\cos[c + d x]} (a^4 + a^4 \cos[c + d x]) \sin[c + d x]}{105 d}$$

Result (type 5, 1451 leaves):

$$\frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{13/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(- \frac{(32 A + 23 B - 20 C + 32 A \cos[2 c] + 33 B \cos[2 c] + 20 C \cos[2 c]) \csc[c] \sec[c]}{40 d} + \frac{(191 A + 112 B + 28 C) \cos[d x] \sin[c]}{336 d} + \right.$$

$$\frac{(4 A + B) \cos[2 d x] \sin[2 c]}{40 d} + \frac{A \cos[3 d x] \sin[3 c]}{112 d} + \frac{(191 A + 112 B + 28 C) \cos[c] \sin[d x]}{336 d} + \frac{C \sec[c] \sec[c + d x]^2 \sin[d x]}{12 d} +$$

$$\left. \frac{\sec[c] \sec[c + d x] (C \sin[c] + 3 B \sin[d x] + 12 C \sin[d x])}{12 d} + \frac{(4 A + B) \cos[2 c] \sin[2 d x]}{40 d} + \frac{A \cos[3 c] \sin[3 d x]}{112 d} \right) -$$

$$\frac{1}{21 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 17 A \cos[c + d x]^6 \csc[c]$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]}$$

$$\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} -$$

$$\frac{1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}}$$

$$4 B \cos [c+d x]^6 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\frac{\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} - 1}{1}$$

$$3 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2}$$

$$5 C \cos [c+d x]^6 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right]$$

$$\sec \left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]]$$

$$\sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} -$$

$$\left(4 A \cos [c+d x]^6 \csc [c] \sec \left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right)$$

$$\left(\left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right)$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) -$$

$$\left(\frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (5 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

$$\left(7 B \cos [c+d x]^6 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \operatorname{Sec}[c+d x])^4 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / (10 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

- **Problem 1213: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^4 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 267 leaves, 10 steps):

$$\frac{56 a^4 (A-C) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d} + \frac{8 a^4 (4 A+5 B+4 C) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} +$$

$$\frac{4 a^4 (A-25 B-41 C) \sqrt{\cos [c+d x]} \sin [c+d x]}{15 d} + \frac{2 a (5 B+8 C) (a+a \cos [c+d x])^3 \sin [c+d x]}{15 d \cos [c+d x]^{3/2}} + \frac{2 C (a+a \cos [c+d x])^4 \sin [c+d x]}{5 d \cos [c+d x]^{5/2}} +$$

$$\frac{2 (5 A+15 B+19 C) (a^2+a^2 \cos [c+d x])^2 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}} - \frac{4 (6 A+25 B+34 C) \sqrt{\cos [c+d x]} (a^4+a^4 \cos [c+d x]) \sin [c+d x]}{15 d}$$

Result (type 5, 1449 leaves):

$$\frac{1}{A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]} \cos [c+d x]^{13/2} \operatorname{Sec}\left[\frac{c}{2}+\frac{d x}{2}\right]^8 (a+a \operatorname{Sec}[c+d x])^4 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)$$

$$\left(-\frac{(23 A-20 B-61 C+33 A \cos [2 c]+20 B \cos [2 c]+5 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{40 d} + \frac{(4 A+B) \cos [d x] \sin [c]}{12 d} + \frac{A \cos [2 d x] \sin [2 c]}{40 d} + \right.$$

$$\left. \frac{(4 A+B) \cos [c] \sin [d x]}{12 d} + \frac{C \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^3 \sin [d x]}{20 d} + \frac{\operatorname{Sec}[c] \operatorname{Sec}[c+d x]^2 (3 C \sin [c]+5 B \sin [d x]+20 C \sin [d x])}{60 d} + \right.$$

$$\left. \frac{\text{Sec}[c] \text{Sec}[c + dx] (5 B \text{Sin}[c] + 20 C \text{Sin}[c] + 15 A \text{Sin}[dx] + 60 B \text{Sin}[dx] + 99 C \text{Sin}[dx])}{60 d} + \frac{A \text{Cos}[2c] \text{Sin}[2dx]}{40 d} \right) -$$

$$\frac{1}{3 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2}} 4 A \text{Cos}[c + dx]^6 \text{Csc}[c]$$

$$\text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \text{Sec}[c + dx])^4$$

$$(A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]}$$

$$\sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{3 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2}}$$

$$5 B \text{Cos}[c + dx]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right]$$

$$\text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \text{Sec}[c + dx])^4 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\frac{1}{3 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2}}$$

$$4 C \text{Cos}[c + dx]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right]$$

$$\text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \text{Sec}[c + dx])^4 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]]$$

$$\sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} -$$

$$\left(7 A \text{Cos}[c + dx]^6 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \text{Sec}[c + dx])^4 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \text{Sin}[dx + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(7 C \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]))$$

- **Problem 1214: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 271 leaves, 10 steps):

$$\begin{aligned}
& - \frac{8 a^4 (7 B + 8 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{5 d} + \frac{8 a^4 (35 A + 28 B + 17 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} - \\
& \frac{4 a^4 (175 A + 287 B + 253 C) \sqrt{\cos[c + d x]} \sin[c + d x]}{105 d} + \frac{2 a (7 B + 8 C) (a + a \cos[c + d x])^3 \sin[c + d x]}{35 d \cos[c + d x]^{5/2}} + \frac{2 C (a + a \cos[c + d x])^4 \sin[c + d x]}{7 d \cos[c + d x]^{7/2}} + \\
& \frac{2 (35 A + 77 B + 73 C) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{105 d \cos[c + d x]^{3/2}} + \frac{4 (175 A + 238 B + 197 C) (a^4 + a^4 \cos[c + d x]) \sin[c + d x]}{105 d \sqrt{\cos[c + d x]}}
\end{aligned}$$

Result (type 5, 1454 leaves):

$$\begin{aligned}
& \frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \cos[c + d x]^{13/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 \\
& \left((A + B \sec[c + d x] + C \sec[c + d x])^2 \left(- \frac{(-20 A - 61 B - 64 C + 20 A \cos[2 c] + 5 B \cos[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{40 d} + \frac{A \cos[d x] \sin[c]}{12 d} + \right. \right. \\
& \frac{A \cos[c] \sin[d x]}{12 d} + \frac{C \sec[c] \sec[c + d x]^4 \sin[d x]}{28 d} + \frac{\sec[c] \sec[c + d x]^3 (5 C \sin[c] + 7 B \sin[d x] + 28 C \sin[d x])}{140 d} + \\
& \left. \frac{\sec[c] \sec[c + d x]^2 (21 B \sin[c] + 84 C \sin[c] + 35 A \sin[d x] + 140 B \sin[d x] + 235 C \sin[d x])}{420 d} + \frac{1}{420 d} \right) - \\
& \left. \sec[c] \sec[c + d x] (35 A \sin[c] + 140 B \sin[c] + 235 C \sin[c] + 420 A \sin[d x] + 693 B \sin[d x] + 672 C \sin[d x]) \right) - \\
& \frac{1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 5 A \cos[c + d x]^6 \operatorname{Csc}[c] \\
& \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 \\
& (A + B \sec[c + d x] + C \sec[c + d x])^2 \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \\
& \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} - \\
& \frac{1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} \\
& 4 B \cos[c + d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] \\
& \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x])^2 \sec[d x - \operatorname{ArcTan}[\cot[c]]] \\
& \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\cot[c]]]} -
\end{aligned}$$

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$$\begin{aligned}
& 21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \\
& 17 C \cos [c + d x]^6 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] \\
& \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec [c + d x])^4 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \\
& \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} + \\
& \left(7 B \cos [c + d x]^6 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec [c + d x])^4 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \operatorname{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \operatorname{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (10 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) + \\
& \left(4 C \cos [c + d x]^6 \csc [c] \sec \left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec [c + d x])^4 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [d x + \operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x + \operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right.
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]))$$

■ **Problem 1215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 274 leaves, 10 steps):

$$\begin{aligned} & - \frac{8 a^4 (21 A + 24 B + 19 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{8 a^4 (28 A + 17 B + 12 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} \\ & + \frac{4 a^4 (287 A + 253 B + 193 C) \sin[c + d x]}{105 d \sqrt{\cos[c + d x]}} + \frac{2 a (9 B + 8 C) (a + a \cos[c + d x])^3 \sin[c + d x]}{63 d \cos[c + d x]^{7/2}} + \frac{2 C (a + a \cos[c + d x])^4 \sin[c + d x]}{9 d \cos[c + d x]^{9/2}} \\ & + \frac{2 (63 A + 117 B + 97 C) (a^2 + a^2 \cos[c + d x])^2 \sin[c + d x]}{315 d \cos[c + d x]^{5/2}} + \frac{4 (21 A + 24 B + 19 C) (a^4 + a^4 \cos[c + d x]) \sin[c + d x]}{45 d \cos[c + d x]^{3/2}} \end{aligned}$$

Result (type 5, 1748 leaves):

$$\begin{aligned} & \frac{1}{A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]} \\ & \cos[c + d x]^{13/2} \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \left(- \frac{(-183 A - 192 B - 152 C + 15 A \cos[2 c]) \csc[c] \sec[c]}{120 d} + \right. \\ & \frac{C \sec[c] \sec[c + d x]^5 \sin[d x]}{36 d} + \frac{\sec[c] \sec[c + d x]^4 (7 C \sin[c] + 9 B \sin[d x] + 36 C \sin[d x])}{252 d} + \\ & \frac{\sec[c] \sec[c + d x]^3 (45 B \sin[c] + 180 C \sin[c] + 63 A \sin[d x] + 252 B \sin[d x] + 427 C \sin[d x])}{1260 d} + \frac{1}{420 d} \\ & \left. \sec[c] \sec[c + d x] (140 A \sin[c] + 235 B \sin[c] + 240 C \sin[c] + 693 A \sin[d x] + 672 B \sin[d x] + 532 C \sin[d x]) + \frac{1}{1260 d} \right. \\ & \left. \sec[c] \sec[c + d x]^2 (63 A \sin[c] + 252 B \sin[c] + 427 C \sin[c] + 420 A \sin[d x] + 705 B \sin[d x] + 720 C \sin[d x]) \right) - \\ & \frac{1}{3 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2}} 4 A \cos[c + d x]^6 \csc[c] \end{aligned}$$

$$\begin{aligned}
& \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 \\
& \frac{(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} - \\
& \frac{1}{21 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} \\
& 17 B \cos[c + d x]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} - \\
& \frac{1}{7 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} \\
& 4 C \cos[c + d x]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \\
& \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \\
& \frac{\sqrt{1 - \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[d x - \text{ArcTan}[\text{Cot}[c]]}} + \\
& \left(7 A \cos[c + d x]^6 \text{Csc}[c] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right. \\
& \left. \left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \right. \\
& \left. \left. \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) - \right. \right.
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (10 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(4 B \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(19 C \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\left(30 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])\right)}$$

- **Problem 1216: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\sqrt{\text{Cos}[c + d x]}} dx$$

Optimal (type 4, 310 leaves, 11 steps):

$$\begin{aligned} & - \frac{8 a^4 (24 A + 19 B + 16 C) \text{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \frac{8 a^4 (187 A + 132 B + 113 C) \text{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{231 d} + \\ & \frac{4 a^4 (913 A + 803 B + 667 C) \text{Sin}[c + d x]}{1155 d \text{Cos}[c + d x]^{3/2}} + \frac{8 a^4 (24 A + 19 B + 16 C) \text{Sin}[c + d x]}{15 d \sqrt{\text{Cos}[c + d x]}} + \\ & \frac{2 a (11 B + 8 C) (a + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{99 d \text{Cos}[c + d x]^{9/2}} + \frac{2 C (a + a \text{Cos}[c + d x])^4 \text{Sin}[c + d x]}{11 d \text{Cos}[c + d x]^{11/2}} + \\ & \frac{2 (33 A + 55 B + 43 C) (a^2 + a^2 \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{231 d \text{Cos}[c + d x]^{7/2}} + \frac{4 (891 A + 946 B + 769 C) (a^4 + a^4 \text{Cos}[c + d x]) \text{Sin}[c + d x]}{3465 d \text{Cos}[c + d x]^{5/2}} \end{aligned}$$

Result (type 5, 1795 leaves):

$$\begin{aligned} & \frac{1}{A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]} \text{Cos}[c + d x]^{13/2} \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \\ & \left(\frac{(24 A + 19 B + 16 C) \text{Csc}[c] \text{Sec}[c]}{15 d} + \frac{C \text{Sec}[c] \text{Sec}[c + d x]^6 \text{Sin}[d x]}{44 d} + \frac{\text{Sec}[c] \text{Sec}[c + d x]^5 (9 C \text{Sin}[c] + 11 B \text{Sin}[d x] + 44 C \text{Sin}[d x])}{396 d} \right. \\ & \frac{\text{Sec}[c] \text{Sec}[c + d x]^4 (77 B \text{Sin}[c] + 308 C \text{Sin}[c] + 99 A \text{Sin}[d x] + 396 B \text{Sin}[d x] + 675 C \text{Sin}[d x])}{2772 d} + \frac{1}{13860 d} \\ & \text{Sec}[c] \text{Sec}[c + d x]^3 (495 A \text{Sin}[c] + 1980 B \text{Sin}[c] + 3375 C \text{Sin}[c] + 2772 A \text{Sin}[d x] + 4697 B \text{Sin}[d x] + 4928 C \text{Sin}[d x]) + \frac{1}{4620 d} \\ & \text{Sec}[c] \text{Sec}[c + d x] (2585 A \text{Sin}[c] + 2640 B \text{Sin}[c] + 2260 C \text{Sin}[c] + 7392 A \text{Sin}[d x] + 5852 B \text{Sin}[d x] + 4928 C \text{Sin}[d x]) + \frac{1}{13860 d} \\ & \left. \text{Sec}[c] \text{Sec}[c + d x]^2 (2772 A \text{Sin}[c] + 4697 B \text{Sin}[c] + 4928 C \text{Sin}[c] + 7755 A \text{Sin}[d x] + 7920 B \text{Sin}[d x] + 6780 C \text{Sin}[d x]) \right) - \\ & \frac{1}{21 d (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2}} 17 A \text{Cos}[c + d x]^6 \text{Csc}[c] \\ & \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] \text{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \text{Sec}[c + d x])^4 \end{aligned}$$

$$\begin{aligned}
& \frac{(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}} - 1} \\
& \frac{7 d (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2}}{4 B \operatorname{Cos}[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]} \\
& \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{\sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} - 1} \\
& \frac{231 d (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2}}{113 C \operatorname{Cos}[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right]} \\
& \frac{\operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}{\sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} + 1} \\
& \left(4 A \operatorname{Cos}[c + dx]^6 \operatorname{Csc}[c] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (a + a \operatorname{Sec}[c + dx])^4 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right\] \operatorname{Sin}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1 + \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\operatorname{Cos}[c] \operatorname{Cos}[dx + \operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1 + \operatorname{Tan}[c]^2}} \sqrt{1 + \operatorname{Tan}[c]^2} \right) - \right.
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (5 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(19 B \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}} \right) / (30 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) +$$

$$\left(8 C \cos[c + d x]^6 \csc[c] \sec\left[\frac{c}{2} + \frac{d x}{2}\right]^8 (a + a \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{(15 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]))}$$

- **Problem 1217: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + d x]^{7/2} (A + B \sec[c + d x] + C \sec[c + d x]^2)}{a + a \sec[c + d x]} dx$$

Optimal (type 4, 210 leaves, 8 steps):

$$\begin{aligned} & - \frac{3(7A - 7B + 5C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} + \frac{5(9A - 7B + 7C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21ad} + \frac{5(9A - 7B + 7C) \sqrt{\cos[c + dx]} \sin[c + dx]}{21ad} \\ & - \frac{(7A - 7B + 5C) \cos[c + dx]^{3/2} \sin[c + dx]}{5ad} + \frac{(9A - 7B + 7C) \cos[c + dx]^{5/2} \sin[c + dx]}{7ad} - \frac{(A - B + C) \cos[c + dx]^{7/2} \sin[c + dx]}{d(a + a \cos[c + dx])} \end{aligned}$$

Result (type 5, 2117 leaves):

$$\begin{aligned} & - \frac{1}{10(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} \\ & 21i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \text{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\ & \left(\left(2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\ & \frac{1}{10(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} 21i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \text{Csc}\left[\frac{c}{2}\right] \\ & \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\ & \left(\left(2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ & \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) - \\
& \frac{1}{2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])} 3 i C \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + d x] + C \sec[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c])} \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c])} \right) \right) + \\
& \frac{1}{(A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) (a + a \sec[c + d x])} \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x]^{3/2} (A + B \sec[c + d x] + C \sec[c + d x]^2) \\
& \left(\frac{4 (5 A - 5 B + 5 C + 16 A \cos[c] - 16 B \cos[c] + 10 C \cos[c]) \operatorname{Csc}[c]}{5 d} + \frac{2 (51 A - 28 B + 28 C) \cos[d x] \sin[c]}{21 d} - \right. \\
& \frac{4 (A - B) \cos[2 d x] \sin[2 c]}{5 d} + \frac{2 A \cos[3 d x] \sin[3 c]}{7 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right])}{d} + \\
& \left. \frac{2 (51 A - 28 B + 28 C) \cos[c] \sin[d x]}{21 d} - \frac{4 (A - B) \cos[2 c] \sin[2 d x]}{5 d} + \frac{2 A \cos[3 c] \sin[3 d x]}{7 d} \right) - \\
& \left(30 A \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& (A + B \sec[c + d x] + C \sec[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}}{\right) / \\
& \left(7 d (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + d x]) \right) + \\
& \left(10 B \cos\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \cos[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + d x] + C \sec[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx]) \right) -$$

$$\left(10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right.$$

$$\left. \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx]) \right)$$

■ **Problem 1218: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{a + a \sec[c + dx]} dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$\frac{3(7A - 5B + 5C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] - (5A - 5B + 3C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} - \frac{3ad}{5ad}$$

$$\frac{(5A - 5B + 3C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3ad} + \frac{(7A - 5B + 5C) \cos[c + dx]^{3/2} \sin[c + dx]}{5ad} - \frac{(A - B + C) \cos[c + dx]^{5/2} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 2063 leaves):

$$\frac{1}{10(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])}$$

$$21iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\begin{aligned}
& \frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} 3iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} 3iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{4(5A - 5B + 5C + 16A \cos[c] - 10B \cos[c] + 10C \cos[c]) \operatorname{Csc}[c]}{5d} - \right. \right. \\
& \quad \frac{8(A - B) \cos[dx] \sin[c]}{3d} + \frac{4A \cos[2dx] \sin[2c]}{5d} - \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} - \\
& \quad \left. \left. \frac{8(A - B) \cos[c] \sin[dx]}{3d} + \frac{4A \cos[2c] \sin[2dx]}{5d} \right) \right) / ((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])) + \\
& \left(10A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx]) \right) - \\
& \left(10B\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right. \\
& \quad \left. \sec\left[\frac{c}{2}\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx]) \right) + \\
& \left(2C\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c+dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right. \\
& \quad \left. \sec\left[\frac{c}{2}\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx]) \right)
\end{aligned}$$

■ **Problem 1219: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A+B\sec[c+dx] + C\sec[c+dx]^2)}{a+a\sec[c+dx]} dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(3A-3B+C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{ad} + \frac{(5A-3B+3C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3ad} + \\
& \frac{(5A-3B+3C) \sqrt{\cos[c+dx]} \sin[c+dx]}{3ad} - \frac{(A-B+C) \cos[c+dx]^{3/2} \sin[c+dx]}{d(a+a\cos[c+dx])}
\end{aligned}$$

Result (type 5, 2008 leaves):

$$- \frac{1}{2(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) (a+a\sec[c+dx])}$$

$$\begin{aligned}
& 3 i A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \operatorname{Sec} [c + d x])} 3 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \operatorname{Sec} [c + d x])} i C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x]^{3/2} (A + B \operatorname{Sec} [c + d x] + C \operatorname{Sec} [c + d x]^2) \left(\frac{4 (A - B + C + 2 A \cos [c] - 2 B \cos [c]) \operatorname{Csc} [c]}{d} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{8 A \cos [d x] \sin [c]}{3 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(A \sin\left[\frac{d x}{2}\right] - B \sin\left[\frac{d x}{2}\right] + C \sin\left[\frac{d x}{2}\right]\right)}{d} + \frac{8 A \cos [c] \sin [d x]}{3 d} \right) \right) \right) / \\
& \left((A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x]) \right) - \\
& \left(10 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right) + \\
& \left(2 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right) - \\
& \left(2 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}} \right) / \\
& \left(d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right)
\end{aligned}$$

- **Problem 1220: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{a + a \operatorname{Sec}[c + d x]} dx$$

Optimal (type 4, 93 leaves, 5 steps):

$$\frac{(3A - B + C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{ad} - \frac{(A - B + C) \sqrt{\cos[c + dx]} \sin[c + dx]}{d(a + a \cos[c + dx])}$$

Result (type 5, 1973 leaves):

$$\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} \\
3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \\
\operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \operatorname{Csc}\left[\frac{c}{2}\right] \\
\operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
\left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right)$$

$$\begin{aligned}
& \left. \sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]} \right) / \left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx]^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{4(A - B + C + 2A \cos[c]) \csc[c]}{d} - \right. \right. \\
& \left. \left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right) \right) / \left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx]) \right) + \\
& \left(2A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx]) \right) - \\
& \left(2B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \right. \\
& \left. \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx]) \right) - \\
& \left(2C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \right. \\
& \left. \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]}} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx]) \right)
\end{aligned}$$

■ **Problem 1221: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])} dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{(A - B + 3 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{a d} + \frac{(A + B - C) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{a d} + \frac{(A - B + 3 C) \operatorname{Sin}[c + d x]}{a d \sqrt{\operatorname{Cos}[c + d x]}} - \frac{(A - B + C) \operatorname{Sin}[c + d x]}{d \sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Cos}[c + d x])}$$

Result (type 5, 2009 leaves):

$$\begin{aligned} & -\frac{1}{2(A + 2C + 2B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2c + 2d x]) (a + a \operatorname{Sec}[c + d x])} \\ & i A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\ & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2c] + i e^{2 i d x} \operatorname{Sin}[2c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2c] + i e^{2 i d x} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) + \\ & \frac{1}{2(A + 2C + 2B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2c + 2d x]) (a + a \operatorname{Sec}[c + d x])} i B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \\ & \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\ & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2c] + i e^{2 i d x} \operatorname{Sin}[2c]} \right) / (3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) - \right. \\ & \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2c] + i e^{2 i d x} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c]) \right) - \\ & \frac{1}{2(A + 2C + 2B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2c + 2d x]) (a + a \operatorname{Sec}[c + d x])} 3 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \\ & \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\ & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x]^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \left(\frac{2(2 C + A \operatorname{Cos}[c] - B \operatorname{Cos}[c] + C \operatorname{Cos}[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \right. \right. \right. \\
& \left. \left. \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] (A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right])}{d} + \frac{8 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \operatorname{Sin}[d x]}{d}\right)\right) \right) \Bigg/ \\
& \left. \left((A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x]) \right) - \right. \\
& \left. \left(2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right) \right. \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\left. \right)} \right) \Bigg/ \\
& \left. \left(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right) - \right. \\
& \left. \left(2 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right) \right. \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\left. \right)} \right) \Bigg/ \\
& \left. \left(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x]) \right) + \right. \\
& \left. \left(2 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right) \right)
\end{aligned}$$

$$\left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx]) \right)$$

■ **Problem 1222: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \sec[c + dx])} dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$\frac{(A - 3B + 3C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{ad} + \frac{(3A - 3B + 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3ad} +$$

$$\frac{(3A - 3B + 5C) \sin[c + dx]}{3ad \cos[c + dx]^{3/2}} - \frac{(A - 3B + 3C) \sin[c + dx]}{ad \sqrt{\cos[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \cos[c + dx]^{3/2} (a + a \cos[c + dx])}$$

Result (type 5, 2052 leaves):

$$\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])}$$

$$+ i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) -$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \Big) -$$

$$\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])} + 3i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right]$$

$$\sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) -$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \\
& \frac{1}{2 \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \left(a + a \operatorname{Sec}[c + d x]\right)} 3 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}\left[\frac{c}{2}\right] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) \right) + \\
& \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x]^{3/2} \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \left(-\frac{2 \left(-2 B + 2 C + A \operatorname{Cos}[c] - B \operatorname{Cos}[c] + C \operatorname{Cos}[c]\right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} - \right. \right. \\
& \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{d x}{2}\right] \left(A \operatorname{Sin}\left[\frac{d x}{2}\right] - B \operatorname{Sin}\left[\frac{d x}{2}\right] + C \operatorname{Sin}\left[\frac{d x}{2}\right]\right)}{d} + \frac{8 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \operatorname{Sin}[d x]}{3 d} + \\
& \left. \left. \frac{8 \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \left(C \operatorname{Sin}[c] + 3 B \operatorname{Sin}[d x] - 3 C \operatorname{Sin}[d x]\right)}{3 d} \right) \right) \left. \right) / \left(\left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \left(a + a \operatorname{Sec}[c + d x]\right) \right) - \\
& \left(2 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\left(d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right) \right)} \right) + \\
& \left(2 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Cos}[c + d x] \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \right) \\
& \operatorname{Sec}\left[\frac{c}{2}\right] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}
\end{aligned}$$

$$\left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} (a + a \sec[c + dx]) \right) -$$

$$\left(10C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \right.$$

$$\left. \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right)$$

$$\left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} (a + a \sec[c + dx]) \right)$$

■ **Problem 1223: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \sec[c + dx])} dx$$

Optimal (type 4, 210 leaves, 8 steps):

$$\frac{3(5A - 5B + 7C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] - (3A - 5B + 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{5ad} +$$

$$\frac{(5A - 5B + 7C) \sin[c + dx]}{5ad \cos[c + dx]^{5/2}} - \frac{(3A - 5B + 5C) \sin[c + dx]}{3ad \cos[c + dx]^{3/2}} + \frac{3(5A - 5B + 7C) \sin[c + dx]}{5ad \sqrt{\cos[c + dx]}} - \frac{(A - B + C) \sin[c + dx]}{d \cos[c + dx]^{5/2} (a + a \cos[c + dx])}$$

Result (type 5, 2111 leaves):

$$\frac{1}{2(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])}$$

$$3iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \cos[c + dx] \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\begin{aligned}
& \frac{1}{2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])} 3 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{10 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])} 21 i C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x] \operatorname{Csc} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{(A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])} \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \cos [c + d x]^{3/2} (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\frac{2 (10 A - 10 B + 16 C + 5 A \cos [c] - 5 B \cos [c] + 5 C \cos [c]) \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c]}{5 d} + \right. \\
& \quad \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right] (A \sin \left[\frac{d x}{2} \right] - B \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{d} + \frac{8 C \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^3 \sin [d x]}{5 d} - \\
& \quad \frac{8 \operatorname{Sec} [c] \operatorname{Sec} [c + d x] (-5 B \sin [c] + 5 C \sin [c] - 15 A \sin [d x] + 15 B \sin [d x] - 24 C \sin [d x])}{15 d} + \\
& \quad \left. \frac{8 \operatorname{Sec} [c] \operatorname{Sec} [c + d x]^2 (3 C \sin [c] + 5 B \sin [d x] - 5 C \sin [d x])}{15 d} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 A \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \cos [c + dx] \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \right) \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad \left(A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2 \right) \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \quad \left(d (A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \operatorname{Sec} [c + dx]) \right) - \\
& \left(10 B \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \cos [c + dx] \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \right) \\
& \quad \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2) \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \operatorname{Sec} [c + dx]) \right) + \\
& \left(10 C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \cos [c + dx] \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \right]^2 \right) \\
& \quad \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2) \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \operatorname{Sec} [c + dx]) \right)
\end{aligned}$$

- **Problem 1224: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + dx]^{7/2} (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2)}{(a + a \operatorname{Sec} [c + dx])^2} dx$$

Optimal (type 4, 258 leaves, 9 steps):

$$\begin{aligned}
& - \frac{7(11A - 8B + 5C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5a^2d} + \frac{5(30A - 21B + 14C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21a^2d} + \\
& \frac{5(30A - 21B + 14C) \sqrt{\cos[c + dx]} \sin[c + dx]}{21a^2d} - \frac{7(11A - 8B + 5C) \cos[c + dx]^{3/2} \sin[c + dx]}{15a^2d} + \\
& \frac{(30A - 21B + 14C) \cos[c + dx]^{5/2} \sin[c + dx]}{7a^2d} - \frac{(11A - 8B + 5C) \cos[c + dx]^{7/2} \sin[c + dx]}{3a^2d(1 + \cos[c + dx])} - \frac{(A - B + C) \cos[c + dx]^{9/2} \sin[c + dx]}{3d(a + a \cos[c + dx])^2}
\end{aligned}$$

Result (type 5, 2174 leaves):

$$\begin{aligned}
& - \frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 77iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 56iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 7iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx}(2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \left(200 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2 \right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(7 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \left(20 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2 \right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) - \\
& \left(40 C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2 \right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \frac{1}{(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} \\
& \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\operatorname{Cos}[c + d x]} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\frac{8 (25 A - 20 B + 15 C + 52 A \operatorname{Cos}[c] - 36 B \operatorname{Cos}[c] + 20 C \operatorname{Cos}[c]) \operatorname{Csc}[c]}{5 d} + \frac{4 (107 A - 56 B + 28 C) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{21 d} \right) -
\end{aligned}$$

$$\frac{8(2A-B)\cos[2dx]\sin[2c]}{5d} + \frac{4A\cos[3dx]\sin[3c]}{7d} - \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3\left(A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right]\right)}{3d} +$$

$$\frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]\left(5A\sin\left[\frac{dx}{2}\right] - 4B\sin\left[\frac{dx}{2}\right] + 3C\sin\left[\frac{dx}{2}\right]\right)}{d} + \frac{4(107A - 56B + 28C)\cos[c]\sin[dx]}{21d} -$$

$$\left. \frac{8(2A-B)\cos[2c]\sin[2dx]}{5d} + \frac{4A\cos[3c]\sin[3dx]}{7d} - \frac{4(A-B+C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2\tan\left[\frac{c}{2}\right]}{3d} \right)$$

■ **Problem 1225: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{5/2} (A+B\sec[c+dx]+C\sec[c+dx]^2)}{(a+a\sec[c+dx])^2} dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$\frac{(56A-35B+20C)\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5a^2d} - \frac{5(3A-2B+C)\operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2d} - \frac{5(3A-2B+C)\sqrt{\cos[c+dx]}\sin[c+dx]}{3a^2d} +$$

$$\frac{(56A-35B+20C)\cos[c+dx]^{3/2}\sin[c+dx]}{15a^2d} - \frac{(3A-2B+C)\cos[c+dx]^{5/2}\sin[c+dx]}{a^2d(1+\cos[c+dx])} - \frac{(A-B+C)\cos[c+dx]^{7/2}\sin[c+dx]}{3d(a+a\cos[c+dx])^2}$$

Result (type 5, 2120 leaves):

$$\frac{1}{(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx])^2} 56iA\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A+B\sec[c+dx]+C\sec[c+dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (-id(1+e^{2idx})\cos[c]+d(-1+e^{2idx})\sin[c]) \right) -$$

$$\frac{1}{(A+2C+2B\cos[c+dx]+A\cos[2c+2dx])(a+a\sec[c+dx])^2} 7iB\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]$$

$$(A+B\sec[c+dx]+C\sec[c+dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c]+i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c]+2i(-1+e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\cos[2c]+ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c]-3d(-1+e^{2idx})\sin[c]) - \right.$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} 4 i C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \right) + \\
& \left(20 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) - \\
& \left(40 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \left(20 C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}
\end{aligned}$$

$$\left(\frac{\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]}}{\left(3d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a\sec[c+dx])^2 \right) + \frac{1}{(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) (a+a\sec[c+dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx])^2} \right) /$$

$$\left(-\frac{8(20A-15B+10C+36A\cos[c]-20B\cos[c]+10C\cos[c])\csc[c]}{5d} - \frac{16(2A-B)\cos[dx]\sin[c]}{3d} + \frac{8A\cos[2dx]\sin[2c]}{5d} + \frac{4\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right])}{3d} - \frac{8\sec\left[\frac{c}{2}\right]\sec\left[\frac{c}{2} + \frac{dx}{2}\right] (4A\sin\left[\frac{dx}{2}\right] - 3B\sin\left[\frac{dx}{2}\right] + 2C\sin\left[\frac{dx}{2}\right])}{d} - \frac{16(2A-B)\cos[c]\sin[dx]}{3d} + \frac{8A\cos[2c]\sin[2dx]}{5d} + \frac{4(A-B+C)\sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)$$

- **Problem 1226: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c+dx]^{3/2} (A+B\sec[c+dx] + C\sec[c+dx]^2)}{(a+a\sec[c+dx])^2} dx$$

Optimal (type 4, 180 leaves, 7 steps):

$$-\frac{(7A-4B+C)\text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} + \frac{(10A-5B+2C)\text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} + \frac{(10A-5B+2C)\sqrt{\cos[c+dx]}\sin[c+dx]}{3a^2 d} - \frac{(7A-4B+C)\cos[c+dx]^{3/2}\sin[c+dx]}{3a^2 d(1+\cos[c+dx])} - \frac{(A-B+C)\cos[c+dx]^{5/2}\sin[c+dx]}{3d(a+a\cos[c+dx])^2}$$

Result (type 5, 2064 leaves):

$$-\frac{1}{(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) (a+a\sec[c+dx])^2} 7iA\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A+B\sec[c+dx] + C\sec[c+dx]^2)$$

$$\left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx}(\cos[c] + i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c] + 2i(-1+e^{2idx})\sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1+e^{2idx}\cos[2c] + ie^{2idx}\sin[2c]} \right) / (3id(1+e^{2idx})\cos[c] - 3d(-1+e^{2idx})\sin[c]) - \right.$$

$$\left. \left(2\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx}(\cos[c] + i\sin[c])^2\right] \sqrt{e^{-idx}(2(1+e^{2idx})\cos[c] + 2i(-1+e^{2idx})\sin[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right) + \\
& \frac{1}{(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} 4 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left((A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \right. \\
& \left. \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c])} - \right. \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right) \right) - \\
& \frac{1}{(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^2} i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left((A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \right. \\
& \left. \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c])} - \right. \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right) \right) - \\
& \left(40 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}}{\left. \right)} \right) \\
& \left(3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^2 \right) + \\
& \left(20 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a \sec[c+dx])^2 \right) - \\
& \left(8C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A+B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{1+\cot[c]^2} (a+a \sec[c+dx])^2 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} (A+B \sec[c+dx] + C \sec[c+dx]^2) \right. \\
& \quad \left(\frac{8(3A-2B+C+4A \cos[c] - 2B \cos[c]) \csc[c]}{d} + \frac{16A \cos[dx] \sin[c]}{3d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (3A \sin\left[\frac{dx}{2}\right] - 2B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{d} \right. \\
& \quad \left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{16A \cos[c] \sin[dx]}{3d} - \frac{4(A-B+C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \left. \right) / \\
& \left((A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \sec[c+dx])^2 \right)
\end{aligned}$$

■ **Problem 1227: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos[c+dx]} (A+B \sec[c+dx] + C \sec[c+dx]^2)}{(a+a \sec[c+dx])^2} dx$$

Optimal (type 4, 144 leaves, 6 steps):

$$\frac{(4A-B) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{a^2 d} - \frac{(5A-2B-C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2 d} - \\
\frac{(5A-2B-C) \sqrt{\cos[c+dx]} \sin[c+dx]}{3a^2 d (1+\cos[c+dx])} - \frac{(A-B+C) \cos[c+dx]^{3/2} \sin[c+dx]}{3d (a+a \cos[c+dx])^2}$$

Result (type 5, 1628 leaves):

$$\frac{1}{(A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \sec[c+dx])^2} 4iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A+B \sec[c+dx] + C \sec[c+dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) - \\
& \frac{1}{(A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^2} i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \csc \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} \right] \\
& (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \Big) + \\
& \left(20 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \sec \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) \Big) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^2 \right) - \\
& \left(8 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \csc \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 \sec \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) \Big) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(4 C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left. (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2) \operatorname{Sec} [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]] \sqrt{1 - \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot} [c]^2} \sin [c] \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \sqrt{1 + \sin [dx - \operatorname{ArcTan} [\operatorname{Cot} [c]]]} \right) / \\
& \quad \left(3 d (A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) \sqrt{1 + \operatorname{Cot} [c]^2} (a + a \operatorname{Sec} [c + dx])^2 \right) + \\
& \quad \left(\cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \sqrt{\cos [c + dx]} (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2) \right. \\
& \quad \left(-\frac{8 (2 A - B + 2 A \cos [c]) \operatorname{Csc} [c]}{d} - \frac{8 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right] (2 A \sin \left[\frac{dx}{2} \right] - B \sin \left[\frac{dx}{2} \right])}{d} + \right. \\
& \quad \left. \frac{4 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^3 (A \sin \left[\frac{dx}{2} \right] - B \sin \left[\frac{dx}{2} \right] + C \sin \left[\frac{dx}{2} \right])}{3 d} + \frac{4 (A - B + C) \operatorname{Sec} \left[\frac{c}{2} + \frac{dx}{2} \right]^2 \operatorname{Tan} \left[\frac{c}{2} \right]}{3 d} \right) \left. \right) / \\
& \quad \left((A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) (a + a \operatorname{Sec} [c + dx])^2 \right)
\end{aligned}$$

- **Problem 1228: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2}{\sqrt{\cos [c + dx]} (a + a \operatorname{Sec} [c + dx])^2} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\begin{aligned}
& -\frac{(A - C) \operatorname{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{a^2 d} + \frac{(2 A + B + 2 C) \operatorname{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{3 a^2 d} + \\
& \frac{(A - C) \sqrt{\cos [c + dx]} \sin [c + dx]}{a^2 d (1 + \cos [c + dx])} - \frac{(A - B + C) \sqrt{\cos [c + dx]} \sin [c + dx]}{3 d (a + a \cos [c + dx])^2}
\end{aligned}$$

Result (type 5, 1620 leaves):

$$\begin{aligned}
& -\frac{1}{(A + 2 C + 2 B \cos [c + dx] + A \cos [2 c + 2 dx]) (a + a \operatorname{Sec} [c + dx])^2} i A \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] (A + B \operatorname{Sec} [c + dx] + C \operatorname{Sec} [c + dx]^2) \\
& \quad \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos [c] + 2 i (-1 + e^{2 i dx}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos [2 c] + i e^{2 i dx} \sin [2 c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \cos [c] - 3 d (-1 + e^{2 i dx}) \sin [c]) -
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left(-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) + \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} i C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left(3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c] \right) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / \left(-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c] \right) \Big) - \\
& \left(8 A \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) - \\
& \left(4 B \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) - \\
& \left(8 C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^4 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad (A + B \sec[c + dx] + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}
\end{aligned}$$

$$\left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^2 \right) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{8(A - C) \csc[c]}{d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{d} - \right. \right.$$

$$\left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} - \frac{4(A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right) \right) /$$

$$\left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2 \right)$$

■ **Problem 1229: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\cos[c + dx]^{3/2} (a + a \sec[c + dx])^2} dx$$

Optimal (type 4, 167 leaves, 7 steps):

$$\frac{(B - 4C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(A + 2B - 5C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} -$$

$$\frac{(B - 4C) \sin[c + dx]}{a^2 d \sqrt{\cos[c + dx]}} + \frac{(A + 2B - 5C) \sin[c + dx]}{3a^2 d \sqrt{\cos[c + dx]} (1 + \cos[c + dx])} - \frac{(A - B + C) \sin[c + dx]}{3d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2}$$

Result (type 5, 1660 leaves):

$$\frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) -$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right.$$

$$\left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 4 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right]$$

$$(A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left(3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left(-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) \Big) - \\
& \left(4 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} \right] \\
& \quad \left(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2 \right) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) \Big) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^2 \right) - \\
& \left(8 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} \right] \\
& \quad \left(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2 \right) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) \Big) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^2 \right) + \\
& \left(20 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} \right] \\
& \quad \left(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2 \right) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) \Big) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^2 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \sqrt{\cos [c + d x]} (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)
\end{aligned}$$

$$\left(\frac{4(2C - B \cos[c] + 2C \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (-B \sin\left[\frac{dx}{2}\right] + 2C \sin\left[\frac{dx}{2}\right])}{d} + \frac{16C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} + \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{3d} \right) / \left((A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2 \right)$$

■ **Problem 1230: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^2} dx$$

Optimal (type 4, 211 leaves, 8 steps):

$$\frac{(A - 4B + 7C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{a^2 d} + \frac{(2A - 5B + 10C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3a^2 d} + \frac{(2A - 5B + 10C) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2}} - \frac{(A - 4B + 7C) \sin[c + dx]}{a^2 d \sqrt{\cos[c + dx]}} - \frac{(A - 4B + 7C) \sin[c + dx]}{3a^2 d \cos[c + dx]^{3/2} (1 + \cos[c + dx])} - \frac{(A - B + C) \sin[c + dx]}{3d \cos[c + dx]^{3/2} (a + a \cos[c + dx])^2}$$

Result (type 5, 2107 leaves):

$$\frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (3i d (1 + e^{2i dx}) \cos[c] - 3d(-1 + e^{2i dx}) \sin[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (-i d (1 + e^{2i dx}) \cos[c] + d(-1 + e^{2i dx}) \sin[c]) \right) - \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^2} 4i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (3i d (1 + e^{2i dx}) \cos[c] - 3d(-1 + e^{2i dx}) \sin[c]) - \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1 + e^{2i dx}) \cos[c] + 2i(-1 + e^{2i dx}) \sin[c])} \sqrt{1 + e^{2i dx} \cos[2c] + i e^{2i dx} \sin[2c]} \right) / (-i d (1 + e^{2i dx}) \cos[c] + d(-1 + e^{2i dx}) \sin[c]) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) + \\
& \frac{1}{\left(\left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \left(a + a \operatorname{Sec}[c + d x]\right)^2\right)} 7 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left(\left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2\right) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right)}{\right. \\
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) - \\
& \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right)}{\right. \\
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) \Bigg) - \\
& \left(8 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right]\right) \\
& \left(\left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right) \\
& \left.\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}\right) \Bigg) / \\
& \left(3 d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^2\right) + \\
& \left(20 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right]\right) \\
& \left(\left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right) \\
& \left.\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}\right) \Bigg) / \\
& \left(3 d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^2\right) - \\
& \left(40 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right]\right) \\
& \left(\left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]\right)^2 \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}\right)
\end{aligned}$$

$$\left(\frac{\sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]}}{\left(3d(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) \sqrt{1+\cot^2[c]} (a+a\sec[c+dx])^2 \right) + \frac{1}{(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) (a+a\sec[c+dx])^2} \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx])^2} \right) - \left(\frac{4(-2B+4C+A\cos[c] - 2B\cos[c] + 3C\cos[c]) \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c]}{d} - \frac{4\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A\sin\left[\frac{dx}{2}\right] - B\sin\left[\frac{dx}{2}\right] + C\sin\left[\frac{dx}{2}\right])}{3d} - \frac{8\sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A\sin\left[\frac{dx}{2}\right] - 2B\sin\left[\frac{dx}{2}\right] + 3C\sin\left[\frac{dx}{2}\right])}{d} + \frac{16C\sec[c] \sec[c+dx]^2 \sin[dx]}{3d} + \frac{16\sec[c] \sec[c+dx] (C\sin[c] + 3B\sin[dx] - 6C\sin[dx])}{3d} - \frac{4(A-B+C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)$$

- **Problem 1231: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A+B\sec[c+dx] + C\sec[c+dx]^2}{\cos[c+dx]^{7/2} (a+a\sec[c+dx])^2} dx$$

Optimal (type 4, 250 leaves, 9 steps):

$$-\frac{(20A-35B+56C) \text{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5a^2d} - \frac{5(A-2B+3C) \text{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3a^2d} + \frac{(20A-35B+56C) \sin[c+dx]}{15a^2d \cos[c+dx]^{5/2}} - \frac{5(A-2B+3C) \sin[c+dx]}{3a^2d \cos[c+dx]^{3/2}} + \frac{(20A-35B+56C) \sin[c+dx]}{5a^2d \sqrt{\cos[c+dx]}} - \frac{(A-2B+3C) \sin[c+dx]}{a^2d \cos[c+dx]^{5/2} (1+\cos[c+dx])} - \frac{(A-B+C) \sin[c+dx]}{3d \cos[c+dx]^{5/2} (a+a\cos[c+dx])^2}$$

Result (type 5, 2164 leaves):

$$-\frac{1}{(A+2C+2B\cos[c+dx] + A\cos[2c+2dx]) (a+a\sec[c+dx])^2} 4iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] (A+B\sec[c+dx] + C\sec[c+dx]^2) \left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i\sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1+e^{2idx} \cos[2c] + ie^{2idx} \sin[2c]} \right) \right) / (3id(1+e^{2idx}) \cos[c] - 3d(-1+e^{2idx}) \sin[c]) - \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i\sin[c])^2\right] \sqrt{e^{-idx} (2(1+e^{2idx}) \cos[c] + 2i(-1+e^{2idx}) \sin[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c])} \right) + \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 7 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left((A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
& \left. \left(\left(2 e^{2ix} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c])} - \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c])} \right) \right) - \\
& \frac{1}{5 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} 56 i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \left((A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
& \left. \left(\left(2 e^{2ix} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c])} - \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c])} \right) \right) + \\
& \left(20 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \Big/ \\
& \left(3 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \sec[c + dx])^2 \right) - \\
& \left(40 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx])^2 \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \left(20C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^2 \right) + \\
& \frac{1}{(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^2} \\
& \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \sqrt{\cos[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(\frac{4 (10A - 20B + 36C + 10A \cos[c] - 15B \cos[c] + 20C \cos[c]) \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c]}{5d} + \right. \\
& \quad \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{3d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (2A \sin\left[\frac{dx}{2}\right] - 3B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right])}{d} + \\
& \quad \frac{16C \sec[c] \sec[c + dx]^3 \sin[dx]}{5d} - \frac{16 \sec[c] \sec[c + dx] (-5B \sin[c] + 10C \sin[c] - 15A \sin[dx] + 30B \sin[dx] - 54C \sin[dx])}{15d} + \\
& \quad \left. \frac{16 \sec[c] \sec[c + dx]^2 (3C \sin[c] + 5B \sin[dx] - 10C \sin[dx])}{15d} + \frac{4 (A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{3d} \right)
\end{aligned}$$

- **Problem 1232: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{(a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 273 leaves, 9 steps):

$$\frac{7(33A - 17B + 7C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} - \frac{(63A - 33B + 13C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} -$$

$$\frac{(63A - 33B + 13C) \sqrt{\cos[c + dx]} \sin[c + dx]}{6a^3d} + \frac{7(33A - 17B + 7C) \cos[c + dx]^{3/2} \sin[c + dx]}{30a^3d} -$$

$$\frac{(A - B + C) \cos[c + dx]^{9/2} \sin[c + dx]}{5d(a + a \cos[c + dx])^3} - \frac{(12A - 7B + 2C) \cos[c + dx]^{7/2} \sin[c + dx]}{15ad(a + a \cos[c + dx])^2} - \frac{(63A - 33B + 13C) \cos[c + dx]^{5/2} \sin[c + dx]}{10d(a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 2257 leaves):

$$\frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3}$$

$$231iA \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} 119iB \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3} 49iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right)} \right) \right) + \\
& \left(84 A \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec}[c + dx] \right. \\
& \left. \left(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2 \right) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) \Big/ \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) - \\
& \left(44 B \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \left. \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) \Big/ \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(52 C \cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]] \right]^2 \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \left. \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) \Big/ \\
& \left(3 d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \operatorname{Sec}[c + dx])^3 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{dx}{2} \right]^6 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right)
\end{aligned}$$

$$\left(\begin{aligned} & - \frac{8 (99 A - 59 B + 29 C + 132 A \cos[c] - 60 B \cos[c] + 20 C \cos[c]) \operatorname{Csc}[c]}{5 d} - \frac{32 (3 A - B) \cos[dx] \sin[c]}{3 d} + \frac{16 A \cos[2 dx] \sin[2 c]}{5 d} - \\ & \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5 d} + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (24 A \sin\left[\frac{dx}{2}\right] - 19 B \sin\left[\frac{dx}{2}\right] + 14 C \sin\left[\frac{dx}{2}\right])}{15 d} - \\ & \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (99 A \sin\left[\frac{dx}{2}\right] - 59 B \sin\left[\frac{dx}{2}\right] + 29 C \sin\left[\frac{dx}{2}\right])}{5 d} - \frac{32 (3 A - B) \cos[c] \sin[dx]}{3 d} + \\ & \frac{16 A \cos[2 c] \sin[2 dx]}{5 d} + \frac{8 (24 A - 19 B + 14 C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} - \frac{4 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \end{aligned} \right) /$$

$$\left(\sqrt{\cos[c + dx]} (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)$$

- **Problem 1233: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^{3/2} (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)}{(a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 234 leaves, 8 steps):

$$\begin{aligned} & - \frac{(119 A - 49 B + 9 C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^3 d} + \frac{(33 A - 13 B + 3 C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6 a^3 d} + \frac{(33 A - 13 B + 3 C) \sqrt{\cos[c + dx]} \sin[c + dx]}{6 a^3 d} \\ & - \frac{(A - B + C) \cos[c + dx]^{7/2} \sin[c + dx]}{5 d (a + a \cos[c + dx])^3} - \frac{(2 A - B) \cos[c + dx]^{5/2} \sin[c + dx]}{3 a d (a + a \cos[c + dx])^2} - \frac{(119 A - 49 B + 9 C) \cos[c + dx]^{3/2} \sin[c + dx]}{30 d (a^3 + a^3 \cos[c + dx])} \end{aligned}$$

Result (type 5, 2206 leaves):

$$\begin{aligned} & - \frac{1}{5 (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3} \\ & 119 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\ & \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ & \quad \left. \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\ & \quad \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \\ & \quad \left. \sqrt{1 + e^{2 i dx} \cos[2 c] + i e^{2 i dx} \sin[2 c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \Big) + \\ & \frac{1}{5 (A + 2 C + 2 B \cos[c + dx] + A \cos[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3} 49 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \end{aligned}$$

$$\begin{aligned}
& \text{Sec}[c + dx] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2 i dx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2c] + i e^{2 i dx} \text{Sin}[2c]} \right) / (3 i d (1 + e^{2 i dx}) \text{Cos}[c] - 3 d (-1 + e^{2 i dx}) \text{Sin}[c]) - \right. \\
& \quad \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2c] + i e^{2 i dx} \text{Sin}[2c]} \right) / (-i d (1 + e^{2 i dx}) \text{Cos}[c] + d (-1 + e^{2 i dx}) \text{Sin}[c]) \right) - \\
& \quad \frac{1}{5 (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2 dx]) (a + a \text{Sec}[c + dx])^3} 9 i C \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{Sec} \left[\frac{c}{2} \right] \\
& \quad \text{Sec}[c + dx] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2 i dx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2c] + i e^{2 i dx} \text{Sin}[2c]} \right) / (3 i d (1 + e^{2 i dx}) \text{Cos}[c] - 3 d (-1 + e^{2 i dx}) \text{Sin}[c]) - \right. \\
& \quad \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \text{Cos}[c] + 2 i (-1 + e^{2 i dx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i dx} \text{Cos}[2c] + i e^{2 i dx} \text{Sin}[2c]} \right) / (-i d (1 + e^{2 i dx}) \text{Cos}[c] + d (-1 + e^{2 i dx}) \text{Sin}[c]) \right) - \\
& \quad \left(44 A \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} \right] \text{Sec}[c + dx] \\
& \quad (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \quad (d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2 dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + dx])^3) + \\
& \quad \left(52 B \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \right) \text{Sec} \left[\frac{c}{2} \right] \\
& \quad \text{Sec}[c + dx] (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) - \\
& \left(4 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 \sec \left[\frac{c}{2} \right] \right. \\
& \quad \left. \sec [c + d x] (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^3 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
& \quad \left(\frac{8 (59 A - 29 B + 9 C + 60 A \cos [c] - 20 B \cos [c]) \operatorname{Csc} [c]}{5 d} + \frac{32 A \cos [d x] \sin [c]}{3 d} + \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^5 (A \sin \left[\frac{d x}{2} \right] - B \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{5 d} \right. \\
& \quad \left. \frac{8 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (59 A \sin \left[\frac{d x}{2} \right] - 29 B \sin \left[\frac{d x}{2} \right] + 9 C \sin \left[\frac{d x}{2} \right])}{5 d} - \frac{8 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (19 A \sin \left[\frac{d x}{2} \right] - 14 B \sin \left[\frac{d x}{2} \right] + 9 C \sin \left[\frac{d x}{2} \right])}{15 d} \right. \\
& \quad \left. \left. \frac{32 A \cos [c] \sin [d x]}{3 d} - \frac{8 (19 A - 14 B + 9 C) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[\frac{c}{2} \right]}{15 d} + \frac{4 (A - B + C) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[\frac{c}{2} \right]}{5 d} \right) \right) / \\
& \left(\sqrt{\cos [c + d x]} (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^3 \right)
\end{aligned}$$

■ **Problem 1234: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^3} dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$\begin{aligned}
& \frac{(49 A - 9 B - C) \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{10 a^3 d} - \frac{(13 A - 3 B - C) \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{6 a^3 d} - \\
& \frac{(A - B + C) \cos [c + d x]^{5/2} \sin [c + d x]}{5 d (a + a \cos [c + d x])^3} - \frac{(8 A - 3 B - 2 C) \cos [c + d x]^{3/2} \sin [c + d x]}{15 a d (a + a \cos [c + d x])^2} - \frac{(13 A - 3 B - C) \sqrt{\cos [c + d x]} \sin [c + d x]}{6 d (a^3 + a^3 \cos [c + d x])}
\end{aligned}$$

Result (type 5, 2175 leaves):

$$\frac{1}{5 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^3}$$

$$\begin{aligned}
& 49 i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1+e^{2 i dx}) \cos[c]-3 d (-1+e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (-i d (1+e^{2 i dx}) \cos[c]+d (-1+e^{2 i dx}) \sin[c]) \right) - \\
& \frac{1}{5 (A+2 C+2 B \cos[c+dx]+A \cos[2 c+2 dx]) (a+a \operatorname{Sec}[c+dx])^3} 9 i B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1+e^{2 i dx}) \cos[c]-3 d (-1+e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (-i d (1+e^{2 i dx}) \cos[c]+d (-1+e^{2 i dx}) \sin[c]) \right) - \\
& \frac{1}{5 (A+2 C+2 B \cos[c+dx]+A \cos[2 c+2 dx]) (a+a \operatorname{Sec}[c+dx])^3} i C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}[c+dx] (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \\
& \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (3 i d (1+e^{2 i dx}) \cos[c]-3 d (-1+e^{2 i dx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1+e^{2 i dx}) \cos[c]+2 i (-1+e^{2 i dx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1+e^{2 i dx} \cos[2 c]+i e^{2 i dx} \sin[2 c]} \right) / (-i d (1+e^{2 i dx}) \cos[c]+d (-1+e^{2 i dx}) \sin[c]) \right) + \\
& \left(52 A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c+dx] \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) - \\
& \left(4B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) - \\
& \left(4C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \quad \left(-\frac{8(29A - 9B - C + 20A \cos[c]) \csc[c]}{5d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (29A \sin\left[\frac{dx}{2}\right] - 9B \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{5d} \right. \\
& \quad \left. \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (14A \sin\left[\frac{dx}{2}\right] - 9B \sin\left[\frac{dx}{2}\right] + 4C \sin\left[\frac{dx}{2}\right])}{15d} \right. \\
& \quad \left. \left. \frac{8(14A - 9B + 4C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) \right) / \\
& \left(\sqrt{\cos[c + dx]} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)
\end{aligned}$$

■ **Problem 1235:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + a \operatorname{Sec}[c + d x])^3} dx$$

Optimal (type 4, 193 leaves, 7 steps):

$$-\frac{(9A + B - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(3A + B + C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{5d(a + a \operatorname{Cos}[c + dx])^3} - \frac{(6A - B - 4C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15ad(a + a \operatorname{Cos}[c + dx])^2} + \frac{(9A + B - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10d(a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 2167 leaves):

$$-\frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} \left(9iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) \right) - \\ \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\ \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) \right) + \\ \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) \right) / \left(3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c] \right) - \\
& \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c] \right) \Big) - \\
& \left(4 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \text{Sec} \left[\frac{c}{2} \right] \text{Sec}[c + d x] \right. \\
& \quad \left(A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2 \right) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^3 \right) - \\
& \left(4 B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \text{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \text{Sec}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^3 \right) - \\
& \left(4 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\text{Cot}[c]]] \right]^2 \text{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \text{Sec}[c + d x] (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin [c] \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + d x])^3 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^6 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \left(\frac{8 (9 A + B - C) \text{Csc}[c]}{5 d} - \frac{8 \text{Sec} \left[\frac{c}{2} \right] \text{Sec} \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (9 A \sin \left[\frac{d x}{2} \right] - 4 B \sin \left[\frac{d x}{2} \right] - C \sin \left[\frac{d x}{2} \right])}{15 d} \right) \right) +
\end{aligned}$$

$$\frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(9 A \operatorname{Sin}\left[\frac{dx}{2}\right] + B \operatorname{Sin}\left[\frac{dx}{2}\right] - C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d}$$

$$\left. \left. \left. \frac{8 (9 A - 4 B - C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} + \frac{4 (A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \right) \right) \right) /$$

$$\left(\sqrt{\operatorname{Cos}[c + dx]} (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3 \right)$$

■ **Problem 1236: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^3} dx$$

Optimal (type 4, 191 leaves, 7 steps):

$$- \frac{(A - B - 9 C) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^3 d} + \frac{(A + B + 3 C) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{6 a^3 d} -$$

$$\frac{(A - B + C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{5 d (a + a \operatorname{Cos}[c + dx])^3} + \frac{(4 A + B - 6 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{15 a d (a + a \operatorname{Cos}[c + dx])^2} + \frac{(A - B - 9 C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10 d (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 2164 leaves):

$$- \frac{1}{5 (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3}$$

$$i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) +$$

$$\frac{1}{5 (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]) (a + a \operatorname{Sec}[c + dx])^3} i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{5 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (a + a \operatorname{Sec}[c + d x])^3} 9 i C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \quad \operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \right) - \\
& \left(4 A \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec}[c + d x] \right. \\
& \quad \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2 \right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^3 \right) - \\
& \left(4 B \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} (a + a \operatorname{Sec}[c + d x])^3 \right) - \\
& \left(4 C \operatorname{Cos} \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left. \operatorname{Sec}[c + d x] (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) +$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{8(A - B - 9C) \csc[c]}{5d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] - 9C \sin\left[\frac{dx}{2}\right])}{5d} \right) \right. +$$

$$\frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (4A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right] - 6C \sin\left[\frac{dx}{2}\right])}{15d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} +$$

$$\left. \frac{8(4A + B - 6C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} - \frac{4(A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) /$$

$$\left(\sqrt{\cos[c + dx]} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)$$

■ **Problem 1237: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\cos[c + dx]^{5/2} (a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$\frac{(A + 9B - 49C) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{(A + 3B - 13C) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{6a^3d} - \frac{(A + 9B - 49C) \sin[c + dx]}{10a^3d \sqrt{\cos[c + dx]}} -$$

$$\frac{(A - B + C) \sin[c + dx]}{5d \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^3} + \frac{(2A + 3B - 8C) \sin[c + dx]}{15ad \sqrt{\cos[c + dx]} (a + a \cos[c + dx])^2} + \frac{(A + 3B - 13C) \sin[c + dx]}{6d \sqrt{\cos[c + dx]} (a^3 + a^3 \cos[c + dx])}$$

Result (type 5, 2205 leaves):

$$\frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3}$$

$$+ i A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \csc\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2e^{2idx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right) + \\
& \frac{1}{5 \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \left(a + a \operatorname{Sec}[c + d x]\right)^3} 9 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}[c + d x] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right) \right) - \\
& \frac{1}{5 \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \left(a + a \operatorname{Sec}[c + d x]\right)^3} 49 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\
& \operatorname{Sec}[c + d x] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right) \right) - \\
& \left(4 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] \right. \\
& \left. \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]}}{\left. \right)} \right) / \\
& \left(3 d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^3 \right) - \\
& \left(4 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\
& \left. \operatorname{Sec}[c + d x] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \left(52C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx] (A + B \sec[c + dx] + C \sec[c + dx])^2 \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^3 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^6 (A + B \sec[c + dx] + C \sec[c + dx])^2 \right. \\
& \quad \left(\frac{4(20C - A \cos[c] - 9B \cos[c] + 29C \cos[c]) \text{Csc}\left[\frac{c}{2}\right] \sec\left[\frac{c}{2}\right] \sec[c]}{5d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 9B \sin\left[\frac{dx}{2}\right] - 29C \sin\left[\frac{dx}{2}\right])}{5d} \right) + \\
& \quad \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] - 6B \sin\left[\frac{dx}{2}\right] + 11C \sin\left[\frac{dx}{2}\right])}{15d} + \\
& \quad \left. \frac{32C \sec[c] \sec[c + dx] \sin[dx]}{d} + \frac{8(A - 6B + 11C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{15d} + \frac{4(A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{5d} \right) / \\
& \left(\sqrt{\cos[c + dx]} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^3 \right)
\end{aligned}$$

- **Problem 1238: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[c + dx] + C \sec[c + dx]^2}{\cos[c + dx]^{7/2} (a + a \sec[c + dx])^3} dx$$

Optimal (type 4, 268 leaves, 9 steps):

$$\frac{(9A - 49B + 119C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right] + (3A - 13B + 33C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{10a^3d} + \frac{6a^3d}{(3A - 13B + 33C) \operatorname{Sin}[c + dx]} - \frac{(9A - 49B + 119C) \operatorname{Sin}[c + dx]}{6a^3d \operatorname{Cos}[c + dx]^{3/2}} - \frac{(A - B + C) \operatorname{Sin}[c + dx]}{5d \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Cos}[c + dx])^3} + \frac{(B - 2C) \operatorname{Sin}[c + dx]}{3ad \operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Cos}[c + dx])^2} - \frac{(9A - 49B + 119C) \operatorname{Sin}[c + dx]}{30d \operatorname{Cos}[c + dx]^{3/2} (a^3 + a^3 \operatorname{Cos}[c + dx])}$$

Result (type 5, 2248 leaves):

$$\frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} \left(9iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \right. \\ \left. \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) - \\ \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} 49iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) + \\ \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^3} 119iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ \operatorname{Sec}[c + dx] (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) \right] / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) \right)$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(3 i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i d x} \left(2 \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]}}{\left(-i d \left(1 + e^{2 i d x}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i d x}\right) \operatorname{Sin}[c]\right)}\right) - \right. \\
& \left. \left(4 A \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + d x] \right. \right. \\
& \left. \left. \left(\left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}\right] \right. \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}}{\left. \right)}\right) / \right. \\
& \left. \left(d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^3\right) + \right. \\
& \left. \left(52 B \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}[c + d x] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}\right] \right. \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}}{\left. \right)}\right) / \right. \\
& \left. \left(3 d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^3\right) - \right. \\
& \left. \left(44 C \operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \right. \\
& \left. \left. \operatorname{Sec}[c + d x] \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}\right] \right. \right. \\
& \left. \left. \frac{\sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]}}}{\left. \right)}\right) / \right. \\
& \left. \left(d \left(A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]\right) \sqrt{1 + \operatorname{Cot}[c]^2} \left(a + a \operatorname{Sec}[c + d x]\right)^3\right) + \right. \\
& \left. \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^6 \left(A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2\right) \left(-\frac{4 \left(-20 B + 60 C + 9 A \operatorname{Cos}[c] - 29 B \operatorname{Cos}[c] + 59 C \operatorname{Cos}[c]\right) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5 d}\right) - \right. \right.
\end{aligned}$$

$$\frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 \left(A \operatorname{Sin}\left[\frac{dx}{2}\right] - B \operatorname{Sin}\left[\frac{dx}{2}\right] + C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 \left(6 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 11 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 16 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{15 d} -$$

$$\frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] \left(9 A \operatorname{Sin}\left[\frac{dx}{2}\right] - 29 B \operatorname{Sin}\left[\frac{dx}{2}\right] + 59 C \operatorname{Sin}\left[\frac{dx}{2}\right]\right)}{5 d} + \frac{32 C \operatorname{Sec}[c] \operatorname{Sec}[c + dx]^2 \operatorname{Sin}[dx]}{3 d} +$$

$$\frac{32 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \left(C \operatorname{Sin}[c] + 3 B \operatorname{Sin}[dx] - 9 C \operatorname{Sin}[dx]\right)}{3 d} - \frac{8 \left(6 A - 11 B + 16 C\right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{15 d} -$$

$$\frac{4 \left(A - B + C\right) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{5 d} \Bigg) \Bigg/ \left(\sqrt{\operatorname{Cos}[c + dx]} \left(A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]\right) \left(a + a \operatorname{Sec}[c + dx]\right)^3\right)$$

■ **Problem 1239: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]^{3/2} \left(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2\right)}{\left(a + a \operatorname{Sec}[c + dx]\right)^4} dx$$

Optimal (type 4, 278 leaves, 9 steps):

$$- \frac{\left(176 A - 57 B + 8 C\right) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{10 a^4 d} + \frac{\left(339 A - 108 B + 17 C\right) \operatorname{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{42 a^4 d} +$$

$$\frac{\left(339 A - 108 B + 17 C\right) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{42 a^4 d} - \frac{\left(43 A - 15 B + C\right) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{42 a^4 d \left(1 + \operatorname{Cos}[c + dx]\right)^2} -$$

$$\frac{\left(176 A - 57 B + 8 C\right) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{30 a^4 d \left(1 + \operatorname{Cos}[c + dx]\right)} - \frac{\left(A - B + C\right) \operatorname{Cos}[c + dx]^{9/2} \operatorname{Sin}[c + dx]}{7 d \left(a + a \operatorname{Cos}[c + dx]\right)^4} - \frac{\left(13 A - 6 B - C\right) \operatorname{Cos}[c + dx]^{7/2} \operatorname{Sin}[c + dx]}{35 a d \left(a + a \operatorname{Cos}[c + dx]\right)^3}$$

Result (type 5, 2319 leaves):

$$- \frac{1}{5 \left(A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]\right) \left(a + a \operatorname{Sec}[c + dx]\right)^4}$$

$$+ \frac{352 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \left(A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2\right)}{\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i dx} \left(2 \left(1 + e^{2 i dx}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i dx}\right) \operatorname{Sin}[c]\right)}\right.}\right.}$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]}\right)\right) \Bigg/ \left(3 i d \left(1 + e^{2 i dx}\right) \operatorname{Cos}[c] - 3 d \left(-1 + e^{2 i dx}\right) \operatorname{Sin}[c]\right) -$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} \left(\operatorname{Cos}[c] + i \operatorname{Sin}[c]\right)^2\right] \sqrt{e^{-i dx} \left(2 \left(1 + e^{2 i dx}\right) \operatorname{Cos}[c] + 2 i \left(-1 + e^{2 i dx}\right) \operatorname{Sin}[c]\right)}\right.}$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]}\right)\right) \Bigg/ \left(-i d \left(1 + e^{2 i dx}\right) \operatorname{Cos}[c] + d \left(-1 + e^{2 i dx}\right) \operatorname{Sin}[c]\right) \Bigg) +$$

$$\frac{1}{5 \left(A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]\right) \left(a + a \operatorname{Sec}[c + dx]\right)^4} + \frac{114 i B \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]}{5 \left(A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx]\right) \left(a + a \operatorname{Sec}[c + dx]\right)^4}$$

$$\begin{aligned}
& \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \frac{1}{5(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \text{Sec}[c + dx])^4} 16iC \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Csc}\left[\frac{c}{2}\right] \text{Sec}\left[\frac{c}{2}\right] \\
& \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \left(904A \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \text{Sec}\left[\frac{c}{2}\right] \text{Sec}[c + dx]^2 \right. \\
& \quad \left. (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(7d(A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + dx])^4 \right) + \\
& \left(288B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \text{Sec}\left[\frac{c}{2}\right] \right. \\
& \quad \left. \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(7 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^4 \right) - \\
& \left(136 C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]]^2 \right] \sec \left[\frac{c}{2} \right] \right. \\
& \quad \left. \sec [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} (a + a \sec [c + d x])^4 \right) + \\
& \left(\cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 (A + B \sec [c + d x] + C \sec [c + d x]^2) \left(\frac{16 (96 A - 37 B + 8 C + 80 A \cos [c] - 20 B \cos [c]) \operatorname{Csc} [c]}{5 d} + \frac{64 A \cos [d x] \sin [c]}{3 d} - \right. \right. \\
& \quad \frac{4 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^7 (A \sin \left[\frac{d x}{2} \right] - B \sin \left[\frac{d x}{2} \right] + C \sin \left[\frac{d x}{2} \right])}{7 d} + \frac{16 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right] (96 A \sin \left[\frac{d x}{2} \right] - 37 B \sin \left[\frac{d x}{2} \right] + 8 C \sin \left[\frac{d x}{2} \right])}{5 d} + \\
& \quad \frac{8 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^5 (33 A \sin \left[\frac{d x}{2} \right] - 26 B \sin \left[\frac{d x}{2} \right] + 19 C \sin \left[\frac{d x}{2} \right])}{35 d} - \\
& \quad \frac{8 \sec \left[\frac{c}{2} \right] \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^3 (629 A \sin \left[\frac{d x}{2} \right] - 363 B \sin \left[\frac{d x}{2} \right] + 167 C \sin \left[\frac{d x}{2} \right])}{105 d} + \frac{64 A \cos [c] \sin [d x]}{3 d} - \\
& \quad \left. \left. \frac{8 (629 A - 363 B + 167 C) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^2 \tan \left[\frac{c}{2} \right]}{105 d} + \frac{8 (33 A - 26 B + 19 C) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^4 \tan \left[\frac{c}{2} \right]}{35 d} - \frac{4 (A - B + C) \sec \left[\frac{c}{2} + \frac{d x}{2} \right]^6 \tan \left[\frac{c}{2} \right]}{7 d} \right) \right) / \\
& (\cos [c + d x]^{3/2} (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^4)
\end{aligned}$$

- **Problem 1240: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2)}{(a + a \sec [c + d x])^4} dx$$

Optimal (type 4, 244 leaves, 8 steps):

$$\begin{aligned}
& \frac{(57 A - 8 B - C) \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{10 a^4 d} - \frac{(108 A - 17 B - 4 C) \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{42 a^4 d} - \frac{(141 A - 29 B - 13 C) \cos [c + d x]^{3/2} \sin [c + d x]}{210 a^4 d (1 + \cos [c + d x])^2} - \\
& \frac{(108 A - 17 B - 4 C) \sqrt{\cos [c + d x]} \sin [c + d x]}{42 a^4 d (1 + \cos [c + d x])} - \frac{(A - B + C) \cos [c + d x]^{7/2} \sin [c + d x]}{7 d (a + a \cos [c + d x])^4} - \frac{(11 A - 4 B - 3 C) \cos [c + d x]^{5/2} \sin [c + d x]}{35 a d (a + a \cos [c + d x])^3}
\end{aligned}$$

Result (type 5, 2286 leaves):

$$\begin{aligned}
& \frac{1}{5 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^4} \\
& 114 i A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{5 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^4} 16 i B \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{5 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a + a \sec [c + d x])^4} 2 i C \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{Sec} \left[\frac{c}{2} \right] \\
& \operatorname{Sec} [c + d x]^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left(288 A \cos \left[\frac{c}{2} + \frac{d x}{2} \right]^8 \operatorname{Csc} \left[\frac{c}{2} \right] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\operatorname{Cot} [c]]]^2 \right] \operatorname{Sec} \left[\frac{c}{2} \right] \operatorname{Sec} [c + d x]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left((A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(7d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(136B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(21d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(32C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \text{Csc}\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(21d (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \sec[c + dx])^4 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left(-\frac{16(37A - 8B - C + 20A \cos[c]) \text{Csc}[c]}{5d} - \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (37A \sin\left[\frac{dx}{2}\right] - 8B \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{5d} + \right. \\
& \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{7d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (26A \sin\left[\frac{dx}{2}\right] - 19B \sin\left[\frac{dx}{2}\right] + 12C \sin\left[\frac{dx}{2}\right])}{35d} + \\
& \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (363A \sin\left[\frac{dx}{2}\right] - 167B \sin\left[\frac{dx}{2}\right] + 41C \sin\left[\frac{dx}{2}\right])}{105d} + \frac{8(363A - 167B + 41C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{105d} - \\
& \left. \left. \frac{8(26A - 19B + 12C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{35d} + \frac{4(A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \tan\left[\frac{c}{2}\right]}{7d} \right) \right) / \\
& \left(\cos[c + dx]^{3/2} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4 \right)
\end{aligned}$$

Problem 1241: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 4, 232 leaves, 8 steps):

$$\begin{aligned} & - \frac{(8A + B) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^4d} + \frac{(17A + 4B + 3C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{42a^4d} - \frac{(83A + B - 15C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{210a^4d(1 + \operatorname{Cos}[c + dx])^2} + \\ & \frac{(8A + B) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10a^4d(1 + \operatorname{Cos}[c + dx])} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^{5/2} \operatorname{Sin}[c + dx]}{7d(a + a \operatorname{Cos}[c + dx])^4} - \frac{(9A - 2B - 5C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{35ad(a + a \operatorname{Cos}[c + dx])^3} \end{aligned}$$

Result (type 5, 1862 leaves):

$$\begin{aligned} & - \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} \\ & 16iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) - \\ & \frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} 2iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \\ & \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\ & \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) - \\ & \left(136A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \right) \end{aligned}$$

$$\begin{aligned}
& \left((A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(32 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(8 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(7 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(\frac{16 (8 A + B) \operatorname{Csc}[c]}{5 d} + \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (8 A \sin\left[\frac{dx}{2}\right] + B \sin\left[\frac{dx}{2}\right])}{5 d} \right. \right. \\
& \left. \left. \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (167 A \sin\left[\frac{dx}{2}\right] - 41 B \sin\left[\frac{dx}{2}\right] - 15 C \sin\left[\frac{dx}{2}\right])}{105 d} - \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{7 d} \right. \right. \\
& \left. \left. \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (19 A \sin\left[\frac{dx}{2}\right] - 12 B \sin\left[\frac{dx}{2}\right] + 5 C \sin\left[\frac{dx}{2}\right])}{35 d} - \frac{8 (167 A - 41 B - 15 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{105 d} \right. \right. \\
& \left. \left. \frac{8 (19 A - 12 B + 5 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{35 d} - \frac{4 (A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \tan\left[\frac{c}{2}\right]}{7 d} \right) \right) / \\
& (\cos[c + dx]^{3/2} (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4)
\end{aligned}$$

■ **Problem 1242:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\operatorname{Cos}[c + dx]^{3/2} (a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$-\frac{(A - C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^4 d} + \frac{(4A + 3B + 4C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{42 a^4 d} + \frac{(41A + 15B - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{210 a^4 d (1 + \operatorname{Cos}[c + dx])^2} +$$

$$\frac{(A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10 a^4 d (1 + \operatorname{Cos}[c + dx])} - \frac{(A - B + C) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{7 d (a + a \operatorname{Cos}[c + dx])^4} - \frac{(A - C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{5 a d (a + a \operatorname{Cos}[c + dx])^3}$$

Result (type 5, 1862 leaves):

$$-\frac{1}{5 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4}$$

$$2 i A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) +$$

$$\frac{1}{5 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} 2 i C \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])}\right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2c] + i e^{2 i dx} \operatorname{Sin}[2c]} \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) -$$

$$\left(32 A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \right.$$

$$\left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(8 B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(7 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) - \\
& \left(32 C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c + dx]^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} (a + a \sec[c + dx])^4 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \quad \left(\frac{16 (A - C) \csc[c]}{5 d} - \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (12 A \sin\left[\frac{dx}{2}\right] - 5 B \sin\left[\frac{dx}{2}\right] - 2 C \sin\left[\frac{dx}{2}\right])}{35 d} + \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{5 d} \right. \\
& \quad \left. \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (41 A \sin\left[\frac{dx}{2}\right] + 15 B \sin\left[\frac{dx}{2}\right] - C \sin\left[\frac{dx}{2}\right])}{105 d} + \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{7 d} \right. \\
& \quad \left. \left. \frac{8 (41 A + 15 B - C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{105 d} - \frac{8 (12 A - 5 B - 2 C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{35 d} + \frac{4 (A - B + C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \tan\left[\frac{c}{2}\right]}{7 d} \right) \right) / \\
& (\cos[c + dx]^{3/2} (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4)
\end{aligned}$$

■ **Problem 1243: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\operatorname{Cos}[c + dx]^{5/2} (a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 4, 234 leaves, 8 steps):

$$\frac{(B + 8C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10 a^4 d} + \frac{(3A + 4B + 17C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{42 a^4 d} + \frac{(15A - B - 83C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{210 a^4 d (1 + \operatorname{Cos}[c + dx])^2} - \frac{(B + 8C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{10 a^4 d (1 + \operatorname{Cos}[c + dx])} - \frac{(A - B + C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{7 d (a + a \operatorname{Cos}[c + dx])^4} + \frac{(5A + 2B - 9C) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{35 a d (a + a \operatorname{Cos}[c + dx])^3}$$

Result (type 5, 1862 leaves):

$$\frac{1}{5 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} \left(2iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) + \right. \\ \left. \frac{1}{5 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} 16iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \right. \\ \left. \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\ \left. \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right. \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right. \\ \left. \left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) - \right. \\ \left. \left(8A \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 \right. \right. \\ \left. \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) \right)$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(7d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{1+\cot^2[c]} (a+a \sec[c+dx])^4 \right) - \\
& \left(32B \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c+dx]^2 (A+B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(21d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{1+\cot^2[c]} (a+a \sec[c+dx])^4 \right) - \\
& \left(136C \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \csc\left[\frac{c}{2}\right] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] \sec\left[\frac{c}{2}\right] \right. \\
& \quad \left. \sec[c+dx]^2 (A+B \sec[c+dx] + C \sec[c+dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(21d (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) \sqrt{1+\cot^2[c]} (a+a \sec[c+dx])^4 \right) + \\
& \left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A+B \sec[c+dx] + C \sec[c+dx]^2) \left(-\frac{16(B+8C) \csc[c]}{5d} + \right. \right. \\
& \quad \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (15A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] - 83C \sin\left[\frac{dx}{2}\right])}{105d} + \frac{8 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (5A \sin\left[\frac{dx}{2}\right] + 2B \sin\left[\frac{dx}{2}\right] - 9C \sin\left[\frac{dx}{2}\right])}{35d} \\
& \quad \frac{4 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^7 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{7d} - \frac{16 \sec\left[\frac{c}{2}\right] \sec\left[\frac{c}{2} + \frac{dx}{2}\right] (B \sin\left[\frac{dx}{2}\right] + 8C \sin\left[\frac{dx}{2}\right])}{5d} + \\
& \quad \left. \left. \frac{8(15A-B-83C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \tan\left[\frac{c}{2}\right]}{105d} + \frac{8(5A+2B-9C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \tan\left[\frac{c}{2}\right]}{35d} - \frac{4(A-B+C) \sec\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \tan\left[\frac{c}{2}\right]}{7d} \right) \right) / \\
& (\cos[c+dx])^{3/2} (A+2C+2B \cos[c+dx] + A \cos[2c+2dx]) (a+a \sec[c+dx])^4
\end{aligned}$$

■ **Problem 1244:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2}{\operatorname{Cos}[c + dx]^{7/2} (a + a \operatorname{Sec}[c + dx])^4} dx$$

Optimal (type 4, 276 leaves, 9 steps):

$$\frac{(A + 8B - 57C) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{10a^4 d} + \frac{(4A + 17B - 108C) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{42a^4 d} -$$

$$\frac{(A + 8B - 57C) \operatorname{Sin}[c + dx]}{10a^4 d \sqrt{\operatorname{Cos}[c + dx]}} + \frac{(13A + 29B - 141C) \operatorname{Sin}[c + dx]}{210a^4 d \sqrt{\operatorname{Cos}[c + dx]} (1 + \operatorname{Cos}[c + dx])^2} + \frac{(4A + 17B - 108C) \operatorname{Sin}[c + dx]}{42a^4 d \sqrt{\operatorname{Cos}[c + dx]} (1 + \operatorname{Cos}[c + dx])} -$$

$$\frac{(A - B + C) \operatorname{Sin}[c + dx]}{7d \sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Cos}[c + dx])^4} + \frac{(3A + 4B - 11C) \operatorname{Sin}[c + dx]}{35ad \sqrt{\operatorname{Cos}[c + dx]} (a + a \operatorname{Cos}[c + dx])^3}$$

Result (type 5, 2316 leaves):

$$\frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4}$$

$$2iA \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) +$$

$$\frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} 16iB \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\operatorname{Sec}[c + dx]^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2)$$

$$\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (3id(1 + e^{2idx}) \operatorname{Cos}[c] - 3d(-1 + e^{2idx}) \operatorname{Sin}[c]) - \right.$$

$$\left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \operatorname{Cos}[c] + 2i(-1 + e^{2idx}) \operatorname{Sin}[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \operatorname{Cos}[2c] + i e^{2idx} \operatorname{Sin}[2c]} \right) / (-id(1 + e^{2idx}) \operatorname{Cos}[c] + d(-1 + e^{2idx}) \operatorname{Sin}[c]) \right) -$$

$$\frac{1}{5(A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) (a + a \operatorname{Sec}[c + dx])^4} 114iC \operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right]^8 \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right]$$

$$\begin{aligned}
& \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \text{Cos}[c] + 2 i (-1 + e^{2idx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \text{Cos}[2c] + i e^{2idx} \text{Sin}[2c]} \right) / (3 i d (1 + e^{2idx}) \text{Cos}[c] - 3 d (-1 + e^{2idx}) \text{Sin}[c]) - \right. \\
& \quad \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \text{Cos}[c] + 2 i (-1 + e^{2idx}) \text{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \text{Cos}[2c] + i e^{2idx} \text{Sin}[2c]} \right) / (-i d (1 + e^{2idx}) \text{Cos}[c] + d (-1 + e^{2idx}) \text{Sin}[c]) \right) - \\
& \left(32 A \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^8 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \text{Sec} \left[\frac{c}{2} \right] \text{Sec}[c + dx]^2 \right. \\
& \quad \left. (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + dx])^4 \right) - \\
& \left(136 B \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^8 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \text{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left. \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(21 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + dx])^4 \right) + \\
& \left(288 C \text{Cos} \left[\frac{c}{2} + \frac{dx}{2} \right]^8 \text{Csc} \left[\frac{c}{2} \right] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]^2 \right] \text{Sec} \left[\frac{c}{2} \right] \right. \\
& \quad \left. \text{Sec}[c + dx]^2 (A + B \text{Sec}[c + dx] + C \text{Sec}[c + dx]^2) \text{Sec}[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(7 d (A + 2 C + 2 B \text{Cos}[c + dx] + A \text{Cos}[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} (a + a \text{Sec}[c + dx])^4 \right) +
\end{aligned}$$

$$\left(\cos\left[\frac{c}{2} + \frac{dx}{2}\right]^8 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\ \left. \left(\frac{8(20C - A \cos[c] - 8B \cos[c] + 37C \cos[c]) \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}[c]}{5d} - \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^3 (A \sin\left[\frac{dx}{2}\right] + 83B \sin\left[\frac{dx}{2}\right] - 237C \sin\left[\frac{dx}{2}\right])}{105d} \right. \right. \\ \left. \left. + \frac{16 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right] (A \sin\left[\frac{dx}{2}\right] + 8B \sin\left[\frac{dx}{2}\right] - 37C \sin\left[\frac{dx}{2}\right])}{5d} + \frac{4 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^7 (A \sin\left[\frac{dx}{2}\right] - B \sin\left[\frac{dx}{2}\right] + C \sin\left[\frac{dx}{2}\right])}{7d} \right. \right. \\ \left. \left. + \frac{8 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^5 (2A \sin\left[\frac{dx}{2}\right] - 9B \sin\left[\frac{dx}{2}\right] + 16C \sin\left[\frac{dx}{2}\right])}{35d} + \frac{64C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \sin[dx]}{d} \right. \right. \\ \left. \left. + \frac{8(A + 83B - 237C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \operatorname{Tan}\left[\frac{c}{2}\right]}{105d} + \frac{8(2A - 9B + 16C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^4 \operatorname{Tan}\left[\frac{c}{2}\right]}{35d} + \frac{4(A - B + C) \operatorname{Sec}\left[\frac{c}{2} + \frac{dx}{2}\right]^6 \operatorname{Tan}\left[\frac{c}{2}\right]}{7d} \right) \right) / \\ (\cos[c + dx]^{3/2} (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (a + a \sec[c + dx])^4)$$

■ **Problem 1248: Result unnecessarily involves higher level functions.**

$$\int \cos[c + dx]^{3/2} \sqrt{a + a \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{2\sqrt{a} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \\ \frac{2a(A+3B) \sin[c+dx]}{3d \sqrt{\cos[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2A \sqrt{\cos[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \sin[c+dx]}{3d}$$

Result (type 5, 192 leaves):

$$\left(4 \sqrt{\cos[c+dx]} (C + B \cos[c+dx] + A \cos[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} \right. \\ \left. \left(-3i C e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i C e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \right. \right. \\ \left. \left. + (2A + 3B + A \cos[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / (3d(A + 2C + 2B \cos[c+dx] + A \cos[2(c+dx)]))$$

■ **Problem 1249: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\cos[c + dx]} \sqrt{a + a \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 3, 139 leaves, 5 steps):

$$\frac{\sqrt{a} (2B + C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d} + \frac{a(2A - C) \sin[c+dx]}{d \sqrt{\cos[c+dx]} \sqrt{a+a \sec[c+dx]}} + \frac{C \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{d \sqrt{\cos[c+dx]}}$$

Result (type 5, 209 leaves):

$$\left(2 (C + B \cos[c+dx] + A \cos[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1 + \sec[c+dx])} \left(-3i(2B + C) e^{\frac{1}{2}i(c+dx)} \cos[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i(2B + C) e^{\frac{3}{2}i(c+dx)} \cos[c+dx] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3(C + 2A \cos[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) / (3d \sqrt{\cos[c+dx]} (A + 2C + 2B \cos[c+dx] + A \cos[2(c+dx)]))$$

■ **Problem 1250: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\frac{\sqrt{a} (8A + 4B + 3C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} + \frac{a(4B + C) \sin[c+dx]}{4d \cos[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}} + \frac{C \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{2d \cos[c+dx]^{3/2}}$$

Result (type 5, 172 leaves):

$$\frac{1}{12d} \sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1 + \sec[c+dx])} \left(-3i(8A + 4B + 3C) e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i(8A + 4B + 3C) e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3 \sec[c+dx] (4B + 3C + 2C \sec[c+dx]) \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1251: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 3, 199 leaves, 6 steps):

$$\frac{\sqrt{a} (8A + 6B + 5C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8d} +$$

$$\frac{a(6B+C) \sin[c+dx]}{12d \cos[c+dx]^{5/2} \sqrt{a+a \sec[c+dx]}} + \frac{a(8A+6B+5C) \sin[c+dx]}{8d \cos[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}} + \frac{c \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{3d \cos[c+dx]^{5/2}}$$

Result (type 5, 194 leaves):

$$\frac{1}{24d} \sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\left(-3i(8A+6B+5C) e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i(8A+6B+5C) e^{\frac{3}{2}i(c+dx)} \right.$$

$$\left. \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + \sec[c+dx] (3(8A+6B+5C) + 2(6B+5C) \sec[c+dx] + 8C \sec[c+dx]^2) \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1252: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+a \sec[c+dx]} (A+B \sec[c+dx] + C \sec[c+dx]^2)}{\cos[c+dx]^{5/2}} dx$$

Optimal (type 3, 247 leaves, 7 steps):

$$\frac{\sqrt{a} (48A + 40B + 35C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{64d} + \frac{a(8B+C) \sin[c+dx]}{24d \cos[c+dx]^{7/2} \sqrt{a+a \sec[c+dx]}} +$$

$$\frac{a(48A+40B+35C) \sin[c+dx]}{96d \cos[c+dx]^{5/2} \sqrt{a+a \sec[c+dx]}} + \frac{a(48A+40B+35C) \sin[c+dx]}{64d \cos[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}} + \frac{c \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{4d \cos[c+dx]^{7/2}}$$

Result (type 5, 334 leaves):

$$\frac{1}{128d} (48A + 40B + 35C) \sqrt{\cos[c+dx]}$$

$$\left(-2i e^{\frac{1}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - \frac{2}{3} i e^{\frac{3}{2}i(c+dx)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] \right)$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])} + \frac{1}{d} \sqrt{\cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{a(1+\sec[c+dx])}$$

$$\left(\frac{1}{4} C \sec[c+dx]^4 \sin\left[\frac{1}{2}(c+dx)\right] + \frac{1}{24} \sec[c+dx]^3 \left(8B \sin\left[\frac{1}{2}(c+dx)\right] + 7C \sin\left[\frac{1}{2}(c+dx)\right] \right) \right) +$$

$$\frac{1}{64} \sec[c+dx] \left(48A \sin\left[\frac{1}{2}(c+dx)\right] + 40B \sin\left[\frac{1}{2}(c+dx)\right] + 35C \sin\left[\frac{1}{2}(c+dx)\right] \right) +$$

$$\frac{1}{96} \sec[c+dx]^2 \left(48A \sin\left[\frac{1}{2}(c+dx)\right] + 40B \sin\left[\frac{1}{2}(c+dx)\right] + 35C \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1256: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{5 / 2} (a+a \sec [c+d x])^{3 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 192 leaves, 6 steps):

$$\frac{2 a^{3 / 2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{d} + \frac{2 a^2 (12 A+20 B+15 C) \sin [c+d x]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+a \sec [c+d x]}} +$$

$$\frac{2 a (3 A+5 B) \sqrt{\cos [c+d x]} \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{15 d} + \frac{2 A \cos [c+d x]^{3 / 2} (a+a \sec [c+d x])^{3 / 2} \sin [c+d x]}{5 d}$$

Result (type 5, 224 leaves):

$$\frac{1}{15 d (A+2 C+2 B \cos [c+d x]+A \cos [2 (c+d x)])} a \sqrt{\cos [c+d x]} (1+\cos [c+d x]) (C+B \cos [c+d x]+A \cos [c+d x]^2) \sec \left[\frac{1}{2} (c+d x)\right]^3$$

$$\sqrt{a (1+\sec [c+d x])} \left(-30 i C e^{\frac{1}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - 10 i C e^{\frac{3}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] +\right.$$

$$\left.(39 A+50 B+30 C+2 (9 A+5 B) \cos [c+d x]+3 A \cos [2 (c+d x)]) \sin \left[\frac{1}{2} (c+d x)\right]\right)$$

■ **Problem 1257: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{3 / 2} (a+a \sec [c+d x])^{3 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 3, 197 leaves, 6 steps):

$$\frac{a^{3 / 2} (2 B+3 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{d} + \frac{a^2 (8 A+6 B-3 C) \sin [c+d x]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+a \sec [c+d x]}} -$$

$$\frac{a (2 A-3 C) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{3 d \sqrt{\cos [c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]} (a+a \sec [c+d x])^{3 / 2} \sin [c+d x]}{3 d}$$

Result (type 5, 242 leaves):

$$\frac{1}{3 d \sqrt{\cos [c+d x]} (A+2 C+2 B \cos [c+d x]+A \cos [2 (c+d x)])} a (1+\cos [c+d x]) (C+B \cos [c+d x]+A \cos [c+d x]^2) \sec \left[\frac{1}{2} (c+d x)\right]^3$$

$$\sqrt{a (1+\sec [c+d x])} \left(-3 i (2 B+3 C) e^{\frac{1}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - i (2 B+3 C) e^{\frac{3}{2} i (c+d x)}\right.$$

$$\left.\cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + (A+3 C+2 (5 A+3 B) \cos [c+d x]+A \cos [2 (c+d x)]) \sin \left[\frac{1}{2} (c+d x)\right]\right)$$

■ **Problem 1258: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\cos[c+dx]} (a + a \sec[c+dx])^{3/2} (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

Optimal (type 3, 203 leaves, 6 steps):

$$\frac{a^{3/2} (8A + 12B + 7C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{4d} +$$

$$\frac{a^2 (8A - 4B - 5C) \sin[c+dx]}{4d \sqrt{\cos[c+dx]} \sqrt{a+a \sec[c+dx]}} + \frac{a (4B + 3C) \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{4d \sqrt{\cos[c+dx]}} + \frac{C (a+a \sec[c+dx])^{3/2} \sin[c+dx]}{2d \sqrt{\cos[c+dx]}}$$

Result (type 5, 256 leaves):

$$\frac{1}{12d \cos[c+dx]^{3/2} (A + 2C + 2B \cos[c+dx] + A \cos[2(c+dx)])}$$

$$a (1 + \cos[c+dx]) (C + B \cos[c+dx] + A \cos[c+dx]^2) \sec\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a(1 + \sec[c+dx])}$$

$$\left(-3i (8A + 12B + 7C) e^{\frac{1}{2}i(c+dx)} \cos[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - i (8A + 12B + 7C) e^{\frac{3}{2}i(c+dx)} \cos[c+dx]^2 \right.$$

$$\left. \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + 3((4B + 7C) \cos[c+dx] + 2(2A + C + 2A \cos[2(c+dx)])) \sin\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1259: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \sec[c+dx])^{3/2} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 3, 201 leaves, 6 steps):

$$\frac{a^{3/2} (24A + 14B + 11C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+a \sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{8d} +$$

$$\frac{a^2 (24A + 30B + 19C) \sin[c+dx]}{24d \cos[c+dx]^{3/2} \sqrt{a+a \sec[c+dx]}} + \frac{a (2B + C) \sqrt{a+a \sec[c+dx]} \sin[c+dx]}{4d \cos[c+dx]^{3/2}} + \frac{C (a+a \sec[c+dx])^{3/2} \sin[c+dx]}{3d \cos[c+dx]^{3/2}}$$

Result (type 5, 261 leaves):

$$\frac{1}{24 d \cos [c+d x]^{5 / 2} (A+2 C+2 B \cos [c+d x]+A \cos [2(c+d x)])}$$

$$a(1+\cos [c+d x])\left(C+B \cos [c+d x]+A \cos [c+d x]^2\right) \sec \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{a(1+\sec [c+d x])}$$

$$\left(-3 i(24 A+14 B+11 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-i(24 A+14 B+11 C) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^3\right. \\ \left.\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(8 C+2(6 B+11 C) \cos [c+d x]+3(8 A+14 B+11 C) \cos [c+d x]^2) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 1260: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2)}{\cos [c+d x]^{3 / 2}} dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{3 / 2}(112 A+88 B+75 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{64 d} + \frac{a^2(48 A+56 B+39 C) \sin [c+d x]}{96 d \cos [c+d x]^{5 / 2} \sqrt{a+a \sec [c+d x]}} +$$

$$\frac{a^2(112 A+88 B+75 C) \sin [c+d x]}{64 d \cos [c+d x]^{3 / 2} \sqrt{a+a \sec [c+d x]}} + \frac{a(8 B+3 C) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{24 d \cos [c+d x]^{5 / 2}} + \frac{C(a+a \sec [c+d x])^{3 / 2} \sin [c+d x]}{4 d \cos [c+d x]^{5 / 2}}$$

Result (type 5, 281 leaves):

$$\frac{1}{192 d \cos [c+d x]^{7 / 2} (A+2 C+2 B \cos [c+d x]+A \cos [2(c+d x)])} a(1+\cos [c+d x])\left(C+B \cos [c+d x]+A \cos [c+d x]^2\right)$$

$$\sec \left[\frac{1}{2}(c+d x)\right]^3 \sqrt{a(1+\sec [c+d x])}\left(-3 i(112 A+88 B+75 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-\right. \\ \left.i(112 A+88 B+75 C) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(48 C+8(8 B+15 C) \cos [c+d x]+2(48 A+88 B+75 C) \cos [c+d x]^2+3(112 A+88 B+75 C) \cos [c+d x]^3) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 1261: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2)}{\cos [c+d x]^{5 / 2}} dx$$

Optimal (type 3, 303 leaves, 8 steps):

$$\frac{a^{3/2} (176 A + 150 B + 133 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{128 d} +$$

$$\frac{a^2 (80 A + 90 B + 67 C) \operatorname{Sin}[c+dx]}{240 d \operatorname{Cos}[c+dx]^{7/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (176 A + 150 B + 133 C) \operatorname{Sin}[c+dx]}{192 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (176 A + 150 B + 133 C) \operatorname{Sin}[c+dx]}{128 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a (10 B + 3 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{40 d \operatorname{Cos}[c+dx]^{7/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{7/2}}$$

Result (type 5, 301 leaves):

$$\frac{1}{1920 d \operatorname{Cos}[c+dx]^{9/2} (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2(c+dx)])}$$

$$a (1 + \operatorname{Cos}[c+dx]) (C + B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^3 \sqrt{a (1 + \operatorname{Sec}[c+dx])}$$

$$\left(-15 i (176 A + 150 B + 133 C) e^{\frac{1}{2} i (c+dx)} \operatorname{Cos}[c+dx]^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - 5 i (176 A + 150 B + 133 C) e^{\frac{3}{2} i (c+dx)} \right.$$

$$\operatorname{Cos}[c+dx]^5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] + (384 C + 48 (10 B + 19 C) \operatorname{Cos}[c+dx] + 8 (80 A + 150 B + 133 C) \operatorname{Cos}[c+dx]^2 +$$

$$\left. 10 (176 A + 150 B + 133 C) \operatorname{Cos}[c+dx]^3 + 15 (176 A + 150 B + 133 C) \operatorname{Cos}[c+dx]^4 \right) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right]$$

■ **Problem 1265: Result unnecessarily involves higher level functions.**

$$\int \operatorname{Cos}[c+dx]^{7/2} (a+a \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{2 a^{5/2} C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{d} +$$

$$\frac{2 a^3 (160 A + 224 B + 245 C) \operatorname{Sin}[c+dx]}{105 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{2 a^2 (40 A + 56 B + 35 C) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{105 d} +$$

$$\frac{2 a (5 A + 7 B) \operatorname{Cos}[c+dx]^{3/2} (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{35 d} + \frac{2 A \operatorname{Cos}[c+dx]^{5/2} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{7 d}$$

Result (type 5, 343 leaves):

$$\left(C \cos [c+d x]^{9/2} \left(-2 i e^{\frac{1}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \frac{2}{3} i e^{\frac{3}{2} i (c+d x)} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] \right) \right. \\ \left. \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right) / (2 d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) + \\ \left(\cos [c+d x]^{9/2} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 (a(1+\operatorname{Sec}[c+d x]))^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\ \left. \left(\frac{5}{8} (3 A+4 B+4 C) \sin \left[\frac{1}{2}(c+d x)\right] + \frac{1}{24} (11 A+10 B+4 C) \sin \left[\frac{3}{2}(c+d x)\right] + \frac{1}{40} (5 A+2 B) \sin \left[\frac{5}{2}(c+d x)\right] + \frac{1}{56} A \sin \left[\frac{7}{2}(c+d x)\right] \right) \right) / \\ (d (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))$$

■ **Problem 1266: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{5/2} (a+a \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 243 leaves, 7 steps):

$$\frac{a^{5/2} (2 B+5 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{d} + \\ \frac{a^3 (64 A+70 B+15 C) \sin [c+d x]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}} - \frac{a^2 (16 A+10 B-15 C) \sqrt{a+a \operatorname{Sec}[c+d x]} \sin [c+d x]}{15 d \sqrt{\cos [c+d x]}} + \\ \frac{2 a (A+B) \sqrt{\cos [c+d x]} (a+a \operatorname{Sec}[c+d x])^{3/2} \sin [c+d x]}{3 d} + \frac{2 A \cos [c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{5/2} \sin [c+d x]}{5 d}$$

Result (type 5, 271 leaves):

$$\frac{1}{60 d \sqrt{\cos [c+d x]} (A+2 C+2 B \cos [c+d x]+A \cos [2(c+d x)])} - a^2 (1+\cos [c+d x])^2 (C+B \cos [c+d x]+A \cos [c+d x]^2) \\ \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\operatorname{Sec}[c+d x])} \left(-30 i (2 B+5 C) e^{\frac{1}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+d x)}\right] - \right. \\ \left. 10 i (2 B+5 C) e^{\frac{3}{2} i (c+d x)} \cos [c+d x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+d x)}\right] + \right. \\ \left. (28 A+10 B+30 C+(181 A+160 B+60 C) \cos [c+d x]+2(14 A+5 B) \cos [2(c+d x)]+3 A \cos [3(c+d x)]) \sin \left[\frac{1}{2}(c+d x)\right] \right)$$

■ **Problem 1267: Result unnecessarily involves higher level functions.**

$$\int \cos [c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{5/2} (8A + 20B + 19C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{4d} +$$

$$\frac{a^3 (56A + 12B - 27C) \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} - \frac{a^2 (8A - 12B - 21C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]}} -$$

$$\frac{a (4A - 3C) (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{6d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{2A \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{3d}$$

Result (type 5, 282 leaves):

$$\frac{1}{24d \operatorname{Cos}[c+dx]^{3/2} (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[c+dx])^2$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-3i (8A + 20B + 19C) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - \right.$$

$$\left. i (8A + 20B + 19C) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx]^2 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + \right.$$

$$\left. (32A + 12B + 6C + 3(2A + 4B + 11C) \operatorname{Cos}[c+dx] + 4(8A + 3B) \operatorname{Cos}[2(c+dx)] + 2A \operatorname{Cos}[3(c+dx)]) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1268: Result unnecessarily involves higher level functions.**

$$\int \sqrt{\operatorname{Cos}[c+dx]} (a+a \operatorname{Sec}[c+dx])^{5/2} (A+B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{5/2} (40A + 38B + 25C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{8d} + \frac{a^3 (24A - 54B - 49C) \operatorname{Sin}[c+dx]}{24d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (24A + 42B + 31C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{a (6B + 5C) (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{12d \sqrt{\operatorname{Cos}[c+dx]}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{3d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 5, 276 leaves):

$$\frac{1}{48d \operatorname{Cos}[c+dx]^{5/2} (A + 2C + 2B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[c+dx])^2$$

$$\operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-3i (40A + 38B + 25C) e^{\frac{1}{2}i(c+dx)} \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2i(c+dx)}\right] - \right.$$

$$\left. i (40A + 38B + 25C) e^{\frac{3}{2}i(c+dx)} \operatorname{Cos}[c+dx]^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2i(c+dx)}\right] + \right.$$

$$\left. (8C + 2(6B + 17C) \operatorname{Cos}[c+dx] + 3(8A + 22B + 25C) \operatorname{Cos}[c+dx]^2 + 48A \operatorname{Cos}[c+dx]^3) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1269: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\sqrt{\operatorname{Cos}[c + d x]}} dx$$

Optimal (type 3, 253 leaves, 7 steps):

$$\frac{a^{5/2} (304 A + 200 B + 163 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{64 d} + \frac{a^3 (432 A + 392 B + 299 C) \operatorname{Sin}[c+dx]}{192 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^2 (16 A + 24 B + 17 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{32 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{a (8 B + 5 C) (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{24 d \operatorname{Cos}[c+dx]^{3/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{4 d \operatorname{Cos}[c+dx]^{3/2}}$$

Result (type 5, 283 leaves):

$$\frac{1}{384 d \operatorname{Cos}[c+dx]^{7/2} (A + 2 C + 2 B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[2(c+dx)])} a^2 (1 + \operatorname{Cos}[c+dx])^2 (C + B \operatorname{Cos}[c+dx] + A \operatorname{Cos}[c+dx]^2) \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5$$

$$\sqrt{a(1 + \operatorname{Sec}[c+dx])} \left(-3 i (304 A + 200 B + 163 C) e^{\frac{1}{2} i (c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i (c+dx)}\right] - \right.$$

$$\left. i (304 A + 200 B + 163 C) e^{\frac{3}{2} i (c+dx)} \operatorname{Cos}[c+dx]^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i (c+dx)}\right] + \right.$$

$$\left. (48 C + 8 (8 B + 23 C) \operatorname{Cos}[c+dx] + (96 A + 272 B + 326 C) \operatorname{Cos}[c+dx]^2 + (528 A + 600 B + 489 C) \operatorname{Cos}[c+dx]^3) \operatorname{Sin}\left[\frac{1}{2}(c+dx)\right] \right)$$

■ **Problem 1270: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + a \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 3, 301 leaves, 8 steps):

$$\frac{a^{5/2} (400 A + 326 B + 283 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+a \operatorname{Sec}[c+dx]}}\right] \sqrt{\operatorname{Cos}[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}}{128 d} + \frac{a^3 (1040 A + 950 B + 787 C) \operatorname{Sin}[c+dx]}{960 d \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} +$$

$$\frac{a^3 (400 A + 326 B + 283 C) \operatorname{Sin}[c+dx]}{128 d \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+a \operatorname{Sec}[c+dx]}} + \frac{a^2 (80 A + 110 B + 79 C) \sqrt{a+a \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{240 d \operatorname{Cos}[c+dx]^{5/2}} +$$

$$\frac{a (2 B + C) (a+a \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{8 d \operatorname{Cos}[c+dx]^{5/2}} + \frac{C (a+a \operatorname{Sec}[c+dx])^{5/2} \operatorname{Sin}[c+dx]}{5 d \operatorname{Cos}[c+dx]^{5/2}}$$

Result (type 5, 305 leaves):

$$\frac{1}{3840 d \cos [c+d x]^{9 / 2} (A+2 C+2 B \cos [c+d x]+A \cos [2(c+d x)])} a^2 (1+\cos [c+d x])^2 (C+B \cos [c+d x]+A \cos [c+d x]^2) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \sqrt{a(1+\sec [c+d x])}$$

$$\left(-15 i(400 A+326 B+283 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-5 i(400 A+326 B+283 C) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(384 C+48(10 B+29 C) \cos [c+d x]+8(80 A+230 B+283 C) \cos [c+d x]^2+10(272 A+326 B+283 C) \cos [c+d x]^3+15(400 A+326 B+283 C) \cos [c+d x]^4) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 1271: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+a \sec [c+d x])^{5 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\cos [c+d x]^{5 / 2}} dx$$

Optimal (type 3, 353 leaves, 9 steps):

$$\frac{a^{5 / 2} (1304 A+1132 B+1015 C) \operatorname{ArcSinh}\left[\frac{\sqrt{a} \tan [c+d x]}{\sqrt{a+a \sec [c+d x]}}\right] \sqrt{\cos [c+d x]} \sqrt{\sec [c+d x]}}{512 d} +$$

$$\frac{a^3 (680 A+628 B+545 C) \sin [c+d x]}{960 d \cos [c+d x]^{7 / 2} \sqrt{a+a \sec [c+d x]}} + \frac{a^3 (1304 A+1132 B+1015 C) \sin [c+d x]}{768 d \cos [c+d x]^{5 / 2} \sqrt{a+a \sec [c+d x]}} + \frac{a^3 (1304 A+1132 B+1015 C) \sin [c+d x]}{512 d \cos [c+d x]^{3 / 2} \sqrt{a+a \sec [c+d x]}} +$$

$$\frac{a^2 (120 A+156 B+115 C) \sqrt{a+a \sec [c+d x]} \sin [c+d x]}{480 d \cos [c+d x]^{7 / 2}} + \frac{a(12 B+5 C)(a+a \sec [c+d x])^{3 / 2} \sin [c+d x]}{60 d \cos [c+d x]^{7 / 2}} + \frac{C(a+a \sec [c+d x])^{5 / 2} \sin [c+d x]}{6 d \cos [c+d x]^{7 / 2}}$$

Result (type 5, 328 leaves):

$$\frac{1}{15360 d \cos [c+d x]^{11 / 2} (A+2 C+2 B \cos [c+d x]+A \cos [2(c+d x)])} a^2 (1+\cos [c+d x])^2 (C+B \cos [c+d x]+A \cos [c+d x]^2) \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5$$

$$\sqrt{a(1+\sec [c+d x])} \left(-15 i(1304 A+1132 B+1015 C) e^{\frac{1}{2} i(c+d x)} \cos [c+d x]^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, -e^{2 i(c+d x)}\right]-5 i(1304 A+1132 B+1015 C) e^{\frac{3}{2} i(c+d x)} \cos [c+d x]^6 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, -e^{2 i(c+d x)}\right]+(1280 C+128(12 B+35 C) \cos [c+d x]+48(40 A+29(4 B+5 C)) \cos [c+d x]^2+8(920 A+1132 B+1015 C) \cos [c+d x]^3+10(1304 A+1132 B+1015 C) \cos [c+d x]^4+15(1304 A+1132 B+1015 C) \cos [c+d x]^5) \sin \left[\frac{1}{2}(c+d x)\right]\right)$$

■ **Problem 1275: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sqrt{a+a \sec [c+d x]}} dx$$

Optimal (type 3, 178 leaves, 7 steps) :

$$\frac{2 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{\sqrt{a} d} - \frac{\sqrt{2} (A-B+C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{\sqrt{a} d} + \frac{2 A \operatorname{Sin}[c+d x]}{d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+a \operatorname{Sec}[c+d x]}}$$

Result (type 3, 388 leaves) :

$$\frac{1}{2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a(1+\operatorname{Sec}[c+d x])} \sqrt{\operatorname{Sin}[c+d x]^2}} \operatorname{Sin}[c+d x] \left(2 \sqrt{2} (A-B) \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}[1+\operatorname{Cos}[c+d x]] - 4 C \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[\sqrt{\operatorname{Cos}[c+d x]} (1+\operatorname{Cos}[c+d x])\right] + \sqrt{2} C \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[(1+\operatorname{Cos}[c+d x])^2\right] - 2 \sqrt{2} A \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[2 \sqrt{1+\operatorname{Cos}[c+d x]} + \sqrt{2-2 \operatorname{Cos}[c+d x]^2}\right] + 2 \sqrt{2} B \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[2 \sqrt{1+\operatorname{Cos}[c+d x]} + \sqrt{2-2 \operatorname{Cos}[c+d x]^2}\right] + 4 C \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[1+\operatorname{Cos}[c+d x] + \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sin}[c+d x]^2}\right] - \sqrt{2} C \sqrt{1+\operatorname{Cos}[c+d x]} \operatorname{Log}\left[3+2 \operatorname{Cos}[c+d x] - \operatorname{Cos}[c+d x]^2 + 2 \sqrt{2} \sqrt{1+\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sin}[c+d x]^2}\right] + 4 A \sqrt{\operatorname{Sin}[c+d x]^2} \right)$$

■ **Problem 1283: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2}{\sqrt{\operatorname{Cos}[c+d x]} (a+a \operatorname{Sec}[c+d x])^{3/2}} dx$$

Optimal (type 3, 189 leaves, 7 steps) :

$$\frac{2 C \operatorname{ArcSinh}\left[\frac{\sqrt{a} \operatorname{Tan}[c+d x]}{\sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{a^{3/2} d} + \frac{(3 A+B-5 C) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{\operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[c+d x]}}\right] \sqrt{\operatorname{Cos}[c+d x]} \sqrt{\operatorname{Sec}[c+d x]}}{2 \sqrt{2} a^{3/2} d} - \frac{(A-B+C) \operatorname{Sin}[c+d x]}{2 d \operatorname{Cos}[c+d x]^{3/2} (a+a \operatorname{Sec}[c+d x])^{3/2}}$$

Result (type 3, 572 leaves) :

$$\begin{aligned}
& - \left(2 \cos \left[\frac{1}{2} (c + dx) \right] \sqrt{\cos [c + dx]} (A + B \sec [c + dx] + C \sec [c + dx]^2) \left(A \sin \left[\frac{1}{2} (c + dx) \right] - B \sin \left[\frac{1}{2} (c + dx) \right] + C \sin \left[\frac{1}{2} (c + dx) \right] \right) \right) / \\
& \quad \left(d (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) (a (1 + \sec [c + dx]))^{3/2} \right) - \\
& \left(\sqrt{2} (3A + B - C) \cos \left[\frac{1}{2} (c + dx) \right]^2 \sqrt{\cos [c + dx]} \sqrt{1 + \cos [c + dx]} \left(\log [1 + \cos [c + dx]] - \log \left[2 \sqrt{1 + \cos [c + dx]} + \sqrt{2 - 2 \cos [c + dx]^2} \right] \right) \right) \\
& \quad \left(A + B \sec [c + dx] + C \sec [c + dx]^2 \right) \sin [c + dx] \Big) / \\
& \left(d \sqrt{1 - \cos [c + dx]^2} (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) (a (1 + \sec [c + dx]))^{3/2} \right) - \\
& \left(2C \cos \left[\frac{1}{2} (c + dx) \right]^2 \sqrt{\cos [c + dx]} \sqrt{1 + \cos [c + dx]} \right. \\
& \quad \left(-\sqrt{2} \log [(1 + \cos [c + dx])^2] + 4 \log [\sqrt{\cos [c + dx]} + \cos [c + dx]^{3/2}] - 4 \log [1 + \cos [c + dx] + \sqrt{1 + \cos [c + dx]} \sqrt{1 - \cos [c + dx]^2}] + \right. \\
& \quad \left. \sqrt{2} \log [3 + 2 \cos [c + dx] - \cos [c + dx]^2 + 2 \sqrt{2} \sqrt{1 + \cos [c + dx]} \sqrt{1 - \cos [c + dx]^2}] \right) (A + B \sec [c + dx] + C \sec [c + dx]^2) \sin [c + dx] \Big) / \\
& \left(d \sqrt{1 - \cos [c + dx]^2} (A + 2C + 2B \cos [c + dx] + A \cos [2c + 2dx]) (a (1 + \sec [c + dx]))^{3/2} \right)
\end{aligned}$$

■ **Problem 1289: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec [c + dx] + C \sec [c + dx]^2}{\sqrt{\cos [c + dx]} (a + a \sec [c + dx])^{5/2}} dx$$

Optimal (type 3, 183 leaves, 5 steps):

$$\frac{(19A + 5B + 3C) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sqrt{\sec [c + dx]} \sin [c + dx]}{\sqrt{2} \sqrt{a + a \sec [c + dx]}} \right] \sqrt{\cos [c + dx]} \sqrt{\sec [c + dx]}}{16 \sqrt{2} a^{5/2} d} - \frac{(A - B + C) \sin [c + dx]}{4 d \cos [c + dx]^{3/2} (a + a \sec [c + dx])^{5/2}} - \frac{(9A - B - 7C) \sin [c + dx]}{16 a d \cos [c + dx]^{3/2} (a + a \sec [c + dx])^{3/2}}$$

Result (type 3, 379 leaves):

$$\left(\cos\left[\frac{1}{2}(c+dx)\right]^5 (A+B\sec[c+dx]+C\sec[c+dx]^2) \left(\sec\left[\frac{1}{2}(c+dx)\right]^4 \left(A\sin\left[\frac{1}{2}(c+dx)\right] - B\sin\left[\frac{1}{2}(c+dx)\right] + C\sin\left[\frac{1}{2}(c+dx)\right] \right) + \frac{1}{2}\sec\left[\frac{1}{2}(c+dx)\right]^2 \left(-13A\sin\left[\frac{1}{2}(c+dx)\right] + 5B\sin\left[\frac{1}{2}(c+dx)\right] + 3C\sin\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) /$$

$$\left(d\sqrt{\cos[c+dx]} (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2} - \right.$$

$$\left. \left((19A+5B+3C)\cos\left[\frac{1}{2}(c+dx)\right]^4 \sqrt{1+\cos[c+dx]} \left(\log[1+\cos[c+dx]] - \log\left[2\sqrt{1+\cos[c+dx]} + \sqrt{2-2\cos[c+dx]^2}\right] \right) \right) \right)$$

$$\left((A+B\sec[c+dx]+C\sec[c+dx]^2)\sin[c+dx] \right) /$$

$$\left(2\sqrt{2}d\sqrt{\cos[c+dx]}\sqrt{1-\cos[c+dx]^2} (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2} \right)$$

■ **Problem 1290: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B\sec[c+dx]+C\sec[c+dx]^2}{\cos[c+dx]^{3/2} (a+a\sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 241 leaves, 8 steps):

$$\frac{2C \operatorname{ArcSinh}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]} + (5A+3B-43C) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{\sec[c+dx]}\sin[c+dx]}{\sqrt{2}\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2}d} + \frac{(A-B+C)\sin[c+dx]}{4d\cos[c+dx]^{5/2}(a+a\sec[c+dx])^{5/2}} + \frac{(5A+3B-11C)\sin[c+dx]}{16ad\cos[c+dx]^{3/2}(a+a\sec[c+dx])^{3/2}}$$

Result (type 3, 625 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^5 (A+B\sec[c+dx]+C\sec[c+dx]^2) \left(\frac{1}{2}\sec\left[\frac{1}{2}(c+dx)\right]^2 \left(5A\sin\left[\frac{1}{2}(c+dx)\right]+3B\sin\left[\frac{1}{2}(c+dx)\right]-11C\sin\left[\frac{1}{2}(c+dx)\right]\right) \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(c+dx)\right]^4 \left(-A\sin\left[\frac{1}{2}(c+dx)\right]+B\sin\left[\frac{1}{2}(c+dx)\right]-C\sin\left[\frac{1}{2}(c+dx)\right]\right)\right)\right) / \\
& \left(d \sqrt{\cos[c+dx]} (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2} \right) + \\
& \left(\cos\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{\sec[c+dx]} (A+B\sec[c+dx]+C\sec[c+dx]^2) \left(-\frac{1}{\sqrt{1-\cos[c+dx]^2}} \sqrt{2} (5A+3B-11C) \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \right. \right. \\
& \quad \left. \left(\log[1+\cos[c+dx]] - \log\left[2\sqrt{1+\cos[c+dx]} + \sqrt{2-2\cos[c+dx]^2}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] - \right. \\
& \quad \left. \frac{1}{\sqrt{1-\cos[c+dx]^2}} 16C \sqrt{\cos[c+dx]} \sqrt{1+\cos[c+dx]} \left(-\sqrt{2} \log[(1+\cos[c+dx])^2] + \right. \right. \\
& \quad \left. \left. 4 \log\left[\sqrt{\cos[c+dx]} + \cos[c+dx]^{3/2}\right] - 4 \log\left[1+\cos[c+dx] + \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] + \sqrt{2} \right. \right. \\
& \quad \left. \left. \log\left[3+2\cos[c+dx] - \cos[c+dx]^2 + 2\sqrt{2} \sqrt{1+\cos[c+dx]} \sqrt{1-\cos[c+dx]^2}\right] \right) \sec\left[\frac{1}{2}(c+dx)\right] \sqrt{\sec[c+dx]} \sin[c+dx] \right) / \\
& \left(4d(A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) (a(1+\sec[c+dx]))^{5/2} \right)
\end{aligned}$$

■ **Problem 1291: Result more than twice size of optimal antiderivative.**

$$\int \frac{A+B\sec[c+dx]+C\sec[c+dx]^2}{\cos[c+dx]^{5/2} (a+a\sec[c+dx])^{5/2}} dx$$

Optimal (type 3, 294 leaves, 9 steps):

$$\begin{aligned}
& \frac{(2B-5C) \operatorname{ArcSinh}\left[\frac{\sqrt{a}\tan[c+dx]}{\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{a^{5/2}d} + \\
& \frac{(3A-43B+115C) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{\sec[c+dx]}\sin[c+dx]}{\sqrt{2}\sqrt{a+a\sec[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\sin[c+dx]}{4d\cos[c+dx]^{7/2}(a+a\sec[c+dx])^{5/2}} + \\
& \frac{(A+7B-15C)\sin[c+dx]}{16ad\cos[c+dx]^{5/2}(a+a\sec[c+dx])^{3/2}} + \frac{(3A-11B+35C)\sin[c+dx]}{16a^2d\cos[c+dx]^{3/2}\sqrt{a+a\sec[c+dx]}}
\end{aligned}$$

Result (type 3, 651 leaves):

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} (c + d x) \right] \right)^5 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \left(16 C \sec [c + d x] \sin \left[\frac{1}{2} (c + d x) \right] + \sec \left[\frac{1}{2} (c + d x) \right]^4 \left(A \sin \left[\frac{1}{2} (c + d x) \right] - B \sin \left[\frac{1}{2} (c + d x) \right] + C \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) + \\
& \left. \frac{1}{2} \sec \left[\frac{1}{2} (c + d x) \right]^2 \left(3 A \sin \left[\frac{1}{2} (c + d x) \right] - 11 B \sin \left[\frac{1}{2} (c + d x) \right] + 19 C \sin \left[\frac{1}{2} (c + d x) \right] \right) \right) \Bigg) / \\
& \left(d \sqrt{\cos [c + d x]} (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \right) + \\
& \left(\cos \left[\frac{1}{2} (c + d x) \right] \right)^5 \sqrt{\sec [c + d x]} (A + B \sec [c + d x] + C \sec [c + d x]^2) \left(- \frac{1}{\sqrt{1 - \cos [c + d x]^2}} \sqrt{2} (3 A - 11 B + 35 C) \sqrt{\cos [c + d x]} \sqrt{1 + \cos [c + d x]} \right. \\
& \left. \left(\log [1 + \cos [c + d x]] - \log \left[2 \sqrt{1 + \cos [c + d x]} + \sqrt{2 - 2 \cos [c + d x]^2} \right] \right) \sec \left[\frac{1}{2} (c + d x) \right] \sqrt{\sec [c + d x]} \sin [c + d x] - \right. \\
& \left. \frac{1}{2 \sqrt{1 - \cos [c + d x]^2}} (32 B - 80 C) \sqrt{\cos [c + d x]} \sqrt{1 + \cos [c + d x]} \left(-\sqrt{2} \log [(1 + \cos [c + d x])^2] + \right. \right. \\
& \left. \left. 4 \log \left[\sqrt{\cos [c + d x]} + \cos [c + d x]^{3/2} \right] - 4 \log \left[1 + \cos [c + d x] + \sqrt{1 + \cos [c + d x]} \sqrt{1 - \cos [c + d x]^2} \right] + \sqrt{2} \right. \right. \\
& \left. \left. \log \left[3 + 2 \cos [c + d x] - \cos [c + d x]^2 + 2 \sqrt{2} \sqrt{1 + \cos [c + d x]} \sqrt{1 - \cos [c + d x]^2} \right] \right) \sec \left[\frac{1}{2} (c + d x) \right] \sqrt{\sec [c + d x]} \sin [c + d x] \right) \Bigg) / \\
& \left(4 d (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) (a (1 + \sec [c + d x]))^{5/2} \right)
\end{aligned}$$

■ **Problem 1294: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{5/2} (a + b \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 4, 116 leaves, 6 steps):

$$\frac{2 (3 a A + 5 b B + 5 a C) \operatorname{EllipticE} \left[\frac{1}{2} (c + d x), 2 \right]}{5 d} + \frac{2 (A b + a B + 3 b C) \operatorname{EllipticF} \left[\frac{1}{2} (c + d x), 2 \right]}{3 d} + \\
\frac{2 (A b + a B) \sqrt{\cos [c + d x]} \sin [c + d x]}{3 d} + \frac{2 a A \cos [c + d x]^{3/2} \sin [c + d x]}{5 d}$$

Result (type 5, 1569 leaves):

$$\left(\cos [c + d x]^{7/2} (a + b \sec [c + d x]) (A + B \sec [c + d x] + C \sec [c + d x]^2) \right. \\
\left. \left(- \frac{4 (3 a A + 5 b B + 5 a C) \cot [c]}{5 d} + \frac{4 (A b + a B) \cos [d x] \sin [c]}{3 d} + \frac{2 a A \cos [2 d x] \sin [2 c]}{5 d} + \frac{4 (A b + a B) \cos [c] \sin [d x]}{3 d} + \right. \right. \\
\left. \left. \frac{2 a A \cos [2 c] \sin [2 d x]}{5 d} \right) \right) / \left((b + a \cos [c + d x]) (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \right) -$$

$$\begin{aligned}
& \left(4 A b \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x]) \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x]) (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 a B \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x]) \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x]) (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 b C \cos [c+d x]^3 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x]) \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b+a \cos [c+d x]) (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(6 a A \cos [c+d x]^3 \operatorname{Csc}[c] (a+b \operatorname{Sec}[c+d x]) (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) \right) /
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (5 d (b + a \cos[c + d x]) (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(2 b B \cos[c + d x]^3 \csc[c] (a + b \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (d (b + a \cos[c + d x]) (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(2 a C \cos[c + d x]^3 \csc[c] (a + b \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) / (d (b + a \cos[c + d x]) (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]))$$

■ **Problem 1295: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{3/2} (a + b \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 106 leaves, 6 steps):

$$\frac{2 (Ab + aB - bC) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{d} + \frac{2 (3bB + a(A + 3C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3d} + \frac{2bC \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2aA \sqrt{\cos[c + d x]} \sin[c + d x]}{3d}$$

Result (type 5, 1904 leaves):

$$\frac{1}{(b + a \cos[c + d x]) (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} i A b \cos[c + d x]^3 \text{Csc}[c] (a + b \sec[c + d x]) (A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) +$$

$$\frac{1}{(b + a \cos[c + d x]) (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} i a B \cos[c + d x]^3 \text{Csc}[c] (a + b \sec[c + d x])$$

$$(A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right.$$

$$\left. \left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \right) -$$

$$\begin{aligned}
& \frac{1}{(b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i b C \cos[c + dx]^3 \csc[c] (a + b \sec[c + dx]) \\
& \left(\frac{(A + B \sec[c + dx] + C \sec[c + dx])^2}{\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \left(\cos[c + dx]^{7/2} (a + b \sec[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx])^2 \left(-\frac{2 (A b + a B - 2 b C + A b \cos[2c] + a B \cos[2c]) \csc[c] \sec[c]}{d} + \right. \right. \\
& \left. \left. \frac{4 a A \cos[dx] \sin[c]}{3 d} + \frac{4 a A \cos[c] \sin[dx]}{3 d} + \frac{4 b C \sec[c] \sec[c + dx] \sin[dx]}{d} \right) \right) / \\
& ((b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) - \\
& \left(4 a A \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx])^2 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 b B \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx])^2 \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a C \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right.
\end{aligned}$$

$$\frac{(A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]}}{\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]}}} \Bigg/$$

$$\left(d (b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right)$$

■ **Problem 1296: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + dx]} (a + b \sec[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 112 leaves, 6 steps):

$$-\frac{2(bB - a(A - C)) \text{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{2(3Ab + 3aB + bC) \text{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2bc \sin[c + dx]}{3d \cos[c + dx]^{3/2}} + \frac{2(bB + aC) \sin[c + dx]}{d \sqrt{\cos[c + dx]}}$$

Result (type 5, 1909 leaves):

$$\frac{1}{(b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i a A \cos[c + dx]^3 \csc[c] (a + b \sec[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \Bigg/ (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \Bigg/ (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\frac{1}{(b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i b B \cos[c + dx]^3 \csc[c] (a + b \sec[c + dx])$$

$$(A + B \sec[c + dx] + C \sec[c + dx]^2)$$

$$\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \Bigg/ (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \Bigg/ (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\begin{aligned}
& \frac{1}{(b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i a C \cos[c + dx]^3 \csc[c] (a + b \sec[c + dx]) \\
& \left(\frac{(A + B \sec[c + dx] + C \sec[c + dx]^2)}{\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) + \\
& \left(\cos[c + dx]^{7/2} (a + b \sec[c + dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2) \left(-\frac{2 (a A - 2 b B - 2 a C + a A \cos[2c]) \csc[c] \sec[c]}{d} + \right. \right. \\
& \left. \left. \frac{4 b C \sec[c] \sec[c + dx]^2 \sin[dx]}{3 d} + \frac{4 \sec[c] \sec[c + dx] (b C \sin[c] + 3 b B \sin[dx] + 3 a C \sin[dx])}{3 d} \right) \right) / \\
& ((b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) - \\
& \left(4 a b \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a b \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx]) (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 b C \cos[c + dx]^3 \csc[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx]) \right.
\end{aligned}$$

$$\left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3d (b + a \operatorname{Cos}[c + dx]) (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right)$$

■ **Problem 1300: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[c + dx]^{7/2} (a + b \operatorname{Sec}[c + dx])^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) dx$$

Optimal (type 4, 202 leaves, 7 steps):

$$\frac{2(6aAb + 3a^2B + 5b^2B + 10abc) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \\ \frac{2(14abB + 7b^2(A + 3C) + a^2(5A + 7C)) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{21d} + \frac{2(4Ab^2 + 14abB + a^2(5A + 7C)) \sqrt{\operatorname{Cos}[c + dx]} \operatorname{Sin}[c + dx]}{21d} + \\ \frac{2a(4Ab + 7aB) \operatorname{Cos}[c + dx]^{3/2} \operatorname{Sin}[c + dx]}{35d} + \frac{2A \sqrt{\operatorname{Cos}[c + dx]} (b + a \operatorname{Cos}[c + dx])^2 \operatorname{Sin}[c + dx]}{7d}$$

Result (type 5, 2361 leaves):

$$\left(\operatorname{Cos}[c + dx]^{9/2} (a + b \operatorname{Sec}[c + dx])^2 (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \left(-\frac{4(6aAb + 3a^2B + 5b^2B + 10abc) \operatorname{Cot}[c]}{5d} + \right. \right. \\ \left. \frac{(23a^2A + 28Ab^2 + 56abB + 28a^2C) \operatorname{Cos}[dx] \operatorname{Sin}[c]}{21d} + \frac{2a(2Ab + aB) \operatorname{Cos}[2dx] \operatorname{Sin}[2c]}{5d} + \frac{a^2A \operatorname{Cos}[3dx] \operatorname{Sin}[3c]}{7d} + \right. \\ \left. \frac{(23a^2A + 28Ab^2 + 56abB + 28a^2C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{21d} + \frac{2a(2Ab + aB) \operatorname{Cos}[2c] \operatorname{Sin}[2dx]}{5d} + \frac{a^2A \operatorname{Cos}[3c] \operatorname{Sin}[3dx]}{7d} \right) \Bigg) / \\ \left((b + a \operatorname{Cos}[c + dx])^2 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \right) - \\ \left(20a^2A \operatorname{Cos}[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^2 \right. \\ \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(21d (b + a \operatorname{Cos}[c + dx])^2 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) -$$

$$\begin{aligned}
& \left(4 A b^2 \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(8 a b B \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^2 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 b^2 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) -
\end{aligned}$$

$$\left(12 a A b \cos [c+d x]^4 \csc [c] (a+b \sec [c+d x])^2 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) /$$

$$(5 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) -$$

$$\left(6 a^2 B \cos [c+d x]^4 \csc [c] (a+b \sec [c+d x])^2 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) /$$

$$(5 d (b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) -$$

$$\left(2 b^2 B \cos [c + d x]^4 \csc [c] (a + b \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \text{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\left(\frac{\frac{\sin [d x + \text{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (d (b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) -$$

$$\left(4 a b C \cos [c + d x]^4 \csc [c] (a + b \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4} \right\}, \cos [d x + \text{ArcTan}[\tan [c]]]^2 \right] \sin [d x + \text{ArcTan}[\tan [c]]] \tan [c] \right) /$$

$$\left(\sqrt{1 - \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2} \sqrt{1 + \tan [c]^2} \right) -$$

$$\left(\frac{\frac{\sin [d x + \text{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1 + \tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan}[\tan [c]]] \sqrt{1 + \tan [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan}[\tan [c]]]} \sqrt{1 + \tan [c]^2}} \right) / (d (b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]))$$

Problem 1301: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos[c + dx]^{5/2} (a + b \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 186 leaves, 7 steps):

$$\frac{2(10abB + 5b^2(A - C) + a^2(3A + 5C)) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{5d} + \frac{2(a^2B + 3b^2B + 2ab(A + 3C)) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} + \frac{2a(2Ab + aB - 6bC) \sqrt{\cos[c + dx]} \sin[c + dx]}{3d} + \frac{2a^2(A - 5C) \cos[c + dx]^{3/2} \sin[c + dx]}{5d} + \frac{2C(b + a \cos[c + dx])^2 \sin[c + dx]}{d \sqrt{\cos[c + dx]}}$$

Result (type 5, 3011 leaves):

$$\frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{3ia^2 A \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +} \\ \frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{iab^2 \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2}{(A + B \sec[c + dx] + C \sec[c + dx]^2)} \\ \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +} \\ \frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{2iabB \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2}{(A + B \sec[c + dx] + C \sec[c + dx]^2)}$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{(b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} i a^2 C \cos [c + d x]^4 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^2 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{(b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} i b^2 C \cos [c + d x]^4 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^2 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \left(\cos [c + d x]^{9/2} (a + b \operatorname{Sec}[c + d x])^2 (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \right. \\
& \quad \left. \left(-1 / (5 d) 2 (3 a^2 A + 5 A b^2 + 10 a b B + 5 a^2 C - 10 b^2 C + 3 a^2 A \cos [2 c] + 5 A b^2 \cos [2 c] + 10 a b B \cos [2 c] + 5 a^2 C \cos [2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c] + \right. \right. \\
& \quad \left. \frac{4 a (2 A b + a B) \cos [d x] \sin [c]}{3 d} + \frac{2 a^2 A \cos [2 d x] \sin [2 c]}{5 d} + \frac{4 a (2 A b + a B) \cos [c] \sin [d x]}{3 d} + \frac{4 b^2 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \sin [d x]}{d} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \frac{2 a^2 A \cos [2 c] \sin [2 d x]}{5 d} \right) \right) / \left((b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \right) - \\
& \left(8 a A b \cos [c+d x]^4 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 (a+b \sec [c+d x])^2 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(4 a^2 B \cos [c+d x]^4 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 (a+b \sec [c+d x])^2 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(4 b^2 B \cos [c+d x]^4 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 (a+b \sec [c+d x])^2 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(8 a b C \cos [c+d x]^4 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]] \right]^2 (a+b \sec [c+d x])^2 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1 + \cot [c]^2} \right)
\end{aligned}$$

Problem 1302: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos[c + dx]^{3/2} (a + b \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 180 leaves, 7 steps):

$$\frac{2(a^2 B - b^2 B + 2ab(A - C)) \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), 2\right]}{d} + \frac{2(6abB + b^2(3A + C) + a^2(A + 3C)) \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), 2\right]}{3d} +$$

$$\frac{2b(3bB + 4aC) \sin[c + dx]}{3d \sqrt{\cos[c + dx]}} + \frac{2a^2(A - C) \sqrt{\cos[c + dx]} \sin[c + dx]}{3d} + \frac{2C(b + a \cos[c + dx])^2 \sin[c + dx]}{3d \cos[c + dx]^{3/2}}$$

Result (type 5, 2779 leaves):

$$\frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{2iaAb \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right.} \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) +$$

$$\frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{ia^2 B \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2}{(A + B \sec[c + dx] + C \sec[c + dx]^2)} \\ \left(\left(2e^{2idx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\ \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\frac{1}{(b + a \cos[c + dx])^2 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{ib^2 B \cos[c + dx]^4 \operatorname{Csc}[c] (a + b \sec[c + dx])^2}{(A + B \sec[c + dx] + C \sec[c + dx]^2)}$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \quad \frac{1}{(b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 2 i a b C \cos [c + d x]^4 \csc [c] (a + b \sec [c + d x])^2 \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \quad \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \quad \left(\cos [c + d x]^{9/2} (a + b \sec [c + d x])^2 (A + B \sec [c + d x] + C \sec [c + d x])^2 \right. \\
& \quad \left(-\frac{2 (2 a A b + a^2 B - 2 b^2 B - 4 a b C + 2 a A b \cos [2 c] + a^2 B \cos [2 c]) \csc [c] \sec [c]}{d} + \frac{4 a^2 A \cos [d x] \sin [c]}{3 d} + \frac{4 a^2 A \cos [c] \sin [d x]}{3 d} + \right. \\
& \quad \left. \frac{4 b^2 C \sec [c] \sec [c + d x]^2 \sin [d x]}{3 d} + \frac{4 \sec [c] \sec [c + d x] (b^2 C \sin [c] + 3 b^2 B \sin [d x] + 6 a b C \sin [d x])}{3 d} \right) / \\
& \quad ((b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) - \\
& \quad \left(4 a^2 A \cos [c + d x]^4 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]] \right]^2 (a + b \sec [c + d x])^2 \right. \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(3 d (b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(4 A b^2 \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]}}\right) / \\
& \quad \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(8 a b B \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]}}\right) / \\
& \quad \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^2 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]}}\right) / \\
& \quad \left(d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 b^2 C \cos [c+d x]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a+b \operatorname{Sec}[c+d x])^2 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]}}\right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^2 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right)
\end{aligned}$$

■ **Problem 1303: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c+dx]} (a+b \sec[c+dx])^2 (A+B \sec[c+dx]+C \sec[c+dx])^2 dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$\begin{aligned} & - \frac{2(10abB - 5a^2(A-C) + b^2(5A+3C)) \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), 2\right]}{5d} + \frac{2(3a^2B + b^2B + 2ab(3A+C)) \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), 2\right]}{3d} \\ & + \frac{2b(5bB + 4aC) \sin[c+dx]}{15d \cos[c+dx]^{3/2}} + \frac{2(5Ab^2 + 10abB + 4a^2C + 3b^2C) \sin[c+dx]}{5d \sqrt{\cos[c+dx]}} + \frac{2C(b+a \cos[c+dx])^2 \sin[c+dx]}{5d \cos[c+dx]^{5/2}} \end{aligned}$$

Result (type 5, 3017 leaves):

$$\begin{aligned} & \frac{1}{(b+a \cos[c+dx])^2 (A+2C+2B \cos[c+dx]+A \cos[2c+2dx])} \\ & \frac{i a^2 A \cos[c+dx]^4 \operatorname{Csc}[c] (a+b \sec[c+dx])^2 (A+B \sec[c+dx]+C \sec[c+dx])^2}{\left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2i dx}) \cos[c]+2i(-1+e^{2i dx}) \sin[c])} \right.} \right.} \\ & \left. \left. \sqrt{1+e^{2i dx} \cos[2c]+i e^{2i dx} \sin[2c]} \right) / (3id(1+e^{2i dx}) \cos[c]-3d(-1+e^{2i dx}) \sin[c]) - \right.} \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2i dx}) \cos[c]+2i(-1+e^{2i dx}) \sin[c])} \right. \right. \\ & \left. \left. \sqrt{1+e^{2i dx} \cos[2c]+i e^{2i dx} \sin[2c]} \right) / (-id(1+e^{2i dx}) \cos[c]+d(-1+e^{2i dx}) \sin[c]) \right) - } \\ & \frac{1}{(b+a \cos[c+dx])^2 (A+2C+2B \cos[c+dx]+A \cos[2c+2dx])} i A b^2 \cos[c+dx]^4 \operatorname{Csc}[c] (a+b \sec[c+dx])^2 \\ & \frac{(A+B \sec[c+dx]+C \sec[c+dx])^2}{\left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2i dx}) \cos[c]+2i(-1+e^{2i dx}) \sin[c])} \right.} \right.} \\ & \left. \left. \sqrt{1+e^{2i dx} \cos[2c]+i e^{2i dx} \sin[2c]} \right) / (3id(1+e^{2i dx}) \cos[c]-3d(-1+e^{2i dx}) \sin[c]) - \right.} \\ & \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2i dx}) \cos[c]+2i(-1+e^{2i dx}) \sin[c])} \right. \right. \\ & \left. \left. \sqrt{1+e^{2i dx} \cos[2c]+i e^{2i dx} \sin[2c]} \right) / (-id(1+e^{2i dx}) \cos[c]+d(-1+e^{2i dx}) \sin[c]) \right) - } \\ & \frac{1}{(b+a \cos[c+dx])^2 (A+2C+2B \cos[c+dx]+A \cos[2c+2dx])} 2iabB \cos[c+dx]^4 \operatorname{Csc}[c] (a+b \sec[c+dx])^2 \\ & \frac{(A+B \sec[c+dx]+C \sec[c+dx])^2}{\left(\left(2 e^{2i dx} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2i dx} (\cos[c]+i \sin[c])^2\right] \sqrt{e^{-i dx} (2(1+e^{2i dx}) \cos[c]+2i(-1+e^{2i dx}) \sin[c])} \right.} \right.} \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]\right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]\right)}\right) - \right. \\
& \frac{1}{(b + a \cos[c + d x])^2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} i a^2 C \cos[c + d x]^4 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^2 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]\right) - \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]\right)}\right) - \right. \right. \\
& \frac{1}{5 (b + a \cos[c + d x])^2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} 3 i b^2 C \cos[c + d x]^4 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^2 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]\right) - \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])}\right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]}}{\left(-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]\right)}\right) + \right. \right. \\
& \frac{1}{(b + a \cos[c + d x])^2 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])} \cos[c + d x]^{9/2} (a + b \operatorname{Sec}[c + d x])^2 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2 \\
& \left(-\frac{2 (5 a^2 A - 10 A b^2 - 20 a b B - 10 a^2 C - 6 b^2 C + 5 a^2 A \cos[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c]}{5 d} + \right. \\
& \left. \frac{4 b^2 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \sin[d x]}{5 d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (3 b^2 C \sin[c] + 5 b^2 B \sin[d x] + 10 a b C \sin[d x])}{15 d} + \frac{1}{15 d} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. 4 \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \left(5 b^2 B \sin[c] + 10 a b C \sin[c] + 15 A b^2 \sin[dx] + 30 a b B \sin[dx] + 15 a^2 C \sin[dx] + 9 b^2 C \sin[dx] \right) \right) - \\
& \left(8 a A b \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + dx])^2 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^2 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^2 B \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + dx])^2 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^2 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 b^2 B \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + dx])^2 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + dx])^2 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(8 a b C \cos[c + dx]^4 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + dx])^2 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) /
\end{aligned}$$

$$\left(3 d (b + a \cos [c + d x])^2 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right)$$

■ **Problem 1306: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c + d x]^{9/2} (a + b \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) dx$$

Optimal (type 4, 296 leaves, 8 steps):

$$\begin{aligned} & \frac{2 (27 a^2 b B + 15 b^3 B + 9 a b^2 (3 A + 5 C) + a^3 (7 A + 9 C)) \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), 2\right]}{15 d} + \\ & \frac{2 (5 a^3 B + 21 a b^2 B + 7 b^3 (A + 3 C) + 3 a^2 b (5 A + 7 C)) \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), 2\right]}{21 d} + \\ & \frac{2 (8 A b^3 + 15 a^3 B + 54 a b^2 B + 9 a^2 b (5 A + 7 C)) \sqrt{\cos [c + d x]} \sin [c + d x]}{63 d} + \frac{2 a (24 A b^2 + 99 a b B + 7 a^2 (7 A + 9 C)) \cos [c + d x]^{3/2} \sin [c + d x]}{315 d} + \\ & \frac{2 (2 A b + 3 a B) \sqrt{\cos [c + d x]} (b + a \cos [c + d x])^2 \sin [c + d x]}{21 d} + \frac{2 A \sqrt{\cos [c + d x]} (b + a \cos [c + d x])^3 \sin [c + d x]}{9 d} \end{aligned}$$

Result (type 5, 3237 leaves):

$$\begin{aligned} & \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \\ & \cos [c + d x]^{11/2} (a + b \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x]^2) \left(- \frac{4 (7 a^3 A + 27 a A b^2 + 27 a^2 b B + 15 b^3 B + 9 a^3 C + 45 a b^2 C) \cot [c]}{15 d} + \right. \\ & \frac{(69 a^2 A b + 28 A b^3 + 23 a^3 B + 84 a b^2 B + 84 a^2 b C) \cos [d x] \sin [c]}{21 d} + \frac{a (19 a^2 A + 54 A b^2 + 54 a b B + 18 a^2 C) \cos [2 d x] \sin [2 c]}{45 d} + \\ & \frac{a^2 (3 A b + a B) \cos [3 d x] \sin [3 c]}{7 d} + \frac{a^3 A \cos [4 d x] \sin [4 c]}{18 d} + \frac{(69 a^2 A b + 28 A b^3 + 23 a^3 B + 84 a b^2 B + 84 a^2 b C) \cos [c] \sin [d x]}{21 d} + \\ & \left. \frac{a (19 a^2 A + 54 A b^2 + 54 a b B + 18 a^2 C) \cos [2 c] \sin [2 d x]}{45 d} + \frac{a^2 (3 A b + a B) \cos [3 c] \sin [3 d x]}{7 d} + \frac{a^3 A \cos [4 c] \sin [4 d x]}{18 d} \right) - \\ & \left(20 a^2 A b \cos [c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (a + b \sec [c + d x])^3 \right. \\ & \frac{(A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan}[\cot [c]]]}{\sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1 + \sin [d x - \operatorname{ArcTan}[\cot [c]]]}}} \left. \right) / \\ & \left(7 d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \end{aligned}$$

$$\begin{aligned}
& \left(4 A b^3 \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(20 a^3 B \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(21 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 a b^2 B \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 a^2 b C \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(4 b^3 C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2\right) (a+b \operatorname{Sec}[c+d x])^3 \\
& \quad \left((A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \quad \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \quad \left(14 a^3 A \cos [c+d x]^5 \operatorname{Csc}[c] (a+b \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2\right) \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) / \\
& \quad \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2} \sqrt{1+\operatorname{Tan}[c]^2} \right) - \\
& \quad \left(\frac{\frac{\sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c]}{\sqrt{1+\operatorname{Tan}[c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \sqrt{1+\operatorname{Tan}[c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]} \sqrt{1+\operatorname{Tan}[c]^2}} \right) / \\
& \quad \left(15 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \right) - \\
& \quad \left(18 a A b^2 \cos [c+d x]^5 \operatorname{Csc}[c] (a+b \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]]\right]^2\right) \sin [d x+\operatorname{ArcTan}[\operatorname{Tan}[c]]] \operatorname{Tan}[c] \right) /
\end{aligned}$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) /$$

$$(5 d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(18 a^2 b B \cos[c + d x]^5 \csc[c] (a + b \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c] \right) /$$

$$\left(\frac{\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2}}{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}} \right) /$$

$$(5 d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(2 b^3 B \cos[c + d x]^5 \csc[c] (a + b \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left(\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \left(d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \right) - \left(6 a^3 C \cos[c + d x]^5 \text{Csc}[c] (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \left(\frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \left(5 d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \right) - \left(6 a b^2 C \cos[c + d x]^5 \text{Csc}[c] (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2 \right] \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\ \left. \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\ \left. \left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \left(d (b + a \text{Cos}[c + d x])^3 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \right) \right)$$

- **Problem 1307: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x]^{7/2} (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 277 leaves, 8 steps):

$$\frac{2 (3 a^3 B + 15 a b^2 B + 5 b^3 (A - C) + 3 a^2 b (3 A + 5 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \\ \frac{2 (21 a^2 b B + 21 b^3 B + 21 a b^2 (A + 3 C) + a^3 (5 A + 7 C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \\ \frac{2 a (21 a b B + 6 b^2 (3 A - 7 C) + a^2 (5 A + 7 C)) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{21 d} + \frac{2 a^2 (11 A b + 7 a B - 35 b C) \text{Cos}[c + d x]^{3/2} \text{Sin}[c + d x]}{35 d} + \\ \frac{2 a (A - 7 C) \sqrt{\text{Cos}[c + d x]} (b + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{7 d} + \frac{2 C (b + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{d \sqrt{\text{Cos}[c + d x]}}$$

Result (type 5, 3915 leaves):

$$\frac{1}{5 (b + a \text{Cos}[c + d x])^3 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} \\ 9 i a^2 A b \text{Cos}[c + d x]^5 \text{Csc}[c] (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \\ \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \right. \\ \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c]) - \right. \\ \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right) \right)$$

$$\begin{aligned}
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) - \\
& \frac{1}{(b + a \operatorname{Cos}[c + dx])^3 (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx])} i b^3 C \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^3 \\
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) \right) / (3 i d (1 + e^{2 i dx}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i dx}) \operatorname{Sin}[c])} \right. \right. \\
& \left. \left. \left. \sqrt{1 + e^{2 i dx} \operatorname{Cos}[2 c] + i e^{2 i dx} \operatorname{Sin}[2 c]} \right) \right) / (-i d (1 + e^{2 i dx}) \operatorname{Cos}[c] + d (-1 + e^{2 i dx}) \operatorname{Sin}[c]) \right) + \\
& \frac{1}{(b + a \operatorname{Cos}[c + dx])^3 (A + 2 C + 2 B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2 c + 2 dx])} \operatorname{Cos}[c + dx]^{11/2} (a + b \operatorname{Sec}[c + dx])^3 \\
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \right. \\
& \left. \left(-\frac{1}{5 d} 2 (9 a^2 A b + 5 A b^3 + 3 a^3 B + 15 a b^2 B + 15 a^2 b C - 10 b^3 C + 9 a^2 A b \operatorname{Cos}[2 c] + 5 A b^3 \operatorname{Cos}[2 c] + 3 a^3 B \operatorname{Cos}[2 c] + \right. \right. \\
& \left. \left. 15 a b^2 B \operatorname{Cos}[2 c] + 15 a^2 b C \operatorname{Cos}[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c] + \frac{a (23 a^2 A + 84 A b^2 + 84 a b B + 28 a^2 C) \operatorname{Cos}[dx] \operatorname{Sin}[c]}{21 d} + \right. \right. \\
& \left. \frac{2 a^2 (3 A b + a B) \operatorname{Cos}[2 dx] \operatorname{Sin}[2 c]}{5 d} + \frac{a^3 A \operatorname{Cos}[3 dx] \operatorname{Sin}[3 c]}{7 d} + \frac{a (23 a^2 A + 84 A b^2 + 84 a b B + 28 a^2 C) \operatorname{Cos}[c] \operatorname{Sin}[dx]}{21 d} + \right. \\
& \left. \frac{4 b^3 C \operatorname{Sec}[c] \operatorname{Sec}[c + dx] \operatorname{Sin}[dx]}{d} + \frac{2 a^2 (3 A b + a B) \operatorname{Cos}[2 c] \operatorname{Sin}[2 dx]}{5 d} + \frac{a^3 A \operatorname{Cos}[3 c] \operatorname{Sin}[3 dx]}{7 d} \right) - \\
& \left(20 a^3 A \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2 \right] (a + b \operatorname{Sec}[c + dx])^3 \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 a A b^2 \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 a^2 b B \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 b^3 B \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(4 a^3 C \cos [c+d x]^5 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x - \operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^3 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x - \operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x - \operatorname{ArcTan}[\cot [c]]]} \right)
\end{aligned}$$

$$\left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(3 d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1+\cot[c]^2} \right) -$$

$$\left(12 a b^2 C \cos[c + d x]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^3 \right.$$

$$\left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \right.$$

$$\left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \text{ArcTan}[\cot[c]]]} \right) /$$

$$\left(d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1+\cot[c]^2} \right)$$

■ **Problem 1308: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + d x]^{5/2} (a + b \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 267 leaves, 8 steps):

$$\frac{2 (15 a^2 b B - 5 b^3 B + 15 a b^2 (A - C) + a^3 (3 A + 5 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} +$$

$$\frac{2 (a^3 B + 9 a b^2 B + b^3 (3 A + C) + 3 a^2 b (A + 3 C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{3 d} + \frac{2 a (a^2 B - 6 b^2 B + 3 a b (A - 5 C)) \sqrt{\cos[c + d x]} \sin[c + d x]}{3 d} +$$

$$\frac{2 a^2 (3 a A - 15 b B - 35 a C) \cos[c + d x]^{3/2} \sin[c + d x]}{15 d} + \frac{2 (b B + 2 a C) (b + a \cos[c + d x])^2 \sin[c + d x]}{d \sqrt{\cos[c + d x]}} + \frac{2 C (b + a \cos[c + d x])^3 \sin[c + d x]}{3 d \cos[c + d x]^{3/2}}$$

Result (type 5, 3868 leaves):

$$\frac{1}{5 (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])}$$

$$3 i a^3 A \cos[c + d x]^5 \csc[c] (a + b \sec[c + d x])^3 (A + B \sec[c + d x] + C \sec[c + d x]^2)$$

$$\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right. \right.$$

$$\left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos[c] - 3 d (-1 + e^{2 i d x}) \sin[c]) -$$

$$\left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos[c] + 2 i (-1 + e^{2 i d x}) \sin[c])} \right.$$

$$\left. \sqrt{1 + e^{2 i d x} \cos[2 c] + i e^{2 i d x} \sin[2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos[c] + d (-1 + e^{2 i d x}) \sin[c]) \Big) +$$

$$\frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} 3 i a A b^2 \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} 3 i a^2 b B \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) -$$

$$\frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} i b^3 B \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3$$

$$\left(\left(2 e^{2 i dx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \right.$$

$$\left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) / (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \right) +$$

$$\frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} i a^3 C \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \quad \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 3 i a b^2 C \cos [c + d x]^5 \csc [c] (a + b \sec [c + d x])^3 \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \quad \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \quad \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^{11/2} (a + b \sec [c + d x])^3 \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \\
& \quad \left(-\frac{1}{5 d} 2 (3 a^3 A + 15 a A b^2 + 15 a^2 b B - 10 b^3 B + 5 a^3 C - 30 a b^2 C + 3 a^3 A \cos [2 c] + 15 a A b^2 \cos [2 c] + 15 a^2 b B \cos [2 c] + 5 a^3 C \cos [2 c]) \right. \\
& \quad \csc [c] \sec [c] + \frac{4 a^2 (3 A b + a B) \cos [d x] \sin [c]}{3 d} + \frac{2 a^3 A \cos [2 d x] \sin [2 c]}{5 d} + \frac{4 a^2 (3 A b + a B) \cos [c] \sin [d x]}{3 d} + \\
& \quad \left. \frac{4 b^3 C \sec [c] \sec [c + d x]^2 \sin [d x]}{3 d} + \frac{4 \sec [c] \sec [c + d x] (b^3 C \sin [c] + 3 b^3 B \sin [d x] + 9 a b^2 C \sin [d x])}{3 d} + \frac{2 a^3 A \cos [2 c] \sin [2 d x]}{5 d} \right) - \\
& \quad \left(4 a^2 A b \cos [c + d x]^5 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan}[\cot [c]]]^2 \right] (a + b \sec [c + d x])^3 \right. \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x])^2 \sec [d x - \text{ArcTan}[\cot [c]]] \sqrt{1 - \sin [d x - \text{ArcTan}[\cot [c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \text{ArcTan}[\cot [c]]]} \sqrt{1 + \sin [d x - \text{ArcTan}[\cot [c]]]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(4 A b^3 \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 (a + b \sec [c + d x])^3 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(4 a^3 B \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 (a + b \sec [c + d x])^3 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(3 d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(12 a b^2 B \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 (a + b \sec [c + d x])^3 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(12 a^2 b C \cos [c + d x]^5 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]] \right]^2 (a + b \sec [c + d x])^3 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) -
\end{aligned}$$

$$\left(4 b^3 C \cos [c+d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x])^3 \right. \\ \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\ \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\ \left(3 d (b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right)$$

■ **Problem 1309: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{3 / 2} (a+b \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) d x$$

Optimal (type 4, 274 leaves, 8 steps):

$$\frac{2\left(5 a^3 B-15 a b^2 B+15 a^2 b(A-C)-b^3(5 A+3 C)\right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d} + \\ \frac{2\left(9 a^2 b B+b^3 B+3 a b^2(3 A+C)+a^3(A+3 C)\right) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d} + \frac{2 b\left(15 A b^2+35 a b B+24 a^2 C+9 b^2 C\right) \sin [c+d x]}{15 d \sqrt{\cos [c+d x]}} + \\ \frac{2 a^2\left(5 a A-5 b B-9 a C\right) \sqrt{\cos [c+d x]} \sin [c+d x]}{15 d} + \frac{2(5 b B+6 a C)(b+a \cos [c+d x])^2 \sin [c+d x]}{15 d \cos [c+d x]^{3 / 2}} + \frac{2 C(b+a \cos [c+d x])^3 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}}$$

Result (type 5, 3871 leaves):

$$\frac{1}{(b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} \\ \frac{3 i a^2 A b \cos [c+d x]^5 \operatorname{Csc}[c] (a+b \operatorname{Sec}[c+d x])^3 (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)} \right. \right. \\ \left. \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]}\right)\right) / (3 i d\left(1+e^{2 i d x}\right) \cos [c]-3 d\left(-1+e^{2 i d x}\right) \sin [c]) - \\ \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)} \right. \\ \left. \sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]}\right) / (-i d\left(1+e^{2 i d x}\right) \cos [c]+d\left(-1+e^{2 i d x}\right) \sin [c])\right) - \\ \frac{1}{(b+a \cos [c+d x])^3 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} i A b^3 \cos [c+d x]^5 \operatorname{Csc}[c] (a+b \operatorname{Sec}[c+d x])^3 \\ (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} i a^3 B \cos [c + d x]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^3 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 3 i a b^2 B \cos [c + d x]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^3 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{(b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 3 i a^2 b C \cos [c + d x]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^3 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \\
& \frac{1}{5 (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} 3 i b^3 C \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^3 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \right) + \\
& \frac{1}{(b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x])} \operatorname{Cos}[c + d x]^{11/2} (a + b \operatorname{Sec}[c + d x])^3 \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(-\frac{1}{5 d} 2 (15 a^2 A b - 10 A b^3 + 5 a^3 B - 30 a b^2 B - 30 a^2 b C - 6 b^3 C + 15 a^2 A b \operatorname{Cos}[2 c] + 5 a^3 B \operatorname{Cos}[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c] + \frac{4 a^3 A \operatorname{Cos}[d x] \operatorname{Sin}[c]}{3 d} + \right. \\
& \quad \frac{4 a^3 A \operatorname{Cos}[c] \operatorname{Sin}[d x]}{3 d} + \frac{4 b^3 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \operatorname{Sin}[d x]}{5 d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (3 b^3 C \operatorname{Sin}[c] + 5 b^3 B \operatorname{Sin}[d x] + 15 a b^2 C \operatorname{Sin}[d x])}{15 d} \\
& \quad \left. \frac{1}{15 d} 4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x] (5 b^3 B \operatorname{Sin}[c] + 15 a b^2 C \operatorname{Sin}[c] + 15 A b^3 \operatorname{Sin}[d x] + 45 a b^2 B \operatorname{Sin}[d x] + 45 a^2 b C \operatorname{Sin}[d x] + 9 b^3 C \operatorname{Sin}[d x]) \right) - \\
& \left(4 a^3 A \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 (a + b \operatorname{Sec}[c + d x])^3 \right. \\
& \quad (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + d x])^3 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(12 a A b^2 \operatorname{Cos}[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \operatorname{Sin}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \right]^2 (a + b \operatorname{Sec}[c + d x])^3 \right.
\end{aligned}$$

$$\begin{aligned}
& \left((A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \operatorname{Cos}[c + dx])^3 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(12 a^2 b B \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^3 \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \operatorname{Cos}[c + dx])^3 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 b^3 B \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^3 \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \operatorname{Cos}[c + dx])^3 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^3 C \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^3 \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \operatorname{Sin}[c] \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \operatorname{Cos}[c + dx])^3 (A + 2C + 2B \operatorname{Cos}[c + dx] + A \operatorname{Cos}[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 a b^2 C \operatorname{Cos}[c + dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^3 \right. \\
& \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \operatorname{Sin}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]}} \right/$$

$$\left(d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right)$$

■ **Problem 1310: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + dx]} (a + b \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 294 leaves, 8 steps):

$$\begin{aligned} & \frac{2 (15 a^2 b B + 3 b^3 B - 5 a^3 (A - C) + 3 a b^2 (5 A + 3 C)) \text{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{5 d} + \\ & \frac{2 (21 a^3 B + 21 a b^2 B + 21 a^2 b (3 A + C) + b^3 (7 A + 5 C)) \text{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{21 d} + \\ & \frac{2 b (35 A b^2 + 63 a b B + 24 a^2 C + 25 b^2 C) \sin[c + dx]}{105 d \cos[c + dx]^{3/2}} + \frac{2 (98 a^2 b B + 21 b^3 B + 24 a^3 C + 21 a b^2 (5 A + 3 C)) \sin[c + dx]}{35 d \sqrt{\cos[c + dx]}} + \\ & \frac{2 (7 b B + 6 a C) (b + a \cos[c + dx])^2 \sin[c + dx]}{35 d \cos[c + dx]^{5/2}} + \frac{2 C (b + a \cos[c + dx])^3 \sin[c + dx]}{7 d \cos[c + dx]^{7/2}} \end{aligned}$$

Result (type 5, 3933 leaves):

$$\begin{aligned} & \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \\ & \frac{i a^3 A \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right.} \\ & \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right/ (3 i d (1 + e^{2 i dx}) \cos[c] - 3 d (-1 + e^{2 i dx}) \sin[c]) - \\ & \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2 i dx} \cos[2c] + i e^{2 i dx} \sin[2c]} \right) \right/ (-i d (1 + e^{2 i dx}) \cos[c] + d (-1 + e^{2 i dx}) \sin[c]) \Big) - \\ & \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \frac{3 i a A b^2 \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3}{(A + B \sec[c + dx] + C \sec[c + dx]^2)} \\ & \left(\left(2 e^{2 i dx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i dx} (\cos[c] + i \sin[c])^2\right] \sqrt{e^{-i dx} (2 (1 + e^{2 i dx}) \cos[c] + 2 i (-1 + e^{2 i dx}) \sin[c])} \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) -} \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) -} \right) \right) - \\
& \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} 3 i a^2 b B \cos[c + dx]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^3 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \\
& \left(\left(2 e^{2ix} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) -} \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) -} \right) \right) \right) - \\
& \frac{1}{5 (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} 3 i b^3 B \cos[c + dx]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^3 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \\
& \left(\left(2 e^{2ix} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) -} \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) -} \right) \right) \right) - \\
& \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} i a^3 C \cos[c + dx]^5 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^3 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx])^2 \\
& \left(\left(2 e^{2ix} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c] \right) -} \right. \right. \\
& \left. \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2 i (-1 + e^{2ix}) \sin[c])} \right. \right. \right. \\
& \left. \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{\left(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c] \right) -} \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c])} \right) - \\
& \frac{1}{5 (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} 9 i a b^2 C \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3 \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(\left(2 e^{2ix} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(3 i d (1 + e^{2ix}) \cos[c] - 3 d (-1 + e^{2ix}) \sin[c])} \right) - \right. \\
& \left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2ix} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-ix} (2 (1 + e^{2ix}) \cos[c] + 2i (-1 + e^{2ix}) \sin[c])} \right. \right. \\
& \left. \left. \frac{\sqrt{1 + e^{2ix} \cos[2c] + i e^{2ix} \sin[2c]}}{(-i d (1 + e^{2ix}) \cos[c] + d (-1 + e^{2ix}) \sin[c])} \right) \right) + \\
& \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^{11/2} (a + b \sec[c + dx])^3 \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(-\frac{2 (5 a^3 A - 30 a A b^2 - 30 a^2 b B - 6 b^3 B - 10 a^3 C - 18 a b^2 C + 5 a^3 A \cos[2c]) \csc[c] \sec[c]}{5 d} + \frac{4 b^3 C \sec[c] \sec[c + dx]^4 \sin[dx]}{7 d} + \right. \\
& \frac{4 \sec[c] \sec[c + dx]^3 (5 b^3 C \sin[c] + 7 b^3 B \sin[dx] + 21 a b^2 C \sin[dx])}{35 d} + \frac{1}{105 d} 4 \sec[c] \sec[c + dx] (35 A b^3 \sin[c] + 105 a b^2 B \sin[c] + \\
& 105 a^2 b C \sin[c] + 25 b^3 C \sin[c] + 315 a A b^2 \sin[dx] + 315 a^2 b B \sin[dx] + 63 b^3 B \sin[dx] + 105 a^3 C \sin[dx] + 189 a b^2 C \sin[dx]) + \\
& \left. \frac{1}{105 d} 4 \sec[c] \sec[c + dx]^2 (21 b^3 B \sin[c] + 63 a b^2 C \sin[c] + 35 A b^3 \sin[dx] + 105 a b^2 B \sin[dx] + 105 a^2 b C \sin[dx] + 25 b^3 C \sin[dx]) \right) - \\
& \left(12 a^2 A b \cos[c + dx]^5 \csc[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \right]^2 (a + b \sec[c + dx])^3 \right. \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\text{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \text{Cot}[c]^2} \right) - \\
& \left(4 A b^3 \cos[c + dx]^5 \csc[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]] \right]^2 (a + b \sec[c + dx])^3 \right. \\
& (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\text{Cot}[c]]]}
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a^3 B \cos[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^3 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a b^2 B \cos[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^3 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a^2 b C \cos[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^3 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^3 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(20 b^3 C \cos[c + d x]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^3 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\cot[c]]]} \right)
\end{aligned}$$

$$\left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]}\right) /$$

$$\left(21 d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right)$$

■ **Problem 1311: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{\cos[c + dx]}} dx$$

Optimal (type 4, 357 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 (15 a^3 B + 27 a b^2 B + 9 a^2 b (5 A + 3 C) + b^3 (9 A + 7 C)) \text{EllipticE}\left[\frac{1}{2} (c + dx), 2\right]}{15 d} + \\ & \frac{2 (21 a^2 b B + 5 b^3 B + 7 a^3 (3 A + C) + 3 a b^2 (7 A + 5 C)) \text{EllipticF}\left[\frac{1}{2} (c + dx), 2\right]}{21 d} + \frac{2 b (63 A b^2 + 99 a b B + 24 a^2 C + 49 b^2 C) \sin[c + dx]}{315 d \cos[c + dx]^{5/2}} + \\ & \frac{2 (54 a^2 b B + 15 b^3 B + 8 a^3 C + 9 a b^2 (7 A + 5 C)) \sin[c + dx]}{63 d \cos[c + dx]^{3/2}} + \frac{2 (15 a^3 B + 27 a b^2 B + 9 a^2 b (5 A + 3 C) + b^3 (9 A + 7 C)) \sin[c + dx]}{15 d \sqrt{\cos[c + dx]}} + \\ & \frac{2 (3 b B + 2 a C) (b + a \cos[c + dx])^2 \sin[c + dx]}{21 d \cos[c + dx]^{7/2}} + \frac{2 C (b + a \cos[c + dx])^3 \sin[c + dx]}{9 d \cos[c + dx]^{9/2}} \end{aligned}$$

Result (type 5, 3345 leaves):

$$\begin{aligned} & \frac{1}{(b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^{11/2} (a + b \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \\ & \left(\frac{4 (45 a^2 A b + 9 A b^3 + 15 a^3 B + 27 a b^2 B + 27 a^2 b C + 7 b^3 C) \csc[c] \sec[c]}{15 d} + \frac{4 b^3 C \sec[c] \sec[c + dx]^5 \sin[dx]}{9 d} + \right. \\ & \frac{4 \sec[c] \sec[c + dx]^4 (7 b^3 C \sin[c] + 9 b^3 B \sin[dx] + 27 a b^2 C \sin[dx])}{63 d} + \frac{1}{315 d} 4 \sec[c] \sec[c + dx]^2 (63 A b^3 \sin[c] + 189 a b^2 B \sin[c] + \\ & 189 a^2 b C \sin[c] + 49 b^3 C \sin[c] + 315 a A b^2 \sin[dx] + 315 a^2 b B \sin[dx] + 75 b^3 B \sin[dx] + 105 a^3 C \sin[dx] + 225 a b^2 C \sin[dx]) + \\ & \frac{1}{315 d} 4 \sec[c] \sec[c + dx]^3 (45 b^3 B \sin[c] + 135 a b^2 C \sin[c] + 63 A b^3 \sin[dx] + 189 a b^2 B \sin[dx] + 189 a^2 b C \sin[dx] + 49 b^3 C \sin[dx]) + \\ & \left. \frac{1}{105 d} 4 \sec[c] \sec[c + dx] (105 a A b^2 \sin[c] + 105 a^2 b B \sin[c] + 25 b^3 B \sin[c] + 35 a^3 C \sin[c] + 75 a b^2 C \sin[c] + \right. \\ & \left. 315 a^2 A b \sin[dx] + 63 A b^3 \sin[dx] + 105 a^3 B \sin[dx] + 189 a b^2 B \sin[dx] + 189 a^2 b C \sin[dx] + 49 b^3 C \sin[dx]) \right) - \\ & \left(4 a^3 A \cos[c + dx]^5 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^3 \right) \end{aligned}$$

$$\begin{aligned}
& \left((A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a A b^2 \cos[c + dx]^5 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a^2 b B \cos[c + dx]^5 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(20 b^3 B \cos[c + dx]^5 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (b + a \cos[c + dx])^3 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a^3 C \cos[c + dx]^5 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^3 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d(b+a\cos[c+dx])^3 (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) \sqrt{1+\cot^2[c]} \right) - \\
& \left(20a^2b^2C\cos[c+dx]^5 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a+b\sec[c+dx])^3 \right. \\
& \quad \left. (A+B\sec[c+dx]+C\sec[c+dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1-\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(7d(b+a\cos[c+dx])^3 (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) \sqrt{1+\cot^2[c]} \right) + \\
& \left(6a^2Ab\cos[c+dx]^5 \operatorname{Csc}[c] (a+b\sec[c+dx])^3 (A+B\sec[c+dx]+C\sec[c+dx]^2) \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2\right] \sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \quad \left. \left(\sqrt{1-\cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2} \sqrt{1+\tan[c]^2} \right) - \right. \\
& \quad \left. \frac{\frac{\sin[dx + \operatorname{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1+\tan[c]^2}} + \frac{2\cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] \sqrt{1+\tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]} \sqrt{1+\tan[c]^2}} \right) / \left(d(b+a\cos[c+dx])^3 (A+2C+2B\cos[c+dx]+A\cos[2c+2dx]) \right) + \\
& \left(6Ab^3\cos[c+dx]^5 \operatorname{Csc}[c] (a+b\sec[c+dx])^3 (A+B\sec[c+dx]+C\sec[c+dx]^2) \right)
\end{aligned}$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \right. \\
\left. (5 d (b + a \text{Cos}[c + d x])^3 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) + \right. \\
\left. 2 a^3 B \text{Cos}[c + d x]^5 \text{Csc}[c] (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right) \\
\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \right]^2 \right) \text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right. \\
\left. \left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2} \right) - \right. \\
\left. \frac{\frac{\text{Sin}[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Tan}[c]^2}} \right) / \left(d (b + a \text{Cos}[c + d x])^3 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \right) + \\
\left. 18 a b^2 B \text{Cos}[c + d x]^5 \text{Csc}[c] (a + b \text{Sec}[c + d x])^3 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / \\
(5 d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) + \\
\left(18 a^2 b C \cos [c + d x]^5 \text{Csc} [c] (a + b \sec [c + d x])^3 (A + B \sec [c + d x] + C \sec [c + d x])^2 \right) \\
\left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \right]^2 \sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c] \right) / \right. \\
\left. \left(\sqrt{1 - \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2} \sqrt{1 + \text{Tan} [c]^2} \right) - \right. \\
\left. \frac{\frac{\sin [d x + \text{ArcTan} [\text{Tan} [c]]] \text{Tan} [c]}{\sqrt{1 + \text{Tan} [c]^2}} + \frac{2 \cos [c]^2 \cos [d x + \text{ArcTan} [\text{Tan} [c]]] \sqrt{1 + \text{Tan} [c]^2}}{\cos [c]^2 + \sin [c]^2}}{\sqrt{\cos [c] \cos [d x + \text{ArcTan} [\text{Tan} [c]]]} \sqrt{1 + \text{Tan} [c]^2}} \right) / \\
(5 d (b + a \cos [c + d x])^3 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])) +$$

$$\left(14 b^3 C \cos[c + dx]^5 \csc[c] (a + b \sec[c + dx])^3 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
\left. \left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[dx + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
\left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
\left. \left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) \right) / \\
(15 d (b + a \cos[c + dx])^3 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))$$

- **Problem 1313: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{9/2} (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 377 leaves, 9 steps):

$$\frac{2 (36 a^3 b B + 60 a b^3 B + 15 b^4 (A - C) + 18 a^2 b^2 (3 A + 5 C) + a^4 (7 A + 9 C)) \text{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{15 d} + \\
\frac{2 (5 a^4 B + 42 a^2 b^2 B + 21 b^4 B + 28 a b^3 (A + 3 C) + 4 a^3 b (5 A + 7 C)) \text{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{21 d} + \\
\frac{2 a (15 a^3 B + 117 a b^2 B + 2 b^3 (31 A - 63 C) + 12 a^2 b (5 A + 7 C)) \sqrt{\cos[c + dx]} \sin[c + dx]}{63 d} + \\
\frac{2 a^2 (162 a b B + 3 b^2 (41 A - 105 C) + 7 a^2 (7 A + 9 C)) \cos[c + dx]^{3/2} \sin[c + dx]}{315 d} + \\
\frac{2 a (5 A b + 3 a B - 21 b C) \sqrt{\cos[c + dx]} (b + a \cos[c + dx])^2 \sin[c + dx]}{21 d} + \\
\frac{2 a (A - 9 C) \sqrt{\cos[c + dx]} (b + a \cos[c + dx])^3 \sin[c + dx]}{9 d} + \frac{2 C (b + a \cos[c + dx])^4 \sin[c + dx]}{d \sqrt{\cos[c + dx]}}$$

Result (type 5, 4114 leaves) :

$$\begin{aligned}
& \frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^{13/2} (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \\
& \left(-\frac{1}{15d} 2 (7a^4 A + 54a^2 A b^2 + 15A b^4 + 36a^3 b B + 60a b^3 B + 9a^4 C + 90a^2 b^2 C - 30b^4 C + 7a^4 A \cos[2c] + 54a^2 A b^2 \cos[2c] + \right. \\
& \quad 15A b^4 \cos[2c] + 36a^3 b B \cos[2c] + 60a b^3 B \cos[2c] + 9a^4 C \cos[2c] + 90a^2 b^2 C \cos[2c]) \csc[c] \sec[c] + \\
& \quad \frac{a (92a^2 A b + 112A b^3 + 23a^3 B + 168a b^2 B + 112a^2 b C) \cos[dx] \sin[c] + a^2 (19a^2 A + 108A b^2 + 72a b B + 18a^2 C) \cos[2dx] \sin[2c]}{21d} + \frac{a^2 (19a^2 A + 108A b^2 + 72a b B + 18a^2 C) \cos[2dx] \sin[2c]}{45d} + \\
& \quad \frac{a^3 (4A b + a B) \cos[3dx] \sin[3c]}{7d} + \frac{a^4 A \cos[4dx] \sin[4c]}{18d} + \\
& \quad \frac{a (92a^2 A b + 112A b^3 + 23a^3 B + 168a b^2 B + 112a^2 b C) \cos[c] \sin[dx]}{21d} + \frac{4b^4 C \sec[c] \sec[c + dx] \sin[dx]}{d} + \\
& \quad \left. \frac{a^2 (19a^2 A + 108A b^2 + 72a b B + 18a^2 C) \cos[2c] \sin[2dx]}{45d} + \frac{a^3 (4A b + a B) \cos[3c] \sin[3dx]}{7d} + \frac{a^4 A \cos[4c] \sin[4dx]}{18d} \right) - \\
& \left(80a^3 A b \cos[c + dx]^6 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \quad \left(21d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(16a A b^3 \cos[c + dx]^6 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \quad \left(3d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(20a^4 B \cos[c + dx]^6 \csc[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]\right]^2 (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} \right) - \\
& \left(8 a^2 b^2 B \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} \right) - \\
& \left(4 b^4 B \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} \right) - \\
& \left(16 a^3 b C \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1+\sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot[c]^2} \right) - \\
& \left(16 a b^3 C \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \operatorname{Sec}[c + dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sqrt{-\sqrt{1+\cot^2[c]} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]] \sqrt{1+\sin[dx - \text{ArcTan}[\cot[c]]]}} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1+\cot^2[c]} \right) - \\
& \left(14 a^4 A \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. \left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right\} \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right. \\
& \left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / \\
& \left(15 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \right) - \\
& \left(36 a^2 A b^2 \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right. \\
& \left. \left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right\} \right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right. \\
& \left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.
\end{aligned}$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) /$$

$$(5 d (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) -$$

$$\left(2 A b^4 \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) / (d (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) -$$

$$\left(24 a^3 b B \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) /$$

$$(5 d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(8 a b^3 B \cos[c + d x]^6 \csc[c] (a + b \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \cos[c]^2 \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) /$$

$$(d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x])) -$$

$$\left(6 a^4 C \cos[c + d x]^6 \csc[c] (a + b \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{\cos[c] \cos[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2} \sqrt{1 + \text{Tan}[c]^2}} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) /$$

$$(5 d (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) -$$

$$\left(12 a^2 b^2 C \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}} \right) / (d (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])) +$$

$$\left(2 b^4 C \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \right)$$

$$\left(\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c] \right) / \right)$$

$$\left(\sqrt{1 - \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{1 + \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]]} \sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}} \sqrt{1 + \text{Tan}[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[d x + \text{ArcTan}[\text{Tan}[c]]] \text{Tan}[c]}{\sqrt{1 + \text{Tan}[c]^2}} + \frac{2 \text{Cos}[c]^2 \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}{\text{Cos}[c]^2 + \text{Sin}[c]^2}}{\sqrt{\text{Cos}[c] \text{Cos}[d x + \text{ArcTan}[\text{Tan}[c]]] \sqrt{1 + \text{Tan}[c]^2}}}}{\right) / \left(d (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \right)$$

- **Problem 1314: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[c + d x]^{7/2} (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) dx$$

Optimal (type 4, 371 leaves, 9 steps):

$$\begin{aligned} & \frac{2 (3 a^4 B + 30 a^2 b^2 B - 5 b^4 B + 20 a b^3 (A - C) + 4 a^3 b (3 A + 5 C)) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \\ & \frac{2 (28 a^3 b B + 84 a b^3 B + 7 b^4 (3 A + C) + 42 a^2 b^2 (A + 3 C) + a^4 (5 A + 7 C)) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \\ & \frac{2 a (28 a^2 b B - 42 b^3 B + 3 a b^2 (13 A - 49 C) + a^3 (5 A + 7 C)) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{21 d} + \\ & \frac{2 a^2 (54 a A b + 21 a^2 B - 105 b^2 B - 350 a b C) \text{Cos}[c + d x]^{3/2} \text{Sin}[c + d x]}{105 d} + \frac{2 a (a A - 7 b B - 21 a C) \sqrt{\text{Cos}[c + d x]} (b + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{7 d} + \\ & \frac{2 (3 b B + 8 a C) (b + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{3 d \sqrt{\text{Cos}[c + d x]}} + \frac{2 C (b + a \text{Cos}[c + d x])^4 \text{Sin}[c + d x]}{3 d \text{Cos}[c + d x]^{3/2}} \end{aligned}$$

Result (type 5, 4776 leaves):

$$\begin{aligned} & \frac{1}{5 (b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} \\ & \frac{12 i a^3 A b \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)}{\left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right.} \right.} \\ & \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c] \right) - \right. \\ & \left. \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \right. \\ & \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \text{Cos}[c] + d (-1 + e^{2 i d x}) \text{Sin}[c] \right) \right) + \\ & \frac{1}{(b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} 4 i a A b^3 \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 \\ & (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \end{aligned}$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{5 (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 3 i a^4 B \cos [c + d x]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^4 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \frac{1}{(b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 6 i a^2 b^2 B \cos [c + d x]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^4 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \frac{1}{(b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} i b^4 B \cos [c + d x]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^4 \\
& (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \\
& \quad \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) + \\
& \frac{1}{(b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2} 4 i a^3 b C \operatorname{Cos}[c + d x]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^4 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \right) - \\
& \frac{1}{(b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2} 4 i a b^3 C \operatorname{Cos}[c + d x]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + d x])^4 \\
& \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(3 i d (1 + e^{2 i d x}) \operatorname{Cos}[c] - 3 d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) - \right. \\
& \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\operatorname{Cos}[c] + i \operatorname{Sin}[c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \operatorname{Cos}[c] + 2 i (-1 + e^{2 i d x}) \operatorname{Sin}[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \operatorname{Cos}[2 c] + i e^{2 i d x} \operatorname{Sin}[2 c]} \right) / \left(-i d (1 + e^{2 i d x}) \operatorname{Cos}[c] + d (-1 + e^{2 i d x}) \operatorname{Sin}[c] \right) \right) + \\
& \frac{1}{(b + a \operatorname{Cos}[c + d x])^4 (A + 2 C + 2 B \operatorname{Cos}[c + d x] + A \operatorname{Cos}[2 c + 2 d x]) (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x])^2} \operatorname{Cos}[c + d x]^{13/2} (a + b \operatorname{Sec}[c + d x])^4 \\
& \left(-\frac{1}{5 d} 2 (12 a^3 A b + 20 a A b^3 + 3 a^4 B + 30 a^2 b^2 B - 10 b^4 B + 20 a^3 b C - 40 a b^3 C + 12 a^3 A b \operatorname{Cos}[2 c] + \right. \\
& \quad 20 a A b^3 \operatorname{Cos}[2 c] + 3 a^4 B \operatorname{Cos}[2 c] + 30 a^2 b^2 B \operatorname{Cos}[2 c] + 20 a^3 b C \operatorname{Cos}[2 c]) \operatorname{Csc}[c] \operatorname{Sec}[c] + \\
& \quad \frac{a^2 (23 a^2 A + 168 A b^2 + 112 a b B + 28 a^2 C) \operatorname{Cos}[d x] \operatorname{Sin}[c]}{21 d} + \frac{2 a^3 (4 A b + a B) \operatorname{Cos}[2 d x] \operatorname{Sin}[2 c]}{5 d} + \frac{a^4 A \operatorname{Cos}[3 d x] \operatorname{Sin}[3 c]}{7 d} + \\
& \quad \left. \frac{a^2 (23 a^2 A + 168 A b^2 + 112 a b B + 28 a^2 C) \operatorname{Cos}[c] \operatorname{Sin}[d x]}{21 d} + \frac{4 b^4 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 \operatorname{Sin}[d x]}{3 d} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c+dx] (b^4 C \sin[c] + 3 b^4 B \sin[dx] + 12 a b^3 C \sin[dx])}{3 d} + \frac{2 a^3 (4 A b + a B) \cos[2c] \sin[2dx]}{5 d} + \frac{a^4 A \cos[3c] \sin[3dx]}{7 d} \right) - \\
& \left(20 a^4 A \cos[c+dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c+dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(21 d (b + a \cos[c+dx])^4 (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(8 a^2 A b^2 \cos[c+dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c+dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c+dx])^4 (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 A b^4 \cos[c+dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c+dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c+dx])^4 (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(16 a^3 b B \cos[c+dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a + b \operatorname{Sec}[c+dx])^4 \right. \\
& \quad \left. (A + B \operatorname{Sec}[c+dx] + C \operatorname{Sec}[c+dx]^2) \operatorname{Sec}[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \cos[c+dx])^4 (A + 2 C + 2 B \cos[c+dx] + A \cos[2c + 2dx]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(16 a b^3 B \cos [c+d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x])^4 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^4 C \cos [c+d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x])^4 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(24 a^2 b^2 C \cos [c+d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x])^4 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right) - \\
& \left(4 b^4 C \cos [c+d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]^2\right] (a+b \operatorname{Sec}[c+d x])^4 \right. \\
& \quad \left. (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) \operatorname{Sec}[d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\operatorname{Cot}[c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\operatorname{Cot}[c]^2} \right)
\end{aligned}$$

■ **Problem 1315: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^4(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 388 leaves, 9 steps):

$$\begin{aligned} & \frac{2\left(20 a^3 b B-20 a b^3 B+30 a^2 b^2(A-C)-b^4(5 A+3 C)+a^4(3 A+5 C)\right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), 2\right]}{5 d}+ \\ & \frac{2\left(a^4 B+18 a^2 b^2 B+b^4 B+4 a b^3(3 A+C)+4 a^3 b(A+3 C)\right) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), 2\right]}{3 d}+ \\ & \frac{2 a\left(5 a^3 B-105 a b^2 B+4 a^2 b(5 A-33 C)-6 b^3(5 A+3 C)\right) \sqrt{\cos [c+d x]} \sin [c+d x]}{15 d}- \\ & \frac{2 a^2\left(50 a b B-a^2(3 A-59 C)+3 b^2(5 A+3 C)\right) \cos [c+d x]^{3 / 2} \sin [c+d x]}{15 d}+\frac{2\left(5 A b^2+15 a b B+16 a^2 C+3 b^2 C\right)(b+a \cos [c+d x])^2 \sin [c+d x]}{5 d \sqrt{\cos [c+d x]}}+ \\ & \frac{2(5 b B+8 a C)(b+a \cos [c+d x])^3 \sin [c+d x]}{15 d \cos [c+d x]^{3 / 2}}+\frac{2 C(b+a \cos [c+d x])^4 \sin [c+d x]}{5 d \cos [c+d x]^{5 / 2}} \end{aligned}$$

Result (type 5, 4960 leaves):

$$\begin{aligned} & \frac{1}{5(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} \\ & \frac{3 i a^4 A \cos [c+d x]^6 \operatorname{Csc}[c](a+b \sec [c+d x])^4(A+B \sec [c+d x]+C \sec [c+d x]^2)}{\left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right.}\right. \\ & \left.\left.\sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]}\right)\right) / \left(3 i d\left(1+e^{2 i d x}\right) \cos [c]-3 d\left(-1+e^{2 i d x}\right) \sin [c]\right)- \\ & \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right.}\right. \\ & \left.\left.\sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]}\right)\right) / \left(-i d\left(1+e^{2 i d x}\right) \cos [c]+d\left(-1+e^{2 i d x}\right) \sin [c]\right)\right)+ \\ & \frac{1}{(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])} \frac{6 i a^2 A b^2 \cos [c+d x]^6 \operatorname{Csc}[c](a+b \sec [c+d x])^4}{(A+B \sec [c+d x]+C \sec [c+d x]^2)} \\ & \left(\left(2 e^{2 i d x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right.}\right. \\ & \left.\left.\sqrt{1+e^{2 i d x} \cos [2 c]+i e^{2 i d x} \sin [2 c]}\right)\right) / \left(3 i d\left(1+e^{2 i d x}\right) \cos [c]-3 d\left(-1+e^{2 i d x}\right) \sin [c]\right)- \\ & \left(2 \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4},-e^{2 i d x}(\cos [c]+i \sin [c])^2\right] \sqrt{e^{-i d x}\left(2\left(1+e^{2 i d x}\right) \cos [c]+2 i\left(-1+e^{2 i d x}\right) \sin [c]\right)}\right.}\right. \end{aligned}$$

$$\begin{aligned}
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} 6iab^2 C \cos[c + dx]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^4 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) - \\
& \frac{1}{5(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} 3ib^4 C \cos[c + dx]^6 \operatorname{Csc}[c] (a + b \operatorname{Sec}[c + dx])^4 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(\left(2 e^{2idx} \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right. \\
& \quad \left. \left(2 \operatorname{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2 (1 + e^{2idx}) \cos[c] + 2i (-1 + e^{2idx}) \sin[c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) + \\
& \frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])} \cos[c + dx]^{13/2} (a + b \operatorname{Sec}[c + dx])^4 \\
& (A + B \operatorname{Sec}[c + dx] + C \operatorname{Sec}[c + dx]^2) \\
& \left(-\frac{1}{5d} 2 (3a^4 A + 30a^2 A b^2 - 10A b^4 + 20a^3 b B - 40a b^3 B + 5a^4 C - 60a^2 b^2 C - 6b^4 C + 3a^4 A \cos[2c] + 30a^2 A b^2 \cos[2c] + 20a^3 b B \cos[2c] + \right.
\end{aligned}$$

$$\begin{aligned}
& 5 a^4 C \cos[2 c] \operatorname{Csc}[c] \operatorname{Sec}[c] + \frac{4 a^3 (4 A b + a B) \cos[d x] \sin[c]}{3 d} + \frac{2 a^4 A \cos[2 d x] \sin[2 c]}{5 d} + \frac{4 a^3 (4 A b + a B) \cos[c] \sin[d x]}{3 d} + \\
& \frac{4 b^4 C \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^3 \sin[d x]}{5 d} + \frac{4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x]^2 (3 b^4 C \sin[c] + 5 b^4 B \sin[d x] + 20 a b^3 C \sin[d x])}{15 d} + \frac{1}{15 d} 4 \operatorname{Sec}[c] \operatorname{Sec}[c + d x] \\
& \left(5 b^4 B \sin[c] + 20 a b^3 C \sin[c] + 15 A b^4 \sin[d x] + 60 a b^3 B \sin[d x] + 90 a^2 b^2 C \sin[d x] + 9 b^4 C \sin[d x] \right) + \frac{2 a^4 A \cos[2 c] \sin[2 d x]}{5 d} \Big) - \\
& \left(16 a^3 A b \cos[c + d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + d x])^4 \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(16 a A b^3 \cos[c + d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + d x])^4 \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(4 a^4 B \cos[c + d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + d x])^4 \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \operatorname{Cot}[c]^2} \sin[c] \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \sqrt{1 + \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \operatorname{Cot}[c]^2} \right) - \\
& \left(24 a^2 b^2 B \cos[c + d x]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]\right]^2 (a + b \operatorname{Sec}[c + d x])^4 \right. \\
& \left. (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) \operatorname{Sec}[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]] \sqrt{1 - \sin[d x - \operatorname{ArcTan}[\operatorname{Cot}[c]]]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 b^4 B \cos[c + dx]^6 \csc[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(16 a^3 b C \cos[c + dx]^6 \csc[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(16 a b^3 C \cos[c + dx]^6 \csc[c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin[dx - \text{ArcTan}[\cot[c]]]^2 \right] (a + b \sec[c + dx])^4 \right. \\
& \quad \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right)
\end{aligned}$$

- **Problem 1316: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[c + dx]^{3/2} (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 384 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 \left(5 a^4 B - 30 a^2 b^2 B - 3 b^4 B + 20 a^3 b (A - C) - 4 a b^3 (5 A + 3 C) \right) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{5 d} + \\
& \frac{2 \left(84 a^3 b B + 28 a b^3 B + 42 a^2 b^2 (3 A + C) + 7 a^4 (A + 3 C) + b^4 (7 A + 5 C) \right) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \\
& \frac{2 b \left(413 a^2 b B + 63 b^3 B + 192 a^3 C + 2 a b^2 (175 A + 101 C) \right) \text{Sin}[c + d x]}{105 d \sqrt{\text{Cos}[c + d x]}} - \\
& \frac{2 a^2 \left(98 a b B - a^2 (35 A - 87 C) + 5 b^2 (7 A + 5 C) \right) \sqrt{\text{Cos}[c + d x]} \text{Sin}[c + d x]}{105 d} + \\
& \frac{2 \left(35 A b^2 + 77 a b B + 48 a^2 C + 25 b^2 C \right) (b + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{105 d \text{Cos}[c + d x]^{3/2}} + \\
& \frac{2 (7 b B + 8 a C) (b + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{35 d \text{Cos}[c + d x]^{5/2}} + \frac{2 C (b + a \text{Cos}[c + d x])^4 \text{Sin}[c + d x]}{7 d \text{Cos}[c + d x]^{7/2}}
\end{aligned}$$

Result (type 5, 4791 leaves):

$$\begin{aligned}
& \frac{1}{(b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} \\
& 4 i a^3 A b \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) \right) / \left(3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c] \right) - \\
& \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) \right) / \left(-i d (1 + e^{2 i d x}) \text{Cos}[c] + d (-1 + e^{2 i d x}) \text{Sin}[c] \right) \Big) - \\
& \frac{1}{(b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x])} 4 i a A b^3 \text{Cos}[c + d x]^6 \text{Csc}[c] (a + b \text{Sec}[c + d x])^4 \\
& (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \\
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) \right) / \left(3 i d (1 + e^{2 i d x}) \text{Cos}[c] - 3 d (-1 + e^{2 i d x}) \text{Sin}[c] \right) - \\
& \left(2 \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\text{Cos}[c] + i \text{Sin}[c])^2\right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \text{Cos}[c] + 2 i (-1 + e^{2 i d x}) \text{Sin}[c])} \right. \\
& \left. \left. \sqrt{1 + e^{2 i d x} \text{Cos}[2 c] + i e^{2 i d x} \text{Sin}[2 c]} \right) \right) / \left(-i d (1 + e^{2 i d x}) \text{Cos}[c] + d (-1 + e^{2 i d x}) \text{Sin}[c] \right) \Big) +
\end{aligned}$$

$$\frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} i a^4 B \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} 6 i a^2 b^2 B \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\frac{1}{5 (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} 3 i b^4 B \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4$$

$$\left(\left(2 e^{2idx} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (3id(1 + e^{2idx}) \cos[c] - 3d(-1 + e^{2idx}) \sin[c]) - \right.$$

$$\left. \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2idx} (\cos[c] + i \sin[c])^2 \right] \sqrt{e^{-idx} (2(1 + e^{2idx}) \cos[c] + 2i(-1 + e^{2idx}) \sin[c])} \right. \right.$$

$$\left. \left. \sqrt{1 + e^{2idx} \cos[2c] + i e^{2idx} \sin[2c]} \right) / (-id(1 + e^{2idx}) \cos[c] + d(-1 + e^{2idx}) \sin[c]) \right) -$$

$$\frac{1}{(b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) (A + B \sec[c + dx] + C \sec[c + dx]^2)} 4 i a^3 b C \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4$$

$$\begin{aligned}
& \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) - \\
& \quad \frac{1}{5 (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} 12 i a b^3 C \cos [c + d x]^6 \csc [c] \\
& \quad (a + b \sec [c + d x])^4 (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \quad \left(\left(2 e^{2 i d x} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (3 i d (1 + e^{2 i d x}) \cos [c] - 3 d (-1 + e^{2 i d x}) \sin [c]) - \right. \\
& \quad \left(2 \text{Hypergeometric2F1} \left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2 i d x} (\cos [c] + i \sin [c])^2 \right] \sqrt{e^{-i d x} (2 (1 + e^{2 i d x}) \cos [c] + 2 i (-1 + e^{2 i d x}) \sin [c])} \right. \\
& \quad \left. \left. \sqrt{1 + e^{2 i d x} \cos [2 c] + i e^{2 i d x} \sin [2 c]} \right) / (-i d (1 + e^{2 i d x}) \cos [c] + d (-1 + e^{2 i d x}) \sin [c]) \right) + \\
& \quad \frac{1}{(b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x])} \cos [c + d x]^{13/2} (a + b \sec [c + d x])^4 \\
& \quad (A + B \sec [c + d x] + C \sec [c + d x]^2) \\
& \quad \left(-\frac{1}{5 d} 2 (20 a^3 A b - 40 a A b^3 + 5 a^4 B - 60 a^2 b^2 B - 6 b^4 B - 40 a^3 b C - 24 a b^3 C + 20 a^3 A b \cos [2 c] + 5 a^4 B \cos [2 c]) \csc [c] \sec [c] + \right. \\
& \quad \frac{4 a^4 A \cos [d x] \sin [c]}{3 d} + \frac{4 a^4 A \cos [c] \sin [d x]}{3 d} + \frac{4 b^4 C \sec [c] \sec [c + d x]^4 \sin [d x]}{7 d} + \\
& \quad \frac{4 \sec [c] \sec [c + d x]^3 (5 b^4 C \sin [c] + 7 b^4 B \sin [d x] + 28 a b^3 C \sin [d x])}{35 d} + \frac{1}{105 d} \\
& \quad 4 \sec [c] \sec [c + d x] (35 A b^4 \sin [c] + 140 a b^3 B \sin [c] + 210 a^2 b^2 C \sin [c] + 25 b^4 C \sin [c] + \\
& \quad 420 a A b^3 \sin [d x] + 630 a^2 b^2 B \sin [d x] + 63 b^4 B \sin [d x] + 420 a^3 b C \sin [d x] + 252 a b^3 C \sin [d x]) + \frac{1}{105 d} \\
& \quad \left. 4 \sec [c] \sec [c + d x]^2 (21 b^4 B \sin [c] + 84 a b^3 C \sin [c] + 35 A b^4 \sin [d x] + 140 a b^3 B \sin [d x] + 210 a^2 b^2 C \sin [d x] + 25 b^4 C \sin [d x]) \right) - \\
& \quad \left(4 a^4 A \cos [c + d x]^6 \csc [c] \text{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \text{ArcTan} [\cot [c]]] \right]^2 (a + b \sec [c + d x])^4 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left((A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(24a^2 A b^2 \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4Ab^4 \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(3d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(16a^3 b B \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(16a^3 b^3 B \cos[c + dx]^6 \operatorname{Csc}[c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \operatorname{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + dx])^4 \right. \\
& \left. (A + B \sec[c + dx] + C \sec[c + dx]^2) \sec[dx - \operatorname{ArcTan}[\cot[c]]] \sqrt{1 - \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[dx - \operatorname{ArcTan}[\cot[c]]]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(3 d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(4 a^4 C \cos[c + d x]^6 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^4 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(8 a^2 b^2 C \cos[c + d x]^6 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^4 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right) - \\
& \left(20 b^4 C \cos[c + d x]^6 \csc[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[d x - \text{ArcTan}[\cot[c]]]^2\right] (a + b \sec[c + d x])^4 \right. \\
& \quad \left. (A + B \sec[c + d x] + C \sec[c + d x]^2) \sec[d x - \text{ArcTan}[\cot[c]]] \sqrt{1 - \sin[d x - \text{ArcTan}[\cot[c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot[c]^2} \sin[c] \sin[d x - \text{ArcTan}[\cot[c]]]} \sqrt{1 + \sin[d x - \text{ArcTan}[\cot[c]]]} \right) / \\
& \left(21 d (b + a \cos[c + d x])^4 (A + 2 C + 2 B \cos[c + d x] + A \cos[2 c + 2 d x]) \sqrt{1 + \cot[c]^2} \right)
\end{aligned}$$

- **Problem 1317: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\cos[c + d x]} (a + b \sec[c + d x])^4 (A + B \sec[c + d x] + C \sec[c + d x]^2) dx$$

Optimal (type 4, 401 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 \left(60 a^3 b B + 36 a b^3 B - 15 a^4 (A - C) + 18 a^2 b^2 (5 A + 3 C) + b^4 (9 A + 7 C) \right) \text{EllipticE}\left[\frac{1}{2} (c + d x), 2\right]}{15 d} + \\
& \frac{2 \left(21 a^4 B + 42 a^2 b^2 B + 5 b^4 B + 28 a^3 b (3 A + C) + 4 a b^3 (7 A + 5 C) \right) \text{EllipticF}\left[\frac{1}{2} (c + d x), 2\right]}{21 d} + \\
& \frac{2 b \left(261 a^2 b B + 75 b^3 B + 64 a^3 C + 2 a b^2 (147 A + 101 C) \right) \text{Sin}[c + d x]}{315 d \text{Cos}[c + d x]^{3/2}} + \\
& \frac{2 \left(1098 a^3 b B + 756 a b^3 B + 192 a^4 C + 21 b^4 (9 A + 7 C) + 7 a^2 b^2 (261 A + 155 C) \right) \text{Sin}[c + d x]}{315 d \sqrt{\text{Cos}[c + d x]}} + \\
& \frac{2 \left(63 A b^2 + 117 a b B + 48 a^2 C + 49 b^2 C \right) (b + a \text{Cos}[c + d x])^2 \text{Sin}[c + d x]}{315 d \text{Cos}[c + d x]^{5/2}} + \\
& \frac{2 (9 b B + 8 a C) (b + a \text{Cos}[c + d x])^3 \text{Sin}[c + d x]}{63 d \text{Cos}[c + d x]^{7/2}} + \frac{2 C (b + a \text{Cos}[c + d x])^4 \text{Sin}[c + d x]}{9 d \text{Cos}[c + d x]^{9/2}}
\end{aligned}$$

Result (type 5, 4150 leaves):

$$\begin{aligned}
& \frac{1}{(b + a \text{Cos}[c + d x])^4 (A + 2 C + 2 B \text{Cos}[c + d x] + A \text{Cos}[2 c + 2 d x]) \text{Cos}[c + d x]^{13/2} (a + b \text{Sec}[c + d x])^4 (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2)} \\
& \left(- \frac{1}{15 d} 2 \left(15 a^4 A - 180 a^2 A b^2 - 18 A b^4 - 120 a^3 b B - 72 a b^3 B - 30 a^4 C - 108 a^2 b^2 C - 14 b^4 C + 15 a^4 A \text{Cos}[2 c] \right) \text{Csc}[c] \text{Sec}[c] + \right. \\
& \frac{4 b^4 C \text{Sec}[c] \text{Sec}[c + d x]^5 \text{Sin}[d x]}{9 d} + \frac{4 \text{Sec}[c] \text{Sec}[c + d x]^4 \left(7 b^4 C \text{Sin}[c] + 9 b^4 B \text{Sin}[d x] + 36 a b^3 C \text{Sin}[d x] \right)}{63 d} + \frac{1}{315 d} \\
& 4 \text{Sec}[c] \text{Sec}[c + d x]^2 \left(63 A b^4 \text{Sin}[c] + 252 a b^3 B \text{Sin}[c] + 378 a^2 b^2 C \text{Sin}[c] + 49 b^4 C \text{Sin}[c] + 420 a A b^3 \text{Sin}[d x] + \right. \\
& \left. 630 a^2 b^2 B \text{Sin}[d x] + 75 b^4 B \text{Sin}[d x] + 420 a^3 b C \text{Sin}[d x] + 300 a b^3 C \text{Sin}[d x] \right) + \frac{1}{315 d} 4 \text{Sec}[c] \text{Sec}[c + d x]^3 \\
& \left(45 b^4 B \text{Sin}[c] + 180 a b^3 C \text{Sin}[c] + 63 A b^4 \text{Sin}[d x] + 252 a b^3 B \text{Sin}[d x] + 378 a^2 b^2 C \text{Sin}[d x] + 49 b^4 C \text{Sin}[d x] \right) + \frac{1}{105 d} \\
& 4 \text{Sec}[c] \text{Sec}[c + d x] \left(140 a A b^3 \text{Sin}[c] + 210 a^2 b^2 B \text{Sin}[c] + 25 b^4 B \text{Sin}[c] + 140 a^3 b C \text{Sin}[c] + 100 a b^3 C \text{Sin}[c] + 630 a^2 A b^2 \text{Sin}[d x] + \right. \\
& \left. 63 A b^4 \text{Sin}[d x] + 420 a^3 b B \text{Sin}[d x] + 252 a b^3 B \text{Sin}[d x] + 105 a^4 C \text{Sin}[d x] + 378 a^2 b^2 C \text{Sin}[d x] + 49 b^4 C \text{Sin}[d x] \right) \Big) - \\
& \left(16 a^3 A b \text{Cos}[c + d x]^6 \text{Csc}[c] \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]^2\right] (a + b \text{Sec}[c + d x])^4 \right. \\
& \left. (A + B \text{Sec}[c + d x] + C \text{Sec}[c + d x]^2) \text{Sec}[d x - \text{ArcTan}[\text{Cot}[c]]] \sqrt{1 - \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \right. \\
& \left. \sqrt{-\sqrt{1 + \text{Cot}[c]^2} \text{Sin}[c] \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \sqrt{1 + \text{Sin}[d x - \text{ArcTan}[\text{Cot}[c]]]} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(d (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(16 a A b^3 \cos [c + d x]^6 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]]^2 \right] (a + b \sec [c + d x])^4 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(3 d (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(4 a^4 B \cos [c + d x]^6 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]]^2 \right] (a + b \sec [c + d x])^4 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(8 a^2 b^2 B \cos [c + d x]^6 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]]^2 \right] (a + b \sec [c + d x])^4 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(d (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) - \\
& \left(20 b^4 B \cos [c + d x]^6 \csc [c] \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [d x - \operatorname{ArcTan} [\cot [c]]]^2 \right] (a + b \sec [c + d x])^4 \right. \\
& \quad \left. (A + B \sec [c + d x] + C \sec [c + d x]^2) \sec [d x - \operatorname{ArcTan} [\cot [c]]] \sqrt{1 - \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1 + \cot [c]^2} \sin [c] \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \sqrt{1 + \sin [d x - \operatorname{ArcTan} [\cot [c]]]} \right) / \\
& \left(21 d (b + a \cos [c + d x])^4 (A + 2 C + 2 B \cos [c + d x] + A \cos [2 c + 2 d x]) \sqrt{1 + \cot [c]^2} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(16 a^3 b C \cos [c+d x]^6 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^4 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(3 d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(80 a^3 b^3 C \cos [c+d x]^6 \csc [c] \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [d x-\operatorname{ArcTan}[\cot [c]]]^2\right] (a+b \sec [c+d x])^4 \right. \\
& \quad \left. (A+B \sec [c+d x]+C \sec [c+d x]^2) \sec [d x-\operatorname{ArcTan}[\cot [c]]] \sqrt{1-\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right. \\
& \quad \left. \sqrt{-\sqrt{1+\cot [c]^2} \sin [c] \sin [d x-\operatorname{ArcTan}[\cot [c]]]} \sqrt{1+\sin [d x-\operatorname{ArcTan}[\cot [c]]]} \right) / \\
& \quad \left(21 d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]) \sqrt{1+\cot [c]^2} \right) - \\
& \left(2 a^4 A \cos [c+d x]^6 \csc [c] (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right. \\
& \quad \left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c] \right) / \right. \\
& \quad \left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right. \\
& \quad \left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}} + \frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / \left. (d (b+a \cos [c+d x])^4 (A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) + \right.
\end{aligned}$$

$$\left(12 a^2 A b^2 \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]\right) /$$

$$\left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / (d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left(6 A b^4 \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left(\text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\tan[c]]]^2\right] \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]\right) /$$

$$\left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) -$$

$$\left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) / (5 d (b + a \cos[c + dx])^4 (A + 2C + 2B \cos[c + dx] + A \cos[2c + 2dx])) +$$

$$\left(8 a^3 b B \cos [c+d x]^6 \csc [c] (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) -$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (d(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))+$$

$$\left(24 a b^3 B \cos [c+d x]^6 \csc [c] (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) -$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (5 d(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x]))+$$

$$\left(2 a^4 C \cos [c+d x]^6 \csc [c] (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (d(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) +$$

$$\left(36 a^2 b^2 C \cos [c+d x]^6 \csc [c] (a+b \sec [c+d x])^4 (A+B \sec [c+d x]+C \sec [c+d x]^2) \right.$$

$$\left. \left(\operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos [d x+\operatorname{ArcTan}[\tan [c]]]^2\right] \sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]\right) / \right.$$

$$\left. \left(\sqrt{1-\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2} \sqrt{1+\tan [c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin [d x+\operatorname{ArcTan}[\tan [c]]] \tan [c]}{\sqrt{1+\tan [c]^2}}+\frac{2 \cos [c]^2 \cos [d x+\operatorname{ArcTan}[\tan [c]]] \sqrt{1+\tan [c]^2}}{\cos [c]^2+\sin [c]^2}}{\sqrt{\cos [c] \cos [d x+\operatorname{ArcTan}[\tan [c]]]} \sqrt{1+\tan [c]^2}} \right) / (5 d(b+a \cos [c+d x])^4(A+2 C+2 B \cos [c+d x]+A \cos [2 c+2 d x])) +$$

$$\left(14 b^4 C \cos[c + dx]^6 \csc[c] (a + b \sec[c + dx])^4 (A + B \sec[c + dx] + C \sec[c + dx]^2) \right.$$

$$\left. \left(\left(\text{HypergeometricPFQ} \left[\left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos[dx + \text{ArcTan}[\tan[c]]] \right]^2 \right) \sin[dx + \text{ArcTan}[\tan[c]]] \tan[c] \right) / \right.$$

$$\left. \left(\sqrt{1 - \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2} \sqrt{1 + \tan[c]^2} \right) - \right.$$

$$\left. \frac{\frac{\sin[dx + \text{ArcTan}[\tan[c]]] \tan[c]}{\sqrt{1 + \tan[c]^2}} + \frac{2 \cos[c]^2 \cos[dx + \text{ArcTan}[\tan[c]]] \sqrt{1 + \tan[c]^2}}{\cos[c]^2 + \sin[c]^2}}{\sqrt{\cos[c] \cos[dx + \text{ArcTan}[\tan[c]]]} \sqrt{1 + \tan[c]^2}} \right) /$$

$$(15 d (b + a \cos[c + dx])^4 (A + 2 C + 2 B \cos[c + dx] + A \cos[2c + 2dx]))$$

■ **Problem 1320: Attempted integration timed out after 120 seconds.**

$$\int \frac{\sqrt{\cos[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{a + b \sec[c + dx]} dx$$

Optimal (type 4, 97 leaves, 6 steps):

$$\frac{2 A \text{EllipticE} \left[\frac{1}{2} (c + dx), 2 \right]}{a d} - \frac{2 (A b - a B) \text{EllipticF} \left[\frac{1}{2} (c + dx), 2 \right]}{a^2 d} + \frac{2 (A b^2 - a (b B - a C)) \text{EllipticPi} \left[\frac{2 a}{a + b}, \frac{1}{2} (c + dx), 2 \right]}{a^2 (a + b) d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1334: Unable to integrate problem.**

$$\int \cos[c + dx]^{9/2} \sqrt{a + b \sec[c + dx]} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 457 leaves, 12 steps):

$$\begin{aligned}
& \frac{2(a^2 - b^2)(16Ab^3 - 75a^3B - 24ab^2B + 6a^2b(6A + 7C)) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] - \frac{1}{315a^4d \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]}}}{315a^4d \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]}} - \frac{1}{315a^4d \sqrt{\frac{b+a\cos[c+dx]}{a+b}}} \\
& \frac{2(16Ab^4 - 57a^3bB - 24ab^3B + 6a^2b^2(4A + 7C) - 21a^4(7A + 9C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b\sec[c+dx]} +}{315a^3d} \\
& \frac{2(8Ab^3 + 75a^3B - 12ab^2B + a^2b(13A + 21C)) \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{315a^3d} - \\
& \frac{2(6Ab^2 - 9abB - 7a^2(7A + 9C)) \cos[c+dx]^{3/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{315a^2d} + \\
& \frac{2(Ab + 9aB) \cos[c+dx]^{5/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{63ad} + \frac{2A \cos[c+dx]^{7/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{9d}
\end{aligned}$$

Result (type 8, 47 leaves):

$$\int \cos[c+dx]^{9/2} \sqrt{a+b\sec[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx]^2) dx$$

■ **Problem 1335: Unable to integrate problem.**

$$\int \cos[c+dx]^{7/2} \sqrt{a+b\sec[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx]^2) dx$$

Optimal (type 4, 360 leaves, 11 steps):

$$\begin{aligned}
& \frac{2(a^2 - b^2)(25a^2A + 8Ab^2 - 14abB + 35a^2C) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] - \frac{1}{105a^3d \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]}}}{105a^3d \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]}} + \frac{1}{105a^3d \sqrt{\frac{b+a\cos[c+dx]}{a+b}}} \\
& \frac{2(8Ab^3 + 63a^3B - 14ab^2B + a^2b(19A + 35C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b\sec[c+dx]} -}{105a^2d} \\
& \frac{2(4Ab^2 - 7abB - 5a^2(5A + 7C)) \sqrt{\cos[c+dx]} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{105a^2d} + \\
& \frac{2(Ab + 7aB) \cos[c+dx]^{3/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{35ad} + \frac{2A \cos[c+dx]^{5/2} \sqrt{a+b\sec[c+dx]} \sin[c+dx]}{7d}
\end{aligned}$$

Result (type 8, 47 leaves):

$$\int \cos[c+dx]^{7/2} \sqrt{a+b\sec[c+dx]} (A+B\sec[c+dx] + C\sec[c+dx]^2) dx$$

■ **Problem 1336: Unable to integrate problem.**

$$\int \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 273 leaves, 10 steps):

$$\frac{2\left(a^2-b^2\right)\left(2 A b-5 a B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]-2\left(2 A b^2-5 a b B-3 a^2(3 A+5 C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}+15 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}{15 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}+\frac{2(A b+5 a B) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a d}+\frac{2 A \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 d}$$

Result (type 8, 47 leaves):

$$\int \cos [c+d x]^{5 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

■ **Problem 1337: Attempted integration timed out after 120 seconds.**

$$\int \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 277 leaves, 13 steps):

$$\frac{2\left(A b^2-a^2(A+3 C)\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]+2 b C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]-2(A b+3 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}+2 A \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}+d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}+\frac{2 A \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1338: Unable to integrate problem.**

$$\int \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 258 leaves, 13 steps):

$$\frac{(2aB + bC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (2bB + aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{(2A - C) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + c \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{d \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \sqrt{\cos[c+dx]}}$$

Result (type 8, 47 leaves):

$$\int \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2) dx$$

■ **Problem 1339: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

Optimal (type 4, 346 leaves, 14 steps):

$$\frac{(8aA + 4bB + 3aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (8Ab^2 + 4abB - a^2C + 4b^2C) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{(4bB + aC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + 4bd \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}{4bd \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{c \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{2d \cos[c+dx]^{3/2}} + \frac{(4bB + aC) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4bd \sqrt{\cos[c+dx]}}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\sqrt{\cos[c+dx]}} dx$$

■ **Problem 1340: Unable to integrate problem.**

$$\int \frac{\sqrt{a+b \sec[c+dx]} (A + B \sec[c+dx] + C \sec[c+dx]^2)}{\cos[c+dx]^{3/2}} dx$$

Optimal (type 4, 447 leaves, 15 steps):

$$\frac{(24 A b^2 + 18 a b B - a^2 C + 16 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{24 b d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} -$$

$$\frac{(2 a^2 b B - 8 b^3 B - a^3 C - 4 a b^2 (2 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{8 b^2 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} -$$

$$\frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]} + \frac{c \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{3 d \operatorname{Cos}[c+dx]^{5/2}}}{24 b^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}}} +$$

$$\frac{(6 b B + a C) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{12 b d \operatorname{Cos}[c+dx]^{3/2}} + \frac{(24 A b^2 + 6 a b B - 3 a^2 C + 16 b^2 C) \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{24 b^2 d \sqrt{\operatorname{Cos}[c+dx]}}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2)}{\operatorname{Cos}[c+dx]^{3/2}} dx$$

■ **Problem 1341: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Cos}[c+dx]^{9/2} (a+b \operatorname{Sec}[c+dx])^{3/2} (A+B \operatorname{Sec}[c+dx]+C \operatorname{Sec}[c+dx]^2) dx$$

Optimal (type 4, 455 leaves, 12 steps):

$$\frac{2 (a^2 - b^2) (8 A b^3 + 75 a^3 B - 18 a b^2 B + a^2 (39 A b + 63 b C)) \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{315 a^3 d \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}} + \frac{1}{315 a^3 d \sqrt{\frac{b+a \operatorname{Cos}[c+dx]}{a+b}}}$$

$$2 (8 A b^4 + 246 a^3 b B - 18 a b^3 B + 21 a^4 (7 A + 9 C) + 3 a^2 b^2 (11 A + 21 C)) \sqrt{\operatorname{Cos}[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+dx]} -$$

$$\frac{2 (4 A b^3 - 75 a^3 B - 9 a b^2 B - 2 a^2 b (44 A + 63 C)) \sqrt{\operatorname{Cos}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{315 a^2 d} +$$

$$\frac{2 (3 A b^2 + 72 a b B + 7 a^2 (7 A + 9 C)) \operatorname{Cos}[c+dx]^{3/2} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{315 a d} +$$

$$\frac{2 (A b + 3 a B) \operatorname{Cos}[c+dx]^{5/2} \sqrt{a+b \operatorname{Sec}[c+dx]} \operatorname{Sin}[c+dx]}{21 d} + \frac{2 A \operatorname{Cos}[c+dx]^{7/2} (a+b \operatorname{Sec}[c+dx])^{3/2} \operatorname{Sin}[c+dx]}{9 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1342: Attempted integration timed out after 120 seconds.**

$$\int \cos[c + dx]^{7/2} (a + b \sec[c + dx])^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 359 leaves, 11 steps):

$$\frac{2(a^2 - b^2)(25a^2A - 6Ab^2 + 21abB + 35a^2C) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{105a^2d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} - \frac{1}{105a^2d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

$$\frac{2(6Ab^3 - 63a^3B - 21ab^2B - 2a^2b(41A + 70C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + 2(3Ab^2 + 42abB + 5a^2(5A + 7C)) \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{105ad} +$$

$$\frac{2(3Ab + 7aB) \cos[c+dx]^{3/2} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{35d} + \frac{2A \cos[c+dx]^{5/2} (a+b \sec[c+dx])^{3/2} \sin[c+dx]}{7d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1343: Attempted integration timed out after 120 seconds.**

$$\int \cos[c + dx]^{5/2} (a + b \sec[c + dx])^{3/2} (A + B \sec[c + dx] + C \sec[c + dx]^2) dx$$

Optimal (type 4, 356 leaves, 14 steps):

$$-\frac{2(3Ab^3 - 5a^3B + 5ab^2B - 3a^2b(A + 5C)) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{15ad \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{2b^2C \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} +$$

$$\frac{2(3Ab^2 + 20abB + 3a^2(3A + 5C)) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]}}{15ad \sqrt{\frac{b+a \cos[c+dx]}{a+b}}} +$$

$$\frac{2(3Ab + 5aB) \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{15d} + \frac{2A \cos[c+dx]^{3/2} (a+b \sec[c+dx])^{3/2} \sin[c+dx]}{5d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1344: Attempted integration timed out after 120 seconds.**

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 340 leaves, 14 steps):

$$\frac{(6 a b B-b^2(2 A-3 C)+2 a^2(A+3 C)) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{b(2 b B+3 a C) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{(8 A b+6 a B-3 b C) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} -$$

$$\frac{b(2 A-3 C) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d \sqrt{\cos [c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{3 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1345: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 353 leaves, 14 steps):

$$\begin{aligned}
& \frac{(8 a^2 B + 4 b^2 B + a b (8 A + 7 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(8 A b^2 + 12 a b B + 3 a^2 C + 4 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(8 a A - 4 b B - 5 a C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{4 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \\
& \frac{(4 b B + 3 a C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{C (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{2 d \sqrt{\operatorname{Cos}[c+d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1346: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{3/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\sqrt{\operatorname{Cos}[c+d x]}} dx$$

Optimal (type 4, 446 leaves, 15 steps):

$$\begin{aligned}
& \frac{(42 a b B + 8 b^2 (3 A + 2 C) + a^2 (48 A + 17 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{24 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(6 a^2 b B + 8 b^3 B - a^3 C + 12 a b^2 (2 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{8 b d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} - \\
& \frac{(24 A b^2 + 30 a b B + 3 a^2 C + 16 b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{24 b d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} + \frac{(2 b B + a C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{4 d \operatorname{Cos}[c+d x]^{3/2}} + \\
& \frac{(24 A b^2 + 30 a b B + 3 a^2 C + 16 b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 b d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{C (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{3 d \operatorname{Cos}[c+d x]^{3/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1347: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \operatorname{Sec}[c + d x])^{3/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2)}{\operatorname{Cos}[c + d x]^{3/2}} dx$$

Optimal (type 4, 551 leaves, 16 steps):

$$\frac{(136 a^2 b B + 128 b^3 B - 3 a^3 C + 12 a b^2 (28 A + 19 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right]}{192 b d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} -$$

$$\left((8 a^3 b B - 96 a b^3 B - 3 a^4 C - 24 a^2 b^2 (2 A + C) - 16 b^4 (4 A + 3 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \right) /$$

$$(64 b^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}) - \frac{1}{192 b^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}}$$

$$(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]} +$$

$$\frac{(8 b B + 3 a C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d \operatorname{Cos}[c+d x]^{5/2}} + \frac{(48 A b^2 + 56 a b B + 3 a^2 C + 36 b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{96 b d \operatorname{Cos}[c+d x]^{3/2}} +$$

$$\frac{(24 a^2 b B + 128 b^3 B - 9 a^3 C + 12 a b^2 (20 A + 13 C)) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{192 b^2 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{C (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{4 d \operatorname{Cos}[c+d x]^{5/2}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1348: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Cos}[c + d x]^{11/2} (a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 565 leaves, 13 steps):

$$\begin{aligned}
& \left(2 (a^2 - b^2) (40 A b^4 + 1254 a^3 b B - 110 a b^3 B + 75 a^4 (9 A + 11 C) + 15 a^2 b^2 (19 A + 33 C)) \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \right) / \\
& \left(3465 a^3 d \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \right) + \frac{1}{3465 a^3 d \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{a + b}}} \\
& 2 (40 A b^5 + 1617 a^5 B + 3069 a^3 b^2 B - 110 a b^4 B + 15 a^2 b^3 (17 A + 33 C) + 15 a^4 b (247 A + 319 C)) \\
& \sqrt{\operatorname{Cos}[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \operatorname{Sec}[c + d x]} - \frac{1}{3465 a^2 d} \\
& 2 (20 A b^4 - 1793 a^3 b B - 55 a b^3 B - 75 a^4 (9 A + 11 C) - 5 a^2 b^2 (205 A + 297 C)) \sqrt{\operatorname{Cos}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x] + \\
& 2 (15 A b^3 + 539 a^3 B + 825 a b^2 B + 5 a^2 b (229 A + 297 C)) \operatorname{Cos}[c + d x]^{3/2} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x] \\
& \frac{3465 a d}{+} \\
& \frac{2 (5 A b^2 + 44 a b B + 3 a^2 (9 A + 11 C)) \operatorname{Cos}[c + d x]^{5/2} \sqrt{a + b \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x]}{231 d} + \\
& \frac{2 (5 A b + 11 a B) \operatorname{Cos}[c + d x]^{7/2} (a + b \operatorname{Sec}[c + d x])^{3/2} \operatorname{Sin}[c + d x]}{99 d} + \frac{2 A \operatorname{Cos}[c + d x]^{9/2} (a + b \operatorname{Sec}[c + d x])^{5/2} \operatorname{Sin}[c + d x]}{11 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1349: Attempted integration timed out after 120 seconds.**

$$\int \operatorname{Cos}[c + d x]^{9/2} (a + b \operatorname{Sec}[c + d x])^{5/2} (A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2) dx$$

Optimal (type 4, 452 leaves, 12 steps):

$$\begin{aligned}
& \frac{2(a^2 - b^2)(10Ab^3 - 75a^3B - 45ab^2B - 6a^2b(19A + 28C)) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{315a^2d\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}} - \frac{1}{315a^2d\sqrt{\frac{b+a\cos[c+dx]}{a+b}}} \\
& \frac{2(10Ab^4 - 435a^3bB - 45ab^3B - 21a^4(7A+9C) - 3a^2b^2(93A+161C))\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]\sqrt{a+b\sec[c+dx]} +}{315ad} \\
& \frac{2(5Ab^3 + 75a^3B + 135ab^2B + a^2b(163A+231C))\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}\sin[c+dx]}{315ad} + \\
& \frac{2(15Ab^2 + 90abB + 7a^2(7A+9C))\cos[c+dx]^{3/2}\sqrt{a+b\sec[c+dx]}\sin[c+dx]}{315d} + \\
& \frac{2(5Ab + 9aB)\cos[c+dx]^{5/2}(a+b\sec[c+dx])^{3/2}\sin[c+dx]}{63d} + \frac{2A\cos[c+dx]^{7/2}(a+b\sec[c+dx])^{5/2}\sin[c+dx]}{9d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1350: Attempted integration timed out after 120 seconds.**

$$\int \cos[c+dx]^{7/2} (a+b\sec[c+dx])^{5/2} (A+B\sec[c+dx] + C\sec[c+dx]^2) dx$$

Optimal (type 4, 441 leaves, 15 steps):

$$\begin{aligned}
& - \left(2(15Ab^4 - 56a^3bB + 56ab^3B + 10a^2b^2(A-7C) - 5a^4(5A+7C)) \sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \right) / \\
& \left(105ad\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]} \right) + \frac{2b^3c\sqrt{\frac{b+a\cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}} + \frac{1}{105ad\sqrt{\frac{b+a\cos[c+dx]}{a+b}}} \\
& \frac{2(15Ab^3 + 63a^3B + 161ab^2B + 5a^2b(29A+49C))\sqrt{\cos[c+dx]}\operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right]\sqrt{a+b\sec[c+dx]} +}{105d} \\
& \frac{2(15Ab^2 + 56abB + 5a^2(5A+7C))\sqrt{\cos[c+dx]}\sqrt{a+b\sec[c+dx]}\sin[c+dx]}{105d} + \\
& \frac{2(5Ab + 7aB)\cos[c+dx]^{3/2}(a+b\sec[c+dx])^{3/2}\sin[c+dx]}{35d} + \frac{2A\cos[c+dx]^{5/2}(a+b\sec[c+dx])^{5/2}\sin[c+dx]}{7d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1351: Attempted integration timed out after 120 seconds.**

$$\int \cos [c+d x]^{5 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 419 leaves, 15 steps):

$$\frac{(10 a^3 B+20 a b^2 B-b^3(16 A-15 C)+4 a^2 b(4 A+15 C)) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{b^2(2 b B+5 a C) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{1}{15 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

$$\frac{(70 a b B+b^2(46 A-15 C)+6 a^2(3 A+5 C)) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} -}{15 d \sqrt{\cos [c+d x]}} +$$

$$\frac{b(16 A b+10 a B-15 b C) \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 d} + \frac{2(A b+a B) \sqrt{\cos [c+d x]}(a+b \sec [c+d x])^{3 / 2} \sin [c+d x]}{5 d} + \frac{2 A \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{5 / 2} \sin [c+d x]}{5 d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1352: Attempted integration timed out after 120 seconds.**

$$\int \cos [c+d x]^{3 / 2}(a+b \sec [c+d x])^{5 / 2}(A+B \sec [c+d x]+C \sec [c+d x]^2) d x$$

Optimal (type 4, 427 leaves, 15 steps):

$$\begin{aligned}
& \frac{(48 a^2 b B + 12 b^3 B + 8 a^3 (A + 3 C) + a b^2 (16 A + 33 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{12 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{b(8 A b^2 + 20 a b B + 15 a^2 C + 4 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{4 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(24 a^2 B - 12 b^2 B + a b (56 A - 27 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{12 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} - \\
& \frac{b(8 a A - 12 b B - 21 a C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{12 d \sqrt{\operatorname{Cos}[c+d x]}} - \\
& \frac{b(4 A - 3 C)(a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{6 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{2 A \sqrt{\operatorname{Cos}[c+d x]}(a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{3 d}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1353: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2) dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$\begin{aligned}
& \frac{(48 a^3 B + 66 a b^2 B + 8 b^3 (3 A + 2 C) + a^2 b (96 A + 59 C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{24 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \\
& \frac{(30 a^2 b B + 8 b^3 B + 5 a^3 C + 20 a b^2 (2 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{8 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} - \frac{1}{24 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} \\
& \frac{(54 a b B - a^2 (48 A - 33 C) + 8 b^2 (3 A + 2 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]} +}{24 d \sqrt{\operatorname{Cos}[c+d x]}} \\
& \frac{(24 A b^2 + 42 a b B + 15 a^2 C + 16 b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{24 d \sqrt{\operatorname{Cos}[c+d x]}} + \\
& \frac{(6 b B + 5 a C) (a+b \operatorname{Sec}[c+d x])^{3/2} \operatorname{Sin}[c+d x]}{12 d \sqrt{\operatorname{Cos}[c+d x]}} + \frac{C (a+b \operatorname{Sec}[c+d x])^{5/2} \operatorname{Sin}[c+d x]}{3 d \sqrt{\operatorname{Cos}[c+d x]}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1354: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a+b \operatorname{Sec}[c+d x])^{5/2} (A+B \operatorname{Sec}[c+d x]+C \operatorname{Sec}[c+d x]^2)}{\sqrt{\operatorname{Cos}[c+d x]}} dx$$

Optimal (type 4, 550 leaves, 16 steps):

$$\begin{aligned}
& \left((472 a^2 b B + 128 b^3 B + 4 a b^2 (132 A + 89 C) + a^3 (384 A + 133 C)) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2a}{a + b}\right] \right) / \\
& \left(192 d \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \right) + \\
& \left((40 a^3 b B + 160 a b^3 B - 5 a^4 C + 120 a^2 b^2 (2A + C) + 16 b^4 (4A + 3C)) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + dx), \frac{2a}{a + b}\right] \right) / \\
& \left(64 b d \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \right) - \frac{1}{192 b d \sqrt{\frac{b + a \cos[c + dx]}{a + b}}} \\
& (264 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (108 A + 71 C)) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2a}{a + b}\right] \sqrt{a + b \sec[c + dx]} + \\
& \frac{(16 A b^2 + 24 a b B + 5 a^2 C + 12 b^2 C) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{32 d \cos[c + dx]^{3/2}} + \\
& \frac{(264 a^2 b B + 128 b^3 B + 15 a^3 C + 4 a b^2 (108 A + 71 C)) \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{192 b d \sqrt{\cos[c + dx]}} + \\
& \frac{(8 b B + 5 a C) (a + b \sec[c + dx])^{3/2} \sin[c + dx]}{24 d \cos[c + dx]^{3/2}} + \frac{C (a + b \sec[c + dx])^{5/2} \sin[c + dx]}{4 d \cos[c + dx]^{3/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1355: Attempted integration timed out after 120 seconds.**

$$\int \frac{(a + b \sec[c + dx])^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\cos[c + dx]^{3/2}} dx$$

Optimal (type 4, 674 leaves, 17 steps):

$$\begin{aligned}
& \left((1330 a^3 b B + 3560 a b^3 B - 15 a^4 C + 256 b^4 (5 A + 4 C) + 4 a^2 b^2 (1180 A + 809 C)) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + d x), \frac{2 a}{a + b}\right] \right) / \\
& \left(1920 b d \sqrt{\cos[c + d x]} \sqrt{a + b \sec[c + d x]} \right) - \\
& \left((10 a^4 b B - 240 a^2 b^3 B - 96 b^5 B - 3 a^5 C - 40 a^3 b^2 (2 A + C) - 80 a b^4 (4 A + 3 C)) \sqrt{\frac{b + a \cos[c + d x]}{a + b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c + d x), \frac{2 a}{a + b}\right] \right) / \\
& \left(128 b^2 d \sqrt{\cos[c + d x]} \sqrt{a + b \sec[c + d x]} \right) - \frac{1}{1920 b^2 d \sqrt{\frac{b + a \cos[c + d x]}{a + b}}} \\
& (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C)) \sqrt{\cos[c + d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c + d x), \frac{2 a}{a + b}\right] \sqrt{a + b \sec[c + d x]} + \\
& \frac{(80 A b^2 + 110 a b B + 15 a^2 C + 64 b^2 C) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{240 d \cos[c + d x]^{5/2}} + \\
& \frac{(590 a^2 b B + 360 b^3 B + 15 a^3 C + 4 a b^2 (260 A + 193 C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x]}{960 b d \cos[c + d x]^{3/2}} + \frac{1}{1920 b^2 d \sqrt{\cos[c + d x]}} \\
& (150 a^3 b B + 2840 a b^3 B - 45 a^4 C + 256 b^4 (5 A + 4 C) + 12 a^2 b^2 (220 A + 141 C)) \sqrt{a + b \sec[c + d x]} \sin[c + d x] + \\
& \frac{(2 b B + a C) (a + b \sec[c + d x])^{3/2} \sin[c + d x]}{8 d \cos[c + d x]^{5/2}} + \frac{C (a + b \sec[c + d x])^{5/2} \sin[c + d x]}{5 d \cos[c + d x]^{5/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1356: Unable to integrate problem.**

$$\int \frac{\cos[c + d x]^{7/2} (A + B \sec[c + d x] + C \sec[c + d x]^2)}{\sqrt{a + b \sec[c + d x]}} dx$$

Optimal (type 4, 380 leaves, 11 steps):

$$\left(2 \left(48 A b^4 - 49 a^3 b B - 56 a b^3 B + 5 a^4 (5 A + 7 C) + 2 a^2 b^2 (16 A + 35 C) \right) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2a}{a+b}\right] \right) /$$

$$\left(105 a^4 d \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \right) - \frac{1}{105 a^4 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}}}$$

$$2 \left(48 A b^3 - 63 a^3 B - 56 a b^2 B + a^2 (44 A b + 70 b C) \right) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2a}{a+b}\right] \sqrt{a + b \sec[c + dx]} +$$

$$\frac{2 \left(24 A b^2 - 28 a b B + 5 a^2 (5 A + 7 C) \right) \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{105 a^3 d} -$$

$$\frac{2 \left(6 A b - 7 a B \right) \cos[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{35 a^2 d} + \frac{2 A \cos[c + dx]^{5/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{7 a d}$$

Result (type 8, 47 leaves):

$$\int \frac{\cos[c + dx]^{7/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

■ **Problem 1357: Unable to integrate problem.**

$$\int \frac{\cos[c + dx]^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

Optimal (type 4, 291 leaves, 10 steps):

$$\frac{2 \left(8 A b^3 - 5 a^3 B - 10 a b^2 B + a^2 b (7 A + 15 C) \right) \sqrt{\frac{b + a \cos[c + dx]}{a + b}} \operatorname{EllipticF}\left[\frac{1}{2}(c + dx), \frac{2a}{a+b}\right]}{15 a^3 d \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]}} +$$

$$\frac{2 \left(8 A b^2 - 10 a b B + 3 a^2 (3 A + 5 C) \right) \sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c + dx), \frac{2a}{a+b}\right] \sqrt{a + b \sec[c + dx]}}{15 a^3 d \sqrt{\frac{b + a \cos[c + dx]}{a + b}}}$$

$$\frac{2 \left(4 A b - 5 a B \right) \sqrt{\cos[c + dx]} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{15 a^2 d} + \frac{2 A \cos[c + dx]^{3/2} \sqrt{a + b \sec[c + dx]} \sin[c + dx]}{5 a d}$$

Result (type 8, 47 leaves):

$$\int \frac{\cos[c + dx]^{5/2} (A + B \sec[c + dx] + C \sec[c + dx]^2)}{\sqrt{a + b \sec[c + dx]}} dx$$

■ **Problem 1358: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos [c+d x]^{3 / 2} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} d x$$

Optimal (type 4, 216 leaves, 9 steps):

$$\frac{2\left(2 A b^2-3 a b B+a^2(A+3 C)\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} - \frac{2(2 A b-3 a B) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{3 a^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2 A \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1359: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} d x$$

Optimal (type 4, 219 leaves, 12 steps):

$$-\frac{2(A b-a B) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 C \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{a d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{\cos [c+d x]} (A+B \sec [c+d x]+C \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} d x$$

■ **Problem 1360: Unable to integrate problem.**

$$\int \frac{A+B \sec [c+d x]+C \sec [c+d x]^2}{\sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} d x$$

Optimal (type 4, 260 leaves, 13 steps):

$$\frac{(2A + C) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (2bB - aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{d \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{c \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + \frac{c \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{bd \sqrt{\cos[c+dx]}}}{bd \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 47 leaves):

$$\int \frac{A + B \sec[c+dx] + C \sec[c+dx]^2}{\sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} dx$$

■ **Problem 1361: Unable to integrate problem.**

$$\int \frac{A + B \sec[c+dx] + C \sec[c+dx]^2}{\cos[c+dx]^{3/2} \sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 350 leaves, 14 steps):

$$\frac{(4bB - aC) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] + (8Ab^2 - 4abB + 3a^2C + 4b^2C) \sqrt{\frac{b+a \cos[c+dx]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+dx), \frac{2a}{a+b}\right]}{4bd \sqrt{\cos[c+dx]} \sqrt{a+b \sec[c+dx]}} + \frac{(4bB - 3aC) \sqrt{\cos[c+dx]} \operatorname{EllipticE}\left[\frac{1}{2}(c+dx), \frac{2a}{a+b}\right] \sqrt{a+b \sec[c+dx]} + \frac{c \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{2bd \cos[c+dx]^{3/2}} + \frac{(4bB - 3aC) \sqrt{a+b \sec[c+dx]} \sin[c+dx]}{4b^2d \sqrt{\cos[c+dx]}}}{4b^2d \sqrt{\frac{b+a \cos[c+dx]}{a+b}}}$$

Result (type 8, 47 leaves):

$$\int \frac{A + B \sec[c+dx] + C \sec[c+dx]^2}{\cos[c+dx]^{3/2} \sqrt{a+b \sec[c+dx]}} dx$$

■ **Problem 1362: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos[c+dx]} (aA + (Ab + aB) \sec[c+dx] + bB \sec[c+dx]^2)}{\sqrt{a+b \sec[c+dx]}} dx$$

Optimal (type 4, 208 leaves, 13 steps):

$$\frac{2 a B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 b B \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} + \frac{2 A \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}}$$

Result (type 8, 56 leaves):

$$\int \frac{\sqrt{\cos [c+d x]} (a A + (A b + a B) \sec [c+d x] + b B \sec [c+d x]^2)}{\sqrt{a+b \sec [c+d x]}} dx$$

■ **Problem 1363: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos [c+d x]^{5/2} (A + B \sec [c+d x] + C \sec [c+d x]^2)}{(a+b \sec [c+d x])^{3/2}} dx$$

Optimal (type 4, 461 leaves, 11 steps):

$$\frac{2 (48 A b^3 - 5 a^3 B - 40 a b^2 B + 6 a^2 b (2 A + 5 C)) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{15 a^4 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} -$$

$$\left(2 (48 A b^4 + 25 a^3 b B - 40 a b^3 B - 6 a^2 b^2 (4 A - 5 C) - 3 a^4 (3 A + 5 C)) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) /$$

$$\left(15 a^4 (a^2 - b^2) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \frac{2 (A b^2 - a (b B - a C)) \cos [c+d x]^{3/2} \sin [c+d x]}{a (a^2 - b^2) d \sqrt{a+b \sec [c+d x]}} +$$

$$\frac{2 (24 A b^3 + 5 a^3 B - 20 a b^2 B - a^2 (9 A b - 15 b C)) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{15 a^3 (a^2 - b^2) d} -$$

$$\frac{2 (6 A b^2 - 5 a b B - a^2 (A - 5 C)) \cos [c+d x]^{3/2} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{5 a^2 (a^2 - b^2) d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1364: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos [c+d x]^{3 / 2}\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{\left(a+b \sec [c+d x]\right)^{3 / 2}} d x$$

Optimal (type 4, 350 leaves, 10 steps):

$$\frac{2\left(8 A b^2-6 a b B+a^2(A+3 C)\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\left(2\left(8 A b^3+3 a^3 B-6 a b^2 B-a^2(5 A b-3 b C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}\right) /$$

$$\left(3 a^3\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}\right) + \frac{2\left(A b^2-a(b B-a C)\right) \sqrt{\cos [c+d x]} \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2\left(4 A b^2-3 a b B-a^2(A-3 C)\right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]}{3 a^2\left(a^2-b^2\right) d}$$

Result (type 1, 1 leaves):

???

■ **Problem 1365: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]}\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{\left(a+b \sec [c+d x]\right)^{3 / 2}} d x$$

Optimal (type 4, 249 leaves, 9 steps):

$$\frac{2\left(2 A b-a B\right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} -$$

$$\frac{2\left(2 A b^2-a b B-a^2(A-C)\right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]}}{a^2\left(a^2-b^2\right) d \sqrt{\frac{b+a \cos [c+d x]}{a+b}}} + \frac{2\left(A b^2-a(b B-a C)\right) \sin [c+d x]}{a\left(a^2-b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{\cos [c+d x]}\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{\left(a+b \sec [c+d x]\right)^{3 / 2}} d x$$

■ **Problem 1366: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 311 leaves, 13 steps):

$$\frac{2 A \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + 2 C \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{a d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{2 C \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} +$$

$$\frac{2(A b^2 - a(b B - a C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{a b (a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} - \frac{2(A b^2 - a(b B - a C)) \operatorname{Sin}[c+d x]}{b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

■ **Problem 1367: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{3/2}} dx$$

Optimal (type 4, 393 leaves, 14 steps):

$$\frac{C \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + (2 b B - 3 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{(2 b B - 3 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{b^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} -$$

$$\frac{(2 A b^2 - 2 a b B + 3 a^2 C - b^2 C) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}}{b^2 (a^2 - b^2) d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}} -$$

$$\frac{2(A b^2 - a(b B - a C)) \operatorname{Sin}[c+d x]}{b (a^2 - b^2) d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{(2 A b^2 - 2 a b B + 3 a^2 C - b^2 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{b^2 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1368: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos [c+d x]^{5 / 2}\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 663 leaves, 12 steps):

$$\begin{aligned} & \left(2 \left(128 A b^5 + 5 a^5 B + 80 a^3 b^2 B - 80 a b^4 B - 4 a^2 b^3 (29 A - 10 C) - a^4 b (17 A + 45 C) \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \right) / \\ & \left(15 a^5 \left(a^2 - b^2 \right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \right) + \\ & \left(2 \left(128 A b^5 - 40 a^5 b B + 140 a^3 b^3 B - 80 a b^5 B + 5 a^4 b^2 (11 A - 15 C) - 4 a^2 b^4 (53 A - 10 C) + 3 a^6 (3 A + 5 C) \right) \right. \\ & \left. \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) / \left(15 a^5 \left(a^2 - b^2 \right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \\ & \frac{2 \left(A b^2 - a (b B - a C) \right) \cos [c+d x]^{3 / 2} \sin [c+d x]}{3 a \left(a^2 - b^2 \right) d (a+b \sec [c+d x])^{3 / 2}} - \frac{2 \left(8 A b^4 + 9 a^3 b B - 5 a b^3 B - 2 a^2 b^2 (6 A - C) - 6 a^4 C \right) \cos [c+d x]^{3 / 2} \sin [c+d x]}{3 a^2 \left(a^2 - b^2 \right)^2 d \sqrt{a+b \sec [c+d x]}} - \\ & \frac{1}{15 a^4 \left(a^2 - b^2 \right)^2 d} 2 \left(64 A b^5 - 5 a^5 B + 65 a^3 b^2 B - 40 a b^4 B + 2 a^4 b (7 A - 20 C) - 2 a^2 b^3 (49 A - 10 C) \right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x] + \\ & \frac{1}{15 a^3 \left(a^2 - b^2 \right)^2 d} 2 \left(48 A b^4 + 50 a^3 b B - 30 a b^3 B + a^4 (3 A - 35 C) - a^2 b^2 (71 A - 15 C) \right) \cos [c+d x]^{3 / 2} \sqrt{a+b \sec [c+d x]} \sin [c+d x] \end{aligned}$$

Result (type 1, 1 leaves):

???

■ **Problem 1369: Attempted integration timed out after 120 seconds.**

$$\int \frac{\cos [c+d x]^{3 / 2}\left(A+B \sec [c+d x]+C \sec [c+d x]^2\right)}{(a+b \sec [c+d x])^{5 / 2}} d x$$

Optimal (type 4, 521 leaves, 11 steps):

$$\frac{2 \left(16 A b^4 + 9 a^3 b B - 8 a b^3 B - 2 a^2 b^2 (8 A - C) - a^4 (A + 3 C) \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^4 \left(a^2 - b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

$$\left(2 \left(16 A b^5 - 3 a^5 B + 15 a^3 b^2 B - 8 a b^4 B - 2 a^2 b^3 (14 A - C) + a^4 (8 A b - 6 b C) \right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) /$$

$$\left(3 a^4 \left(a^2 - b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \frac{2 \left(A b^2 - a (b B - a C) \right) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a \left(a^2 - b^2\right) d (a+b \sec [c+d x])^{3/2}} +$$

$$\frac{2 \left(10 a^2 A b^2 - 6 A b^4 - 7 a^3 b B + 3 a b^3 B + 4 a^4 C \right) \sqrt{\cos [c+d x]} \sin [c+d x]}{3 a^2 \left(a^2 - b^2\right)^2 d \sqrt{a+b \sec [c+d x]}} + \frac{1}{3 a^3 \left(a^2 - b^2\right)^2 d}$$

$$2 \left(8 A b^4 + 8 a^3 b B - 4 a b^3 B + a^4 (A - 5 C) - a^2 b^2 (13 A - C) \right) \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]} \sin [c+d x]$$

Result (type 1, 1 leaves):

???

■ **Problem 1370: Unable to integrate problem.**

$$\int \frac{\sqrt{\cos [c+d x]} \left(A + B \sec [c+d x] + C \sec [c+d x]^2 \right)}{\left(a + b \sec [c+d x] \right)^{5/2}} dx$$

Optimal (type 4, 401 leaves, 10 steps):

$$\frac{2 \left(8 A b^3 + 3 a^3 B - 2 a b^2 B - a^2 b (9 A + C) \right) \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 a^3 \left(a^2 - b^2\right) d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}} +$$

$$\left(2 \left(8 A b^4 + 6 a^3 b B - 2 a b^3 B + 3 a^4 (A - C) - a^2 b^2 (15 A + C) \right) \sqrt{\cos [c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \sec [c+d x]} \right) /$$

$$\left(3 a^3 \left(a^2 - b^2\right)^2 d \sqrt{\frac{b+a \cos [c+d x]}{a+b}} \right) + \frac{2 \left(A b^2 - a (b B - a C) \right) \sin [c+d x]}{3 a \left(a^2 - b^2\right) d \sqrt{\cos [c+d x]} (a+b \sec [c+d x])^{3/2}} -$$

$$\frac{2 \left(4 A b^4 + 5 a^3 b B - a b^3 B - 2 a^4 C - 2 a^2 b^2 (4 A + C) \right) \sin [c+d x]}{3 a^2 \left(a^2 - b^2\right)^2 d \sqrt{\cos [c+d x]} \sqrt{a+b \sec [c+d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{\sqrt{\cos [c+d x]} \left(A + B \sec [c+d x] + C \sec [c+d x]^2 \right)}{\left(a + b \sec [c+d x] \right)^{5/2}} dx$$

■ **Problem 1371: Unable to integrate problem.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 378 leaves, 10 steps):

$$\frac{2 (2 A b^2 + a b B - a^2 (3 A + C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] - \left(2 (2 A b^3 + 3 a^3 B + a b^2 B - 2 a^2 b (3 A + 2 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}\right) / \left(3 a^2 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}\right) - \left(3 a^2 (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}\right) - \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} (a+b \operatorname{Sec}[c+d x])^{3/2}} + \frac{2 (A b^4 + 2 a^3 b B + 2 a b^3 B + a^4 C - 5 a^2 b^2 (A + C)) \operatorname{Sin}[c+d x]}{3 a b (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 8, 47 leaves):

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\sqrt{\operatorname{Cos}[c + d x]} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

■ **Problem 1372: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Cos}[c + d x]^{3/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 447 leaves, 14 steps):

$$\frac{2 (A b^2 - a (b B - a C)) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] - 2 C \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] - \left(2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2} (c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}\right) / \left(3 a b (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}\right) + \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d \operatorname{Cos}[c+d x]^{3/2} (a+b \operatorname{Sec}[c+d x])^{3/2}} - \frac{2 (A b^4 - 4 a b^3 B - 3 a^4 C + a^2 b^2 (3 A + 7 C)) \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}$$

Result (type 1, 1 leaves):

???

■ **Problem 1373: Attempted integration timed out after 120 seconds.**

$$\int \frac{A + B \operatorname{Sec}[c + d x] + C \operatorname{Sec}[c + d x]^2}{\operatorname{Cos}[c + d x]^{5/2} (a + b \operatorname{Sec}[c + d x])^{5/2}} dx$$

Optimal (type 4, 563 leaves, 15 steps):

$$\frac{(2 A b^2 - 2 a b B + 5 a^2 C - 3 b^2 C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticF}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] + (2 b B - 5 a C) \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}} \operatorname{EllipticPi}\left[2, \frac{1}{2}(c+d x), \frac{2 a}{a+b}\right]}{3 b^2 (a^2 - b^2) d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}} + \frac{b^3 d \sqrt{\operatorname{Cos}[c+d x]} \sqrt{a+b \operatorname{Sec}[c+d x]}}{\left(8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C\right) \sqrt{\operatorname{Cos}[c+d x]} \operatorname{EllipticE}\left[\frac{1}{2}(c+d x), \frac{2 a}{a+b}\right] \sqrt{a+b \operatorname{Sec}[c+d x]}} \Bigg/$$

$$\left(3 b^3 (a^2 - b^2)^2 d \sqrt{\frac{b+a \operatorname{Cos}[c+d x]}{a+b}}\right) - \frac{2 (A b^2 - a (b B - a C)) \operatorname{Sin}[c+d x]}{3 b (a^2 - b^2) d \operatorname{Cos}[c+d x]^{5/2} (a + b \operatorname{Sec}[c+d x])^{3/2}} +$$

$$\frac{2 (3 A b^4 + 2 a^3 b B - 6 a b^3 B - 5 a^4 C + a^2 b^2 (A + 9 C)) \operatorname{Sin}[c+d x]}{3 b^2 (a^2 - b^2)^2 d \operatorname{Cos}[c+d x]^{3/2} \sqrt{a+b \operatorname{Sec}[c+d x]}} -$$

$$\frac{(8 A b^4 + 6 a^3 b B - 14 a b^3 B - 15 a^4 C + 26 a^2 b^2 C - 3 b^4 C) \sqrt{a+b \operatorname{Sec}[c+d x]} \operatorname{Sin}[c+d x]}{3 b^3 (a^2 - b^2)^2 d \sqrt{\operatorname{Cos}[c+d x]}}$$

Result (type 1, 1 leaves):

???

Test results for the 470 problems in "4.5.7 (d trig)^m (a+b (c sec)^n)^p.m"

■ **Problem 5: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + f x] (a + b \operatorname{Sec}[e + f x])^2 dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$- \frac{(a+b) \operatorname{ArcTanh}[\operatorname{Cos}[e+f x]]}{f} + \frac{b \operatorname{Sec}[e+f x]}{f}$$

Result (type 3, 84 leaves):

$$-\frac{a \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} - \frac{b \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + fx)\right]\right]}{f} + \frac{a \operatorname{Log}\left[\sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b \operatorname{Log}\left[\sin\left[\frac{1}{2}(e + fx)\right]\right]}{f} + \frac{b \operatorname{Sec}[e + fx]}{f}$$

■ **Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + fx]^3 (a + b \operatorname{Sec}[e + fx]^2) dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{(a + 3b) \operatorname{ArcTanh}[\cos[e + fx]]}{2f} - \frac{(a + b) \operatorname{Cot}[e + fx] \operatorname{Csc}[e + fx]}{2f} + \frac{b \operatorname{Sec}[e + fx]}{f}$$

Result (type 3, 236 leaves):

$$-\frac{a \operatorname{Csc}\left[\frac{1}{2}(e + fx)\right]^2}{8f} - \frac{b \operatorname{Csc}\left[\frac{1}{2}(e + fx)\right]^2}{8f} - \frac{a \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + fx)\right]\right]}{2f} - \frac{3b \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + fx)\right]\right]}{2f} + \frac{a \operatorname{Log}\left[\sin\left[\frac{1}{2}(e + fx)\right]\right]}{2f} + \frac{3b \operatorname{Log}\left[\sin\left[\frac{1}{2}(e + fx)\right]\right]}{2f} + \frac{a \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2}{8f} + \frac{b \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2}{8f} + \frac{b \sin\left[\frac{1}{2}(e + fx)\right]}{f \left(\cos\left[\frac{1}{2}(e + fx)\right] - \sin\left[\frac{1}{2}(e + fx)\right]\right)} - \frac{b \sin\left[\frac{1}{2}(e + fx)\right]}{f \left(\cos\left[\frac{1}{2}(e + fx)\right] + \sin\left[\frac{1}{2}(e + fx)\right]\right)}$$

■ **Problem 7: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + fx]^5 (a + b \operatorname{Sec}[e + fx]^2) dx$$

Optimal (type 3, 81 leaves, 5 steps):

$$-\frac{3(a + 5b) \operatorname{ArcTanh}[\cos[e + fx]]}{8f} - \frac{(3a + 7b) \operatorname{Cot}[e + fx] \operatorname{Csc}[e + fx]}{8f} - \frac{(a + b) \operatorname{Cot}[e + fx] \operatorname{Csc}[e + fx]^3}{4f} + \frac{b \operatorname{Sec}[e + fx]}{f}$$

Result (type 3, 198 leaves):

$$\frac{1}{64f} \left(-2(3a + 7b) \operatorname{Csc}\left[\frac{1}{2}(e + fx)\right]^2 - (a + b) \operatorname{Csc}\left[\frac{1}{2}(e + fx)\right]^4 + 1 / \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2 \right) \right. \\ \left. \left(2 \left(-3(a + 13b) + 4 \cos[e + fx] \left(8b + 3(a + 5b) \operatorname{Log}\left[\cos\left[\frac{1}{2}(e + fx)\right]\right] - 3(a + 5b) \operatorname{Log}\left[\sin\left[\frac{1}{2}(e + fx)\right]\right] \right) \right) \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^2 - \right. \\ \left. (a + b) \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^4 + (4(a + 2b) + (3a + 7b) \cos[e + fx]) \operatorname{Sec}\left[\frac{1}{2}(e + fx)\right]^4 \operatorname{Tan}\left[\frac{1}{2}(e + fx)\right]^2 \right) \right)$$

■ **Problem 18: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + fx] (a + b \operatorname{Sec}[e + fx]^2)^2 dx$$

Optimal (type 3, 52 leaves, 4 steps):

$$-\frac{(a+b)^2 \operatorname{ArcTanh}[\cos[e+fx]]}{f} + \frac{b(2a+b) \operatorname{Sec}[e+fx]}{f} + \frac{b^2 \operatorname{Sec}[e+fx]^3}{3f}$$

Result (type 3, 108 leaves):

$$-\left(4(b+a \cos[e+fx])^2 \left(-b^2 - 3b(2a+b) \cos[e+fx]^2 + 3(a+b)^2 \cos[e+fx]^3 \left(\operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right]\right] - \operatorname{Log}\left[\sin\left[\frac{1}{2}(e+fx)\right]\right]\right)\right)\right) \operatorname{Sec}[e+fx]^3 \Big/ (3f(a+2b+a \cos[2(e+fx)])^2)$$

■ **Problem 19: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^3 (a+b \operatorname{Sec}[e+fx])^2 dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{(a+b)(a+5b) \operatorname{ArcTanh}[\cos[e+fx]]}{2f} - \frac{(3a^2+6ab+5b^2) \cot[e+fx] \operatorname{Csc}[e+fx]}{6f} + \frac{b(6a+5b) \operatorname{Sec}[e+fx]}{3f} + \frac{b^2 \operatorname{Csc}[e+fx]^2 \operatorname{Sec}[e+fx]^3}{3f}$$

Result (type 3, 1021 leaves):

$$\begin{aligned}
& \frac{(-a^2 - 2ab - b^2) \cos[e + fx]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (a + b \sec[e + fx]^2)^2}{2f(a + 2b + a \cos[2e + 2fx])^2} - \frac{2(a^2 + 6ab + 5b^2) \cos[e + fx]^4 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right]\right] (a + b \sec[e + fx]^2)^2}{f(a + 2b + a \cos[2e + 2fx])^2} + \\
& \frac{2(a^2 + 6ab + 5b^2) \cos[e + fx]^4 \operatorname{Log}\left[\sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right] (a + b \sec[e + fx]^2)^2}{f(a + 2b + a \cos[2e + 2fx])^2} + \\
& \frac{2b(12a + 13b) \cos[e + fx]^4 \sec[e] (a + b \sec[e + fx]^2)^2}{3f(a + 2b + a \cos[2e + 2fx])^2} + \frac{(a^2 + 2ab + b^2) \cos[e + fx]^4 \sec\left[\frac{e}{2} + \frac{fx}{2}\right]^2 (a + b \sec[e + fx]^2)^2}{2f(a + 2b + a \cos[2e + 2fx])^2} + \\
& \frac{2b^2 \cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 \sin\left[\frac{fx}{2}\right]}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right])^3} + \\
& \frac{\cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 (b^2 \cos\left[\frac{e}{2}\right] + b^2 \sin\left[\frac{e}{2}\right])}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right])^2} + \\
& \frac{2 \cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 (12ab \sin\left[\frac{fx}{2}\right] + 13b^2 \sin\left[\frac{fx}{2}\right])}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right])} - \\
& \frac{2b^2 \cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 \sin\left[\frac{fx}{2}\right]}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right])^3} + \\
& \frac{\cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 (b^2 \cos\left[\frac{e}{2}\right] - b^2 \sin\left[\frac{e}{2}\right])}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right])^2} - \\
& \frac{2 \cos[e + fx]^4 (a + b \sec[e + fx]^2)^2 (12ab \sin\left[\frac{fx}{2}\right] + 13b^2 \sin\left[\frac{fx}{2}\right])}{3f(a + 2b + a \cos[2e + 2fx])^2 (\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right]) (\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right])}
\end{aligned}$$

■ **Problem 21: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + fx]^2)^2 \sin[e + fx]^6 dx$$

Optimal (type 3, 148 leaves, 7 steps):

$$\begin{aligned}
& \frac{5}{16} (a^2 - 12ab + 8b^2) x - \frac{(3a^2 - 36ab + 8b^2) \cos[e + fx] \sin[e + fx]}{16f} + \\
& \frac{a(a - 12b) \cos[e + fx]^3 \sin[e + fx]}{24f} - \frac{(a^2 - 12ab + 12b^2) \tan[e + fx]}{6f} + \frac{a^2 \sin[e + fx]^6 \tan[e + fx]}{6f} + \frac{b^2 \tan[e + fx]^3}{3f}
\end{aligned}$$

Result (type 3, 499 leaves):

$$\frac{1}{768 f (a + 2b + a \cos[2(e + fx)])^2} (b + a \cos[e + fx])^2 \sec[e] \sec[e + fx]^3$$

$$(360 (a^2 - 12ab + 8b^2) fx \cos[fx] + 360 (a^2 - 12ab + 8b^2) fx \cos[2e + fx] + 120 a^2 fx \cos[2e + 3fx] - 1440 abfx \cos[2e + 3fx] + 960 b^2 fx \cos[2e + 3fx] + 120 a^2 fx \cos[4e + 3fx] - 1440 abfx \cos[4e + 3fx] + 960 b^2 fx \cos[4e + 3fx] - 81 a^2 \sin[fx] + 3444 ab \sin[fx] - 3168 b^2 \sin[fx] - 81 a^2 \sin[2e + fx] - 1164 ab \sin[2e + fx] + 2208 b^2 \sin[2e + fx] - 109 a^2 \sin[2e + 3fx] + 2076 ab \sin[2e + 3fx] - 1936 b^2 \sin[2e + 3fx] - 109 a^2 \sin[4e + 3fx] + 540 ab \sin[4e + 3fx] - 144 b^2 \sin[4e + 3fx] - 21 a^2 \sin[4e + 5fx] + 156 ab \sin[4e + 5fx] - 48 b^2 \sin[4e + 5fx] - 21 a^2 \sin[6e + 5fx] + 156 ab \sin[6e + 5fx] - 48 b^2 \sin[6e + 5fx] + 6 a^2 \sin[6e + 7fx] - 12 ab \sin[6e + 7fx] + 6 a^2 \sin[8e + 7fx] - 12 ab \sin[8e + 7fx] - a^2 \sin[8e + 9fx] - a^2 \sin[10e + 9fx])$$

■ **Problem 24: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + fx])^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b(2a + b) \tan[e + fx]}{f} + \frac{b^2 \tan[e + fx]^3}{3f}$$

Result (type 3, 106 leaves):

$$\left(4 (b + a \cos[e + fx])^2 \sec[e + fx]^3 (3 a^2 fx \cos[e + fx]^3 + b^2 \sec[e] \sin[fx] + 2 b (3 a + b) \cos[e + fx]^2 \sec[e] \sin[fx] + b^2 \cos[e + fx] \tan[e]) \right) / (3 f (a + 2 b + a \cos[2 (e + fx)])^2)$$

■ **Problem 25: Result more than twice size of optimal antiderivative.**

$$\int \csc[e + fx]^2 (a + b \sec[e + fx])^2 dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$-\frac{(a + b)^2 \cot[e + fx]}{f} + \frac{2 b (a + b) \tan[e + fx]}{f} + \frac{b^2 \tan[e + fx]^3}{3 f}$$

Result (type 3, 109 leaves):

$$\left(4 (b + a \cos[e + fx])^2 \sec[e + fx]^3 (b^2 \sec[e] \sin[fx] + \cos[e + fx]^2 (3 (a + b)^2 \cot[e + fx] \csc[e] + b (6 a + 5 b) \sec[e]) \sin[fx] + b^2 \cos[e + fx] \tan[e]) \right) / (3 f (a + 2 b + a \cos[2 (e + fx)])^2)$$

■ **Problem 27: Result more than twice size of optimal antiderivative.**

$$\int \csc[e + fx]^6 (a + b \sec[e + fx])^2 dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$-\frac{(a^2 + 6 a b + 6 b^2) \cot[e + fx]}{f} - \frac{2 (a + b) (a + 2 b) \cot[e + fx]^3}{3 f} - \frac{(a + b)^2 \cot[e + fx]^5}{5 f} + \frac{2 b (a + 2 b) \tan[e + fx]}{f} + \frac{b^2 \tan[e + fx]^3}{3 f}$$

Result (type 3, 353 leaves) :

$$-\frac{1}{1920 f} \operatorname{Csc}[e] \operatorname{Csc}[e+f x]^5 \operatorname{Sec}[e] \operatorname{Sec}[e+f x]^3$$

$$\left(20 a (5 a + 12 b) \operatorname{Sin}[2 e] - 32 (2 a^2 + 9 a b + 12 b^2) \operatorname{Sin}[2 f x] - 24 a^2 \operatorname{Sin}[2 (e+f x)] - 108 a b \operatorname{Sin}[2 (e+f x)] - 54 b^2 \operatorname{Sin}[2 (e+f x)] + \right.$$

$$8 a^2 \operatorname{Sin}[4 (e+f x)] + 36 a b \operatorname{Sin}[4 (e+f x)] + 18 b^2 \operatorname{Sin}[4 (e+f x)] + 8 a^2 \operatorname{Sin}[6 (e+f x)] + 36 a b \operatorname{Sin}[6 (e+f x)] +$$

$$18 b^2 \operatorname{Sin}[6 (e+f x)] - 4 a^2 \operatorname{Sin}[8 (e+f x)] - 18 a b \operatorname{Sin}[8 (e+f x)] - 9 b^2 \operatorname{Sin}[8 (e+f x)] + 8 a^2 \operatorname{Sin}[2 (e+2 f x)] +$$

$$96 a b \operatorname{Sin}[2 (e+2 f x)] + 128 b^2 \operatorname{Sin}[2 (e+2 f x)] + 40 a^2 \operatorname{Sin}[4 e+2 f x] + 8 a^2 \operatorname{Sin}[4 e+6 f x] +$$

$$\left. 96 a b \operatorname{Sin}[4 e+6 f x] + 128 b^2 \operatorname{Sin}[4 e+6 f x] - 4 a^2 \operatorname{Sin}[6 e+8 f x] - 48 a b \operatorname{Sin}[6 e+8 f x] - 64 b^2 \operatorname{Sin}[6 e+8 f x] \right)$$

■ **Problem 28: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]^5}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 98 leaves, 4 steps) :

$$\frac{\sqrt{b} (a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+f x]}{\sqrt{b}}\right]}{a^{7/2} f} - \frac{(a+b)^2 \operatorname{Cos}[e+f x]}{a^3 f} + \frac{(2 a+b) \operatorname{Cos}[e+f x]^3}{3 a^2 f} - \frac{\operatorname{Cos}[e+f x]^5}{5 a f}$$

Result (type 3, 425 leaves) :

$$\frac{1}{1920 a^{7/2} \sqrt{b} f (a+b \operatorname{Sec}[e+f x]^2)}$$

$$\left((a+2 b+a \operatorname{Cos}[2 (e+f x)]) \left(15 (5 a^3 + 64 a^2 b + 128 a b^2 + 64 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \right. \right. \right.$$

$$\left. \left. \left. \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right] \right) + 15 (5 a^3 + 64 a^2 b + 128 a b^2 + 64 b^3) \right.$$

$$\left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] \right) -$$

$$75 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{b}}\right] - 75 a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{b}}\right] -$$

$$\left. 8 \sqrt{a} \sqrt{b} \operatorname{Cos}[e+f x] (89 a^2 + 220 a b + 120 b^2 - 4 a (7 a + 5 b) \operatorname{Cos}[2 (e+f x)] + 3 a^2 \operatorname{Cos}[4 (e+f x)]) \right) \operatorname{Sec}[e+f x]^2$$

■ **Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]^3}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 71 leaves, 4 steps) :

$$\frac{\sqrt{b} (a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right]}{a^{5/2} f} - \frac{(a+b) \cos[e+fx]}{a^2 f} + \frac{\cos[e+fx]^3}{3 a f}$$

Result (type 3, 376 leaves):

$$\frac{1}{48 a^{5/2} \sqrt{b} f (a+b \operatorname{Sec}[e+fx]^2)} \left((a+2b+a \cos[2(e+fx)]) \left(3 (a^2+8ab+8b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a}-i\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2}) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left(\sqrt{a}-\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right] \right) \right) + 3 (a^2+8ab+8b^2) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a}+i\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2}) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left(\sqrt{a}+\sqrt{a+b} \sqrt{(\cos[e]-i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right) \right] \right) - 3 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right] - 3 a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right] + 4 \sqrt{a} \sqrt{b} \cos[e+fx] (-5a-6b+a \cos[2(e+fx)]) \right) \operatorname{Sec}[e+fx]^2$$

■ **Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]}{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 47 leaves, 3 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right]}{a^{3/2} f} - \frac{\cos[e+fx]}{a f}$$

Result (type 3, 329 leaves):

$$\frac{1}{8 a^{3/2} \sqrt{b} f (b + a \cos [e + f x]^2)}$$

$$\left((a + 4 b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right] + \cos [e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right]\right) +$$

$$(a + 4 b) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right] + \cos [e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)\right)\right] -$$

$$a \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \tan \left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right] - a \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \tan \left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right] - 4 \sqrt{a} \sqrt{b} \cos [e + f x] \right) (a + 2 b + a \cos [2(e + f x)])$$

- **Problem 31: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc [e + f x]}{a + b \sec [e + f x]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [e + f x]}{\sqrt{b}}\right]}{\sqrt{a} (a + b) f} - \frac{\operatorname{ArcTanh}[\cos [e + f x]]}{(a + b) f}$$

Result (type 3, 239 leaves):

$$\frac{1}{(a + b) f} \left(\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right] + \cos [e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)}{\sqrt{b}}\right]}{\sqrt{a}} + \right.$$

$$\left. \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2}\right) \sin [e] \tan \left[\frac{f x}{2}\right] + \cos [e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\cos [e] - i \sin [e])^2} \tan \left[\frac{f x}{2}\right]\right)}{\sqrt{b}}\right]}{\sqrt{a}} - \operatorname{Log}\left[\cos \left[\frac{1}{2}(e + f x)\right]\right] + \operatorname{Log}\left[\sin \left[\frac{1}{2}(e + f x)\right]\right] \right)$$

- **Problem 32: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc [e + f x]^3}{a + b \sec [e + f x]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$\frac{\sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos [e + f x]}{\sqrt{b}}\right]}{(a + b)^2 f} - \frac{(a - b) \operatorname{ArcTanh}[\cos [e + f x]]}{2 (a + b)^2 f} - \frac{\cot [e + f x] \csc [e + f x]}{2 (a + b) f}$$

Result (type 3, 371 leaves) :

$$\begin{aligned}
 & - \frac{1}{16 (a+b)^2 f (a+b \operatorname{Sec}[e+f x])^2} (a+2 b+a \operatorname{Cos}[2 (e+f x)]) \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\
 & \quad \left. \left. \left(\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] - 8 \sqrt{a} \sqrt{b} \right. \\
 & \quad \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] \right) + \\
 & \quad a \operatorname{Csc}\left[\frac{1}{2} (e+f x)\right]^2 + b \operatorname{Csc}\left[\frac{1}{2} (e+f x)\right]^2 + 4 a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e+f x)\right]\right] - 4 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e+f x)\right]\right] - \\
 & \quad \left. 4 a \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (e+f x)\right]\right] + 4 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (e+f x)\right]\right] - a \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 - b \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \right) \operatorname{Sec}[e+f x]^2
 \end{aligned}$$

- **Problem 33: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+f x]^5}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 129 leaves, 6 steps) :

$$\frac{a^{3/2} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e+f x]}{\sqrt{b}}\right]}{(a+b)^3 f} - \frac{(3 a^2 - 6 a b - b^2) \operatorname{ArcTanh}[\operatorname{Cos}[e+f x]]}{8 (a+b)^3 f} - \frac{(3 a - b) \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{8 (a+b)^2 f} - \frac{\operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^3}{4 (a+b) f}$$

Result (type 3, 903 leaves) :

$$\begin{aligned}
& \left(a^{3/2} \sqrt{b} \operatorname{ArcTan} \left[\right. \right. \\
& \quad \left. \left. \frac{1}{(2\sqrt{b}) \operatorname{Sec} \left[\frac{fx}{2} \right]} \left(2\sqrt{a} \cos \left[e + \frac{fx}{2} \right] - i\sqrt{a+b} \cos \left[e - \frac{fx}{2} \right] \sqrt{\cos[2e] - i \sin[2e]} + i\sqrt{a+b} \cos \left[e + \frac{fx}{2} \right] \sqrt{\cos[2e] - i \sin[2e]} + \right. \right. \\
& \quad \left. \left. \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin \left[e - \frac{fx}{2} \right] - \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin \left[e + \frac{fx}{2} \right] \right) \right] \\
& \quad \left. \left. \frac{(a+2b+a \cos[2e+2fx]) \operatorname{Sec}[e+fx]^2}{(2(a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2))} + \right. \right. \\
& \quad \left. \left. \left(a^{3/2} \sqrt{b} \operatorname{ArcTan} \left[\frac{1}{(2\sqrt{b}) \operatorname{Sec} \left[\frac{fx}{2} \right]} \left(2\sqrt{a} \cos \left[e + \frac{fx}{2} \right] + i\sqrt{a+b} \cos \left[e - \frac{fx}{2} \right] \sqrt{\cos[2e] - i \sin[2e]} - i\sqrt{a+b} \cos \left[e + \frac{fx}{2} \right] \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\cos[2e] - i \sin[2e]} - \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin \left[e - \frac{fx}{2} \right] + \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin \left[e + \frac{fx}{2} \right] \right) \right] \right) \right. \\
& \quad \left. \left. \frac{(a+2b+a \cos[2e+2fx]) \operatorname{Sec}[e+fx]^2}{(2(a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2))} + \right. \right. \\
& \quad \left. \left. \frac{(-3a+b)(a+2b+a \cos[2e+2fx]) \operatorname{Csc} \left[\frac{e}{2} + \frac{fx}{2} \right]^2 \operatorname{Sec}[e+fx]^2}{64(a+b)^2 f (a+b \operatorname{Sec}[e+fx]^2)} - \right. \right. \\
& \quad \left. \left. \frac{(a+2b+a \cos[2e+2fx]) \operatorname{Csc} \left[\frac{e}{2} + \frac{fx}{2} \right]^4 \operatorname{Sec}[e+fx]^2}{128(a+b) f (a+b \operatorname{Sec}[e+fx]^2)} + \right. \right. \\
& \quad \left. \left. \frac{(-3a^2+6ab+b^2)(a+2b+a \cos[2e+2fx]) \operatorname{Log} \left[\cos \left[\frac{e}{2} + \frac{fx}{2} \right] \right] \operatorname{Sec}[e+fx]^2}{16(a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)} + \right. \right. \\
& \quad \left. \left. \frac{(3a^2-6ab-b^2)(a+2b+a \cos[2e+2fx]) \operatorname{Log} \left[\sin \left[\frac{e}{2} + \frac{fx}{2} \right] \right] \operatorname{Sec}[e+fx]^2}{16(a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)} + \right. \right. \\
& \quad \left. \left. \frac{(3a-b)(a+2b+a \cos[2e+2fx]) \operatorname{Sec} \left[\frac{e}{2} + \frac{fx}{2} \right]^2 \operatorname{Sec}[e+fx]^2}{64(a+b)^2 f (a+b \operatorname{Sec}[e+fx]^2)} + \right. \right. \\
& \quad \left. \left. \frac{(a+2b+a \cos[2e+2fx]) \operatorname{Sec} \left[\frac{e}{2} + \frac{fx}{2} \right]^4 \operatorname{Sec}[e+fx]^2}{128(a+b) f (a+b \operatorname{Sec}[e+fx]^2)} \right. \right. \\
& \quad \left. \left. \right) \right)
\end{aligned}$$

- **Problem 34: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^6}{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 166 leaves, 7 steps):

$$\frac{(5a^3 + 30a^2b + 40ab^2 + 16b^3)x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a^4 f} - \frac{(11a^2 + 18ab + 8b^2) \text{Cos}[e+fx] \text{Sin}[e+fx]}{16a^3 f} + \frac{(3a+2b) \text{Cos}[e+fx]^3 \text{Sin}[e+fx]}{8a^2 f} + \frac{\text{Cos}[e+fx]^3 \text{Sin}[e+fx]^3}{6af}$$

Result (type 3, 357 leaves):

$$\frac{1}{768a^4 \sqrt{b} \sqrt{a+b} f (a+b \text{Sec}[e+fx]^2) \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4} \left((a+2b+a \text{Cos}[2(e+fx)]) \text{Sec}[e+fx]^2 \left(3\sqrt{b} (9a^4 + 136a^3b + 384a^2b^2 + 384ab^3 + 128b^4) \right. \right. \\ \left. \left. \text{ArcTan}\left[\frac{\text{Sec}[fx] (\text{Cos}[2e] - i \text{Sin}[2e]) (- (a+2b) \text{Sin}[fx] + a \text{Sin}[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4} \right] (\text{Cos}[2e] - i \text{Sin}[2e]) + \right. \right. \\ \left. \left. \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4 \left(3a^3 (9a+8b) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}} \right] + 2\sqrt{b} \sqrt{a+b} (-12a^3e + 60a^3fx + 360a^2bfx + \right. \right. \right. \\ \left. \left. \left. 480ab^2fx + 192b^3fx - 3a(15a^2 + 32ab + 16b^2) \text{Sin}[2(e+fx)] + 3a^2(3a+2b) \text{Sin}[4(e+fx)] - a^3 \text{Sin}[6(e+fx)] \right) \right) \right) \right)$$

■ **Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e+fx]^4}{a+b \text{Sec}[e+fx]^2} dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{(3a^2 + 12ab + 8b^2)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a^3 f} - \frac{(5a+4b) \text{Cos}[e+fx] \text{Sin}[e+fx]}{8a^2 f} + \frac{\text{Cos}[e+fx]^3 \text{Sin}[e+fx]}{4af}$$

Result (type 3, 303 leaves):

$$\frac{1}{64 a^3 \sqrt{b} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2$$

$$\left(\sqrt{b} (3 a^3+34 a^2 b+64 a b^2+32 b^3) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) (- (a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x])}{2 \sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4}\right] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) + \right.$$

$$\left. \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \left(a^2 (3 a+2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right] + \right. \right.$$

$$\left. \left. \sqrt{b} \sqrt{a+b} (-2 a^2 e+12 a^2 f x+48 a b f x+32 b^2 f x-8 a(a+b) \operatorname{Sin}[2(e+f x)]+a^2 \operatorname{Sin}[4(e+f x)]) \right) \right)$$

- **Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+f x]^2}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 76 leaves, 5 steps):

$$\frac{(a+2 b) x}{2 a^2} - \frac{\sqrt{b} \sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{a^2 f} - \frac{\operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x]}{2 a f}$$

Result (type 3, 245 leaves):

$$\frac{1}{16 (a+b \operatorname{Sec}[e+f x]^2) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2} \left(\frac{\operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{\sqrt{b} \sqrt{a+b} f} - \right.$$

$$\left. \frac{1}{a^2} \left(-4 (a+2 b) x - \frac{(a^2+8 a b+8 b^2) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) (- (a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x])}{2 \sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4}\right] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e])}{\sqrt{a+b} f \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4} + \right. \right.$$

$$\left. \left. \frac{2 a \operatorname{Cos}[2 f x] \operatorname{Sin}[2 e]}{f} + \frac{2 a \operatorname{Cos}[2 e] \operatorname{Sin}[2 f x]}{f} \right) \right)$$

- **Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \operatorname{Cot}[e+fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves):

$$\left((a + 2b + a \operatorname{Cos}[2(e + fx)]) \operatorname{Sec}[e + fx]^2 \left(\sqrt{a+b} f x \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} + \right. \right. \\ \left. \left. b \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a + 2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) \right) / \\ \left(2 a \sqrt{a+b} f (a + b \operatorname{Sec}[e + fx]^2) \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right)$$

- **Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + fx]^2}{a + b \operatorname{Sec}[e + fx]^2} dx$$

Optimal (type 3, 54 leaves, 3 steps):

$$-\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{3/2} f} - \frac{\operatorname{Cot}[e+fx]}{(a+b) f}$$

Result (type 3, 189 leaves):

$$\left((a + 2b + a \operatorname{Cos}[2(e + fx)]) \operatorname{Sec}[e + fx]^2 \right. \\ \left(b \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a + 2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) + \right. \\ \left. \left. \sqrt{a+b} \operatorname{Csc}[e] \operatorname{Csc}[e + fx] \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \operatorname{Sin}[fx] \right) \right) / \left(2 (a + b)^{3/2} f (a + b \operatorname{Sec}[e + fx]^2) \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right)$$

- **Problem 39: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + fx]^4}{a + b \operatorname{Sec}[e + fx]^2} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$-\frac{a\sqrt{b}\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}f} - \frac{a\operatorname{Cot}[e+fx]}{(a+b)^2f} - \frac{\operatorname{Cot}[e+fx]^3}{3(a+b)f}$$

Result (type 3, 226 leaves):

$$\left((a+2b+a\operatorname{Cos}[2(e+fx)])\operatorname{Sec}[e+fx]^2 \right. \\ \left. \left(3ab\operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx](\operatorname{Cos}[2e]-i\operatorname{Sin}[2e])(-a+2b)\operatorname{Sin}[fx]+a\operatorname{Sin}[2e+fx])}{2\sqrt{a+b}\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4}\right] (\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) + \right. \right. \\ \left. \left. \frac{1}{4}\sqrt{a+b}\operatorname{Csc}[e]\operatorname{Csc}[e+fx]^3\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4(6a\operatorname{Sin}[fx]-3b\operatorname{Sin}[2e+fx]+(-2a+b)\operatorname{Sin}[2e+3fx])\right) \right) \\ \left. \left(6(a+b)^{5/2}f(a+b\operatorname{Sec}[e+fx]^2)\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4 \right) \right) /$$

■ **Problem 40: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]^6}{a+b\operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 105 leaves, 4 steps):

$$-\frac{a^2\sqrt{b}\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{7/2}f} - \frac{a^2\operatorname{Cot}[e+fx]}{(a+b)^3f} - \frac{(2a+b)\operatorname{Cot}[e+fx]^3}{3(a+b)^2f} - \frac{\operatorname{Cot}[e+fx]^5}{5(a+b)f}$$

Result (type 3, 318 leaves):

$$\frac{1}{480(a+b)^{7/2}f(a+b\operatorname{Sec}[e+fx]^2)\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4} (a+2b+a\operatorname{Cos}[2(e+fx)])\operatorname{Sec}[e+fx]^2 \\ \left(240a^2b\operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx](\operatorname{Cos}[2e]-i\operatorname{Sin}[2e])(-a+2b)\operatorname{Sin}[fx]+a\operatorname{Sin}[2e+fx])}{2\sqrt{a+b}\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4}\right] (\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) + \right. \\ \left. \sqrt{a+b}\operatorname{Csc}[e]\operatorname{Csc}[e+fx]^5\sqrt{b}(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4(10(8a^2+b^2)\operatorname{Sin}[fx]-30b(3a+b)\operatorname{Sin}[2e+fx]-40a^2\operatorname{Sin}[2e+3fx]+ \right. \\ \left. 30ab\operatorname{Sin}[2e+3fx]+10b^2\operatorname{Sin}[2e+3fx]+15ab\operatorname{Sin}[4e+3fx]+8a^2\operatorname{Sin}[4e+5fx]-9ab\operatorname{Sin}[4e+5fx]-2b^2\operatorname{Sin}[4e+5fx]) \right) \right)$$

- **Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^5}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 161 leaves, 6 steps):

$$\frac{\sqrt{b} (a + b) (3 a + 7 b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e + f x]}{\sqrt{b}}\right]}{2 a^{9/2} f} - \frac{(a + b) (3 a + 7 b) \cos[e + f x]}{2 a^4 f} +$$

$$\frac{(a + b) (3 a + 7 b) \cos[e + f x]^3}{6 a^3 b f} - \frac{\cos[e + f x]^5}{5 a^2 f} - \frac{(a + b)^2 \cos[e + f x]^5}{2 a^2 b f (b + a \cos[e + f x]^2)}$$

Result (type 3, 454 leaves):

$$\frac{1}{3840 a^{9/2} f} \left(\frac{1}{b^{3/2}} 15 (3 a^4 + 384 a^2 b^2 + 1280 a b^3 + 896 b^4) \right.$$

$$\left. \operatorname{ArcTan}\left[1 / (\sqrt{b}) \left((-\sqrt{a} - i \sqrt{a + b} \sqrt{(\cos[e] - i \sin[e])^2}) \sin[e] \tan\left[\frac{f x}{2}\right] + \cos[e] \left(\sqrt{a} - \sqrt{a + b} \sqrt{(\cos[e] - i \sin[e])^2} \tan\left[\frac{f x}{2}\right] \right) \right) \right] \right] +$$

$$\frac{1}{b^{3/2}} 15 (3 a^4 + 384 a^2 b^2 + 1280 a b^3 + 896 b^4)$$

$$\left. \operatorname{ArcTan}\left[1 / (\sqrt{b}) \left((-\sqrt{a} + i \sqrt{a + b} \sqrt{(\cos[e] - i \sin[e])^2}) \sin[e] \tan\left[\frac{f x}{2}\right] + \cos[e] \left(\sqrt{a} + \sqrt{a + b} \sqrt{(\cos[e] - i \sin[e])^2} \tan\left[\frac{f x}{2}\right] \right) \right) \right] \right] -$$

$$\frac{45 a^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a + b} \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{45 a^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a + b} \tan\left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right]}{b^{3/2}} -$$

$$\left. \left(16 \sqrt{a} \cos[e + f x] (150 a^3 + 1436 a^2 b + 2960 a b^2 + 1680 b^3 + a (125 a^2 + 688 a b + 560 b^2) \cos[2(e + f x)] - \right. \right.$$

$$\left. \left. 2 a^2 (11 a + 14 b) \cos[4(e + f x)] + 3 a^3 \cos[6(e + f x)] \right) \right) / (a + 2 b + a \cos[2(e + f x)]) \right)$$

- **Problem 42: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^3}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 114 leaves, 5 steps):

$$\frac{\sqrt{b} (3a + 5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right]}{2a^{7/2}f} - \frac{(a+2b) \cos[e+fx]}{a^3f} + \frac{\cos[e+fx]^3}{3a^2f} - \frac{b(a+b) \cos[e+fx]}{2a^3f(b+a \cos[e+fx]^2)}$$

Result (type 3, 403 leaves):

$$\frac{1}{384a^{7/2}f} \left(\frac{1}{b^{3/2}} 3(3a^3 + 192ab^2 + 320b^3) \right. \\ \left. \operatorname{ArcTan}\left[1 / (\sqrt{b}) \left(\left(-\sqrt{a} - i\sqrt{a+b} \sqrt{(\cos[e] - i\sin[e])^2} \right) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\cos[e] - i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right) \right] \right) + \\ \frac{1}{b^{3/2}} 3(3a^3 + 192ab^2 + 320b^3) \\ \operatorname{ArcTan}\left[1 / (\sqrt{b}) \left(\left(-\sqrt{a} + i\sqrt{a+b} \sqrt{(\cos[e] - i\sin[e])^2} \right) \sin[e] \tan\left[\frac{fx}{2}\right] + \cos[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\cos[e] - i\sin[e])^2} \tan\left[\frac{fx}{2}\right] \right) \right) \right] \right) - \\ \frac{9a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{9a^3 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \\ \left. \frac{32\sqrt{a} \cos[e+fx] (9a^2 + 56ab + 60b^2 + 4a(2a+5b) \cos[2(e+fx)] - a^2 \cos[4(e+fx)])}{a+2b+a \cos[2(e+fx)]} \right)$$

- **Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]}{(a+b \sec[e+fx])^2} dx$$

Optimal (type 3, 84 leaves, 4 steps):

$$\frac{3\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right]}{2a^{5/2}f} - \frac{3 \cos[e+fx]}{2a^2f} + \frac{\cos[e+fx]^3}{2af(b+a \cos[e+fx]^2)}$$

Result (type 3, 393 leaves):

$$\frac{1}{64 a^{5/2} f (a + b \operatorname{Sec}[e + f x])^2} \left((a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \left(\frac{1}{b^{3/2}} (a^2 + 24 b^2) \operatorname{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right) + \frac{1}{b^{3/2}} (a^2 + 24 b^2) \operatorname{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right) \right) - \frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]}{\sqrt{b}}\right]}{b^{3/2}} - \frac{16 \sqrt{a} \operatorname{Cos}[e + f x] (a + 3 b + a \operatorname{Cos}[2 (e + f x)])}{a + 2 b + a \operatorname{Cos}[2 (e + f x)]} \right) \operatorname{Sec}[e + f x]^4$$

- **Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 99 leaves, 5 steps):

$$\frac{\sqrt{b} (3 a + b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e + f x]}{\sqrt{b}}\right]}{2 a^{3/2} (a + b)^2 f} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[e + f x]]}{(a + b)^2 f} - \frac{b \operatorname{Cos}[e + f x]}{2 a (a + b) f (b + a \operatorname{Cos}[e + f x])^2}$$

Result (type 3, 384 leaves):

$$\frac{1}{8 (a + b)^2 f (a + b \operatorname{Sec}[e + f x])^2} (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \operatorname{Sec}[e + f x]^3 \left(-\frac{2 b (a + b)}{a} + \frac{1}{a^{3/2}} \sqrt{b} (3 a + b) \operatorname{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right) (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \operatorname{Sec}[e + f x] + \frac{1}{a^{3/2}} \sqrt{b} (3 a + b) \operatorname{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right) (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \operatorname{Sec}[e + f x] - 2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Sec}[e + f x] + 2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]\right] \operatorname{Sec}[e + f x] \right)$$

- **Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$\frac{(3a - b) \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cos}[e + f x]}{\sqrt{b}}\right]}{2 \sqrt{a} (a + b)^3 f} - \frac{(a - 3b) \text{ArcTanh}[\text{Cos}[e + f x]]}{2 (a + b)^3 f} + \frac{(a - b) \text{Cos}[e + f x]}{2 (a + b)^2 f (b + a \text{Cos}[e + f x]^2)} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]}{2 (a + b) f (b + a \text{Cos}[e + f x]^2)}$$

Result (type 3, 468 leaves):

$$\frac{1}{32 (a + b)^3 f (a + b \text{Sec}[e + f x]^2)^2} (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x]^3$$

$$\left(-8b(a + b) - \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a + b) \text{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} - i\sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \right) \text{Sin}[e] \text{Tan}\left[\frac{f x}{2}\right] + \right. \right.$$

$$\left. \left. \text{Cos}[e] \left(\sqrt{a} - \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x] - \frac{1}{\sqrt{a}} 4\sqrt{b} (-3a + b) \right.$$

$$\left. \text{ArcTan}\left[1 / (\sqrt{b})\right] \left(\left(-\sqrt{a} + i\sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \right) \text{Sin}[e] \text{Tan}\left[\frac{f x}{2}\right] + \text{Cos}[e] \left(\sqrt{a} + \sqrt{a + b} \sqrt{(\text{Cos}[e] - i \text{Sin}[e])^2} \text{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] \right)$$

$$(a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x] - (a + b) (a + 2b + a \text{Cos}[2(e + f x)]) \text{Csc}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x] -$$

$$4(a - 3b) (a + 2b + a \text{Cos}[2(e + f x)]) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right]\right] \text{Sec}[e + f x] +$$

$$4(a - 3b) (a + 2b + a \text{Cos}[2(e + f x)]) \text{Log}\left[\text{Sin}\left[\frac{1}{2}(e + f x)\right]\right] \text{Sec}[e + f x] + (a + b) (a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}\left[\frac{1}{2}(e + f x)\right]^2 \text{Sec}[e + f x]$$

- **Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 197 leaves, 7 steps):

$$\frac{3 \sqrt{a} (a - b) \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{a} \text{Cos}[e + f x]}{\sqrt{b}}\right]}{2 (a + b)^4 f} - \frac{3 (a^2 - 6ab + b^2) \text{ArcTanh}[\text{Cos}[e + f x]]}{8 (a + b)^4 f} +$$

$$\frac{3a(a - 3b) \text{Cos}[e + f x]}{8 (a + b)^3 f (b + a \text{Cos}[e + f x]^2)} - \frac{(a - 5b) \text{Cot}[e + f x] \text{Csc}[e + f x]}{8 (a + b)^2 f (b + a \text{Cos}[e + f x]^2)} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]^3}{4 (a + b) f (b + a \text{Cos}[e + f x]^2)}$$

Result (type 3, 450 leaves):

$$\frac{1}{256 (a+b)^4 f (a+b \operatorname{Sec}[e+fx])^2}$$

$$(a+2b+a \operatorname{Cos}[2(e+fx)]) \left(96 \sqrt{a} (a-b) \sqrt{b} \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a}-i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right. \right.$$

$$\left. \left. \operatorname{Cos}[e] \left(\sqrt{a}-\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cos}[2(e+fx)]) + 96 \sqrt{a} (a-b) \sqrt{b}$$

$$\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left((-\sqrt{a}+i\sqrt{a+b}) \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a}+\sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right]$$

$$(a+2b+a \operatorname{Cos}[2(e+fx)]) - 2(a+b) (11a^2+43ab-4b^2+4(2a^2-5ab+5b^2) \operatorname{Cos}[2(e+fx)] - 3a(a-3b) \operatorname{Cos}[4(e+fx)])$$

$$\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 - 24(a^2-6ab+b^2) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]\right] +$$

$$24(a^2-6ab+b^2) (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \right) \operatorname{Sec}[e+fx]^4$$

■ **Problem 47: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 3, 267 leaves, 8 steps):

$$\frac{(5a^3+60a^2b+120ab^2+64b^3)x}{16a^5} - \frac{\sqrt{b}(a+b)^{3/2}(3a+8b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^5f} - \frac{(33a^2+82ab+48b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3f(a+b+b \operatorname{Tan}[e+fx]^2)} +$$

$$\frac{(9a+8b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2f(a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]^3}{6af(a+b+b \operatorname{Tan}[e+fx]^2)} - \frac{b(19a^2+52ab+32b^2) \operatorname{Tan}[e+fx]}{16a^4f(a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 2987 leaves):

$$- \left((a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[e+fx]^4 \left(16x + \right. \right.$$

$$\left. \left((-a^3+6a^2b+24ab^2+16b^3) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) (-a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4}\right] (\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) \right) \right) /$$

$$\left(b(a+b)^{3/2} f \sqrt{b (\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4} \right) + \frac{(a^2+8ab+8b^2) ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{b(a+b)f(a+2b+a \operatorname{Cos}[2(e+fx)]) (\operatorname{Cos}[e]-\operatorname{Sin}[e]) (\operatorname{Cos}[e]+\operatorname{Sin}[e])} \left. \right) /$$

$$\begin{aligned}
& (512 a^2 (a + b \operatorname{Sec}[e + f x]^2)^2) + \left(3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \right. \\
& \left. \left(-64 (a + 2 b) x + \left((a^4 - 16 a^3 b - 144 a^2 b^2 - 256 a b^3 - 128 b^4) \operatorname{ArcTan} \left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) (- (a + 2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x])}{2 \sqrt{a + b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right] \right. \right. \right. \\
& \left. \left. \left. (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) \right) \right) / \left(b (a + b)^{3/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) + \frac{16 a \operatorname{Cos}[2 f x] \operatorname{Sin}[2 e]}{f} + \frac{16 a \operatorname{Cos}[2 e] \operatorname{Sin}[2 f x]}{f} - \right. \\
& \left. \left. \frac{(a^3 + 18 a^2 b + 48 a b^2 + 32 b^3) ((a + 2 b) \operatorname{Sin}[2 e] - a \operatorname{Sin}[2 f x])}{b (a + b) f (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \right) \right) / (4096 a^3 (a + b \operatorname{Sec}[e + f x]^2)^2) + \\
& \left. 3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \left(\frac{(a + 2 b) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}} \right]}{(a + b)^{3/2}} - \frac{a \sqrt{b} \operatorname{Sin}[2 (e + f x)]}{(a + b) (a + 2 b + a \operatorname{Cos}[2 (e + f x)])} \right) \right) \\
& \frac{\hspace{10em}}{2048 b^{3/2} f (a + b \operatorname{Sec}[e + f x]^2)^2} - \\
& \left((a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \left(- \frac{a \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}} \right]}{(a + b)^{3/2}} + \frac{\sqrt{b} (a + 2 b) \operatorname{Sin}[2 (e + f x)]}{(a + b) (a + 2 b + a \operatorname{Cos}[2 (e + f x)])} \right) \right) \\
& \frac{\hspace{10em}}{2048 b^{3/2} f (a + b \operatorname{Sec}[e + f x]^2)^2} + \\
& \frac{1}{256 (a + b \operatorname{Sec}[e + f x]^2)^2} \\
& (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^4 \\
& \left(\frac{1}{a + b} (-a^5 + 30 a^4 b + 480 a^3 b^2 + 1600 a^2 b^3 + 1920 a b^4 + 768 b^5) \right. \\
& \left. \left(\left(\operatorname{ArcTan} \left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right] \right. \right. \right. \\
& \left. \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Cos}[2 e] \right) \right) / (8 a^4 b \sqrt{a + b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}) - \\
& \left(i \operatorname{ArcTan} \left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right] \right)
\end{aligned}$$

$$\int \frac{\sin[e + f x]^4}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 191 leaves, 7 steps) :

$$\frac{3(a^2 + 8ab + 8b^2)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\operatorname{ArcTan}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b}}\right]}{2a^4f} - \frac{(5a+6b)\cos[e+fx]\sin[e+fx]}{8a^2f(a+b+b\tan[e+fx]^2)} + \frac{\cos[e+fx]^3\sin[e+fx]}{4af(a+b+b\tan[e+fx]^2)} - \frac{3b(3a+4b)\tan[e+fx]}{8a^3f(a+b+b\tan[e+fx]^2)}$$

Result (type 3, 1354 leaves) :

$$\begin{aligned}
& - \left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(16x + \right. \right. \\
& \left. \left((-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (-(a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\
& \left. \left(b(a+b)^{3/2} f \sqrt{b} (\cos[e] - i \sin[e])^4 \right) + \frac{(a^2 + 8ab + 8b^2) ((a + 2b) \sin[2e] - a \sin[2fx])}{b(a+b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / \\
& \frac{3(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(\frac{(a+2b) \operatorname{ArcTan} \left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} - \frac{a \sqrt{b} \sin[2(e+fx)]}{(a+b)(a+2b+a \cos[2(e+fx)])} \right)}{1024 b^{3/2} f (a + b \sec[e + fx]^2)^2} + \\
& \frac{1}{128 (a + b \sec[e + fx]^2)^2} (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(\frac{1}{a+b} (-a^5 + 30a^4b + 480a^3b^2 + 1600a^2b^3 + 1920ab^4 + 768b^5) \right. \\
& \left. \left(\left(\operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b} \cos[4e] - i b \sin[4e]} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b} \cos[4e] - i b \sin[4e]} \right) \right] \right) \right. \right. \\
& \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(8a^4 b \sqrt{a+b} f \sqrt{b} \cos[4e] - i b \sin[4e] \right) - \\
& \left(i \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b} \cos[4e] - i b \sin[4e]} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b} \cos[4e] - i b \sin[4e]} \right) \right] \right) \\
& \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(8a^4 b \sqrt{a+b} f \sqrt{b} \cos[4e] - i b \sin[4e] \right) \right) + \\
& \frac{1}{8a^4 b (a+b) f (a + 2b + a \cos[2e + 2fx])} \sec[2e] \left(160a^4 b f x \cos[2e] + 1248a^3 b^2 f x \cos[2e] + 3392a^2 b^3 f x \cos[2e] + \right. \\
& 3840a b^4 f x \cos[2e] + 1536b^5 f x \cos[2e] + 80a^4 b f x \cos[2fx] + 464a^3 b^2 f x \cos[2fx] + 768a^2 b^3 f x \cos[2fx] + \\
& 384a b^4 f x \cos[2fx] + 80a^4 b f x \cos[4e + 2fx] + 464a^3 b^2 f x \cos[4e + 2fx] + 768a^2 b^3 f x \cos[4e + 2fx] + \\
& 384a b^4 f x \cos[4e + 2fx] + a^5 \sin[2e] + 34a^4 b \sin[2e] + 224a^3 b^2 \sin[2e] + 576a^2 b^3 \sin[2e] + 640a b^4 \sin[2e] + \\
& 256b^5 \sin[2e] - a^5 \sin[2fx] - 62a^4 b \sin[2fx] - 318a^3 b^2 \sin[2fx] - 512a^2 b^3 \sin[2fx] - 256a b^4 \sin[2fx] - \\
& 30a^4 b \sin[4e + 2fx] - 158a^3 b^2 \sin[4e + 2fx] - 256a^2 b^3 \sin[4e + 2fx] - 128a b^4 \sin[4e + 2fx] - \\
& 12a^4 b \sin[2e + 4fx] - 36a^3 b^2 \sin[2e + 4fx] - 24a^2 b^3 \sin[2e + 4fx] - 12a^4 b \sin[6e + 4fx] - 36a^3 b^2 \sin[6e + 4fx] - \\
& \left. 24a^2 b^3 \sin[6e + 4fx] + 2a^4 b \sin[4e + 6fx] + 2a^3 b^2 \sin[4e + 6fx] + 2a^4 b \sin[8e + 6fx] + 2a^3 b^2 \sin[8e + 6fx] \right) \right)
\end{aligned}$$

- **Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^2}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$\frac{(a + 4 b) x}{2 a^3} - \frac{\sqrt{b} (3 a + 4 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a + b}}\right]}{2 a^3 \sqrt{a + b} f} - \frac{\cos[e + f x] \sin[e + f x]}{2 a f (a + b + b \tan[e + f x]^2)} - \frac{b \tan[e + f x]}{a^2 f (a + b + b \tan[e + f x]^2)}$$

Result (type 3, 825 leaves):

$$\begin{aligned}
& - \left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(16x + \right. \right. \\
& \quad \left. \left((-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (-(a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\
& \quad \left. \left(b (a + b)^{3/2} f \sqrt{b} (\cos[e] - i \sin[e])^4 \right) + \frac{(a^2 + 8ab + 8b^2) ((a + 2b) \sin[2e] - a \sin[2fx])}{b (a + b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / \\
& (128a^2 (a + b \sec[e + fx]^2)^2) - \left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \right. \\
& \quad \left. \left(-64(a + 2b)x + \left((a^4 - 16a^3b - 144a^2b^2 - 256ab^3 - 128b^4) \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (-(a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] \right) \right) \right) / \\
& \quad \left(\cos[2e] - i \sin[2e] \right) \left(b (a + b)^{3/2} f \sqrt{b} (\cos[e] - i \sin[e])^4 \right) + \frac{16a \cos[2fx] \sin[2e]}{f} + \frac{16a \cos[2e] \sin[2fx]}{f} - \\
& \quad \left. \frac{(a^3 + 18a^2b + 48ab^2 + 32b^3) ((a + 2b) \sin[2e] - a \sin[2fx])}{b (a + b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / (256a^3 (a + b \sec[e + fx]^2)^2) + \\
& \frac{(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(\frac{(a + 2b) \operatorname{ArcTan} \left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} - \frac{a \sqrt{b} \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right)}{128b^{3/2} f (a + b \sec[e + fx]^2)^2} + \\
& \frac{(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(-\frac{a \operatorname{ArcTan} \left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{(a+b)^{3/2}} + \frac{\sqrt{b} (a + 2b) \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right)}{256b^{3/2} f (a + b \sec[e + fx]^2)^2}
\end{aligned}$$

- **Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} f} - \frac{b \operatorname{Tan}[e+fx]}{2a (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 240 leaves):

$$\frac{1}{8a^2 (a+b \operatorname{Sec}[e+fx]^2)^2} (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^4 \left(2x (a+2b+a \operatorname{Cos}[2(e+fx)]) + \right. \\ \left. \left(b (3a+2b) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right] (a+2b+a \operatorname{Cos}[2(e+fx)]) \right) \right. \\ \left. \left. (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) \right) / \left((a+b)^{3/2} f \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4 + \frac{b ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{(a+b) f (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \right)$$

■ **Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+fx]^2}{(a+b \operatorname{Sec}[e+fx]^2)^2} dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$- \frac{3\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2(a+b)^{5/2} f} - \frac{3 \operatorname{Cot}[e+fx]}{2(a+b)^2 f} + \frac{\operatorname{Cot}[e+fx]}{2(a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 242 leaves):

$$\left((a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^4 \right. \\ \left. \left(\left(3b \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right] (a+2b+a \operatorname{Cos}[2(e+fx)]) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) \right. \right. \\ \left. \left. \left(\sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4 \right) + 2(a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Csc}[e] \operatorname{Csc}[e+fx] \operatorname{Sin}[fx] + \right. \right. \\ \left. \left. \frac{b ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{a (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \right) \right) \right) / \left(8(a+b)^2 f (a+b \operatorname{Sec}[e+fx]^2)^2 \right)$$

- **Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$-\frac{(3a - 2b) \sqrt{b} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{2(a + b)^{7/2} f} - \frac{(a - b) \text{Cot}[e + f x]}{(a + b)^3 f} - \frac{\text{Cot}[e + f x]^3}{3(a + b)^2 f} - \frac{ab \text{Tan}[e + f x]}{2(a + b)^3 f (a + b + b \text{Tan}[e + f x]^2)}$$

Result (type 3, 637 leaves):

$$\begin{aligned} & -\frac{(a + 2b + a \text{Cos}[2e + 2fx])^2 \text{Cot}[e] \text{Csc}[e + fx]^2 \text{Sec}[e + fx]^4}{12(a + b)^2 f (a + b \text{Sec}[e + fx]^2)^2} + \left((3a - 2b) (a + 2b + a \text{Cos}[2e + 2fx])^2 \right. \\ & \left. \text{Sec}[e + fx]^4 \left(\left(b \text{ArcTan}\left[\text{Sec}[fx] \left(\frac{\text{Cos}[2e]}{2\sqrt{a + b} \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e]} - \frac{i \text{Sin}[2e]}{2\sqrt{a + b} \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e]} \right)} \right. \right. \right. \\ & \left. \left. \left. (-a \text{Sin}[fx] - 2b \text{Sin}[fx] + a \text{Sin}[2e + fx]) \right) \text{Cos}[2e] \right) / \left(8\sqrt{a + b} f \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e] \right) - \right. \\ & \left. \left(i b \text{ArcTan}\left[\text{Sec}[fx] \left(\frac{\text{Cos}[2e]}{2\sqrt{a + b} \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e]} - \frac{i \text{Sin}[2e]}{2\sqrt{a + b} \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e]} \right)} \right. \right. \right. \\ & \left. \left. \left. (-a \text{Sin}[fx] - 2b \text{Sin}[fx] + a \text{Sin}[2e + fx]) \right) \text{Sin}[2e] \right) / \left(8\sqrt{a + b} f \sqrt{b \text{Cos}[4e]} - i b \text{Sin}[4e] \right) \right) \right) / \\ & \left((a + b)^3 (a + b \text{Sec}[e + fx]^2)^2 \right) + \frac{(a + 2b + a \text{Cos}[2e + 2fx])^2 \text{Csc}[e] \text{Csc}[e + fx]^3 \text{Sec}[e + fx]^4 \text{Sin}[fx]}{12(a + b)^2 f (a + b \text{Sec}[e + fx]^2)^2} + \\ & \frac{(a + 2b + a \text{Cos}[2e + 2fx])^2 \text{Csc}[e] \text{Csc}[e + fx] \text{Sec}[e + fx]^4 (a \text{Sin}[fx] - 2b \text{Sin}[fx])}{6(a + b)^3 f (a + b \text{Sec}[e + fx]^2)^2} + \\ & \frac{(a + 2b + a \text{Cos}[2e + 2fx]) \text{Sec}[e + fx]^4 (ab \text{Sin}[2e] + 2b^2 \text{Sin}[2e] - ab \text{Sin}[2fx])}{8(a + b)^3 f (a + b \text{Sec}[e + fx]^2)^2 (\text{Cos}[e] - \text{Sin}[e]) (\text{Cos}[e] + \text{Sin}[e])} \end{aligned}$$

- **Problem 53: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\frac{a(3a-4b)\sqrt{b}\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2(a+b)^{9/2}f} - \frac{(5a^2-10ab-b^2)\operatorname{Cot}[e+fx]}{5(a+b)^4f} - \frac{(10a+3b)\operatorname{Cot}[e+fx]^3}{15(a+b)^3f} - \frac{\operatorname{Cot}[e+fx]^5}{5(a+b)f(a+b+b\operatorname{Tan}[e+fx]^2)} - \frac{b(5a^2+2b^2)\operatorname{Tan}[e+fx]}{10(a+b)^4f(a+b+b\operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 777 leaves):

$$\frac{1}{7680(a+b)^4f(a+b\operatorname{Sec}[e+fx]^2)^2} \left((a+2b+a\operatorname{Cos}[2(e+fx)])\operatorname{Sec}[e+fx]^4 \left(\left(960a(3a-4b)b\operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx](\operatorname{Cos}[2e]-i\operatorname{Sin}[2e])(-(a+2b)\operatorname{Sin}[fx]+a\operatorname{Sin}[2e+fx])}{2\sqrt{a+b}\sqrt{b(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4}} \right] \right) \right. \right. \\ \left. \left. (a+2b+a\operatorname{Cos}[2(e+fx)])(\operatorname{Cos}[2e]-i\operatorname{Sin}[2e]) \right) / \left(\sqrt{a+b}\sqrt{b(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^4} \right) - \operatorname{Csc}[e]\operatorname{Csc}[e+fx]^5\operatorname{Sec}[2e] \right. \\ \left. (10a(16a^2+34ab+123b^2)\operatorname{Sin}[fx] - a(16a^2-223ab+1336b^2)\operatorname{Sin}[3fx] + 240a^3\operatorname{Sin}[2e-fx] + 640a^2b\operatorname{Sin}[2e-fx] - 1460ab^2\operatorname{Sin}[2e-fx] + 240b^3\operatorname{Sin}[2e-fx] - 240a^3\operatorname{Sin}[2e+fx] - 715a^2b\operatorname{Sin}[2e+fx] + 860ab^2\operatorname{Sin}[2e+fx] - 240b^3\operatorname{Sin}[2e+fx] + 160a^3\operatorname{Sin}[4e+fx] + 415a^2b\operatorname{Sin}[4e+fx] + 1830ab^2\operatorname{Sin}[4e+fx] + 165a^2b\operatorname{Sin}[2e+3fx] - 30ab^2\operatorname{Sin}[2e+3fx] + 120b^3\operatorname{Sin}[2e+3fx] - 16a^3\operatorname{Sin}[4e+3fx] + 208a^2b\operatorname{Sin}[4e+3fx] - 1036ab^2\operatorname{Sin}[4e+3fx] + 180a^2b\operatorname{Sin}[6e+3fx] - 330ab^2\operatorname{Sin}[6e+3fx] + 120b^3\operatorname{Sin}[6e+3fx] + 48a^3\operatorname{Sin}[2e+5fx] - 268a^2b\operatorname{Sin}[2e+5fx] + 290ab^2\operatorname{Sin}[2e+5fx] - 24b^3\operatorname{Sin}[2e+5fx] + 48a^3\operatorname{Sin}[6e+5fx] - 223a^2b\operatorname{Sin}[6e+5fx] + 230ab^2\operatorname{Sin}[6e+5fx] - 24b^3\operatorname{Sin}[6e+5fx] - 45a^2b\operatorname{Sin}[8e+5fx] + 60ab^2\operatorname{Sin}[8e+5fx] - 16a^3\operatorname{Sin}[4e+7fx] + 83a^2b\operatorname{Sin}[4e+7fx] - 6ab^2\operatorname{Sin}[4e+7fx] - 15a^2b\operatorname{Sin}[6e+7fx] - 16a^3\operatorname{Sin}[8e+7fx] + 68a^2b\operatorname{Sin}[8e+7fx] - 6ab^2\operatorname{Sin}[8e+7fx]) \right) \right)$$

■ **Problem 54: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^5}{(a+b\operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 214 leaves, 6 steps):

$$\frac{\sqrt{b}(15a^2+70ab+63b^2)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{8a^{11/2}f} - \frac{(3a^2+14ab+13b^2)\operatorname{Cos}[e+fx]}{2a^5f} + \frac{(a+3b)(3a+5b)\operatorname{Cos}[e+fx]^3}{12a^4bf} - \frac{\operatorname{Cos}[e+fx]^5}{5a^3f} - \frac{(a+b)^2\operatorname{Cos}[e+fx]^7}{4a^2bf(b+a\operatorname{Cos}[e+fx]^2)^2} - \frac{b(a+b)(3a+11b)\operatorname{Cos}[e+fx]}{8a^5f(b+a\operatorname{Cos}[e+fx]^2)}$$

Result (type 3, 1641 leaves):

1

$$491\,520\,a^{11/2}\,b^{5/2}\,f\,(a+b\,\text{Sec}[e+f\,x])^3$$

$$\begin{aligned} & (a+2b+a\,\text{Cos}[2(e+f\,x)])\,\text{Sec}[e+f\,x]^6 \left(-900\,a^{11/2}\,b^{3/2}\,\text{Cos}[e+f\,x] - 109\,000\,a^{9/2}\,b^{5/2}\,\text{Cos}[e+f\,x] - 936\,000\,a^{7/2}\,b^{7/2}\,\text{Cos}[e+f\,x] - \right. \\ & 2\,803\,072\,a^{5/2}\,b^{9/2}\,\text{Cos}[e+f\,x] - 3\,763\,200\,a^{3/2}\,b^{11/2}\,\text{Cos}[e+f\,x] - 1\,935\,360\,\sqrt{a}\,b^{13/2}\,\text{Cos}[e+f\,x] - \\ & 900\,a^{11/2}\,b^{3/2}\,\text{Cos}[e+f\,x]\,\text{Cos}[2(e+f\,x)] + 900\,a^{9/2}\,b^{3/2}\,\text{Cos}[e+f\,x]\,(a+2b+a\,\text{Cos}[2(e+f\,x)]) + \\ & 24\,000\,a^{7/2}\,b^{5/2}\,\text{Cos}[e+f\,x]\,(a+2b+a\,\text{Cos}[2(e+f\,x)]) + 43\,200\,a^{5/2}\,b^{7/2}\,\text{Cos}[e+f\,x]\,(a+2b+a\,\text{Cos}[2(e+f\,x)]) + \\ & \left. 225\,a^5\,\text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right]\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 115\,200\,a^2\,b^3 \\ & \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 537\,600\,a\,b^4\,\text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \right. \right. \\ & \left. \left. \text{Cos}[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 483\,840\,b^5 \\ & \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + \\ & 225\,a^5\,\text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 115\,200\,a^2\,b^3 \\ & \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 537\,600\,a\,b^4 \\ & \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 483\,840\,b^5 \\ & \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Sin}[e]\,\text{Tan}\left[\frac{f\,x}{2}\right] + \text{Cos}[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\text{Cos}[e]-i\,\text{Sin}[e])^2}\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right] \right) \\ & (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 - 225\,a^5\,\text{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{b}}\right] (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 - \\ & 225\,a^5\,\text{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b}\,\text{Tan}\left[\frac{1}{2}(e+f\,x)\right]}{\sqrt{b}}\right] (a+2b+a\,\text{Cos}[2(e+f\,x)])^2 + 19\,200\,a^{5/2}\,b^{5/2}\,\text{Cos}[e]\,\text{Cos}[f\,x]\,(a+2b+a\,\text{Cos}[2(e+f\,x)])^2 - \end{aligned}$$

$$\begin{aligned}
& 20\,352 a^{9/2} b^{5/2} \cos[e+fx] \cos[4(e+fx)] - 115\,712 a^{7/2} b^{7/2} \cos[e+fx] \cos[4(e+fx)] - \\
& 129\,024 a^{5/2} b^{9/2} \cos[e+fx] \cos[4(e+fx)] + 2048 a^{9/2} b^{5/2} \cos[e+fx] \cos[6(e+fx)] + 4608 a^{7/2} b^{7/2} \cos[e+fx] \cos[6(e+fx)] - \\
& 384 a^{9/2} b^{5/2} \cos[e+fx] \cos[8(e+fx)] - 19\,200 a^{5/2} b^{5/2} (a+2b+a\cos[2(e+fx)])^2 \sin[e] \sin[fx] - \\
& 32\,496 a^{9/2} b^{5/2} \csc[e+fx] \sin[4(e+fx)] - 252\,080 a^{7/2} b^{7/2} \csc[e+fx] \sin[4(e+fx)] - \\
& \left. 577\,024 a^{5/2} b^{9/2} \csc[e+fx] \sin[4(e+fx)] - 403\,200 a^{3/2} b^{11/2} \csc[e+fx] \sin[4(e+fx)] \right)
\end{aligned}$$

- **Problem 55: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^3}{(a+b\sec[e+fx]^2)^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{5\sqrt{b}(3a+7b)\operatorname{ArcTan}\left[\frac{\sqrt{a}\cos[e+fx]}{\sqrt{b}}\right]}{8a^{9/2}f} - \frac{(a+3b)\cos[e+fx]}{a^4f} + \frac{\cos[e+fx]^3}{3a^3f} + \frac{b^2(a+b)\cos[e+fx]}{4a^4f(b+a\cos[e+fx])^2} - \frac{b(9a+13b)\cos[e+fx]}{8a^4f(b+a\cos[e+fx])^2}$$

Result (type 3, 1392 leaves):

$$\begin{aligned}
& \left(3 \left(\frac{3\operatorname{ArcTan}\left[\frac{\sqrt{a}-\sqrt{a+b}\tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{\sqrt{a}} - \frac{3\operatorname{ArcTan}\left[\frac{\sqrt{a}+\sqrt{a+b}\tan\left[\frac{1}{2}(e+fx)\right]}{\sqrt{b}}\right]}{\sqrt{a}} - \frac{2\sqrt{b}\cos[e+fx](3a+10b+3a\cos[2(e+fx)])}{(a+2b+a\cos[2(e+fx)])^2} \right) \right. \\
& \left. (a+2b+a\cos[2e+2fx])^3 \sec[e+fx]^6 \right) / \left(8192 b^{5/2} f (a+b\sec[e+fx]^2)^3 \right) + \frac{1}{2048 a^{3/2} b^{5/2} f (a+b\sec[e+fx]^2)^3} \left((3a-4b)\operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\
& \left. \left. \left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2} \right) \sin[e]\tan\left[\frac{fx}{2}\right] + \cos[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan\left[\frac{fx}{2}\right] \right) \right) \right] + (3a-4b) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2} \right) \sin[e]\tan\left[\frac{fx}{2}\right] + \cos[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan\left[\frac{fx}{2}\right] \right) \right) \right] \right) + \\
& \left. \frac{2\sqrt{a}\sqrt{b}\cos[e+fx](3a^2+6ab+8b^2+a(3a-4b)\cos[2(e+fx)])}{(a+2b+a\cos[2(e+fx)])^2} \right) (a+2b+a\cos[2e+2fx])^3 \sec[e+fx]^6 - \\
& \frac{1}{49\,152 a^{9/2} b^{5/2} f (a+b\sec[e+fx]^2)^3} \left(-3(3a^4-40a^3b+720a^2b^2+6720ab^3+8960b^4) \right. \\
& \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2} \right) \sin[e]\tan\left[\frac{fx}{2}\right] + \cos[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan\left[\frac{fx}{2}\right] \right) \right) \right] \right) - \\
& \left. 3(3a^4-40a^3b+720a^2b^2+6720ab^3+8960b^4) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{ArcTan}\left[\frac{1}{\sqrt{b}}\left(\left(-\sqrt{a} + i\sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\right)\text{Sin}[e]\text{Tan}\left[\frac{fx}{2}\right] + \text{Cos}[e]\left(\sqrt{a} + \sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\text{Tan}\left[\frac{fx}{2}\right]\right)\right)\right] - \\
& \left(2\sqrt{a}\sqrt{b}\text{Cos}[e+fx]\left(9a^5 - 90a^4b - 10144a^3b^2 - 48672a^2b^3 - 85120ab^4 - 53760b^5 + a\left(9a^4 - 120a^3b - 12432a^2b^2 - 47936ab^3 - \right.\right.\right. \\
& \left.\left.\left.44800b^4\right)\text{Cos}[2(e+fx)] - 128a^2b^2(15a+28b)\text{Cos}[4(e+fx)] + 128a^3b^2\text{Cos}[6(e+fx)]\right)\right) / (a+2b+a\text{Cos}[2(e+fx)])^2 \\
& (a+2b+a\text{Cos}[2e+2fx])^3 \text{Sec}[e+fx]^6 - \frac{1}{16384a^{7/2}f(a+b\text{Sec}[e+fx]^2)^3} 3(a+2b+a\text{Cos}[2e+2fx])^3 \\
& \text{Sec}[e+fx]^6 \left(\frac{1}{b^{5/2}} 3(a^3 - 8a^2b + 80ab^2 + 320b^3) \text{ArcTan}\left[\right.\right. \\
& \left.\left.1 / (\sqrt{b})\left(\left(-\sqrt{a} - i\sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\right)\text{Sin}[e]\text{Tan}\left[\frac{fx}{2}\right] + \text{Cos}[e]\left(\sqrt{a} - \sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\text{Tan}\left[\frac{fx}{2}\right]\right)\right)\right]\right) + \\
& \frac{1}{b^{5/2}} 3(a^3 - 8a^2b + 80ab^2 + 320b^3) \text{ArcTan}\left[1 / (\sqrt{b})\left(\left(-\sqrt{a} + i\sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\right)\text{Sin}[e]\text{Tan}\left[\frac{fx}{2}\right] + \right.\right. \\
& \left.\left.\text{Cos}[e]\left(\sqrt{a} + \sqrt{a+b}\sqrt{(\text{Cos}[e] - i\text{Sin}[e])^2}\text{Tan}\left[\frac{fx}{2}\right]\right)\right)\right] - 512\sqrt{a}\text{Cos}[e]\text{Cos}[fx] + \\
& \left.\frac{8\sqrt{a}(a^3 + 24a^2b + 80ab^2 + 64b^3)\text{Cos}[e+fx]}{b(a+2b+a\text{Cos}[2(e+fx)])^2} + \frac{2\sqrt{a}(3a^3 - 24a^2b - 400ab^2 - 576b^3)\text{Cos}[e+fx]}{b^2(a+2b+a\text{Cos}[2(e+fx)])} + 512\sqrt{a}\text{Sin}[e]\text{Sin}[fx]\right)
\end{aligned}$$

- **Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e+fx]}{(a+b\text{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 116 leaves, 5 steps):

$$\frac{15\sqrt{b}\text{ArcTan}\left[\frac{\sqrt{a}\text{Cos}[e+fx]}{\sqrt{b}}\right]}{8a^{7/2}f} - \frac{15\text{Cos}[e+fx]}{8a^3f} + \frac{\text{Cos}[e+fx]^5}{4af(b+a\text{Cos}[e+fx]^2)^2} + \frac{5\text{Cos}[e+fx]^3}{8a^2f(b+a\text{Cos}[e+fx]^2)}$$

Result (type 3, 656 leaves):

$$\frac{1}{4096 a^{7/2} b^{5/2} f (a + b \operatorname{Sec}[e + f x])^3} (a + 2b + a \operatorname{Cos}[2(e + f x)])^3 \operatorname{Sec}[e + f x]^6 \left(15 (a^3 + 64 b^3) \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \right. \right. \\ \left. \left. \left(\left(-\sqrt{a} - i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] + 15 (a^3 + 64 b^3) \right. \\ \left. \operatorname{ArcTan}\left[\frac{1}{\sqrt{b}} \left(\left(-\sqrt{a} + i \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{f x}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i \operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{f x}{2}\right] \right) \right) \right] \right) + \\ \frac{1}{(a + 2b + a \operatorname{Cos}[2(e + f x)])^2} \sqrt{a} \left(24 a^4 \sqrt{b} \operatorname{Cos}[e + f x] - 24 a^3 b^{3/2} \operatorname{Cos}[e + f x] - 144 a^2 b^{5/2} \operatorname{Cos}[e + f x] + 512 b^{9/2} \operatorname{Cos}[e + f x] - 72 a^3 b^{3/2} \right. \\ \left. \operatorname{Cos}[e + f x] \operatorname{Cos}[2(e + f x)] - 24 a^3 \sqrt{b} \operatorname{Cos}[e + f x] (a + 2b + a \operatorname{Cos}[2(e + f x)]) + 72 a^2 b^{3/2} \operatorname{Cos}[e + f x] (a + 2b + a \operatorname{Cos}[2(e + f x)]) - \right. \\ \left. 1152 b^{7/2} \operatorname{Cos}[e + f x] (a + 2b + a \operatorname{Cos}[2(e + f x)]) - 15 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} - \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right] (a + 2b + a \operatorname{Cos}[2(e + f x)])^2 - \right. \\ \left. 15 a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} + \sqrt{a+b} \operatorname{Tan}\left[\frac{1}{2}(e + f x)\right]}{\sqrt{b}}\right] (a + 2b + a \operatorname{Cos}[2(e + f x)])^2 - 512 b^{5/2} \operatorname{Cos}[e] \operatorname{Cos}[f x] (a + 2b + a \operatorname{Cos}[2(e + f x)])^2 + \right. \\ \left. 512 b^{5/2} (a + 2b + a \operatorname{Cos}[2(e + f x)])^2 \operatorname{Sin}[e] \operatorname{Sin}[f x] + 6 a^4 \sqrt{b} \operatorname{Csc}[e + f x] \operatorname{Sin}[4(e + f x)] \right) \right)$$

- **Problem 57: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$\frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Cos}[e + f x]}{\sqrt{b}}\right]}{8 a^{5/2} (a + b)^3 f} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[e + f x]]}{(a + b)^3 f} - \frac{b \operatorname{Cos}[e + f x]^3}{4 a (a + b) f (b + a \operatorname{Cos}[e + f x])^2} - \frac{b (7 a + 3 b) \operatorname{Cos}[e + f x]}{8 a^2 (a + b)^2 f (b + a \operatorname{Cos}[e + f x])^2}$$

Result (type 3, 447 leaves):

$$\frac{1}{64 (a+b)^3 f (a+b \operatorname{Sec}[e+f x])^3}$$

$$(a+2b+a \cos[2(e+f x)]) \operatorname{Sec}[e+f x]^5 \left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(a+b)(9a+5b)(a+2b+a \cos[2(e+f x)])}{a^2} + \frac{1}{a^{5/2}} \sqrt{b} (15a^2+10ab+3b^2) \right.$$

$$\left. \operatorname{ArcTan}\left[1/\left(\sqrt{b}\right)\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\right)\sin[e]\tan\left[\frac{fx}{2}\right]+ \cos[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan\left[\frac{fx}{2}\right]\right)\right]\right]\right]$$

$$(a+2b+a \cos[2(e+f x)])^2 \operatorname{Sec}[e+f x] + \frac{1}{a^{5/2}} \sqrt{b} (15a^2+10ab+3b^2)$$

$$\operatorname{ArcTan}\left[1/\left(\sqrt{b}\right)\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\right)\sin[e]\tan\left[\frac{fx}{2}\right]+ \cos[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan\left[\frac{fx}{2}\right]\right)\right]\right]$$

$$(a+2b+a \cos[2(e+f x)])^2 \operatorname{Sec}[e+f x] - 8(a+2b+a \cos[2(e+f x)])^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}[e+f x] +$$

$$8(a+2b+a \cos[2(e+f x)])^2 \operatorname{Log}\left[\sin\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}[e+f x]$$

- **Problem 58: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+f x]^3}{(a+b \operatorname{Sec}[e+f x])^3} dx$$

Optimal (type 3, 213 leaves, 7 steps):

$$\frac{\sqrt{b} (15a^2 - 10ab - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \cos[e+fx]}{\sqrt{b}}\right] - (a-5b) \operatorname{ArcTanh}[\cos[e+fx]]}{8a^{3/2}(a+b)^4 f} - \frac{(2a-b)b \cos[e+fx]}{4a(a+b)^2 f (b+a \cos[e+fx])^2} + \frac{(4a^2 - 9ab - b^2) \cos[e+fx]}{8a(a+b)^3 f (b+a \cos[e+fx])^2} - \frac{\cos[e+fx] \cot[e+fx]^2}{2(a+b) f (b+a \cos[e+fx])^2}$$

Result (type 3, 532 leaves):

$$\frac{1}{64 (a+b)^4 f (a+b \operatorname{Sec}[e+f x])^3}$$

$$(a+2b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^5 \left(\frac{8b^2(a+b)^2}{a} - \frac{2b(a+b)(9a+b)(a+2b+a \operatorname{Cos}[2(e+f x)])}{a} - \frac{1}{a^{3/2}} \sqrt{b} (-15a^2+10ab+b^2) \right.$$

$$\left. \operatorname{ArcTan}\left[1/\left(\sqrt{b}\right)\left(\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2}\right)\operatorname{Sin}[e]\operatorname{Tan}\left[\frac{fx}{2}\right]+\operatorname{Cos}[e]\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2}\operatorname{Tan}\left[\frac{fx}{2}\right]\right)\right)\right]\right]$$

$$(a+2b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Sec}[e+f x] - \frac{1}{a^{3/2}} \sqrt{b} (-15a^2+10ab+b^2)$$

$$\operatorname{ArcTan}\left[1/\left(\sqrt{b}\right)\left(\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2}\right)\operatorname{Sin}[e]\operatorname{Tan}\left[\frac{fx}{2}\right]+\operatorname{Cos}[e]\left(\sqrt{a}+\sqrt{a+b}\sqrt{(\operatorname{Cos}[e]-i\operatorname{Sin}[e])^2}\operatorname{Tan}\left[\frac{fx}{2}\right]\right)\right)\right]$$

$$(a+2b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Sec}[e+f x] - (a+b)(a+2b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x] -$$

$$4(a-5b)(a+2b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}[e+f x] + 4(a-5b)(a+2b+a \operatorname{Cos}[2(e+f x)])^2$$

$$\operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right] \operatorname{Sec}[e+f x] + (a+b)(a+2b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2 \operatorname{Sec}[e+f x]$$

■ **Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e+f x]^5}{(a+b \operatorname{Sec}[e+f x])^3} dx$$

Optimal (type 3, 257 leaves, 8 steps):

$$\frac{3\sqrt{b}(5a^2-10ab+b^2)\operatorname{ArcTan}\left[\frac{\sqrt{a}\operatorname{Cos}[e+fx]}{\sqrt{b}}\right]}{8\sqrt{a}(a+b)^5f} - \frac{3(a^2-10ab+5b^2)\operatorname{ArcTanh}[\operatorname{Cos}[e+fx]]}{8(a+b)^5f} + \frac{(a^2-9ab+2b^2)\operatorname{Cos}[e+fx]}{8(a+b)^3f(b+a\operatorname{Cos}[e+fx])^2} +$$

$$\frac{3(a^2-6ab+b^2)\operatorname{Cos}[e+fx]}{8(a+b)^4f(b+a\operatorname{Cos}[e+fx])^2} - \frac{(a-7b)\operatorname{Cot}[e+fx]\operatorname{Csc}[e+fx]}{8(a+b)^2f(b+a\operatorname{Cos}[e+fx])^2} - \frac{\operatorname{Cot}[e+fx]^3\operatorname{Csc}[e+fx]}{4(a+b)f(b+a\operatorname{Cos}[e+fx])^2}$$

Result (type 3, 549 leaves):

$$\frac{1}{1024 (a+b)^5 f (a+b \operatorname{Sec}[e+fx])^3} \\
(a+2b+a \operatorname{Cos}[2(e+fx)]) \left(\frac{1}{\sqrt{a}} 48 \sqrt{b} (5a^2 - 10ab + b^2) \operatorname{ArcTan}\left[1/\left(\sqrt{b}\right) \left((-\sqrt{a} - i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \right) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \right. \right. \\
\left. \left. \operatorname{Cos}[e] \left(\sqrt{a} - \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) (a+2b+a \operatorname{Cos}[2(e+fx)])^2 + \frac{1}{\sqrt{a}} 48 \sqrt{b} (5a^2 - 10ab + b^2) \\
\operatorname{ArcTan}\left[1/\left(\sqrt{b}\right) \left((-\sqrt{a} + i\sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2}) \operatorname{Sin}[e] \operatorname{Tan}\left[\frac{fx}{2}\right] + \operatorname{Cos}[e] \left(\sqrt{a} + \sqrt{a+b} \sqrt{(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^2} \operatorname{Tan}\left[\frac{fx}{2}\right] \right) \right] \right) \\
(a+2b+a \operatorname{Cos}[2(e+fx)])^2 - 2(a+b) (30a^3 + 112a^2b + 182ab^2 - 140b^3 + (35a^3 + 78a^2b - 93ab^2 + 224b^3) \operatorname{Cos}[2(e+fx)] + \\
2(a^3 - 8a^2b + 53ab^2 - 10b^3) \operatorname{Cos}[4(e+fx)] - 3a^3 \operatorname{Cos}[6(e+fx)] + 18a^2b \operatorname{Cos}[6(e+fx)] - 3ab^2 \operatorname{Cos}[6(e+fx)]) \\
\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 - 48(a^2 - 10ab + 5b^2) (a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+fx)\right]\right] + \\
48(a^2 - 10ab + 5b^2) (a+2b+a \operatorname{Cos}[2(e+fx)])^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+fx)\right]\right] \operatorname{Sec}[e+fx]^6$$

■ **Problem 60: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 314 leaves, 9 steps):

$$\frac{5(a+2b)(a^2+16ab+16b^2)x}{16a^6} - \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8a^6f} - \\
\frac{(33a^2+110ab+80b^2)\operatorname{Cos}[e+fx]\operatorname{Sin}[e+fx]}{48a^3f(a+b+b \operatorname{Tan}[e+fx])^2} + \frac{(9a+10b)\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]}{24a^2f(a+b+b \operatorname{Tan}[e+fx])^2} + \\
\frac{\operatorname{Cos}[e+fx]^3\operatorname{Sin}[e+fx]^3}{6af(a+b+b \operatorname{Tan}[e+fx])^2} - \frac{5b(9a^2+32ab+24b^2)\operatorname{Tan}[e+fx]}{48a^4f(a+b+b \operatorname{Tan}[e+fx])^2} - \frac{5b(5a^2+20ab+16b^2)\operatorname{Tan}[e+fx]}{16a^5f(a+b+b \operatorname{Tan}[e+fx])^2}$$

Result (type 3, 2057 leaves):

$$\left(5(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \right. \\
\left. \left(\frac{(3a^2+8ab+8b^2)\operatorname{ArcTan}\left[\frac{\sqrt{b}\operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\operatorname{Cos}[2(e+fx)])\operatorname{Sin}[2(e+fx)]}{(a+b)^2(a+2b+a \operatorname{Cos}[2(e+fx)])^2} \right) \right) \sqrt{\quad}$$

$$\begin{aligned}
& (65\,536\,b^{5/2}\,f\,(a+b\,\text{Sec}[e+f\,x]^2)^3) + \frac{1}{2048\,(a+b\,\text{Sec}[e+f\,x]^2)^3} \\
& (a+2b+a\,\text{Cos}[2e+2f\,x])^3\,\text{Sec}[e+f\,x]^6 \left(\frac{32\,(7a^3+54a^2b+120ab^2+80b^3)\,x}{a^6} - \right. \\
& \frac{1}{(a+b)^2} \left(-3a^8+64a^7b-2240a^6b^2-53\,760a^5b^3-313\,600a^4b^4-802\,816a^3b^5-1\,032\,192a^2b^6-655\,360ab^7-163\,840b^8 \right) \\
& \left(\left(\text{ArcTan}\left[\text{Sec}[f\,x] \left(\frac{\text{Cos}[2e]}{2\sqrt{a+b}\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]}} - \frac{i\,\text{Sin}[2e]}{2\sqrt{a+b}\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]}} \right) \right] \right. \right. \\
& \left. \left. (-a\,\text{Sin}[f\,x]-2b\,\text{Sin}[f\,x]+a\,\text{Sin}[2e+f\,x]) \right] \text{Cos}[2e] \right) / \left(64a^6b^2\sqrt{a+b}\,f\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]} \right) - \\
& \left(i\,\text{ArcTan}\left[\text{Sec}[f\,x] \left(\frac{\text{Cos}[2e]}{2\sqrt{a+b}\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]}} - \frac{i\,\text{Sin}[2e]}{2\sqrt{a+b}\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]}} \right) \right] \right. \\
& \left. \left. (-a\,\text{Sin}[f\,x]-2b\,\text{Sin}[f\,x]+a\,\text{Sin}[2e+f\,x]) \right] \text{Sin}[2e] \right) / \left(64a^6b^2\sqrt{a+b}\,f\sqrt{b\,\text{Cos}[4e]-ib\,\text{Sin}[4e]} \right) \Big) - \\
& \frac{1}{16a^6b\,(a+b)\,f\,(a+2b+a\,\text{Cos}[2e+2f\,x])^2} \text{Sec}[2e] \left(a^7\,\text{Sin}[2e]+74a^6b\,\text{Sin}[2e]+984a^5b^2\,\text{Sin}[2e]+5264a^4b^3\,\text{Sin}[2e]+ \right. \\
& 14\,080a^3b^4\,\text{Sin}[2e]+19\,968a^2b^5\,\text{Sin}[2e]+14\,336ab^6\,\text{Sin}[2e]+4096b^7\,\text{Sin}[2e]-a^7\,\text{Sin}[2f\,x]-72a^6b\,\text{Sin}[2f\,x]- \\
& 840a^5b^2\,\text{Sin}[2f\,x]-3584a^4b^3\,\text{Sin}[2f\,x]-6912a^3b^4\,\text{Sin}[2f\,x]-6144a^2b^5\,\text{Sin}[2f\,x]-2048ab^6\,\text{Sin}[2f\,x] \Big) - \\
& \frac{1}{64a^6b^2\,(a+b)^2\,f\,(a+2b+a\,\text{Cos}[2e+2f\,x])} \text{Sec}[2e] \left(3a^8\,\text{Sin}[2e]-64a^7b\,\text{Sin}[2e]-4480a^6b^2\,\text{Sin}[2e]- \right. \\
& 45\,696a^5b^3\,\text{Sin}[2e]-196\,928a^4b^4\,\text{Sin}[2e]-438\,272a^3b^5\,\text{Sin}[2e]-528\,384a^2b^6\,\text{Sin}[2e]-327\,680ab^7\,\text{Sin}[2e]- \\
& 81\,920b^8\,\text{Sin}[2e]-3a^8\,\text{Sin}[2f\,x]+66a^7b\,\text{Sin}[2f\,x]+4056a^6b^2\,\text{Sin}[2f\,x]+33\,936a^5b^3\,\text{Sin}[2f\,x]+ \\
& 111\,360a^4b^4\,\text{Sin}[2f\,x]+173\,568a^3b^5\,\text{Sin}[2f\,x]+129\,024a^2b^6\,\text{Sin}[2f\,x]+36\,864ab^7\,\text{Sin}[2f\,x] \Big) - \\
& (7a^2+32ab+32b^2) \left(-\frac{6i\,\text{Cos}[2e+2f\,x]}{a^5f} + \frac{6\,\text{Sin}[2e+2f\,x]}{a^5f} \right) - (7a^2+32ab+32b^2) \left(\frac{6i\,\text{Cos}[2e+2f\,x]}{a^5f} + \frac{6\,\text{Sin}[2e+2f\,x]}{a^5f} \right) - \\
& (a+2b) \left(-\frac{6i\,\text{Cos}[4e+4f\,x]}{a^4f} - \frac{6\,\text{Sin}[4e+4f\,x]}{a^4f} \right) - \\
& (a+2b) \left(\frac{6i\,\text{Cos}[4e+4f\,x]}{a^4f} - \frac{6\,\text{Sin}[4e+4f\,x]}{a^4f} \right) - \frac{4\,\text{Sin}[6e+6f\,x]}{3a^3f} \Big) - \\
& \left(15\,(a+2b+a\,\text{Cos}[2e+2f\,x])^3\,\text{Sec}[e+f\,x]^6 \left(-\frac{6a^2\,\text{ArcTan}\left[\frac{\text{Sec}[f\,x]\,(\text{Cos}[2e]-i\,\text{Sin}[2e])\,(-a+2b)\,\text{Sin}[f\,x]+a\,\text{Sin}[2e+f\,x]}{2\sqrt{a+b}\sqrt{b\,(\text{Cos}[e]-i\,\text{Sin}[e])^4}} \right]}{\sqrt{a+b}\sqrt{b\,(\text{Cos}[e]-i\,\text{Sin}[e])^4}} \right) (\text{Cos}[2e]-i\,\text{Sin}[2e]) \right. \right. \\
& \left. \left. (a\,\text{Sec}[2e] \left((-9a^4-16a^3b+48a^2b^2+128ab^3+64b^4) \text{Sin}[2f\,x] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a \left(-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3 \right) \operatorname{Sin}[2 (e + 2 f x)] + \left(3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4 \right) \operatorname{Sin}[4 e + 2 f x] + \\
& \left. \left(9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5 \right) \operatorname{Tan}[2 e] \right) / \left(a^2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \right) \Bigg) / \\
& \left(262144 b^2 (a + b)^2 f (a + b \operatorname{Sec}[e + f x]^2)^3 \right) + \frac{1}{65536 a^4 (a + b \operatorname{Sec}[e + f x]^2)^3} 3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \\
& \operatorname{Sec}[e + f x]^6 \\
& \left(-1536 (a + 2 b) x - \left(3 (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \right. \\
& \left. \left. \operatorname{ArcTan} \left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) (- (a + 2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x])}{2 \sqrt{a + b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right] (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) \right) \right) / \\
& \left(b^2 (a + b)^{5/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) + \frac{4 (a^4 + 32 a^3 b + 160 a^2 b^2 + 256 a b^3 + 128 b^4) \operatorname{Sec}[2 e] ((a + 2 b) \operatorname{Sin}[2 e] - a \operatorname{Sin}[2 f x])}{b (a + b) f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2} + \\
& \frac{256 a \operatorname{Sin}[2 (e + f x)]}{f} + (a (-3 a^5 + 26 a^4 b + 736 a^3 b^2 + 2624 a^2 b^3 + 3200 a b^4 + 1280 b^5) \operatorname{Sec}[2 e] \operatorname{Sin}[2 f x] + \\
& \left. \left(3 a^6 - 24 a^5 b - 920 a^4 b^2 - 4864 a^3 b^3 - 10112 a^2 b^4 - 9216 a b^5 - 3072 b^6 \right) \operatorname{Tan}[2 e] \right) / \left(b^2 (a + b)^2 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \right) \Bigg)
\end{aligned}$$

■ **Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 238 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 (a^2 + 12 a b + 16 b^2) x}{8 a^5} - \frac{3 \sqrt{b} (5 a^2 + 20 a b + 16 b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}} \right]}{8 a^5 \sqrt{a + b} f} - \frac{(5 a + 8 b) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{8 a^2 f (a + b + b \operatorname{Tan}[e + f x]^2)^2} + \\
& \frac{\operatorname{Cos}[e + f x]^3 \operatorname{Sin}[e + f x]}{4 a f (a + b + b \operatorname{Tan}[e + f x]^2)^2} - \frac{b (7 a + 12 b) \operatorname{Tan}[e + f x]}{8 a^3 f (a + b + b \operatorname{Tan}[e + f x]^2)^2} - \frac{3 b (a + 2 b) \operatorname{Tan}[e + f x]}{2 a^4 f (a + b + b \operatorname{Tan}[e + f x]^2)}
\end{aligned}$$

Result (type 3, 3109 leaves):

$$\left(3 (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \right)$$

$$\left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \cos[2(e+fx)]) \sin[2(e+fx)]}{(a+b)^2 (a+2b+a \cos[2(e+fx)])^2} \right) /$$

$$(16384b^{5/2} f (a+b \sec[e+fx]^2)^3) + \left((a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6 \right.$$

$$\left. - \frac{3a(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \cos[2(e+fx)]) \sin[2(e+fx)]}{(a+b)^2 (a+2b+a \cos[2(e+fx)])^2} \right) /$$

$$(16384b^{5/2} f (a+b \sec[e+fx]^2)^3) - \frac{1}{512 (a+b \sec[e+fx]^2)^3}$$

$$3(a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6 \left(\frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right.$$

$$\left. \left(\operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{ib \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right. \right. \right.$$

$$\left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e+fx]) \right] \cos[2e] \right) / \left(64a^3b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) - \right.$$

$$\left. \left(i \operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{ib \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right. \right. \right.$$

$$\left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e+fx]) \right] \sin[2e] \right) / \left(64a^3b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) \right) +$$

$$\frac{1}{128a^3b^2 (a+b)^2 f (a+2b+a \cos[2e+2fx])^2} \sec[2e] (768a^4b^2fx \cos[2e] + 3584a^3b^3fx \cos[2e] + 6912a^2b^4fx \cos[2e] +$$

$$6144ab^5fx \cos[2e] + 2048b^6fx \cos[2e] + 512a^4b^2fx \cos[2fx] + 2048a^3b^3fx \cos[2fx] + 2560a^2b^4fx \cos[2fx] +$$

$$1024ab^5fx \cos[2fx] + 512a^4b^2fx \cos[4e+2fx] + 2048a^3b^3fx \cos[4e+2fx] + 2560a^2b^4fx \cos[4e+2fx] +$$

$$1024ab^5fx \cos[4e+2fx] + 128a^4b^2fx \cos[2e+4fx] + 256a^3b^3fx \cos[2e+4fx] + 128a^2b^4fx \cos[2e+4fx] +$$

$$128a^4b^2fx \cos[6e+4fx] + 256a^3b^3fx \cos[6e+4fx] + 128a^2b^4fx \cos[6e+4fx] - 9a^6 \sin[2e] + 12a^5b \sin[2e] +$$

$$684a^4b^2 \sin[2e] + 2880a^3b^3 \sin[2e] + 5280a^2b^4 \sin[2e] + 4608ab^5 \sin[2e] + 1536b^6 \sin[2e] + 9a^6 \sin[2fx] -$$

$$14a^5b \sin[2fx] - 608a^4b^2 \sin[2fx] - 2112a^3b^3 \sin[2fx] - 2560a^2b^4 \sin[2fx] - 1024ab^5 \sin[2fx] - 3a^6 \sin[4e+2fx] +$$

$$10a^5b \sin[4e+2fx] + 304a^4b^2 \sin[4e+2fx] + 1056a^3b^3 \sin[4e+2fx] + 1280a^2b^4 \sin[4e+2fx] + 512ab^5 \sin[4e+2fx] +$$

$$3a^6 \sin[2e+4fx] - 12a^5b \sin[2e+4fx] - 204a^4b^2 \sin[2e+4fx] - 384a^3b^3 \sin[2e+4fx] - 192a^2b^4 \sin[2e+4fx]) \left. \right) +$$

$$\begin{aligned}
& \frac{1}{512 (a+b \operatorname{Sec}[e+f x])^3} (a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left(\frac{12 (7 a^2+32 a b+32 b^2) x}{a^5} + \right. \\
& \frac{1}{(a+b)^2} (a^7-14 a^6 b+336 a^5 b^2+5600 a^4 b^3+22400 a^3 b^4+37632 a^2 b^5+28672 a b^6+8192 b^7) \\
& \left. \left(\left(3 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right)} \right] \right. \right. \\
& \left. \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \right) \operatorname{Cos}[2 e] \right) / \left(64 a^5 b^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) - \\
& \left(3 i \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]}} \right)} \right] \right. \\
& \left. \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \right) \operatorname{Sin}[2 e] \right) / \left(64 a^5 b^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) \left. \right) + \\
& (\operatorname{Sec}[2 e] (-a^6 \operatorname{Sin}[2 e]-52 a^5 b \operatorname{Sin}[2 e]-500 a^4 b^2 \operatorname{Sin}[2 e]-1920 a^3 b^3 \operatorname{Sin}[2 e]-3520 a^2 b^4 \operatorname{Sin}[2 e]-3072 a b^5 \operatorname{Sin}[2 e]-1024 b^6 \\
& \operatorname{Sin}[2 e]+a^6 \operatorname{Sin}[2 f x]+50 a^5 b \operatorname{Sin}[2 f x]+400 a^4 b^2 \operatorname{Sin}[2 f x]+1120 a^3 b^3 \operatorname{Sin}[2 f x]+1280 a^2 b^4 \operatorname{Sin}[2 f x]+512 a b^5 \operatorname{Sin}[2 f x])) / \\
& (16 a^5 b (a+b) f (a+2 b+a \operatorname{Cos}[2 e+2 f x])^2) + \frac{1}{64 a^5 b^2 (a+b)^2 f (a+2 b+a \operatorname{Cos}[2 e+2 f x])} \\
& \operatorname{Sec}[2 e] (-3 a^7 \operatorname{Sin}[2 e]+42 a^6 b \operatorname{Sin}[2 e]+2192 a^5 b^2 \operatorname{Sin}[2 e]+16480 a^4 b^3 \operatorname{Sin}[2 e]+51200 a^3 b^4 \operatorname{Sin}[2 e]+ \\
& 77824 a^2 b^5 \operatorname{Sin}[2 e]+57344 a b^6 \operatorname{Sin}[2 e]+16384 b^7 \operatorname{Sin}[2 e]+3 a^7 \operatorname{Sin}[2 f x]-44 a^6 b \operatorname{Sin}[2 f x]- \\
& 1900 a^5 b^2 \operatorname{Sin}[2 f x]-10880 a^4 b^3 \operatorname{Sin}[2 f x]-23360 a^3 b^4 \operatorname{Sin}[2 f x]-21504 a^2 b^5 \operatorname{Sin}[2 f x]-7168 a b^6 \operatorname{Sin}[2 f x]) + \\
& (a+2 b) \left(-\frac{12 i \operatorname{Cos}[2 e+2 f x]}{a^4 f} - \frac{12 \operatorname{Sin}[2 e+2 f x]}{a^4 f} \right) + (a+2 b) \left(\frac{12 i \operatorname{Cos}[2 e+2 f x]}{a^4 f} - \frac{12 \operatorname{Sin}[2 e+2 f x]}{a^4 f} \right) + \\
& \frac{2 \operatorname{Sin}[4 e+4 f x]}{a^3 f} \left. \right) - \\
& \left((a+2 b+a \operatorname{Cos}[2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left(-\frac{6 a^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) (-a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4}} \right]}{\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4}} \right) (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) \right. \right. \\
& (a \operatorname{Sec}[2 e] ((-9 a^4-16 a^3 b+48 a^2 b^2+128 a b^3+64 b^4) \operatorname{Sin}[2 f x]+ \\
& a (-3 a^3+2 a^2 b+24 a b^2+16 b^3) \operatorname{Sin}[2 (e+2 f x)]+(3 a^4-64 a^2 b^2-128 a b^3-64 b^4) \operatorname{Sin}[4 e+2 f x])) + \\
& \left. \left. (9 a^5+18 a^4 b-64 a^3 b^2-256 a^2 b^3-320 a b^4-128 b^5) \operatorname{Tan}[2 e] \right) / (a^2 (a+2 b+a \operatorname{Cos}[2 (e+f x)])^2) \right) \left. \right) /
\end{aligned}$$

$$\begin{aligned}
& (8192 b^2 (a+b)^2 f (a+b \operatorname{Sec}[e+fx]^2)^3) + \frac{1}{16384 a^4 (a+b \operatorname{Sec}[e+fx]^2)^3} (a+2b+a \operatorname{Cos}[2e+2fx])^3 \\
& \operatorname{Sec}[e+fx]^6 \\
& \left(-1536 (a+2b) x - \left(3 (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \right. \\
& \left. \left. \operatorname{ArcTan} \left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])}{2 \sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) / \right. \\
& \left(b^2 (a+b)^{5/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) + \frac{4 (a^4 + 32 a^3 b + 160 a^2 b^2 + 256 a b^3 + 128 b^4) \operatorname{Sec}[2e] ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{b (a+b) f (a+2b+a \operatorname{Cos}[2(e+fx)])^2} + \\
& \frac{256 a \operatorname{Sin}[2(e+fx)]}{f} + (a (-3 a^5 + 26 a^4 b + 736 a^3 b^2 + 2624 a^2 b^3 + 3200 a b^4 + 1280 b^5) \operatorname{Sec}[2e] \operatorname{Sin}[2fx] + \\
& \left. \left. (3 a^6 - 24 a^5 b - 920 a^4 b^2 - 4864 a^3 b^3 - 10112 a^2 b^4 - 9216 a b^5 - 3072 b^6) \operatorname{Tan}[2e] \right) / (b^2 (a+b)^2 f (a+2b+a \operatorname{Cos}[2(e+fx)])) \right)
\end{aligned}$$

■ **Problem 62: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e+fx]^2}{(a+b \operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 184 leaves, 7 steps):

$$\begin{aligned}
& \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b} (15a^2 + 40ab + 24b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}} \right]}{8a^4 (a+b)^{3/2} f} - \\
& \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{2af (a+b+b \operatorname{Tan}[e+fx]^2)^2} - \frac{3b \operatorname{Tan}[e+fx]}{4a^2 f (a+b+b \operatorname{Tan}[e+fx]^2)^2} - \frac{b (11a+12b) \operatorname{Tan}[e+fx]}{8a^3 (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}
\end{aligned}$$

Result (type 3, 2187 leaves):

$$\begin{aligned}
& \left(5 (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \right. \\
& \left. \left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}} \right]}{(a+b)^{5/2}} - \frac{a \sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \operatorname{Cos}[2(e+fx)]) \operatorname{Sin}[2(e+fx)]}{(a+b)^2 (a+2b+a \operatorname{Cos}[2(e+fx)])^2} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& (8192 b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^3) + \left((a + 2b + a \operatorname{Cos}[2e + 2fx])^3 \operatorname{Sec}[e + f x]^6 \right. \\
& \left. \left(-\frac{3a(a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + fx]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} + \frac{\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \operatorname{Cos}[2(e + fx)]) \operatorname{Sin}[2(e + fx)]}{(a + b)^2 (a + 2b + a \operatorname{Cos}[2(e + fx)])^2} \right) \right) / \\
& (2048 b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^3) + \frac{1}{32 (a + b \operatorname{Sec}[e + f x]^2)^3} (a + 2b + a \operatorname{Cos}[2e + 2fx])^3 \operatorname{Sec}[e + f x]^6 \\
& \left(-\frac{1}{(a + b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
& \left(\left(\operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}} \right) \right] \right) \right. \\
& \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right] \operatorname{Cos}[2e] \Big/ (64a^3b^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}) - \\
& \left(i \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}} \right) \right] \right) \right. \\
& \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right] \operatorname{Sin}[2e] \Big/ (64a^3b^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - ib \operatorname{Sin}[4e]}) \Big) - \\
& \frac{1}{128a^3b^2 (a + b)^2 f (a + 2b + a \operatorname{Cos}[2e + 2fx])^2} \operatorname{Sec}[2e] (768a^4b^2fx \operatorname{Cos}[2e] + 3584a^3b^3fx \operatorname{Cos}[2e] + 6912a^2b^4fx \operatorname{Cos}[2e] + \\
& 6144ab^5fx \operatorname{Cos}[2e] + 2048b^6fx \operatorname{Cos}[2e] + 512a^4b^2fx \operatorname{Cos}[2fx] + 2048a^3b^3fx \operatorname{Cos}[2fx] + 2560a^2b^4fx \operatorname{Cos}[2fx] + \\
& 1024ab^5fx \operatorname{Cos}[2fx] + 512a^4b^2fx \operatorname{Cos}[4e + 2fx] + 2048a^3b^3fx \operatorname{Cos}[4e + 2fx] + 2560a^2b^4fx \operatorname{Cos}[4e + 2fx] + \\
& 1024ab^5fx \operatorname{Cos}[4e + 2fx] + 128a^4b^2fx \operatorname{Cos}[2e + 4fx] + 256a^3b^3fx \operatorname{Cos}[2e + 4fx] + 128a^2b^4fx \operatorname{Cos}[2e + 4fx] + \\
& 128a^4b^2fx \operatorname{Cos}[6e + 4fx] + 256a^3b^3fx \operatorname{Cos}[6e + 4fx] + 128a^2b^4fx \operatorname{Cos}[6e + 4fx] - 9a^6 \operatorname{Sin}[2e] + 12a^5b \operatorname{Sin}[2e] + \\
& 684a^4b^2 \operatorname{Sin}[2e] + 2880a^3b^3 \operatorname{Sin}[2e] + 5280a^2b^4 \operatorname{Sin}[2e] + 4608ab^5 \operatorname{Sin}[2e] + 1536b^6 \operatorname{Sin}[2e] + 9a^6 \operatorname{Sin}[2fx] - \\
& 14a^5b \operatorname{Sin}[2fx] - 608a^4b^2 \operatorname{Sin}[2fx] - 2112a^3b^3 \operatorname{Sin}[2fx] - 2560a^2b^4 \operatorname{Sin}[2fx] - 1024ab^5 \operatorname{Sin}[2fx] - 3a^6 \operatorname{Sin}[4e + 2fx] + \\
& 10a^5b \operatorname{Sin}[4e + 2fx] + 304a^4b^2 \operatorname{Sin}[4e + 2fx] + 1056a^3b^3 \operatorname{Sin}[4e + 2fx] + 1280a^2b^4 \operatorname{Sin}[4e + 2fx] + 512ab^5 \operatorname{Sin}[4e + 2fx] + \\
& 3a^6 \operatorname{Sin}[2e + 4fx] - 12a^5b \operatorname{Sin}[2e + 4fx] - 204a^4b^2 \operatorname{Sin}[2e + 4fx] - 384a^3b^3 \operatorname{Sin}[2e + 4fx] - 192a^2b^4 \operatorname{Sin}[2e + 4fx]) \Big) - \\
& \left((a + 2b + a \operatorname{Cos}[2e + 2fx])^3 \operatorname{Sec}[e + f x]^6 \left(-\frac{6a^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a + 2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx])}{2\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}}\right]}{\sqrt{a+b} \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4}} \right) (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right. \right. \\
& \left. \left. (a \operatorname{Sec}[2e] ((-9a^4 - 16a^3b + 48a^2b^2 + 128ab^3 + 64b^4) \operatorname{Sin}[2fx] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a \left(-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3 \right) \operatorname{Sin}[2 (e + 2 f x)] + \left(3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4 \right) \operatorname{Sin}[4 e + 2 f x] + \\
& \left. \left. \left. \left(9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5 \right) \operatorname{Tan}[2 e] \right) / \left(a^2 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2 \right) \right) \right) / \\
& \left(4096 b^2 (a + b)^2 f (a + b \operatorname{Sec}[e + f x])^3 \right) - \frac{1}{8192 a^4 (a + b \operatorname{Sec}[e + f x])^3} (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \\
& \operatorname{Sec}[e + f x]^6 \\
& \left(-1536 (a + 2 b) x - \left(3 (a^6 - 8 a^5 b + 120 a^4 b^2 + 1280 a^3 b^3 + 3200 a^2 b^4 + 3072 a b^5 + 1024 b^6) \right. \right. \\
& \left. \left. \operatorname{ArcTan} \left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) (- (a + 2 b) \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x])}{2 \sqrt{a + b} \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right] (\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]) \right) \right) / \\
& \left(b^2 (a + b)^{5/2} f \sqrt{b (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right) + \frac{4 (a^4 + 32 a^3 b + 160 a^2 b^2 + 256 a b^3 + 128 b^4) \operatorname{Sec}[2 e] ((a + 2 b) \operatorname{Sin}[2 e] - a \operatorname{Sin}[2 f x])}{b (a + b) f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^2} + \\
& \frac{256 a \operatorname{Sin}[2 (e + f x)]}{f} + (a (-3 a^5 + 26 a^4 b + 736 a^3 b^2 + 2624 a^2 b^3 + 3200 a b^4 + 1280 b^5) \operatorname{Sec}[2 e] \operatorname{Sin}[2 f x] + \\
& \left. \left. \left. \left(3 a^6 - 24 a^5 b - 920 a^4 b^2 - 4864 a^3 b^3 - 10112 a^2 b^4 - 9216 a b^5 - 3072 b^6 \right) \operatorname{Tan}[2 e] \right) / \left(b^2 (a + b)^2 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)]) \right) \right) \right)
\end{aligned}$$

■ **Problem 63: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}} \right]}{8 a^3 (a + b)^{5/2} f} - \frac{b \operatorname{Tan}[e + f x]}{4 a (a + b) f (a + b + b \operatorname{Tan}[e + f x])^2} - \frac{b (7 a + 4 b) \operatorname{Tan}[e + f x]}{8 a^2 (a + b)^2 f (a + b + b \operatorname{Tan}[e + f x])^2}$$

Result (type 3, 627 leaves):

$$\frac{x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \left((15a^2 + 20ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \right.$$

$$\left. \sec[e + fx]^6 \left(\left(b \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right.$$

$$\left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) -$$

$$\left(i b \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) (-a \sin[fx] - \right.$$

$$\left. 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \Bigg) / \left((a+b)^2 (a + b \sec[e + fx]^2)^3 \right) +$$

$$\left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6 (9a^2 b \sin[2e] + 28ab^2 \sin[2e] + 16b^3 \sin[2e] - 9a^2 b \sin[2fx] - 6ab^2 \sin[2fx]) \right) /$$

$$\left(64a^3 (a+b)^2 f (a + b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) +$$

$$\frac{(a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6 (-a b^2 \sin[2e] - 2b^3 \sin[2e] + a b^2 \sin[2fx])}{16a^3 (a+b) f (a + b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) (\cos[e] + \sin[e])}$$

- **Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + fx]^2}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$-\frac{15\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{8(a+b)^{7/2} f} - \frac{15 \cot[e + fx]}{8(a+b)^3 f} + \frac{\cot[e + fx]}{4(a+b) f (a + b \tan[e + fx]^2)^2} + \frac{5 \cot[e + fx]}{8(a+b)^2 f (a + b \tan[e + fx]^2)}$$

Result (type 3, 987 leaves):

$$\left((a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \left(\left(15b \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right. \right. \right. \\ \left. \left. \left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \right) \cos[2e] \right) / \left(64\sqrt{a+b}f\sqrt{b\cos[4e] - ib\sin[4e]} \right) - \right. \\ \left. \left(15ib \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right. \right. \\ \left. \left. \left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \right) \sin[2e] \right) / \left(64\sqrt{a+b}f\sqrt{b\cos[4e] - ib\sin[4e]} \right) \right) / \right) \\ (a + b)^3 (a + b \sec[e + fx]^2)^3 + \frac{1}{512a^2(a + b)^3 f (a + b \sec[e + fx]^2)^3} (a + 2b + a \cos[2e + 2fx])$$

Csc[e]

Csc[e + fx]

Sec[2e]

Sec[e + fx]^6

$$\begin{aligned} & (-32a^4 \sin[fx] - 64a^3 b \sin[fx] + 22a^2 b^2 \sin[fx] + 80ab^3 \sin[fx] + 16b^4 \sin[fx] + 32a^4 \sin[3fx] + 46a^3 b \sin[3fx] - \\ & 54a^2 b^2 \sin[3fx] - 8ab^3 \sin[3fx] - 48a^4 \sin[2e - fx] - 128a^3 b \sin[2e - fx] - 106a^2 b^2 \sin[2e - fx] + \\ & 80ab^3 \sin[2e - fx] + 16b^4 \sin[2e - fx] + 48a^4 \sin[2e + fx] + 146a^3 b \sin[2e + fx] + 182a^2 b^2 \sin[2e + fx] + \\ & 80ab^3 \sin[2e + fx] + 16b^4 \sin[2e + fx] - 32a^4 \sin[4e + fx] - 82a^3 b \sin[4e + fx] - 54a^2 b^2 \sin[4e + fx] - \\ & 80ab^3 \sin[4e + fx] - 16b^4 \sin[4e + fx] - 8a^4 \sin[2e + 3fx] + 18a^3 b \sin[2e + 3fx] + 54a^2 b^2 \sin[2e + 3fx] + \\ & 8ab^3 \sin[2e + 3fx] + 32a^4 \sin[4e + 3fx] + 73a^3 b \sin[4e + 3fx] + 24a^2 b^2 \sin[4e + 3fx] + 8ab^3 \sin[4e + 3fx] - \\ & 8a^4 \sin[6e + 3fx] - 9a^3 b \sin[6e + 3fx] - 24a^2 b^2 \sin[6e + 3fx] - 8ab^3 \sin[6e + 3fx] + 8a^4 \sin[2e + 5fx] - \\ & 9a^3 b \sin[2e + 5fx] - 2a^2 b^2 \sin[2e + 5fx] + 9a^3 b \sin[4e + 5fx] + 2a^2 b^2 \sin[4e + 5fx] + 8a^4 \sin[6e + 5fx]) \end{aligned}$$

■ **Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\csc[e + fx]^4}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 164 leaves, 6 steps):

$$\begin{aligned} & - \frac{5(3a - 4b)\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{8(a+b)^{9/2} f} - \frac{(a - 2b) \cot[e + fx]}{(a+b)^4 f} - \\ & \frac{\cot[e + fx]^3}{3(a+b)^3 f} - \frac{ab \tan[e + fx]}{4(a+b)^3 f (a+b + b \tan[e + fx]^2)^2} - \frac{(7a - 4b)b \tan[e + fx]}{8(a+b)^4 f (a+b + b \tan[e + fx]^2)} \end{aligned}$$

Result (type 3, 1234 leaves):

$$\begin{aligned}
& \left((3a - 4b) (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \right. \\
& \left. \left(\left(5b \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right] \right. \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(64 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \right. \\
& \left. \left(5i b \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right] \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(64 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) \Bigg) / \\
& \left((a+b)^4 (a+b \sec[e+fx]^2)^3 \right) + \frac{1}{6144 a (a+b)^4 f (a+b \sec[e+fx]^2)^3} (a+2b+a \cos[2e+2fx]) \\
& \operatorname{Csc}[e] \\
& \operatorname{Csc}[e+fx]^3 \\
& \operatorname{Sec}[2e] \\
& \operatorname{Sec}[e+fx]^6 \\
& (-176 a^4 \sin[fx] - 488 a^3 b \sin[fx] - 252 a^2 b^2 \sin[fx] - 504 a b^3 \sin[fx] - 144 b^4 \sin[fx] + 96 a^4 \sin[3fx] + 71 a^3 b \sin[3fx] - \\
& 344 a^2 b^2 \sin[3fx] + 1208 a b^3 \sin[3fx] - 48 b^4 \sin[3fx] - 224 a^4 \sin[2e - fx] - 576 a^3 b \sin[2e - fx] - 124 a^2 b^2 \sin[2e - fx] + \\
& 2184 a b^3 \sin[2e - fx] - 144 b^4 \sin[2e - fx] + 224 a^4 \sin[2e + fx] + 657 a^3 b \sin[2e + fx] + 538 a^2 b^2 \sin[2e + fx] - \\
& 984 a b^3 \sin[2e + fx] - 144 b^4 \sin[2e + fx] - 176 a^4 \sin[4e + fx] - 569 a^3 b \sin[4e + fx] - 666 a^2 b^2 \sin[4e + fx] - \\
& 1704 a b^3 \sin[4e + fx] + 144 b^4 \sin[4e + fx] - 48 a^4 \sin[2e + 3fx] - 111 a^3 b \sin[2e + 3fx] - 360 a^2 b^2 \sin[2e + 3fx] - \\
& 312 a b^3 \sin[2e + 3fx] + 48 b^4 \sin[2e + 3fx] + 96 a^4 \sin[4e + 3fx] + 152 a^3 b \sin[4e + 3fx] - 146 a^2 b^2 \sin[4e + 3fx] + \\
& 728 a b^3 \sin[4e + 3fx] + 48 b^4 \sin[4e + 3fx] - 48 a^4 \sin[6e + 3fx] - 192 a^3 b \sin[6e + 3fx] - 558 a^2 b^2 \sin[6e + 3fx] + \\
& 168 a b^3 \sin[6e + 3fx] - 48 b^4 \sin[6e + 3fx] - 16 a^4 \sin[2e + 5fx] + 598 a^2 b^2 \sin[2e + 5fx] - 48 a b^3 \sin[2e + 5fx] - \\
& 72 a^3 b \sin[4e + 5fx] - 150 a^2 b^2 \sin[4e + 5fx] + 48 a b^3 \sin[4e + 5fx] - 16 a^4 \sin[6e + 5fx] - 27 a^3 b \sin[6e + 5fx] + \\
& 388 a^2 b^2 \sin[6e + 5fx] - 45 a^3 b \sin[8e + 5fx] + 60 a^2 b^2 \sin[8e + 5fx] - 16 a^4 \sin[4e + 7fx] + 83 a^3 b \sin[4e + 7fx] - \\
& 6 a^2 b^2 \sin[4e + 7fx] - 27 a^3 b \sin[6e + 7fx] + 6 a^2 b^2 \sin[6e + 7fx] - 16 a^4 \sin[8e + 7fx] + 56 a^3 b \sin[8e + 7fx])
\end{aligned}$$

■ **Problem 66: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + fx]^6}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{\sqrt{b} (15 a^2 - 40 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{8 (a+b)^{11/2} f} - \frac{(5 a^2 - 20 a b + 2 b^2) \operatorname{Cot}[e+fx]}{5 (a+b)^5 f} - \frac{(10 a+b) \operatorname{Cot}[e+fx]^3}{15 (a+b)^4 f} - \frac{\operatorname{Cot}[e+fx]^5}{5 (a+b) f (a+b+b \tan[e+fx]^2)^2} - \frac{b (5 a^2 + 4 b^2) \tan[e+fx]}{20 (a+b)^4 f (a+b+b \tan[e+fx]^2)^2} - \frac{b (35 a^2 - 40 a b + 24 b^2) \tan[e+fx]}{40 (a+b)^5 f (a+b+b \tan[e+fx]^2)^2}$$

Result (type 3, 908 leaves):

$$\frac{(-4 a \cos[e] + 11 b \cos[e]) (a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^2 \operatorname{Sec}[e+fx]^6}{120 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} - \frac{(a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Cot}[e] \operatorname{Csc}[e+fx]^4 \operatorname{Sec}[e+fx]^6}{40 (a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \left((15 a^2 - 40 a b + 8 b^2) (a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Sec}[e+fx]^6 \left(\left(b \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right] \right. \right. \right. \\ \left. \left. \left. (-a \sin[fx] - 2 b \sin[fx] + a \sin[2 e + f x]) \right) \cos[2 e] \right) / \left(64 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) - \left(i b \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right] \right) \right. \\ \left. \left. (-a \sin[fx] - 2 b \sin[fx] + a \sin[2 e + f x]) \right) \sin[2 e] \right) / \left(64 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) \right) / \left((a+b)^5 (a+b \operatorname{Sec}[e+fx]^2)^3 \right) + \frac{(a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \operatorname{Sec}[e+fx]^6 \sin[fx]}{40 (a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \frac{(a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^3 \operatorname{Sec}[e+fx]^6 (4 a \sin[fx] - 11 b \sin[fx])}{120 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \frac{(a + 2 b + a \cos[2 e + 2 f x])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx]^6 (8 a^2 \sin[fx] - 59 a b \sin[fx] + 23 b^2 \sin[fx])}{(120 (a+b)^5 f (a+b \operatorname{Sec}[e+fx]^2)^3) +} + \frac{(a + 2 b + a \cos[2 e + 2 f x]) \operatorname{Sec}[2 e] \operatorname{Sec}[e+fx]^6 (-a b^2 \sin[2 e] - 2 b^3 \sin[2 e] + a b^2 \sin[2 f x])}{16 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \frac{(a + 2 b + a \cos[2 e + 2 f x])^2 \operatorname{Sec}[2 e] \operatorname{Sec}[e+fx]^6 (9 a^2 b \sin[2 e] + 16 a b^2 \sin[2 e] - 8 b^3 \sin[2 e] - 9 a^2 b \sin[2 f x] + 6 a b^2 \sin[2 f x])}{(64 (a+b)^5 f (a+b \operatorname{Sec}[e+fx]^2)^3)}$$

■ **Problem 67: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \sin[e+fx]^5 dx$$

Optimal (type 3, 139 leaves, 6 steps) :

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f} + \frac{2(5a+b) \operatorname{Cos}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{15a^2 f} - \frac{\operatorname{Cos}[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{5af}$$

Result (type 8, 27 leaves) :

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^5 dx$$

■ **Problem 68: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^3 dx$$

Optimal (type 3, 100 leaves, 5 steps) :

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f} + \frac{\operatorname{Cos}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{3af}$$

Result (type 8, 27 leaves) :

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^3 dx$$

■ **Problem 69: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx] dx$$

Optimal (type 3, 66 leaves, 4 steps) :

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cos}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f}$$

Result (type 8, 25 leaves) :

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx] dx$$

■ **Problem 71: Unable to integrate problem.**

$$\int \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 124 leaves, 7 steps) :

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{(a+2b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{2\sqrt{a+b} f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{2f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 72: Unable to integrate problem.**

$$\int \operatorname{Csc}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 183 leaves, 8 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{f} - \frac{(3a^2 + 12ab + 8b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+fx]}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}\right]}{8(a+b)^{3/2} f} - \frac{(3a+4b) \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8(a+b) f} - \frac{\operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{4f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Csc}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 73: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^6 dx$$

Optimal (type 3, 240 leaves, 9 steps):

$$\frac{(5a^3 - 15a^2b - 5ab^2 - b^3) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16a^{5/2} f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(a-b)(5a+b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16a^2 f} - \frac{(5a-b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24af} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Sin}[e+fx]^6 dx$$

■ **Problem 74: Unable to integrate problem.**

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x]^4 dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$\frac{(3a^2 - 6ab - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right] + \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{8a^{3/2}f} - \frac{(3a - b) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{8af} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]^3 \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{4f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x]^4 dx$$

■ **Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Sin}[e + f x]^2 dx$$

Optimal (type 3, 123 leaves, 7 steps):

$$\frac{(a - b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right] + \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{2\sqrt{a}f} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{2f}$$

Result (type 3, 432 leaves):

$$\frac{1}{4\sqrt{2} f \sqrt{a+2b+a \cos[2e+2fx]}} e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]$$

$$\left(i(-1+e^{2i(e+fx)}) + 2e^{2i(e+fx)} \left(2afx - 2bfx - i(a-b) \operatorname{Log}\left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + \right. \right.$$

$$\left. i(a-b) \operatorname{Log}\left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] - 4\sqrt{a} \sqrt{b} \operatorname{Log}\left[\right. \right.$$

$$\left. \left. \frac{\left(-\sqrt{b}(-1+e^{2i(e+fx)}) + i \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) f}{2b(1+e^{2i(e+fx)})} \right] \right) \left/ \left(\sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \right) \sqrt{a+b \operatorname{Sec}[e+fx]^2}$$

■ **Problem 76: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 68 leaves, 4 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f}$$

Result (type 3, 285 leaves):

$$\begin{aligned}
& - \left((1 + e^{2i(e+fx)}) \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\
& \left. i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \sqrt{b} (-1 + e^{2i(e+fx)}) \operatorname{Log} \left[\frac{-4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f}{1 + e^{2i(e+fx)}} \right] \right) \\
& \left. \sqrt{a + b \operatorname{Sec}[e + fx]^2} \right) / \left(\sqrt{2} (-1 + e^{2i(e+fx)}) \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \sqrt{a + 2b + a \operatorname{Cos}[2(e + fx)]} \right)
\end{aligned}$$

- **Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + fx]^4 \sqrt{a + b \operatorname{Sec}[e + fx]^2} dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \operatorname{Tan}[e + fx]}{\sqrt{a + b \operatorname{Tan}[e + fx]^2}} \right]}{f} - \frac{\operatorname{Cot}[e + fx] \sqrt{a + b + b \operatorname{Tan}[e + fx]^2}}{f} - \frac{\operatorname{Cot}[e + fx]^3 (a + b + b \operatorname{Tan}[e + fx]^2)^{3/2}}{3(a + b)f}$$

Result (type 3, 309 leaves):

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b+ae^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx] \right.$$

$$\left. - \frac{i(2a(1-4e^{2i(e+fx)}+e^{4i(e+fx)})+b(3-10e^{2i(e+fx)}+3e^{4i(e+fx)}))}{(a+b)(-1+e^{2i(e+fx)})^3} - \frac{3\sqrt{b} \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right)$$

$$\left. \sqrt{a+b\operatorname{Sec}[e+fx]^2} \right) / (3f\sqrt{a+2b+a\operatorname{Cos}[2e+2fx]})$$

- **Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^6 \sqrt{a+b\operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$\frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b\operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b\operatorname{Tan}[e+fx]^2}}{f} -$$

$$\frac{2(5a+4b) \operatorname{Cot}[e+fx]^3 (a+b\operatorname{Tan}[e+fx]^2)^{3/2}}{15(a+b)^2 f} - \frac{\operatorname{Cot}[e+fx]^5 (a+b\operatorname{Tan}[e+fx]^2)^{3/2}}{5(a+b)f}$$

Result (type 3, 422 leaves):

$$\frac{1}{15 f \sqrt{a+2 b+a \cos [2 e+2 f x]}} \sqrt{2} e^{i(e+f x)} \sqrt{4 b+a e^{-2 i(e+f x)}\left(1+e^{2 i(e+f x)}\right)^2 \cos [e+f x]}$$

$$\left(-\left(i\left(8 a^2\left(1-6 e^{2 i(e+f x)}+16 e^{4 i(e+f x)}-6 e^{6 i(e+f x)}+e^{8 i(e+f x)}\right)+b^2\left(15-80 e^{2 i(e+f x)}+178 e^{4 i(e+f x)}-80 e^{6 i(e+f x)}+15 e^{8 i(e+f x)}\right)\right)+\right.$$

$$\left. a b\left(25-136 e^{2 i(e+f x)}+318 e^{4 i(e+f x)}-136 e^{6 i(e+f x)}+25 e^{8 i(e+f x)}\right)\right) / \left(\left(a+b\right)^2\left(-1+e^{2 i(e+f x)}\right)^5\right) -$$

$$\frac{15 \sqrt{b} \operatorname{Log}\left[\frac{-4 \sqrt{b}\left(-1+e^{2 i(e+f x)}\right) f+4 i \sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2} f}{1+e^{2 i(e+f x)}}\right]}{\sqrt{4 b e^{2 i(e+f x)}+a\left(1+e^{2 i(e+f x)}\right)^2}} \sqrt{a+b \operatorname{Sec}[e+f x]^2}$$

■ **Problem 80: Unable to integrate problem.**

$$\int (a+b \operatorname{Sec}[e+f x]^2)^{3/2} \sin [e+f x]^5 dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\frac{(3 a-4 b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} + \frac{(3 a-4 b) b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2 a f} -$$

$$\frac{(3 a-4 b) \cos [e+f x]\left(a+b \operatorname{Sec}[e+f x]^2\right)^{3/2}}{3 a f} + \frac{2 \cos [e+f x]^3\left(a+b \operatorname{Sec}[e+f x]^2\right)^{5/2}}{3 a f} - \frac{\cos [e+f x]^5\left(a+b \operatorname{Sec}[e+f x]^2\right)^{5/2}}{5 a f}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+f x]^2)^{3/2} \sin [e+f x]^5 dx$$

■ **Problem 81: Unable to integrate problem.**

$$\int (a+b \operatorname{Sec}[e+f x]^2)^{3/2} \sin [e+f x]^3 dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{(3 a-2 b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} + \frac{(3 a-2 b) b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2 a f} -$$

$$\frac{(3 a-2 b) \cos [e+f x]\left(a+b \operatorname{Sec}[e+f x]^2\right)^{3/2}}{3 a f} + \frac{\cos [e+f x]^3\left(a+b \operatorname{Sec}[e+f x]^2\right)^{5/2}}{3 a f}$$

Result (type 8, 27 leaves) :

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x]^3 dx$$

■ **Problem 82: Unable to integrate problem.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x] dx$$

Optimal (type 3, 100 leaves, 5 steps) :

$$\frac{3 a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} + \frac{3 b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2 f} - \frac{\operatorname{Cos}[e+f x] (a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{f}$$

Result (type 8, 25 leaves) :

$$\int (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \operatorname{Sin}[e + f x] dx$$

■ **Problem 83: Unable to integrate problem.**

$$\int \operatorname{Csc}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 122 leaves, 7 steps) :

$$\frac{\sqrt{b} (3 a + 2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} - \frac{(a+b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{f} + \frac{b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2 f}$$

Result (type 8, 25 leaves) :

$$\int \operatorname{Csc}[e + f x] (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

■ **Problem 84: Unable to integrate problem.**

$$\int \operatorname{Csc}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 161 leaves, 8 steps) :

$$\frac{\sqrt{b} (3 a + 4 b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} - \frac{\sqrt{a+b} (a + 4 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 f} + \frac{b \operatorname{Sec}[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} - \frac{\operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x] (a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{2 f}$$

Result (type 8, 27 leaves) :

$$\int \operatorname{Csc}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

■ **Problem 85: Unable to integrate problem.**

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 218 leaves, 9 steps):

$$\frac{3 \sqrt{b} (a + 2 b) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Sec}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}}\right]}{2 f} - \frac{3 (a^2 + 8 a b + 8 b^2) \text{ArcTanh}\left[\frac{\sqrt{a + b} \text{Sec}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}}\right]}{8 \sqrt{a + b} f} + \frac{3 (a + 4 b) \text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]^2}}{8 f} - \frac{3 (a + 2 b) \text{Csc}[e + f x]^2 \text{Sec}[e + f x] \sqrt{a + b \text{Sec}[e + f x]^2}}{8 f} - \frac{\text{Cot}[e + f x] \text{Csc}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^{3/2}}{4 f}$$

Result (type 8, 27 leaves):

$$\int \text{Csc}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^{3/2} dx$$

■ **Problem 86: Unable to integrate problem.**

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Sin}[e + f x]^6 dx$$

Optimal (type 3, 298 leaves, 10 steps):

$$\frac{(5 a^3 - 45 a^2 b + 15 a b^2 + b^3) \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{16 a^{3/2} f} + \frac{(3 a - 5 b) \sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{2 f} - \frac{(5 a^2 - 26 a b + b^2) \text{Tan}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{16 a f} + \frac{(5 a^2 - 40 a b + 3 b^2) \text{Sin}[e + f x]^2 \text{Tan}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{48 a f} + \frac{(5 a - 3 b) \text{Sin}[e + f x]^4 \text{Tan}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{24 f} - \frac{\text{Cos}[e + f x] \text{Sin}[e + f x]^5 (a + b \text{Tan}[e + f x]^2)^{3/2}}{6 f}$$

Result (type 8, 27 leaves):

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Sin}[e + f x]^6 dx$$

■ **Problem 87: Unable to integrate problem.**

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Sin}[e + f x]^4 dx$$

Optimal (type 3, 217 leaves, 9 steps):

$$\frac{3(a^2 - 6ab + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} - \frac{3(a-3b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} +$$

$$\frac{3(a-b) \operatorname{Sin}[e+fx]^2 \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4f}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^4 dx$$

- **Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Sin}[e+fx]^2 dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$\frac{\sqrt{a} (a-3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{(3a-b)\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} +$$

$$\frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{f} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{2f}$$

Result (type 3, 493 leaves):

$$\frac{1}{2\sqrt{2} f (a + 2b + a \cos[2e + 2fx])^{3/2}}$$

$$e^{-i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e + fx]^3 \left(\frac{i(-1 + e^{2i(e+fx)})(-4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2)}{(1 + e^{2i(e+fx)})^2} + \right.$$

$$\left. \left(2e^{2i(e+fx)} \left(2\sqrt{a}(a-3b)fx - i\sqrt{a}(a-3b) \operatorname{Log}\left[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right] + \right. \right.$$

$$\left. \left. i\sqrt{a}(a-3b) \operatorname{Log}\left[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right] + 2\sqrt{b}(-3a+b) \right. \right.$$

$$\left. \left. \operatorname{Log}\left[\frac{\left(\sqrt{b}(-1 + e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right) f}{b(-3a+b)(1 + e^{2i(e+fx)})} \right] \right) \right) / \left(\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right) (a + b \sec[e + fx]^2)^{3/2}$$

- **Problem 89: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{f} + \frac{\sqrt{b}(3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{2f} + \frac{b \tan[e+fx] \sqrt{a+b \tan[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\frac{1}{f (a + 2b + a \cos[2e + 2fx])^{3/2}} \sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e + fx]^3$$

$$\left(-\frac{i b (-1 + e^{2i(e+fx)})}{(1 + e^{2i(e+fx)})^2} + \frac{1}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \left(2 a^{3/2} f x - i a^{3/2} \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] + \right. \right.$$

$$i a^{3/2} \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}] -$$

$$3 a \sqrt{b} \operatorname{Log}\left[\frac{-2 \sqrt{b} (-1 + e^{2i(e+fx)}) f + 2i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f}{b (3a + b) (1 + e^{2i(e+fx)})} \right] -$$

$$\left. \left. b^{3/2} \operatorname{Log}\left[\frac{-2 \sqrt{b} (-1 + e^{2i(e+fx)}) f + 2i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f}{b (3a + b) (1 + e^{2i(e+fx)})} \right] \right) \right) (a + b \operatorname{Sec}[e + fx]^2)^{3/2}$$

- **Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e + fx]^2 (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

Optimal (type 3, 105 leaves, 5 steps):

$$\frac{3 \sqrt{b} (a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + fx]}{\sqrt{a + b \operatorname{Tan}[e + fx]^2}}\right]}{2 f} + \frac{3 b \operatorname{Tan}[e + fx] \sqrt{a + b \operatorname{Tan}[e + fx]^2}}{2 f} - \frac{\operatorname{Cot}[e + fx] (a + b \operatorname{Tan}[e + fx]^2)^{3/2}}{f}$$

Result (type 3, 310 leaves):

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b+ae^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\ \left. - \frac{i(2a(1+e^{2i(e+fx)})^2+b(3+2e^{2i(e+fx)}+3e^{4i(e+fx)}))}{(-1+e^{2i(e+fx)})(1+e^{2i(e+fx)})^2} - \frac{3\sqrt{b}(a+b) \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \\ \left. (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / (f(a+2b+a \cos[2e+2fx])^{3/2})$$

- **Problem 91: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^4 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 172 leaves, 6 steps):

$$\frac{\sqrt{b}(3a+5b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b(3a+5b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2(a+b)f} - \\ \frac{(3a+5b) \operatorname{Cot}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{3(a+b)f} - \frac{\operatorname{Cot}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{3(a+b)f}$$

Result (type 3, 369 leaves):

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\ \left. - \left(i \left(4a (1+e^{2i(e+fx)})^2 (1-4e^{2i(e+fx)} + e^{4i(e+fx)}) + b (15-20e^{2i(e+fx)} - 22e^{4i(e+fx)} - 20e^{6i(e+fx)} + 15e^{8i(e+fx)}) \right) \right) / \right. \\ \left. \left((-1+e^{2i(e+fx)})^3 (1+e^{2i(e+fx)})^2 - \frac{3\sqrt{b}(3a+5b) \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \right. \\ \left. (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / (3f(a+2b+a \cos[2e+2fx])^{3/2})$$

- **Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^6 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 209 leaves, 7 steps):

$$\frac{\sqrt{b} (3a+7b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b(3a+7b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2(a+b)f} - \\ \frac{(3a+7b) \operatorname{Cot}[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{3(a+b)f} - \frac{2 \operatorname{Cot}[e+fx]^3 (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{3(a+b)f} - \frac{\operatorname{Cot}[e+fx]^5 (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{5(a+b)f}$$

Result (type 3, 512 leaves):

$$\frac{1}{15 f (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^{3/2}} \sqrt{2} e^{i (e + f x)} \sqrt{4 b + a e^{-2 i (e + f x)} (1 + e^{2 i (e + f x)})^2} \operatorname{Cos}[e + f x]^3$$

$$\left(- \frac{1}{(a + b) (-1 + e^{2 i (e + f x)})^5 (1 + e^{2 i (e + f x)})^2} i (16 a^2 (1 + e^{2 i (e + f x)})^2 (1 - 6 e^{2 i (e + f x)} + 16 e^{4 i (e + f x)} - 6 e^{6 i (e + f x)} + e^{8 i (e + f x)}) + \right.$$

$$b^2 (105 - 350 e^{2 i (e + f x)} + 231 e^{4 i (e + f x)} + 412 e^{6 i (e + f x)} + 231 e^{8 i (e + f x)} - 350 e^{10 i (e + f x)} + 105 e^{12 i (e + f x)}) +$$

$$a b (115 - 402 e^{2 i (e + f x)} + 317 e^{4 i (e + f x)} + 708 e^{6 i (e + f x)} + 317 e^{8 i (e + f x)} - 402 e^{10 i (e + f x)} + 115 e^{12 i (e + f x)}) \left. - \right.$$

$$\frac{15 \sqrt{b} (3 a + 7 b) \operatorname{Log}\left[\frac{-4 \sqrt{b} (-1 + e^{2 i (e + f x)}) f + 4 i \sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2} f}{1 + e^{2 i (e + f x)}}\right]}{\sqrt{4 b e^{2 i (e + f x)} + a (1 + e^{2 i (e + f x)})^2}} \left. \right) (a + b \operatorname{Sec}[e + f x]^2)^{3/2}$$

■ **Problem 96: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 43 leaves, 3 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{\sqrt{a+b} f}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Csc}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 97: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 87 leaves, 5 steps) :

$$\frac{a \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 (a + b)^{3/2} f} - \frac{\operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^2}}{2 (a + b) f}$$

Result (type 8, 27 leaves) :

$$\int \frac{\text{Csc}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 98: Unable to integrate problem.**

$$\int \frac{\text{Csc}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$\frac{3 a^2 \text{ArcTanh}\left[\frac{\sqrt{a+b} \text{Sec}[e+f x]}{\sqrt{a+b \text{Sec}[e+f x]^2}}\right]}{8 (a+b)^{5/2} f} - \frac{(5 a+2 b) \text{Cot}[e+f x] \text{Csc}[e+f x] \sqrt{a+b \text{Sec}[e+f x]^2}}{8 (a+b)^2 f} - \frac{\text{Cot}[e+f x]^3 \text{Csc}[e+f x] \sqrt{a+b \text{Sec}[e+f x]^2}}{4 (a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Csc}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 193 leaves, 7 steps):

$$\frac{5 (a+b)^3 \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e+f x]}{\sqrt{a+b \text{Tan}[e+f x]^2}}\right]}{16 a^{7/2} f} - \frac{(33 a^2 + 40 a b + 15 b^2) \text{Cos}[e+f x] \text{Sin}[e+f x] \sqrt{a+b \text{Tan}[e+f x]^2}}{48 a^3 f} +$$

$$\frac{(9 a+5 b) \text{Cos}[e+f x]^3 \text{Sin}[e+f x] \sqrt{a+b \text{Tan}[e+f x]^2}}{24 a^2 f} + \frac{\text{Cos}[e+f x]^3 \text{Sin}[e+f x]^3 \sqrt{a+b \text{Tan}[e+f x]^2}}{6 a f}$$

Result (type 3, 2258 leaves):

$$\frac{5 \text{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} \text{Sin}[e+f x]}{\sqrt{a+2 b+a \text{Cos}[2 (e+f x)]}}\right] \sqrt{a+2 b+a \text{Cos}[2 (e+f x)]} \text{Sec}[e+f x]}{64 \sqrt{2} \sqrt{a} f \sqrt{a+b \text{Sec}[e+f x]^2}} -$$

$$\frac{1}{1536 \sqrt{2} a^{7/2} f \sqrt{a+b \text{Sec}[e+f x]^2}} e^{-i (13 e+5 f x)} \sqrt{4 b+a e^{-2 i (e+f x)} (1+e^{2 i (e+f x)})^2} \sqrt{a+2 b+a \text{Cos}[2 e+2 f x]}$$

$$\left((1+e^{14 i e}) \left(i \sqrt{a} (-60 b^2 e^{4 i f x} (-e^{12 i e} + e^{2 i f x}) + 10 a b e^{2 i f x} (-e^{10 i e} - 6 e^{4 i f x} + 6 e^{2 i (6 e+f x)} + e^{2 i (e+3 f x)}) + \right. \right.$$

$$\left. \left. a^2 (2 e^{8 i e} - 11 e^{6 i f x} + 11 e^{4 i (3 e+f x)} - 5 e^{2 i (5 e+f x)} + 5 e^{2 i (e+4 f x)} - 2 e^{2 i (2 e+5 f x)}) \right) \right) -$$

$$\begin{aligned}
& \left(6 (a^3 + 12 a^2 b + 30 a b^2 + 20 b^3) e^{6 i f x} \left(-i \operatorname{Log} \left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] + e^{14 i e} \left(2 f x + i \operatorname{Log} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] \right) \right) \right) / \left(\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right) + \\
& (-1 + e^{14 i e}) \left(-i \sqrt{a} (60 b^2 e^{4 i f x} (e^{12 i e} + e^{2 i f x}) - 10 a b e^{2 i f x} (e^{10 i e} - 6 e^{4 i f x} - 6 e^{2 i (6 e+f x)} + e^{2 i (e+3 f x)}) + \right. \\
& \quad \left. a^2 (2 e^{8 i e} + 11 e^{6 i f x} + 11 e^{4 i (3 e+f x)} - 5 e^{2 i (5 e+f x)} - 5 e^{2 i (e+4 f x)} + 2 e^{2 i (2 e+5 f x)}) \right) + \\
& \left(6 (a^3 + 12 a^2 b + 30 a b^2 + 20 b^3) e^{6 i f x} \left(i \operatorname{Log} \left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] + e^{14 i e} \left(2 f x + i \operatorname{Log} \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] \right) \right) \right) / \left(\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right) \right)
\end{aligned}$$

$$\operatorname{Sec}[e + f x] + \frac{1}{128 \sqrt{2} a^{3/2} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \sqrt{a + b \operatorname{Sec}[e + f x]^2}} 9 i$$

$$e^{-i (e+f x)}$$

$$\sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2}$$

$$\sqrt{a + 2 b + a \operatorname{Cos}[2 e + 2 f x]}$$

$$\left(-\sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \right.$$

$$\left. \sqrt{a} e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} - 2 i a e^{2 i (e+f x)} f x - 4 i b e^{2 i (e+f x)} f x - \right.$$

$$\left. (a + 2 b) e^{2 i (e+f x)} \operatorname{Log} \left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] + \right.$$

$$\left. (a + 2 b) e^{2 i (e+f x)} \operatorname{Log} \left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \right] \right) \operatorname{Sec}[e + f x] +$$

1

$$256 \sqrt{2} a^{5/2} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f \sqrt{a + b \operatorname{Sec}[e + f x]^2}$$

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$$e^{-3 i (e+f x)}$$

$$\begin{aligned}
& \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \\
& \sqrt{a + 2b + a \operatorname{Cos}[2e + 2fx]} \\
& \left(i a^{3/2} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} - 3 i a^{3/2} e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} - \right. \\
& 6 i \sqrt{a} b e^{2i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + 3 i a^{3/2} e^{4i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \\
& 6 i \sqrt{a} b e^{4i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} - i a^{3/2} e^{6i(e+fx)} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \\
& 4 a^2 e^{4i(e+fx)} f x + 24 a b e^{4i(e+fx)} f x + 24 b^2 e^{4i(e+fx)} f x - \\
& \left. 2 i (a^2 + 6 a b + 6 b^2) e^{4i(e+fx)} \operatorname{Log}\left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) + \\
& \left. 2 i (a^2 + 6 a b + 6 b^2) e^{4i(e+fx)} \operatorname{Log}\left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) \Bigg] \operatorname{Sec}[e + fx]
\end{aligned}$$

- **Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[e + fx]^4}{\sqrt{a + b \operatorname{Sec}[e + fx]^2}} dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\frac{3(a+b)^2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8 a^{5/2} f} - \frac{(5a+3b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8 a^2 f} + \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4 a f}$$

Result (type 3, 1286 leaves):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{a}\sin[e+fx]}{\sqrt{a+2b+a\cos[2e+2fx]}}\right]\sqrt{a+2b+a\cos[2e+2fx]}\sec[e+fx]}{8\sqrt{2}\sqrt{a}f\sqrt{a+b\sec[e+fx]^2}} + \\
& \frac{1}{32\sqrt{2}a^{3/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f\sqrt{a+b\sec[e+fx]^2}} - 3ie^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \\
& \sqrt{a+2b+a\cos[2e+2fx]}\left(-\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + \sqrt{a}e^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} - \right. \\
& \left. 2iae^{2i(e+fx)}fx - 4ib e^{2i(e+fx)}fx - (a+2b)e^{2i(e+fx)}\text{Log}\left[e^{-2ie}\left(a+2b+ae^{2i(e+fx)} + \sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right) + \\
& \left.(a+2b)e^{2i(e+fx)}\text{Log}\left[e^{-2ie}\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)} + \sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right)\sec[e+fx] + \\
& \frac{1}{64\sqrt{2}a^{5/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f\sqrt{a+b\sec[e+fx]^2}} e^{-3i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} \\
& \sqrt{a+2b+a\cos[2e+2fx]}\left(ia^{3/2}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} - 3ia^{3/2}e^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} - \right. \\
& 6i\sqrt{a}be^{2i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + 3ia^{3/2}e^{4i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + 6i\sqrt{a}be^{4i(e+fx)} \\
& \left.\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} - ia^{3/2}e^{6i(e+fx)}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + 4a^2e^{4i(e+fx)}fx + 24abe^{4i(e+fx)}fx + \right. \\
& \left. 24b^2e^{4i(e+fx)}fx - 2i(a^2+6ab+6b^2)e^{4i(e+fx)}\text{Log}\left[e^{-2ie}\left(a+2b+ae^{2i(e+fx)} + \sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right) + \\
& \left. 2i(a^2+6ab+6b^2)e^{4i(e+fx)}\text{Log}\left[e^{-2ie}\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)} + \sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right)\sec[e+fx]
\end{aligned}$$

- **Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e+fx]^2}{\sqrt{a+b\sec[e+fx]^2}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2 a^{3/2} f} - \frac{\operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2 a f}$$

Result (type 3, 558 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]}}\right] \sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]} \operatorname{Sec}[e+fx]}{4 \sqrt{2} \sqrt{a} f \sqrt{a+b \operatorname{Sec}[e+fx]^2}} +$$

$$\frac{1}{8 \sqrt{2} a^{3/2} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} f \sqrt{a+b \operatorname{Sec}[e+fx]^2}} i e^{-i(e+fx)} \sqrt{4 b + a e^{-2 i(e+fx)} (1 + e^{2 i(e+fx)})^2}$$

$$\sqrt{a+2 b+a \operatorname{Cos}[2 e+2 f x]} \left(-\sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} + \sqrt{a} e^{2 i(e+fx)} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} - \right.$$

$$\left. 2 i a e^{2 i(e+fx)} f x - 4 i b e^{2 i(e+fx)} f x - (a+2 b) e^{2 i(e+fx)} \operatorname{Log}\left[e^{-2 i e} \left(a+2 b+a e^{2 i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} \right) \right] \right) +$$

$$(a+2 b) e^{2 i(e+fx)} \operatorname{Log}\left[e^{-2 i e} \left(a+a e^{2 i(e+fx)} + 2 b e^{2 i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i(e+fx)} + a (1 + e^{2 i(e+fx)})^2} \right) \right] \right) \operatorname{Sec}[e+fx]$$

■ **Problem 102: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

■ **Problem 109: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e+fx]}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\text{Sec}[e+fx]}{\sqrt{a+b\text{Sec}[e+fx]^2}}\right]}{(a+b)^{3/2}f} - \frac{b\text{Sec}[e+fx]}{a(a+b)f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Csc}[e+fx]}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 110: Unable to integrate problem.**

$$\int \frac{\text{Csc}[e+fx]^3}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$-\frac{(a-2b)\text{ArcTanh}\left[\frac{\sqrt{a+b}\text{Sec}[e+fx]}{\sqrt{a+b\text{Sec}[e+fx]^2}}\right]}{2(a+b)^{5/2}f} - \frac{\text{Cot}[e+fx]\text{Csc}[e+fx]}{2(a+b)f\sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{3b\text{Sec}[e+fx]}{2(a+b)^2f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Csc}[e+fx]^3}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 111: Unable to integrate problem.**

$$\int \frac{\text{Csc}[e+fx]^5}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 177 leaves, 7 steps):

$$-\frac{3a(a-4b)\text{ArcTanh}\left[\frac{\sqrt{a+b}\text{Sec}[e+fx]}{\sqrt{a+b\text{Sec}[e+fx]^2}}\right]}{8(a+b)^{7/2}f} - \frac{5a\text{Cot}[e+fx]\text{Csc}[e+fx]}{8(a+b)^2f\sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{\text{Cot}[e+fx]^3\text{Csc}[e+fx]}{4(a+b)f\sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{(13a-2b)b\text{Sec}[e+fx]}{8(a+b)^3f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Csc}[e+fx]^5}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e+fx]^6}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 242 leaves, 8 steps) :

$$\frac{5 (a+b)^2 (a+7b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16 a^{9/2} f} - \frac{(a+b) (33a+35b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48 a^3 f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} +$$

$$\frac{(9a+7b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24 a^2 f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]^3}{6 a f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} - \frac{b (81 a^2 + 190 a b + 105 b^2) \operatorname{Tan}[e+fx]}{48 a^4 f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 3, 3051 leaves) :

$$\left(3 e^{-3 i (e+fx)} \sqrt{4 b+a e^{-2 i (e+fx)} (1+e^{2 i (e+fx)})^2} (a+2 b+a \operatorname{Cos}[2 e+2 f x])^{3/2} \right.$$

$$\left(i a^{7/2} + i a^{5/2} b - 5 i a^{7/2} e^{2 i (e+fx)} - 15 i a^{5/2} b e^{2 i (e+fx)} - 10 i a^{3/2} b^2 e^{2 i (e+fx)} - 13 i a^{7/2} e^{4 i (e+fx)} - \right.$$

$$104 i a^{5/2} b e^{4 i (e+fx)} - 210 i a^{3/2} b^2 e^{4 i (e+fx)} - 120 i \sqrt{a} b^3 e^{4 i (e+fx)} + 13 i a^{7/2} e^{6 i (e+fx)} + 104 i a^{5/2} b e^{6 i (e+fx)} +$$

$$210 i a^{3/2} b^2 e^{6 i (e+fx)} + 120 i \sqrt{a} b^3 e^{6 i (e+fx)} + 5 i a^{7/2} e^{8 i (e+fx)} + 15 i a^{5/2} b e^{8 i (e+fx)} + 10 i a^{3/2} b^2 e^{8 i (e+fx)} -$$

$$i a^{7/2} e^{10 i (e+fx)} - i a^{5/2} b e^{10 i (e+fx)} + 24 a^3 e^{4 i (e+fx)} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} f x +$$

$$144 a^2 b e^{4 i (e+fx)} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} f x + 240 a b^2 e^{4 i (e+fx)} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} f x +$$

$$120 b^3 e^{4 i (e+fx)} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} f x - 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4 i (e+fx)} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2}$$

$$\left. \operatorname{Log}\left[e^{-2 i e} \left(a+2 b+a e^{2 i (e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} \right) \right] + 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4 i (e+fx)} \right.$$

$$\left. \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} \operatorname{Log}\left[e^{-2 i e} \left(a+a e^{2 i (e+fx)} + 2 b e^{2 i (e+fx)} + \sqrt{a} \sqrt{4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2} \right) \right] \right] \operatorname{Sec}[e+fx]^3 \Big/$$

$$\left(512 \sqrt{2} a^{7/2} (a+b) (4 b e^{2 i (e+fx)} + a (1+e^{2 i (e+fx)})^2) f (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) +$$

$$\left(i e^{-5 i (e+fx)} \sqrt{4 b+a e^{-2 i (e+fx)} (1+e^{2 i (e+fx)})^2} (a+2 b+a \operatorname{Cos}[2 e+2 f x])^{3/2} \right.$$

$$\left(-2 a^{9/2} - 2 a^{7/2} b + 7 a^{9/2} e^{2 i (e+fx)} + 21 a^{7/2} b e^{2 i (e+fx)} + 14 a^{5/2} b^2 e^{2 i (e+fx)} - 27 a^{9/2} e^{4 i (e+fx)} - 167 a^{7/2} b e^{4 i (e+fx)} - \right.$$

$$280 a^{5/2} b^2 e^{4 i (e+fx)} - 140 a^{3/2} b^3 e^{4 i (e+fx)} - 63 a^{9/2} e^{6 i (e+fx)} - 790 a^{7/2} b e^{6 i (e+fx)} - 2830 a^{5/2} b^2 e^{6 i (e+fx)} - 3780 a^{3/2} b^3 e^{6 i (e+fx)} -$$

$$1680 \sqrt{a} b^4 e^{6 i (e+fx)} + 63 a^{9/2} e^{8 i (e+fx)} + 790 a^{7/2} b e^{8 i (e+fx)} + 2830 a^{5/2} b^2 e^{8 i (e+fx)} + 3780 a^{3/2} b^3 e^{8 i (e+fx)} + 1680 \sqrt{a} b^4 e^{8 i (e+fx)} +$$

$$27 a^{9/2} e^{10 i (e+fx)} + 167 a^{7/2} b e^{10 i (e+fx)} + 280 a^{5/2} b^2 e^{10 i (e+fx)} + 140 a^{3/2} b^3 e^{10 i (e+fx)} - 7 a^{9/2} e^{12 i (e+fx)} - 21 a^{7/2} b e^{12 i (e+fx)} -$$

$$\begin{aligned}
& 14 a^{5/2} b^2 e^{12 i (e+f x)} + 2 a^{9/2} e^{14 i (e+f x)} + 2 a^{7/2} b e^{14 i (e+f x)} - 120 i a^4 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - \\
& 1200 i a^3 b e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - 3600 i a^2 b^2 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - \\
& 4200 i a b^3 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - 1680 i b^4 e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f x - \\
& 60 (a^4 + 10 a^3 b + 30 a^2 b^2 + 35 a b^3 + 14 b^4) e^{6 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \\
& \text{Log}\left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] + 60 (a^4 + 10 a^3 b + 30 a^2 b^2 + 35 a b^3 + 14 b^4) e^{6 i (e+f x)} \\
& \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \text{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \Bigg) \text{Sec}[e + f x]^3 \Bigg) / \\
& (1536 \sqrt{2} a^{9/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2) f (a + b \text{Sec}[e + f x]^2)^{3/2}) - \\
& \left(e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} (a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \right. \\
& \left(-3 i a^{3/2} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} - 4 i \sqrt{a} b \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \right. \\
& 3 i a^{3/2} e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + 4 i \sqrt{a} b e^{2 i (e+f x)} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} + \\
& 4 a^2 f x + 4 a b f x + 8 a^2 e^{2 i (e+f x)} f x + 24 a b e^{2 i (e+f x)} f x + 16 b^2 e^{2 i (e+f x)} f x + 4 a^2 e^{4 i (e+f x)} f x + 4 a b e^{4 i (e+f x)} f x - \\
& 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2) \text{Log}\left[e^{-2 i e} \left(a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] + \\
& \left. 2 i (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2) \text{Log}\left[e^{-2 i e} \left(a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}\right)\right] \right) \Bigg) \\
& \text{Sec}[e + f x]^3 \Bigg) / (64 \sqrt{2} a^{3/2} (a + b) (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)^{3/2} f (a + b \text{Sec}[e + f x]^2)^{3/2}) + \\
& \frac{3 (a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{256 (a + b) f \sqrt{a + 2 b + a \text{Cos}[2 (e + f x)]} (a + b \text{Sec}[e + f x]^2)^{3/2}}
\end{aligned}$$

■ **Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 175 leaves, 7 steps):

$$\frac{3(a+b)(a+5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8a^{7/2}f} - \frac{5(a+b) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{8a^2 f \sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{\operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{4af \sqrt{a+b \operatorname{Tan}[e+fx]^2}} - \frac{b(13a+15b) \operatorname{Tan}[e+fx]}{8a^3 f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 3, 2543 leaves):

$$\left(i e^{-i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \operatorname{Cos}[2e+2fx])^{3/2} \right. \\ \left(-2a^{5/2} - 2a^{3/2}b - 7a^{5/2}e^{2i(e+fx)} - 30a^{3/2}be^{2i(e+fx)} - 24\sqrt{a}b^2e^{2i(e+fx)} + 7a^{5/2}e^{4i(e+fx)} + 30a^{3/2}be^{4i(e+fx)} + \right. \\ \left. 24\sqrt{a}b^2e^{4i(e+fx)} + 2a^{5/2}e^{6i(e+fx)} + 2a^{3/2}be^{6i(e+fx)} - 12ia^2e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - \right. \\ \left. 36iab e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - 24ib^2e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - \right. \\ \left. 6(a^2+3ab+2b^2)e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) + \\ \left. 6(a^2+3ab+2b^2)e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2be^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) \operatorname{Sec}[e+fx]^3 \Big/ \\ \left(128\sqrt{2}a^{5/2}(a+b)(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2) f (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) + \\ \left(e^{-3i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \operatorname{Cos}[2e+2fx])^{3/2} \right. \\ \left(i a^{7/2} + i a^{5/2}b - 5i a^{7/2}e^{2i(e+fx)} - 15i a^{5/2}be^{2i(e+fx)} - 10i a^{3/2}b^2e^{2i(e+fx)} - 13i a^{7/2}e^{4i(e+fx)} - \right. \\ \left. 104i a^{5/2}be^{4i(e+fx)} - 210i a^{3/2}b^2e^{4i(e+fx)} - 120i \sqrt{a}b^3e^{4i(e+fx)} + 13i a^{7/2}e^{6i(e+fx)} + 104i a^{5/2}be^{6i(e+fx)} + \right. \\ \left. 210i a^{3/2}b^2e^{6i(e+fx)} + 120i \sqrt{a}b^3e^{6i(e+fx)} + 5i a^{7/2}e^{8i(e+fx)} + 15i a^{5/2}be^{8i(e+fx)} + 10i a^{3/2}b^2e^{8i(e+fx)} - \right. \\ \left. i a^{7/2}e^{10i(e+fx)} - i a^{5/2}be^{10i(e+fx)} + 24a^3e^{4i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx + \right.$$

$$\begin{aligned}
& 144 a^2 b e^{4i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f x + 240 a b^2 e^{4i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f x + \\
& 120 b^3 e^{4i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f x - 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \\
& \text{Log}\left[e^{-2i e} \left(a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right)\right] + 12 i (a^3 + 6 a^2 b + 10 a b^2 + 5 b^3) e^{4i(e+fx)} \\
& \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \text{Log}\left[e^{-2i e} \left(a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right)\right] \Big) \text{Sec}[e + f x]^3 \Big) / \\
& \left(128 \sqrt{2} a^{7/2} (a + b) (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) f (a + b \text{Sec}[e + f x]^2)^{3/2} \right) - \\
& \left(3 e^{i(e+fx)} \sqrt{4 b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \right. \\
& \left. \left(-3 i a^{3/2} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} - 4 i \sqrt{a} b \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \right. \right. \\
& \left. \left. 3 i a^{3/2} e^{2i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + 4 i \sqrt{a} b e^{2i(e+fx)} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} + \right. \right. \\
& \left. \left. 4 a^2 f x + 4 a b f x + 8 a^2 e^{2i(e+fx)} f x + 24 a b e^{2i(e+fx)} f x + 16 b^2 e^{2i(e+fx)} f x + 4 a^2 e^{4i(e+fx)} f x + 4 a b e^{4i(e+fx)} f x - \right. \right. \\
& \left. \left. 2 i (a + b) (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) \text{Log}\left[e^{-2i e} \left(a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right)\right] \right) + \right. \\
& \left. \left. 2 i (a + b) (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) \text{Log}\left[e^{-2i e} \left(a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right)\right] \right) \right) \\
& \text{Sec}[e + f x]^3 \Big) / \left(128 \sqrt{2} a^{3/2} (a + b) (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2) f (a + b \text{Sec}[e + f x]^2)^{3/2} \right) + \\
& \frac{3 (a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{128 (a + b) f \sqrt{a + 2 b + a \text{Cos}[2 (e + f x)]} (a + b \text{Sec}[e + f x]^2)^{3/2}}
\end{aligned}$$

■ **Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[e + f x]^2}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 121 leaves, 6 steps):

$$\frac{(a + 3b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}\right]}{2 a^{5/2} f} - \frac{\operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{2 a f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}} - \frac{3 b \operatorname{Tan}[e + f x]}{2 a^2 f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 3, 1522 leaves):

$$\begin{aligned}
& \left(i e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{3/2} \right. \\
& \left. \left(-2a^{5/2} - 2a^{3/2}b - 7a^{5/2}e^{2i(e+fx)} - 30a^{3/2}be^{2i(e+fx)} - 24\sqrt{a}b^2e^{2i(e+fx)} + 7a^{5/2}e^{4i(e+fx)} + 30a^{3/2}be^{4i(e+fx)} + \right. \right. \\
& \left. \left. 24\sqrt{a}b^2e^{4i(e+fx)} + 2a^{5/2}e^{6i(e+fx)} + 2a^{3/2}be^{6i(e+fx)} - 12ia^2e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - \right. \right. \\
& \left. \left. 36iab e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - 24ib^2e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} fx - \right. \right. \\
& \left. \left. 6(a^2+3ab+2b^2)e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) + \right. \\
& \left. 6(a^2+3ab+2b^2)e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right. \\
& \left. \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2be^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) \operatorname{Sec}[e+fx]^3 \Big/ \\
& \left(32\sqrt{2} a^{5/2} (a+b) (4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2) f (a+b \operatorname{Sec}[e+fx])^{3/2} \right) - \\
& \left(e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{3/2} \right. \\
& \left. \left(-3ia^{3/2} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} - 4i\sqrt{a}b \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + \right. \right. \\
& \left. \left. 3ia^{3/2}e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + 4i\sqrt{a}be^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} + \right. \right. \\
& \left. \left. 4a^2fx + 4abfx + 8a^2e^{2i(e+fx)}fx + 24abe^{2i(e+fx)}fx + 16b^2e^{2i(e+fx)}fx + 4a^2e^{4i(e+fx)}fx + 4abe^{4i(e+fx)}fx - \right. \right. \\
& \left. \left. 2i(a+b) (4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2) \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) + \right. \\
& \left. 2i(a+b) (4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2) \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2be^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \right] \right) \Big/ \\
& \operatorname{Sec}[e+fx]^3 \Big/ \left(32\sqrt{2} a^{3/2} (a+b) (4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2} f (a+b \operatorname{Sec}[e+fx])^{3/2} \right) + \\
& \frac{(a+2b+a \cos[2e+2fx])^{3/2} \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{16(a+b) f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \operatorname{Sec}[e+fx])^{3/2}}
\end{aligned}$$

■ **Problem 115: Unable to integrate problem.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps) :

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{a^{3/2} f} - \frac{b \operatorname{Tan}[e + f x]}{a (a + b) f \sqrt{a + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 8, 18 leaves) :

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 122: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 127 leaves, 6 steps) :

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{(a+b)^{5/2} f} - \frac{b \operatorname{Sec}[e+f x]}{3 a (a+b) f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \frac{b (5 a+2 b) \operatorname{Sec}[e+f x]}{3 a^2 (a+b)^2 f \sqrt{a+b \operatorname{Sec}[e+f x]^2}}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Csc}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

■ **Problem 123: Unable to integrate problem.**

$$\int \frac{\operatorname{Csc}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 171 leaves, 7 steps) :

$$-\frac{(a-4 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{2 (a+b)^{7/2} f} - \frac{\operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]}{2 (a+b) f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \frac{5 b \operatorname{Sec}[e+f x]}{6 (a+b)^2 f (a+b \operatorname{Sec}[e+f x]^2)^{3/2}} - \frac{(13 a-2 b) b \operatorname{Sec}[e+f x]}{6 a (a+b)^3 f \sqrt{a+b \operatorname{Sec}[e+f x]^2}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\operatorname{Csc}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

■ **Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[e + f x]^5}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 234 leaves, 8 steps):

$$\begin{aligned} & - \frac{(3 a^2 - 24 a b + 8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}\right]}{8 (a+b)^{9/2} f} - \frac{(5 a - 2 b) \operatorname{Cot}[e + f x] \operatorname{Csc}[e + f x]}{8 (a+b)^2 f (a + b \operatorname{Sec}[e + f x]^2)^{3/2}} \\ & - \frac{\operatorname{Cot}[e + f x]^3 \operatorname{Csc}[e + f x]}{4 (a+b) f (a + b \operatorname{Sec}[e + f x]^2)^{3/2}} - \frac{(23 a - 12 b) b \operatorname{Sec}[e + f x]}{24 (a+b)^3 f (a + b \operatorname{Sec}[e + f x]^2)^{3/2}} - \frac{5 (11 a - 10 b) b \operatorname{Sec}[e + f x]}{24 (a+b)^4 f \sqrt{a + b \operatorname{Sec}[e + f x]^2}} \end{aligned}$$

Result (type 6, 1709 leaves):

$$\begin{aligned} & - \left(\left(7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Cot}[e + f x]^4 \operatorname{Csc}[e + f x]^3 \right) / \right. \\ & \left(20 \sqrt{2} f (a + b \operatorname{Sec}[e + f x]^2)^{5/2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\ & \left. \left(5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] - 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \right) \right. \\ & \left. \left. 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Sin}[e + f x]^2 \right) \right) \\ & \left(- \left(7 a^2 \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Cos}[e + f x]^4 \operatorname{Cot}[e + f x] \right) / \right. \\ & \left(4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{7/2} \left(5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] - 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \right. \right. \right. \\ & \left. \left. \left. \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] + 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Sin}[e + f x]^2 \right) \right) \right) + \\ & \left(7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Cos}[e + f x]^2 \operatorname{Cot}[e + f x] \right) / \\ & \left(10 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left(5 (a+b) \operatorname{AppellF1}\left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] - 4 a \operatorname{AppellF1}\left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \right. \right. \right. \\ & \left. \left. \left. \frac{9}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] + 7 a \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e + f x]^2, \frac{(a+b) \operatorname{Csc}[e + f x]^2}{a}\right] \operatorname{Sin}[e + f x]^2 \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& \left(7 a \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x]^3 \right) / \left(10 \sqrt{2} (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \right. \\
& \quad \left(5 (a+b) \operatorname{AppellF1} \left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] - 4 a \operatorname{AppellF1} \left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \right. \right. \\
& \quad \left. \left. \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] + 7 a \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Sin}[e+f x]^2 \right) \left. \right) - \\
& \left(7 a \operatorname{Cos}[e+f x]^2 \operatorname{Cot}[e+f x]^2 \left(-1 / (7 a) 25 (a+b) f \operatorname{AppellF1} \left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{20}{7} f \operatorname{AppellF1} \left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) \right) / \\
& \left(20 \sqrt{2} f (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \left(5 (a+b) \operatorname{AppellF1} \left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] - 4 a \operatorname{AppellF1} \left[\frac{7}{2}, -1, \right. \right. \\
& \quad \left. \left. \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] + 7 a \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Sin}[e+f x]^2 \right) \right) \left. \right) + \\
& \left(7 a \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cos}[e+f x]^2 \operatorname{Cot}[e+f x]^2 \right. \\
& \quad \left(5 (a+b) \left(-\frac{1}{9 a} 49 (a+b) f \operatorname{AppellF1} \left[\frac{9}{2}, -2, \frac{9}{2}, \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{28}{9} \right. \right. \\
& \quad \left. \left. f \operatorname{AppellF1} \left[\frac{9}{2}, -1, \frac{7}{2}, \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) - \right. \\
& \quad \left. 4 a \left(-\frac{1}{9 a} 35 (a+b) f \operatorname{AppellF1} \left[\frac{9}{2}, -1, \frac{7}{2}, \frac{11}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{14}{9} \right. \right. \\
& \quad \left. \left. f \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \operatorname{Hypergeometric2F1} \left[\frac{5}{2}, \frac{9}{2}, \frac{11}{2}, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \right) \right) + \\
& \quad 14 a f \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] + \\
& \quad 7 a \left(-\frac{1}{7 a} 25 (a+b) f \operatorname{AppellF1} \left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 + \frac{20}{7} \right. \\
& \quad \left. \left. f \operatorname{AppellF1} \left[\frac{7}{2}, -1, \frac{5}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] \operatorname{Cot}[e+f x] \operatorname{Csc}[e+f x]^2 \right) \operatorname{Sin}[e+f x]^2 \right) \left. \right) / \\
& \left(20 \sqrt{2} f (a+b-a \operatorname{Sin}[e+f x]^2)^{5/2} \left(5 (a+b) \operatorname{AppellF1} \left[\frac{7}{2}, -2, \frac{7}{2}, \frac{9}{2}, \operatorname{Csc}[e+f x]^2, \frac{(a+b) \operatorname{Csc}[e+f x]^2}{a} \right] - 4 a \operatorname{AppellF1} \left[\frac{7}{2}, -1, \frac{5}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 560 e^{2i(9e+fx)} + 840 e^{2i(e+2fx)} + 7 e^{2i(4e+5fx)} - 4 a^4 b^3 (-6 e^{14ie} + 430 e^{4ifx} + 3987 e^{20ie+6ifx} + 2610 e^{22ie+8ifx} + 84 e^{2i(8e+fx)} + \\
& 3987 e^{4i(e+2fx)} + 1684 e^{2i(9e+2fx)} + 2610 e^{2i(e+3fx)} + 84 e^{4i(2e+3fx)} + 1684 e^{2i(3e+5fx)} + 430 e^{2i(12e+5fx)} - 6 e^{2i(5e+7fx)}) - \\
& 6 a^5 b^2 (-2 e^{14ie} + 80 e^{4ifx} + 407 e^{20ie+6ifx} + 318 e^{22ie+8ifx} + 14 e^{2i(8e+fx)} + 407 e^{4i(e+2fx)} + 191 e^{2i(9e+2fx)} + \\
& 318 e^{2i(e+3fx)} + 14 e^{4i(2e+3fx)} + 191 e^{2i(3e+5fx)} + 80 e^{2i(12e+5fx)} - 2 e^{2i(5e+7fx)}) - \\
& 12 a^3 b^4 (-e^{14ie} + 175 e^{4ifx} + 3750 e^{20ie+6ifx} + 1835 e^{22ie+8ifx} + 35 e^{2i(8e+fx)} + 3750 e^{4i(e+2fx)} + 1214 e^{2i(9e+2fx)} + \\
& 1835 e^{2i(e+3fx)} + 35 e^{4i(2e+3fx)} + 1214 e^{2i(3e+5fx)} + 175 e^{2i(12e+5fx)} - e^{2i(5e+7fx)}) + \\
& \frac{1}{24 \sqrt{2} a^4 b^2 (a+b)^2 (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^2 f} i e^{-i(17e+3fx)} (1 + e^{18ie}) \sqrt{4 b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \\
& (13440 b^7 e^{4ie+6ifx} (-e^{16ie} + e^{2ifx}) + a^7 e^{4ifx} (-1 + e^{18ie}) (1 + e^{2i(e+fx)})^3 - \\
& 6 a^6 b e^{4ifx} (-1 + e^{18ie}) (3 + 8 e^{2i(e+fx)} + 8 e^{4i(e+fx)} + 3 e^{6i(e+fx)}) + \\
& 2240 a b^6 e^{2i(e+2fx)} (-2 e^{16ie} + 3 e^{2ifx} + 2 e^{4ie+6ifx} - 3 e^{4i(5e+fx)} - 21 e^{2i(9e+fx)} + 21 e^{2i(e+2fx)}) + \\
& 24 a^2 b^5 e^{2ifx} (-7 e^{16ie} + 35 e^{2ifx} + 2710 e^{4ie+6ifx} - 840 e^{22ie+6ifx} + 560 e^{6ie+8ifx} - 35 e^{8i(3e+fx)} - 2710 e^{4i(5e+fx)} - \\
& 560 e^{2i(9e+fx)} + 840 e^{2i(e+2fx)} + 7 e^{2i(4e+5fx)}) - 12 a^3 b^4 (-e^{14ie} - 175 e^{4ifx} + 3750 e^{20ie+6ifx} + 1835 e^{22ie+8ifx} + 35 e^{2i(8e+fx)} - \\
& 3750 e^{4i(e+2fx)} + 1214 e^{2i(9e+2fx)} - 1835 e^{2i(e+3fx)} - 35 e^{4i(2e+3fx)} - 1214 e^{2i(3e+5fx)} + 175 e^{2i(12e+5fx)} + e^{2i(5e+7fx)}) - \\
& 6 a^5 b^2 (-2 e^{14ie} - 80 e^{4ifx} + 407 e^{20ie+6ifx} + 318 e^{22ie+8ifx} + 14 e^{2i(8e+fx)} - 407 e^{4i(e+2fx)} + 191 e^{2i(9e+2fx)} - \\
& 318 e^{2i(e+3fx)} - 14 e^{4i(2e+3fx)} - 191 e^{2i(3e+5fx)} + 80 e^{2i(12e+5fx)} + 2 e^{2i(5e+7fx)}) - \\
& 4 a^4 b^3 (-6 e^{14ie} - 430 e^{4ifx} + 3987 e^{20ie+6ifx} + 2610 e^{22ie+8ifx} + 84 e^{2i(8e+fx)} - 3987 e^{4i(e+2fx)} + 1684 e^{2i(9e+2fx)} - \\
& 2610 e^{2i(e+3fx)} - 84 e^{4i(2e+3fx)} - 1684 e^{2i(3e+5fx)} + 430 e^{2i(12e+5fx)} + 6 e^{2i(5e+7fx)}) + \\
& \left(5 (3 a^2 + 14 a b + 14 b^2) e^{i(-17e+fx)} (1 + e^{18ie}) \sqrt{4 b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \right. \\
& \left. \left(-i \operatorname{Log} \left[e^{-2ie} \left(a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \right. \\
& \left. \left. e^{18ie} \left(2 f x + i \operatorname{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) \right) \right) / \\
& \left(\sqrt{2} a^{9/2} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) - \left(5 (3 a^2 + 14 a b + 14 b^2) e^{i(-17e+fx)} (-1 + e^{18ie}) \right. \\
& \left. \sqrt{4 b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \left(i \operatorname{Log} \left[e^{-2ie} \left(a + 2 b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right. \right. \\
& \left. \left. e^{18ie} \left(2 f x + i \operatorname{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2 b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \right) \right) \right) / \\
& \left(\sqrt{2} a^{9/2} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f \right) \operatorname{Sec}[e + f x]^5 -
\end{aligned}$$

$$\begin{aligned}
& \left(i e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left. \begin{aligned}
& -25 a^{7/2} - 58 a^{5/2} b - 32 a^{3/2} b^2 - 15 a^{7/2} e^{2i(e+fx)} - \\
& 108 a^{5/2} b e^{2i(e+fx)} - 192 a^{3/2} b^2 e^{2i(e+fx)} - 96 \sqrt{a} b^3 e^{2i(e+fx)} + \\
& 15 a^{7/2} e^{4i(e+fx)} + 108 a^{5/2} b e^{4i(e+fx)} + 192 a^{3/2} b^2 e^{4i(e+fx)} + \\
& 96 \sqrt{a} b^3 e^{4i(e+fx)} + 25 a^{7/2} e^{6i(e+fx)} + 58 a^{5/2} b e^{6i(e+fx)} + \\
& 32 a^{3/2} b^2 e^{6i(e+fx)} - 24 i a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 48 i a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 24 i b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 12 a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 24 a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 12 b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 12 a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 24 a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 12 b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] \Big] \\
& \operatorname{Sec}[e+fx]^5 \Big/ \left(384 \sqrt{2} a^{5/2} (a+b)^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2 \right) \\
& f \\
& \left((a+b \operatorname{Sec}[e+fx]^2)^{5/2} \right) - \left(i \right. \\
& \left. e^{-i(e+fx)} \right. \\
& \left. \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \right. \\
& \left. (a+2b+a \cos[2e+2fx])^{5/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-12 a^{9/2} - 24 a^{7/2} b - 12 a^{5/2} b^2 - 113 a^{9/2} e^{2i(e+fx)} - 532 a^{7/2} b e^{2i(e+fx)} - 740 a^{5/2} b^2 e^{2i(e+fx)} - \right. \\
& 320 a^{3/2} b^3 e^{2i(e+fx)} - 87 a^{9/2} e^{4i(e+fx)} - 690 a^{7/2} b e^{4i(e+fx)} - 2040 a^{5/2} b^2 e^{4i(e+fx)} - \\
& 2400 a^{3/2} b^3 e^{4i(e+fx)} - 960 \sqrt{a} b^4 e^{4i(e+fx)} + 87 a^{9/2} e^{6i(e+fx)} + 690 a^{7/2} b e^{6i(e+fx)} + \\
& 2040 a^{5/2} b^2 e^{6i(e+fx)} + 2400 a^{3/2} b^3 e^{6i(e+fx)} + 960 \sqrt{a} b^4 e^{6i(e+fx)} + 113 a^{9/2} e^{8i(e+fx)} + \\
& 532 a^{7/2} b e^{8i(e+fx)} + 740 a^{5/2} b^2 e^{8i(e+fx)} + 320 a^{3/2} b^3 e^{8i(e+fx)} + 12 a^{9/2} e^{10i(e+fx)} + \\
& 24 a^{7/2} b e^{10i(e+fx)} + 12 a^{5/2} b^2 e^{10i(e+fx)} - 120 i a^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - \\
& 480 i a^2 b e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - \\
& 600 i a b^2 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - 240 i b^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - \\
& 60 a^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \\
& 240 a^2 b e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \\
& 300 a b^2 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \\
& 120 b^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\
& 60 a^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\
& 240 a^2 b e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\
& 300 a b^2 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\
& 120 b^3 e^{2i(e+fx)} (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \Big) \\
& \text{Sec}[e + f x]^5 \Big) / \left(1536 \sqrt{2} a^{7/2} (a + b)^2 (4 b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^2 \right. \\
& f \\
& \left. (a + b \text{Sec}[e + f x]^2)^{5/2} \right) + \\
& \frac{(2 a + 3 b + a \text{Cos}[2(e + f x)]) (a + 2 b + a \text{Cos}[2 e + 2 f x])^{5/2} \text{Sec}[e + f x]^4 \text{Tan}[e + f x]}{256 (a + b)^2 f (a + 2 b + a \text{Cos}[2(e + f x)])^{3/2} (a + b \text{Sec}[e + f x]^2)^{5/2}} -
\end{aligned}$$

$$\frac{(b + (3a + 2b) \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \tan[e + fx]}{384 (a + b)^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}}$$

■ **Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[e + fx]^2}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\frac{(a + 5b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + fx]}{\sqrt{a + b \tan[e + fx]^2}}\right]}{2 a^{7/2} f} - \frac{\cos[e + fx] \sin[e + fx]}{2 a f (a + b \tan[e + fx]^2)^{3/2}} - \frac{5 b \tan[e + fx]}{6 a^2 f (a + b \tan[e + fx]^2)^{3/2}} - \frac{b (13 a + 15 b) \tan[e + fx]}{6 a^3 (a + b) f \sqrt{a + b \tan[e + fx]^2}}$$

Result (type 3, 3247 leaves):

$$\begin{aligned} & - \left(i e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2b + a \cos[2e + 2fx])^{5/2} \left(-25 a^{7/2} - 58 a^{5/2} b - 32 a^{3/2} b^2 - 15 a^{7/2} e^{2i(e+fx)} - \right. \right. \\ & 108 a^{5/2} b e^{2i(e+fx)} - 192 a^{3/2} b^2 e^{2i(e+fx)} - 96 \sqrt{a} b^3 e^{2i(e+fx)} + 15 a^{7/2} e^{4i(e+fx)} + 108 a^{5/2} b e^{4i(e+fx)} + 192 a^{3/2} b^2 e^{4i(e+fx)} + \\ & 96 \sqrt{a} b^3 e^{4i(e+fx)} + 25 a^{7/2} e^{6i(e+fx)} + 58 a^{5/2} b e^{6i(e+fx)} + 32 a^{3/2} b^2 e^{6i(e+fx)} - 24 i a^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - \\ & 48 i a b (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - 24 i b^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} f x - \\ & 12 a^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \\ & 24 a b (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \\ & 12 b^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\ & 12 a^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\ & 24 a b (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \\ & 12 b^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] \Big) \\ & \left. \sec[e + fx]^5 \right) / \left((128 \sqrt{2} a^{5/2} (a + b)^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)^2 f (a + b \sec[e + fx]^2)^{5/2} \right) \Big) + \end{aligned}$$

$$\begin{aligned}
& \left(i e^{-i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left. -12 a^{9/2} - 24 a^{7/2} b - 12 a^{5/2} b^2 - 113 a^{9/2} e^{2i(e+fx)} - 532 a^{7/2} b e^{2i(e+fx)} - 740 a^{5/2} b^2 e^{2i(e+fx)} - 320 a^{3/2} b^3 e^{2i(e+fx)} - \right. \\
& 87 a^{9/2} e^{4i(e+fx)} - 690 a^{7/2} b e^{4i(e+fx)} - 2040 a^{5/2} b^2 e^{4i(e+fx)} - 2400 a^{3/2} b^3 e^{4i(e+fx)} - 960 \sqrt{a} b^4 e^{4i(e+fx)} + 87 a^{9/2} e^{6i(e+fx)} + \\
& 690 a^{7/2} b e^{6i(e+fx)} + 2040 a^{5/2} b^2 e^{6i(e+fx)} + 2400 a^{3/2} b^3 e^{6i(e+fx)} + 960 \sqrt{a} b^4 e^{6i(e+fx)} + 113 a^{9/2} e^{8i(e+fx)} + \\
& 532 a^{7/2} b e^{8i(e+fx)} + 740 a^{5/2} b^2 e^{8i(e+fx)} + 320 a^{3/2} b^3 e^{8i(e+fx)} + 12 a^{9/2} e^{10i(e+fx)} + 24 a^{7/2} b e^{10i(e+fx)} + 12 a^{5/2} b^2 e^{10i(e+fx)} - \\
& 120 i a^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - 480 i a^2 b e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 600 i a b^2 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - 240 i b^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 60 a^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 240 a^2 b e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 300 a b^2 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 120 b^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 60 a^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 240 a^2 b e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 300 a b^2 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 120 b^3 e^{2i(e+fx)} (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \text{Log} \left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] \Big) \\
& \text{Sec}[e+fx]^5 \Big) / \left(384 \sqrt{2} a^{7/2} (a+b)^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2 f (a+b \text{Sec}[e+fx]^2)^{5/2} \right) + \\
& \frac{5 (2a+3b+a \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \text{Sec}[e+fx]^4 \text{Tan}[e+fx]}{384 (a+b)^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \text{Sec}[e+fx]^2)^{5/2}} -
\end{aligned}$$

$$\frac{(b + (3a + 2b) \cos[2(e + fx)]) (a + 2b + a \cos[2e + 2fx])^{5/2} \sec[e + fx]^4 \tan[e + fx]}{384 (a + b)^2 f (a + 2b + a \cos[2(e + fx)])^{3/2} (a + b \sec[e + fx]^2)^{5/2}}$$

■ **Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e + fx]}{\sqrt{a + b \tan[e + fx]^2}}\right]}{a^{5/2} f} - \frac{b \tan[e + fx]}{3a(a + b) f (a + b + b \tan[e + fx]^2)^{3/2}} - \frac{b(5a + 3b) \tan[e + fx]}{3a^2(a + b)^2 f \sqrt{a + b + b \tan[e + fx]^2}}$$

Result (type 6, 1927 leaves):

$$\left(3(a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] \cos[e + fx]^4 \sin[e + fx] \right) /$$

$$\left(4\sqrt{2} f (a + b \sec[e + fx]^2)^{5/2} (a + b - a \sin[e + fx]^2)^{5/2} \right.$$

$$\left. \left(3(a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] + \left(5a \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] - \right. \right. \right.$$

$$\left. \left. 4(a + b) \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] \right) \sin[e + fx]^2 \right)$$

$$\left(\left(15a(a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] \cos[e + fx]^5 \sin[e + fx]^2 \right) / \left(4\sqrt{2} (a + b - a \sin[e + fx]^2)^{7/2} \right. \right.$$

$$\left. \left(3(a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] + \left(5a \text{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] - \right. \right. \right.$$

$$\left. \left. 4(a + b) \text{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] \right) \sin[e + fx]^2 \right) +$$

$$\left(3(a + b) \text{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a + b}\right] \cos[e + fx]^5 \right) / \left(4\sqrt{2} (a + b - a \sin[e + fx]^2)^{5/2} \right)$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \sin[e+fx]^2 \right) / \left(\sqrt{2} (a+b - a \sin[e+fx]^2)^{5/2} \right. \\
& \quad \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) + \\
& \left(3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
& \left(4 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \right. \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \sin[e+fx] \right. \\
& \quad \left. \left(2 f \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \right. \\
& \quad \left. \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \right.
\end{aligned}$$

$$\frac{1}{f(1+m)} \text{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right]$$

$$(\text{Cos}[e+fx]^2)^{\frac{1}{2}+p} (a+b \text{Sec}[e+fx]^2)^p (d \text{Sin}[e+fx])^m \left(\frac{a+b-a \text{Sin}[e+fx]^2}{a+b}\right)^{-p} \text{Tan}[e+fx]$$

Result (type 6, 3356 leaves):

$$\left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \text{Cos}[e+fx] \right.$$

$$\left. (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^p (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx] (d \text{Sin}[e+fx])^m \left(\frac{\text{Tan}[e+fx]}{\sqrt{\text{Sec}[e+fx]^2}}\right)^m \right) /$$

$$\left(f(1+m) \left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] - \right.$$

$$\left. \left(-2bp \text{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \right.$$

$$\left. \left. (a+b)(2+m) \text{AppellF1}\left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right) \text{Tan}[e+fx]^2 \right)$$

$$\left(\left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-1+p} \right. \right.$$

$$\left. \left. \left(\frac{\text{Tan}[e+fx]}{\sqrt{\text{Sec}[e+fx]^2}}\right)^m \right) / \left((1+m) \left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] - \right. \right.$$

$$\left. \left(-2bp \text{AppellF1}\left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] + \right. \right.$$

$$\left. \left. (a+b)(2+m) \text{AppellF1}\left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] \right) \text{Tan}[e+fx]^2 \right) \right)$$

$$\left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^p \right.$$

$$\left. \text{Sin}[e+fx]^2 \left(\frac{\text{Tan}[e+fx]}{\sqrt{\text{Sec}[e+fx]^2}}\right)^m \right) / \left((1+m) \left((a+b)(3+m) \text{AppellF1}\left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b}\right] - \right.$$

$$\begin{aligned}
& \left(-2 b p \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] + \right. \\
& \quad \left. (a+b)(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \tan[e+f x]^2 \right) + \\
& \left(2(a+b)(3+m) p \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] (a+2b+a \cos[2(e+f x)])^p (\sec[e+f x]^2)^p \right. \\
& \quad \left. \sin[e+f x]^2 \left(\frac{\tan[e+f x]}{\sqrt{\sec[e+f x]^2}} \right)^m \right) / \left((1+m) \left((a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left(-2 b p \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b)(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \tan[e+f x]^2 \right) \right) - \\
& \left(2 a(a+b)(3+m) p \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \cos[e+f x] \right. \\
& \quad \left. (a+2b+a \cos[2(e+f x)])^{-1+p} (\sec[e+f x]^2)^p \sin[e+f x] \sin[2(e+f x)] \left(\frac{\tan[e+f x]}{\sqrt{\sec[e+f x]^2}} \right)^m \right) / \\
& \left((1+m) \left((a+b)(3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left(-2 b p \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b)(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \tan[e+f x]^2 \right) \right) + \\
& \left((a+b)(3+m) \cos[e+f x] (a+2b+a \cos[2(e+f x)])^p (\sec[e+f x]^2)^p \sin[e+f x] \left(\frac{\tan[e+f x]}{\sqrt{\sec[e+f x]^2}} \right)^m \right. \\
& \quad \left. \left(1 / ((a+b)(3+m)) 2 b(1+m) p \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{2+m}{2}, 1-p, 1 + \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \sec[e+f x]^2 \tan[e+f x] - \right. \right. \\
& \quad \left. \left. 1 / (3+m)(1+m)(2+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{2+m}{2}, -p, 1 + \frac{3+m}{2}, -\tan[e+f x]^2, -\frac{b \tan[e+f x]^2}{a+b} \right] \sec[e+f x]^2 \tan[e+f x] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1+m) \left((a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left(-2bp \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) + \\
& \left((a+b) m (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \quad \left. (\sec[e+fx]^2)^p \sin[e+fx] \left(\frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^{-1+m} \left(\frac{\sqrt{\sec[e+fx]^2} - \tan[e+fx]^2}{\sqrt{\sec[e+fx]^2}} \right) \right) \Big/ \\
& \left((1+m) \left((a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left(-2bp \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \right) - \\
& \left((a+b) (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \quad \left. (\sec[e+fx]^2)^p \sin[e+fx] \left(\frac{\tan[e+fx]}{\sqrt{\sec[e+fx]^2}} \right)^m \left(-2 \left(-2bp \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (a+b) (2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{4+m}{2}, -p, \frac{5+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \sec[e+fx]^2 \tan[e+fx] + (a+b) (3+m) \right. \\
& \quad \left. \left(\frac{1}{(a+b) (3+m)} - 2b (1+m) p \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, \frac{2+m}{2}, 1-p, 1 + \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \quad \left. \left. \frac{1}{3+m} (1+m) (2+m) \operatorname{AppellF1} \left[1 + \frac{1+m}{2}, 1 + \frac{2+m}{2}, -p, 1 + \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) \right) - \\
& \tan[e+fx]^2 \left(-2bp \left(-\frac{1}{(a+b) (5+m)} - 2b (3+m) (1-p) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, \frac{2+m}{2}, 2-p, 1 + \frac{5+m}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \frac{1}{5+m} (2+m) (3+m) \operatorname{AppellF1} \left[1 + \frac{3+m}{2}, 1 + \frac{2+m}{2}, 1-p, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& b (-1 + 2p) \operatorname{AppellF1} \left[-\frac{1}{2} - p, -\frac{1}{2}, -p, \frac{1}{2} - p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] \tan[e + f x]^2 \Bigg) \Bigg) + \\
& 2^p p \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-\frac{1}{2}+p} \left(\frac{a+b+b \tan[e + f x]^2}{1 + \tan[e + f x]^2} \right)^{-1+p} \left(\frac{2b \sec[e + f x]^2 \tan[e + f x]}{1 + \tan[e + f x]^2} - \right. \\
& \left. \frac{2 \sec[e + f x]^2 \tan[e + f x] (a+b+b \tan[e + f x]^2)}{(1 + \tan[e + f x]^2)^2} \right) \\
& \left(- \left(2(a+b) \operatorname{AppellF1} \left[1, \frac{1}{2}, -p, 2, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] \right) / \right. \\
& \left(4(a+b) \operatorname{AppellF1} \left[1, \frac{1}{2}, -p, 2, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] + \left(2bp \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-p, 3, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{b \tan[e + f x]^2}{a+b} \right] - (a+b) \operatorname{AppellF1} \left[2, \frac{3}{2}, -p, 3, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] \right) \tan[e + f x]^2 \right) + \\
& \left(b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2} - p, -\frac{1}{2}, -p, \frac{1}{2} - p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] (1 + \tan[e + f x]^2) \right) / \\
& \left((1+2p) \left(-2(a+b)p \operatorname{AppellF1} \left[\frac{1}{2} - p, -\frac{1}{2}, 1-p, \frac{3}{2} - p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] - \right. \right. \\
& \left. \left. b \operatorname{AppellF1} \left[\frac{1}{2} - p, \frac{1}{2}, -p, \frac{3}{2} - p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] + b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2} - p, -\frac{1}{2}, -p, \frac{1}{2} - \right. \right. \right. \\
& \left. \left. \left. p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] \tan[e + f x]^2 \right) \right) \Bigg) + 2^p \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-\frac{1}{2}+p} \\
& \left(\frac{a+b+b \tan[e + f x]^2}{1 + \tan[e + f x]^2} \right)^p \left(- \left(2(a+b) \left(\frac{bp \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-p, 3, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] \sec[e + f x]^2 \tan[e + f x]}{a+b} - \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} \operatorname{AppellF1} \left[2, \frac{3}{2}, -p, 3, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) / \right. \\
& \left(4(a+b) \operatorname{AppellF1} \left[1, \frac{1}{2}, -p, 2, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] + \left(2bp \operatorname{AppellF1} \left[2, \frac{1}{2}, 1-p, 3, -\tan[e + f x]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{b \tan[e + f x]^2}{a+b} \right] - (a+b) \operatorname{AppellF1} \left[2, \frac{3}{2}, -p, 3, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b} \right] \right) \tan[e + f x]^2 \right) + \\
& \left(2b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2} - p, -\frac{1}{2}, -p, \frac{1}{2} - p, -\cot[e + f x]^2, -\frac{(a+b) \cot[e + f x]^2}{b} \right] \sec[e + f x]^2 \tan[e + f x] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left((1+2p) \left(-2(a+b)p \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] - \right. \right. \\
& \quad b \operatorname{AppellF1} \left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] + \\
& \quad \left. \left. b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Tan}[e+fx]^2 \right) \right) + \\
& \left(b(-1+2p) \left(-1 / \left(b \left(\frac{1}{2}-p \right) \right) \right)^2 (a+b) \left(-\frac{1}{2}-p \right) p \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, \right. \right. \\
& \quad \left. \left. -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 - 1 / \left(\left(\frac{1}{2}-p \right) \left(-\frac{1}{2}-p \right) \right) \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 \right) (1+\operatorname{Tan}[e+fx]^2) \right) / \\
& \left((1+2p) \left(-2(a+b)p \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, 1-p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] - \right. \right. \\
& \quad b \operatorname{AppellF1} \left[\frac{1}{2}-p, \frac{1}{2}, -p, \frac{3}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] + \\
& \quad \left. \left. b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Tan}[e+fx]^2 \right) \right) - \\
& \left(b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] (1+\operatorname{Tan}[e+fx]^2) \right) \\
& \left(-2(a+b)p \left(\frac{1}{b \left(\frac{3}{2}-p \right)} 2(a+b) \left(\frac{1}{2}-p \right) (1-p) \operatorname{AppellF1} \left[\frac{3}{2}-p, -\frac{1}{2}, 2-p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 - \frac{1}{\frac{3}{2}-p} \left(\frac{1}{2}-p \right) \operatorname{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, 1-p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \operatorname{Cot}[e+fx] \right. \right. \\
& \quad \left. \left. \operatorname{Csc}[e+fx]^2 \right) - b \left(-\frac{1}{b \left(\frac{3}{2}-p \right)} 2(a+b) \left(\frac{1}{2}-p \right) p \operatorname{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, 1-p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 + \frac{1}{\frac{3}{2}-p} \left(\frac{1}{2}-p \right) \operatorname{AppellF1} \left[\frac{3}{2}-p, \frac{3}{2}, -p, \frac{5}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[e+fx] \operatorname{Csc}[e+fx]^2 \right) + 2b(-1+2p) \operatorname{AppellF1} \left[-\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{1}{2}-p, -\operatorname{Cot}[e+fx]^2, -\frac{(a+b)\operatorname{Cot}[e+fx]^2}{b} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Sec}[e + f x]^2 \text{Tan}[e + f x] + b (-1 + 2 p) \left(-\frac{1}{b \left(\frac{1}{2} - p\right)} {}_2F_1\left(a + b, \left(-\frac{1}{2} - p\right), p, \text{AppellF1}\left[\frac{1}{2} - p, -\frac{1}{2}, 1 - p, \frac{3}{2} - p, \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] \text{Cot}[e + f x] \text{Csc}[e + f x]^2 - \frac{1}{\frac{1}{2} - p} \left(-\frac{1}{2} - p\right) \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1}\left[\frac{1}{2} - p, \frac{1}{2}, -p, \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] \text{Cot}[e + f x] \text{Csc}[e + f x]^2 \right] \text{Tan}[e + f x]^2 \right) \right) \Bigg/ \\
& \left((1 + 2 p) \left(-2 (a + b) p \text{AppellF1}\left[\frac{1}{2} - p, -\frac{1}{2}, 1 - p, \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] - b \right. \right. \\
& \quad \left. \text{AppellF1}\left[\frac{1}{2} - p, \frac{1}{2}, -p, \frac{3}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] + b (-1 + 2 p) \right. \\
& \quad \left. \left. \text{AppellF1}\left[-\frac{1}{2} - p, -\frac{1}{2}, -p, \frac{1}{2} - p, -\text{Cot}[e + f x]^2, -\frac{(a + b) \text{Cot}[e + f x]^2}{b}\right] \text{Tan}[e + f x]^2 \right)^2 \right) + \\
& \left(2 (a + b) \text{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \left(2 \left(2 b p \text{AppellF1}\left[2, \frac{1}{2}, 1 - p, 3, -\text{Tan}[e + f x]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] - (a + b) \text{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \right) \text{Sec}[e + f x]^2 \text{Tan}[e + f x] + \right. \\
& \quad \left. 4 (a + b) \left(\frac{b p \text{AppellF1}\left[2, \frac{1}{2}, 1 - p, 3, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{a + b} - \right. \right. \\
& \quad \left. \left. \frac{1}{2} \text{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) + \\
& \text{Tan}[e + f x]^2 \left(2 b p \left(-\frac{1}{3 (a + b)} 4 b (1 - p) \text{AppellF1}\left[3, \frac{1}{2}, 2 - p, 4, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \\
& \quad \left. \frac{2}{3} \text{AppellF1}\left[3, \frac{3}{2}, 1 - p, 4, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) - \\
& \quad (a + b) \left(\frac{1}{3 (a + b)} 4 b p \text{AppellF1}\left[3, \frac{3}{2}, 1 - p, 4, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \\
& \quad \left. \left. 2 \text{AppellF1}\left[3, \frac{5}{2}, -p, 4, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b}\right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) \Bigg/
\end{aligned}$$

$$\left(4 (a+b) \operatorname{AppellF1}\left[1, \frac{1}{2}, -p, 2, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(2 b p \operatorname{AppellF1}\left[2, \frac{1}{2}, 1-p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - (a+b) \operatorname{AppellF1}\left[2, \frac{3}{2}, -p, 3, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right)^2 \Bigg) \Bigg)$$

■ **Problem 137: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$\frac{\operatorname{AppellF1}\left[\frac{3}{2}, 2, -p, \frac{5}{2}, \operatorname{Sec}[e+fx]^2, -\frac{b \operatorname{Sec}[e+fx]^2}{a}\right] \operatorname{Sec}[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^p \left(1 + \frac{b \operatorname{Sec}[e+fx]^2}{a}\right)^{-p}}{3 f}$$

Result (type 6, 2081 leaves):

$$\begin{aligned} & - \left(\left(b (-3+2p) \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \right. \right. \\ & \quad \left. \left. (a+2b+a \cos[2(e+fx)])^p \operatorname{Csc}[e+fx]^3 (\operatorname{Sec}[e+fx]^2)^{\frac{1}{2}+p} (a+b \operatorname{Sec}[e+fx]^2)^p \right) / \right. \\ & \quad \left(f (-1+2p) \left(2 (a+b)^p \operatorname{AppellF1}\left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \right. \right. \\ & \quad \left. \left. b \left(\operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \right. \right. \right. \\ & \quad \left. \left. \left. (3-2p) \operatorname{AppellF1}\left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \tan[e+fx]^2 \right) \right) \right) \\ & \quad \left(- \left(b (-3+2p) (a+2b+a \cos[2(e+fx)])^p \left(-1 / \left(b \left(\frac{3}{2}-p \right) \right) 2 (a+b) \left(\frac{1}{2}-p \right)^p \operatorname{AppellF1}\left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \cot[e+fx] \operatorname{Csc}[e+fx]^2 - 1 / \left(\frac{3}{2}-p \right) \left(\frac{1}{2}-p \right) \right. \right. \right. \\ & \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] \cot[e+fx] \operatorname{Csc}[e+fx]^2 \right) (\operatorname{Sec}[e+fx]^2)^{\frac{1}{2}+p} \right) / \right. \\ & \quad \left((-1+2p) \left(2 (a+b)^p \operatorname{AppellF1}\left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \right. \right. \\ & \quad \left. \left. b \left(\operatorname{AppellF1}\left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot[e+fx]^2, -\frac{(a+b) \cot[e+fx]^2}{b}\right] + \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. (3-2p) \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \right) \right) + \\
& \left(2 a b p (-3+2 p) \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] (a+2 b+a \cos [2(e+f x)])^{-1+p} \right. \\
& \quad \left. (\sec [e+f x]^2)^{\frac{1}{2}+p} \sin [2(e+f x)] \right) / \\
& \left((-1+2 p) \left(2(a+b) p \operatorname{AppellF1} \left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + \right. \right. \\
& \quad \left. b \left(\operatorname{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + (3-2 p) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \right) \right) - \\
& \left(2 b \left(\frac{1}{2}+p \right) (-3+2 p) \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] (a+2 b+a \cos [2(e+f x)])^p \right. \\
& \quad \left. (\sec [e+f x]^2)^{\frac{1}{2}+p} \tan [e+f x] \right) / \left((-1+2 p) \left(2(a+b) p \operatorname{AppellF1} \left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + \right. \right. \\
& \quad \left. b \left(\operatorname{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] + (3-2 p) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \tan [e+f x]^2 \right) \right) \right) + \\
& \left(b (-3+2 p) \operatorname{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] (a+2 b+a \cos [2(e+f x)])^p \right. \\
& \quad \left. (\sec [e+f x]^2)^{\frac{1}{2}+p} \left(2(a+b) p \left(\frac{1}{b \left(\frac{5}{2}-p \right)} 2(a+b)(1-p) \left(\frac{3}{2}-p \right) \operatorname{AppellF1} \left[\frac{5}{2}-p, -\frac{1}{2}, 2-p, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{7}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \cot [e+f x] \csc [e+f x]^2 - \frac{1}{\frac{5}{2}-p} \left(\frac{3}{2}-p \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{5}{2}-p, \frac{1}{2}, 1-p, \frac{7}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \cot [e+f x] \csc [e+f x]^2 \right) \right) \right) + \\
& \quad \left. b \left(-\frac{1}{b \left(\frac{5}{2}-p \right)} 2(a+b) \left(\frac{3}{2}-p \right) p \operatorname{AppellF1} \left[\frac{5}{2}-p, \frac{1}{2}, 1-p, \frac{7}{2}-p, -\cot [e+f x]^2, -\frac{(a+b) \cot [e+f x]^2}{b} \right] \cot [e+f x] \csc [e+f x]^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\frac{5}{2}-p} \left(\frac{3}{2}-p \right) \text{AppellF1} \left[\frac{5}{2}-p, \frac{3}{2}, -p, \frac{7}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] \text{Cot}[e+fx] \text{Csc}[e+fx]^2 + 2(3-2p) \\
& \text{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] + (3-2p) \left(-\frac{1}{b\left(\frac{3}{2}-p\right)} 2(a+ \right. \\
& \quad \left. b) \left(\frac{1}{2}-p \right) p \text{AppellF1} \left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] \text{Cot}[e+fx] \text{Csc}[e+fx]^2 - \frac{1}{\frac{3}{2}-p} \right. \\
& \quad \left. \left(\frac{1}{2}-p \right) \text{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] \text{Cot}[e+fx] \text{Csc}[e+fx]^2 \right) \text{Tan}[e+fx]^2 \left. \right) \left. \right) \left. \right) \left. \right) / \\
& \left((-1+2p) \left(2(a+b) p \text{AppellF1} \left[\frac{3}{2}-p, -\frac{1}{2}, 1-p, \frac{5}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] + \right. \right. \\
& \quad \left. \left. b \left(\text{AppellF1} \left[\frac{3}{2}-p, \frac{1}{2}, -p, \frac{5}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (3-2p) \text{AppellF1} \left[\frac{1}{2}-p, -\frac{1}{2}, -p, \frac{3}{2}-p, -\text{Cot}[e+fx]^2, -\frac{(a+b)\text{Cot}[e+fx]^2}{b} \right] \text{Tan}[e+fx]^2 \right) \right) \right) \right) \left. \right) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

■ **Problem 138: Result more than twice size of optimal antiderivative.**

$$\int (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx]^4 dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$\frac{1}{5f} \text{AppellF1} \left[\frac{5}{2}, 3, -p, \frac{7}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Tan}[e+fx]^5 (a+b+b \text{Tan}[e+fx]^2)^p \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 5878 leaves): Display of huge result suppressed!

■ **Problem 139: Result more than twice size of optimal antiderivative.**

$$\int (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx]^2 dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$\frac{1}{3f} \text{AppellF1} \left[\frac{3}{2}, 2, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Tan}[e+fx]^3 (a+b+b \text{Tan}[e+fx]^2)^p \left(1 + \frac{b \text{Tan}[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 3781 leaves):

$$\left(3(a+b)(a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-2+p} (a+b \text{Sec}[e+fx]^2)^p \text{Sin}[e+fx]^2 \text{Tan}[e+fx] \right)$$

$$\begin{aligned}
& \left(\text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] / \left(-3(a+b) \text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-b p \text{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + 2(a+b) \text{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \right) + \left(\text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \right) / \\
& \quad \left(3(a+b) \text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] + 2 \left(b p \text{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) / \\
& \left(f \left(3(a+b) (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{-1+p} \left(\text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] / \right. \right. \right. \\
& \quad \left(-3(a+b) \text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + 2 \left(-b p \text{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + 2(a+b) \text{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \right) + \\
& \quad \left(\text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \right) / \left(3(a+b) \text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] + 2 \left(b p \text{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) - \\
& 6 a (a+b) p (a+2b+a \text{Cos}[2(e+fx)])^{-1+p} (\text{Sec}[e+fx]^2)^{-2+p} \text{Sin}[2(e+fx)] \text{Tan}[e+fx] \\
& \left(\text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] / \left(-3(a+b) \text{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-b p \text{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + 2(a+b) \text{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \right) + \left(\text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \text{Sec}[e+fx]^2 \right) / \\
& \quad \left(3(a+b) \text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] + 2 \left(b p \text{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, \right. \right. \\
& \quad \left. \left. -\text{Tan}[e+fx]^2 \right] - (a+b) \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e+fx]^2}{a+b}, -\text{Tan}[e+fx]^2 \right] \right) \text{Tan}[e+fx]^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 6 (a+b) (-2+p) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-2+p} \tan[e+fx]^2 \left(\operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) / \\
& \left(-3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 \left(-b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \right) / \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) \right) + 3 (a+b) (a+2b+a \cos[2(e+fx)])^p \\
& (\sec[e+fx]^2)^{-2+p} \tan[e+fx] \left(\left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{4}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \\
& \left(-3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 \left(-b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
& \left(2 \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] \right) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. -\tan[e+fx]^2\right] - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) + \\
& \left(\sec[e+fx]^2 \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \left(4 \left(-b p \operatorname{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \quad \left. \left. 3 (a+b) \left(\frac{2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{4}{3} \operatorname{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) + \right. \\
& \quad \left. 2 \operatorname{Tan}[e+f x]^2 \left(-b p \left(-\frac{1}{5 (a+b)} 6 b (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 2, 2-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \right. \\
& \quad \left. \left. \frac{12}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 3, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \right. \\
& \quad \left. 2 (a+b) \left(\frac{1}{5 (a+b)} 6 b p \operatorname{AppellF1} \left[\frac{5}{2}, 3, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \quad \left. \left. \frac{18}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 4, -p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) \right) / \\
& \left(-3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + 2 \left(-b p \operatorname{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + 2 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 - \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \left(4 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (a+b) \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \\
& \left. \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \\
& 2 \operatorname{Tan}[e+f x]^2 \left(b p \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \left. \left. \frac{1}{5 (a+b)} 6 b (1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2-p, 1, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) - \right. \\
& (a+b) \left(\frac{1}{5 (a+b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \\
& \left. \left. \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \Bigg) \Bigg) \Bigg) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[e+f x]^2\right] - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 \right) \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 140: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Tan}[e+f x] (a+b+b \operatorname{Tan}[e+f x]^2)^p \left(1+\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 2137 leaves):

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right. \\
& \left. \operatorname{Cos}[e+f x] (a+2 b+a \operatorname{Cos}[2 (e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right) \Bigg) / \\
& \left(f \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \right. \\
& \left. \left. -\operatorname{Tan}[e+f x]^2\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 \\
& \left(\left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-1+p} \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x]^2 \right) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(6 (a+b)^p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x]^2 \right) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(6 a (a+b)^p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Cos}[e+f x] (a+2b+a \operatorname{Cos}[2(e+f x)])^{-1+p} \right. \\
& \left. (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \operatorname{Sin}[2(e+f x)] \right) / \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \right. \\
& \left. 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \right) \\
& \operatorname{Tan}[e+f x]^2 \left. \right) + \left(3 (a+b) \operatorname{Cos}[e+f x] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \right. \\
& \left. \operatorname{Sin}[e+f x] \right) \frac{2 b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} -
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right] \right) \right) / \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) - \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Cos}[e+fx] (a+2b+a \operatorname{Cos}[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p \operatorname{Sin}[e+fx] \right. \\
& \quad \left. \left(4 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] + 3(a+b) \left(\frac{2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{3(a+b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \right) + \\
& 2 \operatorname{Tan}[e+fx]^2 \left(b p \left(-\frac{6}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] - \right. \right. \\
& \quad \left. \left. \frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 2-p, 1, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) - \right. \\
& \quad \left. (a+b) \left(\frac{6 b p \operatorname{AppellF1} \left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{5(a+b)} - \right. \right. \\
& \quad \left. \left. \frac{12}{5} \operatorname{AppellF1} \left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \right) \right) \right) \right) / \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+fx]^2}{a+b}, -\operatorname{Tan}[e+fx]^2 \right] \right) \operatorname{Tan}[e+fx]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 152: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^5 (a + b \text{Sec}[e + f x]^2) dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{(6a + 5b) \text{ArcTanh}[\text{Sin}[e + f x]]}{16f} + \frac{(6a + 5b) \text{Sec}[e + f x] \text{Tan}[e + f x]}{16f} + \frac{(6a + 5b) \text{Sec}[e + f x]^3 \text{Tan}[e + f x]}{24f} + \frac{b \text{Sec}[e + f x]^5 \text{Tan}[e + f x]}{6f}$$

Result (type 3, 445 leaves):

$$\begin{aligned} & - \frac{3a \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8f} - \frac{5b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{16f} + \\ & \frac{3a \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8f} + \frac{5b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{16f} + \frac{b}{48f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6} + \\ & \frac{a}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} + \frac{b}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} + \frac{3a}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} + \\ & \frac{5b}{32f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] - \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{b}{48f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6} - \frac{a}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} - \\ & \frac{b}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4} - \frac{3a}{16f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} - \frac{5b}{32f \left(\text{Cos}\left[\frac{1}{2}(e + f x)\right] + \text{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2} \end{aligned}$$

■ **Problem 155: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[e + f x] (a + b \text{Sec}[e + f x]^2) dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{b \text{ArcTanh}[\text{Sin}[e + f x]]}{f} + \frac{a \text{Sin}[e + f x]}{f}$$

Result (type 3, 92 leaves):

$$- \frac{b \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] - \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b \text{Log}\left[\text{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] + \text{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{a \text{Cos}[f x] \text{Sin}[e]}{f} + \frac{a \text{Cos}[e] \text{Sin}[f x]}{f}$$

■ **Problem 165: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^5 (a + b \text{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{(48 a^2 + 80 a b + 35 b^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{128 f} + \frac{(48 a^2 + 80 a b + 35 b^2) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{128 f} +$$

$$\frac{(48 a^2 + 80 a b + 35 b^2) \operatorname{Sec}[e + f x]^3 \operatorname{Tan}[e + f x]}{192 f} + \frac{b (10 a + 7 b) \operatorname{Sec}[e + f x]^5 \operatorname{Tan}[e + f x]}{48 f} + \frac{b \operatorname{Sec}[e + f x]^7 (a + b - a \operatorname{Sin}[e + f x]^2) \operatorname{Tan}[e + f x]}{8 f}$$

Result (type 3, 803 leaves):

$$-\frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} - \frac{5 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} - \frac{35 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{128 f} +$$

$$\frac{3 a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} + \frac{5 a b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{8 f} + \frac{35 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right]}{128 f} +$$

$$\frac{128 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^8}{a^2} + \frac{24 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6}{a b} + \frac{192 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6}{15 b^2} +$$

$$\frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{3 a^2} + \frac{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{5 a b} + \frac{256 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{35 b^2} +$$

$$\frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{b^2} + \frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{a b} + \frac{256 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{5 b^2} -$$

$$\frac{128 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^8}{a^2} - \frac{24 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6}{a b} - \frac{192 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^6}{15 b^2} -$$

$$\frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{3 a^2} - \frac{8 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{5 a b} - \frac{256 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^4}{35 b^2} -$$

$$\frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{b^2} - \frac{16 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{a b} - \frac{256 f \left(\operatorname{Cos}\left[\frac{1}{2}(e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2}(e + f x)\right]\right)^2}{5 b^2}$$

■ **Problem 166: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 129 leaves, 5 steps):

$$\frac{(8 a^2 + 12 a b + 5 b^2) \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{16 f} + \frac{(8 a^2 + 12 a b + 5 b^2) \operatorname{Sec}[e + f x] \operatorname{Tan}[e + f x]}{16 f} +$$

$$\frac{b (8 a + 5 b) \operatorname{Sec}[e + f x]^3 \operatorname{Tan}[e + f x]}{24 f} + \frac{b \operatorname{Sec}[e + f x]^5 (a + b - a \operatorname{Sin}[e + f x]^2) \operatorname{Tan}[e + f x]}{6 f}$$

Result (type 3, 601 leaves):

$$\begin{aligned}
& - \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{2f} - \frac{3ab \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{4f} - \frac{5b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right]}{16f} + \\
& \frac{a^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{2f} + \frac{3ab \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{4f} + \frac{5b^2 \operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right]}{16f} + \\
& \frac{b^2}{b^2} + \frac{ab}{ab} + \frac{b^2}{b^2} + \\
& \frac{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^6}{a^2} + \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{3ab} + \frac{16f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{5b^2} - \\
& \frac{4f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{b^2} + \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{ab} + \frac{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] - \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{b^2} - \\
& \frac{48f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^6}{a^2} - \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{3ab} - \frac{16f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^4}{5b^2} - \\
& \frac{4f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{a^2} - \frac{8f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{3ab} - \frac{32f \left(\cos\left[\frac{1}{2}(e+fx)\right] + \sin\left[\frac{1}{2}(e+fx)\right]\right)^2}{5b^2}
\end{aligned}$$

■ **Problem 169: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^3 (a+b \operatorname{Sec}[e+fx]^2)^2 dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{f} + \frac{a(a+2b) \operatorname{Sin}[e+fx]}{f} - \frac{a^2 \operatorname{Sin}[e+fx]^3}{3f}$$

Result (type 3, 134 leaves):

$$\begin{aligned}
& - \frac{b^2 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] - \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \frac{b^2 \operatorname{Log}\left[\cos\left[\frac{e}{2} + \frac{fx}{2}\right] + \sin\left[\frac{e}{2} + \frac{fx}{2}\right]\right]}{f} + \\
& \frac{2ab \cos[fx] \operatorname{Sin}[e]}{f} + \frac{2ab \cos[e] \operatorname{Sin}[fx]}{f} + \frac{3a^2 \operatorname{Sin}[e+fx]}{4f} + \frac{a^2 \operatorname{Sin}[3(e+fx)]}{12f}
\end{aligned}$$

■ **Problem 170: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^5 (a+b \operatorname{Sec}[e+fx]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a+b)^2 \operatorname{Sin}[e+fx]}{f} - \frac{2a(a+b) \operatorname{Sin}[e+fx]^3}{3f} + \frac{a^2 \operatorname{Sin}[e+fx]^5}{5f}$$

Result (type 3, 111 leaves):

$$\frac{b^2 \cos[fx] \operatorname{Sin}[e]}{f} + \frac{b^2 \cos[e] \operatorname{Sin}[fx]}{f} + \frac{5a^2 \operatorname{Sin}[e+fx]}{8f} + \frac{3ab \operatorname{Sin}[e+fx]}{2f} + \frac{5a^2 \operatorname{Sin}[3(e+fx)]}{48f} + \frac{ab \operatorname{Sin}[3(e+fx)]}{6f} + \frac{a^2 \operatorname{Sin}[5(e+fx)]}{80f}$$

■ **Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \sec[e + f x]^6 (a + b \sec[e + f x]^2)^2 dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$\frac{(a+b)^2 \tan[e+fx]}{f} + \frac{2(a+b)(a+2b)\tan[e+fx]^3}{3f} + \frac{(a^2+6ab+6b^2)\tan[e+fx]^5}{5f} + \frac{2b(a+2b)\tan[e+fx]^7}{7f} + \frac{b^2 \tan[e+fx]^9}{9f}$$

Result (type 3, 261 leaves):

$$\begin{aligned} & \frac{8a^2 \tan[e+fx]}{15f} + \frac{32ab \tan[e+fx]}{35f} + \frac{128b^2 \tan[e+fx]}{315f} + \frac{4a^2 \sec[e+fx]^2 \tan[e+fx]}{15f} + \\ & \frac{16ab \sec[e+fx]^2 \tan[e+fx]}{35f} + \frac{64b^2 \sec[e+fx]^2 \tan[e+fx]}{315f} + \frac{a^2 \sec[e+fx]^4 \tan[e+fx]}{5f} + \frac{12ab \sec[e+fx]^4 \tan[e+fx]}{35f} + \\ & \frac{16b^2 \sec[e+fx]^4 \tan[e+fx]}{105f} + \frac{2ab \sec[e+fx]^6 \tan[e+fx]}{7f} + \frac{8b^2 \sec[e+fx]^6 \tan[e+fx]}{63f} + \frac{b^2 \sec[e+fx]^8 \tan[e+fx]}{9f} \end{aligned}$$

■ **Problem 172: Result more than twice size of optimal antiderivative.**

$$\int \sec[e + f x]^4 (a + b \sec[e + f x]^2)^2 dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$\frac{(a+b)^2 \tan[e+fx]}{f} + \frac{(a+b)(a+3b)\tan[e+fx]^3}{3f} + \frac{b(2a+3b)\tan[e+fx]^5}{5f} + \frac{b^2 \tan[e+fx]^7}{7f}$$

Result (type 3, 190 leaves):

$$\begin{aligned} & \frac{2a^2 \tan[e+fx]}{3f} + \frac{16ab \tan[e+fx]}{15f} + \frac{16b^2 \tan[e+fx]}{35f} + \frac{a^2 \sec[e+fx]^2 \tan[e+fx]}{3f} + \frac{8ab \sec[e+fx]^2 \tan[e+fx]}{15f} + \\ & \frac{8b^2 \sec[e+fx]^2 \tan[e+fx]}{35f} + \frac{2ab \sec[e+fx]^4 \tan[e+fx]}{5f} + \frac{6b^2 \sec[e+fx]^4 \tan[e+fx]}{35f} + \frac{b^2 \sec[e+fx]^6 \tan[e+fx]}{7f} \end{aligned}$$

■ **Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \sec[e + f x]^2 (a + b \sec[e + f x]^2)^2 dx$$

Optimal (type 3, 53 leaves, 3 steps):

$$\frac{(a+b)^2 \tan[e+fx]}{f} + \frac{2b(a+b)\tan[e+fx]^3}{3f} + \frac{b^2 \tan[e+fx]^5}{5f}$$

Result (type 3, 116 leaves):

$$\frac{a^2 \tan[e + f x]}{f} + \frac{4 a b \tan[e + f x]}{3 f} + \frac{8 b^2 \tan[e + f x]}{15 f} +$$

$$\frac{2 a b \sec[e + f x]^2 \tan[e + f x]}{3 f} + \frac{4 b^2 \sec[e + f x]^2 \tan[e + f x]}{15 f} + \frac{b^2 \sec[e + f x]^4 \tan[e + f x]}{5 f}$$

■ **Problem 174: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + f x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b (2 a + b) \tan[e + f x]}{f} + \frac{b^2 \tan[e + f x]^3}{3 f}$$

Result (type 3, 106 leaves):

$$\left(4 (b + a \cos[e + f x]^2)^2 \sec[e + f x]^3 \right. \\ \left. (3 a^2 f x \cos[e + f x]^3 + b^2 \sec[e] \sin[f x] + 2 b (3 a + b) \cos[e + f x]^2 \sec[e] \sin[f x] + b^2 \cos[e + f x] \tan[e]) \right) / (3 f (a + 2 b + a \cos[2 (e + f x)]^2))$$

■ **Problem 178: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[c + d x]^2)^3 dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$a^3 x + \frac{b (3 a^2 + 3 a b + b^2) \tan[c + d x]}{d} + \frac{b^2 (3 a + 2 b) \tan[c + d x]^3}{3 d} + \frac{b^3 \tan[c + d x]^5}{5 d}$$

Result (type 3, 268 leaves):

$$\frac{1}{480 d} \sec[c] \sec[c + d x]^5 (150 a^3 d x \cos[d x] + 150 a^3 d x \cos[2 c + d x] + 75 a^3 d x \cos[2 c + 3 d x] + 75 a^3 d x \cos[4 c + 3 d x] + 15 a^3 d x \cos[4 c + 5 d x] + 15 a^3 d x \cos[6 c + 5 d x] + 540 a^2 b \sin[d x] + 420 a b^2 \sin[d x] + 160 b^3 \sin[d x] - 360 a^2 b \sin[2 c + d x] - 180 a b^2 \sin[2 c + d x] + 360 a^2 b \sin[2 c + 3 d x] + 300 a b^2 \sin[2 c + 3 d x] + 80 b^3 \sin[2 c + 3 d x] - 90 a^2 b \sin[4 c + 3 d x] + 90 a^2 b \sin[4 c + 5 d x] + 60 a b^2 \sin[4 c + 5 d x] + 16 b^3 \sin[4 c + 5 d x])$$

■ **Problem 179: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[c + d x]^2)^4 dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$a^4 x + \frac{b (2 a + b) (2 a^2 + 2 a b + b^2) \tan[c + d x]}{d} + \frac{b^2 (6 a^2 + 8 a b + 3 b^2) \tan[c + d x]^3}{3 d} + \frac{b^3 (4 a + 3 b) \tan[c + d x]^5}{5 d} + \frac{b^4 \tan[c + d x]^7}{7 d}$$

Result (type 3, 455 leaves):

$$\frac{1}{13440d} \operatorname{Sec}[c] \operatorname{Sec}[c+dx]^7$$

$$(3675 a^4 dx \operatorname{Cos}[dx] + 3675 a^4 dx \operatorname{Cos}[2c+dx] + 2205 a^4 dx \operatorname{Cos}[2c+3dx] + 2205 a^4 dx \operatorname{Cos}[4c+3dx] + 735 a^4 dx \operatorname{Cos}[4c+5dx] + 735 a^4 dx \operatorname{Cos}[6c+5dx] + 105 a^4 dx \operatorname{Cos}[6c+7dx] + 105 a^4 dx \operatorname{Cos}[8c+7dx] + 16800 a^3 b \operatorname{Sin}[dx] + 18480 a^2 b^2 \operatorname{Sin}[dx] + 11200 a b^3 \operatorname{Sin}[dx] + 3360 b^4 \operatorname{Sin}[dx] - 12600 a^3 b \operatorname{Sin}[2c+dx] - 10920 a^2 b^2 \operatorname{Sin}[2c+dx] - 4480 a b^3 \operatorname{Sin}[2c+dx] + 12600 a^3 b \operatorname{Sin}[2c+3dx] + 15120 a^2 b^2 \operatorname{Sin}[2c+3dx] + 9408 a b^3 \operatorname{Sin}[2c+3dx] + 2016 b^4 \operatorname{Sin}[2c+3dx] - 5040 a^3 b \operatorname{Sin}[4c+3dx] - 2520 a^2 b^2 \operatorname{Sin}[4c+3dx] + 5040 a^3 b \operatorname{Sin}[4c+5dx] + 5880 a^2 b^2 \operatorname{Sin}[4c+5dx] + 3136 a b^3 \operatorname{Sin}[4c+5dx] + 672 b^4 \operatorname{Sin}[4c+5dx] - 840 a^3 b \operatorname{Sin}[6c+5dx] + 840 a^3 b \operatorname{Sin}[6c+7dx] + 840 a^2 b^2 \operatorname{Sin}[6c+7dx] + 448 a b^3 \operatorname{Sin}[6c+7dx] + 96 b^4 \operatorname{Sin}[6c+7dx])$$

■ **Problem 180: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^5}{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$-\frac{(2a-b) \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{2b^2 f} + \frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}}\right]}{b^2 \sqrt{a+b} f} + \frac{\operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx]}{2bf}$$

Result (type 3, 2519 leaves):

$$\frac{(2a-b)(a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] - \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right] \operatorname{Sec}[e+fx]^2}{4b^2 f (a+b \operatorname{Sec}[e+fx]^2)} +$$

$$\frac{(-2a+b)(a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e}{2} + \frac{fx}{2}\right] + \operatorname{Sin}\left[\frac{e}{2} + \frac{fx}{2}\right]\right] \operatorname{Sec}[e+fx]^2}{4b^2 f (a+b \operatorname{Sec}[e+fx]^2)} + \frac{1}{4b^2 \sqrt{a+b} f (a+b \operatorname{Sec}[e+fx]^2) \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]}}$$

$$+ \frac{i a^{3/2} \operatorname{ArcTan}\left[\left(-i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3e] + i b \operatorname{Cos}[3e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + a \operatorname{Sin}[3e] + b \operatorname{Sin}[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] - 2i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]\right)}{(a \operatorname{Cos}[e] + 3b \operatorname{Cos}[e] + a \operatorname{Cos}[3e] + b \operatorname{Cos}[3e] + a \operatorname{Cos}[e+2fx] + a \operatorname{Cos}[3e+2fx] - 3i a \operatorname{Sin}[e] - i b \operatorname{Sin}[e] - i a \operatorname{Sin}[3e] - i b \operatorname{Sin}[3e] - i a \operatorname{Sin}[e+2fx] + i a \operatorname{Sin}[3e+2fx]) \operatorname{Cos}[e] (a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[e+fx]^2 - (a^{3/2} \operatorname{ArcTanh}\left[\left(2(a+b) \operatorname{Sin}[e]\right) / \left(-2i a \operatorname{Cos}[e] - 2i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} + \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx]\right)}{\operatorname{Cos}[e] (a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[e+fx]^2} \right]}{\left(4b^2 \sqrt{a+b} f (a+b \operatorname{Sec}[e+fx]^2) \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]}\right)} +$$

$$\frac{\left(a^{3/2} \operatorname{Cos}[e] (a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Log}\left[a+2a \operatorname{Cos}[2e] + 2b \operatorname{Cos}[2e] - a \operatorname{Cos}[2e+2fx] - 2i a \operatorname{Sin}[2e] - 2i b \operatorname{Sin}[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]} \operatorname{Sin}[2e+fx]\right] \operatorname{Sec}[e+fx]^2\right)}{\left(8b^2 \sqrt{a+b} f (a+b \operatorname{Sec}[e+fx]^2) \sqrt{\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]}\right)} -$$

$$\frac{\left(a^{3/2} \operatorname{Cos}[e] (a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Log}\left[-a-2a \operatorname{Cos}[2e] - 2b \operatorname{Cos}[2e] + a \operatorname{Cos}[2e+2fx] + 2i a \operatorname{Sin}[2e] + 2i b \operatorname{Sin}[2e] +$$

$$\begin{aligned}
& \left. \left(2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[fx] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[2e+fx] \right) \sec[e+fx]^2 \right) / \\
& \left(8b^2\sqrt{a+b}f(a+b\sec[e+fx]^2)\sqrt{\cos[2e]-i\sin[2e]} \right) + \frac{1}{4b^2\sqrt{a+b}f(a+b\sec[e+fx]^2)\sqrt{\cos[2e]-i\sin[2e]}} \\
& a^{3/2}\text{ArcTan}\left[\left(-ia\cos[e] - ib\cos[e] + ia\cos[3e] + ib\cos[3e] + a\sin[e] + b\sin[e] - \sqrt{a}\sqrt{a+b}\cos[e-fx]\sqrt{\cos[2e]-i\sin[2e]} + \right. \right. \\
& \quad \left. \left. \sqrt{a}\sqrt{a+b}\cos[3e+fx]\sqrt{\cos[2e]-i\sin[2e]} + a\sin[3e] + b\sin[3e] - i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[e-fx] - \right. \right. \\
& \quad \left. \left. 2i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[e+fx] + i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[3e+fx] \right) \right] / \\
& \left(a\cos[e] + 3b\cos[e] + a\cos[3e] + b\cos[3e] + a\cos[e+2fx] + a\cos[3e+2fx] - 3ia\sin[e] - ib\sin[e] - \right. \\
& \quad \left. ia\sin[3e] - ib\sin[3e] - ia\sin[e+2fx] + ia\sin[3e+2fx] \right) (a+2b+a\cos[2e+2fx])\sec[e+fx]^2\sin[e] + \\
& \left(ia^{3/2}\text{ArcTan}\left[\left(2(a+b)\sin[e] \right) / \left(-2ia\cos[e] - 2ib\cos[e] - \sqrt{a}\sqrt{a+b}\cos[e-fx]\sqrt{\cos[2e]-i\sin[2e]} + \sqrt{a}\sqrt{a+b}\cos[3e+fx] \right. \right. \right. \\
& \quad \left. \left. \sqrt{\cos[2e]-i\sin[2e]} - i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[e-fx] + i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[3e+fx] \right) \right] \right) \\
& \left(a+2b+a\cos[2e+2fx] \right) \sec[e+fx]^2\sin[e] \Big/ \left(4b^2\sqrt{a+b}f(a+b\sec[e+fx]^2)\sqrt{\cos[2e]-i\sin[2e]} \right) - \\
& \left(ia^{3/2}(a+2b+a\cos[2e+2fx])\text{Log}\left[a+2a\cos[2e]+2b\cos[2e]-a\cos[2e+2fx]-2ia\sin[2e]-2ib\sin[2e]+ \right. \right. \\
& \quad \left. \left. 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[fx] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[2e+fx] \right] \right) \\
& \sec[e+fx]^2\sin[e] \Big/ \left(8b^2\sqrt{a+b}f(a+b\sec[e+fx]^2)\sqrt{\cos[2e]-i\sin[2e]} \right) + \\
& \left(ia^{3/2}(a+2b+a\cos[2e+2fx])\text{Log}\left[-a-2a\cos[2e]-2b\cos[2e]+a\cos[2e+2fx]+2ia\sin[2e]+2ib\sin[2e]+ \right. \right. \\
& \quad \left. \left. 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[fx] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e]-i\sin[2e]}\sin[2e+fx] \right] \sec[e+fx]^2\sin[e] \right) \Big/ \\
& \left(8b^2\sqrt{a+b}f(a+b\sec[e+fx]^2)\sqrt{\cos[2e]-i\sin[2e]} \right) + \frac{(a+2b+a\cos[2e+2fx])\sec[e+fx]^2}{8bf(a+b\sec[e+fx]^2)\left(\cos\left[\frac{e}{2}+\frac{fx}{2}\right]-\sin\left[\frac{e}{2}+\frac{fx}{2}\right]\right)^2} - \\
& \frac{(a+2b+a\cos[2e+2fx])\sec[e+fx]^2}{8bf(a+b\sec[e+fx]^2)\left(\cos\left[\frac{e}{2}+\frac{fx}{2}\right]+\sin\left[\frac{e}{2}+\frac{fx}{2}\right]\right)^2}
\end{aligned}$$

■ **Problem 181: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^3}{a+b\sec[e+fx]^2} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sin[e+fx]]}{bf} - \frac{\sqrt{a}\text{ArcTanh}\left[\frac{\sqrt{a}\sin[e+fx]}{\sqrt{a+b}}\right]}{b\sqrt{a+b}f}$$

Result (type 3, 1022 leaves):

$$\begin{aligned}
& \frac{1}{8 b \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x])^2 \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \\
& (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \left(-\sqrt{a} \operatorname{Cos}[e] \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]- \right. \right. \\
& \quad \left. \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] + \right. \\
& \quad \left. \sqrt{a} \operatorname{Cos}[e] \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+ \right. \right. \\
& \quad \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] -2 i \sqrt{a} \operatorname{ArcTan}\left[\right. \right. \\
& \quad \left. \left. \left(2 \operatorname{Sin}[e] \left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} + \right. \right. \right. \\
& \quad \left. \left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right) \right] \right) / \\
& \quad \left. \left. \left(i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+ \right. \right. \\
& \quad \left. \left. b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x] \right) \right] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) - \\
& 4 \sqrt{a+b} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x) \right]-\operatorname{Sin}\left[\frac{1}{2}(e+f x) \right] \right] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} +4 \sqrt{a+b} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x) \right]+\operatorname{Sin}\left[\frac{1}{2}(e+f x) \right] \right] \\
& \quad \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} +i \sqrt{a} \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+ \right. \\
& \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sin}[e]- \\
& i \sqrt{a} \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
& \quad \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sin}[e]+ \\
& \left. 2 \sqrt{a} \operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]} \right] (i \operatorname{Cos}[e]+\operatorname{Sin}[e]) \right)
\end{aligned}$$

■ **Problem 182: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{\sqrt{a} \sqrt{a+b} f}$$

Result (type 3, 653 leaves):

$$\begin{aligned}
& \frac{1}{8 \sqrt{a} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \\
& (a+2 b+a \operatorname{Cos}[2(e+f x)]) \left(-2 i \operatorname{ArcTan} \left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]}} \right] + 2 i \operatorname{ArcTan} \left[\right. \right. \\
& \left. \left. \left(2 \operatorname{Sin}[e] \left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \right. \right. \right. \right. \\
& \left. \left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right) \right) \right] \right) / \\
& \left(i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+ \right. \\
& \left. a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x] \right) + \\
& \operatorname{Log} \left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
& \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] - \\
& \operatorname{Log} \left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \\
& \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \left. \right) \operatorname{Sec}[e+f x]^2 (\operatorname{Cos}[e]-i \operatorname{Sin}[e])
\end{aligned}$$

- **Problem 183: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+f x]}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b \operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}} \right]}{a^{3/2} \sqrt{a+b} f} + \frac{\operatorname{Sin}[e+f x]}{a f}$$

Result (type 3, 941 leaves):

1

$$\begin{aligned}
& 8 a^{3/2} \sqrt{a+b} f (a+b \operatorname{Sec}[e+f x]^2) \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \\
& (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^2 \left(-b \operatorname{Cos}[e] \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]- \right. \right. \\
& \quad \left. \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] + \right. \\
& \quad \left. b \operatorname{Cos}[e] \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+ \right. \right. \\
& \quad \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] + \right. \\
& \quad \left. i b \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \right. \right. \\
& \quad \left. \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sin}[e]-i b \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+ \right. \right. \\
& \quad \left. \left. 2 i b \operatorname{Sin}[2 e]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sin}[e]+ \right. \\
& \quad \left. 2 b \operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} \right] (i \operatorname{Cos}[e]+\operatorname{Sin}[e])+\operatorname{ArcTan}\left[\right. \right. \\
& \quad \left. \left. \left(2 \operatorname{Sin}[e] \left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \right. \right. \right. \\
& \quad \left. \left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right) \right) \right] / \\
& \quad \left. \left. (i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+ \right. \right. \\
& \quad \left. \left. b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x]) \right] (-2 i b \operatorname{Cos}[e]-2 b \operatorname{Sin}[e])+4 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[e+f x] \right)
\end{aligned}$$

- **Problem 186: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]^6}{a+b \operatorname{Sec}[e+f x]^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{a^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{b^{5/2} \sqrt{a+b} f} - \frac{(a-b) \operatorname{Tan}[e+f x]}{b^2 f} + \frac{\operatorname{Tan}[e+f x]^3}{3 b f}$$

Result (type 3, 224 leaves):

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \right. \\ \left. - 3a^2 \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] (\cos[2e] - i \sin[2e]) + \right. \\ \left. \sqrt{a+b} \sec[e + fx] \sqrt{b} (i \cos[e] + \sin[e])^4 (\sec[e] (-3a + 2b + b \sec[e + fx]^2) \sin[fx] + b \sec[e + fx] \tan[e]) \right) \Bigg) / \\ \left(6b^2 \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b} (\cos[e] - i \sin[e])^4 \right)$$

■ **Problem 187: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^4}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{a \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{b^{3/2} \sqrt{a+b} f} + \frac{\tan[e + fx]}{bf}$$

Result (type 3, 192 leaves):

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \right. \\ \left. + a \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] (\cos[2e] - i \sin[2e]) + \right. \\ \left. \sqrt{a+b} \sec[e] \sec[e + fx] \sqrt{b} (i \cos[e] + \sin[e])^4 \sin[fx] \right) \Bigg) / \left(2b \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b} (\cos[e] - i \sin[e])^4 \right)$$

■ **Problem 189: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \cot[e + fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves) :

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^2 \left(\sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + \right. \right. \\ \left. \left. b \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\ \left(2a \sqrt{a+b} f (a + b \sec[e + fx]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

■ **Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^5}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 102 leaves, 5 steps) :

$$\frac{\operatorname{ArcTanh}[\sin[e + fx]]}{b^2 f} - \frac{\sqrt{a} (2a + 3b) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \sin[e + fx]}{\sqrt{a+b}} \right]}{2b^2 (a+b)^{3/2} f} - \frac{a \sin[e + fx]}{2b (a+b) f (a+b - a \sin[e + fx]^2)}$$

Result (type 3, 2333 leaves) :

$$- \frac{(a + 2b + a \cos[2e + 2fx])^2 \operatorname{Log} \left[\cos \left[\frac{e}{2} + \frac{fx}{2} \right] - \sin \left[\frac{e}{2} + \frac{fx}{2} \right] \right] \sec[e + fx]^4}{4b^2 f (a + b \sec[e + fx]^2)^2} + \\ \frac{(a + 2b + a \cos[2e + 2fx])^2 \operatorname{Log} \left[\cos \left[\frac{e}{2} + \frac{fx}{2} \right] + \sin \left[\frac{e}{2} + \frac{fx}{2} \right] \right] \sec[e + fx]^4}{4b^2 f (a + b \sec[e + fx]^2)^2} + \\ \frac{1}{(a+b) (a + b \sec[e + fx]^2)^2} (-2a^2 - 3ab) (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \\ \left(\left(i \operatorname{ArcTan} \left[\left(-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e - fx] \sqrt{\cos[2e] - i \sin[2e]} + \right. \right. \right. \right. \\ \left. \left. \left. \sqrt{a} \sqrt{a+b} \cos[3e + fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - fx] - \right. \right. \right. \\ \left. \left. \left. 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e + fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + fx] \right) \right] / \right. \\ \left. (a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e + 2fx] + a \cos[3e + 2fx] - 3i a \sin[e] - i b \sin[e] - \right. \\ \left. i a \sin[3e] - i b \sin[3e] - i a \sin[e + 2fx] + i a \sin[3e + 2fx]) \right] \cos[e] \Big/ \left(16 \sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \\ \left(\operatorname{ArcTan} \left[\left(-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e - fx] \sqrt{\cos[2e] - i \sin[2e]} + \right. \right. \right. \\ \left. \left. \left. \sqrt{a} \sqrt{a+b} \cos[3e + fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - fx] - \right. \right. \right. \\ \left. \left. \left. 2i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e + fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + fx] \right) \right] / \right. \\ \left. (a \cos[e] + 3b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e + 2fx] + a \cos[3e + 2fx] - 3i a \sin[e] - i b \sin[e] - \right.$$

$$\begin{aligned}
& \left. \frac{1}{(a+b) (a+b \operatorname{Sec}[e+fx])^2} \left((2a+3b) (a+2b+a \cos[2e+2fx])^2 \operatorname{Sec}[e+fx]^4 \right. \right. \\
& \left. \left(\left(\sqrt{a} \operatorname{ArcTanh} \left[\frac{(2(a+b) \sin[e])}{(-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + \right. \right. \right. \\
& \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \cos[e] \right) / \left(16b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
& \left. \left(i \sqrt{a} \operatorname{ArcTanh} \left[\frac{(2(a+b) \sin[e])}{(-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + \right. \right. \right. \\
& \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \sin[e] \right) / \left(16b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
& \left. \frac{1}{(a+b) (a+b \operatorname{Sec}[e+fx])^2} (-2a^2 - 3ab) (a+2b+a \cos[2e+2fx])^2 \operatorname{Sec}[e+fx]^4 \right. \\
& \left. \left(\left(\cos[e] \log \left[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right] \right) / \left(32\sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \\
& \left. \left(i \log \left[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right] \sin[e] \right) / \left(32\sqrt{a} b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
& \left. \frac{1}{(a+b) (a+b \operatorname{Sec}[e+fx])^2} (2a+3b) (a+2b+a \cos[2e+2fx])^2 \right. \\
& \left. \operatorname{Sec}[e+fx]^4 \right. \\
& \left. \left(\left(\sqrt{a} \cos[e] \log \left[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e+2fx] + 2ia \sin[2e] + 2ib \sin[2e] + \right. \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right] \right) / \\
& \left(32b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left(i \sqrt{a} \log \left[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e+2fx] + 2ia \sin[2e] + \right. \right. \\
& \left. \left. \left. 2ib \sin[2e] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right) \right] \sin[e] \right) / \\
& \left. \left(32b^2 \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) - \frac{a(a+2b+a \cos[2e+2fx]) \operatorname{Sec}[e+fx]^3 \tan[e+fx]}{4b(a+b) f (a+b \operatorname{Sec}[e+fx])^2}
\end{aligned}$$

- **Problem 194: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^3}{(a+b \operatorname{Sec}[e+fx])^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right]}{2\sqrt{a}(a+b)^{3/2}f} + \frac{\sin[e+fx]}{2(a+b)f(a+b-a\sin[e+fx]^2)}$$

Result (type 3, 798 leaves):

$$\frac{1}{32\sqrt{a}(a+b)^{3/2}f(a+b\sec[e+fx]^2)^2\sqrt{(\cos[e]-i\sin[e])^2}}(a+2b+a\cos[2(e+fx)])\sec[e+fx]^3$$

$$\left(-2i\text{ArcTan}\left[\frac{(a+b)\sin[e]}{(a+b)\cos[e]-\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}(\cos[2e]+i\sin[2e])\sin[e+fx]}}\right](a+2b+a\cos[2(e+fx)])\sec[e+fx]\right.$$

$$(\cos[e]-i\sin[e])+(a+2b+a\cos[2(e+fx)])\log[a+2(a+b)\cos[2e]-a\cos[2(e+fx)]-2ia\sin[2e]-2ib\sin[2e]+$$

$$2\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\sin[fx]+2\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\sin[2e+fx]]\sec[e+fx](\cos[e]-i\sin[e])-$$

$$(a+2b+a\cos[2(e+fx)])\log[-a-2(a+b)\cos[2e]+a\cos[2(e+fx)]+2ia\sin[2e]+2ib\sin[2e]+2\sqrt{a}\sqrt{a+b}$$

$$\sqrt{(\cos[e]-i\sin[e])^2}\sin[fx]+2\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\sin[2e+fx]]\sec[e+fx](\cos[e]-i\sin[e])+2\text{ArcTan}\left[$$

$$\left(2\sin[e]\left(ia+ib+i(a+b)\cos[2e]+\sqrt{a}\sqrt{a+b}\cos[fx]\sqrt{(\cos[e]-i\sin[e])^2}-\sqrt{a}\sqrt{a+b}\cos[2e+fx]\sqrt{(\cos[e]-i\sin[e])^2}+$$

$$a\sin[2e]+b\sin[2e]-i\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\sin[fx]-i\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\sin[2e+fx]\right)\right]/$$

$$(i(a+3b)\cos[e]+i(a+b)\cos[3e]+ia\cos[e+2fx]+ia\cos[3e+2fx]+3a\sin[e]+b\sin[e]+$$

$$a\sin[3e]+b\sin[3e]+a\sin[e+2fx]-a\sin[3e+2fx])\left.] \right.$$

$$(a+2b+a\cos[2(e+fx)])\sec[e+fx](i\cos[e]+\sin[e])+8\sqrt{a}\sqrt{a+b}\sqrt{(\cos[e]-i\sin[e])^2}\tan[e+fx]$$

■ **Problem 195: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]}{(a+b\sec[e+fx]^2)^2} dx$$

Optimal (type 3, 83 leaves, 3 steps):

$$\frac{(2a+b)\text{ArcTanh}\left[\frac{\sqrt{a}\sin[e+fx]}{\sqrt{a+b}}\right]}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin[e+fx]}{2a(a+b)f(a+b-a\sin[e+fx]^2)}$$

Result (type 3, 819 leaves):

$$\frac{1}{32 a^{3/2} (a+b)^{3/2} f (a+b \operatorname{Sec}[e+f x])^2 \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^3$$

$$\left(-2 i (2 a+b) \operatorname{ArcTan}\left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]} \right] \right.$$

$$(a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) +$$

$$(2 a+b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log}\left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+ \right.$$

$$2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \left. \right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) -$$

$$(2 a+b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log}\left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+ \right.$$

$$2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \left. \right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) +$$

$$2(2 a+b) \operatorname{ArcTan}\left[\left(2 \operatorname{Sin}[e] \left(i a+i b+i(a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} - \right. \right. \right.$$

$$\left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} +a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]- \right. \right.$$

$$\left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right) \right] / (i(a+3 b) \operatorname{Cos}[e]+i(a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+$$

$$i a \operatorname{Cos}[3 e+2 f x]+3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x]) \left. \right]$$

$$(a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (i \operatorname{Cos}[e]+\operatorname{Sin}[e]) -8 \sqrt{a} b \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Tan}[e+f x] \left. \right)$$

- **Problem 196: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+f x]}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 101 leaves, 5 steps):

$$-\frac{b(4 a+3 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{2 a^{5/2} (a+b)^{3/2} f} + \frac{\operatorname{Sin}[e+f x]}{a^2 f} + \frac{b^2 \operatorname{Sin}[e+f x]}{2 a^2 (a+b) f (a+b-a \operatorname{Sin}[e+f x]^2)}$$

Result (type 3, 945 leaves):

$$\frac{1}{32 a^{5/2} (a+b)^{3/2} f (a+b \operatorname{Sec}[e+f x])^2 \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^3 \left(-2 i b (4 a+3 b) \operatorname{ArcTan} \left[\right. \right.$$

$$\left. \left. \begin{aligned} & \left(2 \operatorname{Sin}[e] \left(i a+i b+i (a+b) \operatorname{Cos}[2 e]+\sqrt{a} \sqrt{a+b} \operatorname{Cos}[f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}-\sqrt{a} \sqrt{a+b} \operatorname{Cos}[2 e+f x]} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2}+ \right. \right. \\ & \left. \left. a \operatorname{Sin}[2 e]+b \operatorname{Sin}[2 e]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]-i \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right) \right] / \\ & \left(i (a+3 b) \operatorname{Cos}[e]+i (a+b) \operatorname{Cos}[3 e]+i a \operatorname{Cos}[e+2 f x]+i a \operatorname{Cos}[3 e+2 f x]+3 a \operatorname{Sin}[e]+b \operatorname{Sin}[e]+a \operatorname{Sin}[3 e]+ \right. \\ & \left. b \operatorname{Sin}[3 e]+a \operatorname{Sin}[e+2 f x]-a \operatorname{Sin}[3 e+2 f x] \right) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) - \\ & b (4 a+3 b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log} \left[a+2(a+b) \operatorname{Cos}[2 e]-a \operatorname{Cos}[2(e+f x)]-2 i a \operatorname{Sin}[2 e]-2 i b \operatorname{Sin}[2 e]+ \right. \\ & \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) + \\ & b (4 a+3 b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Log} \left[-a-2(a+b) \operatorname{Cos}[2 e]+a \operatorname{Cos}[2(e+f x)]+2 i a \operatorname{Sin}[2 e]+2 i b \operatorname{Sin}[2 e]+ \right. \\ & \left. 2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+2 \sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[2 e+f x] \right] \operatorname{Sec}[e+f x] (\operatorname{Cos}[e]-i \operatorname{Sin}[e]) + \\ & 8 \sqrt{a} (a+b)^{3/2} \operatorname{Cos}[f x] (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[e]+ \\ & 2 b (4 a+3 b) \operatorname{ArcTan} \left[\frac{(a+b) \operatorname{Sin}[e]}{(a+b) \operatorname{Cos}[e]-\sqrt{a} \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} (\operatorname{Cos}[2 e]+i \operatorname{Sin}[2 e]) \operatorname{Sin}[e+f x]} \right] \\ & (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] (i \operatorname{Cos}[e]+\operatorname{Sin}[e]) + \\ & 8 \sqrt{a} (a+b)^{3/2} \operatorname{Cos}[e] (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x] \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Sin}[f x]+ \\ & \left. 8 \sqrt{a} b^2 \sqrt{a+b} \sqrt{(\operatorname{Cos}[e]-i \operatorname{Sin}[e])^2} \operatorname{Tan}[e+f x] \right) \end{aligned} \right]$$

- **Problem 199: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{a(3a+4b) \operatorname{ArcTan} \left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}} \right]}{2 b^{5/2} (a+b)^{3/2} f} + \frac{\operatorname{Tan}[e+f x]}{b^2 f} + \frac{a^2 \operatorname{Tan}[e+f x]}{2 b^2 (a+b) f (a+b+b \operatorname{Tan}[e+f x]^2)}$$

Result (type 3, 248 leaves):

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \right. \\ \left. \left(\left(a (3a + 4b) \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (a + 2b + a \cos[2(e + fx)]) \right. \right. \right. \\ \left. \left. \left. (\cos[2e] - i \sin[2e]) \right) \right) / \left((a + b)^{3/2} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \right. \\ \left. \left. 2 (a + 2b + a \cos[2(e + fx)]) \sec[e] \sec[e + fx] \sin[fx] + \frac{a (- (a + 2b) \sin[2e] + a \sin[2fx])}{(a + b) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / (8b^2 f (a + b \sec[e + fx]^2)^2)$$

- **Problem 201: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^2}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{2 \sqrt{b} (a + b)^{3/2} f} + \frac{\tan[e + fx]}{2 (a + b) f (a + b + b \tan[e + fx]^2)}$$

Result (type 3, 211 leaves):

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \right. \\ \left. \left(- \left(\operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (a + 2b + a \cos[2(e + fx)]) (\cos[2e] - i \sin[2e]) \right) \right) / \right. \\ \left. \left(\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} + \frac{- (a + 2b) \sin[2e] + a \sin[2fx]}{a (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / (8 (a + b) f (a + b \sec[e + fx]^2)^2)$$

- **Problem 202: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{3/2} f} - \frac{b \operatorname{Tan}[e+fx]}{2a (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 240 leaves):

$$\frac{1}{8a^2 (a+b \operatorname{Sec}[e+fx]^2)^2} (a+2b+a \operatorname{Cos}[2(e+fx)]) \operatorname{Sec}[e+fx]^4 \left(2x (a+2b+a \operatorname{Cos}[2(e+fx)]) + \right. \\ \left. \left[b (3a+2b) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) (- (a+2b) \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4} \right] (a+2b+a \operatorname{Cos}[2(e+fx)]) \right] \right. \\ \left. (\operatorname{Cos}[2e] - i \operatorname{Sin}[2e]) \right) \Bigg/ \left((a+b)^{3/2} f \sqrt{b} (\operatorname{Cos}[e] - i \operatorname{Sin}[e])^4 \right) + \frac{b ((a+2b) \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx])}{(a+b) f (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} \Bigg)$$

■ **Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+fx]^6}{(a+b \operatorname{Sec}[e+fx]^2)^2} dx$$

Optimal (type 3, 278 leaves, 8 steps):

$$\frac{(5a^3 - 12a^2b + 24ab^2 - 64b^3)x}{16a^5} + \frac{b^{7/2} (9a + 8b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^5 (a+b)^{3/2} f} + \frac{(15a^2 - 26ab + 48b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \\ \frac{(5a - 8b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2 f (a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{\operatorname{Cos}[e+fx]^5 \operatorname{Sin}[e+fx]}{6af (a+b+b \operatorname{Tan}[e+fx]^2)} + \frac{b (5a^3 - 7a^2b + 12ab^2 + 32b^3) \operatorname{Tan}[e+fx]}{16a^4 (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 921 leaves):

$$\begin{aligned}
& \frac{(5a^3 - 12a^2b + 24ab^2 - 64b^3) x (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4}{64a^5 (a + b \sec[e + fx]^2)^2} + \\
& \frac{(15a^2 - 32ab + 48b^2) \cos[2fx] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[2e]}{256a^4 f (a + b \sec[e + fx]^2)^2} + \\
& \frac{(3a - 4b) \cos[4fx] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[4e]}{256a^3 f (a + b \sec[e + fx]^2)^2} + \left((9a + 8b) (a + 2b + a \cos[2e + 2fx])^2 \right. \\
& \left. \sec[e + fx]^4 \left(- \left(b^4 \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(8a^5 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) + \right. \\
& \left. \left(i b^4 \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \\
& \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(8a^5 \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) / \\
& \left((a + b) (a + b \sec[e + fx]^2)^2 \right) + \frac{\cos[6fx] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[6e]}{768a^2 f (a + b \sec[e + fx]^2)^2} + \\
& \frac{(15a^2 - 32ab + 48b^2) \cos[2e] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[2fx]}{256a^4 f (a + b \sec[e + fx]^2)^2} + \\
& \frac{(a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^4 (-a b^4 \sin[2e] - 2b^5 \sin[2e] + a b^4 \sin[2fx])}{8a^5 (a + b) f (a + b \sec[e + fx]^2)^2 (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} + \\
& \frac{(3a - 4b) \cos[4e] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[4fx]}{256a^3 f (a + b \sec[e + fx]^2)^2} + \\
& \frac{\cos[6e] (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \sin[6fx]}{768a^2 f (a + b \sec[e + fx]^2)^2}
\end{aligned}$$

- **Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e + fx]^5}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right]}{8 \sqrt{a} (a+b)^{5/2} f} + \frac{\sin[e+fx]}{4 (a+b) f (a+b-a \sin[e+fx])^2} + \frac{3 \sin[e+fx]}{8 (a+b)^2 f (a+b-a \sin[e+fx])^2}$$

Result (type 3, 2171 leaves):

$$\frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6$$

$$\left(\left(3 i \operatorname{ArcTan}\left[\frac{-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx]}{\left(a \cos[e] + 3 b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e+2fx] + i a \sin[3e+2fx] \right)} \right] \cos[e] \right) / \left(128 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) +$$

$$\left(3 \operatorname{ArcTan}\left[\frac{-i a \cos[e] - i b \cos[e] + i a \cos[3e] + i b \cos[3e] + a \sin[e] + b \sin[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} + a \sin[3e] + b \sin[3e] - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] - 2 i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e+fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx]}{\left(a \cos[e] + 3 b \cos[e] + a \cos[3e] + b \cos[3e] + a \cos[e+2fx] + a \cos[3e+2fx] - 3 i a \sin[e] - i b \sin[e] - i a \sin[3e] - i b \sin[3e] - i a \sin[e+2fx] + i a \sin[3e+2fx] \right)} \right] \sin[e] \right) / \left(128 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) +$$

$$\frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6$$

$$\left(- \left(3 \operatorname{ArcTanh}\left[(2(a+b) \sin[e]) \right] / \left(-2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \cos[e] \right) / \left(128 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) +$$

$$\left(3 i \operatorname{ArcTanh}\left[(2(a+b) \sin[e]) \right] / \left(-2 i a \cos[e] - 2 i b \cos[e] - \sqrt{a} \sqrt{a+b} \cos[e-fx] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a+b} \cos[3e+fx] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e-fx] + i \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e+fx] \right) \right] \sin[e] \right) / \left(128 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) +$$

$$\frac{1}{(a+b)^2 (a+b \sec[e+fx])^3} (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6$$

$$\left(\left(3 \cos[e] \operatorname{Log}\left[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right] \right) / \left(256 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) -$$

$$\left(3 i \operatorname{Log}\left[a+2a \cos[2e] + 2b \cos[2e] - a \cos[2e+2fx] - 2 i a \sin[2e] - 2 i b \sin[2e] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2 \sqrt{a} \sqrt{a+b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e+fx] \right] \sin[e] \right) / \left(256 \sqrt{a} \sqrt{a+b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) +$$

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6$$

$$\left(- \left(3 \operatorname{Cos}[e] \operatorname{Log} \left[-a-2a \operatorname{Cos}[2e]-2b \operatorname{Cos}[2e]+a \operatorname{Cos}[2e+2fx]+2ia \operatorname{Sin}[2e]+2ib \operatorname{Sin}[2e]+2\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right. \right. \right.$$

$$\left. \left. \operatorname{Sin}[fx]+2\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[2e+fx] \right] \right) / \left(256\sqrt{a}\sqrt{a+b}f\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) +$$

$$\left(3i \operatorname{Log} \left[-a-2a \operatorname{Cos}[2e]-2b \operatorname{Cos}[2e]+a \operatorname{Cos}[2e+2fx]+2ia \operatorname{Sin}[2e]+2ib \operatorname{Sin}[2e]+2\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[fx]+ \right. \right.$$

$$\left. \left. 2\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[2e+fx] \right] \operatorname{Sin}[e] \right) / \left(256\sqrt{a}\sqrt{a+b}f\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \Bigg) +$$

$$\frac{(a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[e+fx]^5 \operatorname{Tan}[e+fx]}{8(a+b) f (a+b \operatorname{Sec}[e+fx])^3} + \frac{3(a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[e+fx]^5 \operatorname{Tan}[e+fx]}{32(a+b)^2 f (a+b \operatorname{Sec}[e+fx])^3}$$

■ **Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^3}{(a+b \operatorname{Sec}[e+fx])^3} dx$$

Optimal (type 3, 125 leaves, 4 steps):

$$\frac{(4a+b) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}} \right]}{8a^{3/2} (a+b)^{5/2} f} - \frac{b \operatorname{Sin}[e+fx]}{4a(a+b) f (a+b-a \operatorname{Sin}[e+fx])^2} + \frac{(4a+b) \operatorname{Sin}[e+fx]}{8a(a+b)^2 f (a+b-a \operatorname{Sin}[e+fx])^2}$$

Result (type 3, 2214 leaves):

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (4a+b) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6$$

$$\left(\left(i \operatorname{ArcTan} \left[\left(-ia \operatorname{Cos}[e]-ib \operatorname{Cos}[e]+ia \operatorname{Cos}[3e]+ib \operatorname{Cos}[3e]+a \operatorname{Sin}[e]+b \operatorname{Sin}[e]-\sqrt{a}\sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + \right. \right. \right. \right.$$

$$\left. \left. \sqrt{a}\sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + a \operatorname{Sin}[3e]+b \operatorname{Sin}[3e]-i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[e-fx] - \right. \right.$$

$$\left. \left. 2i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[e+fx]+i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[3e+fx] \right) \right] /$$

$$(a \operatorname{Cos}[e]+3b \operatorname{Cos}[e]+a \operatorname{Cos}[3e]+b \operatorname{Cos}[3e]+a \operatorname{Cos}[e+2fx]+a \operatorname{Cos}[3e+2fx]-3ia \operatorname{Sin}[e]-ib \operatorname{Sin}[e]-$$

$$ia \operatorname{Sin}[3e]-ib \operatorname{Sin}[3e]-ia \operatorname{Sin}[e+2fx]+ia \operatorname{Sin}[3e+2fx]) \operatorname{Cos}[e] \Bigg) / \left(128a^{3/2}\sqrt{a+b}f\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) +$$

$$\left(\operatorname{ArcTan} \left[\left(-ia \operatorname{Cos}[e]-ib \operatorname{Cos}[e]+ia \operatorname{Cos}[3e]+ib \operatorname{Cos}[3e]+a \operatorname{Sin}[e]+b \operatorname{Sin}[e]-\sqrt{a}\sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + \right. \right. \right. \right.$$

$$\left. \left. \sqrt{a}\sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + a \operatorname{Sin}[3e]+b \operatorname{Sin}[3e]-i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[e-fx] - \right. \right.$$

$$\left. \left. 2i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[e+fx]+i\sqrt{a}\sqrt{a+b}\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]}\operatorname{Sin}[3e+fx] \right) \right] /$$

$$(a \operatorname{Cos}[e]+3b \operatorname{Cos}[e]+a \operatorname{Cos}[3e]+b \operatorname{Cos}[3e]+a \operatorname{Cos}[e+2fx]+a \operatorname{Cos}[3e+2fx]-3ia \operatorname{Sin}[e]-ib \operatorname{Sin}[e]-$$

$$ia \operatorname{Sin}[3e]-ib \operatorname{Sin}[3e]-ia \operatorname{Sin}[e+2fx]+ia \operatorname{Sin}[3e+2fx]) \operatorname{Sin}[e] \Bigg) / \left(128a^{3/2}\sqrt{a+b}f\sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \Bigg) +$$

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx])^3} (-4a-b) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6$$

$$\left(\left(\operatorname{ArcTanh} \left[(2(a+b) \operatorname{Sin}[e]) \right] / \left(-2ia \operatorname{Cos}[e]-2ib \operatorname{Cos}[e]-\sqrt{a}\sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + \right. \right. \right.$$

$$\begin{aligned}
& \frac{\sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] +}{i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx] \Big] \operatorname{Cos}[e]} \Bigg/ \left(128 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) - \\
& \left(i \operatorname{ArcTanh} \left[(2(a+b) \operatorname{Sin}[e]) \Big/ \left(-2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} + \right. \right. \right. \\
& \left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3e+fx] \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[e-fx] + \right. \right. \right. \\
& \left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[3e+fx] \right) \right] \operatorname{Sin}[e] \Big/ \left(128 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \right) + \\
& \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (4a+b) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \\
& \left(\left(\operatorname{Cos}[e] \operatorname{Log} \left[a+2a \operatorname{Cos}[2e]+2b \operatorname{Cos}[2e]-a \operatorname{Cos}[2e+2fx]-2ia \operatorname{Sin}[2e]-2ib \operatorname{Sin}[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[fx] + \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[2e+fx] \right) \right] \Big/ \left(256 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) - \right. \\
& \left. \left(i \operatorname{Log} \left[a+2a \operatorname{Cos}[2e]+2b \operatorname{Cos}[2e]-a \operatorname{Cos}[2e+2fx]-2ia \operatorname{Sin}[2e]-2ib \operatorname{Sin}[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[fx] + \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[2e+fx] \right] \operatorname{Sin}[e] \right) \Big/ \left(256 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \right) + \\
& \frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+fx]^2)^3} (-4a-b) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \\
& \operatorname{Sec}[e+fx]^6 \\
& \left(\left(\operatorname{Cos}[e] \operatorname{Log} \left[-a-2a \operatorname{Cos}[2e]-2b \operatorname{Cos}[2e]+a \operatorname{Cos}[2e+2fx]+2ia \operatorname{Sin}[2e]+2ib \operatorname{Sin}[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sin}[fx]+2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[2e+fx] \right) \right] \Big/ \left(256 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) - \right. \\
& \left. \left(i \operatorname{Log} \left[-a-2a \operatorname{Cos}[2e]-2b \operatorname{Cos}[2e]+a \operatorname{Cos}[2e+2fx]+2ia \operatorname{Sin}[2e]+2ib \operatorname{Sin}[2e]+2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[fx] + \right. \right. \right. \\
& \left. \left. \left. 2\sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \operatorname{Sin}[2e+fx] \right] \operatorname{Sin}[e] \right) \Big/ \left(256 a^{3/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2e]-i \operatorname{Sin}[2e]} \right) \right) + \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[e+fx]^6 (4a \operatorname{Sin}[e+fx]+b \operatorname{Sin}[e+fx])}{32 a (a+b)^2 f (a+b \operatorname{Sec}[e+fx]^2)^3} - \\
& \frac{b (a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[e+fx]^5 \operatorname{Tan}[e+fx]}{8 a (a+b) f (a+b \operatorname{Sec}[e+fx]^2)^3}
\end{aligned}$$

- **Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]}{(a+b \operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$\frac{(8a^2+8ab+3b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}} \right]}{8a^{5/2} (a+b)^{5/2} f} - \frac{b \operatorname{Cos}[e+fx]^2 \operatorname{Sin}[e+fx]}{4a (a+b) f (a+b-a \operatorname{Sin}[e+fx]^2)^2} - \frac{3b (2a+b) \operatorname{Sin}[e+fx]}{8a^2 (a+b)^2 f (a+b-a \operatorname{Sin}[e+fx]^2)}$$

Result (type 3, 2256 leaves):

$$\frac{\sin[fx] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[2e+fx]}{\left(256a^{5/2}\sqrt{a+b}f\sqrt{\cos[2e] - i\sin[2e]}\right) - \left(i\log\left[-a - 2a\cos[2e] - 2b\cos[2e] + a\cos[2e+2fx] + 2ia\sin[2e] + 2ib\sin[2e] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[fx] + 2\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[2e+fx]\right]\sin[e]\right) / \left(256a^{5/2}\sqrt{a+b}f\sqrt{\cos[2e] - i\sin[2e]}\right) + \frac{(a+2b+a\cos[2e+2fx])^2 \sec[e+fx]^6 (-8ab\sin[e+fx] - 5b^2\sin[e+fx])}{32a^2(a+b)^2f(a+b\sec[e+fx]^2)^3} + \frac{b^2(a+2b+a\cos[2e+2fx])\sec[e+fx]^5 \tan[e+fx]}{8a^2(a+b)f(a+b\sec[e+fx]^2)^3}$$

■ **Problem 209: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]}{(a+b\sec[e+fx]^2)^3} dx$$

Optimal (type 3, 156 leaves, 6 steps):

$$-\frac{3b(4(a+b)^2 + (2a+b)^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sin[e+fx]}{\sqrt{a+b}}\right]}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin[e+fx]}{a^3f} - \frac{b^3\sin[e+fx]}{4a^3(a+b)f(a+b-a\sin[e+fx]^2)^2} + \frac{3b^2(4a+3b)\sin[e+fx]}{8a^3(a+b)^2f(a+b-a\sin[e+fx]^2)^2}$$

Result (type 3, 2382 leaves):

$$\frac{\cos[fx](a+2b+a\cos[2e+2fx])^3 \sec[e+fx]^6 \sin[e]}{8a^3f(a+b\sec[e+fx]^2)^3} + \frac{1}{(a+b)^2(a+b\sec[e+fx]^2)^3} (8a^2b+12ab^2+5b^3)(a+2b+a\cos[2e+2fx])^3 \sec[e+fx]^6 \left(-3i \operatorname{ArcTan}\left[\frac{-ia\cos[e] - ib\cos[e] + ia\cos[3e] + ib\cos[3e] + a\sin[e] + b\sin[e] - \sqrt{a}\sqrt{a+b}\cos[e-fx]\sqrt{\cos[2e] - i\sin[2e]} + \sqrt{a}\sqrt{a+b}\cos[3e+fx]\sqrt{\cos[2e] - i\sin[2e]} + a\sin[3e] + b\sin[3e] - i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[e-fx] - 2i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[e+fx] + i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[3e+fx]}{\left(128a^{7/2}\sqrt{a+b}f\sqrt{\cos[2e] - i\sin[2e]}\right) - (a\cos[e] + 3b\cos[e] + a\cos[3e] + b\cos[3e] + a\cos[e+2fx] + a\cos[3e+2fx] - 3ia\sin[e] - ib\sin[e] - ia\sin[3e] - ib\sin[3e] - ia\sin[e+2fx] + ia\sin[3e+2fx])\cos[e]} \right] - \left(3 \operatorname{ArcTan}\left[\frac{-ia\cos[e] - ib\cos[e] + ia\cos[3e] + ib\cos[3e] + a\sin[e] + b\sin[e] - \sqrt{a}\sqrt{a+b}\cos[e-fx]\sqrt{\cos[2e] - i\sin[2e]} + \sqrt{a}\sqrt{a+b}\cos[3e+fx]\sqrt{\cos[2e] - i\sin[2e]} + a\sin[3e] + b\sin[3e] - i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[e-fx] - 2i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[e+fx] + i\sqrt{a}\sqrt{a+b}\sqrt{\cos[2e] - i\sin[2e]}\sin[3e+fx]}{\left(128a^{7/2}\sqrt{a+b}f\sqrt{\cos[2e] - i\sin[2e]}\right) + (a\cos[e] + 3b\cos[e] + a\cos[3e] + b\cos[3e] + a\cos[e+2fx] + a\cos[3e+2fx] - 3ia\sin[e] - ib\sin[e] - ia\sin[3e] - ib\sin[3e] - ia\sin[e+2fx] + ia\sin[3e+2fx])\sin[e]} \right] \right) + \frac{1}{(a+b)^2(a+b\sec[e+fx]^2)^3} (8a^2b+12ab^2+5b^3)(a+2b+a\cos[2e+2fx])^3$$

$$\begin{aligned}
& \frac{\text{Sec}[e + f x]^6}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} \left(\left(3 \text{ArcTanh} \left[\frac{(2(a + b) \sin[e])}{(-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a + b} \cos[e - f x] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a + b} \cos[3e + f x] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - f x] + i \sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + f x])} \right] \cos[e] \right) / \left(128 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \right. \\
& \left. \left(3i \text{ArcTanh} \left[\frac{(2(a + b) \sin[e])}{(-2ia \cos[e] - 2ib \cos[e] - \sqrt{a} \sqrt{a + b} \cos[e - f x] \sqrt{\cos[2e] - i \sin[2e]} + \sqrt{a} \sqrt{a + b} \cos[3e + f x] \sqrt{\cos[2e] - i \sin[2e]} - i \sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[e - f x] + i \sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[3e + f x])} \right] \sin[e] \right) / \left(128 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
& \frac{1}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} (8a^2 b + 12a b^2 + 5b^3) (a + 2b + a \cos[2e + 2fx])^3 \\
& \frac{\text{Sec}[e + f x]^6}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} \left(- \left(3 \cos[e] \log \left[a + 2a \cos[2e] + 2b \cos[2e] - a \cos[2e + 2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right] \right) / \left(256 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) + \right. \\
& \left. \left(3i \log \left[a + 2a \cos[2e] + 2b \cos[2e] - a \cos[2e + 2fx] - 2ia \sin[2e] - 2ib \sin[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right] \sin[e] \right) / \left(256 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \\
& \frac{1}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} (8a^2 b + 12a b^2 + 5b^3) (a + 2b + a \cos[2e + 2fx])^3 \\
& \frac{\text{Sec}[e + f x]^6}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} \left(\left(3 \cos[e] \log \left[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e + 2fx] + 2ia \sin[2e] + 2ib \sin[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right] \right) / \right. \\
& \left(256 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) - \left(3i \log \left[-a - 2a \cos[2e] - 2b \cos[2e] + a \cos[2e + 2fx] + 2ia \sin[2e] + 2ib \sin[2e] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[fx] + 2\sqrt{a} \sqrt{a + b} \sqrt{\cos[2e] - i \sin[2e]} \sin[2e + fx] \right] \sin[e] \right) / \\
& \left(256 a^{7/2} \sqrt{a + b} f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) + \frac{\cos[e] (a + 2b + a \cos[2e + 2fx])^3 \text{Sec}[e + f x]^6 \sin[fx]}{8a^3 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \frac{3(a + 2b + a \cos[2e + 2fx])^2 \text{Sec}[e + f x]^6 (4a b^2 \sin[e + fx] + 3b^3 \sin[e + fx])}{32a^3 (a + b)^2 f (a + b \text{Sec}[e + f x]^2)^3} - \\
& \frac{b^3 (a + 2b + a \cos[2e + 2fx]) \text{Sec}[e + f x]^5 \tan[e + fx]}{8a^3 (a + b) f (a + b \text{Sec}[e + f x]^2)^3}
\end{aligned}$$

■ **Problem 211: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 214 leaves, 6 steps) :

$$-\frac{b^3 (80 a^2 + 140 a b + 63 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Sin}[e+f x]}{\sqrt{a+b}}\right]}{8 a^{11/2} (a+b)^{5/2} f} + \frac{(a^2 - 3 a b + 6 b^2) \operatorname{Sin}[e+f x]}{a^5 f} -$$

$$\frac{(2 a - 3 b) \operatorname{Sin}[e+f x]^3}{3 a^4 f} + \frac{\operatorname{Sin}[e+f x]^5}{5 a^3 f} - \frac{b^5 \operatorname{Sin}[e+f x]}{4 a^5 (a+b) f (a+b-a \operatorname{Sin}[e+f x])^2} + \frac{b^4 (20 a + 17 b) \operatorname{Sin}[e+f x]}{8 a^5 (a+b)^2 f (a+b-a \operatorname{Sin}[e+f x])^2}$$

Result (type 3, 2670 leaves) :

$$\frac{(5 a^2 - 18 a b + 48 b^2) \operatorname{Cos}[f x] (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e+f x]^6 \operatorname{Sin}[e]}{64 a^5 f (a+b \operatorname{Sec}[e+f x])^3} +$$

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+f x])^3} (-80 a^2 b^3 - 140 a b^4 - 63 b^5) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e+f x]^6$$

$$\left(\left(i \operatorname{ArcTan}\left[\left(-i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3 e] + i b \operatorname{Cos}[3 e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right. \right.$$

$$\left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + a \operatorname{Sin}[3 e] + b \operatorname{Sin}[3 e] - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-f x] - \right. \right. \right.$$

$$\left. \left. \left. 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e+f x] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+f x] \right) / \right.$$

$$\left. \left. \left. (a \operatorname{Cos}[e] + 3 b \operatorname{Cos}[e] + a \operatorname{Cos}[3 e] + b \operatorname{Cos}[3 e] + a \operatorname{Cos}[e+2 f x] + a \operatorname{Cos}[3 e+2 f x] - 3 i a \operatorname{Sin}[e] - i b \operatorname{Sin}[e] - \right. \right. \right.$$

$$\left. \left. \left. i a \operatorname{Sin}[3 e] - i b \operatorname{Sin}[3 e] - i a \operatorname{Sin}[e+2 f x] + i a \operatorname{Sin}[3 e+2 f x]) \right] \operatorname{Cos}[e] \right) / \left(128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) +$$

$$\left(\operatorname{ArcTan}\left[\left(-i a \operatorname{Cos}[e] - i b \operatorname{Cos}[e] + i a \operatorname{Cos}[3 e] + i b \operatorname{Cos}[3 e] + a \operatorname{Sin}[e] + b \operatorname{Sin}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right.$$

$$\left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + a \operatorname{Sin}[3 e] + b \operatorname{Sin}[3 e] - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-f x] - \right. \right. \right.$$

$$\left. \left. \left. 2 i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e+f x] + i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+f x] \right) / \right.$$

$$\left. \left. \left. (a \operatorname{Cos}[e] + 3 b \operatorname{Cos}[e] + a \operatorname{Cos}[3 e] + b \operatorname{Cos}[3 e] + a \operatorname{Cos}[e+2 f x] + a \operatorname{Cos}[3 e+2 f x] - 3 i a \operatorname{Sin}[e] - i b \operatorname{Sin}[e] - \right. \right. \right.$$

$$\left. \left. \left. i a \operatorname{Sin}[3 e] - i b \operatorname{Sin}[3 e] - i a \operatorname{Sin}[e+2 f x] + i a \operatorname{Sin}[3 e+2 f x]) \right] \operatorname{Sin}[e] \right) / \left(128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) +$$

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+f x])^3} (80 a^2 b^3 + 140 a b^4 + 63 b^5) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3$$

$$\operatorname{Sec}[e+f x]^6$$

$$\left(\left(\operatorname{ArcTanh}\left[(2 (a+b) \operatorname{Sin}[e]) / \left(-2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right. \right.$$

$$\left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-f x] + \right. \right. \right.$$

$$\left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+f x] \right) \right] \operatorname{Cos}[e] \right) / \left(128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) -$$

$$\left(i \operatorname{ArcTanh}\left[(2 (a+b) \operatorname{Sin}[e]) / \left(-2 i a \operatorname{Cos}[e] - 2 i b \operatorname{Cos}[e] - \sqrt{a} \sqrt{a+b} \operatorname{Cos}[e-f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} + \right. \right. \right.$$

$$\left. \left. \left. \sqrt{a} \sqrt{a+b} \operatorname{Cos}[3 e+f x] \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} - i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[e-f x] + \right. \right. \right.$$

$$\left. \left. \left. i \sqrt{a} \sqrt{a+b} \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \operatorname{Sin}[3 e+f x] \right) \right] \operatorname{Sin}[e] \right) / \left(128 a^{11/2} \sqrt{a+b} f \sqrt{\operatorname{Cos}[2 e] - i \operatorname{Sin}[2 e]} \right) +$$

$$\frac{1}{(a+b)^2 (a+b \operatorname{Sec}[e+f x])^3} (-80 a^2 b^3 - 140 a b^4 - 63 b^5) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3$$

$$\begin{aligned}
& \frac{\text{Sec}[e + f x]^6}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} \left(\left(\text{Cos}[e] \text{Log} \left[a + 2 a \text{Cos}[2 e] + 2 b \text{Cos}[2 e] - a \text{Cos}[2 e + 2 f x] - 2 i a \text{Sin}[2 e] - 2 i b \text{Sin}[2 e] + 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[f x] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[2 e + f x] \right] \right) / \left(256 a^{11/2} \sqrt{a + b} f \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \right) - \right. \\
& \quad \left. \left(i \text{Log} \left[a + 2 a \text{Cos}[2 e] + 2 b \text{Cos}[2 e] - a \text{Cos}[2 e + 2 f x] - 2 i a \text{Sin}[2 e] - 2 i b \text{Sin}[2 e] + 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[f x] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[2 e + f x] \right] \text{Sin}[e] \right) / \left(256 a^{11/2} \sqrt{a + b} f \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \right) \right) + \\
& \quad \frac{1}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} (80 a^2 b^3 + 140 a b^4 + 63 b^5) (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \\
& \quad \frac{\text{Sec}[e + f x]^6}{(a + b)^2 (a + b \text{Sec}[e + f x]^2)^3} \left(\left(\text{Cos}[e] \text{Log} \left[-a - 2 a \text{Cos}[2 e] - 2 b \text{Cos}[2 e] + a \text{Cos}[2 e + 2 f x] + 2 i a \text{Sin}[2 e] + 2 i b \text{Sin}[2 e] + 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \right. \right. \right. \\
& \quad \left. \left. \left. \text{Sin}[f x] + 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[2 e + f x] \right] \right) / \left(256 a^{11/2} \sqrt{a + b} f \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \right) - \right. \\
& \quad \left. \left(i \text{Log} \left[-a - 2 a \text{Cos}[2 e] - 2 b \text{Cos}[2 e] + a \text{Cos}[2 e + 2 f x] + 2 i a \text{Sin}[2 e] + 2 i b \text{Sin}[2 e] + 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[f x] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \sqrt{a} \sqrt{a + b} \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \text{Sin}[2 e + f x] \right] \text{Sin}[e] \right) / \left(256 a^{11/2} \sqrt{a + b} f \sqrt{\text{Cos}[2 e] - i \text{Sin}[2 e]} \right) \right) + \\
& \quad \frac{(5 a - 12 b) \text{Cos}[3 f x] (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \text{Sin}[3 e]}{384 a^4 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \quad \frac{\text{Cos}[5 f x] (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \text{Sin}[5 e]}{640 a^3 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \quad \frac{(5 a^2 - 18 a b + 48 b^2) \text{Cos}[e] (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \text{Sin}[f x]}{64 a^5 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \quad \frac{(5 a - 12 b) \text{Cos}[3 e] (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \text{Sin}[3 f x]}{384 a^4 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \quad \frac{\text{Cos}[5 e] (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \text{Sin}[5 f x]}{640 a^3 f (a + b \text{Sec}[e + f x]^2)^3} + \\
& \quad \frac{(a + 2 b + a \text{Cos}[2 e + 2 f x])^2 \text{Sec}[e + f x]^6 (20 a b^4 \text{Sin}[e + f x] + 17 b^5 \text{Sin}[e + f x])}{32 a^5 (a + b)^2 f (a + b \text{Sec}[e + f x]^2)^3} - \\
& \quad \frac{b^5 (a + 2 b + a \text{Cos}[2 e + 2 f x]) \text{Sec}[e + f x]^5 \text{Tan}[e + f x]}{8 a^5 (a + b) f (a + b \text{Sec}[e + f x]^2)^3}
\end{aligned}$$

■ **Problem 213: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 123 leaves, 4 steps) :

$$\frac{(a+4b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8b^{3/2}(a+b)^{5/2}f} - \frac{a \operatorname{Tan}[e+fx]}{4b(a+b)f(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \frac{(a+4b) \operatorname{Tan}[e+fx]}{8b(a+b)^2f(a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 539 leaves) :

$$\left((-a-4b)(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6 \right. \\ \left. \left(\left(\operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b}\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b}\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]}} \right) \right] \right. \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \operatorname{Cos}[2e] \right] / \left(64b\sqrt{a+b}f\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]} \right) - \right. \\ \left. \left(i \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b}\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b}\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]}} \right) \right] \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \operatorname{Sin}[2e] \right] / \left(64b\sqrt{a+b}f\sqrt{b \operatorname{Cos}[4e]-ib \operatorname{Sin}[4e]} \right) \right) \right) / \\ \left((a+b)^2(a+b \operatorname{Sec}[e+fx]^2)^3 \right) + \frac{(a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[e+fx]^6 (-a \operatorname{Sin}[2e] - 2b \operatorname{Sin}[2e] + a \operatorname{Sin}[2fx])}{16a(a+b)f(a+b \operatorname{Sec}[e+fx]^2)^3 (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])} + \\ \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[e+fx]^6 (a \operatorname{Sin}[2e] + 4b \operatorname{Sin}[2e] - a \operatorname{Sin}[2fx] + 2b \operatorname{Sin}[2fx])}{64b(a+b)^2f(a+b \operatorname{Sec}[e+fx]^2)^3 (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e])}$$

■ **Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^2}{(a+b \operatorname{Sec}[e+fx]^2)^3} dx$$

Optimal (type 3, 106 leaves, 4 steps) :

$$\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\operatorname{Tan}[e+fx]}{4(a+b)f(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \frac{3 \operatorname{Tan}[e+fx]}{8(a+b)^2f(a+b+b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 265 leaves) :

$$\left((a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^6 \right. \\ \left. - \left(3 \operatorname{ArcTan} \left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (a + 2b + a \cos[2(e + fx)])^2 (\cos[2e] - i \sin[2e]) \right) \right) / \\ \left(\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4} \right) + \frac{4b(a+b) \sec[2e] ((a+2b) \sin[2e] - a \sin[2fx])}{a^2} + \\ \left. \frac{(a + 2b + a \cos[2(e + fx)]) \sec[2e] (- (5a^2 + 16ab + 8b^2) \sin[2e] + a(5a + 2b) \sin[2fx])}{a^2} \right) / (64(a+b)^2 f (a+b \sec[e + fx]^2)^3)$$

- **Problem 215: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\frac{x}{a^3} - \frac{\sqrt{b} (15a^2 + 20ab + 8b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}} \right]}{8a^3 (a+b)^{5/2} f} - \frac{b \tan[e + fx]}{4a(a+b) f (a+b + b \tan[e + fx]^2)^2} - \frac{b(7a + 4b) \tan[e + fx]}{8a^2 (a+b)^2 f (a+b + b \tan[e + fx]^2)}$$

Result (type 3, 627 leaves):

$$\frac{x (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6}{8a^3 (a + b \sec[e + fx]^2)^3} + \left((15a^2 + 20ab + 8b^2) (a + 2b + a \cos[2e + 2fx])^3 \right. \\ \left. \sec[e + fx]^6 \left(\left(b \operatorname{ArcTan}[\sec[fx]] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right. \right. \right. \\ \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) \right) / \left(64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) - \\ \left(i b \operatorname{ArcTan}[\sec[fx]] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right) (-a \sin[fx] - \\ 2b \sin[fx] + a \sin[2e + fx]) \sin[2e] \right) / \left(64a^3 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]} \right) \Bigg) / \left((a+b)^2 (a + b \sec[e + fx]^2)^3 \right) + \\ \left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^6 (9a^2 b \sin[2e] + 28ab^2 \sin[2e] + 16b^3 \sin[2e] - 9a^2 b \sin[2fx] - 6ab^2 \sin[2fx]) \right) / \\ \left(64a^3 (a+b)^2 f (a + b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right) + \\ \left((a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^6 (-ab^2 \sin[2e] - 2b^3 \sin[2e] + ab^2 \sin[2fx]) \right) \\ \left. \right) / \left(16a^3 (a+b) f (a + b \sec[e + fx]^2)^3 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)$$

- **Problem 217: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + fx]^4}{(a + b \sec[e + fx]^2)^3} dx$$

Optimal (type 3, 269 leaves, 8 steps):

$$\frac{3(a^2 - 4ab + 16b^2)x}{8a^5} - \frac{3b^{5/2}(21a^2 + 36ab + 16b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{8a^5 (a+b)^{5/2} f} + \frac{(3a - 8b) \cos[e + fx] \sin[e + fx]}{8a^2 f (a + b + b \tan[e + fx]^2)^2} + \\ \frac{\cos[e + fx]^3 \sin[e + fx]}{4af (a + b + b \tan[e + fx]^2)^2} + \frac{b(3a^2 - 7ab - 12b^2) \tan[e + fx]}{8a^3 (a+b) f (a + b + b \tan[e + fx]^2)^2} + \frac{3b(a + 2b)(a^2 - 4ab - 4b^2) \tan[e + fx]}{8a^4 (a+b)^2 f (a + b + b \tan[e + fx]^2)^2}$$

Result (type 3, 1430 leaves):

$$\left((21 a^2 + 36 a b + 16 b^2) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \right. \\ \left. \left(\left(3 b^3 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right)} \right] \right. \right. \\ \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Cos}[2 e] \right) / \left(64 a^5 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) - \\ \left(3 i b^3 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right)} \right] \right. \\ \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Sin}[2 e] \right) / \left(64 a^5 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) \right) / \\ \left((a+b)^2 (a+b \operatorname{Sec}[e+f x])^3 \right) + \frac{1}{2048 a^5 (a+b)^2 f (a+b \operatorname{Sec}[e+f x])^3} (a+2b+a \operatorname{Cos}[2e+2fx])$$

$\operatorname{Sec}[2 e]$

$\operatorname{Sec}[e+f x]^6$

$$\begin{aligned} & (144 a^6 f x \operatorname{Cos}[2 e] + 96 a^5 b f x \operatorname{Cos}[2 e] + 912 a^4 b^2 f x \operatorname{Cos}[2 e] + 6720 a^3 b^3 f x \operatorname{Cos}[2 e] + 16512 a^2 b^4 f x \operatorname{Cos}[2 e] + \\ & 16896 a b^5 f x \operatorname{Cos}[2 e] + 6144 b^6 f x \operatorname{Cos}[2 e] + 96 a^6 f x \operatorname{Cos}[2 f x] + 480 a^4 b^2 f x \operatorname{Cos}[2 f x] + 4416 a^3 b^3 f x \operatorname{Cos}[2 f x] + \\ & 6912 a^2 b^4 f x \operatorname{Cos}[2 f x] + 3072 a b^5 f x \operatorname{Cos}[2 f x] + 96 a^6 f x \operatorname{Cos}[4 e + 2 f x] + 480 a^4 b^2 f x \operatorname{Cos}[4 e + 2 f x] + \\ & 4416 a^3 b^3 f x \operatorname{Cos}[4 e + 2 f x] + 6912 a^2 b^4 f x \operatorname{Cos}[4 e + 2 f x] + 3072 a b^5 f x \operatorname{Cos}[4 e + 2 f x] + 24 a^6 f x \operatorname{Cos}[2 e + 4 f x] - \\ & 48 a^5 b f x \operatorname{Cos}[2 e + 4 f x] + 216 a^4 b^2 f x \operatorname{Cos}[2 e + 4 f x] + 672 a^3 b^3 f x \operatorname{Cos}[2 e + 4 f x] + 384 a^2 b^4 f x \operatorname{Cos}[2 e + 4 f x] + \\ & 24 a^6 f x \operatorname{Cos}[6 e + 4 f x] - 48 a^5 b f x \operatorname{Cos}[6 e + 4 f x] + 216 a^4 b^2 f x \operatorname{Cos}[6 e + 4 f x] + 672 a^3 b^3 f x \operatorname{Cos}[6 e + 4 f x] + \\ & 384 a^2 b^4 f x \operatorname{Cos}[6 e + 4 f x] + 816 a^3 b^3 \operatorname{Sin}[2 e] + 2848 a^2 b^4 \operatorname{Sin}[2 e] + 3968 a b^5 \operatorname{Sin}[2 e] + 1792 b^6 \operatorname{Sin}[2 e] + 44 a^6 \operatorname{Sin}[2 f x] + \\ & 104 a^5 b \operatorname{Sin}[2 f x] - 180 a^4 b^2 \operatorname{Sin}[2 f x] - 1696 a^3 b^3 \operatorname{Sin}[2 f x] - 3264 a^2 b^4 \operatorname{Sin}[2 f x] - 1664 a b^5 \operatorname{Sin}[2 f x] + \\ & 44 a^6 \operatorname{Sin}[4 e + 2 f x] + 104 a^5 b \operatorname{Sin}[4 e + 2 f x] - 180 a^4 b^2 \operatorname{Sin}[4 e + 2 f x] - 608 a^3 b^3 \operatorname{Sin}[4 e + 2 f x] - 192 a^2 b^4 \operatorname{Sin}[4 e + 2 f x] + \\ & 128 a b^5 \operatorname{Sin}[4 e + 2 f x] + 38 a^6 \operatorname{Sin}[2 e + 4 f x] + 60 a^5 b \operatorname{Sin}[2 e + 4 f x] - 170 a^4 b^2 \operatorname{Sin}[2 e + 4 f x] - 640 a^3 b^3 \operatorname{Sin}[2 e + 4 f x] - \\ & 400 a^2 b^4 \operatorname{Sin}[2 e + 4 f x] + 38 a^6 \operatorname{Sin}[6 e + 4 f x] + 60 a^5 b \operatorname{Sin}[6 e + 4 f x] - 170 a^4 b^2 \operatorname{Sin}[6 e + 4 f x] - 368 a^3 b^3 \operatorname{Sin}[6 e + 4 f x] - \\ & 176 a^2 b^4 \operatorname{Sin}[6 e + 4 f x] + 12 a^6 \operatorname{Sin}[4 e + 6 f x] + 8 a^5 b \operatorname{Sin}[4 e + 6 f x] - 20 a^4 b^2 \operatorname{Sin}[4 e + 6 f x] - 16 a^3 b^3 \operatorname{Sin}[4 e + 6 f x] + \\ & 12 a^6 \operatorname{Sin}[8 e + 6 f x] + 8 a^5 b \operatorname{Sin}[8 e + 6 f x] - 20 a^4 b^2 \operatorname{Sin}[8 e + 6 f x] - 16 a^3 b^3 \operatorname{Sin}[8 e + 6 f x] + a^6 \operatorname{Sin}[6 e + 8 f x] + \\ & 2 a^5 b \operatorname{Sin}[6 e + 8 f x] + a^4 b^2 \operatorname{Sin}[6 e + 8 f x] + a^6 \operatorname{Sin}[10 e + 8 f x] + 2 a^5 b \operatorname{Sin}[10 e + 8 f x] + a^4 b^2 \operatorname{Sin}[10 e + 8 f x]) \end{aligned}$$

■ **Problem 218: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x])^3} dx$$

Optimal (type 3, 352 leaves, 9 steps):

$$\begin{aligned}
& \frac{(5a^3 - 18a^2b + 48ab^2 - 160b^3)x}{16a^6} + \frac{b^{7/2}(99a^2 + 176ab + 80b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8a^6(a+b)^{5/2}f} + \\
& \frac{(15a^2 - 34ab + 80b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3f(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \frac{5(a-2b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2f(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \frac{\operatorname{Cos}[e+fx]^5 \operatorname{Sin}[e+fx]}{6af(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \\
& \frac{b(15a^3 - 29a^2b + 64ab^2 + 120b^3) \operatorname{Tan}[e+fx]}{48a^4(a+b)f(a+b+b \operatorname{Tan}[e+fx]^2)^2} + \frac{b(5a^4 - 8a^3b + 17a^2b^2 + 116ab^3 + 80b^4) \operatorname{Tan}[e+fx]}{16a^5(a+b)^2f(a+b+b \operatorname{Tan}[e+fx]^2)}
\end{aligned}$$

Result (type 3, 1770 leaves):

$$\left((99 a^2 + 176 a b + 80 b^2) (a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \right. \\ \left. - \left(b^4 \operatorname{ArcTan}\left[\sec[f x] \left(\frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right] \right. \right. \\ \left. \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \cos[2 e] \right) / \left(64 a^6 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) + \\ \left(i b^4 \operatorname{ArcTan}\left[\sec[f x] \left(\frac{\cos[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} - \frac{i \sin[2 e]}{2 \sqrt{a+b} \sqrt{b \cos[4 e] - i b \sin[4 e]}} \right) \right] \right. \\ \left. (-a \sin[f x] - 2 b \sin[f x] + a \sin[2 e + f x]) \right] \sin[2 e] \right) / \left(64 a^6 \sqrt{a+b} f \sqrt{b \cos[4 e] - i b \sin[4 e]} \right) \Bigg) / \\ \left((a+b)^2 (a+b \sec[e+f x])^3 \right) + \frac{1}{12288 a^6 (a+b)^2 f (a+b \sec[e+f x])^3} (a+2b+a \cos[2e+2fx])$$

$\sec[2e]$

$\sec[e+fx]^6$

$$\begin{aligned} & (720 a^7 f x \cos[2 e] + 768 a^6 b f x \cos[2 e] + 1296 a^5 b^2 f x \cos[2 e] - 8352 a^4 b^3 f x \cos[2 e] - 64128 a^3 b^4 f x \cos[2 e] - \\ & 158976 a^2 b^5 f x \cos[2 e] - 165888 a b^6 f x \cos[2 e] - 61440 b^7 f x \cos[2 e] + 480 a^7 f x \cos[2 f x] + 192 a^6 b f x \cos[2 f x] + \\ & 96 a^5 b^2 f x \cos[2 f x] - 4608 a^4 b^3 f x \cos[2 f x] - 41856 a^3 b^4 f x \cos[2 f x] - 67584 a^2 b^5 f x \cos[2 f x] - 30720 a b^6 f x \cos[2 f x] + \\ & 480 a^7 f x \cos[4 e + 2 f x] + 192 a^6 b f x \cos[4 e + 2 f x] + 96 a^5 b^2 f x \cos[4 e + 2 f x] - 4608 a^4 b^3 f x \cos[4 e + 2 f x] - \\ & 41856 a^3 b^4 f x \cos[4 e + 2 f x] - 67584 a^2 b^5 f x \cos[4 e + 2 f x] - 30720 a b^6 f x \cos[4 e + 2 f x] + 120 a^7 f x \cos[2 e + 4 f x] - \\ & 192 a^6 b f x \cos[2 e + 4 f x] + 408 a^5 b^2 f x \cos[2 e + 4 f x] - 1968 a^4 b^3 f x \cos[2 e + 4 f x] - 6528 a^3 b^4 f x \cos[2 e + 4 f x] - \\ & 3840 a^2 b^5 f x \cos[2 e + 4 f x] + 120 a^7 f x \cos[6 e + 4 f x] - 192 a^6 b f x \cos[6 e + 4 f x] + 408 a^5 b^2 f x \cos[6 e + 4 f x] - \\ & 1968 a^4 b^3 f x \cos[6 e + 4 f x] - 6528 a^3 b^4 f x \cos[6 e + 4 f x] - 3840 a^2 b^5 f x \cos[6 e + 4 f x] - 6048 a^3 b^4 \sin[2 e] - 21312 a^2 b^5 \sin[2 e] - \\ & 29952 a b^6 \sin[2 e] - 13824 b^7 \sin[2 e] + 262 a^7 \sin[2 f x] + 524 a^6 b \sin[2 f x] - 26 a^5 b^2 \sin[2 f x] + 1728 a^4 b^3 \sin[2 f x] + \\ & 14976 a^3 b^4 \sin[2 f x] + 28416 a^2 b^5 \sin[2 f x] + 14592 a b^6 \sin[2 f x] + 262 a^7 \sin[4 e + 2 f x] + 524 a^6 b \sin[4 e + 2 f x] - \\ & 26 a^5 b^2 \sin[4 e + 2 f x] + 1728 a^4 b^3 \sin[4 e + 2 f x] + 6912 a^3 b^4 \sin[4 e + 2 f x] + 5376 a^2 b^5 \sin[4 e + 2 f x] + 768 a b^6 \sin[4 e + 2 f x] + \\ & 238 a^7 \sin[2 e + 4 f x] + 304 a^6 b \sin[2 e + 4 f x] - 250 a^5 b^2 \sin[2 e + 4 f x] + 1556 a^4 b^3 \sin[2 e + 4 f x] + 5904 a^3 b^4 \sin[2 e + 4 f x] + \\ & 3744 a^2 b^5 \sin[2 e + 4 f x] + 238 a^7 \sin[6 e + 4 f x] + 304 a^6 b \sin[6 e + 4 f x] - 250 a^5 b^2 \sin[6 e + 4 f x] + 1556 a^4 b^3 \sin[6 e + 4 f x] + \\ & 3888 a^3 b^4 \sin[6 e + 4 f x] + 2016 a^2 b^5 \sin[6 e + 4 f x] + 87 a^7 \sin[4 e + 6 f x] + 46 a^6 b \sin[4 e + 6 f x] - 9 a^5 b^2 \sin[4 e + 6 f x] + \\ & 192 a^4 b^3 \sin[4 e + 6 f x] + 160 a^3 b^4 \sin[4 e + 6 f x] + 87 a^7 \sin[8 e + 6 f x] + 46 a^6 b \sin[8 e + 6 f x] - 9 a^5 b^2 \sin[8 e + 6 f x] + \\ & 192 a^4 b^3 \sin[8 e + 6 f x] + 160 a^3 b^4 \sin[8 e + 6 f x] + 13 a^7 \sin[6 e + 8 f x] + 16 a^6 b \sin[6 e + 8 f x] - 7 a^5 b^2 \sin[6 e + 8 f x] - \\ & 10 a^4 b^3 \sin[6 e + 8 f x] + 13 a^7 \sin[10 e + 8 f x] + 16 a^6 b \sin[10 e + 8 f x] - 7 a^5 b^2 \sin[10 e + 8 f x] - 10 a^4 b^3 \sin[10 e + 8 f x] + \\ & a^7 \sin[8 e + 10 f x] + 2 a^6 b \sin[8 e + 10 f x] + a^5 b^2 \sin[8 e + 10 f x] + a^7 \sin[12 e + 10 f x] + 2 a^6 b \sin[12 e + 10 f x] + a^5 b^2 \sin[12 e + 10 f x] \end{aligned}$$

■ **Problem 219: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \sec[c+dx])^4} dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\frac{x}{a^4} - \frac{\sqrt{b} (35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[c+dx]}{\sqrt{a+b}}\right]}{16 a^4 (a+b)^{7/2} d} - \frac{b \operatorname{Tan}[c+dx]}{6 a (a+b) d (a+b+b \operatorname{Tan}[c+dx])^2} - \frac{b (11 a+6 b) \operatorname{Tan}[c+dx]}{24 a^2 (a+b)^2 d (a+b+b \operatorname{Tan}[c+dx])^2} - \frac{b (19 a^2+22 a b+8 b^2) \operatorname{Tan}[c+dx]}{16 a^3 (a+b)^3 d (a+b+b \operatorname{Tan}[c+dx])^2}$$

Result (type 3, 1411 leaves):

$$\left((35 a^3 + 70 a^2 b + 56 a b^2 + 16 b^3) (a + 2 b + a \operatorname{Cos}[2 c + 2 d x])^4 \operatorname{Sec}[c + d x]^8 \right. \\ \left. \left(\left(b \operatorname{ArcTan}\left[\operatorname{Sec}[d x]\right] \left(\frac{\operatorname{Cos}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} - \frac{i \operatorname{Sin}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} \right) \right. \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[d x] - 2 b \operatorname{Sin}[d x] + a \operatorname{Sin}[2 c + d x]) \right) \operatorname{Cos}[2 c] \right) / \left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]} \right) - \right. \\ \left. \left(i b \operatorname{ArcTan}\left[\operatorname{Sec}[d x]\right] \left(\frac{\operatorname{Cos}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} - \frac{i \operatorname{Sin}[2 c]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]}} \right) \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[d x] - 2 b \operatorname{Sin}[d x] + a \operatorname{Sin}[2 c + d x]) \right) \operatorname{Sin}[2 c] \right) / \left(256 a^4 \sqrt{a+b} d \sqrt{b \operatorname{Cos}[4 c] - i b \operatorname{Sin}[4 c]} \right) \right) \right) / \left(3072 a^4 (a+b)^3 d (a+b \operatorname{Sec}[c+dx])^4 \right) + \frac{1}{3072 a^4 (a+b)^3 d (a+b \operatorname{Sec}[c+dx])^4} (a + 2 b + a \operatorname{Cos}[2 c + 2 d x])$$

$\operatorname{Sec}[2 c]$

$\operatorname{Sec}[c + d x]^8$

$$(480 a^6 d x \operatorname{Cos}[2 c] + 3168 a^5 b d x \operatorname{Cos}[2 c] + 8928 a^4 b^2 d x \operatorname{Cos}[2 c] + 14112 a^3 b^3 d x \operatorname{Cos}[2 c] + 13248 a^2 b^4 d x \operatorname{Cos}[2 c] + 6912 a b^5 d x \operatorname{Cos}[2 c] + 1536 b^6 d x \operatorname{Cos}[2 c] + 360 a^6 d x \operatorname{Cos}[2 d x] + 2232 a^5 b d x \operatorname{Cos}[2 d x] + 5688 a^4 b^2 d x \operatorname{Cos}[2 d x] + 7272 a^3 b^3 d x \operatorname{Cos}[2 d x] + 4608 a^2 b^4 d x \operatorname{Cos}[2 d x] + 1152 a b^5 d x \operatorname{Cos}[2 d x] + 360 a^6 d x \operatorname{Cos}[4 c + 2 d x] + 2232 a^5 b d x \operatorname{Cos}[4 c + 2 d x] + 5688 a^4 b^2 d x \operatorname{Cos}[4 c + 2 d x] + 7272 a^3 b^3 d x \operatorname{Cos}[4 c + 2 d x] + 4608 a^2 b^4 d x \operatorname{Cos}[4 c + 2 d x] + 1152 a b^5 d x \operatorname{Cos}[4 c + 2 d x] + 144 a^6 d x \operatorname{Cos}[2 c + 4 d x] + 720 a^5 b d x \operatorname{Cos}[2 c + 4 d x] + 1296 a^4 b^2 d x \operatorname{Cos}[2 c + 4 d x] + 1008 a^3 b^3 d x \operatorname{Cos}[2 c + 4 d x] + 288 a^2 b^4 d x \operatorname{Cos}[2 c + 4 d x] + 144 a^6 d x \operatorname{Cos}[6 c + 4 d x] + 720 a^5 b d x \operatorname{Cos}[6 c + 4 d x] + 1296 a^4 b^2 d x \operatorname{Cos}[6 c + 4 d x] + 1008 a^3 b^3 d x \operatorname{Cos}[6 c + 4 d x] + 288 a^2 b^4 d x \operatorname{Cos}[6 c + 4 d x] + 24 a^6 d x \operatorname{Cos}[4 c + 6 d x] + 72 a^5 b d x \operatorname{Cos}[4 c + 6 d x] + 72 a^4 b^2 d x \operatorname{Cos}[4 c + 6 d x] + 24 a^3 b^3 d x \operatorname{Cos}[4 c + 6 d x] + 24 a^6 d x \operatorname{Cos}[8 c + 6 d x] + 72 a^5 b d x \operatorname{Cos}[8 c + 6 d x] + 72 a^4 b^2 d x \operatorname{Cos}[8 c + 6 d x] + 24 a^3 b^3 d x \operatorname{Cos}[8 c + 6 d x] + 870 a^5 b \operatorname{Sin}[2 c] + 4292 a^4 b^2 \operatorname{Sin}[2 c] + 8792 a^3 b^3 \operatorname{Sin}[2 c] + 9936 a^2 b^4 \operatorname{Sin}[2 c] + 5824 a b^5 \operatorname{Sin}[2 c] + 1408 b^6 \operatorname{Sin}[2 c] - 870 a^5 b \operatorname{Sin}[2 d x] - 3792 a^4 b^2 \operatorname{Sin}[2 d x] - 6432 a^3 b^3 \operatorname{Sin}[2 d x] - 4608 a^2 b^4 \operatorname{Sin}[2 d x] - 1248 a b^5 \operatorname{Sin}[2 d x] + 435 a^5 b \operatorname{Sin}[4 c + 2 d x] + 2124 a^4 b^2 \operatorname{Sin}[4 c + 2 d x] + 3972 a^3 b^3 \operatorname{Sin}[4 c + 2 d x] + 3072 a^2 b^4 \operatorname{Sin}[4 c + 2 d x] + 864 a b^5 \operatorname{Sin}[4 c + 2 d x] - 435 a^5 b \operatorname{Sin}[2 c + 4 d x] - 1374 a^4 b^2 \operatorname{Sin}[2 c + 4 d x] - 1248 a^3 b^3 \operatorname{Sin}[2 c + 4 d x] - 384 a^2 b^4 \operatorname{Sin}[2 c + 4 d x] + 87 a^5 b \operatorname{Sin}[6 c + 4 d x] + 366 a^4 b^2 \operatorname{Sin}[6 c + 4 d x] + 408 a^3 b^3 \operatorname{Sin}[6 c + 4 d x] + 144 a^2 b^4 \operatorname{Sin}[6 c + 4 d x] - 87 a^5 b \operatorname{Sin}[4 c + 6 d x] - 116 a^4 b^2 \operatorname{Sin}[4 c + 6 d x] - 44 a^3 b^3 \operatorname{Sin}[4 c + 6 d x])$$

■ **Problem 228: Unable to integrate problem.**

$$\int \sec[e + f x]^5 \sqrt{a + b \sec[e + f x]^2} \, dx$$

Optimal (type 4, 372 leaves, 11 steps):

$$\begin{aligned} & - \frac{(2a^2 - 3ab - 8b^2) \sin[e + f x] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}}{15b^2 f} + \frac{1}{15b^2 f \sqrt{1 - \frac{a \sin[e + f x]^2}{a + b}}} \\ & (2a^2 - 3ab - 8b^2) \sqrt{\cos[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a + b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} - \\ & \left((a - 8b)(a + b) \sqrt{\cos[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a + b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \sqrt{1 - \frac{a \sin[e + f x]^2}{a + b}} \right) / \\ & (15bf(a + b - a \sin[e + f x]^2)) + \frac{(a + 4b) \sec[e + f x] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \tan[e + f x]}{15bf} + \\ & \frac{\sec[e + f x]^3 \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \tan[e + f x]}{5f} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sec[e + f x]^5 \sqrt{a + b \sec[e + f x]^2} \, dx$$

■ **Problem 229: Unable to integrate problem.**

$$\int \sec[e + f x]^3 \sqrt{a + b \sec[e + f x]^2} \, dx$$

Optimal (type 4, 288 leaves, 10 steps):

$$\frac{(a+2b) \operatorname{Sin}[e+fx] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)}}{3bf} -$$

$$\frac{(a+2b) \sqrt{\operatorname{Cos}[e+fx]^2} \operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]], \frac{a}{a+b}] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)}}{3bf \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}}} +$$

$$\left(2(a+b) \sqrt{\operatorname{Cos}[e+fx]^2} \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]], \frac{a}{a+b}] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)} \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}} \right) /$$

$$(3f(a+b-a \operatorname{Sin}[e+fx]^2)) + \frac{\operatorname{Sec}[e+fx] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)} \operatorname{Tan}[e+fx]}{3f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Sec}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 230: Unable to integrate problem.**

$$\int \operatorname{Sec}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 4, 218 leaves, 10 steps):

$$\frac{\operatorname{Sin}[e+fx] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)}}{f} -$$

$$\frac{\sqrt{\operatorname{Cos}[e+fx]^2} \operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]], \frac{a}{a+b}] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)}}{f \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}}} + \frac{1}{f(a+b-a \operatorname{Sin}[e+fx]^2)}$$

$$(a+b) \sqrt{\operatorname{Cos}[e+fx]^2} \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]], \frac{a}{a+b}] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)} \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 232: Unable to integrate problem.**

$$\int \operatorname{Cos}[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 4, 246 leaves, 9 steps) :

$$\frac{\cos[e+fx]^2 \sin[e+fx] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)}}{3f} +$$

$$\frac{(2a+b) \sqrt{\cos[e+fx]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)}}{3af \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} -$$

$$\left(b(a+b) \sqrt{\cos[e+fx]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}} \right) /$$

$$(3af(a+b-a \sin[e+fx]^2))$$

Result (type 8, 27 leaves) :

$$\int \cos[e+fx]^3 \sqrt{a+b \sec[e+fx]^2} dx$$

■ **Problem 233: Unable to integrate problem.**

$$\int \cos[e+fx]^5 \sqrt{a+b \sec[e+fx]^2} dx$$

Optimal (type 4, 338 leaves, 10 steps) :

$$\frac{2(2a-b) \cos[e+fx]^2 \sin[e+fx] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)}}{15af} +$$

$$\frac{\cos[e+fx]^2 \sin[e+fx] (a+b-a \sin[e+fx]^2) \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)}}{5af} + \frac{1}{15a^2 f \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} -$$

$$(8a^2 + 3ab - 2b^2) \sqrt{\cos[e+fx]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)} -$$

$$\left(2(2a-b)b(a+b) \sqrt{\cos[e+fx]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] \sqrt{\sec[e+fx]^2 (a+b-a \sin[e+fx]^2)} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}} \right) /$$

$$(15a^2 f (a+b-a \sin[e+fx]^2))$$

Result (type 8, 27 leaves) :

$$\int \cos[e+fx]^5 \sqrt{a+b \sec[e+fx]^2} dx$$

- **Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^6 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 186 leaves, 6 steps):

$$\frac{(a + b) (a^2 - 2 a b + 5 b^2) \text{ArcTanh}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{16 b^{5/2} f} + \frac{(a^2 - 2 a b + 5 b^2) \text{Tan}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{16 b^2 f} - \frac{(3 a - 5 b) \text{Tan}[e + f x] (a + b + b \text{Tan}[e + f x]^2)^{3/2}}{24 b^2 f} + \frac{\text{Sec}[e + f x]^2 \text{Tan}[e + f x] (a + b + b \text{Tan}[e + f x]^2)^{3/2}}{6 b f}$$

Result (type 3, 407 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \text{Cos}[e + f x] \right. \\ \left. -1 / (1 + e^{2i(e+fx)})^6 i \sqrt{b} (-1 + e^{2i(e+fx)}) (-3a^2 (1 + e^{2i(e+fx)})^4 + 4ab (1 + e^{2i(e+fx)})^2 (1 + 4e^{2i(e+fx)} + e^{4i(e+fx)}) + \right. \\ \left. b^2 (15 + 100e^{2i(e+fx)} + 298e^{4i(e+fx)} + 100e^{6i(e+fx)} + 15e^{8i(e+fx)})) - \right. \\ \left. \frac{3(a^3 - a^2b + 3ab^2 + 5b^3) \text{Log}\left[\frac{-4\sqrt{b}(-1 + e^{2i(e+fx)})f + 4i\sqrt{4be^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}f}{1 + e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}} \right) \\ \left. \sqrt{a + b \text{Sec}[e + f x]^2} \right) / \left(24 \sqrt{2} b^{5/2} f \sqrt{a + 2b + a \text{Cos}[2e + 2fx]} \right)$$

- **Problem 235: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[e + f x]^4 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$-\frac{(a-3b)(a+b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{8b^{3/2}f} - \frac{(a-3b)\tan[e+fx]\sqrt{a+b\tan[e+fx]^2}}{8bf} + \frac{\tan[e+fx](a+b+b\tan[e+fx]^2)^{3/2}}{4bf}$$

Result (type 3, 322 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \cos[e+fx] \left(-\frac{i\sqrt{b}(-1+e^{2i(e+fx)})(a(1+e^{2i(e+fx)})^2+b(3+14e^{2i(e+fx)}+3e^{4i(e+fx)}))}{(1+e^{2i(e+fx)})^4} + \frac{(a^2-2ab-3b^2)\operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \sqrt{a+b\sec[e+fx]^2} \right) / \left(4\sqrt{2}b^{3/2}f\sqrt{a+2b+a\cos[2e+2fx]} \right)$$

- **Problem 236: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^2 \sqrt{a+b\sec[e+fx]^2} dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{(a+b)\operatorname{ArcTanh}\left[\frac{\sqrt{b}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{2\sqrt{b}f} + \frac{\tan[e+fx]\sqrt{a+b\tan[e+fx]^2}}{2f}$$

Result (type 3, 257 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \cos[e+fx] \left(-\frac{i(-1+e^{2i(e+fx)})}{(1+e^{2i(e+fx)})^2} - \frac{(a+b)\operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{b}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \sqrt{a+b\sec[e+fx]^2} \right) / \left(\sqrt{2}f\sqrt{a+2b+a\cos[2e+2fx]} \right)$$

- **Problem 237: Unable to integrate problem.**

$$\int \sqrt{a+b\sec[e+fx]^2} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a+b} \operatorname{Sec}[e+fx]^2 dx$$

- **Problem 238: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2\sqrt{a}f} + \frac{\cos[e+fx] \sin[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 322 leaves):

$$\left(e^{-i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx] \right. \\ \left. \left(-i(-1+e^{2i(e+fx)}) + \left(2(a+b) e^{2i(e+fx)} \left(2fx - i \operatorname{Log}\left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + \right. \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}\left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \right) \right) / \\ \left(\sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \sqrt{a+b \operatorname{Sec}[e+fx]^2} / \left(4\sqrt{2} f \sqrt{a+2b+a \cos[2e+2fx]} \right)$$

- **Problem 239: Unable to integrate problem.**

$$\int \cos[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\frac{(3a-b)(a+b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8a^{3/2}f} + \\ \frac{(3a-b) \cos[e+fx] \sin[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8af} + \frac{\cos[e+fx]^3 \sin[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4af}$$

Result (type 8, 27 leaves):

$$\int \cos[e + f x]^4 \sqrt{a + b \sec[e + f x]^2} dx$$

■ **Problem 240: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^6 \sqrt{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\frac{(a+b)(5a^2 - 2ab + b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{16 a^{5/2} f} + \frac{(3a-b)(5a+3b) \cos[e+fx] \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{48 a^2 f} + \frac{(5a+b) \cos[e+fx]^3 \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{24 a f} + \frac{\cos[e+fx]^5 \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{6 f}$$

Result (type 6, 1902 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{10} \sqrt{a+2b+a \cos[2(e+fx)]} \sqrt{a+b \sec[e+fx]^2} \sin[e+fx] \right) /$$

$$\left(f \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right.$$

$$\left. \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right.$$

$$\left. \sin[e+fx]^2 \right) \left(\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^5 \sqrt{a+2b+a \cos[2(e+fx)]} \right) / \right.$$

$$\left. \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \right.$$

$$\left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) -$$

$$\left(12(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^3 \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx]^2 \right) /$$

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right.$$

$$\begin{aligned}
& 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \Bigg) + \left(3 (a+b) \cos[e+fx]^4 \right. \\
& \left. \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx] \left(-\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \left. \left. \frac{4}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \Bigg) / \\
& \left(f \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right. \right. \\
& \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \Bigg) - \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, -\frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \sqrt{a+2b+a \cos[2(e+fx)]} \sin[e+fx] \right. \\
& \left. -2 f \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right) \\
& \cos[e+fx] \sin[e+fx] + 3 (a+b) \left(-\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \\
& \left. \frac{4}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -1, -\frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \Bigg) - \\
& \sin[e+fx]^2 \left(a \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5 (a+b)} - \right. \right.
\end{aligned}$$

Optimal (type 4, 450 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{2(a+2b)(a^2-4ab-4b^2)\sin[e+fx]\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}}{35b^2f} + \frac{1}{35b^2f\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}} \\
 & 2(a+2b)(a^2-4ab-4b^2)\sqrt{\cos[e+fx]^2}\operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]],\frac{a}{a+b}\right]\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)} - \\
 & \left((a+b)(a^2-16ab-16b^2)\sqrt{\cos[e+fx]^2}\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]],\frac{a}{a+b}\right] \right. \\
 & \left. \sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}} \right) / (35bf(a+b-a\sin[e+fx]^2)) + \\
 & \frac{(a^2+11ab+8b^2)\sec[e+fx]\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\tan[e+fx]}{35bf} + \\
 & \frac{2(4a+3b)\sec[e+fx]^3\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\tan[e+fx]}{35f} + \\
 & \frac{b\sec[e+fx]^5\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\tan[e+fx]}{7f}
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sec[e+fx]^5(a+b\sec[e+fx]^2)^{3/2} dx$$

■ **Problem 242: Unable to integrate problem.**

$$\int \sec[e+fx]^3(a+b\sec[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 371 leaves, 11 steps):

$$\frac{(3a^2 + 13ab + 8b^2) \operatorname{Sin}[e + fx] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)}}{15bf} - \frac{1}{15bf \sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}}}$$

$$(3a^2 + 13ab + 8b^2) \sqrt{\operatorname{Cos}[e + fx]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} +$$

$$\left((a+b)(9a+8b) \sqrt{\operatorname{Cos}[e + fx]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} \sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}} \right) /$$

$$(15f(a+b - a \operatorname{Sin}[e + fx]^2)) + \frac{2(3a+2b) \operatorname{Sec}[e + fx] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} \operatorname{Tan}[e + fx]}{15f} +$$

$$\frac{b \operatorname{Sec}[e + fx]^3 \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} \operatorname{Tan}[e + fx]}{5f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Sec}[e + fx]^3 (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

■ **Problem 243: Unable to integrate problem.**

$$\int \operatorname{Sec}[e + fx] (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

Optimal (type 4, 290 leaves, 10 steps):

$$\frac{2(2a+b) \operatorname{Sin}[e + fx] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)}}{3f} - \frac{1}{3f \sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}}}$$

$$2(2a+b) \sqrt{\operatorname{Cos}[e + fx]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} +$$

$$\left((a+b)(3a+2b) \sqrt{\operatorname{Cos}[e + fx]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} \sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}} \right) /$$

$$(3f(a+b - a \operatorname{Sin}[e + fx]^2)) + \frac{b \operatorname{Sec}[e + fx] \sqrt{\operatorname{Sec}[e + fx]^2 (a + b - a \operatorname{Sin}[e + fx]^2)} \operatorname{Tan}[e + fx]}{3f}$$

Result (type 8, 25 leaves):

$$\int \operatorname{Sec}[e + fx] (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

■ **Problem 244: Unable to integrate problem.**

$$\int \cos[e + f x] (a + b \sec[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 224 leaves, 9 steps):

$$\frac{b \sin[e + f x] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}}{f} +$$

$$\frac{(a - b) \sqrt{\cos[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}}{f \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}} + \frac{1}{f (a + b - a \sin[e + f x]^2)}$$

$$b (a + b) \sqrt{\cos[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}$$

Result (type 8, 25 leaves):

$$\int \cos[e + f x] (a + b \sec[e + f x]^2)^{3/2} dx$$

■ **Problem 246: Unable to integrate problem.**

$$\int \cos[e + f x]^5 (a + b \sec[e + f x]^2)^{3/2} dx$$

Optimal (type 4, 319 leaves, 10 steps):

$$-\frac{2(a - 3(a + b)) \cos[e + f x]^2 \sin[e + f x] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}}{15 f} +$$

$$\frac{a \cos[e + f x]^4 \sin[e + f x] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}}{5 f} + \frac{1}{15 a f \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}}$$

$$(8 a^2 + 13 a b + 3 b^2) \sqrt{\cos[e + f x]^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} -$$

$$\left(b (a + b) (4 a + 3 b) \sqrt{\cos[e + f x]^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}\right] \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}} \right) /$$

$$(15 a f (a + b - a \sin[e + f x]^2))$$

Result (type 8, 27 leaves):

$$\int \cos [e + f x]^5 (a + b \sec [e + f x]^2)^{3/2} dx$$

- **Problem 247: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [e + f x]^6 (a + b \sec [e + f x]^2)^{3/2} dx$$

Optimal (type 3, 243 leaves, 7 steps) :

$$\frac{(a+b)^2 (3a^2 - 10ab + 35b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{128 b^{5/2} f} + \frac{(a+b) (3a^2 - 10ab + 35b^2) \tan[e+fx] \sqrt{a+b \tan[e+fx]^2}}{128 b^2 f} + \frac{(3a^2 - 10ab + 35b^2) \tan[e+fx] (a+b \tan[e+fx]^2)^{3/2}}{192 b^2 f} - \frac{(3a-7b) \tan[e+fx] (a+b \tan[e+fx]^2)^{5/2}}{48 b^2 f} + \frac{\sec[e+fx]^2 \tan[e+fx] (a+b \tan[e+fx]^2)^{5/2}}{8 b f}$$

Result (type 3, 512 leaves) :

$$\frac{1}{96 \sqrt{2} b^{5/2} f (a + 2b + a \cos[2e + 2fx])^{3/2}} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3$$

$$\left(-\frac{1}{(1 + e^{2i(e+fx)})^8} i \sqrt{b} (-1 + e^{2i(e+fx)}) (-9a^3 (1 + e^{2i(e+fx)})^6 + 3a^2 b (1 + e^{2i(e+fx)})^4 (5 + 18e^{2i(e+fx)} + 5e^{4i(e+fx)}) + \right.$$

$$a b^2 (1 + e^{2i(e+fx)})^2 (145 + 948e^{2i(e+fx)} + 2758e^{4i(e+fx)} + 948e^{6i(e+fx)} + 145e^{8i(e+fx)}) +$$

$$b^3 (105 + 910e^{2i(e+fx)} + 3591e^{4i(e+fx)} + 8644e^{6i(e+fx)} + 3591e^{8i(e+fx)} + 910e^{10i(e+fx)} + 105e^{12i(e+fx)})) -$$

$$\left. \frac{3(a+b)^2 (3a^2 - 10ab + 35b^2) \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f + 4i\sqrt{4be^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}} \right) (a+b \sec[e+fx]^2)^{3/2}$$

- **Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec [e + f x]^4 (a + b \sec [e + f x]^2)^{3/2} dx$$

Optimal (type 3, 165 leaves, 6 steps) :

$$\begin{aligned}
 & - \frac{(a-5b)(a+b)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}\right]}{16 b^{3/2} f} - \frac{(a-5b)(a+b) \operatorname{Tan}[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{16 b f} \\
 & \frac{(a-5b) \operatorname{Tan}[e+fx] (a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}}{24 b f} + \frac{\operatorname{Tan}[e+fx] (a+b+b \operatorname{Tan}[e+fx]^2)^{5/2}}{6 b f}
 \end{aligned}$$

Result (type 3, 400 leaves):

$$\begin{aligned}
 & \left(e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\
 & \left. -1 / (1+e^{2i(e+fx)})^6 i \sqrt{b} (-1+e^{2i(e+fx)}) (3a^2 (1+e^{2i(e+fx)})^4 + 2ab (1+e^{2i(e+fx)})^2 (11+50e^{2i(e+fx)} + 11e^{4i(e+fx)}) + \right. \\
 & \quad \left. b^2 (15+100e^{2i(e+fx)} + 298e^{4i(e+fx)} + 100e^{6i(e+fx)} + 15e^{8i(e+fx)})) + \right. \\
 & \left. \frac{3(a-5b)(a+b)^2 \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \\
 & (12\sqrt{2} b^{3/2} f (a+2b+a \cos[2e+2fx])^{3/2})
 \end{aligned}$$

■ **Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx]^2 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 111 leaves, 5 steps):

$$\frac{3(a+b)^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}\right]}{8\sqrt{b} f} + \frac{3(a+b) \operatorname{Tan}[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{8f} + \frac{\operatorname{Tan}[e+fx] (a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4f}$$

Result (type 3, 313 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \right.$$

$$\left. - \frac{i(-1 + e^{2i(e+fx)}) (5a(1 + e^{2i(e+fx)})^2 + b(3 + 14e^{2i(e+fx)} + 3e^{4i(e+fx)}))}{(1 + e^{2i(e+fx)})^4} - \frac{3(a+b)^2 \operatorname{Log}\left[\frac{-4\sqrt{b}(-1 + e^{2i(e+fx)})f + 4i\sqrt{4be^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}f}{1 + e^{2i(e+fx)}}\right]}{\sqrt{b}\sqrt{4be^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}} \right.$$

$$\left. (a + b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \left(2\sqrt{2}f(a + 2b + a \cos[2e + 2fx])^{3/2} \right)$$

- **Problem 250: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} (3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves):

$$\frac{1}{f (a + 2 b + a \cos [2 e + 2 f x])^{3/2}} \sqrt{2} e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \cos [e + f x]^3$$

$$\left(-\frac{i b (-1 + e^{2 i (e+f x)})}{(1 + e^{2 i (e+f x)})^2} + \frac{1}{\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}} \left(2 a^{3/2} f x - i a^{3/2} \log [a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] + \right. \right.$$

$$i a^{3/2} \log [a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}] -$$

$$3 a \sqrt{b} \log \left[\frac{-2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f}{b (3 a + b) (1 + e^{2 i (e+f x)})} \right] -$$

$$\left. \left. b^{3/2} \log \left[\frac{-2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f}{b (3 a + b) (1 + e^{2 i (e+f x)})} \right] \right) (a + b \sec [e + f x]^2)^{3/2}$$

- **Problem 251: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [e + f x]^2 (a + b \sec [e + f x]^2)^{3/2} dx$$

Optimal (type 3, 124 leaves, 7 steps):

$$\frac{\sqrt{a} (a + 3 b) \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + b \tan [e + f x]^2}} \right]}{2 f} + \frac{b^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \tan [e + f x]}{\sqrt{a + b \tan [e + f x]^2}} \right]}{f} + \frac{a \cos [e + f x] \sin [e + f x] \sqrt{a + b \tan [e + f x]^2}}{2 f}$$

Result (type 3, 466 leaves):

$$\frac{1}{2\sqrt{2} f (a + 2b + a \cos[2e + 2fx])^{3/2}}$$

$$e^{-i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \left(-i a (-1 + e^{2i(e+fx)}) + \frac{1}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \right)$$

$$2 e^{2i(e+fx)} \left(2 a^{3/2} f x + 6 \sqrt{a} b f x - i \sqrt{a} (a + 3b) \operatorname{Log} \left[e^{-2ie} \left(a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] + \right.$$

$$\left. i \sqrt{a} (a + 3b) \operatorname{Log} \left[e^{-2ie} \left(a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \right] - \right.$$

$$\left. 4 b^{3/2} \operatorname{Log} \left[-\frac{e^{3ie} \left(\sqrt{b} (-1 + e^{2i(e+fx)}) - i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) f}{2 b^2 (1 + e^{2i(e+fx)})} \right] \right] \left(a + b \operatorname{Sec}[e+fx]^2 \right)^{3/2}$$

- **Problem 252: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^4 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 125 leaves, 5 steps):

$$\frac{3(a+b)^2 \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}} \right]}{8\sqrt{a} f} + \frac{3(a+b) \cos[e+fx] \sin[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{8f} + \frac{\cos[e+fx]^3 \sin[e+fx] (a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}}{4f}$$

Result (type 3, 369 leaves):

$$\left(e^{-3i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \cos[e+fx]^3 \left(-i(-1+e^{2i(e+fx)}) (10b e^{2i(e+fx)} + a(1+8e^{2i(e+fx)} + e^{4i(e+fx)})) + \right. \right. \\ \left. \left. \left(12(a+b)^2 e^{4i(e+fx)} \left(2fx - i \operatorname{Log} \left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log} \left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \right) \right) / \\ \left(\sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \left(a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2} / \left(16\sqrt{2} f (a+2b+a \cos[2e+2fx])^{3/2} \right)$$

- **Problem 253: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^6 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 193 leaves, 6 steps):

$$\frac{(5a-b)(a+b)^2 \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{16a^{3/2} f} + \frac{(5a-b)(a+b) \cos[e+fx] \sin[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16af} + \\ \frac{(5a-b) \cos[e+fx]^3 \sin[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{3/2}}{24af} + \frac{\cos[e+fx]^5 \sin[e+fx] (a+b \operatorname{Tan}[e+fx]^2)^{5/2}}{6af}$$

Result (type 3, 453 leaves):

$$\frac{1}{96\sqrt{2} a^{3/2} f (a+2b+a \cos[2e+2fx])^{3/2}} e^{-5i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \cos[e+fx]^3 \\ \left(-i\sqrt{a}(-1+e^{2i(e+fx)}) (6b^2 e^{4i(e+fx)} + ab e^{2i(e+fx)} (7+58e^{2i(e+fx)} + 7e^{4i(e+fx)}) + a^2 (1+9e^{2i(e+fx)} + 46e^{4i(e+fx)} + 9e^{6i(e+fx)} + e^{8i(e+fx)})) + \right. \\ \left. \left(12(5a-b)(a+b)^2 e^{6i(e+fx)} \left(2fx - i \operatorname{Log} \left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] + i \operatorname{Log} \left[a+a e^{2i(e+fx)} + \right. \right. \right. \\ \left. \left. \left. 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right) \right) \right) / \left(\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \left(a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2}$$

- **Problem 254: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[c+dx]^2)^{5/2} dx$$

Optimal (type 3, 166 leaves, 8 steps):

$$\frac{a^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[c+dx]}{\sqrt{a+b \operatorname{Tan}[c+dx]^2}}\right]}{d} + \frac{\sqrt{b} (15 a^2 + 10 a b + 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[c+dx]}{\sqrt{a+b \operatorname{Tan}[c+dx]^2}}\right]}{8 d} +$$

$$\frac{b (7 a + 3 b) \operatorname{Tan}[c+dx] \sqrt{a+b \operatorname{Tan}[c+dx]^2}}{8 d} + \frac{b \operatorname{Tan}[c+dx] (a+b \operatorname{Tan}[c+dx]^2)^{3/2}}{4 d}$$

Result (type 3, 706 leaves):

$$\frac{1}{\sqrt{2} d (a + 2 b + a \operatorname{Cos}[2 c + 2 d x])^{5/2}}$$

$$e^{i(c+dx)} \sqrt{4 b + a e^{-2 i(c+dx)} (1 + e^{2 i(c+dx)})^2} \operatorname{Cos}[c+dx] \left[- \frac{i b (-1 + e^{2 i(c+dx)}) (9 a (1 + e^{2 i(c+dx)})^2 + b (3 + 14 e^{2 i(c+dx)} + 3 e^{4 i(c+dx)}))}{(1 + e^{2 i(c+dx)})^4} + \right.$$

$$\left. \frac{1}{\sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}} \left(8 a^{5/2} dx - 4 i a^{5/2} \operatorname{Log}\left[a + 2 b + a e^{2 i(c+dx)} + \sqrt{a} \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2} \right] + \right.$$

$$4 i a^{5/2} \operatorname{Log}\left[a + a e^{2 i(c+dx)} + 2 b e^{2 i(c+dx)} + \sqrt{a} \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2} \right] -$$

$$15 a^2 \sqrt{b} \operatorname{Log}\left[\frac{-4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}}{b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)})} \right] -$$

$$10 a b^{3/2} \operatorname{Log}\left[\frac{-4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}}{b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)})} \right] -$$

$$\left. \left. 3 b^{5/2} \operatorname{Log}\left[\frac{-4 \sqrt{b} d (-1 + e^{2 i(c+dx)}) + 4 i d \sqrt{4 b e^{2 i(c+dx)} + a (1 + e^{2 i(c+dx)})^2}}{b (15 a^2 + 10 a b + 3 b^2) (1 + e^{2 i(c+dx)})} \right] \right] \right) (a + b \operatorname{Sec}[c+dx]^2)^{5/2}$$

■ **Problem 255: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (1 + \operatorname{Sec}[x]^2)^{3/2} dx$$

Optimal (type 3, 42 leaves, 6 steps) :

$$2 \operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2}}\right] + \operatorname{ArcTan}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2 + \operatorname{Tan}[x]^2}}\right] + \frac{1}{2} \operatorname{Tan}[x] \sqrt{2 + \operatorname{Tan}[x]^2}$$

Result (type 3, 109 leaves) :

$$\frac{1}{(3 + \operatorname{Cos}[2x])^{3/2}} (1 + \operatorname{Cos}[x]^2) \operatorname{Sec}[x] \sqrt{1 + \operatorname{Sec}[x]^2} \\ \left(4 \sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \operatorname{Sin}[x]}{\sqrt{3 + \operatorname{Cos}[2x]}}\right] \operatorname{Cos}[x]^2 - 2i \sqrt{2} \operatorname{Cos}[x]^2 \operatorname{Log}\left[\sqrt{3 + \operatorname{Cos}[2x]} + i \sqrt{2} \operatorname{Sin}[x]\right] + \sqrt{3 + \operatorname{Cos}[2x]} \operatorname{Sin}[x] \right)$$

■ **Problem 256: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{1 + \operatorname{Sec}[x]^2} dx$$

Optimal (type 3, 24 leaves, 5 steps) :

$$\operatorname{ArcSinh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2}}\right] + \operatorname{ArcTan}\left[\frac{\operatorname{Tan}[x]}{\sqrt{2 + \operatorname{Tan}[x]^2}}\right]$$

Result (type 3, 57 leaves) :

$$\frac{\sqrt{2} \left(\operatorname{ArcSin}\left[\frac{\operatorname{Sin}[x]}{\sqrt{2}}\right] + \operatorname{ArcTanh}\left[\frac{\sqrt{2} \operatorname{Sin}[x]}{\sqrt{3 + \operatorname{Cos}[2x]}}\right] \right) \operatorname{Cos}[x] \sqrt{1 + \operatorname{Sec}[x]^2}}{\sqrt{3 + \operatorname{Cos}[2x]}}$$

■ **Problem 257: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e + fx]^5}{\sqrt{a + b \operatorname{Sec}[e + fx]^2}} dx$$

Optimal (type 4, 330 leaves, 10 steps) :

$$\frac{2(a-b) \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] (a+b-a \operatorname{Sin}[e + fx]^2) - (a-2b) \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + fx]], \frac{a}{a+b}\right] \sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}}}{3b^2 f \sqrt{\operatorname{Cos}[e + fx]^2} \sqrt{\operatorname{Sec}[e + fx]^2} (a+b-a \operatorname{Sin}[e + fx]^2)} - \frac{3bf \sqrt{\operatorname{Cos}[e + fx]^2} \sqrt{\operatorname{Sec}[e + fx]^2} (a+b-a \operatorname{Sin}[e + fx]^2)}{\sqrt{1 - \frac{a \operatorname{Sin}[e + fx]^2}{a+b}}} \\ \frac{2(a-b) \operatorname{Sec}[e + fx] (a+b-a \operatorname{Sin}[e + fx]^2) \operatorname{Tan}[e + fx]}{3b^2 f \sqrt{\operatorname{Sec}[e + fx]^2} (a+b-a \operatorname{Sin}[e + fx]^2)} + \frac{\operatorname{Sec}[e + fx]^3 (a+b-a \operatorname{Sin}[e + fx]^2) \operatorname{Tan}[e + fx]}{3bf \sqrt{\operatorname{Sec}[e + fx]^2} (a+b-a \operatorname{Sin}[e + fx]^2)}$$

Result (type 8, 27 leaves) :

$$\int \frac{\operatorname{Sec}[e + f x]^5}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 258: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 4, 170 leaves, 7 steps) :

$$\frac{\sqrt{a} \sqrt{a+b} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}}\right], \frac{a+b}{a}\right] \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}}}{b f \sqrt{\operatorname{Cos}[e+fx]^2} \sqrt{\operatorname{Sec}[e+fx]^2 (a+b - a \operatorname{Sin}[e+fx]^2)}} + \frac{\operatorname{Sec}[e+fx] (a+b - a \operatorname{Sin}[e+fx]^2) \operatorname{Tan}[e+fx]}{b f \sqrt{\operatorname{Sec}[e+fx]^2 (a+b - a \operatorname{Sin}[e+fx]^2)}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\operatorname{Sec}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 260: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 4, 105 leaves, 5 steps) :

$$\frac{\sqrt{a+b} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a} \operatorname{Sin}[e+fx]}{\sqrt{a+b}}\right], \frac{a+b}{a}\right] \sqrt{1 - \frac{a \operatorname{Sin}[e+fx]^2}{a+b}}}{\sqrt{a} f \sqrt{\operatorname{Cos}[e+fx]^2} \sqrt{\operatorname{Sec}[e+fx]^2 (a+b - a \operatorname{Sin}[e+fx]^2)}}$$

Result (type 8, 25 leaves) :

$$\int \frac{\operatorname{Cos}[e + f x]}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 261: Unable to integrate problem.**

$$\int \frac{\operatorname{Cos}[e + f x]^3}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 4, 255 leaves, 9 steps) :

$$\frac{\sin[e+fx] (a+b-a\sin[e+fx]^2)}{3af\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}} + \frac{2(a-b)\operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] (a+b-a\sin[e+fx]^2)}{3a^2f\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}} -$$

$$\frac{(a-2b)b\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}}{3a^2f\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e+fx]^3}{\sqrt{a+b\sec[e+fx]^2}} dx$$

■ **Problem 262: Unable to integrate problem.**

$$\int \frac{\cos[e+fx]^5}{\sqrt{a+b\sec[e+fx]^2}} dx$$

Optimal (type 4, 345 leaves, 10 steps):

$$\frac{4(a-b)\sin[e+fx](a+b-a\sin[e+fx]^2)}{15a^2f\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}} + \frac{\cos[e+fx]^2\sin[e+fx](a+b-a\sin[e+fx]^2)}{5af\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}} +$$

$$\frac{(8a^2-7ab+8b^2)\operatorname{EllipticE}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right] (a+b-a\sin[e+fx]^2)}{15a^3f\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}} -$$

$$\frac{b(4a^2-3ab+8b^2)\operatorname{EllipticF}\left[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}\right]\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}}{15a^3f\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e+fx]^5}{\sqrt{a+b\sec[e+fx]^2}} dx$$

■ **Problem 263: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[e+fx]^6}{\sqrt{a+b\sec[e+fx]^2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{(3a^2 - 2ab + 3b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8b^{5/2}f} - \frac{3(a-b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8b^2f} + \frac{\operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4bf}$$

Result (type 3, 326 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right. \\ \left. - \frac{i\sqrt{b}(-1+e^{2i(e+fx)})(-3a(1+e^{2i(e+fx)})^2+b(3+14e^{2i(e+fx)}+3e^{4i(e+fx)}))}{(1+e^{2i(e+fx)})^4} - \frac{(3a^2-2ab+3b^2) \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \operatorname{Sec}[e+fx] \right) / \left(8\sqrt{2}b^{5/2}f\sqrt{a+b \operatorname{Sec}[e+fx]^2} \right)$$

■ **Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e+fx]^4}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$-\frac{(a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2b^{3/2}f} + \frac{\operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2bf}$$

Result (type 3, 266 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \sqrt{a+2b+a \cos[2e+2fx]} \right. \\ \left. - \frac{i\sqrt{b}(-1+e^{2i(e+fx)})}{(1+e^{2i(e+fx)})^2} + \frac{(a-b) \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \operatorname{Sec}[e+fx] \Big/ \left(2\sqrt{2} b^{3/2} f \sqrt{a+b \operatorname{Sec}[e+fx]^2} \right)$$

■ **Problem 265: Unable to integrate problem.**

$$\int \frac{\operatorname{Sec}[e+fx]^2}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{\sqrt{b} f}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Sec}[e+fx]^2}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

■ **Problem 266: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

■ **Problem 267: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^2}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 87 leaves, 4 steps):

$$\frac{(a - b) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{2 a^{3/2} f} + \frac{\cos[e + f x] \sin[e + f x] \sqrt{a + b \tan[e + f x]^2}}{2 a f}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e + f x]^2}{\sqrt{a + b \sec[e + f x]^2}} dx$$

■ **Problem 268: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^4}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 143 leaves, 6 steps):

$$\frac{(3 a^2 - 2 a b + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{8 a^{5/2} f} + \frac{3(a - b) \cos[e + f x] \sin[e + f x] \sqrt{a + b \tan[e + f x]^2}}{8 a^2 f} + \frac{\cos[e + f x]^3 \sin[e + f x] \sqrt{a + b \tan[e + f x]^2}}{4 a f}$$

Result (type 6, 1840 leaves):

$$\left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \cos[e + f x]^8 \sin[e + f x] \right) /$$

$$\left(f \sqrt{a + 2 b + a \cos[2(e + f x)]} \sqrt{a + b \sec[e + f x]^2} \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] + \right. \right.$$

$$\left. \left. \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] - 4(a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b}\right] \right) \right)$$

$$\begin{aligned}
& \left. \left(\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \right) / \right. \right. \\
& \left. \left(\sqrt{a+2b+a \cos[2(e+fx)]} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\
& \left(12 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \sin[e+fx]^2 \right) / \left(\sqrt{a+2b+a \cos[2(e+fx)]} \right. \\
& \quad \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
& \left(3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left(\frac{a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
& \left(f \sqrt{a+2b+a \cos[2(e+fx)]} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \sin[e+fx] \right. \\
& \quad \left. \left(2 f \left(a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \cos[e+fx] \sin[e+fx] + 3(a+b) \left(\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3(a+b)} - \right. \\
& \left. \frac{4}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \\
& \sin[e+fx]^2 \left(a \left(\frac{9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5(a+b)} - \right. \right. \\
& \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) - \right. \\
& \left. 4(a+b) \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5(a+b)} - \frac{1}{8 a^3} 9(a+b)^3 f \cot[e+fx] \right) \right) \\
& \operatorname{Csc}[e+fx]^4 \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}} \left(-\frac{2 a \sin[e+fx]^2}{a+b} - \frac{4 a^2 \sin[e+fx]^4}{3(a+b)^2} + \frac{2 \sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} \right) \\
& \left(f \sqrt{a+2b+a \cos[2(e+fx)]} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \right. \right. \right. \\
& \left. \left. \left. \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right)^2 + \\
& \left(3 a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \sin[e+fx] \sin[2(e+fx)] \right) / \\
& \left((a+2b+a \cos[2(e+fx)])^{3/2} \right. \\
& \left. \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right.
\end{aligned}$$

$$4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \Bigg) \Bigg) \Bigg)$$

- **Problem 269: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^6}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\frac{(a-b)(5a^2+2ab+5b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{16a^{7/2}f} + \frac{(15a^2-14ab+15b^2) \cos[e+fx] \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{48a^3f} +$$

$$\frac{5(a-b) \cos[e+fx]^3 \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{24a^2f} + \frac{\cos[e+fx]^5 \sin[e+fx] \sqrt{a+b \tan[e+fx]^2}}{6af}$$

Result (type 6, 1739 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{12} \sin[e+fx]\right) /$$

$$\left(f \sqrt{a+2b+a \cos[2(e+fx)]} \sqrt{a+b \sec[e+fx]^2} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] +\right.\right.$$

$$\left.\left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right]\right) \right)$$

$$\sin[e+fx]^2 \left(\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^7 \right) / \right.$$

$$\left. \left(\sqrt{a+2b+a \cos[2(e+fx)]} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right]\right) \sin[e+fx]^2 \right) \right) -$$

$$\left(18(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^5 \sin[e+fx]^2 \right) / \left(\sqrt{a+2b+a \cos[2(e+fx)]}\right)$$

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] -\right.$$

$$\begin{aligned}
& 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^2 \Bigg) + \\
& \left(3 (a+b) \cos[e+fx]^6 \sin[e+fx] \left(\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \left. \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
& \left(f \sqrt{a+2b+a \cos[2(e+fx)]} \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right) - \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^6 \sin[e+fx] \right. \\
& \left. \left(2 f \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 6 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right) \right. \\
& \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left(\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \left. \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) + \\
& \sin[e+fx]^2 \left(a \left(\frac{9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5 (a+b)} - \right. \right. \\
& \left. \left. \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) - \right. \\
& \left. 6 (a+b) \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{3}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5 (a+b)} - \right. \right. \\
& \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{1}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \Bigg) /
\end{aligned}$$

$$\left(f \sqrt{a + 2b + a \cos[2(e + fx)]} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] - 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] \right) \sin[e + fx]^2 \right)^2 \right) + \left(3a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] \cos[e + fx]^6 \sin[e + fx] \sin[2(e + fx)] \right) / \left((a + 2b + a \cos[2(e + fx)])^{3/2} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{1}{2}, \frac{3}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] - 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{1}{2}, \frac{5}{2}, \sin[e + fx]^2, \frac{a \sin[e + fx]^2}{a+b}\right] \right) \sin[e + fx]^2 \right) \right) \right)$$

■ **Problem 270: Unable to integrate problem.**

$$\int \frac{\sec[e + fx]^5}{(a + b \sec[e + fx]^2)^{3/2}} dx$$

Optimal (type 4, 289 leaves, 10 steps):

$$\frac{a(2a+b)\sin[e+fx]}{b^2(a+b)f\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}} - \frac{(2a+b)\operatorname{EllipticE}[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}](a+b-a\sin[e+fx]^2)}{b^2(a+b)f\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}} + \frac{\operatorname{EllipticF}[\operatorname{ArcSin}[\sin[e+fx]], \frac{a}{a+b}]\sqrt{1-\frac{a\sin[e+fx]^2}{a+b}}}{bf\sqrt{\cos[e+fx]^2}\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}} + \frac{\sec[e+fx]\tan[e+fx]}{bf\sqrt{\sec[e+fx]^2(a+b-a\sin[e+fx]^2)}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sec[e + fx]^5}{(a + b \sec[e + fx]^2)^{3/2}} dx$$

■ **Problem 272: Unable to integrate problem.**

$$\int \frac{\sec[e + fx]}{(a + b \sec[e + fx]^2)^{3/2}} dx$$

Optimal (type 4, 229 leaves, 9 steps):

$$\frac{\text{Sin}[e + f x]}{(a + b) f \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} -$$

$$\frac{\text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] (a + b - a \text{Sin}[e + f x]^2)}{a (a + b) f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}} + \frac{\text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}}{a f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 273: Unable to integrate problem.**

$$\int \frac{\text{Cos}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 240 leaves, 9 steps):

$$- \frac{b \text{Sin}[e + f x]}{a (a + b) f \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} +$$

$$\frac{(a + 2b) \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] (a + b - a \text{Sin}[e + f x]^2)}{a^2 (a + b) f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}} - \frac{2b \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}}{a^2 f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cos}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 274: Unable to integrate problem.**

$$\int \frac{\text{Cos}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 335 leaves, 10 steps):

$$\begin{aligned}
& - \frac{b \cos[e + f x]^2 \sin[e + f x]}{a(a+b) f \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} + \frac{(a+4b) \sin[e + f x] (a+b - a \sin[e + f x]^2)}{3 a^2 (a+b) f \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} + \\
& \frac{(2 a^2 - 3 a b - 8 b^2) \operatorname{EllipticE}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{3 a^3 (a+b) f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}} - \\
& \frac{(a-8b) b \operatorname{EllipticF}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{3 a^3 f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e + f x]^3}{(a+b \sec[e + f x]^2)^{3/2}} dx$$

■ **Problem 275: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^5}{(a+b \sec[e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 436 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b \cos[e + f x]^4 \sin[e + f x]}{a(a+b) f \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} + \\
& \frac{(4 a^2 - 5 a b - 24 b^2) \sin[e + f x] (a+b - a \sin[e + f x]^2)}{15 a^3 (a+b) f \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} + \frac{(a+6b) \cos[e + f x]^2 \sin[e + f x] (a+b - a \sin[e + f x]^2)}{5 a^2 (a+b) f \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} + \\
& \frac{(8 a^3 - 9 a^2 b + 16 a b^2 + 48 b^3) \operatorname{EllipticE}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{15 a^4 (a+b) f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}} - \\
& \frac{4 b (a^2 - 2 a b + 12 b^2) \operatorname{EllipticF}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{15 a^4 f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a+b - a \sin[e + f x]^2)}} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e + f x]^5}{(a+b \sec[e + f x]^2)^{3/2}} dx$$

- **Problem 276: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^6}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$-\frac{(3a - b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{2b^{5/2}f} - \frac{a \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{b(a + b)f \sqrt{a + b \operatorname{Tan}[e + f x]^2}} + \frac{(3a + b) \operatorname{Tan}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{2b^2(a + b)f}$$

Result (type 3, 375 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2b + a \operatorname{Cos}[2e + 2fx])^{3/2} \right. \\ \left. - \frac{i \sqrt{b} (-1 + e^{2i(e+fx)}) (4b^2 e^{2i(e+fx)} + 3a^2 (1 + e^{2i(e+fx)})^2 + ab (1 + 6e^{2i(e+fx)} + e^{4i(e+fx)}))}{(a + b) (1 + e^{2i(e+fx)})^2 (4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2)} + \right. \\ \left. \frac{(3a - b) \operatorname{Log}\left[\frac{-4\sqrt{b} (-1 + e^{2i(e+fx)}) f + 4i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} f}{1 + e^{2i(e+fx)}}\right]}{\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2}} \right) \operatorname{Sec}[e + f x]^3 \Big/ \left(4\sqrt{2} b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^{3/2} \right)$$

- **Problem 277: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^4}{(a + b \operatorname{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{b^{3/2}f} - \frac{a \operatorname{Tan}[e + f x]}{b(a + b)f \sqrt{a + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 3, 289 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} (a + 2b + a \cos[2e + 2fx])^{3/2} \right.$$

$$\left. \frac{i a \sqrt{b} (-1 + e^{2i(e+fx)})}{(a+b)(4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2)} - \frac{\text{Log}\left[\frac{-4\sqrt{b}(-1 + e^{2i(e+fx)})f + 4i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}f}{1 + e^{2i(e+fx)}}\right]}{\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}} \right)$$

$$\left. \text{Sec}[e + fx]^3 \right) / (2\sqrt{2} b^{3/2} f (a + b \text{Sec}[e + fx]^2)^{3/2})$$

■ **Problem 279: Unable to integrate problem.**

$$\int \frac{1}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e+fx]}{\sqrt{a+b \text{Tan}[e+fx]^2}}\right]}{a^{3/2} f} - \frac{b \text{Tan}[e + fx]}{a(a+b) f \sqrt{a+b \text{Tan}[e + fx]^2}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

■ **Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + fx]^2}{(a + b \text{Sec}[e + fx]^2)^{3/2}} dx$$

Optimal (type 3, 131 leaves, 6 steps):

$$\frac{(a - 3b) \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e+fx]}{\sqrt{a+b \text{Tan}[e+fx]^2}}\right]}{2 a^{5/2} f} + \frac{\cos[e + fx] \sin[e + fx]}{2 a f \sqrt{a + b \text{Tan}[e + fx]^2}} + \frac{b(a + 3b) \text{Tan}[e + fx]}{2 a^2 (a + b) f \sqrt{a + b \text{Tan}[e + fx]^2}}$$

Result (type 6, 2059 leaves) :

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^6 \sin[e+fx] \right) / \\
& \left(2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \sec[e+fx]^2)^{3/2} \right. \\
& (a+b-a \sin[e+fx]^2) \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \\
& \left. \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right) \\
& \sin[e+fx]^2 \left(\left(3 a (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \sin[e+fx]^2 \right) / \right. \\
& \left. \left(\sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2)^2 \right. \right. \\
& \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
& \left. \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \right) / \left(2 \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \right. \\
& \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
& \left. \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) - \\
& \left(6 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \sin[e+fx]^2 \right) / \\
& \left(\sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, \right. \right. \right. \\
& \left. \left. \left. -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
& \left(3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left(\frac{a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{a+b} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right] \right) \right) \right) / \\
& \left(2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \right. \right. \\
& \left. \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \\
& \left. \left. \sin[e+fx]^2 \right) \right) - \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \sin[e+fx] \right. \\
& \left. \left(2 f \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right) \right. \\
& \left. \cos[e+fx] \sin[e+fx] + 3(a+b) \left(\frac{a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \right. \\
& \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) + \\
& \sin[e+fx]^2 \left(3 a \left(\frac{3 a f \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \right. \\
& \left. \frac{12}{5} f \operatorname{AppellF1} \left[\frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) - \\
& \left. 4(a+b) \left(\frac{9 a f \operatorname{AppellF1} \left[\frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{5(a+b)} - \right. \right. \\
& \left. \left. \frac{6}{5} f \cos[e+fx] \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{a \sin[e+fx]^2}{a+b} \right] \sin[e+fx] \right) \right) \right) \right) / \left(2 f \sqrt{a+2b+a \cos[2(e+fx)]} \right. \\
& \left. (a+b-a \sin[e+fx]^2) \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(3 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \right. \right. \right. \\
& \left. \left. \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) \right) +
\end{aligned}$$

$$\left(3 a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \sin[e+fx] \sin[2(e+fx)] \right) /$$

$$\left(2 (a+2b+a \cos[2(e+fx)])^{3/2} (a+b-a \sin[e+fx]^2) \right.$$

$$\left. \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(3 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right.$$

$$\left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right)$$

■ **Problem 281: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^4}{(a+b \sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 194 leaves, 7 steps):

$$\frac{3(a^2 - 2ab + 5b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{8a^{7/2}f} + \frac{(3a-5b) \cos[e+fx] \sin[e+fx]}{8a^2 f \sqrt{a+b \tan[e+fx]^2}} +$$

$$\frac{\cos[e+fx]^3 \sin[e+fx]}{4af \sqrt{a+b \tan[e+fx]^2}} + \frac{(a-3b)b(3a+5b) \tan[e+fx]}{8a^3(a+b)f \sqrt{a+b \tan[e+fx]^2}}$$

Result (type 6, 2046 leaves):

$$\left((a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^{10} \sin[e+fx] \right) /$$

$$\left(2f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \sec[e+fx]^2)^{3/2} (a+b-a \sin[e+fx]^2) \right.$$

$$\left. \left((a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - \right. \right. \right.$$

$$\left. \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right)$$

$$\left(\left(a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^7 \sin[e+fx]^2 \right) / \left(\sqrt{a+2b+a \cos[2(e+fx)]} \right. \right.$$

$$\left. \left. (a+b-a \sin[e+fx]^2)^2 \left((a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \right. \right. \right. \right.$$

$$\begin{aligned}
& 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \Bigg) + \\
& \sin[e+f x]^2 \left(a \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{7}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x]}{a+b} - \right. \right. \\
& \left. \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) - \\
& \left. 2 (a+b) \left(\frac{9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x]}{5 (a+b)} - \right. \right. \\
& \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{3}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) \right) \Bigg) \Bigg) / \left(2 f \sqrt{a+2 b+a \cos[2 (e+f x)]} \right. \\
& \left. (a+b-a \sin[e+f x]^2) \left((a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b} \right] - 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \right) \sin[e+f x]^2 \right)^2 \Bigg) + \\
& \left(a (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x]^6 \sin[e+f x] \sin[2 (e+f x)] \right) \Bigg) / \\
& \left(2 (a+2 b+a \cos[2 (e+f x)])^{3/2} (a+b-a \sin[e+f x]^2) \right. \\
& \left. \left((a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{3}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] + \left(a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b} \right] - \right. \right. \right. \\
& \left. \left. \left. 2 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{3}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \right) \sin[e+f x]^2 \right) \right) \Bigg) \Bigg)
\end{aligned}$$

- **Problem 282: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+f x]^6}{(a+b \sec[e+f x]^2)^{3/2}} dx$$

Optimal (type 3, 271 leaves, 8 steps):

$$\frac{(5a^3 - 9a^2b + 15ab^2 - 35b^3) \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16a^{9/2}f} + \frac{(15a^2 - 22ab + 35b^2) \operatorname{Cos}[e+fx] \operatorname{Sin}[e+fx]}{48a^3f\sqrt{a+b \operatorname{Tan}[e+fx]^2}} +$$

$$\frac{(5a - 7b) \operatorname{Cos}[e+fx]^3 \operatorname{Sin}[e+fx]}{24a^2f\sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{\operatorname{Cos}[e+fx]^5 \operatorname{Sin}[e+fx]}{6af\sqrt{a+b \operatorname{Tan}[e+fx]^2}} + \frac{b(15a^3 - 17a^2b + 25ab^2 + 105b^3) \operatorname{Tan}[e+fx]}{48a^4(a+b)f\sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 6, 2068 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^{14} \operatorname{Sin}[e+fx]\right) /$$

$$\left(2f\sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b \operatorname{Sec}[e+fx]^2)^{3/2}\right.$$

$$(a+b-a \operatorname{Sin}[e+fx]^2) \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] +\right.$$

$$\left.3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right]\right)$$

$$\operatorname{Sin}[e+fx]^2 \left(\left(3a(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^9 \operatorname{Sin}[e+fx]^2\right) /$$

$$\left(\sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b-a \operatorname{Sin}[e+fx]^2)^2\right.$$

$$\left.3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \left(3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] -\right.\right.$$

$$\left.8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right]\right) \operatorname{Sin}[e+fx]^2 \left.+\right)$$

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^9\right) / \left(2\sqrt{a+2b+a \operatorname{Cos}[2(e+fx)]} (a+b-a \operatorname{Sin}[e+fx]^2)\right.$$

$$\left.3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \left(3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] -\right.\right.$$

$$\left.8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right]\right) \operatorname{Sin}[e+fx]^2 \left. -\right)$$

$$\left(12(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \operatorname{Cos}[e+fx]^7 \operatorname{Sin}[e+fx]^2\right) /$$

$$\begin{aligned}
& \left(\sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(3a \operatorname{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right) + \\
& \left(3(a+b) \cos[e+fx]^8 \sin[e+fx] \left(\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \right. \\
& \quad \left. \left. \frac{8}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \right) / \\
& \left(2 f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b-a \sin[e+fx]^2) \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\
& \quad \left. \left(3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right) \\
& \quad \left. \sin[e+fx]^2 \right) - \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{3}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^8 \sin[e+fx] \right) \\
& \left(2 f \left(3a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 8(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right) \\
& \quad \cos[e+fx] \sin[e+fx] + 3(a+b) \left(\frac{a f \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \\
& \quad \left. \frac{8}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{3}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \\
& \quad \sin[e+fx]^2 \left(3a \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -4, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \right. \\
& \quad \left. \left. \frac{24}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) - \right. \\
& \quad \left. 8(a+b) \left(\frac{9 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5(a+b)} - \right. \right.
\end{aligned}$$

■ **Problem 285: Unable to integrate problem.**

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 327 leaves, 10 steps):

$$\frac{2 (2 a + b) \text{Sin}[e + f x]}{3 a (a + b)^2 f \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} - \frac{b \text{Sin}[e + f x]}{3 a (a + b) f (a + b - a \text{Sin}[e + f x]^2) \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} -$$

$$\frac{2 (2 a + b) \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] (a + b - a \text{Sin}[e + f x]^2)}{3 a^2 (a + b)^2 f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}} +$$

$$\frac{(3 a + 2 b) \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}}{3 a^2 (a + b) f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Sec}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

■ **Problem 286: Unable to integrate problem.**

$$\int \frac{\text{Cos}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 349 leaves, 10 steps):

$$- \frac{2 b (3 a + 2 b) \text{Sin}[e + f x]}{3 a^2 (a + b)^2 f \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} - \frac{b \text{Cos}[e + f x]^2 \text{Sin}[e + f x]}{3 a (a + b) f (a + b - a \text{Sin}[e + f x]^2) \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}} +$$

$$\frac{(3 a^2 + 13 a b + 8 b^2) \text{EllipticE}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] (a + b - a \text{Sin}[e + f x]^2)}{3 a^3 (a + b)^2 f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)} \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}} -$$

$$\frac{b (9 a + 8 b) \text{EllipticF}[\text{ArcSin}[\text{Sin}[e + f x]], \frac{a}{a+b}] \sqrt{1 - \frac{a \text{Sin}[e + f x]^2}{a+b}}}{3 a^3 (a + b) f \sqrt{\text{Cos}[e + f x]^2} \sqrt{\text{Sec}[e + f x]^2 (a + b - a \text{Sin}[e + f x]^2)}}$$

Result (type 8, 25 leaves):

$$\int \frac{\cos[e + f x]}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

■ **Problem 287: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^3}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 441 leaves, 11 steps):

$$\begin{aligned} & - \frac{2 b (4 a + 3 b) \cos[e + f x]^2 \sin[e + f x]}{3 a^2 (a + b)^2 f \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}} - \\ & \frac{b \cos[e + f x]^4 \sin[e + f x]}{3 a (a + b) f (a + b - a \sin[e + f x]^2) \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}} + \frac{(a^2 + 11 a b + 8 b^2) \sin[e + f x] (a + b - a \sin[e + f x]^2)}{3 a^3 (a + b)^2 f \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}} + \\ & \frac{2 (a + 2 b) (a^2 - 4 a b - 4 b^2) \operatorname{EllipticE}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{3 a^4 (a + b)^2 f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}} - \\ & \frac{b (a^2 - 16 a b - 16 b^2) \operatorname{EllipticF}[\operatorname{ArcSin}[\sin[e + f x]], \frac{a}{a+b}]}{3 a^4 (a + b) f \sqrt{\cos[e + f x]^2} \sqrt{\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)}} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cos[e + f x]^3}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

■ **Problem 288: Unable to integrate problem.**

$$\int \frac{\cos[e + f x]^5}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

Optimal (type 4, 559 leaves, 12 steps):

$$\begin{aligned}
& - \frac{2 b (5 a + 4 b) \operatorname{Cos}[e + f x]^4 \operatorname{Sin}[e + f x]}{3 a^2 (a + b)^2 f \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)}} - \frac{b \operatorname{Cos}[e + f x]^6 \operatorname{Sin}[e + f x]}{3 a (a + b) f (a + b - a \operatorname{Sin}[e + f x]^2) \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)}} + \\
& \frac{2 (2 a^3 - 3 a^2 b - 42 a b^2 - 32 b^3) \operatorname{Sin}[e + f x] (a + b - a \operatorname{Sin}[e + f x]^2)}{15 a^4 (a + b)^2 f \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)}} + \frac{(3 a^2 + 61 a b + 48 b^2) \operatorname{Cos}[e + f x]^2 \operatorname{Sin}[e + f x] (a + b - a \operatorname{Sin}[e + f x]^2)}{15 a^3 (a + b)^2 f \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)}} + \\
& \left((8 a^4 - 11 a^3 b + 27 a^2 b^2 + 184 a b^3 + 128 b^4) \operatorname{EllipticE}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] (a + b - a \operatorname{Sin}[e + f x]^2) \right) / \\
& \left(15 a^5 (a + b)^2 f \sqrt{\operatorname{Cos}[e + f x]^2} \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}} \right) - \\
& \frac{b (4 a^3 - 9 a^2 b + 120 a b^2 + 128 b^3) \operatorname{EllipticF}\left[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}\right] \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^2}{a + b}}}{15 a^5 (a + b) f \sqrt{\operatorname{Cos}[e + f x]^2} \sqrt{\operatorname{Sec}[e + f x]^2 (a + b - a \operatorname{Sin}[e + f x]^2)}}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Cos}[e + f x]^5}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

- **Problem 289: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[e + f x]^6}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{b^{5/2} f} - \frac{a \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 b (a + b) f (a + b + b \operatorname{Tan}[e + f x]^2)^{3/2}} - \frac{a (3 a + 5 b) \operatorname{Tan}[e + f x]}{3 b^2 (a + b)^2 f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 3, 357 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{5/2} \right. \\ \left. \frac{i a \sqrt{b} (-1+e^{2i(e+fx)}) (24b^2 e^{2i(e+fx)} + 3a^2 (1+e^{2i(e+fx)})^2 + ab(5+26e^{2i(e+fx)}+5e^{4i(e+fx)}))}{(a+b)^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2} \right. \\ \left. \frac{3 \operatorname{Log}\left[\frac{-4\sqrt{b}(-1+e^{2i(e+fx)})f+4i\sqrt{4b e^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}f}{1+e^{2i(e+fx)}}\right]}{\sqrt{4b e^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}} \right) \operatorname{Sec}[e+fx]^5 \Big/ \left(12\sqrt{2} b^{5/2} f (a+b \operatorname{Sec}[e+fx]^2)^{5/2}\right)$$

■ **Problem 292: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \operatorname{Sec}[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \operatorname{Tan}[e+fx]}{3a(a+b)f(a+b+b \operatorname{Tan}[e+fx]^2)^{3/2}} - \frac{b(5a+3b) \operatorname{Tan}[e+fx]}{3a^2(a+b)^2 f \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 6, 1927 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \operatorname{Sin}[e+fx] \right) \Big/ \\ \left(4\sqrt{2} f (a+b \operatorname{Sec}[e+fx]^2)^{5/2} (a+b-a \operatorname{Sin}[e+fx]^2)^{5/2} \right. \\ \left. \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] + \left(5a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] - \right. \right. \right. \\ \left. \left. \left. 4(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e+fx]^2, \frac{a \operatorname{Sin}[e+fx]^2}{a+b}\right] \right) \operatorname{Sin}[e+fx]^2 \right) \right)$$

$$\left(\left(15 a (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \sin[e+fx]^2 \right) / \left(4 \sqrt{2} (a+b-a \sin[e+fx]^2)^{7/2} \right) \right. \\
\left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
\left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) + \\
\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \right) / \left(4 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right) - \\
\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
\left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \sin[e+fx]^2 \right) / \left(\sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right) \\
\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
\left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) + \\
\left(3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
\left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
\left(4 \sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \right. \right. \right. \\
\left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \right.$$

$$\left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx]^4 \sin[e+fx] \right.$$

$$\left. 2 f \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] - 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right.$$

$$\cos[e+fx] \sin[e+fx] + 3 (a+b) \left(\frac{5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right.$$

$$\left. \frac{4}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) +$$

$$\sin[e+fx]^2 \left(5 a \left(\frac{21 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5 (a+b)} - \right. \right.$$

$$\left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) -$$

$$4 (a+b) \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{a+b} - \right.$$

$$\left. \left(6 (a+b)^3 f \cot[e+fx] \csc[e+fx]^4 \left(-1 + \frac{a \sin[e+fx]^2}{a+b} \right)^2 \frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e+fx]}{\sqrt{a+b}}\right] \sin[e+fx]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e+fx]^2}{a+b}}} + \right. \right.$$

$$\left. \left. \left. \left. \left. \frac{a^2 \sin[e + f x]^4}{3 (a + b)^2 \left(-1 + \frac{a \sin[e + f x]^2}{a + b}\right)^2} + \frac{a \sin[e + f x]^2}{(a + b) \left(-1 + \frac{a \sin[e + f x]^2}{a + b}\right)} \right) \right) \right) \right) \right) \left/ \left(a^3 \left(1 - \frac{a \sin[e + f x]^2}{a + b} \right)^{3/2} \right) \right) \right) \right) \left/ \left(4 \sqrt{2} f (a + b - a \sin[e + f x]^2)^{5/2} \left(3 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] - 4 (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \right) \sin[e + f x]^2 \right)^2 \right) \right) \right) \right)$$

■ **Problem 293: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e + f x]^2}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\frac{(a - 5b) \operatorname{ArcTan} \left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}} \right]}{2 a^{7/2} f} + \frac{\cos[e + f x] \sin[e + f x]}{2 a f (a + b \tan[e + f x]^2)^{3/2}} + \frac{b (3a + 5b) \tan[e + f x]}{6 a^2 (a + b) f (a + b \tan[e + f x]^2)^{3/2}} + \frac{b (3a^2 + 22ab + 15b^2) \tan[e + f x]}{6 a^3 (a + b)^2 f \sqrt{a + b \tan[e + f x]^2}}$$

Result (type 6, 1775 leaves):

$$\left(3 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \cos[e + f x]^8 \sin[e + f x] \right) \left/ \left(4 \sqrt{2} f (a + b \sec[e + f x]^2)^{5/2} (a + b - a \sin[e + f x]^2)^{5/2} \right) \right. \\ \left(3 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] - 6 (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \right) \sin[e + f x]^2 \right) \\ \left(\left(15 a (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a + b} \right] \cos[e + f x]^7 \sin[e + f x]^2 \right) \right) \left/ \left(4 \sqrt{2} (a + b - a \sin[e + f x]^2)^{7/2} \right) \right)$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) + \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^7 \right) / \left(4 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right. \\
& \quad \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \right. \\
& \quad \left. \left. 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
& \left(9 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^5 \sin[e+fx]^2 \right) / \\
& \left(2 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) + \\
& \left(3 (a+b) \cos[e+fx]^6 \sin[e+fx] \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. 2 f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
& \left(4 \sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \right. \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^6 \sin[e+fx] \right. \\
& \quad \left. \left(2 f \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \right. \\
& \quad \left. \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 2 f \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) + \\
& \sin[e+f x]^2 \left(5 a \left(\frac{21 a f \operatorname{AppellF1}\left[\frac{5}{2}, -3, \frac{9}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x]}{5(a+b)} - \right. \right. \\
& \left. \frac{18}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{7}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) - \\
& \left. 6(a+b) \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{7}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x]}{a+b} - \right. \right. \\
& \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{5}{2}, \frac{7}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x] \sin[e+f x] \right) \right) \right) \Bigg) / \\
& \left(4 \sqrt{2} f (a+b-a \sin[e+f x]^2)^{5/2} \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -3, \frac{5}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -3, \right. \right. \right. \\
& \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] - 6(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{5}{2}, \frac{5}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \right) \sin[e+f x]^2 \right)^2 \right) \Bigg)
\end{aligned}$$

- **Problem 294: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+f x]^4}{(a+b \sec[e+f x]^2)^{5/2}} dx$$

Optimal (type 3, 261 leaves, 8 steps):

$$\begin{aligned}
& \frac{(3 a^2 - 10 a b + 35 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{a+b \tan[e+f x]^2}}\right]}{8 a^{9/2} f} + \frac{(3 a - 7 b) \cos[e+f x] \sin[e+f x]}{8 a^2 f (a+b \tan[e+f x]^2)^{3/2}} + \\
& \frac{\cos[e+f x]^3 \sin[e+f x]}{4 a f (a+b \tan[e+f x]^2)^{3/2}} + \frac{b(9 a^2 - 18 a b - 35 b^2) \tan[e+f x]}{24 a^3 (a+b) f (a+b \tan[e+f x]^2)^{3/2}} + \frac{b(9 a^3 - 15 a^2 b - 145 a b^2 - 105 b^3) \tan[e+f x]}{24 a^4 (a+b)^2 f \sqrt{a+b \tan[e+f x]^2}}
\end{aligned}$$

Result (type 6, 1777 leaves):

$$\left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+f x]^2, \frac{a \sin[e+f x]^2}{a+b}\right] \cos[e+f x]^{12} \sin[e+f x] \right) /$$

$$\begin{aligned}
& \left(4 \sqrt{2} f (a + b \operatorname{Sec}[e + f x]^2)^{5/2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \\
& \quad \left. \left. 8 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \\
& \left(\left(15 a (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^9 \operatorname{Sin}[e + f x]^2 \right) / \left(4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{7/2} \right. \right. \\
& \quad \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \\
& \quad \left. \left. 8 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \right) + \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^9 \right) / \left(4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right. \\
& \quad \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \\
& \quad \left. \left. 8 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \right) - \\
& \left(3 \sqrt{2} (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^7 \operatorname{Sin}[e + f x]^2 \right) / \\
& \quad \left((a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - 8 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \right) + \\
& \left(3 (a + b) \operatorname{Cos}[e + f x]^8 \operatorname{Sin}[e + f x] \right) \left(\frac{5 a f \operatorname{AppellF1}\left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x]}{3 (a + b)} - \right. \\
& \quad \left. \frac{8}{3} f \operatorname{AppellF1}\left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] \right) \right) / \\
& \left(4 \sqrt{2} f (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -4, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 8(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \right. \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^8 \sin[e+fx] \right. \\
& \left. \left(2f \left(5a \operatorname{AppellF1} \left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 8(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \right. \\
& \left. \left. \cos[e+fx] \sin[e+fx] + 3(a+b) \left(\frac{5af \operatorname{AppellF1} \left[\frac{3}{2}, -4, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3(a+b)} - \right. \right. \right. \\
& \left. \left. \left. \frac{8}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) + \right. \\
& \left. \sin[e+fx]^2 \left(5a \left(\frac{21af \operatorname{AppellF1} \left[\frac{5}{2}, -4, \frac{9}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{5(a+b)} - \right. \right. \right. \\
& \left. \left. \left. \frac{24}{5} f \operatorname{AppellF1} \left[\frac{5}{2}, -3, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) - \right. \\
& \left. 8(a+b) \left(\frac{3af \operatorname{AppellF1} \left[\frac{5}{2}, -3, \frac{7}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{a+b} - \right. \right. \\
& \left. \left. \left. \frac{18}{5} f \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{5}{2}, \frac{7}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) \right) \right) \Big/ \\
& \left(4\sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -4, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5a \operatorname{AppellF1} \left[\frac{3}{2}, -4, \right. \right. \right. \\
& \left. \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 8(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 295: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[e+fx]^6}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 332 leaves, 9 steps):

$$\frac{5(a-3b)(a^2+7b^2)\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{16a^{11/2}f} + \frac{(5a^2-10ab+21b^2)\cos[e+fx]\sin[e+fx]}{16a^3f(a+b+b\tan[e+fx]^2)^{3/2}} + \frac{(5a-9b)\cos[e+fx]^3\sin[e+fx]}{24a^2f(a+b+b\tan[e+fx]^2)^{3/2}} +$$

$$\frac{\cos[e+fx]^5\sin[e+fx]}{6af(a+b+b\tan[e+fx]^2)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan[e+fx]}{48a^4(a+b)f(a+b+b\tan[e+fx]^2)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan[e+fx]}{48a^5(a+b)^2f\sqrt{a+b+b\tan[e+fx]^2}}$$

Result(type 6, 1776 leaves):

$$\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\cos[e+fx]^{16}\sin[e+fx]\right) /$$

$$\left(4\sqrt{2}f(a+b\sec[e+fx]^2)^{5/2}(a+b-a\sin[e+fx]^2)^{5/2}\right.$$

$$\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] + 5\left(a\text{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] -\right.$$

$$\left.2(a+b)\text{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\right)\sin[e+fx]^2\left.)\right)$$

$$\left(\left(15a(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\cos[e+fx]^{11}\sin[e+fx]^2\right) / \left(4\sqrt{2}(a+b-a\sin[e+fx]^2)^{7/2}\right.\right.$$

$$\left.\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] + 5\left(a\text{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] -\right.\right.$$

$$\left.\left.2(a+b)\text{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\right)\right)\sin[e+fx]^2\left.)\right) +$$

$$\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\cos[e+fx]^{11}\right) / \left(4\sqrt{2}(a+b-a\sin[e+fx]^2)^{5/2}\right.$$

$$\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] + 5\left(a\text{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] -\right.$$

$$\left.\left.2(a+b)\text{AppellF1}\left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\right)\right)\sin[e+fx]^2\left.)\right) -$$

$$\left(15(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right]\cos[e+fx]^9\sin[e+fx]^2\right) /$$

$$\left(2\sqrt{2}(a+b-a\sin[e+fx]^2)^{5/2}\left(3(a+b)\text{AppellF1}\left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] + 5\left(a\text{AppellF1}\left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a\sin[e+fx]^2}{a+b}\right] -\right.\right.\right.$$

$$\left(4 \sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -5, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + 5 \left(a \operatorname{AppellF1} \left[\frac{3}{2}, -5, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 2 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -4, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right)^2 \right) \right)$$

- **Problem 296: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+b \sec[c+dx]^2)^{7/2}} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{a} \tan[c+dx]}{\sqrt{a+b \tan[c+dx]^2}} \right]}{a^{7/2} d} - \frac{b \tan[c+dx]}{5 a (a+b) d (a+b+b \tan[c+dx]^2)^{5/2}} - \frac{b (9 a+5 b) \tan[c+dx]}{15 a^2 (a+b)^2 d (a+b+b \tan[c+dx]^2)^{3/2}} - \frac{b (33 a^2+40 a b+15 b^2) \tan[c+dx]}{15 a^3 (a+b)^3 d \sqrt{a+b+b \tan[c+dx]^2}}$$

Result (type 6, 1777 leaves):

$$\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx]^6 \sin[c+dx] \right) / \left(8 \sqrt{2} d (a+b \sec[c+dx]^2)^{7/2} (a+b-a \sin[c+dx]^2)^{7/2} \right. \\ \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right) \right) \\ \left(\left(21 a (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx]^7 \sin[c+dx]^2 \right) / \left(8 \sqrt{2} (a+b-a \sin[c+dx]^2)^{9/2} \right. \right. \\ \left. \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right) \right) + \right. \\ \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx]^7 \right) / \left(8 \sqrt{2} (a+b-a \sin[c+dx]^2)^{7/2} \right) \right)$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right) - \\
& \left(9 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx]^5 \sin[c+dx]^2 \right) / \\
& \left(4 \sqrt{2} (a+b-a \sin[c+dx]^2)^{7/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right) + \\
& \left(3 (a+b) \cos[c+dx]^6 \sin[c+dx] \left(\frac{7 a d \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. 2 d \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx] \right) \right) / \\
& \left(8 \sqrt{2} d (a+b-a \sin[c+dx]^2)^{7/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \right. \right. \right. \\
& \quad \left. \left. \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx]^6 \sin[c+dx] \right. \\
& \quad \left. \left(2 d \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \right) \right. \\
& \quad \left. \cos[c+dx] \sin[c+dx] + 3 (a+b) \left(\frac{7 a d \operatorname{AppellF1} \left[\frac{3}{2}, -3, \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. 2 d \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx] \right) \right) + \\
& \sin[c+dx]^2 \left(7 a \left(\frac{27 a d \operatorname{AppellF1} \left[\frac{5}{2}, -3, \frac{11}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx]}{5 (a+b)} - \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left. \frac{18}{5} d \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx] \right) - \\ & 6 (a+b) \left(\frac{21 a d \operatorname{AppellF1} \left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx]}{5 (a+b)} - \right. \\ & \left. \frac{12}{5} d \operatorname{AppellF1} \left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \cos[c+dx] \sin[c+dx] \right) \Bigg) \Bigg) \Bigg) \Bigg) / \\ & \left(8 \sqrt{2} d (a+b-a \sin[c+dx]^2)^{7/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -3, \frac{7}{2}, \frac{3}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] + \left(7 a \operatorname{AppellF1} \left[\frac{3}{2}, -3, \right. \right. \right. \right. \\ & \left. \left. \left. \frac{9}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] - 6 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[c+dx]^2, \frac{a \sin[c+dx]^2}{a+b} \right] \right) \sin[c+dx]^2 \right)^2 \right) \Bigg) \Bigg) \Bigg) \Bigg) \end{aligned}$$

■ **Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1 + \sec[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\operatorname{ArcTan} \left[\frac{\tan[x]}{\sqrt{2 + \tan[x]^2}} \right]$$

Result (type 3, 47 leaves):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{2} \sin[x]}{\sqrt{3 + \cos[2x]}} \right] \sqrt{3 + \cos[2x]} \sec[x]}{\sqrt{2} \sqrt{1 + \sec[x]^2}}$$

■ **Problem 298: Result more than twice size of optimal antiderivative.**

$$\int (d \sec[e+fx])^m (a+b \sec[e+fx]^2)^p dx$$

Optimal (type 6, 111 leaves, ? steps):

$$\frac{1}{f m} \operatorname{AppellF1} \left[\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \sec[e+fx]^2, -\frac{b \sec[e+fx]^2}{a} \right] \\ \cot[e+fx] (d \sec[e+fx])^m (a+b \sec[e+fx]^2)^p \left(1 + \frac{b \sec[e+fx]^2}{a} \right)^{-p} \sqrt{-\tan[e+fx]^2}$$

Result (type 6, 2195 leaves):

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \\
& \quad \left. (a+2b+a \cos[2(e+fx)])^p (d \sec[e+fx])^m (\sec[e+fx]^2)^{-1+\frac{m}{2}+p} (a+b \sec[e+fx]^2)^p \tan[e+fx] \right) / \\
& \left(f \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (-2+m) \operatorname{AppellF1} \left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \\
& \left(\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{m}{2}+p} \right) / \right. \\
& \quad \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (-2+m) \operatorname{AppellF1} \left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
& \left(6 a (a+b)^p \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-1+\frac{m}{2}+p} \right. \\
& \quad \left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left. \left. (a+b) (-2+m) \operatorname{AppellF1} \left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
& \left(6 (a+b) \left(-1 + \frac{m}{2} + p \right) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \quad \left. (\sec[e+fx]^2)^{-1+\frac{m}{2}+p} \tan[e+fx]^2 \right) / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{m}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (-2+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \left(3 (a+b) (a+2b+a \cos[2(e+fx)])^p \right.
\end{aligned}$$

$$\begin{aligned}
& (\operatorname{Sec}[e + f x]^2)^{-1 + \frac{m}{2} + p} \operatorname{Tan}[e + f x] \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 (a + b)} - \right. \\
& \left. \frac{2}{3} \left(1 - \frac{m}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \Bigg) / \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + (a + b) (-2 + m) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \right) \operatorname{Tan}[e + f x]^2 \Bigg) - \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^p (\operatorname{Sec}[e + f x]^2)^{-1 + \frac{m}{2} + p} \right. \\
& \operatorname{Tan}[e + f x] \left(2 \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + \right. \right. \\
& \left. \left. (a + b) (-2 + m) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \right) \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] + \right. \\
& \left. 3 (a + b) \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 (a + b)} - \right. \right. \\
& \left. \left. \frac{2}{3} \left(1 - \frac{m}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, 2 - \frac{m}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) + \right. \\
& \operatorname{Tan}[e + f x]^2 \left(2 b p \left(-\frac{1}{5 (a + b)} 6 b (1 - p) \operatorname{AppellF1}\left[\frac{5}{2}, 1 - \frac{m}{2}, 2 - p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \\
& \left. \left. \frac{6}{5} \left(1 - \frac{m}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - \frac{m}{2}, 1 - p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) + \right. \\
& \left. (a + b) (-2 + m) \left(\frac{1}{5 (a + b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 2 - \frac{m}{2}, 1 - p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \\
& \left. \left. \frac{6}{5} \left(2 - \frac{m}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, 3 - \frac{m}{2}, -p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) \Bigg) \Bigg) / \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 1 - \frac{m}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{m}{2}, 1 - p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, \right. \right. \right.
\end{aligned}$$

$$\left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) (-2+m) \operatorname{AppellF1} \left[\frac{3}{2}, 2-\frac{m}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx]^2 \right)^2 \right) \right)$$

■ **Problem 299: Result more than twice size of optimal antiderivative.**

$$\int \sec[e+fx]^3 (a+b \sec[e+fx]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[\frac{1}{2}, 2+p, -p, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right]$$

$$\left(\cos[e+fx]^2 \right)^p \sin[e+fx] \left(\sec[e+fx]^2 (a+b - a \sin[e+fx]^2) \right)^p \left(1 - \frac{a \sin[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 1989 leaves):

$$\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right.$$

$$\left. (a+2b+a \cos[2(e+fx)])^p \sec[e+fx]^3 (\sec[e+fx]^2)^{\frac{1}{2}+p} (a+b \sec[e+fx]^2)^p \tan[e+fx] \right) /$$

$$\left(f \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2b^p \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right. \right.$$

$$\left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right)$$

$$\left(\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{\frac{3}{2}+p} \right) / \right.$$

$$\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2b^p \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right. \right.$$

$$\left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) -$$

$$\left(6a(a+b)^p \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{\frac{1}{2}+p} \right.$$

$$\left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2b^p \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \right. \right. \right.$$

$$\left. \left. \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) +$$

$$\begin{aligned}
& \left(6 (a+b) \left(\frac{1}{2} + p \right) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] (a+2b+a \text{Cos}[2(e+fx)])^p \right. \\
& \quad \left. (\text{Sec}[e+fx]^2)^{\frac{1}{2}+p} \text{Tan}[e+fx]^2 \right) / \left(3 (a+b) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left(2 b p \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left. (a+b) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \left. + \left(3 (a+b) (a+2b+a \text{Cos}[2(e+fx)])^p \right. \right. \\
& \quad \left. \left. (\text{Sec}[e+fx]^2)^{\frac{1}{2}+p} \text{Tan}[e+fx] \left(\frac{2 b p \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx]}{3 (a+b)} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{3} \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) \right) \right) / \\
& \quad \left(3 (a+b) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \left(2 b p \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + (a+b) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Tan}[e+fx]^2 \right) - \\
& \quad \left(3 (a+b) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] (a+2b+a \text{Cos}[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{\frac{1}{2}+p} \right. \\
& \quad \left. \text{Tan}[e+fx] \left(2 \left(2 b p \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] + \right. \right. \right. \\
& \quad \left. \left. (a+b) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \right) \text{Sec}[e+fx]^2 \text{Tan}[e+fx] + \right. \\
& \quad \left. 3 (a+b) \left(\frac{2 b p \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx]}{3 (a+b)} + \right. \right. \\
& \quad \left. \left. \frac{1}{3} \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] \right) \right) + \\
& \quad \left. \text{Tan}[e+fx]^2 \left(2 b p \left(-\frac{1}{5 (a+b)} 6 b (1-p) \text{AppellF1} \left[\frac{5}{2}, -\frac{1}{2}, 2-p, \frac{7}{2}, -\text{Tan}[e+fx]^2, -\frac{b \text{Tan}[e+fx]^2}{a+b} \right] \text{Sec}[e+fx]^2 \text{Tan}[e+fx] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \Big) + \\ & (a+b) \left(\frac{6 b p \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{5 (a+b)} - \right. \\ & \left. \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \Big) \Big) \Big) \Big) \Big) / \\ & \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\ & \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right)^2 \Big) \Big) \end{aligned}$$

■ **Problem 300: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 1+p, -p, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \\ & (\cos[e+fx]^2)^p \sin[e+fx] (\operatorname{Sec}[e+fx]^2 (a+b - a \sin[e+fx]^2))^p \left(1 - \frac{a \sin[e+fx]^2}{a+b}\right)^{-p} \end{aligned}$$

Result (type 6, 1995 leaves):

$$\begin{aligned} & \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right. \\ & \left. (a+2b+a \cos[2(e+fx)])^p \operatorname{Sec}[e+fx] (\operatorname{Sec}[e+fx]^2)^{-\frac{1}{2}+p} (a+b \operatorname{Sec}[e+fx]^2)^p \tan[e+fx] \right) \Big) \Big) / \\ & \left(f \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \right. \\ & \left. \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \right) \Big) \Big) \\ & \left(\left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{\frac{1}{2}+p} \right) \right) \Big) \Big) / \end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
& \left(6 a (a+b) p \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-\frac{1}{2}+p} \right. \\
& \quad \left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \\
& \quad \tan[e+fx]^2 \right) + \left(6 (a+b) \left(-\frac{1}{2} + p \right) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \quad \left. (\sec[e+fx]^2)^{-\frac{1}{2}+p} \tan[e+fx]^2 \right) / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \quad \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \\
& \quad \tan[e+fx]^2 \right) + \left(3 (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \right. \\
& \quad \left. \tan[e+fx] \right) \frac{\left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \right. \\
& \quad \left. 3 (a+b) \right)}{\left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] - \right. \\
& \quad \left. 3 (a+b) \right)} / \left(3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx] \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \right. \\
& \quad \left. \tan[e+fx] \right) \left(2 \left(2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \\
& 3(a+b) \left(\frac{2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{3(a+b)} - \right. \\
& \left. \frac{1}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
& \tan[e+fx]^2 \left(2bp \left(-\frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \left. \left. \frac{3}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - \right. \\
& \left. (a+b) \left(\frac{6bp \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{5(a+b)} - \right. \right. \\
& \left. \left. \frac{9}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) \Big/ \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) \right) \Big)
\end{aligned}$$

■ **Problem 301: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 101 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, p, -p, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \\
& (\cos[e+fx]^2)^p \sin[e+fx] (\operatorname{Sec}[e+fx]^2 (a+b - a \sin[e+fx]^2))^p \left(1 - \frac{a \sin[e+fx]^2}{a+b}\right)^{-p}
\end{aligned}$$

Result (type 6, 1983 leaves):

$$\begin{aligned}
& - \left(\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{3}{2}+p} \right. \right. \\
& \left. \left. (a+b \sec[e+fx]^2)^p \sin[e+fx] \right) / \left(f \left(-3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
& \left. \left(-2bp \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \right. \\
& \left. \left. 3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) \\
& \left(- \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-\frac{1}{2}+p} \right) / \right. \\
& \left(-3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(-2bp \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \\
& \left. \left. -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) + \\
& \left(6a (a+b)^p \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-\frac{3}{2}+p} \right. \\
& \left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(-3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \left(-2bp \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, 1-p, \right. \right. \right. \\
& \left. \left. \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \\
& \left(6 (a+b) \left(-\frac{3}{2} + p \right) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \left. (\sec[e+fx]^2)^{-\frac{3}{2}+p} \tan[e+fx]^2 \right) / \left(-3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \left(-2bp \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right. \\
& \left. \left. 3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) - \left(3 (a+b) (a+2b+a \cos[2(e+fx)])^p \right. \\
& \left. (\sec[e+fx]^2)^{-\frac{3}{2}+p} \tan[e+fx] \right) \left(\frac{2bp \operatorname{AppellF1} \left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \sec[e+fx]^2 \tan[e+fx]}{3 (a+b)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] \right] \right) \right) \right) \right) / \\
& \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + 3(a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) + \\
& \left(3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (\text{Sec}[e+fx]^2)^{\frac{3}{2}+p} \right. \\
& \quad \left. \tan[e+fx] \left(2 \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 3(a+b) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \text{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \quad \left. \left. 3(a+b) \frac{\left(2bp \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] \right)}{3(a+b)} - \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] \right) \right) + \\
& \quad \tan[e+fx]^2 \left(-2bp \left(-\frac{1}{5(a+b)} 6b(1-p) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 2-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \quad \left. \left. \frac{9}{5} \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] \right) + 3(a+b) \right. \\
& \quad \left. \left(\frac{1}{5(a+b)} 6bp \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \quad \left. \left. 3 \text{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \text{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \right) \right) \right) / \\
& \left(-3(a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + 3(a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \right) \right) \right)
\end{aligned}$$

Problem 302: Result more than twice size of optimal antiderivative.

$$\int \cos[e + f x]^3 (a + b \sec[e + f x]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, -1+p, -p, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right]$$

$$\left(\cos[e + f x]^2\right)^p \sin[e + f x] \left(\sec[e + f x]^2 (a + b - a \sin[e + f x]^2)\right)^p \left(1 - \frac{a \sin[e + f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 1987 leaves):

$$\begin{aligned} & - \left(\left(3 (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (\sec[e + f x]^2)^{-\frac{7}{2}+p} \right. \right. \\ & \left. \left. (a+b \sec[e + f x]^2)^p \sin[e + f x] \right) / \left(f \left(-3 (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + \right. \right. \right. \\ & \left. \left. \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + \right. \right. \right. \\ & \left. \left. \left. 5 (a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \right) \tan[e + f x]^2 \right) \right) \\ & \left(- \left(3 (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (\sec[e + f x]^2)^{-\frac{3}{2}+p} \right) / \right. \\ & \left. \left(-3 (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \\ & \left. \left. \left. -\frac{b \tan[e + f x]^2}{a+b}\right] + 5 (a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \right) \tan[e + f x]^2 \right) + \right. \\ & \left. \left(6a (a+b)^p \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e + f x]^2)^{-\frac{5}{2}+p} \right. \right. \\ & \left. \left. \sin[2(e+fx)] \tan[e + f x] \right) / \left(-3 (a+b) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + \left(-2bp \text{AppellF1}\left[\frac{3}{2}, \frac{5}{2}, 1-p, \right. \right. \right. \right. \\ & \left. \left. \left. p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + 5 (a+b) \text{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \right) \tan[e + f x]^2 \right) - \right. \\ & \left. \left(6 (a+b) \left(-\frac{5}{2} + p \right) \text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(\text{Sec}[e + f x]^2 \right)^{-\frac{5}{2}+p} \text{Tan}[e + f x]^2 \Bigg/ \left(-3 (a + b) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] + \right. \\
& \left(-2 b p \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] + \right. \\
& \left. 5 (a + b) \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \right) \text{Tan}[e + f x]^2 \Bigg) - \left(3 (a + b) (a + 2 b + a \text{Cos}[2 (e + f x)])^p \right. \\
& \left. \left(\text{Sec}[e + f x]^2 \right)^{-\frac{5}{2}+p} \text{Tan}[e + f x] \left(\frac{2 b p \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{3 (a + b)} - \right. \right. \\
& \left. \left. \frac{5}{3} \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) \Bigg/ \\
& \left(-3 (a + b) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] + \left(-2 b p \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, \right. \right. \right. \\
& \left. \left. -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] + 5 (a + b) \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \right) \text{Tan}[e + f x]^2 \Bigg) + \\
& \left(3 (a + b) \text{AppellF1} \left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] (a + 2 b + a \text{Cos}[2 (e + f x)])^p \left(\text{Sec}[e + f x]^2 \right)^{-\frac{5}{2}+p} \right. \\
& \text{Tan}[e + f x] \left(2 \left(-2 b p \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] + 5 (a + b) \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \right) \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \\
& \left. 3 (a + b) \left(\frac{2 b p \text{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1 - p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{3 (a + b)} - \frac{5}{3} \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) + \\
& \text{Tan}[e + f x]^2 \left(-2 b p \left(-\frac{1}{5 (a + b)} 6 b (1 - p) \text{AppellF1} \left[\frac{5}{2}, \frac{5}{2}, 2 - p, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \\
& \left. \left. 3 \text{AppellF1} \left[\frac{5}{2}, \frac{7}{2}, 1 - p, \frac{7}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) + 5 (a + b) \right)
\end{aligned}$$

$$\left(\frac{1}{5(a+b)} 6 b p \operatorname{AppellF1} \left[\frac{5}{2}, \frac{7}{2}, 1-p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \frac{21}{5} \operatorname{AppellF1} \left[\frac{5}{2}, \frac{9}{2}, -p, \frac{7}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \Bigg) \Bigg) \Bigg) /$$

$$\left(-3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{5}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \left(-2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{5}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + 5(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg) \Bigg) \Bigg)$$

■ **Problem 303: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e+f x]^5 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[\frac{1}{2}, -2+p, -p, \frac{3}{2}, \operatorname{Sin}[e+f x]^2, \frac{a \operatorname{Sin}[e+f x]^2}{a+b} \right]$$

$$\left(\operatorname{Cos}[e+f x]^2 \right)^p \operatorname{Sin}[e+f x] \left(\operatorname{Sec}[e+f x]^2 (a+b - a \operatorname{Sin}[e+f x]^2) \right)^p \left(1 - \frac{a \operatorname{Sin}[e+f x]^2}{a+b} \right)^{-p}$$

Result (type 6, 1997 leaves):

$$- \left(\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Cos}[e+f x]^4 (a+2b+a \operatorname{Cos}[2(e+f x)])^p \left(\operatorname{Sec}[e+f x]^2 \right)^{-\frac{7}{2}+p} (a+b \operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right) \Bigg) \Bigg) /$$

$$\left(f \left(-3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \left(-2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right) + 7(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg) /$$

$$\left(-3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p \left(\operatorname{Sec}[e+f x]^2 \right)^{-\frac{5}{2}+p} \right) \Bigg) \Bigg) /$$

$$\left(-3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \left(-2 b p \operatorname{AppellF1} \left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \right)$$

$$\begin{aligned}
& -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}] + 7(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Tan}[e+f x]^2) + \\
& \left(6 a(a+b)^p \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] (a+2 b+a \operatorname{Cos}[2(e+f x)])^{-1+p} (\operatorname{Sec}[e+f x]^2)^{-\frac{7}{2}+p} \right. \\
& \left. \operatorname{Sin}[2(e+f x)] \operatorname{Tan}[e+f x]\right) / \left(-3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \left(-2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2\right) - \\
& \left(6(a+b) \left(-\frac{7}{2}+p\right) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] (a+2 b+a \operatorname{Cos}[2(e+f x)])^p \right. \\
& \left. (\operatorname{Sec}[e+f x]^2)^{-\frac{7}{2}+p} \operatorname{Tan}[e+f x]^2\right) / \left(-3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
& \left. \left(-2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \right. \\
& \left. \left. 7(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2\right) - \left(3(a+b)(a+2 b+a \operatorname{Cos}[2(e+f x)])^p \right. \\
& \left. (\operatorname{Sec}[e+f x]^2)^{-\frac{7}{2}+p} \operatorname{Tan}[e+f x] \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3(a+b)} - \right. \right. \\
& \left. \left. \frac{7}{3} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]\right)\right) / \\
& \left(-3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \left(-2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2\right) + \\
& \left(-2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + \right. \\
& \left. 7(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Tan}[e+f x]^2) + \\
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] (a+2 b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-\frac{7}{2}+p} \right. \\
& \left. \operatorname{Tan}[e+f x] \left(2 \left(-2 b p \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] + 7(a+b) \right. \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right.
\end{aligned}$$

$$\begin{aligned}
& 3(a+b) \left(\frac{2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{3(a+b)} - \frac{7}{3} \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
& \tan[e+fx]^2 \left(-2bp \left(-\frac{1}{5(a+b)} 6b(1-p) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{7}{2}, 2-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \left. \left. \frac{21}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + 7(a+b) \right. \\
& \left. \left(\frac{1}{5(a+b)} 6bp \operatorname{AppellF1}\left[\frac{5}{2}, \frac{9}{2}, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \left. \left. \frac{27}{5} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{11}{2}, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \Bigg) \Bigg) \Bigg) / \\
& \left(-3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{7}{2}, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \left(-2bp \operatorname{AppellF1}\left[\frac{3}{2}, \frac{7}{2}, 1-p, \frac{5}{2}, -\tan[e+fx]^2, \right. \right. \right. \\
& \left. \left. \left. -\frac{b \tan[e+fx]^2}{a+b}\right] + 7(a+b) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{9}{2}, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 307: Result more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1}\left[\frac{1}{2}, 1, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx] (a+b+b \tan[e+fx]^2)^p \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 2137 leaves):

$$\begin{aligned}
& \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right. \\
& \left. \cos[e+fx] (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^p (a+b \operatorname{Sec}[e+fx]^2)^p \sin[e+fx] \right) \Bigg) / \\
& \left(f \left(3(a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + 2 \left(bp \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Tan}[e+f x]^2 \\
& \left(\left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-1+p} \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x]^2 \right) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(6 (a+b)^p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x]^2 \right) / \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(6 a (a+b)^p \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Cos}[e+f x] (a+2b+a \operatorname{Cos}[2(e+f x)])^{-1+p} \right. \\
& \left. (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \operatorname{Sin}[2(e+f x)] \right) / \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \right. \\
& \left. 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \right) \\
& \operatorname{Tan}[e+f x]^2 \left. \right) + \left(3 (a+b) \operatorname{Cos}[e+f x] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \right. \\
& \left. \operatorname{Sin}[e+f x] \right) \frac{2 b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} -
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right] \right) \right) \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \right. \\
& \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[e+f x] \right. \right. \\
& \quad \left. \left. \left(4 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \right) \right. \right. \\
& \quad \left. \left. \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + 3 (a+b) \left(\frac{2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) + \right. \\
& \quad 2 \operatorname{Tan}[e+f x]^2 \left(b p \left(-\frac{6}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a+b)} 6 b (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 2-p, 1, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) - \right. \\
& \quad \left. \left. (a+b) \left(\frac{6 b p \operatorname{AppellF1} \left[\frac{5}{2}, 1-p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{5 (a+b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{12}{5} \operatorname{AppellF1} \left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \right) \right) \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 308: Result more than twice size of optimal antiderivative.**

$$\int \cos[e + f x]^2 (a + b \sec[e + f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \tan[e + f x] (a + b + b \tan[e + f x]^2)^p \left(1 + \frac{b \tan[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 6, 1914 leaves):

$$\begin{aligned} & \left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right. \\ & \quad \left. \cos[e + f x] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} (a + b \sec[e + f x]^2)^p \sin[e + f x] \right) / \\ & \left(f \left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + 2 \left(b^p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \right. \\ & \quad \left. \left. 2 (a + b) \text{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \tan[e + f x]^2 \right) \\ & \left(\left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-1+p} \right) / \right. \\ & \quad \left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + 2 \left(b^p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - \right. \right. \\ & \quad \left. \left. 2 (a + b) \text{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \tan[e + f x]^2 - \right. \\ & \quad \left(6 a (a + b)^p \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] (a + 2b + a \cos[2(e + f x)])^{-1+p} (\sec[e + f x]^2)^{-2+p} \right. \\ & \quad \left. \sin[2(e + f x)] \tan[e + f x] \right) / \left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \\ & \quad \left. 2 \left(b^p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - 2 (a + b) \text{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \right. \\ & \quad \left. \tan[e + f x]^2 \right) + \left(6 (a + b) (-2 + p) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] (a + 2b + a \cos[2(e + f x)])^p \right. \\ & \quad \left. (\sec[e + f x]^2)^{-2+p} \tan[e + f x]^2 \right) / \left(3 (a + b) \text{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \\ & \quad \left. 2 \left(b^p \text{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] - 2 (a + b) \text{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b}\right] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \tan[e + f x]^2 \Big) + \left(3 (a + b) (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} \right. \\
& \tan[e + f x] \left[\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x]}{3 (a + b)} - \right. \\
& \left. \left. \frac{4}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x] \right] \right) / \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] - \right. \right. \\
& \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \right) \tan[e + f x]^2 \right) - \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-2+p} \right. \\
& \tan[e + f x] \left[4 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] - \right. \right. \\
& \left. \left. 2 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \right) \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \left. 3 (a + b) \left[\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x]}{3 (a + b)} - \right. \right. \\
& \left. \left. \frac{4}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x] \right] \right) + \\
& 2 \tan[e + f x]^2 \left(b p \left(-\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 2, 2-p, \frac{7}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \frac{7}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x] \right) - \right. \\
& \left. 2 (a + b) \left[\frac{6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 3, 1-p, \frac{7}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a+b}\right] \sec[e + f x]^2 \tan[e + f x]}{5 (a + b)} - \right. \right.
\end{aligned}$$

$$\left. \left. \left. \left. \frac{18}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 4, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right] \right) \right) \right) \right) /$$

$$\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 2, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 2, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right.$$

$$\left. \left. 2(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 3, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right)^2 \right)$$

■ **Problem 309: Result more than twice size of optimal antiderivative.**

$$\int \cos[e+fx]^4 (a+b \operatorname{Sec}[e+fx]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \tan[e+fx] (a+b+b \tan[e+fx]^2)^p \left(1 + \frac{b \tan[e+fx]^2}{a+b} \right)^{-p}$$

Result (type 6, 1912 leaves):

$$\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right.$$

$$\left. \cos[e+fx]^3 (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-3+p} (a+b \operatorname{Sec}[e+fx]^2)^p \sin[e+fx] \right) /$$

$$\left(f \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right. \right.$$

$$\left. \left. 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right)$$

$$\left(\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^p (\operatorname{Sec}[e+fx]^2)^{-2+p} \right) / \right.$$

$$\left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] - \right. \right.$$

$$\left. \left. 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] \right) \tan[e+fx]^2 \right) -$$

$$\left(6a(a+b)^p \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\operatorname{Sec}[e+fx]^2)^{-3+p} \right.$$

$$\left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b} \right] + \right.$$

$$\begin{aligned}
& 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \\
& \operatorname{Tan}[e+f x]^2 + \left(6(a+b)(-3+p) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p \right. \\
& \left. (\operatorname{Sec}[e+f x]^2)^{-3+p} \operatorname{Tan}[e+f x]^2 \right) / \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \right. \\
& \left. 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \right. \\
& \left. \operatorname{Tan}[e+f x]^2 \right) + \left(3(a+b)(a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-3+p} \right. \\
& \left. \operatorname{Tan}[e+f x] \left(\frac{2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3(a+b)} - \right. \right. \\
& \left. \left. 2 \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - \right. \right. \\
& \left. \left. 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(3(a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^{-3+p} \right. \\
& \left. \operatorname{Tan}[e+f x] \left(4 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - \right. \right. \\
& \left. \left. 3(a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] + \right. \\
& \left. 3(a+b) \left(\frac{2 b p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3(a+b)} - \right. \right. \\
& \left. \left. 2 \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Tan}[e + f x]^2 \left(b^p \left(-\frac{1}{5(a+b)} 6 b (1-p) \operatorname{AppellF1} \left[\frac{5}{2}, 3, 2-p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \\
& \left. \frac{18}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 4, 1-p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) - \\
& 3 (a+b) \left(\frac{6 b^p \operatorname{AppellF1} \left[\frac{5}{2}, 4, 1-p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{5 (a+b)} - \right. \\
& \left. \frac{24}{5} \operatorname{AppellF1} \left[\frac{5}{2}, 5, -p, \frac{7}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 3, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 3, 1-p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] - \right. \right. \\
& \left. \left. 3 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 4, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Tan}[e + f x]^2 \right)^2 \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cos}[e + f x]^6 (a + b \operatorname{Sec}[e + f x]^2)^p dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{1}{f} \operatorname{AppellF1} \left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Tan}[e + f x] (a + b + b \operatorname{Tan}[e + f x]^2)^p \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right)^{-p}$$

Result (type 6, 1914 leaves):

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \right. \\
& \left. \operatorname{Cos}[e + f x]^5 (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^p (\operatorname{Sec}[e + f x]^2)^{-4+p} (a + b \operatorname{Sec}[e + f x]^2)^p \operatorname{Sin}[e + f x] \right) \Bigg) / \\
& \left(f \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] - \right. \right. \right. \\
& \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] \operatorname{Tan}[e + f x]^2 \right) \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(\left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a+b} \right] (a + 2 b + a \operatorname{Cos}[2 (e + f x)])^p (\operatorname{Sec}[e + f x]^2)^{-3+p} \right) \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
& \left(6 a (a+b) p \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^{-4+p} \right. \\
& \quad \left. \sin[2(e+fx)] \tan[e+fx] \right) / \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
& \quad \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \right. \\
& \quad \left. \tan[e+fx]^2 \right) + \left(6 (a+b) (-4+p) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p \right. \\
& \quad \left. (\sec[e+fx]^2)^{-4+p} \tan[e+fx]^2 \right) / \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + \right. \\
& \quad \left. 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \right. \\
& \quad \left. \tan[e+fx]^2 \right) + \left(3 (a+b) (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-4+p} \right. \\
& \quad \left. \tan[e+fx] \right) \frac{\left(2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] - \right. \\
& \quad \left. \frac{8}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right)}{3 (a+b)} - \\
& \quad \left. \left. \left. \frac{8}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \sec[e+fx]^2 \tan[e+fx] \right) \right) / \right. \\
& \quad \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \right) \tan[e+fx]^2 \right) - \\
& \quad \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (a+2b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{-4+p} \right. \\
& \quad \left. \tan[e+fx] \right) \left(4 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] + \\
& 3 (a+b) \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{3 (a+b)} - \right. \\
& \left. \frac{8}{3} \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) + \\
& 2 \tan[e+fx]^2 \left(b p \left(-\frac{1}{5 (a+b)} 6 b (1-p) \operatorname{AppellF1}\left[\frac{5}{2}, 4, 2-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] - \right. \right. \\
& \left. \left. \frac{24}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 5, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) - \right. \\
& \left. 4 (a+b) \left(\frac{6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 5, 1-p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx]}{5 (a+b)} - \right. \right. \\
& \left. \left. 6 \operatorname{AppellF1}\left[\frac{5}{2}, 6, -p, \frac{7}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \operatorname{Sec}[e+fx]^2 \tan[e+fx] \right) \right) \Big/ \\
& \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, 4, -p, \frac{3}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 4, 1-p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] - \right. \right. \\
& \left. \left. 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, 5, -p, \frac{5}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] \tan[e+fx]^2 \right) \right)
\end{aligned}$$

■ **Problem 327: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^2 dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{b(2a+b) \operatorname{Log}[\cos[e+fx]]}{f} + \frac{(a+b)^2 \operatorname{Log}[\sin[e+fx]]}{f} + \frac{b^2 \operatorname{Sec}[e+fx]^2}{2f}$$

Result (type 3, 180 leaves):

$$\begin{aligned}
& \frac{1}{4f} (2b^2 + 2ia^2fx + 4iabfx + 2ib^2fx - 4i(a+b)^2 \operatorname{ArcTan}[\tan[e+fx]] \cos[e+fx]^2 - \\
& 4ab \operatorname{Log}[\cos[e+fx]] - 2b^2 \operatorname{Log}[\cos[e+fx]] + a^2 \operatorname{Log}[\sin[e+fx]^2] + 2ab \operatorname{Log}[\sin[e+fx]^2] + b^2 \operatorname{Log}[\sin[e+fx]^2] + \\
& \cos[2(e+fx)] (-2b(2a+b) \operatorname{Log}[\cos[e+fx]] + (a+b)^2 (2ifx + \operatorname{Log}[\sin[e+fx]^2]))) \operatorname{Sec}[e+fx]^2
\end{aligned}$$

- **Problem 328: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{(a+b)^2 \operatorname{Csc}[e+fx]^2}{2f} - \frac{b^2 \operatorname{Log}[\operatorname{Cos}[e+fx]]}{f} - \frac{(a^2-b^2) \operatorname{Log}[\operatorname{Sin}[e+fx]]}{f}$$

Result (type 3, 163 leaves):

$$\frac{1}{4f} \operatorname{Csc}[e+fx]^2 \left(-2a^2 - 4ab - 2b^2 - 2i a^2 f x + 2i b^2 f x - 2b^2 \operatorname{Log}[\operatorname{Cos}[e+fx]] - a^2 \operatorname{Log}[\operatorname{Sin}[e+fx]^2] + b^2 \operatorname{Log}[\operatorname{Sin}[e+fx]^2] + \right. \\ \left. \operatorname{Cos}[2(e+fx)] \left(2b^2 \operatorname{Log}[\operatorname{Cos}[e+fx]] + (a^2-b^2) \left(2i f x + \operatorname{Log}[\operatorname{Sin}[e+fx]^2] \right) \right) + 4i (a^2-b^2) \operatorname{ArcTan}[\operatorname{Tan}[e+fx]] \operatorname{Sin}[e+fx]^2 \right)$$

- **Problem 329: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^5 (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 51 leaves, 4 steps):

$$\frac{a(a+b) \operatorname{Csc}[e+fx]^2}{f} - \frac{(a+b)^2 \operatorname{Csc}[e+fx]^4}{4f} + \frac{a^2 \operatorname{Log}[\operatorname{Sin}[e+fx]]}{f}$$

Result (type 3, 132 leaves):

$$\left((b+a \operatorname{Cos}[e+fx]^2)^2 \left(-4i a^2 \operatorname{ArcTan}[\operatorname{Tan}[e+fx]] \operatorname{Cos}[e+fx]^4 + 4a(a+b) \operatorname{Cos}[e+fx]^2 \operatorname{Cot}[e+fx]^2 - \right. \right. \\ \left. \left. (a+b)^2 \operatorname{Cot}[e+fx]^4 + 2a^2 \operatorname{Cos}[e+fx]^4 \left(2i f x + \operatorname{Log}[\operatorname{Sin}[e+fx]^2] \right) \right) \operatorname{Sec}[e+fx]^4 \right) / (f(a+2b+a \operatorname{Cos}[2(e+fx)]))^2$$

- **Problem 330: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 \operatorname{Tan}[e + f x]^6 dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-a^2 x + \frac{a^2 \operatorname{Tan}[e+fx]}{f} - \frac{a^2 \operatorname{Tan}[e+fx]^3}{3f} + \frac{a^2 \operatorname{Tan}[e+fx]^5}{5f} + \frac{b(2a+b) \operatorname{Tan}[e+fx]^7}{7f} + \frac{b^2 \operatorname{Tan}[e+fx]^9}{9f}$$

Result (type 3, 275 leaves):

$$\frac{1}{315f(a+2b+a \operatorname{Cos}[2(e+fx)])^2} \\ 4(b+a \operatorname{Cos}[e+fx]^2)^2 \operatorname{Sec}[e+fx]^9 \left(315a^2 f x \operatorname{Cos}[e+fx]^9 - 35b^2 \operatorname{Sec}[e] \operatorname{Sin}[fx] - 5(18a-19b)b \operatorname{Cos}[e+fx]^2 \operatorname{Sec}[e] \operatorname{Sin}[fx] - \right. \\ 3(21a^2-90ab+25b^2) \operatorname{Cos}[e+fx]^4 \operatorname{Sec}[e] \operatorname{Sin}[fx] + (231a^2-270ab+5b^2) \operatorname{Cos}[e+fx]^6 \operatorname{Sec}[e] \operatorname{Sin}[fx] - \\ (483a^2-90ab-10b^2) \operatorname{Cos}[e+fx]^8 \operatorname{Sec}[e] \operatorname{Sin}[fx] - 35b^2 \operatorname{Cos}[e+fx] \operatorname{Tan}[e] - 5(18a-19b)b \operatorname{Cos}[e+fx]^3 \operatorname{Tan}[e] - \\ \left. 3(21a^2-90ab+25b^2) \operatorname{Cos}[e+fx]^5 \operatorname{Tan}[e] + (231a^2-270ab+5b^2) \operatorname{Cos}[e+fx]^7 \operatorname{Tan}[e] \right)$$

■ **Problem 331: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 \operatorname{Tan}[e + f x]^4 dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$a^2 x - \frac{a^2 \operatorname{Tan}[e + f x]}{f} + \frac{a^2 \operatorname{Tan}[e + f x]^3}{3 f} + \frac{b (2 a + b) \operatorname{Tan}[e + f x]^5}{5 f} + \frac{b^2 \operatorname{Tan}[e + f x]^7}{7 f}$$

Result (type 3, 395 leaves):

$$\frac{1}{13440 f} \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^7$$

$$\begin{aligned} & (3675 a^2 f x \operatorname{Cos}[f x] + 3675 a^2 f x \operatorname{Cos}[2 e + f x] + 2205 a^2 f x \operatorname{Cos}[2 e + 3 f x] + 2205 a^2 f x \operatorname{Cos}[4 e + 3 f x] + 735 a^2 f x \operatorname{Cos}[4 e + 5 f x] + \\ & 735 a^2 f x \operatorname{Cos}[6 e + 5 f x] + 105 a^2 f x \operatorname{Cos}[6 e + 7 f x] + 105 a^2 f x \operatorname{Cos}[8 e + 7 f x] - 5320 a^2 \operatorname{Sin}[f x] + 1680 a b \operatorname{Sin}[f x] + 840 b^2 \operatorname{Sin}[f x] + \\ & 4480 a^2 \operatorname{Sin}[2 e + f x] - 1260 a b \operatorname{Sin}[2 e + f x] + 420 b^2 \operatorname{Sin}[2 e + f x] - 3780 a^2 \operatorname{Sin}[2 e + 3 f x] + 924 a b \operatorname{Sin}[2 e + 3 f x] - 168 b^2 \operatorname{Sin}[2 e + 3 f x] + \\ & 2100 a^2 \operatorname{Sin}[4 e + 3 f x] - 840 a b \operatorname{Sin}[4 e + 3 f x] - 420 b^2 \operatorname{Sin}[4 e + 3 f x] - 1540 a^2 \operatorname{Sin}[4 e + 5 f x] + 168 a b \operatorname{Sin}[4 e + 5 f x] + \\ & 84 b^2 \operatorname{Sin}[4 e + 5 f x] + 420 a^2 \operatorname{Sin}[6 e + 5 f x] - 420 a b \operatorname{Sin}[6 e + 5 f x] - 280 a^2 \operatorname{Sin}[6 e + 7 f x] + 84 a b \operatorname{Sin}[6 e + 7 f x] + 12 b^2 \operatorname{Sin}[6 e + 7 f x]) \end{aligned}$$

■ **Problem 332: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 \operatorname{Tan}[e + f x]^2 dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$-a^2 x + \frac{a^2 \operatorname{Tan}[e + f x]}{f} + \frac{b (2 a + b) \operatorname{Tan}[e + f x]^3}{3 f} + \frac{b^2 \operatorname{Tan}[e + f x]^5}{5 f}$$

Result (type 3, 281 leaves):

$$-\frac{1}{480 f} \operatorname{Sec}[e] \operatorname{Sec}[e + f x]^5 (150 a^2 f x \operatorname{Cos}[f x] + 150 a^2 f x \operatorname{Cos}[2 e + f x] + 75 a^2 f x \operatorname{Cos}[2 e + 3 f x] + 75 a^2 f x \operatorname{Cos}[4 e + 3 f x] + 15 a^2 f x \operatorname{Cos}[4 e + 5 f x] + 15 a^2 f x \operatorname{Cos}[6 e + 5 f x] - 180 a^2 \operatorname{Sin}[f x] + 80 a b \operatorname{Sin}[f x] - 20 b^2 \operatorname{Sin}[f x] + 120 a^2 \operatorname{Sin}[2 e + f x] - 120 a b \operatorname{Sin}[2 e + f x] - 60 b^2 \operatorname{Sin}[2 e + f x] - 120 a^2 \operatorname{Sin}[2 e + 3 f x] + 40 a b \operatorname{Sin}[2 e + 3 f x] + 20 b^2 \operatorname{Sin}[2 e + 3 f x] + 30 a^2 \operatorname{Sin}[4 e + 3 f x] - 60 a b \operatorname{Sin}[4 e + 3 f x] - 30 a^2 \operatorname{Sin}[4 e + 5 f x] + 20 a b \operatorname{Sin}[4 e + 5 f x] + 4 b^2 \operatorname{Sin}[4 e + 5 f x])$$

■ **Problem 333: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^2 dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$a^2 x + \frac{b (2 a + b) \operatorname{Tan}[e + f x]}{f} + \frac{b^2 \operatorname{Tan}[e + f x]^3}{3 f}$$

Result (type 3, 106 leaves):

$$\frac{(4 (b + a \cos [e + f x])^2 \sec [e + f x]^3 (3 a^2 f x \cos [e + f x]^3 + b^2 \sec [e] \sin [f x] + 2 b (3 a + b) \cos [e + f x]^2 \sec [e] \sin [f x] + b^2 \cos [e + f x] \tan [e]))}{(3 f (a + 2 b + a \cos [2 (e + f x)])^2)}$$

- **Problem 334: Result more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^2 (a + b \sec [e + f x]^2)^2 dx$$

Optimal (type 3, 36 leaves, 4 steps) :

$$-a^2 x - \frac{(a + b)^2 \cot [e + f x]}{f} + \frac{b^2 \tan [e + f x]}{f}$$

Result (type 3, 82 leaves) :

$$-\left(4 (b + a \cos [e + f x])^2 \sec [e + f x] (a^2 f x \cos [e + f x] - ((a + b)^2 \cot [e + f x] \csc [e] + b^2 \sec [e]) \sin [f x])\right) / (f (a + 2 b + a \cos [2 (e + f x)])^2)$$

- **Problem 335: Result more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^4 (a + b \sec [e + f x]^2)^2 dx$$

Optimal (type 3, 45 leaves, 4 steps) :

$$a^2 x + \frac{(a^2 - b^2) \cot [e + f x]}{f} - \frac{(a + b)^2 \cot [e + f x]^3}{3 f}$$

Result (type 3, 160 leaves) :

$$\frac{1}{24 f}$$

$$\csc [e] \csc [e + f x]^3 (9 a^2 f x \cos [f x] - 9 a^2 f x \cos [2 e + f x] - 3 a^2 f x \cos [2 e + 3 f x] + 3 a^2 f x \cos [4 e + 3 f x] - 12 a^2 \sin [f x] + 12 b^2 \sin [f x] - 12 a^2 \sin [2 e + f x] - 12 a b \sin [2 e + f x] + 8 a^2 \sin [2 e + 3 f x] + 4 a b \sin [2 e + 3 f x] - 4 b^2 \sin [2 e + 3 f x])$$

- **Problem 336: Result more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^6 (a + b \sec [e + f x]^2)^2 dx$$

Optimal (type 3, 65 leaves, 4 steps) :

$$-a^2 x - \frac{a^2 \cot [e + f x]}{f} + \frac{(a^2 - b^2) \cot [e + f x]^3}{3 f} - \frac{(a + b)^2 \cot [e + f x]^5}{5 f}$$

Result (type 3, 256 leaves) :

$$\frac{1}{480 f} \left(\text{Csc}[e] \text{Csc}[e + f x]^5 \left(-150 a^2 f x \text{Cos}[f x] + 150 a^2 f x \text{Cos}[2 e + f x] + 75 a^2 f x \text{Cos}[2 e + 3 f x] - 75 a^2 f x \text{Cos}[4 e + 3 f x] - 15 a^2 f x \text{Cos}[4 e + 5 f x] + 15 a^2 f x \text{Cos}[6 e + 5 f x] + 280 a^2 \text{Sin}[f x] + 120 a b \text{Sin}[f x] + 20 b^2 \text{Sin}[f x] + 180 a^2 \text{Sin}[2 e + f x] - 60 b^2 \text{Sin}[2 e + f x] - 140 a^2 \text{Sin}[2 e + 3 f x] + 20 b^2 \text{Sin}[2 e + 3 f x] - 90 a^2 \text{Sin}[4 e + 3 f x] - 60 a b \text{Sin}[4 e + 3 f x] + 46 a^2 \text{Sin}[4 e + 5 f x] + 12 a b \text{Sin}[4 e + 5 f x] - 4 b^2 \text{Sin}[4 e + 5 f x] \right) \right)$$

■ **Problem 337: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^5}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 69 leaves, 4 steps):

$$\frac{(a + 2 b) \text{Log}[\text{Cos}[e + f x]]}{b^2 f} - \frac{(a + b)^2 \text{Log}[b + a \text{Cos}[e + f x]^2]}{2 a b^2 f} + \frac{\text{Sec}[e + f x]^2}{2 b f}$$

Result (type 3, 180 leaves):

$$\frac{1}{8 a b^2 f (a + b \text{Sec}[e + f x]^2)} \left((a + 2 b + a \text{Cos}[2 (e + f x)]) \left(2 a b + 2 a (a + 2 b) \text{Log}[\text{Cos}[e + f x]] - a^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] - 2 a b \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] - b^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + \text{Cos}[2 (e + f x)] \left(2 a (a + 2 b) \text{Log}[\text{Cos}[e + f x]] - (a + b)^2 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] \right) \right) \text{Sec}[e + f x]^4 \right)$$

■ **Problem 342: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^5}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{(2 a + 3 b) \text{Csc}[e + f x]^2}{2 (a + b)^2 f} - \frac{\text{Csc}[e + f x]^4}{4 (a + b) f} + \frac{b^3 \text{Log}[b + a \text{Cos}[e + f x]^2]}{2 a (a + b)^3 f} + \frac{(a^2 + 3 a b + 3 b^2) \text{Log}[\text{Sin}[e + f x]]}{(a + b)^3 f}$$

Result (type 3, 464 leaves):

$$\frac{1}{32 a (a + b)^3 f (-1 + \text{Cot}[e]^2) (a + b \text{Sec}[e + f x]^2)} \left(\text{Cos}[2 e] (a + 2 b + a \text{Cos}[2 (e + f x)]) \text{Csc}[e]^2 \text{Csc}[e + f x]^4 \text{Sec}[e + f x]^2 \left(4 a^3 + 12 a^2 b + 8 a b^2 + 6 i a^3 f x + 18 i a^2 b f x + 18 i a b^2 f x + 2 i a^3 f x \text{Cos}[4 (e + f x)] + 6 i a^2 b f x \text{Cos}[4 (e + f x)] + 6 i a b^2 f x \text{Cos}[4 (e + f x)] + 3 b^3 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + b^3 \text{Cos}[4 (e + f x)] \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + 3 a^3 \text{Log}[\text{Sin}[e + f x]^2] + 9 a^2 b \text{Log}[\text{Sin}[e + f x]^2] + 9 a b^2 \text{Log}[\text{Sin}[e + f x]^2] + a^3 \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 3 a^2 b \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 3 a b^2 \text{Cos}[4 (e + f x)] \text{Log}[\text{Sin}[e + f x]^2] + 4 \text{Cos}[2 (e + f x)] \left(-b^3 \text{Log}[a + 2 b + a \text{Cos}[2 (e + f x)]] + a (a^2 (-2 - 2 i f x) + 3 b^2 (-1 - 2 i f x) + a b (-5 - 6 i f x) - (a^2 + 3 a b + 3 b^2) \text{Log}[\text{Sin}[e + f x]^2]) \right) \right) - 16 i a (a^2 + 3 a b + 3 b^2) \text{ArcTan}[\text{Tan}[e + f x]] \text{Sin}[e + f x]^4 \right)$$

- **Problem 343: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^6}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 83 leaves, 7 steps):

$$-\frac{x}{a} + \frac{(a+b)^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a b^{5/2} f} - \frac{(a+2b) \tan[e+fx]}{b^2 f} + \frac{\tan[e+fx]^3}{3 b f}$$

Result (type 3, 229 leaves):

$$\frac{1}{6 (a + b \sec[e + f x]^2)} (a + 2b + a \cos[2(e + f x)]) \sec[e + f x]^2 \left(-\frac{3x}{a} - \frac{3(a+b)^{5/2} \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4}\right]}{a b^2 f \sqrt{b} (\cos[e] - i \sin[e])^4} (\cos[2e] - i \sin[2e]) - \frac{(3a + 7b) \sec[e] \sec[e + f x] \sin[fx]}{b^2 f} + \frac{\sec[e] \sec[e + f x]^3 \sin[fx]}{b f} + \frac{\sec[e + f x]^2 \tan[e]}{b f} \right)$$

- **Problem 344: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^4}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 59 leaves, 6 steps):

$$\frac{x}{a} - \frac{(a+b)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a b^{3/2} f} + \frac{\tan[e+fx]}{b f}$$

Result (type 3, 206 leaves):

$$\left((a + 2b + a \cos[2(e + f x)]) \sec[e + f x]^2 \left((a+b)^2 \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a+2b) \sin[fx] + a \sin[2e+fx])}{2\sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4}\right]} (\cos[2e] - i \sin[2e]) + \sqrt{a+b} \sqrt{b} (i \cos[e] + \sin[e])^4 (b f x + a \sec[e] \sec[e + f x] \sin[fx]) \right) \right) / \left(2 a b \sqrt{a+b} f (a + b \sec[e + f x]^2) \sqrt{b} (\cos[e] - i \sin[e])^4 \right)$$

- **Problem 345: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 46 leaves, 5 steps) :

$$-\frac{x}{a} + \frac{\sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{a \sqrt{b} f}$$

Result (type 3, 184 leaves) :

$$-\left((a + 2b + a \cos[2(e + f x)]) \sec[e + f x]^2 \left(\sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + \right. \right. \\ \left. \left. (a + b) \operatorname{ArcTan}\left[\frac{\sec[f x] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[f x] + a \sin[2e + f x])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\ \left(2 a \sqrt{a+b} f (a + b \sec[e + f x]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

- **Problem 346: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{a + b \sec[e + f x]^2} dx$$

Optimal (type 3, 45 leaves, 3 steps) :

$$\frac{x}{a} + \frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{a+b} \cot[e+fx]}{\sqrt{b}}\right]}{a \sqrt{a+b} f}$$

Result (type 3, 182 leaves) :

$$\left((a + 2b + a \cos[2(e + f x)]) \sec[e + f x]^2 \left(\sqrt{a+b} f x \sqrt{b (\cos[e] - i \sin[e])^4} + \right. \right. \\ \left. \left. b \operatorname{ArcTan}\left[\frac{\sec[f x] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[f x] + a \sin[2e + f x])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\ \left(2 a \sqrt{a+b} f (a + b \sec[e + f x]^2) \sqrt{b (\cos[e] - i \sin[e])^4} \right)$$

- **Problem 347: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^2}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 62 leaves, 6 steps):

$$-\frac{x}{a} + \frac{b^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a (a+b)^{3/2} f} - \frac{\text{Cot}[e+fx]}{(a+b) f}$$

Result (type 3, 204 leaves):

$$-\left((a + 2b + a \text{Cos}[2(e + f x)]) \text{Sec}[e + f x]^2 \right. \\ \left. \left(b^2 \text{ArcTan}\left[\frac{\text{Sec}[fx] (\text{Cos}[2e] - i \text{Sin}[2e]) (- (a + 2b) \text{Sin}[fx] + a \text{Sin}[2e + fx])}{2 \sqrt{a+b} \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4} \right] (\text{Cos}[2e] - i \text{Sin}[2e]) + \right. \right. \\ \left. \left. \sqrt{a+b} \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4 ((a+b)fx - a \text{Csc}[e] \text{Csc}[e+fx] \text{Sin}[fx]) \right) \right) \Bigg) / \\ \left(2a (a+b)^{3/2} f (a+b \text{Sec}[e+fx]^2) \sqrt{b} (\text{Cos}[e] - i \text{Sin}[e])^4 \right)$$

- **Problem 348: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^4}{a + b \text{Sec}[e + f x]^2} dx$$

Optimal (type 3, 86 leaves, 7 steps):

$$\frac{x}{a} - \frac{b^{5/2} \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e+fx]}{\sqrt{a+b}}\right]}{a (a+b)^{5/2} f} + \frac{(a+2b) \text{Cot}[e+fx]}{(a+b)^2 f} - \frac{\text{Cot}[e+fx]^3}{3(a+b) f}$$

Result (type 3, 587 leaves):

$$\begin{aligned}
& \frac{x (a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^2}{2a (a + b \sec[e + fx]^2)} - \frac{(a + 2b + a \cos[2e + 2fx]) \cot[e] \csc[e + fx]^2 \sec[e + fx]^2}{6(a + b) f (a + b \sec[e + fx]^2)} + \\
& \left((a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^2 \left(\left(b^3 \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \right. \\
& \quad \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(2a \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) - \right. \\
& \quad \left. \left(i b^3 \operatorname{ArcTan}\left[\sec[fx]\right] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - i b \sin[4e]}} \right) \right. \right. \\
& \quad \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(2a \sqrt{a+b} f \sqrt{b \cos[4e] - i b \sin[4e]} \right) \right) / \\
& \left((a + b)^2 (a + b \sec[e + fx]^2) \right) + \frac{(a + 2b + a \cos[2e + 2fx]) \csc[e] \csc[e + fx]^3 \sec[e + fx]^2 \sin[fx]}{6(a + b) f (a + b \sec[e + fx]^2)} + \\
& \frac{(a + 2b + a \cos[2e + 2fx]) \csc[e] \csc[e + fx] \sec[e + fx]^2 (-4a \sin[fx] - 7b \sin[fx])}{6(a + b)^2 f (a + b \sec[e + fx]^2)}
\end{aligned}$$

■ **Problem 349: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + fx]^6}{a + b \sec[e + fx]^2} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$-\frac{x}{a} + \frac{b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{a (a + b)^{7/2} f} - \frac{(a^2 + 3ab + 3b^2) \cot[e + fx]}{(a + b)^3 f} + \frac{(a + 2b) \cot[e + fx]^3}{3 (a + b)^2 f} - \frac{\cot[e + fx]^5}{5 (a + b) f}$$

Result (type 3, 671 leaves):

$$\frac{1}{960 a (a+b)^3 f (a+b \operatorname{Sec}[e+f x])^2} \left(\frac{480 b^4 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[f x] (\cos[2 e]-i \sin[2 e]) (-(a+2 b) \sin[f x]+a \sin[2 e+f x])}{2 \sqrt{a+b} \sqrt{b (\cos[e]-i \sin[e])^4}}\right] (\cos[2 e]-i \sin[2 e])}{\sqrt{a+b} \sqrt{b (\cos[e]-i \sin[e])^4}} + \right.$$

$$\left. \begin{aligned} & \operatorname{Csc}[e] \operatorname{Csc}[e+f x]^5 (-150 (a+b)^3 f x \cos[f x] + 150 (a+b)^3 f x \cos[2 e+f x] + 75 a^3 f x \cos[2 e+3 f x] + 225 a^2 b f x \cos[2 e+3 f x] + \\ & 225 a b^2 f x \cos[2 e+3 f x] + 75 b^3 f x \cos[2 e+3 f x] - 75 a^3 f x \cos[4 e+3 f x] - 225 a^2 b f x \cos[4 e+3 f x] - \\ & 225 a b^2 f x \cos[4 e+3 f x] - 75 b^3 f x \cos[4 e+3 f x] - 15 a^3 f x \cos[4 e+5 f x] - 45 a^2 b f x \cos[4 e+5 f x] - \\ & 45 a b^2 f x \cos[4 e+5 f x] - 15 b^3 f x \cos[4 e+5 f x] + 15 a^3 f x \cos[6 e+5 f x] + 45 a^2 b f x \cos[6 e+5 f x] + 45 a b^2 f x \cos[6 e+5 f x] + \\ & 15 b^3 f x \cos[6 e+5 f x] + 280 a^3 \sin[f x] + 780 a^2 b \sin[f x] + 680 a b^2 \sin[f x] + 180 a^3 \sin[2 e+f x] + 540 a^2 b \sin[2 e+f x] + \\ & 480 a b^2 \sin[2 e+f x] - 140 a^3 \sin[2 e+3 f x] - 420 a^2 b \sin[2 e+3 f x] - 400 a b^2 \sin[2 e+3 f x] - 90 a^3 \sin[4 e+3 f x] - \\ & 240 a^2 b \sin[4 e+3 f x] - 180 a b^2 \sin[4 e+3 f x] + 46 a^3 \sin[4 e+5 f x] + 132 a^2 b \sin[4 e+5 f x] + 116 a b^2 \sin[4 e+5 f x] \end{aligned} \right)$$

■ **Problem 354: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^3}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 111 leaves, 4 steps):

$$-\frac{b^3}{2 a^2 (a+b)^2 f (b+a \cos[e+f x])^2} - \frac{\operatorname{Csc}[e+f x]^2}{2 (a+b)^2 f} - \frac{b^2 (3 a+b) \operatorname{Log}[b+a \cos[e+f x]^2]}{2 a^2 (a+b)^3 f} - \frac{(a+3 b) \operatorname{Log}[\sin[e+f x]]}{(a+b)^3 f}$$

Result (type 3, 306 leaves):

$$-\frac{1}{8 a^2 (a+b)^3 f (a+2 b+a \cos[2 (e+f x)])} \left(\operatorname{Csc}[e+f x]^2 (4 a^4 + 12 a^3 b + 8 a^2 b^2 + 4 a b^3 + 4 b^4 + 3 a^2 b^2 \operatorname{Log}[a+2 b+a \cos[2 (e+f x)]] + 13 a b^3 \operatorname{Log}[a+2 b+a \cos[2 (e+f x)]] + \right.$$

$$4 b^4 \operatorname{Log}[a+2 b+a \cos[2 (e+f x)]] + 2 a^4 \operatorname{Log}[\sin[e+f x]] + 14 a^3 b \operatorname{Log}[\sin[e+f x]] + 24 a^2 b^2 \operatorname{Log}[\sin[e+f x]] -$$

$$a \cos[4 (e+f x)] (b^2 (3 a+b) \operatorname{Log}[a+2 b+a \cos[2 (e+f x)]] + 2 a^2 (a+3 b) \operatorname{Log}[\sin[e+f x]]) +$$

$$\left. 4 \cos[2 (e+f x)] (a^4 + a^3 b - a b^3 - b^4 - b^3 (3 a+b) \operatorname{Log}[a+2 b+a \cos[2 (e+f x)]] - 2 a^2 b (a+3 b) \operatorname{Log}[\sin[e+f x]]) \right)$$

■ **Problem 355: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^5}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$\frac{b^4}{2 a^2 (a+b)^3 f (b+a \cos[e+f x])^2} + \frac{(a+2 b) \operatorname{Csc}[e+f x]^2}{(a+b)^3 f} -$$

$$\frac{\operatorname{Csc}[e+f x]^4}{4 (a+b)^2 f} + \frac{b^3 (4 a+b) \operatorname{Log}[b+a \cos[e+f x]^2]}{2 a^2 (a+b)^4 f} + \frac{(a^2+4 a b+6 b^2) \operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^4 f}$$

Result (type 3, 292 leaves):

$$\frac{1}{16 a^2 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^2} (a+2 b+a \cos[2(e+f x)])$$

$$(4 b^4 (a+b) + 4 i a^2 (a^2+4 a b+6 b^2) f x (a+2 b+a \cos[2(e+f x)]) - 4 i a^2 (a^2+4 a b+6 b^2) \operatorname{ArcTan}[\operatorname{Tan}[e+f x]] (a+2 b+a \cos[2(e+f x)]) +$$

$$4 a^2 (a+b) (a+2 b) (a+2 b+a \cos[2(e+f x)]) \operatorname{Csc}[e+f x]^2 - a^2 (a+b)^2 (a+2 b+a \cos[2(e+f x)]) \operatorname{Csc}[e+f x]^4 +$$

$$2 b^3 (4 a+b) (a+2 b+a \cos[2(e+f x)]) \operatorname{Log}[a+2 b+a \cos[2(e+f x)]] +$$

$$2 a^2 (a^2+4 a b+6 b^2) (a+2 b+a \cos[2(e+f x)]) \operatorname{Log}[\operatorname{Sin}[e+f x]^2]) \operatorname{Sec}[e+f x]^4$$

■ **Problem 356: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[e+f x]^6}{(a+b \operatorname{Sec}[e+f x]^2)^2} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$-\frac{x}{a^2} - \frac{(3 a-2 b) (a+b)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{2 a^2 b^{5/2} f} + \frac{(3 a+b) \operatorname{Tan}[e+f x]}{2 a b^2 f} - \frac{(a+b) \operatorname{Tan}[e+f x]^3}{2 a b f (a+b+b \operatorname{Tan}[e+f x]^2)}$$

Result (type 3, 593 leaves):

$$\begin{aligned}
& - \frac{x (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4}{4a^2 (a + b \sec[e + fx]^2)^2} + \frac{1}{(a + b \sec[e + fx]^2)^2} (3a - 2b)(a + b)^2 (a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \\
& \left(\left(\operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right] (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \right) \right. \\
& \quad \left. \cos[2e] \right) / \left(8a^2 b^2 \sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) - \\
& \left(i \operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right] \right. \\
& \quad \left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e + fx]) \right) \sin[2e] \Big/ \left(8a^2 b^2 \sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) + \\
& \frac{(a + 2b + a \cos[2e + 2fx])^2 \sec[e] \sec[e + fx]^5 \sin[fx]}{4b^2 f (a + b \sec[e + fx]^2)^2} + \left((a + 2b + a \cos[2e + 2fx]) \sec[e + fx]^4 \right. \\
& \quad \left. (-a^3 \sin[2e] - 4a^2 b \sin[2e] - 5ab^2 \sin[2e] - 2b^3 \sin[2e] + a^3 \sin[2fx] + 2a^2 b \sin[2fx] + ab^2 \sin[2fx]) \right) / \\
& \left(8a^2 b^2 f (a + b \sec[e + fx]^2)^2 (\cos[e] - \sin[e]) (\cos[e] + \sin[e]) \right)
\end{aligned}$$

■ **Problem 357: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + fx]^4}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{(a - 2b) \sqrt{a+b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{2a^2 b^{3/2} f} - \frac{(a+b) \tan[e+fx]}{2abf(a+b+b\tan[e+fx]^2)}$$

Result (type 3, 249 leaves):

$$\begin{aligned}
& \frac{1}{8a^2 (a + b \sec[e + fx]^2)^2} (a + 2b + a \cos[2(e + fx)]) \sec[e + fx]^4 \\
& \left(2x (a + 2b + a \cos[2(e + fx)]) + \left((-a^2 + ab + 2b^2) \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i\sin[2e]) (-a + 2b) \sin[fx] + a \sin[2e + fx]}{2\sqrt{a+b}\sqrt{b(\cos[e] - i\sin[e])^4}} \right] \right) \right. \\
& \quad \left. (a + 2b + a \cos[2(e + fx)]) (\cos[2e] - i\sin[2e]) \right) \Big/ \left(b \sqrt{a+b} f \sqrt{b(\cos[e] - i\sin[e])^4} \right) + \frac{(a+b) ((a+2b) \sin[2e] - a \sin[2fx])}{bf(\cos[e] - \sin[e])(\cos[e] + \sin[e])}
\end{aligned}$$

- **Problem 358: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{x}{a^2} + \frac{(a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a+b}}\right]}{2 a^2 \sqrt{b} \sqrt{a+b} f} + \frac{\tan[e + f x]}{2 a f (a + b + b \tan[e + f x]^2)}$$

Result (type 3, 388 leaves):

$$\begin{aligned} & - \left((a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(16x + \right. \right. \\ & \left. \left((-a^3 + 6a^2b + 24ab^2 + 16b^3) \operatorname{ArcTan}\left[\frac{\sec[fx] (\cos[2e] - i \sin[2e]) (- (a + 2b) \sin[fx] + a \sin[2e + fx])}{2 \sqrt{a+b} \sqrt{b} (\cos[e] - i \sin[e])^4} \right] (\cos[2e] - i \sin[2e]) \right) \right) / \\ & \left. \left(b (a + b)^{3/2} f \sqrt{b} (\cos[e] - i \sin[e])^4 \right) + \frac{(a^2 + 8ab + 8b^2) ((a + 2b) \sin[2e] - a \sin[2fx])}{b (a + b) f (a + 2b + a \cos[2(e + fx)]) (\cos[e] - \sin[e]) (\cos[e] + \sin[e])} \right) \right) / \\ & (64 a^2 (a + b \sec[e + fx]^2)^2) + \frac{(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \left(\frac{(a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + fx]}{\sqrt{a+b}}\right]}{(a+b)^{3/2}} - \frac{a \sqrt{b} \sin[2(e + fx)]}{(a+b) (a + 2b + a \cos[2(e + fx)])} \right)}{64 b^{3/2} f (a + b \sec[e + fx]^2)^2} \end{aligned}$$

- **Problem 359: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \sec[e + f x]^2)^2} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{x}{a^2} - \frac{\sqrt{b} (3a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a+b}}\right]}{2 a^2 (a + b)^{3/2} f} - \frac{b \tan[e + f x]}{2 a (a + b) f (a + b + b \tan[e + f x]^2)}$$

Result (type 3, 240 leaves):

$$\frac{1}{8 a^2 (a+b \operatorname{Sec}[e+f x])^2} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \operatorname{Sec}[e+f x]^4 \left(2 x (a+2 b+a \operatorname{Cos}[2(e+f x)]) + \right. \\ \left. \left(b (3 a+2 b) \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[f x] (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) (-a+2 b) \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]}{2 \sqrt{a+b} \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4} \right] (a+2 b+a \operatorname{Cos}[2(e+f x)]) \right) \right. \\ \left. (\operatorname{Cos}[2 e]-i \operatorname{Sin}[2 e]) \right) / \left((a+b)^{3/2} f \sqrt{b} (\operatorname{Cos}[e]-i \operatorname{Sin}[e])^4 \right) + \frac{b((a+2 b) \operatorname{Sin}[2 e]-a \operatorname{Sin}[2 f x])}{(a+b) f (\operatorname{Cos}[e]-\operatorname{Sin}[e]) (\operatorname{Cos}[e]+\operatorname{Sin}[e])} \Bigg)$$

■ **Problem 360: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^2}{(a+b \operatorname{Sec}[e+f x])^2} dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$-\frac{x}{a^2} + \frac{b^{3/2} (5 a+2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b}}\right]}{2 a^2 (a+b)^{5/2} f} - \frac{(2 a-b) \operatorname{Cot}[e+f x]}{2 a (a+b)^2 f} - \frac{b \operatorname{Cot}[e+f x]}{2 a (a+b) f (a+b+b \operatorname{Tan}[e+f x]^2)}$$

Result (type 3, 564 leaves):

$$-\frac{x (a+2 b+a \operatorname{Cos}[2 e+2 f x])^2 \operatorname{Sec}[e+f x]^4}{4 a^2 (a+b \operatorname{Sec}[e+f x])^2} + \left((5 a+2 b) (a+2 b+a \operatorname{Cos}[2 e+2 f x])^2 \right. \\ \left. \operatorname{Sec}[e+f x]^4 \left(- \left(b^2 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) \right] \right. \right. \right. \\ \left. \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \right) \operatorname{Cos}[2 e] \right) / \left(8 a^2 \sqrt{a+b} f \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e] \right) + \\ \left(i b^2 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e]} \right) \right] \right. \\ \left. (-a \operatorname{Sin}[f x]-2 b \operatorname{Sin}[f x]+a \operatorname{Sin}[2 e+f x]) \right) \operatorname{Sin}[2 e] \Bigg) / \left(8 a^2 \sqrt{a+b} f \sqrt{b} \operatorname{Cos}[4 e]-i b \operatorname{Sin}[4 e] \right) \Bigg) / \\ ((a+b)^2 (a+b \operatorname{Sec}[e+f x])^2) + \frac{(a+2 b+a \operatorname{Cos}[2 e+2 f x])^2 \operatorname{Csc}[e] \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x]^4 \operatorname{Sin}[f x]}{4 (a+b)^2 f (a+b \operatorname{Sec}[e+f x])^2} + \\ \frac{(a+2 b+a \operatorname{Cos}[2 e+2 f x]) \operatorname{Sec}[e+f x]^4 (-a b^2 \operatorname{Sin}[2 e]-2 b^3 \operatorname{Sin}[2 e]+a b^2 \operatorname{Sin}[2 f x])}{8 a^2 (a+b)^2 f (a+b \operatorname{Sec}[e+f x])^2 (\operatorname{Cos}[e]-\operatorname{Sin}[e]) (\operatorname{Cos}[e]+\operatorname{Sin}[e])}$$

- **Problem 361: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^2} dx$$

Optimal (type 3, 160 leaves, 8 steps):

$$\frac{x}{a^2} - \frac{b^{5/2} (7a + 2b) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{7/2} f} + \frac{(2a^2 + 6ab - b^2) \text{Cot}[e + f x]}{2a (a+b)^3 f} - \frac{(2a - 3b) \text{Cot}[e + f x]^3}{6a (a+b)^2 f} - \frac{b \text{Cot}[e + f x]^3}{2a (a+b) f (a + b + b \text{Tan}[e + f x]^2)}$$

Result (type 3, 1896 leaves):

$$\left((7a + 2b)(a + 2b + a \cos[2e + 2fx])^2 \sec[e + fx]^4 \right. \\ \left. \left(\left(b^3 \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right] \right. \right. \right. \\ \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \right) / \left(8a^2 \sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) - \right. \\ \left. \left(i b^3 \operatorname{ArcTan} \left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b\cos[4e] - ib\sin[4e]}} \right) \right] \right. \right. \\ \left. \left. \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \right) / \left(8a^2 \sqrt{a+b} f \sqrt{b\cos[4e] - ib\sin[4e]} \right) \right) \right) / \\ \left((a + b)^3 (a + b \sec[e + fx]^2)^2 \right) + \frac{1}{384 a^2 (a + b)^3 f (a + b \sec[e + fx]^2)^2} (a + 2b + a \cos[2e + 2fx])$$

Csc[e]

Csc[e + fx]³

Sec[2e]

Sec[e + fx]⁴

$$\begin{aligned} & (-6a^4 f x \cos[fx] - 54a^3 b f x \cos[fx] - 126a^2 b^2 f x \cos[fx] - 114ab^3 f x \cos[fx] - 36b^4 f x \cos[fx] + 3a^4 f x \cos[3fx] - \\ & 3a^3 b f x \cos[3fx] - 27a^2 b^2 f x \cos[3fx] - 33ab^3 f x \cos[3fx] - 12b^4 f x \cos[3fx] + 6a^4 f x \cos[2e - fx] + \\ & 54a^3 b f x \cos[2e - fx] + 126a^2 b^2 f x \cos[2e - fx] + 114ab^3 f x \cos[2e - fx] + 36b^4 f x \cos[2e - fx] + 6a^4 f x \cos[2e + fx] + \\ & 54a^3 b f x \cos[2e + fx] + 126a^2 b^2 f x \cos[2e + fx] + 114ab^3 f x \cos[2e + fx] + 36b^4 f x \cos[2e + fx] - 6a^4 f x \cos[4e + fx] - \\ & 54a^3 b f x \cos[4e + fx] - 126a^2 b^2 f x \cos[4e + fx] - 114ab^3 f x \cos[4e + fx] - 36b^4 f x \cos[4e + fx] - 3a^4 f x \cos[2e + 3fx] + \\ & 3a^3 b f x \cos[2e + 3fx] + 27a^2 b^2 f x \cos[2e + 3fx] + 33ab^3 f x \cos[2e + 3fx] + 12b^4 f x \cos[2e + 3fx] + 3a^4 f x \cos[4e + 3fx] - \\ & 3a^3 b f x \cos[4e + 3fx] - 27a^2 b^2 f x \cos[4e + 3fx] - 33ab^3 f x \cos[4e + 3fx] - 12b^4 f x \cos[4e + 3fx] - 3a^4 f x \cos[6e + 3fx] + \\ & 3a^3 b f x \cos[6e + 3fx] + 27a^2 b^2 f x \cos[6e + 3fx] + 33ab^3 f x \cos[6e + 3fx] + 12b^4 f x \cos[6e + 3fx] - 3a^4 f x \cos[2e + 5fx] - \\ & 9a^3 b f x \cos[2e + 5fx] - 9a^2 b^2 f x \cos[2e + 5fx] - 3ab^3 f x \cos[2e + 5fx] + 3a^4 f x \cos[4e + 5fx] + 9a^3 b f x \cos[4e + 5fx] + \\ & 9a^2 b^2 f x \cos[4e + 5fx] + 3ab^3 f x \cos[4e + 5fx] - 3a^4 f x \cos[6e + 5fx] - 9a^3 b f x \cos[6e + 5fx] - 9a^2 b^2 f x \cos[6e + 5fx] - \\ & 3ab^3 f x \cos[6e + 5fx] + 3a^4 f x \cos[8e + 5fx] + 9a^3 b f x \cos[8e + 5fx] + 9a^2 b^2 f x \cos[8e + 5fx] + 3ab^3 f x \cos[8e + 5fx] - \\ & 12a^4 \sin[fx] - 60a^3 b \sin[fx] - 96a^2 b^2 \sin[fx] + 18b^4 \sin[fx] + 4a^4 \sin[3fx] + 36a^3 b \sin[3fx] + 80a^2 b^2 \sin[3fx] - \\ & 6ab^3 \sin[3fx] + 6b^4 \sin[3fx] + 4a^4 \sin[2e - fx] + 76a^3 b \sin[2e - fx] + 144a^2 b^2 \sin[2e - fx] + 18b^4 \sin[2e - fx] - \\ & 4a^4 \sin[2e + fx] - 76a^3 b \sin[2e + fx] - 144a^2 b^2 \sin[2e + fx] + 6ab^3 \sin[2e + fx] + 18b^4 \sin[2e + fx] - 12a^4 \sin[4e + fx] - \\ & 60a^3 b \sin[4e + fx] - 96a^2 b^2 \sin[4e + fx] - 6ab^3 \sin[4e + fx] - 18b^4 \sin[4e + fx] - 12a^4 \sin[2e + 3fx] - 24a^3 b \sin[2e + 3fx] + \\ & 6ab^3 \sin[2e + 3fx] - 6b^4 \sin[2e + 3fx] + 4a^4 \sin[4e + 3fx] + 36a^3 b \sin[4e + 3fx] + 80a^2 b^2 \sin[4e + 3fx] - 3ab^3 \sin[4e + 3fx] - \\ & 6b^4 \sin[4e + 3fx] - 12a^4 \sin[6e + 3fx] - 24a^3 b \sin[6e + 3fx] + 3ab^3 \sin[6e + 3fx] + 6b^4 \sin[6e + 3fx] + 8a^4 \sin[2e + 5fx] + \\ & 20a^3 b \sin[2e + 5fx] + 3ab^3 \sin[2e + 5fx] - 3ab^3 \sin[4e + 5fx] + 8a^4 \sin[6e + 5fx] + 20a^3 b \sin[6e + 5fx]) \end{aligned}$$

■ **Problem 362:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cot[e + fx]^6}{(a + b \sec[e + fx]^2)^2} dx$$

Optimal (type 3, 207 leaves, 9 steps) :

$$-\frac{x}{a^2} + \frac{b^{7/2} (9a + 2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{2a^2 (a+b)^{9/2} f} - \frac{(2a^3 + 8a^2b + 12ab^2 - b^3) \operatorname{Cot}[e+fx]}{2a (a+b)^4 f} +$$

$$\frac{(2a^2 + 6ab - 3b^2) \operatorname{Cot}[e+fx]^3}{6a (a+b)^3 f} - \frac{(2a - 5b) \operatorname{Cot}[e+fx]^5}{10a (a+b)^2 f} - \frac{b \operatorname{Cot}[e+fx]^5}{2a (a+b) f (a+b + b \operatorname{Tan}[e+fx]^2)}$$

Result (type 3, 3028 leaves) :

$$\left((9a + 2b) (a + 2b + a \operatorname{Cos}[2e + 2fx])^2 \operatorname{Sec}[e + fx]^4 \right.$$

$$\left. - \left(b^4 \operatorname{ArcTan}\left[\operatorname{Sec}[fx]\right] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right) \right. \right.$$

$$\left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right] \operatorname{Cos}[2e] \right) / \left(8a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) +$$

$$\left(i b^4 \operatorname{ArcTan}\left[\operatorname{Sec}[fx]\right] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2\sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right) \right.$$

$$\left. \left. (-a \operatorname{Sin}[fx] - 2b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e + fx]) \right] \operatorname{Sin}[2e] \right) / \left(8a^2 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) \Bigg) /$$

$$\left((a+b)^4 (a+b \operatorname{Sec}[e+fx]^2)^2 \right) + \frac{1}{7680 a^2 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^2} (a + 2b + a \operatorname{Cos}[2e + 2fx])$$

Csc[e]
 Csc[e + fx]⁵
 Sec[2e]
 Sec[e + fx]⁴

$$(75 a^5 f x \operatorname{Cos}[fx] + 900 a^4 b f x \operatorname{Cos}[fx] + 2850 a^3 b^2 f x \operatorname{Cos}[fx] + 3900 a^2 b^3 f x \operatorname{Cos}[fx] + 2475 a b^4 f x \operatorname{Cos}[fx] +$$

$$600 b^5 f x \operatorname{Cos}[fx] - 15 a^5 f x \operatorname{Cos}[3fx] + 240 a^4 b f x \operatorname{Cos}[3fx] + 1110 a^3 b^2 f x \operatorname{Cos}[3fx] + 1740 a^2 b^3 f x \operatorname{Cos}[3fx] +$$

$$1185 a b^4 f x \operatorname{Cos}[3fx] + 300 b^5 f x \operatorname{Cos}[3fx] - 75 a^5 f x \operatorname{Cos}[2e - fx] - 900 a^4 b f x \operatorname{Cos}[2e - fx] - 2850 a^3 b^2 f x \operatorname{Cos}[2e - fx] -$$

$$3900 a^2 b^3 f x \operatorname{Cos}[2e - fx] - 2475 a b^4 f x \operatorname{Cos}[2e - fx] - 600 b^5 f x \operatorname{Cos}[2e - fx] - 75 a^5 f x \operatorname{Cos}[2e + fx] - 900 a^4 b f x \operatorname{Cos}[2e + fx] -$$

$$2850 a^3 b^2 f x \operatorname{Cos}[2e + fx] - 3900 a^2 b^3 f x \operatorname{Cos}[2e + fx] - 2475 a b^4 f x \operatorname{Cos}[2e + fx] - 600 b^5 f x \operatorname{Cos}[2e + fx] + 75 a^5 f x \operatorname{Cos}[4e + fx] +$$

$$900 a^4 b f x \operatorname{Cos}[4e + fx] + 2850 a^3 b^2 f x \operatorname{Cos}[4e + fx] + 3900 a^2 b^3 f x \operatorname{Cos}[4e + fx] + 2475 a b^4 f x \operatorname{Cos}[4e + fx] +$$

$$600 b^5 f x \operatorname{Cos}[4e + fx] + 15 a^5 f x \operatorname{Cos}[2e + 3fx] - 240 a^4 b f x \operatorname{Cos}[2e + 3fx] - 1110 a^3 b^2 f x \operatorname{Cos}[2e + 3fx] -$$

$$1740 a^2 b^3 f x \operatorname{Cos}[2e + 3fx] - 1185 a b^4 f x \operatorname{Cos}[2e + 3fx] - 300 b^5 f x \operatorname{Cos}[2e + 3fx] - 15 a^5 f x \operatorname{Cos}[4e + 3fx] +$$

$$240 a^4 b f x \operatorname{Cos}[4e + 3fx] + 1110 a^3 b^2 f x \operatorname{Cos}[4e + 3fx] + 1740 a^2 b^3 f x \operatorname{Cos}[4e + 3fx] + 1185 a b^4 f x \operatorname{Cos}[4e + 3fx] +$$

$$300 b^5 f x \operatorname{Cos}[4e + 3fx] + 15 a^5 f x \operatorname{Cos}[6e + 3fx] - 240 a^4 b f x \operatorname{Cos}[6e + 3fx] - 1110 a^3 b^2 f x \operatorname{Cos}[6e + 3fx] -$$

$$1740 a^2 b^3 f x \operatorname{Cos}[6e + 3fx] - 1185 a b^4 f x \operatorname{Cos}[6e + 3fx] - 300 b^5 f x \operatorname{Cos}[6e + 3fx] + 45 a^5 f x \operatorname{Cos}[2e + 5fx] +$$

$$120 a^4 b f x \operatorname{Cos}[2e + 5fx] + 30 a^3 b^2 f x \operatorname{Cos}[2e + 5fx] - 180 a^2 b^3 f x \operatorname{Cos}[2e + 5fx] - 195 a b^4 f x \operatorname{Cos}[2e + 5fx] -$$

$$60 b^5 f x \operatorname{Cos}[2e + 5fx] - 45 a^5 f x \operatorname{Cos}[4e + 5fx] - 120 a^4 b f x \operatorname{Cos}[4e + 5fx] - 30 a^3 b^2 f x \operatorname{Cos}[4e + 5fx] + 180 a^2 b^3 f x \operatorname{Cos}[4e + 5fx] +$$

$$\begin{aligned}
& 195 a b^4 f x \operatorname{Cos}[4 e + 5 f x] + 60 b^5 f x \operatorname{Cos}[4 e + 5 f x] + 45 a^5 f x \operatorname{Cos}[6 e + 5 f x] + 120 a^4 b f x \operatorname{Cos}[6 e + 5 f x] + \\
& 30 a^3 b^2 f x \operatorname{Cos}[6 e + 5 f x] - 180 a^2 b^3 f x \operatorname{Cos}[6 e + 5 f x] - 195 a b^4 f x \operatorname{Cos}[6 e + 5 f x] - 60 b^5 f x \operatorname{Cos}[6 e + 5 f x] - \\
& 45 a^5 f x \operatorname{Cos}[8 e + 5 f x] - 120 a^4 b f x \operatorname{Cos}[8 e + 5 f x] - 30 a^3 b^2 f x \operatorname{Cos}[8 e + 5 f x] + 180 a^2 b^3 f x \operatorname{Cos}[8 e + 5 f x] + \\
& 195 a b^4 f x \operatorname{Cos}[8 e + 5 f x] + 60 b^5 f x \operatorname{Cos}[8 e + 5 f x] - 15 a^5 f x \operatorname{Cos}[4 e + 7 f x] - 60 a^4 b f x \operatorname{Cos}[4 e + 7 f x] - 90 a^3 b^2 f x \operatorname{Cos}[4 e + 7 f x] - \\
& 60 a^2 b^3 f x \operatorname{Cos}[4 e + 7 f x] - 15 a b^4 f x \operatorname{Cos}[4 e + 7 f x] + 15 a^5 f x \operatorname{Cos}[6 e + 7 f x] + 60 a^4 b f x \operatorname{Cos}[6 e + 7 f x] + \\
& 90 a^3 b^2 f x \operatorname{Cos}[6 e + 7 f x] + 60 a^2 b^3 f x \operatorname{Cos}[6 e + 7 f x] + 15 a b^4 f x \operatorname{Cos}[6 e + 7 f x] - 15 a^5 f x \operatorname{Cos}[8 e + 7 f x] - 60 a^4 b f x \operatorname{Cos}[8 e + 7 f x] - \\
& 90 a^3 b^2 f x \operatorname{Cos}[8 e + 7 f x] - 60 a^2 b^3 f x \operatorname{Cos}[8 e + 7 f x] - 15 a b^4 f x \operatorname{Cos}[8 e + 7 f x] + 15 a^5 f x \operatorname{Cos}[10 e + 7 f x] + \\
& 60 a^4 b f x \operatorname{Cos}[10 e + 7 f x] + 90 a^3 b^2 f x \operatorname{Cos}[10 e + 7 f x] + 60 a^2 b^3 f x \operatorname{Cos}[10 e + 7 f x] + 15 a b^4 f x \operatorname{Cos}[10 e + 7 f x] - \\
& 10 a^5 \operatorname{Sin}[f x] + 860 a^4 b \operatorname{Sin}[f x] + 3120 a^3 b^2 \operatorname{Sin}[f x] + 3600 a^2 b^3 \operatorname{Sin}[f x] - 300 b^5 \operatorname{Sin}[f x] + 46 a^5 \operatorname{Sin}[3 f x] - 508 a^4 b \operatorname{Sin}[3 f x] - \\
& 2324 a^3 b^2 \operatorname{Sin}[3 f x] - 3120 a^2 b^3 \operatorname{Sin}[3 f x] + 75 a b^4 \operatorname{Sin}[3 f x] - 150 b^5 \operatorname{Sin}[3 f x] - 240 a^5 \operatorname{Sin}[2 e - f x] - 1840 a^4 b \operatorname{Sin}[2 e - f x] - \\
& 4840 a^3 b^2 \operatorname{Sin}[2 e - f x] - 5040 a^2 b^3 \operatorname{Sin}[2 e - f x] - 300 b^5 \operatorname{Sin}[2 e - f x] + 240 a^5 \operatorname{Sin}[2 e + f x] + 1840 a^4 b \operatorname{Sin}[2 e + f x] + \\
& 4840 a^3 b^2 \operatorname{Sin}[2 e + f x] + 5040 a^2 b^3 \operatorname{Sin}[2 e + f x] - 75 a b^4 \operatorname{Sin}[2 e + f x] - 300 b^5 \operatorname{Sin}[2 e + f x] - 10 a^5 \operatorname{Sin}[4 e + f x] + 860 a^4 b \operatorname{Sin}[4 e + f x] + \\
& 3120 a^3 b^2 \operatorname{Sin}[4 e + f x] + 3600 a^2 b^3 \operatorname{Sin}[4 e + f x] + 75 a b^4 \operatorname{Sin}[4 e + f x] + 300 b^5 \operatorname{Sin}[4 e + f x] - 240 a^4 b \operatorname{Sin}[2 e + 3 f x] - \\
& 900 a^3 b^2 \operatorname{Sin}[2 e + 3 f x] - 1200 a^2 b^3 \operatorname{Sin}[2 e + 3 f x] - 75 a b^4 \operatorname{Sin}[2 e + 3 f x] + 150 b^5 \operatorname{Sin}[2 e + 3 f x] + 46 a^5 \operatorname{Sin}[4 e + 3 f x] - \\
& 508 a^4 b \operatorname{Sin}[4 e + 3 f x] - 2324 a^3 b^2 \operatorname{Sin}[4 e + 3 f x] - 3120 a^2 b^3 \operatorname{Sin}[4 e + 3 f x] + 60 a b^4 \operatorname{Sin}[4 e + 3 f x] + 150 b^5 \operatorname{Sin}[4 e + 3 f x] - \\
& 240 a^4 b \operatorname{Sin}[6 e + 3 f x] - 900 a^3 b^2 \operatorname{Sin}[6 e + 3 f x] - 1200 a^2 b^3 \operatorname{Sin}[6 e + 3 f x] - 60 a b^4 \operatorname{Sin}[6 e + 3 f x] - 150 b^5 \operatorname{Sin}[6 e + 3 f x] - \\
& 48 a^5 \operatorname{Sin}[2 e + 5 f x] - 32 a^4 b \operatorname{Sin}[2 e + 5 f x] + 340 a^3 b^2 \operatorname{Sin}[2 e + 5 f x] + 864 a^2 b^3 \operatorname{Sin}[2 e + 5 f x] - 60 a b^4 \operatorname{Sin}[2 e + 5 f x] + \\
& 30 b^5 \operatorname{Sin}[2 e + 5 f x] - 90 a^5 \operatorname{Sin}[4 e + 5 f x] - 300 a^4 b \operatorname{Sin}[4 e + 5 f x] - 300 a^3 b^2 \operatorname{Sin}[4 e + 5 f x] + 60 a b^4 \operatorname{Sin}[4 e + 5 f x] - \\
& 30 b^5 \operatorname{Sin}[4 e + 5 f x] - 48 a^5 \operatorname{Sin}[6 e + 5 f x] - 32 a^4 b \operatorname{Sin}[6 e + 5 f x] + 340 a^3 b^2 \operatorname{Sin}[6 e + 5 f x] + 864 a^2 b^3 \operatorname{Sin}[6 e + 5 f x] - \\
& 15 a b^4 \operatorname{Sin}[6 e + 5 f x] - 30 b^5 \operatorname{Sin}[6 e + 5 f x] - 90 a^5 \operatorname{Sin}[8 e + 5 f x] - 300 a^4 b \operatorname{Sin}[8 e + 5 f x] - 300 a^3 b^2 \operatorname{Sin}[8 e + 5 f x] + \\
& 15 a b^4 \operatorname{Sin}[8 e + 5 f x] + 30 b^5 \operatorname{Sin}[8 e + 5 f x] + 46 a^5 \operatorname{Sin}[4 e + 7 f x] + 172 a^4 b \operatorname{Sin}[4 e + 7 f x] + 216 a^3 b^2 \operatorname{Sin}[4 e + 7 f x] + \\
& 15 a b^4 \operatorname{Sin}[4 e + 7 f x] - 15 a b^4 \operatorname{Sin}[6 e + 7 f x] + 46 a^5 \operatorname{Sin}[8 e + 7 f x] + 172 a^4 b \operatorname{Sin}[8 e + 7 f x] + 216 a^3 b^2 \operatorname{Sin}[8 e + 7 f x]
\end{aligned}$$

■ **Problem 366: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cot}[e + f x]}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$-\frac{b^3}{4 a^3 (a+b) f (b+a \operatorname{Cos}[e+f x]^2)^2} + \frac{b^2 (3 a+2 b)}{2 a^3 (a+b)^2 f (b+a \operatorname{Cos}[e+f x]^2)} + \frac{b (3 a^2+3 a b+b^2) \operatorname{Log}[b+a \operatorname{Cos}[e+f x]^2]}{2 a^3 (a+b)^3 f} + \frac{\operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^3 f}$$

Result (type 3, 253 leaves):

$$\begin{aligned}
& \frac{1}{32 a^3 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} (a+2 b+a \operatorname{Cos}[2(e+f x)]) \\
& (-4 b^3 (a+b)^2 + 4 b^2 (a+b) (3 a+2 b) (a+2 b+a \operatorname{Cos}[2(e+f x)]) + 4 i b (3 a^2+3 a b+b^2) f x (a+2 b+a \operatorname{Cos}[2(e+f x)])^2 - \\
& 2 i b (3 a^2+3 a b+b^2) \operatorname{ArcTan}[\operatorname{Tan}[2(e+f x)]] (a+2 b+a \operatorname{Cos}[2(e+f x)])^2 + b (3 a^2+3 a b+b^2) (a+2 b+a \operatorname{Cos}[2(e+f x)])^2 \\
& \operatorname{Log}[(a+2 b+a \operatorname{Cos}[2(e+f x)])^2] + 4 a^3 (a+2 b+a \operatorname{Cos}[2(e+f x)])^2 \operatorname{Log}[\operatorname{Sin}[e+f x]]) \operatorname{Sec}[e+f x]^6
\end{aligned}$$

■ **Problem 367: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e + f x]^3}{(a + b \operatorname{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 154 leaves, 4 steps) :

$$\frac{b^4}{4 a^3 (a+b)^2 f (b+a \cos [e+f x])^2} - \frac{b^3 (2 a+b)}{a^3 (a+b)^3 f (b+a \cos [e+f x])^2} -$$

$$\frac{\operatorname{Csc}[e+f x]^2}{2 (a+b)^3 f} - \frac{b^2 (6 a^2+4 a b+b^2) \operatorname{Log}[b+a \cos [e+f x]^2]}{2 a^3 (a+b)^4 f} - \frac{(a+4 b) \operatorname{Log}[\sin [e+f x]]}{(a+b)^4 f}$$

Result (type 3, 1045 leaves) :

$$\frac{b^4 (a+2 b+a \cos [2 e+2 f x]) \operatorname{Sec}[e+f x]^6}{8 a^3 (a+b)^2 f (a+b \operatorname{Sec}[e+f x]^2)^3} - \frac{b^3 (2 a+b) (a+2 b+a \cos [2 e+2 f x])^2 \operatorname{Sec}[e+f x]^6}{4 a^3 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} -$$

$$\frac{i (-6 a^2 b^2-4 a b^3-b^4) \operatorname{ArcTan}[\tan [2 e+2 f x]] (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6}{16 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3} -$$

$$\frac{(a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Csc}[e+f x]^2 \operatorname{Sec}[e+f x]^6}{16 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} +$$

$$\frac{(-6 a^2 b^2-4 a b^3-b^4) (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Log}[(a+2 b+a \cos [2 e+2 f x])^2] \operatorname{Sec}[e+f x]^6}{32 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3} +$$

$$\frac{(-a-4 b) (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Log}[\sin [e+f x]] \operatorname{Sec}[e+f x]^6}{8 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3} +$$

$$\frac{1}{(a+b \operatorname{Sec}[e+f x]^2)^3} x (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left(\frac{a \operatorname{Cot}[e]}{8 (a+b)^4} + \frac{b \operatorname{Cot}[e]}{2 (a+b)^4} - \frac{3 i b^2 \cos [e]^2}{4 a (a+b)^4 (\cos [e]^2-\sin [e]^2)} - \right.$$

$$\frac{i b^3 \cos [e]^2}{2 a^2 (a+b)^4 (\cos [e]^2-\sin [e]^2)} - \frac{i b^4 \cos [e]^2}{8 a^3 (a+b)^4 (\cos [e]^2-\sin [e]^2)} - \frac{3 b^2 \cos [e] \sin [e]}{2 a (a+b)^4 (\cos [e]^2-\sin [e]^2)} - \frac{b^3 \cos [e] \sin [e]}{a^2 (a+b)^4 (\cos [e]^2-\sin [e]^2)} -$$

$$\frac{b^4 \cos [e] \sin [e]}{4 a^3 (a+b)^4 (\cos [e]^2-\sin [e]^2)} + \frac{3 i b^2 \sin [e]^2}{4 a (a+b)^4 (\cos [e]^2-\sin [e]^2)} + \frac{i b^3 \sin [e]^2}{2 a^2 (a+b)^4 (\cos [e]^2-\sin [e]^2)} +$$

$$\frac{i b^4 \sin [e]^2}{8 a^3 (a+b)^4 (\cos [e]^2-\sin [e]^2)} - \frac{i (a+a \cos [2 e]+i a \sin [2 e])}{8 (a+b)^4 (-1+\cos [2 e]+i \sin [2 e])} - \frac{i (b+b \cos [2 e]+i b \sin [2 e])}{2 (a+b)^4 (-1+\cos [2 e]+i \sin [2 e])} -$$

$$\left. \frac{3 i (-b^2+b^2 \cos [4 e]+i b^2 \sin [4 e])}{4 a (a+b)^4 (1+\cos [4 e]+i \sin [4 e])} - \frac{i (-b^3+b^3 \cos [4 e]+i b^3 \sin [4 e])}{2 a^2 (a+b)^4 (1+\cos [4 e]+i \sin [4 e])} - \frac{i (-b^4+b^4 \cos [4 e]+i b^4 \sin [4 e])}{8 a^3 (a+b)^4 (1+\cos [4 e]+i \sin [4 e])} \right)$$

■ **Problem 368: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e+f x]^5}{(a+b \operatorname{Sec}[e+f x]^2)^3} dx$$

Optimal (type 3, 192 leaves, 4 steps) :

$$-\frac{b^5}{4 a^3 (a+b)^3 f (b+a \cos [e+f x])^2} + \frac{b^4 (5 a+2 b)}{2 a^3 (a+b)^4 f (b+a \cos [e+f x])^2} + \frac{(2 a+5 b) \operatorname{Csc}[e+f x]^2}{2 (a+b)^4 f} - \frac{\operatorname{Csc}[e+f x]^4}{4 (a+b)^3 f} + \frac{b^3 (10 a^2+5 a b+b^2) \operatorname{Log}[b+a \cos [e+f x]^2]}{2 a^3 (a+b)^5 f} + \frac{(a^2+5 a b+10 b^2) \operatorname{Log}[\operatorname{Sin}[e+f x]]}{(a+b)^5 f}$$

Result (type 3, 1286 leaves) :

$$-\frac{b^5 (a+2 b+a \cos [2 e+2 f x]) \operatorname{Sec}[e+f x]^6}{8 a^3 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} + \frac{b^4 (5 a+2 b) (a+2 b+a \cos [2 e+2 f x])^2 \operatorname{Sec}[e+f x]^6}{8 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3} - \frac{i (a^2+5 a b+10 b^2) \operatorname{ArcTan}[\operatorname{Tan}[e+f x]] (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6}{8 (a+b)^5 f (a+b \operatorname{Sec}[e+f x]^2)^3} - \frac{i (10 a^2 b^3+5 a b^4+b^5) \operatorname{ArcTan}[\operatorname{Tan}[2 e+2 f x]] (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6}{16 a^3 (a+b)^5 f (a+b \operatorname{Sec}[e+f x]^2)^3} + \frac{(2 a+5 b) (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Csc}[e+f x]^2 \operatorname{Sec}[e+f x]^6}{16 (a+b)^4 f (a+b \operatorname{Sec}[e+f x]^2)^3} - \frac{(a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Csc}[e+f x]^4 \operatorname{Sec}[e+f x]^6}{32 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} + \frac{(10 a^2 b^3+5 a b^4+b^5) (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Log}[(a+2 b+a \cos [2 e+2 f x])^2] \operatorname{Sec}[e+f x]^6}{32 a^3 (a+b)^5 f (a+b \operatorname{Sec}[e+f x]^2)^3} + \frac{(a^2+5 a b+10 b^2) (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Log}[\operatorname{Sin}[e+f x]^2] \operatorname{Sec}[e+f x]^6}{16 (a+b)^5 f (a+b \operatorname{Sec}[e+f x]^2)^3} + \frac{1}{(a+b \operatorname{Sec}[e+f x]^2)^3} x (a+2 b+a \cos [2 e+2 f x])^3 \operatorname{Sec}[e+f x]^6 \left(\frac{i a^2}{8 (a+b)^5} + \frac{5 i a b}{8 (a+b)^5} + \frac{5 i b^2}{4 (a+b)^5} - \frac{a^2 \operatorname{Cot}[e]}{8 (a+b)^5} - \frac{5 a b \operatorname{Cot}[e]}{8 (a+b)^5} - \frac{5 b^2 \operatorname{Cot}[e]}{4 (a+b)^5} + \frac{5 i b^3 \cos [e]^2}{4 a (a+b)^5 (\cos [e]^2 - \sin [e]^2)} + \frac{5 i b^4 \cos [e]^2}{8 a^2 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} + \frac{i b^5 \cos [e]^2}{8 a^3 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} + \frac{5 b^3 \cos [e] \sin [e]}{2 a (a+b)^5 (\cos [e]^2 - \sin [e]^2)} + \frac{5 b^4 \cos [e] \sin [e]}{4 a^2 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} - \frac{b^5 \cos [e] \sin [e]}{4 a^3 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} - \frac{5 i b^3 \sin [e]^2}{4 a (a+b)^5 (\cos [e]^2 - \sin [e]^2)} - \frac{5 i b^4 \sin [e]^2}{8 a^2 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} - \frac{i b^5 \sin [e]^2}{8 a^3 (a+b)^5 (\cos [e]^2 - \sin [e]^2)} + \frac{i (a^2+5 a b+a^2 \cos [2 e]+5 a b \cos [2 e]+i a^2 \sin [2 e]+5 i a b \sin [2 e])}{8 (a+b)^5 (-1+\cos [2 e]+i \sin [2 e])} + \frac{5 i (b^2+b^2 \cos [2 e]+i b^2 \sin [2 e])}{4 (a+b)^5 (-1+\cos [2 e]+i \sin [2 e])} + \frac{5 i (-b^3+b^3 \cos [4 e]+i b^3 \sin [4 e])}{4 a (a+b)^5 (1+\cos [4 e]+i \sin [4 e])} + \frac{5 i (-b^4+b^4 \cos [4 e]+i b^4 \sin [4 e])}{8 a^2 (a+b)^5 (1+\cos [4 e]+i \sin [4 e])} + \frac{i (-b^5+b^5 \cos [4 e]+i b^5 \sin [4 e])}{8 a^3 (a+b)^5 (1+\cos [4 e]+i \sin [4 e])} \right)$$

- **Problem 369: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^6}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 147 leaves, 7 steps):

$$-\frac{x}{a^3} + \frac{\sqrt{a+b} (3a^2 - 4ab + 8b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{8a^3 b^{5/2} f} - \frac{(a+b) \tan[e+fx]^3}{4abf (a+b+b \tan[e+fx]^2)^2} - \frac{(3a-4b)(a+b) \tan[e+fx]}{8a^2 b^2 f (a+b+b \tan[e+fx]^2)}$$

Result (type 3, 760 leaves):

$$\frac{1}{(a+b \sec[e+fx]^2)^3} (-3a^3 + a^2b - 4ab^2 - 8b^3) (a+2b+a \cos[2e+2fx])^3 \sec[e+fx]^6$$

$$\left(\left(\operatorname{ArcTan}\left[\sec[fx]\left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b}\cos[4e]-ib\sin[4e]} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b}\cos[4e]-ib\sin[4e]}\right)\right] (-a\sin[fx] - 2b\sin[fx] + a\sin[2e+fx]) \right) \right.$$

$$\left. \cos[2e] \right) / \left(64a^3 b^2 \sqrt{a+b} f \sqrt{b\cos[4e]-ib\sin[4e]} \right) -$$

$$\left(i \operatorname{ArcTan}\left[\sec[fx]\left(\frac{\cos[2e]}{2\sqrt{a+b}\sqrt{b}\cos[4e]-ib\sin[4e]} - \frac{i\sin[2e]}{2\sqrt{a+b}\sqrt{b}\cos[4e]-ib\sin[4e]}\right)\right] \right.$$

$$\left. (-a\sin[fx] - 2b\sin[fx] + a\sin[2e+fx]) \sin[2e] \right) / \left(64a^3 b^2 \sqrt{a+b} f \sqrt{b\cos[4e]-ib\sin[4e]} \right) +$$

$$\frac{1}{128a^3 b^2 f (a+b \sec[e+fx]^2)^3} (a+2b+a \cos[2e+2fx]) \sec[2e] \sec[e+fx]^6$$

$$(-24a^2 b^2 f x \cos[2e] - 64a b^3 f x \cos[2e] - 64b^4 f x \cos[2e] - 16a^2 b^2 f x \cos[2fx] - 32a b^3 f x \cos[2fx] - 16a^2 b^2 f x \cos[4e+2fx] -$$

$$32a b^3 f x \cos[4e+2fx] - 4a^2 b^2 f x \cos[2e+4fx] - 4a^2 b^2 f x \cos[6e+4fx] + 9a^4 \sin[2e] + 15a^3 b \sin[2e] - 18a^2 b^2 \sin[2e] -$$

$$72a b^3 \sin[2e] - 48b^4 \sin[2e] - 9a^4 \sin[2fx] - 13a^3 b \sin[2fx] + 28a^2 b^2 \sin[2fx] + 32a b^3 \sin[2fx] + 3a^4 \sin[4e+2fx] -$$

$$a^3 b \sin[4e+2fx] - 20a^2 b^2 \sin[4e+2fx] - 16a b^3 \sin[4e+2fx] - 3a^4 \sin[2e+4fx] + 3a^3 b \sin[2e+4fx] + 6a^2 b^2 \sin[2e+4fx])$$

- **Problem 370: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^4}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 137 leaves, 7 steps):

$$\frac{x}{a^3} + \frac{(a^2 - 4ab - 8b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{8a^3 b^{3/2} \sqrt{a+b} f} - \frac{(a+b) \tan[e+fx]}{4abf (a+b+b \tan[e+fx]^2)^2} + \frac{(a-4b) \tan[e+fx]}{8a^2 b f (a+b+b \tan[e+fx]^2)}$$

Result (type 3, 1744 leaves):

$$\begin{aligned}
& \left((a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \right. \\
& \left. \left(\frac{(3a^2 + 8ab + 8b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} - \frac{a\sqrt{b} (3a^2 + 16ab + 16b^2 + 3a(a+2b) \cos[2(e+fx)]) \sin[2(e+fx)]}{(a+b)^2 (a+2b+a \cos[2(e+fx)])^2} \right) \right) / \\
& (1024b^{5/2} f (a+b \sec[e+fx]^2)^3) - \left((a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \right. \\
& \left. \left(-\frac{3a(a+2b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b}}\right]}{(a+b)^{5/2}} + \frac{\sqrt{b} (3a^3 + 14a^2b + 24ab^2 + 16b^3 + a(3a^2 + 4ab + 4b^2) \cos[2(e+fx)]) \sin[2(e+fx)]}{(a+b)^2 (a+2b+a \cos[2(e+fx)])^2} \right) \right) / \\
& (2048b^{5/2} f (a+b \sec[e+fx]^2)^3) + \frac{1}{32(a+b \sec[e+fx]^2)^3} (a + 2b + a \cos[2e + 2fx])^3 \sec[e + fx]^6 \\
& \left(\frac{1}{(a+b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
& \left(\left(\operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right] \right) \right. \\
& \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \cos[2e] \Big/ (64a^3b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]}) - \\
& \left(i \operatorname{ArcTan}\left[\sec[fx] \left(\frac{\cos[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} - \frac{i \sin[2e]}{2\sqrt{a+b} \sqrt{b \cos[4e] - ib \sin[4e]}} \right) \right] \right) \\
& \left. (-a \sin[fx] - 2b \sin[fx] + a \sin[2e + fx]) \right) \sin[2e] \Big/ (64a^3b^2 \sqrt{a+b} f \sqrt{b \cos[4e] - ib \sin[4e]}) \Big) + \\
& \frac{1}{128a^3b^2 (a+b)^2 f (a+2b+a \cos[2e+2fx])^2} \sec[2e] (768a^4b^2fx \cos[2e] + 3584a^3b^3fx \cos[2e] + 6912a^2b^4fx \cos[2e] + \\
& 6144ab^5fx \cos[2e] + 2048b^5fx \cos[2e] + 512a^4b^2fx \cos[2fx] + 2048a^3b^3fx \cos[2fx] + 2560a^2b^4fx \cos[2fx] + \\
& 1024ab^5fx \cos[2fx] + 512a^4b^2fx \cos[4e+2fx] + 2048a^3b^3fx \cos[4e+2fx] + 2560a^2b^4fx \cos[4e+2fx] + \\
& 1024ab^5fx \cos[4e+2fx] + 128a^4b^2fx \cos[2e+4fx] + 256a^3b^3fx \cos[2e+4fx] + 128a^2b^4fx \cos[2e+4fx] + \\
& 128a^4b^2fx \cos[6e+4fx] + 256a^3b^3fx \cos[6e+4fx] + 128a^2b^4fx \cos[6e+4fx] - 9a^6 \sin[2e] + 12a^5b \sin[2e] + \\
& 684a^4b^2 \sin[2e] + 2880a^3b^3 \sin[2e] + 5280a^2b^4 \sin[2e] + 4608ab^5 \sin[2e] + 1536b^6 \sin[2e] + 9a^6 \sin[2fx] - \\
& 14a^5b \sin[2fx] - 608a^4b^2 \sin[2fx] - 2112a^3b^3 \sin[2fx] - 2560a^2b^4 \sin[2fx] - 1024ab^5 \sin[2fx] - 3a^6 \sin[4e+2fx] + \\
& 10a^5b \sin[4e+2fx] + 304a^4b^2 \sin[4e+2fx] + 1056a^3b^3 \sin[4e+2fx] + 1280a^2b^4 \sin[4e+2fx] + 512ab^5 \sin[4e+2fx] +
\end{aligned}$$

$$\left. \begin{aligned} & 3 a^6 \sin[2 e + 4 f x] - 12 a^5 b \sin[2 e + 4 f x] - 204 a^4 b^2 \sin[2 e + 4 f x] - 384 a^3 b^3 \sin[2 e + 4 f x] - 192 a^2 b^4 \sin[2 e + 4 f x] \right) - \\ & \left((a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \left(- \frac{6 a^2 \operatorname{ArcTan}\left[\frac{\sec[f x] (\cos[2 e] - i \sin[2 e]) (- (a + 2 b) \sin[f x] + a \sin[2 e + f x])}{2 \sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}}\right]}{\sqrt{a+b} \sqrt{b (\cos[e] - i \sin[e])^4}} \right) (\cos[2 e] - i \sin[2 e]) \right. \right. \\ & \left. \left. + (a \sec[2 e] \left((-9 a^4 - 16 a^3 b + 48 a^2 b^2 + 128 a b^3 + 64 b^4) \sin[2 f x] + a (-3 a^3 + 2 a^2 b + 24 a b^2 + 16 b^3) \sin[2 (e + 2 f x)] + \right. \right. \right. \\ & \left. \left. \left. (3 a^4 - 64 a^2 b^2 - 128 a b^3 - 64 b^4) \sin[4 e + 2 f x] \right) + (9 a^5 + 18 a^4 b - 64 a^3 b^2 - 256 a^2 b^3 - 320 a b^4 - 128 b^5) \tan[2 e] \right) / \right. \\ & \left. \left. \left. (a^2 (a + 2 b + a \cos[2 (e + f x)])^2) \right) \right) \right) / (2048 b^2 (a + b)^2 f (a + b \sec[e + f x]^2)^3) \end{aligned} \right) -$$

- **Problem 371: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{x}{a^3} + \frac{(3 a^2 + 12 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a+b}}\right]}{8 a^3 \sqrt{b} (a + b)^{3/2} f} + \frac{\tan[e + f x]}{4 a f (a + b + b \tan[e + f x]^2)^2} + \frac{(3 a + 4 b) \tan[e + f x]}{8 a^2 (a + b) f (a + b + b \tan[e + f x]^2)}$$

Result (type 3, 1745 leaves):

$$\left((a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \right. \\ \left. \left(\frac{(3 a^2 + 8 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} - \frac{a \sqrt{b} (3 a^2 + 16 a b + 16 b^2 + 3 a (a + 2 b) \cos[2 (e + f x)]) \sin[2 (e + f x)]}{(a + b)^2 (a + 2 b + a \cos[2 (e + f x)])^2} \right) \right) / \\ (1024 b^{5/2} f (a + b \sec[e + f x]^2)^3) + \left((a + 2 b + a \cos[2 e + 2 f x])^3 \sec[e + f x]^6 \right. \\ \left. \left(- \frac{3 a (a + 2 b) \operatorname{ArcTan}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a+b}}\right]}{(a + b)^{5/2}} + \frac{\sqrt{b} (3 a^3 + 14 a^2 b + 24 a b^2 + 16 b^3 + a (3 a^2 + 4 a b + 4 b^2) \cos[2 (e + f x)]) \sin[2 (e + f x)]}{(a + b)^2 (a + 2 b + a \cos[2 (e + f x)])^2} \right) \right) /$$

$$\begin{aligned}
& (2048 b^{5/2} f (a + b \operatorname{Sec}[e + f x]^2)^3) + \frac{1}{32 (a + b \operatorname{Sec}[e + f x]^2)^3} (a + 2b + a \operatorname{Cos}[2e + 2fx])^3 \operatorname{Sec}[e + f x]^6 \\
& \left(-\frac{1}{(a + b)^2} (3a^5 - 10a^4b + 80a^3b^2 + 480a^2b^3 + 640ab^4 + 256b^5) \right. \\
& \quad \left(\left(\operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b}\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]}} - \frac{i\operatorname{Sin}[2e]}{2\sqrt{a+b}\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]}} \right) \right] \right. \right. \\
& \quad \left. \left. (-a\operatorname{Sin}[fx] - 2b\operatorname{Sin}[fx] + a\operatorname{Sin}[2e + fx]) \right] \operatorname{Cos}[2e] \right) / \left(64a^3b^2\sqrt{a+b}f\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]} \right) - \\
& \quad \left(i\operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2\sqrt{a+b}\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]}} - \frac{i\operatorname{Sin}[2e]}{2\sqrt{a+b}\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]}} \right) \right] \right. \\
& \quad \left. \left. (-a\operatorname{Sin}[fx] - 2b\operatorname{Sin}[fx] + a\operatorname{Sin}[2e + fx]) \right] \operatorname{Sin}[2e] \right) / \left(64a^3b^2\sqrt{a+b}f\sqrt{b\operatorname{Cos}[4e] - ib\operatorname{Sin}[4e]} \right) \Big) - \\
& \frac{1}{128a^3b^2(a+b)^2f(a+2b+a\operatorname{Cos}[2e+2fx])^2} \operatorname{Sec}[2e] (768a^4b^2fx\operatorname{Cos}[2e] + 3584a^3b^3fx\operatorname{Cos}[2e] + 6912a^2b^4fx\operatorname{Cos}[2e] + \\
& 6144ab^5fx\operatorname{Cos}[2e] + 2048b^6fx\operatorname{Cos}[2e] + 512a^4b^2fx\operatorname{Cos}[2fx] + 2048a^3b^3fx\operatorname{Cos}[2fx] + 2560a^2b^4fx\operatorname{Cos}[2fx] + \\
& 1024ab^5fx\operatorname{Cos}[2fx] + 512a^4b^2fx\operatorname{Cos}[4e+2fx] + 2048a^3b^3fx\operatorname{Cos}[4e+2fx] + 2560a^2b^4fx\operatorname{Cos}[4e+2fx] + \\
& 1024ab^5fx\operatorname{Cos}[4e+2fx] + 128a^4b^2fx\operatorname{Cos}[2e+4fx] + 256a^3b^3fx\operatorname{Cos}[2e+4fx] + 128a^2b^4fx\operatorname{Cos}[2e+4fx] + \\
& 128a^4b^2fx\operatorname{Cos}[6e+4fx] + 256a^3b^3fx\operatorname{Cos}[6e+4fx] + 128a^2b^4fx\operatorname{Cos}[6e+4fx] - 9a^6\operatorname{Sin}[2e] + 12a^5b\operatorname{Sin}[2e] + \\
& 684a^4b^2\operatorname{Sin}[2e] + 2880a^3b^3\operatorname{Sin}[2e] + 5280a^2b^4\operatorname{Sin}[2e] + 4608ab^5\operatorname{Sin}[2e] + 1536b^6\operatorname{Sin}[2e] + 9a^6\operatorname{Sin}[2fx] - \\
& 14a^5b\operatorname{Sin}[2fx] - 608a^4b^2\operatorname{Sin}[2fx] - 2112a^3b^3\operatorname{Sin}[2fx] - 2560a^2b^4\operatorname{Sin}[2fx] - 1024ab^5\operatorname{Sin}[2fx] - 3a^6\operatorname{Sin}[4e+2fx] + \\
& 10a^5b\operatorname{Sin}[4e+2fx] + 304a^4b^2\operatorname{Sin}[4e+2fx] + 1056a^3b^3\operatorname{Sin}[4e+2fx] + 1280a^2b^4\operatorname{Sin}[4e+2fx] + 512ab^5\operatorname{Sin}[4e+2fx] + \\
& 3a^6\operatorname{Sin}[2e+4fx] - 12a^5b\operatorname{Sin}[2e+4fx] - 204a^4b^2\operatorname{Sin}[2e+4fx] - 384a^3b^3\operatorname{Sin}[2e+4fx] - 192a^2b^4\operatorname{Sin}[2e+4fx]) \Big) - \\
& \left((a + 2b + a \operatorname{Cos}[2e + 2fx])^3 \operatorname{Sec}[e + f x]^6 \left(-\frac{6a^2 \operatorname{ArcTan}\left[\frac{\operatorname{Sec}[fx] (\operatorname{Cos}[2e] - i\operatorname{Sin}[2e]) (- (a+2b)\operatorname{Sin}[fx] + a\operatorname{Sin}[2e+fx])}{2\sqrt{a+b}\sqrt{b(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^4}} \right]}{\sqrt{a+b}\sqrt{b(\operatorname{Cos}[e] - i\operatorname{Sin}[e])^4}} \right] (\operatorname{Cos}[2e] - i\operatorname{Sin}[2e]) \right. \right. \\
& \quad \left. \left. (a \operatorname{Sec}[2e] \left((-9a^4 - 16a^3b + 48a^2b^2 + 128ab^3 + 64b^4) \operatorname{Sin}[2fx] + a(-3a^3 + 2a^2b + 24ab^2 + 16b^3) \operatorname{Sin}[2(e + 2fx)] + \right. \right. \right. \\
& \quad \left. \left. \left. (3a^4 - 64a^2b^2 - 128ab^3 - 64b^4) \operatorname{Sin}[4e + 2fx] \right) + (9a^5 + 18a^4b - 64a^3b^2 - 256a^2b^3 - 320ab^4 - 128b^5) \operatorname{Tan}[2e] \right) / \right. \\
& \quad \left. \left. \left. (a^2 (a + 2b + a \operatorname{Cos}[2(e + fx)])^2) \right) \right) \right) / (2048b^2(a+b)^2f(a+b\operatorname{Sec}[e+fx]^2)^3)
\end{aligned}$$

■ **Problem 372:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 144 leaves, 6 steps):

$$\frac{x}{a^3} - \frac{\sqrt{b} (15 a^2 + 20 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{5/2} f} - \frac{b \operatorname{Tan}[e + f x]}{4 a (a + b) f (a + b + b \operatorname{Tan}[e + f x])^2} - \frac{b (7 a + 4 b) \operatorname{Tan}[e + f x]}{8 a^2 (a + b)^2 f (a + b + b \operatorname{Tan}[e + f x])^2}$$

Result (type 3, 627 leaves):

$$\frac{x (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6}{8 a^3 (a + b \operatorname{Sec}[e + f x])^3} + \left((15 a^2 + 20 a b + 8 b^2) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \right. \\ \left. \operatorname{Sec}[e + f x]^6 \left(\left(b \operatorname{ArcTan}\left[\operatorname{Sec}[f x]\right] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right. \right. \right. \\ \left. \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Cos}[2 e] \right) \right) / \left(64 a^3 \sqrt{a + b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) - \\ \left(i b \operatorname{ArcTan}\left[\operatorname{Sec}[f x]\right] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) (-a \operatorname{Sin}[f x] - \right. \\ \left. 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right) \operatorname{Sin}[2 e] \right) / \left(64 a^3 \sqrt{a + b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) \left. \right) / \left((a + b)^2 (a + b \operatorname{Sec}[e + f x])^3 \right) + \\ \left((a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^2 \operatorname{Sec}[e + f x]^6 (9 a^2 b \operatorname{Sin}[2 e] + 28 a b^2 \operatorname{Sin}[2 e] + 16 b^3 \operatorname{Sin}[2 e] - 9 a^2 b \operatorname{Sin}[2 f x] - 6 a b^2 \operatorname{Sin}[2 f x]) \right) / \\ \left(64 a^3 (a + b)^2 f (a + b \operatorname{Sec}[e + f x])^3 (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e]) \right) + \\ \left(a + 2 b + a \operatorname{Cos}[2 e + 2 f x] \right) \operatorname{Sec}[e + f x]^6 (-a b^2 \operatorname{Sin}[2 e] - 2 b^3 \operatorname{Sin}[2 e] + a b^2 \operatorname{Sin}[2 f x]) \\ \left. \right) / \left(16 a^3 (a + b) f (a + b \operatorname{Sec}[e + f x])^3 (\operatorname{Cos}[e] - \operatorname{Sin}[e]) (\operatorname{Cos}[e] + \operatorname{Sin}[e]) \right)$$

■ **Problem 373: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[e + f x]^2}{(a + b \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 181 leaves, 8 steps):

$$-\frac{x}{a^3} + \frac{b^{3/2} (35 a^2 + 28 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{7/2} f} - \frac{(8 a^2 - 11 a b - 4 b^2) \operatorname{Cot}[e + f x]}{8 a^2 (a + b)^3 f} - \\ \frac{b \operatorname{Cot}[e + f x]}{4 a (a + b) f (a + b + b \operatorname{Tan}[e + f x])^2} - \frac{b (9 a + 4 b) \operatorname{Cot}[e + f x]}{8 a^2 (a + b)^2 f (a + b + b \operatorname{Tan}[e + f x])^2}$$

Result (type 3, 2089 leaves):

$$\left((35 a^2 + 28 a b + 8 b^2) (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^3 \operatorname{Sec}[e + f x]^6 \right. \\ \left. - \left(b^2 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right] \right. \right. \\ \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right] \operatorname{Cos}[2 e] \right) / \left(64 a^3 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) + \\ \left(i b^2 \operatorname{ArcTan}\left[\operatorname{Sec}[f x] \left(\frac{\operatorname{Cos}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} - \frac{i \operatorname{Sin}[2 e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]}} \right) \right] \right. \\ \left. \left. (-a \operatorname{Sin}[f x] - 2 b \operatorname{Sin}[f x] + a \operatorname{Sin}[2 e + f x]) \right] \operatorname{Sin}[2 e] \right) / \left(64 a^3 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4 e] - i b \operatorname{Sin}[4 e]} \right) \Bigg) / \\ ((a+b)^3 (a+b \operatorname{Sec}[e+f x]^2)^3) + \frac{1}{512 a^3 (a+b)^3 f (a+b \operatorname{Sec}[e+f x]^2)^3} (a+2b+a \operatorname{Cos}[2e+2fx])$$

Csc[e]

Csc[e + f x]

Sec[2 e]

Sec[e + f x]^6

$$\begin{aligned} & (8 a^5 f x \operatorname{Cos}[f x] + 56 a^4 b f x \operatorname{Cos}[f x] + 184 a^3 b^2 f x \operatorname{Cos}[f x] + 296 a^2 b^3 f x \operatorname{Cos}[f x] + 224 a b^4 f x \operatorname{Cos}[f x] + 64 b^5 f x \operatorname{Cos}[f x] - \\ & 12 a^5 f x \operatorname{Cos}[3 f x] - 68 a^4 b f x \operatorname{Cos}[3 f x] - 132 a^3 b^2 f x \operatorname{Cos}[3 f x] - 108 a^2 b^3 f x \operatorname{Cos}[3 f x] - 32 a b^4 f x \operatorname{Cos}[3 f x] - \\ & 8 a^5 f x \operatorname{Cos}[2 e - f x] - 56 a^4 b f x \operatorname{Cos}[2 e - f x] - 184 a^3 b^2 f x \operatorname{Cos}[2 e - f x] - 296 a^2 b^3 f x \operatorname{Cos}[2 e - f x] - 224 a b^4 f x \operatorname{Cos}[2 e - f x] - \\ & 64 b^5 f x \operatorname{Cos}[2 e - f x] - 8 a^5 f x \operatorname{Cos}[2 e + f x] - 56 a^4 b f x \operatorname{Cos}[2 e + f x] - 184 a^3 b^2 f x \operatorname{Cos}[2 e + f x] - 296 a^2 b^3 f x \operatorname{Cos}[2 e + f x] - \\ & 224 a b^4 f x \operatorname{Cos}[2 e + f x] - 64 b^5 f x \operatorname{Cos}[2 e + f x] + 8 a^5 f x \operatorname{Cos}[4 e + f x] + 56 a^4 b f x \operatorname{Cos}[4 e + f x] + 184 a^3 b^2 f x \operatorname{Cos}[4 e + f x] + \\ & 296 a^2 b^3 f x \operatorname{Cos}[4 e + f x] + 224 a b^4 f x \operatorname{Cos}[4 e + f x] + 64 b^5 f x \operatorname{Cos}[4 e + f x] + 12 a^5 f x \operatorname{Cos}[2 e + 3 f x] + 68 a^4 b f x \operatorname{Cos}[2 e + 3 f x] + \\ & 132 a^3 b^2 f x \operatorname{Cos}[2 e + 3 f x] + 108 a^2 b^3 f x \operatorname{Cos}[2 e + 3 f x] + 32 a b^4 f x \operatorname{Cos}[2 e + 3 f x] - 12 a^5 f x \operatorname{Cos}[4 e + 3 f x] - \\ & 68 a^4 b f x \operatorname{Cos}[4 e + 3 f x] - 132 a^3 b^2 f x \operatorname{Cos}[4 e + 3 f x] - 108 a^2 b^3 f x \operatorname{Cos}[4 e + 3 f x] - 32 a b^4 f x \operatorname{Cos}[4 e + 3 f x] + \\ & 12 a^5 f x \operatorname{Cos}[6 e + 3 f x] + 68 a^4 b f x \operatorname{Cos}[6 e + 3 f x] + 132 a^3 b^2 f x \operatorname{Cos}[6 e + 3 f x] + 108 a^2 b^3 f x \operatorname{Cos}[6 e + 3 f x] + \\ & 32 a b^4 f x \operatorname{Cos}[6 e + 3 f x] - 4 a^5 f x \operatorname{Cos}[2 e + 5 f x] - 12 a^4 b f x \operatorname{Cos}[2 e + 5 f x] - 12 a^3 b^2 f x \operatorname{Cos}[2 e + 5 f x] - 4 a^2 b^3 f x \operatorname{Cos}[2 e + 5 f x] - \\ & 4 a^5 f x \operatorname{Cos}[4 e + 5 f x] + 12 a^4 b f x \operatorname{Cos}[4 e + 5 f x] + 12 a^3 b^2 f x \operatorname{Cos}[4 e + 5 f x] + 4 a^2 b^3 f x \operatorname{Cos}[4 e + 5 f x] - 4 a^5 f x \operatorname{Cos}[6 e + 5 f x] - \\ & 12 a^4 b f x \operatorname{Cos}[6 e + 5 f x] - 12 a^3 b^2 f x \operatorname{Cos}[6 e + 5 f x] - 4 a^2 b^3 f x \operatorname{Cos}[6 e + 5 f x] + 4 a^5 f x \operatorname{Cos}[8 e + 5 f x] + 12 a^4 b f x \operatorname{Cos}[8 e + 5 f x] + \\ & 12 a^3 b^2 f x \operatorname{Cos}[8 e + 5 f x] + 4 a^2 b^3 f x \operatorname{Cos}[8 e + 5 f x] - 32 a^5 \operatorname{Sin}[f x] - 64 a^4 b \operatorname{Sin}[f x] - 30 a^2 b^3 \operatorname{Sin}[f x] - 120 a b^4 \operatorname{Sin}[f x] - \\ & 48 b^5 \operatorname{Sin}[f x] + 32 a^5 \operatorname{Sin}[3 f x] + 64 a^4 b \operatorname{Sin}[3 f x] + 26 a^3 b^2 \operatorname{Sin}[3 f x] + 86 a^2 b^3 \operatorname{Sin}[3 f x] + 32 a b^4 \operatorname{Sin}[3 f x] - \\ & 48 a^5 \operatorname{Sin}[2 e - f x] - 128 a^4 b \operatorname{Sin}[2 e - f x] - 128 a^3 b^2 \operatorname{Sin}[2 e - f x] - 30 a^2 b^3 \operatorname{Sin}[2 e - f x] - 120 a b^4 \operatorname{Sin}[2 e - f x] - \\ & 48 b^5 \operatorname{Sin}[2 e - f x] + 48 a^5 \operatorname{Sin}[2 e + f x] + 128 a^4 b \operatorname{Sin}[2 e + f x] + 102 a^3 b^2 \operatorname{Sin}[2 e + f x] - 86 a^2 b^3 \operatorname{Sin}[2 e + f x] - \\ & 136 a b^4 \operatorname{Sin}[2 e + f x] - 48 b^5 \operatorname{Sin}[2 e + f x] - 32 a^5 \operatorname{Sin}[4 e + f x] - 64 a^4 b \operatorname{Sin}[4 e + f x] + 26 a^3 b^2 \operatorname{Sin}[4 e + f x] + 86 a^2 b^3 \operatorname{Sin}[4 e + f x] + \\ & 136 a b^4 \operatorname{Sin}[4 e + f x] + 48 b^5 \operatorname{Sin}[4 e + f x] - 8 a^5 \operatorname{Sin}[2 e + 3 f x] - 26 a^3 b^2 \operatorname{Sin}[2 e + 3 f x] - 86 a^2 b^3 \operatorname{Sin}[2 e + 3 f x] - \\ & 32 a b^4 \operatorname{Sin}[2 e + 3 f x] + 32 a^5 \operatorname{Sin}[4 e + 3 f x] + 64 a^4 b \operatorname{Sin}[4 e + 3 f x] - 13 a^3 b^2 \operatorname{Sin}[4 e + 3 f x] - 36 a^2 b^3 \operatorname{Sin}[4 e + 3 f x] - \\ & 16 a b^4 \operatorname{Sin}[4 e + 3 f x] - 8 a^5 \operatorname{Sin}[6 e + 3 f x] + 13 a^3 b^2 \operatorname{Sin}[6 e + 3 f x] + 36 a^2 b^3 \operatorname{Sin}[6 e + 3 f x] + 16 a b^4 \operatorname{Sin}[6 e + 3 f x] + \\ & 8 a^5 \operatorname{Sin}[2 e + 5 f x] + 13 a^3 b^2 \operatorname{Sin}[2 e + 5 f x] + 6 a^2 b^3 \operatorname{Sin}[2 e + 5 f x] - 13 a^3 b^2 \operatorname{Sin}[4 e + 5 f x] - 6 a^2 b^3 \operatorname{Sin}[4 e + 5 f x] + 8 a^5 \operatorname{Sin}[6 e + 5 f x]) \end{aligned}$$

■ **Problem 374: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[e + f x]^4}{(a + b \text{Sec}[e + f x]^2)^3} dx$$

Optimal (type 3, 230 leaves, 9 steps):

$$\frac{x}{a^3} - \frac{b^{5/2} (63 a^2 + 36 a b + 8 b^2) \text{ArcTan}\left[\frac{\sqrt{b} \text{Tan}[e + f x]}{\sqrt{a + b}}\right]}{8 a^3 (a + b)^{9/2} f} + \frac{(8 a^3 + 32 a^2 b - 15 a b^2 - 4 b^3) \text{Cot}[e + f x]}{8 a^2 (a + b)^4 f} - \frac{(8 a^2 - 39 a b - 12 b^2) \text{Cot}[e + f x]^3}{24 a^2 (a + b)^3 f} - \frac{b \text{Cot}[e + f x]^3}{4 a (a + b) f (a + b + b \text{Tan}[e + f x]^2)^2} - \frac{b (11 a + 4 b) \text{Cot}[e + f x]^3}{8 a^2 (a + b)^2 f (a + b + b \text{Tan}[e + f x]^2)}$$

Result (type 3, 3340 leaves):

$$\left((63 a^2 + 36 a b + 8 b^2) (a + 2 b + a \text{Cos}[2 e + 2 f x])^3 \text{Sec}[e + f x]^6 \right. \\ \left. \left(\left(b^3 \text{ArcTan}\left[\text{Sec}[f x] \left(\frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right] \right. \right. \right. \\ \left. \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Cos}[2 e] \right) \right) / \left(64 a^3 \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) - \\ \left(i b^3 \text{ArcTan}\left[\text{Sec}[f x] \left(\frac{\text{Cos}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} - \frac{i \text{Sin}[2 e]}{2 \sqrt{a + b} \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]}} \right) \right] \right. \\ \left. \left. (-a \text{Sin}[f x] - 2 b \text{Sin}[f x] + a \text{Sin}[2 e + f x]) \right) \text{Sin}[2 e] \right) / \left(64 a^3 \sqrt{a + b} f \sqrt{b \text{Cos}[4 e] - i b \text{Sin}[4 e]} \right) \right) / \\ \left((a + b)^4 (a + b \text{Sec}[e + f x]^2)^3 \right) + \frac{1}{6144 a^3 (a + b)^4 f (a + b \text{Sec}[e + f x]^2)^3} (a + 2 b + a \text{Cos}[2 e + 2 f x]) \\ \text{Csc}[e] \\ \text{Csc}[e + f x]^3 \\ \text{Sec}[2 e] \\ \text{Sec}[e + f x]^6 \\ (-36 a^6 f x \text{Cos}[f x] - 336 a^5 b f x \text{Cos}[f x] - 1560 a^4 b^2 f x \text{Cos}[f x] - 3600 a^3 b^3 f x \text{Cos}[f x] - 4260 a^2 b^4 f x \text{Cos}[f x] - 2496 a b^5 f x \text{Cos}[f x] - \\ 576 b^6 f x \text{Cos}[f x] + 36 a^6 f x \text{Cos}[3 f x] + 240 a^5 b f x \text{Cos}[3 f x] + 408 a^4 b^2 f x \text{Cos}[3 f x] - 48 a^3 b^3 f x \text{Cos}[3 f x] - \\ 732 a^2 b^4 f x \text{Cos}[3 f x] - 672 a b^5 f x \text{Cos}[3 f x] - 192 b^6 f x \text{Cos}[3 f x] + 36 a^6 f x \text{Cos}[2 e - f x] + 336 a^5 b f x \text{Cos}[2 e - f x] + \\ 1560 a^4 b^2 f x \text{Cos}[2 e - f x] + 3600 a^3 b^3 f x \text{Cos}[2 e - f x] + 4260 a^2 b^4 f x \text{Cos}[2 e - f x] + 2496 a b^5 f x \text{Cos}[2 e - f x] + \\ 576 b^6 f x \text{Cos}[2 e - f x] + 36 a^6 f x \text{Cos}[2 e + f x] + 336 a^5 b f x \text{Cos}[2 e + f x] + 1560 a^4 b^2 f x \text{Cos}[2 e + f x] + 3600 a^3 b^3 f x \text{Cos}[2 e + f x] + \\ 4260 a^2 b^4 f x \text{Cos}[2 e + f x] + 2496 a b^5 f x \text{Cos}[2 e + f x] + 576 b^6 f x \text{Cos}[2 e + f x] - 36 a^6 f x \text{Cos}[4 e + f x] - 336 a^5 b f x \text{Cos}[4 e + f x] - \\ 1560 a^4 b^2 f x \text{Cos}[4 e + f x] - 3600 a^3 b^3 f x \text{Cos}[4 e + f x] - 4260 a^2 b^4 f x \text{Cos}[4 e + f x] - 2496 a b^5 f x \text{Cos}[4 e + f x] - \\ 576 b^6 f x \text{Cos}[4 e + f x] - 36 a^6 f x \text{Cos}[2 e + 3 f x] - 240 a^5 b f x \text{Cos}[2 e + 3 f x] - 408 a^4 b^2 f x \text{Cos}[2 e + 3 f x] + \\ 48 a^3 b^3 f x \text{Cos}[2 e + 3 f x] + 732 a^2 b^4 f x \text{Cos}[2 e + 3 f x] + 672 a b^5 f x \text{Cos}[2 e + 3 f x] + 192 b^6 f x \text{Cos}[2 e + 3 f x] +$$

$$\begin{aligned}
& 36 a^6 f x \cos[4 e + 3 f x] + 240 a^5 b f x \cos[4 e + 3 f x] + 408 a^4 b^2 f x \cos[4 e + 3 f x] - 48 a^3 b^3 f x \cos[4 e + 3 f x] - \\
& 732 a^2 b^4 f x \cos[4 e + 3 f x] - 672 a b^5 f x \cos[4 e + 3 f x] - 192 b^6 f x \cos[4 e + 3 f x] - 36 a^6 f x \cos[6 e + 3 f x] - \\
& 240 a^5 b f x \cos[6 e + 3 f x] - 408 a^4 b^2 f x \cos[6 e + 3 f x] + 48 a^3 b^3 f x \cos[6 e + 3 f x] + 732 a^2 b^4 f x \cos[6 e + 3 f x] + \\
& 672 a b^5 f x \cos[6 e + 3 f x] + 192 b^6 f x \cos[6 e + 3 f x] - 12 a^6 f x \cos[2 e + 5 f x] - 144 a^5 b f x \cos[2 e + 5 f x] - \\
& 456 a^4 b^2 f x \cos[2 e + 5 f x] - 624 a^3 b^3 f x \cos[2 e + 5 f x] - 396 a^2 b^4 f x \cos[2 e + 5 f x] - 96 a b^5 f x \cos[2 e + 5 f x] + \\
& 12 a^6 f x \cos[4 e + 5 f x] + 144 a^5 b f x \cos[4 e + 5 f x] + 456 a^4 b^2 f x \cos[4 e + 5 f x] + 624 a^3 b^3 f x \cos[4 e + 5 f x] + \\
& 396 a^2 b^4 f x \cos[4 e + 5 f x] + 96 a b^5 f x \cos[4 e + 5 f x] - 12 a^6 f x \cos[6 e + 5 f x] - 144 a^5 b f x \cos[6 e + 5 f x] - \\
& 456 a^4 b^2 f x \cos[6 e + 5 f x] - 624 a^3 b^3 f x \cos[6 e + 5 f x] - 396 a^2 b^4 f x \cos[6 e + 5 f x] - 96 a b^5 f x \cos[6 e + 5 f x] + \\
& 12 a^6 f x \cos[8 e + 5 f x] + 144 a^5 b f x \cos[8 e + 5 f x] + 456 a^4 b^2 f x \cos[8 e + 5 f x] + 624 a^3 b^3 f x \cos[8 e + 5 f x] + \\
& 396 a^2 b^4 f x \cos[8 e + 5 f x] + 96 a b^5 f x \cos[8 e + 5 f x] - 12 a^6 f x \cos[4 e + 7 f x] - 48 a^5 b f x \cos[4 e + 7 f x] - \\
& 72 a^4 b^2 f x \cos[4 e + 7 f x] - 48 a^3 b^3 f x \cos[4 e + 7 f x] - 12 a^2 b^4 f x \cos[4 e + 7 f x] + 12 a^6 f x \cos[6 e + 7 f x] + 48 a^5 b f x \cos[6 e + 7 f x] + \\
& 72 a^4 b^2 f x \cos[6 e + 7 f x] + 48 a^3 b^3 f x \cos[6 e + 7 f x] + 12 a^2 b^4 f x \cos[6 e + 7 f x] - 12 a^6 f x \cos[8 e + 7 f x] - 48 a^5 b f x \cos[8 e + 7 f x] - \\
& 72 a^4 b^2 f x \cos[8 e + 7 f x] - 48 a^3 b^3 f x \cos[8 e + 7 f x] - 12 a^2 b^4 f x \cos[8 e + 7 f x] + 12 a^6 f x \cos[10 e + 7 f x] + \\
& 48 a^5 b f x \cos[10 e + 7 f x] + 72 a^4 b^2 f x \cos[10 e + 7 f x] + 48 a^3 b^3 f x \cos[10 e + 7 f x] + 12 a^2 b^4 f x \cos[10 e + 7 f x] - \\
& 128 a^6 \sin[f x] - 440 a^5 b \sin[f x] - 1152 a^4 b^2 \sin[f x] - 1920 a^3 b^3 \sin[f x] + 228 a^2 b^4 \sin[f x] + 1320 a b^5 \sin[f x] + \\
& 432 b^6 \sin[f x] + 48 a^6 \sin[3 f x] + 104 a^5 b \sin[3 f x] + 640 a^4 b^2 \sin[3 f x] + 1511 a^3 b^3 \sin[3 f x] - 528 a^2 b^4 \sin[3 f x] + \\
& 264 a b^5 \sin[3 f x] + 144 b^6 \sin[3 f x] - 32 a^6 \sin[2 e - f x] + 384 a^5 b \sin[2 e - f x] + 2048 a^4 b^2 \sin[2 e - f x] + 3072 a^3 b^3 \sin[2 e - f x] + \\
& 228 a^2 b^4 \sin[2 e - f x] + 1320 a b^5 \sin[2 e - f x] + 432 b^6 \sin[2 e - f x] + 32 a^6 \sin[2 e + f x] - 384 a^5 b \sin[2 e + f x] - \\
& 2048 a^4 b^2 \sin[2 e + f x] - 2919 a^3 b^3 \sin[2 e + f x] + 642 a^2 b^4 \sin[2 e + f x] + 1416 a b^5 \sin[2 e + f x] + 432 b^6 \sin[2 e + f x] - \\
& 128 a^6 \sin[4 e + f x] - 440 a^5 b \sin[4 e + f x] - 1152 a^4 b^2 \sin[4 e + f x] - 2073 a^3 b^3 \sin[4 e + f x] - 642 a^2 b^4 \sin[4 e + f x] - \\
& 1416 a b^5 \sin[4 e + f x] - 432 b^6 \sin[4 e + f x] - 144 a^6 \sin[2 e + 3 f x] - 672 a^5 b \sin[2 e + 3 f x] - 960 a^4 b^2 \sin[2 e + 3 f x] + \\
& 153 a^3 b^3 \sin[2 e + 3 f x] + 528 a^2 b^4 \sin[2 e + 3 f x] - 264 a b^5 \sin[2 e + 3 f x] - 144 b^6 \sin[2 e + 3 f x] + 48 a^6 \sin[4 e + 3 f x] + \\
& 104 a^5 b \sin[4 e + 3 f x] + 640 a^4 b^2 \sin[4 e + 3 f x] + 1664 a^3 b^3 \sin[4 e + 3 f x] - 66 a^2 b^4 \sin[4 e + 3 f x] - 408 a b^5 \sin[4 e + 3 f x] - \\
& 144 b^6 \sin[4 e + 3 f x] - 144 a^6 \sin[6 e + 3 f x] - 672 a^5 b \sin[6 e + 3 f x] - 960 a^4 b^2 \sin[6 e + 3 f x] + 66 a^2 b^4 \sin[6 e + 3 f x] + \\
& 408 a b^5 \sin[6 e + 3 f x] + 144 b^6 \sin[6 e + 3 f x] + 80 a^6 \sin[2 e + 5 f x] + 480 a^5 b \sin[2 e + 5 f x] + 832 a^4 b^2 \sin[2 e + 5 f x] + \\
& 294 a^2 b^4 \sin[2 e + 5 f x] + 96 a b^5 \sin[2 e + 5 f x] - 48 a^6 \sin[4 e + 5 f x] - 120 a^5 b \sin[4 e + 5 f x] - 294 a^2 b^4 \sin[4 e + 5 f x] - \\
& 96 a b^5 \sin[4 e + 5 f x] + 80 a^6 \sin[6 e + 5 f x] + 480 a^5 b \sin[6 e + 5 f x] + 832 a^4 b^2 \sin[6 e + 5 f x] - 51 a^3 b^3 \sin[6 e + 5 f x] - \\
& 132 a^2 b^4 \sin[6 e + 5 f x] - 48 a b^5 \sin[6 e + 5 f x] - 48 a^6 \sin[8 e + 5 f x] - 120 a^5 b \sin[8 e + 5 f x] + 51 a^3 b^3 \sin[8 e + 5 f x] + \\
& 132 a^2 b^4 \sin[8 e + 5 f x] + 48 a b^5 \sin[8 e + 5 f x] + 32 a^6 \sin[4 e + 7 f x] + 104 a^5 b \sin[4 e + 7 f x] + 51 a^3 b^3 \sin[4 e + 7 f x] + \\
& 18 a^2 b^4 \sin[4 e + 7 f x] - 51 a^3 b^3 \sin[6 e + 7 f x] - 18 a^2 b^4 \sin[6 e + 7 f x] + 32 a^6 \sin[8 e + 7 f x] + 104 a^5 b \sin[8 e + 7 f x]
\end{aligned}$$

■ **Problem 375: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cot[e + f x]^6}{(a + b \sec[e + f x]^2)^3} dx$$

Optimal (type 3, 285 leaves, 10 steps):

$$\begin{aligned}
& -\frac{x}{a^3} + \frac{b^{7/2} (99 a^2 + 44 a b + 8 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b}}\right]}{8 a^3 (a+b)^{11/2} f} - \\
& \frac{(8 a^4 + 40 a^3 b + 80 a^2 b^2 - 19 a b^3 - 4 b^4) \operatorname{Cot}[e+fx]}{8 a^2 (a+b)^5 f} + \frac{(8 a^3 + 32 a^2 b - 51 a b^2 - 12 b^3) \operatorname{Cot}[e+fx]^3}{24 a^2 (a+b)^4 f} - \\
& \frac{(8 a^2 - 75 a b - 20 b^2) \operatorname{Cot}[e+fx]^5}{40 a^2 (a+b)^3 f} - \frac{b \operatorname{Cot}[e+fx]^5}{4 a (a+b) f (a+b+b \operatorname{Tan}[e+fx]^2)^2} - \frac{b (13 a + 4 b) \operatorname{Cot}[e+fx]^5}{8 a^2 (a+b)^2 f (a+b+b \operatorname{Tan}[e+fx]^2)}
\end{aligned}$$

Result (type 3, 976 leaves):

$$\begin{aligned}
& -\frac{x (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Sec}[e+fx]^6}{8 a^3 (a+b \operatorname{Sec}[e+fx]^2)^3} + \frac{(11 a \operatorname{Cos}[e] + 26 b \operatorname{Cos}[e]) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^2 \operatorname{Sec}[e+fx]^6}{120 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} - \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Cot}[e] \operatorname{Csc}[e+fx]^4 \operatorname{Sec}[e+fx]^6}{40 (a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \left((99 a^2 + 44 a b + 8 b^2) (a+2b+a \operatorname{Cos}[2e+2fx])^3 \right. \\
& \left. \operatorname{Sec}[e+fx]^6 \left(-\left(b^4 \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right)} \right) \right. \right. \\
& \left. \left. (-a \operatorname{Sin}[fx] - 2 b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \operatorname{Cos}[2e] \right) / \left(64 a^3 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) + \\
& \left(i b^4 \operatorname{ArcTan}\left[\operatorname{Sec}[fx] \left(\frac{\operatorname{Cos}[2e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} - \frac{i \operatorname{Sin}[2e]}{2 \sqrt{a+b} \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]}} \right)} \right] \right. \\
& \left. (-a \operatorname{Sin}[fx] - 2 b \operatorname{Sin}[fx] + a \operatorname{Sin}[2e+fx]) \right) \operatorname{Sin}[2e] \right) / \left(64 a^3 \sqrt{a+b} f \sqrt{b \operatorname{Cos}[4e] - i b \operatorname{Sin}[4e]} \right) \Big) / \\
& \left((a+b)^5 (a+b \operatorname{Sec}[e+fx]^2)^3 \right) + \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^5 \operatorname{Sec}[e+fx]^6 \operatorname{Sin}[fx]}{40 (a+b)^3 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx]^3 \operatorname{Sec}[e+fx]^6 (-11 a \operatorname{Sin}[fx] - 26 b \operatorname{Sin}[fx])}{120 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^3 \operatorname{Csc}[e] \operatorname{Csc}[e+fx] \operatorname{Sec}[e+fx]^6 (23 a^2 \operatorname{Sin}[fx] + 106 a b \operatorname{Sin}[fx] + 173 b^2 \operatorname{Sin}[fx])}{(120 (a+b)^5 f (a+b \operatorname{Sec}[e+fx]^2)^3) +} \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx]) \operatorname{Sec}[2e] \operatorname{Sec}[e+fx]^6 (a b^5 \operatorname{Sin}[2e] + 2 b^6 \operatorname{Sin}[2e] - a b^5 \operatorname{Sin}[2fx])}{16 a^3 (a+b)^4 f (a+b \operatorname{Sec}[e+fx]^2)^3} + \\
& \frac{(a+2b+a \operatorname{Cos}[2e+2fx])^2 \operatorname{Sec}[2e] \operatorname{Sec}[e+fx]^6}{(-21 a^2 b^4 \operatorname{Sin}[2e] - 52 a b^5 \operatorname{Sin}[2e] - 16 b^6 \operatorname{Sin}[2e] + 21 a^2 b^4 \operatorname{Sin}[2fx] + 6 a b^5 \operatorname{Sin}[2fx])} / \left(64 a^3 (a+b)^5 f (a+b \operatorname{Sec}[e+fx]^2)^3 \right)
\end{aligned}$$

■ **Problem 376: Unable to integrate problem.**

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^5 dx$$

Optimal (type 3, 111 leaves, 7 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} - \frac{(a+2b)(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3b^2 f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{5/2}}{5b^2 f}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^5 dx$$

■ **Problem 377: Unable to integrate problem.**

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^3 dx$$

Optimal (type 3, 80 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f} + \frac{(a+b \operatorname{Sec}[e+f x]^2)^{3/2}}{3bf}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x]^3 dx$$

■ **Problem 378: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \operatorname{Sec}[e + f x]^2} \operatorname{Tan}[e + f x] dx$$

Optimal (type 3, 54 leaves, 5 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{f}$$

Result (type 3, 307 leaves):

$$\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 \cos[e+fx] \right. \\ \left. \left(\frac{2}{1+e^{2i(e+fx)}} + \left(i \sqrt{a} \left(2fx + i \operatorname{Log} \left[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] + \right. \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log} \left[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] \right) \right) \right) \Bigg/ \\ \left(\sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \sqrt{a+b \operatorname{Sec}[e+fx]^2} \Bigg/ \left(\sqrt{2} f \sqrt{a+2b+a \cos[2e+2fx]} \right)$$

■ **Problem 379: Unable to integrate problem.**

$$\int \cot[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 70 leaves, 7 steps) :

$$\frac{\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}} \right]}{f} - \frac{\sqrt{a+b} \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}} \right]}{f}$$

Result (type 8, 25 leaves) :

$$\int \cot[e+fx] \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 380: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^3 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 109 leaves, 8 steps) :

$$-\frac{\sqrt{a} \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}} \right]}{f} + \frac{(2a+b) \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}} \right]}{2\sqrt{a+b} f} - \frac{\cot[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves) :

$$\frac{1}{\sqrt{2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]}}$$

$$e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx] \left(\frac{1+e^{2i(e+fx)}}{(-1+e^{2i(e+fx)})^2} - \frac{1}{\sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}} \right.$$

$$\left. \left(-2i \sqrt{a} \sqrt{a+b} f x + (2a+b) \operatorname{Log}[1-e^{2i(e+fx)}] + \sqrt{a} \sqrt{a+b} \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}] + \right. \right.$$

$$\left. \sqrt{a} \sqrt{a+b} \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}] - \right.$$

$$\left. 2a \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}] - \right.$$

$$\left. b \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2}] \right) \sqrt{a+b} \operatorname{Sec}[e+fx]^2$$

■ **Problem 381: Unable to integrate problem.**

$$\int \operatorname{Cot}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 161 leaves, 9 steps):

$$\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} - \frac{(8a^2 + 12ab + 3b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{3/2} f} +$$

$$\frac{(4a+3b) \operatorname{Cot}[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{8(a+b) f} - \frac{\operatorname{Cot}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{4 f}$$

Result (type 8, 27 leaves):

$$\int \operatorname{Cot}[e+fx]^5 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 382: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^6 dx$$

Optimal (type 3, 219 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right] - (a^3 + 5 a^2 b + 15 a b^2 - 5 b^3) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{16 b^{5/2} f}{16 b^2 f} \\
& \frac{(a-b)(a+5b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16 b^2 f} + \frac{(a-5b) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24 b f} + \frac{\operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6 f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^6 dx$$

■ **Problem 383: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^4 dx$$

Optimal (type 3, 165 leaves, 9 steps):

$$\begin{aligned}
& \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right] - (a^2 + 6 a b - 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{8 b^{3/2} f}{8 b^3/2 f} + \\
& \frac{(a-3b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8 b f} + \frac{\operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4 f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^4 dx$$

■ **Problem 384: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} \operatorname{Tan}[e+fx]^2 dx$$

Optimal (type 3, 118 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right] + (a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{2 \sqrt{b} f}{2 \sqrt{b} f} + \frac{\operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2 f}
\end{aligned}$$

Result (type 3, 526 leaves):

$$\frac{1}{\sqrt{2} f \sqrt{a+2b+a \cos[2e+2fx]}}$$

$$e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx] \left(-\frac{i(-1+e^{2i(e+fx)})}{(1+e^{2i(e+fx)})^2} + \frac{1}{\sqrt{b} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}} \right)$$

$$\left(-2\sqrt{a}\sqrt{b}fx + i\sqrt{a}\sqrt{b} \operatorname{Log}\left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] - i\sqrt{a}\sqrt{b} \operatorname{Log}\left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right] \right)$$

$$\sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} - a \operatorname{Log}\left[\frac{2\left(\sqrt{b}(-1+e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) f}{(a-b)(1+e^{2i(e+fx)})} \right] +$$

$$b \operatorname{Log}\left[\frac{2\left(\sqrt{b}(-1+e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) f}{(a-b)(1+e^{2i(e+fx)})} \right] \Bigg) \sqrt{a+b \operatorname{Sec}[e+fx]^2}$$

■ **Problem 385: Unable to integrate problem.**

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f}$$

Result (type 8, 18 leaves):

$$\int \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 386: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+fx]^2 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 69 leaves, 6 steps) :

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{\operatorname{Cot}[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{f}$$

Result (type 3, 306 leaves) :

$$\left(e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \operatorname{Cos}[e+fx] \right. \\ \left. \left(-\frac{2i}{-1+e^{2i(e+fx)}} + \left(\sqrt{a} \left(-2fx+i \operatorname{Log}\left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] - \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}\left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] \right) \right) \right) / \\ \left(\sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \left(\sqrt{a+b \operatorname{Sec}[e+fx]^2} \right) / \left(\sqrt{2} f \sqrt{a+2b+a \operatorname{Cos}[2e+2fx]} \right)$$

■ **Problem 387: Unable to integrate problem.**

$$\int \operatorname{Cot}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 114 leaves, 7 steps) :

$$\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(3a+2b) \operatorname{Cot}[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{3(a+b)f} - \frac{\operatorname{Cot}[e+fx]^3 \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{3f}$$

Result (type 8, 27 leaves) :

$$\int \operatorname{Cot}[e+fx]^4 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

■ **Problem 388: Unable to integrate problem.**

$$\int \operatorname{Cot}[e+fx]^6 \sqrt{a+b \operatorname{Sec}[e+fx]^2} dx$$

Optimal (type 3, 167 leaves, 8 steps) :

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(15a^2+25ab+8b^2) \operatorname{Cot}[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{15(a+b)^2 f} \\ - \frac{(b-5(a+b)) \operatorname{Cot}[e+fx]^3 \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{15(a+b)f} - \frac{\operatorname{Cot}[e+fx]^5 \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{5f}$$

Result (type 8, 27 leaves) :

$$\int \text{Cot}[e + f x]^6 \sqrt{a + b \text{Sec}[e + f x]^2} dx$$

■ **Problem 389: Unable to integrate problem.**

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Tan}[e + f x]^5 dx$$

Optimal (type 3, 135 leaves, 8 steps) :

$$-\frac{a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{a \sqrt{a+b \text{Sec}[e+f x]^2}}{f} + \frac{(a+b \text{Sec}[e+f x]^2)^{3/2}}{3 f} - \frac{(a+2 b)(a+b \text{Sec}[e+f x]^2)^{5/2}}{5 b^2 f} + \frac{(a+b \text{Sec}[e+f x]^2)^{7/2}}{7 b^2 f}$$

Result (type 8, 27 leaves) :

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Tan}[e + f x]^5 dx$$

■ **Problem 390: Unable to integrate problem.**

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Tan}[e + f x]^3 dx$$

Optimal (type 3, 104 leaves, 7 steps) :

$$\frac{a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{a \sqrt{a+b \text{Sec}[e+f x]^2}}{f} - \frac{(a+b \text{Sec}[e+f x]^2)^{3/2}}{3 f} + \frac{(a+b \text{Sec}[e+f x]^2)^{5/2}}{5 b f}$$

Result (type 8, 27 leaves) :

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Tan}[e + f x]^3 dx$$

■ **Problem 391: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \text{Sec}[e + f x]^2)^{3/2} \text{Tan}[e + f x] dx$$

Optimal (type 3, 78 leaves, 6 steps) :

$$-\frac{a^{3/2} \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{a \sqrt{a+b \text{Sec}[e+f x]^2}}{f} + \frac{(a+b \text{Sec}[e+f x]^2)^{3/2}}{3 f}$$

Result (type 3, 343 leaves) :

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\ \left. \left(\frac{8(b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2)}{(1+e^{2i(e+fx)})^3} + \left(3i a^{3/2} \left(2fx + i \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] \right) + \right. \right. \right. \\ \left. \left. \left. i \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] \right] \right) \right) / \\ \left(\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2} \right) \left(a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2} / (3f(a+2b+a \cos[2e+2fx])^{3/2})$$

- **Problem 392: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 91 leaves, 8 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{f} - \frac{(a+b)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{f} + \frac{b \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{f}$$

Result (type 3, 506 leaves):

$$\frac{1}{f(a+2b+a \cos[2e+2fx])^{3/2}} \sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \left(\frac{2b}{1+e^{2i(e+fx)}} + \frac{1}{\sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}} \right. \\ \left. \left(-2i a^{3/2} f x + 2(a+b)^{3/2} \operatorname{Log}[1-e^{2i(e+fx)}] + a^{3/2} \operatorname{Log}[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] + \right. \right. \\ \left. \left. a^{3/2} \operatorname{Log}[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] - \right. \right. \\ \left. \left. 2a \sqrt{a+b} \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] - \right. \right. \\ \left. \left. 2b \sqrt{a+b} \operatorname{Log}[a+b+a e^{2i(e+fx)} + b e^{2i(e+fx)} + \sqrt{a+b} \sqrt{4b e^{2i(e+fx)} + a(1+e^{2i(e+fx)})^2}] \right) \right) \left(a+b \operatorname{Sec}[e+fx]^2 \right)^{3/2}$$

- **Problem 393: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^3 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$-\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} + \frac{(2a-b) \sqrt{a+b} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{2f} - \frac{(a+b) \cot [e+f x]^2 \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{2f}$$

Result (type 3, 622 leaves):

$$\frac{1}{f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}}$$

$$\sqrt{2} e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \operatorname{Cos}[e + f x]^3 \left(\frac{(a+b) (1 + e^{2 i (e+f x)})}{(-1 + e^{2 i (e+f x)})^2} - \frac{1}{\sqrt{a+b} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}} \right)$$

$$\left(-2 i a^{3/2} \sqrt{a+b} f x + (2 a^2 + a b - b^2) \operatorname{Log}[1 - e^{2 i (e+f x)}] + a^{3/2} \sqrt{a+b} \operatorname{Log}\left[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] + \right.$$

$$a^{3/2} \sqrt{a+b} \operatorname{Log}\left[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] -$$

$$2 a^2 \operatorname{Log}\left[a + b + a e^{2 i (e+f x)} + b e^{2 i (e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] -$$

$$a b \operatorname{Log}\left[a + b + a e^{2 i (e+f x)} + b e^{2 i (e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] +$$

$$\left. b^2 \operatorname{Log}\left[a + b + a e^{2 i (e+f x)} + b e^{2 i (e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] \right) (a + b \operatorname{Sec}[e + f x]^2)^{3/2}$$

- **Problem 394: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot [e + f x]^5 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 159 leaves, 9 steps):

$$\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{f} - \frac{(8 a^2 + 4 a b - b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{8 \sqrt{a+b} f} +$$

$$\frac{(4 a - b) \operatorname{Cot}[e+f x]^2 \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{8 f} - \frac{(a+b) \operatorname{Cot}[e+f x]^4 \sqrt{a+b \operatorname{Sec}[e+f x]^2}}{4 f}$$

Result (type 3, 684 leaves) :

$$\frac{1}{2 \sqrt{2} f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}} e^{i(e+f x)} \sqrt{4 b + a e^{-2 i(e+f x)} (1 + e^{2 i(e+f x)})^2} \operatorname{Cos}[e+f x]^3$$

$$\left(- \frac{(1 + e^{2 i(e+f x)}) (b (1 + 6 e^{2 i(e+f x)} + e^{4 i(e+f x)}) + a (6 - 4 e^{2 i(e+f x)} + 6 e^{4 i(e+f x)}))}{(-1 + e^{2 i(e+f x)})^4} + \frac{1}{\sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}} \right.$$

$$\left. \left(-8 i a^{3/2} \sqrt{a+b} f x + (8 a^2 + 4 a b - b^2) \operatorname{Log}[1 - e^{2 i(e+f x)}] + 4 a^{3/2} \sqrt{a+b} \operatorname{Log}[a + 2 b + a e^{2 i(e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}] + \right.$$

$$4 a^{3/2} \sqrt{a+b} \operatorname{Log}[a + a e^{2 i(e+f x)} + 2 b e^{2 i(e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}] -$$

$$8 a^2 \operatorname{Log}[a + b + a e^{2 i(e+f x)} + b e^{2 i(e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}] -$$

$$4 a b \operatorname{Log}[a + b + a e^{2 i(e+f x)} + b e^{2 i(e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}] +$$

$$\left. \left. b^2 \operatorname{Log}[a + b + a e^{2 i(e+f x)} + b e^{2 i(e+f x)} + \sqrt{a+b} \sqrt{4 b e^{2 i(e+f x)} + a (1 + e^{2 i(e+f x)})^2}] \right) \right) (a + b \operatorname{Sec}[e+f x]^2)^{3/2}$$

■ **Problem 395: Unable to integrate problem.**

$$\int (a + b \operatorname{Sec}[e+f x]^2)^{3/2} \operatorname{Tan}[e+f x]^6 dx$$

Optimal (type 3, 290 leaves, 11 steps) :

$$\begin{aligned}
& - \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(3a^4 + 20a^3b + 90a^2b^2 - 60ab^3 - 5b^4) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{128b^{5/2}f} \\
& + \frac{(3a^3 + 17a^2b - 55ab^2 - 5b^3) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{128b^2f} + \frac{(3a^2 - 50ab - 5b^2) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{192bf} \\
& + \frac{(9a+b) \operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{48f} + \frac{b \operatorname{Tan}[e+fx]^7 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Tan}[e+fx]^6 dx$$

■ **Problem 396: Unable to integrate problem.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Tan}[e+fx]^4 dx$$

Optimal (type 3, 214 leaves, 10 steps):

$$\begin{aligned}
& \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(a-b)(a^2 + 10ab + b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{16b^{3/2}f} + \\
& \frac{(a^2 - 8ab - b^2) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{16bf} + \frac{(7a+b) \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{24f} + \frac{b \operatorname{Tan}[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{6f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Tan}[e+fx]^4 dx$$

■ **Problem 397: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \operatorname{Tan}[e+fx]^2 dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\begin{aligned}
& - \frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{(3a^2 - 6ab - b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{8\sqrt{b}f} + \\
& \frac{(5a+b) \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{8f} + \frac{b \operatorname{Tan}[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{4f}
\end{aligned}$$

Result (type 3, 702 leaves):

$$\begin{aligned}
& \frac{1}{2\sqrt{2} f (a + 2b + a \cos[2(e + fx)])^{3/2}} \\
& e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e + fx]^3 \left(- \frac{i(-1 + e^{2i(e+fx)}) (5a(1 + e^{2i(e+fx)})^2 - b(1 - 6e^{2i(e+fx)} + e^{4i(e+fx)}))}{(1 + e^{2i(e+fx)})^4} + \right. \\
& \frac{1}{\sqrt{b} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}} \left(-8a^{3/2} \sqrt{b} f x + 4i a^{3/2} \sqrt{b} \operatorname{Log}\left[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right] - \right. \\
& 4i a^{3/2} \sqrt{b} \operatorname{Log}\left[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right] - \\
& 3a^2 \operatorname{Log}\left[\frac{4\left(\sqrt{b}(-1 + e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}\right) f}{(3a^2 - 6ab - b^2)(1 + e^{2i(e+fx)})} \right] + \\
& 6ab \operatorname{Log}\left[\frac{4\left(\sqrt{b}(-1 + e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}\right) f}{(3a^2 - 6ab - b^2)(1 + e^{2i(e+fx)})} \right] + \\
& \left. \left. b^2 \operatorname{Log}\left[\frac{4\left(\sqrt{b}(-1 + e^{2i(e+fx)}) - i\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}\right) f}{(3a^2 - 6ab - b^2)(1 + e^{2i(e+fx)})} \right] \right) \right) (a + b \operatorname{Sec}[e + fx]^2)^{3/2}
\end{aligned}$$

- **Problem 398: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + fx]^2)^{3/2} dx$$

Optimal (type 3, 118 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} + \frac{\sqrt{b} (3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{2f} + \frac{b \operatorname{Tan}[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{2f}$$

Result (type 3, 527 leaves) :

$$\frac{1}{f (a + 2 b + a \operatorname{Cos}[2 e + 2 f x])^{3/2}} \sqrt{2} e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} \operatorname{Cos}[e + f x]^3$$

$$\left(-\frac{i b (-1 + e^{2 i (e+f x)})}{(1 + e^{2 i (e+f x)})^2} + \frac{1}{\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2}} \left(2 a^{3/2} f x - i a^{3/2} \operatorname{Log}\left[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] + \right. \right.$$

$$i a^{3/2} \operatorname{Log}\left[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] -$$

$$3 a \sqrt{b} \operatorname{Log}\left[\frac{-2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f}{b (3 a + b) (1 + e^{2 i (e+f x)})} \right] -$$

$$\left. \left. b^{3/2} \operatorname{Log}\left[\frac{-2 \sqrt{b} (-1 + e^{2 i (e+f x)}) f + 2 i \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} f}{b (3 a + b) (1 + e^{2 i (e+f x)})} \right] \right) \right) (a + b \operatorname{Sec}[e + f x]^2)^{3/2}$$

- **Problem 399: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e + f x]^2 (a + b \operatorname{Sec}[e + f x]^2)^{3/2} dx$$

Optimal (type 3, 111 leaves, 8 steps) :

$$-\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f} + \frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Tan}[e+f x]}{\sqrt{a+b \operatorname{Tan}[e+f x]^2}}\right]}{f} - \frac{(a+b) \operatorname{Cot}[e+f x] \sqrt{a+b \operatorname{Tan}[e+f x]^2}}{f}$$

Result (type 3, 410 leaves) :

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b + a e^{-2i(e+fx)} (1 + e^{2i(e+fx)})^2} \cos[e+fx]^3 \right.$$

$$\left. - \frac{2i(a+b)}{-1 + e^{2i(e+fx)}} + \left(i a^{3/2} \operatorname{Log} \left[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] - i a^{3/2} \operatorname{Log} \left[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \right. \right. \right.$$

$$\left. \left. \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right] - 2 \left(a^{3/2} f x + b^{3/2} \operatorname{Log} \left[\frac{\left(\sqrt{b} (-1 + e^{2i(e+fx)}) - i \sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) f}{b^2 (1 + e^{2i(e+fx)})} \right] \right) \right) /$$

$$\left(\sqrt{4b e^{2i(e+fx)} + a (1 + e^{2i(e+fx)})^2} \right) \left((a + b \operatorname{Sec}[e+fx]^2)^{3/2} \right) / \left(f (a + 2b + a \cos[2e + 2fx])^{3/2} \right)$$

- **Problem 400: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx]^4 (a + b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$\frac{a^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b+b \operatorname{Tan}[e+fx]^2}} \right]}{f} + \frac{(3a-b) \cot[e+fx] \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{3f} - \frac{(a+b) \cot[e+fx]^3 \sqrt{a+b+b \operatorname{Tan}[e+fx]^2}}{3f}$$

Result (type 3, 354 leaves):

$$\left(\sqrt{2} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} \cos[e+fx]^3 \right. \\ \left. \left(\frac{8i(b e^{2i(e+fx)} + a(1 - e^{2i(e+fx)} + e^{4i(e+fx)}))}{(-1 + e^{2i(e+fx)})^3} + \left(3a^{3/2} \left(2fx - i \operatorname{Log}[a + 2b + a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}] \right) + \right. \right. \\ \left. \left. i \operatorname{Log}[a + a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2}] \right) \right) \Bigg/ \\ \left(\sqrt{4b e^{2i(e+fx)} + a(1 + e^{2i(e+fx)})^2} \right) \left(a + b \operatorname{Sec}[e+fx]^2 \right)^{3/2} \Bigg/ (3f(a + 2b + a \cos[2e + 2fx])^{3/2})$$

■ **Problem 401: Unable to integrate problem.**

$$\int \cot[e+fx]^6 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{a^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}}\right]}{f} - \frac{(15a^2 + 10ab - 2b^2) \cot[e+fx] \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{15(a+b)f} + \\ \frac{(5a-b) \cot[e+fx]^3 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{15f} - \frac{(a+b) \cot[e+fx]^5 \sqrt{a+b \operatorname{Tan}[e+fx]^2}}{5f}$$

Result (type 8, 27 leaves):

$$\int \cot[e+fx]^6 (a+b \operatorname{Sec}[e+fx]^2)^{3/2} dx$$

■ **Problem 402: Unable to integrate problem.**

$$\int \frac{\operatorname{Tan}[e+fx]^5}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \operatorname{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{(a+2b) \sqrt{a+b \operatorname{Sec}[e+fx]^2}}{b^2 f} + \frac{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}}{3b^2 f}$$

Result (type 8, 27 leaves):

$$\int \frac{\operatorname{Tan}[e+fx]^5}{\sqrt{a+b \operatorname{Sec}[e+fx]^2}} dx$$

■ **Problem 403: Unable to integrate problem.**

$$\int \frac{\tan[e + f x]^3}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 56 leaves, 5 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} + \frac{\sqrt{a+b \sec[e+f x]^2}}{b f}$$

Result (type 8, 27 leaves) :

$$\int \frac{\tan[e + f x]^3}{\sqrt{a + b \sec[e + f x]^2}} dx$$

■ **Problem 404: Unable to integrate problem.**

$$\int \frac{\tan[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 33 leaves, 4 steps) :

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f}$$

Result (type 8, 25 leaves) :

$$\int \frac{\tan[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

■ **Problem 405: Unable to integrate problem.**

$$\int \frac{\cot[e + f x]}{\sqrt{a + b \sec[e + f x]^2}} dx$$

Optimal (type 3, 70 leaves, 7 steps) :

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+f x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b} f}$$

Result (type 8, 25 leaves) :

$$\int \frac{\text{Cot}[e + f x]}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 406: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} + \frac{(2a+3b) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2(a+b)^{3/2} f} - \frac{\text{Cot}[e+fx]^2 \sqrt{a+b \text{Sec}[e+fx]^2}}{2(a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^3}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 407: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{\sqrt{a} f} - \frac{(8a^2 + 20ab + 15b^2) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{5/2} f} + \frac{(4a+7b) \text{Cot}[e+fx]^2 \sqrt{a+b \text{Sec}[e+fx]^2}}{8(a+b)^2 f} - \frac{\text{Cot}[e+fx]^4 \sqrt{a+b \text{Sec}[e+fx]^2}}{4(a+b) f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^5}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 408: Unable to integrate problem.**

$$\int \frac{\text{Tan}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 173 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{\sqrt{a} f} + \frac{(3a^2 + 10ab + 15b^2) \text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{8b^{5/2} f} \\
& \frac{(3a + 7b) \tan[e+fx] \sqrt{a+b \tan[e+fx]^2}}{8b^2 f} + \frac{\tan[e+fx]^3 \sqrt{a+b \tan[e+fx]^2}}{4bf}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^6}{\sqrt{a+b \sec[e+fx]^2}} dx$$

■ **Problem 409: Unable to integrate problem.**

$$\int \frac{\tan[e+fx]^4}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{\sqrt{a} f} - \frac{(a+3b) \text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{2b^{3/2} f} + \frac{\tan[e+fx] \sqrt{a+b \tan[e+fx]^2}}{2bf}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^4}{\sqrt{a+b \sec[e+fx]^2}} dx$$

■ **Problem 410: Unable to integrate problem.**

$$\int \frac{\tan[e+fx]^2}{\sqrt{a+b \sec[e+fx]^2}} dx$$

Optimal (type 3, 80 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{\sqrt{a} f} + \frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{\sqrt{b} f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^2}{\sqrt{a+b \sec[e+fx]^2}} dx$$

■ **Problem 411: Unable to integrate problem.**

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps) :

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f}$$

Result (type 8, 18 leaves) :

$$\int \frac{1}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 412: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + f x]^2}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 74 leaves, 6 steps) :

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} - \frac{\operatorname{Cot}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{(a + b) f}$$

Result (type 8, 27 leaves) :

$$\int \frac{\operatorname{Cot}[e + f x]^2}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

■ **Problem 413: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e + f x]^4}{\sqrt{a + b \operatorname{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 119 leaves, 7 steps) :

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} + \frac{(3a + 5b) \operatorname{Cot}[e + f x] \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{3(a + b)^2 f} - \frac{\operatorname{Cot}[e + f x]^3 \sqrt{a + b \operatorname{Tan}[e + f x]^2}}{3(a + b) f}$$

Result (type 8, 27 leaves) :

$$\int \frac{\text{Cot}[e + f x]^4}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 414: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

Optimal (type 3, 172 leaves, 8 steps) :

$$\begin{aligned} & - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + f x]}{\sqrt{a + b \text{Tan}[e + f x]^2}}\right]}{\sqrt{a} f} - \frac{(15 a^2 + 40 a b + 33 b^2) \text{Cot}[e + f x] \sqrt{a + b \text{Tan}[e + f x]^2}}{15 (a + b)^3 f} + \\ & \frac{(5 a + 9 b) \text{Cot}[e + f x]^3 \sqrt{a + b \text{Tan}[e + f x]^2}}{15 (a + b)^2 f} - \frac{\text{Cot}[e + f x]^5 \sqrt{a + b \text{Tan}[e + f x]^2}}{5 (a + b) f} \end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{\text{Cot}[e + f x]^6}{\sqrt{a + b \text{Sec}[e + f x]^2}} dx$$

■ **Problem 415: Unable to integrate problem.**

$$\int \frac{\text{Tan}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 88 leaves, 6 steps) :

$$\begin{aligned} & - \frac{\text{ArcTanh}\left[\frac{\sqrt{a + b \text{Sec}[e + f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} + \frac{(a + b)^2}{a b^2 f \sqrt{a + b \text{Sec}[e + f x]^2}} + \frac{\sqrt{a + b \text{Sec}[e + f x]^2}}{b^2 f} \end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{\text{Tan}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 63 leaves, 5 steps) :

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} - \frac{a+b}{a b f \sqrt{a+b \text{Sec}[e+f x]^2}}$$

Result (type 6, 1695 leaves):

$$\begin{aligned} & \left(3 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \text{Sin}[e+f x]^3 \text{Tan}[e+f x]^4 \right) / \\ & \left(4 \sqrt{2} f (a+b \text{Sec}[e+f x]^2)^{3/2} (a+b-a \text{Sin}[e+f x]^2)^{3/2} \left(6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left(3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \right) \text{Sin}[e+f x]^2 \right) \\ & \left(\left(9 a (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \text{Sin}[e+f x]^5 \right) / \right. \\ & \quad \left(4 \sqrt{2} (a+b-a \text{Sin}[e+f x]^2)^{5/2} \left(6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + \left(3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \right. \right. \right. \\ & \quad \left. \left. \left. \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \right) \text{Sin}[e+f x]^2 \right) \right) + \\ & \left(9 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \text{Sin}[e+f x]^3 \right) / \left(4 \sqrt{2} (a+b-a \text{Sin}[e+f x]^2)^{3/2} \right. \\ & \quad \left(6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + \left(3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left. (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \right) \text{Sin}[e+f x]^2 \right) \right) + \\ & \left(3 (a+b) \text{Sin}[e+f x]^3 \left(\frac{2 a f \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \text{Cos}[e+f x] \text{Sin}[e+f x]}{a+b} + \right. \right. \\ & \quad \left. \frac{2}{3} f \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \text{Cos}[e+f x] \text{Sin}[e+f x] \right) \text{Tan}[e+f x] \right) / \\ & \left(4 \sqrt{2} f (a+b-a \text{Sin}[e+f x]^2)^{3/2} \left(6 (a+b) \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left(3 a \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] + (a+b) \text{AppellF1}\left[3, \frac{3}{2}, \frac{3}{2}, 4, \text{Sin}[e+f x]^2, \frac{a \text{Sin}[e+f x]^2}{a+b}\right] \right) \right) \end{aligned}$$

$$\int \frac{\text{Tan}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 57 leaves, 5 steps) :

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} + \frac{1}{a f \sqrt{a+b \text{Sec}[e+f x]^2}}$$

Result (type 3, 425 leaves) :

$$-\frac{(a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \text{Sec}[e + f x]^2}{8 b f \sqrt{a + 2 b + a \text{Cos}[2 (e + f x)]} (a + b \text{Sec}[e + f x]^2)^{3/2}} +$$

$$\left(e^{i (e+f x)} \sqrt{4 b + a e^{-2 i (e+f x)} (1 + e^{2 i (e+f x)})^2} (a + 2 b + a \text{Cos}[2 e + 2 f x])^{3/2} \left(\frac{\sqrt{a} (a + 4 b) (1 + e^{2 i (e+f x)})}{b (4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2)} + \right. \right.$$

$$\left. \left(4 i f x - 2 \text{Log}\left[a + 2 b + a e^{2 i (e+f x)} + \sqrt{a} \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] - 2 \text{Log}\left[a + a e^{2 i (e+f x)} + 2 b e^{2 i (e+f x)} + \sqrt{a} \right. \right. \right.$$

$$\left. \left. \sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right] \right) / \left(\sqrt{4 b e^{2 i (e+f x)} + a (1 + e^{2 i (e+f x)})^2} \right) \text{Sec}[e + f x]^3 / \left(8 \sqrt{2} a^{3/2} f (a + b \text{Sec}[e + f x]^2)^{3/2} \right)$$

■ **Problem 418: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 100 leaves, 8 steps) :

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{3/2} f} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{(a+b)^{3/2} f} - \frac{b}{a (a+b) f \sqrt{a+b \text{Sec}[e+f x]^2}}$$

Result (type 8, 25 leaves) :

$$\int \frac{\text{Cot}[e + f x]}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 419: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 153 leaves, 9 steps) :

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{3/2}f} + \frac{(2a+5b)\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{\text{Cot}[e+fx]^2}{2(a+b)f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e+fx]^3}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 420: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e+fx]^5}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 213 leaves, 10 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{8(a+b)^{7/2}f} + \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\text{Sec}[e+fx]^2}} + \frac{(4a+9b)\text{Cot}[e+fx]^2}{8(a+b)^2f\sqrt{a+b\text{Sec}[e+fx]^2}} - \frac{\text{Cot}[e+fx]^4}{4(a+b)f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e+fx]^5}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 421: Unable to integrate problem.**

$$\int \frac{\text{Tan}[e+fx]^6}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 172 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\text{Tan}[e+fx]}{\sqrt{a+b+b\text{Tan}[e+fx]^2}}\right]}{a^{3/2}f} - \frac{(3a+5b)\text{ArcTanh}\left[\frac{\sqrt{b}\text{Tan}[e+fx]}{\sqrt{a+b+b\text{Tan}[e+fx]^2}}\right]}{2b^{5/2}f} - \frac{(a+b)\text{Tan}[e+fx]^3}{abf\sqrt{a+b+b\text{Tan}[e+fx]^2}} + \frac{(3a+2b)\text{Tan}[e+fx]\sqrt{a+b+b\text{Tan}[e+fx]^2}}{2ab^2f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Tan}[e+fx]^6}{(a+b\text{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 422: Unable to integrate problem.**

$$\int \frac{\tan[e + f x]^4}{(a + b \sec[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{a^{3/2} f} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{b^{3/2} f} - \frac{(a + b) \tan[e + f x]}{a b f \sqrt{a + b + b \tan[e + f x]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e + f x]^4}{(a + b \sec[e + f x]^2)^{3/2}} dx$$

■ **Problem 423: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e + f x]^2}{(a + b \sec[e + f x]^2)^{3/2}} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a + b \tan[e + f x]^2}}\right]}{a^{3/2} f} + \frac{\tan[e + f x]}{a f \sqrt{a + b + b \tan[e + f x]^2}}$$

Result (type 3, 764 leaves):

$$\begin{aligned}
& - \left(\left(e^{i(e+fx)} \sqrt{4b+a} e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2 (a+2b+a \cos[2e+2fx])^{3/2} \right. \right. \\
& \quad \left(-3ia^{3/2} \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 - 4i\sqrt{a}b \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 + \right. \\
& \quad \left. 3ia^{3/2}e^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 + 4i\sqrt{a}be^{2i(e+fx)} \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 + \right. \\
& \quad \left. 4a^2fx + 4abfx + 8a^2e^{2i(e+fx)}fx + 24abe^{2i(e+fx)}fx + 16b^2e^{2i(e+fx)}fx + 4a^2e^{4i(e+fx)}fx + 4abe^{4i(e+fx)}fx - \right. \\
& \quad \left. 2i(a+b) (4be^{2i(e+fx)}+a) (1+e^{2i(e+fx)})^2 \right) \operatorname{Log} \left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 \right) \right] + \\
& \quad \left. 2i(a+b) (4be^{2i(e+fx)}+a) (1+e^{2i(e+fx)})^2 \right) \operatorname{Log} \left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2be^{2i(e+fx)} + \sqrt{a} \sqrt{4be^{2i(e+fx)}+a} (1+e^{2i(e+fx)})^2 \right) \right] \right] + \\
& \quad \left. \operatorname{Sec}[e+fx]^3 \right) / \left(8\sqrt{2}a^{3/2}(a+b) (4be^{2i(e+fx)}+a) (1+e^{2i(e+fx)})^2 \right)^{3/2} f (a+b \operatorname{Sec}[e+fx]^2)^{3/2} \right) + \\
& \quad \frac{(a+2b+a \cos[2e+2fx])^{3/2} \operatorname{Sec}[e+fx]^2 \operatorname{Tan}[e+fx]}{8(a+b) f \sqrt{a+2b+a \cos[2(e+fx)]} (a+b \operatorname{Sec}[e+fx]^2)^{3/2}}
\end{aligned}$$

■ **Problem 424: Unable to integrate problem.**

$$\int \frac{1}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$\frac{\operatorname{ArcTan} \left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+b \operatorname{Tan}[e+fx]^2}} \right]}{a^{3/2} f} - \frac{b \operatorname{Tan}[e+fx]}{a(a+b) f \sqrt{a+b \operatorname{Tan}[e+fx]^2}}$$

Result (type 8, 18 leaves):

$$\int \frac{1}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

■ **Problem 425: Unable to integrate problem.**

$$\int \frac{\operatorname{Cot}[e+fx]^2}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{a^{3/2}f} - \frac{b\cot[e+fx]}{a(a+b)f\sqrt{a+b\tan[e+fx]^2}} - \frac{(a-b)\cot[e+fx]\sqrt{a+b\tan[e+fx]^2}}{a(a+b)^2f}$$

Result (type 8, 27 leaves):

$$\int \frac{\cot[e+fx]^2}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

■ **Problem 426: Unable to integrate problem.**

$$\int \frac{\cot[e+fx]^4}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{a^{3/2}f} - \frac{b\cot[e+fx]^3}{a(a+b)f\sqrt{a+b\tan[e+fx]^2}} +$$

$$\frac{(3a-b)(a+3b)\cot[e+fx]\sqrt{a+b\tan[e+fx]^2}}{3a(a+b)^3f} - \frac{(a-3b)\cot[e+fx]^3\sqrt{a+b\tan[e+fx]^2}}{3a(a+b)^2f}$$

Result (type 8, 27 leaves):

$$\int \frac{\cot[e+fx]^4}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

■ **Problem 427: Unable to integrate problem.**

$$\int \frac{\cot[e+fx]^6}{(a+b\sec[e+fx]^2)^{3/2}} dx$$

Optimal (type 3, 241 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{a^{3/2}f} - \frac{b\cot[e+fx]^5}{a(a+b)f\sqrt{a+b\tan[e+fx]^2}} - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot[e+fx]\sqrt{a+b\tan[e+fx]^2}}{15a(a+b)^4f} +$$

$$\frac{(5a^2+14ab-15b^2)\cot[e+fx]^3\sqrt{a+b\tan[e+fx]^2}}{15a(a+b)^3f} - \frac{(a-5b)\cot[e+fx]^5\sqrt{a+b\tan[e+fx]^2}}{5a(a+b)^2f}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{3/2}} dx$$

■ **Problem 428: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 97 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} + \frac{(a+b)^2}{3 a b^2 f (a+b\text{Sec}[e+fx]^2)^{3/2}} + \frac{\frac{1}{a^2} - \frac{1}{b^2}}{f \sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 6, 1699 leaves):

$$\begin{aligned} & \left((a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^5 \text{Tan}[e+fx]^6 \right) / \\ & \left(3 \sqrt{2} f (a+b \text{Sec}[e+fx]^2)^{5/2} (a+b-a \text{Sin}[e+fx]^2)^{5/2} \left(8 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left(5 a \text{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + (a+b) \text{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \right) \text{Sin}[e+fx]^2 \right) \\ & \left(\left(5 a (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^7 \right) / \right. \\ & \quad \left(3 \sqrt{2} (a+b-a \text{Sin}[e+fx]^2)^{7/2} \left(8 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \left(5 a \text{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \right. \right. \right. \\ & \quad \left. \left. \left. \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + (a+b) \text{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \right) \text{Sin}[e+fx]^2 \right) \right) + \\ & \left(5 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Sin}[e+fx]^5 \right) / \left(3 \sqrt{2} (a+b-a \text{Sin}[e+fx]^2)^{5/2} \right. \\ & \quad \left(8 (a+b) \text{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \left(5 a \text{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] + \right. \right. \\ & \quad \left. \left. (a+b) \text{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \right) \text{Sin}[e+fx]^2 \right) \right) + \\ & \left((a+b) \text{Sin}[e+fx]^5 \left(\frac{15 a f \text{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \text{Sin}[e+fx]^2, \frac{a \text{Sin}[e+fx]^2}{a+b}\right] \text{Cos}[e+fx] \text{Sin}[e+fx]}{4 (a+b)} + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left. \frac{3}{4} f \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \tan[e+fx] \Big/ \\
& \left(3 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left(8 (a+b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \right. \right. \\
& \quad \left. \left(5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right. \\
& \quad \left. \sin[e+fx]^2 \right) - \left((a+b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^5 \right. \\
& \quad \left. \left(2 f \left(5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \right. \right. \\
& \quad \left. \cos[e+fx] \sin[e+fx] + 8 (a+b) \left(\frac{15 a f \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{4 (a+b)} + \right. \right. \\
& \quad \left. \left. \frac{3}{4} f \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \right. \\
& \quad \left. \sin[e+fx]^2 \left(5 a \left(\frac{28 a f \operatorname{AppellF1}\left[5, \frac{1}{2}, \frac{9}{2}, 6, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{5 (a+b)} + \right. \right. \\
& \quad \left. \frac{4}{5} f \operatorname{AppellF1}\left[5, \frac{3}{2}, \frac{7}{2}, 6, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) + \right. \\
& \quad \left. (a+b) \left(\frac{4 a f \operatorname{AppellF1}\left[5, \frac{3}{2}, \frac{7}{2}, 6, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx]}{a+b} + \right. \right. \\
& \quad \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[5, \frac{5}{2}, \frac{5}{2}, 6, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \cos[e+fx] \sin[e+fx] \right) \right) \right) \tan[e+fx] \Big/ \\
& \left(3 \sqrt{2} f (a+b - a \sin[e+fx]^2)^{5/2} \left(8 (a+b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + (a+b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right)^2 \right) +
\end{aligned}$$

$$\left((a+b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \sin[e+fx]^3 \tan[e+fx]^2 \right) / \left(3 \sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right. \\ \left. \left(8 (a+b) \operatorname{AppellF1}\left[3, \frac{1}{2}, \frac{5}{2}, 4, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] + \left(5 a \operatorname{AppellF1}\left[4, \frac{1}{2}, \frac{7}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) + \right. \right. \\ \left. \left. (a+b) \operatorname{AppellF1}\left[4, \frac{3}{2}, \frac{5}{2}, 5, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b}\right] \right) \sin[e+fx]^2 \right) \right)$$

- **Problem 429: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]^3}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sec[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2} f} - \frac{a+b}{3 a b f (a+b \sec[e+fx]^2)^{3/2}} - \frac{1}{a^2 f \sqrt{a+b \sec[e+fx]^2}}$$

Result (type 3, 613 leaves):

$$-\frac{(a+3b+a \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \sec[e+fx]^4}{48 b^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \sec[e+fx]^2)^{5/2}} + \\ \frac{(a+b+(a-2b) \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \sec[e+fx]^4}{96 b^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \sec[e+fx]^2)^{5/2}} - \\ \frac{1}{96 \sqrt{2} a^{5/2} f (a+b \sec[e+fx]^2)^{5/2}} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{5/2} \\ \left(-\left(\sqrt{a} (1+e^{2i(e+fx)}) (-96 b^3 e^{2i(e+fx)} + a^3 (1+e^{2i(e+fx)})^2 - 32 a b^2 (1+e^{2i(e+fx)})^2 - 6 a^2 b (1+e^{2i(e+fx)} + e^{4i(e+fx)})) \right) / \right. \\ \left. \left(b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2 \right) + \left(24 i f x - 12 \operatorname{Log}\left[a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] - \right. \\ \left. 12 \operatorname{Log}\left[a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right] \right) / \left(\sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right) \sec[e+fx]^5$$

- **Problem 430: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 83 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2}f} + \frac{1}{3af(a+b\text{Sec}[e+fx]^2)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 3, 613 leaves):

$$\begin{aligned} & -\frac{(a+3b+a\text{Cos}[2(e+fx)])(a+2b+a\text{Cos}[2e+2fx])^{5/2}\text{Sec}[e+fx]^4}{48b^2f(a+2b+a\text{Cos}[2(e+fx)])^{3/2}(a+b\text{Sec}[e+fx]^2)^{5/2}} + \\ & \frac{(a+b+(a-2b)\text{Cos}[2(e+fx)])(a+2b+a\text{Cos}[2e+2fx])^{5/2}\text{Sec}[e+fx]^4}{32b^2f(a+2b+a\text{Cos}[2(e+fx)])^{3/2}(a+b\text{Sec}[e+fx]^2)^{5/2}} + \\ & \frac{1}{96\sqrt{2}a^{5/2}f(a+b\text{Sec}[e+fx]^2)^{5/2}} e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} (a+2b+a\text{Cos}[2e+2fx])^{5/2} \\ & \left(-\left(\sqrt{a}(1+e^{2i(e+fx)})(-96b^3e^{2i(e+fx)}+a^3(1+e^{2i(e+fx)})^2-32ab^2(1+e^{2i(e+fx)})^2-6a^2b(1+e^{2i(e+fx)}+e^{4i(e+fx)}))\right) \right) / \\ & \left(b^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^2 + \left(24ifx-12\text{Log}\left[a+2b+ae^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right] - \right. \right. \\ & \left. \left. 12\text{Log}\left[a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right] \right) \right) / \left(\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2} \right) \text{Sec}[e+fx]^5 \end{aligned}$$

■ **Problem 431: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e+fx]}{(a+b\text{Sec}[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 137 leaves, 9 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a}}\right]}{a^{5/2}f} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\text{Sec}[e+fx]^2}}{\sqrt{a+b}}\right]}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f(a+b\text{Sec}[e+fx]^2)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\text{Sec}[e+fx]^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{\text{Cot}[e+fx]}{(a+b\text{Sec}[e+fx]^2)^{5/2}} dx$$

■ **Problem 432: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 200 leaves, 10 steps):

$$\begin{aligned} & - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{5/2} f} + \frac{(2 a + 7 b) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{2 (a+b)^{7/2} f} \\ & - \frac{(3 a - 2 b) b}{6 a (a+b)^2 f (a+b \text{Sec}[e+f x]^2)^{3/2}} - \frac{\text{Cot}[e+f x]^2}{2 (a+b) f (a+b \text{Sec}[e+f x]^2)^{3/2}} - \frac{b (a^2 - 6 a b - 2 b^2)}{2 a^2 (a+b)^3 f \sqrt{a+b \text{Sec}[e+f x]^2}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^3}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

■ **Problem 433: Unable to integrate problem.**

$$\int \frac{\text{Cot}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 268 leaves, 11 steps):

$$\begin{aligned} & \frac{\text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a}}\right]}{a^{5/2} f} - \frac{(8 a^2 + 36 a b + 63 b^2) \text{ArcTanh}\left[\frac{\sqrt{a+b \text{Sec}[e+f x]^2}}{\sqrt{a+b}}\right]}{8 (a+b)^{9/2} f} + \frac{b (12 a^2 + 39 a b - 8 b^2)}{24 a (a+b)^3 f (a+b \text{Sec}[e+f x]^2)^{3/2}} \\ & + \frac{(4 a + 11 b) \text{Cot}[e+f x]^2}{8 (a+b)^2 f (a+b \text{Sec}[e+f x]^2)^{3/2}} - \frac{\text{Cot}[e+f x]^4}{4 (a+b) f (a+b \text{Sec}[e+f x]^2)^{3/2}} + \frac{b (4 a^3 + 15 a^2 b - 32 a b^2 - 8 b^3)}{8 a^2 (a+b)^4 f \sqrt{a+b \text{Sec}[e+f x]^2}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\text{Cot}[e + f x]^5}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

■ **Problem 434: Unable to integrate problem.**

$$\int \frac{\text{Tan}[e + f x]^6}{(a + b \text{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 157 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} + \frac{\text{ArcTanh}\left[\frac{\sqrt{b} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{b^{5/2} f} - \frac{(a+b) \tan[e+fx]^3}{3 a b f (a+b+b \tan[e+fx]^2)^{3/2}} + \frac{\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tan[e+fx]}{f \sqrt{a+b+b \tan[e+fx]^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\tan[e+fx]^6}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

- **Problem 435: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\tan[e+fx]^4}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 120 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{(a+b) \tan[e+fx]}{3 a b f (a+b+b \tan[e+fx]^2)^{3/2}} + \frac{(a-3b) \tan[e+fx]}{3 a^2 b f \sqrt{a+b+b \tan[e+fx]^2}}$$

Result (type 3, 1414 leaves):

$$\begin{aligned}
& \left(i e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)} (1+e^{2i(e+fx)})^2} (a+2b+a \cos[2e+2fx])^{5/2} \right. \\
& \left. -25 a^{7/2} - 58 a^{5/2} b - 32 a^{3/2} b^2 - 15 a^{7/2} e^{2i(e+fx)} - 108 a^{5/2} b e^{2i(e+fx)} - 192 a^{3/2} b^2 e^{2i(e+fx)} - \right. \\
& 96 \sqrt{a} b^3 e^{2i(e+fx)} + 15 a^{7/2} e^{4i(e+fx)} + 108 a^{5/2} b e^{4i(e+fx)} + 192 a^{3/2} b^2 e^{4i(e+fx)} + 96 \sqrt{a} b^3 e^{4i(e+fx)} + \\
& 25 a^{7/2} e^{6i(e+fx)} + 58 a^{5/2} b e^{6i(e+fx)} + 32 a^{3/2} b^2 e^{6i(e+fx)} - 24 i a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 48 i a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - 24 i b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} f x - \\
& 12 a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 24 a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] - \\
& 12 b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+2b+a e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 12 a^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 24 a b (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] + \\
& 12 b^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^{3/2} \operatorname{Log}\left[e^{-2ie} \left(a+a e^{2i(e+fx)} + 2b e^{2i(e+fx)} + \sqrt{a} \sqrt{4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2} \right) \right] \Big] \\
& \operatorname{Sec}[e+fx]^5 \Big) / \left(96 \sqrt{2} a^{5/2} (a+b)^2 (4b e^{2i(e+fx)} + a (1+e^{2i(e+fx)})^2)^2 f (a+b \operatorname{Sec}[e+fx]^2)^{5/2} \right) + \\
& \frac{(2a+3b+a \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \operatorname{Sec}[e+fx]^4 \operatorname{Tan}[e+fx]}{48 (a+b)^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \operatorname{Sec}[e+fx]^2)^{5/2}} \\
& \frac{(b+(3a+2b) \cos[2(e+fx)]) (a+2b+a \cos[2e+2fx])^{5/2} \operatorname{Sec}[e+fx]^4 \operatorname{Tan}[e+fx]}{32 (a+b)^2 f (a+2b+a \cos[2(e+fx)])^{3/2} (a+b \operatorname{Sec}[e+fx]^2)^{5/2}}
\end{aligned}$$

■ **Problem 436: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[e+fx]^2}{(a+b \operatorname{Sec}[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 119 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a}\tan[e+fx]}{\sqrt{a+b\tan[e+fx]^2}}\right]}{a^{5/2}f} + \frac{\tan[e+fx]}{3af(a+b\tan[e+fx]^2)^{3/2}} + \frac{(2a+3b)\tan[e+fx]}{3a^2(a+b)f\sqrt{a+b\tan[e+fx]^2}}$$

Result (type 3, 1414 leaves):

$$-\left(\left(i e^{i(e+fx)} \sqrt{4b+a e^{-2i(e+fx)}(1+e^{2i(e+fx)})^2} (a+2b+a\cos[2e+2fx])^{5/2} \left(-25a^{7/2}-58a^{5/2}b-32a^{3/2}b^2-15a^{7/2}e^{2i(e+fx)}-108a^{5/2}b e^{2i(e+fx)}-192a^{3/2}b^2 e^{2i(e+fx)}-96\sqrt{a}b^3 e^{2i(e+fx)}+15a^{7/2}e^{4i(e+fx)}+108a^{5/2}b e^{4i(e+fx)}+192a^{3/2}b^2 e^{4i(e+fx)}+96\sqrt{a}b^3 e^{4i(e+fx)}+25a^{7/2}e^{6i(e+fx)}+58a^{5/2}b e^{6i(e+fx)}+32a^{3/2}b^2 e^{6i(e+fx)}-24ia^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}fx-48iab(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}fx-24ib^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}fx-12a^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]-24ab(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]-12b^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+2b+a e^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]+12a^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+a e^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]+24ab(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+a e^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]+12b^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^{3/2}\text{Log}\left[e^{-2ie}\left(a+a e^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2}\right)\right]\right)\right) + \frac{\text{Sec}[e+fx]^5}{\left(96\sqrt{2}a^{5/2}(a+b)^2(4be^{2i(e+fx)}+a(1+e^{2i(e+fx)})^2)^2f(a+b\text{Sec}[e+fx]^2)^{5/2}\right)} + \frac{(2a+3b+a\cos[2(e+fx)])(a+2b+a\cos[2e+2fx])^{5/2}\text{Sec}[e+fx]^4\tan[e+fx]}{48(a+b)^2f(a+2b+a\cos[2(e+fx)])^{3/2}(a+b\text{Sec}[e+fx]^2)^{5/2}} - \frac{(b+(3a+2b)\cos[2(e+fx)])(a+2b+a\cos[2e+2fx])^{5/2}\text{Sec}[e+fx]^4\tan[e+fx]}{96(a+b)^2f(a+2b+a\cos[2(e+fx)])^{3/2}(a+b\text{Sec}[e+fx]^2)^{5/2}}$$

- **Problem 437: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \operatorname{Sec}[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 125 leaves, 6 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e + f x]}{\sqrt{a + b \operatorname{Tan}[e + f x]^2}}\right]}{a^{5/2} f} - \frac{b \operatorname{Tan}[e + f x]}{3 a (a + b) f (a + b + b \operatorname{Tan}[e + f x]^2)^{3/2}} - \frac{b (5 a + 3 b) \operatorname{Tan}[e + f x]}{3 a^2 (a + b)^2 f \sqrt{a + b + b \operatorname{Tan}[e + f x]^2}}$$

Result (type 6, 1927 leaves):

$$\left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^4 \operatorname{Sin}[e + f x] \right) /$$

$$\left(4 \sqrt{2} f (a + b \operatorname{Sec}[e + f x]^2)^{5/2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right.$$

$$\left. \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right. \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right)$$

$$\left(\left(15 a (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^5 \operatorname{Sin}[e + f x]^2 \right) / \left(4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{7/2} \right. \right.$$

$$\left. \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) \right) +$$

$$\left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \operatorname{Cos}[e + f x]^5 \right) / \left(4 \sqrt{2} (a + b - a \operatorname{Sin}[e + f x]^2)^{5/2} \right.$$

$$\left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] - \right. \right.$$

$$\left. \left. 4 (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a + b}\right] \right) \operatorname{Sin}[e + f x]^2 \right) -$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^3 \sin[e+fx]^2 \right) / \left(\sqrt{2} (a+b-a \sin[e+fx]^2)^{5/2} \right. \\
& \quad \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - \right. \right. \\
& \quad \left. \left. 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) + \\
& \left(3 (a+b) \cos[e+fx]^4 \sin[e+fx] \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) / \\
& \left(4 \sqrt{2} f (a+b-a \sin[e+fx]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] + \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \right. \right. \right. \\
& \quad \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \sin[e+fx]^2 \right) - \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx]^4 \sin[e+fx] \right. \\
& \quad \left. \left(2 f \left(5 a \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] - 4 (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \right) \right. \right. \\
& \quad \left. \left. \cos[e+fx] \sin[e+fx] + 3 (a+b) \left(\frac{5 a f \operatorname{AppellF1} \left[\frac{3}{2}, -2, \frac{7}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx]}{3 (a+b)} - \right. \right. \right. \\
& \quad \left. \left. \frac{4}{3} f \operatorname{AppellF1} \left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e+fx]^2, \frac{a \sin[e+fx]^2}{a+b} \right] \cos[e+fx] \sin[e+fx] \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \sin[e + f x]^2 \left(5 a \left(\frac{21 a f \operatorname{AppellF1}\left[\frac{5}{2}, -2, \frac{9}{2}, \frac{7}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] \cos[e + f x] \sin[e + f x]}{5 (a+b)} - \right. \right. \\
& \left. \left. \frac{12}{5} f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] \cos[e + f x] \sin[e + f x] \right) - \right. \\
& \left. 4 (a+b) \left(\frac{3 a f \operatorname{AppellF1}\left[\frac{5}{2}, -1, \frac{7}{2}, \frac{7}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] \cos[e + f x] \sin[e + f x]}{a+b} - \right. \right. \\
& \left. \left(6 (a+b)^3 f \cot[e + f x] \csc[e + f x]^4 \left(-1 + \frac{a \sin[e + f x]^2}{a+b} \right)^2 \left(\frac{\sqrt{a} \operatorname{ArcSin}\left[\frac{\sqrt{a} \sin[e + f x]}{\sqrt{a+b}}\right] \sin[e + f x]}{\sqrt{a+b} \sqrt{1 - \frac{a \sin[e + f x]^2}{a+b}}} + \right. \right. \\
& \left. \left. \frac{a^2 \sin[e + f x]^4}{3 (a+b)^2 \left(-1 + \frac{a \sin[e + f x]^2}{a+b} \right)^2} + \frac{a \sin[e + f x]^2}{(a+b) \left(-1 + \frac{a \sin[e + f x]^2}{a+b} \right)} \right) \right) \left/ \left(a^3 \left(1 - \frac{a \sin[e + f x]^2}{a+b} \right)^{3/2} \right) \right) \right) \left/ \right) \\
& \left(4 \sqrt{2} f (a+b - a \sin[e + f x]^2)^{5/2} \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -2, \frac{5}{2}, \frac{3}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] + \left(5 a \operatorname{AppellF1}\left[\frac{3}{2}, -2, \right. \right. \right. \\
& \left. \left. \left. \frac{7}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] - 4 (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -1, \frac{5}{2}, \frac{5}{2}, \sin[e + f x]^2, \frac{a \sin[e + f x]^2}{a+b}\right] \right) \sin[e + f x]^2 \right)^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 438: Unable to integrate problem.**

$$\int \frac{\cot[e + f x]^2}{(a + b \sec[e + f x]^2)^{5/2}} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \cot[e+fx]}{3 a (a+b) f (a+b \tan[e+fx]^2)^{3/2}} - \\
& \frac{b (7 a+3 b) \cot[e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b \tan[e+fx]^2}} - \frac{(a-3 b) (3 a+b) \cot[e+fx] \sqrt{a+b \tan[e+fx]^2}}{3 a^2 (a+b)^3 f}
\end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{\cot[e+fx]^2}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

■ **Problem 439: Unable to integrate problem.**

$$\int \frac{\cot[e+fx]^4}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 236 leaves, 9 steps) :

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \cot[e+fx]^3}{3 a (a+b) f (a+b \tan[e+fx]^2)^{3/2}} - \frac{b (3 a+b) \cot[e+fx]^3}{a^2 (a+b)^2 f \sqrt{a+b \tan[e+fx]^2}} + \\
& \frac{(a-b) (3 a^2+14 a b+3 b^2) \cot[e+fx] \sqrt{a+b \tan[e+fx]^2}}{3 a^2 (a+b)^4 f} - \frac{(a^2-10 a b-3 b^2) \cot[e+fx]^3 \sqrt{a+b \tan[e+fx]^2}}{3 a^2 (a+b)^3 f}
\end{aligned}$$

Result (type 8, 27 leaves) :

$$\int \frac{\cot[e+fx]^4}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

■ **Problem 440: Unable to integrate problem.**

$$\int \frac{\cot[e+fx]^6}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

Optimal (type 3, 315 leaves, 10 steps) :

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+b \tan[e+fx]^2}}\right]}{a^{5/2} f} - \frac{b \cot[e+fx]^5}{3 a (a+b) f (a+b \tan[e+fx]^2)^{3/2}} - \\
& \frac{b (11 a+3 b) \cot[e+fx]^5}{3 a^2 (a+b)^2 f \sqrt{a+b \tan[e+fx]^2}} - \frac{(15 a^4+70 a^3 b+128 a^2 b^2-70 a b^3-15 b^4) \cot[e+fx] \sqrt{a+b \tan[e+fx]^2}}{15 a^2 (a+b)^5 f} + \\
& \frac{(5 a^3+19 a^2 b-65 a b^2-15 b^3) \cot[e+fx]^3 \sqrt{a+b \tan[e+fx]^2}}{15 a^2 (a+b)^4 f} - \frac{(a^2-20 a b-5 b^2) \cot[e+fx]^5 \sqrt{a+b \tan[e+fx]^2}}{5 a^2 (a+b)^3 f}
\end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{\cot[e+fx]^6}{(a+b \sec[e+fx]^2)^{5/2}} dx$$

■ **Problem 441: Result more than twice size of optimal antiderivative.**

$$\int (a+b \sec[e+fx]^2)^p (d \tan[e+fx])^m dx$$

Optimal (type 6, 105 leaves, 4 steps):

$$\frac{1}{d f (1+m)} \text{AppellF1}\left[\frac{1+m}{2}, 1, -p, \frac{3+m}{2}, -\tan[e+fx]^2, -\frac{b \tan[e+fx]^2}{a+b}\right] (d \tan[e+fx])^{1+m} (a+b \tan[e+fx]^2)^p \left(1 + \frac{b \tan[e+fx]^2}{a+b}\right)^{-p}$$

Result (type 6, 2929 leaves):

$$\begin{aligned}
& \left((a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right. \\
& \left. (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^p (a+b \sec[e+fx]^2)^p \sin[e+fx] \tan[e+fx]^m (d \tan[e+fx])^m \right) / \\
& \left(f (1+m) \left((a+b) (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \right. \right. \\
& \left. \left. 2 \left(b^p \text{AppellF1}\left[\frac{3+m}{2}, 1-p, 1, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right. \\
& \left. \left. (a+b) \text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) \\
& \left(\left((a+b) m (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, 1, \frac{3+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \cos[e+fx] \right. \right. \\
& \left. \left. (a+2 b+a \cos[2(e+fx)])^p (\sec[e+fx]^2)^{1+p} \sin[e+fx] \tan[e+fx]^{-1+m} \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2] - (a + b) \operatorname{AppellF1}\left[\frac{3 + m}{2}, -p, 2, \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2 \Big) \Big) + \\
& \left((a + b) (3 + m) \cos[e + f x] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^p \sin[e + f x] \tan[e + f x]^m \right. \\
& \left(1 / ((a + b) (3 + m)) 2b (1 + m)^p \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, 1 - p, 1, 1 + \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \\
& \left. 1 / (3 + m) 2 (1 + m) \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, -p, 2, 1 + \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) \Big) \Big) / \\
& \left((1 + m) \left((a + b) (3 + m) \operatorname{AppellF1}\left[\frac{1 + m}{2}, -p, 1, \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3 + m}{2}, 1 - p, 1, \frac{5 + m}{2}, \right. \right. \right. \\
& \left. \left. \left. -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3 + m}{2}, -p, 2, \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \Big) - \\
& \left((a + b) (3 + m) \operatorname{AppellF1}\left[\frac{1 + m}{2}, -p, 1, \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \cos[e + f x] (a + 2b + a \cos[2(e + f x)])^p \right. \\
& \left. (\sec[e + f x]^2)^p \sin[e + f x] \tan[e + f x]^m \left(4 \left(b^p \operatorname{AppellF1}\left[\frac{3 + m}{2}, 1 - p, 1, \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] - \right. \right. \right. \\
& \left. \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3 + m}{2}, -p, 2, \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right) \sec[e + f x]^2 \tan[e + f x] + (a + b) (3 + m) \left(\frac{1}{(a + b) (3 + m)} \right. \right. \right. \\
& \left. \left. \left. 2b (1 + m)^p \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, 1 - p, 1, 1 + \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \frac{1}{3 + m} 2 (1 + m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[1 + \frac{1 + m}{2}, -p, 2, 1 + \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) + 2 \tan[e + f x]^2 \left(b^p \left(-\frac{1}{5 + m} \right. \right. \right. \\
& \left. \left. \left. 2 (3 + m) \operatorname{AppellF1}\left[1 + \frac{3 + m}{2}, 1 - p, 2, 1 + \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \frac{1}{(a + b) (5 + m)} \right. \right. \right. \\
& \left. \left. \left. 2b (3 + m) (1 - p) \operatorname{AppellF1}\left[1 + \frac{3 + m}{2}, 2 - p, 1, 1 + \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) - (a + b) \right. \right. \\
& \left. \left(\frac{1}{(a + b) (5 + m)} 2b (3 + m)^p \operatorname{AppellF1}\left[1 + \frac{3 + m}{2}, 1 - p, 2, 1 + \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{1}{5 + m} 4 (3 + m) \operatorname{AppellF1}\left[1 + \frac{3 + m}{2}, -p, 3, 1 + \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \Big) \Big) \Big) / \\
& \left((1 + m) \left((a + b) (3 + m) \operatorname{AppellF1}\left[\frac{1 + m}{2}, -p, 1, \frac{3 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + \right. \right. \\
& \left. \left. 2 \left(b^p \operatorname{AppellF1}\left[\frac{3 + m}{2}, 1 - p, 1, \frac{5 + m}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] - (a + b) \right) \right) \right)
\end{aligned}$$

$$\text{AppellF1}\left[\frac{3+m}{2}, -p, 2, \frac{5+m}{2}, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \tan[e+fx]^2 \Bigg)^2 \Bigg) \Bigg)$$

- **Problem 445: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cot[e+fx] (a+b \sec[e+fx]^2)^p dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$\frac{\text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{a+b \sec[e+fx]^2}{a+b}\right] (a+b \sec[e+fx]^2)^{1+p}}{2(a+b) f (1+p)} + \frac{\text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1 + \frac{b \sec[e+fx]^2}{a}\right] (a+b \sec[e+fx]^2)^{1+p}}{2 a f (1+p)}$$

Result (type 6, 2055 leaves):

$$\left((a+2b+a \cos[2(e+fx)])^p \cot[e+fx] (\sec[e+fx]^2)^p \right. \\ \left. (a+b \sec[e+fx]^2)^p \left(\frac{\left(1 + \frac{(a+b) \cot[e+fx]^2}{b}\right)^{-p} \text{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \cot[e+fx]^2}{b}\right]}{p} - \right. \right. \\ \left. \left(2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sin[e+fx]^2 \right) / \right. \\ \left. \left(2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] + \left(b^p \text{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] - \right. \right. \right. \\ \left. \left. \left. (a+b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \right) \tan[e+fx]^2 \right) \right) \Bigg) \Bigg) / \\ \left(2 f \left(-a^p (a+2b+a \cos[2(e+fx)])^{-1+p} (\sec[e+fx]^2)^p \sin[2(e+fx)] \right. \right. \\ \left. \left(\frac{\left(1 + \frac{(a+b) \cot[e+fx]^2}{b}\right)^{-p} \text{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \cot[e+fx]^2}{b}\right]}{p} - \right. \right. \\ \left. \left. \left(2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e+fx]^2}{a+b}, -\tan[e+fx]^2\right] \sin[e+fx]^2 \right) \right) / \right)$$

$$\begin{aligned}
& \left(2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \left(b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& p (a+2 b+a \operatorname{Cos}[2 (e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x] \left(\frac{\left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \operatorname{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b}\right]}{p} - \right. \\
& \left. \left(2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sin}[e+f x]^2 \right) \Bigg) / \right. \\
& \left. \left(2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \left(b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \frac{1}{2} (a+2 b+a \operatorname{Cos}[2 (e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \left(\frac{1}{b} 2 (a+b) \operatorname{Cot}[e+f x] \left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-1-p} \operatorname{Csc}[e+f x]^2 \right. \\
& \quad \left. \operatorname{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b}\right] + 2 \left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \operatorname{Csc}[e+f x] \right. \\
& \quad \left. \left(\left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^p - \operatorname{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b}\right] \right) \operatorname{Sec}[e+f x] - \right. \\
& \left. \left(4 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] \right) \Bigg) / \right. \\
& \left. \left(2 (a+b) \operatorname{AppellF1}\left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + \left(b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \right. \\
& \quad \left. \left. (a+b) \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(2 (a+b) \operatorname{Sin}[e+f x]^2 \left(\frac{b p \operatorname{AppellF1}\left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{a+b} - \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \Bigg) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \left(b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \left(2 \left(b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Sec}[e+f x]^2 \right. \\
& \quad \left. \operatorname{Tan}[e+f x] + 2 (a+b) \left(\frac{b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{a+b} - \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) + \\
& \operatorname{Tan}[e+f x]^2 \left(b p \left(-\frac{4}{3} \operatorname{AppellF1} \left[3, 1-p, 2, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \quad \left. \frac{1}{3 (a+b)} 4 b (1-p) \operatorname{AppellF1} \left[3, 2-p, 1, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) - \\
& \quad (a+b) \left(\frac{1}{3 (a+b)} 4 b p \operatorname{AppellF1} \left[3, 1-p, 2, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \\
& \quad \left. \frac{8}{3} \operatorname{AppellF1} \left[3, -p, 3, 4, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) \Bigg) / \\
& \left(2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \left(b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) \Bigg)
\end{aligned}$$

■ **Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+f x]^3 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 5, 157 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\text{Cot}[e + f x]^2 (a + b \text{Sec}[e + f x]^2)^{1+p}}{2(a+b)f} + \frac{(a+b-bp) \text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{a+b \text{Sec}[e+fx]^2}{a+b}\right] (a+b \text{Sec}[e+fx]^2)^{1+p}}{2(a+b)^2 f (1+p)} \\
& \frac{\text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1 + \frac{b \text{Sec}[e+fx]^2}{a}\right] (a+b \text{Sec}[e+fx]^2)^{1+p}}{2af(1+p)}
\end{aligned}$$

Result (type 6, 2951 leaves):

$$\begin{aligned}
& \left(2^{-1+p} \text{Cot}[e + f x]^3 (a + b \text{Sec}[e + f x]^2)^p (1 + \text{Tan}[e + f x]^2)^p \right. \\
& \left(\frac{a + b + b \text{Tan}[e + f x]^2}{1 + \text{Tan}[e + f x]^2} \right)^p \left(- \left(2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \text{Tan}[e + f x]^2 \right) / \right. \\
& \left. \left((1 + \text{Tan}[e + f x]^2) \left(-2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + \left(-bp \text{AppellF1}\left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + (a+b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) + \\
& \frac{1}{(-1+p)p} \text{Cot}[e + f x]^2 \left(1 + \frac{(a+b) \text{Cot}[e + f x]^2}{b} \right)^{-p} \left(p \text{Hypergeometric2F1}\left[1-p, -p, 2-p, -\frac{(a+b) \text{Cot}[e + f x]^2}{b}\right] - \right. \\
& \left. (-1+p) \text{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \text{Cot}[e + f x]^2}{b}\right] \text{Tan}[e + f x]^2 \right) \left. \right) / \\
& \left(f \left(2^p p \text{Sec}[e + f x]^2 \text{Tan}[e + f x] (1 + \text{Tan}[e + f x]^2)^{-1+p} \left(\frac{a + b + b \text{Tan}[e + f x]^2}{1 + \text{Tan}[e + f x]^2} \right)^p \right. \right. \\
& \left. \left(- \left(2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \text{Tan}[e + f x]^2 \right) / \right. \right. \\
& \left. \left((1 + \text{Tan}[e + f x]^2) \left(-2(a+b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + \left(-bp \text{AppellF1}\left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] + (a+b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \text{Tan}[e + f x]^2}{a+b}, -\text{Tan}[e + f x]^2\right] \right) \text{Tan}[e + f x]^2 \right) \right) + \\
& \frac{1}{(-1+p)p} \text{Cot}[e + f x]^2 \left(1 + \frac{(a+b) \text{Cot}[e + f x]^2}{b} \right)^{-p} \left(p \text{Hypergeometric2F1}\left[1-p, -p, 2-p, -\frac{(a+b) \text{Cot}[e + f x]^2}{b}\right] - \right. \\
& \left. (-1+p) \text{Hypergeometric2F1}\left[-p, -p, 1-p, -\frac{(a+b) \text{Cot}[e + f x]^2}{b}\right] \text{Tan}[e + f x]^2 \right) \left. \right) + \\
& 2^{-1+p} p (1 + \text{Tan}[e + f x]^2)^p \left(\frac{a + b + b \text{Tan}[e + f x]^2}{1 + \text{Tan}[e + f x]^2} \right)^{-1+p} \left(\frac{2b \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{1 + \text{Tan}[e + f x]^2} - \frac{2 \text{Sec}[e + f x]^2 \text{Tan}[e + f x] (a + b + b \text{Tan}[e + f x]^2)}{(1 + \text{Tan}[e + f x]^2)^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(- \left(2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^2 \right) / \right. \\
& \quad \left((1+\operatorname{Tan}[e+f x]^2) \left(-2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \left(-b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) + \\
& \quad \frac{1}{(-1+p) p} \operatorname{Cot}[e+f x]^2 \left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \left(p \operatorname{Hypergeometric2F1} \left[1-p, -p, 2-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] - \right. \\
& \quad \left. (-1+p) \operatorname{Hypergeometric2F1} \left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \operatorname{Tan}[e+f x]^2 \right) \right) + \\
& 2^{-1+p} (1+\operatorname{Tan}[e+f x]^2)^p \left(\frac{a+b+b \operatorname{Tan}[e+f x]^2}{1+\operatorname{Tan}[e+f x]^2} \right)^p \left(\frac{1}{(-1+p) p} \operatorname{Cot}[e+f x]^2 \left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^{-p} \right. \\
& \quad \left(-2 (1-p) p \operatorname{Csc}[e+f x] \left(\left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^p - \operatorname{Hypergeometric2F1} \left[1-p, -p, 2-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \right) \operatorname{Sec}[e+f x] - \right. \\
& \quad \left. 2 (-1+p) p \left(\left(1 + \frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right)^p - \operatorname{Hypergeometric2F1} \left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \right) \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \\
& \quad \left. \left. 2 (-1+p) \operatorname{Hypergeometric2F1} \left[-p, -p, 1-p, -\frac{(a+b) \operatorname{Cot}[e+f x]^2}{b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) + \right. \\
& \quad \left(4 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]^3 \right) / \\
& \quad \left((1+\operatorname{Tan}[e+f x]^2)^2 \left(-2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \left(-b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) - \\
& \quad \left(4 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) / \\
& \quad \left((1+\operatorname{Tan}[e+f x]^2) \left(-2 (a+b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + \left(-b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + (a+b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) \right) - \\
& \quad \left(2 (a+b) \operatorname{Tan}[e+f x]^2 \left(\frac{b p \operatorname{AppellF1} \left[2, 1-p, 1, 3, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] - \right. \right. \\
& \quad \left. \left. a+b \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]\right] \right) \right) \right) \right) \right) / \\
& \left((1 + \tan[e + f x]^2) \left(-2(a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + \left(-b p \text{AppellF1}\left[2, 1 - p, 1, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + (a + b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right) \tan[e + f x]^2 \right) \right) + \\
& \frac{1}{b(-1 + p)} 2(a + b) \cot[e + f x]^3 \left(1 + \frac{(a + b) \cot[e + f x]^2}{b} \right)^{-1 - p} \text{Csc}[e + f x]^2 \left(p \text{Hypergeometric2F1}\left[1 - p, -p, 2 - p, \right. \right. \\
& \quad \left. \left. -\frac{(a + b) \cot[e + f x]^2}{b} \right] - (-1 + p) \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \cot[e + f x]^2}{b} \right] \tan[e + f x]^2 \right) - \frac{1}{(-1 + p)p} \\
& 2 \cot[e + f x] \left(1 + \frac{(a + b) \cot[e + f x]^2}{b} \right)^{-p} \text{Csc}[e + f x]^2 \left(p \text{Hypergeometric2F1}\left[1 - p, -p, 2 - p, -\frac{(a + b) \cot[e + f x]^2}{b} \right] - \right. \\
& \quad \left. (-1 + p) \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\frac{(a + b) \cot[e + f x]^2}{b} \right] \tan[e + f x]^2 \right) + \\
& \left(2(a + b) \text{AppellF1}\left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2 \left(2 \left(-b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan[e + f x]^2\right] + (a + b) \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \right) \sec[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. 2(a + b) \left(\frac{b p \text{AppellF1}\left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]}{a + b} - \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) \right) + \\
& \tan[e + f x]^2 \left(-b p \left(-\frac{4}{3} \text{AppellF1}\left[3, 1 - p, 2, 4, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{1}{3(a + b)} 4 b(1 - p) \text{AppellF1}\left[3, 2 - p, 1, 4, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) \right) + \\
& (a + b) \left(\frac{1}{3(a + b)} 4 b p \text{AppellF1}\left[3, 1 - p, 2, 4, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \\
& \quad \left. \frac{8}{3} \text{AppellF1}\left[3, -p, 3, 4, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \right) \right) /
\end{aligned}$$

$$\left((1 + \tan[e + f x]^2) \left(-2 (a + b) \operatorname{AppellF1} \left[1, -p, 1, 2, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + \left(-b p \operatorname{AppellF1} \left[2, 1 - p, 1, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + (a + b) \operatorname{AppellF1} \left[2, -p, 2, 3, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right)^2 \right) \right)$$

■ **Problem 447: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[e + f x]^2)^p \tan[e + f x]^4 dx$$

Optimal (type 6, 88 leaves, 4 steps):

$$\frac{1}{5 f} \operatorname{AppellF1} \left[\frac{5}{2}, 1, -p, \frac{7}{2}, -\tan[e + f x]^2, -\frac{b \tan[e + f x]^2}{a + b} \right] \tan[e + f x]^5 (a + b + b \tan[e + f x]^2)^p \left(1 + \frac{b \tan[e + f x]^2}{a + b} \right)^{-p}$$

Result (type 6, 2777 leaves):

$$\begin{aligned} & \left((a + 2 b + a \cos[2 (e + f x)])^p (\sec[e + f x]^2)^p (a + b \sec[e + f x]^2)^p \right. \\ & \quad \tan[e + f x]^5 \left(\left(9 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \cos[e + f x]^2 \right) / \right. \\ & \quad \left(3 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \\ & \quad \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \left(1 + \frac{b \tan[e + f x]^2}{a + b} \right)^{-p} \\ & \quad \left. \left(-3 \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b} \right] + \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b} \right] \tan[e + f x]^2 \right) \right) \right) / \\ & \left(3 f \left(\frac{1}{3} (a + 2 b + a \cos[2 (e + f x)])^p (\sec[e + f x]^2)^{1+p} \left(\left(9 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \cos[e + f x]^2 \right) / \right. \right. \right. \\ & \quad \left(3 (a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \\ & \quad \left. \left. -\tan[e + f x]^2 \right) - (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \left(1 + \frac{b \tan[e + f x]^2}{a + b} \right)^{-p} \\ & \quad \left. \left(-3 \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b} \right] + \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b} \right] \tan[e + f x]^2 \right) \right) - \\ & \quad \frac{2}{3} a p (a + 2 b + a \cos[2 (e + f x)])^{-1+p} (\sec[e + f x]^2)^p \sin[2 (e + f x)] \tan[e + f x] \end{aligned}$$

$$\begin{aligned}
& \left(\left(9 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x]^2 \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} \\
& \left(-3 \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \frac{2}{3} p (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x]^2 \\
& \left(\left(9 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x]^2 \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} \\
& \left(-3 \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \frac{1}{3} (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x] \\
& \left(- \left(18 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(9 (a+b) \operatorname{Cos}[e+f x]^2 \left(\frac{2 b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\tan[e + f x]^2 - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \tan[e + f x]^2 - \\
& \frac{1}{a + b} 2 b p \sec[e + f x]^2 \tan[e + f x] \left(1 + \frac{b \tan[e + f x]^2}{a + b}\right)^{-1-p} \left(-3 \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}\right] + \right. \\
& \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}\right] \tan[e + f x]^2\right) - \\
& \left(9 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \cos[e + f x]^2 \left(4 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, \right. \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right]\right) \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \left. 3 (a + b) \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]}{3 (a + b)} - \right. \right. \\
& \left. \left. \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]\right) + \right. \\
& \left. 2 \tan[e + f x]^2 \left(b p \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \right. \\
& \left. \left. \frac{1}{5 (a + b)} 6 b (1 - p) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - p, 1, \frac{7}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]\right) - \right. \\
& \left. (a + b) \left(\frac{1}{5 (a + b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x] - \right. \right. \\
& \left. \left. \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] \sec[e + f x]^2 \tan[e + f x]\right)\right)\right) / \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, \right. \right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2\right]\right) \tan[e + f x]^2\right)^2 + \\
& \left(1 + \frac{b \tan[e + f x]^2}{a + b}\right)^{-p} \left(2 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}\right] \sec[e + f x]^2 \tan[e + f x] - \right. \\
& \left. 3 \operatorname{Csc}[e + f x] \sec[e + f x] \left(-\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}\right] + \left(1 + \frac{b \tan[e + f x]^2}{a + b}\right)^p\right) + \right.
\end{aligned}$$

$$3 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \left(-\operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right)^p \right) \right) \right) \right)$$

■ **Problem 448: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Sec}[e + f x]^2)^p \operatorname{Tan}[e + f x]^2 dx$$

Optimal (type 6, 88 leaves, 4 steps):

$$\frac{1}{3f} \operatorname{AppellF1}\left[\frac{3}{2}, 1, -p, \frac{5}{2}, -\operatorname{Tan}[e + f x]^2, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Tan}[e + f x]^3 (a + b + b \operatorname{Tan}[e + f x]^2)^p \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right)^{-p}$$

Result (type 6, 2465 leaves):

$$\begin{aligned} & \left((a + 2b + a \operatorname{Cos}[2(e + f x)])^p (\operatorname{Sec}[e + f x]^2)^p (a + b \operatorname{Sec}[e + f x]^2)^p \right. \\ & \operatorname{Tan}[e + f x]^3 \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right)^{-p} \right. \\ & \left. \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Cos}[e + f x]^2 \right) / \right. \\ & \left. \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \right. \right. \right. \\ & \left. \left. \left. (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) \right) / \\ & \left(f \left((a + 2b + a \operatorname{Cos}[2(e + f x)])^p (\operatorname{Sec}[e + f x]^2)^{1+p} \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right)^{-p} \right. \right. \right. \\ & \left. \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Cos}[e + f x]^2 \right) / \right. \\ & \left. \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, \right. \right. \right. \\ & \left. \left. \left. -\operatorname{Tan}[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Tan}[e + f x]^2 \right) \right) - \\ & 2ap (a + 2b + a \operatorname{Cos}[2(e + f x)])^{-1+p} (\operatorname{Sec}[e + f x]^2)^p \operatorname{Sin}[2(e + f x)] \operatorname{Tan}[e + f x] \left(\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \right. \\ & \left. \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right)^{-p} - \left(3(a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Cos}[e + f x]^2 \right) / \right) \end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& 2^p (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x]^2 \left(\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} - \right. \\
& \left. \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x]^2 \right) \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \operatorname{Tan}[e+f x] \left(-\frac{1}{a+b} 2 b^p \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \right. \\
& \quad \left. \operatorname{Tan}[e+f x] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-1-p} + \left(6 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] \right) \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \left(3 (a+b) \operatorname{Cos}[e+f x]^2 \left(\frac{2 b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \operatorname{Csc}[e+f x] \operatorname{Sec}[e+f x] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} \left(-\operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] + \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^p \right) +
\end{aligned}$$

$$\begin{aligned}
& \left. \cos[e + f x] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^p (a + b \sec[e + f x]^2)^p \sin[e + f x] \right) / \\
& \left(f \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \right. \\
& \quad \left. \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) \right) \\
& \left(\left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^{-1+p} \right) / \right. \\
& \quad \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) - \\
& \quad \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^p \sin[e + f x]^2 \right) / \\
& \quad \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) + \\
& \quad \left(6(a + b) p \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] (a + 2b + a \cos[2(e + f x)])^p (\sec[e + f x]^2)^p \sin[e + f x]^2 \right) / \\
& \quad \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - \right. \right. \\
& \quad \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \tan[e + f x]^2 \right) - \\
& \quad \left(6 a (a + b) p \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \cos[e + f x] (a + 2b + a \cos[2(e + f x)])^{-1+p} \right. \\
& \quad \left. (\sec[e + f x]^2)^p \sin[e + f x] \sin[2(e + f x)] \right) / \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] + \right. \\
& \quad \left. 2 \left(b p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] - (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + f x]^2}{a + b}, -\tan[e + f x]^2 \right] \right) \right)
\end{aligned}$$

$$\left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right)^2 \Bigg)$$

■ **Problem 450: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Cot}[e+f x]^2 (a+b \operatorname{Sec}[e+f x]^2)^p dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$-\frac{1}{f} \operatorname{AppellF1}\left[-\frac{1}{2}, 1, -p, \frac{1}{2}, -\operatorname{Tan}[e+f x]^2, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \operatorname{Cot}[e+f x] (a+b+b \operatorname{Tan}[e+f x]^2)^p \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p}$$

Result (type 6, 2469 leaves):

$$\begin{aligned} & \left((a+2b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Cot}[e+f x]^3 (\operatorname{Sec}[e+f x]^2)^p \right. \\ & (a+b \operatorname{Sec}[e+f x]^2)^p \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p} \right. \\ & \left. \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sin}[e+f x]^2 \right) / \right. \\ & \left. \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \right. \\ & \left. \left. (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \right) \Bigg) / \right. \\ & \left. \left(f \left(2^p (a+2b+a \operatorname{Cos}[2(e+f x)])^p (\operatorname{Sec}[e+f x]^2)^p \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p} \right. \right. \right. \right. \\ & \left. \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \operatorname{Sin}[e+f x]^2 \right) / \right. \\ & \left. \left(3 (a+b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] + 2 \left(b^p \operatorname{AppellF1}\left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] - \right. \right. \right. \\ & \left. \left. -\operatorname{Tan}[e+f x]^2 \right) - (a+b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2\right] \right) \operatorname{Tan}[e+f x]^2 \Bigg) - \right. \\ & \left. (a+2b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Csc}[e+f x]^2 (\operatorname{Sec}[e+f x]^2)^p \left(-\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b}\right)^{-p} \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \right) / \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \right. \\
& \quad \left. \left. - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& 2 a p (a+2 b+a \operatorname{Cos}[2(e+f x)])^{-1+p} \operatorname{Cot}[e+f x] (\operatorname{Sec}[e+f x]^2)^p \operatorname{Sin}[2(e+f x)] \left(-\operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \right. \\
& \quad \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} - \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \right) / \left(3 (a+b) \right. \\
& \quad \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] - \right. \\
& \quad \left. (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \left. \right) + (a+2 b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Cot}[e+f x] \\
& (\operatorname{Sec}[e+f x]^2)^p \left(\frac{2 b^p \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-1-p}}{a+b} - \right. \\
& \quad \left(6 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Cos}[e+f x] \operatorname{Sin}[e+f x] \right) / \\
& \quad \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \right. \\
& \quad \left. \left. - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \\
& \quad \left(3 (a+b) \operatorname{Sin}[e+f x]^2 \left(\frac{2 b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \\
& \quad \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right. \right. \\
& \quad \left. \left. - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \text{Csc}[e + f x] \text{Sec}[e + f x] \left(1 + \frac{b \text{Tan}[e + f x]^2}{a + b} \right)^{-p} \left(\text{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] - \left(1 + \frac{b \text{Tan}[e + f x]^2}{a + b} \right)^p \right) + \\
& \left(3 (a + b) \text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sin}[e + f x]^2 \left(4 \left(b p \text{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\text{Tan}[e + f x]^2 \right] - (a + b) \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \right) \text{Sec}[e + f x]^2 \text{Tan}[e + f x] + \right. \right. \\
& \quad \left. \left. 3 (a + b) \left(\frac{2 b p \text{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x]}{3 (a + b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2}{3} \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) + \right. \right. \\
& \quad \left. \left. 2 \text{Tan}[e + f x]^2 \left(b p \left(-\frac{6}{5} \text{AppellF1} \left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{5 (a + b)} 6 b (1 - p) \text{AppellF1} \left[\frac{5}{2}, 2 - p, 1, \frac{7}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) - \right. \right. \\
& \quad \left. \left. (a + b) \left(\frac{1}{5 (a + b)} 6 b p \text{AppellF1} \left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{12}{5} \text{AppellF1} \left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \text{Sec}[e + f x]^2 \text{Tan}[e + f x] \right) \right) \right) \right) \right) / \\
& \left(3 (a + b) \text{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] + 2 \left(b p \text{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\text{Tan}[e + f x]^2 \right] - (a + b) \text{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \text{Tan}[e + f x]^2}{a + b}, -\text{Tan}[e + f x]^2 \right] \right) \text{Tan}[e + f x]^2 \right)^2 \right) \right) \right)
\end{aligned}$$

■ **Problem 451: Result more than twice size of optimal antiderivative.**

$$\int \text{Cot}[e + f x]^4 (a + b \text{Sec}[e + f x]^2)^p dx$$

Optimal (type 6, 88 leaves, 4 steps):

$$-\frac{1}{3f} \text{AppellF1} \left[-\frac{3}{2}, 1, -p, -\frac{1}{2}, -\text{Tan}[e + f x]^2, -\frac{b \text{Tan}[e + f x]^2}{a + b} \right] \text{Cot}[e + f x]^3 (a + b + b \text{Tan}[e + f x]^2)^p \left(1 + \frac{b \text{Tan}[e + f x]^2}{a + b} \right)^{-p}$$

Result (type 6, 3033 leaves):

$$\begin{aligned}
& \left((a + 2b + a \cos[2(e + fx)])^p \cot[e + fx]^7 (\sec[e + fx]^2)^p (a + b \sec[e + fx]^2)^p \right. \\
& \left(\left(9(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \sin[e + fx]^2 \tan[e + fx]^2 \right) / \right. \\
& \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] - \right. \right. \\
& \left. \left. (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \right) \tan[e + fx]^2 \right) - \left(1 + \frac{b \tan[e + fx]^2}{a + b} \right)^{-p} \\
& \left. \left(\operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] - 3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] \tan[e + fx]^2 \right) \right) \Big) / \\
& \left(3f \left(\frac{2}{3} \right)^p (a + 2b + a \cos[2(e + fx)])^p \cot[e + fx]^2 (\sec[e + fx]^2)^p \right. \\
& \left(\left(9(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \sin[e + fx]^2 \tan[e + fx]^2 \right) / \right. \\
& \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \right. \right. \\
& \left. \left. - \tan[e + fx]^2 \right) - (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \right) \tan[e + fx]^2 \right) - \left(1 + \frac{b \tan[e + fx]^2}{a + b} \right)^{-p} \\
& \left. \left(\operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] - 3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] \tan[e + fx]^2 \right) \right) - \\
& (a + 2b + a \cos[2(e + fx)])^p \cot[e + fx]^2 \csc[e + fx]^2 (\sec[e + fx]^2)^p \\
& \left(\left(9(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \sin[e + fx]^2 \tan[e + fx]^2 \right) / \right. \\
& \left(3(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \right. \right. \\
& \left. \left. - \tan[e + fx]^2 \right) - (a + b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \right) \tan[e + fx]^2 \right) - \left(1 + \frac{b \tan[e + fx]^2}{a + b} \right)^{-p} \\
& \left. \left(\operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] - 3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \tan[e + fx]^2}{a + b} \right] \tan[e + fx]^2 \right) \right) - \\
& \frac{2}{3} a^p (a + 2b + a \cos[2(e + fx)])^{-1+p} \cot[e + fx]^3 (\sec[e + fx]^2)^p \sin[2(e + fx)] \\
& \left(\left(9(a + b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \tan[e + fx]^2}{a + b}, -\tan[e + fx]^2 \right] \sin[e + fx]^2 \tan[e + fx]^2 \right) / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) - \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-p} \\
& \left(\operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - 3 \operatorname{Hypergeometric2F1} \left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] \operatorname{Tan}[e+f x]^2 \right) \Bigg) + \\
& \frac{1}{3} (a+2b+a \operatorname{Cos}[2(e+f x)])^p \operatorname{Cot}[e+f x]^3 (\operatorname{Sec}[e+f x]^2)^p \\
& \left(\left(18 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sin}[e+f x]^2 \operatorname{Tan}[e+f x] \right) / \right. \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(18 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Tan}[e+f x]^3 \right) / \Bigg) \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \left(9 (a+b) \operatorname{Sin}[e+f x]^2 \operatorname{Tan}[e+f x]^2 \left(\frac{2 b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x]}{3 (a+b)} - \right. \right. \\
& \quad \left. \left. \frac{2}{3} \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \right) \right) / \Bigg) \\
& \left(3 (a+b) \operatorname{AppellF1} \left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] + 2 \left(b^p \operatorname{AppellF1} \left[\frac{3}{2}, 1-p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, \right. \right. \right. \\
& \quad \left. \left. \left. -\operatorname{Tan}[e+f x]^2 \right] - (a+b) \operatorname{AppellF1} \left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b}, -\operatorname{Tan}[e+f x]^2 \right] \right) \operatorname{Tan}[e+f x]^2 \right) + \\
& \frac{1}{a+b} 2 b^p \operatorname{Sec}[e+f x]^2 \operatorname{Tan}[e+f x] \left(1 + \frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right)^{-1-p} \left(\operatorname{Hypergeometric2F1} \left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e+f x]^2}{a+b} \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Tan}[e + f x]^2 \Big) - \\
& \left(9 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sin}[e + f x]^2 \operatorname{Tan}[e + f x]^2 \left(4 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Sec}[e + f x]^2 \right. \right. \\
& \quad \left. \left. \operatorname{Tan}[e + f x] + 3 (a + b) \left(\frac{2 b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x]}{3 (a + b)} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2}{3} \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) + \right. \\
& \quad \left. 2 \operatorname{Tan}[e + f x]^2 \left(b p \left(-\frac{6}{5} \operatorname{AppellF1}\left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \frac{1}{5 (a + b)} 6 b (1 - p) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - p, 1, \frac{7}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) - \right. \\
& \quad \left. (a + b) \left(\frac{1}{5 (a + b)} 6 b p \operatorname{AppellF1}\left[\frac{5}{2}, 1 - p, 2, \frac{7}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{12}{5} \operatorname{AppellF1}\left[\frac{5}{2}, -p, 3, \frac{7}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \right) \right) \right) \Big) \Big) \Big) \Big) / \\
& \left(3 (a + b) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] + 2 \left(b p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - p, 1, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}[e + f x]^2\right] - (a + b) \operatorname{AppellF1}\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}, -\operatorname{Tan}[e + f x]^2\right] \right) \operatorname{Tan}[e + f x]^2 \Big)^2 - \\
& \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b} \right)^{-p} \left(-6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] - \right. \\
& \quad \left. 3 \operatorname{Sec}[e + f x]^2 \operatorname{Tan}[e + f x] \left(\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] - \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b} \right)^p \right) \right) - \\
& \quad \left. 3 \operatorname{Csc}[e + f x] \operatorname{Sec}[e + f x] \left(-\operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, -p, -\frac{1}{2}, -\frac{b \operatorname{Tan}[e + f x]^2}{a + b}\right] + \left(1 + \frac{b \operatorname{Tan}[e + f x]^2}{a + b} \right)^p \right) \right) \Big) \Big) \Big) \Big)
\end{aligned}$$

■ **Problem 457: Result is not expressed in closed-form.**

$$\int \frac{\tan[e + f x]^5}{a + b \sec[e + f x]^3} dx$$

Optimal (type 3, 219 leaves, 11 steps):

$$-\frac{(a^{2/3} + 2b^{2/3}) \operatorname{ArcTan}\left[\frac{b^{1/3} - 2a^{1/3} \cos[e + f x]}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} a^{1/3} b^{4/3} f} - \frac{(a^{2/3} - 2b^{2/3}) \operatorname{Log}[b^{1/3} + a^{1/3} \cos[e + f x]]}{3 a^{1/3} b^{4/3} f} +$$

$$\frac{(a^{2/3} - 2b^{2/3}) \operatorname{Log}[b^{2/3} - a^{1/3} b^{1/3} \cos[e + f x] + a^{2/3} \cos[e + f x]^2]}{6 a^{1/3} b^{4/3} f} - \frac{\operatorname{Log}[b + a \cos[e + f x]^3]}{3 a f} + \frac{\sec[e + f x]}{b f}$$

Result (type 7, 251 leaves):

$$\frac{1}{3 a b f}$$

$$\left(3 b \operatorname{Log}\left[\sec\left[\frac{1}{2}(e + f x)\right]^2\right] - \operatorname{RootSum}\left[-8 a + 12 a \#1 - 6 a \#1^2 + a \#1^3 - b \#1^3 \&, 1 / (4 a - 4 a \#1 + a \#1^2 - b \#1^2)\right] \left(-4 a^2 \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] +\right.\right.$$

$$4 a b \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] + 2 a^2 \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1 - 8 a b \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1 +$$

$$\left. a b \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1^2 - b^2 \operatorname{Log}\left[1 - \#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1^2\right) \& \left. + 3 a \sec[e + f x] \right)$$

■ **Problem 458: Result is not expressed in closed-form.**

$$\int \frac{\tan[e + f x]^3}{a + b \sec[e + f x]^3} dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{b^{1/3} - 2a^{1/3} \cos[e + f x]}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} a^{1/3} b^{2/3} f} - \frac{\operatorname{Log}[b^{1/3} + a^{1/3} \cos[e + f x]]}{3 a^{1/3} b^{2/3} f} + \frac{\operatorname{Log}[b^{2/3} - a^{1/3} b^{1/3} \cos[e + f x] + a^{2/3} \cos[e + f x]^2]}{6 a^{1/3} b^{2/3} f} + \frac{\operatorname{Log}[b + a \cos[e + f x]^3]}{3 a f}$$

Result (type 7, 242 leaves):

$$\frac{1}{3 a f} \left(-3 \operatorname{Log}\left[\sec\left[\frac{1}{2}(e + f x)\right]^2\right] + \operatorname{RootSum}\left[-a - b + 3 a \#1 - 3 b \#1 - 3 a \#1^2 - 3 b \#1^2 + a \#1^3 - b \#1^3 \&, \right.\right.$$

$$\left. \left(-a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] - b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] - 4 a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1 - 2 b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1 +\right.\right.$$

$$\left. \left. a \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1^2 - b \operatorname{Log}\left[-\#1 + \tan\left[\frac{1}{2}(e + f x)\right]^2\right] \#1^2\right) / (a - b - 2 a \#1 - 2 b \#1 + a \#1^2 - b \#1^2) \& \right)$$

■ **Problem 460: Result is not expressed in closed-form.**

$$\int \frac{\cot[e + f x]}{a + b \sec[e + f x]^3} dx$$

Optimal (type 3, 295 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3}-2a^{1/3}\cos[e+fx]}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3})f} + \frac{\operatorname{Log}[1-\cos[e+fx]]}{2(a+b)f} + \frac{\operatorname{Log}[1+\cos[e+fx]]}{2(a-b)f} - \frac{(a^{2/3}+b^{2/3})b^{2/3}\operatorname{Log}[b^{1/3}+a^{1/3}\cos[e+fx]]}{3a^{1/3}(a^2-b^2)f} + \\
 & \frac{(a^{2/3}+b^{2/3})b^{2/3}\operatorname{Log}[b^{2/3}-a^{1/3}b^{1/3}\cos[e+fx]+a^{2/3}\cos[e+fx]^2]}{6a^{1/3}(a^2-b^2)f} - \frac{b^2\operatorname{Log}[b+a\cos[e+fx]^3]}{3a(a^2-b^2)f}
 \end{aligned}$$

Result (type 7, 290 leaves):

$$\begin{aligned}
 & \frac{1}{3a(a-b)(a+b)f} \left(3 \left(a(a+b)\operatorname{Log}\left[\cos\left[\frac{1}{2}(e+fx)\right]\right] + b^2\operatorname{Log}\left[\sec\left[\frac{1}{2}(e+fx)\right]^2\right] + a(a-b)\operatorname{Log}\left[\sin\left[\frac{1}{2}(e+fx)\right]\right] \right) - \right. \\
 & \left. b\operatorname{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\ \&, \left(-4a^2\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + \right. \right. \right. \\
 & \left. \left. 4ab\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right] + 2a^2\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\#1 - 2ab\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\#1 + \right. \right. \\
 & \left. \left. ab\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\#1^2 - b^2\operatorname{Log}\left[1-\#1+\tan\left[\frac{1}{2}(e+fx)\right]^2\right]\#1^2\right) / \left(4a-4a\#1+a\#1^2-b\#1^2\ \&\right) \right)
 \end{aligned}$$

■ **Problem 461: Result is not expressed in closed-form.**

$$\int \frac{\cot[e+fx]^3}{a+b\sec[e+fx]^3} dx$$

Optimal (type 3, 393 leaves, 11 steps):

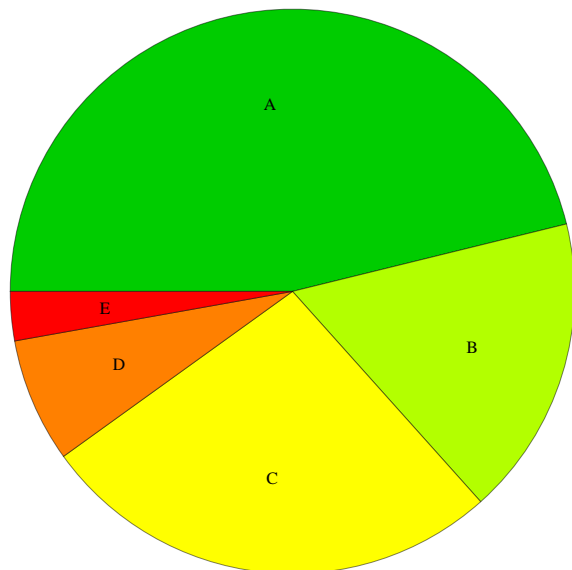
$$\begin{aligned}
 & \frac{b^{4/3}(a^2-3a^{2/3}b^{4/3}+2b^2)\operatorname{ArcTan}\left[\frac{b^{1/3}-2a^{1/3}\cos[e+fx]}{\sqrt{3}b^{1/3}}\right]}{\sqrt{3}a^{1/3}(a^2-b^2)^2f} - \frac{1}{4(a+b)f(1-\cos[e+fx])} - \frac{1}{4(a-b)f(1+\cos[e+fx])} - \\
 & \frac{(2a+5b)\operatorname{Log}[1-\cos[e+fx]]}{4(a+b)^2f} - \frac{(2a-5b)\operatorname{Log}[1+\cos[e+fx]]}{4(a-b)^2f} - \frac{b^{4/3}(a^2+3a^{2/3}b^{4/3}+2b^2)\operatorname{Log}[b^{1/3}+a^{1/3}\cos[e+fx]]}{3a^{1/3}(a^2-b^2)^2f} + \\
 & \frac{b^{4/3}(a^2+3a^{2/3}b^{4/3}+2b^2)\operatorname{Log}[b^{2/3}-a^{1/3}b^{1/3}\cos[e+fx]+a^{2/3}\cos[e+fx]^2]}{6a^{1/3}(a^2-b^2)^2f} - \frac{b^2(2a^2+b^2)\operatorname{Log}[b+a\cos[e+fx]^3]}{3a(a^2-b^2)^2f}
 \end{aligned}$$

Result (type 7, 336 leaves):

$$\begin{aligned}
& \frac{1}{24 f} \left(-\frac{3 \operatorname{Csc}\left[\frac{1}{2}(e+f x)\right]^2}{a+b} + \frac{12(-2 a+5 b) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(e+f x)\right]\right]}{(a-b)^2} - \frac{12(2 a+5 b) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(e+f x)\right]\right]}{(a+b)^2} + \right. \\
& \left. \frac{1}{\left(a\left(a^2-b^2\right)^2\right) 8 b^2} \left(3\left(2 a^2+b^2\right) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2\right] + (-a+b) \operatorname{RootSum}\left[-8 a+12 a \# 1-6 a \# 1^2+a \# 1^3-b \# 1^3 \&, \right. \right. \\
& \left. \left. \frac{1}{\left(4 a-4 a \# 1+a \# 1^2-b \# 1^2\right)} \left(8 a^2 \operatorname{Log}\left[1-\# 1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - 4 a b \operatorname{Log}\left[1-\# 1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] - 6 a^2 \operatorname{Log}\left[1-\# 1+\right. \right. \right. \\
& \left. \left. \left. \operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \# 1+2 a^2 \operatorname{Log}\left[1-\# 1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \# 1^2+b^2 \operatorname{Log}\left[1-\# 1+\operatorname{Tan}\left[\frac{1}{2}(e+f x)\right]^2\right] \# 1^2\right) \& \right) \right) - \frac{3 \operatorname{Sec}\left[\frac{1}{2}(e+f x)\right]^2}{a-b} \right)
\end{aligned}$$

Summary of Integration Test Results

5052 integration problems



A - 2332 optimal antiderivatives

B - 870 more than twice size of optimal antiderivatives

C - 1348 unnecessarily complex antiderivatives

D - 360 unable to integrate problems

E - 142 integration timeouts